A Comprehensive Guide to GATE - 2024

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PART - I

- Reasoning
- Quantitative Aptitude
- Verbal Reasoning

Chapter 1

Number and Letter Series

CHAPTER HIGHLIGHTS

- *Number Series*
- *Difference Series*
- *Product Series*
- *Squares/Cubes Series*
- *Miscellaneous Series*
- *Combination Series*
- *Letter Series*

Introduction

Number and Letter Series form an important part of the Reasoning section in various competitive examinations. There are two or three broad categories of questions that appear in various exams from this particular chapter.

In the first category of questions, a series of numbers/letters is given with one number/letter (or two numbers/letters) missing, represented by a blank or a question mark. The given series of numbers/letters will be such that each one follows its predecessor in a certain way, i.e. according to a definite pattern. Students are required to find out the way in which the series is formed and, hence, work out the missing number/numbers or letter/letters to complete the series. For the purpose of our discussion, we will refer to this category of questions as Number Series Type I or Letter Series Type I questions. Under Type I questions, there are a large variety of patterns that are possible, and the student requires a proper understanding of various patterns to be able to do well in these types of questions.

In the second category of questions, a series of numbers/letters is given, and the student is required to count how many numbers/letters in that series satisfy a given condition and mark that as the answer. For the purpose of our understanding, we will refer to this category of questions as Number Series Type II or Letter Series Type II questions. These questions will mainly involve counting of numbers/letters satisfying a given condition.

Number Series

For better understanding, we will classify this into the following broad categories.

1. Difference series
2. Product series
3. Squares/cubes series
4. Miscellaneous series
5. Combination series

Difference Series

The difference series can be further classified as follows.

1. Number series with a constant difference.
2. Number series with an increasing or decreasing difference.

In the number series with a constant difference, there is always a constant difference between two consecutive numbers. For example, the numbers of the series 1, 4, 7, 10, 13, … are such that any number is obtained by adding a constant figure of 3 to the preceding term of the series.

If we have to find the next number in the aforementioned series, we need to add a 3 to the last term 13. Thus, 16 is the next term of the series.

Under the series with constant difference, we can have series of odd numbers or series of even numbers also.

In the series with increasing/decreasing difference, the difference between consecutive terms keeps increasing (or decreasing, as the case may be). For example, let us try to find out the next number in the series 2, 3, 5, 8, 12, 17, 23, …

Here, the difference between the first two terms of the series is 1; the difference between the second and third terms is 2; the difference between the third and the fourth terms is 3; and so on. That is, the difference between any pair of consecutive terms is one more than the difference between the first number of this pair and the number immediately preceding this number. Here, since the difference between 17 and 23 is 6, the next difference should be 7. So, the number that comes after 23 should be (23 + 7) = 30.

We can also have a number series where the difference is in decreasing order (unlike in the previous example where the difference is increasing). For example, let us find out the next term of the series 10, 15, 19, 22, 24, …

10, 15, 19, 22, 24

+5 +4 +3 +2

Here, the differences between 1st and 2nd, 2nd and 3rd, 3rd and 4th numbers, etc., are 5, 4, 3, 2, and so on. Since the difference between 22 and 24 is 2, the next difference should be 1. So, the number that comes after 24 should be 25.

Product Series

A product series is usually a number series where the terms are obtained by a process of multiplication. Here also, there can be different types of series. We will look at these through examples.

Consider the series 2, 4, 8, 16, 32, 64, …

2, 4, 8, 16, 32, 64

× 2 × 2 × 2 × 2 × 2

Here, each number in the series is multiplied by 2 to get the next term. So, the term that comes after 64 is 128. So, each term is multiplied by a fixed number to get the next term. Similarly, we can have a series where we have numbers obtained by **dividing** the previous term with a constant number. For example, in the series 64, 32, 16, 8, …, each number is obtained by dividing the previous number by 2 (or in other words, by multiplying the previous term by $\frac{1}{2}$). So, here, the next term will be 4 (obtained by dividing 8 with 2).

Consider the series 4, 20, 80, 240, …

4 20, 80, 240,

+5 +4 +3

Here, the first term is multiplied by 5 to get the second term; the second term is multiplied by 4 to get the third term; the third term is multiplied by 3 to get the fourth term. Hence, to get the fifth term, we have to multiply the fourth term by 2, i.e. the fifth term is 480. So, each term is multiplied by a decreasing factor (or it could also be an increasing factor) to get the next term. That is, with whatever number a particular term is multiplied to get the next term, this latest term is multiplied by a number different from the previous multiplying factor to get the next term of the series. All the multiplying factors follow a certain pattern (normally of increasing or decreasing order).

Consider the series 2, 6, 12, 20, 30, …

2, 6, 12, 20, 30

+4 +6 +8 +10

This can be looked at a series of increasing differences. The differences of consecutive pairs of terms are 4 (between 2 and 6), 6 (between 6 and 12), 8 (between 12 and 20), 10 (between 20 and 30), and so on. Hence, the difference between 30 and the next term should be 12, and, so, the next term will be 42. But, this series can also be looked at as a product series.

2,	6,	12,	20,	30
↓	↓	↓	↓	↓
1×2	2×3	3×4	4×5	5×6

The first term is the product of 1 and 2; the second term is the product of 2 and 3; the third term is the product of 3 and 4; the fourth term is the product of 4 and 5; the fifth term is the product of 5 and 6. Hence, the next term will be the product of 6 and 7, that is 42.

Squares/Cubes Series

There can be series where all the terms are related to the squares of numbers or cubes of numbers. With squares/cubes of numbers as the basis, there can be many variations in the pattern of the series. Let us look at various possibilities of series based on squares/cubes.

Each term of the series may be the square of a natural number, such as 1, 4, 9, 16, …

1,	4,	9,	16
↓	↓	↓	↓
1^2	2^2	3^2	4^2

The numbers are squares of 1, 2, 3, 4 …, respectively. The number which follows 16 (which is the square of 4) will be 25 (which is the square of 5).

The terms of the series may be the squares of odd numbers (e.g. 1, 9, 25, 49, …) or even numbers (e.g. 4, 16, 36, 64, …).

The terms of the series could be such that a number and its square are both given one after the other and such pairs are given in some specific pattern. For example, take the series 2, 4, 3, 9, 4, 16, …

2, 4, 3, 9, 4, 16,

+1 +1

Here, 2 is followed by its square 4; then comes the number 3 (which is one more than 2) followed by its square 9 and so on. Hence, the next number in the series is 5, and the one after that is its square, i.e. 25.

Similarly, each term could be the square root of its predecessor. For example, in the series 81, 9, 64, 8, 49, 7, 36, …, 81 is the square of 9, 64 the square of 8, and so on. Therefore, the next number which follows in the series should be the square root of 36, i.e. 6.

The terms of the series could be the squares of natural numbers increased or reduced by certain number. For example, in the series 3, 8, 15, 24, …

$$\begin{array}{cccc} 3, & 8, & 15, & 24 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2^2-1 & 3^2-1 & 4^2-1 & 5^2-1 \end{array}$$

We have {Squares of natural numbers – 1} as the terms. The first term is $2^2 - 1$; the second term is $3^2 - 1$; the third term is $4^2 - 1$, and so on. Hence, the next term will be $6^2 - 1$, i.e. 35. [Please note that the above series can also be looked at as a series with increasing differences. The differences between the 1st and 2nd terms, the 2nd and 3rd terms, and so on are 5, 7, 9, and so on. Hence, the next difference should be 11 giving us the next term as 35.] There could also be a series with {squares of natural numbers + some constant}.

Like we have seen series with squares of numbers, we can have similar series with cubes of numbers. For example, take the series 1, 8, 27, 64, …

$$\begin{array}{cccc} 1, & 8, & 27, & 64 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1^3 & 2^3 & 3^3 & 4^3 \end{array}$$

Here, all the terms are cubes of natural numbers. So, the next term will be 5^3, i.e. 125.

Consider the series 2, 9, 28, 65, …

$$\begin{array}{cccc} 2, & 9, & 28, & 65 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1^3+1 & 2^3+1 & 3^3+1 & 4^3+1 \end{array}$$

Here, the terms are {Cubes of natural numbers + 1}. The first term is $1^3 + 1$; the second term is $2^3 + 1$; the third term is $3^3 + 1$, and so on. Hence, the next term will be $5^3 + 1$, i.e. 125.

Miscellaneous Series

There are series that do not come under the other patterns and are of general nature but are important and are fairly common. Even here, some times, there can be a specific pattern in some cases.

Take the series 3, 5, 7, 11, 13, … . This is a series of consecutive PRIME NUMBERS. It is an important series and the student should look out for this as one of the patterns. The next term in this series is 17.

There can also be variations using prime numbers. Take the series 9, 25, 49, 121, … . In this series, the terms are squares of prime numbers. Hence, the next term is 13^2, i.e. 169.

Take the series 15, 35, 77, … . The first term is 3×5; the second term is 5×7; the third term is 7×11; here, the terms are product of two consecutive prime numbers. So, the next term will be the product of 11 and 13, i.e. 143.

Take the series 8, 24, 48, 120, 168, … Here, the 2nd term is 3 times the first term and the 3rd term is 2 times the 2nd term, but after that, it does not follow this pattern any more. If you look at the terms carefully, you will find that the terms are {one less than squares of prime numbers}. Hence, the next term will be $17^2 - 1$, i.e. 288.

Consider the series 1, 4, 9, 1, 6, 2, 5, 3, …

At first sight, there is nothing we can say about the series. This is actually a series formed by squares of natural numbers. However, if any of the squares is in two or more digits, each of the digits is written as a separate term of the series. Thus, the first terms are 1, 4, and 9, the squares of 1, 2, and 3, respectively. After this, we should get 16 (which is the square of 4). Since this has two digits 1 and 6, these two digits are written as two different terms 1 and 6 in the series. Similarly, the next square 25 is written as two different terms 2 and 5 in the series. So, the next square 36 should be written as two terms 3 and 6. Of these, 3 is already given. So, the next term of the series is 6.

Consider the series 1, 1, 2, 3, 5, 8, …

$$\begin{array}{cccccc} 1, & 1, & 2, & 3, & 5, & 8 \\ & & \downarrow & \downarrow & \downarrow & \downarrow \\ & & 1+1 & 1+2 & 2+3 & 3+5 \end{array}$$

Here, each term, starting with the third number, is the sum of the two preceding terms. After taking the first two terms as given (1 and 1), then onwards, to get any term, we need to add the two terms that come immediately before that position. Hence, to get the next term of the series, we should take the two preceding terms 5 and 8 and add them up to get 13. So, the next term of the series is 13. The term after this will be 21 (= 8 + 13).

Combination Series

A number series which has more than one type of (arithmetic) operation performed or more than one series combined together is a combination series. The series that are combined can be two series of the same type or could be different types of series as described earlier. Let us look at some examples.

First, let us look at those series that are formed by more than one arithmetic operation performed on the terms to get the subsequent terms.

Consider the series: 2, 6, 10, 3, 9, 13, 4, 12, … Here, the first term 2 is multiplied by 3 to get the second term, and 4 is added to get the third term. The next term is 3 (one more than the first term 2), and it is multiplied by 3 to get 9 (which is the next term) and then 4 is added to get the next term 13. The next term 4 (which is one more than 3), which is multiplied with 3 to get 12. Then, 4 is added to this to get the next number 16.

Consider the series: 1, 2, 6, 21, 88, …. Here, we can observe that 88 is close to 4 times 21. It is in fact $21 \times 4 + 4$. So, if we now look at the previous term 21, it is related to

the previous term 6 as $6 \times 3 + 3$. Now we get the general pattern: to get any term, multiply the previous term with k and then add k where k is a natural number with values in increasing order from 1. So, to get the second term, the first term has to be multiplied with 1 and then 1 is added. To get the third term, the second term is multiplied with 2 and then 2 is added and so on. Hence, after 88, the next term is $88 \times 5 + 5$, i.e. 445.

Now, let us look at a series that is formed by combining two (or more) different series. The two (or more) series can be of the same type or of different types described earlier.

Consider the series: 8, 12, 9, 13, 10, 14, …. Here the 1st, 3rd, 5th, … terms, which are 8, 9, 10, …, form one series whereas the 2nd, 4th, 6th, etc. terms, which are 12, 13, 14, form another series. Here, both series that are being combined are two simple constant difference series. Therefore, the missing number will be the next term of the first series 8, 9, 10, …, which is equal to 11.

Consider the series: 0, 7, 2, 17, 6, 31, 12, 49, 20, …. Here, the series consisting of 1st, 3rd, 5th, … terms (i.e. the series consisting of the odd terms), which is 0, 2, 6, 12, 20, … is combined with another series consisting of 2nd, 4th, 6th, … terms (i.e. the series consisting of the even terms) which is 7, 17, 31, 49, … . The first series has the differences in increasing order 2, 4, 6, 8, 10, and so on. The second series also has the difference in increasing order 10, 14, 18, …. Since, the last term 20 belongs to the first series, a number from the second series should follow next. The next term of the second series will be obtained by adding 22 to 49, that is 71.

Consider the series: 1, 1, 2, 4, 3, 9, 4, 16, …. Here, one series consisting of odd terms, which is 1, 2, 3, 4, …, is combined with the series of even terms which is 1, 4, 9, 16, …. The first series is a series of natural numbers. The second series is the squares of natural numbers. Hence, the next term is 5.

Consider the series: 1, 1, 4, 8, 9, 27, …. Here, the series of squares of natural numbers is combined with the series of cubes of natural numbers. The next term in the series will be 4.

Consider the series: 2, 4, 5, 9, 9, 16, 14, ?, 20, …. Here, we have to find out the term that should come in place of the question mark. The odd terms form one series 2, 5, 9, 14, 20, … where the difference is increasing. The differences are 3, 4, 5, 6, … This series is combined with the series of even terms 4, 9, 16, … where the terms are squares of numbers 2, 3, 4, … . Hence, the term that should come in place of the question mark is the next term of the second series which is 5^2, i.e. 25.

A general approach to the number Series: The best way of approaching the number series questions is to first observe the difference between terms. If the difference is constant, it is a constant difference series. If the difference is increasing or decreasing by a constant number, then it is a series with a constant increasing or decreasing difference. If there is no constant increasing or decreasing difference, then try out the product series approach. For this, first divide the second term with the first term, third with the second, and so on. If the numbers obtained are the same, then it is a product series. Alternatively, try writing each term of the series as a product of two factors and see if there is any pattern that can be observed. If still there is no inference, but the difference is increasing or decreasing in a rapid manner, then check out the square series. If the increase is very high, and it is not a square series, then try out the cube series.

If the difference is alternately decreasing and increasing (or increasing for some time and alternately decreasing), then it should most probably be a mixed series. Therefore, test out the series with alternate numbers. If still the series is not solved, try out the general series.

Letter Series

The questions here are similar to the questions in Number Series Type I. Instead of numbers, we have letters of the alphabet given here. We have to first identify the pattern that the series of letters follow. Then, we have to find the missing letter based on the pattern already identified. Ins number series, we saw different patterns that the numbers in the series can follow—like squares, cubes. In letter series, obviously, patterns like squares, cubes will not be possible. In letter series, in general, we have a series with constant or increasing or decreasing differences. The position of the letters in the English alphabet is considered to be the value of the alphabet in questions on letter series. Also, when we are counting, after we count from *A* to *Z*, we again start with *A*, i.e. we treat the letters as being cyclic in nature. Like in number series, in this type of letter series also, we can have a 'combination' of series, i.e. two series are combined and given. We need to identify the pattern in the two series to find out the missing letter. Sometimes, there will be some special types of series also. Let us look at a few examples to understand questions on letter series.

Solved Examples

Example 1

Find the next letter in the series

D, G, J, M, P, ______.

(A) Q (B) R (C) S (D) T

Solution

Three letters are added to each letter to get the next letter in the series.

i.e. $D^{+3}, G^{+3}, J^{+3}, M^{+3}, P^{+3}, \underline{S}$

P + 3 and P = 16 and 16 + 3 = 19 and the 19th letter in the alphabet is S.

Example 2

Find the next letter in the series

A, B, D, H, ______.

(A) L (B) N (C) R (D) P

Solution

Each letter in the given series is multiplied with 2 to get the next letter in the series.

$A \times 2 \Rightarrow 1 \times 2 = 2$ and the 2nd letter is B, $B \times 2 \Rightarrow 2 \times 2 = 4$ and the 4th letter is D.

Similarly, $H \times 2 \Rightarrow 8 \times 2 = 16$ and the 16th letter is P.

Example 3

What is the next letter in the series?

B, D, G, K, P, ______

(A) S (B) V (C) W (D) X

Solution

$B^{+2}, D^{+3}, G^{+4}, K^{+5}, P^{+6}$, ______

$P + 6 = 16 + 6 = 22$ and the 22nd letter is V.

Example 4

I, X, J, W, K, V, L, ______.

(A) M (B) U (C) S (D) T

Solution

The given series is an alternate series.

I^{+1}, J^{+1}, K^{+1}, L is one series and X^{-1}, W^{-1}, V^{-1}, ______ is the other series.

$X - 1 = W$, $W - 1 = V$ and $V - 1 = 22 - 1 = 21$ and the 21st letter is U.

Example 5

97, 83, 73, 67, 59, ______

(A) 53 (B) 49 (C) 47 (D) 51

Solution

The given numbers are alternate prime numbers in decreasing order, starting with 97. Hence, the next number in the series is 47.

Example 6

75, 291, 416, 480, 507, ______

(A) 515 (B) 532 (C) 511 D) 521

Solution

$75^{+216}, 291^{+125}, 416^{+64}, 480^{+27}, 507$, ______

The differences are cubes of consecutive natural numbers in decreasing order. Hence, the next number in the series in $507 + (B)^3 = 515$.

Exercises

Direction for questions 1 to 25: Complete the following series.

1. 17, 19, 23, 29, 31, 37, ______
(A) 41 (B) 43 (C) 40 (D) 42

2. 225, 196, 169, ______, 121, 100, 81
(A) 156 (B) 144 (C) 136 (D) 125

3. 64, 125, 216, 343, ______
(A) 64 (B) 424 (C) 317 (D) 512

4. 54, 66, 82, 102, 126, ______
(A) 146 (B) 130 (C) 154 (D) 144

5. 7, 11, 20, 36, 61, ______, 146
(A) 25 (B) 91 (C) 97 (D) 92

6. 8, 16, 48, 96, 288, 576, ______
(A) 1152 (B) 1728
(C) 1052 (D) 1428

7. 125, 375, 377, 1131, 1133, ______
(A) 3399 (B) 1136
(C) 1135 (D) 1234

8. 12, 35, 106, 317, 952, ______
(A) 2851 (B) 2855
(C) 1851 (D) 1849

9. 2, 4, 7, 35, 42, 462, ______
(A) 5016 (B) 470 (C) 4712 (D) 475

10. $13\frac{1}{3}, 15, \frac{120}{7}, 20, 24$, ______
(A) 30 (B) 36 (C) 40 (D) $37^1/_3$

11. 6, 15, 35, 77, 143, 221, ______
(A) 357 (B) 437 (C) 323 (D) 383

12. 29, 29, 27, 23, 25, 19, 23, 17, ______, ______
(A) 19, 13 (B) 19, 15 (C) 21, 13 (D) 19, 13

13. 24, 625, 26, 729, 28, 841, ______
(A) 30 (B) 29 (C) 900 (D) 961

14. 3731, 2923, 1917, 1311, ______
(A) 117 (B) 119 (C) 917 (D) 75

15. 11, 28, 327, 464, ______
(A) 525 (B) 5625 (C) 5125 (D) 5250

16. 6, 24, 60, 120, 210, ______
(A) 336 (B) 343 (C) 368 (D) 322

17. 132, 182, 306, 380, 552, 870, ______
(A) 930 (B) 1010 (C) 992 (D) 1142

18. KPD, LOE, MNF, NMG, ______
(A) ONF (B) OLH (C) MLH (D) MNH

19. BEP, CIQ, DOR, FUS, GAT, ______
(A) HEV (B) HIT (C) IET (D) IEU

20. GKF, IPC, LTY, PWT, UYN, ______
(A) ABZ (B) XBX
(C) XAH (D) AZG

21. QLR, JPD, RNU, GNC, SPX, DLB, ______
(A) TRA (B) AJA
(C) BTU (D) KJE

22. GTB, CYV, YDP, ______, QND
(A) DIV (B) UIJ
(C) DDV (D) UVV

23. ABDH, BDHP, CFLX, DHPF, ______
(A) EKNT (B) TNEK
(C) EJTN (D) JNTE

24. TCFK, RADI, OXAF, JSVA, ______
(A) DMPU (B) DMOT
(C) CMOT (D) CLOT

25. KJAM, GGWJ, ______, YAOD, UXKA
(A) CDUI (B) DFTC
(C) DCTF (D) CDSG

Answer Keys

1. A	**2.** B	**3.** D	**4.** C	**5.** C	**6.** B	**7.** A	**8.** B	**9.** D	**10.** A
11. C	**12.** C	**13.** A	**14.** D	**15.** C	**16.** A	**17.** C	**18.** B	**19.** A	**20.** D
21. A	**22.** B	**23.** C	**24.** D	**25.** D					

Chapter 2

Analogies

CHAPTER HIGHLIGHTS

- *Analogy*
- *Number Analogies*
- *Letter Analogies*
- *Verbal Analogies*

Analogy

Analogy means 'similarity' or 'similar relationship'. In questions on number or letter analogies, a pair, that has a certain relationship between them, is given. This number/letter pair is followed by a third number/letter. The student is expected to identify the relationship between the pair given and find out a FOURTH number such that the relationship between the third and the fourth is similar to the relationship that exists between the first and the second. (In some cases, it may not be the fourth one that has to be found out. The fourth one will be given and the student has to find out one of the other three, whichever is not given).

Number Analogies

Typical relationships between the numbers in a given pair can be any of the following:

- One number is a multiple of the other.
- One number is the square or square root of the other.
- One number is the cube or cube root of the other.
- The two numbers are squares of two other numbers which themselves are related. For example, the two numbers are squares of two consecutive integers or squares of two consecutive even integers or squares of two consecutive odd integers.
- The two numbers are such that they are obtained by subtracting a certain number from the squares or cubes of the two related numbers.
- The two numbers are such that they are obtained by adding a certain number to the squares or cubes of the two related numbers.
- The two numbers can be consecutive, even, odd, or prime numbers.

There can be many more combinations that one can think of but the student has to note an important point in solving questions on number analogies. In number series-related questions, since a series of numbers (more than two numbers) will be given, the relationship or pattern can be identified uniquely. In number analogies, since only two numbers are given, it may be possible to think of more than one relationship existing between the two numbers in the given pair. But, it should be kept in mind that generally, simple addition of one number or subtraction of one number is not what is given in number analogies. The questions try to test the insight that the student has got into the relationship between the numbers.

Let us take a few examples and understand the questions on number analogies.

Solved Examples

Example 1

Find the missing number 25 : 36 : : 49 : _____.

(A) 61 (B) 63 (C) 65 (D) 60

Solution

When the numbers in the question are considered the students tend to consider 25 and 36 as squares of two consecutive natural numbers. But, the answer choices does not consist of an answer suitable to the above logic. Hence, it is important that, the student keeps the answer choices in view in arriving at the logic.

$$25 + 11 = 36$$

Similarly, $49 + 11 = 60$

Example 2

Find the missing number 27 : 51 : : 83 : _____.

(A) 102 (B) 117 (C) 123 (D) 138

Solution

The given analogy can be written as

$$5^2 + 2 : 7^2 + 2 : : 9^2 + 2 : \_\_\_\_\_.$$

5 and 7 are successive odd numbers.

Similarly, next odd number to 9 is 11 and $11^2 + 2 = 121 + 2 = 123$.

Example 3

Find the missing number.

11 : 25 : : 17 : _____.

(A) 33 (B) 28 (C) 41 (D) 37

Solution

$$11 \times 2 + 3 = 22 + 3 = 25$$

Similarly, $17 \times 2 + 3 = 34 + 3 = 37$.

Letter Analogies

The questions in this area are similar to verbal analogies. Here, the questions are based on the relationship between two groups of letters (instead of two words as in verbal analogies). Typically, three sets of letters are given followed by a question mark (where a fourth set of letters is supposed to inserted). The student has to find the relation or order in which the letters have been grouped together in the first two sets of letters on the left hand side of the symbol : : and then find a set of letters to fit in place of the question mark so that the third and the fourth set of letters will also have the same relationship as the first and the second. The sequence or order in which the letters are grouped can be illustrated by the following examples.

Example 4

BDEG : DFGI : : HKMO : _____.

(A) ILNP (B) JMOP

(C) JMOQ (D) JNOQ

Solution

Two letters are added to each letter to get the next letters in the analogy.

B	D	E	G;	Similarly,	H	K	M	O
+2	+2	+2	+2		+2	+2	+2	+2
D	F	G	I		J	M	O	Q

Example 5

ACDF : CGJN : : BEHI : _____.

(A) DJNQ (B) DINQ

(C) DINR (D) DHNQ

Solution

A	C	D	F;	Similarly,	B	E	H	I
+2	+4	+6	+8		+2	+4	+6	+8
C	G	J	N		D	I	N	Q

Example 6

SUWY : LPTX : : PRTV : _____.

(A) INRU (B) INQU

(C) IMRU (D) IMQU

Solution

S	U	W	Y;	Similarly,	P	R	T	V
–7	–5	–3	–1		–7	–5	–3	–1
L	P	T	X		I	M	Q	U

Example 7

BCDE : DFHH : : FGHI : _____.

(A) LJPL (B) LKPL

(C) JKPJ (D) IKPL

Solution

B	C	D	E;	Similarly,	F	G	H	I
×2	+3	×2	+3		×2	+3	×2	+3
D	G	H	H		L	J	P	L

Verbal Analogies

Here, the questions are based on relationship between two words. In these kind of questions, three words are followed by a blank space, which the student has to fill up in such a way that the third and the fourth words have the same relationship between them as the first and the second words have. The following examples help in understanding the concepts.

Example 8

Gum : Stick : : Needle : _____

(A) Cloth (B) Prick

(C) Taylor (D) Stitch

Solution

Gum is used to stick and needle is used to stitch.

Example 9

Socks : Feet : _____ : Hands

(A) Arms (B) Shirt
(C) Gloves (D) Fingers

Solution

Socks are worn on feet, similarly gloves are worn on hands.

Example 10

Soft : Hard : : Cold : _____

(A) Hot (B) Shirt
(C) Gloves (D) Fingers

Solution

Soft and hard are antonyms; similarly, the antonym of cold is hot.

Exercises

Direction for questions 1 to 25: Find the missing term.

1. 97 : 89 : : 43 : _______
(A) 37 (B) 31 (C) 39 (D) 41

2. 196 : 256 : : 324 : _______
(A) 361 (B) 400 (C) 411 (D) 484

3. 121 : 169 : : 361 : _______
(A) 529 (B) 400 (C) 484 (D) 576

4. 125 : 343 : : 343 : _______
(A) 512 (B) 1331 (C) 1728 (D) 81

5. 4 : 256 : : 5 : _______
(A) 625 (B) 1025 (C) 525 (D) 875

6. 12 : 144 : : 18 : _______
(A) 160 (B) 180 (C) 190 (D) 150

7. 25 : 21 : : 59 : _______
(A) 42 (B) 46 (C) 76 (D) 56

8. 8 : 72 : : 10 : _______
(A) 95 (B) 106 (C) 99 (D) 90

9. 8 : 0.125 : : 4 : _______
(A) 0.5 (B) 0.4 (C) 0.35 (D) 0.25

10. 11 : 143 : : 19 : _______
(A) 443 (B) 450 (C) 420 (D) 437

11. 568 : 352 : : 732 : _______
(A) 516 (B) 496 (C) 526 (D) 536

12. 6 : 222 : : 9 : _______
(A) 738 (B) 720 (C) 729 (D) 744

13. 5 : 120 : : 8 : _______
(A) 520 (B) 504 (C) 448 (D) 512

14. 16 : 68 : : 36 : _______
(A) 216 (B) 210
(C) 222 (D) 226

15. 10 : 95 : : 16 : _______
(A) 218 (B) 318 (C) 248 (D) 102

16. 3829 : 3851 : : 2987 : _______
(A) 301 (B) 3007
(C) 3017 (D) 3023

17. 47 : 121 : : 89 : _______
(A) 183 (B) 187 (C) 193 (D) 195

18. NATURE : PEVASI : : ISOMERS : _______
(A) OTUNJTV (B) OTUNIST
(C) PUVNJST (D) OVTNJST

19. BAD : BBL : : JDFE : _______
(A) JHRI (B) JHPX
(C) JFTV (D) JHRT

20. FIELD : LRJXH : : CRICKET : _______
(A) FHRDXLJ (B) FJPDTLN
(C) FJRDXAL (D) FJRFVJN

21. TAP : SUZBOQ : : RED : _______
(A) QTDGDE (B) PSDEDF
(C) QSDFCE (D) QRDGBE

22. Train : Track : : Bus : _______
(A) Driver (B) Road
(C) Petrol (D) Passengers

23. Earth : Planet : : Carrot : _______
(A) Vegetable (B) Plant
(C) Cooking (D) Nut

24. Wood : Carpenter : : Iron : _______
(A) Goldsmith (B) Instrument
(C) Melting (D) Blacksmith

25. Pen : Write : : Knife : _______
(A) Vegetable (B) Cut
(C) Sharp (D) Shoot

Answer Keys

1. D	**2.** B	**3.** A	**4.** B	**5.** A	**6.** D	**7.** D	**8.** D	**9.** D	**10.** D
11. A	**12.** A	**13.** B	**14.** C	**15.** C	**16.** A	**17.** B	**18.** B	**19.** D	**20.** D
21. C	**22.** B	**23.** A	**24.** D	**25.** B					

Chapter 3

Odd Man Out

CHAPTER HIGHLIGHTS

- *Odd Man Out*
- *Alphabet Classification*
- *Word Classification*
- *Number Classification*

Introduction

Finding the odd man out from the given alternatives is a very common type of questions that one comes across in different competitive examinations. In the questions on odd man out, all the items—except one—follow a certain pattern (in their formation) or belong to a group. The item that does not follow the pattern or does not belong to the group has to be marked as the answer choice.

The problems of this variety often fall under the category of CLASSIFICATION. When a given set of elements is classified under a single head, one of the items will not fall into that group to which the rest belong, i.e. it will not have the common property, which the others will have. Hence, it becomes the odd man out.

Questions on classification can be asked in any form. Some of the commonly asked ones are given below.

Alphabet Classification

In this type, a group of jumbled letters typically consisting of three letters, (but can be four or two or just a single letter) are put together. The pattern or order in which they are grouped is to be studied and we need to find out which groups have the same pattern or relationship between the letters. There will be one choice, which will have a pattern different from the rest and that is our answer.

Solved Examples

Example 1

Find the odd one among the following.

(A) ZW (B) TQ
(C) SP (D) NL

Solution

$$Z^{-3}W,\ T^{-3}Q,\ S^{-3}P,\ N^{-2}L$$

Hence, NL is the odd one.

Example 2

Find the odd one among the following.

(A) CFD (B) GJH
(C) KNM (D) JMK

Solution

$$C^{+3}F^{-2}D,\ G^{+3}J^{-2}H,\ K^{+3}N^{-1}M,\ J^{+3}M^{-2}K$$

Hence, KNM is the odd one.

Word Classification

Here, different items are classified based on common properties like names, places, professions, and parts of speech. A few examples are illustrated further.

Example 3

Find the odd one among the following.

(A) Mercury (B) Moon
(C) Jupiter (D) Saturn

Solution

All others except Moon are planets, whereas Moon is a satellite.

Example 4

Find the odd one among the following.

(A) SORE (B) SOTLU
(C) NORGAE (D) MEJNIAS

Solution

The words are jumbled. The actual words are ROSE, LOTUS, ORANGE, and JASMINE. All, except ORANGE, are flowers, whereas ORANGE is a fruit.

Example 5

(A) Cow (B) Goat
(C) Horse (D) Dog

Solution

All except dog are herbivorous animals.

Example 6

(A) Shoe (B) Spectacle
(C) Scissor (D) Shirt

Solution

All except shirt are in prises.

Number Classification

In this case, we need to choose the odd number from the given alternatives. The numbers may belong to a particular set, i.e. they may be odd, even, prime, rational, squares, cubes, and they may also be coded into binary digits (involving 0's and 1's), etc. and only one of the choices will not follow the rule that others do and that is our answer. A few illustrations are given further.

Example 7

Find the odd one among the following.

(A) 17 (B) 27
(C) 37 (D) 47

Solution

All the given numbers except 27 are prime numbers, whereas 27 is a composite number.

Example 8

(A) 441 (B) 289
(C) 361 (D) 343

Solution

The given numbers can be written as $(21)^2, (17)^2, (19)^2, (7)^3$. All except 343 are the squares, whereas 343 is a cube.

Example 9

(A) 10 (B) 50
(C) 120 (D) 290

Solution

All expect 120 can be expressed as $n^2 + 1$

$$10 = 3^2 + 1,\ 50 = 7^2 + 1$$

$290 = 17^2 + 1$ but $120 = 11^2 - 1$.

Example 10

(A) 235 (B) 352
(C) 523 (D) 253

Solution

All expect 352 are odd numbers but whereas 352 is an even number.

Exercises

Direction for questions 1 to 25: Find the odd man out.

1. (A) 16 (B) 28
(C) 36 (D) 64
2. (A) 27 (B) 37
(C) 47 (D) 67
3. (A) 8 (B) 27
(C) 64 (D) 125
4. (A) 42624 (B) 37573
(C) 84284 (D) 93339
5. (A) 30 (B) 630
(C) 10 (D) 520
6. (A) 8 : 9 (B) 25 : 25
(C) 64 : 81 (D) 16 : 16
7. (A) $\frac{3}{\sqrt{4+25}}$ (B) $\frac{7}{\sqrt{36+64}}$
(C) $\frac{11}{\sqrt{49+169}}$ (D) $\frac{5}{\sqrt{9+49}}$

8. (A) $13\frac{17}{23}$ (B) $41\frac{45}{49}$
(C) $71\frac{73}{79}$ (D) $83\frac{89}{97}$

9. (A) 4422 (B) 2442
(C) 4242 (D) 2244

10. (A) 350 (B) 70
(C) 30 (D) 520

11. (A) N (B) O
(C) B (D) K

12. (A) $E\frac{V}{R}$ (B) $O\frac{L}{B}$
(C) $I\frac{R}{V}$ (D) $U\frac{B}{L}$

13. (A) ABD (B) BDH
(C) CEJ (D) DFL

14. (A) BCDE (B) FGHI
(C) RSTU (D) WXYZ

15. (A) DFRTH (B) ABEJM
(C) NBEJM (D) DHKVY

16. (A) Cat (B) Dog
(C) Tiger (D) Elephant

17. (A) Chameleon (B) Crocodile
(C) Turtle (D) Allegator

18. (A) Trivandrum (B) Hyderabad
(C) Calicut (D) Bangalore

19. (A) Part (B) Trap
(C) Cart (D) Dart

20. (A) Rocket (B) Star
(C) Planet (D) Comet

21. (A) Skin (B) Tongue
(C) Leg (D) Nose

22. (A) Baseball (B) Boxing
(C) Chess (D) Wrestling

23. (A) Walk (B) Talk
(C) Drink (D) Plank

24. (A) Ganga (B) Nagarjuna sagar
(C) Yamuna (D) Sutlez

25. (A) HEWAT (B) CERI
(C) ROWAJ (D) EECRALS

Answer Keys

1. B	**2.** A	**3.** C	**4.** C	**5.** B	**6.** B	**7.** B	**8.** B	**9.** C	**10.** B
11. B	**12.** D	**13.** A	**14.** D	**15.** A	**16.** D	**17.** A	**18.** C	**19.** B	**20.** A
21. C	**22.** A	**23.** D	**24.** B	**25.** D					

Chapter 4

Coding and Decoding

CHAPTER HIGHLIGHTS

- *Coding*
- *Decoding*

Coding and Decoding

Before looking at the different types of questions and some of the codes that can be used with the help of examples, let us first understand what we mean by ***coding*** and ***decoding***. When we say ***coding***, a particular code or pattern is used to express a word in English language as a different word or in a different form. The coded word itself does not make any sense unless we know the pattern or code that has been followed. ***Decoding*** refers to the process of arriving at the equivalent English word from the code word given.

In the questions, a particular code is given and on the basis of this given code, we have to find out how another word (in English language) can be coded. The correct code for the given word has to be selected from the answer choices on the basis of the code given in the question.

Solved Examples

Example 1

In a certain code language, if the word 'PARTNER' is coded as OZQSMDQ, then what is the code for the word 'SEGMENT' in that language?

(A) TFHNFOU (B) RDFLDMS
(C) RDELDMS (D) RDFEDNS

Solution

Word : P A R T N E R
Logic :–1 –1 –1 –1 –1 –1 –1
Code : O Z Q S M D Q

Similarly, the code for SEGMENT is

Word : S E G M E N T
Logic : –1 –1 –1 –1 –1 –1 –1
Code : R D F L D M S

Example 2

In a certain code language, if the word RECTANGLE' is coded as TGEVCPING, then how is the word 'RHOMBUS' coded in that language?

(A) TJOQDWV (B) TJQNDWU
(C) TJQODWU (D) TJQOEWU

Solution

Word : R E C T A N G L E
Logic : +2 +2 +2 +2 +2 +2 +2 +2 +2
Code : T G E V C P I N G

Similarly, the code for RHOMBUS is

Word : R H O M B U S
Logic : +2 +2 +2 +2 +2 +2 +2
Code: T J Q O D W U

Example 3

In a certain code language, if the word 'SPHERE' is coded as EREHPS, then how is the word 'EXHIBITION' coded in that language?

(A) NOTITBIHXE (B) NOITIDIHXE
(C) NOITIBIHWE (D) NOITIBIHXE

Solution

Word: S P H E R E
Logic: The letters in the given word are reversed.
Code: E R E H P S

Similarly, the code for EXHIBITION, is

Word: E X H I B I T I O N
Logic: The letters in the given word are reversed.
Code: N O I T I B I H X E

Example 4

In a certain code language, if the word 'REJECTION' is coded as SGMIHZPWW, then how is the word 'MECHANIC' coded in that language?

(A) NGFLFTPK (B) NGPLFTPK
(C) NGFKFTPK (D) NGPTPKIL

Solution

Word:	R	E	J	E	C	T	I	O	N
Logic :	+1	+2	+3	+4	+5	+6	+7	+8	+9
Code :	S	G	M	I	H	Z	P	W	W

Similarly, the code for MECHANIC is

Word :	M	E	C	H	A	N	I	C
Logic :	+1	+2	+3	+4	+5	+6	+7	+8
Code :	N	G	F	L	F	T	P	K

Example 5

In a certain code language, if the word 'PLAYER' is coded as AELPRY, then how is the word 'MANAGER' coded in that language?

(A) AEAGMNR (B) AAGEMNR
(C) AAEGMNR (D) AAEGNMR

Solution

Word: P L A Y E R
Logic: The letters in the word are arranged in the increasing order of their value as in the alphabet.
Code: A E L P R Y

Similarly, the code for MANAGER is AAEGMNR.

Example 6

In a certain code language, if the number 1 is assigned to all the letters in odd numbered places in the alphabet and the remaining letters are assigned the number 2, then what is the code for the word 'INDIAN'?

(A) 121212 (B) 111222
(C) 112212 (D) 122112

Solution

The code for the word INDIAN is 122112.

Example 7

In a certain code language, if CRICKET is coded as 3923564, ROCKET is coded as 913564, and KETTLE is coded as 564406, then how is LITTLE coded in that language?

(A) 244060 (B) 024406
(C) 020446 (D) 200446

Solution

As we observe that the letters and their corresponding codes are given in an order, i.e. the code for C is 3, R is 9, I is 2, and so on. Hence, the code for LITTLE is 024406.

Direction for questions 8 to 10: In a certain code language, the codes for some words are as follow.

NATION	-	agvnab
REMOTE	-	rzgrbe
STAIR	-	efgnv
FORMAL	-	bensyz
COMMON	-	zabzpb
FOR	-	ebs

On the basis of the above coding pattern, answer the following questions.

8. What is the code for 'SCREEN'?
(A) fepcra (B) fpersa
(C) fpreba (D) fperra

9. What is the code for 'RATION'?
(A) ensvba (B) engvba
(C) engrba (D) engvca

10. What is the code for 'CREATOR'?
(A) prengbc (B) persbgc
(C) perngbe (D) pebryc

Solutions for questions 8 to 10:

The given words and their codes are as follow

(A)	NATION	-	agvnab
(B)	REMOTE	-	rzgrbi
(C)	STAIR	-	efgnv
(D)	FORMAL	-	bensyz
(E)	COMMON	-	zabzpb
(6)	FOR	-	ebs

In the 1st word, the letter N is repeated and so is the code *a*. Hence, for N, the code is a. Similarly, from the 2nd word, the code for E is '*r*'. In 1st and 6th words, the letter *o* is common and so is the code *b*. Hence, the code for *o* is *b*. In the 5th word, the letter *m* is repeated and so is the code *z*. Hence, the code for *m* is *z*. Similarly, the codes for the remaining letters can be determined.

The letters and their respective codes are as follows:

Letter	A	C	E	F	I	L	M	N	O	R	S	T
Code letter	n	p	R	s	v	y	z	a	b	e	f	G

8. The code for 'SCREEN' is fperra.
Hence, the correct option is (D).

9. The code for 'RATION' is engvba.
Hence, the correct option is (B).

10. The code for 'CREATOR' is perngbe.
Hence, the correct option is (C).

Exercises

Direction for questions 1 to 12: Select the correct alternative from the given choices.

1. In a certain code language, if the word CIRCUMSTANCE is coded as CRUSACICMTNE, then how is the word HAPPINESS coded in that language?
(A) HPEISAPNS (B) HPISEAPNS
(C) HPIESPANS (D) HPIESAPNS

2. In a certain code language, if the word REGISTRATION is coded as TSIGERNOITAR, then how is the word ACCURATE coded in that language?
(A) UCCAETAR (B) UACCETAR
(C) UCACETAR (D) UCCATEAR

3. In a certain code language, if the word LIBERAL is coded as MJCFSBM, then how is the word REDUCTION coded in that language?
(A) EDCTBSHNM (B) SFEVDUJPO
(C) SFEVCTJPO (D) SFDUCTJPO

4. In a certain code language, if the word STRUCTURE is coded as TVUYHZBZN, then how is the word REMEDY coded in that language?
(A) SGPIJE (B) SGPEJD
(C) SGPIHE (D) SGPIIE

5. In a certain code language, if the word SEARCH is coded as IDSBFT, then how is the word FURNISH coded in that language?
(A) ITKNSVG (B) ITJORWG
(C) ITJOSVG (D) ITHNRVG

6. In a certain code language 'two' is called 'three', 'three' is called 'four', 'four' is called 'one', 'one' is called 'five', 'five', is called 'six', and 'six' is called 'nine', then what in the code language is the sum of one and three?
(A) six (B) two (C) nine (D) one

7. In a certain code language if 'pink' means 'black', 'black' means 'white', 'white' means 'yellow', 'yellow' means 'orange', 'orange' means 'red', and 'red' means 'green', then which colour stands for peace in that code?
(A) Red (B) Black
(C) Orange (D) Green

8. In a certain code language, if MENTION = 49 and NEUROTIC = 64, then MARVELLOUS = ?
(A) 81 (B) 88 (C) 64 (D) 100

9. In a certain code language, if CABINET = 70 and BEAUTY = 60, then PRODUCTION = ?
(A) 90 (B) 100 (C) 110 (D) 120

10. In a certain code language, if IMPEND = 61 and DISH = 40, then FRUIT = ?
(A) 86 (B) 68 (C) 74 (D) 76

11. In a certain code language, if BUG = 90 and ALMS = 180, then CADET = ?
(A) 153 (B) 165 (C) 175 (D) 148

12. In a certain code language, if INFER = 25 and JERSEY = 28, then CHOICE =?
(A) 34 (B) 39 (C) 41 (D) 47

Direction for questions 13 to 15: These questions are based on the following data.

In a certain code language, if the word ROUTINE is coded as JMPRRLJ and the word FIDELITY is coded as LGHCXGNW, then how will you code the following words in that language?

13. PREVAIL
(A) FPLRDGX (B) FPJTBGX
(C) FTJBNKX (D) FPJVBIX

14. LANGUAGE
(A) XYBDPXNC (B) XYBDPXMC
(C) XYCEPXNC (D) XYBEPYNC

15. TOBACCO
(A) NMDXEAF (B) NMDYEBF
(C) NMCYFBD (D) NMDYFAD

Direction for questions 16 to 20: For the following groups of letters given in column I, the codes are given in column II. Answer the following questions by finding the codes for the groups from the given columns.

	Column I	Column II
(A)	lit kit bit dit	b r p d
(B)	fit git mit kit	t d s v
(C)	rit bit git tit	x p v w
(D)	nit dit fit rit	r s x j

16. What is the code for lit?
(A) v (B) r (C) p (D) b

17. What is the code for tit?
(A) w (B) x (C) p (D) v

18. What is the code for rit?
(A) j (B) s (C) r (D) x

19. What is the code for nit?
(A) x (B) s (C) j (D) r

20. What is the code for kit?
(A) r (B) p (C) x (D) d

Direction for questions 21 to 25: For the words given in column I, the codes are given in column II. Answer the following questions by finding the codes for the letters from the words and their codes given in the columns.

	Column I	Column II
(A)	PRETEND	4396408
(B)	COMMON	615715
(C)	HOUSE	4*2&1
(D)	SUPPORT	3*21839
(E)	DRUM	5*08

21. What is the code for the word PROTECT?
(A) 3895479 (B) 3846978
(C) 3819479 (D) 3814978

22. What is the code for the word HORMONE?
(A) &385364 (B) &176561
(C) &175184 (D) &185164

23. What is the code for the word EMPEROR?
(A) 5495717 (B) 4534818
(C) 3453919 (D) 4537178

24. What is the code for the word DETHRONE?
(A) 049&7264
(B) 049&8164
(C) 059&7164
(D) 059&8164

25. What is the code for the word COMPOUND?
(A) 71531*60 (B) 72532*80
(C) 91531*70 (D) 72542*60

Answer Keys

1. D	**2.** A	**3.** B	**4.** D	**5.** C	**6.** D	**7.** B	**8.** D	**9.** B	**10.** C
11. B	**12.** A	**13.** B	**14.** D	**15.** D	**16.** D	**17.** A	**18.** D	**19.** C	**20.** D
21. C	**22.** D	**23.** B	**24.** B	**25.** A					

Chapter 5

Blood Relations

CHAPTER HIGHLIGHTS

- *Blood Relations*
- *Grandparents*
- *Parents and In-Laws*
- *Siblings, Spouse, and In-Laws*
- *Children and In-Laws*
- *Grand Children*

Blood Relations

There are two types of questions based on blood relations that are given in different competitive examinations. For the sake of convenience, we will refer to the two types of questions as Type I and Type II. (Please note that the questions on blood relationships are not categorised as above in the actual exam papers. It is being done purely from the point of better understanding.)

In the exams, the success of a candidate in the questions on blood relations depends upon his knowledge about various blood relations. Some of the relationships given below help in solving the problems.

The easiest and non-confusing way to solve these types of problems would be to draw a family tree diagram and increase the levels in the hierarchy as shown below:

1st stage :	GRANDPARENTS	→ (Grandfather, Grandmother, Granduncle, Grandaunt)
2nd stage :	PARENTS and IN-LAWS	→ (Father, Mother, Uncle, Aunt, Father-in-law, Mother-in-law)
3rd stage :	SIBLINGS, SPOUSE, and IN-LAWS	→ (Brother, Sister, Cousin, Wife, Husband, Brother-in-law, Sister-in-law)
4th stage :	CHILDREN and IN-LAWS	→ (Son, Daughter, Niece, Nephew, Son-in-law, Daughter-in-law)
5th stage :	GRAND CHILDREN	→ (Grandson, Granddaughter)

Mother's or Father's son	:	Brother
Mother's or Father's daughter	:	Sister
Mother's or Father's brother	:	Uncle
Mother's or Father's sister	:	Aunt
Mother's or Father's mother	:	Grandmother
Mother's or Father's father	:	Grandfather
Grandmother's brother	:	Granduncle
Grandmother's sister	:	Grandaunt
Grandfather's brother	:	Granduncle
Grandfather's sister	:	Grandaunt

Sister's or Brother's son	:	Nephew
Sister's or Brother's daughter	:	Niece
Uncle or Aunt's son or daughter	:	Cousin
Son's wife	:	Daughter-in-law
Daughter's husband	:	Son-in-law
Husband's or Wife's sister	:	Sister-in-law
Husband's or Wife's brother	:	Brother-in-law
Sister's husband	:	Brother-in-law
Brother's wife	:	Sister-in-law
Children of same parents	:	Siblings (could be all brothers, all sisters or some brothers and some sisters)
Children	:	Son, Daughter
Children's Children	:	Grandchildren (Grandson, Granddaughter)

In addition, remember the word spouse, which means either husband or wife.

Grandfather and grandmother will come in the first stage; mother, father, uncle, and aunt will come in the second stage; sister, brother, and cousin will come at the third stage; son, daughter, niece, and nephew will come in the fourth stage; and finally, granddaughters and grandsons will come. These stages are made from the point of view of an individual.

In Type-I questions, the relationship between two people is given through a roundabout way of relating them through other people. We have to go through the series of relationships and finally determine the relationship between the two people given in the question. The relationship can be given as a simple statement or as a statement made by a person. In the first example given, a person is involved in making a statement, whereas in the second question, there is no person involved in making a statement.

Solved Examples

Example 1

A's father's mother-in-law's only daughter's son is B. How is A related to B?

(A) Brother
(B) Sister
(C) Nephew
(D) Cannot be determined

Solution

A's father's mother-in-law's only daughter is A's mother. A's mother's son is A's brother.
But A can be either brother or sister to B.

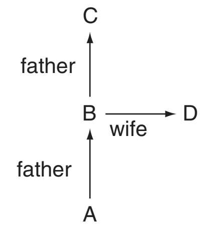
Example 2

If A's father is B, C is the father of B, and D is A's mother, then how is C related to D?

(A) Father (B) Grandfather
(C) Father-in-law (D) Uncle

Solution

A's father is B and mother is D. Therefore, D is B's wife and C is the father of B. Hence, C is D's father-in-law.



Example 3

A + B means A is the son of B.
A – B means A is the daughter of B.
A × B means A is the father of B.
A ÷ B means A is the mother of B.

If M × N + O – P ÷ Q, then how is M related to Q?

(A) Husband (B) Cousin
(C) Brother-in-law (D) Uncle

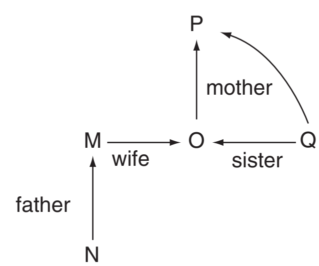
Solution

M × N + O – P ÷ Q means M is the father of N, N is the son of O, O is the daughter of P, P is the mother of Q.

M is the father of N and N is the son of O means M is the husband of O.

O is the daughter of P and P is the mother of Q means O is the sister of Q.

M is the husband of O and O is the sister of Q means M is the brother-in-law of Q.



Example 4

A + B means A is the son of B.
A – B means A is the daughter of B.
A × B means A is the father of B.
A ÷ B means A is the mother of B.

Which of the following means S is the son-in-law of P?

(A) P + Q ÷ R × S – T
(B) P × Q ÷ R – S + T
(C) P + Q × R – S ÷ T
(D) P × Q – R ÷ S × T

Solution

P + Q ÷ R × S – T means P is the son of Q. Q is the mother of R, R is the father of S, and S is the daughter of T. Hence, S is the nephew of P.

P × Q ÷ R – S + T means P is the father of Q, Q is the mother of R, R is the daughter of S, and S is the son of T. Hence, S is the son-in-law of P.

P + Q × R – S ÷ T means P is the son of Q, Q is the father of R, R is the daughter of S, and S is the mother of T. Hence, S is the mother of P.

P × Q – R ÷ S × T means P is the father of Q, Q is the daughter of R, R is the mother of S, and S is the father of T. Hence, S is the son of P.

Example 5

Pointing to a person, Raju said, 'He is the only brother of my father's mother's daughter'. How is the person related to Raju?

(A) Brother (B) Father
(C) Uncle (D) Nephew

Solution

Raju's father's mother's daughter is Raju's father's sister. Raju's father's sister's only brother is Raju's father. Hence, the person is Raju's father.

Example 6

A's mother's father is the husband of B's mother. How is A related to B, if A and B are both male.

(A) Uncle (B) Father
(C) Nephew (D) Son

Solution

A's mother's father is the husband of B's mother. That means A's mother is the sister of B. Hence, all the nephew of B.

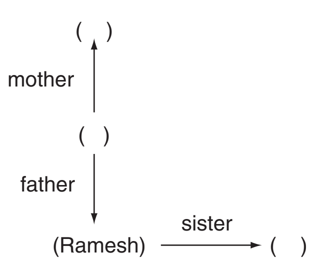
Example 7

Pointing to a photograph Ramesh said, 'she is the sister of my father's mother's only child's son.' How is the person in the photograph related to Ramesh?

(A) Sister (B) Aunt
(C) Mother (D) Cousin

Solution

My father's mother's only child is my father. My father's son's sister is in the photograph. Hence, she is Ramesh's sister.



Example 8

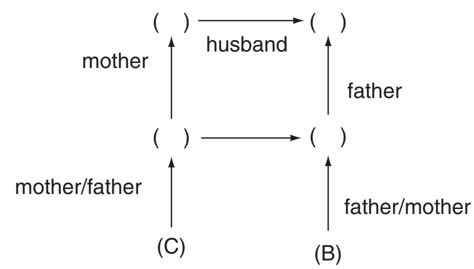
B's father's father is the husband of C's mother's mother. How is B related to C?

(A) Brother
(B) Sister
(C) Cousin
(D) Cannot be determined

Solution

B's father's father is B's grandfather.

B's grandfather is the husband of C's mother's mother, i.e. grandmother. It is possible that B and C are sibling's and the persons mentioned are their paternal/maternal grand parents. It is also possible that B and C are cousins. Hence, the relationship cannot be determined.



Direction for questions 9 and 10:

A × B means A is the daughter of B
A ∗ B means A is the son of B
A + B means A is the mother of B
A – B means A is the brother of B
A ÷ B means A is the sister of B
A = B means A is the father of B

Example 9

Which of the following means S is the nephew of P?

(A) P – Q ÷ R = S (B) P – Q × R ∗ S
(C) S ∗ R – Q ÷ P (D) P + Q – R × S

Solution

P – Q ÷ R = S means P is the brother of Q, Q is the sister of R, and R is the father of S. As we do not know whether S is the son or daughter of R, we cannot determine that S is the nephew.

P – Q × R ∗ S means P is the brother of Q, Q is the daughter of R, and R is the son of S. Here, S is the grand-father of P.

S ∗ R – Q ÷ P means S is the son of R. R is the brother of Q, and Q is the sister of P. Hence, S is the nephew of P.

Example 10

Which of the following means T is the husband of V?

(A) T = S – R ∗ V (B) T + S – R × V
(C) R × V – T ∗ S (D) R + S – T × V

Solution

T = S – R ∗ V means T is the father of S, S is the brother of R, and R is the son of V. Therefore, T is the husband of V.

Exercises

Direction for questions 1 to 15: Select the correct alternative from the given choices.

1. A person who is the husband of my son's sister is my
   (A) Nephew (B) Son-in-law
   (C) Son (D) Brother
2. Y is the daughter of X's brother's wife's father-in-law. Y is X's ________.
   (A) Niece (B) Daughter
   (C) Sister (D) Sister-in-law
3. Showing a photograph P said,' She is my mother's mother's son's daughter'. How is the person in the photograph related to P?
   (A) Sister (B) Cousin
   (C) Niece (D) Mother
4. How is my father's mother's only daughter-in-law's sister related to me?
   (A) Aunt (B) Sister
   (C) Cousin (D) Niece
5. How is my grandmother's only child's husband's mother related to me?
   (A) Mother (B) Grandmother
   (C) Aunt (D) Sister
6. How is Ramu's mother-in-law's only daughter's son related to Ramu?
   (A) Nephew (B) Brother
   (C) Son (D) Uncle
7. How is my son's mother's daughter related to me?
   (A) Niece (B) Granddaughter
   (C) Daughter (D) Aunt
8. How is my father's brother's only sibling's mother related to me?
   (A) Mother (B) Cousin
   (C) Daughter (D) Grandmother
9. A is the father of B. C is the son of D. E is the brother of C while D is the sister of B. How is B related to E?
   (A) Uncle (B) Aunt
   (C) Mother (D) Either (A) or (B)
10. My mother's sister's son's father's mother-in-law is related to me as
    (A) Mother (B) Grandmother
    (C) Mother-in-law (D) Aunt
11. How is David's father's only daughter-in-law's son's wife related to David?
    (A) Daughter (B) Daughter-in-law
    (C) Niece (D) Granddaughter
12. How is Ravi's mother's father's son related to Ravi's father?
    (A) Cousin (B) Uncle
    (C) Brother-in-law (D) Son-in-law
13. Divya's father, pointing towards a person, said, 'He is the brother of my father's only sibling'. How is the person related to Divya?
    (A) Father (B) Uncle
    (C) Brother (D) Grandfather
14. Tinku, introducing a person to Rinku, said 'He is the father of your sister's son and he is also my mother's husband'. How is Tinku's father related to Rinku's mother?
    (A) Nephew (B) Uncle
    (C) Son-in-law (D) Father
15. A is B's father, B is C's daughter, E is D's only sibling. C is D's only daughter. How is B related to E's niece?
    (A) Niece (B) Granddaughter
    (C) Daughter (D) Mother

Direction for questions 16 to 20: Use the relations defined below and answer the following questions.

$A + B$ means A is the mother of B.
$A - B$ means A is the sister of B.
$A \times B$ means A is the father of B.
$A \div B$ means A is the son of B.
$A = B$ means A is the brother of B.
$A \neq B$ means A is the daughter of B.

16. Which of the following means P is the aunt of Q?
    (A) $P - R \div Q$ (B) $P + R \times Q$
    (C) $P \neq R \times Q$ (D) $P - R + Q$
17. Which of the following means, S is the son of T's daughter?
    (A) $T \times M + S + N$ (B) $T \times M + S = N$
    (C) $T + M \times S - N$ (D) $S \div M \div T - N$
18. Which of the following means W is the uncle of Z?
    (A) $W \times A - B + Z$ (B) $W = A + B - Z$
    (C) $W = A + B + Z$ (D) $W \times A \times B = Z$
19. Which of the following means C is the grandfather of both D and E?
    (A) $C \times A \div D - E$ (B) $C + A + D \neq E$
    (C) $C \div A \neq D = E$ (D) $C \times A \times D - E$
20. Which of the following means I is the mother of L?
    (A) $I + B - C \neq D \times L$ (B) $I \neq B + C \times L$
    (C) $I + B \times C \neq D - L$ (D) $I + B - C \times L$

Direction for questions 21 to 25: These questions are based on the information given below.

A, B, C, D, E, and F are six members of a family. A is the mother of B, who is the husband of D. F is the brother of one of the parents of C. D is the daughter-in-law of E and has no siblings. C is the son of D.

21. How is C related to A?
    (A) Nephew
    (B) Son-in-law
    (C) Grandson
    (D) Father

22. How is F related to D?
(A) Cousin
(B) Brother-in-law
(C) Brother
(D) Father

23. How is E related to F?
(A) Mother
(B) Son
(C) Father
(D) Father-in-law

24. If F is married to G, then how is G related to B?
(A) Sister
(B) Sister-in-law
(C) Cousin
(D) Mother

25. How many male members are there in the family?
(A) Two
(B) Three
(C) Four
(D) Cannot be determined

Answer Keys

1. B	**2.** C	**3.** B	**4.** A	**5.** B	**6.** C	**7.** B	**8.** D	**9.** D	**10.** B
11. B	**12.** C	**13.** D	**14.** C	**15.** C	**16.** D	**17.** B	**18.** B	**19.** D	**20.** A
21. C	**22.** B	**23.** C	**24.** B	**25.** C					

Chapter 6

Venn Diagrams

CHAPTER HIGHLIGHTS

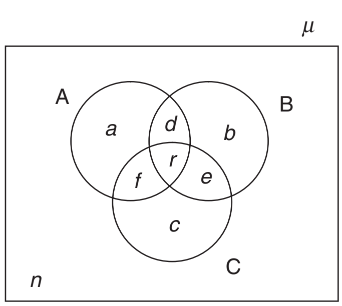
- *Venn Diagrams*
- *Venn Diagram Type I*
- *Venn Diagram Type II*

Venn Diagrams

Venn Diagrams are diagrammatic representation of sets, using geometrical figures like circles, triangles, and rectangles. Each geometrical figure represents a group as shown in the examples. The area common to two or more figures represent those elements which are common to two or more groups. There are various models in Venn diagrams, which we will discuss with examples.

Venn Diagram Type I

In this type, two, three, or four different groups could be given with some elements common to two or more groups. Let us observe the diagram given.



Here, A, B, and C are three different groups, and the various regions can be explained as follows:

Only A = a
Only B = b
Only C = c
A and B only = d
B and C only = e
C and A only = f
All the three (A, B, and C) = r
Both A and B = $d + r$
Both B and C = $e + r$
Both C and D = $f + r$
Neither A, nor B, nor C = n
A, B, or C and none = μ
Also, $\mu = (A \cup B \cup C) + n$

Here, the rectangle represents the sample space, which consists of three groups A, B, and C, and also n, which is the number of people belonging to neither A, nor B, nor C.

Some more formulae are as given under:

1. $A' = (b + e + c) + n$; where A' = A complement (not in A)
2. $B' = (a + f + c) + n$; where B' = B complement (not in B)
3. $C' = (a + d + b) + n$; where C' = C complement (not in C)
4. $A - B = A - (A \cap B)$
5. $A \Delta B = (A - B) \cup (B - A)$
6. Number of people (or things) belonging to at least one out of the three groups = $A \cup B \cup C$
   $= \underbrace{(a + b + c)}_{\text{exactly one}} + \underbrace{(d + e + f)}_{\text{exactly two}} + \underbrace{r}_{\text{exactly three}}$
7. $A + B + C = (A \cup B \cup C) + (d + e + f) + 2r$
   $= (a + b + c) + 2\,(d + e + f) + 3r$

Solved Examples

Direction for questions 1 to 3: These questions are based on the data given.

In a class of 165 students, 45 students are passed in Maths a well as in English, whereas 60 students are failed in Maths and 65 students are failed in English.

Example 1: How many students are passed in exactly one subject?
(A) 160 (B) 100
(C) 115 (D) 165

Example 2: How many students are failed in both the subjects?
(A) 25 (B) 20 (C) 45 (D) 5

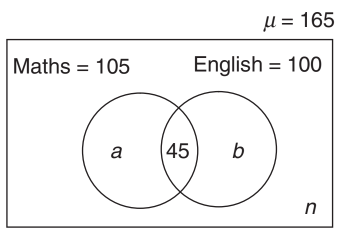
Example 3: How many students are failed only in Maths?
(A) 55 (B) 60 (C) 65 (D) 70

Solutions for questions 1 to 3:
Since, 60 students are failed in Maths.

∴ 165 – 60 = 105 students passed in Maths similarly, 65 students failed in English.

∴ 165 – 65 = 100 students passed in English.

The respective Venn diagrams is as follows, which shows the number of students who passed the subject.



45 students passed in Maths as well as English.

∴ $a = 105 - 45 = 60$ students passed only in Maths and $b = 100 - 45 = 55$ students passed only English.

Number of students passed in atleast one subject = 60 + 55 + 45 = 160.

Hence, $n = 165 - 160 = 5$ students failed in both.

Solution 1: $a + b = 60 + 55 = 115$ students passed exactly in one subject.

Hence, the correct option is (C).

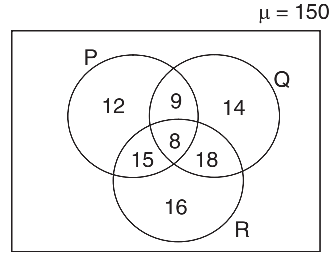
Solution 2: 5 students failed in both the subjects.

Hence, the correct option is (D).

Solution 3: As 55 students passed only in English which implies that 55 students failed only in Maths.

Hence, the correct option is (A).

Direction for questions 4 to 7: These questions are based on the given diagram.



Example 4: How many elements are there in Q′ (complement of Q)?
(A) 100 (B) 49 (C) 101 (D) 50

Example 5: How many elements are there in $P' \cap Q' \cap R'$?
(A) 35 (B) 8 (C) 58 (D) 48

Example 6: How many elements are there in R?
(A) 16 (B) 57 (C) 41 (D) 8

Example 7: How many elements are there in $P \cap (Q \cup R)$?
(A) 32 (B) 48 (C) 54 (D) 44

Solutions for questions 4 to 7:

Solution 4: Number of elements in Q′ = (μ) – (number of elements in Q)
= 150 – (14 + 18 + 8 + 9)
⇒ 150 – 49 = 101.

Hence, the correct option is (C).

Solution 5: Number of elements in $P' \cap Q' \cap R'$

$$= \mu - (P \cup Q \cup R)$$
$$= 150 - (12 + 15 + 9 + 8 + 18 + 16 + 14)$$
$$= 150 - 92 = 58.$$

Hence, the correct option is (C).

Solution 6: Number of elements in R = 16 + 15 + 8 + 18 = 57

Hence, the correct option is (B).

Solution 7: Number of elements in $Q \cup R$ =

(14 + 9 + 8 + 15 + 18 + 16)

Number of elements in P = (12 + 9 + 15 + 8)

$P \cap (Q \cup R)$ is the region common to P and $Q \cup R$

Number of elements in $P \cap (Q \cup R)$

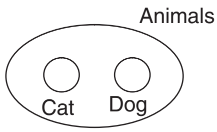
= 9 + 8 +15 = 32.

Hence, the correct option is (A).

Venn Diagram Type II

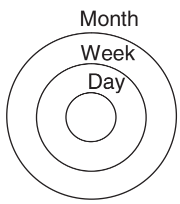
In this type, Venn diagrams are used to establish relationship between the given groups. In other words, two or more groups are given, and the Venn diagram, which most correctly establishes a relation between them, has to be chosen out of the various Venn diagrams given in the choices. Let us look at some of the examples given.

Examples: Animals, Cat, Dog



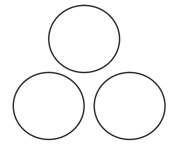
Here, in animals we have many species of which cat and dog are two different kinds of species, having nothing in common. So the above diagram is the most appropriate representation of the given groups.

Examples: Month, week, day



We know that day is a part of the week and week is a part of the month. So, the given diagram is the most appropriate representation of the given groups.

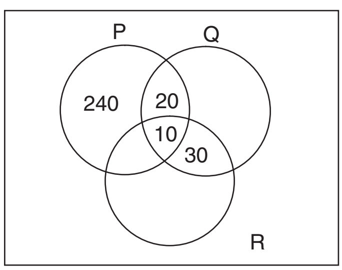
Examples: Mars, Earth, Jupiter



We know that Mars, Earth, and Jupiter are three independent entities having nothing in common. So the above-given diagram is the most appropriate representation of the given groups.

Exercises

Direction for questions 1 to 5: There are 1500 students in a college. Each student can be a member of three student communities namely P, Q, and R. Now, using the data mentioned and the diagram given, answer the questions that follow.



- Total members in community P is 300.
- Total members in community Q is 420.
- Total members in community R is 490.

1. How many students are part of only community R?
(A) 360 (B) 420 (C) 210 (D) 350

2. How many students is not part of any community?
(A) 390 (B) 420 (C) 410 (D) 490

3. How many students are part of at least two communities?
(A) 10 (B) 30
(C) 80 (D) 90

4. How many students are part of at least one community?
(A) 1000 (B) 1090
(C) 1110 (D) 1100

5. How many students are part of exactly two communities?
(A) 90 (B) 80
(C) 100 (D) 120

Direction for questions 6 to 10: These questions are based on the data given.

In a class of 95 students, 40 play cricket, 50 play football, and 10 play both cricket and football.

6. How many students play only football?
(A) 45 (B) 30 (C) 40 (D) 28

7. How many students play at least one game?
(A) 80 (B) 70 (C) 60 (D) 50

8. How many students play only cricket?
(A) 30 (B) 35 (C) 40 (D) 25

9. How many students play exactly one game?
(A) 85 (B) 80 (C) 70 (D) 75

10. How may students play neither cricket nor football?
(A) 12 (B) 15 (C) 18 (D) 20

Direction for questions 11 to 15: Study the following data and answer the questions given.

In a certain college, 37% of the students write EAMCET exam, 47% of the students write IIT-JEE exam, and 50% of the students write AIEEE exam. Also known that, 11% of the students write both EAMCET and IIT-JEE, 11% of the students write both EAMCET and AIEEE, 15% of the students write both IIT-JEE and AIEEE, while 15 students write all the three exams. Each student in the college writes at least one of the three exams.

11. How many students appear for the exams from the college?
(A) 400 (B) 200
(C) 500 (D) 600

12. How many students write exactly two exams?
(A) 120 (B) 110 (C) 140 (D) 150

13. The number of students who write only EAMCET as a percentage of the number of students who write only AIEEE is

(A) $33\frac{1}{3}\%$ (B) $66\frac{2}{3}\%$

(C) $33\frac{2}{3}\%$ (D) $66\frac{1}{3}\%$

14. How many students write exactly one exam?

(A) 345 (B) 395 (C) 198 (D) 398

15. What is the ratio of the number of students who write only AIEEE to that of those who write only IIT JEE?

(A) 3 : 2 (B) 2 : 3 (C) 8 : 9 (D) 9 : 8

Direction for questions 16 to 20: These questions are based on the data given.

In a library maintained by a student, there are books on different subjects. It was found that 35 books are on sports, 45 books are on business, and 15 books are on current affairs;14 books are on at least two subjects among sports, business, and current affairs; 3 books have sports, business as well as current affairs in them. Every book in the library is assumed to contain at least one of sports, business, or current affairs in them.

16. How many books are there, which contain information regarding only one subject?

(A) 58 (B) 64 (C) 60 (D) 62

17. What are the total number of books in his library?

(A) 78 (B) 72 (C) 68 (D) 80

18. How many books contained information regarding exactly two subjects?

(A) 11 (B) 10 (C) 9 (D) 14

19. How many books are there, which contain information regarding at most two subjects?

(A) 11 (B) 64 (C) 72 (D) 75

20. If the number of books on only sports is equal to 26, then how many books are there in the library, which are on both business and current affairs but not sports?

(A) 5 (B) 3 (C) 2 (D) 8

Direction for questions 21 to 25: These questions are based on the data given.

In a colony, it is known that three brands of mobile phones are used, namely Nokia, Sony Ericsson, and Motorola. 70 families use only one brand, 47 families use exactly two brands, and 8 use all the three brands. It is assumed that each family uses at least one of these three brands.

21. How many families are there in the colony?

(A) 75 (B) 100 (C) 105 (D) 125

22. How many families use at least two brands?

(A) 117 (B) 55 (C) 47 (D) 125

23. If 10 families stop using Nokia and start using Motorola, then what is the maximum number of families who use exactly two brands?

(A) 57 (B) 37 (C) 47 (D) 67

24. What is the ratio of the number of families which use exactly one brand to that which use at least one brand?

(A) 14 : 25 (B) 14 : 11

(C) 11 : 25 (D) 11 : 14

25. How many families do not use all the three brands?

(A) 125 (B) 117 (C) 0 (D) 8

Answer Keys

1. B	**2.** A	**3.** D	**4.** C	**5.** B	**6.** C	**7.** A	**8.** A	**9.** C	**10.** B
11. C	**12.** C	**13.** B	**14.** A	**15.** D	**16.** B	**17.** A	**18.** A	**19.** D	**20.** A
21. D	**22.** B	**23.** A	**24.** A	**25.** B					

Chapter 7

Seating Arrangements

CHAPTER HIGHLIGHTS

- *Seating Arrangement*
- *Linear Sequencing*
- *Circular Arrangement*

Linear Sequencing

Linear sequencing is essentially arranging the items given in a sequence (in a single line). The questions of this type are also referred to as 'seating arrangement'. The word 'seating arrangement' should not be misconstrued—it should not be treated as consisting of questions involving only persons sitting as per specified conditions. Essentially, these questions involve arranging subjects (people or things) satisfying the given conditions. The arrangement is done only on one 'axis' and, hence, the position of the subjects assumes importance here in terms of order: first position, second position, etc.

Let us look at the examples:

Direction for questions 1 to 5: Read the data given below carefully and answer the questions that follow.

Seven persons Paul, Queen, Rax, Sam, Tom, Unif, and Vali are sitting in a row facing us. Rax and Sam sit next to each other. There must be exactly four persons between Queen and Vali. Sam sits to the immediate right of Queen.

Solved Examples

Example 1: If Paul and Tom are separated exactly by two persons, then who sits to the immediate left of Vali?

(A) Paul (B) Tom (C) Unif (D) Rax

Example 2: If Queen is not sitting at either extreme of the row, then who among the following has as many persons on his left as on his right?

(A) Sam (B) Unif (C) Rax (D) Vali

Example 3: If Queen sits at one extreme, then who is at the other extreme?

(A) Paul (B) Tom

(C) Vali (D) Cannot be determined

Example 4: Tom sits to the right of Queen, and Paul is separated from Tom by exactly three persons. Then, who is sitting to the immediate left of Vali?

(A) Unif (B) Paul (C) Tom (D) Rax

Example 5: In how many different ways can the seven persons sit in a row?

(A) 3 (B) 2 (C) 10 (D) 12

Solutions for questions 1 to 5:

Let us write down the conditions given in short form and then represent them pictorially. Also, let us treat the left of the persons sitting as 'left' and their right as 'right' for interpreting the conditions.

Rax and Sam sit next to each other → RS or SR.

There are exactly 4 persons between Queen and Vali → Q — — — — V or V — — — — Q.

Sam sits to the immediate right of Queen → SQ.

Now, let us analyse the data/conditions that we are given and then put the three conditions together. Let us number the seats from *our* left to right as Seat 1 to Seat 7.

Since S is to the right of Q and since R and S have to be next to each other, R can come only to the immediate right of S. Thus, R, S, and Q, will be in the order RSQ. Since there are four persons between Q and V, Q can be placed in seats 1, 2, 6,' or 7. But if Q is in Seat 1 or 2, then, there are no seats for R and S. Hence, there are only two seats available for Q. Let us fix the positions of R, S, and V in each of these two positions of Q and write them down. The directions left and Right are as shown below.

◀ R ▶ L

Arrangement I:

1	**2**	**3**	**4**	**5**	**6**	**7**
	V			R	S	Q

Arrangement II:

1	**2**	**3**	**4**	**5**	**6**	**7**
	V			R	S	Q

These are the only two arrangements possible for the four persons V, R, S, and Q. The other three persons Paul, Tom, and Unif can sit in the three vacant seats in any order, as no information is given about them. Now let us look at each of the questions.

Example 1: Paul and Tom are separated by exactly two persons.

Arrangement I is the only one possible as in Arrangement II, Paul and Tom cannot have exactly two persons between them. So, we have the arrangement as follows:

T/P, V, U, P/T, R, S, Q

So, Unif must be sitting to the immediate left of Vali.

Hence, the correct option is (C).

Example 2: If Queen is not at the extreme right, then only Arrangement II above is possible. The person who has as many persons on his left as on his right can only be the person who is sitting in the middle seat, i.e., seat 4. In this arrangement, Rax is sitting in seat 4.

Hence, the correct option is (C).

Example 3: 'Queen sits at one extreme' means that we should look at arrangement I. In this arrangement, any one out of the three persons Paul, Tom, and Unif can be in seat 1, i.e. extreme right.

Hence, the correct option is (D).

Example 4: If Tom and Paul are separated by exactly three persons, then only Arrangement II is possible. So, Tom and Paul have to be in seats 3 and 7, Since, we are also given that Tom is to the right of Queen, Tom has to be in seat 3 and Paul, in seat 7. So, the arrangement must be as follows:

V, U, T, R, S, Q, P

The person sitting to the immediate left of Vali is Unif.

Hence, the correct option is (A).

Example 5: We have two possible arrangements—Arrangement I and Arrangement II that we looked at already. In each arrangement, the remaining three people can sit in the remaining three seats in 6 ways. Thus, a total of 12 ways of seating the seven persons is possible.

Hence, the correct option is (D).

Direction for questions 6 to10: Read the data given carefully and answer the questions that follow.

Seven boys—Rajan, Shyam, Vardhan, Mithra, Vimal, Raj and Kishan—are sitting in a row. Shyam sits to the immediate left of Vardhan and third to the right of Rajan, whereas Mithra, who sits at the left extreme, is next to Kishan.

Example 6: Who is sitting to the immediate right of Shyam?

(A) Mithra (B) Kishan
(C) Vimal (D) Vardhan

Example 7: If Vardhan and Kishan exchange places with each other without changing the rest of the arrangement that is already done, who will be sitting to the immediate left of Rajan?

(A) Kishan (B) Raj
(C) Vimal (D) Vardhan

Example 8: If only Shyam sits between Raj and Vardhan, who is exactly in the middle of the row?

(A) Raj (B) Vardhan
(C) Vimal (D) Rajan

Example 9: Which of the following cannot confirm the seating arrangement of all the boys?

(A) Raj is to the immediate right of Rajan, whereas Vimal is to the left of Shyam.
(B) Mithra and Raj have two persons between them.
(C) Raj and Kishan have two persons between them.
(D) Rajan and Shyam have two persons in between them.

Example 10: After arranging all the boys as per the conditions given in the data, if Rajan now exchanges his place with Mithra, and Vardhan exchanges his place with Vimal, then how many persons will be there between Vimal and Rajan?

(A) Three (B) Two
(C) Five (D) Cannot be determined

Solutions for question 6 to10:

Let us denote left and right as shown below:

◀ L ▶ R

Now, let us represent the data given in pictorial form (we use R for Raj and Rn for Rajan; Va for Vardhan; Vi for Vimal; S for Shyam; M for Mithra; and K for Kishan).

Mithra sits at the left extreme—next to Kishan → M K — — — — — —.

Shyam sits to the immediate left of Vardhan and third to the right of Rajan → Rn — — S Va.

Putting both the arrangements together, Va can go only to extreme right position. Thus, we have the arrangement as M K Rn — — S Va.

Raj and Vimal occupy the two vacant seats between Rajan and Shyam.

Example 6: From the seating arrangement figure, Vardhan is to the immediate right of Shyam.

Hence, the correct option is (D).

Example 7: If Kishan and Vardhan exchange places, as can be seen from the arrangement, the person to the immediate left of Rajan will be Vardhan.

Hence, the correct option is (D).

Example 8: If Shyam sits between Raj and Vardhan, then the seating arrangement is as follows: Mithra, Kishan, Rajan, Vimal, Raj, Shyam, Vardhan. Then, Vimal will be exactly in the middle of the row.

Hence, the correct option is (D).

Example 9: Statement (A) makes the arrangement as:
Mithra, Kishan, Rajan, Raj, Vimal, Shyam, Vardhan

Statement (B) gives the seating arrangement as:
Mithra, Kishan, Rajan, Raj, Vimal, Shyam, Vardhan.

Statement (C) makes the seating arrangement as:
Mithra, Kishan, Rajan, Vimal, Raj, Shyam, Vardhan.

So, only statement (D) cannot make the seating arrangement unique while others can.

Hence, the correct option is (D).

Important point to note is that, on the basis of the given data, we know that the places of only Raj and Vimal have not been fixed. Hence, if there is an additional statement that we are considering to determine the arrangement uniquely, it should have at least one of the two people Raj and Vimal. In this case, choice (D) does not have either one of the two names, and, hence, this statement cannot help us determine the arrangement uniquely. So, this becomes the answer choice.

Hence, the correct option is (D).

Example 10: The arrangement is

M K Rn R/Vi Vi/R S Va

Rajan exchanges his place with Mithra, and Vimal with Vardhan, then we have the following arrangement:

Rn K M R/Va Va/R S Vi.

While we still do not know the exact position of Vardhan (or which place Vimal sits), we can see that there are five persons between Rajan and Vimal.

Hence, the correct option is (C).

In addition to the questions that we saw, where a set of questions are based on the data given, there are also 'stand-alone' questions. In these questions, on the basis of the data given, only one question is asked. Given below is an example of this type.

Direction for question 11: Select the correct alternative from the given choices.

Example 11: Four persons A, B, C, and D arrive to attend a meeting. D arrives 10 minutes after B and 20 minutes before A, who arrives 10 minutes before C. Who is the first person to arrive at the meeting?

(A) A (B) B (C) C (D) D

Solution for question 11:

C arrived after A. A arrived after D. D arrived after B. This implies that B arrived first.

Hence, the correct option is (B).

CIRCULAR ARRANGEMENT

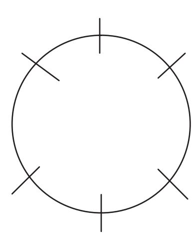
Questions on circular arrangement involve seating of people around a table or arrangement of things in a circular manner (e.g. different coloured beads strung to form a necklace).

In case of people sitting around a table, the table could be of any shape—rectangular, square, circular, or any other.

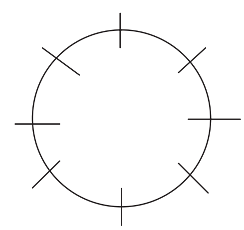
The data given in such sets of questions specify the positions of some or all of the individuals (or things) in the arrangement. The positions are specified through conditions involving specified persons sitting (or not sitting) opposite each other or a particular person sitting to the right or left of another person, etc.

Once you read the data, first draw the shape specified in the data and then draw the slots in the seating arrangement.

Six people around a circular table



Eight people around a circular table



Statements like 'A and B are sitting farthest from each other' or 'A and B sit across the table' imply that A and B sit opposite each other.

On the other hand, you should remember that, unlike in straight-line arrangement, the words 'immediately' and 'directly' do not play any role in circular arrangement. In general, there is no left side or right side (unless we are talking of 'immediate right' or 'immediate left').

So, if it is given that C sits to right of B, then it is clear that C must be to the immediate right of B. Go 'anti-clockwise' if anybody's right has to be located, and go 'clockwise' if somebody's left has to be located.

Let us take some examples.

Direction for questions 12 to 16: Read the following information and answer the questions that follow.

P, Q, R, S, and T sit around a table.

P sits two seats to the left of R and Q sits two seats to the right of R.

12. If S sits in between Q and R, who sits to the immediate right of P?

(A) T (B) S (C) Q (D) R

13. Which of the following cannot be the correct seating arrangement of the five persons in either the clockwise direction or the anticlockwise direction?

(A) P, Q, R, S, T (B) P, S, R, T, Q
(C) P, Q, S, R, T (D) P, T, R, S, Q

14. If S is not sitting next to Q, who is sitting between Q and S?

(A) R (B) P
(C) T (D) Both (R) and (P)

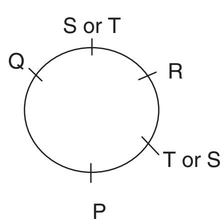
15. If a new person U joins the group such that the initial conditions for the seating arrangement should be observed and also a new condition that U does not sit next to R be satisfied, then which of the following statements is true?

(A) U sits to the immediate right of S.
(B) U sits to the immediate left of T.
(C) U sits to the immediate left of P.
(D) Either (A) or (B) above

16. If a new person U joins the group such that the initial conditions for the seating arrangement should be observed and also a new condition that U does not sit next to P, S or T be satisfied, then who will be the neighbours of P (one on either side)?

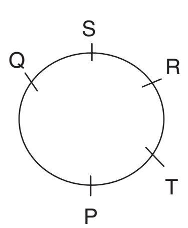
(A) S and T (B) S and Q
(C) T and R (D) R and Q

Solutions for questions 12 to 16:



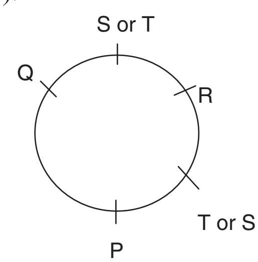
P sits two seats to the left of R, and Q sits two seats to the right of R. We can represent this information in the diagram below.

12. If S sits between Q and R, then the arrangement is as follows.



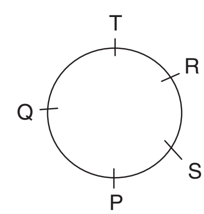
As can be seen from the diagram, T is to the immediate right of P.
Hence, the correct option is (A).

13. We will take each choice and see whether it fits in the arrangement that we represented through a diagram in the analysis of the data (the same diagram is reproduced below).



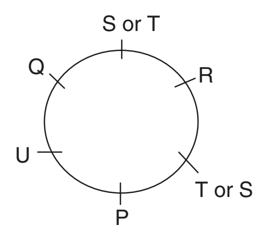
We can see that the arrangement given in choice (A) is not possible,' and, hence, the answer choice is (A).
Hence, the correct option is (A).

14. If S is not next to Q, then the seating arrangement is fixed as follows.



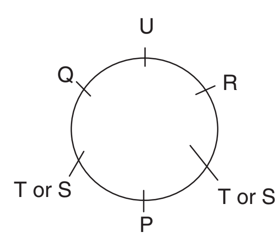
Now P is between Q and S.
Hence, the correct option is (B).

15. On the basis of the diagram that we drew, we find that to accommodate U we have to create a new slot between P and Q.



Hence, choice (C) is the correct answer.

16. We create a new slot for the sixth person. But since U will not sit next to P, S, or T, he will have to sit between R and Q. The arrangement will then look as follows:



As we can see from the diagram, the neighbours of P will be T and S.
Hence, the correct option is (A).

Direction for questions 17 to 21: Read the following information and answer the questions that follow.

There are 10 persons at a round table conference, consisting of a Professor, a Lawyer, a Doctor, a Scientist, an Accountant, a Grocer, two Computer Specialists, and two Marketing Executives. The Professor sits opposite to the Lawyer. The Scientist and the Doctor sit opposite each other. The two Marketing Executives sit opposite each other with one of them sitting to the immediate left of the scientist. The Professor sits to the immediate right of the Scientist.

17. If the two Computer Specialists sit opposite each other but neither of them is immediately next to any Marketing Executive, who sits to the immediate right of the professor?
(A) Computer Specialist
(B) Marketing Executive
(C) Grocer
(D) Accountant

18. If the Grocer and Accountant do not sit opposite each other, then which of the following must be TRUE?
(A) The Computer Specialist cannot sit beside the Lawyer.
(B) One of the Computer Specialists is next to a Marketing Executive.
(C) The Professor cannot have the Scientist and a Computer Specialist on his either side.
(D) The Computer Specialists must sit next to one another.

19. If the Grocer and the Accountant each have a Marketing Executive as his immediate neighbour, then which of the following is definitely FALSE?
(A) The two Computer Specialists are opposite each other.
(B) A Computer Specialist is an immediate neighbour of the Scientist.
(C) The Grocer is next to a Computer Specialist.
(D) A Computer Specialist is an immediate neighbour of the Lawyer.

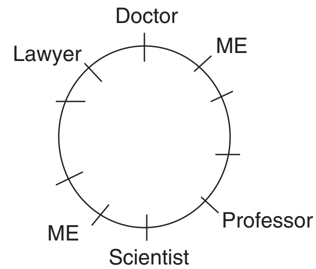
20. If a Computer Specialist is the immediate neighbour of a Marketing Executive and the Grocer is the immediate neighbour of the Lawyer, how many different kinds of seating arrangements are possible? (Assume that the two Computer Specialists are indistinguishable from each other and the two Marketing Executives are indistinguishable from each other.)
(A) 3 (B) 6 (C) 16 (D) 8

21. The maximum number of persons you can count if you start counting with the Scientist and end with a Marketing Executive (excluding both) is
(A) 0 (B) 8 (C) 5 (D) 6

Solutions for questions 17 to 21:

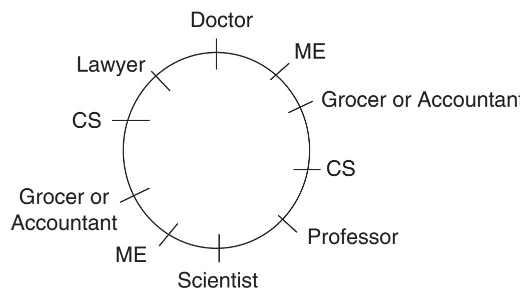
The Professor sits to the immediate right of the Scientist and opposite to the Lawyer. The Scientist sits opposite to the Doctor and one Marketing Executive is to the immediate left of the Scientist.

Choosing to place the Scientist in one of the 10 seats, we have the arrangement as follows.



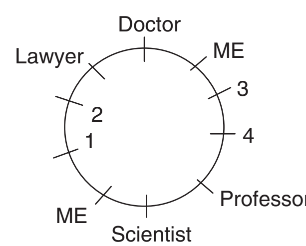
The vacant seats are one each for the two Computer Specialists, one for the Grocer and one for the Accountant.

17. The two Computer Specialists sit opposite each other. Neither of them is next to any Marketing Executive. So, the arrangement must be as follows



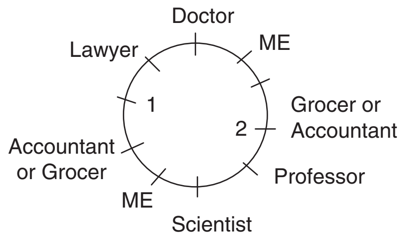
So, a Computer Specialist sits to the immediate right of the professor.
Hence, the correct option is (A).

18. The Grocer and the Accountant do not sit opposite each other. Then, the arrangements can be as follows:



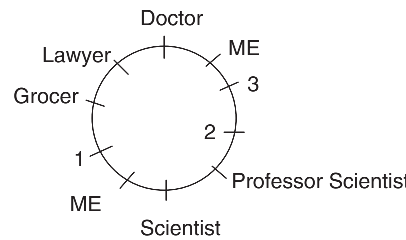
The Grocer and the Accountant can occupy the following pairs of seats: 3 and 4, 1 and 4, 1 and 2, or 2 and 3. Then, the two Computer Specialists may occupy one of the pairs of seats 1 and 2, 2 and 3, 3 and 4, or 4 and 1. We check for the choices given in the question, one by one, and find that whichever combination is taken, there is a Computer Specialist in seat 1 or seat 3, both of, which are next to the Marketing Executives seats. So, choice (B), which states that one of the Computer Specialists is next to a Marketing Executive, is true.
Hence, the correct option is (B).

19. The Grocer and the Accountant have one each of the Marketing Executives on their immediate side. So, the arrangement must be as follows.



Now the Computer Specialists must sit in chairs 1 and 2 only. But, no Computer Specialist can be the immediate neighbour of the Scientist. Choice (B) is definitely FALSE. (Note that choices (A), (C), and (D) are true). Hence, choice (B) is the correct answer.
Hence, the correct option is (B).

20. Given that the Grocer is the immediate neighbour of the Lawyer, we have the three slots numbered 1, 2, and 3 (in the following diagram) free for the two Computer Specialists and the Accountant. Since a Computer Specialist has to be next to a Marketing Executive, he should be in slot 1 or 3. By fixing the Accountant in any one of the three slots 1, 2, or 3, we can ensure that there is a Computer Specialist next to a Marketing Executive. Hence, there are three possible seating arrangements.



Hence, the correct option is (A).

21. Based on the seating arrangement that we discussed, the number of persons between the Scientist and a Marketing Executive can be 3 or 8 (counted clockwise) or 0 or 5 (counted anticlockwise). Maximum number that can be counted is 8.
Hence, the correct option is (B).

Exercises

Direction for questions 1 to 3: These questions are based on the following information.

Five boys Anil, Charan, David, John and Kamal sit in a row facing north, not necessarily in the same order.

I. John sits exactly in between Anil and David.
II. John sits exactly in between Charan and Kamal.

1. Who sits exactly at the middle of the row?
(A) John
(B) Kamal
(C) David
(D) Cannot be determined

2. In how many different ways these five boys can sit?
(A) 2 (B) 4 (C) 8 (D) 16

3. If Anil sits to the immediate left of John and if a boy sits to the immediate right of Kamal then who is that boy?
(A) David
(B) Anil
(C) Charan
(D) None of these

Direction for questions 4 to 6: These questions are based on the following information.

Seven girls A, B, C, D, E, F, and G sit in a row facing north, not necessarily in the same order. It is also known that,

I. Two girls sit in between B and F.
II. Three girls sit in between C and G.
III. Four girls sit in between A and D.

4. Who sits exactly at the middle of the row?
(A) B
(B) D
(C) E
(D) Cannot be determined

5. If B sits to the immediate right of D then who sits in between A and E?
(A) F
(B) C
(C) G
(D) Cannot be determined

6. If F and G sit on either sides of E then who sits at the right end of the row?
(A) A
(B) C
(C) D
(D) Cannot be determined

Direction for questions 7 to 9: These questions are based on the following information.

Five persons P, Q, R, S, and T sit in a row facing North not necessarily in the same order. The following information is known about them:

I. Either P or S sits at the one end of the row.
II. Either Q and T or S and T sit on either sides of P.
III. R sits to the left of S and to the immediate left of Q.

7. In how many different ways can these five people sit?
(A) 2 (B) 3
(C) 1 (D) 4

8. If Q sits to the immediate left T then who sits exactly at the middle of the row?
(A) P (B) R
(C) T (D) Cannot be determined

9. If P is not sitting adjacent to S, then who sits to the immediate right of Q?
(A) Q (B) P
(C) R (D) Cannot be determined

Direction for questions 10 to 12: These questions are based on the following information.

Each of six persons Pavan, Raman, Kiran, Charan, Shravan and Rajan stay in a different floor of a six-storied (1^{st}, 2^{nd}, 3^{rd}, 4^{th}, 5^{th}, and 6^{th} from bottom to top, respectively) building.

I. Raman stays above Kiran but below Charan.
II. Pavan stays below Rajan but above Shravan.
III. Kiran stays above Pavan but below Raman who stays above Rajan.

10. Who stays in the 2^{nd} floor?
(A) Pavan
(B) Shravan
(C) Rajan
(D) Cannot be determined

11. Who stays in the 4^{th} floor?
(A) Raman
(B) Rajan
(C) Kiran
(D) Cannot be determined

12. If one person stays in between Pavan and Kiran then who stays in the 3^{rd} floor?
(A) Shravan (B) Pavan
(C) Rajan (D) Charan

Direction for questions 13 to 15: These questions are based on the following information.

There are five buildings of different heights in a row. These houses are painted with a different colour among red, blue, white, green, and yellow such that each house is painted with exactly one colour.

The following information is know about them:

I. Yellow and green buildings are on either sides of the white building.
II. The shortest building is painted in red colour but it is neither at any end of the row nor adjacent to the tallest building.
III. The white building is exactly in between the tallest and the second tallest buildings.

13. Which among the following buildings is definitely at one end of the row?
(A) Yellow building
(B) Green building
(C) The tallest building
(D) The third tallest building

14. Which among the following is definitely false?
(A) The white building is the third tallest
(B) The third tallest and the shortest buildings are together
(C) Blue and yellow buildings are at either ends of the row.
(D) Yellow and green buildings are at either ends of the row.

15. If the yellow building is to the immediate left of the third tallest building, then what could be the order of these buildings in the descending order of their heights?
(A) blue, yellow, red, white, green
(B) blue, green, white, yellow, red
(C) green, white, blue, yellow, red
(D) green, yellow, white, blue, red

Direction for questions 16 to 18: These questions are based on the following information.

Each of the six persons—John, Ted, Humpty, Dumpty, Jack and Jill, is from one different country among India, Japan, China, Australia, America, and England and are sitting around a circular table, may not be in the same order. John, who is from China, is sitting adjacent to American, who is not Humpty. Ted is not an Indian, and Chinese is not sitting adjacent to Indian. The person from England is sitting one place away to the left of the Australian. Humpty is sitting opposite the Indian, who is adjacent to the Japanese. Australian and Dumpty are sitting opposite each other. Jack is not from India and Ted is not from Japan but both are not adjacent to each other.

16. Who among them is from India?
(A) Jill (B) Dumpty
(C) Humpty (D) None of these

17. If Jack is the Japanese, then who is sitting opposite the American?
(A) Jill (B) Ted
(C) Jack (D) Dumpty

18. Which country does Humpty belong to?
(A) Japan (B) Australia
(C) America (D) England

Direction for questions 19 to 22: These questions are based on the following information.

Eight persons—Ram, Ramesh, Mohan, Sohan, Seema, Saroj, Sakshi, and Saloni, are sitting around a circular table. Each of them is one among doctor, engineer, dancer, singer, teacher, lawyer, Accountant, and Pilot, not necessarily in the given order. Further more it is known that

I. Pilot is sitting opposite Ramesh, who is adjacent to the accountant.
II. Dancer is sitting opposite the lawyer and is not adjacent to Sakshi who is not sitting adjacent to the lawyer.
III. Saloni is sitting opposite the engineer, Ramesh is not a lawyer or doctor or engineer.

IV. Sakshi, the singer, is sitting one place away to the right of Saroj.
V. Seema is sitting opposite the lawyer and Ram is sitting opposite the dancer.
VI. Ramesh is sitting three places to the right of singer. Mohan is neither the accountant nor adjacent to the dancer.

19. Who among the following is the doctor?
(A) Ramesh
(B) Saloni
(C) Saroj
(D) Cannot be determined

20. What is the profession of Mohan?
(A) Accountant
(B) Pilot
(C) Engineer
(D) Cannot be determined

21. Who is sitting opposite Ramesh?
(A) Seema (B) Sakshi
(C) Saroj (D) None of these

22. Who is sitting opposite the accountant?
(A) Sakshi (B) Mohan
(C) Seema (D) Saroj

Direction for questions 23 to 25: These questions are based on the following information.

Eight persons—Arun, Pankaj, Rohan, Veda, Suman, Shanu, Dimple, and Pinky, are sitting around a circular table for a group discussion. Suman is not sitting opposite Pinky, and Shanu is sitting three places away to the right of Pankaj. Dimple is sitting in between Pankaj and Suman. Rohan is sitting adjacent to Pankaj who is sitting opposite Arun.

23. Who is sitting opposite Dimple?
(A) Piniky
(B) Shanu
(C) Rohan
(D) Cannot be determined

24. Who is sitting opposite Veda?
(A) Suman
(B) Pinky
(C) Shanu
(D) Cannot be determined

25. If Rohan is sitting to the left of Veda, then who is sitting opposite Shanu?
(A) Rohan
(B) Dimple
(C) Suman
(D) Cannot be determined

Answer Keys

1. A	**2.** C	**3.** B	**4.** C	**5.** D	**6.** D	**7.** A	**8.** C	**9.** B	**10.** A
11. D	**12.** C	**13.** C	**14.** D	**15.** D	**16.** B	**17.** A	**18.** B	**19.** B	**20.** C
21. C	**22.** A	**23.** D	**24.** A	**25.** B					

Chapter 8

Puzzles

CHAPTER HIGHLIGHTS

- *Distribution*
- *Order Sequence*
- *Selections*
- *Questions on Routes/Networks*

Puzzles

In this type of problems, you have to match two or more 'variables'. (Variable means a 'subject' as used in the discussion of linear arrangement.) In double line-up, the data given may talk of four people living in four houses each of a different colour. What we need to find out is the colour of the house of each of the four persons. There is no first position or second position of the houses.

Sometimes, double line-up is also called as 'distribution'. An example of data given for this variety of questions is:

'Each of the four persons A, B, C, and D wears a different coloured shirt—red, pink, blue, and white. A has a red shirt and D does not have a pink shirt'.

From the statement, it becomes clear that no person among A, B, C, and D can have shirts of two different colours among red, pink, blue, and white.

As discussed in the questions on single line-up, questions can be solved easily by representing the given data pictorially. In case of double line-up, it will help us if we represent the data in the form of a matrix or a table.

Let us see how to draw a matrix for the data given.

Names	Colours			
	Red	Pink	Blue	White
A	✓			
B				
C				
D		×		

As it is given that A has red shirt, it is clear that he does not have any other colour shirt. Similarly B, C, D do not have red shirt. So, in all the other cells in the row belonging to A, we put a cross ('×'). Then, the table will look as follows:

Names	Colours			
	Red	Pink	Blue	White
A	✓	×	×	×
B	×			
C	×			
D	×	×		

In this manner, we can fill up the cells on the basis of the data given to us. Once, we use up all the data, we will draw any conclusions that can be drawn and then answer the questions given in the set.

Let us Take a Few Examples

Direction for questions 1 to 5: These questions are based on the following information.

P, Q, R, S, T, U, V, an d W are eight employees of a concern. Each is allotted a different locker, out of eight lockers numbered 1 to 8 in a cupboard. The lockers are arranged in four rows with two lockers in each row.

Lockers 1 and 2 are in the top row from left to right, respectively, while lockers 7 and 8 are in the bottom row—arranged from left to right, respectively. Lockers 3 and 4 are in the second row from the top—arranged from right to left, respectively. So are lockers 5 and 6—arranged from right to

left, respectively—in the second row from the bottom. P has been allotted locker 1 while V has been allotted locker 8. T's locker is just above that of Q which is just above that of R, whereas W's locker is in the bottom row.

Solved Examples

Example 1: Which of the following cannot be the correct locker number–occupant pair?
(A) 3-Q (B) 7-W (C) 4-U (D) 6-R

Example 2: If U's locker is not beside Q's locker, whose locker is just above that of W?
(A) U (B) S (C) R (D) Q

Example 3: Which of these pairs cannot have lockers that are diagonally placed?
(A) P-Q (B) S-R
(C) U-R (D) Either (B) or (C)

Example 4: Which of the following groups consists only occupants of odd numbered lockers?
(A) Q, R, W (B) R, V, W
(C) T, R, Q (D) P, T, Q

Example 5: If U's locker is in the same row as that of R, and S exchanges his locker with V, then who is the new neighbour of V in the same row? (Assume that nothing else is disturbed from the original arrangement)
(A) P (B) Q (C) R (D) U

Solutions for questions 1 to 5:
Let us first try to locate the lockers in the cupboard as per the conditions given. Then, we will do the allotment to the persons.

Lockers 1 and 2 are in the top row and lockers 7 and 8 are in the bottommost row. In these two rows, the lockers are numbered from left to right. In the other two rows, the lockers are numbered from right to left.

L	R	
1	2	Top Row
4	3	
6	5	
7	8	Bottom Row

Now let us look at the conditions given for the allotment of the lockers.

P has locker 1. V has locker 8.

1-P	2
4	3
6	5
7	8-V

Locker of W is in the bottom row → W's locker must be 7.

1-P	2
4	3
6	5
7-W	8-V

T's locker is just above that of Q, which is just above that of R → The lockers of T, Q, and R must be 2, 3, and 5, respectively (there are no other group of lockers which satisfy this condition).

1-P	2-T
4	3-Q
6	5-R
7-W	8-V

S and U have lockers 4 and 6 left for them.

Thus, on the basis of the data given to us, we can show the final arrangement of lockers as below:

1-P	2-T
4-S/U	3-Q
6-U/S	5-R
7-W	8-V

Now we can answer the questions easily on the basis of the above.

Example 1: By looking at the final arrangement of lockers above, we find that choice (D) does not represent the correct combination of locker number-occupant pair.

Hence, the correct option is (D).

Example 2: If U's locker is not beside Q's locker, then U's locker must be locker 6. So, it is U's locker that will be immediately above W's.

Hence, the correct option is (A).

Example 3: R's locker is in the same row as that of exactly one of S or U and diagonally placed to the other one. Hence, 'either S-R or U-R' is the answer.

Hence, the correct option is (D).

Example 4: The odd-numbered lockers 1, 3, 5, and 7, which belong to P, Q, R, and W, respectively. Of the choices, we find that Q, R, W appear in choice (A). Hence, this is the correct choice.

Hence, the correct option is (A).

Example 5: U's locker is in the same row as that of R, which means that locker 6 belongs to U. So, locker 4 belongs to S. Now, V and S exchange lockers. Then, the new neighbour of V is Q.

Hence, the correct option is (B).

Direction for questions 6 to 10: These questions are based on the following information.

There are four trees—lemon, coconut, mango, and neem—each at a different corner of a rectangular plot.

A well is located at one corner and a cabin at another corner. Lemon and coconut trees are on either side of the gate, which is located at the centre of the side opposite to the side, at whose extremes, the well and the cabin are located. The mango tree is not at the corner where the cabin is located.

Example 6: Which of the following pairs can be diagonally opposite to each other in the plot?
(A) Neem tree and lemon tree
(B) Cabin and neem tree
(C) Mango tree and well
(D) Coconut tree and lemon tree

Example 7: If the lemon tree is diagonally opposite to the well, then the coconut tree is diagonally opposite to the
(A) Mango tree (B) Well
(C) Cabin (D) Gate

Example 8: If the coconut tree and the neem tree cannot be at adjacent corners of the plot, then which of the following will necessarily have to be at diagonally opposite corners of the plot?
(A) Coconut tree and well
(B) Lemon tree and cabin
(C) Lemon tree and coconut tree
(D) Lemon tree and well

Example 9: Which of the following must be TRUE?
(A) Cabin and well are not at adjacent corners.
(B) Cabin and coconut tree cannot be at the adjacent corners.
(C) Neem tree and well are at adjacent corners.
(D) Neem tree and well are not at adjacent corners.

Example 10: Which of the following is definitely FALSE?
(A) Mango tree is adjacent to the well at one corner.
(B) Neem tree is adjacent to the cabin at one corner.
(C) Coconut tree is at the corner adjacent to the well.
(D) Lemon tree is not on the same side of the plot as the gate.

Solutions for questions 6 to 10:
Lemon and coconut trees are on either sides of the gate.

Lemon Or Coconut	Gate	Coconut Or Lemon

The well and the cabin are at either end of the wall opposite to the gate.

Mango tree and cabin are not at the same corner. So, neem tree and well are not at the same corner. This means that mango tree and the well are at the same corner and neem tree and the cabin are at the same corner.

(Well, Mango) Or (Cabin, Neem) Lemon or Coconut	(Cabin, Neem) or (Well, Mango) Coconut or Lemon

Gate

Example 6: Let us take each choice and check with the above diagram to see if it is possible or not.

Neem and lemon trees can be diagonally opposite each other. Hence, this is the correct answer choice. (In an exam, you do not need to check the other choices since the first choice is correct. But, for the sake of clarity and proper understanding, we will check all the choices.)

From the diagram given, we can see that cabin and neem tree cannot be located diagonally opposite each other.

Mango tree and well cannot be located diagonally opposite to each other.

Coconut and lemon trees cannot be located diagonally opposite each other.

Hence, the correct option is (A).

Example 7: If lemon tree is diagonally opposite to the well, then we can have the following two possible arrangements.

Neem Cabin		Well Mango
Lemon	Gate	Coconut

OR

Well Mango Coconut	Gate	Cabin Neem Lemon

The coconut tree is diagonally opposite the cabin and neem.

Hence, the correct option is (C).

Example 8: Since coconut and neem trees cannot be at adjacent corners, the following arrangements are possible.

(Well, Mango)	(Cabin, Neem)
Coconut	Lemon

OR

(Cabin, Neem)	(Well, Mango)
Lemon	Coconut

From these diagrams, we find that choice (D) is the correct answer.

Hence, the correct option is (D).

Example 9: We can check each statement with the diagram that we drew initially. We find that the statement in choice (C) which says that neem tree and the well are at the adjacent corners is true. Hence, choice (C) is the correct answer.
Hence, the correct option is (C).

Example 10: We check each statement with the diagram that we drew initially to find out which of the statements has to be false.

We find that choice (D) has to be false.

Hence, the correct option is (D).

Direction for question 11: Select the correct answer from the given choices.

Example 11: A, B, C, and D play four different games among baseball, cricket, kabaddi and volleyball. A does not play baseball or cricket. B does not play kabaddi or volleyball. C plays volleyball and D plays either baseball or volleyball. Who plays cricket?

(A) A (B) B (C) C (D) D

Solution for question 11:

C plays volleyball. A does not play cricket and D does not play cricket as he plays either baseball or volleyball.

∴ B should play cricket.

Hence, the correct option is (B).

Order Sequence

The term 'order sequence' is self-explanatory. In questions for this category, you will be asked to deal with relative positions of subjects. The absolute values of the subjects is not what you should be interested in. It is comparison between different subjects that you have to deal with. The data also specify the relationships like 'A is greater than B' or 'C is not less than D', and so on. You have to decide the positions of the subjects in ascending or descending order on the parameters given. The subjects of comparison can be people or things.

In short, data will be given to compare the quality or quantity. The parameters on which the subjects are compared can be heights or weights of people, the money with them, complexion, sizes of things, etc.

In such questions, you will come across typical statements like 'A is taller than B,' 'B is not shorter than C', and so on. You may use the following symbols to symbolically represent the conditions given and, then later, represent all the subjects pictorially.

Greater than	>
Less than	<
Greater than or equal	≥
Less than or equal	≤.

'Not greater than' is the same as 'less than or equal to'. Similarly, 'not less than' is the same as 'greater than or equal to'.

Words like 'Who, And, Which, But' used in the data play a significant role in analysing the data. 'And' and 'But' play the same role, whereas 'Who' and 'Which' play the same role.

Let us take one statement.

'A is taller than B, who is shorter than C and taller than D but shorter than E, who is taller than F and G but shorter than H'.

By using appropriate symbols, the above statement can be represented as follows.

$$A > B; B < C; B > D; B < E; E > F; E > G; E < H$$

Questions on the given data can be as follows.

1. Who is the tallest?
2. Who is the shortest?
3. Who is the second tallest in the group? etc.

Let us take some examples.

Direction for questions 12 to 16: Read the information given and answer the questions that follow.

A, B, C, D, and E are five cars while P, Q, and R are three motorcycles. A is the fastest of the cars and R is the slowest of the motorcycles. C is costlier than D and Q but cheaper than B. Among cars, A is not the costliest. D is cheaper than E and there is no car whose cost lies between the cost of these two. E is faster than three of the cars and all the motorcycles. Q is costlier than R but cheaper than P, who is faster than Q.

Example 12: Which of the following cars cannot stand exactly in the middle position among cars as far as their cost is concerned?

(A) A (B) C
(C) E (D) D

Example 13: Which of the following statements is true about the motorcycles?

(A) P is the costliest as well as the fastest motorcycle.
(B) The fastest motorcycle is not the costliest motorcycle.
(C) The slowest motorcycle is also the cheapest motorcycle.
(D) Both (A) and (C)

Example 14: If P is costlier than E, how many cars are cheaper than P?

(A) 1
(B) 2
(C) 3
(D) Cannot be determined

Example 15: If P is cheaper than A, which is not costlier than E, which of these is the cheapest of all the cars and motorcycles put together?

(A) R
(B) Q
(C) E
(D) Cannot be determined

Example 16: Which of these is the slowest of the cars, if B and C are faster than D?

(A) B (B) D (C) E (D) A

Solutions for questions 12 to 16:

Let us first write down all the comparisons given for costs and speeds. Then we will tabulate them.

Speed

A → fastest car

E → Faster than three of the cars ➔ E is the second fastest car

R → slowest motorcycle

P > Q

Cost

C > D

C > Q

B > C

A → Not the costliest among cars

E > D → No other car lies between these two

Q > R

P > Q

Now let us tabulate these data.

Speed

Cars

Fastest	A	E				Slowest

Motorcycles

Fastest	P	Q	R	Slowest

Cost

Cars

Costliest	B C E D	Cheapest

Here, we know that A is not the costliest car but we do not know where it will fit in. It can come anywhere after B except between E and D.

Motorcycles

Costliest	P	Q	R	Cheapest

In addition, we have to also keep in mind that C > Q in cost. (From this, we can conclude that B > Q, B > R, C > R in cost.)

Example 12: In terms of cost of the cars, A can come between B and C or between C and E or to the right of E. In each of the cases, the middle car will be C, A, and E, respectively. Hence, among the cars given, D cannot be in the middle.

Hence, the correct option is (D).

Example 13: By looking at the tables, we can make out that choices (A) and (C) are both correct, and, hence, the correct answer is choice (D).

Hence, the correct option is (D).

Example 14: If P is costlier than E, we can also conclude that it is costlier than D but we cannot conclude anything about the relationship between the cost of P and that of B, C and A.

Hence, the correct option is (D).

Example 15: Since A is not costlier than E, it means that A is at the same level of E or cheaper than E. We cannot conclude which of these two positions A is in. Hence, we cannot conclude which the cheapest of all the vehicles is. {Please note that if A is the cheapest car, then R will be the cheapest of all the vehicles. However, if A is at the same level as E in cost, then there is a possibility of R or D being the cheapest of all the vehicles.}

Hence, the correct option is (D).

Example 16: If B and C are faster than D, then the order will be as follows:

1	2	3	4	5
A	E	B/C	C/B	D

Hence, D is the slowest of all the cars.

Hence, the correct option is (B).

Direction for questions 17 to 21: Read the information given and answer the questions that follow.

J, K, L, M, and N are five boys in a class. They are ranked in the order of heights—from the tallest to the shortest—and in order of cleverness—from the cleverest to the dullest. K is taller than N, but not as clever as J and L, whereas M is the cleverest of all but shorter than J. While L is shorter than M but taller than K, L is not as clever as J. No two persons got the same ranks in any of these parameters.

Example 17: Who is the third in the order of heights?

(A) J (B) N (C) K (D) L

Example 18: If N is not the last in at least one of the two comparisons, which of the following is the dullest of all the five?

(A) K (B) L (C) M (D) J

Example 19: If L is the third in order of cleverness, who is the dullest of all?

(A) M

(B) N

(C) L

(D) Cannot be determined

Example 20: Who among the following is cleverer as well as taller than K?

(A) L and J only (B) N

(C) L and N (D) J, L and M

Example 21: How many people are definitely shorter than K?

(A) 1 (B) 2

(C) 3 (D) None of these

Solutions for questions 17 to 21:
Let us first write down all the conditions given and then tabulate the data.

Cleverness
J > K
L > K
M is the cleverest.
J > L

Height
K > N
J > M
M > L
L > K

Now, let us put together all the information we have.

Cleverness

Cleverest	M J L K	Dullest

We do not know where N will come in the order of cleverness but he will definitely be after M.
Height

Tallest	J M L K N	Shortest

Example 17: From the table, we can clearly see that L is ranked third in order of heights.
Hence, the correct option is (D).

Example 18: N is the last in terms of height. Since we are given that he is not the last in at least one of the lists, he cannot be the last in cleverness. So, K is the dullest of all.
Hence, the correct option is (A).

Example 19: If L is the third in the order of cleverness, as can be seen from the table, either N or K can be the dullest.
Hence, the correct option is (D).

Example 20: By looking at the tables we made and from the answer choices, we find that L, J, and M are taller as well as cleverer than K.
Hence, the correct option is (D).

Example 21: Only N is shorter than K.
Hence, the correct option is (A).

Direction for question 22: Select the correct alternative from the given choices.

Example 22: P, Q, R, S, and T are five girls competing in a running race. R and P have at least two girls ahead of each of them. T and P do not have more than one girl behind each of them. Who arrives at the finishing line after two girls as well as before two other girls, if no two girls finish the race at the same time?
(A) Q (B) S (C) T (D) R

Solution for question 22:
R and P have at least two girls before them ➔ R and P have to be in two out of 3rd, 4th, and 5th positions.
T and P have not more than one girl behind each of them ➔ T and P have to be in the 4th or 5th positions.
These two statements together mean that R will have to be in the third position.
Hence, the correct option is (D).

Selections

In this category of questions, a small group of items or persons has to be selected from a larger group satisfying the given conditions. The conditions will specify as to when a particular item or person can be included or cannot be included in the subgroup. For example, the condition may specify that two particular persons should always be together or that two particular persons should not be together.
Sometimes, the conditions given for selection or non-selection of items or persons may be based on logical connectives if-then, either-or, unless, etc. You should be careful in interpreting the logical connectives used in the conditions.

Direction for questions 23 to 27: These questions are based on the following information.
Amit, Bittu, Chintu, Dumpy, Falgun, Hitesh, Ronit, Purav, and Saurav are nine players from among whom three teams consisting of 4 members, 3 members, and 2 members, respectively, must be formed subject to the following conditions.
Chintu must have three more players with him while Dumpy must have only two more with him.
Chintu and Saurav cannot be in the same team.
Purav and Bittu cannot be in the same team.
Ronit and Hitesh must be in the same team.

Example 23: If Dumpy, Falgun, Purav form the team of 3 members, then which of the following must be TRUE?
(A) Hitesh must be in a team with Bittu.
(B) Saurav must form a two-member team with Amit or Chintu.
(C) Saurav must form a two-member team with Bittu or Amit.
(D) Chintu should form a team of 4 members with Hitesh, Ronit, and Amit.

Example 24: If Dumpy takes Amit as a part of his three-member team, which of the following must go into Chintu's team?
(A) Bittu and Hitesh (B) Hitesh and Ronit
(C) Purav and Ronit (D) Purav and Falgun

Example 25: If Chintu and Falgun are together and Saurav is in the team of two members, then how many sets of different teams are possible?
(A) 4 (B) 3 (C) 2 (D) 1

Example 26: If Chintu does not have Purav in his team and the two member team consists of Saurav and Amit, then Chintu should take
(A) Hitesh, Bittu, and Ronit.
(B) Bittu but not Ronit.
(C) Bittu and Falgun.
(D) Hitesh and Ronit.

Example 27: If Purav is in the same team as Chintu and Falgun, then Saurav must be in the same team as
(A) Bittu (B) Bittu and Amit.
(C) Amit (D) Bittu and Dumpy.

Solutions for questions 23 to 27:
It is given that:
Chintu must form a team of 4 members only
Dumpy must form a team of 3 members only.
Since Chintu and Dumpy are in two different teams, let us, for convenience, denote the two teams as the respective teams of these two persons. Let us call the team with four members as the first team and the team with three members as the second team. The third team should have two persons.

Number of members		
4	3	2
Chintu	Dumpy	Saurav
	Saurav	

Now let us take the other conditions and fill them up in the table.
Chintu and Saurav cannot be in the same team.
➔ Saurav will be in the second or the third team.

Purav and Bittu cannot be in the same team.
Hitesh and Ronit must be in the same team.
We cannot represent these two conditions right now in the table but we will use them as we go along.

Example 23: If Dumpy, Falgun, Purav form the team of 3 members, then Saurav should be in the third team.
Since Hitesh and Ronit must be in the same team, they have to be in the first team. That leaves only Amit or Bittu to be with Saurav in the third team.
Hence, the correct option is (C).
(Also, note that we can eliminate choice (B) easily.)

Example 24: Dumpy takes Amit as a member of his team.
If we take Hitesh and Ronit as the two members of the third team, then Saurav has to be in the second team, in which case we will have both Purav and Bittu coming into the same team—the first team—which is not possible.
Since Saurav cannot be in Chintu's team and Purav and Bittu cannot be in the same team, the three people required for Chintu's team will **have to be** Hitesh and Ronit check-font Falgun or Purav or Bittu.
Hence, the correct option is (B).

Example 25: Let us analyse the conditions. It is given that Chintu and Falgun are together, whereas Saurav is in the team of two members. Let us fill up these details in the box that we made and then see in how many ways we can fill up the remaining cells in the box.

Chintu	Dumpy	Saurav
Falgun		

First let us look at Hitesh and Ronit who must be in the same team.
They can go into the first team or the second team. Let us consider these two cases.

Case 1: Hitesh and Ronit go into the first team.
Then, one out of Bittu and Purav will to go into the third team and the other into the second team. This gives rise to two ways of forming the teams: one with Bittu in the second team and the other with Bittu in the third team.

Case 2: Hitesh and Ronit go into the second team.
In this case too, one out of Bittu and Purav will go into the third team and the other into the second team. Hence, this will also give rise to two ways of forming the teams.
Hence, there are total four ways of forming the teams.
Hence, the correct option is (A).

Example 26: Let us use the table that we built in the initial analysis and fill up the details that we have in this problem. Since the two member team is already formed and Chintu does not take Purav, hence Purav will have to go into the second team.

Chintu	Dumpy	Saurav
	Purav	Amit

Since Ronit and Hitesh have to be in the same team, they should go into the first team. Since Bittu cannot go with Purav, he should also be in the first team. This leaves Falgun for the second team. Thus, we can fill up the table as follows:

Chintu	Dumpy	Saurav
Ronit	Purav	Amit
Hitesh	Falgun	
Bittu		

Hence, the correct option is (A).

Example 27: If Purav is with Chintu and Falgun, then Bittu cannot be with them. Since Ronit and Hitesh should be together, the only other person left is Amit. These four members form the first team.

If Hitesh and Ronit together form the two member team, then Bittu and Saurav will be part of the three member team.

Instead, if Hitesh and Ronit are in the three-member team, then Saurav and Bittu will form the two-member team.

In either case, Saurav and Bittu are together in one team. Hence, the correct option is (A).

Direction for questions 28 to 31: These questions are based on the following information.

A, B, C, D, E, F, and G are seven players. They form two teams of two players each and one team of three players. A and B cannot be in the same team. B and C cannot be in the same team whereas E and F must be in the same team. G and D cannot be in the same team.

Example 28: If C, D, and A form a team of three players, which of the following can be the members of one of the other teams?

(A) A and E (B) G and B
(C) E and F (D) Both (B) and (C)

Example 29: If E, F, and G form a team of three members, then in how many ways can the remaining two teams of two players each be formed?

(A) 2 (B) 4 (C) 3 (D) 1

Example 30: If D and A are not in the same team, then altogether in how many ways can the teams of two members be formed?

(A) 4 (B) 7 (C) 8 (D) 5

Example 31: If B, E, and F form a team of three members, which of the following cannot be the two teams of two members each?

(A) AC, GD (B) AD, CG
(C) AG, CD (D) Both (A) and (B)

Solutions for questions 28 to 31:

Let Team I be of 3 players, Team II be of 2 players, and Team III be of 2 players.

It is given that A and B cannot be together. We will represent it as $A \times B$.

Similarly, we have $B \times C$ and $G \times D$.

E and F must be in the same team. So, E and F can form a team of 2 members on their own or can form a team of 3 members with another person.

Let us now take up the questions and work them out.

Example 28: Given that C, D, A form a team of 3 members, one of the other teams **has** to have E and F together. Hence, B and G should form one team.

Hence, the correct option is (D).

Example 29: Given that E, F, G form a team of 3 players. Since A and B or B and C cannot be in the same team, we must necessarily have A and C together in one team and B and D in the other team. So the teams can be formed only in one way.

Hence, the correct option is (D).

Example 30: Given that A and D are not in the same team. Hence, $A \times B$, $B \times C$, $G \times D$, and $A \times D$.

We already know that E and F must be in the same team. They may form a team of 3 members or they themselves be a team of 2 members. Let us consider these two possibilities and then fill up the other teams. They can be formed as follows:

	Team I	Team II	Team III
1.	A E F	B D	C G
2.	A E F	C D	B G
3.	B E F	A G	C D
4.	C E F	A G	B D
5.	D E F	A C	B G
6.	G E F	A C	B D
7.	A C G	B D	E F

Thus the teams can be formed in 7 ways.

Hence, the correct option is (B).

Example 31: If B, E, F form a team of 3 members, then the two members teams must be formed from A, C, D, G. The teams can be AD and CG or AG and CD. As D and G cannot form a team, AC and GD cannot be formed. Choice (A). {Please note that we can answer this question from the answer choices—from choice (A), we find that G and D are together in one team which is not possible. Thus, choice (A) is the answer.}

Hence, the correct option is (A).

Direction for question 32: Select the correct alternative from the given choices.

Example 32: At least two boys out of A, B, C, and D and at least two girls out of P, Q, R, and S have to be chosen to form a group of 5 members.

Neither A nor C can go with Q.

Neither P nor S can go with B.

Q and R cannot be together.

Which of the following is an acceptable team?

(A) ARCQP
(B) ASQPD
(C) ASQRP
(D) PSRAD

Solution for question 32:

The required group of 5 members must be formed with at least two boys from A, B, C, D and at least 2 girls from P, Q, R, S.

Answers 1, 2, and 3 can be ruled out as A and Q cannot be together.

In choice (D), P, S, R, A, D can be together without violating any of the given conditions.

Hence, the correct option is (D).

Questions on routes/networks involve different points or locations between which there is some movement or communication. The way the movement or communication is effected is described in the data/conditions. Sometimes, these are also referred to as 'maps' because the routes given resemble a map.

The data given in these types of questions may not always have the word 'route' or 'network' in them but a network is indicated by some sort of connectivity between two 'points'. The way the statements are worded is important. The wording includes statements like

1. Some poles are connected through wires.
2. Some towers send signals to one another.
3. Some cabins, market, cities, etc. are connected via. passages or roads and so on.

The connectivity between the two 'points' can be only one-way or two-way. In one-way connectivity, the flow will be in only one direction, whereas in two-way connectivity, the flow will be in both directions between the points.

Read the data carefully and then draw the diagram or network. The words 'from' and 'to' play an important role in these questions, and, hence, care should be taken while interpreting the data. While drawing a diagram, arrow marks can be used very effectively to indicate the direction of connectivity as explained below.

1. If the statement mentions that there is a one-way route from city A to B then it can be represented as follows.

   $A \longrightarrow B$

2. If the statement mentions that cities X and Y have roads on which you can travel in either direction, it means that it is a two-way connectivity. Then it can be represented as follows.

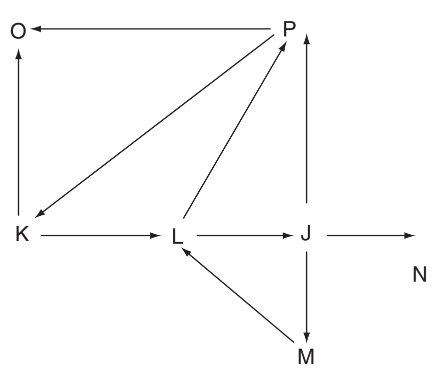
   $X \longleftrightarrow Y$

3. If the statement mentions that all the projected roads are one-ways and there is a route from point K to P and then from P to K, then it should be represented as follows.

   $P \rightleftarrows K$

   i.e. from P to K is one route and from K to P is another route.

   Now, consider the following network.



In this network, let us say that a person starts from J and he wants to reach K. We want to find out the number of distinct routes he can take without touching any point twice. Starting from J, the possible directions of movement are from J to N, J to M, and J to P, but if he goes from J to N then coming back or travelling in some other direction is not possible. Hence, the person has only two options for movement from J (J to M and J to P). Now, if he goes to M, he has to go to L from M. At L, it appears that he has two options—he can go to P or to J. But, since he started from J and as he cannot touch any point twice, he cannot go to J. So, there is only one option at L—that is going to P. So, to reach P from J, there are two options—one directly to P from J and the other via. M and L. Once he reaches P, he has only one way of reaching K—along the diagonal PK. If he goes to O from P, then he cannot travel to K from O (the route is one way in the O to K). Thus, the total number of ways from J to K is two (JPK and JMLPK).

Thus, one has to look at all the possible routes carefully in the above-discussed manner.

In the aforementioned example, if the route between J and L is two-way and then we have to find out the number of ways to reach O stating from J, the routes we have will be as follows:

JPO, JPKO, JMLPO, JMLPKO, JLPO, JLPKO

This gives us a total of six distinct ways of reaching O from J.

Direction for questions 33 to 37: Read the following information and answer the questions given below.

P, Q, R, S, T, U, V are seven places on a map. The following places are connected by two-way roads: P and Q; P and U; R and U; R and S; U and V; S and T; Q and R; T and V. No other road exists.

Example 33: The shortest route (the route with the least number of intermediate places) from P to V is

(A) P-R-V (B) P-T-V
(C) P-Q-R-U-V (D) P-U-V

Example 34: How many distinct routes exist from S to U (without touching any place more than once)?

(A) 3 (B) 2 (C) 1 (D) 4

Example 35: The route covering the maximum number of places and going from P to R does not pass through

(A) U (B) T (C) S (D) Q

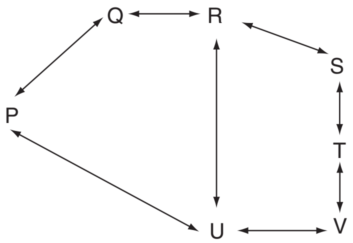
Example 36: If U to V and S to R are only one-way routes, then which of the following places lose contact with P?

(A) V
(B) T
(C) S
(D) No place loses contact with P

Example 37: If the number of places to which a place is connected directly considered as the measure of importance, then which of the following places is of the highest importance?

(A) Q (B) P (C) R (D) S

Solutions for questions 33 to 37:
The route map of the places is as follows:



Example 33: As is seen, P-U-V is the shortest way (with only one intermediate point) from P to V.
Hence, the correct option is (D).

Example 34: To travel from S to U, the routes available are: S-R-U; S-T-V-U, and S-R-Q-P-U—a total of 3 routes.
Hence, the correct option is (A).

Example 35: First let us write down the route from P to R with the maximum number of intermediate points. By observation, we find that it is P-U-V-T-S-R. It does not touch Q.
Hence, the correct option is (D).

Example 36: If U to V and S to R are only one-way routes, from the figure, we find that all places can still be reached from P. Hence, none of the places loses contact with P.
Hence, the correct option is (D).

Example 37: For each of the places given in the choices, Q, P, R, and S, let us see how many places are directly connected.
Q is directly connected to 2 places.
P is directly connected to 2 places.
R is directly connected to 3 places.
S is directly connected to 2 places.
Hence, the correct option is (C).

Direction for questions 38 to 41: Read the following information and answer the questions given.
Five cities P, Q, R, S, and T are connected by different modes of transport as follows:
P and Q are connected by boat as well as by rail.
S and R are connected by bus and by boat.
Q and T are connected only by air.
P and R are connected only by boat.
T and R are connected by rail and by bus.

Example 38: Which of the following pair of cities are connected by any of the routes directly [without going through any other city]?
(A) P and T (B) T and S
(C) Q and R (D) None of these

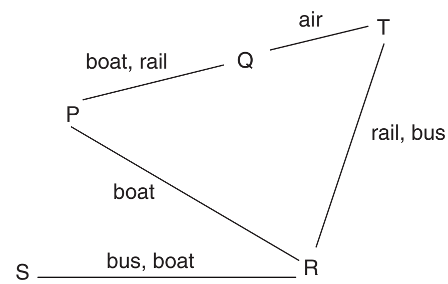
Example 39: Which mode of transport would help one to reach R starting from Q but without changing the mode of transport?
(A) Boat (B) Rail (C) Bus (D) Air

Example 40: If a person visits each of the places starting from P and gets back to P, which of the following places must he visit twice?
(A) Q (B) R
(C) S (D) T

Example 41: Between which two cities among the pairs of cities given below are there maximum travel options available? (If there is more than one route possible between two cities, consider the route with least number of cities enroute.)
(A) Q and S (B) P and R
(C) P and T (D) Q and R

Solutions for questions 38 to 41:

Example 38:



P and T are connected through Q.
T and S are connected through R
Q and R are connected through T or P. Q and S are connected through R and P or T. So, none of the pairs in the choices are directly connected.
Hence, the correct option is (D).

Example 39: From Q to P, he can reach by boat.
From P to R also he can travel by boat.
So, a person should travel by boat to reach R from Q without changing the mode of transport.
Hence, the correct option is (A).

Example 40: If a person wants to visit all the places and again return to P, then he can go in the order of $P \to Q \to T \to R \to S$ and then
$S \to R \to P$ (OR)
$P \to R \to S$ and then
$S \to R \to T \to Q \to P$
He must visit R twice.
Hence, the correct option is (B).

Example 41: One has to travel between any of the two cities with a restriction that if there is more than one possible route, he has to go by the least number of cities enroute. It is better to take the pair of cities given in each of the choices.
Choice (A) Q and S: A person can go from Q to T to R to S (or) Q to P to R to S or vice versa. In both the routes, there are two cities enroute. We can calculate the number of options in the entire route by multiplying the options available in each segment of the route.

For the route Q-T-R-S, the number of options = $1 \times 2 \times 2 = 4$

For the route Q-P-R-S, the number of options = $2 \times 1 \times 2 = 4$

Choice (B) P and R: The route between P and R has only one mode of travel, that is boat.

Choice (C) P and T: A person can go by PQT in either way or by PRT in either way. If it is by PQT, then the options are boat–air or rail–air, i.e. two ways. If it is by PRT, then the options are boat-rail or boat-bus in either way. Hence, they are only two options.

Choice (D) Q and R: If a person travels between Q and R, then he can go by QTR or by QPR. If he goes by QTR, then the options are air–rail or air–bus, that gives two options or if he goes by QPR, then the options are rail–boat or boat–boat, that will again give us two options.

So, it is very clear that Q and S have maximum number of travel options available between them.

Hence, the correct option is (A).

Direction for question 42: Select the correct alternative from the given choices.

Example 42: Four computers P, Q, A, and B are interconnected for the transmission of data. A and B each can send data to both P and Q but B cannot receive data from A. P and Q can have data flow in both directions between them but they cannot transmit the data so received to B but can otherwise send the data directly to B. Which of the following routes can be followed if B has to receive data from A?

I. AQPB II. APB

III. AQB IV. APQB

(A) I and III (B) II and III

(C) III and IV (D) All four

Solution for question 42:

AQPB and APQB can be eliminated because P and Q cannot send the data to B.

The paths AQB and APB do not violate any conditions and hence can be possible routes to send data from A to B.

Hence, the correct option is (B).

Exercises

Direction for questions 1 to 3: These questions are based on the following information.

Each of the seven delegates A through G came to India to attend a conference from seven different countries—China, Japan, Malaysia, England, Australia, Germany, and Poland.

(i) China, Japan, and Malaysia are the only Asian countries.

(ii) A and B are from Asian countries, whereas D is neither from England nor from Australia.

(iii) E and F are from non Asian countries but neither of them came from either Australia or England.

(iv) C is not from England and the person from Poland is not F.

(v) A is from China.

1. Who is from Germany?

(A) E (B) C (C) F (D) G

2. Who is from Malaysia?

(A) B (B) D

(C) A (D) Either (A) or (B)

3. Which country did G come?

(A) England

(B) Australia

(C) Poland

(D) Cannot be determined

Direction for questions 4 to 6: These questions are based on the following information.

Bingo, Pingo, Tingo, Hingo, and Mingo are five friends, each of whom is working in a different company among C_1, C_2, C_3, C_4 and C_5 and they belongs to the same city but a different locality—l_1, l_2, l_3, l_4, and l_5.

(i) The persons who are working with C_1 and C_2 are from l_3 and l_4

(ii) Bingo is from l_5 but does not work for C_5.

(iii) Tingo is not from l_4 but works for C_2.

(iv) Pingo works neither for C_5 nor in C_3 and is not form l_2.

(v) The person working for C_3 is from l_1.

(vi) Mingo does not work for C_3.

4. For which company does Hingo work?

(A) C_3 (B) C_4 (C) C_5 (D) C_2

5. Who is from l_4?

(A) Mingo (B) Hingo

(C) Tingo (D) Pingo

6. Who works for C_4?

(A) Bingo (B) Mingo

(C) Pingo (D) Hingo

Direction for questions 7 to 9: These questions are based on the following information.

A team of three is to be selected from six persons Amar, Bhavan, Chetan, Dawan, Ekta, and Farheen under the following constraints:

(i) If Amar or Bhavan is selected, then Chetan must not be selected.

(ii) If Chetan or Dawan is selected, then at least one of Ekta and Farheen must be selected.

7. If Dawan is selected, then who must not be selected?

(A) Amar

(B) Bhavan

(C) Chetan

(D) None of these

8. If Amar is selected, then in how many ways the team can be selected?
(A) 5 (B) 6 (C) 4 (D) 7

9. If Bhavan is selected, then who must be selected?
(A) Dawan
(B) Ekta
(C) Farheen
(D) Either (B) or (C)

Direction for questions 10 to 12: These questions are based on the following information.

Three girls Anjali, Bharathi, and Chandrika and four boys Kiran, Lala, Manoj, and Naveen are to be divided into two teams under the following constraints.

(i) Each team must have at least one girl and at least one boy and at least three persons in total.
(ii) If Anjali and Bharathi are selected in a team, then the team must have only one boy.
(iii) Kiran and Lala cannot be in the same team.
(iv) Chandrika and Naveen can be in the same team, only if Bharathi is selected in that team.

10. If Kiran and Chandrika are in the same team, then in how many ways can the other team be selected?
(A) 6 (B) 3 (C) 4 (D) 5

11. If Manoj is not in the same team as Bharathi, then in how many ways can the teams be selected?
(A) 3 (B) 4 (C) 5 (D) 6

12. If three boys are selected into one team, then in how many ways can the teams be selected?
(A) 4 (B) 5 (C) 3 (D) 6

Direction for questions 13 to 15: These questions are based on the following information.

Seven persons—P, Q, R, S, T, U and V, who are of different ages, are comparing their ages. We know the following information.

(i) P is younger than R, who is not older than S.
(ii) S is younger than only two persons.
(iii) Q is not the oldest but older than fourth youngest person.
(iv) T is older than only U.

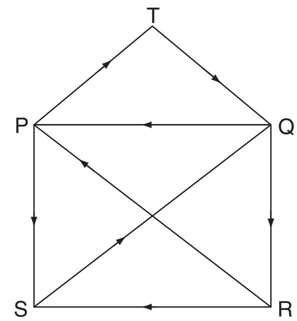
13. Who is oldest?
(A) S (B) T (C) U (D) V

14. Who is the third youngest?
(A) V (B) P (C) R (D) S

15. Who is the fourth eldest?
(A) R (B) P (C) S (D) V

Direction for questions 16 to 19: These questions are based on the diagram given.

Five cities P, Q, R, S, and T are connected by one-way rail routes as shown. One takes one hour duration to travel between any two directly connected cities.



At station S, for every 2 hours, one train departs and the departure time of the first train is 6:00 a.m. Similarly, at station R, for every 3 hours, one train departs in each route and the departure time of the first train is 4 a.m.

At station Q, one train departs for every 2 hour in each route and the departure time of the first train is 7 a.m.

At station P, train departs for every 1 hour in each route and the departure time of the first train is 8 a.m.

At station T, for every 3 hours, one train departs and the departure time of the first train is 5:30 a.m.

16. What is the least time will it take to reach P from R, if one takes the longest route without visiting any station more than once?
(A) 6 hours (B) 3 hours
(C) 5 hours (D) 4 hours

17. If a person reaches Q at 1:00 p.m. from R, which of the following can be the time at what time he must have started from R if that person takes the shortest route?
(A) 10:00 a.m. (B) 9:00 a.m.
(C) 11:00 a.m. (D) None of these

18. A person wants to travel from R to T and he takes the longest route without visiting any station more than once. If he starts at 4:00 a.m., then for how much time he has to wait for the trains altogether in all stations before reaching T.
(A) 2 hours (B) 3 hours
(C) 2 hours (D) 1 hour

19. If a person starts from P at 10:00 a.m. to reach S and he takes the longest route without visiting any station more than once then at what time will he be reachings?
(A) 1:00 p.m. (B) 4:00 p.m.
(C) 5:00 p.m. (D) 2:00 p.m.

Direction for question 20: This question is based on the information given.

Five cities Ahmedabad, Bangalore, Calicut, Delhi, and Indore are connected by one-way routes from Ahmedabad to Bangalore, Delhi to Ahmedabad, Indore to Delhi, Delhi to Calicut, Ahmedabad to Calicut, Bangalore to Calicut, Calicut to Indore, Indore to Bangalore, and Ahmedabad to Indore.

20. In how many ways a person can travel from Delhi to Indore without visiting any city more than once?
(A) 5 (B) 3 (C) 6 (D) 4

Direction for questions 21 and 22: Select the correct alternative from the given choices.

21. There are 15 identical coins out of which fourteen are of equal weights and one coin lighter than each of the other coins. What is the minimum number of weighings required using a common balance to definitely identify the counterfeit coin?

(A) 3 (B) 4
(C) 5 (D) None of these

22. Beside a lake, there are three temples and a flower garden. Whenever some flowers are dipped into the lake, the flowers gets triplet. A person brought some flowers from the garden and dipped then into the lake. He placed x flowers in front of the first temple and dipped the remaining flowers into the lake. He placed x flowers in front of the second temple and dipped the remaining flowers into the lake. Now, he placed x flowers in front of the third temple and has no flowers. Which of the following numbers can be the value of x?

(A) 9 (B) 18 (C) 27 (D) 36

Direction for questions 23: These questions are based on the following letter – multiplication in which each letter is represents a unique non-zero digit.

A	B	C		
×	C	B	A	
C	D	E	F	C

Also, it is known that D = 3C and F = 4B

23. What is the value of D?

(A) 3
(B) 6
(C) 9
(D) Cannot be determined

Direction for questions 24 and 25: These questions are based on the following data.

Each individual of a city called 'Josh' belongs exactly to one of the two types, viz., Yes-type or No-type. Yes-type people always give the true reply, while the No-type always lies. Answer the following questions based on the information.

24. You met three residents A, B, and C, of the city and asked them, 'who among you are married?' and got the following replies.

A: I am married to B.
B: I am married to C.
C: I am not married to A.

If it is further known that A is married to one of B and C and there is exactly one married couple among the three, then which of the following is definitely true?

(A) C is married to A.
(B) B is married to A.
(C) A is of Yes-type.
(D) B is of No-type.
(E) A is of No-type.

25. You approached three inhabitants A, B, and C of the city and asked them, 'Who is of No-type among you?', and got the following replies.

A: B is of No-type.
B: C is of No-type.
C: A is of No-type.

It can be concluded that:

(A) A is a No-type.
(B) B is a No-type.
(C) C is a No-type.
(D) Either A or B is of No-type.
(E) Data inconsistent.

Answer Keys

1. C	**2.** D	**3.** A	**4.** A	**5.** D	**6.** A	**7.** D	**8.** A	**9.** D	**10.** D
11. D	**12.** A	**13.** D	**14.** B	**15.** A	**16.** B	**17.** A	**18.** D	**19.** C	**20.** D
21. A	**22.** C	**23.** C	**24.** D	**25.** C					

Chapter 9

Clocks and Calendars

CHAPTER HIGHLIGHTS

- Clocks
- Calendar
- Leap and Non-leap Year
- Counting the Number of Odd Days

Clocks

The hour hand and the minute hand of a clock move in relation to each other continuously, and at any given point of time, they make an angle between 0° and 180° with each other.

If the time shown by the clock is known, the angle between the hands can be calculated. Similarly, if the angle between two hands is known, the time shown by the clock can be found out.

When we say angle between the hands, we normally refer to the acute/obtuse angles (upto 180°) between the two hands and not the reflex angle (> 180°).

For solving the problems on clocks, the following points will be helpful.

- Minute hand covers 360° in 1 hour, i.e. in 60 minutes. Hence, MINUTE HAND COVERS 6° PER MINUTE.
- Hour hand covers 360° in 12 hours. Hence, hour hand covers 30° per hour. Hence, HOUR HAND COVERS 1/2° PER MINUTE.

The following additional points also should be remembered. In a period of 12 hours, the hands make an angle of

- 0° with each other (i.e. they coincide with each other), 11 times.
- 180° with each other (i.e., they point exactly in opposite directions), 11 times.
- 90° or any other angle with each other, 22 times.

NOTE

We can also solve the problems on clocks using the method of 'Relative Velocity'.

In 1 minute, minute hand covers 6° and hour hand covers 1/2°.

Therefore, relative velocity = 6 – 1/2 = $5\frac{1}{2}^{\circ}$ per minute. Alternately, in 1 hour, the minute hand covers 60 minute divisions, whereas the hour hand covers 5 minute divisions.

∴ Relative speed = 60 – 5 = 55 minutes per hour.

However, adopting the approach of actual angles covered is by far the simplest and does not create any confusion.

Points to Note

- Any angle other than (0° and 180°) is made 22 times in a period of 12 hours.
- In a period of 12 hours, there are 11 coincidences of the two hands, when the two hands are in a straight line facing opposite directions.
- The time gap between any two coincidences is 12/11 hours or $65\frac{5}{11}$ minutes.
- If the hands of a clock (which do not show the correct time) coincide every p minutes, then

If $p > 65\frac{5}{11}$, then the watch is going slow or losing time.

If $p < 65\frac{5}{11}$, then the watch is going fast or gaining time.

To calculate the angle 'θ' between the hands of a clock, we use the following formula (where m = minutes and h = hours)

1. $\theta = \frac{11}{2}m - 30h \left(\text{when } \frac{11}{2}m > 30h \right)$
2. $\theta = 30h - \frac{11}{2}m \left(\text{when } 30h > \frac{11}{2}m \right)$

Solved Examples

Example 1

What is the angle between the minute hand and the hour hand of a clock at 3 hours 40 minutes?

(A) 20° (B) 70° (C) 90° (D) 130°

Solution

The angle between the hands can be calculated by $\theta = \left|\frac{11}{2}m - 30h\right|$, where m is minutes and h is hours. Here, $m = 40$ and $h = 3$

$$\therefore \quad \theta = \left|\frac{11}{2} \times 40 - 30 \times 3\right| = |220 - 90| = 130°$$

The angle between the two hands is 130°.

Example 2

Find the time between 2 and 3 O'clock at which the minute hand and the hour hand make an angle of 60° with each other.

Solution

In the formula $\theta = \left|\frac{11}{2}m - 30h\right|$,

$$\theta = 60° \text{ and } h = 2$$

$$\therefore \quad 60 = \frac{11}{2}m - 30 \times 2$$

$$\frac{11}{2}m = 120$$

$$m = \frac{240}{11} = 21\frac{9}{11}\,m \text{ past } 2$$

(or) $$60 = 30 \times 2 - \frac{11}{2}m$$

$$\therefore \quad \frac{11}{2}m = 0$$

$$m = 0$$

Therefore, the angle between the hour hand and the minute hand is 60° at 2 O'clock and at $21\frac{9}{11}$ minutes past 2 O'clock.

Example 3

Find the time between 2 and 3 O'clock at which the minute hand and the hour hand overlap.

Solution

When the two hands overlap, the angle between them is 0°.

$$\theta = \left|\frac{11}{2}m - 30h\right|$$

$$\therefore \quad \theta = 0° \quad \text{and} \quad h = 2$$

$$\frac{11}{2}m = 30 \times 2$$

$$m = \frac{120}{11} = 10\frac{10}{11} \text{ min past 2.}$$

Example 4

Find the time between 2 and 3 O'clock at which the minute hand and the hour hand are perpendicular to each other.

Solution

When two hands are perpendicular, $\theta = 90°$ and $h = 2$

$$\therefore \quad \theta = \left(\frac{11}{2}m - 30h\right) \quad \text{or} \quad \left(30h - \frac{11}{2}m\right)$$

$$90 = \frac{11}{2}m - 30 \times 2; \quad \frac{11}{2}m = 150$$

$$m = \frac{300}{11} = 27\frac{3}{11} \text{ minutes past 2}$$

(or) $$90 = 30 \times 2 - \frac{11}{2}m$$

$$\Rightarrow \quad \frac{11}{2}m = -30$$

As m cannot be negative, this case is not possible.

So, the hands are perpendicular to each other only once i.e. at $27\frac{3}{11}$ minutes past 2 O'clock.

Example 5

Find the time between 2 and 3 O'clock at which the minute hand and the hour hand are on the same straight line but are pointing in opposite directions.

Solution

When two hands are pointing opposite directions and are on a straight line the angle between them would be 180°. i.e. $\theta = 180°$ and $h = 2$.

$$180° = \frac{11}{2}m - 30h; \quad \frac{11}{2}m = 180 + 60 = 240$$

$$m = \frac{480}{11} = 43\frac{7}{11}$$

So, at $43\frac{7}{11}$ minutes past 2 O'clock the hands will be at 180°.

CALENDAR

Suppose you are asked to find the day of the week on 30th June, 1974, it would be a tough job to find it if you do not know the method. The method of finding the day of the week lies in the number of 'odd days'.

NOTE

Every 7^{th} day will be the same day count wise, i.e. if today is Monday, then the 7^{th} day counting from Tuesday onwards will once again be Monday. Odd days is the days remaining after completion of an exact number of weeks. Odd days is the reminder obtained on dividing the total number of days with seven.

Example: 52 days ÷ 7 = 3 odd days.

Leap and Non-leap Year: A non-leap year has 365 days, whereas a leap year has one extra day because of 29 days in the month of February. Every year which is divisible by 4 is called a leap year. Leap year consists of 366 days (52 complete weeks + 2 days), the extra two days are the odd days. So, a leap year has two odd days.

An non-leap year consists of 365 days (52 complete weeks + 1 day). The extra one day is the odd day.

NOTE

Every century, year which is a multiple of 400, is a leap year. A century year which is not divisible by 400 is a non-leap year.

Example: 400, 800, 1200, 1600 … are leap years.
500, 700, 900, 1900 … are non-leap years.

Counting the number of Odd Days: 100 years consist of 24 leap years + 76 ordinary years. (100 years when divided by 4, we get 25. But at the 100^{th} year is not a leap year, hence only 24 leap years).

$$= 2 \times 24 \text{ odd days} + 1 \times 76 \text{ odd days} = 124 \text{ days}$$

$$= 17 \text{ weeks} + 5 \text{ days}$$

The extra 5 days are the odd days.

So, 100 years contain 5 odd days.

Similarly, for 200 years we have 10 extra days (1 week + 3 days).

∴ 200 years contains 3 odd days.

Similarly, 300 years contain 1 odd day and 400 years contain 0 odd days.

Counting of number of odd days, when only one date is given: Here, we take January 1^{st} 1 AD as the earlier date and we assume that this day is a Monday. We take its previous day, i.e. Sunday as the reference day. After this the above-mentioned method is applied to count the number of odd days and find the day of the week for the given date.

Counting number of odd days, when two dates are given: Any month which has 31 days has 3 odd days.

(∵ 31 ÷ 7 leaves 3 as remainder) and any month which has 30 days has 2 odd days (30 ÷ 7 leaves 2 as remainder).

Then, the total number of odd days are calculated by adding the odd days for each month. The value so obtained is again divided by 7 to get the final number of odd days. The day of the week of the second date is obtained by adding the odd days to the day of the week of the earlier date.

Example 6

If you were born on 14^{th} April, 1992, which was a Sunday, then on which day of the week does your birthday fall in 1993?

(A) Monday (B) Tuesday
(C) Wednesday (D) Cannot be determined

Solution

14^{th} April 1992 to 14^{th} April 1993 is a complete year, which has 365 days. Hence, the number of odd days from 14^{th} April 1992 to 14^{th} April 1993 is 1.

Hence, 14^{th} April 1993 is one day after Sunday, i.e. Monday.

Example 7

If 1^{st} Jan, 1992, is a Tuesday then on which day of the week will 1^{st} Jan, 1993, fall?

(A) Wednesday (B) Thursday
(C) Friday (D) Saturday

Solution

Since 1992 is a leap year there are 2 odd days.

Hence, 1^{st} January 1992 is two days after Tuesday, i.e., Thursday.

Example 8

If 1^{st} April, 2003 was Monday, then which day of the week will 25^{th} December of the same year be?

(A) Tuesday (B) Wednesday
(C) Thursday (D) Friday

Solution

The number of days from 1^{st} April to 25^{th} December

(29 + 31 + 30 + 31 + 31 + 30 + 31 + 30 + 25) days = 268 days

$$= \frac{268}{7} = 38 + 2 \text{ odd days.}$$

Hence, 25^{th} December is two days after Monday, i.e. Wednesday.

Example 9

Which year will have the same calendar as that of 2005?

(A) 2006 (B) 2007
(C) 2008 (D) 2011

Solution

Year: 2005 + 2006 + 2007 + 2008 + 2009 + 2010

Odd days : 1+ 1+ 1+ 2+ 1 + 1

Total number of odd days from 2005 to 2010 are 7 ≅ 0 odd days.

Hence, 2011 will have the same calendar as that of 2005.

Example 10

What day of the week was 18th April 1901?

(A) Monday (B) Tuesday
(C) Wednesday (D) Thursday

Solution

18th April 1901 ⇒ (1600 + 300) years + 1st January to 18th April 1901.

1600 years have – 0 odd days

300 years have – 1 odd day

The number of days from 1st January, 1901 to 18th April 1901 is (31 + 28 + 31 + 18) days

108 days ≅ 3 odd days

∴ Total number of odd days = 3 + 1 = 4

Hence, 18th April 1901 is Thursday.

Exercises

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. What is the angle covered by the minute hand in 22 minutes?
(A) 66° (B) 110° (C) 132° (D) 220°

2. By how many degrees will the minute hand move, in the same time, in which the hour hand moves 6°?
(A) 54° (B) 84° (C) 72° (D) 60°

3. What is the angle between the hands of the clock, when it shows 40 minutes past 6?
(A) 40° (B) 70° (C) 80° (D) 90°

4. What is the angle between the two hands of a clock when the time is 25 minutes past 7 O'clock?
(A) $62\frac{1}{2}°$ (B) $66\frac{1}{2}°$
(C) $72\frac{1}{2}°$ (D) $69\frac{1}{2}°$

5. At what time between 9 and 10 O'clock, will the two hands of the clock coincide?
(A) $43\frac{3}{11}$ minutes past 9 O'clock
(B) $45\frac{6}{11}$ minutes past 9 O'clock
(C) $49\frac{1}{11}$ minutes past 9 O'clock
(D) $49\frac{6}{11}$ minutes past 9 O'clock

6. At what time between 4 and 5 O'clock are the two hands of a clock in the opposite directions?
(A) $52\frac{3}{11}$ minutes past 4 O'clock
(B) $54\frac{6}{11}$ minutes past 4 O'clock
(C) $51\frac{7}{11}$ minutes past 4 O'clock
(D) $53\frac{9}{11}$ minutes past 4 O'clock

7. The angle between the two hands of a clock is 20° and the hour hand is in between 2 and 3. What is the time shown by the clock?
(A) $7\frac{3}{11}$ minutes past 2
(B) $14\frac{6}{11}$ minutes past 2
(C) $15\frac{5}{11}$ minutes past 2
(D) Both (A) and (B)

8. Which of the following can be the time shown by the clock, when the hour hand is in between 4 and 5 and the angle between the two hands of the clock is 60°?
(A) $16\frac{4}{11}$ min past 4 (B) $18\frac{9}{11}$ min past 4
(C) $32\frac{8}{11}$ min past 4 (D) $36\frac{5}{11}$ min past 4

9. How many times, the two hands of a clock will be at 30° with each other in a day?
(A) 36 (B) 40 (C) 44 (D) 48

10. If the time in a clock is 10 hours 40 minutes, then what time does its mirror image show?
(A) 2 hours 20 minutes
(B) 1 hour 15 minutes
(C) 1 hour 10 minutes
(D) 1 hour 20 minutes

11. There are two clocks on a wall, both set to show the correct time at 5:00 p.m. The clocks lose 2 minutes and 3 minutes respectively in an hour. When the clock which loses 2 minutes in one hour shows 9:50 p.m. on the same day, then what time does the other clock show?
(A) 9:30 p.m. (B) 9:40 p.m.
(C) 9:45 p.m. (D) 10:15 p.m.

12. A watch that gains uniformly was observed to be 1 minute slow at 8:00 a.m. on a day. At 6:00 p.m. on the same day it was 1 minute fast. At what time did the watch show the correct time?
(A) 12:00 noon (B) 1:00 p.m.
(C) 2:00 p.m. (D) 3:00 p.m.

13. A watch, which gains uniformly, was observed to be 6 minutes slow at 9:00 a.m. on a Tuesday and 3 minutes fast at 12:00 noon on the subsequent Wednesday. When did the watch show the correct time?
(A) 9:00 p.m. on Tuesday
(B) 12:00 a.m. on Wednesday
(C) 3:00 a.m. on Wednesday
(D) 6:00 a.m. on Wednesday

14. The number of odd days in a non-leap year is
(A) 0 (B) 1 (C) 2 (D) 3

15. What will be next leap year after 2096?
(A) 2100 (B) 2101 (C) 2104 (D) 2108

16. If 21st March 2000 was a Monday, what day of the week will 21st March 2003 be?
(A) Tuesday (B) Friday
(C) Thursday (D) Wednesday

17. If 5th January 2001 was a Friday then what day of the weak will 25th December 2001 be?
(A) Monday (B) Tuesday
(C) Wednesday (D) Thursday

18. If 14th February 2001 was a Wednesday, then what day of the week will 14th February 2101 be (i.e. after a century)?
(A) Friday (B) Saturday
(C) Sunday (D) Monday

19. If 8th February 1995 was a Wednesday, then what day of the week will 8th February 1994 be?
(A) Wednesday (B) Thursday
(C) Tuesday (D) Monday

20. If holidays are declared only on Sundays and in a particular year 12th March is a Sunday, is 23rd September in that year a holiday?
(A) Yes
(B) No
(C) Yes, if it is a leap year.
(D) No, if it is a leap year.

21. Which day of the week was 1601, Jan 15?
(A) Monday (B) Tuesday
(C) Wednesday (D) Thursday

22. In a year, if 23rd November is a Friday then what day of the week will 14th March in that year be?
(A) Monday
(B) Wednesday
(C) Sunday
(D) Cannot be determined

23. The calendar of which of the following years is the same as that of the year 2001?
(A) 2005 (B) 2006 (C) 2007 (D) 2008

24. Pankaj met his friend three days ago. He told his friend that he has his last exam five days later. He met his friend again, three days after the last exam. Six days after he met his friend after the last exam, they left for a vacation. The day on which they left for a vacation is a Saturday. What is today?
(A) Saturday (B) Tuesday
(C) Sunday (D) Cannot be determined

25. Five days ago Shweta lost her phone. Two days after loosing the phone she lodged a complaint with the police. Six days after lodging the complaint she bought a new phone. Four days after buying a new phone, i.e. on a Thursday she found her old phone. On which day did she loose her phone?
(A) Friday (B) Saturday
(C) Thursday (D) None of these

Previous Years' Questions

1. The Palghat Gap (or Palakkad Gap), a region about 30 km wide in the southern part of the Western Ghats in India, is lower than the hilly terrain to its north and south. The exact reasons for the formation of this gap are not clear. It results in the neighbouring regions of Tamil Nadu getting more rainfall from the South West monsoon and the neighboring regions of Kerala having higher summer temperatures. **[GATE, 2014]**
What can be inferred from this passage?
(A) The Palghat gap is caused by high rainfall and high temperatures in Southern Tamil Nadu and Kerala.
(B) The regions in Tamil Nadu and Kerala that are near the Palghat Gap are near the low–lying.
(C) The low terrain of the Palghat Gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala.
(D) Higher summer temperatures result in higher rainfall near the Palghat Gap area.

2. Geneticists say that they are very close to confirming the genetic roots of psychiatric illnesses such as depression and Schizophrenia, and consequently, that doctors will be able to eradicate these diseases through early identification and gene therapy.
On which of the following assumptions does the statement above rely? **[GATE, 2014]**
(A) Strategies are now available for eliminating psychiatric illnesses.
(B) Certain psychiatric illnesses have a genetic basis.
(C) All human diseases can be traced back to genes and how they are expressed.
(D) In the future, genetics will become the only relevant field for identifying psychiatric illness.

3. The old city of Koenigsberg, which had a German majority population before World War 2, is now called Kaliningrad. After the events of the war, Kaliningrad is now a Russian territory and has a predominantly Russian population. It is bordered by the Baltic Sea on the north and the countries of Poland to the South and West Lithuania to the east respectively. Which of the statements below can be inferred from this passage? **[GATE, 2014]**

(A) Kaliningrad was historically Russian in its ethnic make up.
(B) Kaliningrad is part of Russia despite it not being contiguous with the rest of Russia.
(C) Koenigsberg was renamed Kaliningrad, as that was its original Russian name.
(D) Poland and Lithuania are on the route from Kalinigrad to the rest of Russia.

4. The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in north India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region.
Which one of the following statements, if true, does not contradict this conclusion? **[GATE, 2014]**

(A) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent.
(B) More cases of dengue are now reported because of an increase in the Municipal Office's administrative efficiency.
(C) Many more cases of dengue are being diagnosed this year since the introduction of a new and effective diagnostic test
(D) The number of people with malarial fever (also contracted from mosquito bites) has increased this year.

5. At what time between 6 am and 7 am, will the minute hand and hour hand of a clock make an angle closest to 60°? **[GATE, 2014]**

(A) 6:22 am (B) 6:27 am
(C) 6:38 am (D) 6:45 am

6. Which number does not belong in the series below?
2, 5, 10, 17, 26, 37, 50, 64 **[GATE, 2014]**

(A) 17 (B) 37
(C) 64 (D) 26

7. A dance programme is scheduled for 10:00 am. Some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is **[GATE, 2014]**

(A) Students should come at 9:00 am and parents should come at 10:00 am.
(B) Participating students should come at 9:00 am accompanied by a parent, and other parents and students should come by 10:00 am.
(C) Students who are not participating should come by 10:00 am and they should not bring their parents. Participating students should come at 9:00 am.
(D) Participating students should come before 9:00 am. Parents who accompany them should come at 9:00 am. All others should come at 10:00 am.

8. By the beginning of the 20th century, several hypotheses were being proposed, suggesting a paradigm shift in our understanding, of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun.
Which of the following inference(s) may be drawn from the above passage?

I. Our understanding of the universe changes based on the positions of stars.
II. Paradigm shifts usually occur at the beginning of centuries.
III. Stars are important objects in the universe.
IV. Experimental evidence was important in confirming this paradigm shift. **[GATE, 2014]**

(A) I, II and IV (B) III only
(C) I and IV (D) IV only

9. After several defeats in wars, Robert Bruce went in exile and wanted to commit suicide. Just before committing suicide, he came across a spider attempting tirelessly to have its net. Time and again, the spider failed but that did not deter it to refrain from making attempts. Such attempts by the spider made Bruce curious. Thus, Bruce started observing the near-impossible goal of the spider to have the net. Ultimately, the spider succeeded in having its net despite several failures. Such act of the spider encouraged Bruce not to commit suicide. And then, Bruce went back again and won many a battle, and the rest is history.
Which one of the following assertions is best supported by the above information? **[GATE, 2013]**

(A) Failure is the pillar of success.
(B) Honesty is the best policy.
(C) Life begins and ends with adventures.
(D) No adversity justifies giving up hope.

10. Given the sequence of terms AD, CG, FK, JP, ? the next term is **[GATE, 2012]**

(A) OV (B) OW
(C) PV (D) PW

11. Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.
Based on the above passage which topic would not be included in a unit on bereavement? **[GATE, 2012]**
(A) How to write a letter of condolence
(B) What emotional stages are passed through in the healing process
(C) What the leading causes of death are
(D) How to give support to a grieving friend.

12. 25 persons are in a room. 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. The number of persons playing neither hockey nor football is ______. **[GATE, 2010]**
(A) 2 (B) 17
(C) 13 (D) 3

13. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.
Unemployed : Worker **[GATE, 2010]**
(A) Fallow : Land
(B) Unaware : Sleeper
(C) Wit : Jester
(D) Renovated : House

14. Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e., brothers and sisters). All were born on 1st January, in different years. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:
I. Hari's age + Gita's age > Irfan's age + Saira's age
II. The age difference between Gita and Saira is 1 year. However Gita is not the oldest and Saira is not the youngest.
III. There are no twins.
Which of the following in a possible order in which they were born? **[GATE, 2010]**
(A) HSIG
(B) SGHI
(C) IGSH
(D) IHSG

Answer Keys

Exercises

1. C	**2.** C	**3.** A	**4.** C	**5.** C	**6.** B	**7.** D	**8.** C	**9.** C	**10.** D
11. C	**12.** B	**13.** C	**14.** B	**15.** C	**16.** C	**17.** B	**18.** D	**19.** C	**20.** B
21. A	**22.** B	**23.** C	**24.** B	**25.** B					

Previous Years' Questions

1. C	**2.** B	**3.** B	**4.** D	**5.** A	**6.** C	**7.** B	**8.** D	**9.** D	**10.** A
11. C	**12.** D	**13.** A	**14.** B						

Test

Logical Ability

Time: 25 Minutes

Direction for questions 1 to 7: Complete the following series.

1. 11, 26, 51, 76, ____
   (A) 101 (B) 115
   (C) 125 (D) 133
2. 23, 57, 1113, 1719, ____
   (A) 2329 (B) 2931
   (C) 3137 (D) 3743
3. VIQ, TAC, WJR, VCE, XKS, XEG, _____
   (A) YGL (B) ZFH
   (C) YLT (D) YNR
4. 25 : 343 : : 49 : _____
   (A) 121 (B) 343
   (C) 512 (D) 1331
5. BIDM : DLPR : : HSBC : _____
   (A) PVEH (B) PXDH
   (C) PVHH (D) RVHD
6. 2Y5 : 4W9 : : 3J6 : _____
   (A) 4W9 (B) 6L4
   (C) 8C1 (D) 6N4
7. Aeroplane : Pilot : : Ship : _____
   (A) Driver (B) Chef
   (C) Captain (D) Marshal

Direction for questions 8 to 11: Find the odd man out.

8. (A) 38 – 121 (B) 48 – 144
   (C) 68 – 196 (D) 98 – 361
9. (A) BDGC (B) DHKR
   (C) FLOH (D) EJMZ
10. (A) 6V12 (B) 2H4
    (C) 9F18 (D) 3R6
11. (A) Mercury (B) Mars
    (C) Moon (D) Venus

Direction for questions 12 to 14: Choose the correct alternative from the given choices.

12. In a certain code language the word PRIVATE is coded as AEIPRTV then how is the word PRESENT coded in that language?
    (A) EEPNRST (B) EENPRST
    (C) EPSNERT (D) EENRPST
13. In a certain code, if the word CHLORATE is written as DFOKWUAW then how is the word PHOSPHATE written in that code?
    (A) QFRUOBHLN (B) QFROBUHLN
    (C) QFHROUBLN (D) QFROUBHLN
14. In a certain code, if the word PRESSURE is written as KIVHHFIV then how is the word SOLUTION written in that code?
    (A) HLOUTRLM (B) HLPGFRLM
    (C) HLOFGRLM (D) HLOGTROM

Direction for questions 15 and 16: These questions are based on the following information.

Six persons—P through U—are standing in a queue in the increasing order of their heights so that the shortest is at the front of the queue and the tallest is at the back. Furthermore,

(i) U is the shortest.
(ii) Exactly two persons are taller than T.
(iii) P is taller than S and exactly two persons stand between P and S.
(iv) Q is taller than P.

15. Who is the second tallest person?
    (A) T (B) R
    (C) S (D) P
16. Who is/are the persons in between P and R?
    (A) Only T (B) Q and S
    (C) T and U (D) Only S

Direction for questions 17 to 19: These questions are based on the following data.

Eight persons A, B, C, D, E, F, G, and H attended a conference and are sitting around a circular table. Among them, there are CEOs of 4 companies who came along with one assistant each. Each CEO has his assistant sitting to his right.

(i) Assistants of C and A are sitting opposite each other.
(ii) E, who is the assistant of B, is sitting opposite F.
(iii) E was not sitting adjacent to A.
(iv) G is neither adjacent nor opposite to D.

17. Who is to the left of A?
    (A) D (B) G
    (C) F (D) H
18. Who is the assistant of C?
    (A) D (B) G
    (C) E (D) H
19. If H is opposite to G, then D is to the right of ____.
    (A) A (B) B
    (C) F (D) H

Direction for questions 20 to 24: These questions are based on the following data.

In a class, 50 students failed in Maths.

40 students failed in Physics.

30 students failed in Chemistry.

10 students failed in Physics and Chemistry.

10 students failed in Maths and Physics.

No student failed in both Maths and Chemistry.

None of the students failed in all the three subjects.

20. How many students failed atleast in one subject?
(A) 50 (B) 100
(C) 75 (D) 125

21. What is the ratio of the number of students who failed in Maths and Physics to that who failed in Physics and Chemistry?
(A) 1 : 2 (B) 2 : 1
(C) 1 : 1 (D) 4 : 3

22. How many students failed in exactly two subjects?
(A) 10 (B) 20
(C) 30 (D) 40

23. The number of students who failed in only Maths, in only Physics and in only Chemistry respectively is
(A) 40, 20, 20 (B) 20, 40, 20
(C) 20, 20, 40 (D) 50, 40, 30

24. Which of the following statements is true?
(A) The number of students who failed in only Maths equals to that of the students who failed in only Physics.
(B) The number of students who failed in only Maths equals to that of the students who failed in only Physics or in only Chemistry.
(C) The number of students who failed in all the three subjects is 10.
(D) None of these.

Direction for question 25 to 30: Select the correct alternative from the given choices.

25. In the following addition each latter represents a different digit from of O to 9. which of the following is a possible number represented by FAN ?

	H	A	N
F	A	F	N
5	5	8	8

(A) 434 (B) 534
(C) 345 (D) 135

26. At what time between 10 O' clock and 11 O' clock are the hands of the clock together?
(A) $54^6/_{11}$ past 10 (B) $27^8/_{11}$ past 10
(C) Both A and B (D) None of these

27. How many times the hands of a clock are at right angles in a day?
(A) 24 (B) 22
(C) 44 (D) 48

28. On a particular day if it is found that a clock is showing 10 minutes less at 1:00 pm and 5 minutes more at 6:00 pm on the same day. At what time did the clock show the correct time?
(A) 3 hr 20 min (B) 4 hr 20 min
(C) 5 hr 40 min (D) 6 hr 40 min

29. If 21st March 2000 was a Monday, which day of the week will be 21st March 2003?
(A) Tuesday (B) Friday
(C) Thursday (D) Wednesday

30. The movie of my favorite hero is going to be released on Wednesday. To watch the movie on the first day of release, I booked my ticket the day before yesterday. If I watch the movie on the fourth day from today, on which day of the week did I book my ticket?
(A) Wednesday (B) Thursday
(C) Friday (D) Saturday

Answer Keys

1. B	**2.** A	**3.** C	**4.** D	**5.** C	**6.** B	**7.** C	**8.** D	**9.** A	**10.** A
11. C	**12.** B	**13.** D	**14.** C	**15.** D	**16.** A	**17.** C	**18.** D	**19.** D	**20.** B
21. C	**22.** B	**23.** A	**24.** B	**25.** B	**26.** A	**27.** C	**28.** B	**29.** C	**30.** B

Logical Ability Test I

Number of Questions: 35 **Time: 35 min**

Directions for questions 1 to 5: Complete the following series.

1. 4, 27, 25, 343, 121, ______ .
(1) 169 (B) 2197
(C) 3197 (D) 2457

2. 11, 25, 77, 157, 473, ______.
(A) 978 (B) 1421
(C) 949 (D) 1431

3. 12, 30, 56, 132, 182, ______.
(A) 240 (B) 300
(C) 316 (D) 306

4. 53, 61, 71, 79, 89, ______.
(A) 91 (B) 93
(C) 101 (D) 95

5. 19, 58, 175, 526, ______.
(A) 1578 (B) 1238
(C) 1458 (D) 1579

Directions for questions 6 to 10: Find the missing term.

6. 24 : 576 : : 32 : ______ .
(A) 961 (B) 1000
(C) 1225 (D) 1024

7. 4 : 27 : : 25 : ______.
(A) 64 (B) 216
(C) 125 (D) 36

8. BILK : DLPP : HMTO : ______.
(A) JOWQ (B) JRWS
(C) JPVS (D) JPXT

9. Cricket : Game : : Kangaroo : ______.
(A) Animal (B) Team
(C) Bird (D) Fish

10. Driver : Bus : : ______ : Horse
(A) Saddle (B) Jockey
(C) Horseman (D) Cowboy

Directions for questions 11 to 15: Find the odd man out.

11. (A) 11 (B) 21
(C) 31 (D) 41

12. (A) 3527 (B) 2357
(C) 5723 (D) 7532

13. (A) Brown (B) Green
(C) Yellow (D) Red

14. (A) June (B) May
(C) November (D) September

15. (A) Radish (B) Carrot
(C) Potato (D) Cabbage

Directions for questions 16 to 20: Select the correct alternative from the given choices.

16. If ‘CENTURY’ is coded as ‘AGLVSTW’, then what is the code for ‘SACHIN’?
(A) QCAFKL (B) UCEFGL
(C) QCAJGP (D) UCAJGP

17. If the code for ‘AMBITION’ is ‘GSHOZOUT’, then which of the following is coded as ‘VXOTZUAZ’?
(A) PRINTOUT (B) PRINTING
(C) PREDATOR (D) PROFOUND

18. If ‘PRESIDENT’ is coded as ‘KIVHRWVMG’, then ‘MAHENDAR’ is coded as ______.
(A) NZTVMWZI (B) NZTUMWZI
(C) NZSVMWZI (D) NZSUMWZI

19. In a code language, if pen is called pencil, pencil is called eraser, eraser is called paper, paper is called book, book is called table, table is called chair and chair is called desk, then on which of the following do we sit? (according to that language)
(A) Table (B) Paper
(C) Desk (D) Book

20. In a code language, if shirt means shoe, shoe means wallet, wallet means spectacle, spectacle means fan, fan means cabin and cabin means card, then which of the following do we use when we want some air? (according to that language)
(A) Wallets (B) Spectacles
(C) Fans (D) Cabins

Directions for questions 21 to 25: In a certain code language, the codes for sentences given in column I are given in column II. Each word has a unique code. Answer the questions based on these codes.

Column I	**Column II**
earth gets heat from sun	pep tep nep mep wep
moon gets light from sun	hep kep tep pep nep
sun gave energy to plants	bep pep dep zep lep
human gets food from plants	qep tep nep dep rep
heat and light gave life	hep fep sep wep bep
life needs food, food needs light	fep qep gep qep gep hep

21. What is the code for the word ‘food’?
(A) gep (B) fep
(C) qep (D) pep

22. What is the code for the word ‘sun’?
(A) pep (B) nep
(C) mep (D) wep

23. Which word is coded as ‘lep’?
(A) gave (B) energy
(C) to (D) Cannot be determined

24. what is the code for 'earth sun and moon'?
(A) mep kep sep pep (B) mep tep nep sep
(C) kep qep sep mep (D) sep pep rep tep

25. What can be the meaning of 'fep gep zep sep hep'?
(A) life needs energy and light
(B) sun gave light and energy
(C) human needs sun and moon
(D) plants need sun and moon

Directions for questions 26 to 30: These questions are based on the following data.

A, *B*, *C*, *D*, *E*, *F* and *G* are the seven members in a family. Among them, there are two couples and each couple has exactly two children. *B*, who is married, has no siblings and he is not married to *E*, a female, who is also married. *D* is the father of *G*. *F*, the youngest in the family, has a paternal uncle. *A* is unmarried while *F* and *C* are of the same gender. *A* and *G* are of different gender.

26. How is *F* related to *A*?
(A) Daughter (B) Niece
(C) Nephew (D) Son

27. How is *C* related to *E*?
(A) Mother (B) Daughter
(C) Daughter-in-law (D) Mother-in-law

28. How is *A* related to *E*?
(A) Brother-in-law (B) Husband
(C) Brother (D) Father-in-law

29. Which among the following is the complete group of females in the family?
(A) *E*, *F* and *G* (B) *A*, *C*, *E* and *F*
(C) *C*, *E*, *B* and *F* (D) *C*, *E*, *F* and *G*

30. How is *C* related to *G*?
(A) Mother (B) Father
(C) Uncle (D) Grandmother

Directions for questions 31 to 35: These questions are based on the following data.

Five artists - a violinist, a pianist, a singer, a dancer and an actress-have to present their work one after the other, not necessarily in that order. The five artists are Anu, Gowri, Radhika, Sudha and Mythili. Also
(i) Mythili presents her work after the singer - not necessarily immediately.
(ii) the dancer presents her work immediately after Radhika.
(iii) Gowri, the violinist, plays second.
(iv) Radhika's item is not immediately next to Gowri's.
(v) Anu is not an actress.
(vi) Radhika is the singer.

31. Who is the dancer?
(A) Mythili (B) Sudha
(C) Anu (D) Radhika

32. If the actress plays first, who plays third?
(A) Anu (B) Mythili
(C) The dancer (D) Radhika

33. Who is the actress?
(A) Anu (B) Sudha
(C) Mythili (D) Radhika

34. The order in which the artists present their programmes is
(A) pianist, violinist, actress, singer, dancer.
(B) actress, violinist, pianist, dancer, singer.
(C) actress, violinist, dancer, pianist, singer.
(D) Cannot be determined.

35. If the actress plays third, when does Sudha play?
(A) Immediately after Radhika.
(B) Immediately before the singer.
(C) After the dancer.
(D) After Gowri but not immediately.

Directions for questions 36 to 39: These questions are based on the following data.

A man goes to work in his car on all days except Sundays. There are 4 different parking spaces near his office. Out of these, the cellar and the ground floor are the closest to his office while the garage and the parking lot are the farthest. Whenever he comes to office, he parks his car in one of the four parking spaces. It is known that
(i) the parking lot is open on all days of the week but he can afford it for only 2 days a week.
(ii) the Garage is open on Mondays, Tuesdays and Thursdays but he can use it only once a week.
(iii) he can use the cellar for 2 days of the week but he cannot use it on Tuesdays, Thursdays and Saturdays.
(iv) he can use the ground floor for one day of the week, but not on Mondays, Wednesdays, and Fridays.
(v) as he is always late on Mondays, he likes to park his car close to his office.

36. If he parks his car on the ground floor on Tuesday and in the parking lot on Wednesday, then where should he park it on Friday?
(A) Cellar (B) Parking lot
(C) Ground floor (D) Garage

37. If he uses the garage on Tuesday and the parking lot on Thursday, where does he park his car on Wednesday?
(A) Cellar (B) Parking lot
(C) Ground floor (D) Either (A) or (B)

38. If he uses the garage on Tuesday and parking lot on Saturday, then which is the place he uses on Thursday?
(A) Ground floor or Parking lot
(B) Cellar
(C) Ground floor
(D) Parking lot

39. If he uses the parking lot on Tuesday, then what must he use on Thursday?
(A) Parking lot
(B) Garage or Ground floor
(C) Ground floor
(D) Garage

Directions for questions 40 to 42: These questions are based on the following information.

Eight persons – Anand, Brijesh, Chandak, Dweepesh, Sayan, Jagat Rupak and Palak – are sitting around a square table such that two persons are sitting along each side. The following information is known about them.

(i) Jagat, who is sitting to the immediate right of Rupak, is sitting opposite Chandak who is sitting to the immediate right of Brijesh.

(ii) Sayan is sitting opposite Dweepesh, who sits along the same side as Brijesh.

(iii) Palak is not sitting along the same side as Sayan.

40. Who is sitting along the same side as Chandak?
(A) Anand (B) Palak
(C) Sayan (D) Rupak

41. Who is sitting opposite Rupak?
(A) Palak (B) Anand
(C) Brijesh (D) Data inadequate

42. Who is sitting to the immediate right of Sayan?
(A) Anand (B) Rupak
(C) Chandak (D) Data inadequate

Directions for questions 43 to 45: These questions are based on the following information.

Three people are to be selected from a group of six people – M, N, P, Q, R and S under the following constraints.

(i) If M is not selected, then N is selected.

(ii) If P is not selected, then Q is selected.

(iii) If R is not selected, then S is selected.

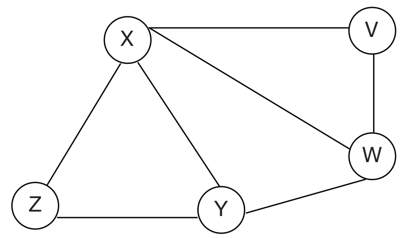
43. In how many ways can the team be selected?
(A) 2 (B) 4
(C) 8 (D) None of these

44. Who must be there in the team?
(A) N (B) Q
(C) S (D) None of these

45. Which of the following is a possible team?
(A) P, R, S
(B) P, Q, S
(C) M, Q, S
(D) More than one of the above

Directions for questions 46 and 47: Answer the questions on the basis of the following information.

Shown below is the layout of the major cities of a state and the rail tracks, connecting those cities.



Five trains – T_1, T_2, T_3, T_4 and T_5 run only on two days (Saturday and Sunday), along the following routes, between these cities.

$T_1 : Y – X – V$
$T_2 : Z – Y – X – V$
$T_3 : Z – Y – W – V$
$T_4 : Z – X – W – V$
$T_5 : Z – X – W$

Route $Y – W$ cannot be used on Sunday. On any day, no two trains are scheduled to run on the same track connecting two adjacent cities.

Each train should run exactly once in these two days.

46. T_4 can run
(A) only on Saturday
(B) only on Sunday
(C) on either day
(D) only if $W – Y$ route is used on Sunday.

47. Which of the following is NOT true?
(A) T_2 and T_4 can be scheduled to run on the same day.
(B) T_5 cannot be scheduled to run on Sunday.
(C) T_3 can be scheduled to run on Saturday.
(D) T_4 and T_1 can be scheduled to run on the same day.

Directions for questions 48 to 50: These questions are based on the data given below.

Six persons – A, B, C, D, E and F – stand in a row. A is to the left of B. C is to the right of D. E and F have two persons standing between them and neither of these two persons is C or A.

48. What is the total number of possible arrangements?
(A) 2 (B) 4
(C) 6 (D) 5

49. Who among the following stand at the extreme ends of the row?
(A) E and F (B) E and C
(C) A and C (D) F and A

50. If A sits to the immediate left of E, then who sits to the immediate right of B?
(A) D
(B) F
(C) C
(D) Cannot be determined

ANSWER KEYS

1. B	**2.** C	**3.** D	**4.** C	**5.** D	**6.** D	**7.** B	**8.** D	**9.** A	**10.** B
11. B	**12.** D	**13.** A	**14.** B	**15.** D	**16.** C	**17.** A	**18.** C	**19.** C	**20.** B
21. C	**22.** A	**23.** D	**24.** A	**25.** A	**26.** B	**27.** D	**28.** A	**29.** D	**30.** D
31. A	**32.** A	**33.** B	**34.** D	**35.** B	**36.** A	**37.** D	**38.** C	**39.** D	**40.** B
41. A	**42.** B	**43.** C	**44.** D	**45.** C	**46.** B	**47.** D	**48.** B	**49.** C	**50.** D

HINTS AND EXPLANATIONS

1. The given series can be expressed as follows.
$2^2, 3^3, 5^2, 7^3, 11^2$ where 2, 3, 5, 7, 11 are prime numbers.
The next in the series is $13^3 = 2197$ Choice (B)

2. The given series can be expressed as follows.
$(11 \times 2) + 3 = 25$; $(25\ 3) + 2 = 77$; $(77 \times 2) + 3 = 157$
$(157 \times 3) + 2, = 473$; $(473\ 2) + 3 = 949$ Choice (C)

3. The given series can be expressed as follows.
$3^2 + 3, 5^2 + 5, 7^2 + 7, 11^2 + 11, 13^2 + 13$ with 3, 5, 7, 11, 13 being prime numbers. The next number in the series is $17^2 + 17 = 289 + 17 = 306$ Choice (D)

4. The given series is the series of alternate prime numbers. The next in the series is 101. Choice (C)

5. The given series can be expressed as
$6 \times 3 + 1 = 19$; $19 \times 3 + 1 = 58$; $58 \times 3 + 1 = 175$
$175 \times 3 + 1 = 526$; $526 \times 3 + 1 = 1579$ Choice (D)

6. $24 : (24)^2 : : 32 : \underline{(32)^2}$
Square of the first number is the second number.
$(32)^2 = 1024$. Choice (D)

7. 4 : 27 : : 25 : ______
$(2)^2 : (3)^3 : : (5)^2 : \underline{(6)^3}$
This is of the form $(n)^2 : (n + 1)^3$.
$(6)^3 = 216$ is the next number. Choice (B)

8. BILK : DLPP : : HMTO : ______

B	I	*L*	*K*	
+2	+3	+4	+5	
D	*L*	*P*	*P*	
Similarly, H		*M*	*T*	*O*
+2	+3	+4	+5	
J	*P*	*X*	*T*	

Hence, JPXT is the next term. Choice (D)

9. Cricket is a type of game.
Similarly, kangaroo is type of animal. Choice (A)

10. Bus is driven by a driver.
Similarly, jockey rides a horse. Choice (B)

11. All the given numbers except 21 are prime numbers where as 21 is a composite number. Choice (B)

12. All the given numbers except 7532 are odd numbers whereas 7532 is an even number. Choice (D)

13. All the given colours except brown are the colours in (rainbow) VIBGYOR. Choice (A)

14. All the given months except May have 30 days where as in May there are 31 days. Choice (B)

15. All except Cabbage, grow under the soil. Choice (D)

16.

Word:	C	E	N	T	U	R	Y
	↑	↑	↑	↑	↑	↑	↑
Logic:	–2	+2	–2	+2	–2	+2	–2
	↓	↓	↓	↓	↓	↓	↓
Code:	A	G	L	V	S	T	W

Similarly,

Word:	S	A	C	H	I	N
	↑	↑	↑	↑	↑	↑
Logic:	–2	+2	–2	+2	–2	+2
	↓	↓	↓	↓	↓	↓
Code:	Q	C	A	J	G	P

∴ QCAJGP is the code for ‘SACHIN’. Choice (C)

17.

Word:	A	M	B	I	T	I	O	N
	↑	↑	↑	↑	↑	↑	↑	↑
Logic:	+6	+6	+6	+6	+6	+6	+6	+6
	↓	↓	↓	↓	↓	↓	↓	↓
Code:	G	S	H	O	Z	O	U	T

Similarly,

Code:	V	X	O	T	Z	U	A	Z
	↑	↑	↑	↑	↑	↑	↑	↑
Logic:	–6	–6	–6	–6	–6	–6	–6	–6
	↓	↓	↓	↓	↓	↓	↓	↓
Word:	P	R	I	N	T	O	U	T

‘PRINTOUT’ is coded as ‘VXOTZUAZ’ Choice (A)

18. The code for the letter whose place value is ‘n’ is the letter whose place value is $(27 - n)$.
∴ MAHENDAR is coded as NZSVMWZI.
Choice (C)

19. We sit on a chair and chair is called desk. Choice (C)

20. We use fans when we want air and spectacles means fan. Choice (B)

Solutions for questions 21 to 25:

The given statements and their codes are as follows.
(1) earth gets heat from sun – pep tep nep mep wep
(2) moon gets light from sun – hep kep tep pep nep
(3) sun gave energy to plants – bep pep dep zep lep

(4) human gets food from plants – qep tep nep dep rep
(5) heat and light gave life – hep fep sep wep bep
(6) life needs food, food needs light – fep qep gep qep gep hep

From (6), the words ‘food’ and ‘needs’ are repeated and the codes ‘qep’ and ‘gep’ are repeated. And now from (4) and (6) as only the word food is repeated the code for ‘food’ is ‘qep’ and hence the code for ‘needs’ is ‘gep’.

From (2) and (6) the word ‘light’ and the code ‘hep’ are common.

Hence, the code for ‘light’ is ‘hep’. The code for the remaining word in (6), i.e., ‘life’ is ‘fep’.

The words ‘gets’ and ‘from’ are common for (1), (2) and (4).

Similarly the codes ’tep’ and ‘nep’ are common. But the codes for ‘gets’ and ‘from’ cannot be individually obtained. Except the word ‘and’ and the code ‘sep’ in (5), all other words and codes are used in at least one of the other sentences. Hence, the code for ‘and’ is ‘sep’.

By using comparison and elimination procedures we can find the codes for other words.

Word	earth	sun	heat	gets/ from	moon	light	gave	energy/ to	plants	human	and	life	needs	food
Code	mep	pep	wep	nep/tep	kep	hep	bep	zep/lep	dep	rep	sep	fep	gep	qep

21. The code for ‘food’ is ‘qep’. Choice (C)

22. The code for ‘sun’ is ‘pep’. Choice (A)

23. Either ‘energy’ or ‘to’ is coded as ‘lep’. Choice (D)

24. The code for ‘earth sun and moon’ is ‘mep kep sep pep’. Choice (A)

25. The meaning of ‘fep gep zep sep hep’ can be ‘life needs energy and light’ . Choice (A)

Solutions for questions 26 to 30:

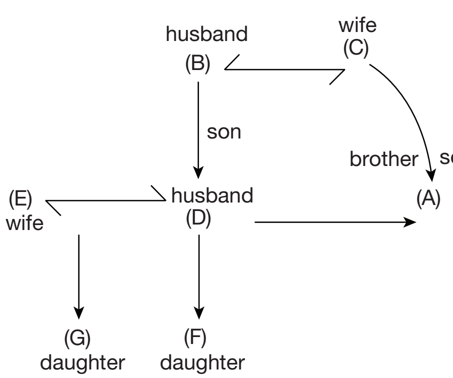
It is given that there are seven members in the family. *A*, *B*, *C*, *D*, *E*, *F* and *G*.

There are two couples in the family and each couple has exactly two children.

F, the youngest in the family has a paternal uncle which implies that *F*’s father has a brother. *B*, who is a male has no siblings and he is married. Hence, *B* will come in the first generation and he has two children and a spouse.

E is married and is a female. Hence, *E* is the mother of *F* and she is married to *D*. *G* is the child of *D* and *A* is the brother of *D*. Hence, *C* is the wife of *B*.

F and *C* are of same gender. Hence, *F* is female. *A* and *G* are of different gender. Since *A* is male, *G* is female. The given information can be represented in the diagram as follows:



26. *F* is *A*’s brother’s daughter. Hence, *F* is the niece of *A*. Choice (B)

27. *C* is *E*’s husband’s mother. Hence, *C* is the mother-in-law of *E*. Choice (D)

28. *A* is *E*’s husband’s brother. Hence, *A* is the brother-in-law of *E*. Choice (A)

29. *C*, *E*, *F* and *G* are the females. Choice (D)

30. *C* is the grandmother of *G*. Choice (D)

Solutions for questions 31 to 35:

From the given information, we have

Gowri, the violinist, plays second. Radhika is a singer who does not come immediately after Gowri, Radhika cannot be 3rd since the dancer presents her work immediately after Radhika. Radhika cannot be 1st or 5th so, Mythili comes in the 5th place. Radhika has to be 4th and dancer 5th.

So, Anu is a Pianist and Sudha is an actress and they come in the first and the third places – not necessarily in that order. Thus we have

Order	Artist	Profession
1		
2	Gowri	Violinist
3		
4	Radhika	Singer
5	Mythili	Dancer

31. Mythili is the dancer. Choice (A)

32. If actress plays first, then Anu plays third. Choice (A)

33. Sudha is the actress. Choice (B)

34. It is not clear whether the actress or the pianist plays first. Hence, cannot be determined. Choice (D)

35. If the actress (Sudha) plays third, then Sudha plays immediately before the singer.(Radhika) Choice (B)

Solutions for questions 36 to 39:

Let us tabulate the days of the week when different slots are available.

	Cellar	Ground Floor	Garage	Parking lot
Monday		X	✓	
Tuesday	X		✓	
Wednesday		X		
Thursday	X		✓	
Friday		X		
Saturday	X			
Can use for	2 days of the week	Only one day	Only one day	2 days of the week

In addition, on Mondays he should park the car closest to his office, i.e., cellar or ground floor. But since ground floor cannot be used on Mondays, only the cellar can be used on Mondays. Hence, garage cannot be used on Mondays. Now, we can answer the questions.

36. If ground floor is used on Tuesday & parking lot on Wednesday; then we have

	Cellar	Ground Floor	Garage	Parking lot
Monday	✓	X	X	X
Tuesday	X	✓	X	
Wednesday	X	X	X	✓
Thursday	X	X	✓	
Friday		X	X	
Saturday	X	X	X	
Total	2	1	1	2

As can be seen, he has to park his car in the cellar on Friday. Otherwise he won't be able to use the cellar 2 times a week. Choice (A)

37. On Wednesday, he can park his car in the cellar or in the parking lot. Choice (D)

	Mon	Tue	Wed	Thur	Fri	Sat
Cellar	✓	X		X		X
Ground Floor	X	X	X	X	X	✓
Garage	X	✓	X	X	X	X
Parking Lot	X	X		✓		

38

	Mon	Tue	Wed	Thur	Fri	Sat	Total
Cellar	✓	X		X		X	2
Ground Floor	X	X	X	✓	X	X	1
Garage	X	✓	X	X	X	X	1
Parking Lot	X	X				✓	2

As can be seen, on Thursday only ground floor can be used (because, if he uses the parking lot on Thursday, then no day is available for the ground floor).

Choice (C)

39.

	Mon	Tue	Wed	Thur	Fri	Sat
Cellar	✓	X				
Ground Floor	X	X	X		X	
Garage	X	X	X		X	X
Parking Lot	X	✓				

Now, garage has only Thursday and no other day left. Hence, garage should be used on Thursday.

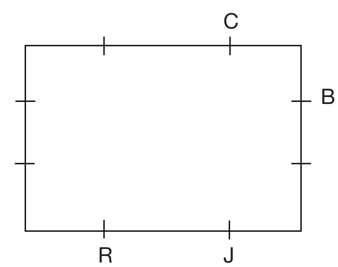
Choice (D)

Solutions for questions 40 to 42:

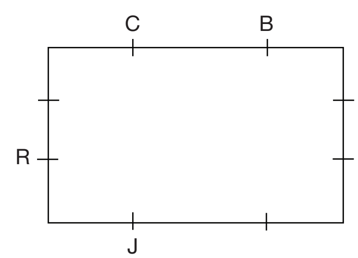
Let us represent the persons by the first letters of each name.

From (i), we get the following possibilities

Case (*a*),

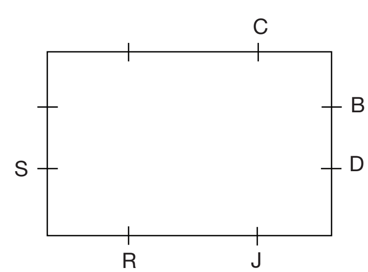


Case (*b*)

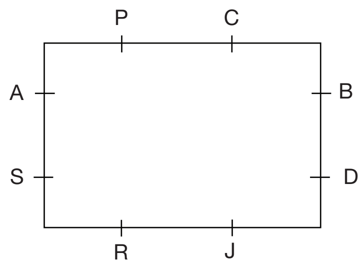


From (ii) as Brijesh and Dweepesh are sitting along the same side, case (*b*) is not possible.

From (ii) we get



From (iii) the possibility is as follows.



40. Palak is sitting along the same side as Chandak. Choice (B)

41. Palak is sitting opposite Rupak. Choice (A)

42. Rupak is sitting to the immediate right of Sayan. Choice (B)

Solutions for questions 43 to 45:

From (i), M and N can be selected as follows.
→ Only M is selected.
→ Only N is selected.
→ Both M and N are selected.
This implies, at least one between M and N must be selected.
Similarly, from (ii) and (iii)
At least one among P and Q must be selected.
At least one among R and S must be selected.
As only 3 persons are to be selected, both M and N, both P and Q and both R and S cannot be selected.
∴ Exactly one among M and N, exactly one among P and Q and exactly one among R and S must be selected.
⇒ $M / N, P / Q, R / S$

43. The number of possible ways to select the team.
$= 2 \times 2 \times 2 = 8$ Choice (C)

44. There is no such person who must always be there in the team. Choice (D)

45. M, Q, S is a possible team. Choice (C)

Solutions for questions 46 and 47:

Given that the route $Y – W$ cannot be used on Sunday. Hence T_3 can be scheduled to run on Saturday. As it is given that, on any day, no two trains are scheduled to run on the same track connecting two adjacent cities, $Z – Y$ and $W – V$ should not run on Saturday ($\because T_3$ is covering the route).
As T_2 is covering $Y – X – V$ on Sunday, T_1 has to be scheduled on Saturday. Similarly, T_5 is to scheduled on Saturday.
Finally, T_1, T_5, T_3 are to be scheduled on Saturdays and T_2 and T_4 are to be scheduled on Sundays.

46. T_4 should on run Sunday. Choice (B)

47. From the choices,
T_4 and T_1 can be scheduled on two different days. Choice (D)

Solutions for question 48 to 50:

The data is as given below:
(i) Six persons – A, B, C, D, E and F stand in a row.
Left of
(ii) A B
Right
(iii) D C
(iv) $\frac{E}{F}$ —— $\frac{F}{E}$

x C/A (neither C nor A)
Let us make all the possible arrangements as per the above data.

–	E/F	–	–	F/E	–
1	2	3	4	5	6

Positions 2 and 5 would be occupied by E or F. A cannot be at 3 and 4 (condition (iv)) and also A cannot be at 6 (condition (ii)). Hence, A must be at position 1. Similarly, C must be at position 6. Hence, we will get the following arrangement:

$\underline{A}$ $\underline{E/F}$ $\underline{B/D}$ $\underline{D/B}$ $\underline{F/E}$ $\underline{C}$

Therefore, the total number of arrangements are 4.

48. Total possible arrangements are four. Choice (B)

49. A and C stand at extreme ends. Choice (C)

50. The arrangement will be as shown below:
$\underline{A}$ $\underline{E}$ $\underline{B/D}$ $\underline{D/B}$ $\underline{F}$ $\underline{C}$
Hence, the person sitting to the immediate right of B cannot be determined. Choice (D)

Logical Ability Test 2

Number of Questions: 35 **Time: 35 min**

Directions for questions 1 to 4: These questions are based on the following data.

Out of a group of 315 students who went to Mumbai, 125 visited Essel World, 140 visited Lumbini Garden and 160 visited Film Nagar. Twenty Five of them visited all the three places while 200 visited exactly one of the three places. The number of students, who visited exactly 2 out of the three places, is five times as many as those who have not visited any of the places.

1. How many did not visit any of the three places?
(A) 75 (B) 25
(C) 125 (D) 15

2. How many students visited not more than one place?
(A) 200 (B) 180
(C) 250 (D) 215

3. If the number of students who visited at least one of the two places, Lumbini Gardens and Film Nagar is 255, then how many students visited only Essel World?
(A) 45 (B) 25
(C) 125 (D) 75

4. If the number of students who visited at least one of the two places, Lumbini Gardens and Film Nagar is 255, then how many students visited only one of the two places, Lumbini Gardens and Film Nagar (and not any of the other two places)?
(A) 215 (B) 125
(C) 155 (D) 175

Directions for questions 5 to 9: These questions are based on the following data.

In a colony, a survey was conducted regarding the ownership of three different types of vehicles – car, scooter and bicycle.

-- The number of residents owning all three vehicles is the same as those owning none.
-- The number of residents owning any two out of the three vehicles is the same as those owning any other two which in turn is the same as those owning none of the three.
-- The number of residents owning scooters alone is the same as those owning cars alone and each in turn is twice those owning bicycles alone.
-- Half the number of residents who own a bicycle also own at least one of the other two vehicles.

5. If the number of residents who own only bicycles is 150, then what is the total number of residents in the colony?
(A) 500 (B) 1000
(C) 750 (D) 1250

6. What percentage of the residents, who own a car, also own at least another vehicle?
(A) $66^{2}/_{3}$% (B) $13^{2}/_{3}$%
(C) $28^{4}/_{7}$% (D) $33^{1}/_{3}$%

7. If 15 residents do not own any of the three vehicles, then how many residents are there in the colony?
(A) 100 (B) 200
(C) 300 (D) 400

8. What percentage of the colony residents own exactly one type of vehicle?
(A) 15% (B) 25%
(C) 55% (D) 75%

9. What percentage of the residents own a scooter or a car but not a bicycle?
(A) 65% (B) 55%
(C) 75% (D) 45%

Directions for questions 10 to 12: These questions are based on the following data.

There are three trade unions – Viram, Vishram and Be-kam – and three thousand six hundred workers in a company. Becoming a member of a trade union is optional. A worker can be a member of more than one of the three trade unions also.

There are 500 workers who are members of at least two trade unions while Vishram has 1400 members. There are 100 workers who are members of only Viram and Be-kam, whereas 200 Vishram members also are Be-kam members; 550 workers are members of only Be-kam, whereas 20% of Viram members are members of exactly one more union. An eighth of all the workers in the company are members of exactly two unions.

10. How many workers are members of only Viram or only Be-kam?
(A) 3200 (B) 2700
(C) 1400 (D) 1700

11. If 10 workers give up their Be-kam membership and take up Vishram membership, then how many workers will now have membership of all the three unions?
(A) 40 (B) 50
(C) 60 (D) 45

12. How many workers are members of Vishram but not members of Be-kam?
(A) 400 (B) 800
(C) 1200 (D) 1600

Directions for questions 13 to 15: These questions are based on the following data.

In a school, 60% of the students passed in English, and 25% of the students who passed in English passed in the foreign language also, whereas $66^{2}/_{3}$% of the students who passed

in the foreign language failed in English. Twenty students failed in both English and the foreign language.

13. What percent of the students passed in exactly one of the two subjects – English and the foreign language?
(A) 15% (B) 65%
(C) 45% (D) 75%

14. The students who failed in exactly one subject are allowed to take a re-exam and it was found that the number of students who passed in both the subjects increased by 20%. What is the least value for the percentage of students in the school who pass only in English?
(A) 42% (B) 46%
(C) 34% (D) 28%

15. All the students, who failed in one or more subjects, are given grace marks and it was found that the number of students passing in exactly one subject went up by 4 and the number of students who failed in both the subjects dropped by 40%. What percent of the school now pass in both subjects?
(A) 40% (B) 15%
(C) 12% (D) 17%

Directions for questions 16 to 35: Select the correct alternative from the given choices.

16. The angle between the two hands of a clock at 9:00 a.m. is 90°. What will the angle between them be one minute later?
(A) 84.5° (B) 95.5°
(C) 101° (D) 79°

17. The angle between the two hands of a clock at 5:00 p.m. is 150°, what will the angle between them be one minute later?
(A) 144.5° (B) 155.5°
(C) 161° (D) 139°

18. What is the angle between the two hands of a clock at 7:28 p.m.?
(A) 56° (B) 58°
(C) 60° (D) 63°

19. At what time between 6 and 7 O' clock will the angle between the two hands of a clock be 50°?
(A) 6 hr.$41^9/_{11}$ min (B) 6 hr. $23^7/_{11}$ min
(C) 6 hr.$^2/_{11}$ min (D) Either (A) or (B)

20. At which of the following times between 8 and 9 O' clock, will the angle between the hands of the clock be 120°?
(A) 8 hr.$21^9/_{11}$ min (B) 8 hr. $16^9/_{11}$ min
(C) 8 hr. $34^6/_{11}$ min (D) 8 hr. $29^3/_{11}$ min

21. A watch, which gains uniformly was observed to be 10 minutes slow at 12 noon and 5 minutes fast at 6:00 p.m. on the same day. When did the watch show the correct time?
(A) 2:00 p.m. (B) 3:00 p.m.
(C) 4:00 p.m. (D) 5:00 p.m.

22. A watch, which loses uniformly was observed to be 5 minutes fast at 5:00 p.m. and 4 minutes slow at 8:00 p.m. on the same day. When did the watch show the correct time?
(A) 6:20 p.m. (B) 6:40 p.m.
(C) 7:00 p.m. (D) 6:30 p.m.

23. Two clocks show the correct time at 3:00 p.m. One clock gains 4 minutes in an hour, while the other loses 4 minutes. At 6:00 p.m. on the same day, by how much time will the two clocks differ?
(A) 8 minutes (B) 16 minutes
(C) 12 minutes (D) 24 minutes

24. The minutes hand of a clock overtakes the hours hand after every 60 minutes of correct time. How much time does the clock gain or lose in eleven hours of normal time?
(A) Gains 11 minutes (B) Loses 11 minutes
(C) Gains 60 minutes (D) Loses 60 minutes

25. If the time in a clock is 8 hours 20 minutes, then what time does its mirror image show?
(A) 3 hours 20 minutes (B) 3 hours 50 minutes
(C) 3 hours 40 minutes (D) 4 hours 40 minutes

26. If 22nd April, 1982 was a Thursday, then what day of the week was 3rd November, 1982?
(A) Monday (B) Wednesday
(C) Friday (D) Sunday

27. If 30th June, 1989 was a Friday, then what day of the week was 17th September, 1993?
(A) Monday (B) Wednesday
(C) Friday (D) Sunday

28. If 10th April, 1963 was a Wednesday, then what day of the week was 23rd August, 1959?
(A) Sunday (B) Monday
(C) Friday (D) Tuesday

29. If today is Sunday, then what day of the week will be the 426th day from today?
(A) Saturday (B) Friday
(C) Tuesday (D) Wednesday

30. If today is Wednesday, what day will it be, 1 year and 10 days from today?
(A) Sunday
(B) Friday
(C) Monday
(D) Cannot be determined

31. In a year, if two successive months start with the same day of the week, then the year is
(A) a century year.
(B) a leap year.
(C) a non-leap year.
(D) only a non-leap century year.

32. If a year has 53 Sundays, how many Saturdays will be there in that year?

(A) 52 (B) 53
(C) 54 (D) Cannot be determined

33. Three days ago I met my friend and asked him to lend me his maths book. He promised that he will lend it on the eighth day from that day. If today is Thursday, on which day will he lend me the book?
(A) Friday (B) Tuesday
(C) Monday (D) Sunday

34. There are three more days to go for my cousin to arrive. Eight days ago when I talked to him he said he would write his last exam three days later and that he would leave three days after his last exam. If his last exam is on a Saturday, on which day of the week will my cousin arrive?
(A) Saturday (B) Friday
(C) Tuesday (D) Sunday

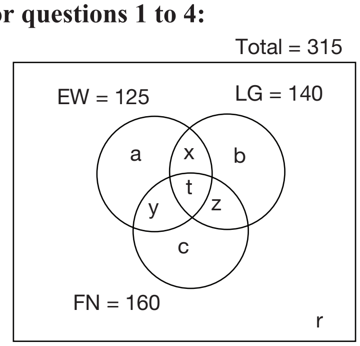
35. If the second Sunday of a month falls on the 8th, what is the date of the fourth day after the third Saturday of that month?
(A) 2 (B) 15
(C) 25 (D) 21

Answer Keys

1. D	**2.** D	**3.** A	**4.** C	**5.** B	**6.** D	**7.** C	**8.** D	**9.** A	**10.** D
11. B	**12.** C	**13.** D	**14.** A	**15.** D	**16.** B	**17.** A	**18.** A	**19.** D	**20.** A
21. C	**22.** B	**23.** D	**24.** C	**25.** C	**26.** B	**27.** C	**28.** A	**29.** A	**30.** D
31. C	**32.** D	**33.** B	**34.** D	**35.** C					

Hints and Explanations

Solutions for questions 1 to 4:



EW - Essel World
LG - Lumbini Gardens
FN - Film Nagar
25 visited all three $\Rightarrow t = 25$
125 visited *EW* $\Rightarrow a + x + y + t = 125$
$\Rightarrow \quad a + x + y = 100$ …. (1)
140 visited *LG* $\Rightarrow b + x + t + z = 140$
$\Rightarrow \quad b + x + z = 115$ …. (2)
160 visited *FN* $\Rightarrow c + y + z + t = 160$
$\Rightarrow \quad c + y + z = 135$ …. (3)
Number of students who visited exactly 2 places = 5 times those who did not visit any.
$\Rightarrow \quad x + y + z = 5r$
200 students visited exactly one place = $a + b + c = 200$.

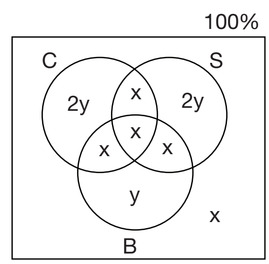
1. Total no of students = 315
$\Rightarrow \quad a + b + c + x + y + z + t + r = 315$
$\Rightarrow \quad 200 + 5r + 25 + r = 315$
$6r = 90 \Rightarrow r = 15$
$\therefore$ 15 did not visit any of the three places. Choice (D)

2. Number of students who did not visit more than one place = Number of students who visited exactly one place + those who did not visit any = $a + b + c + r$ = 200 + 15 = 215. Choice (D)

3. Number of students who visited at least one of *LG* and *FN*
$= 255 \Rightarrow b + x + z + t + y + c = 255$ … (4)
We know $a + b + c + x + y + z + t$
$= 315 - r = 300$ …… (5)
(5) – (4) $\Rightarrow a = 45$ visited only *EW*. Choice (A)

4. Number of students who visited only one among *LG* and *FN* $= b + c = 200 - a = 200 - 45 = 155$.
Choice (C)

Solutions for questions 5 to 9:



The first three statements can be represented as shown in the diagram and hence, $5x + 5y = 100$
(because we have taken x & y as percentages)
or $x + y = 20$ …. (1)
From the fourth condition, we get $(y + 3x) = B$
i.e., $y = 3x$ …. (2)
From equations (1) and (2).
We get $x = 5\%$ and $y = 15\%$.

5. $y = 15\% = 150$.
Hence, total number % of residents
$= x = \dfrac{150 \times 100}{15} = 1000.$ Choice (B)

6. $\dfrac{3x}{3x + 2y} = \dfrac{15}{45} = \dfrac{1}{3} = 33^{1}/_{3}\%.$ Choice (D)

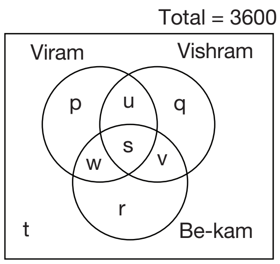
7. $x = 5\% = 15$. Hence total $= \dfrac{15}{0.05} = 300$. Choice (C)

8. $2y + 2y + y = 75\%$. Choice (D)

9. $2y + 2y + x = 65\%$. Choice (A)

Solutions for questions 10 to 12:

Represent in various segments as in the following diagram,



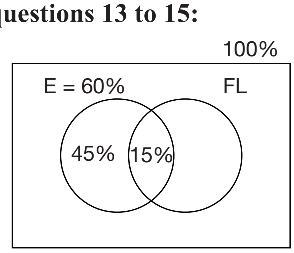
Members of at least two unions $= u + v + w + s = 500$ ….. (1)
Vishram members $= q + s + u + v = 1400$ ……… (2)
Only Viram and Be-kam $= w = 100$ ……. (3)
Vishram and Be-kam $= s + v = 200$ ….. (4)
Only Be-kam $= r = 550$ …… (5)
Members of Viram who are members of only one more union $= w + u = 20\%$ of $(p + u + s + w)$.. (6)
$u + v + w = {}^1/_8$ (Total workers) $= 450$. ….. (7)
From (1), (3) and (4), $u = 200$.
From equations (2),
$q = 1400 - u - (s + v)$
$= 1400 - 200 - 200 = 1000$
From (7), $v = 450 - 200 - 100 = 150$
From (4), $s = 200 - 150 = 50$
From (6), $p = 1150$
$t = 3600 - (p + q + r + s + u + v + w)$
$= 3600 - (1150 + 1000 + 550 + 50 + 200 + 150 + 100)$
$= 3600 - (3200) = 400$.
Now, we have all the figures and the questions can be answered.

10. $p + r = 1150 + 550 = 1700$. Choice (D)

11. Since 10 workers have given up their Be-kam membership and taken Vishram membership, it means these 10 workers were initially Be-kam members but not Vishram members, i.e., they must be a part of r or w. When they give up Be-kam and take up Vishram, they will move to q or u respectively. So, s does not undergo any change at all. Hence, 50 is the answer. Choice (B)

12. $q + u = 1000 + 200 = 1200$. Choice (C)

Solutions for questions 13 to 15:



-- 25% of 60%, i.e., 15% of the school passed in both English and Foreign Language.

-- Since $66^2/_3\%$ of the students who passed in Foreign Language failed in English, $33^1/_3\%$ of students who passed in Foreign Language passed in English also, i.e., ${}^1/_3$ Foreign Language = 15%

$\Rightarrow$ Foreign Language = 45%

So, we have only passed in (English) $= 60 - 15 = 45\%$,
Only Foreign Language pass $= 45 - 15 = 30\%$
Passed both in English & Foreign Language = 15%.
A total of 90% passed in at least one of the subjects. So, 10% failed in both.

Number of students in the school $= \dfrac{20}{0.10} = 200$.

13. $45\% + 30\% = 75\%$. Choice (D)

14. 20% of 15% = 3% increase in pass in both subjects. Hence, least value of pass only in English will come when all the new people who pass in both subjects are from the group which passed only in English. i.e., $45 - 3 = 42\%$. Choice (A)

15. 40% of 20 students = 8 students. Out of this, 4 students passed in exactly one subject. Hence, the remaining 4 students (which is 2% of the school strength) pass in both subjects. So, pass in both the subjects $= 15 + 2 = 17\%$. Choice (D)

Solutions for questions 16 to 35:

16. As the minute hand is 5.5°/minute faster than the hour hand and the minute hand is ahead of the hour hand, the angle after one minute, i.e., at 9:01 a.m., will be $90° + 5.5° = 95.5°$. Choice (B)

17. The angle between the hands will change by 5.5°/min. In this case, since the minute hand is behind the hour hand the angle will decrease by 5.5°. Hence, the angle at $5{:}01 = 150 - 5.5 = 144.5°$. Choice (A)

18. The angle $= \left|\dfrac{11}{2}(28) - 30(7)\right| = 56°$. Choice (A)

19. $\theta = \left|\dfrac{11}{2}(m) - 30(h)\right| \Rightarrow m = (30h \pm \theta)\left(\dfrac{2}{11}\right)$.

Here $h = 6$ and $\theta = 50°$

$\Rightarrow \quad m = (180 \pm 50°)\left(\dfrac{2}{11}\right) = \dfrac{460}{11}$ or $\dfrac{260}{11}$.

The two hands are 50° apart at $6\,\text{hr.}\ 41\dfrac{9}{11}\ \text{min}$ and at

$6\ hr.\ 23\dfrac{7}{11}\ \text{min}$. Choice (D)

20. $m = (30h \pm \theta)\left(\dfrac{2}{11}\right)$

Here $h = 8$ and $\theta = 120°$

$\Rightarrow \quad m = (240 \pm 120)\left(\dfrac{2}{11}\right) \Rightarrow 65\dfrac{5}{11}$ or $21\dfrac{9}{11}$

If $m = 65\frac{5}{11}$, then the time will be more than 9 O'clock.

$\therefore$ The time is 8 hr. $21\frac{9}{11}$ min Choice (A)

21. In 6 hours, the watch gained 15 minutes.
$\therefore$ In order to gain 10 minutes, (the initial difference) it takes 4 hours.
$\therefore$ At 4:00 p.m. it shows the correct time. Choice (C)

22. In 3 hours, the watch loses 9 minutes.
$\therefore$ In order to lose 5 minutes, (the initial difference) it takes 5 x 3/9 hours, i.e., 1 hour 40 minutes.
$\therefore$ At 6 : 40 p.m., it shows the correct time. Choice (B)

23. In one hour, the clocks will differ by (4 + 4) minutes i.e., 8 minutes.
$\therefore$ In three hours they will differ by 24 minutes. Choice (D)

24. As $60 < 65\frac{5}{11}$, the watch is gaining time.

The gain is $65\frac{5}{11} - 60 = 5\frac{5}{11}$ minutes per hour.

$\therefore$ In eleven hours, the watch will gain $11\left(\frac{60}{11}\right)$ = 60 minutes. Choice (C)

25. Actual time + mirror image time = 12 hours
$\therefore$ Mirror image time = 12 – 8 : 20 = 3 : 40 = 3 hours 40 minutes. Choice (C)

26. It is given that 22nd April, 1982 was a Thursday.
Number of days from 22nd April 1982 to 3rd November 1982. Month: April + May + June + July + August + September + October + November
In each month number of odd days :
1 + 3 + 2 + 3 + 3 + 2 + 3 + 3 = 6 odd days
6^{th} day after Thursday is Wednesday. Choice (B)

27. Number of odd days from 30th June, 1989 to 30th June, 1993 are five.
Number of days from 30th June, 1993 to 17th September, are
Month: July + August + September
Odd days : 3 + 3 + 3 = 9
Number of odd days = 9 + 5 =14 = 0 odd days
Hence, 17th September 1993 was a Friday. Choice (C)

28. Number of days from 10th April, 1963 to 23rd August, 1963
Month: April + May + June + July + August
Days: 20 + 31 + 30 + 31 + 23 = 135
Number of odd days in 135 days = 135/7 = 20 days
23rd August 1963 is 2 days to Wednesday i.e., Friday
Number of odd days form 23rd August 1963 to 23rd August 1959 are five odd days.
Hence, 23rd August 1959 is five days back to Friday is Sunday. Choice (A)

29. Number of odd days in 426 days
= 426/7 = 60 complete weeks + 6 odd days.
6th day after Sunday is a Saturday. Choice (A)

30. Whether the given year is a leap year or a non-leap year, is not given, hence the answer cannot be determined. Choice (D)

31. Two successive months can start with the same day of the week, only if the previous month does not have any odd days. This is possible only when February has 28 days. Hence, the year is a non-leap year. Choice (C)

32. A leap year has 52 weeks and two additional days, while a non-leap year has 52 weeks and one additional day. In any year each day of the week occurs at least 52 times.
A non-leap year starts and ends with the same day of the week. In a leap year the first two days of the year repeat. As it is not known whether the 53rd Sunday is the last day of the year or not, the number of Saturdays could be 52 or 53. Choice (D)

33. Today is a Thursday. Hence, the required day
= Thursday – 3 (ago) + 8 (from 3days ago)
= Thursday + 5 = Tuesday. Choice (B)

34. Eight days ago my cousin said that his last exam is three days later i.e. five days ago from today. He is going to arrive three days later from today. i.e. eight days after the last exam. As the last exam is on a Saturday, he is arriving on a Sunday. Choice (D)

35. Since, 8th is the second Sunday, 7th is the first Saturday. Hence, the third Saturday is on 21st and four days later it is 25th. Choice (C)

Chapter 1

Grammar

CHAPTER HIGHLIGHTS

- *Grammar*
- *Nouns, Pronouns, Articles*
- *Verbs, Auxiliaries, Modals*
- *Adjectives, Adverbs*
- *Prepositions, Conjunctions*
- *Active/Passive Voice, Direct/Indirect speech*
- *Verbal Phrases*
- *Sentence Completion*

Introduction

The Verbal Ability section of the book includes inputs followed by Practice Exercises. It comprises inputs in the following sections:

1. Grammar
2. Sentence Corrections
3. Vocabulary

Grammar

The questions under this category are based on English Grammar. These questions test the student's knowledge of grammar and ability to use English Language correctly. The grammar section includes

1. Nouns, Pronouns, Articles
2. Verbs, Auxiliaries, Modals
3. Adjectives, Adverbs
4. Prepositions, Conjunctions
5. Active/Passive Voice, Direct/Indirect speech
6. Verbal Phrases

Sentence Completion

This section includes sentence correction and sentence rephrasing questions. Questions in this section test a student's ability to comprehend a given context and indentify grammatical errors.

Section I: Grammar

Overview

A. Sentence

A group of words that makes complete sense is a sentence.

Examples:

1. Birds fly.
2. It is a good college.
3. The children of that school wear blue uniform.
4. This is the theory that throws light on superconductivity.

B. Parts of a sentence

A sentence can be divided into two parts:
(1) subject and (2) predicate
Here are some sentences divided into the two parts.

	A (subject)	B (predicate)
1.	Birds	fly.
2.	Mohan	is an engineer.
3.	He	teaches Engineering Graphics.
4.	This intelligent girl	has passed her examination in the first division.
5.	The President of India	has awarded him a gold medal.

NOTES

1. The words underlined in Part A are subject words and those underlined in Part B are verbs.
2. The subject is a noun or a pronoun with or without other words going with it.
3. The predicate is the verb with or without other words going with it (other words may be object, complement, adverb, adverb phrase, etc.).
4. The subject of the sentence usually comes first but occasionally the predicate precedes.

Example:

- Down went the Royal Francis.

Sample Exercise–1

Underline the subject parts in the following sentences.

1. The train arrived late.
2. He was the captain of his team.
3. There was a large crowd on the platform to receive the team.
4. Birds of the same feather flock together.
5. The Prime Minister with all his Cabinet members has submitted his resignation to the President.

Sample Exercise–2

Underline the predicate parts in the following sentences.

1. Aeroplanes fly over my house frequently.
2. They are shortly moving out of their house.
3. He is suffering from a fever.
4. The college is equipped with new furniture.
5. The Secretary and the members of the club have been discussing the issue since morning.

Now study the following sentences:

Group A

1. There is a beautiful garden in front of my house.
2. It has been raining heavily since morning.
3. There is a phone call for your friend.

NOTE

The words underlined in these sentences are not real subjects. They are said to be grammatical or dummy subjects.

Now study the following sentences:

Group B

1. Here come my friends and their parents.
2. There go the principal and the members of the teaching staff.

NOTE

In these sentences, you notice that the predicate part precedes the subject part. This is called S.P and P.P inversion. This construction is allowed in standard English. This is used for emphasis.

Parts of Speech: Identification

English words (according to their functions in a sentence) can be classified into parts of speech as follows:

1. Noun
2. Pronoun
3. Adjective
4. Verb
5. Adverb
6. Preposition
7. Conjunction
8. Interjection

Noun

A noun is a word used as the name of a person, place, animal, thing, etc.

Examples: Rajitha, Hyderabad, lion, table, etc.

Pronoun

A word used instead of a noun.

Examples: he, she, it, they, etc.

Adjective

A word that qualifies or describes a noun or a pronoun.

Examples:

1. Our team played a good game.
   adj. noun
2. Every sentence that he spoke was listened to with
   adj. noun
   great attention.
3. All these proposals are good.
   noun adj.

NOTE

Some words are used either as pronouns or as adjectives as in the following examples.

Pronoun	Adjective
a. These are good students.	1. These students are good.
b. Each of these books is worth reading.	2. Each book has some special features worth reading.
c. Some of the problems are difficult to solve	3. Some problems are difficult to solve.
d. Either of you can participate in the competition.	4. Either book is good for reference.

Verb

A verb expresses what the subject of a sentence is or does, or has or what is done to it.

1. Dr. Rao teaches English.
   verb
2. He has taught the subject for twenty-five years.
   verb
3. He is a capable teacher.
   verb
4. She has a car.
   verb

Adverb

A word that modifies an adjective or a verb or another adverb.

1. I am deeply grateful to you for your timely help.
   adv. adj.
2. This horse runs very fast.
   verb adv. adv.
3. He read the passage quickly.
   verb adv

Preposition

A preposition is a word used with a noun or a pronoun to show how the person or thing denoted by the noun or pronoun stands in relation to something else.

1. Work in the college begins at 10 a.m.
2. He wrote the document with a pen.
3. The Professor gave a lecture on superconductivity.
4. I washed my plate after I had eaten.

In the above sentences, the words underlined are called prepositions.

Conjunction (Connector)

A conjunction is a word that joins words, phrases or sentences.

1. Delhi and Kolkata are densely populated cities.
2. She must be either the President or the Secretary of YMCA.
3. The professor read the essay and was impressed by it.
4. He is not only intelligent but also industrious.

Interjection

A word that expresses a strong or sudden feeling such as surprise, joy, fear, sorrow, etc. It is not grammatically connected with the rest of the sentence. Usually, an exclamatory mark is put after it.

Examples: Ah!, Hurrah!, Well!, Dear!, Oh!

1. Alas! she is dead.
2. Hurrah! we won the match.

NOTE

As words are divided into different parts of speech, depending on the work they do in the sentences, it is not possible to say to which part of speech a word belongs unless we see what function it performs in the sentence.

Phrases and Clauses

Phrase

A phrase is a group of words without a finite verb. There are three types of phrases.

1. Noun phrase
   A fifty-year-old man is seen in the garden.
2. Adjective phrase
   This chain is made of gold.
3. Adverb phrase
   Please place the chemicals in the storeroom.

NOTE

A 'finite verb' is a word that denotes tense and a 'non-finite' is a verb that does not.

Examples:

1. He has written some books. (finite verb) (present perfect tense)
2. Speaking on the occasion, the Minister said prohibition would be lifted soon. (non-finite verb)

There are three types of non-finites in English: INFINITIVE, GERUND, PARTICIPLE.
(*Please note that there is a separate chapter on non-finites.)

Clause

A clause is a group of words that contains a 'finite verb'. Clauses are of two types:

1. Main clause
2. Subordinate clause

A main clause is also called a principal clause or an independent clause because it makes complete sense.

A subordinate clause, which can also be called a dependent clause, is divided into three types:

NOUN CLAUSE, ADJECTIVE CLAUSE, ADVERB CLAUSE

Now study the uses (functions) of these three types of clauses.

1. **THE NOUN CLAUSE** is used as

   (a) the subject of a verb.
   That he will be appointed to the job is certain.
   subject verb

   (b) the object of a verb.
   We knew that the plan was impractical.
   verb object

   (c) the complement of a verb.
   Our hope is that he will secure first rank.
   v. complement

   (d) the complement of an adjective.
   We were certain that he would pass.
   adj. complement

   I am hopeful that he will agree to our proposal.
   adj complement

NOTE

When the noun clause is an object or a complement, the conjunction that may be omitted.

Examples:
I believed he was right. (I believed that he was right)
verb object

I was sure he was right. (I was sure that he was right)
adj. compl.

   (e) in apposition to the noun before it.
   The suggestion, that the meeting should be adjourned, was unanimously accepted.
   N N.Cl (in apposition)

   (f) Wh - clauses as Noun Clauses
   Wh - clauses perform all the five functions that the 'that' clauses do. In addition to these, they can function as the objects of prepositions.

   Examples:
   Subject: What you say is true. Where he lives is a mystery.

   Object: I don't know what he would say to this proposal.

   Subject Complement: The question is who will bell the cat.

   In apposition: My question, who is responsible for the loss, has not been answered.

   Adjectival Complement: I wasn't sure whose fault it was.

   Object of a Preposition: We were not aware of what they were doing.

   (g) If/whether clauses as noun clauses:
   - I asked him if/whether he can speak French.
   - I don't know if/whether he has been appointed or not.
   - I don't care whether he attends the meeting or not.
   - Whether or not he comes doesn't worry my boss.

2. **ADJECTIVE CLAUSES:** (also called relative clauses). It does the work of an adjective. It qualifies a noun or a pronoun in the main clause.

   (a) The young man who was here a little while ago is the President of the union.
   antecedent adj. cl

   (b) I have finished reading the novel (that) you gave me.
   ant. adj. cl

   (c) This is the story (which) my friend wrote.
   ant. adj. cl

   Note: ( ) indicates optional use, i.e., may or may not be used.

NOTES

1. The words who, that, and which introducing the relative clauses are relative pronouns.
2. Who is used to refer to persons; that is used to refer to persons or things; and which is used to refer to things, animals, etc.
3. The noun or noun phrase, which a relative pronoun refers to, is called its antecedent (ant.)

Special Note 1: The object form 'whom' can also function as a relative pronoun introducing adjective clauses.

Example: This is the person whom you want to meet.

Special Note 2: When the adjective clause defines a place, time, or reason, we use where, when, and why in place of the relative adverbs.

Examples:
1. We visited the place where Mr. Gandhi was assassinated. (place)
2. At a time when prices are high, we must reduce our expenditure. (time)
3. Could you tell me the reason why he has not been appointed? (reason)

NOTE

The possessive form whose can also introduce a relative clause. 'Whose' can be used in speaking of persons, animals, and non-living things.

Example: The doctor whose car I bought wants to buy it back.

3. **ADVERBIAL CLAUSES:** Adverbial clauses are of different kinds. They modify the verbs to which they are connected in meaning, i.e. they do the work of an adverb.
   (a) Adverbial clauses of time:
   (The conjunctions used are : when, as, while, till/until, before, after, as soon as, whenever, since, as long as, etc.)

   Example: Some persons in the audience were asleep while the chief guest was speaking.

   (b) Adverbial clause of place:
   You may go wherever you like.

   (c) Adverbial clause of reason or cause :
   (as, because, since, now that)
   You can be punctual to school now that you have a school bus.

   (d) Adverbial clause of purpose:
   (so that, that, in order that)
   The police officer went in plain clothes so that no one might recognize him.

   (e) Adverbial clause of result or effect:
   (that, so that, that)
   It rained very heavily for two days so that the rivers and lakes were flooded.

   (f) Adverbial clause of condition:
   (if/unless)
   If he apologizes to me, I shall forgive him.

   (g) Adverbial clause of concession:
   (though, although, even though)
   Though he is intelligent, he cannot argue well.

   (h) Adverbial clause of manner:
   You may complete the work as you please.

Sample Exercise–3

In each of the following sentences, you will find a word/a group of words underlined. Write (P) or (C) in the bracket to denote phrase or clause respectively, as the case may be.

1. Mangoes are expensive this year. ( )
2. I found this book in the university library. ( )
3. This is the novel that Saul Bellow has written. ( )
4. Smoke from vehicle exhausts causes air pollution. ( )
5. He bought the house that belonged to his friend. ( )

Sample Exercise–4

Given below are a few sentences in which clauses are underlined. Identify the type of clause noun clause, adjective clause and adverb clause) in each sentence and write its name in the brackets provided against each sentence.

1. He reached the place when the sun had set. ( )
2. Because you have done this, you must explain ( )
3. If you can do it fast, please do so. ( )
4. The table that has a broken leg is from the staff room. ( )
5. He expects that he would get through the examination. ( )
6. He is very rich, yet he is unhappy. ( )
7. Although he is an industrious student, he failed in the examination. ( )
8. The children clapped as the clown entered the ring. ( )
9. People started running out as soon as the accident took place. ( )
10. Please answer the call for me in case I go out. ( )

Conditionals

There is a great variety of conditional sentences.

The two main divisions are detailed further:

Type A

Those with clauses that contain a condition that may or may not be fulfilled.

These are called clauses of open condition or factual condition.

Example: What shall we do if it rains?

In the above-mentioned example, the speaker does not declare that the condition will be realized or that it will not be realized.

He leaves the question open or unanswered.

The speaker here merely puts forward the possibility that it may rain.

More examples:

- Don't come unless I tell you to come.
- We shall go provided the weather is fine.

Type B

Those with clauses in which a theoretical condition is put forward.

These are clauses in which the condition is combined with improbability or unreality.

- He would come, if he had time.
- If you were a bird, you could fly.

Conditional clauses may be introduced by if (even if, if only), so long as, suppose or supposing (that), on condition (that), provided (that), and, for a negative condition, unless (which means the same as if ….not but is more emphatic). In case is used to introduce a contingency or possibility against which a precaution is needed.

- If you have enough money, why don't you buy a house?
- So long as you return the book faithfully, I will lend it to you with pleasure.
- You'd better take an umbrella with you in case it rains.

If–Conditionals

Type–I

Look at the following example.

If he comes to the library, I will give him these books.
pres. fut.
(It is possible that he will come)

NOTE

This is an open condition, i.e. this expresses a possible situation.

In the aforementioned example, the present tense is used in the if–clause (or conditional clause), and the future tense is used in the result clause.

Type–II

If she practised well, she could win the game.
(She doesn't practise well)

If she came to my house, I would give her your message.
(She doesn't come to my house)

NOTE

This is an unlikely condition. This is called unreal condition used for impossible or unreal situations.

The past tense is used in the 'if–clause' and would, could, or might + the simple past tense verb is used in the main clause.

Type–III

If she had come to my house, I would have given her your message. Would + have + past participle
past perf. (She didn't come to my house)

NOTE

This is a closed condition.

When the past perfect is used in the 'if–clause', would, could, or might + have + the past participle are used in the main clause.

We can summarize the usage of the above-discussed conditionals as follows:

If–clause		Main clause
If + present	....	future (result)
If + past	....	would + verb (result)
If + past perfect	....	would + have + past participle (result)

NOTES

(A) In general, avoid using would in the if–clause.

(B) If the if–conditionals express a

(i) doubt or supposition or

(ii) a wish or desire, the form of the verb to be used is 'were'.

Examples: If he were rich, he would go to the USA for higher studies.

If I were a bird, I would fly to England without a visa.

Examples of Errors in the Usage of Conditionals

The following examples will make clear some of the common mistakes committed in the usage of conditionals.

1. WRONG: If I will win the contest, I will buy a new house.

   RIGHT: If I win the contest, I will buy a new house.

2. WRONG: If I had been there, I would make a speech.

   RIGHT: If I were there, I would make a speech.

   If I had been there, I would have made a speech.

3. WRONG: If I would have a degree from that University, I would get a good job.

   RIGHT: If I had a degree from that University, I would get a good job.

Types of Sentences (Structural)

A sentence, for the purpose of analysis, can be examined from the point of view of its structure. Depending on the number of clauses it contains, a sentence can be called SIMPLE, COMPLEX, or COMPOUND.

Let us look at each of these types of sentences with examples.

1. Simple Sentence

A simple sentence is one that contains only one subject and one predicate.

(The subject: Person or thing about which something is said).

(The predicate: What is said about the subject).

In other words, a simple sentence contains only one main clause. It does not have subordinate clauses.

Examples:

1. My father is a senior manager in this organization.
   subject (s). The predicate (p)
2. Chandigarh is a planned city.
   s p
3. Microbiology is an interesting subject for me.
   s p

Note that there is only one finite verb in each of the above sentences. That means there is only one clause. One clause sentence is a simple sentence.

2. Compound Sentence

A compound sentence is one that contains two or more main or principal clauses. It may or may not have subordinate clauses.

In other words, a compound sentence consists of two or more independent sentences joined together by a coordinating conjunction.

Example: He is poor yet he is happy.

The above sentence consists of two parts.
(i) He is poor (ii) He is happy

These two are joined by the co-ordinating conjunction 'yet'.

In the above example, each part contains a subject and a predicate of its own, i.e. each part is called a clause.

Each clause makes good sense by itself and, hence, can stand independent of the other. Hence, each is called a principal clause or a main clause.

Study the following examples.

Examples:

1. You may either stay in the hostel or stay with your relatives.
2. The flowers were blooming, the birds were singing, and spring was in the air.

Analysis: In sentence 1, there are two main clauses.

In sentence 2, there are three main clauses.

So, each of them is a compound sentence.

Some more examples of compound sentences:

1. I went to his farmhouse several times, but I was unable to meet him.
2. He is very intelligent, yet he has failed in the examination.
3. Many doctors are attending on him, still he is not out of danger.
4. He is very tired, for he has been working since morning.
5. There is no quorum, so the meeting was adjourned.

NOTE

In the aforementioned examples, the underlined words are called coordinating conjunctions. Coordinating conjunctions join main clauses.

The following is a list of some coordinating conjunctions

and, as well as, also, too, both … and, not only … but also, now, well, either … or, neither … nor, otherwise, or else, still, yet, only, however, whereas, but, nevertheless, therefore, for, so then, so then.

3. Complex Sentence

A complex sentence is one that contains only one main clause and one or more subordinate clauses. Study the following examples.

Example 1: If you work hard, you will pass.

In the aforementioned example, the clause 'you will pass' makes good sense by itself and, hence, can stand by itself. This is called the principal or independent clause.

The clause 'If you work hard,' cannot stand independently, it depends on the clause, 'you will pass'. It is, therefore, called a dependent or a subordinate clause.

Example 2: As soon as the meeting began, a member said that he wanted to raise a point of order.

Analysis: This sentence has only one main clause and two subordinate clauses. Therefore, this is a complex sentence.

Subordinate clause 1: That he wanted to raise a point of order.

Subordinate clause 2: As soon as the meeting began.

Main clause: A member said

Some more examples of complex sentences:

1. The children rode an elephant when they visited the zoo.
2. I think that the president will not accept this proposal.

3. They were so tired that they simply had to sit down to take rest.
4. He knew that he had to get someone to cut down the trees in the garden.
5. If you want a passport, you must consult the passport issuing authority in the city.

In the aforementioned examples, the underlined words are called subordinating conjunctions that introduce subordinate classes.

The following is a list of some subordinating conjunctions: after, before, since, as soon as, while, until, as, so long as, till, in order to, lest, so that, that, because, since, supposing, unless, whether, on condition, so … that, than, no less than, as much as, so far as, according as, if, though, although, however, notwithstanding, even if, whatever, whichever.

Sample Exercise–5

Identify the type of sentence—simple, complex, or compound.

1. As soon as the boy saw the cobra, he began to run.
2. The South Pole is too cold for human beings to live in.
3. A hundred per cent in English is a very high scores, and it is almost impossible for a student to achieve this.
4. On account of mismanagement, the company suffered a great loss.
5. In spite of being supplied with the best weapons, the army could not win the battle.
6. The rooms were so selected as to enable me to reach the place of my interview.
7. It is in the field of education that the seed of secularism has to be sown at the earliest stage possible so that the plant can be carefully nurtured as it grows.
8. Our hope that the clouds would disperse was cheering.
9. To most people, a long journey by motorcar is exhausting and disagreeable.
10. Although mountains are undoubtedly impressive, they have a kind of dreadful monotony that makes people like me feel hostile to their beauty.

Types of Sentences (Semantic)

On the basis of the meaning (semantic), sentences are categorised into:

1. Assertive sentences
2. Imperative sentences
3. Interrogative sentences
4. Exclamatory sentences
5. Affirmative sentences
6. Negative sentences

Assertive Sentences (or) Declarative Sentences

Sentences that state facts, describe things, or report events are called assertive or declarative sentences.

Examples:
Dr. Mohan teaches English.
Varanasi is on the banks of the Ganga.
Hyderabad is the capital of Andhra Pradesh.

Imperative Sentences

Sentences that express an order, a request, an advice, a direction, a suggestion, a command, etc. are imperative sentences. These sentences start with the verb, and the subject (you) is understood.

Examples:

Take these books to the library.

Get me a cup of coffee, please.

Be neither a borrower nor a lender.

NOTE

There are some sentences that express 'commands' in an indirect way. Let is used at the beginning of these sentences. Subject is not omitted but expressed in these sentences.

Examples:

Let the manager talk to the director first.

Let's start at once.

Let the boys assemble in the auditorium for the seminar.

Interrogative Sentences

Sentences that are used to make enquiries and to ask questions are interrogative or question sentences.

Examples:

Can I help you?

What shall we have for lunch?

Is the bus late this morning?

Exclamatory Sentences

Sentences that express surprise, admiration, pity, sorrow, and other feelings in an emphatic way are called exclamatory sentences.

Examples:

How fast time flies!

What a beautiful view we have from here!

What a wonderful piece of art this is!

What a tragedy!

Affirmative Sentences

You have seen that all the above-given sentences as examples are statements, commands, questions, and exclamations. They affirm something in a positive way. They are affirmative sentences.

Negative Sentences

Look at the following sentences.

Examples:

He is not a rich man.

I cannot complete this work today.

They don't understand our difficulties.

I have never watched a test match.

All these are also statements, but they use words such as NOT and NEVER and are negative in meaning. They are called negative sentences.

NOTE

Commands can also be negative.

Example: Don't touch those glass exhibits.

Sentences of this kind are called negative commands or prohibitions. There can also be questions that are negative:

Hasn't he got a car?

Why didn't you come to the class in time?

Sentences of this kind are called negative questions.

Section II: Sentence Correction

Correct Use of Nouns

A noun is a naming word. It is the name of a person, place, thing, animal, concept, river, etc. Nouns are divided into five main kinds.

1. **Common Noun:** A name given in common to every person or thing of the same class is called a common noun.
   Examples:
   - Hyderabad is a big city.
   - She is a good girl.
2. **Proper Noun:** The name of a particular person or place is called a proper noun.
   Examples:
   - King Solomon was a wise king.
   - Kalidas was a great poet.
3. **Collective Noun:** It is the name of a number of persons or things taken together and spoken of as a whole.
   Examples:
   - A herd of cattle is grazing in the field.
   - A bunch of keys is on the table.
4. **Abstract Noun:** The name of a quality, a state of mind or a stage of life is called an abstract noun.
   Examples:
   - Kindness is a virtue.
   - Childhood is the happiest stage of one's life.
5. **Material Noun:** All those nouns, which are used to refer to metals and materials, are called material nouns.
   Examples:
   - This chain is made of gold.
   - This house is built of brick.

In the aforementioned examples 'gold' and 'brick' are material nouns.

Nouns are further classified into two types depending on whether they can be counted or not.

1. **Countable nouns:** Countable nouns are nouns that we can count: e.g. boy—one boy, two boys, three boys, etc. **Note:** The classes of nouns known as 'common nouns' (boy, table, etc.) and 'collective nouns' (flock, regiment, crowd, troop, etc.) can be included in the larger class of countable nouns.
2. **Uncountable Nouns:** (also called non-count nouns) Uncountable nouns (NCNs) name things that we cannot count or usually don't count. The classes of nouns known as material nouns and abstract nouns can be included in this category.

Uncountable nouns fall into the following groups:

1. Things that are considered in mass or quantity and not in numbers: sugar, sand, hair, dust etc.
2. **Materials:** wood, cloth, silver, gold, iron, etc.
3. **Liquids:** water, milk, oil, ink, honey, etc.
4. **Gases:** steam, mist, nitrogen, air, etc.
5. **Natural phenomena:** heat, cold, electricity, lightning, etc.
6. **Abstract nouns:** stages of life, states of mind, qualities, processes, actions, ideas, etc.
   courage, happiness, freedom, attention, childhood, honesty, enjoyment.

   Note: A number of words used to refer to qualities, states, actions, etc. are however countable. These may be considered exceptions.

   Examples: joys, sorrows, dreams, miseries, etc.
7. **Branches of learning:** Economics, Mathematics, Politics, etc.

   Note: There are some uncountable nouns, which we refer to in parts. When we do so, we use them with expressions such as a piece of, a grain of and a kilo of. Here is a list of such expressions:

a piece of chalk	a piece of information
a cake of soap	a piece of furniture
a block of ice	a loaf of bread
a piece of advice	a log of wood, etc.

These expressions have their plural forms: pieces of chalk, cakes of soap, etc.

Proper nouns are not categorized either as countable nouns or as uncountable nouns.

Now, study carefully some rules related to the correct use of nouns:

Rule 1: A proper noun becomes a common noun when it is used in the plural form or when an article is placed before it.

Examples:

There are five Gandhis in our college.

Vijay Kumar is the Milton of your college.

Kalidas is the Shakespeare of India.

Rule 2: A collective noun takes a singular verb when the whole group is considered as one unit.

Examples:

The committee consists of five members.

The family living next door has come from Chennai.

NOTE

If we refer to the members of the committee or family separately, the collective noun takes a plural verb, but it (the collective noun) remains singular in form, e.g. The committee have taken their seats.

Rule 3: An abstract noun can also be used as a common noun by placing an article before it.

Example: Rajani is a beauty (a beautiful girl).

Rule 4: An abstract noun can also be used in the sense of a collective noun. When an abstract noun is used as a collective noun, it takes a plural verb.

Example: Youth are the pillars of the nation. (collective noun)

Rule 5: When a material noun denotes a mass of matter, it is not used in the plural form.

Examples:

Their house is built of bricks and stones. (incorrect)

Their house is built of brick and stone. (correct)

Rule 6: Some nouns have the same form for the plural as well as for the singular. The following are some nouns that belong to this category.

Example: sheep, species, deer, aircraft, offspring, yoke, space craft and salmon.

Rule 7: Some nouns are used in the singular only. The following are some nouns that belong to this category.

Example: advice, corn, grain, justice, clothing, information, poetry, scenery, machinery and hair.

NOTE

Advice is the noun form, and advise is the verb form.

The advice of the doctor is that I should not smoke. (noun)

The doctor advised me not to smoke. (verb)

Similarly, practice is noun, and practise is verb.

Rule 8: There are some nouns that are used only in the plural form.

Examples:

scissors	shears	spectacles
trousers	pantaloons	shorts
bowels	intestines	biceps
genitals	gymnastics	premises
proceeds	riches	thanks
nuptials	outskirts	gallows
doldrums	earnings	

Rule 9: Some nouns are apparently plural in form but are singular in use.

Example: news, economics, physics, measles, mumps, ethics.

NOTE

Summons is singular and summonses plural.

Rule 10: The following nouns are always used in plural.

Example: cattle, police, people, vermin, poultry.

Rule 11: When a plural noun denotes a specific amount, length, weight, quantity, etc. considered as a whole, the verb must be in the singular form.

Examples:

Ten thousand rupees is a large amount.

Six kilometres is not a long distance for a runner like you.

Rule 12: Inanimate objects (lifeless objects), which are remarkable for beauty, gentleness, gracefulness, tenderness, weakness, and so on, are treated as feminine gender.

Example: the moon, the earth, spring, virtue, peace, charity, fame, nature, hope, justice, mercy, pity, fortune, truth, victory, defeat, modesty, liberty.

NOTE

Poets refer to them as feminine gender.

Rule 13: Inanimate things that are known for strength, courage, violence, superiority, and so forth are treated as masculine.

Example: the sun, death, summer, winter, war, anger, fear, thunder.

NOTE

The possessive case of nouns is formed when we put's to the noun (apostrophe s)

Examples:

- The girl's dress.
- The boy's pen.

The possessive case is used with the nouns of living things but not with inanimate objects.

- Table's leg (incorrect)
- Leg of the table (correct)

The possessive is used with the names of personified objects.
- Death's icy hands

Rule 14: When the noun is plural and ends in s, the possessive case is formed by adding only an apostrophe (after s).

Examples:
- Girls' hostel
- Birds' nests

Rule 15: When the noun is plural but does not end in s, the possessive case is formed by adding 's.

Examples:
- Men's club
- Children's park
- Women's day

Rule 16: When a noun or a title consists of several words, the possessive case is formed by adding 's to the last word.

Examples:
- The Prime Minister of India's speech.
- The King of Bhutan's visit to India….

Rule 17: When two or more proper nouns are joined by 'and' and a common possession is meant, the possessive case will be formed by adding ('s) to the last noun.

Example: Amar and Kumar's house is really beautiful.

NOTE

This sentence denotes that the house belongs to both Amar and Kumar.

Rule 18: When there are two nouns in apposition, the possessive case will be formed by adding ('s) to the second noun.

Examples:
1. Penicillin, Flemming's discovery, has saved the lives of many people.
2. Mohan, the professor's son, is very intelligent.

Rule 19: The possessive case of a compound noun will be formed by adding ('s) to the last word.

Examples:
Sister-in-law's house.
Commander-in-chief's office.

Rule 20: If the last syllable of a singular noun ends in '-s' or –'ce' and the noun is followed by the word 'sake', the possessive case of the noun will be formed by adding (') only. However, modern day English accepts the use of 's' after apostrophe as the word is pronounced so, e.g. goodness's sake.

Examples:
for goodness' sake, for justice' sake.
for conscience' sake, for Jesus' sake.

Sample Exercise–6

Correct the following sentences where necessary.

1. This scissor will not cut such a hard wire.
2. I have seen many cannons in the military parade ground.
3. The beggar is begging for alm.
4. He has lost all his luggages on his way home.
5. The company wants to dispose of the equipment.
6. She took great pain to write the essay properly.
7. The first innings is over just now.
8. These sceneries of Kashmir are beautiful.
9. My brother-in-laws will come to our house next week.
10. These golds have been brought from South Africa.

Correct Use of Articles

There are two types of articles:

1. The Indefinite Article
2. The Definite Article

The Indefinite Article:	A and An (Please note that 'An' is a variant of 'A'.)
The Definite Article:	The

NOTE

1. A or An is used before a singular countable noun.
2. The choice between A and An wholly depends on the pronunciation of the word.
3. In English, the 'spelling of a word' is different from its 'pronunciation'.
4. 'An' is used before the word, the pronunciation of which starts with a vowel sound.
5. There are 44 sounds (each is given a symbol from the International Phonetic Alphabet).
6. Out of the 44 sounds, there are 20 vowel and 24 consonant sounds.
7. Please note that, strictly speaking, A, E, I, O, U are not vowels but just letters of the English Alphabet consisting of 26 letters.
8. The indefinite article 'a' is used before
   (a) a word beginning with a letter that has a consonant sound, e.g.: a book, a man, a pen.
   (b) a word that begins with a letter (like O) with the sound like 'wa' e.g.: a one-rupee note, a one-eyed man.
   (c) a word beginning with 'u' or 'eu' giving the consonant sound 'yu'.
   Examples: a university, a European.
9. The indefinite article 'an' is used before:
   (a) a word beginning with a letter that has a vowel sound.
   Examples: an apple, an egg, an umbrella, etc.
   (b) a word beginning with 'h' but the pronunciation of which starts with a vowel sound.
   Examples: an heir, an hour, an honest man.

(c) an abbreviation, the first letter of which has a vowel sound
Examples: an MLA, an MP, an MCom, an SDO, an FRCS, an X-mas gift.

Some Important Rules Regarding the Use of Articles

Rule 1: A or an is not used before:

1. plural nouns: a books, a universities, a dinners (wrong)
2. uncountable nouns: an advice, an information (wrong)
3. names of meals:

Examples:
Let us have dinner. (correct)
Let us have a dinner. (incorrect)

NOTE

An indefinite article can be used before names of meals when these are preceded by an adjective.

Example: She gave me a good breakfast at 8 a.m.

Rule 2: A or an is used in the following ways:

Before the word 'most' when it is used in the sense of very or much or exceedingly.

Examples:
Sreedhar is a most intelligent student.
This is a most unfortunate event.

NOTE

It must be borne in mind that 'man' or 'woman', when used in a general sense to represent 'mankind' as a whole, never takes an article.

Example: Man is mortal.

Rule 3: With a noun complement. This includes names of professions.

Examples:
He is an actor.
She is a good dancer.
It was an accident.

* The words underlined are noun complements. They are also called subject complements.

Rule 4: In certain phrases: a cold, a pain, on an average, make a noise, make an effort, make a mistake, a fever, etc.

Rule 5: With certain numbers: a hundred, a thousand, a million, etc.

Rule 6: 'A' can be used before Mr./ Miss/ Mrs. + name

Example: a Mr. Bose, a Mrs. Bose, etc.

NOTE

A Mr. Bose denotes a person who is called Bose and implies that he is a stranger to the speaker.

Mr. Bose without 'a', implies that the speaker knows Mr. Bose or knows of his existence.

Rule 7: Before a proper noun to make it a common noun.

Examples:
Mohan is a Newton. (Mohan is a great scientist.)
Amaresh is a Shakespeare. (Amaresh is a great dramatist.)

Rule 8: Before certain uncountable nouns preceded by nouns + of

Example: a piece of advice, a bit of news, a drop of water.

Rule 9: After the words many, rather, such, quite, etc. in certain structures.

Examples:
Many a friend of mine is attending the party.
Such a show cannot be arranged now.
He is rather a fool to take such decisions.

Rule 10: In certain expressions of quantity.

Example: a lot of, a couple, a great many, a good deal of, a good many, a great deal of, a few, a little.

THE is used in the following ways:

Rule 11: Before a noun denoting a hospital, temple, school, college, prison, etc. if its purpose or use is not referred to or if used not in its primary purpose.

Example: He has gone to the hospital to visit a friend.

Rule 12:
Before an adjective in the superlative degree.

Example: Michael is the tallest boy in the class.

Rule 13: With nouns that refer to things that are unique.

Example: the sun, the moon, the sky, the earth, the equator.

Rule 14: Before an abstract noun or a material if it is used with an adjunct (a qualifying clause)

(or when there is a particular reference to the material noun).

Example: The gold you have brought from the USA is of good quality.

Rule 15: Before a singular countable noun that picks out one individual, object, etc. as representative of a class.

Example: The tiger is a ferocious animal.

NOTE

All the following sentences mean the same thing.

The tiger is a fierce animal.

A tiger is a fierce animal.

Tigers are fierce animals.

Rule 16: The is used before certain adjectives to give a plural meaning.

Examples:
rich, poor, dead, sick, healthy, deaf, blind

the rich = rich people

the poor = poor people

Rule 17: With certain adjectives indicating nationality.

Example: the Dutch, the Spanish, the Chinese, the Burmese.

Rule 18: Before 'only' and ordinal numbers, such as first, second, millionth.

Examples:
All the students of the first year are invited.

The second ranker is my son.

Rule 19: Before a noun when special emphasis is needed.

Example: This is the novel I am talking about.

Rule 20: Before a common noun to give it the meaning of an abstract noun.

Example: At last, the father in him prevailed and excused him.

Rule 21: Before an adjective in the comparative degree, when the selection of one out of only two persons, places, or things is meant.

Example: Nalini is the more beautiful of the two girls in the class.

Rule 22: In special comparatives.

Examples:
The more you earn, the more you spend.

The more, the better.

The higher you go, the cooler you feel.

Rule 23: Before musical instruments.

Example: Rajani can play the piano very well.

Rule 24: Before the proper names of certain well-known or sacred books.

Example: the Mahabharat, the Ramayana, the Gita, the Bible, the Quran.

Rule 25: Before the names of certain countries each of which is a union of smaller units.

Example: the USA, the UAE, the U.K. and, etc.

Rule 26: Before the words like north south when these are used as nouns.

Example: the north of India, the Middle East, the West Asia

Rule 27: Before some proper nouns consisting of adjectives and noun or noun + of + noun.

Example: the State Bank of India, the National Museum

Rule 28: The + singular noun + clause or phrase can be used to distinguish one person from another of the same name.

Examples:
We have two Reddys, which Reddy do you want?

I want the Reddy who signed the letter.

Rule 29: Before the names of political parties.

Example: The Congress, The BJP.

Rule 30: When it is clear from the context that a particular person, place, or thing is meant.

Examples:
I talked to the principal yesterday.

The students are playing in the garden.

Rule 31: Before the names of the historical or public buildings.

Example: the Taj Mahal, the Red Fort, the Rashtrapati Bhavan.

Rule 32: Before the names of rivers.

Example: the Krishna, the Ganga, the Yamuna.

Rule 33: Before the names of seas.

Example: the Arabian Sea, the Red Sea.

Rule 34: Before the name of Oceans.

Example: the Indian Ocean, the Atlantic Ocean.

Rule 35: Before the names of certain chains of Mountains.

Example: the Himalayas, the Alps.

Rule 36: Before the names of deserts

Example: the Sahara, the Thar.

Rule 37: Before the names of newspapers, magazines.

Example: the Hindustan Times, the Statesman, the Deccan Chronicle.

Rule 38: Before groups of islands.

Example: the Andamans, the West Indies.

Rule 39: Before the names of certain countries.

Example: the Yemen, the Sudan, the Hagues.

Rule 40: Before the dates of months.

Example: the 23rd October, 1949, the 15th August, 1947, etc.

THE is omitted in the following cases:

Rule 41: Before a common noun used in the widest sense.

Examples:
In general, woman is kind-hearted.

Man is mortal.

Rule 42: Before material nouns.

Example: Gold is a precious metal.

NOTE

The may be used with a material noun if it is used with an adjunct that makes it definite.

Example: The gold we use in India is all imported.

Rule 43: Usually before proper nouns.

Example: Hyderabad is the capital of Andhra Pradesh.

Rule 44: Before abstract nouns used in a general sense.

Examples:

Honesty is the best policy.

Sincerity pays dividends.

NOTE

If an abstract noun is qualified by an adjective or an adjectival phrase or clause, it may have the article 'the' before it.

Example: The wisdom of Moses is biblically significant.

Rule 45: Before the words—father, mother, aunt, uncle, etc., in general sense.

Example: Father is very angry today.

Rule 46: Before predicative nouns denoting a position that is normally held at one time by one person only.

Examples:
Mr. Kiran was elected chairman of the committee.

He became principal of our college in 1996.

Rule 47: Before plural nouns that are used to denote a class.

Examples:
Historians study old monuments for research purposes.

Camels are useful animals in deserts.

Rule 48: Before plural nouns used in general sense.

Example: Members are requested to pay their subscriptions in time.

Rule 49: Before the names of meals.

Examples:
I had lunch at 2.30 p.m.

Let's have dinner at 9.00 p.m.

NOTE

'The' can be used when meals are preceded by an adjective or a clause or phrase particularising them.

Examples:
The dinner given by our colony association yesterday was not properly arranged.

What about the wedding dinner tonight?

Rule 50: Before common nouns used in pairs.

Examples:
He worked day and night to prepare for the Civil Service exam.

Both husband and wife are supposed to take part in this competition.

Rule 51: No article is used before the nouns in the following expressions:

to catch fire	to send word	to give ear
to lose heart	to leave home	to take offence
at home	in hand	in debt
by day	at sunrise	at noon
on demand	by land	by air
at night	on foot	at ease

Rule 52: 'The' is not used before the words—bed, hospital, sea, temple, prison, court, school, university, college, etc. when these places are visited or used for their primary purpose.

We go to bed to sleep.

We go to hospital as doctors.

We go to university for higher studies.

NOTE

When these places are visited or used for the other reasons or purposes, 'the' should be used.

Example: Sometimes I go to the prison to give lectures on morality, social behaviour, etc.

The articles a, an, or the can be used as follows in some cases:

Rule 53: When two or more adjectives qualify the same noun, the article is used before the first adjective only.

Example: He has a black and white coat.

Rule 54: When two or more connected nouns refer to the same person or thing, the article is placed before the first noun only.

Example: The chief accounts officer and financial adviser is supposed to certify your documents.

Rule 55: In expressing a comparison, if two nouns refer to the same person or thing, article 'a' is used before the first noun only.

Example: He is a better teacher than administrator.

Articles are omitted in the following cases:

English	⇒	the English language
the English	⇒	the English people

Examples:
We speak English at home.

The English and the French have fought various wars.

He has gone to market	⇒	He has gone to make purchases.
He has gone to the market	⇒	He has gone to the place where there is a market, not necessarily to make purchases.
To see light	⇒	to look at light
To see the light	⇒	to be born

Examples:
I see light at the end of the tunnel. (metaphoric use which means there is hope)

His proposals for a new international airport never saw the light of the day.

The article 'the' is omitted before the nouns in the phrases below:

He invited me to lunch/tea/dinner.

NOTE

lunch/tea/dinner are used in general sense.

Correct use of Pronouns

A pronoun is a word used in the place of a noun.

Types of pronouns:

Personal Pronouns

Person	Singular	Plural
First person	I	We
Second person	You	You
Third person	He, She, It	They

Forms of pronouns: (cases)

Nominative case (subject)	Accusative case (object)	Possessive	
I	Me	My	Mine
We	Us	Our	Ours
You	You	Your	Yours
He	Him	His	His
She	Her	Her	Hers
It	It	Its	Its
They	Them	Their	Theirs

NOTE

The forms my, our, your, his, her, its, and their are determiners. They can also be used as possessive adjectives.

Possessive Pronouns

Example: This book is his.

Reflexive Pronouns

Examples:

1. She washed herself at a well.
2. They killed themselves for no reason.
3. I must blame myself for this.

Emphatic Pronouns

Example: I offered to drive the car myself. She herself made the mistake.

Demonstrative Pronouns

Examples:

1. That is Dr. Rao's house.
2. Those are some new buildings.
3. This is your passport.
4. These are good books.

Indefinite Pronouns

Example:

1. Some are born great.
2. Anyone can take a horse to the pond, but no one can make it drink.

Other indefinite pronouns are:
somebody, anybody, anything, something, (the) other, others, another, nobody, nothing etc.

Pronouns of Number

Examples:

1. Of the ten guests, three were men.
2. Many of them are not good books.

Pronouns of Quantity

Examples: more, much, little, etc.

Distributive Pronouns

Look at the pronouns underlined in the sentences given below:

Examples:

1. Everyone has his own problems.
2. Either of these applicants can be employed.
3. Neither of the students can get through the examination.
4. Each of those players deserves a prize.
5. Everybody will be given a chance to participate in this match.

These words, which are underlined, are distributive pronouns. They are called so because they refer to persons considered individually. The distributive pronouns are in the singular number and are therefore followed by singular verbs.

NOTES

1. Neither is a negative word that is used to refer to two persons or things.
2. None is also a negative word that is used to refer to more than two persons or things.
3. The usage: Each one of the boys … (is wrong because 'one' becomes redundant).

Reciprocal Pronouns

Look at these sentences:

Examples:

1. The two girls helped each other in every respect.
2. The political parties quarrelled with one another.

Each other and one another express a mutual or reciprocal relationship. They are considered as single units and are called reciprocal pronouns or 'compound personal pronouns'.

NOTE

Each other is usually used to refer to two persons or things and one another to more than two persons or things.

Interrogative Pronouns

The words underlined in the following sentences are called interrogative pronouns because they are used in questions.

Examples:

1. Who is the president of India?
2. Whose is this calculator?
3. To whom did you give the parcel?
4. Which is the road to the hospital?
5. What have you to do now?
6. Whom did you borrow this book from?
7. Whom did she refer to in her lecture?

Relative Pronouns

Look at these sentences:

Examples:

1. The gentleman who is speaking is our principal.
2. This is the gentleman whose guest I was in Bombay.

The words underlined are pronouns standing for the nouns used before them. The nouns are called antecedents, and the pronouns are called relative pronouns. They relate the adjective clauses to the main clauses.

Rules for use of Pronouns

Now let us look at some aspects related to pronouns that will be helpful in answering questions in Sentence Correction.

Rule 1: Some important uses of the pronoun 'it' are given below with examples.

1. To introduce a sentence
   Example: It is not certain that the president will come.
2. To give emphasis to the noun or pronoun that follows
   Example: It was you who began the quarrel with us.
3. As an indefinite nominative of an impersonal verb.
   Examples:
   It rains.
   It is snowing outside.
4. In sentences showing distance
   Example: It is not far to walk.
5. In sentences indicating time.
   Example: It is ten O'clock now.
6. To introduce a phrase.
   Example: It is decided to declare a holiday today.
7. In exclamatory sentences.
   Example: What a beautiful book it is!
8. To introduce a that clause.
   Example: It is said that smoking is injurious to health.
9. As a sort of object in order to avoid repetition.
   Example: Let us fight it (the issue) out.

Rule 2: While confessing a fault (or expressing a negative idea), the sequence of the personal pronouns should be as follows:

Example: I, you, and he are in the wrong and will be punished.

Sequence: First person first, second person next, and third person last.

Rule 3: While expressing a positive idea or praise, the sequence of the personal pronouns should be as follows:

Example: You, he, and I will get an award for the good work we have done.

Sequence: Second person, third person, and first person.

In standard English, however, the sequence of third second and first is also accepted (i.e.) He, you and I ______.

Rule 4: When two singular nouns joined by and denote the same person or thing, the pronoun used for them must be singular in number. The definite article the is placed before the first noun.

Example: The accounts officer and treasurer should be careful in his work of keeping accounts.

Rule 5: When two singular nouns are joined by 'and' and are preceded by each or every, the pronoun must be in singular number.

Example: Every student and every teacher took his or her seat.

Rule 6: When a personal pronoun is connected by a conjunction with some other word in the objective case, it must be in the objective or accusative case.

Example: These clothes are for you and me.

Rule 7: When a singular noun and a plural noun are combined by or, either ... or, neither ... nor, the singular noun usually comes first in the sentence, and the pronoun must be in the plural number, corresponding to the plural noun that is closer to it.

Example: Either the manager or his subordinates failed in their duty in sending the official message.

Rule 8: The personal pronouns—yours, ours, hers, theirs and its—are written without the apostrophe (').

Examples:

Your's sincerely. (wrong)
Yours sincerely. (right)

NOTE

'It's' means 'It is' and not belonging to it.

Example: It's a mad dog which bites its tail.

Rule 9: When a personal pronoun is used as a complement to the verb to be, it (the pronoun) must be in the nominative case.

Example: It was he, who could solve the problem easily.

Rule 10: A pronoun should be used in the objective case in a sentence beginning with let.

Examples:
Let him go to his office immediately.

Let her submit the records in time.

Rule 11: One can be used to talk about people in general. The pronoun that follows one should be one's. (not his/her)*

Examples:
One should do his duty. (wrong)

One should do one's duty. (correct)

NOTE

But there is a controversy here. In American English, one can be followed by his or her. Students taking TOEFL or GRE or GMAT or SAT should keep this in mind. However, usually one need not worry about this problem. If a sentence begins with one, be sure that you or they DOES NOT follow. Hence, it is never correct to say:

If one takes this exam without studying, you are likely to fail.

Additional example: One should never tell his secrets to a gossip if he wishes them to remain secret. (correct)

Rule 12: A relative pronoun must always be placed as near its antecedent as possible. Also, it must always agree with its antecedent in number, gender, and person.

Example:
This is the manager who abused the clerk.
ant. rel. pr.

Rule 13: Generally, the relative pronoun in the objective case is omitted.

Example: The student (whom is omitted) you wanted to punish is absent today.

Rule 14: The pronouns who, whom, and whose are generally used for persons.

Who is used in the nominative case.

Whom is used in the objective case.

Whose is used in the possessive case.

Examples:
1. Sarita is the student who got an award.
2. They are the thieves whom the police caught.
3. This is the student whose certificates are lost.

Rule 15: When the relative pronoun is in different cases, one in the nominative and the other in the objective, it must be mentioned twice, once for each verb.

Example: The girl, who is my daughter and whom you met in the library yesterday, left for Mumbai this morning.

In the aforementioned example, the subject of the sentence 'the girl' is also the object of the sentence. Hence, both 'who' and 'whom' are used.

Rule 16: Uses of WHICH:

'Which' is used in the following ways:

1. For infants, small animals, and objects
   Examples:
   This is the baby which was lost in the theatre.
   This is the dog which my friend bought from the Kennel's club.
2. When selection is expressed.
   Example: Which of these television sets do you want to purchase?
3. To refer to a sentence.
   Example: He was said to be drunk, which was not true.

Rule 17: Uses of THAT:

'That' is used in the following ways:

1. For persons, lifeless things, and small animals in the singular or in the plural number.
   Examples:
   This is the girl that failed in the exam.
   This is the radio that I bought yesterday.
2. As a substitute for a singular noun already mentioned.
   (pay attention to this use, particularly in the comparative degree)
   Examples:
   The weather of Hyderabad is far better than Chennai (wrong).
   The weather of Hyderabad is far better than that of Chennai.

3. After a noun phrase used as direct object.
   Example:
   I vividly remember the night that she came.
   (Here that is used in the sense of when)

Rule 18: 'Either' and 'neither' are used in speaking of two persons or places or things etc.

Examples:
Neither Mahesh nor Mohan is intelligent.
(negative meaning is implied)

Either Mahesh or Mohan is expected to get a prize.
(positive meaning is implied)

Rule 19: Uses of each other and one another

1. Each other is used for two persons or things or places etc.
   Example: These two students love each other.
2. One another is used for more than two persons or things.
   Example: Those four countries always disagree with one another.

Rule 20: Each can come in three different positions in a sentence.

1. Each of the students got a prize. (initial)
2. The students got a prize each. (end)
3. The students were each given a prize. (middle)

Rule 21: ONE is used in the following ways :

1. For people in general.
   Examples:
   One must try to do one's duty.
   One must not be proud of oneself.
2. In place of a noun previously mentioned.
   Examples:
   Give me a banana which is a fresh one.
   Give me bananas which are fresh ones.

Rule 22: As regards anybody, everyone, everybody, etc. the pronoun of the masculine or feminine gender should be used according to the context.

Example: Everyone of the boys got his hall ticket.

Sample Exercise–7

Rewrite the following sentences after making the necessary corrections.

1. She availed of the opportunity.
2. Those two companies always help one another.
3. India and Pakistan should cooperate with one another in this matter.
4. Dear students, enjoy during the holidays.
5. The committee were divided in its opinion regarding this issue.
6. The team, after taking its bath, have gone for practice of cricket.
7. My opinion is the same as your.
8. We were told to let only you and she enter.
9. This is the scientist which won the first prize.
10. His problems are the same as my.

Correct Use of Adjectives

An adjective is a word which qualifies a noun or a pronoun.

Example:
She is a good teacher.
adj. noun

An adjective can be used in two ways:
(1) attributively (2) predicatively

An adjective is used attributively if it is placed immediately before the noun it qualifies.

Example:
She is a good teacher.
adj. noun

An adjective is used predicatively if it is used after the verb.

Example:
Bimala is intelligent.
v. adj.

An adjective has three forms: positive, comparative, and superlative. These are the three degrees of comparison:

Look at the following examples:

Superlative degree:	Mohan is the best student in the class.
Comparative degree:	Mohan is better than any other student in the class.
Positive degree:	No other student in the class is as good as Mohan.

- The positive degree of an adjective is the adjective in its simple form. It is used to refer to the mere existence of a particular quality.
- The comparative degree of an adjective denotes a higher degree of the quality than the positive, and is used, when two things are compared.
- The superlative degree denotes the highest degree of the quality.

NOTES

1. The usages 'as … as' and 'so … as' appear in positive degree.
Comparative degree has 'adjective + er ... than'.
Superlative degree has 'the + adjective in the superlative form'.
Note the use of 'the' before the adjective in the Superlative (superlative form).

2. There is no superlative degree if the comparison involves only two objects, persons, or place.

Examples:
Of the two sisters, she is the best. (wrong)
Of the two sisters, she is the better. (correct)
When selection of one of the two persons or things (sets of things) of the same kind is meant, the comparative degree is preceded by 'the' and is followed by 'of'.

Example:
She is the better of the two sisters.
Let us look at some important rules in the use of adjectives:

Rule 1: Most adjectives form their comparatives by the addition of -r or -er and their superlative by the addition of -st or -est (depending on the spelling) to the positive.

Examples:

Positive	Comparative	Superlative
Great	Greater	Greatest
Brave	Braver	Bravest

Rule 2: Some adjectives (usually having more than two syllables in the spelling) form their comparative by using the adverb more with the positive and the superlative by using the adverb most with the positive.

Examples:

Positive	Comparative	Superlative
beautiful	more beautiful	most beautiful
interesting	more interesting	most interesting

Rule 3: (very important) When two qualities in the same person or thing are compared, the comparative degree is formed by using more, instead of -r or -er with the positive.

Example:
Reddy is more wise than intelligent.

(This sentence means that Reddy is both wise and intelligent. But his wisdom is greater than his intelligence.)

Rule 4: When an object is compared with the rest of the group, the latter term of comparison must exclude the former by using 'any other'.

Examples:
Gold is more precious than any other metal. (correct)

Gold is more precious than any metal. (wrong)

Rule 5: There are some words which are used only in the positive and cannot be used in the comparitive or in the superlative.

Example: interior, exterior, ulterior, major, minor.

- This is more inferior than that. (wrong)
- This is inferior to that. (right)

Rule 6: The following are used as comparative adjectives and are followed by to and not than:

Example: superior, senior, junior, inferior, prior, anterior, posterior, prefer.

Rule 7: The adjectives—empty, excellent, circular, extreme, chief, entire, complete, perfect, final, last, unique, universal, round, square, triangular, eternal, etc., are not used in the comparative or superlative degree.

Rule 8: Some adjectives can be used only in the positive and in the superlative. They are:

Positive	Superlative
top	topmost
northern	northernmost
southern	southernmost
eastern	easternmost
western	westernmost

Rule 9: The adjective 'preferable' is used as a comparative. It is followed by to. It is not used with more.

Examples:
This is more preferable than that. (wrong)
This is preferable to that. (correct)

Rule 10: The comparative adjectives ending in 'or' are followed by the preposition to.

Inferior, superior, prior, anterior, posterior, senior, junior

Examples:
He is superior to me. (correct)
He is superior than me. (incorrect)

Rule 11: Sometimes the words much, less, far, etc. are used before the comparatives to denote emphasis or excess.

Example: Ramakrishna is by far the ablest lecturer in the college.

Rule 12: Two adjectives that refer to the same noun or pronoun joined by a conjunction must be in the same degree of comparison.

Example: Gandhiji is the noblest and wisest of all national leaders.

Rule 13: Certain adjectives, when preceded by 'the' become nouns in plural and require a plural verb if they are used as subjects.

These are rich, poor, needy, aged, blind, dead, meek, wicked, etc.

Examples: The rich (rich people) usually hate the poor (poor people).

Rule 14: Some adjectives often confused:

1. FARTHER - FURTHER
   Farther denotes more distant in space,
   Examples:
   Sheela lives at the farther end of this street.
   Further means additional; more

Examples:
There is nothing further to say about him.
For further details, please contact the director.

2. Many, a great many, a good many
All these take a plural noun and a plural verb after them.

Examples:
1. A great many people died in the cyclone.
2. My sister has a good many friends.

3. Use of many + a + noun (singular) + verb (singular) structure :

Examples:
Many a delegate is attending this seminar.
(meaning: Many delegates are attending this seminar)

4. MODIFIERS: FEW, LITTLE, MUCH, and MANY
Few is used with plural nouns. Little is used with singular non-count nouns.

Without articles, few and little usually have rather negative meanings. They often suggest 'not as much/many as one would like', or 'not as much/many as expected' or a similar idea.

With articles 'few' and 'little', i.e. a few, and a little, are more positive in meaning. Their meaning is closer to 'some'. They often suggest ideas like 'better than nothing' or 'more than expected'.

Examples:
(a) There are few books on 'Thermodynamics' in the library; so, you have to purchase them.
(b) You don't need to go shopping. There are a few eggs in the fridge.
(c) I've got a little bread and cheese. It'll be enough for supper.

Difference between fewer and less: In theory, fewer (the comparative of few) is used before countable nouns and less (the comparative of little) before uncountable words.

Examples:
(a) There are fewer exploited workers than there used to be.
(b) My friend earns lesser than a postman does.

Note the following:

PD		CD		SD
few	-	fewer	-	fewest
little	-	less/lesser	-	least

* fewer and less (lesser) are followed by than as they are the comparative forms.

NOTE

'Many' like 'few' is used before a count noun.

Example:
Many/few students.
CN

'Much', like 'less' is used before a non-count or a material noun.

Example:
Much/less water.
NCN

5. MODIFIERS—TOO, VERY, ENOUGH
Compare the meanings and patterns of too, very, and enough.

(a) Very means to a high degree but does not suggest impossibility.
Mary is very intelligent.

(b) Too suggests impossibility or undesirable degree.
too + adj + to + v
She is too sick to come to class today

Note: It is better not to use 'too' with a positive adjective.
Instead of 'too' it is better to use 'very'.

(c) Enough suggests possibility or sufficient degree.
He is tall enough to play basketball.

NOTES

1. Be careful to put enough after the adjective.
2. Be careful to put to+v (infinitive) after enough.
3. In patterns 1 and 2, do not use any other structure word after the adjective or adverb except to.
4. In the aforementioned patterns, adverbs can be used in the same position as adjectives.
Example:
Mary sings very well.
5. Enough can come before or after a noun to express sufficiency.

Examples:
He had money enough to buy a new car.
N
OR
He had enough money to buy a new car.
N

6. MODIFIERS—NEGATION
There are some words that have negative meanings even though they do not appear to be negative, for example: hardly, scarcely, rarely, seldom, without, and only. Do not use another negative word with these words.

Examples:
(a) He had scarcely enough money to purchase books.
(b) They went to bed without dinner.

Note: Scarcely and hardly are followed by when and not than. No sooner is followed by than.

These expressions mean 'as soon as'.

Examples:

(a) As soon as I reached the station, the train left.

(b) No sooner had I reached the station, than the train left.

No sooner + had + subject + past participle + ….

(c) No sooner did I reach the station, than the train left.

No sooner + did + subject + present tense verb + ….

Similarly: Hardly/Scarcely + had + subject + past participle … when ….

OR

Scarcely/Hardly + did + subject + present tense verb … when ….

Rule 15: MODIFIERS—ADJECTIVES AFTER VERBS OF SENSATION

These verbs of sensation are generally followed by adjectives* not by adverbs: feel, look, seem, appear, smell, taste, and sound.

(* These are also called predicate adjectives)

He feels bad	(correct)
He feels badly	(wrong)
The soup smells delicious	(correct)
The soup smells deliciously	(wrong)

Rule 16: MODIFIERS—HYPHENATED OR COMPOUND ADJECTIVES

Nouns are sometimes found as part of hyphenated or compound adjectives (adjectives of more than one word joined by hyphens). These nouns are never plural.

I bought a four hundred year-old painting in Germany.

HYPHENATED ADJ. N

Some more examples:

1. B.Com is a three-year degree course.
2. This is a ten-kilometre-long distance.
3. The professor has delivered a two-hour lecture today.

Rule 17: MODIFIERS—CARDINAL AND ORDINAL NUMBERS

There are two kinds of numbers, cardinal and ordinal.

Examples:

CARDINAL	ORDINAL
one	first
two	second
three	third
four	fourth
twenty-one	twenty-first

The following patterns are used to designate items in a series:

1. Ordinal numbers are used in this pattern:
   THE + ORDINAL + NOUN
   The first book of the series is about verbs.
2. Cardinal numbers are used in this pattern:
   NOUN + CARDINAL
   Book one of the series is about verbs.

NOTES

1. Use the with ordinal numbers.
2. Do not use the with cardinal numbers.
3. Be careful to use the correct word order for each pattern.

Rule 18: MODIFIERS: NOUN ADJECTIVES

The first noun (N) in the following pattern is used as an adjective.

All these are language students.
N N

NOTES

1. When nouns are used as adjectives, they do not have plural or possessive forms.
2. Exceptions: The following nouns always end in -s but are singular in number when they are used as names of courses or sciences : Physics, Mathematics, etc.

He is an Economics teacher.
N N

BUT The current economic situation is extremely uncertain.

NOTES

'Economic' is an adjective here.
'Economical' means frugal, thrifty.

SAMPLE EXERCISE–8

Correct the mistakes relating to the use of adjectives in the following sentences where necessary.

1. The two first chapters of this book are very important.
2. Suvarna is greater than any student in the class.
3. Shakuntala is better than any drama in Sanskrit.
4. His house is further from the station than yours.
5. The later suggestion is better than the former.
6. These flowers smell sweetly.
7. Little learning is a dangerous thing.
8. Her birthday is the most happiest event this year.
9. Usually the rich people hate poor.
10. Go and meet the concerned clerk in the university.
11. Honour is dear to me than life.
12. Little act would have saved the situation.
13. The tiger is the ferocious of all animals in the forest.
14. My knife is the sharpest of the two.

Correct use of Adverbs

An adverb is a word which modifies a verb, an adjective, or another adverb in a given sentence.

Examples:

She is reading very fast.
verb adv.

She is a very intelligent girl.
adv. adj.

The horse runs very quickly.
adv. adv.

(**Note:** usually the adverbs end with -ly.)

Rules Pertaining to Use of Adverbs

Rule 1: In order to make our meaning clear, an adverb must be placed as near as possible to the word it modifies.

Examples:

She has only three dollars with her.
adv. adj.

He says he often visits my place.

He often says he visits my place.

NOTE

Other adverbs like only are: just, nearly, hardly, almost, and scarcely.

Read the following examples:

Wrong : We only have four hours to finish this paper.
Right : We have only four hours to finish this paper.
Wrong : She just wants to take one class. (not anything else)
Right : She wants to take just one class. (not even second)
Wrong : That building nearly costs sixty thousand rupees.
Right : That building costs nearly sixty thousand rupees.

Rule 2: When an adverb modifies an intransitive verb, it usually follows it.

Examples:

- She sang melodiously.
- She writes neatly.

Rule 3: When a verb consists of an auxiliary and a main verb, the adverb is placed between the auxiliary and the main verb.

Wrong : I have told him often not to come late.

Right : I have often told him not to come late

Rule 4: When an adverb modifies an adjective or another adverb, the adverb usually comes before it.

Examples:

The cuckoo sings quite sweetly.
adv.

Do not speak so fast.
adv.

NOTE

The adverb 'enough' is always placed after the word it modifies.

- Why don't you speak loud enough to be heard.

Rule 5: The word 'only' should be placed before the word it is intended to modify.

Examples:

- Only she gave me this book. (i.e. she and nobody else)
- She only told me the truth. (and nothing else)
- I solved only two problems. (correct)
- I only solved two problems. (incorrect)

Rule 6: POSITION OF ADVERBS

The position of adverbs is often determined by shades of meaning, for which rules cannot be given, but some generalizations can be made.

Adverbs of frequency: always, often, rarely, never, ever, generally, usually, sometimes, occasionally, etc.

- If the verb is in the simple tense form, the adverb is usually placed between the subject and the verb, preferably before the verb it modifies.
  He always goes to college on foot.
  He often visits the US.
  His brother never takes alcohol.
- When the verb is some form of 'be' (is, am, was, are), the adverb follows the verb:
  They are always late.
  He is never punctual.
  If you are ever in trouble, please meet me.

If the verb is a compound one, the adverb is usually placed after the auxiliary:

I shall never forget his help.

He will always behave properly.

In negative sentences, the adverb of frequency follows not.

They are not often late.

In interrogative sentences, the adverb of frequency follows the subject immediately:

Does he often go fishing?

Has he ever travelled by air?

At times 'often' may be placed at the end to emphasize it. This is mainly confined to negative statements and questions.

He does not see his friend often, as he lives in a remote village,

'Never' is sometimes placed at the beginning to emphasize it. Then, the verb and subject are inverted as in a question.

I never saw such an accident.

Never did I see such an accident.

(S–V becomes V–S here)

Rule 7: USE OF HARD, HARDLY, SCARCE, SCARCELY

1. Hard as an adverb means 'diligently' . It usually follows the verb.
   He works hard to make both ends meet.
2. Hardly when used as an adverb means scarcely, barely It conveys a negative meaning.
   Hardly (scarcely) had he reached the station, when the train left.
   Note: Hardly and scarcely are followed by when, not than. No sooner is followed by than not when. (This is very important.)
3. Scarce as an adjective means 'not plentiful', hard to find, not often found.
   Coal has become scarce in England.
   Scarcely as an adverb is almost synonymous with 'hardly'.
   I can scarcely hear you.
   They have scarcely enough money to look after their children.

Rule 8: SPLIT INFINITIVE

The infinitive is to + the simple form of the verb (V).
Do not put an adverb between to and verb.

1. He refused to do the work quickly.
   to+v Adv.
2. They have decided to repeat the experiments carefully.
   to+v adv.

Wrong: He wanted to carefully read the directions.
Right: He wanted to read the directions carefully.

Rule 9: DANGLING MODIFIER

The subject of the main clause must be the same as the understood subject of the introductory phrase. In other words, the introductory phrase modifies the subject of the main clause.

Examples:

1. Looking at his watch, Mr.Vijaykumar got up and left.
   Introductory Ph. Subject
   Who looked at his watch? Mr. Vijaykumar
   Who got up and left? Mr. Vijaykumar
2. Traveling to Bombay, Nalini injured her leg.
   Introductory Ph. Subject

NOTE

Both these sentences are right. In both these sentences, the subject of the introductory phrase and the subject of the main clause are same.

Wrong: When only a baby, my mother took me to the circus.
Right: When only a child, I was taken to the circus by my mother.

Sample Exercise–9

Direction for questions 1 to 4: Fill in the blanks with suitable adverbs.

1. The speaker pauses ________ in order to heighten the beauty of his speech.
(A) hastily (B) briefly
(C) lustrously (D) incandescently

2. He always speaks ________ unmindful of the seriousness of situation.
(A) facetiously (B) soberly
(C) solemnly (D) thoughtfully

3. Though they are married, they are living ________.
(A) apart (B) away
(C) differently (D) together

4. The politician was careless and ________ made promises.
(A) uncommunicatively (B) inconsiderately
(C) uncommittedly (D) unceremoniously

Correct Use of the Verb

Structure of the Verb Phrase

A verb indicates the action done by the subject or the state of being of the subject.

Examples:

1. He has completed the work. (action)
2. Radha is an intelligent student. (state)
   In every sentence you find a verb phrase (VP).
   A verb phrase may have a single word, two words or a group of words.

Examples:

1. The Sun rises in the east.
   1
2. He has ordered tea for us.
   1 2
3. She has been teaching English since 1965.
   1 2 3
4. She will have been typing when I meet her at 10 a.m.
   1 2 3 4

Verbs —
- Main Verb
- Auxiliary Verb

Main Verb – Basic forms —
- Simple Present Tense - Sing
- Simple Past Tense - Sang
- Present Participle - Singing
- Past Participle - Sung

Auxiliary Verbs and Their Forms

	Primary Auxiliaries	Modal Auxiliaries
BE	be, is, are, am, was, were, being, been	can, might, may, must, will, needn't
HAVE	have, has, had, having	shall, daren't, should, ought
DO	do, does, did, doing, done	would, used to, could

NOTE

The verb phrase in any sentence is constituted by using only the main verb form or one of the basic forms of the main verb and the primary auxiliaries or modal auxiliaries or both the primary auxiliaries and the modal auxiliaries.

Examples:

1. I teach English.
   MV
2. I am teaching English now.
   PA MV
3. I will have been teaching English.
   MA PA PA MV

Special Note: Every verb phrase has a particular structure acceptable according to standard written English.

The verb phrases in the following sentences are wrong.

1. I will the work
2. I will did the work
3. I going there
4. I can done the work

Verbs: Transitive and Intransitive

The verbs that do not require or do not have objects in the sentence in which they are used are called intransitive verbs (IV).

In the following sentences, complements are used, not objects.

Examples:

This book costs ten rupees
IV comp.

The play lasted an hour
IV comp.

The birds fly in the sky
IV adv. ph.

The verbs that require or have objects are called transitive verbs

Example:

The professor gave them some assignments
v object

NOTE

There are a number of verbs which can be used with or without objects, that is, 'transitively' or 'intransitively.' Here are some examples of verbs used both intransitively and transitively.

Intransitive use	Transitive use
My father is reading.	He is reading the newspaper.
The play ended at five.	Rain ended the play.

Verb—Voice

There are two voices:

1. Active voice
2. Passive voice

Notice the change in the following sentences.

1. She has done the work. (A.V)
   The work has been done by her. (P.V)
2. These engineers can draw good designs. (A.V)
   Good designs can be drawn by these engineers. (P.V)
3. The Principal read the report. (A.V)
   The report was read by the Principal. (P.V)

NOTES

1. Depending on the emphasis we want to lay, we use a particular voice in a particular context.
Examples:
Some people dug a well to provide water to the village (A.V)
A well was dug to provide water to the village (P.V)
Here we would prefer to use the second rather than the first sentence, because what we want to refer to is not the action of the Panchayat but the result of the action, namely, a well, being provided in order to supply water to the village. Who dug the well is not the main idea in our minds.

2. Intransitive verbs do not have passive voice, i.e. we cannot transform a sentence from active to passive if the verb is intransitive.

For example, the sentence 'Birds fly in the sky' cannot be transformed into passive voice.

A more detailed note on voice is included in a later chapter.

Verb—Form and Use of the Tenses

There are 12 tense-structures in English.

PRESENT

Simple Present	:	I teach English.
Present Continuous	:	I am teaching English.
Present Perfect	:	I have taught English
Present Perfect Continuous	:	I have been teaching English.

PAST

Simple Past	:	I taught English.
Past Continuous	:	I was teaching English
Past Perfect	:	I had taught English.
Past Perfect Continuous	:	I had been teaching English

FUTURE

Simple Future	:	I will teach English.
Future Continuous	:	I will be teaching English.
Future Perfect	:	I will have taught English.
Future Perfect Continuous	:	I will have been teaching English.

Uses of Tenses—Some Important Rules

Simple present tense is used to express habitual actions, permanent or verifiable truths, or facts (scientific or universal).

1. He goes to church every Sunday.
2. Water boils at 100°c.
3. Calcutta stands on the banks of the Hoogli.
4. The sun rises in the east.

The simple present tense is used to express a planned future action or a series of such planned actions.

1. We leave Hyderabad at 8:30 p.m. next Tuesday and arrive in New Delhi at 11 a.m. on Thursday.

The present continuous tense is used to describe an action that is in progress at the time of speaking.

2. The children are playing in the garden now.

The present continuous tense is also used to describe an action that is in progress and will be continued, but not necessarily going on at the moment of speaking.

3. The college authorities are building a new hostel.

The present continuous tense can also express an action that has been arranged to take place in the near future and one's immediate plans.

1. I am meeting the CM tomorrow morning.
2. We are going to a movie this evening.

There are a number of verbs that are not normally used in the present continuous tense. These are:

1. verbs of perception :
   see, hear, smell, notice, etc.
2. verbs used to express feelings or states of mind:
   want, desire, wish, refuse, forgive, care, hate, like, admire, love, etc.
3. verbs involving the process of thinking:
   feel, know, mean, remember, forget, recall, etc.
4. verbs denoting possession:
   have, own, belong, possess, etc.
5. verbs such as:
   contain, consist, keep, seem, cost

When some of the above-listed verbs are used in the present continuous tense, their meanings change.

1. I see several mistakes in this book.
   I am seeing the principal at 3 p.m.
   (will be meeting)
2. We hear several rumours about the Minister.
   The judge is hearing the case tomorrow. (conducting the trial)
3. I have a house at Malakpet.
   I am having my breakfast. (act of eating)

The following sentences are wrong:

1. I am loving that girl. (love ✓)
2. They are understanding the lesson. (understand ✓)
3. I am slowly understanding you. (wrong)
   I have begun to understand you. (right)
4. I am having a telephone at my residence. (have ✓)

The present perfect tense is used to indicate an action that has just been completed.

Examples:
I have finished my work. (just now)

The present perfect tense is also used to represent a past action continuing to the present.

Examples:
We have lived in Hyderabad for ten years. (I.e. we are still living in Hyderabad.)

Do not use present perfect tense (has or have) when time is specified (e.g. last year, 1994) in the sentence.

Examples:
I have seen this film last year. (incorrect)

I saw this film last year. (correct)

Also note that for and since are commonly used with the present perfect tense. 'For' shows length of time and 'since' shows some point of time in the past as being the starting point of the action or event.

Examples:
1. My friend has lived in Hyderabad for twenty years. (He still lives here)
2. These monuments have been here since 1650 A.D. (still they are here)

The following are some time-expressions that go with the simple past and some that go with the present perfect.

Simple Past Tense	Present Perfect Tense
yesterday	so far
a week ago	since
recently	lately

Note the difference in meaning between the following two sentences:

Dr. Mohan lived in Delhi for fifteen years. (in the past)

Dr. Mohan has lived in Delhi for fifteen years.
(still Dr. Mohan lives in Delhi)

Simple past tense indicates an action completed sometime in the past.

Example: I lived in Hyderabad for ten years. (in the past)

Past Progressive Tense:

1. The past progressive (continuous) tense expresses an action that was in progress at a point of time in the past, having begun before that point and probably continuing after it.
   Examples:
   (a) I was reading the newspaper at 7 a.m. this morning.
   (b) I was having my breakfast at 9 a.m.
2. The past progressive tense expresses an action continuing over a period of time in the past.
   Example: The students in the hostel were listening to the cricket commentary the whole of yesterday.
3. The past continuous tense describes two or more actions going on at the same time; often the conjunction 'while' is used to connect the clauses.
   Example: While some boys were reading in the library, the others were playing.

Past Perfect Tense:
This tense is used when we wish to emphasize the sequence of the two actions in the past and when the earlier action has some relation to the later action or situation.

Examples:
1. On reaching the school, I found that I had forgotten to bring my English textbook.
2. When we reached the theatre, the play had begun already.
3. I borrowed some money from a friend of mine, since I had lost my purse.

The Future Tense:
There are several ways of expressing future time in English.

One of the most common ways to express this is to use shall or will with the bare forms of the verb : shall come, will go, etc. But there are other ways of expressing the future :

Example: Our cricket team leaves for Bombay this evening. They play two matches in Bombay. They play one at Poona. They return next Monday.

In the aforementioned sentences, the present simple tense is used to express a series of intended or planned actions in future.

1. The PM is visiting the city tomorrow.
2. The college team is playing a match with the city team next Sunday.

The present continuous tense is used in these sentences to express a future event. Usually, the time is mentioned (tomorrow, next sunday, etc.), and it is in the near future. Note that the verbs go and come are not usually used with going to. We don't say, for instance, He is going to go to Bombay tomorrow; we say, He is going to Bombay tomorrow.

Special expressions to indicate future:

1. The train is about to leave.
2. The President is about to speak.

This construction be about to + verb expresses events that are likely to happen in a very short while.

1. I am to be at a meeting at 5 o'clock.
2. There is to be an enquiry into the railway accident.

In these sentences, the form be + to + the base form of the verb is used to express a duty or necessity or planned course of action in the future.

Subject and Verb Concord (Agreement)

General rule: The verb must agree with its subject in number and person. In other words, the verb must be of the same number and person as the subject.

NOTE

In the correction of sentences section of many competitive examinations, the S–V concord is usually tested.

Rule 1: When two subjects are joined by 'and', the verb is plural.
My friend and his father are in India.

Rule 2: When two singular nouns joined by and refer to the same person or thing, the verb is singular.

The secretary and treasurer has been arrested.

The District Magistrate and Collector is on leave today.

NOTES

1. Article 'the' is used only once when the two nouns refer to the same person or thing.
2. If the two nouns refer to different persons or things, article 'the' is used before each noun. In such cases, the verb will be in the plural form.

Example: The secretary and the president have been given warm welcome.

Rule 3: If two different singular nouns express one idea, the verb should be in the singular form.

Bread and milk is good for breakfast.

Rice and curry is my favourite dish.

This is the long and the short of the matter.

Rule 4:
When two singular subjects are practically synonymous, the verb should be in the singular form.

The law and order situation in the state is under control.

His power and influence is on the decline.

Power and position has no charm for my friend.

Peace and prosperity is the need of the day.

Rule 5: If two singular subjects (combined by and) are preceded by each or every, the verb should be in the singular.

1. Every boy and girl was present in the class yesterday.
2. Every man and every woman has the right to express his or her view.

Rule 6: When the subjects joined by 'either–or' or 'neither–nor' are of different persons, the verb will agree in person and number with the noun nearest to it. Also, the plural subject must be placed nearest to the verb. (This is very important and)

Either Radha or Rajani has done this mischief.

Neither Mohini nor Ragini is beautiful.

* Either the chief minister or the cabinet ministers are responsible for this problem.

Neither you nor he is to take up this task.

Either you or I am responsible for this mistake.

Rule 7: If connectives such as with, together with, as well as, accompanied by are used to combine two subjects, the verb agrees with the subject mentioned first.

The President of India together with his personal secretaries is invited to this function.

The actress, along with her manager and some friends, is attending the function.

Mr. Michael, accompanied by his wife and children, is arriving tonight by train.

NOTE

If the conjunction and is used instead, the verb would then be plural.

Compare (i) Saritha and Rajitha are our professor's daughters.

Rule 8: When 'not only … but also' is used to combine two subjects, the verb agrees with the subject close to it.

Not only Harish, but also his brothers were arrested.

Rule 9: None / No

None can take either a singular or a plural verb depending on the noun which follows it;

Structure: none + of the + non-count noun + singular verb

None of the counterfeit money has been found.

Structure: none + of the + plural count noun + plural verb

None of the students have finished the exam yet.

No can take either a singular or plural verb depending on the noun which follows it.

Structure: No + singular noun + singular verb.
Example: No example is relevant to this case.

Structure: No + plural noun + plural verb
Example: No examples are relevant to this case.

Rule 10: Many words indicating a number of people or animals are singular. The following nouns are usually singular. In some cases, they are plural if the sentence indicates that the individual members are acting separately.

congress	family	group
committee	class	organisation
team	army	club
crowd	government	jury
minority	public	

Examples of collective nouns used with a singular verb:
The committee has met, and it has accepted the proposal.
The family was happy at the news.
The crowd was wild with excitement.
The Congress has initiated a new plan to combat inflation.
Our team is certain to win the match.
Some collective nouns are used in plural:

Examples:
The committee have arrived by different trains.
The family were fighting over inheritance.
The family living next door often quarrel among themselves

Rule 11: Majority can be singular or plural. If it is alone it is usually singular, if it is followed by a plural noun, it is usually plural.

The majority believes that the country can progress.

V

The majority of the lecturers believe that the student has not copied in the examination.

V

Rule 12: A number of/the number of observe the two structures:

1. A number of + plural noun + plural verb.
2. The number of + plural noun + singular verb.

Examples:
1. A number of students are going to the class picnic.
2. The number of days in a week is seven.
3. The number of residents who have been living in this colony is quite small.
4. A number of the applicants have already been interviewed.

Rule 13: Collective nouns indicating time, money, and measurements used as a whole are singular and take a singular verb.

Twenty-five rupees is not such a big amount for him.
Two miles is too much for this man to run.

Rule 14: When a lot of, a great deal of, plenty of, most of, and some of refer to number, a plural verb is used.

Examples:
A lot of people were present in the gallery.
Some of the students were absent.

NOTE

If these expressions refer to an amount or an uncountable noun, the verb is in the singular number.

Examples:
A lot of work has to be completed before we go.
A great deal of work has been finished.

Rule 15: When the percentage or a part of something is mentioned with plural meaning the plural verb is used.

Examples:
30% of Indian women are literate.

Three-quarters of the food has been eaten.

(Here the reference is to the food as a whole.)

Rule 16: 'Barracks', 'headquarters', 'whereabouts' 'alms', etc. take a singular verb, as well as the plural verb.

The headquarters of the UNO is/are New York.

Rule 17: In sports, while referring to the players, the name of the country is followed by plural verb.

England have won the World Cup.
V

Rule 18: When the word 'enemy' is used in the sense of 'armed forces' of a nation with which one's country is at war, we have to use the plural verb:

The enemy were forced to retreat.

Sample Exercise–10

Direction for questions 1 to 4: Fill in the blanks with 10 appropriate verbs.

1. I am sure that you will soon ________ to this environment.
(A) get used (B) be used
(C) be habituated (D) be linked

2. I ________ him from taking unnecessary loans.
(A) distinguished (B) dissuaded
(C) diverted (D) dissented

3. He ________ his studies.
(A) has left (B) is learning
(C) deserts (D) has discontinued

4. This big book ________ 30 chapters.
(A) is comprising of (B) consists
(C) comprises of (D) comprises

Non-Finites

Verbs can be classified as

1. Finites and
2. Non-finites.

A finite verb is a verb that denotes tense.

Example:
He goes to school.

'Goes' represents the simple present tense.

Similarly 'went', 'gone' are the other forms of the verb. These have been discussed in a later chapter.

Non-finites are verb forms that do not give complete (finite) meaning. They have none of the tenses, have no number, and take no modals (auxiliary verbs). There are four types of non-finites:

1. Infinitive
2. Gerund
3. Present participle
4. Past participle

Look at these sentences:
1. She wants to buy a new house. (Infinitive)
2. Buying a new house needs money. (Gerund)
3. I found him looking for a new house. (Present participle)
4. Wounded in the war, the soldier was brought to the medical camp. (Past participle)

The Infinitives

1. Look at the forms of the infinitive in the following sentences:
(a) Rajitha wanted to learn photography.
(b) She is reported to be learning it.
(c) She is reported to have learnt painting already.
(d) She is known to have been learning music for the last five years.

In these sentences, you see four forms of the infinitive. The following are the structures of these four infinitives.

Sentence (a)	to + verb
Sentence (b)	to be + verb -ing (present participle)
Sentence (c)	to have + past participle
Sentence (d)	to have + past participle of be + verb -ing

Some of these forms have the corresponding passive voice.

to learn—to be learnt:
There is a great deal to be learnt in photography.
To have asked—to have been asked:
He was happy to have been asked to deliver the convocation address.

2. The aforementioned infinitives have the word to before them.
   But look at these sentences:
   (a) The mother made the child drink the medicine.
   (b) Then she let the child go out and play.

'To' is very commonly used with the infinite, but it is not a necessary sign of the infinitive. The following verbs take the infinitive without 'to'.

1. The principal verbs: can, must, let, make, bid, dare, please, need, etc.
2. Verbs denoting some kind of perception: see, hear, feel, watch, know, think, etc.

Examples:

1. Make her do the work.
2. You need not go there.
3. Please go there.
4. Let him stand there.

The infinitive without 'to' is used after had better, had rather, had sooner, would rather, sooner than, rather than.

Examples:

1. You had better meet your uncle.
2. He would rather fight than give up.
3. I had rather write than play.
4. She had sooner run than walk.

SAMPLE EXERCISE–11

Rewrite the following sentences using to, wherever necessary, before the verbs given in the brackets.

1. Dr. Rao asked his daughter _____ home before 9 o' clock. (come)
2. If you have planned _____ the airport before five, you had better _____ at once. (reach, start)
3. He wouldn't let anyone _____ his bicycle. (use)
4. We wanted _____ the house where Mahatma Gandhi was born. (see)
5. He would rather _____ than _____ a lie. (die, tell)

THE GERUND

The gerund and the present participle have the same form, verb + ing : swimming, walking, cutting, etc. (the gerund is also called the verbal noun)

Uses of the gerund:

1. As the subject of a verb:
   Swimming is a good exercise.
   Smoking is injurious to health.
   Speaking is easier than writing.
   Looking after children needs a lot of patience.
2. As the object of a verb:
   He likes reading detective novels.
   Would you mind returning these books to him?
   I hate waiting at bus stop.
3. As the object of a preposition:
   The minister inaugurated the school by lighting a lamp.
   He was fined for being drunk.
   He is thinking of resigning his job.
   They resumed their journey after resting for an hour.
4. As the subject complement:
   Talking to him is wasting time.
   Seeing is believing.

NOTE

When a noun or pronoun is placed before a gerund, it must be in the possessive case.

Example:

- I was pleased at Robert coming here (Incorrect)
- I was pleased at Robert's coming here (Correct)
- They insisted upon me writing the essay. (Incorrect)
- They insisted upon my writing the essay. (Correct)

But the possessive case cannot be used with the gerund in the following cases.

1. When the noun denotes a lifeless thing.

Examples:

There is no danger of the wall's falling down. (Incorrect)

There is no danger of the wall falling down. (Correct)

NOTES

1. The following verbs can take either an infinitive or a gerund as an object.

Continue regret	try	propose
forget	neglect	remember learn

2. The following verbs can be followed by the infinitive (to + V) as the direct object.

agree	forbid	learn	offer
care	forget	plan	decide
hope	pretend	deserve	intend
mean	refuse		fail

3. The following verbs can be followed by the gerund (V + ing) as the direct object.

admit	deny	postpone	appreciate
enjoy	practise	avoid	finish
stop	cannot help	keep	suggest
consider			

4. The following verb phrases (verb + preposition) can be followed by the gerund (V + ing). Remember that gerunds, not infinitives, follow prepositions in general.

decide on	think about	keep on	think of
plan on	put off	look forward to	

The Participle

Look at the words underlined in these sentences.

1. The crying child gets milk.
2. We found a man running for the bus.
3. Inaugurating the seminar, the Education Minister spoke about the prohibition policy.

The words underlined are participles. There are two types of participles.

1. The present participle
2. The past participle

The form of the present participle:

Verb + ing	Example:	cry + ing

The form of the past participle:

V + ed	Example:	wound + ed
	(OR)	
V + en	Example:	drunk + en

Wrong use of participles:

1. Whenever a participle is used, the noun or pronoun to which it refers, must also be mentioned.
   Look at the following sentence:
   While taking a bath, the shampoo bottle fell.
   What is wrong with this sentence?

The sentence, as it stands means that the 'shampoo bottle' was taking a bath. In other words, there is no noun or pronoun to which the participle 'taking' refers. Hence, the sentence should be re-written, as 'while I was taking bath, the shampoo bottle fell.'

Given below are a few more examples.

1. Being a rainy day, I did not go out. (Incorrect)
   It being a rainy a day, I did not go out. (Correct)
2. Crossing the road, a truck hit her. (Incorrect)
   While she was crossing the road, a truck hit her. (Correct)

NOTES

1. Sometimes the spellings of the past participles are not different in the case of some verbs.
Example: come, hit, etc.
2. Sometimes, the present participle and the past participle can function like adjectives.

Examples:

1. The wounded soldier was carried to the medical camp.
   past. part.
2. A rolling stone gathers no moss.
   pre. part.

Correct Use of Prepositions

A preposition is a word placed before a noun or a pronoun or a noun equivalent and shows some relationship between that and some other word in the sentence.

1. Mohan gave a lecture on patriotism.
   prep N
2. The prize is given to her
   prep. pr. noun
3. His objection is to what all you say.
   prep. noun eq. (clause)

The words underlined are called prepositions because they normally take position before (pre) a noun. The preposition is said to 'govern' the noun that follows it. The noun is said to be the 'object' of the preposition.

Besides single-word prepositions, there are also phrases which do the work of prepositions and are called 'phrase prepositions.'

Types of Prepositions:

1. Single-word prepositions
   in, on, after, at, with, under, above, etc. (These are simple prepositions.)
2. Phrase prepositions (complex prepositions)

Some types of complex preposition structures with examples are given below.

1. Adverb + preposition
   along with, apart from, as for, as to, away from, onto, out of, together with, upto, such as, instead of
2. Verb/adjective/conjunction, etc. + preposition
   except for, owing to, due to, but for, because of.
3. Preposition + noun + preposition
   by means of, on account of, in comparison with, in accordance with, in view of, in spite of,

NOTE

On certain occasions, it is both necessary and correct to end a sentence with a preposition. English is a flexible language and defies pedantic rules of grammar.

Now, look at the following sentences:
This is the boy I gave the book to.
This is the house that I was born in.
In these sentences, the preposition is used at the end of a sentence.

Object of the Preposition

The noun or noun-equivalent (pronoun, adverb, gerund, infinitive, adverbial clause, or any clause that can be used as the object of the preposition) before which the preposition is placed is called its object.

Examples:

The glass is on the table.	(noun)
I depend on him.	(pronoun)
Go away from here.	(adverb)
He is fond of playing.	(gerund)
She was well till a few days ago.	(adverbial phrase)
I shall see it for what it is worth.	(adverbial clause)

Rules Pertaining to the Use of Prepositions

Rule 1: A preposition is placed at the end of a sentence in the following ways:

1. When the relative pronoun is 'that':
   Example: Here is the pen that you are looking for.
2. If a preposition governs a relative pronoun:
   Example: This is the student whom I spoke about.
3. When the relative pronoun is understood:
   Example: This is the person you spoke to.
4. If a preposition governs an interrogative pronoun or an interrogative adverb:
   Example: What are you looking at?
5. When the preposition is used with the infinitive placed at the end of the sentence:
   Example: Do you have a chair to sit on?
6. When the object governed by the preposition is placed first:
   Examples: This I insist on.
   He is known all the world over.

Rule 2: A preposition can also be used at the beginning of an interrogative sentence.

Examples:
In which city do you live?
To whom are you referring?

Rule 3: There are many words which can be used as prepositions or as adverbs. The most important of these are about, above, across, along, before, below, behind, besides, by, down, in, near, off, over, past, round, through, under, up, etc.

Examples:

Preposition	Adverb
He was here before ten.	*He has done this before.*
He was behind us.	*She is long way behind.*
The shop is just round the corner.	*Come round and see me this afternoon.*

Rule 4: There are some words with prepositions, which require gerunds after them.

Examples:

refrain from hurting	abstain from drinking
prevent from working	aid at doing
persist in disobeying	addicted to gambling
succeed in doing	bent upon doing
disqualified from doing	averse to playing
knack of getting	expert in inventing
prohibit from entering	desist from talking
tired of writing	capable of teaching
pretext for delaying	desirous of going
fond of playing	insist on going

Rule 5: The verbs that are placed immediately after prepositions are usually in the gerund form.

Examples:
Manohar insisted on buying a television.
She left the hotel without paying the bill.

Rule 6: Some words with prepositions can take the gerund as well as the infinitive.

Examples:
He is afraid of going out alone at night.
He is afraid to go out alone at night.

Rule 7: Some intransitive verbs become transitive by placing prepositions after them.

Examples: laugh at, listen to, depend on/upon, prevail on/upon, etc.

Rule 8: In and Within

1. In refers to the end of a period of time usually in the future, e.g. He will return in a month. (at the end of one month)
2. Within means before the end of a period of time.
   (at any time before the specified period)
   He will return within a month. (he may come after two weeks also)

Rule 9: In and Into

1. 'In' indicates rest or motion inside anything.

   Examples:
   She is in the garden. (rest, inside)
   She is walking in the garden (motion inside)
2. 'Into' means motion towards the inside of anything.

   Examples:
   I walked into the garden.
   Thieves broke into my friend's house yesterday.

Rule 10: On and Upon
1. On is used:
   (i) in speaking of things at rest.
   He sat on a big stone.
   (ii) before the names of days and dates.
   On Friday, on the 2nd of August, etc.
   (iii) to denote support for and concern about somebody or something.
   He lives on his maternal uncle's wealth.
   I wrote books on politics.
2. Upon is used:
   in speaking of things in motion.
   The tiger sprang upon the goat.

Rule 11: Beside and Besides

Beside means 'by the side of'

Example:
My house is beside the Kali temple.

Besides means in addition to or moreover

Examples:
- There are four professors in the department besides the head of the department.
- Besides English we are taught French.

Rule 12: Between and Among

Between is used for two persons or things

Examples: Distribute these sweets between the two children.

There is a good understanding between him and her.

Among is used for more than two persons or things.

Example: The boys were fighting among themselves in the absence of their teacher.

Rule 13: By is used to mean:

1. according to; from the evidence of
   By my watch, it is 10.30.
2. to denote the doer of an action in the passive voice.
   The thief was beaten by the policeman.

Rule 14: Prepositions from, since, for with reference to time.

From, since indicate a point of time.

1. I have not seen her since Monday.
   I have not seen her from August.
   For indicates a length or period of time.
2. I have not seen her for six months.

SAMPLE EXERCISE–12

Direction for questions 1 to 5: Fill in the blanks with appropriate prepositions.

1. The aggrieved party demands that the CBI should investigate _____ the case.
(A) into (B) in
(C) at (D) no preposition required

2. He is persistently prevailing _____ his wife to resign her job.
(A) to (B) upon
(C) at (D) over

3. Though she searched _____ the ring everywhere, it was in vain.
(A) for (B) about
(C) out (D) no preposition required

4. The speaker has not yet entered _____ the assembly hall.
(A) into (B) in
(C) to (D) no preposition required

5. Which pen would you like to write _____?
(A) for
(B) with
(C) in
(D) no preposition required

CORRECT USE OF CONJUNCTIONS

A conjunction is a word which connects words, phrases, clauses or sentences. It also brings about relationship between the elements which are thus joined. There are two types of conjunctions:

1. Co-ordinate conjunctions
2. Subordinate conjunctions

A co-ordinate conjunction joins two clauses or sentences of equal rank. Also, it joins two words of equal grammatical rank.

And, but, for, nor, or, but, otherwise, else, also, either–or, neither–nor, etc. are the chief co-ordinate conjunctions.

Examples:
He went to the hospital and met the doctor.
Dr. Rao and Dr. Reddy are best friends.
The co-ordinate conjunctions are of four kinds:

1. **Cumulative Conjunction:** A conjunction which adds one statement or fact to another is a cumulative conjunction.

   Example:
   The professor as well as the lecturer has accepted to conduct the examination next week.

   The following are the cumulative conjunctions:
   Not only ----- but also,
   Both ----- and,
   as well as
   too, also, moreover etc.

2. **Alternative Conjunctions:** A conjunction of this kind expresses a choice between two alternatives.
   Either–or, neither–nor, otherwise, else, etc. are alternative conjunctions.

   Examples:
   She is good neither at games nor at studies.
   Work hard, otherwise you will fail.

3. **Adversative Conjunctions:** An adversative conjunction expresses a contrast between two facts or statements.
   Only, however, but, still, yet, whereas, nevertheless, etc. are adversative conjunctions.

   Examples:
   She was angry, but she kept quiet.
   She hates me, yet I love her.

4. **Illative Conjunctions:** Such a conjunction shows that a statement or fact is proved or inferred from another.
   Therefore, hence, so, consequently, for, etc. are illative conjunctions.

   Example:
   He is honest and amiable, hence he is revered.

5. **Subordinate Conjunctions:** They are the conjunctions that connect a clause to another on which it depends for its full meaning.
   The adverbial clauses are usually connected to the main clauses by means of the subordinate conjunctions.

The chief subordinating conjunctions are after, because, if, another, though, till, etc.

The following are some sentences in which important subordinate conjunctions are used.

1. The patient had died before the doctor arrived.
2. We eat so that we may live.
3. He behaved in such a manner that all disliked him.

The following compound expressions also can be used as conjunctions.

Examples: In order that, on condition that, even if, so that, provided that, as though, as well as, as if.

NOTE

There are some words that are used as both conjunctions and prepositions.

Conjunctions	Prepositions
We went home after he came to the office	*We went home after sunset.*
I went to bed early, for I was tired.	*I shall do it for him.*

The following conjunctions are used in pairs and hence are called correlative conjunctions.

Either … or
Neither … nor
Both … and
Whether … or
Not only … but also

NOTE

When conjunctions are used as correlatives, each of the correlated words should be placed immediately before the words to be connected.

For example

- He not only visited Bombay but also Pune. (wrong)
- He visited not only Bombay but also Pune. (correct)
- He is neither good at Mathematics nor at Science. (incorrect)
- He is good at neither Mathematics nor Sceince. (correct)

Sample Exercise–13

Direction for questions 1 to 10: Fill in the blanks with appropriate conjunctions.

1. I don't know how to rate him; he is neither innocent ________ intelligent.
(A) And (B) Nor
(C) Or (D) But

2. I am ready to help him, ________ he may not accept it.
(A) Therefore (B) However
(C) Even (D) Despite

3. In functions, better be first as far as food is concerned ________ you might starve.
(A) Lest (B) Otherwise
(C) So that (D) Hence

4. He says that he is speaking the truth but I don't think ________.
(A) Still (B) Although
(C) So (D) No conjunction

5. ________his ill health, he has successfully completed the given task.
(A) Despite (B) Besides
(C) Because of (D) Owing to

6. Beyond doubt he is rich, ________ he could afford to study abroad.
(A) As well as (B) Though
(C) Still (D) Hence

7. ________ it was very hot, we could not walk further.
(A) Because (B) As such
(C) Although (D) Though

8. Many people waste power and water ________ many others are completely deprived of it.
(A) When (B) Because
(C) While (D) For

9. The weather is very hot, ________ it is middle of May.
(A) While (B) Since
(C) Supposing (D) Hence

10. He is very poor ________ he managed to get a good education.
(A) Where (B) Nevertheless
(C) Despite (D) Inspite

Notes on Phrasal Verbs

Phrasal verbs: Certain verbs when followed by certain prepositions or adverbs acquire a new significance.

They are said to be phrasal verbs.

Given below are a few phrasal verbs with meanings and usage. Study them carefully.

1. Back up: to support; to sustain.
   You need to back up your statement with correct data.
2. Bear down: to overthrow; to overcome.

When people could not endure the atrocities of the government in power, they bore it down.

3. Bear with: to tolerate.
 I cannot bear with his peevish nature any further.
4. Break out: to escape from a place or a situation.
 I think Archana needs to break out of her daily routine and do something exciting.
5. Break in/into: to enter a building by force.
 Burglars broke into my neighbour's house while they were not at home.
6. Call for: to demand.
 The Reading Comprehension section calls for good reading skills from the students.
7. Call up: to recall.
 The beautiful scenery around called up memories of my childhood.
8. Come by: to obtain.
 How did you come by that beautifully embroidered skirt?
9. Come upon: to find by accident.
 I came upon some very interesting facts about our planet in this Encyclopaedia.
10. Go about: to start working on something, to tackle.
 Can you just explain to me how to go about solving this problem?
11. Go by: to be guided by.
 This is a good example to go by while solving the problems.
12. Hang back: to hesitate to proceed.
 Don't hang back, you can do it as well as anyone else can do.
13. Hang on: to depend on.
 Your success hangs on how much effort you are going to put in.
14. Fall back: to recede, or to retreat.
 Having seen the armed guards, the civilians fell back.
15. Fall off: to withdraw; to drop off.
 A true friend does not fall off in adversity.
16. Live for: to devote one's life to.
 All through his life he lived for the amelioration of the poor and the destitute.
17. Live on: to depend on.
 Most people of this region live on rice and fish.
18. Lay about: to attack some one violently.
 A group of ruffians laid about him with sticks while he was returning home.
19. Lay out: to plan how something should look and arrange it in this way.
 The ancient Chinese laid out beautiful gardens with lawns and flower beds.
20. Make away with: to squander.
 After his father's demise, he made away with the entire property.

Parallelism

1. Items in a series must be parallel: that is, they must have the same grammatical form.

(a) NOUNS He likes music, art, and history.
N N N

(b) GERUNDS He likes swimming, dancing, and riding.
V+ING V+ING V+ING

(c) ADJECTIVES He is tall, dark, and handsome.
ADJ ADJ ADJ

(d) ADJECTIVES They wanted to paint the living room, to lay a new carpet, and to buy a new sofa.
INFIN. INFIN. INFIN.

(e) PAST TENSE The Romans conquered, colonized, and governed much of the world.
PAST PAST PAST

(f) PAST PERFECT TENSE

He had finished the game, had taken a shower, and had eaten lunch by the time I went to his house.

NOTE

The auxiliary 'had' may be omitted in the second and the third verb phrases.

2. The structure joined by and, but, as, or then, or although must have the same grammatical form.

(a) AND
He enjoyed the music of Spain and the sculpture of France.
N PHRASE N PHRASE

(b) BUT
That verb form is not active but passive
ADJ ADJ

(c) AS
Taking the bus can be as costly as taking a plane.
V+ING V+ING

(d) OR
He wanted to borrow a car or to hire one while his car was being repaired.
INFIN. INFIN.

(e) THAN
Eating in a restaurant is more fun than cooking at home.
V+ING V+ING

(f) ALTHOUGH
Although he liked to eat good food, he did not like to pay a high price for it.
INFIN. INFIN. INFIN.

Question Tags

Question Tags are a feature of most languages, but English differs from many of them. Consequently, the following error is sometimes found.

Examples:

You are going to the cinema this evening, isn't it? (Incorrect)

You are going to the cinema this evening, aren't you? (Correct)

The following rules pertaining to the use of question tags should be borne in mind to avoid making errors:

1. Use the same auxiliary verb as in the main clause.
2. If the sentence has no auxiliary verb, use do, did, or does
   Examples:
   John sings very well, doesn't he?
   John does not sing very well, does he?
3. If the given sentence is positive, the tag is negative. If the given sentence is negative, the tag is positive.
   Examples:
   Positive Statement Negative Tag
   Rajini is a music teacher, isn't she?

   Negative Statement Positive Tag
   Rajini is not a music teacher, is she?
4. Don't change the tense.
   The tense of the verb in the tag should be the same as the one in the statement.
   Examples:
   Mahesh did not accept the job, did he ? (correct)
   Mahesh did not accept the job, doesn't he? (incorrect).
5. Both the main sentence and the tag should have the same subject. The tag must contain the subject form of the pronoun.
   Example: You are teaching them grammar, aren't you?
6. If the main sentence consists of the forms like it is, there are and there is; the question tag also takes there or it.
   Example: There are only twenty boys in this class, aren't there?

Observe the following: (All of the following are correct)

1. Dr. Rao is a professor of English, isn't he?
2. My cousin cannot run fast, can he?
3. She mustn't see the film, must she?
4. We used to live in Nigeria, didn't we?
5. You couldn't swim a mile, could you?
6. Mr. Brown will be our new principal, won't he?
7. I am not a good player, am I?
8. I am a good player, aren't I?

ACTIVE-PASSIVE VOICE

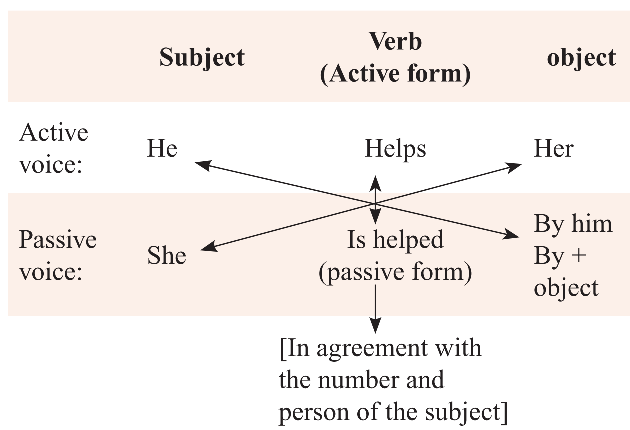
In English, the active voice is more common than the passive voice, although the passive voice is acceptable and even preferred at times. It is the context that is to be taken into consideration while deciding upon the type of construction: active or passive. While both types of construction may be grammatically correct, one of them could be more elegant. The following are some of the points of observation.

1. When the subject of the sentence is the doer of the action, denoted by the verb, then the sentence is said to be in active voice.*
   * **Example:** The hunter shot the tiger.
   s v o
2. When the subject of the sentence is the receiver of the action denoted by the verb, then it is said to be in the passive voice.
   The tiger was shot by the hunter.
   s v o

   Note: The subject of the verb in the active voice is made the object of the verb in the passive voice and is introduced with the preposition 'by'. Sometimes, the word 'by' is omitted when the agent is not required to be mentioned.

   Example: He was elected the leader of the group.

This can be shown graphically:



3. The passive voice is preferred when the doer of the action is unknown or unimportant.
   The cure for cancer will probably be discovered by some unknown scientist in a laboratory.
   ('the cure for cancer' is the main idea in the sentence and not 'the unknown scientist')
   That church was built in the 16th century.
   (who built the church is unimportant and hence not mentioned)

4. When discussing history, the passive voice is often used.
   Example:
   The war was fought over gold.
5. The active voice is used when the subject is more important than the object.
   We watched the news.
   Archimedes discovered the laws of floating bodies.
6. Avoid using active and passive in the same sentence.
   The food was cooked and served.
   passive passive
   Susan cooked the dinner and washed the dishes.
   active active
7. Use one verb instead of two whenever possible.
   Saritha enjoys good food and music.
   (Saritha enjoys good food and music is also enjoyed by her.)

The following table will help you to note the changes from the active voice to the passive voice:

	Tense	Active	Passive
1.	*Simple present tense*	*He writes a letter*	*A letter is written by him*
2.	*Present continuous*	*He is writing a letter*	*A letter is being written by him.*
3.	*Present perfect tense*	*He has written a letter*	*A letter has been written by him.*
4.	*Present prefect continuous tense*	*He has been writing a letter*	*No passive form.*
5.	*Simple past tense*	*He wrote a letter*	*A letter was written by him.*
6.	*Past continuous tense*	*He was writing a letter*	*A letter was being written by him.*
7.	*Past perfect tense*	*He had written a letter.*	*A letter had been written by him.*
8.	*Past perfect continuous tense*	*He had been writing a letter.*	*No passive form.*
9.	*Simple future*	*He will write a letter.*	*A letter will be written by him.*
10.	*Future continuous tense*	*He will be writing a letter.*	*No passive form.*
11.	*Future perfect tense*	*He will have written a letter.*	*A letter will have been written by him.*
12.	*Future perfect continuous tense*	*He will have been writing a letter.*	*No passive form.*

NOTE

Sentences with intransitive verbs cannot be changed to passive voice because such verbs do not take objects after them.

Example:
The sky is blue.
He goes to school. (intransitive verb)

Additional Examples

In the following examples, we cannot say that the first sentence is absolutely wrong. In certain contexts, it may even be preferred. However, generally speaking, the corrected sentence (the second sentence) is preferrable. We are calling the first sentence AWKWARD and the second sentence BETTER.

1. AWKWARD: Ice cream was eaten at the party by the children.
   BETTER: The children ate ice cream at the party.
2. AWKWARD: Some people painted pictures of animals on ancient cave walls.
   BETTER: Pictures of animals were painted on ancient cave walls.
3. AWKWARD: Henry likes swimming and golfing is also liked by him
   BETTER: Henry likes swimming and golfing.

Direct and Indirect Speech

Speech can be reported in two ways

1. The actual words of the speaker can be repeated. This is called DIRECT SPEECH
   Example: Meena said, 'I don't want to play now'.
2. We can report what the speaker said without quoting his exact words. This is called INDIRECT SPEECH.
   Example: Meena said that she didn't want to play then.

The verb that introduces the reported speech is called the reporting verb and the words that are put within inverted commas are called the reported speech.

Direct speech is always enclosed within inverted commas and always begins with a capital letter.

It is separated from the reporting verb (said) by a comma.

Rules for changing direct speech into indirect speech.

1. No inverted commas are used in indirect speech, and no comma is used after the reporting verb.

2. The tense of the reporting verb is never changed.
3. The conjunction 'that' is used after the reporting verb in the case of statements.
4. When the reporting verb is in the past tense, the present tenses of the direct speech are changed into the corresponding past tenses.

   (a) Simple present becomes simple past

   Examples:
   Raju said, 'I am happy.'
   Raju said that he was happy.

   (b) Present continuous becomes past continuous

   Examples:
   Madhav said, 'My father is sleeping'. Madhav said that his father was sleeping.

   (c) Present perfect becomes past perfect.

   Examples:
   Umesh said, 'I have failed in my duty'. Umesh said that he had failed in his duty.

   (d) 'Shall' of the future tense changes to 'should', 'will' changes to 'would' or 'should'.

   Example:
   He said to me, 'I will meet you tomorrow'.
   He told me that he would meet me the next day.

Exceptions

1. If the reporting verb is in the present or in the future tense, the tense of the verb in the reported speech does not change.

   Examples:
   He says, 'I will never come back'.
   He says that he will never come back.
   My father will say, 'I cannot buy you a car'.
   My father will say that he cannot buy me a car.
2. If the reported speech expresses a universal truth or a habitual fact its tense remains unchanged.

   Examples:
   The teacher said, 'Honesty is the best policy'.
   The teacher said that honesty is the best policy.
   Mother said, 'Mahesh drinks milk before going to bed every night'.
   Mother said that Mahesh drinks milk before going to bed every night.
3. When the reported speech contains a time clause and both the main verb and the verb in the time clause are in the simple past, the verbs remain unchanged.

   Examples:
   He said, 'The bus did not move till all the passengers were seated'.
   He said that the bus did not move till all the passengers were seated.

NOTE

If the main verb is in the simple past tense and the verb in the time clause is in the past continuous tense, it is usual to change the main verb to past perfect and leave the verb in the time clause unchanged.

   Examples:
   She said, 'Dhiren fell while he was crossing the road'.
   She said that Dhiren had fallen while he was crossing the road.
4. If the reported speech describes a state of affairs that still exists when the speech is reported, its tense remains unchanged.

   Examples:
   Ram said, 'My wife keeps complaining of aches and pains'.
   Ram said that his wife keeps complaining of aches and pains.
5. When the verb in the spoken sentence is in the past perfect tense, there is no change in the tense of the verb in the reported speech.

   Examples:
   He said, 'I had waited there for half an hour before the bus arrived'.
   He said that he had waited there for half an hour before the bus arrived.
6. (a) The simple past in direct speech becomes past perfect in indirect.

   Examples:
   The student said,' I wrote a letter yesterday'.
   The student said that he had written a letter the previous day.

   (b) Past continuous tense of direct speech is changed to past perfect continuous.

   Examples:
   Seema said, 'We were watching TV the whole evening'.
   Seema said that they had been watching TV all evening.
7. The pronouns of the direct speech are changed where necessary so that it is clear who said what to whom, about whom.

   Examples:
   He said, 'I don't understand you'.
   He said he didn't understand me.

   I said to him, 'I don't trust you'.
   I told him that I didn't trust him

NOTE

'Said' followed by 'to' changes to 'told'.

   They said, 'We will come again tomorrow'. They said that they would come again the next day.
   I said, 'I have done my duty'.
   I said that I had done my duty.

The teacher said, 'You have done well'.
The teacher said that I had done well.
He said, 'You should be regular to class'.
He said that I should be regular to class.

8. Words expressing nearness in terms of time and place become words of distance.

Direct	Indirect
this/these	that/those
here	there
now	then
ago	before
thus	so
today	that day
tomorrow	the next day
yesterday	the day before the previous day
last night	the night before/the previous night
next week	the following week

Examples:
He said, 'These are the books I am looking for'.
He said that those were the books he was looking for.

9. **Reporting different kinds of sentences**
   (a) **Statements**
   In indirect statements, the conjunction 'that' is placed after the reporting verb. However, it is often omitted. Indirect statements are introduced by such verbs as 'say', 'tell', 'reply', and 'inform'
   Examples:
   He said, 'I am feeling very weak'.
   He said that he was feeling very weak.
   She said to me, 'You must obey your parents'.
   She told me that I must obey my parents.
   Sheela said, 'No, I will not be able to come'.
   Sheela replied that she would not be able to come.
   Madhav said, 'The last train leaves at 10 o'clock'.
   Madhav informed me that the last train leaves at 10 o' clock.

   (b) **Questions**
   In reporting questions, the indirect speech is introduced by verbs such as 'asked', 'inquired', etc. In indirect questions, the question form changes to the statement form. That is, the verb is placed after the subject.
   Examples:
   He said, 'Where is the letter?'
   He asked where the letter was.
   If the question in direct speech begins with a question word (e.g. who, what, which, when), this word serves as a link between the reporting verb and the reported question.
   'Why did she come?' Sekhar said.
   Sekhar asked why she had come.
   If the question has no question word and can be answered with a 'yes' or 'no', the conjunction 'whether' or 'if' is placed after the reported verb.
   Examples:
   'Do you know French?' she said.
   She asked me if I knew French.

   (c) **Commands and Requests**
   In reporting commands and requests, the indirect speech is introduced by a verb expressing command or request and the Imperative mood is changed into the infinitive.
   In indirect commands and requests, a verb such as 'tell', 'ask', 'order', 'command', and 'request' is followed by the person addressed and the 'to infinitive'.
   Examples:
   He said to his servant, 'Bring me a glass of milk'.
   He ordered his servant to bring him a glass of milk.
   'Can you post this letter for me?' she said.
   She requested me to post that letter for her.

   (d) **Exclamations**
   In reporting exclamations and wishes, the indirect speech is introduced by a verb that expresses an exclamation or a wish such as exclaimed, praised, blamed, applauded, called to witness, wished and desired.
   Examples:
   'What a beautiful house you have!' my friend said.
   My friend exclaimed that I had a very beautiful house.
   Alice said, 'How clever you are!'
   Alice exclaimed that I was very clever.
   'Bravo! You have done well', the Principal said.
   The Principal applauded the boy for doing well.

Here are some more reporting verbs for statements.

declare	inform	maintain
state	propose	announce

When the spoken sentence is by way of a reply to a question, we can use 'answer', 'reply', 'respond', etc.

Some more verbs for reporting imperative sentences:

order	tell	pray
command	advise	suggest

Diction (Words often confused)

Diction is the choice of words. In English, there are many words which are usually confused. Some words have similar meanings but cannot be used interchangeably; that is, a choice must be made according to the grammatical situation. The following are some pairs of words that are often confused in their usage.

1.	Advice	: (noun)	The teacher gave much advice to the students.
	Advise	: (verb)	The doctor advised me to take nutritious food.
2.	Adapt	: (make suitable)	Novels are adapted for the stage.
	Adopt	: (take a child as one's own)	He adopted a son.
3.	Allusion	: (reference)	The allusion that I am stingy is a mistake.
	Illusion	: (false notion)	I do not have illusions about his ability.
4.	Amiable	: (pleasant and good-tempered)	Radha is quite an amiable person.
	Amicable	: (friendly)	The dispute is quite serious and therefore cannot be settled amicably.
5.	Apposite	: (relevant)	His speech was apposite to the occasion.
	Opposite	: (contrary)	Heavy is the opposite of light.
6.	Beside	: (by the side of, close)	His mother sat beside him.
	Besides	: (in addition to)	Is anyone else coming besides you?
7.	Childish	: (silly)	I don't like his childish behaviour.
	Childlike	: (innocent)	Gandhiji always put on a childlike smile on his lips.
8.	Confidant	: (person with whom one trusts with secrets)	Nehruji was a confidant of Gandhiji in political matters.
	Confident	: (to be sure)	I am confident of success in the examination.
9.	Continual	: (very frequent)	He had continual arguments with his wife.
	Continuous	: (going on without a break)	There was continuous rain yesterday.
10.	Deny	: (ascertain that something is wrong)	The minister denied the allegation that he had taken bribe.
	Refuse	: (decline to take something that is offered or to do something that one is asked to do)	He refused the money given as bribe.

Exercises

Grammar—Nouns/Articles/Pronouns

Basic

Direction for questions 1 to 5: Fill in the blanks with the correct pair of words or phrases so that the sentence is grammatically correct and meaningful.

1. ______ man loves ______ money.
(A) The . . . . the
(B) A . . . .the
(C) The . . . .no article
(D) No article required . . . . no article required

2. ______ student of the class ______ to finish the work by tomorrow.
(A) All . . . .has
(B) Each . . . .have
(C) Every . . . .were
(D) Each . . . .has

3. ______ platinum is ______ any other metal.
(A) The . . . .costlier than
(B) No article required . . . . costlier than
(C) The . . . .more costlier than
(D) No article required . . . . more costlier than

4. The ______ of my professors inspired me to ______ hard and win the competition.
(A) Advise . . . .practise
(B) Advice . . . .practise
(C) Advise . . . .practice
(D) Advice. . . . practice

5. In India, ______ power to commute a death sentence is vested ______ the president.
(A) The. . . . for
(B) No article required . . . . by
(C) No article required . . . .on
(D) The. . . . in

Direction for questions 6 to 10: Fill in the blanks with suitable articles.

6. ______ British ruled India for four hundred years.
(A) No article (B) A
(C) An (D) The

7. The teacher ordered the students not to make ______ noise.
(A) A (B) An
(C) No article (D) The

8. Women in many Middle Eastern countries are deprived of ______ freedom even to this day.
(A) A (B) An
(C) The (D) No article

9. Besides being a scholar, he is also ______ eminent orator.
(A) The (B) No article
(C) An (D) A

10. ______ Rome is one of the ancient cities of the world.
(A) An
(B) No article
(C) A
(D) The

Advanced

Direction for questions 11 to 14: Correct the following sentences where necessary.

11. The doctor has given me many informations about the disease.

12. The magistrate has issued a summon asking me to attend his court on Monday.

13. Our office has purchased new furnitures for our department.

14. Four thiefs broke into my cousin's house yesterday.

Direction for questions 15 to 20: Rewrite the following sentences after making the necessary corrections.

15. Good students like you and he should study regularly.

16. You, he and I are in the wrong.

17. Only you and him can do this work fast.

18. She helped everyone of those boys in doing their work.

19. Every teacher and every student should do their duty.

20. Neither of the boys have submitted their records.

Grammar—Verbs/Tenses/Auxiliary

Basic

Direction for questions 21 to 25: Fill in the blanks with suitable verbs.

21. We ______ with the photograph trying to hang it, till it fell to the floor and the frame broke.
(A) Wrested
(B) Wrestled
(C) Strived
(D) Wrecked

22. The young boy was lying in the middle of the road, and ______ in pain but nobody picked him up.
(A) Shaking
(B) Squirming
(C) Twisting
(D) Writhing

23. I took some flowers and chocolates with me for I was anxious to ______ for my thoughtlessness.
(A) Atone (B) Compensate
(C) Propitiate (D) Redeem

24. The ship was scheduled to ______ a couple of days later when it hit an iceberg and sank.
(A) Port (B) Dock
(C) Wharf (D) Moor

25. He ______ on his old feet but there was nobody to extend a helping hand.
(A) Tottered (B) Drooled
(C) Doddered (D) Lurched

Direction for questions 26 to 30: Fill in the blanks with appropriate verbs.

26. Sixty miles ______ a very long distance to drive.
(A) Are (B) Were
(C) Is (D) Has been

27. We hope that they ______ good health.
(A) Kept (B) Are enjoying
(C) Keep (D) Indulge in

28. The Prime Minister ______ to the US next week.
(A) Is flown (B) Fly
(C) Flied (D) Will fly

29. Did you ______ him any letter last week?
(A) Write for (B) Write to
(C) Write (D) Wrote

30. If I ______ you, I would have served them to the best of my ability.
(A) Were (B) Was
(C) Am (D) Will be like

Advanced

Direction for questions 31 to 35: Fill in the blanks in the given sentences so as to make sense. Select the correct word from the answer choices and mark its number as the answer.

31. In our country, agriculture must _____ pace with industrial development.
(A) Take (B) Make
(C) Loose (D) Keep

32. The people of South Asia _____ behind in economic development because of tensions and conflicts.
(A) Has been left (B) Had been left
(C) Left (D) Will be left

33. The motivation to _____ comes from a burning desire to achieve a purpose.
(A) Meet (B) Start
(C) Succeed (D) Idolize

34. He does not _____ the ugly aspects of human nature from his picture of life.
(A) Eliminate (B) Include
(C) Excuse (D) Extricate

35. After the shipwreck, they were _____ on the island for three days
(A) Stuck (B) Sleeping
(C) Marooned (D) Guided

Direction for questions 36 to 40: Correct the mistakes relating to the use of verbs in the sentences given below.

36. The 'Arabian Nights' are an interesting book.
37. The cost of all essential commodities have gone up.
38. Bread and milk are my only food for today.
39. Sanjay as well as his brothers have gone home.
40. The notorious dacoit with his followers have escaped.

Grammar—Adjectives/Adverbs

Basic

Direction for questions 41 to 45: Fill in the blanks with suitable adjectives.

41. In the face of public censure, the government was compelled to take a more ______ stand.
(A) Staid (B) Acquiescent
(C) Beneficial (D) Belligerent

42. The ultra modern building looked ______ in such a quaint old village.
(A) Competent (B) Devious
(C) Incongruous (D) Decorous

43. He was already well _____ when we reached the party a little late.
(A) Advanced (B) Sloshed
(C) Incapacitated (D) Slurred

44. I keep away from _____ people who always preach about morals, values and principles to everyone.
(A) Spiritual (B) Sacrosanct
(C) Sanctimonious (D) Devout

45. The preacher's _____ remarks were well received by the audience.
(A) Sententious (B) Sensuous
(C) Sequestrable (D) Sheathed

Direction for questions 45 to 50: Fill in the blanks with suitable adverbs.

46. They struggled ______ all through the wind and rain and finally reached their destination.
(A) Madly (B) Skillfully
(C) Manfully (D) Roughly

47. After winning the match, he ______ held the trophy aloft for all to see.
(A) Urgently (B) Surreptitiously
(C) Vicariously (D) Victoriously

48. As many people repose faith in him, he is ________ a leader of the masses.
(A) Irrefutably
(B) Arguably
(C) Interestingly
(D) Officially

49. In spite of the big hoardings and TV commercials, there were ________ any people in the theatre for the first show of the picture.
(A) Realistically (B) Hardly
(C) Reasonably (D) Discreetly

50. He was appointed the spokesman of the party ________ to focus on the ideology of the party.
(A) Pre-eminently
(B) Conspicuously
(C) Outstandingly
(D) Uniquely

Advanced

Direction for questions 51 to 57: Correct the mistakes relating to the use of adjectives in the following sentences where necessary.

51. I am more wiser than that student.
52. The climate of Hyderabad is better than Madras.
53. Many a student is attending the meeting today.
54. My friend is senior than me by 4 years.
55. Death is preferable than disgrace.
56. Kolkata is a worth-seeing city.
57. Shakuntala is better than any drama in Sanskrit.

Direction for questions 58 to 60: Correct the mistakes relating to the use of adverbs in the following sentences where necessary.

58. His house is further from the station than yours.
59. She was so quiet that hardly he noticed her.
60. No sooner did I go to the railway station when the train left.

Grammar—Conjunctions/Prepositions

Basic

Direction for questions 61 to 65: Fill in the blanks with suitable prepositions.

61. There is no tax _____ printed books.
(A) On (B) In
(C) With (D) Towards

62. India is a noble, affluent land, teeming _____ natural wealth.
(A) At (B) About
(C) For (D) With

63. After a very long and tiring walk, we rested ______ a tree.
(A) Below (B) Under
(C) Down (D) Behind

64. He died _____ cancer at a very young age.
(A) Of (B) With
(C) By (D) Off

65. _______ all your shortcomings, I still love you.
(A) With (B) Through
(C) For (D) Besides

Direction for questions 66 to 70: Fill in the blanks with suitable conjunctions.

66. Her son hasn't come back from abroad, he hasn't written to her ______.
(A) Neither (B) Either
(C) Even (D) Though

67. He ______ plays some games in the evenings.
(A) Hardly (B) Regularly
(C) Immediately (D) Cautiously

68. You will be allowed to enter by the gate ______ you bribe the gateman.
(A) Provided
(B) Nevertheless
(C) While
(D) Whereas

69. I started early ______ I might reach in time.
(A) So that (B) Lest
(C) For (D) Since

70. He went to bed early ______ to wake up early in the morning.
(A) Such that (B) As though
(C) Such as (D) So as

Advanced

Direction for questions 71 to 75: Correct the errors relating to the use of conjunctions in each of the following sentences.

71. It had been raining as he left the house.

72. Your action was either just or fair.

73. I shall not come unless I am not invited.

74. Unless you have no objection, I shall see you tomorrow.

75. God made the world so man made the town.

Direction for questions 76 to 80: Correct the error relating to the use of prepositions in each of the following sentences.

76. What is the time with your watch?

77. You must travel with a bus to reach in time.

78. He is suffering with malaria.

79. He has been working since five years.

80. It has been raining from morning.

Grammar—Active/Passive, Direct/Indirect

Basic

Direction for questions 81 and 82: Change the following sentences from direct to indirect speech.

81. Whenever you call him, he says, 'Indeed, I am thinking of calling you shortly'.
(A) Whenever you call him, he says that he was thinking of calling you just then.
(B) Whenever you call him, he said that he was thinking of calling you.
(C) Whenever you call him, he says that he is thinking of calling you just then.
(D) Whenever you call him, he says that he had been thinking of calling him just then.

82. Great people never say, 'We are busy and we don't have time'.
(A) Great people never said that they are busy and they don't have time.
(B) Great people never say that they are busy and they don't have time.
(C) Great people will never say that they are being busy and they don't have time.
(D) Great people would never say that they are busy and they don't have any time.

Direction for questions 83 to 85: Change the following sentences from indirect to direct speech.

83. My friend asked me affectionately what he could do for me when I was in trouble.
(A) When I was in troubles, my friend asked me, 'What could I do for you dear?'
(B) When I was in troubles, my friend had affectionately asked me, 'What I can do for you?'
(C) When I was in trouble, my friend asked me, 'What can I do for you dear?'
(D) My friend has asked me when I was in troubles, 'What can I do for you?'

84. The employee asked the employer whether there was possibility of a good hike in the salary.
(A) The employee asked the employer, 'Is there any possibility of a good hike in my salary?'
(B) The employee told the employer, 'Whether there is any possibility of a good hike in my salary?'
(C) The employee said to the employer, 'If there is any possibility of a good hike in my salary?'
(D) The employee said to the employer, 'Was there any possibility of a good hike in my salary?'

85. The doctor advised the patient to give up drinking.
(A) The doctor said to the patient, 'You should give up drinking'.
(B) The doctor warned the patient, 'Give up drinking'.
(C) The doctor requested the patient, 'Please give up drinking'.
(D) The doctor advised the patient, 'Give up drinking'.

Direction for questions 86 and 87: Change the following sentences from active to passive.

86. Buy this book.
(A) This book should be bought.
(B) This book shall be bought.
(C) This book is bought.
(D) Let this book be bought

87. Bring fruits immediately.
(A) Fruits may be brought immediately.
(B) Let fruits be brought immediately.
(C) Fruits might be bought immediately.
(D) All the above are correct.

Direction for questions 88 to 90: Change the following sentences from passive to active voice.

88. This law must be objected to by everyone.
(A) Everyone must object this law.
(B) Everyone had to object to this law.
(C) Everyone must object that law.
(D) Everyone must object to this law.

89. When is her household work done by her?
(A) When does she do her household work?
(B) When she does her household work?
(C) When has she done her household work?
(D) When she had done her household work?

90. Jawaharlal Nehru was said to have lived in great luxury in his childhood.
(A) People say that Jawaharlal Nehru had lived in great luxury in his childhood.
(B) People say that Jawaharlal Nehru lived in great luxury in his childhood.
(C) People say that Jawaharlal Nehru was lived in great luxury in his childhood.
(D) People had said that Jawaharlal Nehru lived in great luxury in his childhood.

Advanced

Direction for questions 91 and 92: Select the correct passive form of the given sentences.

91. We use this room only on special occasions.
(A) Only on special occasions, we use this room.
(B) This room will be used only on special occasions.
(C) This room is used by us only on special occasions.
(D) Only this room is used on special occasions by us.

92. Salesmen from all regions attended the meeting.
(A) The meeting has been attended by salesmen from all regions.
(B) Salesmen have attended the meeting from all regions.
(C) The meeting was attended by salesmen from all regions.
(D) The meeting by salesmen was attended from all regions.

Direction for questions 93 to 95: Select the correct active form of the given sentences.

93. This polish can be applied to any surface.
(A) You can apply this polish to any surface.
(B) Any surface can be applied by this polish.
(C) This polish can apply to any surface.
(D) You have applied this polish to any surface.

94. Was the window pane broken by you?
(A) Do you break the window pane?
(B) Did you break the window pane?
(C) You broke the window pane, did you?
(D) The window pane was broken by you.

95. The electric wires have been cut.
(A) They cut the electric wires.
(B) Someone had cut the electric wires.
(C) Someone has cut the electric wires.
(D) They have been cutting the electric wires.

Direction for questions 96 and 97: Select the correct reported speech of the given sentence.

96. The hotel manager asked the stranger, 'Where do you come from?'
(A) The hotel manager asked the stranger whether he knew where he came from.
(B) The hotel manager asked the stranger where he came from.
(C) The hotel manager enquired where did the stranger come from.
(D) The hotel manager asked the stranger where he had came.

97. She said, 'Sorry, I won't do this again'.
(A) She appologized and said that she wouldn't do that again.

(B) She said sorry and that she won't do it again.
(C) She said that she was sorry and that she would not do this again.
(D) She apologized and said that she shall not do it again.

Direction for questions 98 to 100: Select the correct direct speech of the given sentence.

98. The teacher asked them if they were making good progress.
(A) The teacher asked them 'Were they making good progress?'
(B) The teacher told them, 'Were you making good progress?'
(C) The teacher asked them, 'Are you making good progress?'
(D) The teacher asked them, 'If you are making good progress?'

99. He suggested that they should come early the next day and complete the work.
(A) He said, 'They should come early tomorrow and complete the work'.
(B) He said, 'Let us come early tomorrow and complete the work'.
(C) He said 'We may come early tomorrow to complete the work'.
(D) He said, 'Come early tomorrow and complete the work'.

100. The teacher asked the new boy what his name was, where he came from, and which school he had attended last.
(A) The teacher asked the new boy, 'What is your name? Where do you come from? Which school did you attend last?
(B) The teacher told the new boy, 'What is your name? Where are you coming from? Which school do you attend last?'
(C) The teacher said to the new boy, 'What is his name? Where does he come from? Which school he attended last?'
(D) The teacher asked the new boy, 'What your name is? Where you come from? Which school you attended last?'

Grammar—Phrasal Verbs

Basic

Direction for questions 101 to 110: Fill in the blanks in each sentence with the suitable word.

101. Unscrupulous builders may soon find it impossible to hide _____ terms that conceal the real floor area of the apartments they are selling.
(A) Away (B) Behind
(C) In (D) Out

102. The worker is patting _____ the bumps on the floor so that he can lay the marble properly.
(A) Up (B) On
(C) Down (D) Off

103. No sooner had the government come to know about the farmer's agitation than it sent the paramilitary forces to snuff it_____.
(A) Down (B) Out
(C) Off (D) About

104. The army of Yangon moved along the border to eliminate terrorist training camps and flush the cadres _____.
(A) In (B) Into
(C) Out of (D) Out

105. Given his special role in hounding _____ Saddam Hussein, it is Mr. Blair's coy reaction that is most interesting.
(A) To (B) At
(C) On (D) Out

106. According to Daniel Archibugi, it is impossible to deal _____ a democratic fashion with undemocratic governments.
(A) In (B) With
(C) Into (D) Out

107. South Korea is unable to shrug ____ its US oriented historical baggage of the 1950s.
(A) Away (B) Off
(C) Up (D) From

108. In Nafis' plays, we watch women writhing ____ pain in the patriarchal stranglehold.
(A) With (B) From
(C) In (D) Away

109. The media reported about victims of road accidents lying on the road and everyone hurrying by paying no attention to lives ebbing_____.
(A) Out (B) Away
(C) Off (D) In

110. The onus is clearly on the leadership of both the political parties to build on the spirit rather than to fritter it _____.
(A) On (B) Up
(C) Down (D) Away

Advanced

***Direction for questions** 111 **to** 120:* Select the appropriate phrasal verb from the given options.

111. She is a very friendly girl who can _____ with everyone.
(A) Get along (B) Get off
(C) Get ahead (D) Get away

112. Though I knew she was unscrupulous, I was completely _____ by her charming face and started believing her words.
(A) Taken in (B) Taken aback
(C) Taken over (D) Taken off

113. She is a woman of many talents and I have _____ her.
(A) Taken on (B) Taken after
(C) Taken to (D) Taken up with

114. The prisoner _____ of the prison but was soon recaptured.
(A) Broke out
(B) Broke through
(C) Ran out
(D) Rushed out

115. They had to _____ the party because of heavy rain.
(A) Call back (B) Cancel out
(C) Call off (D) Wipe out

116. These machines are programmed to _____ automatically in case of an emergency.
(A) Shut away (B) Tune out
(C) Shut down (D) Turn down

117. Most of the water bodies in this area have _____ which has lead to serve food and water shortage.
(A) Dried out (B) Dried up
(C) Dried off (D) Dried away

118. Our manager gave us the general plan and we _____ the minor details.
(A) Worked on
(B) Worked to
(C) Worked over
(D) Worked in

119. She _____ in any crowd because she is very beautiful and attractive.
(A) Stands aside (B) Stands out
(C) Stamps on (D) Stamps out

120. My friend, who considers himself an excellent driver, was put off when he saw so many cars easily _____ us.
(A) Catching up with
(B) Catching at
(C) Taking over
(D) Taking on

Answer Keys

Exercises

1. D	**2.** D	**3.** B	**4.** B	**5.** D	**6.** D	**7.** A	**8.** D	**9.** C	**14.** B
21. B	**22.** D	**23.** B	**24.** B	**25.** C	**26.** C	**27.** B	**28.** D	**29.** B	**30.** A
31. D	**32.** B	**33.** C	**34.** A	**35.** C	**1.** B	**42.** C	**43.** B	**44.** A	**45.** A
46. C	**47.** D	**48.** A	**49.** B	**50.** A	**61.** A	**62.** D	**63.** B	**64.** A	**65.** C
66. B	**67.** B	**68.** A	**69.** A	**70.** D	**81.** C	**82.** B	**83.** C	**84.** A	**85.** A
86. D	**87.** B	**88.** D	**89.** A	**90.** B	**91.** C	**92.** C	**93.** A	**94.** B	**95.** C
96. B	**97.** A	**98.** C	**99.** B	**100.** A	**101.** B	**102.** C	**103.** B	**104.** D	**105.** D
106. A	**107.** B	**108.** C	**109.** B	**110.** D	**111.** A	**112.** A	**113.** C	**114.** A	**115.** C
116. C	**117.** B	**118.** A	**119.** B	**120.** A					

Chapter 2

Vocabulary

CHAPTER HIGHLIGHTS

- *Vocabulary*
- *Synonyms*
- *Antonyms*
- *Analogy*
- *Reverse Analogy*
- *Verbal Reasoning*
- *Critical Reasoning*
- *Logical Reasoning*

Vocabulary

Common Roots

In this chapter, some of the most commonly used roots are given. Corresponding to the root in the first column, its meaning is given in the second column. Given in the third column are words (and their meanings) that contain the corresponding root given in the first column.

The verbal questions are based on the meanings of words or their opposites. These question types are designed to test the students vocabulary. This section includes

1. Synonyms
2. Antonyms

Verbal Analogies

The word 'Analogy' means a comparison. Vocabulary plays an important part here. This question type involves identifying the logical relationship between the words. This section includes

1. Analogy
2. Reverse Analogy

Verbal Reasoning

The ability to isolate the key issue and to identify irrelevant issues is important.

Logical reasoning is important. This section includes

1. Critical Reasoning
2. Logical Reasoning

Root	Meaning	Example	Meaning
ac, acr	*sharp*	*ACRIMONIOUS* *ACERBITY* *ACIDULATE*	*bitter, caustic* *bitterness of temper* *to make somewhat acid or sour*
aev, ev	*age, era*	*PRIMEVAL* *COEVAL* *MEDIEVAL or* *MEDIAEVAL*	*of the first age* *of the same age or era* *of the middle ages*
belli	*war*	*BELLICOSE* *BELLIGERENT* *REBELLIOUS*	*inclined to fight* *inclined to wage war* *resisting authority*

Root	Meaning	Example	Meaning
ben, bon	good	BENEFACTOR	one who does good deeds
		BENEVOLENCE	charity (wishing good)
		BONUS	something extra above regular pay
biblio	book	BIBLIOGRAPHY	list of books
		BIBLIOPHILE	lover of books
		BIBLE	The Book
cad, cas	to fall	DECADENT	deteriorating
		CADENCE	intonation, musical movement
		CASCADE	waterfall
carn	flesh	CARNIVOROUS	flesh-eating
		CARNAGE	destruction of life
		CARNAL	fleshly
ced, cess	to yield, to go	RECEDE	go back, withdraw
		ANTECEDENT	that which goes before
		PROCESS	go forward
chron	time	CHRONOLOGY	time-table of events
		ANACHRONISM	a thing out of time sequence
		CHRONICLE	register events in order of time
cracy	rule	THEOCRACY	that constitution of state in which god is regarded as sole sovereign
		GERONTOCRACY	government by old men.
		PLUTOCRACY	government by the wealthy
		BUREAUCRACY	a system of government by officials, responsible only to their departmental chief
		GYNAECOCRACY	government by women
		OCHLOCRACY	mob rule
		KAKISTOCRACY	government by the worst
		TIMOCRACY	a form of government in which property is the quality for office
cred, credit	to believe	INCREDULOUS	not believing, skeptical
		CREDULITY	gullibility
		CREDENCE	belief
culpa	fault, blame	EXCULPATE	free from blame
		CULPABLE	which can be punished
		CULPRIT	one who is at fault
dem	people	DEMOCRACY	rule of the people
		DEMAGOGUE	(false) leader of the people
		EPIDEMIC	widespread (among the people)
ego		EGOIST	person who is self-interested
		EGOTIST	selfish person
		EGOCENTRIC	revolving about self
err	to wander	ERROR	mistake
		ERRATIC	not reliable, wandering
		KNIGHT-ERRANT	wandering knight

Root	Meaning	Example	Meaning
eu	*good, well, beautiful*	*EUPEPTIC* *EULOGIZE* *EUPHEMISM*	*having good digestion* *praise* *substitution of pleasant way of saying something blunt*
gen	*to give birth to*	*GENE* *CONGENITAL* *GENEALOGY* *PSYCHOGENIC*	*unit or factor of heredity* *existing from birth* *study of the family tree* *born in or caused by the mind*
grad, gress	*go, step*	*DIGRESS* *REGRESS* *GRADUAL*	*to astray (from the main point)* *go backwards* *step by step, by degrees*
loqu, locut	*to talk*	*SOLILOQUY* *LOQUACIOUS* *ELOCUTION*	*speech by one individual* *talkative* *speech*
luc	*light*	*ELUCIDATE* *LUCID* *TRANSLUCENT*	*enlighten* *clear* *allowing some light to pass through*
magn	*great*	*MAGNIFY* *MAGNANIMITY*	*enlarge* *generosity, greatness of soul*
mal	*bad*	*MALEVOLENT* *MALEDICTION* *MALEFACTOR* *MALIGNANT*	*wishing evil* *curse* *evil-doer* *disposed to do harm*
mob, mot, mov	*move*	*MOBILIZE* *MOTILITY* *IMMOVABLE*	*cause to move* *ability to move* *not able to be moved*
pater, patr	*father*	*PATRIOTISM* *PATRIARCH* *PATERNITY*	*love of one's country (fatherland)* *male ruler of a family, group or state* *fatherhood*
path	*disease, feeling*	*PATHOLOGY* *APATHETIC* *ANTIPATHY*	*study of diseased tissue* *lacking feeling; indifferent* *hostile feeling*
phil	*to love*	*PHILANTHROPIST* *ANGLOPHILE* *PHILANDERER* *PHILOSOPHER* *PHILATELIST*	*benefactor, lover of humanity* *lover of everything English* *one involved in brief love affairs* *a lover of wisdom* *one who loves collecting stamps*
phoebe	*fear*	*ASTRAPHOBIA* *BALLISTROPHOBIA* *CLAUSTROPHOBIA* *ERGOPHOBIA* *NYCTOPHOBIA* *OCHLOPHOBIA* *PANOPHOBIA* *PHOTOPHOBIA* *SITOPHOBIA* *XENOPHOBIA* *TRICHOPHOBIA*	*fear of celestial space* *fear of missiles* *fear of closed place* *fear of work* *fear of night* *fear of mob* *a form of melancholia marked by groundless fears* *fear of light* *fear of food* *fear of strangers* *fear of hair*

Root	Meaning	Example	Meaning
poten	*able, powerful*	*OMNIPOTENT* *POTENTATE* *IMPOTENT*	*all-powerful* *powerful person* *powerless*
psych	*mind*	*PSYCHOLOGY* *PSYCHOSIS* *PSYCHOPATH*	*study of the mind* *mental disorder* *mentally ill person*
sacr	*holy*	*SACRILEGIOUS* *SACRAMENT*	*impious, violating something holy* *religious act*
somn	*sleep*	*INSOMNIA* *SOMNOLENT* *SOMNAMBULIST*	*inability to sleep* *sleepy* *sleepwalker*
therm	*heat*	*THERMOSTAT* *DIATHERMY*	*instrument that regulates heat* *sending heat through body tissues*
tract	*drag, pull*	*DISTRACT* *INTRACTABLE*	*pull (one's attention) away* *stubborn, unable to be dragged*
vac	*empty*	*VACUOUS* *EVACUATE*	*lacking content, empty-headed* *compel to empty an area*
verb	*word*	*VERBOSE* *VERBIAGE* *VERBATIM*	*wordy* *excessive use of words* *word for word*
voc, vocat	*to call*	*AVOCATION* *PROVOCATION* *INVOCATION*	*calling, minor occupation* *calling or rousing the anger of* *calling in prayer*
vol	*wish*	*MALEVOLENT* *VOLUNTARY*	*wishing someone ill* *of one's own will*
volv, volut	*to roll*	*CONVOLUTION*	*coiled state*

Exercises

Sentence Completion

Basic

Direction for questions 1 to 10: Each question gives a sentence with a part of the sentence underlined. Four alternatives for the underlined part are given. One of them is correct. Identify the correct one and mark its letter as the answer.

1. As they approaching the house, the colonel's wife calls him for tea.
   (A) As they approaching the house
   (B) As they will be approaching the house
   (C) As they approach the house
   (D) As they approach to the house

2. A writer always is having his own vision of life.
   (A) Always is having
   (B) Every time is having
   (C) Always has
   (D) Every time will have

3. With only two weeks before the party, the last place I would wanted to be was in the hospital recovering from surgery.
   (A) Would wanted to be
   (B) Wanted to be

(C) Would want myself to be
(D) Wanted me to be

4. We had been almost halfway there when the left rear tyre blew out.
(A) Had been almost
(B) Were almost
(C) Were going to be almost
(D) Were reaching

5. After I grew up and went into business, I always have had a soft spot for kids without bikes.
(A) Always have had (B) Always had had
(C) Always had (D) Have always had

6. Gayatri informed one of the directors that she has sent the material for printing.
(A) Shall be sending (B) Sends
(C) Had sent (D) Had been sending

7. Sosin suggested that not to talk about her selection in IAS, till she received the orders.
(A) Not to be talking
(B) That we should not talk
(C) That we would not talk
(D) That she will not talk

8. I would rather work twelve hours a day in the office than sitting and watching the idiot box at home.
(A) Than to sit and watch
(B) Instead of sitting and watching
(C) Than to sitting and watching
(D) Than sit and watch

9. He informed me that he will visit my house in a day or two.
(A) He would visit my house in a day or two.
(B) He visits my house in a day or two.
(C) He could visit my house in the coming two days.
(D) He couldn't visit my house in a day or two.

10. I shall wait here until the bus has arrived.
(A) Until the bus may arrive.
(B) Until the bus arrives.
(C) Until the bus will arrive.
(D) Until the bus has arrived.

Advanced

Direction for questions 11 to 20: Identify the incorrect sentence or sentences:

11. a. Civilization is not inherited.
b. It had to be learned and earned by each generation anew.
c. If the transmission is interrupted for one century, civilization will die.
d. And we will be savages again.
(A) a and b (B) b and d
(C) c and d (D) b and c

12. a. Orchids can be found all over India.
b. Arunachal Pradesh, Sikkim, and Meghalaya are especially blessed by these blooms.
c. More than 800 of India's 1200 species flourish in the North eastern states.
d. Infact, the state flowers of both Arunachal Pradesh and Assam is the foxtail orchid.
(A) a and c (B) b and d
(C) Only b (D) Only c

13. a. Water blessed our nation this year and also cursed it.
b. Downpours ended California's drought, and a dry summer seared South Carolina.
c. The Midwest still reels from flood.
d. Water is on everyones mind.
(A) Only b (B) b and d
(C) a, b and c (D) b, c and d

14. a. Flowers have always played a significant part in religious ceremony.
b. Their perfumes have a distinct affect on the mind and its moods.
c. The Arabians imported their perfumes from India, as they still do.
d. Kannauj, in Uttar Pradesh, is the heart of the perfume industry.
(A) Only a (B) a and b
(C) Only c (D) c and d

15. a. The country boasts about more than 400 airports.
b. But just 62 of them are in active use.
c. Mumbai and Delhi airports alone account over 40% of the passenger traffic.
d. The facilities at the airports are woefully short.
(A) Only a (B) a and c
(C) a and b (D) a, c, and d

16. a. The news about coffee is surprisingly good.
b. Moderate intake of coffee does not pose a healthy threat.
c. The only serious charge against caffeine is that it may addict.
d. Its absence can result in headaches, fatigue etc.
(A) Only a (B) Only c
(C) b and c (D) a and d

17. a. The plane banked over the Amazonian rainforest.
b. Prance gazed down the dark green canopy below.
c. It was an expanse of trees, almost unbroken.
d. It had more variety of plants and animal life than any other place on earth.
(A) b and d (B) a and c
(C) a, b, and c (D) b, c, and d

18. a. I watched the match on TV with my son.
b. It was about so exciting as watching a moth sleep.
c. I considered myself lucky if he grunted a monosyllable response.
d. Even eye contact was off question.
(A) a and b (B) a, b, and c
(C) b, c, and d (D) b and d

19. a. They had to do what no drug enforcement agents had ever done.
b. They had to infiltrate the shadowy world of the Chinese drug cartel.
c. They had to lure Johnny Kon, its crafty leader, in the open.
d. The shrewd Kon was more a match for these inexperienced investigators.
(A) a and b
(B) c and d
(C) a, c, and d
(D) b, c, and d

20. a. Shende lost the use for his arms in childhood.
b. Yet he is an expert harmonium player.
c. He is a music teacher also at Mumbai's municipal school.
d. Shende plays the harmonium deftly with his toes.
(A) a and c (B) a and b
(C) c and d (D) b and d

Synonyms

Basic

Direction for questions 21 to 35: Each question gives a word followed by four choices. From the choices, select the most suitable synonym (word which means the same) for the main word and mark its number as the answer.

21. ABSTRACT
(A) Peculiar (B) Summary
(C) Normal (D) Diagnostic

22. COMPLACENT
(A) Disappointed (B) Evasive
(C) Satisfied (D) Flimsy

23. EXTERMINATE
(A) Evict (B) Dissociate
(C) Accuse (D) Annihilate

24. CHAGRIN
(A) Current (B) Annoyance
(C) Accuse (D) Intoxicate

25. DUNK
(A) Douse (B) Stale
(C) Wet (D) Smell

26. DISTRAUGHT
(A) Agitated (B) Tranquil
(C) Rugged (D) Uneven

27. LISSOME
(A) Pretty (B) Petite
(C) Lithe (D) Tiny

28. PALLIATE
(A) Alleviate (B) Denude
(C) Mollify (D) Approbate

29. RESILIENT
(A) Stretchable (B) Spirited
(C) Rigid (D) Buoyant

30. IMPETUOUS
(A) Cat Like (B) Rash
(C) Impertinent (D) Jovial

31. ARRAIGN
(A) Exonerate (B) Initiate
(C) Accuse (D) Impart

32. APOCRYPHAL
(A) Huge (B) Destructive
(C) Mythological (D) Dubious

33. ABDUCT
(A) Ransack (B) Surround
(C) Destroy (D) Kidnap

34. ABODE
(A) Sanctuary (B) Asylum
(C) Reformatory (D) Dwelling

35. EPITOMIZE
(A) Disappoint (B) Distend
(C) Embody (D) Generate

Advanced

Direction for questions 36 to 50: In each question given below, identify the word that is similar in meaning (synonyms) to the question word.

36. REPRIEVE
(A) Save (B) Forgive
(C) Victimize (D) Cure

37. TRIBUTE
(A) Declaration (B) Accolade
(C) Criticism (D) Anger

38. OUTFIT
(A) Costume (B) Career
(C) Behaviour (D) Employment

39. ACCREDIT
(A) Undermine (B) Legalize
(C) Enlighten (D) Recognize

40. CAUSTIC
(A) Rude (B) Sarcastic
(C) Brazen (D) Polite

41. DECREE
(A) Request (B) Worship
(C) Appeal (D) Command

42. ARBITRATE
(A) Compare (B) Attach
(C) Believe (D) Judge

43. AFFIDAVIT
(A) Document (B) Oath
(C) Deposition (D) Will

44. ASPERITY
(A) Parity
(B) Ascendancy
(C) Harshness
(D) Accuracy

45. VAPID
(A) Quick (B) Restless
(C) Insipid (D) Decisive

46. FUSTY
(A) Freak (B) Musty
(C) Delicate (D) Roomy

47. EXPROPRIATE
(A) Embiggle (B) Seize
(C) Wreck (D) Outsmart

48. FORLORN
(A) Forgotten (B) Lamented
(C) Courageous (D) Woebegone

49. THRIFTLESS
(A) Penurious (B) Mendacious
(C) Profligate (D) Tackless

50. COMBUSTION
(A) Ignition (B) Explain
(C) Shrinkage (D) Strength

Antonyms

Basic

Direction for questions 51 to 65: Each question has a word followed by four choices. From the choices, identify the one which is opposite in meaning (antonym) to the main word and mark its number as your answer.

51. IMMACULATE
(A) Unclean (B) Immobile
(C) Motherly (D) Accumulated

52. IGNOMINIOUS
(A) Crooked (B) Dishonest
(C) Ominous (D) Laudable

53. MELLOWED
(A) Ripened (B) Sensational
(C) Immature (D) Ridiculous

54. GROUCHY
(A) Cantankerous (B) Grumpy
(C) Contented (D) Surly

55. IMPUDENT
(A) Rude (B) Courteous
(C) Impertinent (D) Insolent

56. MITIGATE
(A) Soothe (B) Abate
(C) Assuage (D) Aggravate

57. EXPOSTULATION
(A) Immigration (B) Susceptibility
(C) Elegance (D) Approbation

58. DISPARAGE
(A) Evolve (B) Expose
(C) Appreciate (D) Converge

59. OSSIFICATION
(A) Liquefaction (B) Flexibility
(C) Vigour (D) Viability

60. PEDESTRIAN
(A) Mobile (B) Regular
(C) Exciting (D) Imaginative

61. TUMULTUOUS
(A) Halcyon (B) Mope
(C) Cold (D) Conniving

62. ANACHRONISM
(A) Obsession (B) Vogue
(C) Radicalism (D) Pre-emption

63. BESEECH
(A) Charge (B) Cajole
(C) Command (D) Request

64. BATTERY
(A) Slimy (B) Individual
(C) Dense (D) Clean

65. BLANDISHMENTS
(A) Smack (B) Reproof
(C) Scuttle (D) Extradition

Advanced

Direction for questions 66 to 80: In each of the questions given below, identify the word that is opposite (antonym) in meaning to the question word.

66. DEMENTED
(A) Abundant (B) Opulent
(C) Sane (D) Divine

67. BRUSQUE
(A) Abrupt (B) Polite
(C) Art (D) Gruff

68. BUCOLIC
(A) Pastoral (B) Rural
(C) Rustic (D) Urban

69. CONTRITE
(A) Apologetic (B) Penitent
(C) Defiant (D) Ecstatic

70. CONTROVERT
(A) Change (B) Accept
(C) Control (D) Deny

71. COWER
(A) Come Out (B) Cringe
(C) Smother (D) Blush

72. CREDIBLE
(A) Legal (B) Unbelievable
(C) Legendary (D) Logical

73. CRESTFALLEN
(A) Abashed (B) Cheerful
(C) Broken (D) Treble

74. JEOPARDY
(A) Vivacity (B) Temerity
(C) Consummation (D) Security

75. MUSTY
(A) Desolate (B) Uncertain
(C) Fragrant (D) Clear

76. HAZY
(A) Bright (B) Misty
(C) Indolent (D) Spirited

77. PROVIDENTIAL
(A) Difficult (B) Spartan
(C) Inopportune (D) Usurping

78. ABATE
(A) Appease (B) Subside
(C) Intensify (D) Curtail

79. ABSTEMIOUS
(A) Disciplined (B) Uninteresting
(C) Unrestricted (D) Intemperate

80. SPONTANEOUS
(A) Voluntary (B) Automatic
(C) Coercive (D) Irritable

Analogies

Basic

Direction for questions 81 to 95: In each of the following questions a pair of words in capitals is given followed by four numbered pairs of words. Select from the choices the pair that exhibits the same relationship as the capitalized pair of words and mark the letter as your answer.

81. MONEY : MERCENARY
(A) Fame : Soldier
(B) Love : Mother
(C) Pass : Student
(D) Gold : Midas

82. POLYMER : CELL
(A) Coin : Money
(B) Food : Wheat
(C) Chain : Link
(D) Fibre : Plastic

83. POETRY : RHYME
(A) Chair : Table
(B) Mystery : Suspense
(C) Shakespeare : Drama
(D) Clothes : Paper

84. COMPUTER : RAM
(A) Book : Page
(B) Cloud : Rain
(C) Table : Tablecloth
(D) Sky : Blue

85. BELLS : JINGLE
(A) Crow : Caw
(B) Anklets : Tinkle
(C) Pig : Snuff
(D) Clock : Time

86. COGENT : CONVINCING
(A) Insane : Distinguished
(B) Laconic : Pithy
(C) Illogical : Reasonable
(D) Jovial : Abstruse

87. RETROSPECTION : PAST
(A) Syllogism : Logic
(B) Idiosyncrasy : Coherence
(C) Prognostication : Future
(D) Transience : Rigidity

88. BENEVOLENT : GRASPING
(A) Repulsive : Pushing
(B) Euphonious : Discordant
(C) Churlish : Impolite
(D) Rebellious : Disorderly

89. FRIGHTEN : PETRIFY
(A) Enamour : Protect
(B) Sneer : Appreciate
(C) Abbreviate : Interest
(D) Humiliate : Mortify

90. MUNIFICENT : STINGY
(A) Inclement : Merciless
(B) Incorrigible : Recalcitrant
(C) Articulate : Obscure
(D) Egregious : Outstanding

91. CRAVENLY : COWARD
(A) Synthetic : Savant
(B) Menial : Composer
(C) Brazen : Drunkard
(D) Duplicitous : Cheat

92. DIFFIDENT : SHY
(A) Lacklustre : Dull
(B) Covering : Encompass
(C) Infuriating : Regale
(D) Overhauling : Desert

93. RIGOUR : ONEROUS
(A) Vigour : Tentative
(B) Tedium : Boring
(C) Narration : Timely
(D) Simplicity : Rudimentary

94. ESOTERIC : SCHOLARLY
(A) Meandering : Accurate
(B) Dogmatic : Anomalous
(C) Honourable : Shameless
(D) Profuse : Plentiful

95. ADVOCATE : PROPAGATE
(A) Sap : Deliberate
(B) Exact : Demand
(C) Afflict : Ameliorate
(D) Deteriorate : Roughen

Advanced

Direction for questions 96 to 110: In each of the following questions a capitalized pair of words if given followed by four pairs of numbered words. Three of the numbered pairs exhibit the same relation between the words as the capitalized pair of words. Identify the numbered pair that does not exhibit the same relationship as the capitalized pair and mark its number as your answer.

96. IMPORT : EXPORT
(A) Inhale : Exhale
(B) Include : Exclude
(C) Inspire : Expire
(D) Increase : Decrease

97. MILK : MILKMAID
(A) Letters : Postman
(B) Biscuit : Baker
(C) Newspaper : Newspaper Boy
(D) Sweets : Street Vendor

98. ORNITHOLOGIST : BIRDS
(A) Entomologist : Insects
(B) Herpetologist : Reptiles
(C) Psychologist : Mind
(D) Ichthyology : Fossils

99. DISHONOUR : DEMEAN
(A) Demolish : Annihilate
(B) Glorify : Extol
(C) Consecrate : Sanctify
(D) Articulate : Obfuscate

100. LEPROSY : BACTERIA
(A) Summer : May
(B) Cloud : Evaporation
(C) Violence : Anger
(D) Day : Sun

101. MILK : CHEESE
(A) Water : Ice
(B) Cotton : Clothes
(C) Rainbow : Rain
(D) Tobacco : Cigar

102. PRINCIPAL : SCHOOL
(A) President : India
(B) Monarch : Britain
(C) God : Universe
(D) Buddha : Burma

103. FASTIDIOUS : FINICKY
(A) Parsimonious : Miserly
(B) Asinine : Unintelligent
(C) Precocious : Talented
(D) Prudent : Injudicious

104. INSIPID : ORIGINAL
(A) Ambidextrous : Unskilled
(B) Foppish : Affected
(C) Tranquil : Disturbed
(D) Winding : Succinct

105. ATTRITION : WEAR
(A) Rupture : Tear
(B) Mammon : Money
(C) Sacrilege : Honesty
(D) Melody : Rhythm

106. ANTIQUATED : ANCIENT
(A) Compassionate : Lustful
(B) Fortuitous : Coincidental
(C) Pristine : Pure
(D) Spiteful : Wicked

107. EMANCIPATE : LIBERATE
(A) Misconstrue : Misinterpret
(B) Commiserate : Sympathize
(C) Sequestrate : Align
(D) Ape : Imitate

108. COSMIC : UNIVERSE
(A) Sidereal : Stars
(B) Arboreal : Trees
(C) Aquatic : Water
(D) Despotic : Depot

109. BOUNTIFUL : PLENTITUDE
(A) Bizarre : Absurdity
(B) Consecutive : Succession
(C) Depraved : Wickedness
(D) Somnolent : Sombreness

110. STYMIE : OBSTRUCT
(A) Parry : Fight
(B) Ponder : Mull
(C) Imbue : Soak
(D) Intervene : Mediatez

Logical Reasoning

Basic

Direction for questions 111 to 115: Select the correct alternative from the given choices.

111. If you find the next version of software products from Microsoft coming with the feminine touch, don't be surprised because the world's largest software major is planning to add more women to its development centre in Hyderabad. Microsoft India Development Centre (MSIDC), the second largest development centre of the company outside its headquarters in the US is planning to kick-start a diversity event, a women's recruitment drive, this Women's Day. As part of the programme, the company has invited applications from girl students who are doing final year computer science engineering across the country.

Which of the following could be the reason behind Microsoft's decision to recruit girl students?

(A) The women recruitment initiative is a pilot project which, if successful, could be replicated at the company's other centres in the country.

(B) Though, due to social conditions, women may opt for engineering colleges near their hometown, rather than IITs, their competency levels are on par with men.

(C) A diverse workforce will be able to anticipate the needs of the diverse customers spread across the world.

(D) This kind of recruitment drive will encourage more women to enrol for engineering course.

112. Its not just media agencies that are obsessed with numbers—even the creative side does not shy away from romancing with digits. Whether it is instructions (Ujala : chaar boondan wala) or just plain information (Complan's 23 vital nutrients), a lot of contemporary advertising serves up a tidal wave of figures. Whether its personal care products, refrigerators, washing machine, tea or tooth paste, everyone is using numbers as a USP. While the claims work for Maggi's 2-minutes noodles in many cases, it is veering towards a game of one -upmanship. Baffling concept innundate the viewers—12 times more volume mascara, natural care tea with 'paanch ayurvedic tatva', a fridge with 6th sense cooling, and a tooth paste that claims to fight 10 problems.

Which of the following does not point to the advantage of using numbers?

(A) People don't believe advertising; so, if you can give a quantified claim, it adds credibility to your brand.

(B) Numbers in advertisements cause disillusionment when they are not borne out.

(C) When numbers are put into context, they prove to be symbolic and evocative, as in the case of Britannia cheese, which says one slice in equal to two glasses of milk.

(D) The number definitely adds a competitive advantage: if one fairness cream claims results in 'six weeks' another will take a mere '30 days'.

113. Art shows in India by Indian artists settled abroad have increased over the past few years and are on the rise.

Which of the following does not account for the above occurrence?

(A) This trend is being fuelled by the booming Indian economy—people have spare funds which they can invest in art.

(B) These artists fetch higher price in India than abroad, probably because their acceptance back home is greater than it is in the oversea art scene.

(C) While acquisition of Indian art overseas continues to grow, purchases are still triggered by the Indian diaspora.

(D) A large cross section of wealthy businessmen travelling abroad drop by at galleries and museums and this develops an understanding of and love for art.

114. Andreas Gursky bestrides the art world like a colossus. For once, in the contemporary art scene, form, content, and price come together like a rare planetary alignment. Yet critics conclude that A. Gursky's photographs are famous mainly because of their massive price tags.

Which one of the following prove his critics wrong?

(A) His photographs are rendered with a pin sharp focus that seems to defy the laws of physics.

(B) The hammer prices of Gursky's works simply reflect the herd instincts of billionaires for unique things.

(C) His pictures depict his ability to freeze a transient moment for ever.

(D) The photographic aesthetic embodied in his work is based on an uncanny dichotomy that makes it singular to every viewer.

115. A visit to the abattoir usually transforms a non-vegetarian into a vegetarian. Similarly, a visit to the Bose Institute in Kolkatta is sure to make a tourist lead an austere life thereafter, even, perhaps, going to the extent of becoming a mineral eater.

Which of the following supports the above conclusion?

(A) The visitor can watch the graphic reaction of the plant when a leaf is plucked.

(B) One marvels at the intricacies of the composition of a plant.

(C) One realizes the fact that it takes years of sunlight, water, and earth to help a tree flourish.

(D) The onlooker gets a very clear picture of the intricate process of photosynthesis.

Advanced

Direction for questions 116 and 117: In each of the following questions, a statement is followed by three assumptions. An assumption is an idea or opinion taken for granted. Find out which of the given assumptions is/are implicit considering that everything given in the statement is true.

116. Statement: We have come a long way from the barter system to cowrie currency to a complex monetary system to womb renting and sperm selling. These are innovative methods of making ends meet in hard times.

Assumptions:

I. Innovative methods are needed to make ends meet in hard times.
II. Barter system is not an innovative method.
III. Womb renting and sperm selling are innovative methods.

(A) All are implicit (B) Only I and II
(C) Only II (D) Only III

117. Statement: Why the brouhaha over dynastic politics? Have not many children followed their parents' profession? May be, genes play a part in their choice of profession.

Assumption:

I. Genes play a part in the choice of profession.
II. Choosing politics as profession because the parents are in politics is the same as choosing any other profession because the parents are in that profession.
III. There is nothing wrong with dynastic politics.

(A) Only III (B) Only II and III
(C) Only II (D) Only I and II

Direction for questions 118 and 119: Each question given below consists of a statement followed by two conclusions numbered I and II. You have to assume everything in the statement to be true and then consider the two conclusions together and decide which of them logically follows beyond a reasonable doubt from the information given in the statement. Give your answer as

(A) If only conclusion I follows.
(B) If only conclusion II follows.
(C) If neither I nor II follows.
(D) If both I and II follow.

118. Statement: 'Had I been asked to choose a portfolio, I would have probably chosen the ministry of environment and forest', Home Minister.

Conclusions:

I. It is easier to handle the environment and forest portfolio rather than the home ministry.
II. The person did not become a home minister by choice.

119. Statement: 'Getting angry makes me mentally tough',–Mr. X.

Conclusions:

I. Mr. X does not get angry.
II. Mr. X is not mentally tough.

Direction for questions 120 to 122: In making decisions about important questions, it is desirable to be able to distinguish between 'strong' arguments and 'weak' arguments. So far, as they relate to the question, 'strong' arguments are those which are both important and directly related to the question. 'Weak' arguments are those which are of minor importance and may not be directly related to the question or may be related to a trivial aspect of the question.

Each question below is followed by two arguments numbered I and II. You have to decide which of the arguments is a 'strong' argument and which is a 'weak' argument. Mark your answer as

(A) If only argument I is strong.
(B) If only argument II is strong.
(C) If neither I nor II is strong.
(D) If both I and II are strong.

120. Statement: Should the Indian police be equipped with more sophisticated weapons?

Arguments:

I. No, they do not know how to use them.
II. Yes, the recent incidents in USA have proved that terrorists have very sophisticated weapons.

121. Statement: Is financial freedom necessary for women?

Arguments:

I. No, their husbands give them sufficient money.
II. No, women have a very important role to play at home in developing a good society.

122. Statement: Should history be removed from school curriculum?

Arguments:

I. No, it helps students to know about their past.
II. No, history provides information which helps in building the future.

Direction for questions 123 and 124: In each question below is given a statement followed by some courses of action. A course of action is a step or administrative decision to be taken for improvement follow-up or further action with regard to the problem, policy, etc. on the basis of the information given in the statement. Assume everything in the given statement to be true, and then decide which of the courses of action logically follow for pursuing.

123. Statement:

The competition among corporate hospitals has not done any good to the people.

Course of action:

I. Permission should be given to open more corporate hospitals.

II. The government should review the norms for corporate hospitals so that they take up some social responsibility.
III. All the corporate offices should be closed down.
(A) Only III follows
(B) Only II follows
(C) Either I or III follows
(D) Only II and III follows

124. Statement:
It is reported that 61 per cent of crimes including murder are committed by those below 25 years. Unemployment and rising prices drive the youth into taking up a life of crime.

Courses of action:
I. Criminals aged below 25 should be given employment.
II. Pocket money given to the young should be increased.
III. Economics should be taught at schools, so that the young understand the mechanism of price changes and creation of employment.

(A) Only I and III follows
(B) Only III follows
(C) Only I and II follows
(D) None follows

Previous Years' Questions

1. Which of the following options is the closest in meaning to the word below: **[GATE, 2010]**
Circuitous
(A) Cycle (B) Indirect
(C) Confusing (D) Crooked

2. Choose the most appropriate work from the options given below to complete the following sentence:
If we manage to _______ our natural resources, we would leave a better planet for out children. **[GATE, 2010]**
(A) uphold (B) restrain
(C) cherish (D) conserve

3. Choose the most appropriate word form the option given below to complete the following sentence:
His rather casual remarks on politic _______ his lack of seriousness about the subject. **[GATE, 2010]**
(A) masked (B) belied
(C) betrayed (D) suppressed

4. Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for the cause.
Which of the following statements best sums up the meaning of the above passage: **[GATE, 2010]**
(A) Modern warfare has resulted in civil strife.
(B) Chemical agents are useful in modern warfare.
(C) Use of chemical agents in warfare would be undesirable.
(D) People in military establishments like to use chemical agents in war.

5. Which of the following options is the closest in the meaning to the word below: **[GATE, 2011]**
Inexplicable
(A) Incomprehensible (B) Indelible
(C) Inextricable (D) Infallible

6. Choose the word from the options given below that is most nearly opposite in meaning to the given word: **[GATE, 2011]**
Amalgamate
(A) Merge (B) Split
(C) Collect (D) Separate

7. Choose the most appropriate word from the options given below to complete the following sentence.
If you are trying to make a strong impression on your audience, your cannot do so by being understated, tentative or _______. **[GATE, 2011]**
(A) hyperbolic (B) restrained
(C) argumentative (D) indifferent

8. Choose the most appropriate word(s) from the options given below to complete the following sentence.
I contemplated _______ Singapore for my vacation but decided against it. **[GATE, 2011]**
(A) to visit (B) having to visit
(C) visiting (D) for a visit

9. Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.
Based on the above passage which topic would not be included in a unit on bereavement? **[GATE, 2011]**
(A) How to write a letter of condolence
(B) What emotional stages are passed through in the healing process
(C) What the leading causes of death are
(D) How to given support to a grieving friend

10. Choose the most appropriate alternative from the options given below to complete the following sentence:
Despite several _____ the mission succeeded in its attempt to resolve the conflict. **[GATE, 2012]**
(A) attempts (B) setbacks
(C) meetings (D) delegations

11. Choose the most appropriate alternative from the options given below to complete the following sentence:

Suresh's dog is the one ______ was hurt in the stampede. **[GATE, 2012]**

(A) that (B) which
(C) who (D) whom

12. Choose the grammatically **INCORRECT** sentence: **[GATE, 2012]**

(A) They gave us the money back less the service charge of three hundred rupees.
(B) This country's expenditure is not less than that of Bangladesh.
(C) The committee initially asked for a funding of fifty lakh rupees, but later settled for a lesser sum.
(D) This country's expenditure on educational reforms is very less.

13. Which one of the following options is the closest in meaning to the word given below? **Mitigate** **[GATE, 2012]**

(A) Diminish (B) Divulge
(C) Dedicate (D) Denote

14. Wanted temporary, part time persons for the post of Field Interviewer to conduct personal interviews to collect and collect economic data requirements: High school pass, must be available for day, Evening and Saturday work. Transportation paid, expenses reimbursed.

Which one of the following is the best inference from the above advertisement? **[GATE, 2012]**

(A) Gender–discriminatory
(B) Xenophobic
(C) Not designed to make the post attractive
(D) Not gender–discriminatory

15. <u>The professor</u> <u>ordered to</u> <u>the students</u> <u>to go out of the class</u>

Which of the above underlined parts of the sentence is grammatically incorrect? **[GATE, 2013]**

(A) I (B) II
(C) III (D) IV

16. Which of the following options is the closest in meaning to the word given below: **[GATE, 2013]**

Primeval

(A) Modern
(B) Historic
(C) Primitive
(D) Antique

17. Friendship, no matter how ______ it is, has its limitations

(A) cordial
(B) intimate
(C) secret
(D) pleasant

18. Abhishek is elder to Savar

Saver is younger to Anshul

Which of the given conclusion is logically valid and is inferred from the above statements? **[GATE, 2013]**

(A) Abhishek is elder to Anshul
(B) Anshul is elder to Abhishek
(C) Abhishek and Anshul are of the same age
(D) No conclusion follows

19. A student is required to demonstrate a high level of <u>comprehension</u> of the subject, especially in the social sciences.

The word closest in meaning to <u>comprehension</u> is **[GATE, 2014]**

(A) understating
(B) meaning
(C) concentration
(D) stability

20. Choose the most appropriate word from the options given below to complete the following sentence. One of his biggest ______ was his ability to forgive. **[GATE, 2014]**

(A) vice (B) virtues
(C) choices (D) strength

21. Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently. **[GATE, 2014]**

Which one of the statements below is logically valid and can be inferred from the above sentences?

(A) Rajan has decided to work only in a group.
(B) Rajan and Sajan were formed into a group against their wishes.
(C) Sajan had decided to give into Rajan request to work with him.
(D) Rajan had believed that Sajan and he would be working together.

22. Choose the most appropriate word from the options given below to complete the following sentence.

A person suffering from Alzheimer's disease ______ short term memory loss. **[GATE, 2014]**

(A) experienced
(B) has experienced
(C) is experiencing
(D) experiences

23. Choose the most appropriate word from the options given below to complete the following sentence. ______ is the key to their happiness; they are satisfied with what they have. **[GATE, 2014]**

(A) Contentment
(B) Ambition
(C) Perseverance
(D) Hunger

24. Which of the following options is the closest in meaning to the sentence below?

'As a woman, I have no country'. **[GATE, 2014]**

(A) Women have no country.
(B) Women are not citizens of any country.
(C) Women's solidarity knows no national boundaries.
(D) Women of all countries have legal rights.

25. Moving into a world of big data will require us to change our thinking about the merits of exactitude. To apply the conventional mindset of measurement to the digital, connected world of the twenty–first century is to miss a crucial point. As mentioned earlier, the obsession with exactness is an artifact of the information deprived analog era. When data was sparse, every data point was critical and thus great care was taken to avoid letting any point bias the analysis.

From 'BIG DATA' viktor mayer-schonberger and Kenneth cukier

The main point of the paragraph is; **[GATE, 2014]**

(A) The twenty first century is a digital world
(B) Big data is obsessed with exactness
(C) Exactitude is not critical in dealing with big data
(D) Sparse data leads to a bias in the analysis

26. Extreme focus on syllabus and studying for tests has become such a dominant concern of Indian students that they close their minds to anything ______ to the requirement of the exam. **[GATE, 2015]**

(A) related (B) extraneous
(C) outside (D) useful

27. The Tamil version of ______ Join Abraham-starrer Madras Café ______ cleared by the Censor Board with no cuts last week, but the film's distributors ______ no takers among the exhibitors for a release in Tamil Nadu ______ this Friday. **[GATE, 2015]**

(A) MR., was, found, on (B) a, was, found, at
(C) the, was found, on (D) a, being, find at

28. Alexander turned his attention towards India, since he had conquered Persia.

Which one of the statements below is logically valid and can be inferred from the above sentence?
[GATE, 2015]

(A) Alexander would not have turned his attention towards India had the not conquered Persia.
(B) Alexander was not ready to rest on his laurels, and wanted to march to India.
(C) Alexander was completely in control of his army and could command it to move towards India.
(D) Since Alexander's kingdom extended to Indian borders after the conquest of Persia, he was keen to move further.

29. Most experts feel that in spite of possessing all the technical skills required to be a batsman of the highest order, he is unlikely to be so due to lack of requisite temperament. He was guilty of throwing away his wicket several times after working hard to lay a strong foundation. His critics pointed out that until he addressed this problem, success at the highest level will continue to elude him.

Which of the statement(s) below is/are logically valid and can be inferred from the above passage?

(i) He was already a successful batsman at the highest level.
(ii) He has to improve his temperament in order to become a great batsman.
(iii) He failed to make many of his good starts count.
(iv) Improving his technical skills will guarantee success. **[GATE, 2015]**

(A) (iii) and (iv)
(B) (ii) and (iii)
(C) (i), (ii) and (iii)
(D) (ii) only

30. Choose the most appropriate word from the options given below to complete the following sentence.

The official answered ______ that the complaints of the citizen would be looked into. **[GATE, 2015]**

(A) respectably
(B) respectfully
(C) reputably
(D) respectively

31. Choose the statement where underlined word is used correctly. **[GATE, 2015]**

(A) The minister insured the victims that everything would be all right.
(B) He ensured that the company will not have to bear any loss.
(C) The actor got himself ensured against any accident.
(D) The teacher insured students of good results.

32. Which word is not a synonym for the word *vernacular*? **[GATE, 2015]**

(A) regional
(B) indigenous
(C) indigent
(D) colloquial

33. The word similar in meaning to 'dreary' is **[GATE, 2015]**

(A) cheerful
(B) dreamy
(C) hard
(D) dismal

Answer Keys

Exercises

1. C	**2.** C	**3.** C	**4.** B	**5.** C	**6.** C	**7.** B	**8.** D	**9.** A	**10.** B
11. D	**12.** B	**13.** B	**14.** B	**15.** B	**16.** C	**17.** A	**18.** C	**19.** C	**20.** A
21. B	**22.** C	**23.** D	**24.** B	**25.** A	**26.** A	**27.** C	**28.** A	**29.** D	**30.** D
31. C	**32.** D	**33.** D	**34.** D	**35.** C	**36.** B	**37.** B	**38.** A	**39.** D	**40.** B
41. D	**42.** D	**43.** C	**44.** C	**45.** C	**46.** B	**47.** B	**48.** B	**49.** C	**50.** C
51. A	**52.** D	**53.** C	**54.** C	**55.** B	**56.** D	**57.** D	**58.** C	**59.** B	**60.** D
61. A	**62.** B	**63.** C	**64.** B	**65.** B	**66.** C	**67.** B	**68.** D	**69.** C	**70.** B
71. A	**72.** B	**73.** B	**74.** D	**75.** C	**76.** A	**77.** C	**78.** C	**79.** D	**80.** C
81. D	**82.** C	**83.** B	**84.** A	**85.** B	**86.** B	**87.** C	**88.** B	**89.** D	**90.** C
91. D	**92.** A	**93.** B	**94.** D	**95.** B	**96.** C	**97.** B	**98.** D	**99.** D	**100.** A
101. C	**102.** D	**103.** D	**104.** B	**105.** C	**106.** A	**107.** C	**108.** D	**109.** D	**110.** A
111. C	**112.** B	**113.** C	**114.** D	**115.** A	**116.** D	**117.** D	**118.** B	**119.** C	**120.** C
121. C	**122.** B	**123.** B	**124.** D						

Previous Years' Questions

1. B	**2.** D	**3.** C	**4.** C	**5.** A	**6.** D	**7.** B	**8.** C	**9.** C	**10.** B
11. A	**12.** D	**13.** A	**14.** D	**15.** B	**16.** C	**17.** B	**18.** A	**19.** A	**20.** B
21. D	**22.** D	**23.** A	**24.** C	**25.** A	**26.** B	**27.** C	**28.** A	**29.** B	**30.** B
31. B	**32.** C	**33.** D							

Test

Verbal Ability

Time: 25 Minutes

Direction for questions 1 to 4: In these questions, you are to analyse the situation on which each question is based and then select the answer choice that is the most appropriate response to the question.

1. The residents of a particular locality were administered a particular medicine of a company continuously for three months. The locality which had reported high incidence of Hepatitis B before the administration of the medicine reported no cases at the end of this period. The company claimed that this was because their medicine helped the residents develop immunity against Hepatitis B.

Which of the following, if true, most seriously weakens the claim of the company?

(A) A few cases of Hepatitis B were reported in the locality a month after the company started administering the medicine to the residents.
(B) The incidence of Hepatitis B is found to be very high during winter, which lasts for about four months in the region, and winter set in three months back.
(C) Research shows that any medicine capable of helping develop immunity against Hepatitis B takes about six months to show results.
(D) Usually newborn children are more affected by Hepatitis B, and there were no children born in the locality in the past three months.

2. Ms. E, a serious protagonist of environmental protection, wins a mayoral election to city 'C' against Mr. D. The success of Ms. E, against Mr. D in the election could be attributed to her concern for the environment in all of the following cases EXCEPT when

(A) The citizens of 'C' are highly environment conscious.
(B) The city administration has powers to act against activities resulting in environmental degradation.
(C) Ms. E is also known to possess excellent leadership qualities.
(D) Mr. D is a champion of environmental protection.

3. The life expectancy of the population of a country has increased from forty five (twenty five years ago) to seventy now—due to increased medical facilities and better health consciousness in people. However, an insurance company in the country has not lowered the premium substantially despite the life expectancy going up.

Which of the following, if true, would rationalize the action of the insurance company in not lowering the insurance premium?

(A) The increase in life expectancy has been significant only in the past five years.
(B) There have been fewer deaths due to cancer in recent years, whereas twenty five years back it was claiming a number of lives.
(C) The average age of the population of the country remained at around forty five only due to the increased birth rate in the past twenty five years.
(D) The country had been at war with all its neighbours during the past twenty five years, resulting in heavy casualties among its soldiers.

4. A company found that the attrition rate among its employees, especially among the skilled, has been particularly high in the last three months. As the company had introduced night shifts three months ago, it was concluded that this was the reason for the attrition. The employees working in night shifts were, therefore paid per cent extra wages.

Which of the following would strongly support the above conclusion?

(A) The wages of the employees compared to those of employees in other companies of comparable size in the same locality are 25% higher.
(B) The productivity of employees working in the night shift is higher.
(C) The number of night shift employees is less than that of the day shift employees.
(D) The attrition rate is higher in the case of night shift employees.

Direction for questions 5 to 10: Each question has a word followed by four choices. From the choices, identify the one which is opposite in meaning (antonym) to the main word and mark its number as your answer.

5. IGNOMINIOUS
(A) Crooked (B) Dishonest
(C) Ominous (D) Laudable

6. MELLOWED
(A) Ripened (B) Sensational
(C) Immature (D) Ridiculous

7. ROBUST
(A) Sturdy (B) Feeble
(C) Muscular (D) Ridiculous

8. VINDICTIVE
(A) Forgiving (B) Relentless
(C) Revengeful (D) Disproving

9. ALTRUISM
(A) Kindness (B) Selfishness
(C) Tenderness (D) Benevolence

10. BOISTEROUS
(A) Serene (B) Rowdy
(C) Violent (D) Tumultuous

Direction for questions 11 to 18: Fill in the blanks in the given sentences so as to make sense. Select the correct word from the answer choices and mark its number as the answer.

11. The angry officer called _____ an explanation from his subordinate for the error.
(A) on (B) for
(C) out (D) in

12. I shall keep _____ nothing from you regarding my family affairs as you are a close friend.
(A) on (B) out
(C) back (D) in

13. Mulk Raj Anand is a novelist with a/an _____.
(A) attribute (B) mission
(C) idea (D) need

14. The motivation to _____ comes from a burning desire to achieve a purpose.
(A) meet (B) start
(C) succeed (D) idolize

15. Rainfall _____ the work being done by the fire-fighters at the site of the tragedy.
(A) protected (B) cherished
(C) hindered (D) illuminated

16. The Americans appreciated the _____ made by Pervez Musharraf.
(A) posture (B) texture
(C) gesture (D) resources

17. She ______ the tray on a table next to the bed.
(A) slowed (B) kept
(C) set (D) shook

18. We have ______ some good times together, she and I.
(A) looked (B) laughed
(C) smiled (D) seen

Direction for questions 19 to 24: Each question gives a word followed by four choices. From the choices, select the most suitable synonym (word which means the same) for the main word and mark its number as the answer.

19. ABSTRACT
(A) Peculiar (B) Summary
(C) Normal (D) Diagnostic

20. COMPLACENT
(A) Disappointed (B) Evasive
(C) Satisfied (D) Flimsy

21. EXTERMINATE
(A) Evict (B) Dissociate
(C) Accuse (D) Annihilate

22. UMPTEEN
(A) Numerous (B) Youthful
(C) Rainy (D) Convergent

23. OSTENSIBLE
(A) Strong (B) Desirable
(C) Apparent (D) Fateful

24. STALWART
(A) Watchful (B) Sturdy
(C) Delicate (D) Encomium

Direction for questions 25 to 26: In each of the following questions, a pair of words in capitals is given followed by four numbered pairs of words. Select from the choices the pair which exhibits the same relationship as the capitalised pair of words and mark the number as your answer.

25. PROPERTY : MORTGAGE
(A) Money : Lend
(B) Equity : Interest
(C) Inventory : Merchandise
(D) Security : Price

26. NEGOTIABLE : CHEQUE
(A) Frozen : Asset
(B) Oil : Seed
(C) Bank : Money
(D) Security : Price

Direction for questions 27 to 30: For the word given on the top of the table, match the dictionary definitions given in the left-hand columns (A, B, C, D) with their corresponding usage given in the right-hand column (E, F, G, H). Out of the four numbered choices given in the boxes below the table, identify the one that has all definitions and usages correctly matched.

27. RAISE

A.	Levy or collect or bring together	E.	The girl has been raised in a very conservative environment.
B.	Cause to be heard or considered	F.	We decided to raise some money to help the flood victims.
C.	Increase the amount or value or strength of	G.	I will raise this issue at the right moment.
D.	Bring up; educate	H.	The doctor raised the potency of the drug before administering it to the patient.

(A)

A	G
B	H
C	F
D	E

(B)

A	F
B	G
C	H
D	E

(C)

A	G
B	E
C	H
D	F

(D)

A	F
B	H
C	G
D	E

28. SENSE

A.	One of the five powers (sight, taste, smell, touch, hearing)	E.	'When will he ever come to his senses?' she asked exasperated
B.	Show or find the meaning of	F.	I could sense that something was amiss in the function.
C.	The ability to think clearly	G.	I cannot make sense of what he says.
D.	Be vaguely aware	H.	We perceive the world through our senses.

(A)

A	H
B	G
C	F
D	E

(B)

A	G
B	E
C	F
D	H

(C)

A	E
B	F
C	H
D	G

(D)

A	H
B	G
C	E
D	F

29. MATCH

A.	Corresponding in some essential respect with what has been mentioned.	E.	The match will start at 9:00 a.m.
B.	Be as good or equal to	F.	Australia finally met its match in the finals.
C.	A contest or competition	G.	Geetanjali would be a nice match for Govind.
D.	A person viewed in regard to his or her eligibility for marriage	H.	If you buy two shirts, we will give any two matching ties free of cost.

(A)

A	H
B	F
C	E
D	G

(B)

A	G
B	E
C	H
D	F

(C)

A	H
B	G
C	E
D	F

(D)

A	G
B	H
C	F
D	E

30. MASS

A.	The Eucharist, especially in the Roman Catholic Church	E.	During peak traffic hours, the entire city seems to be just a mass of humanity.
B.	A dense aggregation of objects	F.	The Mass conducted at the Church was good for the salvation of the soul.
C.	The ordinary people; the majority	G.	Science students must know that mass and weight are entirely different.
D.	The quantity of material that something contains	H.	The masses are bothered only about the problems of day-to-day living and therefore, have no time for politics or sports.

(A)

A	H
B	G
C	F
D	E

(B)

A	G
B	H
C	E
D	F

(C)

A	F
B	E
C	H
D	G

(D)

A	E
B	F
C	H
D	G

Answer Keys

1. C	**2.** D	**3.** D	**4.** D	**5.** D	**6.** C	**7.** B	**8.** A	**9.** B	**10.** A
11. B	**12.** C	**13.** B	**14.** C	**15.** C	**16.** C	**17.** B	**18.** D	**19.** B	**20.** C
21. D	**22.** D	**23.** C	**24.** B	**25.** A	**26.** A	**27.** B	**28.** D	**29.** B	**30.** C

Verbal Ability Test I

Number of Questions: 40 **Time: 30 min**

Directions for questions 1 to 5: Each of the given sentences has four underlined parts. One of them has a mistake. Mark the number of the wrong part as answer.

1. In the Sub-Saharan countries incidences of Sunstrokes correlates (A) / positively withthe level of (B) (C) / Solar radiation. (D)

2. Either you transfer (A) / the data which was demanded (B) / nor file a report explaining why you did not (C) / submit the overall annual figures. (D)

3. Neither the judge (A) / nor I am (B) / ready toannounce (C) / who the winner is. (D)

4. He went about (A) / the bad phase (B) / in his career with (C) / philosophical clam (D)

5. A score of apple is (A) / purchased (B) / by him (C) / for his consumption. (D)

Directions for questions 6 to 10: Read each sentence to find out whether there is any error in it. The error, if any, will be in one part of the sentence. Identify the error and mark the number of the erroneous part as your answer.

6. All the members (A) / of the club (B) / was present (C) / at the special meeting. (D)

7. Over the course of the twentieth century/ (A) the internal combustion engine / (B) has replaced the horse (C) / to the basic means of transport. (D)

8. We must (A) / never give up with (B) /trying to enhance/ (C) the quality of life. (D)

9. It will profit a man nothing (A) / if he was (B) / to gain the world (C) / and lose his own soul. (D)

10. Injustice and discrimination (A) / can never be (B) / tolerated by (C) / no one. (D)

11. Alas! (A) / How lovely (B) / and fragrant (C) / these flowers are! (D)

12. Your story is (A) / so ridiculous that (B) / it could not/ (C) be believed. (D)

13. Government schools (A) / has too many students (B) / in a class (C) / for a teacher to control. (D)

14. Either of (A) / the methods (B) / lead to the (C) / same result. (D)

15. The doctor has (A) / advised him (B) / to avoid the sugar (C) / in his milk. (D)

Directions for questions 16 to 20: Each sentence given below is divided into four parts. One of them has an error. Mark the number of the incorrect part as your answer.

16. The church accepts this popular sentiment / (A) gives it a religious significance (B) /and crystallizes / (C) in a system (D)

17. The government of the Tudors / (A) were masters in the art of disguising / (B) common place, and sometimes sordid, motives / (C) beneath a glittering façade of imposing principles. (D)

18. The increasing reluctance of the sun to rise,/ (A) the extra nip in the breeze / (B) the patten of shed leaves dropping - all the evidences of fall / (C) drifting in winter were clearer each day. (D)

19. Because of it's hardness / this steel is / used principally/ for making razors.
(A) (B) (C) (D)

20. Laying aside all hindrance / thrusting away all private aims / devote yourself unswerving and unflinchingly / to the vigorous and successful prosecution of this war
(A) (B) (C) (D)

Directions for questions 21 to 25: In each question below, two sentences are given. These two sentences are to be combined into a single sentence without changing their meaning. Three probable starters of the combined sentence are given which are denoted by (A), (B) and (C). Any one or more or none of them may be correct. Find out the correct starters(s) and accordingly select your answer from among the given five answer choices.

21. I am a layman. I do not know how a computer works.
(A) Being a layman, I ………..
(B) As I am a layman, I ……….
(C) While I am a layman, I……..
(A) A and B (B) B and C
(C) A and C (D) B only
(E) A only

22. He always listened to good advice. He rose to a good position in life.
(A) Though he listened to good advice ……..
(B) As he rose to good position in life ……..
(C) Since he listened to good advice …….
(A) A and B (B) B and C
(C) A only (D) B only
(E) C only

23. The soldiers saw the camp of the enemy. They started attacking the enemy.
(A) While the soldiers saw the enemy camp ……
(B) When the soldiers saw the camp …….
(C) Because the soldiers saw the camp …….
(A) B and C (B) A and C
(C) B only (D) A only
(E) C only

24. The weather is warm. I like to go for swimming now.
(A) If the weather is …………
(B) Whenever the weather is ………
(C) As the weather is …….
(A) A and B (B) A only
(C) B only (D) C only
(E) None of these

25. We cannot achieve our goals. We lack confidence in our abilities.
(A) When we lack confidence ………..
(B) However we can achieve our goals if we lack …..
(C) Whenever we achieve our goals ………
(A) A and B (B) A only
(C) B only (D) C only
(E) B and C

Directions for questions 26 to 30: Each sentence has a missing part. Choose the best option from those given below the statement to make up the missing part.

26. Given the long and porous border between the two countries and, more important, the links _______.
(A) among Nepalese and Indian rebel groups, New Delhi cannot afford ignoring the Maoists threat of Nepal
(B) between Nepalese and Indian rebel groups, New Delhi cannot afford the ignorance of the Maoists threat in Nepal
(C) among Nepalese and Indian rebel groups, New Delhi cannot afford to ignore the Maoists threat of Nepal
(D) between Nepalese and Indian rebel groups, New Delhi cannot afford to ignore the Maoists threat in Nepal

27. At a time when Beijing's officially scripted anti-Japanese protests are bound to prompt a rethink in Japan about the advisability of continued investment in China, India should be __________.
(A) persuading aggressively Japanese business to shifting at least some of their mammoth investments to its secure location
(B) aggressively persuading Japanese business to shifting at least some of their mammoth investments to its secure location
(C) aggressively persuading Japanese business to shift at least some of their mammoth investments to its secure location
(D) persuading aggressively Japanese business to shift at least some of their mammoth investments to its secure location

28. ________, that they could compete successfully, even with the higher techniques of production, which were being established in England.
(A) So efficient and highly organized were Indian methods of production, and such was the skill of India's artisans and craftsmen
(B) So efficiently and highly organized were Indian methods of production, and such were the skill of India's artisans and craftsmen
(C) So efficient and highly organized were Indian methods of production, and such was the skill of India's artisan and craftsman
(D) So efficiently and highly organized were Indian methods of production, and such were the skill of India's artisan and craftsman

Directions for questions 29 to 33: In the following questions, two sentences are given. There may be an error in the sentence(s). Mark as your answer

(A) if there is an error only in the first sentence;
(B) if there is an error only in the second sentence;
(C) if there are errors in both the sentences and
(D) if there is no error in either of the two sentences.

29. I. He said that he will come, but he didn't.
II. I admit, 'No news is good news now - a -days'.

30. I. I and my friend like to play tennis in grass court.
II. Each of the mistakes have to be corrected before printing.

31. I. I cannot see anything wrong with the plan.
II. You may read the book if you have enough time.

32. I. There is a little truth in what we have heard.
II. You are not going to the theatre, isn't it?

33. I. He informed me before he had posted the letter yesterday.
II. No one is as happy as he.

Directions for questions 34 to 40: A sentence is given in four different forms. Only one of them is correct grammatically. Mark the number of the correct one as the answer.

34. (A) My sister likes painting, dancing and to cooking.
(B) My sister likes painting, dancing and to cook.
(C) My sister like painting, dancing and cooking.
(D) My sister likes painting, dancing and cooking.

35. (A) If you want to play well, you must practise.
(B) If you want to play well, one must practise.
(C) If one want to play well, you must practise.
(D) If one wants to play well, he must practise.

36. (A) Einstein was more cleverer than any other scientist.
(B) Einstein was more cleverer than any scientist.
(C) Einstein was cleverer than any other scientist.
(D) Einstein was cleverer than any scientist.

37. (A) An argument developed among his sister and him.
(B) An argument developed between his sister and he.
(C) An argument developed between his sister and his.
(D) An argument developed between his sister and him.

38. (A) The youngster will benefit from the experience.
(B) The youngster will benefit by the experience.
(C) The youngster will benefit of the experience.
(D) The youngster will benefit out of the experience.

39. (A) There were lesser children in the class than expected.
(B) There were fewer children in the class than expected.
(C) There were a little children in the class than expected.
(D) There were a small children in the class than expected.

40. (A) The child can't hardly wait till its birthday.
(B) The child can wait till it's birthday.
(C) The child can hardly wait till its birthday.
(D) The child can wait hardly till its birthday.

Answer Keys

1. B	**2.** C	**3.** D	**4.** A	**5.** A	**6.** C	**7.** D	**8.** B	**9.** B	**10.** D
11. A	**12.** C	**13.** B	**14.** C	**15.** C	**16.** C	**17.** B	**18.** D	**19.** A	**20.** C
21. A	**22.** D	**23.** C	**24.** D	**25.** B	**26.** D	**27.** C	**28.** A	**29.** A	**30.** C
31. D	**32.** B	**33.** A	**34.** D	**35.** A	**36.** C	**37.** D	**38.** A	**39.** B	**40.** C

Hints and Explanations

1. Indences correlate. Choice (B)

2. 'Nor' must be replaced by 'or'. Choice (C)

3. 'Who the winner is' is redundeant. Use only 'the winner'. Choice (D)

4. 'Went about' is the wrong phrase in the context of the given sentence. Choice (A)

5. "A score of apples' is the correct phrase, as 'score' means twenty or a set of twenty. Choice (A)

6. Choice (C)

7. Choice (D)

8. Choice (B)

9. Choice (B)

10. Choice (D)

11. Choice (A)

12. Choice (C)

13. Choice (B)

14. Choice (C)

15. Choice (C)

16. The third part of the sentence should be 'and crystallizes it'. Crystallizes is a verb which takes an object. Choice (C)

17. Part 2 of the sentence is faulty as the preposition used should be 'of' and not 'in'. One is the 'master of the art' not in the art. Choice (B)

18. The fourth part of the sentence is faulty, as 'drifting into winter' is correct - it indicates motion. 'In' does not indicate motion. Choice (D)

19. In the first part of the sentence 'its' is in the genitive case, so there need not be an apostrophe, 'Because of its hardness' is correct. Choice (A)

20. The third part of the sentence should read 'devote yourself unswervingly and unflinchingly' to maintain parallelism in construction. Choice (C)

21. The two sentences can be combined into a simple sentence by changing the verb 'am' into '*v* + ing' (i.e.) 'being'. Hence 'A' is possible. The conjunction 'as' also can be used, as it shows reason. But 'C' cannot be used as the sentence cannot be combined with the conjunction 'while'. Choice (A)

22. The sentence donates a positive meaning. Hence it cannot begin with 'though'. The conjunction 'as' and 'since' can begin the sentence. But here 'B' begins with the second sentence which changes the meaning of the given sentence. Choice (E)

23. The sentences cannot give the same meaning if the conjunctions 'while' or 'because' are used. It shows a particular time and hence 'when' would be the right way to begin the sentence. Choice (C)

24. 'If' or 'whenever' can also begin the sentence. But here it is not a general statement. It specifies that particular time as 'now' is used. Hence 'as' is the most appropriate way to begin this sentence. Choice (D)

25. The sentence is about 'not achieving'. 'B' and 'C' talk about 'achieving' and hence cannot be appropriate. Choice (B)

Solutions for questions 26 to 28:

26. 'Between' is better than 'among' since only two – the Indian and Nepalese – rebel groups are mentioned. (Between is used for two or more, among for three or more). We are talking of the threat <u>in</u> Nepal not <u>of</u> Nepal. New Delhi cannot ignore (disregard intentionally) not ignorance (lacking knowledge). Choice (D)

27. The adverb 'aggressively' qualifies 'persuading' and hence must precede it (rules out choices 1 and 4). Choice 2 is wrong because 'to shifting' is incorrect. Choice (C)

28. Since the verb at the beginning of the sentence is 'were' (plural) it must be 'efficient and highly organized'. If it were 'efficiently' then both (efficiently and highly) qualify 'organized' and the verb would be was (rules out 2 and 4). We are talking of artisans and craftsmen (plural again not singular) Choice (A)

29. The past tense should be used. The first sentence should be "He said that he would come, but he didn't". Choice (A)

30. The second person must come first.
Statement 1 should be "My friend and I like to play tennis on grass court."
Statement 2 Each of the mistakes has to be corrected before printing. Choice (C)

31. No error in both the sentences. Choice (D)

32. You are not going to the theatre, are you? Choice (B)

33. He informed me before he posted the letter yesterday. Choice (A)

34. The gerund form should be maintained throughout a sentence. Choice (D)

35. 'You' must be followed by 'you'. 'One' must be followed 'one'. Therefore, Choice (A) is right and the other choices are incorrect. Choice (A)

36. ……. Cleverer than any other……… Choice (C)

37. 'Between' should be followed by the objective case 'him' and not the subjective case 'he'. Choice (D)

38. You benefit from something. Correct preposition. Choice (A)

39. In case of numbers we use 'fewer'. 'Less/Lesser' are used in the case of weight and 'little/small' in the case of size. Choice (B)

40. 'Hardly' indicates 'cannot'. So, 'the child cannot wait for its birthday' is being intended. Choice (C)

VERBAL ABILITY TEST 2

Number of Questions: 40 **Time: 30 min**

Directions for questions 1 to 5: In each of the following questions, 3 words are related in some way. Find the "odd man" out.

1. (A) Expert (B) Professional (C) Civilized (D) Maestro

2. (A) Blue (B) Crimson (C) Ruby (D) Scarlet

3. (A) Commentary (B) Critique (C) Authority (D) Review

4. (A) Decahedron (B) Decade (C) Decagon (D) Decibel

5. (A) Grave (B) Coffin (C) Tomb (D) Monument

Directions for questions 6 to 15: In each question given below, identify the word which is similar in meaning (synonym) to the question word.

6. ABATE
(A) improve (B) decrease
(C) subside (D) sharpen
(E) sweep

7. RENOUNCE
(A) relinquish (B) withdraw
(C) forgive (D) punish
(E) accept

8. ABDUCT
(A) ransack (B) surround
(C) induce (D) destroy
(E) kidnap

9. DETEST
(A) hate (B) rebel
(C) neglect (D) pretend
(E) captivate

10. CLEANSE
(A) polish (B) flow
(C) absolve (D) reveal
(E) revolve

11. ABODE
(A) sanctuary (B) residence
(C) reformatory (D) dwelling
(E) shelter

12. RESCIND
(A) withhold (B) countermand
(C) hamper (D) suppress
(E) encroach

13. EPITOMIZE
(A) disappoint (B) distend
(C) exemplify (D) generate
(E) lengthen

14. REPEAL
(A) continue (B) prolong
(C) promote (D) reject
(E) abrogate

15. ABSCOND
(A) run away (B) give away
(C) move away (D) forbid
(E) waste away

Directions for questions 16 to 25: In each of the questions given below, identify the word which is opposite (antonym) in meaning to the question word.

16. FUSION
(A) union (B) participation
(C) isolation (D) marriage
(E) gloom

17. SOMNOLENT
(A) drowsy (B) lively
(C) cheerful (D) joyous
(E) active

18. MELLIFLUOUS
(A) harmonious (B) pitchable
(C) discordant (D) internal
(E) external

19. PRODIGAL
(A) generous (B) revisable
(C) frugal (D) pauper
(E) rich

20. DISASTER
(A) puzzle (B) success
(C) omen (D) festival
(E) fiasco

21. ANIMATE
(A) truthful (B) false
(C) active (D) dull
(E) lazy

22. WRETCHED
(A) filthy (B) tidy
(C) neat (D) clear
(E) scenic

23. SQUANDER
(A) waste (B) liberate
(C) presume (D) donate
(E) economize

24. PERTURBED
(A) servable (B) controllable
(C) composed (D) decided
(E) resolved

25. SLUGGISH
(A) lethargic (B) indolent
(C) apathetic (D) intelligent
(E) exuberant

Directions for questions 26 to 29: In each of the following questions, an idiomatic expression and its five possible meanings are given. Pick out the correct meaning of the idiomatic expression and mark the number of that meaning as your answer.

26. To be on the fiddle
(A) to work on something important
(B) to be doing something dishonest to get money
(C) to constantly find fault with others
(D) to have a less important position than somebody or something else alive
(E) to always keep oneself busy

27. As the crow flies
(A) in a very swift manner
(B) in a straight line
(C) in a very precise manner
(D) very short distance away
(E) in a clumsy way

28. In a melting pot
(A) in a very difficult situation
(B) to take an important decision
(C) to be prone to bad influences
(D) in a helpless situation
(E) in the process of changing

29. To open somebody's eyes
(A) to keep a watch on someone
(B) to be more observant and quick to notice things
(C) to refuse to listen to others
(D) to become close or friendly with someone
(E) to make someone realize or understand something

Directions for questions 30 to 34: In each question, a sentence is given with an idiom (underlined). Four possible meanings of the idiom are also given. Identify the correct meaning and mark the number of the correct choice as answer.

30. The officer was guilty of <u>sharp practices</u> and so he was dismissed.
(A) angry behaviour
(B) disobeying
(C) questionable dealings.
(D) constant quarrel

31. We were <u>kept on tenterhooks</u> while the judges were deciding the winners.
(A) in anxiety (B) in trouble
(C) on the bridge (D) in the witness box

32. The dispute among the students <u>came to a head</u> and the principal declared a holiday.
(A) came to one man's decision
(B) reached a crisis
(C) did not stop
(D) started all over again

33. After getting a job he had no difficulty in <u>keeping the wolf out of the door.</u>
(A) being physically safe
(B) getting the door clear
(C) avoiding starvation
(D) keeping the job intact

34. He asked me not <u>to thrust my nose into his affairs</u>.
(A) smell anything wrong
(B) meddle officiously
(C) forget
(D) combine

Directions for questions 35 to 40: In each of the following questions four numbered choices are given. Three of them belong to the same category. Mark the number of the 'odd man' as your answer.

35. (A) Parents (B) Love
(C) Guidance (D) Punishment

36. (A) Periphery (B) Perfunctory
(C) Cursory (D) Superficial

37. (A) Prune (B) Abridge
(C) Trim (D) Spruce

38. (A) Overfly (B) Overeat
(C) Overdressed (D) Overdose

39. (A) Serendipity (B) Fortuity
(C) Guess (D) Coincidence

40. (A) Vague (B) Abstract
(C) Intangible (D) Empirical

Answer Keys

1. C	**2.** A	**3.** C	**4.** D	**5.** D	**6.** B	**7.** A	**8.** D	**9.** A	**10.** C
11. D	**12.** B	**13.** C	**14.** D	**15.** A	**16.** C	**17.** D	**18.** C	**19.** C	**20.** B
21. D	**22.** B	**23.** D	**24.** C	**25.** D	**26.** B	**27.** B	**28.** D	**29.** A	**30.** C
31. A	**32.** B	**33.** C	**34.** B	**35.** A	**36.** A	**37.** D	**38.** A	**39.** C	**40.** D

Hints and Explanations

1. Expert, professional and maestro talk about the skill or knowledge of a person; 'civilised' talks about the behaviour of a person. Choice (C)
2. Crimson, ruby and scarlet are shades of red. Choice (A)
3. Commentary, critique, review refer to the expression of opinion. Choice (C)
4. Choices 1, 2 and 3 are related to 'ten' (ie) 'deca'. In 'decibel', 'deci' means one-tenth. Choice (D)
5. Grave, tomb and coffin are related to death. Choice (D)
6. The words abate and decrease are synonymous. Choice (B)
7. The word relinquish is synonymous with renounce (give up, discard). Choice (A)
8. Abduct and kidnap are synonymous. Choice (E)
9. Detest means to loathe or hate. Choice (A)
10. Cleanse (rid of something unpleasant or unwanted) is synonymous with absolve (declare free from guilt, blame or sin). Choice (C)
11. The word abode which is a formal or literary term means a house or home, 'dwelling' is its closest synonym. The word 'residence' can be ruled out because it refers only to a person's home, whereas the word 'abode' has a wider connotation. For eg. the abode of animals, the abode of god but it is absurd to say the residence of god or the residence of animals. Choice (D)
12. Rescind means to cancel (a law, order or agreement) the word countermand, which also the means the same, is its synonym. Choice (B)
13. The word epitomize means to be a perfect example of. The word 'exemplify' is its closest synonym. Choice (C)
14. Repeal means to officially cancel (a law of act of parliament) the word 'abrogate' also means the same. Choice (E)
15. Abscond means to leave quickly and secretly to escape from custody or avoid arrest. Choice (A)
16. Fusion (the process of joining two or more things to form a whole) and isolation are antonymous. Choice (C)
17. Active (alert) is an antonym of somnolent (sleepy; drowsy). Choice (E)
18. The word mellifluous means pleasingly smooth or musical to hear. The word discordant (cacophonous, harsh) is its antonym. Choice (C)
19. Prodigal (wasteful, extravagant) and frugal (thrifty, economical) are antonyms. Choice (C)
20. Success is an antonym of the word disaster which also means failure. Choice (B)
21. Animate (alive) and dull are antonyms. Choice (D)
22. The words wretched (miserable) and tidy are antonyms. Choice (B)
23. Squander means to waste (money, time, etc) in a reckless or foolish way. Economize is its antonym. Choice (E)
24. Composed (calm) is an antonym of the word perturbed (alarmed). Choice (C)
25. Exuberant (lively, cheerful) and sluggish (inactive) are antonyms. Choice (E)
26. "To be on the fiddle" means doing something dishonest to get money. Choice (B)
27. The idiom, 'as the crow flies' means 'in a straight line'. Choice (B)
28. The idiom, 'in a melting pot' means 'in the process of changing'. Choice (E)
29. 'To open somebody's eyes' is to cause or make someone realize or understand something. Choice (E)
30. Choice (C)
31. Choice (A)
32. Choice (B)
33. Choice (C)
34. Choice (B)
35. Parents are not concepts as the other three are. Choice (A)
36. Periphery. The other three are associated with casual attitude. Choice (A)
37. The first three refer to cutting short. Choice (D)
38. Overfly is to fly above a place. In all the other three over has the meaning of excessive. Choice (A)
39. Guess. The others refers to a favourable chance. Choice (C)
40. Empirical is verifiable the others are not. Choice (D)

VERBAL ABILITY TEST 3

Number of Questions: 40 **Time: 30 min**

Directions for questions 1 to 10: In each question the word at the top is used in four different ways. Select the option in which the usage of the word is INCORRECT or INAPPROPRIATE.

1. COMMUNICATE

A.	The deaf and dumb communicate by means of sign language.
B.	The excitement was palpable and communicated itself to the crowd.
C.	People living in the suburbs have to communicate a long distance every day.
D.	A contagious disease is communicated through physical contact.

2. OPPOSITE

A.	Being an adolescent, Mrinal felt shy talking to members of the opposite sex.
B.	I expected the bride to be shy and quiet, but she was just the opposite.
C.	The Raos live further down, on the opposite side of the road.
D.	The bank is opposite to the supermarket.

3. RAW

A.	Dostovsky's novels often portray life in the raw.
B.	Women labourers often get raw deal from contractors.
C.	Her own experiences provided the raw material for her first novel.
D.	Being marooned on the island forced them to eat raw meat.

4. PICTURE

A.	There have been a number of changes recently - let me put you in picture.
B.	Atticus always told the old lady that she looked a picture.
C.	Ever since he went into pictures the couple have been drifting apart.
D.	From the reports, the picture for the service sector is encouraging.

5. MASTER

A.	Vivek realised that he was expected to master Japanese before leaving for Japan.
B.	Tilak has a masters in Business Administration.
C.	The master bedroom was spacious and comfortable.
D.	"Don't be led by others, be your own master," Sampath said severely.

6. CLOSE

A.	The soldiers advanced in a close formation.
B.	Over the next few months we have to keep a close eye on sales.
C.	Alind closed down in the nineties.
D.	The police often close ranks when one of their officers is accused.

7. BACKGROUND

A.	The name 'TIME' is written in red on a white background.
B.	The film has good background music by Rehman.
C.	The elections in Sri Lanka took place on a background of violence.
D.	The Director asked for more background on the company's financial position.

8. DEFENCE

A.	Whenever Roja was criticized, her brother leapt to her defence.
B.	Why don't you give a chance for the body's natural defence mechanism to protect it?
C.	No cost is too high when it is for the defence of the country.
D.	News of an imminent attack forced the troops onto the defence.

9. CHANGE

A.	The property changed hands several times in the last decade.
B.	Expecting a change of heart from that stubborn mule is useless.
C.	Some of my old dresses will have to be changed to fit me now.
D.	Marriage has changed Sania for better.

10. EARTH

A.	Yasho was the happiest person on earth when she won the gold medal.
B.	Aditya flung his bike on the earth and rushed inside when he saw smoke emanating form the house.
C.	The good earth always gives back several times what you put in.
D.	Be sure to earth household electrical gadgets so that they are safe to handle.

Directions for questions 11 to 20: In each of the following questions, a paragraph with a 'blank' is given. From the four choices, select the sentence, which can go into the blank to make the paragraph logically coherent.

11. India has the distinction of becoming a country with a billion people. Thus, it becomes the second largest populated country in the world. (____.)
(A) The global population has almost touched the six billion mark.
(B) Statistically speaking, every sixth person in the world is an Indian.
(C) Forced population control is not desirable.
(D) It is difficult to judge how many forests have been encroached upon.

12. In the armed forces, before independence, Muslims constituted around 35 percent of the total strength. (____.) Why so few Muslims?
(A) Most of them were recruited from Punjab and nearby areas.
(B) Today, it has gone down to just a mere two percent out of a total close to a million.
(C) This is basically due to a lack of lobby.
(D) This can be attributed to a change in human behaviour.

13. It would be a Herculean task to remodel our settlements to keep them clean through proper waste disposal systems. As such we have already realized that, recycling of waste liquids and solids would be a saner approach. (____.)
(A) Now we are very used to human interference with nature.
(B) We may not be able to survive utilising the available resources.
(C) But organising such measures will involve considerable time, effort, management and education.
(D) Many of our organisation lack this foresight.

14. When a bird hits an aircraft, it can cause potentially catastrophic damage. (____.) So a team at Britain's Defence Evaluation and Research Agency plans to use crystals that glow when fractured to warn of such unseen damage.
(A) This makes visual inspection of damage unreliable.
(B) This is one of the greatest dangers of information technology.
(C) This is the ease with which communication goes on these days.
(D) But in planes made of carbon composites, such damage may be impossible to spot.

15. The General Electric Company is setting up India's first multi-disciplinary research centre. (____.) It will contribute to the development of multi-disciplinary engineering capabilities in India.
(A) Named the GE India Technology centre, it is also the largest of its kind.
(B) It plans to recruit 500 research scientists.
(C) This will help develop GE's global business.
(D) The project will be over by December, 2000.

16. One major change in careers is that one can work from home. (_____.) So far, only work relating to Information Technology has been thus affected. It is expected that many other careers will afford this flexibility in the future.
(A) Therefore one should develop a confident, outgoing personality.
(B) There is no such thing as a permanent job.
(C) New technologies ensure that geographical distance is not a hindrance to one's work.
(D) While it is true that people will switch jobs faster than ever before, one must be loyal to one's organisation.

17. (____.) There are several cave paintings, stone engravings and carved figures which bear this out. The Neanderthal man attempted this too, but his drawings of the tools he used show that they were rather crude.
(A) Prehistoric man used sophisticated tools for drawing and carving figures.
(B) The Cro-Magnon man, who was the forerunner of modern man, earned his daily bread through paintings.
(C) The Cro-Magnon man, who was the forerunner of modern man, was the first fine artist in the history of man's evolution.
(D) Prehistoric man pursued painting and carving figures as a hobby.

18. The natural atmosphere which man has inherited from the past, has been deteriorating under the impact of industrialization. Factories pump millions of tons of dust into the air, vehicles spread fumes and sprays are used to kill agricultural pests – all combine to change the ideal picture. (____.) The situation near big cities and heavily industrialized areas has become particularly bad, and the air is not fit for breathing.
(A) Movement of vehicular traffic on the roads should be restricted.
(B) The pollution of air has become a matter of great concern because it continues to increase as civilization spreads.
(C) Society will have to move towards stricter pollution control.
(D) The atmosphere should be protected as it is a great and irreplaceable resource for living.

19. In recent times, the number of working women has increased considerably in urban areas. With more and more women opting for career-oriented courses, offices and business establishments are flooded with applications from qualified women. (____.) Women are working side by side with men in all walks of life.
(A) A working woman's life is not a bed of roses.
(B) In fact, there are very few workplaces today which do not have single women.

(C) It is possible to maintain a good standard of living only if the woman contributes to the family income.
(D) Even in small towns and villages, most women are employed.

20. For several thousands of years, the moon has been the only satellite of the earth. Today, however, the earth has many other satellites – all made by man. (____.) However, some of them will still be going around the earth thousands of years from now.
(A) Artificial satellites do not fall because they are not affected by earth's gravity.
(B) They travel in an orbit around the earth.
(C) As they speed along, they tend to go straight off into space.
(D) These artificial satellites are very much smaller than the moon.

Directions for questions 21 to 30: In the following passage there are blanks, each of which has been numbered. These numbers are printed below the passage and against each, five words are suggested, one of which fits the blank appropriately. Find the appropriate word in each case and mark its number as your answer.

In most developed and developing nations, the illicit trade in live wild animals is (21). Each year millions of (22) are wrenched from their natural habitats by people (23) to make quick money, then routed through a ragtag chain of middlemen and international dealers to meet the (24) demand of private collectors in Saudi Arabia, pet shops in Germany, Japan and the U.S.; zoos and circuses in Eastern Europe and folk healers in Asia. According to a wildlife expert, it is the third biggest (25) business, after drugs and arms.

Though many exotic species can be purchased (26) trade in animals and birds that are in (27) danger of extinction is (28) under the United Nations Convention on International Trade in Endangered Species (CITES), which has been signed by 120 nations. The treaty also regulates trade in other species that are seriously threatened, but its provisions are widely (29), even in signatory countries. Tigers have all but (30) from China and are fast disappearing from India and Siberia.

21. (A) diminishing (B) unfavourable (C) miserable (D) flourishing

22. (A) creatures (B) categories (C) people (D) characters

23. (A) hopeless (B) agreeable (C) desperate (D) susceptible

24. (A) fulfilled (B) imperative (C) unavoidable (D) insatiable

25. (A) licensed (B) virtual (C) authentic (D) illegal

26. (A) inequitably (B) immorally (C) legally (D) profitably

27. (A) imminent (B) preliminary (C) precursory (D) terrible

28. (A) authorized (B) recommended (C) sanctioned (D) banned

29. (A) defended (B) ignored (C) cherished (D) maintained

30. (A) departed (B) depleted (C) withdrawn (D) vanished

Directions for questions 31 to 40: In each of the following questions a pair of words in capitals is given followed by four numbered pairs of words. Select from the choices the pair which exhibits the same relationship as the capitalised pair of words and mark the number as your answer.

31. COGENT : CONVINCING
(A) Insane : Distinguished
(B) Laconic : Pithy
(C) Illogical : Reasonable
(D) Jovial : Abstruse

32. RETROSPECTION : PAST
(A) Syllogism : Logic
(B) Idiosyncrasy : Coherence
(C) Prognostication : Future
(D) Transience : Rigidity

33. EULOGISE : LAMBAST
(A) Mystify : Narrate
(B) Dissemble : Besmirch
(C) Invigorate : Debilitate
(D) Malinger : Adhere

34. LION : PRIDE
(A) Rabbit : Burrow (B) Pup : Litter
(C) Whale : Consort (D) Sow : Sty

35. PENURIOUS : AFFLUENCE
(A) Interrogation : Accusation
(B) Garnishment : Command
(C) Taciturn : Verbosity
(D) Condemnation : Mischief

36. MACHIAVELLIAN : DECEIT
(A) Amphibious : Plants
(B) Acquisition : Assumption
(C) Acquittal : Suit
(D) Naïve : Gullibility

37. BENEVOLENT : GRASPING
(A) Repulsive : Pushing
(B) Euphonious : Discordant
(C) Churlish : Impolite
(D) Rebellious : Disorderly

38. FRIGHTEN : PETRIFY
(A) Enamour : Protect
(B) Sneer : Appreciate
(C) Abbreviate : Interest
(D) Humiliate : Mortify

39. MUNIFICENT : STINGY
(A) Inclement : Merciless
(B) Incorrigible : Recalcitrant
(C) Articulate : Obscure
(D) Egregious : Outstanding

40. EUPHORIC : ECSTASY
(A) Modified : Version
(B) Redundant : Relevant
(C) Licentious : Sentiment
(D) Cryptic : Enigma

Answer Keys

1. C	**2.** D	**3.** B	**4.** A	**5.** B	**6.** A	**7.** C	**8.** D	**9.** C	**10.** B
11. B	**12.** B	**13.** C	**14.** D	**15.** A	**16.** C	**17.** A	**18.** B	**19.** D	**20.** D
21. D	**22.** A	**23.** C	**24.** D	**25.** D	**26.** C	**27.** A	**28.** D	**29.** B	**30.** D
31. B	**32.** C	**33.** C	**34.** B	**35.** C	**36.** D	**37.** B	**38.** D	**39.** C	**40.** D

Hints and Explanations

1. In sentence 3 the intended word is 'commute' (travel regularly by bus, train etc between your place of work and home) and not "communicate" (to exchange information, ideas etc). Choice (C)

2. In sentence 4 opposite is a preposition meaning "on the other side of a particular area from something" and does not require 'to'. Choice (D)

3. In sentence 2 it should be 'a raw deal'. The idiom means 'the fact of somebody being treated unfairly'. Choice (B)

4. Sentence 1 should read … in the picture. The idiomatic expression 'to put somebody in the picture' means to give somebody the information they need in order to understand a situation. In sentence 2 'to look a picture' means to look very beautiful or special. Choice (A)

5. When 'masters' refers to a university degree it takes the apostrophe, hence 'Master's in Business Administration'. Choice (B)

6. In sentence 1 'close' means 'without space'. Hence it is 'close formation' not '<u>a</u> close formation'. To "close ranks" (sentence 4) means to work closely together to defend themselves. Choice (A)

7. In sentence 3, background refers to the past. It should be '. . . <u>against</u> a background of violence' but not 'on'. Choice (C)

8. Sentence 4 should read ' . . . onto the defensive' - an idiom that means acting in a way that shows that you expect to be attacked or criticized. Choice (D)

9. 'Change' is a very general term that is used to describe any act of making something different. In sentence 3 the right word is 'alter' not 'change'. You 'alter' something by making a difference in its appearance, character or use. You cannot use 'change' here. Choice (C)

10. To talk about our planet we use 'earth'. Earth is also used to refer to the soil. As a verb it means making an electrical equipment safe by connecting it to the ground by a wire. But when we refer to the hard surface we walk on, we use 'ground' for outside and 'floor' for inside. In sentence 2 it should be 'ground' not earth. Choice (B)

11. The topic is about India. Global population, population, forests are irrelevant. Hence, option B is the only relevant choice. Choice (B)

12. Option B is the only statement that leads to the question, "Why so few Muslims?" Choice (B)

13. The first line has the words "Herculean task" indicating that the task is not easy. Option C brings out the aspect that such measures will require a lot in terms of time, effort, management and education. Option C is the right answer. Choice (C)

14. Choice D is the only relevant statement. Since, the damage could be impossible to spot, an agency is coming up with a new alternative (as suggested by the last line). Choice (D)

15. Stylistically, the flow of the sentences is as follows: GE is going to set up a research center. Named . . ., it is . . . largest of its kind. It will (this research center will) contribute to . . . Hence, option A is the right answer, as it fits the blank perfectly. Choice (A)

16. Sentence 3 is the ideal choice because it supports the previous sentence and explains the subsequent sentences. Choice (C)

17. The second line states, "There were . . . Figures which <u>bear this out</u>. He attempted to create his daily life <u>through this</u>. The Neanderthal man attempted <u>this too</u>, but <u>his drawings</u> of the tools . . . So, we need a statement in the blank which is a general statement on the pre-historic man and drawing/painting. Our choice narrows down to (A) and (D). Out of these two choices, option A is better because it contrasts the sophisticated tools used by pre historic man with crude ones used by Neanderthal man. Choice (A)

18. Sentence 2 explains how air pollution has increased with the spread of civilization. This sentence explains the last sentence of the paragraph. Choice (B)

19. Choice (D) best supports the argument put forth in the paragraph. Choice (D)

20. Choice (D) speaks about artificial satellites, which though smaller than the moon will revolve around the earth even after several years. The sentence emphasizes the power of artificial satellites.

Choice (D)

21. The illicit trade in wild animals is prospering or growing or 'flourishing' in most developed and developing nations. The passage goes on to explain how it is 'flourishing'. All the other choices are negative terms which are inappropriate in this context.

Choice (D)

22. Animals live in their natural habitat. These 'creatures' are wrenched or harshly pulled away from their natural habitat. Choice (A)

23. Who are the people who wrench these creatures away from their natural habitat? They are those who are 'desperately' looking for quick money. The 'desperate' desire to make quick money drives these people to get themselves involved in this illicit trade. Choice (C)

24. The paragraph goes on to give a list of people who are interested in this trade – from private collectors to folk healers. Hence there is always a heavy demand or an 'insatiable' (impossible to satisfy) demand for these animals. Choice (D)

25. But this trade in live wild animals is an 'illegal' business and is rated next to drugs and arms business at the global level. It cannot be 'authentic' or 'licensed' or 'legal' as the second para goes on to explain the directives of the CITES treaty made by various nations in order to protect the endangered species from extinction. Choice (D)

26. Some of these exotic species can be purchased. The use of 'though' indicates that this purchase is considered 'legal' with reference to certain species of animals and birds which are available in plenty. Further the sentence goes on to say that trade in some species of birds and animals that are near extinction is not allowed under the UN convention. Hence 'legally' is the most appropriate word. Choice (C)

27. There are certain species of animals and birds that are in 'imminent' (impending) danger of extinction. Trading of such animals and birds are considered illegal. Choice (A)

28. It is considered illegal because it has been 'banned' by the CITES, a world organisation established in order to protect endangered species of birds and animals globally. Choice (D)

29. The treaty not only 'bans' the trade of certain endangered species but also regulates trade in other species that are threatened. But still the illegal trade of all these animals continues to flourish even in the 120 countries that have signed the treaty. This means that the treaty and its directives are 'ignored' even by these countries. All the other choices are positive and conveys an opposite meaning and hence are incorrect. Choice (B)

30. The last sentence says that tigers are fast disappearing from India and Siberia. Preceding this, it says tigers have already disappeared from China. Hence 'vanished' is the most appropriate choice. 'Deplete' or 'withdrawn' or 'depart' shows that they are decreasing. But the sentence suggests that tigers have already disappeared from China. Hence choice D is most appropriate. Choice (D)

31. Cogent and convincing are synonyms as are laconic and pithy. Choice (B)

32. Retrospection is analysis of something related to the past. Prognostication means prediction, which is related to the future. Choice (C)

33. Eulogise means praise, whereas lambast indicates criticism. Hence, option (C) is the answer as this choice also has a pair of antonyms. 'Invigorate' means 'to energise' and 'debilitate' means to 'weaken'.

Choice (C)

34. A group of lions is referred to as a 'pride', whereas a group of pups (puppies) is called 'litter'. Choice (B)

35. Antonyms Choice (C)

36. A machiavellian person indulges in deceit. A naïve person shows gullibility. Choice (D)

37. Antonymous relationship Choice (B)

38. Relationship of degree. Petrify indicates extreme fright. Mortify indicates extreme humiliation. Choice (D)

39. A munificent person is generous (not stingy) Likewise, any argument which is articulate is very clear (not obscure). Choice (C)

40. Cryptic and enigma are also synonymous.
In both the first is an adjective and the second a noun.
Choice (D)

Verbal Ability Test 4

Number of Questions: 40 **Time: 30 min**

Directions for questions 1 to 8: Four alternative summaries are given below each text. Choose the option that best captures the essence of the text.

1. An experiment is an observation that can be repeated, isolated and varied. The more frequently you can repeat an observation, the more likely are you to see clearly what is there and to describe accurately what you have seen. The more strictly you can isolate an observation, the easier does your task of observation become, and the less danger is there of being led astray by irrelevant circumstances, or of placing emphasis on the wrong point. The more widely you can vary an observation, the more clearly will the uniformity of experience stand out and the better is your chance of discovering laws.

(A) It is essential that scientific experiments be repeated, isolated and varied because repetition increases accuracy, isolation facilitates observation and variety increases the chance of discovering laws.

(B) Isolation and repetition of experiments help in reducing the possibility of error in observation. The lesser the margin of error, the better is the chance of discovering new laws.

(C) Scientific experiments should be isolated, repeated and varied. Repetition helps to see clearly and isolation reduces the chance of being led astray. But it is uniformity in the variety that gives a better chance of discovering laws.

(D) An experiment should be isolated, repeated and varied. They help in observing things clearly and accurately. They also increase the chance of discovering new laws.

2. Aristotle in his little treatise on 'Definitions' suggests that every good definition has two parts, stands on two solid feet: first it assigns the object in question to a class or group whose general characteristics are also its own - so man is, first of all, an animal : and secondly, it indicates wherein the object differs from all the other members in its class - so man, in Aristotelian system, is a rational animal, his 'specific difference' is that unlike all other animals he is rational. Aristotle drops an object into the ocean of its class, then takes it out all dripping with generic meaning with the marks of its kind and group; while its individuality and difference shine out all the more clearly for this juxtaposition with other objects that resemble it so much and are so different.

(A) A good definition is like a man standing on two solid feet. It not only helps in assigning an object to a generic group but also shows the differences and dissimilarities to make it shine out of the group.

(B) Aristotle suggests that man as an object has to be defined first as an animal and then shown to be different from other animals in his rationality. Such an interpretation helps in revealing the unique characteristic of an object.

(C) In order to define an object, Aristotle first assigns it to its class and then points out its dissimilarities. Such a comparison makes its individuality more conspicuous.

(D) According to Aristotle, any good definition of an object has two parts, one that identifies it with its group and the other that shows its uniqueness. A definition that includes both these will help in identifying the individuality of the object.

3. Sociologists are often criticized for their use of jargon, their apparent predilection to develop new words while at the same time giving new and often strange meanings to old and familiar terms. The charges are often justified. Equally often they go beyond reason. Systematic discussion is impossible if one does not work with more or less precisely defined terms. Without a technical language, scientific communication becomes cumbersome and inefficient. In the humanities too, the desire to be more precise in analysis leads to the elaboration of technical terms.

(A) Sociologists are often criticized for the use of strange jargon and for giving extended meaning to familiar terms. But they are justified in doing so as such usage makes their analysis more precise and systematic.

(B) Critics are justified in their charge against the sociologists for their use of strange jargon or for giving extended meaning to familiar terms. But the sociologists give the example of the scientists to justify their stand.

(C) In a desire to be more precise in their analysis sociologists develop new words or give strange meaning to old terms. Their justification is that only such extension of meaning can help them in systematic discussions.

(D) While critics are justified in accusing sociologists of using strange jargons and for giving extended meaning to familiar terms, sociologist use such terms in their desire to be more precise and systematic in their discussions and analysis.

4. How big is your footprint? We're not talking about whether you walk around in dainty Cinderalla's slippers or size 18 Wellington boots, but rather the size of the damaging dent that you leave in the Earth's environment

and its resources while going about your daily life. One way of finding out if your impact on the world is like teetering on tiptoes or stomping about in steel-capped boots is to measure your personal ecological and carbon footprints. A carbon foot print is a measurement of the effect you have on the climate in terms of the total amount of greenhouse gases that your actions cause to be produced, while an ecological footprint is a measure of the amount of productive land required to support your resource demands and to absorb the waste you produce.

(A) Is your footprint on the environment like walking on tiptoe or stomping around in steel boots? That is what is measured by ecological footprint - the land you require for resources to meet your needs, the waste you dump and the greenhouse gases that you cause.

(B) How much we impact our environment is measured by carbon footprint and ecological footprint - the former a measure of the greenhouse gases we are responsible for and the latter the land we need for resources and waste.

(C) If one walks around daintily like Cinderella, one harms the environment less than if one tramps around in boots. The greenhouse gases one emits, the waste one generates and the resources one needs, all measure the damage one inflicts on one's surrounding.

(D) The effect our actions have on our environment is measured by carbon footprint, the amount of land we need to sustain ourselves, the waste and greenhouse gases generated by us are noted to arrive at this figure.

5. It's clear that in many countries protected areas are seriously compromised. The dramatic rise of hunting for bushmeat in west and central Africa over the past 30 years has been well documented, as has the explosion in the past decade of illegal logging in southeast Asia. But elsewhere, protected areas face more insidious threats. The Dong Hua Sao National Biodiversity Conservation Area in Laos, for example, has suffered a gradual erosion of its boundaries at the hands of coffee growers keen to take advantage of its rich volcanic soil. The reasons for such pressures are varied and complex, but they often include population growth, land pressure, poverty, corruption and poor law enforcement.

(A) While hunting and logging are generally considered the main threat to protected areas, a greater evil is the gradual occupation of land by people living around who are attracted by its fertile land.

(B) Population growth, poverty, and corruption have led people in Laos to gradually occupy the land under conservation for biodiversity. Thus it is a greater challenge than the rise in hunting for bushmeat in west and central Africa or logging in southeast Asia.

(C) Protected areas are threatened by hunting and logging as also by subtle actions like poaching on its land for cultivation. The main reason for the threat are population pressure on land, poverty and corruption.

(D) Poaching by coffee growers of protected land is as much a threat as hunting and logging in Asia. The villain of the piece is, however, corruption and poor law enforcement.

6. Two decades after the event, and the word 'Chernobyl' still carries a lot of baggage but then, we're still 25,000 years away from being clean. The meltdown at Reactor 4 of the Soviet nuclear power station was caused by an unnecessary low-power test, an experiment to determine whether the reactor could restart itself with all external power shut off. The result was a black fireball that blew the reactor's roof off and spread radiation across much of the Northern Hemisphere. Equally toxic was the Soviet government's initial response : a three-day silence as to the dangers of the situation. Protective foam was sprayed around the nearby towns, Chernobyl and Pripyat, whose combined population was 135,000, but otherwise, life proceeded as normal - children played in the foam, marched in the May Day parade and breathed in the contaminated air. While the official death toll remains at 41, who knows how many fatalities resulted from the delay in evacuating these towns?

(A) Twenty years after Chernobyl, caused by an experiment, we are still emotionally involved and haven't been able to wipe out the radiation that spread over the Northern Hemisphere. No one knows how many deaths have resulted from the Soviet government's refusal to evacuate the people in the region.

(B) Chernobyl was caused by an unnecessary test that affected millions. The Soviet government's low-keyed response led to many more deaths though officially it is only 41. We are still emotionally swayed by the memory and are far from cleaning up the mess.

(C) The Soviet government's reaction to Chernobyl was as poisonous as the gas it emitted. The government's apathy led to millions of avertable deaths and we still do not know how to stop the radiation from continuing and spreading.

(D) Chernobyl spread radiation across the Northern Hemisphere and affected so many that two decades later we are still emotionally affected by the memory. The Soviet government put up a brave front by not evacuating the people and spraying foam to contain the radiation.

7. Down syndrome babies are generally born to older mothers. The probability of having a Down syndrome baby grows rapidly and exponentially as the age of the

mother increases, from 1 in 2,300 at the age of twenty to 1 in 100 at forty. It is for this reason alone that Down embryos are the principal victims or their mothers the principal users, of genetic screening. In most countries amniocentesis is now offered to - perhaps even imposed on - all older mothers to check whether the foetus carries an extra chromosome. If it does, the mother is offered or cajoled into an abortion. The reason given is that despite the happy demeanor of these children, most people would rather not be parent of a Down child. If you are of one opinion, you see this as a manifestation of benign science, miraculously preventing the birth of cruelly incapacitated people at no suffering. If you are of another opinion you see the officially encouraged murder of a sacred human life in the dubious name of human perfection and to the disrespect of disability. You see, in effect, eugenics still in action, more than fifty years after it was grotesquely discredited by Nazi atrocities.

(A) Since Down syndrome babies are born to women over forty, they are often forced to have amniocentesis. This has been controversial. Some see the prevention of the birth of a disabled child as kindness while others think the state is indulging in eugenics as did the Nazis.

(B) The probability of having a child with Down syndrome increases as the age of the mother increases. So older women take the amniocentesis test and abort a Down embryo even though this amounts to a murder. The killing of the disabled is similar to the practices of the Nazis.

(C) The amniocentesis has been a controversial test since it is used to identify and do away with a Down embryo in older women. While some think it is kindness to prevent the birth and unnecessary suffering that a Down child suffers, others think they are murdering the disabled to selectively breed a healthy generation.

(D) Mothers over forty are more likely to have a Down child. They are persuaded or forced to have amniocentesis. The abortion of Down embryo can be seen as an expression of benevolent science or as officially sanctioned murder of disabled, that is eugenics fifty years after the Nazis.

8. “I place economy among the first and most important of Republican virtues and public debt as the greatest of the dangers to be feared”. So wrote Thomas Jefferson in 1816 in a letter to William Plumer, the governor of New Hampshire. However, contrary to the wisdom of the great man, national debt is by no means an inherently bad thing. Borrowing money is a historically tried-and-tested method of expanding the productive capacity of an economy, if not a pre-requisite. The USA would go on to amass an enormous debt, but this was used relatively sensibly to bring about a more-than-proportionate growth in the size of its economy.

(A) Though Jefferson warned against public debt, the USA accumulated enormous debt as it considered it to be an essential prerequisite for an economy to grow.

(B) As the USA amassed an enormous debt, Jefferson felt public debt to be an evil that should be avoided. But it is essential for an economy to grow.

(C) Though Jefferson called public debt ‘the greatest of dangers’ yet the example of the USA reaffirms it to be a proven method of bringing about growth in the economy.

(D) Jefferson said public debt would lead to the weakening of an economy but his own country showed that it is one of the means of achieving a more than proportionate progress.

Directions for questions 9 to 18: A number of sentences are given below which, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a letter. Choose the most logical order of sentences from among the four given choices to construct a coherent paragraph.

9. (a) Here is the world’s newest temple of modern art, an artistic triumph in itself, and combinig grandeur, originality and stunning power.

(b) It can be compared to a cathedral, in that it occupies such a vast space as inspires awe.

(c) One of the unique elements of this edifice is a monstrous steel spider that must surely have been inspired by a horror movie.

(d) The place is a huge physical entity, vast, and stupendous.

(A) abdc (B) cdba
(C) adbc (D) cadb

10. (a) In the South, freshly grated coconut is the usual gamish.

(b) In other parts, fresh coriander leaves provide the mandatory finishing touch.

(c) Besides the tempering exercise, the fat-laden coconut helps to release fat-soluble carotence, from a carrot for example.

(d) Traditional gamishes are another characteristic feature of our cooking.

(A) cabd (B) dacb
(C) acdb (D) cbda

11. (a) Every ceramic object is a sculpture in miniature, and constitutes a study in several kinds of paradox.

(b) The fragility of material culture is nowhere more manifest than it is in the art of pottery.

(c) At the formal level, the solidity of the ceramic object conveys a sense of permanence and enduring value.

(d) At the same time, the baked earth, from which it is made, renders it fragile, frangible, a hostage to chance and threat of damage.

(A) bacd (B) abcd
(C) bcda (D) acdb

12. (a) It is more than 200 years since Tipu Sultan's mysterious death occurred while defending his fort in Srirangapattana near Mysore in India.
(b) As a warrior, he was a formidable and implacable enemy.
(c) His reputation in the annals of British history puts him in the same class as Chenghis Khan, Attila the Hun and Emperor Napoleon.
(d) But his legend in India and in the West, is still going stron.

(A) abcd (B) adcb
(C) acbd (D) adbc

13. (a) Puranic literature describes the progression of the cosmos which passes through various yugas, each of which is measured in precise numerical terms.
(b) But in the cosmic scheme of Time, it does not amount to much.
(c) Ten centuries may be a substantial time-span in human reckoning.
(d) In that calculation, our millennium, just over, does not form even a small fraction of the Kaliyuga, the last of the four cosmic cycles.

(A) cdba (B) abcd
(C) adbc (D) cbad

14. (a) My boss asked me to call on one of the Ministers he knew there.
(b) I was leaving for Delhi on work as usual.
(c) This meeting was supposed to sort out the mess he was in, created by a new amendment to excise duty provisions on export of cotton.
(d) At Delhi I sought an appointment with the Minister concerned.

(A) dabc (B) bdca
(C) bacd (D) cdab

15. (a) Since then sponsorship and endorsements started playing a big role, with the profit motive becoming all pervasive.
(b) Recent reports trace the corruption of Olympic movement to the total commercialisation of games in 1984.
(c) Sordid behind the scenes going on in the IOC have been finally exposed.
(d) Salt Lake city which is bidding for Winter Olympics bent all the norms and even bribed IOC members.

(A) cdba (B) cabd
(C) cdab (D) dcba

16. (a) Which pump out an enormous quantity of smoke, poisonous gases and other wastes in the surrounding environs.
(b) The land to accommodate huge population was obtained by cutting down trees.
(c) Area cleared by chopping down forests was used to set up industries, thermal plants etc.
(d) Acre after acre of forest is cleared in one day leaving the atmosphere susceptible to pollution and the earth poorer.
(e) The speed of cutting down forest has increased ever since.

(A) cabde (B) ecdba
(C) bcaed (D) dcbae

17. (a) No place in the house seemed secure.
(b) I remembered the agonies of my own childhood when my sister discovered I was writing poems and began to tease me by chanting them in public.
(c) When my daughter began to write her memories, at the age of four, I decided that she must have a place to keep them, if only a section of a bureau drawer or as it happened, a box with a key.
(d) It seems to me that we can't learn too early in life to respect the privacy of the individual.
(e) I tried desperately to hide the notebook of poems.

(A) bcdae (B) dcbea
(C) cedba (D) aecbd

18. (a) My father and mother were the complements of each other.
(b) Her face was responsive, my father's impassive.
(c) My mother was fragile, my father robust.
(d) My mother was not intellectual her natural propensity was intuitive. Her face illustrated that saying "Appearances are deceptive: for it did not show the immense strength of her moral convictions.
(e) My mother's face rippled to emotions as waters to the wind.

(A) acbed (B) cabde
(C) ecbad (D) dbcea

Directions for questions 19 to 23: In each question below, a paragraph or a statement is given followed by four statements. Classify each of the four statements as per the following categories and from among the answer choices, select the one that gives the sequence of letters that matches with your categorization. Categorise the statement as

(A) if it is a CONCLUDING ASSERTION
(B) if it is a SUPPORTING REASON
(C) if it is a STATEMENT OF ARGUMENT
(D) if it is a COUNTER ARGUMENT

19. With globalisation making its presence felt, a number of foreign influences are gradually seeping into our culture. And these are felt in food as well. Moreover, with the break-up of the joint family system and more and more working couples setting up homes on their own, people prefer eating out to returning home to cook after a hectic day's work. Capitalising on this new trend,

established as well as upcoming restaurateurs feel that constant improvisation and innovation is the key to a successful restaurant.

(a) Most restaurateurs feel that sticking to the cuisine of the 60's evokes feelings of nostalgia and improves sales.
(b) Customers want to experiment and wish to tryout cuisine from other countries.
(c) Similar changes have been noticed in the entertainment industry where viewers now give preference to comedies and action parked adventure movies.
(d) Restaurants which improvise and innovate will definitely be successful.

(A) cbbd (B) bcda
(C) cbca (D) bbcb

20. Organization of the Petroleum Exporting Countries (OPEC) has used output curbs to maintain average oil prices over the past three years in its \$ 22 to \$ 28 target range. OPEC was worried that rising supplies from rival non-OPEC nations and another year of modest demand growth could cause a downward price spiral. It is particularly vulnerable to a price fall during the second quarter when demand eases. Forecasts from the Paris based International Energy Agency backed this outlook. It is estimated that if OPEC keeps pumping unchecked it will overwhelm world demand next year by 1.8 million BPb causing a huge stock build-up.

(a) The existence of a similar cartel in the global coal market would have definitely pushed up the price of coal.
(b) A price range of \$ 22 - \$ 28 ensures that the producers make the minimum possible profit out of their operations required to keep their economy in shape.
(c) Any excess production by OPEC would be mopped up in an energy hungry global market and thus OPEC need not worry about excess production.
(d) OPEC countries should concentrate on developing their non oil based economy so as to be able to absorb any adverse shocks as a result of decreasing oil reverses.

(A) cbdc (B) bcca
(C) abcd (D) bcda

21. The recent elections have thrown up so many critical weaknesses in the American electoral system - inconsistent voting procedures from state to state, early declaration of the winner by the media before the polls closed and partisanship of breathtaking proportions that intensified after the balloting. Surely a set of statutory rules about the counting and recounting of votes could have saved the US all this embarrassment.

(a) The adoption of a set of rules to be uniformly applied all over the country would most certainly eradicate the weakness in the electoral system.
(b) Countries which had adopted similar procedures have now started reaping the benefits.
(c) The media should be banned from pronouncing the winner before the polls close so as to prevent any sort of malpractice from taking place.
(d) Different voting procedures in various states and the massive media involvement ensures free and fair polls and thus are invaluable.

(A) abbb (B) abcd
(C) abdd (D) abbd

22. Memory may have been found to be affected by genes, but no gene has been found to have even a small causative relationship with normal 'intelligence'. Intelligence has never been successfully defined or measured, unless one refers to highly specialized, arbitrary, narrowly defined and largely learned skills measured by standard IQ tests. Why should genetic intelligence have evolved along the lines of IQ tests when better measures could have been evolved is something inexplicable.

(a) Genes that influence 'intelligence' are masked by chemicals produced in the brain and are thus difficult to detect.
(b) IQ tests are the most accurate way of measuring intelligence exhibited by humans and no other measure is required.
(c) IQ tests are not the best measure of intelligence as these tests can be taken quite successfully by people who have practiced hard enough.
(d) IQ levels in a human being are function of the nutrition that one receives in childhood.

(A) ddbc (B) ddda
(C) dbaa (D) dbad

23. After Rwanda and Kosovo, another dark page of modern history is being written in East Timor by the Indonesian army. It's a shame the international community did not react faster in condemning Indonesia and preventing the genocide, which has been on-going since the mid 70's in East Timor. Above all political and economic considerations, the international community should never forget that the brave people of East Timor are struggling and dying for something that is so dear to all of us - freedom.

(a) The international community has not found any evidence of wrong doing by the Indonesian army and hence has been averse to condemning it.
(b) The East Timorians are waging a battle not for freedom but because of the fact that most of them are being forced to convert to Christianity.
(c) Genocide in any place under any pretext is abominable and it is the responsibility of the international forum to put a stop to it.
(d) History has proved that freedom is a concept for which people have sacrificed their lives and hold it in high esteem.

(A) dcbb (B) dcaa
(C) dcab (D) abcc

Directions for questions 24 to 28: Each of the following questions has a paragraph from which a sentence has been deleted. From the given options, choose the sentence that completes the paragraph in the most appropriate way.

24. This is a rare moment in the history of economic globalisation. Policy making across the developed and developing world has never been so uniform and homogenised. Whether it is G-7 or G-20 economies the only buzz word among policy makers is 'inject more liquidity". President George Bush himself made the unusal gesture of walking into the meeting of G-20 countries, seeking their active support in mitigating the impact of the financial meltdown. ____________
(A) At a macro level, the attempt by the G-7 governments is to ensure that the big asset bubble built over the past six years must not be allowed to burst.
(B) There are clear pointers that the global financial crises will result in loss of demand across the board.
(C) The thirst for more money seems unending.
(D) Never before have the heads of state met only to discuss how to put in extra cash in the global financial system.

25. At the heart of the wellness concept is the most romantic notion of all: the suggestion that we might postpone, halt and even reverse the aging process itself. While some scientists are doing research on ageing and on human mortality, many wellness buffs-and even some reasonably accredited scientists-are promising that we may soon penetrate the secret of the ageing process and with this knowledge, actually extend Homo Sapiens' normal life span. ________________
(A) While some may search for a magic elixir, others stress on diet and exercise as the keys.
(B) The death barrier may actually be broken.
(C) Wellness is a kind of superstate of mental and physical well being.
(D) Those who remain impervious to these romantic hopes have other incentives or coercions.

26. To test the role of cognition in hypocrisy, scientists had volunteers assign themselves an easy task and a stranger an onerous one. But before judging the fairness of their actions they had to memorize seven numbers. This play keeps the brain's thinking regions too tied up to think about anything else, and it worked: hypocrisy vanished. People judged their own behaviour as harshly as they did others'-strong evidence that moral hypocrisy requires a high-order cognitive process. ____________
(A) When 'people like us' torture, it is justified; when people unlike us do, it is an atrocity.
(B) When the thinking part of the brain is otherwise engaged, we are left with gut-level reaction and instinctively condemn bad behaviour.
(C) Scientists have long wondered whether hypocrisy is driven by emotion or reason.
(D) When we judge our own transgressions less harshly than we judge the same transgressions in others it may be because we have this instinct to preserve our self-image.

27. Celebrity has become the primary commodity of popular culture. Fans used to fall for a specific album or film, but now the public tends to base its consumption on the aura of celebrity attached to any given product. Singers can act in films and actors can record albums, not thanks to any special talent but because their brand is big enough to transcend categories. ____________
(A) Witness the birth of the celebrity luxury fashion brand.
(B) Fashion magazines have all but abandoned the practice of putting models on the cover of their magazines.
(C) Celebrities have wised up to their incredibly powerful market potential, moving from endorsing someone else's high end products to producing their own.
(D) The most successful start-ups have been those by celebrities with iconic personal style.

28. The growth stories of China and India have always been different–China is well known for being the world's factory, while India's new wealth has been built on services. But the result is the same. Over the next twenty years 213 million Chinese households and 123 million Indian ones will begin to have discretionary income. ____________
(A) If both countries continue roughly on their current growth paths we will witness the creation of massive new consumer markets.
(B) The speed of the change will rival Japan's economic miracle of the 1950s.
(C) That will lead to an Asian shopping spree of historic proportions.
(D) The projection is that incomes will grow eight fold cutting China's poverty rate to just 16 percent.

Directions for questions 29 to 40: Select the correct alternative from the given choices.

29. Manufacturers of a food drink claim that their product is a complete planned food containing all the nutrients necessary for the health and growth of a child. However, it is better that children are given a well-balanced diet consisting of a variety of foods for health and growth.
Which of the following, if true would best support the position above?
(A) The flavour of the food drink does not appeal to many children.

(B) Children who are used to taking the food drink do not take natural foods rich in these nutrients.
(C) The nutrients contained in a well-balanced diet are more easily absorbed by the body for effective use.
(D) The contention of the manufacturers that the food drinks are complete planned foods is yet to be proved right.

30. A recent study conducted with school children aged ten to fifteen showed that a plan where these children were provided with a proper meal at school ensured that they were less likely to remain absent from the school than other children. Thus providing a proper meal at school plays a role in reducing student absenteeism. So such a plan must be introduced in schools to reduce absenteeism.
Under which of the following conditions will the breakfast plan work best?
(A) In a locality where student absenteeism is significant.
(B) In a school where there are more boys, (who are more likely to remain absent), than girls.
(C) In schools, where a majority of students have both parents working.
(D) In schools where the students are wards of those who cannot afford a square meal a day.

31. It is generally said that radiations emitted by cell phones cause immediate damage to the membrane in the ear and results in hearing impairment. Hence one is advised to avoid using a cell phone to prevent total loss of hearing.
Which of the following, if true, seriously weakens the above argument?
(A) Use of electronic gadgets such as computers also impairs hearing besides affecting eyesight.
(B) Electronic gadgets such as i pods do not damage hearing in users.
(C) A majority of those who use hearing-aids are found to have never used cell phones in their lives.
(D) It is found in a survey that ninety per cent of employees working as telephone operators have been regular users of cell phones for years.

32. Last year, Alpha University implemented a procedure that encouraged students to evaluate the teaching effectiveness of their professors.
Which of the following principles can the above plan be taken to be based upon?
(A) The effectiveness of any service depends on how useful it actually is to the receiver of the service.
(B) The effectiveness of any service is measured by the perception of the receiver of the service regarding its utility.
(C) The receiver of a service should have the freedom to choose the mode of delivery.
(D) The service provided should take care of the ultimate beneficiary of the services.

33. A recent study indicates that Venadial, a new medicine currently produced in the country *X* only, actually reduces cholesterol levels. Venadial, derived from the resin of pine trees, works by activating a metabolic response that is not yet well understood. However, cholesterol levels fell fifteen to twenty per cent among participants in the study who consumed Venadial daily and reduced the risk of heart attack by one-third. Therefore company *A* in country *Y* should obtain the exclusive right to sell Venadial in *Y*. The profits are sure to increase within a short span of time.
Which of the following, if possible must first be done in order to translate the above optimism into reality?
(A) A publicity campaign informing the public of cholesterol and its permissible levels.
(B) A publicity campaign advertising the role of Venadial in reducing heart attacks.
(C) Conducting research on the metabolic response to Venadial which is not yet well understood.
(D) Convince the government to allow import without restraint.

34. Software companies have grown by leaps and bounds. The employees required to man these companies far outnumber the available candidates with academic skills relevant to the jobs that they are required to perform.
Which of the following plans, if possible, can allow such companies to meet their manpower requirements?
(A) The institutions can enhance the pay of the existing employees to encourage them to take up additional work.
(B) The institutions can scale down their operations to the extent where the requirements can be met with the number of available persons with the needed academic skills.
(C) The institutions can recruit even those who do not have the necessary skills and give them training to enable them to perform their jobs.
(D) The institutions can enter into agreements with the academic institutions like colleges and universities to absorb their students in toto.

35. Smoking is a serious health concern as it affects not only those who smoke, but also others who don't, as they inhale the polluted air. The Government is concerned about this. So it has decided to rope in film celebrities for its campaign against smoking, through documentary films. Which of the following, if true, most strengthens the plan of the Government in its campaign against smoking?
(A) The general public abides by the advice of film celebrities on the screen.

(B) The film celebrities are role models for the common man.
(C) Film celebrities earn huge sums of money and it is in the fitness of things that they contribute to a social cause.
(D) It is easier to produce such films with film personalities as they are already well versed in acting.

36. It is true that there are international laws against drug traffic. But if the Government takes stern action to plug its entry in one place, drugs will enter the country through other sources.
Which of the following is most like the argument above in its logical structure?
(A) It is true that the officials of the police department are expected to be incorruptible. But if they are paid poorly, they may resort to graft to sustain a decent living.
(B) It is true that most of the terrorists are religious fundamentalists. But there are several persons who are deeply religious but they are not terrorists.
(C) It is true that there are severe laws against violation of copy rights. But some company would have done it if the defendant had not done it first.
(D) It is true that adoption of third degree methods against prisoners to extract confessions is against law. But it cannot be helped in certain cases where the prisoner does not reveal all the truth.

37. Balance of trade refers to the difference between the exports and imports of a country. It is said to be positive when exports are more than imports and negative if imports are more than exports. The exports of a country '*C*' increased over the previous years' exports. However the balance of trade has become negative though it was positive last year.
Which of the following cannot be true if the above statements are true?
(A) The imports during this year are more than last year's.
(B) Exports during the previous year were more than imports.
(C) Last year's imports were more than this year's exports.
(D) The imports during this year are more than the exports.

38. The town administration of a hill station where tourists far outnumber the local residents, found that the tourists were using non biodegradable plastic cups, plates and polyethylene bags in the place of substitutes made out of paper which are environmentally preferable. In order to reduce the use of non biodegradable goods but at the same time not to annoy the tourists the administration encouraged the sale of both plastic and paper made goods, but the paper-made goods were sold at substantially very low rates compared to the plastic goods. Which of the following, if true, would undermine the purpose for which paper - made goods are sold at very low rates by the administration?
(A) The shop keepers were found to sell the plastic goods at a price higher than that fixed by the administration.
(B) The tourists were found to possess a higher value for aestheticism (which they find in plastic goods compared to the paper goods) than for costs.
(C) The tourists themselves were well aware of the desirability of using biodegradable goods as against non biodegradable ones.
(D) A similar strategy adopted in another hill station in the same country did not produce desired results immediately.

39. Company "Trendy" is engaged in the manufacture of products which face stiff competition from others in the market. Constant change in the products, their design, packaging, etc is the key to success in the industry. Trendy has been allocating a significant portion of the resources over the years to research and development which takes care of devolvement of products and innovations in designs etc. However, the allocation has fallen from a whopping ten percent to a mere 6 percent of the total resources, in the current year, because of which it is feared that the sales might drop significantly this year. Which of the following is an assumption made in the conclusion drawn above?
(A) Ten per cent of the total resources is enough allocation for research development activities, to for achieve the desired levels of sales.
(B) The allocation of resources to research and development efforts determines the extent of innovation in products and product designs.
(C) Sales of the company will also fall from ten per cent to six per cent.
(D) The production of goods by the company would also drop significantly in the current year.

40. Which of the following, if the dictum 'more the resources for R&D, more will be the innovations' be true, would most weaken the conclusion drawn in question 5 above?
(A) The total resources allocated by the company to all the activities put together were double those of the previous year.
(B) The sales of the same products by the other companies also drop in the year.
(C) The market share of the company for the products it manufactures increases during the current year.
(D) The resources allocated to marketing during this year increased substantially over the previous year.

Answer Keys

1. A	**2.** C	**3.** D	**4.** B	**5.** C	**6.** B	**7.** D	**8.** C	**9.** C	**10.** B
11. D	**12.** D	**13.** D	**14.** C	**15.** A	**16.** C	**17.** B	**18.** A	**19.** C	**20.** A
21. B	**22.** A	**23.** C	**24.** D	**25.** B	**26.** B	**27.** C	**28.** C	**29.** D	**30.** D
31. D	**32.** B	**33.** B	**34.** C	**35.** A	**36.** C	**37.** C	**38.** B	**39.** B	**40.** A

Hints and Explanations

1. The main points are:
(1) An experiment is repeated to increase accuracy.
(2) It is isolated to make the task of observation easier.
(3) Variety in experiments gives a better chance of discovering laws.
Choice A covers all the important points and hence can be called its summary. Choice B does not say anything about 'variety'. Choice C is not as clear or comprehensible as choice A. Choice D does not specify what helps in what aspect. Choice (A)

2. The main points are:
(1) Aristotle says any good definition of an object has two parts.
(2) The first is to assign the object to its class or genre with which it has common features.
(3) The second is to indicate where and how it is unique.
(4) Juxtaposing it with objects of the same kind helps in identifying its individuality.
Choice A does not mention 'Aristotle' and Choice B focuses on the definition of 'man' in particular and then says 'object'. Hence A and B can be ruled out. Choice C summarises the passage. The idea given in the last sentence of choice D is incorrect. Choice (C)

3. The main points are:
(1) Sociologists use strange jargon or give extended meaning to familiar terms.
(2) They are criticized for doing this.
(3) But they have their reasons.
(4) They do it in a desire to be more precise and systematic in their analysis.
Choice A is incorrect as it says' . . . They are justified to do so . . .' which is not stated in the passage. It is the critics 'who are justified'. The second sentence in choice B makes it incorrect. 'The charges/criticism . . .' is not mentioned in choice C. Choice D precisely summarizes the passage. Choice (D)

4. The main points in the text are:
(1) The damage we inflict on our surrounding is measured by carbon footprint and ecological footprint.
(2) Carbon footprint measures the greenhouse gases that is emitted because of us.
(3) Ecological footprint measures the land we need for resources and for dumping waste.
Choice A misses carbon footprint.
Choice B is the right answer.
The first sentence in choice C makes an absurd suggestion.
Choice D mixes up the concept of ecological footprint and carbon footprint. Choice (B)

5. The main points in the text are:
(1) Hunting and logging are a threat to protected areas.
(2) A more subtle threat is illegal occupation of forest land by cultivators.
(3) The reason for all the above is pressure of population on land, poverty, corruption and poor law enforcement.
Choice A is wrong because it calls poaching 'a greater evil', not stated in the passage.
Choice B becomes too specific by mentioning Laos, west and central Africa and southeast Asia. Choice D is again specific as it talks of coffee growers. Choice C is appropriate. Choice (C)

6. The main points in the text are:
(1) Chernobyl still evokes poignant memories after 20 years and we are far from cleaning up the mess.
(2) It was caused by an unnecessary experiment.
(3) The Soviet government did not act immediately and we do not know how many deaths were caused by the failure of the government to evacuate the people.
The original para does not mention that the Soviet government refused to evacuate people. It was reluctant and delayed the process. So, choice A is a distortion. Choice C distorts in calling the radiation 'a gas', and it leaves out many important points. Choice D distorts in saying 'the Soviet government put up a brave front' – not stated in the text. Choice B is concise and correct.
Choice (B)

7. The main points in the text are:
(1) Older women, those over forty, are more likely to have a child with Down syndrome.
(2) As such they are offered, or forced to have, amniocentesis.
(3) The abortion of a Down embryo can be seen as a benign aspect of science or as officially encouraged murder.

(4) It is eugenics, fifty years after the Nazis.

Choice A is not the best summary because it is not amniocentesis but abortion that is controversial. Choice B is not apt since it says older women take the test and abort, as if the initiative is theirs. Choice C makes the test controversial. Choice D is apt. Choice (D)

8. The main points in the text are :

(1) Jefferson warned against public debt.

(2) But it is a tried-and-tested method. The USA being an example.

Choice A distorts the meaning when it says the USA considered it to be a prerequisite. Choice B distorts in saying that because the USA amassed debt Jefferson warned against it. Choice C is an apt summary. Choice D appears to be right but Jefferson never said (as per the passage) that public debt would weaken economy. Choice (C)

9. A and C are our possible starting parts. 'One of the' in C indicates that it is a continuation of 'a them' rather than a starting part and hence C is rejected as our starting part which leaves A as the only possible starting part. "The place' in D relates to 'the newest temple' in A and hence D follows A. 'Vast space' in B connects with 'huge physical entity' in D and hence B follows D. Choice (C)

10. As per the choices A, C and D are our possible starting parts. 'Besides' in C rules it out as our starting part and out of A and D, D is a better starting part as it is a more generalized statement than A. 'Usal garnish' in A follows 'garnishes' in D. 'Fat-laden coconut' in C links up with 'freshly greated cocounut' in A and hence C follows A. B provides a contrast to A and hence follows C. Choice (B)

11. The 'paradoz' associated with ceramic sculptures is elaborated in C and D. C says that the ceramic object's solidity gives a sense of permanence but this is contrasted by D which states that the baked earth form which this sculpture is made makes it more prone to damage on account of its fragile nature. "fragility in B connets with 'fragile, frangible' in D and hence B follows D. Choice (D)

12. A is our starting part 'but' in D offers a contrast by emphasizing the fact that though 200 years have elapsed since Tipu's death yet his fame is growing and hence D follows A. B describes Tipu and hence follows D. 'Reputation' in C is a logical extension of 'formidable and implacable' in B. Choice (D)

13. A and C are our possible starting part of which C is better as it is a very general statement. 'It' in B refers to the 'ten centuries' in C and hence B follows C. 'In that calculation' in D refers to the 'yugas' in A and hence D follows A. Choice (D)

14. B is the opening sentence (it makes a generalized statement). A follows - it explains what the work (mentioned in B) is. C follows A – 'This meeting' in C linking it 'to call on….' In A. D concludes with what he did.

Choice (C)

15. C is a better opening sentence than D, as it is a generalized statement. D follows C – it explains what the 'sordid' going on mentioned in C are. A follows B – 'sponsorship and endorsements' in A linking it to 'commercialization' in B. Choice (A)

16. B is the opening sentence. C follows B – 'Areas cleared' in C linking it to 'land..... obtained by cutting down trees' in B. A follows C – 'set up industries ….. which pump out …..'. E and D follow – The speed of cutting down has increased and its consequences.

Choice (C)

17. D is the opening sentence – 'privacy' is the central idea of the para. C follow D giving an example to elaborate the need for privacy. B follows C – the author's child hood experience compared to his daughter's. E and A follow to complete the author's reminiscence.

Choice (B)

18. A is the opening sentence – it sets out the basic idea of the para. C follows A – explaining (very briefly) how his parents were the complements of each other. B follows C – one more instance of complementing. E and D follow elaborating the qualities of his mother.

Choice (A)

19. The passage states that globalisation has made its impact on the food tastes of the masses, it goes on to state that people prefer innovation in cooking.

A - states a point that runs counter to the argument given in the passage regarding the changing tastes of the masses hence it is a COUNTER ARGUMENT.

B - gives one more reason for the changing states and hence it is a SUPPORTING ARGUMENT.

C - states a point that is not mentioned in the passage and hence this is a STATEMENT OF ARGUMENT.

D - categorically states that restaurants which cater to the changed tastes will be definitely successful and hence it is a CONCLUDING ASSERTION.

Choice (C)

20. The passage talks about the efforts made by OPEC to maintain he level of prices at a certain level and also the scenario that emerges as a result of the efforts that it makes to achieve this objective.

A - talks about a hypothetical case of how the coal prices would have benefited from the existence of such a cartel. Hence this is a STATEMENT OF ARGUMENT.

B - gives a reason as to why the OPEC nations would want to maintain the oil prices in a specific range and hence it is a SUPPORTING REASON.

C - This point runs against the view of the passage that excess production by Opec would result in a decrease of the general price of oil and hence it is a COUNTER ARGUMENT.

D - talks about a possible remedy to the problem and as such it is a new argument that has been proposed. Hence, it is a STATEMENT OF ARGUMENT.

Choice (A)

21. The passage talks about the weakness by the American electoral system and gives a few examples to substantiate this.

A - states categorically that the adoption of a system of rules across the country would correct the system and stress the root hence, this is an ASSERTION.

B - gives a SUPPORTING REASON as to how other countries have benefited by implementing the rules.

C - the role played by the media has been specified in the passage but the question of banning it has not been discussed and as such C which proposes that the media be banned from announcing results is a STATEMENT OF ARGUMENT.

D - gives a reason in support of the measures being proposed in the passage to remedy the situation and as such it is a SUPPORTING REASON.

Choice (B)

22. The passage states the relation between memory and genes and says that intelligence has not been defined clearly and talks about how ineffective IQ tests are in measuring intelligence.

A - this contradicts the first line of the passage and hence this is a COUNTER ARGUMENT.

B - also contradicts what is stated in the passage and hence it is a COUNTER ARGUMENT.

C - provides support to the view given in the passage and hence it is a SUPPORTING REASON.

D - states a new point not mentioned hitherto in the passage and hence it is a STATEMENT OF ARGUMENT. Choice (A)

23. The passage laments the apathy shown by the international community towards the genocide being perpetrated in East Timor by the Indonesian army in order to the quell the demand for freedom.

A - contradicts what is stated in the passage and hence it is a COUNTER ARGUMENT.

B - introduces a new angle to the problem stated and hence it is a STATEMENT OF ARGUMENT.

C - firmly states that genocide is an aberration or civil society and must be stopped and hence this is an assertion.

D - substantiates what is given in the passage and hence it is a supporting reason. Choice (C)

24. The central idea of the paragraph is the notion that the meeting of G-20 nations to discuss how to inject more liquidity was a rare or unusual event. This idea reaches its natural conclusion in 4. Choice (D)

25. The paragraph harps on the fact that wellness can reverse the ageing process. The penultimate sentence refers to 'extending life span'. This leads to the conclusion that death may be conquered which is presented in option B. Choice D begins another idea and so cannot conclude this paragraph. Choice (B)

26. The focus of the paragraph is on the role of cognition in hypocrisy and the revelation that when the mind is busy thinking about other things morality becomes a gut-reaction. This conclusion is brought out in option (B) which summarizes the result of the experiment and is hence a good conclusion. Choice (B)

27. The paragraph focuses on the idea that celebrity status sells. Awareness of celebrities regarding their brand value has led to the stars producing their own products instead of endorsing those of others. This is the logical conclusion found in option C. Option A can continue the para rather than conclude it. Option D can begin a new paragraph. Choice (3)

28. The central idea of the paragraph is the growth track of India and China and the fact that this will lead to greater disposable incomes in both countries. Sentence 3 which states the consequence of such incomes is the logical conclusion of the idea. Sentence 1 can be ruled out because the 'if' in the sentences raises doubts about the growth pattern and thus contradicts what has been stated earlier. Choice (3)

29. **Argument evaluation:**

Situation: Manufacturers claim that their food drinks contain all the nutrients necessary for a child. But a balanced diet containing variety of foods is better.

Reasoning: A well balanced diet is preferable to a food drink because it acts better for the child.

(A) 'Flavour' is irrelevant to the argument.

(B) Children's preference is not the issue.

(C) Also outside the scope of the argument.

(D) Correct. It properly identifies the statement which supports the argument.

D is correct. Choice (D)

30. **Evaluation of plan:**

Situation: Providing a meal to school going children is likely to reduce absenteeism.

Reasoning: A proper meal provided to school going children reduces absenteeism. This plan will best work only where the absenteeism has been due to non-availability of a good meal to the children.

(A) The absenteeism may be due to other reasons and hence the plan to provide as usual may not work.

(B) The composition of the students whether they are boys or girls is beyond the scope of the argument.
(C) 'Working parents' doesn't provide ground for providing breakfast and hence reducing absenteeism.
(D) Correct. It properly identifies the condition under which the breakfast plan will succeed. Choice (D)

31. Argument evaluation:
Situation: Radiations from cell phones cause hearing defects. Hence cell phones must be avoided.
Reasoning: The argument is that cell phones should not be used as they cause heaving defects. Hence the statement which tells us that cell phones do not affect the ears weakens the argument.
(A) The effect of using 'computers' is beyond the scope of the argument.
(B) Use of a `Ipod' is not the question.
(C) It is not necessary that all those who are defective in hearing must use hearing aids.
(D) Correct. It properly identifies the statement which weakens the argument. Since the telephone operators' job involves having, the fact that their hearing is not impaired despite using cell phones regularly weakens the argument. Choice (D)

32. Evaluation of a plan:
Situation: The students of Alpha University were asked to evaluate the teaching effectiveness of their professors.
Reasoning: The university asks the students, the recipients of the service, to evaluate the effectiveness of teaching of the professors, the providers of services. So the plan is based on the principle that the effectiveness of a service rendered is measured by how the receiver views it.
(A) The plan doesn't measure the actual effectiveness of teaching which must have been done by evaluating the students not the teachers.
(B) Correct. The statement properly identifies the principle on which the plan is based.
(C) It is irrelevant to the argument.
(D) This may be the purpose of the plan but not the principle on which it is based.
B is correct. Choice (B)

33. Evaluation of a plan:
Situation: Venadial is effective in reducing cholesterol levels and the risk of heart attack according to a study. Company *A* in country *Y* should sell the medicine, as its sole selling agent, to earn profit in a short span of time.
Reasoning: To realise the optimism expressed above people must be made aware of the effectiveness of Venadial in reducing heart attacks.
(A) Knowing the permissible cholesterol level will not induce the public to go for the medicine.
(B) Correct. It properly identifies the course to be adopted.
(C) Research on the metabolic response to Venadial is irrelevant to the issue.
(D) This is also irrelevant to the argument.
B is correct. Choice (B)

34. Evaluation of a plan:
Situation: Software companies find the persons required outnumbering the availability. The plan is to meet the requirement.
Reasoning: The available candidates with necessary academic skills is insufficient to meet the requirement. So the only way is to recruit even those without the necessary academic skills, but impart them training to make them fit for the jobs.
(A) It is irrelevant to the discussion as 'pay' is the issue.
(B) The question is of a plan to meet the enhanced demand. So scaling down operations is not the answer.
(C) Correct. It properly identifies the correct plan to meet the increased need.
(D) As it is already stated that the number of candidates with necessary skills is insufficient, entering into agreement to absorb the students in to will not help.
C is correct. Choice (C)

35. Evaluation of plan:
Situation: The Government wants to involve film celebrities in anti smoking campaigns as it is concerned about the health hazards that smoking causes to the public.
Reasoning: The Government plans to involve film celebrities in its anti smoking campaign. This plan will be successful if the general public heeds the film celebrities words on the screen.
(A) Correct. It properly identifies the situation when the Government's plans will be successful.
(B) The film celebrities may be smokers in real life and hence if the public emulates their lives, the plan will not be successful.
(C) 'Why the film celebrities need to be involved in social causes' is beyond the scope of the argument.
(D) It may be helpful in production of the film but it does not strengthens the achievement of the purpose of the plan. Choice (A)

36. Argument constructions:
Situation: Though laws exist against drug trafficking, it is bound to exist. Even if the Government plugs one source, as it will enter through others.
Reasoning: The situation is one where there is a law against something. But even if it is curbed at one end, it is bound to be committed by someone else.
(A) It is not similar. It provides a reason for why the police may be corrupt despite laws.
(B) It only says that all religious persons need not be terrorists.

(C) Correct. It properly identifies the situation which is a similar argument to the one given.

(D) It gives an explanation as to why third degree methods become inevitable. Choice (C)

37. Argument construction:

Situation: Last year exports were more than imports. This year imports were more than exports. Also exports during this year were more than last year's exports.

Reasoning: Last year E, > I, (E – Exports and I – Imports). This year $I_2 > E_2$. Also $E_2 > E_1$. So $I_2 > E_2 > E_1 > I_1$. Any statement contradicting the cannot be an inference.

(A) It is correct because $I_2 > I_1$ so it is not the answer.

(B) It is correct as $E_1 > I_1$, so it is not the correct choice.

(C) Correct. It properly identifies a situation which cannot be inferred. This statement implies $I_1 > E_2$. This is not possible as $E_2 > I_1$.

(D) It can be inferred as $I_2 > E_2$.

C is correct. Choice (C)

38. Evaluation of a plan:

Situation: The Administration wants to encourage the use of biodegradable paper products by encouraging their sale at very low prices.

Reasoning: The strategy will fail to work if the users do not take the bait, i.e. they don't give importance to price differentials.

(A) It should in effect actually help the administration in realising it s purpose.

(B) Correct. It properly identifies a situation where the tourists may not give any importance to the price differentials.

(C) It should actually strengthen not undermine the administrations purpose.

(D) It is said that the results were not produced only in the short run.

B is correct. Choice (B)

39. Argument construction:

Situation: Trendy' is a company engaged in the manufacture of goods which need constant innovation in products and their designs. It is feared that a drop in the percentage of allocation of resources of R&D will reduce sales.

Reasoning: Unless the proportion of resources allocated to R&D is going to determine the extent of innovation, the conclusion cannot be drawn as above.

(A) It cannot be an assumption as there is no mention from which it can be assumed that a particular percentage of resources has to be allocated for research and development to achieve the desired results.

(B) Correct. It correctly identifies the assumption that unless the proportion of resources allocated to the research and development to the total resources determines the extent of innovation, it cannot be held that sales will be affected.

(C) It is not an assumption.

(D) It is outside the scope of the argument.

B is correct. Choice (B)

40. Argument evaluation:

Situation: As above

Reasoning: Allocation of resources are held to directly govern the sales. Then the argument will be weakened if there had been no real decrease in allocation of resources to R&D.

(A) Correct. If the total resources are doubled, then six percent of it will be more than ten percent of the pervious years total resources. Hence there has been a real increase in allocation to R&D which can if anything only increase the sales.

(B) It is outside the scope of the argument.

(C) It may be true but it does not mean that the sales have improved, as the sales of other companies also might have dropped.

(D) Resources allocated to marketing is not the crux the argument.

A is correct. Choice (A)

Verbal Ability Test 5

Number of Questions: 20 **Time: 30 min**

Directions for question 1: The question has a sentence with three blanks followed by four pairs of words as choices. From among the choices, select the pair of words that can best complete the given sentence

1. It is now recognized in many countries that family and social environment of the children have a ______ impact on their performance than the quality of teaching at school and children from poor families score significantly ______ in vocabulary, communication skills, simple arithmetic and the ability to concentrate compared to children from ______ income households.
(A) meagre...higher...higher
(B) lower......lower....greater
(C) prime.....faster.....lower
(D) greater....lower.....higher

Directions for questions 2 to 11: Fill in the blanks in the given sentences so as to make sense. Select the correct word from the answer choices and mark its number as the answer.

2. If the items in a diary are reliably listed and the list is ________ reviewed then certainly nothing should be forgotten.
(A) perversely (B) phlegmatically
(C) languidly (D) conscientiously

3. The child had a _____ birth defect in the form of a cleft palate.
(A) hereditary (B) compulsive
(C) congenial (D) congenital

4. The people of South Asia had been left behind in economic development _____ of tensions and conflicts.
(A) in view of (B) because
(C) want (D) full

5. Financial difficulties ________ him to discontinue his studies, and take up a job.
(A) enchanted (B) compelled
(C) entreated (D) dictated

6. Sixty houses were totally gutted ______the fire accident.
(A) at (B) beyond
(C) in (D) off

7. The Kaveri community in Bihar, which has for long _______ with poverty and backwardness, is yet to join the mainstream because of its nomadic lifestyle.
(A) suffered (B) exhausted
(C) struggled (D) figured

8. Alzheimer's disease is not usually _______ until significant memory loss is evident.
(A) detected (B) observed
(C) revealed (D) disclosed

9. When the dilapidated structure collapsed, the _____ of the people could be heard from a distance.
(A) gossip (B) wails
(C) eviction (D) bombardment

10. He does not _____ the ugly aspects of human nature in his picture of life.
(A) eliminate (B) include
(C) exclude (D) extricate

11. The airline has increased its frequency _____ nine flights a week to eleven flights a week.
(A) between (B) of
(C) from (D) for

Directions for question 12 to 17: The question has a sentence with two blanks followed by four pairs of words as choices. From among the choices, select the pair of words that can best complete the given sentence.

12. I am not a _____, I am a _____.
(A) 'have been' . . . ''will be'
(B) 'has been' . . . 'will be'
(C) 'was been' . . . 'will being'
(D) 'had been' . . . 'will be'

13. During his childhood, Williams ______ his parents ______ teachers.
(A) never obeyed . . . . or
(B) never had obeyed . . . . or
(C) never obeyed . . . .nor
(D) never had obeyed . . . . nor

14. The two men were very ______ and rather looked the same but had ______ views on important aspects of life.
(A) close . . . wide (B) similar . . . diverse
(C) different . . . complex (D) familiar . . . similar

15. I am not a ____, I am a ____.
(A) 'have been' . . . 'will be'
(B) 'had been' . . . 'will be'
(C) 'was been' . . . 'will being'
(D) 'has been' . . . 'will be'

16. The two thieves _______ arrested on the charge of stealing from _______ house
(A) was . . . an (B) am . . . a
(C) were. . . an (D) were. . . a

17. The country has no need to test its nuclear arsenal in the absence of an external _____ which is precisely why we have a voluntary _____ on testing.
(A) provocation . . . regression
(B) collaboration . . . relapse
(C) wherewithal . . . setback
(D) aggression . . . moratorium

Directions for question 18 to 20: Read the following paragraph and answer the question that follows:

18. People are throwing themselves off the Golden Gate Bridge at the rate of two a month, which makes it the most popular place in the world for suicide. Many San Franciscans think that the solution is to put up a simple barrier. The Psychiatric Foundation of California, which has proposed to construct a barrier to act as a deterrent, has heard several arguments against a barrier; the most persistent has been that people would simply kill themselves somewhere else, so why bother? Here the word 'deterrent' is closest in meaning to?

(A) Impediment (B) Assistance
(C) Indemnity (D) Restitution

19. In "ambush marketing" campaigns, companies try to promote their brands at sporting events without paying sponsorship fees. Ambush marketers have replaced hooligans as the villains of sporting events, because they undermine official sponsors, who are the main source of revenue in some sports. The IPL organizers have responded by taking control of all prominent advertising sites in the city, including those at train stations and airports, and their use will be limited to official sponsors only. Nevertheless, preventing ambushes is difficult.
Which of the following is closest in meaning to 'ambush'?

(A) Deception (B) Integrity
(C) Virtue (D) Equity

20. Studies of promising young prodigies frequently reveal that most turned out to be failures, in both their professional and personal lives, in later years. Researchers also found that most of the prodigies were diagnosed with symptoms of paranoia and schizophrenia at a later stage. The researchers concluded that mental illnesses inhibited the prodigies from realizing their full potential in later life.
Here, the word 'prodigies' is closest in meaning to

(A) genius (B) Maestros
(C) Adept (D) Apprentice

Answer Keys

1. D	**2.** D	**3.** D	**4.** B	**5.** B	**6.** C	**7.** C	**8.** A	**9.** B	**10.** B
11. C	**12.** B	**13.** A	**14.** B	**15.** D	**16.** D	**17.** D	**18.** A	**19.** A	**20.** A

Hints and Explanations

Explanatory notes for question 1:

1. The sentence compares the social environment and the quality of teaching on the performance of children. Hence the comparative degree of an adjective has to be used in the first blank. 'Lower', and 'greater' are comparative degree adjectives and ore apt. Hence choices A and C can be ignored. According to the passage, children from low income social background are likely to score '*lower'* than their affluent counterparts with *higher* income background. All these words are found in choice D making it the right answer. Choice (D)

Explanatory notes for questions 2 to 11:

2. The context demands 'proper listing' and 'meticulous review' of the list. Among all the options only option D conveys this meaning. Options B and C refer to 'laziness' and 'perversely' refers to 'contrary to what is accepted'. The correct option is D 'conscientiously' which refers to 'being careful and thorough in one's work'. Choice (D)

3. Birth defects are congenital (present from birth) and not hereditary (genetic), compulsive (uncontrollable), or congenial (affable; friendly). Choice (D)

4. The reason the people of South Asia have been left behind in tensions and conflicts. Choice (B)

5. Compel means to force. Financial difficulties forced him to drop his studies and take up a job. Choice (B)

6. **'**In' is the correct preposition to be used as it expresses a period of time during which an event happens. Choice (C)

7. The sentence is about a community in Bihar which has not progressed. The reason for this is its poverty and backwardness. Exhausted means extremely tired. This is a wrong choice. 'Figured' means to understand. 'Sustained' means continued for a long time. The word 'suffered' means to experience physical or mental pain. So these words are not suitable when used in this particular context. 'Struggled' means to use a lot of effort to defeat someone, prevent something, or achieve something. The community has been struggling for a long time to join the mainstream. Choice (C)

8. Detected, which means to discover or identify the presence or existence of something, fits the blank appropriately. Choice (A)

9. Wails are prolonged high-pitched cries of pain or grief thus the word is apt here. Choice (B)

10. The preposition with choices (A), (B), and (D) is 'from' (exclude "from"). Only 'include' will collocate with the preposition 'in'. Choice (B)

11. The frequency is increased from something to something. Hence 'from' is the appropriate preposition in the blank. Choice (C)

Explanatory notes for questions 12 to 17:

12. The sentence implies that the focus should be on what the speaker 'will be' in the future and not what he was in the past. I has - been is a person considered to be outmoded, past his/her prime or no longer of any importance. Choice (B)

13. The correct option is 1. "Never……or" is more apt then "Never……nor". Similarly, there are no two actions taking place here. Therefore, the sentence should be in simple past.

Choice (A)

14. The words apt in the context are 'similar' and diverse; note the use of the conjunction 'but' to link these opposite words. Choice (B)

15. A 'has been' is a person or thing considered to be no longer of any importance. The speaker say that he/she does have it him/her to remain relevant. Choice (D)

16. Since 'thieves' is plural, 'very' should be 'were'. Since the word 'house' begins with a consonant, the article before it should be '*a*' Choice (D)

17. The word 'voluntary' rules out 'regression' 'relapse' and 'setback'. Only 'moratorium' makes sense.

Choice (D)

Explanatory notes for questions 18 to 20:

18. The word 'deterrent' means an obstacle. This is closest in meaning to 'impediment'. The rest of the options are antonyms or inapt in the given concept. Choice (A)

19. The word 'ambush' means to be in a concealed position, waiting to make a move. Choice (A)

20. The meaning of the word 'prodigy' is a genius.

Choice (A)

Chapter 1

Simple Equations

CHAPTER HIGHLIGHTS

- *One Equation in One Unknown*
- *Two Equations in Two Unknowns*
- *Three Equations in Three Unknowns*
- *Additional Cases in Linear Equations*

Introduction

There will be linear equations of one or two unknowns invariably in every problem. A linear equation is one where each variable occurs only in its first power and not in any higher powers. Sometimes, we get three equations in three unknowns. In general, we need as many equations as the variables we will have to solve for. So, for solving for the values of two unknowns, we need two equations (or two conditions given in the problem) and for solving for the values of three unknowns, we need three equations (and hence the problem should give three conditions from which we can frame three equations). Solving the equations by itself is not a difficult task. The most important part of the problem is framing the equation/equations. Once the equations are framed, solving them is very easy. In this chapter, we will deal with problems involving as many equations (of first degree) as the number of unknowns. Later on, we will look at equations of second degree (quadratic equations) and linear equations where the number of equations will be less than that of the number of variables (under the chapter special equations).

One Equation in One Unknown

An equation like $2x + 4 = 26$ is an equation in one unknown. We have only one variable x whose value we have to find out. The steps in solving this are:

Step 1: Take all quantities added to (or subtracted from) the x term (term with the unknown) to the right side with a change of sign.
i.e., $2x = 26 - 4 = 22$.

Step 2: Take the co-efficient of x from left-hand side and divide right-hand side with this term to get the value of x:
i.e. $x = 22/2 = 11$. Therefore, $x = 11$.

Two Equations in Two Unknowns

A set of equations like

$$2x + 3y = 8 \qquad (1)$$

$$5x + 4y = 13 \qquad (2)$$

is called a system of simultaneous equations in two unknowns. Here, we have two variables (or unknowns) x and y whose values we have to find out. This can be done using the two given equations. The steps for this are as follows:

Step 1: Using both the equations, we first eliminate one variable (so that we can then have one equation in one unknown).
For this purpose, we multiply equation (1) with 5 (the co-efficient of x in the second equation) and multiply equation (2) with 2 (the co-efficient of x in the first equation) to eliminate x. Thus, we have

$$(1) \times 5 \Rightarrow 10x + 15y = 40 \qquad (3)$$

$$(2) \times 2 \Rightarrow 10x + \ \ 8y = 26 \qquad (4)$$

Now, subtracting equation (4) from equation (3) we have

$$7y = 14 \qquad (5)$$

This is one equation in one unknown.

Step 2: Solve for the value of one variable from the equation (in one unknown) obtained from Step I above. Therefore, $y = 2$.

Step 3: Substitute this value of the variable in one of the two equations to get the value of the second variable.

Substituting the value of y in equation (1) or equation (2), we get $x = 1$. Therefore the values of x and y that satisfy the given set of equations are $x = 1$ and $y = 2$.

Three Equations in Three Unknowns

A set of equations like

$$x + 2y + 3z = 14 \qquad (6)$$

$$2x + y + 2z = 10 \qquad (7)$$

$$3x + 3y + 4z = 21 \qquad (8)$$

is a system of three equations in three unknowns.

Here we have three unknowns x, y and z which we have to solve for from the three given equations. The procedure for the same is as follows:

Step 1: Take two out of the three equations [say, eqn. (6) and (7)] and eliminate one variable (say x) so that we get an equation in two unknowns (y and z in this case).
For this purpose, take equations (6) and (7). Multiply equation (6) by 2 and subtract equation (7) from it.

Equation (6) × 2 ⇒ $2x + 4y + 6z = 28$

$$2x + y + 2z = 10$$

$$3y + 4z = 18 \qquad (9)$$

Step 2: Repeat Step 1 for two other equations [say equations (7) and (8)] and eliminate the same variable (x in this case) so that we get one more equation in two unknowns (y and z).
For this purpose, take equations (7) and (8). Multiply equation (7) by 3 and from that subtract equation (8) multiplied by 2.

Equation (7) × 3 ⇒ $6x + 3y + 6z = 30$

Equation (8) × 2 ⇒ $6x + 6y + 8z = 42$

$$-3y - 2z = -12 \qquad (10)$$

Step 3: Now the equations in two unknowns that have been obtained from the above two steps have to be solved as discussed previously (in TWO EQUATIONS IN TWO UNKNOWNS) to get the values of two of the three variables (y and z in this case).
In this case, solving equations (9) and (10), we get $y = 2$ and $z = 3$.

Step 4: Substitute these values of the two variables in one of the three equations to get the value of the third variable.
Substitute the value of y and z in equation (6) to get the value of $x = 1$.
Thus, the values of the three variables x, y and z that satisfy the three given equations are $x = 1$; $y = 2$ and $z = 3$

Solved Examples

Example 1

The cost of 3 tables and 4 chairs is ₹2500. The cost of 4 tables and 3 chairs is ₹2400. Find the costs of each table and each chair.

Solution

Let the cost of each table be ₹x.
Let the cost of each chair be ₹y.

$$3x + 4y = 2500 \qquad (1)$$

$$4x + 3y = 2400 \qquad (2)$$

Method 1:
Multiplying (1) by 3 and subtracting it from (2) multiplied by 4, we get

$$7x = 2100$$

$$x = 300$$

Substituting $x = 300$ in (1),

$$y = 400$$

Method 2:
Adding both the equations (1) and (3), we get $7(x + y) = 4900$

$$x + y = 700 \qquad (3)$$

subtracting (2) from (1),

$$-x + y = 100 \qquad (4)$$

Adding (3) and (4), $2y = 800$

$$y = 400$$

Substituting $y = 400$ in either (3) or (4), $x = 300$

Example 2

Raju bought 6 pens, 5 erasers, and 4 sharpeners for ₹32. Had he bought 4 pens, 3 erasers, and 5 sharpeners, his total expenditure would have been ₹23. Had he bought 7 pens, 2 erasers, and 6 sharpeners, his total expenditure would have been ₹31. Find the cost of 1 pen, 1 eraser, and 2 sharpeners.

Solution

Let the prices of each pen, each eraser, and each sharpener be ₹p, ₹e, and ₹s respectively.

$$6p + 5e + 4s = 32 \qquad (1)$$

$$4p + 3e + 5s = 23 \qquad (2)$$

$$7p + 2e + 6s = 31 \qquad (3)$$

Multiplying (1) by 2 and subtracting from (2) multiplied by 3,

$$-e + 7s = 5 \qquad (4)$$

Multiplying (3) by 4 and subtracting it from (2) multiplied by 7,

$$13e + 11s = 37 \tag{5}$$

Multiplying (4) by 13 and adding it to (5), $102s = 102$

$$s = 1$$

Substituting $s = 1$ in (4),

$$e = 2$$

Substituting values of e and s in (1), $p = 3$.

Example 3

In a two digit number, the digits differ by 2. 10 times the number exceeds 5 times the sum of the number formed by reversing its digits and the sum of its digits by 90. Find the number.

Solution

Let the number be xy. Hence, the value of the number is $10x + y$.

$$x - y = 2 \quad \text{or} \quad y - x = 2 \tag{6}$$

$$10\,(10x + y) - 5\,(10y + x + x + y) = 90$$

$$90x - 45y = 90$$

$$2x - y = 2;\ 2x - (x \pm 2) = 2$$

$$x = 4 \quad \text{or} \quad 0$$

As x cannot be 0, $x = 4$

$\therefore \qquad y = 6$

$\therefore$ the number is 46.

Example 4

The age of a man 15 years ago was 5 times his son's age. His age 10 years ago was thrice his son's age. After how may years will their combined age become 80 years?

Solution

Let the present age of the man and his son be f years and s years, respectively.

$$f - 15 = 5\,(s - 15) \quad \Rightarrow \quad f = 5s - 60$$

$$f - 10 = 3\,(s - 10) \quad \Rightarrow \quad f = 3s - 20$$

$$f = 5s - 60 = 3s - 20$$

$$s = 20,\ f = 40$$

Their combined present age is 60 years. For the combined present age to become 80 years, the age of each of them must increase by 10 years.

$\therefore$ Their combined age will become 80 years after 10 years.

Example 5

If the numerator and the denominator of a fraction are both increased by 1, the fraction becomes $\frac{3}{5}$. If both are decreased by 1, it becomes $\frac{5}{9}$. Find the fraction.

Solution

Let the fraction be $\frac{x}{y}$.

$$\frac{x+1}{y+1} = \frac{3}{5}$$

$$\Rightarrow \qquad 5x + 5 = 3y + 3$$

$$5x + 2 = 3y \tag{1}$$

$$\frac{x-1}{y-1} = \frac{5}{9}$$

$$\Rightarrow \qquad 9x - 9 = 5y - 5$$

$$9x - 4 = 5y \tag{2}$$

Multiplying (1) by 5 and subtract it from (2) after multiplying by 3,

$$5\,(5x + 2) = 3\,(9x - 4)$$

$$\Rightarrow \qquad x = 11$$

substituting $x = 11$ in (1), $y = 19$

$\therefore$ The fraction $= \frac{11}{19}$

Additional Cases in Linear Equations

1. If the number of equations is less than the number of unknowns, then we say the variables are 'indeterminate' or we have an 'indeterminate' system of equations. Here, we cannot uniquely determine the values of all the variables. There will be infinite sets of solutions that satisfy the equations.
   For example, if we take the following two equations in three unknowns,

   $$x + y + 2z = 8$$

   $$2x - y + 3z = 13$$

   this system of equations have infinite number of solutions and no unique solution is possible. For any value we take for x, we can find a corresponding set of values for y and z.
2. However, even in case of indeterminate equations, say, of three variables, it is possible that the value of one of the variables may be uniquely determined, i.e. if we have two equations and three unknowns, we may be still able to determine the value of one variable uniquely but the other two variables will

have infinite number of values. This will happen if the ratio of the co-efficients of two variables in one equation is the same as the ratio of the co-efficients of the same two variables in the second equation.
This depends on the equations given. Example 8 will clarify this aspect.

Example 6

Tarun bought 2 shirts, 4 trousers, and 5 pairs of shoes for ₹3600. Had he bought 6 shirts, 5 trousers, and 15 pairs of shoes, his total expenditure would have been ₹8700. Find the price of each trouser.

Solution

Let the prices of each shirt, each trouser and each pair of shoe be ₹x, ₹y, and ₹z, respectively.

$$2x + 4y + 5z = 3600 \quad (1)$$

$$6x + 5y + 15z = 8700 \quad (2)$$

Multiplying (1) by 3 and subtracting (2) from it, $7y = 2100$

$\therefore \quad y = 300$

3. Even in case of indeterminate equations, when some additional conditions are either implicitly built into the problem or explicitly imposed by specifying some constraints on the values of the variables, we may some times be able to determine the values of the variables uniquely or find out a finite set of values that the variables may take. Such problems are separately considered under the chapter 'SPECIAL EQUATIONS.'
4. Sometimes, even if we have equations less in number than the number of variables (i.e., indeterminate equations), while we cannot find out the values of ALL the variables uniquely, it may be possible to find out the value of some specific combination of the variables.

Example 7

The cost of 3 dosas, 5 idlis, and 7 vadas is ₹154. The cost of 5 dosas, 8 idlis, and 11 vadas is ₹246. Find the total cost of one idli, one dosa, and one vada.

Solution

Let the cost of each dosa, each idli, and each vada be ₹d, ₹i, and ₹v, respectively.

$$3d + 5i + 7v = 154 \quad (1)$$

$$5d + 8i + 11v = 246 \quad (2)$$

Multiplying (1) by 3 and subtracting it, from twice (2), $d + i + v = 30$

5. Sometimes, even if we have three equations in three unknown, we may not be able to uniquely determine the values of the variables if the equations are not 'INDEPENDENT,' i.e. one of the given equations can be written as a 'linear combination' of the other two equations.

For example, let us take the following system of three equations in three unknowns.

$$3x + 5y + 7z = 12 \quad (3)$$

$$x - 3y + 9z = 16 \quad (4)$$

$$9x + 8y + 31z = 54 \quad (5)$$

If we try to solve these equations, we will find that we cannot get a unique solution. That is because these equations are not independent. In this case, equation (5) can be obtained by multiplying equation (3) by 2.5 and equation (4) by 1.5 and adding them.

If there are three equations l_1, l_2, and l_3 in three unknowns, we say that they are linearly dependent if one of the three equations can be written as a linear combination of the other two, i.e. $l_3 = l_1 + kl_2$ where k is any constant.

In such a case, the system of equations will have infinite number of solutions.

If it is not possible to write the three equations in the form above, then they are linearly independent and the system of equations will have a unique solution.

6. Sometimes, we can have 'inconsistent' equations. For example, if we know that $x + 2y = 4$, then the value of $2x + 4y$ has to be 8. The expression $(2x + 4y)$ cannot take any other value. If it is given any other value, there will be inconsistency in the data because then we will effectively be saying that $x + 2y = 4$ and at the same time $x + 2y \neq 4$.

So, if we have the system of equations $x + 2y = 4$ and $2x + 4y = k$, this system of equations will be consistent ONLY If the value of $k = 8$. For any other value of k, the system of equations will be inconsistent.

In the above system of equations, when $k = 8$, there will be infinite number of solutions (and not a unique solution).

Example 8

Find the value of k for which the following system of equations will be consistent.

$$2x - 5y = 10 \quad \text{and} \quad 6x - 15y = k$$

Solution

In the given system of equations, the ratio of the coefficients of x equals the ratio of the coefficients of y.

$\therefore$ They would be consistent only if this ratio equals the ratio of the constant terms.

$\therefore$ If $\frac{10}{k} = \frac{2}{6} = \frac{-5}{-15}$ i.e.

if $k = 30$, the given system of equations would be consistent.

Exercises

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. P, Q, and R are successive even natural numbers in ascending order. Five times R is eight more than seven times P. Find Q.
(A) 6 (B) 8 (C) 12 (D) 14

2. Divide 1 kg weight into two parts such that the sum of the parts is $5/4^{th}$ the difference.
(A) 550 g, 450 g (B) 200 g, 800 g
(C) 900 g, 100 g (D) 400 g, 600 g

3. A is greater than B by $1/3^{rd}$ the sum of A and B. If B is increased by 40, it becomes greater than twice A by 10. Find A, B.
(A) 30, 20 (B) 60, 30
(C) 20, 10 (D) 20, 40

4. Ajay was asked to find $(2/9)^{th}$ of a number. He instead multiplied the number by (9/2) and obtained an answer which was 4235 more than the correct answer. Find the number.
(A) 900 (B) 945
(C) 990 (D) 810

5. An amount of ₹5,600 is divided among A, B, and C. The sum of the shares of B and C is equal to thrice the share of A. The sum of the shares of A and C is equal to nine-fifths the share of B. What is the share of C?
(A) ₹1,400 (B) ₹2,400
(C) ₹2,200 (D) ₹2,000

6. Four times the sum of the digits of a two-digit number is 18 less than the number and is also 9 less than the number formed by reversing its digits. Find the product of its digits.
(A) 12 (B) 20 (C) 30 (D) 42

7. Six years ago, Ram's age was four times Shyam's age. Six years hence, Ram's age will be thrice Shyam's age. After how many years from now will their combined age be 150 years?
(A) 21 (B) 9 (C) 36 (D) 18

8. The sum of the ages of Bharat and Sharat is twice the sum of their ages seven years ago. What is the product of their present ages, if the sum of the squares of their ages is 400?
(A) 192 (B) 180 (C) 200 (D) 164

9. Ashok has a total of 30 notes in denominations of ₹20 and ₹5. The total value of the notes with him is ₹300. Find the number of ₹20 notes with him.
(A) 5 (B) 10 (C) 8 (D) 6

10. A fraction is such that the numerator is five less than the denominator. Also four times, the numerator is one more than the denominator. Find the fraction.
(A) 4/9 (B) 3/8 (C) 2/7 (D) 7/12

11. The digits of a two digit number differ by 3. Find the difference of the number and the number formed by reversing its digits.
(A) 18 (B) 27 (C) 36 (D) 45

12. Two chocolates, three milk shakes and four cakes cost ₹190. Four chocolates and eight cakes cost ₹320. Find the cost of a milkshake (in ₹).
(A) 10 (B) 20
(C) 30 (D) Cannot be determined

13. Three consecutive even integers are such that one-third of the second number is equal to one-fourth of the third number. Find the three numbers.
(A) 4, 6, 8 (B) 8, 10, 12
(C) 12, 14, 16 (D) 2, 4, 6

14. Amar, Bhavan, Chetan, and Dinesh have a total of ₹150 with them. Amar has one-fourth of the total amount with the others. Find the amount with Amar (in ₹).
(A) 20 (B) 25 (C) 30 (D) 37.5

15. Ramesh is thrice as old as Suresh. Two years hence, Ramesh will be twice as old as Suresh. Find Ramesh's present age (in years).
(A) 2 (B) 3 (C) 4 (D) 6

16. Nalini has an amount of ₹20 in coins of denominations of 50 paise and ₹1. If she has a total of 30 coins with her, how many ₹1 coins does she have?
(A) 20 (B) 10 (C) 15 (D) 30

17. A two-digit number is one more than six times the sum of its digits and also five more than forty six times the difference of its digits. Find the number.
(A) 79 (B) 97 (C) 49 (D) 94

18. Find the value of k if the equations $3x + (k/3 + 2)y = 1$ and $kx + 2ky = 4$ have infinite solutions.
(A) 9 (B) 6 (C) 18 (D) 12

19. Cost of two pens, five pencils, and seven erasers is ₹37. Cost of seven pens, one eraser, and two pencils is ₹49. What is the cost of nine pencils and fortyseven pens?
(A) ₹184 (B) ₹276
(C) ₹284 (D) None of these

20. The sum of two numbers is 250. The difference of their squares is 12,500. Find the larger number.
(A) 130 (B) 140 (C) 150 (D) 160

21. Five three-digit numbers including N, were to be added. While adding, the reverse of N was added by mistake instead of N. Hence, the sum increased by 11 times the sum of the digits of N. Eight times the difference of N's units and hundreds digits is 6 more than twice its hundreds digit. Find its tens digit.
(A) 4 (B) 6 (C) 8 (D) 2

22. The cost of two pens, one eraser, and three sharpeners, is ₹23. The cost of six pens, three erasers, and one sharpener is ₹45. The cost of fourteen pens, seven erasers, and twenty one sharpeners is ₹161. Find the cost of each pen (in ₹).
(A) 3
(B) 4
(C) 5
(D) Cannot be determined

23. A child went to a shop to buy a pen, a pencil and a ruler where costs are integral values (in ₹) and are in decreasing order. Each item costs at least ₹4. The total cost is ₹15 and the cost of a pencil is ₹5. How many pencils can he purchase with the amount required to purchase ten rulers?
(A) 10 (B) 12 (C) 8 (D) 9

24. Nitya and Satya have some marbles with them. Nitya says to Satya, 'If you give one marble to me, we will have equal number of marbles'. Satya says to Nitya, 'If you give me one marble, I will have twice the number of marbles you have'. How many marbles do Nitya and Satya have respectively?
(A) 4, 6 (B) 5, 7
(C) 6, 4 (D) 7, 5

25. John covers 10 km per hour more than Peter while driving. On doubling his speed, Peter covers 15 km per hour more than John who is driving at his normal speed. What is John's speed?
(A) 40 km/hr
(B) 25 km/hr
(C) 45 km/hr
(D) 35 km/hr

Answer Keys

1. B	**2.** C	**3.** C	**4.** C	**5.** C	**6.** B	**7.** B	**8.** A	**9.** B	**10.** C
11. B	**12.** A	**13.** A	**14.** C	**15.** D	**16.** B	**17.** B	**18.** D	**19.** D	**20.** C
21. B	**22.** D	**23.** C	**24.** B	**25.** D					

Chapter 2

Ratio–Proportion–Variation

CHAPTER HIGHLIGHTS

- *Ratio*
- *Proportion*
- *Variation*
- *Direct Variation*
- *Inverse Variation*
- *Joint Variation*

Ratio

If the values of two quantities A and B are 4 and 6, respectively, then we say that they are in the ratio 4:6 (read as 'four is to six'). Ratio is the relation which one quantity bears to another of the same kind, the comparison being made by considering what multiple, part or parts, one quantity is of the other. The ratio of two quantities 'a' and 'b' is represented as a:b and read as 'a is to b'. Here, 'a' is called antecedent, 'b' is the consequent. Since the ratio expresses the number of times one quantity contains the other, it's *an abstract* quantity.

Ratio of any number of quantities is expressed after removing any common factors that ALL the terms of the ratio have. For example, if there are two quantities having values of 4 and 6, their ratio is 4:6, i.e. 2:3 after taking the common factor 2 between them out. Similarly, if there are three quantities 6, 8, and 18, there is a common factor among all three of them. So, dividing each of the three terms by 2, we get the ratio as 3:4:9.

If two quantities whose values are A and B, respectively, are in the ratio a:b, since we know that some common factor $k(> 0)$ would have been removed from A and B to get the ratio a:b, we can write the original values of the two quantities (i.e. A and B) as ak and bk respectively. For example, if the salaries of two persons are in the ratio 7:5, we can write their individual salaries as $7k$ and $5k$, respectively.

A ratio a:b can also be expressed as a/b. So, if two items are in the ratio 2:3, we can say that their ratio is 2/3. If two terms are in the ratio 2, it means that they are in the ratio of 2/1, i.e. 2:1.

'A ratio is said to be a ratio of greater or less inequality or of equality according as antecedent is greater than, less than or equal to consequent'. In other words,

1. The ratio a:b where $a > b$ is called a ratio of greater inequality (example 3:2)
2. The ratio a:b where $a < b$ is called a ratio of less inequality (example 3:5)
3. The ratio a:b where $a = b$ is called a ratio of equality (example 1:1)

From this, we can find that a ratio of greater inequality is diminished and a ratio of less inequality is increased by adding same quantity to both terms, i.e. in the ratio a:b, when we add the same quantity x (positive) to both the terms of the ratio, we have the following results

$$\begin{array}{lll} \text{if } a < b & \text{then} & (a+x):(b+x) > a:b \\ \text{if } a > b & \text{then} & (a+x):(b+x) < a:b \\ \text{if } a = b & \text{then} & (a+x):(b+x) = a:b \end{array}$$

This idea can also be helpful in questions on Data Interpretation when we need to compare fractions to find the larger of two given fractions.

If two quantities are in the ratio a:b, then the first quantity will be $a/(a + b)$ times the total of the two quantities and the second quantity will be equal to $b/(a + b)$ times the total of the two quantities.

Solved Examples

Example 1

If $a:b = 3:4$, find $3a + 4b:4a + 5b$.

Solution

$3a + 4b:4a + 5b$

$$= \frac{3a+4b}{4a+5b} = \frac{\frac{3a+4b}{b}}{\frac{4a+5b}{b}} = \frac{3\left(\frac{a}{b}\right)+4}{4\left(\frac{a}{b}\right)+5} = \frac{3\left(\frac{3}{4}\right)+4}{4\left(\frac{3}{4}\right)+5} = \frac{25}{32}$$

Example 2

The ratio of the number of marbles with Ram and Shyam is 19:13. If Ram gives Shyam 30 marbles, both will have equal number of marbles. Find the number of marbles with Ram.

Solution

Let the number of marbles with Ram and Shyam be $19x$ and $13x$, respectively. Total number of marbles with them = $32x$

If Ram gives Shyam 30 marbles, each will have $\frac{32x}{2} = 16x$ marbles.

$\therefore \quad 19x - 16x = 30$

$$x = 10$$

$$19x = 190$$

Example 3

1400 is divided into 4 parts such that half of the first part, one third of the second part, one fourth of the third part, and $\frac{1}{12}$th of the last part are all equal. Find the 4 parts.

Solution

Let the first part, second part, third part, and fourth part be a, b, c, and d respectively.

$$\frac{1}{2}a = \frac{1}{3}b = \frac{1}{4}c = \frac{1}{12}d$$

$$d = 6a,\ b = \frac{3}{2}a,\ c = 2a$$

Given, $\quad a + b + c + d = 1400$

$\Rightarrow \quad a + \frac{3}{2}a + 2a + 6a = 1400$

$\Rightarrow \quad a = \frac{400}{3}$

$\therefore \quad b = 200,\ c = \frac{800}{3},\ d = 800$

Alternative method:

As b is common to both ratios and since it is divisible by 3 (from the first ratio) and it is divisible by 2 (from the second ratio), it is divisible by LCM (3, 2), i.e. 6. Hence if $b = 6$, $a = 4$, and $c = 9$

$\therefore \quad a:b:c = 4:6:9$

Example 4

There are 2 classes A and B. If 10 students leave class A and join class B, then the ratio of the number of students in class A and class B would reverse. Find the difference in the numbers of students in class A and class B.

Solution

Let the numbers of students in class A and class B be ax and bx, respectively.

Given, $\quad \frac{ax - 10}{bx + 10} = \frac{b}{a}$

$$a^2x - 10a = b^2x + 10b$$

$\Rightarrow \quad a^2x - b^2x - 10a - 10b = 0$

$\Rightarrow \quad (ax - bx - 10)(a + b) = 0$

$\therefore \quad ax - bx = 10$

Example 5

A husband's age exceeds that of his wife by 6 years. 10 years ago, the ratio of their ages was 5:4. Find the present age of the husband.

Solution

Let the present age of the husband be x years.

$\Rightarrow \quad$ Present age of the wife = $(x - 6)$ years.

10 years ago, the ages of the husband and the wife will be $(x - 10)$ years and $(x - 16)$ years, respectively.

Given $\quad x - 10 = \frac{5}{4}(x - 16)$

$\therefore \quad x = 40$

Alternative method:

Let the age of the husband 10 years ago be $5x$ years. Age of his wife at that time = $4x$ years.

The husband would then also be 6 years, older than his wife.

$\therefore \quad 5x = 4x + 6 \quad \Rightarrow \quad x = 6$

Hence, the present age of the husband

$$= 5x + 10, \quad \text{i.e. } 40 \text{ years}$$

Proportion

When two ratios are equal, then the four quantities involved in the two ratios are said to be proportional, i.e. if $a/b = c/d$, then a, b, c, and d are proportional.

This is represented as $a{:}b{::}\ c{:}d$ and is read as 'a is to b (is) as c is to d'.

When a, b, c, and d are in proportion, then a and d are called the EXTREMES and b and c are called the MEANS. We also have the relationship:

Product of the MEANS = Product of the EXTREMES, i.e. $b\,c = adv$

If $\quad a:b = c:d$ then

$$b:a = d:c \quad \text{(A)}$$

$$a:c = b:d \quad \text{(B)}$$

$$(a + b):b = (c + d):d \quad \text{(C)}$$

(obtained by adding 1 to both sides of the given relationship)

$$(a-b):b=(c-d):d \qquad \text{(D)}$$

(obtained by subtracting 1 from both sides of the given relationship)

$$(a+b):(a-b)=(c+d):(c-d) \qquad \text{(E)}$$

{obtained by dividing relationship (C) above by (D)}

Relationship (A) above is called INVERTENDO;
Relationship (B) is called ALTERNENDO;
Relationship (C) is called COMPONENDO;
Relationship (D) is called DIVIDENDO;
Relationship (E) is called COMPONENDO –DIVIDENDO.

The last relationship, i.e. COMPONENDO–DIVIDENDO is very helpful in simplifying problems. By this rule, whenever we know $a/b = c/d$, then we can write

$$(a+b)/(a-b)=(c+d)/(c-d).$$

The converse of this is also true—whenever we know that $(a+b)/(a-b)=(c+d)/(c-d)$, then we can conclude that $a/b = c/d$.

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}\ldots$, then each of these ratios is equal to $\frac{a+c+e+\ldots}{b+d+f+\ldots}$.

Variation

Two quantities A and B may be such that as one quantities changes in value, the other quantity also changes in value **bearing certain relationship** to the change in the value of the first quantity.

Direct Variation

One quantity A is said to vary directly as another quantity B if the two quantities depend upon each other in such a manner that if B is increased in a certain ratio, A also increases in the same ratio and if B is decreased in a certain ratio, A also decreases in the same ratio.

This is denoted as $A \,\alpha\, B$ (A varies directly as B).
If $A \,\alpha\, B$ then $A = kB$, where k is a constant. It is called the constant of proportionality.

For example, when the quantity of sugar purchased by a housewife doubles from the normal quantity, the total amount she spends on sugar also doubles, i.e. the quantity and the total amount increase (or decrease) in the same ratio.

From the above definition of direct variation, we can see that when two quantities A and B vary directly with each other, then $A/B = k$ or the ratio of the two quantities is a constant. Conversely, when the ratio of two quantities is a constant, we can conclude that they vary directly with each other.

If X varies directly with Y and we have two sets of values of the variables X and Y – X_1 corresponding to Y_1 and X_2 corresponding to Y_2, then, since $X \,\alpha\, Y$, we can write down

$$\frac{X_1}{Y_1}=\frac{X_2}{Y_2} \quad \text{or} \quad \frac{X_1}{X_2}=\frac{Y_1}{Y_2}$$

Inverse Variation

A quantity A is said to vary inversely as another quantity B if the two quantities depend upon each other in such a manner that if B is increased in a certain ratio, A gets decreased in the same ratio and if B is decreased in a certain ratio, then A gets increased in the same ratio.

It is the same as saying that A varies directly with $1/B$. It is denoted as $A \,\alpha\, 1/B$, i.e. $A = k/B$ where k is k the constant of proportionality.

For example, as the number of men doing a certain work increases, the time taken to do the work decreases and conversely, as the number of men decreases, the time taken to do the work increases.

From the definition of inverse variation, we can see that when two quantities A and B vary inversely with each other, then $AB = a$ constant, i.e. the product of the two quantities is a constant. Conversely, if the product of two quantities is a constant, we can conclude that they vary inversely with each other.

If X varies inversely with Y and we have two sets of values of X and Y – X_1 corresponding to Y_1 and X_2 corresponding to Y_2, then since X and Y are inversely related to each other, we can write down

$$X_1Y_1 = X_2Y_2 \quad \text{or} \quad \frac{X_1}{X_2}=\frac{Y_2}{Y_1}$$

Joint Variation

If there are three quantities A, B, and C such that A varies with B when C is constant and varies with C when B is constant, then A is said to vary jointly with B and C when both B and C are varying, i.e. $A \,\alpha\, B$ when C is constant and $A \,\alpha\, C$ when B is a constant; $\Rightarrow A \,\alpha\, BC$.

$A \,\alpha\, BC \Rightarrow A = kBC$ where k is the constant of proportionality.

Example 6

Find the value(s) of x if $\frac{2x+5}{x+1}=\frac{x+2}{x-1}$.

Solution

$$(2x+5)(x-1)=(x+2)(x+1)$$

$$2x^2+5x-2x-5=x^2+2x+x+2$$

$$\Rightarrow \quad x^2 = 7$$

$$\therefore \quad x = \pm\sqrt{7}$$

Example 7

X varies directly with $Y^2 + 18$. When $Y = 18$, $X = 18$. Find Y when $X = 1$.

Solution

$$\frac{X_1}{X_2} = \frac{Y_1^2+18}{Y_2^2+18}\ ;\ \frac{18}{1} = \frac{18^2+18}{Y_2^2+18}$$

$$Y_2^2 + 18 = 19$$

$$Y_2 = \pm 1$$

In these types of problems on variation, there are typically three parts:

1. The relationship between different variables is defined to frame an equation involving the variables and the constant of proportionality.
2. One set of values of all the values of all the variables is given to enable us find the value of the constant of proportionality.
3. The values of all but one variable of a second set are given, and we are asked to find the value of the one variable whose value is not given.

Example 8

The ratio of the monthly incomes of A and B is 4:3. The ratio of their monthly expenditures is 5:4. If A saves one fourth of his monthly income, find the ratio of their monthly savings.

Solution

Let the monthly incomes of A and B be ₹$4x$ and ₹$3x$, respectively.

Let the monthly expenditures of A and B be ₹$5y$ and ₹$4y$ respectively. Monthly savings of A = ₹$(4x - 5y)$. Monthly savings of B = ₹$(3x - 4y)$.

Given that $4x - 5y = \frac{1}{4}(4x)$

$$3x = 5y.$$

$\therefore$ Monthly saving of $B = 3x - 4y = 5y - 4y$, i.e. ₹y.

Required ratio $= \frac{1}{4}(4x) : y = x{:}y = 5{:}3$

Example 9

If x varies directly with y, check whether $x^3 + y^3$ varies directly with $x^3 - y^3$.

Solution

Let $\frac{x}{y} = k$, where k is a constant

$$x = k\,y$$

$$x^3 + y^3 = y^3\,(k^3 + 1)$$

$$x^3 - y^3 = y^3\,(k^3 - 1)$$

$$\frac{x^3 + y^3}{x^3 - y^3} = \frac{k^3+1}{k^3-1} \text{ a constant}$$

$\therefore$ $(x^3 + y^3)$ varies directly with $(x^3 - y^3)$

Example 10

The monthly expenses of Raja on his car are partly constant and partly vary with the number of kilometres he travels in a month. If he travels 100 km in a month his total car expenses will be ₹3,500. If he travels 200 km in a month, his total car expenses will be ₹4,000. If he travels 250 km in a month, what will be his total car expenses.

Solution

Let his total car expenses be ₹T. Let the fixed expense be ₹F. Let the variable expense be ₹V.

$$T = F + V$$

If he travels D km in a month, $\frac{V}{D} = k$, where k is a constant.

$\therefore$
$$T = F + k\,D$$

Given that

$$3500 = F + 100k \quad (1)$$

$$4000 = F + 200k \quad (2)$$

Solving (1) and (2),

$$F = 3000 \text{ and } k = 5.$$

Total car expenses if he travels 250 km

$$= F + 250k = ₹4250.$$

The problems involving ratio and proportion are just different forms of the models of the basic problems we saw above. For example, the problem we just solved above might be reframed bringing in mangoes, bananas, baskets, etc. Here, practice and perseverance pay you a lot. In entrance exams, there will be either direct problems on ratio, proportion, and variation or indirect problems of application of these concepts just discussed to areas like time and work or time and distance.

Exercises

Direction for questions 1 to 18: Select the correct alternative from the given choices.

1. If $p:q = 5:4$ and $p = a + b$ and $q = a - b$, find $a:b$.
   (A) 1:9 (B) 9:1
   (C) 5:4 (D) 4:5
2. The number of marbles with A and B are in the ratio of 10:11. Which of the following cannot be a possible number of marbles with A and B together?
   (A) 189 (B) 210 (C) 231 (D) 153
3. The ratio of the number of students in three classes A, B, and C is 3:7:8. If ten students, are transferred from C to B, B will have 80 students. Find the total number of students in the three classes.
   (A) 150 (B) 160 (C) 180 (D) 210
4. Three positive numbers p, q, and r satisfy $\frac{q+r}{p} = \frac{p+r}{q} = \frac{p+q}{r} = K$. $K =$ ______.
   (A) 3/2 (B) 5/2 (C) 3 (D) 2
5. What must be subtracted from p and added to q so that the ratio of the resultants becomes 1:3?
   (A) $\frac{p+q}{3}$ (B) $\frac{3p-q}{4}$
   (C) $\frac{p-q}{p+q}$ (D) $\frac{q-3p}{4}$
6. Vipin's present age is twice the age of Kishore one year ago. What is the sum of their present ages (in years), if the ratio of the sum of their present ages to the difference of their present ages is 19:5?
   (A) 21 (B) 19 (C) 24 (D) 34
7. Three different types of balls priced at ₹5, ₹8, and ₹13 per piece are displayed in three different boxes by a trader. Mr. Paul bought from this shop all three types of balls spending a total sum of ₹768. The numbers of the balls he bought, taken in the order in which the prices are mentioned above, are in the ratio 5:4:3. How many balls of the costliest variety did he buy?
   (A) 104 (B) 64 (C) 48 (D) 24
8. If $a:b = 2:3$ $b:c = 4:3$ and $c:d = 2:3$, then find $a:b:c:d$.
   (A) 8:12:9:27
   (B) 16:24:18:27
   (C) 18:27:36:8
   (D) 12:18:15:20
9. The weights of Bimal and Basu are in the ratio 2:3 and the weights of Basu and Bali are in the ratio 4:3. What is Basu's weight (in kg) if the sum of the weights of Bimal, Basu, and Bali is 203 kg?
   (A) 84 (B) 76
   (C) 49 (D) 65
10. If $3x - 4y + 2z = 0$ and $4x - 2y - z = 0$, find $x:z:y$.
    (A) 8:10:11 (B) 8:11:40
    (C) 11:40:8 (D) 8:40:11
11. If $a + b - c:b + c - a:a + c - b = 5:6:7$, then find $a:b:c$.
    (A) 12:13:11 (B) 12:11:13
    (C) 13:12:11 (D) 13:11:12
12. Which of the following represents a possible value of $p : q$ satisfying $\frac{20p^2 - 40pq}{pq + 4q^2} = 20$?
    (A) 3:1 (B) 1:4
    (C) 4:1 (D) 5:1
13. Ninety three is divided into two parts such that thrice the first part and twice the second part are in the ratio 25:4. Find the first part.
    (A) 60 (B) 75
    (C) 50 (D) 70
14. If three is subtracted from the numerator and five is added to the denominator of a fraction, the new fraction formed is 1/2. If two is added to the numerator of the initial fraction, the ratio of the new numerator to the denominator becomes 1:1. Find the original fraction.
    (A) 11/13 (B) 18/23
    (C) 13/15 (D) 13/11
15. The ratio of the number of students in classes A, B, and C is 3:7:8. If 10 students leave C and join B, the ratio of the number of students in B and C would be reversed. Find the total number of students in the classes A, B, and C.
    (A) 144 (B) 162 (C) 180 (D) 198
16. A person has with him a certain number of weighing stones of 100 g, 500 g, and 1 kg in the ratio of 3 : 5 : 1. If a maximum of 5 kg can be measured using weighing stones of 500 g alone, then what is the number of 100 g stones he has?
    (A) 6 (B) 3 (C) 9 (D) 5
17. The ratio of the prices of tea last year and this year is 5:6. The ratio of the prices of coffee last year and this year is 7:8. The sum of prices of a kg of tea and a kg of coffee this year is ₹48. Find the price of tea (in ₹) last year if it was $\frac{20}{21}$ of the price of coffee last year.
    (A) 15 (B) 20 (C) 25 (D) 10
18. Ajay and Vijay wrote a test. The sum of Ajay's score and twice Vijay's score is 310. The sum of Vijay's score and twice Ajay's score is 290. Find the ratio of the scores of Ajay and Vijay.
    (A) 9:11 (B) 13:17
    (C) 11:19 (D) 7:13

Direction for questions 19 and 20: These questions are based on the data given below.

A test of 60 minutes contains questions on Mathematics and English only. The time taken to solve a Mathematics question is twice the time taken to answer an English question and the ratio of time taken to solve all Mathematics questions to time taken to answer all English questions is 8/7.

19. What is the ratio of the number of English questions to that of Mathematics?
(A) 11/7 (B) 7/4
(C) 9/4 (D) 7/5

20. If the total number of questions is 22, how many English questions can be answered in 18 minutes?
(A) 8 (B) 10 (C) 11 (D) 9

Direction for questions 21 to 25: Select the correct alternative from the given choices.

21. The pressure of a gas varies directly with the temperature when the volume is constant and varies inversely with the volume when temperature is constant. If the present temperature is 100 K, what will be the increase in temperature if the pressure triples and the volume doubles?
(A) 200 K (B) 600 K
(C) 500 K (D) 100 K

22. For a body starting from rest, the distance travelled (d) is directly proportional to the square of the time elapsed from the start (t). When $t = 4$ s, $d = 48$ m. What is the value of d (in metres) at $t = 7$ s?
(A) 128 (B) 150
(C) 115 (D) 147

23. 'A' varies directly as the sum of two quantities 'B' and 'C'. 'B' in turn varies directly as 'x' and 'C' varies inversely as 'x'. When $x = 1$ or 2, $A = 3$. Find the value of A when $x = 4$.
(A) 5 (B) 4.5
(C) 5.5 (D) 6

24. The extension of a spring from its rest position is directly proportional to the force acting on the spring. An additional force applied on the already stretched spring produces a further extension, which is twice that of the initial extension. What is the ratio of the additional force to the initial force?
(A) 3:1 (B) 2:1
(C) 4:1 (D) 1:3

25. The kinetic energy of a body is directly proportional to the square of its speed when the mass is kept constant and is directly proportional to mass when its speed is kept constant. A body with a mass of 2 kg and a speed of 10 m/s has a kinetic energy of 100 joules. What is the kinetic energy of a body whose mass is 20 kg and speed is 1 m/s?
(A) 100 joules (B) 1000 joules
(C) 10 joules (D) 20 joules

Answer Keys

1. B	**2.** D	**3.** C	**4.** D	**5.** B	**6.** B	**7.** D	**8.** B	**9.** A	**10.** A
11. B	**12.** C	**13.** B	**14.** C	**15.** C	**16.** A	**17.** B	**18.** A	**19.** B	**20.** D
21. C	**22.** D	**23.** B	**24.** B	**25.** C					

Chapter 3

Numbers

CHAPTER HIGHLIGHTS

- *Rule of Signs*
- *Classification of Real Numbers*
- *Perfect Numbers*
- *Hierarchy of Arithmetic Operations*
- *Number of Factors of a Number*
- *Involution and Evolution*
- *LCM and HCF Models*
- *Successive Division*
- *Factorial*
- *Number Systems*
- *Conversions*
- *Binary Arithmetic*

Numbers

NUMBERS is one of the most important topics required for competitive entrance exams—particularly, the MBA entrance exams. In this chapter, we have put together a number of models of problems—mainly based on various problems that have been appearing in different exams.

ADDITION is the process of finding out single number or fraction equal to two or more quantities taken together.

SUBTRACTION is the process of finding out the quantity left when a smaller quantity (number/fraction) is reduced from a larger one.

MULTIPLICATION signifies repeated addition. If a number has to be repeatedly added then that number is multiplicand. The number of multiplicands considered for addition is multiplier. The sum of repetition is product. For example, in the multiplication $3 \times 4 = 12$, 3 is the multiplicand, 4 is the multiplier and 12 is product.

DIVISION is a reversal of multiplication. In this we find how often a given number called divisor is contained in another given number called dividend. The number expressing this is called the quotient and the excess of the dividend over the product of the divisor and the quotient is called remainder.

For example, in the division 32/5, 32 is dividend, 5 is divisor, 6 is quotient, and 2 is remainder.

Rule of Signs

The product of two terms with like signs is positive; the product of two terms with unlike signs is negative.

Example: $-1 \times -1 = +1$;

$+1 \times -1 = -1$;

$+1 \times +1 = +1$;

$-1 \times +1 = -1$;

Classification of Real Numbers

Real numbers are classified into rational and irrational numbers.

Rational Numbers: A number which can be expressed in the form p/q where p and q are integers and $q \neq 0$ is called a rational number.

For example, 4 is a rational number since 4 can be written as 4/1 where 4 and 1 are integers and the denominator $1 \neq 0$. Similarly, the numbers 3/4, –2/5, etc. are also rational numbers.

Recurring decimals are also rational numbers. A recurring decimal is a number in which one or more digits at the end of a number after the decimal point repeats endlessly (For example, 0.333…, 0.111111…, 0.166666…, etc. are all recurring decimals). Any recurring decimal can be expressed as a fraction of the form p/q, and hence it is a rational number. We will study in another section in this chapter the way to convert recurring decimals into fractions.

Between any two numbers, there can be infinite number of other rational numbers.

Irrational Numbers: Numbers which are not rational but which can be represented by points on the number line are called irrational numbers. Examples for irrational numbers are $\sqrt{2}$, $\sqrt{3}$, $\sqrt[4]{5}$, $\sqrt[3]{9}$, etc.

Numbers like π, e are also irrational numbers.

Between any two numbers, there are infinite number of irrational numbers.

Another way of looking at rational and irrational numbers is terminating decimals and recurring decimals are both rational numbers.

Any non-terminating, non-recurring decimal is an irrational number.

Integers: All integers are rational numbers. Integers are classified into negative integers, zero, and positive integers. Positive integers can be classified as prime numbers and composite numbers. In problems on numbers, we very often use the word 'number' to mean an 'integer.'

Prime Numbers: A number other than 1 which does not have any factor apart from one and itself is called a prime number.

Examples for prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, etc.

There is no general formula that can give prime numbers. **Every prime number greater than 3 can be written in the form of $(6k + 1)$ or $(6k - 1)$ where k is an integer.** For the proof of this, refer to 4th point under 'Some important points to note' given later on in this chapter.

Composite Numbers: Any number other than 1, which is not a prime number is called a composite number. In other words, a composite number is a number which has factors other than one and itself.

Examples for composite numbers are 4, 6, 8, 9, 10, 14, 15, etc.

NOTE

The number 1 is neither prime nor composite.

The only prime number that is even is 2.

There are 15 prime numbers between 1 and 50 and 10 prime numbers between 50 and 100. So, there are a total of 25 prime numbers between 1 and 100.

Relative Primes: Two numbers are said to be relative primes or co-primes if they do not have any common factor other than 1. For example, the numbers 15 and 16 do not have any common factors, and, hence, they are relative primes. Please note that none of the two numbers may individually be prime and still they can be relative primes. Unity is a relative prime to all numbers.

Multiples: If one number is divisible exactly by a second number, then the first number is said to be a multiple of the second number. For example, 15 is a multiple of 5; 24 is a multiple of 4.

Factors: If one number divides a second number exactly, then the first number is said to be a factor of the second number. For example, 5 is a factor of 15; 3 is a factor of 18. Factors are also called sub-multiples or divisors.

Even and odd numbers: Numbers divisible by 2 are called even numbers whereas numbers that are not divisible by 2 are called odd numbers.

Examples for even numbers are 2, 4, 6, 8, 10, etc. Examples for odd numbers are 1, 3, 5, 7, 9, etc.

NOTE

Every even number ends in 0, 2, 4, 6, or 8.

The sum of any number of even numbers is always even.

The sum of odd number of odd numbers (i.e. the sum of 3 odd numbers, the sum of 5 odd numbers, etc.) is always odd whereas the sum of even number of odd numbers (i.e., the sum of 2 odd numbers, the sum of 4 odd numbers, etc.) is always even.

The product of any number of odd numbers is always odd.

The product of any number of numbers where there is at least one even number is even.

Perfect Numbers

A number is said to be a perfect number if the sum of ALL its factors excluding itself (but including 1) is equal to the number itself.

For example, 6 is a perfect number because the factors of 6, i.e. 1, 2, and 3 add up to the number 6 itself.

Other examples of perfect numbers are 28, 496, 8128, etc.

Rules for Divisibility

In a number of situations, we will need to find the factors of a given number. Some of the factors of a given number can, in a number of situations, be found very easily either by observation or by applying simple rules. We will look at some rules for divisibility of numbers.

Divisibility by 2: A number divisible by 2 will have an even number as its last digit (128, 246, 2346, etc.).

Divisibility by 3: A number is divisible by 3 if the sum of its digits is a multiple of 3.

For example, take the number 9123, the sum of the digits is 9 + 1 + 2 + 3 = 15, which is a multiple of 3. Hence, the given number 9123 is divisible by 3. Similarly 342, 789, etc. are all divisible by 3. If we take the number 74549, the sum of the digits is 29 which is not a multiple of 3. Hence, the number 74549 is not divisible by 3.

Divisibility by 4: A number is divisible by 4 if the number formed with its last two digits is divisible by 4.

For example, if we take the number 178564, the last two digits form 64. Since this number 64 is divisible by 4, the number 178564 is divisible by 4.

If we take the number 476854, the last two digits form 54 which is not divisible by 4 and hence the number 476854 is not divisible by 4.

Divisibility by 5: A number is divisible by 5 if its last digit is 5 or zero (15, 40, etc.).

Divisibility by 6: A number is divisible by 6 if it is divisible both by 2 and 3 (18, 42, 96, etc.).

Divisibility by 7: If the difference between the number of tens in the number and twice the units digit is divisible by 7, then the given number is divisible by 7. Otherwise, it is not divisible by 7.

Take the units digit of the number, double it and subtract this figure from the remaining part of the number. If the result so obtained is divisible by 7, then the original number is divisible by 7. If that result is not divisible by 7, then the number is not divisible by 7.

For example, let us take the number 595. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 59. If 10 (which is the units digit doubled) is subtracted from 59 we get 49. Since this result 49 is divisible by 7, the original number 595 is also divisible by 7.

Similarly, if we take 967, doubling the units digit gives 14 which when subtracted from 96 gives a result of 82. Since 82 is not divisible by 7, the number 967 is not divisible by 7.

If we take a larger number, the same rule may have to be repeatedly applied till the result comes to a number which we can make out by observation whether it is divisible by 7. For example, take 456745, We will write down the figures in various steps as shown below.

Col(1) Number	Col(2) Twice the units digit	Col(3) Remaining part of the number	Col(3) – Col(2)
456745	10	45674	45664
45664	8	4566	4558
4558	16	455	439
439	18	43	25

Since 25 in the last step is not divisible by 7, the original number 456745 is not divisible by 7.

Divisibility by 8: A number is divisible by 8, if the number formed by the last 3 digits of the number is divisible by 8.

For example, the number 3816 is divisible by 8 because the last three digits form the number 816, which is divisible by 8. Similarly, the numbers 14328, 18864 etc. are divisible by 8. If we take the number 48764, it is not divisible by 8 because the last three digits' number 764 is not divisible by 8.

Divisibility by 9: A number is divisible by 9 if the sum of its digits is a multiple of 9.

For example, if we take the number 6318, the sum of the digits of this number is 6 + 3 + 1 + 8 which is 18. Since this sum 18 is a multiple of 9, the number 6318 is divisible by 9. Similarly, the numbers 729, 981, etc. are divisible by 9. If we take the number 4763, the sum of the digits of this number is 20 which is not divisible by 9. Hence, the number 4763 is not divisible by 9.

Divisibility by 10: A number divisible by 10 should end in zero.

Divisibility by 11: A number is divisible by 11 if the sum of the alternate digits is the same or they differ by multiples of 11—that is, the difference between the sum of digits in odd places in the number and the sum of the digits in the even places in the number should be equal to zero or a multiple of 11.

For example, if we take the number 132, the sum of the digits in odd places is 1 + 2 = 3 and the sum of the digits in even places is 3. Since these two sums are equal, the given number is divisible by 11.

If we take the number 785345, the sum of the digits in odd places is 16 and the sum of the digits in even places is also 16. Since these two sums are equal, the given number is divisible by 11.

Divisibility by numbers like 12, 14, 15 can be checked out by taking factors of the number which are relatively prime and checking the divisibility of the given number by each of the factors. For example, a number is divisible by 12 if it is divisible both by 3 and 4.

Recurring Decimals: A decimal in which a digit or a set of digits is repeated continuously is called a recurring decimal. Recurring decimals are written in a shortened form, the digits which are repeated being marked by dots placed over the first and the last of them, thus

$$\frac{8}{3} = 2.666..... = 2.\dot{6} \text{ or } 2.\overline{6};$$

$$\frac{1}{7} = 0.142857142857142857... = 0.142857$$

In case of 1/7, where the set of digits 142857 is recurring, the dot is placed on top of the first and the last digits of the set or alternatively, a bar is placed over the entire set of the digits that recur.

A recurring decimal like $0.\overline{3}$ is called a pure recurring decimal because all the digits after the decimal point are recurring.

A recurring decimal like $0.1\overline{6}$ (which is equal to 0.16666...) is called a mixed recurring because some of the digits after the decimal are not recurring (in this case, only the digit 6 is recurring and the digit 1 is not recurring).

A recurring decimal is also called a 'circulator'. The digit, or set of digits, which is repeated is called the 'period' of the decimal. In the decimal equivalent to 8/3, the period is 6 and in 1/7 it is 142857.

As already discussed, all recurring decimals are rational numbers as they can be expressed in the form p/q, where p and q are integers. The general rule for converting recurring decimals into fractions will be considered later. Let us first consider a few examples so that we will be able to understand the rule easily.

Solved Examples

Example 1

Express $0\cdot\overline{4}$ in the form of a fraction.

Solution

Let $x = 0\cdot\overline{4} = 0.444 \ldots$ (1)

$$10x = 4.444 \ldots = 4.\overline{4} \quad (2)$$

Subtracting (1) from (2),

$$9x = 4$$

$$\Rightarrow \quad x = \frac{4}{9}.$$

Example 2

Express $0\cdot\overline{63}$ in the form of a fraction.

Solution

Let $x = 0\cdot\overline{63} = 0.636363 \ldots$ (3)

$$100x = 63.636363 \ldots = 63\cdot\overline{63} \quad (4)$$

Subtracting (3) from (4),

$$99x = 63$$

$$\Rightarrow \quad x = \frac{7}{11}.$$

We can now write down the rule for converting a pure recurring decimal into a fraction as follows:

A pure recurring decimal is equivalent to a vulgar fraction which has the number formed by the recurring digits (called the period of the decimal) for its numerator, and for its denominator, the number which has for its digits as many nines as there are digits in the period.

Thus, $0.\overline{37}$ can be written as equal to $\frac{37}{99}$; $0.\overline{225}$ can be written as equal to $\frac{225}{999} = \frac{25}{111}$;

$$0.\overline{63} = \frac{63}{99} = \frac{7}{11}.$$

A mixed recurring decimal becomes the sum of a whole number and a pure recurring decimal, when it is multiplied by suitable power of 10 which will bring the decimal point to the left of the first recurring figure. We can then find the equivalent vulgar fraction by the process as explained in case of a pure recurring decimal.

Now we can write the rule to express a mixed recurring decimal into a (vulgar) fraction as below:

In the numerator, write the entire given number formed by the (recurring and non-recurring parts) and subtract from it the part of the decimal that is not recurring. In the denominator, write as many nines as the period (i.e. as many nines as the number of digits recurring) and then place next to it as many zeroes as there are digits without recurring in the given decimal.

i.e.,
$$0.1\overline{56} = \frac{156-1}{990} = \frac{155}{990} = \frac{31}{198}$$

$$0.7\overline{3} = \frac{73-7}{90} = \frac{66}{90} = \frac{11}{15}$$

Number of Factors of a Number

If N is a composite number such that $N = a^p \,.\, b^q \,.\, c^r \ldots$ where a, b, c are prime factors of N and p, q, r ... are positive integers, then the number of factors of N is given by the expression

$$(p + 1)\,(q + 1)\,(r + 1) \ldots$$

For example $140 = 2^2 \times 5^1 \times 7^1$.

Hence, 140 has (2 + 1)(1 + 1)(1 + 1), i.e. 12 factors.

Please note that the figure arrived at by using the above formula includes 1 and the given number N also as factors. So if you want to find the number of factors the given number has excluding 1 and the number itself, we find out $(p + 1)\,(q + 1)\,(r + 1)$ and then subtract 2 from that figure.

In the above example, the number 140 has 10 factors excluding 1 and itself.

Number of Ways of Expressing a Given Number as a Product of Two Factors

The given number N (which can be written as equal to $a^p.\, b^q.\, c^r \ldots$ where a, b, c are prime factors of N and p, q, r … are positive integers) can be expressed as the product of two factors in different ways.

The number of ways in which this can be done is given by the expression $1/2\ \{(p + 1)(q + 1)(r + 1)\ldots\}$

So, 140 can be expressed as a product of two factors in 12/2 or 6 ways {because $(p + 1)\,(q + 1)\,(r + 1)$ in the case of 140 is equal to 12}

If p, q, r, etc. are all even, then the product $(p + 1)\,(q + 1)\,(r + 1) \ldots$ becomes odd and the above rule will not be valid since we cannot take 1/2 of an odd number to get the number of ways. If p, q, r, … are all even, it means that the number N is a perfect square. This situation arises in the specific cases of perfect squares because a perfect square can also be written as {square root × square root}. So, two different cases arise in case of perfect squares depending on whether we would like to consider writing the number as {square root x square root} also as one of the ways.

Thus, to find out the number of ways in which a perfect square can be expressed as a product of 2 factors, we have the following two rules.

1. as a product of two DIFFERENT factors: $\frac{1}{2}\{(p+1)(q+1)(r+1)\ldots-1\}$ ways (excluding $\sqrt{N}\times\sqrt{N}$).
2. as a product of two factors (including $\sqrt{N}\times\sqrt{N}$) in $\frac{1}{2}\{(p+1)(q+1)(r+1)\ldots+1\}$ways.

Example 3

Find the number of factors of 3025.

Solution

$3025 = (5)(605) = (5)(5)(121) = 5^2\,11^2$
Number of factors of 3025

$$= (2+1)(2+1) = 9$$

Example 4

In how many ways can 22500 be written as a product of two different factors?

Solution

$22500 = 150^2 = ((2)(5)(3)(5))^2 = 2^2 5^4 3^2$
Number of ways

$$= \frac{1}{2}\{(2+1)(4+1)(2+1)-1\} = 22.$$

Sum of all the Factors of a Number

If a number $N = a^p \,.\, b^q \,.\, c^r \ldots$ where $a, b, c\ldots$ are prime numbers and $p, q, r\ldots$ are positive integers, then, the sum of all the factors of N (including 1 and the number itself) is:

$$\left(\frac{a^{p+1}-1}{a-1}\right)\cdot\left(\frac{b^{q+1}-1}{b-1}\right)\cdot\left(\frac{c^{r+1}-1}{c-1}\right)\ldots$$

The above can be verified by an example.

Consider the number 48, when resolved into prime factors, $48 = 2^4 \times 3^1$. Here $a = 2$, $b = 3$, $p = 4$, $q = 1$.

Hence, sum of all the factors

$$= \left(\frac{2^{4+1}-1}{2-1}\right)\left(\frac{3^{1+1}-1}{3-1}\right) = \frac{31}{1}\times\frac{8}{2} = 124$$

The list of factors of 48 is:

1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

If these factors are added, the sum is 124 and tallies with the above result.

Product of all the Factors of a Number

We shall now consider another kind of question which has also appeared frequently in exams. These questions refer to the 'structure' of numbers, i.e. the prime factors of a number and the canonical representation of a number. We shall begin by working out the product of all the factors of a given number.

Example 5

What is the product of all the factors of 180?

Solution

$180 = 4(45) = 2^2 3^2 5^1$. There are $(2+1)(2+1)(1+1)$ or 18 factors.

If the given number is not a perfect square, at least one of the indices is odd and the number of factors is even. We can form pairs such that the product of the two numbers in each pair is the given number (180 in this example).

$\therefore$ The required product is 180^9.

In general, if $N = p^a\, q^b\, r^c$ (where at least one of a, b, c is odd), the product of all the factors of N is $N^{\frac{d}{2}}$, where d is the number of factors of N and is given by $(a+1)(b+1)(c+1)$.

Example 6

Let us see what happens when N is a perfect square, say 36.

We want the product of all the factors of 36.

Solution

$36 = 2^2 3^2$ (there are 9 factors)

$$1(36) = 2(18) = 3(12) = 4(9) = 6(6)$$

$\therefore$ The product of all the factors is 36^4 (6).

In general, let $N = p^a\, q^b\, r^c$ where each of a, b, c is even.

There are $(a+1)(b+1)(c+1)$ say d factors. We can form $\frac{d-1}{2}$ pairs and we would be left with one lone factor, i.e. $\sqrt{N}$. The product of all these factors is $N^{\frac{d-1}{2}}(\sqrt{N}) = N^{\frac{d}{2}}$

$\therefore$ Whether or not N is a perfect square, the product of all its factors is $N^{\frac{d}{2}}$, where d is the number of factors of N.

Number of Ways of Writing a Number as Product of Two Co-primes

Using the same notation and convention used earlier.

If $N = a^p \,.\, b^q \,.\, c^r\ldots$, then, the number of ways of writing N as a product of 2 co-primes is 2^{n-1}, where 'n' is the number of distinct prime factors of the given number N.

Taking the example of 48, which is $2^4 \times 3^1$, the value of 'n' is 2 because only two distinct prime factors (i.e. 2 and 3 only) are involved.

Hence, the number of ways $= 2^{2-1} = 2^1 = 2$ i.e., 48 can be written as product of 2 coprimes, in two different ways. They are (1 and 48) and (3, 16).

Number of Co-primes to *N*, That are Less than *N*

If N is a number that can be written as $a^p \,.\, b^q \,.\, c^r\ldots$, then, the number of co-primes of N, which are less than N, represented by $\phi(N)$ is,

$$N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\ldots$$

For example if, 48 is considered,

$$N = a^p \,.\, b^q \,.\, c^r \ldots$$

i.e., $48 = 2^4 \,.\, 3^1$.

Hence, $a = 2$, $b = 3$, $p = 4$, $q = 1$.

$$\phi(48) = 48\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)$$

$$= 48\times\frac{1}{2}\times\frac{2}{3} = 16.$$

NOTE

If numbers less than 48 are listed, and co-primes to 48 are picked up, the count of co-primes will be 16.

Sum of Co-primes to *N* That Are Less Than *N*

The sum of the co-primes of N, that are less than N is $\frac{N}{2}\,.\,\phi(N)$. If we consider the above example, already we have $\phi(48) = 16$.

Hence, sum of co-primes of 48 that are less than $48 = \frac{N}{2}\,.\,\phi(N) = \frac{48}{2}\times 16 = 384$.

NOTE

After picking out the co-primes of 48 that are less than 48, they can be added and the sum can be verified.

Least Common Multiple (LCM) and Highest Common Factor (HCF)

Least common multiple (LCM) of two or more numbers is the least number that is divisible by each of these numbers (i.e. leaves no remainder or remainder is zero). The same can be algebraically defined as 'LCM of two or more expressions is the expression of the lowest dimension which is divisible by each of them, i.e. leaves no remainder or remainder is zero'.

Highest common factor (HCF) is the largest factor of two or more given numbers. The same can be defined algebraically as 'HCF of two or more algebraical expressions is the expression of highest dimension which divides each of them without remainder'.

HCF is also called GCD (greatest common divisor).

$$\boxed{\textit{Product of two numbers} = LCM \times HCF}$$

$$\boxed{\textit{LCM is a multiple of HCF}}$$

For finding **LCM and HCF of fractions,** first reduce each fraction to its simplest form, i.e. cancel out any common factors between the denominator and numerator and then apply appropriate formula from the following:

$$\textit{HCF of fractions} = \frac{\textit{HCF of numerators}}{\textit{LCM of denominators}}$$

$$\textit{LCM of fractions} = \frac{\textit{LCM of numerators}}{\textit{HCF of denominators}}$$

LCM and HCF Models

LCM—Model 1: In this model of problem, you will need to find out the smallest number (or number in a specified range like the largest five-digit number) which when divided by 2 or more other numbers (i.e. divisors) leaves the same remainder in all cases.

The basic distinguishing feature of this model of problems is that the remainder will be the **same** in all the cases (and that remainder will also be given).

The smallest such number will be the remainder itself. The next higher number that satisfies the given conditions is the LCM of the given numbers (i.e. divisors) plus the remainder given, i.e. add the remainder (which is the same in all cases) to the LCM of the given numbers (i.e. divisors).

To find any larger number that satisfies a given condition, we will first need to find out a multiple of the LCM in that range and add the remainder to this multiple of the LCM.

The general rule can be written as follows:

Any number which when divided by p, q, or r leaving the same remainder s in each case will be of the form ***k (LCM of p, q, and r)*** $+$ ***s*** *where* $k = 0, 1, 2, \ldots$

If we take $k = 0$*, then we get the smallest such number.*

Example 7

Find the HCF of 1363 and 1457.

Solution

```
         1
1363)1457
     1363          14
       94)1362
          1316    2
            47)94
               94
               --
                0
               --
```

$\therefore$ HCF (1367, 1457) = 47

Example 8

Find the smallest number which when divided by 5 or 11 leaves a remainder of 4 and is greater than the remainder.

Solution

Set of such numbers are of the form K [LCM (5, 11)] + 4 where K is a whole number.

We get the required number when $K = 1$

∴ Smallest number = LCM (5, 11) + 4

$$= 55 + 4 = 59.$$

LCM—Model 2: In this model, the remainders in the divisions given will not be the same but the difference between the divisor and the remainder (i.e. the complement of the remainder) will be the same in each case. For example, you may be asked to find out 'the smallest number which when divided by 4 or 6 gives respective remainders of 3 and 5'. Here, the remainders are not the same as in LCM —Model 1; but the difference between the divisor and the remainder is same in each case. In the first case, the difference between the divisor and the remainder is 1(= 4 – 3). In the second case, also the difference between the divisor and the remainder is 1(= 6 – 5).

The smallest such number is LCM minus constant difference (the constant difference being the difference between the divisor and the corresponding remainder in all cases).

Similarly, any multiple of the LCM minus the constant remainder also will satisfy the same condition.

In the aforementioned example, the LCM of 4 and 6 is 12, and hence the required number is 11 (which is equal to 12 – 1).

The general rule can be written as follows:

Any number which when divided by p, q, or r leaving respective remainders of s, t, and u where (p – s) = (q – t) = (r – u) = v (say) will be of the form ***k (LCM of p, q, and r)*** *– v*

The smallest such number will be obtained by substituting k = 1.

Example 9

Find the smallest number which when divided by 9 and 11 leaves remainders of 7 and 9, respectively.

Solution

Required number = LCM (9, 11) – 2 = 97.

Example 10

Find the largest four-digit number which when divided by 9 and 11 leaves remainders of 7 and 9, respectively.

Solution

Required number must be in the form LCM (9, 11) $k - 2$, i.e. $99k - 2$, where k is the largest natural number satisfying $99k - 2 < 10000$.

$$\therefore \quad k < 101\frac{1}{33}$$

$$\therefore \quad k = 101$$

∴ Largest number = 9997.

LCM—Model 3: In this model, the remainders will not be the same and even the differences between each of the given divisors and the corresponding remainders also will not remain the same.

Let us take an example and see how to solve this type of problem.

Example 11

Find the smallest number which leaves a remainder of 7 when divided by 11 and leaves a remainder of 12 when divided by 13.

Solution

Let the number be in the forms $11k_1 + 7$ and $13k_2 + 12$ where k_1 and k_2 have the least possible values.

$$11k_1 + 7 = 13k_2 + 12$$

$$k_1 = k_2 + \frac{2k_2 + 5}{11}$$

As k_1 is an integer, $2k_2 + 5$ must be divisible by 11.

Hence $k_2 = 3$.

∴ Smallest number = 51.

HCF—Model 1: In this model, we have to identify the largest number that exactly divides the given dividends (which are obtained by subtracting the respective remainders from the given numbers).

The largest number with which the numbers p, q, or r are divided giving remainders of s, t, and u, respectively, will be the ***HCF of the three numbers (p – s), (q – t), and (r – u).***

Let us understand this model with an example.

Example 11

Find the largest number which leaves remainders of 2 and 3 when it divides 89 and 148, respectively.

Solution

Largest number = HCF (89 – 2, 148 – 3) = 29

HCF—Model 2: In this model, the problem will be as follows:

'Find the largest number with which if we divide the numbers p, q and r, the remainders are the same'.

Take the difference between any two pairs out of the three given numbers. Let us say we take the two differences $(p - q)$ and $(p - r)$. The HCF of these numbers will be the required number.

Here, the required number = HCF of $(p \sim q)$ and $(p \sim r)$ = HCF of $(p \sim q)$ and $(q \sim r)$ = HCF of $(q \sim r)$ and $(p \sim r)$

Let us take an example and look at this model.

Example 12

Find the largest number which divides 444, 804, and 1344 leaving the same remainder in each case.

Solution

Largest number

$$= \text{HCF } (804 - 444, 1344 - 804)$$

$$= \text{HCF } (360, 540) = 180.$$

Successive Division

If the quotient of a division is taken and this is used as the dividend in the next division, such a division is called 'successive division'. A successive division process can continue upto any number of steps—until the quotient in a division becomes zero for the first time, i.e. the quotient in the first division is taken as dividend and divided in the second division; the quotient in the second division is taken as the dividend in the third division; the quotient in the third division is taken as the dividend in the fourth division and so on.

If we say that 2479 is divided successively by 3, 5, 7, and 2, then the quotients and remainders are as follows in the successive division.

Dividend	Divisor	Quotient	Remainder
2479	3	826	1
826	5	165	1
165	7	23	4
23	2	11	1

Here we say that when 2479 is successively divided by 3, 5, 7, and 2 the respective remainders are 1, 1, 4 and 2.

Example 13

A number when divided by 6 and 4 successively leaves remainders of 5 and 2, respectively. Find the remainder when the largest such two digit number is divided by 9.

Solution

Let the quotients obtained when the number is divided by 6 and 4 successively be q_1 and q_2, respectively.

$$\text{Number} = 6q_1 + 5$$

Its successive division, the quotient obtained for each division starting from the first, forms the dividend for the next division.

$\therefore \quad q_1 = 4q_2 + 2$

$\therefore \quad \text{number} = 6\,(4q_2 + 2) + 5 = 24q_2 + 17$

Largest two-digit number satisfying the given conditions is obtained when $24q_2 + 17 < 100$ and q_2 is maximum, i.e. $q_2 < 3\frac{11}{24}$ and it is maximum, i.e. $q_2 = 3$.

$\therefore \quad$ number = 89. required remainder = 8

Alternative method:

Divisors: 6 × 4

Remainders: 5 2

The smallest number satisfying the given conditions is found using the following method. Each divisor and the remainder it leaves are written as shown above. Starting with the last remainder, each remainder is multiplied with the previous divisor and added to that divisor's remainder. This procedure is carried out until the divisor's remainder is the first remainder.

Smallest possible value of the number

$$= (6)\,(2) + 5 = 17$$

General form of the number $= k(6 \times 4) + 17 = 24k + 17$ where k is any whole number.

The number would be the largest two-digit number when $24\,k + 17 < 100$ and k is maximum, i.e. $k < 3\frac{11}{24}$ and k is maximum, i.e. $k = 3$.

$\therefore$ Largest two-digit number = 89

$\therefore$ Required remainder = 8

Example 14

A number when divided by 3, 5, and 6 successively leaves remainders of 1, 3, and 2, respectively. Find the number of possible values it can assume which are less than 1000.

Solution

Let the quotients obtained when the number is divided by 3, 5, and 6 successively be q_1, q_2, and q_3, respectively.

$$\text{Number} = 3q_1 + 1$$

$$q_1 = 5q_2 + 3$$

$$q_3 = 6q_1 + 2$$

$\therefore$ number $= 3\,(5q_2 + 3) = 1$

$$= 3\,(5(6q_1 + 2) + 3) + 1 = 90\,q_1 + 40$$

$$90\,q_1 + 40 < 1000$$

$$q_1 < 10\frac{2}{3}$$

$\therefore$ q_1 has 11 possibilities, i.e. 0 to 10.

Alternative method:

Divisors: 3 × 5 × 6

Remainders: 1 3 2

Smallest possible value of the number

$$= ([(5 \times 2) + 3] \times 3) + 1 = 40$$

General form of the number $= k \times (3 \times 5 \times 6) + 40 = 90k + 40$, where k is any whole number.

If $\quad 90k + 40 < 1000,\ k < 10\frac{2}{3}$

$\therefore$ k has 11 possibilities (i.e. 0 to 10).

FACTORIAL

Factorial is defined for any positive integer. It is denoted by $\angle$ or !. Thus, 'Factorial n' is written as $n!$ or $\angle n$. $n!$ is defined as the product of all the integers from 1 to n.

Thus $n! = 1, 2, 3, \ldots (n-1)\,n$.

0! is defined to be equal to 1.

0! = 1 and 1! is also equal to 1.

Largest Power of a Number in *N*!

There is a specific model of problems relating to factorial which appeared about 3 to 4 times in CAT papers. This involves finding the largest power of a number contained in the factorial of a given number. Let us understand this type of problem with the help of an example.

Example 15

Find the number of zeros that 324! ends with.

Solution

The largest power of A in B! can be found using the method below when A is composite.

The largest power of each prime factor of A in B! is found. The minimum of these results is the required power.

In the given problem, $10 = (2)(5)$. The required power is the minimum of the largest power of 2 in 324! and the largest power of 5 in 324!. Using the approach shown in the previous example, the largest power of 2 in 324! is 32!. From the previous example, the largest power of 5 in 324! is 78.

$\therefore$ Required power = min (321, 781) = 78.

Alternative method:

Largest power of 10 = Largest power of (2) (5)

As 5 > 2, the largest power of 5 which can divide 324! < the largest power of 2 which can divide 324!

$\therefore$ Largest power of (2) (5) which can divide 324! = largest power of 5 which can divide 324! is 10^{78}.

$\therefore$ 324! ends with 78 zeros.

Some Important Points to Note

Please note the following points also which will be very useful in solving problems on numbers.

1. When any two consecutive integers are taken, one of them is odd and the other is even. Hence, the product of any two consecutive integers is always even, i.e. divisible by 2.
   Two consecutive integers can be written in the form of n and $n - 1$ or n and $n + 1$. Hence, any number of the form $n(n-1)$ or $n(n+1)$ will always be even.
2. Out of any 3 consecutive integers, one of them is divisible by 3 and at least one of the three is definitely even. Hence, the product of any 3 consecutive integers is always divisible by 6.
   Three consecutive integers can be of the form $(n-1)$, n, and $(n+1)$. The product of 3 consecutive integers will be of the form $(n-1)n(n+1)$ or $n(n^2-1)$ or $(n^3 - \text{n})$. Hence, any number of the form $(n-1)n(n+1)$ or $n(n^2 - 1)$ or $(n^3 - n)$ will always be divisible by 6.
3. Out of any n consecutive integers, exactly one number will be divided by n and the product of n consecutive integers will be divisible by n!
4. Any prime number greater than 3 can be written in the form of $6k + 1$ or $6k - 1$. The explanation is:
   Let p be any prime number greater than 3. Consider the three consecutive integers $(p-1)$, p, and $(p+1)$.
   Since p is a prime number greater than 3, p CANNOT be even. Since p is odd, both $(p-1)$ and $(p+1)$ will be even, i.e. both are divisible by 2.
   Also, since, out of any three consecutive integers, one number will be divisible by 3, one of the three numbers $(p-1)$, p, or $(p+1)$ will be divisible by 3. But, since p is prime number—that too greater than 3—p cannot be divisible by 3. Hence, either $(p-1)$ or $(p+1)$, one of them—and only one of them—is definitely divisible by 3.
   If $(p-1)$ is divisible by 3, since it is also divisible by 2, it will be divisible by 6, i.e. it will be of the form $6k$. If $(p-1)$ is of the form $6k$, then p will be of the form $(6k+1)$.
   If $(p+1)$ is divisible by 3, since it is also divisible by 2, it will be divisible by 6, i.e. it will be of the form $6k$. If $(p+1)$ is of the form $6k$, then p will be of the form $(6k-1)$.
   Hence any prime number greater than 3 will be of the form $(6k+1)$ or $(6k-1)$.

Example 16

Find the HCF of $\frac{3}{5}, \frac{6}{10}$, and $\frac{9}{20}$.

Solution

To find the LCM or HCF of fractions, first empress all the fractions in their simplest term.

$$\text{HCF (fractions)} = \frac{\text{HCF(numerators)}}{\text{LCM}\left(\text{denominators}\right)}$$

$$= \frac{\text{HCF}(3,3,9)}{\text{LCM}(5,5,20)} = \frac{3}{20}$$

Example 17

Find the LCM of $\frac{3}{5}, \frac{6}{10}$, and $\frac{9}{20}$.

Solution

To find the LCM or HCF of fractions, first empress all the fractions in their simplest term.

LCM (fractions)

$$= \frac{\text{LCM (numerators)}}{\text{HCF}(\text{denominators})} = \frac{\text{LCM}(3,3,9)}{\text{HCF}(5,5,20)} = \frac{9}{5}$$

Number Systems

The numbers that are commonly used are the decimal numbers, which involve ten symbols, namely 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. If we consider the number 526 in the decimal system, it means $5 \times 10^2 + 2 \times 10^1 + 6 \times 10^0$. Likewise, 85.67 means $8 \times 10^1 + 5 \times 10^0 + 6 \times 10^{-1} + 7 \times 10^{-2}$. The role played by '10' in the decimal system is termed as the 'base' of the system. In this chapter, we see the numbers expressed in various other bases.

Base: It is a number which decides the place value of a symbol or a digit in a number. Alternatively, it is the number of distinct symbols that are used in that number system.

NOTES

1. The base of a number system can be any integer greater than 1.
2. Base is also termed as radix or scale of notation.

The following table lists some number systems along with their respective base and symbols.

Number System	Base	Symbols
Binary	2	0, 1
Septenary	7	0, 1, 2, 3, 4, 5, 6
Octal	8	0, 1, 2, 3, 4, 5, 6, 7
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Duo-decimal	12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B
Hexa-decimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

$A = 10, B = 11, C = 12, D = 13, E = 14, F = 15$. Some books denote ten as 'E' and eleven as 'e'.

Representation: Let N be any integer, r be the base of the system, and $a_0, a_1, a_2, \ldots, a_n$ be the required digits by which N is expressed. Then, $N = a_n r^n + a_{n-1} r^{n-1} + a_{n-2} r^{n-2} + \ldots + a_1 r + a_0$, where $0 \le a_i < r$.

We now look into some representations and their meaning in decimal system.

Examples:

1. $(100011)_2$
$= 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
$= 32 + 0 + 0 + 2 + 1 = (35)_{10}$

2. $(1741)_8$
$= 1 \times 8^3 + 7 \times 8^2 + 4 \times 8^1 + 1 \times 8^0$
$= 512 + 448 + 32 + 1 = 993_{10}$

3. $(A3D)_{16}$
$= A \times 16^2 + 3 \times 16^1 + D \times 16^0$
$= 10 \times 256 + 48 + 13 = 2621_{10}$

Conversions

1. **Decimal to binary:**

(a) $(253)_{10} = (11111101)_2$

Working:

$$\begin{array}{r|l}
2 & 253 \\ \hline
2 & 126 - 1 \\ \hline
2 & 63 - 0 \\ \hline
2 & 31 - 1 \\ \hline
2 & 15 - 1 \\ \hline
2 & 7 - 1 \\ \hline
2 & 3 - 1 \\ \hline
 & 1 - 1 \\ \hline
\end{array}$$

NOTE

The remainders are written from bottom to top.

(b) $(37.3125)_{10} = (100101.0101)_2$

Working: The given decimal number has 2 parts:

(i) Integral part 37,
(ii) Fractional part 0.3125.

(i) Conversion of integral part:

$$\begin{array}{r|ll}
2 & 37 & \\ \hline
2 & 18 & -1 \\ \hline
2 & 9 & -0 \\ \hline
2 & 4 & -1 \\ \hline
2 & 2 & -0 \\ \hline
1 & & -0 \\ \hline
\end{array}$$

$\therefore (36)_{10} = (100100)_2$

(ii) Conversion of the fractional part:

Multiply the decimal part with 2 successively and take the integral part of all the products starting from the first.

	Binary digits
$0.3125 \times 2 = 0.6250$	0
$0.6250 \times 2 = 1.2500$	1
$0.2500 \times 2 = 0.500$	0
$0.5000 \times 2 = 1.0$	1

$\therefore (0.3125)_{10} = (0.0101)_2$

NOTE

We should stop multiplying the fractional part by 2, once we get 0 as a fraction or the fractional part is non-terminating. It can be decided depending on the number of digits in the fractional part required.

2. **Binary to decimal:**
   (i) $(101011011)_2 = (347)_{10}$

   Working: $(101011001)_2$

   $= 1 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

   $= 256 + 0 + 64 + 0 + 16 + 8 + 0 + 2 + 1$

   $= (347)_{10}$

   (ii) $(0.11001)_2 = (0.78125)_{10}$

   Working: $(0.11001)_2$

   $= 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5}$

   $= 1/2 + 1/4 + 1/32 = 25/32 = (0.78125)_{10}$

3. **Decimal to octal:**
   (i) $(2595)_{10} = (5043)_8$

   Working:

$$\begin{array}{r|ll} 8 & 2595 & \\ \hline 8 & 324 & -3 \\ \hline 8 & 40 & -4 \\ \hline & 5 & -0 \end{array}$$

   $\therefore (2595)_{10} = (5043)_8$

4. **Octal to decimal:**
   (i) $(4721)_8 = (2513)_{10}$

   Working: $(4721)_8$

   $= 4 \times 8^3 + 7 \times 8^2 + 2 \times 8^1 + 1 \times 8^0$

   $= 2048 + 448 + 16 + 1 = (2513)_{10}$

   (ii) $(365.74)_8 = (245.9375)_{10}$

   Working:

   (a) Integral part:

   $(365)_8 = 3 \times 8^2 + 6 \times 8^1 + 5 \times 8^0$

   $= 192 + 48 + 5 = 245$

   $\therefore (365)_8 = (245)_{10}$

   (b) Fractional part:

   $(0.74)_8 = 7 \times 8^{-1} + 4 \times 8^{-2}$

   $= \dfrac{56+4}{64} = \dfrac{60}{64} = 0.9375$

   $\therefore (365.74)_8 = (245.9375)_{10}$

5. **Decimal to hexa-decimal:**
   (i) $(47239)_{10} = (B887)_{16}$

   Working:

$$\begin{array}{r|ll} 16 & 47239 & \\ \hline 16 & 2952 & -7 \\ \hline 16 & 184 & -8 \\ \hline & 11 & -8 \end{array}$$

   Recall: 11 is B, in hexa-decimal system.

   $\therefore (47239)_{10} = (B887)_{16}$

   (ii) $(30014)_{10} = (753E)_{16}$

   Working:

$$\begin{array}{r|ll} 16 & 30014 & \\ \hline 16 & 1875 & -14 = E \\ \hline 16 & 117 & -3 \\ \hline & 7 & -5 \end{array}$$

   $\therefore (30014)_{10} = (753E)_{16}$

6. **Hexa-decimal to decimal:**

   $(52B)_{16} = (1323)_{10}$

   Working: $(52B)_{16}$

   $= 5 \times 16^2 + 2 \times 16^1 + B \times 16^0$

   $= 1280 + 32 + 11 = (1323)_{10}$

   $\therefore (52B)_{16} = (1323)_{10}$

7. **Decimal to duo-decimal or duodenary (base 12):**

   $(948)_{10} = (66C)_{12}$

   Working:

$$\begin{array}{r|ll} 12 & 948 & \\ \hline 12 & 78 & -12 \text{ or } C \\ \hline & 6 & -6 \end{array}$$

   $\therefore (948)_{10} = (66C)_{12}$

8. **Duo-decimal to decimal:**

   $(5BC)_{12} = (864)_{10}$

   Working: $(5BC)_{12}$

   $= 5 \times 12^2 + B \times 12^1 + C \times 12^0$

   $= 720 + 132 + 12 = (864)_{10}$

9. **Binary to octal:**

   8 being the base of octal system and 2 being the base of binary system, there is a close relationship between both the systems. One can just club three digits of a binary number into a single block and write the decimal equivalent of each group (left to right).

   Example:

   (i) $(100101111)_2 = (100)_2\ (101)_2\ (111)_2$

   $= (457)_8$

   $\therefore (100101111)_2 = (457)_8$

   (ii) $(11111110)_2 = (011)_2\ (111)_2\ (110)_2$

   $= (376)_8$

   $\therefore (11111110)_2 = (376)_8$

NOTE

Introduce leading zeros to form a block of 3 without changing the magnitude of the number.

10. **Binary to hexa-decimal:**

    This is similar to the method disscused for octal; instead of clubbing 3, we club 4 digits.

Example:

$(10111110)_2 = (1011)_2\ (1110)_2 = (11)\ (14)$

$= (BE)_{16}$

$\therefore\ (10111110)_2 = (BE)_{16}$

NOTE

If the number of digits is not a multiple of 4, introduce leading zeros as done earlier for octal conversion.

Binary Arithmetic

Addition: Elementary Rules

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 10$ (1 will be regarded as carry

$1 + 1 + 1 = 11$ as we do in decimal system)

Examples of Binary Addition

1. $(110101)_2 + (110)_2$

$$\begin{array}{rl} 1\ \ \ \ & \rightarrow \text{ carry} \\ 1\,1\,0\,1\,0\,1 & \\ 0\,0\,0\,1\,1\,0 & \text{(Introduce leading zeros)} \\ \hline 1\,1\,1\,0\,1\,1 & \\ \hline \end{array}$$

2. $(101111)_2 + (111011)_2$

$$\begin{array}{rl} 1\,1\,1\,1\,1\ & \rightarrow \text{ carry} \\ 1\,0\,1\,1\,1\,1 & \\ 1\,1\,1\,0\,1\,1 & \\ \hline 1\,1\,0\,1\,0\,1\,0 & \\ \hline \end{array}$$

3. $(110)_2 + (100)_2 + (010)_2$

$$\begin{array}{rl} 1\ \ \ \ \ \ & \rightarrow \text{ carry} \\ 1\,1\,0 & \\ 1\,0\,0 & \\ 0\,1\,0 & \\ \hline 11\,0\,0 & \\ \hline \end{array}$$

Subtraction: Subtract 1101 from 11010.

1.

$$\begin{array}{rr} & 2\ \ \ \ \ \ \ \\ & 0\,0\,2\,0\,2 \\ \hline & 1\,1\,0\,1\,0 \\ & -\,1\,1\,0\,1 \\ \hline \text{result} \rightarrow & 1\,1\,0\,1 \\ \hline \end{array}$$

Explanation: Say $N = 11010$,

As 1 cannot be subtracted from 0, we borrow 2 from the next place. This gives $2 - 1 = 1$, as the right most digit of the result. The penultimate digit of N would become 0. A similar calculation gives the 3^{rd} digit of the result from the right as 1 and the 4^{th} digit of N from the right becomes 0.

We now borrow a 2 from the 5^{th} digit of N, this makes the 4^{th} digit of N as 2, thereby resulting in $2 - 1 = 1$ as the 4^{th} digit of the result.

2. Subtract 11011 from 111001

$$\begin{array}{rl} 2\,2\,1\ \ \ & \\ 0\,0\,2\,2 & \rightarrow \text{ Borrow} \\ \hline 1\,1\,1\,0\,0\,1 & \\ -1\,1\,0\,1\,1 & \\ \hline 1\,1\,1\,1\,0 & \end{array}$$

Example 18

If $(624)_7 = 312_k$ then find K.

Solution

$(624)_7 = (6)\ (7^2) + (2)\ (7) + (4)\ (7)^0 = 312.$

$$(312)_k = 3k^2 + k + 2$$

Given $(312)_k = (624)_7$

$$3k^2 + k + 2 = 312$$

$$3k^2 + k - 310 = 0$$

$$(k - 10)\ (3k + 31) = 0$$

$$k > 0$$

$$\therefore \quad k = 10.$$

Example 19

Find the hexadecimal equivalent of the number $(234567)_8$.

Solution

$(234567)_8$

$$= (10\ \ 011\ \ 100\ \ 101\ \ 110\ \ 111)_2$$

$$= (0001\ \ 0011\ \ 1001\ \ 0111\ \ 0111)_2$$

$$= (1\ \ 3\ \ 9\ \ 7\ \ 7)_{16}$$

$$= (13977)_{16}.$$

Example 20

A non-zero number in base 8 is such that twice the number is the number formed by reversing its digits. Find it.

Solution

Let the number be $(xy)_8$,
where $0 \le x, y < 8$.

The number formed by reversing its digits is $(yx)_8$.

$$2(xy)_8 = (yx)_8$$

$$2(8x + y) = 8y + x$$

$$\frac{x}{y} = \frac{2}{5}$$

$x = 2$ and $y = 5$ is the only possibility.

$\therefore \quad (xy)_8 = (25)_8.$

Exercises

Direction for questions 1 to 50: Select the correct alternative from the given choices.

1. If x and y are irrational numbers, then $x + y - xy$ is ________.
   (A) a real number (B) a complex number
   (C) a rational number (D) an irrational number

2. Which of the following is a prime number?
   (A) 851 (B) 589
   (C) 429 (D) 307

3. Which of the following pairs of numbers are not twin primes?
   (A) 131 and 133
   (B) 191 and 193
   (C) 157 and 159
   (D) More than one of above

4. Which of the following is divisible by 11?
   (A) 8787878
   (B) 7777777
   (C) 1234567
   (D) More than one of the above

5. What is the least natural number that should be added to 52341693 so that the sum is a multiple of 8?
   (A) 3 (B) 9 (C) 5 (D) 7

6. The product of 7 consecutive natural numbers is always divisible by
   (A) 5040 (B) 10080
   (C) 3430 (D) 6860

7. How many odd natural numbers have the same parity as their factorials?
   (A) 1 (B) 2 (C) 0 (D) 3

8. N is a perfect number. What is the ratio of the sum of the factors of N and N?
   (A) 1 (B) 2 (C) 3 (D) 4

9. $0.2\overline{55} =$
   (A) $\frac{23}{90}$ (B) $\frac{23}{99}$
   (C) $\frac{253}{990}$ (D) $\frac{253}{900}$

10. $0.3\overline{21} =$
    (A) $\frac{53}{165}$ (B) $\frac{106}{333}$
    (C) $\frac{10}{11}$ (D) None of these

11. $0.32\dot{1} =$
    (A) $\frac{289}{900}$ (B) $\frac{289}{990}$
    (C) $\frac{32}{99}$ (D) $\frac{16}{45}$

12. The least natural number that must be added to 599 so that the sum is a perfect cube is
    (A) 120 (B) 125 (C) 130 (D) 135

13. There are 15 consecutive odd numbers. The sum of the first ten of those odd numbers is 200. What is the sum of the last five odd numbers?
    (A) 125 (B) 175 (C) 150 (D) 200

14. Find the number of prime factors of 19019.
    (A) 1 (B) 2 (C) 3 (D) 4

15. If $N = 2^a \times 3^b \times 5^c$, how many numbers (in terms of N) are less than N and are co-prime to it?
    (A) $\frac{2}{15}N$ (B) $\frac{4}{15}N$
    (C) $\frac{8}{15}N$ (D) $\frac{2}{5}N$

16. Which of the following numbers is divisible by 40 and 72?
    (A) 7560 (B) 3840 (C) 5670 (D) 3780

17. What is the least whole number that should be added to 723111 to make the resultant is a multiple of 11?
    (A) 4 (B) 8 (C) 7 (D) 3

18. (a) Prime factorize: 9000
    (A) $2^2 \times 3^2 \times 5^2$ (B) $2^4 \times 3 \times 5^2$
    (C) $2^3 \times 3^2 \times 5^3$ (D) $2^3 \times 3 \times 5^4$

    (b) Prime factorize: 1936
    (A) $2^2 \times 3 \times 11^3$ (B) $2^3 \times 11^3$
    (C) $2^4 \times 11^2$ (D) $2^2 \times 3^2 \times 11^2$

    (c) Write 3969 as a product of prime factors.
    (A) $3^5 \times 7$ (B) $3^3 \times 7^3$
    (C) $3^4 \times 7^2$ (D) $3^2 \times 7^4$

    (d) Write 14553 as a product of prime numbers
    (A) $3 \times 7^3 \times 11$ (B) $3^2 \times 7 \times 11^3$
    (C) $3^3 \times 7^2 \times 11$ (D) $3 \times 7^2 \times 11^2$

19. Simplify the following:
    (a) $248 \times 555 + 148 \times 445$
    (A) 203500 (B) 302500
    (C) 205300 (D) 305200

    (b) $4\frac{1}{2} + 3\frac{1}{5} - 2\frac{1}{10} - 4\frac{1}{20}$
    (A) $1\frac{1}{10}$ (B) $1\frac{11}{20}$
    (C) $1\frac{1}{5}$ (D) $1\frac{11}{40}$

    (c) $\dfrac{(3.37)^3 + 10.11(6.63)^2 + 19.89(3.37)^2 + (6.63)^3}{(3.37)^2 + 2 \times (6.63)(3.37) + (6.63)^2}$
    (A) 3.26 (B) 6.74 (C) 10 (D) 8

20. Find the square root of 17689
    (A) 143 (B) 137 (C) 133 (D) 147

21. The number of positive integers which are co-prime to 349247 is _______.
(A) 4 (B) 5
(C) 3 (D) infinite

22. The sum of the first N natural numbers is equal to x^2 where x is an integer less than 100. What are the values that N can take?
(A) 1, 9, 27 (B) 1, 7, 26
(C) 1, 8, 48 (D) 1, 8, 49

23. What is the unit's place of $(5^n + 4^{2n} + 7^{4n})^{4n}$?
(A) 4 (B) 8 (C) 2 (D) 6

24. What is the highest power of 5 in 240!?
(A) 58 (B) 17 (C) 116 (D) 39

25. The least possible number which when successively divided by 10, 7, and 6 leaves remainders of 8, 4, and 5 respectively is
(A) 256 (B) 148 (C) 398 (D) 198

26. The LCM and HCF of a pair of numbers is 1232 and 14, respectively. How many such pairs are possible?
(A) 3 (B) 2 (C) 1 (D) None

27. Find the square root of 12345654321.
(A) 1111 (B) 11111
(C) 111111 (D) 1111111

28. There are four prime numbers written in ascending order. The product of the first three prime numbers is 2431 and that of the last three is 4199. Find the greatest of them.
(A) 17 (B) 19 (C) 23 (D) 13

29. Find the minimum number of coins required to pay three persons 69 paise, 105 paise, and 85 paise, respectively, using coins in the denominations of 2 paise, 5 paise, 10 paise, 25 paise, and 50 paise.
(A) 9 (B) 10 (C) 14 (D) 11

30. If a, b, and c are prime numbers satisfying $a = b - 2 = c - 4$. How many possible combinations exist for a, b, and c?
(A) 4 (B) 3 (C) 2 (D) 1

31. Let p, q, and r be distinct positive integers that are odd. Which of the following statements cannot always be true?
(A) $pq^2 r^3$ is odd.
(B) $(p + q)^2 r^3$ is even
(C) $(p - q + r)^2 (q + r)$ is even.
(D) If p, q, and r are consecutive odd integers, the remainder of their product when divided by 4 is 3.

32. If *abcde* is a five-digit number the difference of *abcde* and *acdbe* would always be divisible by which of the following for all values of a, b, c, d and e?
(A) 9
(B) 18
(C) 99
(D) Both (A) and (B)

33. Find the value of the expression below

$$\frac{(0.68)^3 + (0.67)^3 - (0.5)^3 + (0.68)(0.67)(1.5)}{(0.68)^2 + (0.67)^2 + (0.5)^2 - (0.68)(0.67) + (0.67)(0.5) + (0.68)(0.5)}$$

(A) 1.85 (B) 0.51 (C) 0.49 (D) 0.85

34. Find the sum of all possible distinct remainders which are obtained when squares of a prime numbers are divided by 6.
(A) 7 (B) 8 (C) 9 (D) 10

35. The least number, which when successively divided by 2, 3, and 7 leaves respective remainders of 1, 2, and 3, is
(A) 56 (B) 130
(C) 68 (D) 23

36. Find the GCD of the numbers p and q where $p = 2^3 . 3^2 . 7^2 . 11^6$ and $q = 2^2 . 3^1 . 5^4 . 11^2 . 13^2$.
(A) 776 (B) 1452
(C) 1164 (D) 2028

37. Which of the following sets of numbers are relative primes?
(a) 57,61
(b) 396,455
(c) 693,132
(d) 6561,1024
(e) 384,352
(A) (c), (e) (B) (a), (b), (d)
(C) (a), (c), (d) (D) (b), (e)

38. (a) Find the units digit of 8^{173}.
(A) 2 (B) 4 (C) 8 (D) 6

(b) What is the last digit of $518^{163} + 142^{157}$?
(A) 2 (B) 4 (C) 6 (D) 8

(c) Find the last digit of $1567^{143} \times 1239^{197} \times 2566^{1027}$
(A) 2 (B) 3 (C) 4 (D) 6

39. If n is a positive integer, then $43^{5n} - 21^{5n}$ is always divisible by
(A) 11 (B) 18 (C) 25 (D) 64

40. Find the greatest number which when divides 6850 and 2575 leaving respective remainders of 50 and 25.
(A) 425 (B) 850 (C) 1700 (D) 1275

41. Find the least number which when divided by 12, 18, and 33 leaves a remainder of 5 in each case.
(A) 394 (B) 396 (C) 391 (D) 401

42. Find the smallest number that must be added to 1994 such that a remainder of 28 is left when the number is divided by 38 and 57.
(A) 66 (B) 68 (C) 86 (D) 98

43. Find the greatest number which divides 3300 and 3640 leaving respective remainders of 23 and 24.
(A) 13 (B) 113 (C) 339 (D) 226

44. Find the greatest number which divides 68, 140, and 248 leaving the same remainder in each case.
(A) 36 (B) 18 (C) 72 (D) 108

45. Five bells toll at intervals of 5, 6, 10, 12, and 15 seconds respectively. If they toll together at the same time, after how many seconds will they toll together again, for the first time?
(A) 300 (B) 120 (C) 60 (D) 30

46. If three numbers are in the ratio 3 : 4 : 5, and their LCM is 480, then find the sum of the three numbers.
(A) 96 (B) 72 (C) 84 (D) 108

47. If $(121)_8 = (x)_2$, then $x =$
(A) 101001 (B) 1010011
(C) 1010001 (D) 1011001

48. If $(ACD)_{16} = (x)_{10}$, then $x =$
(A) 2765 (B) 6725 (C) 5672 (D) 7625

49. Find the digit in the unit's place, in the product of $(25)^7 \times (37)^{12} \times (123)^9$.
(A) 1 (B) 5 (C) 3 (D) 9

50. What is the remainder when 3^{86} is divided by 6?
(A) 2 (B) 3 (C) 4 (D) 0

Answer Keys

1. A	**2.** D	**3.** D	**4.** A	**5.** A	**6.** A	**7.** A	**8.** B	**9.** A	**10.** A
11. A	**12.** C	**13.** B	**14.** D	**15.** B	**16.** A	**17.** C	**18.** (a) C	(b) C	(c) C (d) C
19. (a) A	(b) B	(c) C	**20.** C	**21.** D	**22.** D	**23.** D	**24.** A	**25.** C	**26.** B
27. C	**28.** B	**29.** D	**30.** D	**31.** D	**32.** D	**33.** D	**34.** B	**35.** D	**36.** B
37. B	**38.** (a) C	(b) B	(c) A	**39.** A	**40.** B	**41.** D	**42.** C	**43.** B	**44.** A
45. C	**46.** A	**47.** C	**48.** A	**49.** B	**50.** B				

Chapter 4

Percentage, Profit and Loss

CHAPTER HIGHLIGHTS

- *Percentage*
- *Percentage Points*
- *Profit and Loss*
- *Partnerships*
- *Stocks and Shares*

PERCENTAGE

'Percent' implies 'for every hundred'. This concept is developed to make the comparison of fractions easier by equalising the denominators of all fractions to hundred.

For example, 7/11 as percentage is represented as $\frac{7}{11} = \frac{7 \times 100}{11 \times 100} = \frac{(7 \times 100)/11}{100} = \frac{63.63}{100} = 63.63\%$

Percentages can also be represented as decimal fractions. In such a case it is effectively equivalent to the proportion of the original quantity.

For example, 20% is the same as $\frac{20}{100}$, i.e. 0.2.

Any percentage can be expressed as a decimal fraction by dividing the percentage figure by 100, and conversely, any decimal fraction can be converted to percentage by multiplying it by 100.

PERCENTAGE INCREASE or DECREASE of a quantity is the ratio expressed in percentage of the actual INCREASE or DECREASE of the quantity to the original amount of the quantity, i.e.,

$$\text{PERCENTAGE INCREASE} = \frac{\text{Actual increase}}{\text{Original quantity}} \times 100$$

$$\text{PERCENTAGE DECREASE} = \frac{\text{Actual decrease}}{\text{Original quantity}} \times 100$$

For example, if the production of rice went up from 225 MT in 1993 to 242 MT in 1994, then the percentage increase in rice production from 1993 to 1994 is calculated as follows:

Actual increase = 242 – 225 = 17 MT

Percentage increase

$$= \frac{\text{Quality increase from 1993 to 1994}}{\text{Actual production of rice in 1993}} \times 100$$

$$= \frac{17}{225} \times 100 = 7\frac{5}{9}\%$$

Ratio of any two quantities also can be expressed as percentage.

For example, if the ratio of A and B is 3 : 2, we can say the ratio of A : B is 60% : 40%.

Whenever there is any percentage increase or decrease on a quantity, we can directly calculate the new value of the quantity instead of calculating the actual increase/decrease and then adding to/subtracting from the original quantity.

For example, if the increase on a value of 350 is 15%, the new quantity is $1.15 \times 350 = 402.5$ (where $1.15 = 1 + 0.15$, 0.15 being the decimal equivalent of 15%).

If the production in 1994 is given as 400 MT and the increase from 1993 to 1994 is given to be 25%, then the production in 1993 will be equal to 400/1.25 = 320 MT (where $1.25 = 1 + 0.25$, 0.25 being the decimal equivalent of 25%).

Similarly, if there is a decrease of 12% on a quantity of 225, then the new quantity will be equal to 225×0.88 (where $0.88 = 1 - 0.12$, 0.12 being the decimal equivalent of 12%).

If the production in 1994 is given as 400 MT and it is a decrease of 13% from 1993, then the production in 1993 will be equal to 400/0.87 (where $0.87 = 1 - 0.13$, 0.13 being the decimal equivalent of 13%).

On the basis of percentage increase, we can write down how many times the old value gives the new value. For example, if the percentage increase is 100%, then we can conclude that the new value is 2 times the old value. If the percentage increase is 300%, the new value is 4 times the old value. If the percentage increase is 450%, then the new

value is 5.5 times the old value. In general, if the percentage increase is $p\%$, then the new value is $\left(\frac{p}{100}+1\right)$ times the old value.

Conversely, if we know how many times the old value gives the new value, we can find out the percentage increase in the old value to get the new value. For example, if the new value is 3 times the old value, the percentage increase in the old value to get the new value is 200%. If the new value is 4.25 times the old value, then the percentage increase is 325%. In general, if the new value is k times the old value, then the percentage increase is $(k-1) \times 100$.

Solved Examples

Example 1

Rice production in a country increased by 25% from 2000 to 2004. It increased by 20% from 2004 to 2008. Find the percentage increase in the rice production from 2000 to 2008.

Solution

Let the rice production in 2000 be 100 tonnes.
Rice production in 2004

$$= 100 + \frac{25}{100}(100) = 125 \text{ tonnes}$$

Rice production in 2008

$$= 125 + \frac{20}{100}(125) = 150 \text{ tonnes}$$

$\therefore$ Percentage increase in the rice production from 2000 to 2008 is $\frac{150-100}{100} \times 100$ i.e. 50%.

Example 2

The population of a country increased by 10% from 2001 to 2002. It increased by 20% from 2002 to 2003. It increased by 30% from 2003 to 2004. Find the simple average yearly percentage increase in the population from 2001 to 2004.

Solution

Let the population of the country in 2001 be 100 million.
Population in 2002 = 110 million.
Population in 2003 = 110 × 1.2 = 132 million.
Population in 2004 = 132 × 1.3
= 171 × 6 million.
Simple average percentage increase

$$= \frac{\frac{171.6-100}{100}}{3}(100) = 23\frac{13}{15}$$

Example 3

The price of a Swiss watch was ₹10000 in 2001. Due to devaluation of the rupee, it becomes ₹12000 in 2002. Find the percentage increase in its price from 2001 to 2002.

Solution

Percentage increase

$$= \frac{\text{Final price} - \text{Initial price}}{\text{Initial price}}(100)$$

$$= \frac{12000-10000}{10000}(100) = 20\%.$$

Example 4

64% of a number is 416. Find 85% of that number.

Solution

Let the number be x

$$\frac{64}{100}x = 416$$

$$x = 650$$

$$\frac{85}{100}x = 552.5.$$

Alternative method:

$$64\% = 416$$

$$85\% = \frac{85}{64}(416) = 552.5.$$

Example 5

The ratio of the salaries of A and B is $2:2\frac{1}{7}$. By what percentage is B's salary greater than A's salary?

Solution

Let the salary of A be $2x$

$\Rightarrow$ Salary of B = ₹$2\frac{1}{7}x$

$\therefore$ The salary of B is more than the salary of A by $\frac{\frac{1}{7}x}{2x}(100) = 7\frac{1}{7}\%$.

Example 6

The height of a triangle as well as its base are increased by 30%. Find the percentage increase in its area.

Solution

Let the original height as well as the base be 100 cm. Original area

$$= \frac{(100)(100)}{2} = \frac{100^2}{2}$$

New height = New base = 130 cm

$$\text{New area} = \frac{(130)(130)}{2}$$

$$= \frac{1}{2}\left((1\cdot3)(100)^2\right) = \frac{1\cdot69\left(100^2\right)}{2}$$

$$= 1\cdot69 \text{ (original area)}$$

$\therefore$ the area increased by 69%.

Example 7

If the price of tea goes up by $33\frac{1}{3}\%$, what should be the percentage by which its consumption must be reduced so that the expenditure on it remains unchanged?

Solution

Let the original price be ₹300 per kg and the original consumption be 100 kg. Original expenditure = New expenditure = ₹30000.

$$\text{New price} = 300\left(1+\frac{1}{3}\right) = ₹400.$$

$$\text{New consumption} = \frac{30000}{400} \text{ i.e., 75 kg.}$$

Percentage reduction in consumption

$$= \frac{25}{100}(100) = 25\%$$

In the above three examples, if the percentage given initially is x, what is asked to be found is $\frac{100x}{(100+x)}$.

We can generalize each of the three cases as below:

If the value of an item goes up/down by x%, the percentage reduction/increment to be now made to bring it back to the original level is $\frac{100x}{(100 \pm x)}$%.

If A is x% more/less than B, then B is $\frac{100x}{(100 \pm x)}$ *% less/more than A.*

If the price of an item goes up/down by x%, then the quantity consumed should be reduced/increased by $\frac{100x}{(100 \pm x)}$ *% so that the total expenditure remains the same.*

Percentage Points

The concept of 'percentage points' is important in the usage of percentages. Percentage points is the difference of two percentage figures.

Let us understand this with an example.

Suppose that rice forms 20% of total food grain production in Year I and 30% of total food grain production in Year II.

If we are asked to find out the percentage increase in the production of rice, calculating percentage increase from 20 to 30 as $\frac{30-20}{20}\times100$ and saying it is 50% increase is NOT correct. With the available data, we cannot find out the percentage increase in the production of rice from Year I to Year II. We can only say that the production of rice as a percentage of total food grain production went up by 10 PERCENTAGE POINTS (the 10 being the increase from 20 to 30—both percentage figures).

We can see by taking the following figures that the percentage increase in rice production need not be 50%.

	Year I	Year II
Rice	1000	960
Total foodgrains	5000	3200
Rice as percent of Total foodgrains	20%	30%

Here, while rice is 20% of total food grains in Year I and 30% of total food grains in Year II, we find that the actual production of rice has not even increased—it decreased from 1000 in Year I to 960 in Year II.

Profit and Loss

In any business/commercial environment the most important concern is about the profit/loss of the transaction conducted.

The SELLING PRICE (SP) and the COST PRICE (CP) of an article determine the profit or loss made on the particular transaction.

The computation is done as follows:

$$\text{Profit} = \text{Sale Price} - \text{Cost Price} = \text{SP} - \text{CP}$$

$$\text{Percentage Profit} = \frac{\text{SP}-\text{CP}}{\text{CP}}\times100$$

$$= \frac{\text{Profit}}{\text{CP}}\times100$$

$$\text{Loss} = \text{CP} - \text{SP}$$

$$\text{Percentage Loss} = \frac{\text{Loss}}{\text{CP}}\times100.$$

It is customary to express profit/loss as percentage of cost price. However, in some problems, it may specifically be given that profit/loss percentage has been calculated on the selling price or the student may be asked to calculate the profit/loss percentage on the selling price. Unless such specific directions are given, the profit/loss percentage is always to be calculated on the cost price.

Given profit/loss percentage along with SP, CP can be found out, and similarly, given profit/loss percentage along with CP, SP can be found out by using the concepts discussed

at the beginning of this chapter (where, if percentage increase or decrease is given, we can find out the new value from the old value or the old value from the new value).

The following simple rules can be remembered for this purpose.

Given the cost price (CP) and profit percentage p%, the selling price will be given by $SP = CP \times \frac{(100+p)}{100}$

Given the cost price (CP) and loss percentage p%, the selling price will be given by $SP = CP \times \frac{(100-p)}{100}$

Given the selling price (SP) and profit percentage p%, the cost price will be given by $CP = SP \times \frac{100}{(100+p)}$

Given the selling price (SP) and loss percentage p%, the cost price will be given by $CP = SP \times \frac{100}{(100-p)}$

When two articles are SOLD at the same price (i.e., their SP is the same) such that there is a PROFIT of $p\%$ on one article and a LOSS of $p\%$ on the other (i.e. common profit or loss percentage), then, irrespective of what the SP actually is, the net result of the transaction is LOSS. This percentage loss is given by

$$\text{Loss percentage} = \frac{(\text{Common profit or loss})^2}{100} = \frac{p^2}{100}$$

MARKED PRICE or LIST PRICE is the price that is indicated or marked on the product or it is the price which is given in the price list. This is the price at which the product is intended to be sold. However, there can be some DISCOUNT given on this price, and consequently, the actual SELLING PRICE of the product may be less than the MARKED PRICE.

SELLING PRICE = MARKED PRICE – DISCOUNT

The amount of discount given can also be expressed as a percentage. DISCOUNT is always expressed as a percentage of the MARKED PRICE or the LIST PRICE.

DISCOUNT percent

$$= \frac{\text{Marked price} - \text{Selling price}}{\text{Marked price}} \times 100$$

$$= \frac{\text{Discount}}{\text{Marked price}} \times 100$$

Certain discount is given on an article whose selling price is SP. If further discounts are given on this discounted price, such discounts are referred to as successive discounts. If the successive discounts are $p\%$, $q\%$, and $r\%$, on a product whose selling price is SP, then the effective price after all the discounts is given by

$$\textit{Discounted price} = SP \times \frac{(100-p)(100-q)(100-r)}{100 \times 100 \times 100}$$

Example 8

Alok bought a watch for ₹250 and sold it for ₹300. Find his profit percentage.

Solution

Given that

$$\text{SP} = ₹300 \quad \text{CP} = ₹250$$

$$\Rightarrow \quad \text{Profit} = \text{SP} - \text{CP} = ₹300 - ₹250 = ₹50$$

$$\text{Profit } \% = \frac{50}{250}(100) = 20\%.$$

Example 9

Anand gained 20% by selling a book at ₹30. Find his gain percentage if he sells it for ₹36.

Solution

Let his cost price be ₹x

Given, $\left(1+\frac{20}{100}\right)x = 30$

$\Rightarrow \quad x = 25$

Gain percent when sold at ₹36

$$= \frac{36-25}{25}(100) = 44\%.$$

Example 10

Ajay calculated his profit/loss percentage on his selling prices. Find his actual profit/loss percentage if he calculated

(i) his profit percentage to be 25%

(ii) his loss percentage to be 25%

Solution

Let his selling price be ₹100

(i) Profit = ₹25 ⇒ CP = SP – Profit
= ₹75

Actual profit percentage

$$= \frac{25}{75}(100) = 33\frac{1}{3}\%$$

(ii) Loss = ₹25

⇒ CP = SP + Loss
= ₹125

Actual loss percentage

$$= \frac{25}{125}(100) = 20\%.$$

Example 11

The profit made by selling 5 m of a cloth equals the selling price of 2 m of that cloth. Find the profit percentage made.

Solution

SP (5 m) = CP (5 m) + Profit (5 m)
As Profit (5 m) = SP (2 m),

$$\text{SP (5 m)} = \text{CP (5 m)} + \text{SP (2 m)}$$

$$\Rightarrow \quad \text{SP (3 m)} = \text{CP (5 m)}$$

$$\Rightarrow \quad \frac{\text{S.P.}}{\text{C.P.}} = \frac{5}{3}$$

$$\therefore \quad \text{Profit \%} = \frac{5-3}{3} \times 100 = 66\frac{2}{3}\%.$$

Example 12

A trader promised his customers to sell at cost price. But he cheats his customers by giving 100 g less for every kg that he sells. Find his profit percentage.

Solution

Let the cost of each gram to the trader be ₹1. Cost price of 1000 g = Selling price of 900 g.
Cost of 900 g = ₹900
Selling price of 900 g = ₹1000

$$\text{Profit percentage} = \frac{1000-900}{900}(100)$$

$$= 11\frac{1}{9}\%.$$

Partnerships

Two or more people can get together to do business by pooling their resources. The money put in by each of the partners is called his 'INVESTMENT' or 'CAPITAL'.

All the people who have invested money in the partnership are called PARTNERS.

While two or more partners would have invested money, it is not necessary that all of them should be involved in the day-to-day running of the business. The partners involved in the day-to-day activities of the business are called 'working partners', and the others are called 'sleeping partners' or 'dormant partners'.

The profits left after paying the working partners' remuneration/commission are shared amongst all the partners.

Sometimes, the partners also take interest on their investments, and only the remaining profits are shared by the partners.

Sharing of profits among the partners also depends on the understanding between the partners. However, if no special scheme of sharing the profits is specified (in a problem), then the profits are shared based on the investments of the partners. There are three different possibilities that exist here.

1. If the partners invest DIFFERENT amounts each for the SAME period of time, then the profits at the end of the year are shared in the ratio of their investments.
2. If the partners invest the SAME amounts for DIFFERENT periods of time, then the profits at the end of the year are shared in the ratio of the time periods for which their respective investments have been in business.
3. If the partners invest DIFFERENT amounts and the time periods for which their investments are in the business are also DIFFERENT, then the profits at the end of the year are shared in the ratio of the products of (investment x time period) calculated for each partner.

There can be problems that are modelled along the sharing of profits in partnerships. An example of this type is where a particular facility (like renting a tractor for ploughing their fields by three different people) is used by more than one party and the rent has to be shared by all the concerned parties—similar to sharing of profits in a partnership.

Example 13

Sachin started a business with ₹20000, and after 4 months, Sunil joined him with ₹40000. Sachin received ₹39000 as his annual profit share that included a salary of 16% of the annual profit. Find the annual profit share of Sachin.

Solution

Let the total annual profit be ₹x
Ratio of the part of the total annual profit which is shared in the ratio of the (investments × time) = (20000) (12) : (40000) (8) = 3 : 4.

Hence, the share of Sachin

$$= \frac{3}{7}\left(\frac{84}{100}x\right) + \frac{16x}{100} = 39000$$

$$x = 75000$$

∴ Annual profit share of Sachin

= ₹75000 – ₹39000 = ₹36000.

Example 14

A started a business with ₹20000. After 3 months, B joined him with ₹40000. After some more months, C joined them with ₹100000. B received ₹18000 out of the total annual profit of ₹55000. How many months after A started the business did C join?

Solution

Let us say C joined after x months.
Profit is shared in the ratio

$$(20000)\,(12) : (40000)\,(9) : 100000\,(12-x)$$

$$= 24 : 36 : 10\,(12-x)$$

$$\text{Given} \quad \frac{36}{180-10x} = \frac{18}{55} = \frac{36}{110}$$

$$180 - 10x = 110$$

$$\therefore \quad x = 7.$$

Example 15

Mohan started a business with ₹20000. After 4 months, Sachin joined him with ₹30000. At the beginning of the fifth month, Mohan added ₹10000. Find the ratio in which the annual profit will be shared.

Solution

Ratio of the profit share

$$= [(20000)\ (12) + (10000)\ (8)] : [30000\ (8)] = 4 : 3.$$

EXERCISES

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. Ganesh owns $83\frac{1}{3}\%$ of a property. Three fourths of the part of it he owns is worth ₹5 lakhs. Find the value of the property (in ₹ lakhs).
(A) 7.2 (B) 8 (C) 6.4 (D) 8.8

2. The salaries of two persons are equal. If the salary of one of them is increased by 20% and the salary of the other is decreased by 20%, find the percentage change in the total salary of the two persons.
(A) 4% increase B) 4% decrease
(C) 0% (D) None of these

3. The price of a TV is decreased by 20%. By what percent must it be increased to bring it back to the original price?
(A) 25% (B) 20%
(C) $16\frac{2}{3}\%$ (D) 15%

4. The ratio of two numbers is 5/6 : 2/3. By what percentage is the second number more/less than the first number?
(A) 20% less (B) 25% more
(C) 25% less (D) 20% more

5. In a test, Mohan's mark was 25% more than Sohan's mark. Mohan got the minimum mark required to pass the test. The pass mark was 35. Find Sohan's mark.
(A) 21 (B) 26 (C) 27 (D) 28

6. In 2004, the price of a shampoo bottle increases by 10% with respect to that in 2003. By what percentage is its price in 2003 less than that in 2004?
(A) 10% (B) $9\frac{1}{11}\%$
(C) 11% (D) $10\frac{1}{11}\%$

7. Due to inflation the total cost of monthly household items has gone up by 20%, but the salary of the family increased by only 10%. Initially, the family used to spend 20% of the salary on household items. What percentage of the present salary should the family spend to buy the same quantities of household items?
(A) 10% (B) $20\frac{2}{11}\%$
(C) $22\frac{2}{11}\%$ (D) $21\frac{9}{11}\%$

8. If the area of a rectangle is increased by 32% and its breadth is increased by 10%, what is the percentage increase in its perimeter?
(A) 10%
(B) 12%
(C) 20%
(D) Cannot be determined

9. *A*'s salary is 20% less than *B*'s salary. If *C*'s salary is ₹10000 and it is 25% more than *B*'s salary, then what is *A*'s salary?
(A) ₹6000 (B) ₹9600
(C) ₹8000 (D) ₹6400

10. School *A* has 30% more students than school *B*. If 120 more students join school *B*, the two schools will have the same number of students. What is the sum of the number of students in school *A* and school *B* initially?
(A) 600 (B) 400 (C) 800 (D) 920

11. The price of petrol increased by 2% in a certain week and increased by 4% in the next week. Find the net percentage increase in the price of petrol over these two weeks.
(A) 6.12% (B) 6.08%
(C) 6.16% (D) 6.20%

12. Rahul got 150 marks in a test. He scored 25% more marks than the pass mark in it. Rajesh got 165 marks in it. By what percent did his mark exceed the pass mark?
(A) 40% (B) 37.5%
(C) 45% (D) 32.5%

13. In a college of total strength 1000, 30% of the students are girls. There are 600 PGs and 120 more male UGs than female UGs. What percent of the males are the female UGs?
(A) 20% (B) 15% (C) 25% (D) 10%

14. A machine costs ₹4,00,000. It depreciates by 18% in value in the first year, 16.5% in the second year, 15% in the third year and so on. Find the amount by which it depreciates in the seventh year (in ₹) (Assume all percentages apply to the original cost of the equipment).
(A) 32000 (B) 28000
(C) 36000 (D) 40000

15. The monthly income of Ram increased by 26%. His expenditure which is 70% of his monthly income increased by 20%. His savings must have increased by
(A) 40% (B) 30% (C) 50% (D) 25%

16. A shopkeeper sells an item for ₹60 at a profit of 20%. At what price (in ₹) should he sell it to gain 30%?
(A) 63 (B) 65 (C) 68 (D) 70

17. The profit made on selling 5 m of a cloth equals the cost price of 2 m of that cloth. Find the profit percentage in selling each m of the cloth.
(A) $66\frac{2}{3}\%$ (B) 50%
(C) 40% (D) $28\frac{4}{7}\%$

18. A company manufactures a product for ₹50. It sold it to a dealer for ₹60. The dealer sold it to a shopkeeper for ₹75. The shopkeeper sold it to a customer for ₹100. Find the profit percentage of the company.
(A) $16\frac{2}{3}\%$ (B) 25%
(C) 20% (D) $33\frac{1}{3}\%$

19. The cost price of 80 articles is ₹12.50 per article. Twenty of them were sold for ₹18 each. At what price should each of the remaining articles be sold so as to get an overall profit of ₹4.50 per article?
(A) ₹15 (B) ₹$16\frac{2}{3}$
(C) ₹$17\frac{1}{3}$ (D) ₹18

20. Rohit marked his goods 40% above his cost price. He sold it after a discount at 12% profit. Find his discount percentage.
(A) 20% (B) 25% (C) 15% (D) 30%

21. Two successive discounts of 30% and 10% are equal to a single discount of
(A) 33% (B) 35% (C) 37% (D) 36%

22. *P* and *Q* started a business in which *P* invested ₹10000 and *Q* invested ₹20000. They received a profit of ₹9600 at the end of a year. Find *Q*'s share in profit (in ₹).
(A) 8000 (B) 6400
(C) 4800 (D) 3200

23. Ramesh and Suresh started a business. Ramesh invested ₹9000 for ten months and Suresh invested ₹6000 for a year. If the profit at the end of a year was ₹4500, find Suresh's share.
(A) ₹3600 (B) ₹2700
(C) ₹2500 (D) ₹2000

24. Kanchan has bought 50 articles. He sells 20% of the articles and makes a profit of ₹1200, which is also equal to the cost of 5 articles. If the selling price for all 50 articles is the same, what is the value of the remaining articles at the selling price?
(A) ₹14400 (B) ₹9600
(C) ₹18000 (D) ₹15000

25. If the discount and profit percentage are both 20% by what percent is the marked price above the cost price?
(A) 40% (B) 50% (C) 60% (D) 70%

26. A car dealer sold a car at a discount of ₹100000. Even after the discount, he made a profit of 15%. What is the marked price of the car, if the marked price is 25% more than the cost price?
(A) ₹1000000 (B) ₹1115000
(C) ₹2500000 (D) ₹1250000

27. Simon gets a discount of 25% on purchasing 100 VCD's from Samuel. He sells them and makes a profit equal to the undiscounted price of 25 VCD's. What is the gain percentage?
(A) 25% (B) 30%
(C) 66.66% (D) 33.33%

28. Ashwin bought an article at ₹200 and marked it at ₹300. He offered a discount and then sold it his profit/loss percentage and discount percentage are in the ratio 3 : 2. Find his profit/loss percentage.
(A) 29% profit (B) 25% profit
(C) 20% loss (D) 25% loss

29. Feroze marks up an article by 30% and sells it at a discount of 20% to Sohail. Sohail marks up the price of the article to a certain amount which happens to be 20% more than Feroze's cost price. What is the maximum discount Sohail can offer without going into loss?
(A) 30% (B) 20%
(C) $16\frac{2}{3}\%$ (D) $13\frac{1}{3}\%$

30. Gopal, Hari, and Karthik started a business with investments of ₹8000, ₹12000, and ₹16000 respectively. Hari and Karthik left the business after *x* months. Out of the annual profit share, Gopal got more than Hari but less than Karthik. If *x* is an integer, find the ratio of Gopal's, Hari's and Karthik's shares.
(A) 27 : 21 : 28 (B) 24 : 21 : 28
(C) 30 : 27 : 36 (D) 32 : 30 : 40

Answer Keys

1. B	**2.** C	**3.** A	**4.** A	**5.** D	**6.** B	**7.** D	**8.** D	**9.** D	**10.** D
11. B	**12.** B	**13.** A	**14.** C	**15.** A	**16.** B	**17.** C	**18.** C	**19.** B	**20.** A
21. C	**22.** B	**23.** D	**24.** A	**25.** B	**26.** D	**27.** D	**28.** B	**29.** D	**30.** B

Chapter 5

Simple Interest and Compound Interest

CHAPTER HIGHLIGHTS

- *Interest*
- *Simple Interest*
- *Compound Interest*
- *Compounding More Than Once a Year*
- *Present Value*
- *Repayment in Equal Instalments*

Interest

Interest is money paid to the lender by the borrower for using his money for a specified period of time. Various terms and their general representation are as follows:

1. INTEREST
   Money paid by borrower for using the lender's money. Denoted by I.
2. PRINCIPAL
   The original sum borrowed. Denoted by P.
3. TIME
   Time for which money is borrowed. Denoted by n. (n is expressed in number of periods, which is normally one year.)
4. RATE OF INTEREST
   Rate at which interest is calculated on the original sum. Denoted by r and is expressed as a percentage or decimal fraction.
5. AMOUNT
   Sum of principal and interest. Denoted by A.

Simple Interest

When interest is calculated every year (or every time period) on the original principal, i.e. the sum at the beginning of first year, such interest is called simple interest.

Here, year after year, even though the interest gets accumulated and is due to the lender, this accumulated interest is not taken into account for the purpose of calculating interest for latter years.

$$\text{Simple Interest} = \frac{Pnr}{100}$$

where P, n, r are as explained above.

$$\text{Total Amount } A = P + \frac{Pnr}{100} = P\left(1 + \frac{nr}{100}\right)$$

Compound Interest

Under compound interest, the interest is added to the principal at the end of each period to arrive at the new principal for the next period.

In other words, the amount at the end of first year (or period) will become the principal for the second year (or period); the amount at the end of second year (or period) becomes the principal for the third year (or period); and so on.

If P denotes the principal at the beginning of Period 1, then, principal at the beginning of Period 2

$$= P\left(1 + \frac{r}{100}\right)$$

$$= PR = \text{Amount at the end of Period 1,}$$

where $R = \left\{1 + \left(\frac{r}{100}\right)\right\}$

P at the beginning of Period 3 = $P\left(1+\frac{r}{100}\right)^2$

$= PR^2 =$ Amount at the end of Period 2

P at the beginning of Period $(n + 1)$

$$= P\left(1+\frac{r}{100}\right)^n = PR^n$$

= Amount at the end of Period n

Hence, the amount after n years (periods) = $PR^n = A$

$$\text{Interest} = I = A - P = P\,[R^n - 1]$$

The following table gives an example of how simple interest and compound interest operate, i.e. how the principal is for various years under simple interest and compound interest. A principal at the beginning of 1st year, of ₹100 and a rate of 10% p.a. are considered. The details are worked out for three years and shown below.

(All figures pertaining to principal, interest, and amount are in rupees)

	Under Simple Interest				Under Compound Interest			
Year	Principal at the beginn. of the year	Interest for the year	Interest till the end of the year	Amount at the end of the year	Principal at the beginn. of the year	Interest for the year	Interest till the end of the year	Amount at the end of the year
1	100	10	10	110	100	10	10	110
2	100	10	20	120	110	11	21	121
3	100	10	30	130	121	12.1	33.1	133.1

As can be seen from the table,

In case of simple interest,

1. The principal remains the same every year.
2. The interest for any year is the same as that for any other year.

In case of compound interest,

1. The amount at the end of an year is the principal for the next year.
2. The interest for different years is not the same.

The compound interest for the first year (where compounding is done every year) is the same as the simple interest for one year.

Compounding More Than Once a Year

We just looked at calculating the amount and interest when the compounding is done once a year. But, compounding can also be done more frequently than once a year. For example, the interest can be added to the principal every six months or every four months and so on.

If the interest is added to the principal every six months, we say that compounding is done twice a year. If the interest is added to the principal every four months, we say that compounding is done thrice a year. If the interest is added to the principal every three months, we say that compounding is done four times a year.

The formula that we discussed above for calculating the amount will essentially be the same, i.e.

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n$$

where r = rate % *per annum* and n = number of *years*, but the rate will *not* be for ONE YEAR but for the time period over which compounding is done and the power to which the term inside the bracket is raised (n in the above case) will *not* be the number of years but the number of years multiplied by the number of times compounding is done per year (this product is referred to as the total number of time periods).

For example, if a sum of ₹10000 is lent at the rate of 10% per annum and the compounding is done for every four months (thrice a year), then the amount will be equal to

$$10000\left(1+\frac{10}{3}\times\frac{1}{100}\right)^{2\times 3}$$

Here, the dividing factor of 3 in the rate and the multiplying factor of 3 in the power (multiplying the number of years)—both shown by arrow marks—are nothing but the NUMBER OF TIMES compounding is done in a year.

> *If compounding is done k times a year (i.e. once every 12/k months), at the rate of r% p.a. then in n years, the principal of P will amount to* = $P\left(1+\frac{r}{k\cdot 100}\right)^{kn}$

When compounding is done more than once a year, the rate of interest given in the problem is called NOMINAL RATE OF INTEREST.

We can also calculate a rate of interest which will yield simple interest in one year equal to the interest obtained under the compound interest at the given nominal rate

of interest. The rate of interest so calculated is called EFFECTIVE RATE OF INTEREST.

If the number of times compounding is done in a year is increased to infinity, we say that the compounding is done EVERY MOMENT and then the amount is given by $P \cdot e^{nr/100}$, where r is the rate % p.a. and n is the number of years.

The following points should also be noted, which are helpful in solving problems.

The difference between the compound interest and simple interest on a certain sum for two years is equal to the interest calculated for one year on one year's simple interest.

In mathematical terms, the difference between compound interest and simple interest for two years will be equal to $P(r/100)^2$, which can be written as $P(r/100)\,(r/100)$. In this, $Pr/100$ is the simple interest for one year, and when this is multiplied by $r/100$ again, it gives interest for one year on $Pr/100$, i.e. interest for one year on one year's simple interest.

The difference between the compound interest for k years and the compound interest for (k + 1) years is the interest for one year on the amount at the end of k^{th} year.

This can also be expressed in terms of the amount as follows:

The difference between the amount for k years and the amount for (k + 1) years under compound interest is the interest for one year on the amount at the end of the k^{th} year.

The difference between the compound interest for the k^{th} year and the compound interest for the $(k + 1)^{th}$ year is equal to the interest for one year on the compound interest for the k^{th} year.

Present Value

Consider a given sum P and a rate of interest r.

We have seen that interest is cost of using the money over a period of time. That means a sum at the beginning of a period is always higher than the same amount after a period greater than or equal to 1.

Let the sum P that is being considered at a rate of interest r% p.a., becomes Y at the end of Year 1 and Z at the end of Year 2 (i.e. Y and Z are the amounts at the end of first and second years, respectively, on a principal of P).

Then, we can say that what is P today is equal to Y at the end of one year and equal to Z at the end of the second year. In other words, if an amount of Y were to come at the end of one year from now, its value today is equal to P. Similarly, if an amount of Z were to come at the end of two years from now, its value today is equal to P.

So, P is the PRESENT VALUE of Y coming at the end of one year and P is the PRESENT VALUE of Z coming at the end of two years.

Similarly, if we consider n years (or n periods in general), and X is the amount that P will become in n periods, then we say that P is the PRESENT VALUE of X coming at the end of n periods.

If we consider a series of payments Y_1 at the end of first year, Y_2 at the end of second year, and so on, the present value of the series of payments will then be equal to the sum of the present values of each of the payments calculated separately. If Z_1 is the present value of Y_1, Z_2 is the present value of Y_2, and so on, then the present value of the series of payments $Y_1, Y_2, \ldots$ is equal to $Z_1 + Z_2 + \ldots$

Present value can be looked at both under simple interest and compound interest.

If an amount of Y whose present value is P_1 comes at the end of Year 1 and an amount of Z whose present value is P_2 comes at the end of Year 2, then the present value of both the amounts together will be equal to $(P_1 + P_2)$, i.e. the present value of the stream of payments that come at different points of time is equal to the sum of the present values of the individual amounts coming in at various points of time.

Present Value under Simple Interest: The principal P is amounting to X in n periods. From this, we know that

$$X = P\left(1 + \frac{nr}{100}\right)$$

$$\Rightarrow \qquad P = \frac{X}{\left(1 + \frac{nr}{100}\right)}$$

Hence, in general, the present value P of an amount X coming (or due) after n periods is given by

$$\boxed{P = \frac{X}{\left(1 + \frac{nr}{100}\right)}}$$

where r is the rate percent per time period.

Present Value under Compound Interest: The principal P is amounting to X in n periods. From this, we know that

$$X = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow \qquad P = \frac{X}{\left(1 + \frac{r}{100}\right)^n}$$

Hence, in general, the present value P of an amount X coming (or due) after n periods is given by

$$\boxed{P = \frac{X}{\left(1 + \frac{r}{100}\right)^n}}$$

where r is the rate percent per time period.

Repayment in Equal Instalments—Compound Interest

If a sum P borrowed is repaid in n equal instalments compound interest being calculated at $r\%$ per period of instalment, we can find out the value of each instalment. Let us consider the case of n equal ANNUAL instalments. (Even if the instalments are not annual, but monthly, the approach will remain the same except that the rate of interest taken should then be the rate per month and not rate per annum.)

Let each instalment (i.e. the amount paid at the end of each year) be X.

Instalment X paid after Year 1 gives a present value of $\dfrac{X}{\left(1+\dfrac{r}{100}\right)}$.

Instalment X, paid at the end of Year 2 gives a present value of $\dfrac{X}{\left(1+\dfrac{r}{100}\right)^2}$.

Similarly, instalment X paid for nth period (at the end of year n) gives a present value of $\dfrac{X}{\left(1+\dfrac{r}{100}\right)^n}$.

The sum of all these present values would be equal to the loan amount P (because only if the amount borrowed is equal to the amount repaid can we say that the loan is repaid).

$$\frac{X}{\left(1+\frac{r}{100}\right)}+\frac{X}{\left(1+\frac{r}{100}\right)^2}+\ldots+\frac{X}{\left(1+\frac{r}{100}\right)^n}=P$$

Call $\quad \dfrac{1}{\left(1+\dfrac{r}{100}\right)}=k$

$\Rightarrow \quad k=\dfrac{100}{100+r}$

The above equation can then be rewritten as

$$X\{k+k^2+\ldots+k^n\}=P$$

The terms within the brackets form a G.P. with first term k and common ratio k.

The sum of this G.P. $=\dfrac{k\,(k^n-1)}{(k-1)}$;

Thus $\quad \dfrac{X\cdot k\,(k^n-1)}{(k-1)}=P$

$\Rightarrow \quad X=\dfrac{P\,(k-1)}{k\left(k^n-1\right)}$

$$=\frac{\left[P\left\{\frac{100}{100+r}\right\}-1\right]}{\left[\left\{\frac{100}{100+r}\right\}\right]\left[\left\{\frac{100}{100+r}\right\}^n-1\right]}=\frac{P\cdot r}{100\left[1-\left\{\frac{100}{100+r}\right\}^n\right]}$$

$$\boxed{\textit{Each instalment}=\frac{P\cdot r}{100\left[1-\left\{\frac{100}{100+r}\right\}^n\right]}}$$

Solved Examples

Example 1

Find the simple interest on a sum of ₹1000 at 10% p.a. for 4 years.

Solution

Simple interest $=\dfrac{Pnr}{100}$

$$\text{Interest}=\frac{(1000)(4)(10)}{100}=₹\,400.$$

Example 2

A some of ₹4000 becomes ₹4500 in 2 years under simple interest. In how many years will ₹5000 become ₹5625 under simple interest at the same rate of interest?

Solution

Let the rate of interest be $R\%$ p.a.
Interest on ₹4000 = ₹500

$$500=(4000)\left(\frac{R}{100}\right)(2)$$

$$R=6.25\%$$

Interest on ₹5000 = ₹625
Let the required time be T years.

$$625=(5000)\left(\frac{6\cdot 25}{100}\right)T$$

$\Rightarrow \quad T=2.$

Example 3

Find the value that ₹1000 would amount to under compound interest at 20% p.a., interest being compounded annually in 3 years.

Solution

$$\text{Amount}=P\left(1+\frac{R}{100}\right)^N=1000\left(1+\frac{20}{100}\right)^3=₹1728$$

Example 4

Find the sum that would amount to ₹6600 under simple interest in 4 years at 8% p.a.

Solution

Let the sum be ₹P.

Given that $P\left(1+4\left(\dfrac{8}{100}\right)\right)=6600$

$$P=5000.$$

Example 5

If a sum triples in 4 years under simple interest, find the time that it would take to become 5 times itself at the same rate of interest.

Solution

If the sum triples, the interest obtained will be twice the sum. This takes 4 years. If the sum becomes 5 times, the interest must be four times the sum.

∴ This takes a total of 8 years.

Example 6

A sum triples in 4 years under compound interest at a certain rate of interest, interest being compounded annually. Find the time it would take to become 9 times itself.

Solution

The sum triples in 4 years. If it becomes 9 times itself, it has tripled twice.

∴ This takes 8 years.

Let the sum of ₹P, triple in 4 years at R% p.a.

$$\Rightarrow \quad P\left(1+\frac{R}{100}\right)^4 = 3P$$

$$\Rightarrow \quad \left(1+\frac{R}{100}\right)^4 = 3 \qquad (1)$$

Let it take K years to become 9 times.

$$P\left(1+\frac{R}{100}\right)^K = 9P$$

$$\Rightarrow \quad \left(1+\frac{R}{100}\right)^K = 9$$

$$\Rightarrow \quad \left[\left(1+\frac{R}{100}\right)^4\right]^{\frac{K}{4}} = 3^2 \text{ from } \qquad (2),$$

$$3^{\frac{K}{4}} = 3^2$$

$$\Rightarrow \quad \frac{K}{4} = 2$$

$$\therefore \quad K = 8.$$

Example 7

If ₹4000 is lent at 10% p.a, interest being compounded annually, find the interest for the fourth year.

Solution

Interest for the fourth year = Amount at the end of the first 4 years – Amount at the end of the first 3 years

$$= 4000\left(1+\frac{10}{100}\right)^4 - 4000\left(1+\frac{10}{100}\right)^3$$

$$= 4000\ (1.4641 - 1.3310)$$

$$= 4000\ (0.1331) \quad \text{i.e. ₹ } 532.40.$$

Example 8

The interest on a sum is compounded every 3 months. If the rate of interest is 40% p.a., find the effective rate of interest per annum.

Solution

Let the sum be ₹100

Amount at the end of a year

$$= 100\left(1+\frac{40}{4(100)}\right)^4 = ₹146.41$$

∴ effective rate of interest = 46.41%.

Example 9

The compound interest and the simple interest on a sum at certain rate of interest for 2 years are ₹2760 and ₹2400, respectively. Find the sum and the rate of interest.

Solution

Let the sum be ₹P and let the rate of interest be R% p.a.

Difference between the compound interest and the simple interest = ₹360

$$\therefore \quad P\left(\frac{R}{100}\right)^2 = 360 \qquad (5)$$

$$P(2)\left(\frac{R}{100}\right) = 2400$$

$$\Rightarrow \quad \frac{PR}{100} = 1200 \qquad (6)$$

$$\therefore \quad \frac{PR}{100}\left(\frac{R}{100}\right) = 1200\left(\frac{R}{100}\right) = 360$$

$$R = 30$$

Substituting R in (5) or (6),

$$P = 4000.$$

Exercises

Direction for questions 1 to 20: Select the correct alternative from the given choices.

1. Find the amount obtained by investing ₹24,000 at 18% p.a. simple interest for five years
   (A) ₹21,600 (B) ₹44,000
   (C) ₹45,600 (D) ₹48,000

2. The simple interest for the second year on a certain sum at a certain rate of interest is ₹1000. Find the sum of the interest accrued on it for the 6th, 7th, and 8th years.
   (A) ₹3200 (B) ₹3000
   (C) ₹3300 (D) ₹3630

3. In how many years will a sum of money become sixteen times itself at 30% p.a. simple interest?
(A) 25 (B) 40 (C) 30 (D) 50

4. A sum of money becomes ten times itself at simple interest. If the time period (in years) is numerically equal to the rate of interest, find the annual rate of interest.
(A) 25% (B) 20% (C) 30% (D) 90%

5. An amount of ₹2400 is due after six years under simple interest at 10% p.a. Find its present value (in ₹).
(A) 2000 (B) 1600 (C) 1800 (D) 1500

6. If ₹3000 amounts to ₹3630 in two years under compound interest, interest being compounded annually, what is the annual rate of interest?
(A) 10% (B) 21% (C) 11% (D) 10.5%

7. ₹5000 is invested for two years under compound interest at 10% p.a., interest being compounded annually. Find the interest earned (in ₹).
(A) 500 (B) 1000 (C) 2100 (D) 1050

8. A sum under compound interest, interest being compounded annually amounts to ₹6000 in two years and ₹7200 in three years. Find the rate of interest.
(A) 10% p.a. (B) 20% p.a.
(C) 18% p.a (D) 15% p.a

9. The compound interest on a sum for the third year is ₹2420, interest being compounded annually. The interest on it for the fourth year is ₹2662. Find the rate of interest.
(A) 10% p.a. (B) 11% p.a.
(C) 12% p.a (C) 13% p.a

10. A sum of money becomes four times itself in eight years at compound interest. In how many years will the same sum become sixteen times itself?
(A) 64 (B) 32 (C) 44 (D) 16

11. A sum becomes 2.197 times of itself in three years at compound interest. Find the rate of interest.
(A) 30% (B) 13% (C) 39.9% (D) 235

12. Find the interest (in ₹) earned in the first year on ₹200 at 20% p.a. compound interest, interest compounded every six months.
(A) 40 (B) 42 (C) 44 (D) 48

13. Find the effective rate of interest if the rate of interest is 40% p.a., and the interest is compounded quarterly?
(A) 42% p.a (B) 40% p.a.
(C) 44% p.a (D) 46.41% p.a.

14. Ashok borrowed a total of ₹84000 from two banks at compound interest, interest being compounded annually. One of the banks charged interest at 10% p.a. while the other charged interest at 20% p.a. If Ashok paid ₹13200 as the total interest after a year, find the difference of the sums he borrowed (in ₹).
(A) 24000 (B) 48000
(C) 54000 (D) 12000

15. If the annual rate of simple interest at which a sum is lent for two years increases by 10 percentage points, the interest realized would be ₹4000 more. Find the sum (in ₹).
(A) 20000 (B) 10000
(C) 8000 (D) 16000

16. If a sum was ₹10000 more it would fetch ₹4000 extra as simple interest, if it was lent at a certain rate of interest for two years. Find the annual rate of interest.
(A) 5% (B) 10% (C) 20% (D) 25%

17. A sum was invested under compound interest, interest being compounded annually. It fetches ₹14400 as interest in the second year and ₹17280 as interest in the third year. Find the annual rate of interest.
(A) 10% (B) 15%
(C) 20% (D) 25%

18. A sum takes T_1 years to double at R_1% p.a. simple interest. If it is lent at R_2% p.a. compound interest, interest being compounded annually, it would take the same time to double. Which of the following is always true if $T_1 > 1$?
(A) $R_1 > R_2$ (B) $0.5R_2 < R_1 < R_2$
(C) $R_1 = R_2$ (D) $R_2/3 < R_1 < R_2$

19. A sum takes two years to become 40% more under simple interest at a certain rate of interest. If it was lent at the same interest rate for the same time under compound interest, interest being compounded annually, it would amount to x% more than itself. Find x.
(A) 36 (B) 48
(C) 40 (D) 44

20. A sum was divided into two equal parts. One part was lent at 20% p.a. simple interest. The other part was lent at 20% p.a. compound interest, interest being compounded annually. The difference in the interests fetched by the parts in the second year is ₹400. Find the difference in the interests fetched by the parts in the fourth year (in ₹).
(A) 1414 (B) 1442
(C) 1456 (D) 1484

Answer Keys

1. C	2. B	3. D	4. C	5. D	6. A	7. D	8. B	9. A	10. D
11. A	12. B	13. D	14. D	15. A	16. C	17. C	18. A	19. D	20. C

Chapter 6

Averages, Mixtures, and Alligations

CHAPTER HIGHLIGHTS

- *Averages*
- *Mixtures*
- *Alligations*

Averages

'Average is a very simple but effective way of representing an entire group by a single value'.

'Average' of a group is defined as:

$$\text{Average} = \frac{\text{Sum of all items in the group}}{\text{Number of items in the group}}$$

'Sum of all the items in the group' means 'sum of the values of all the items in the group'.

A batsman's performance can be expressed as the average number of runs scored per innings rather than giving the scores in individual innings. For example, let us say a cricketer scored the following runs in 9 different innings in a year: 35, 56, 124, 29, 0, 87, 98, 45, and 75. Then his average score (per innings) for the year is

$$\frac{35 + 56 + 124 + 29 + 0 + 87 + 98 + 45 + 75}{9} = 61$$

Similarly, if there are 60 students in a class, instead of talking of the height of each individual student, we can talk of 'average' height of the class. The average height of the class of students is equal to the sum of the heights of all the students of the class divided by the number of students in the class.

Average is also called the 'mean' or mean value of all the values.

Points to Remember

1. If the value of each item is increased by the same value p, then the average of the group or items will also increase by p.
2. If the value of each item is decreased by the same value p, then the average of the group or items will also decrease by p.
3. If the value of each item is multiplied by the same value p, then the average of the group or items will also get multiplied by p.
4. If the value of each item is divided by the same value p ($p \neq 0$), then the average of the group or items will also get divided by p.
5. The average of a group of items will always lie between the smallest value in the group and largest value in the group, i.e. the average will be greater than the smallest value and less than the largest value in the group.

An Easy Method to Calculate Averages

As already discussed, the average of a group of items whose values are given can be found out by the rule given at the beginning of this section. However, in most of the cases, we do not need to perform such elaborate additions and divisions. The calculation of averages can be simplified greatly by taking some arbitrary number (P) as a starting point, take

the deviations (differences) of the given items (Q_i) from this arbitrary number, find the average of all these deviations ($Q_i - P$) and algebracially add it to the arbitrary number (P) to give the correct average of the given items.

If there are n items and they are denoted by Q_1, Q_2, Q_3, …, Q_n, then the average of these n items is given by

$$\text{Average} = P + \frac{1}{n}\sum_{i=1}^{n}(Q_i - P)$$

For example, the cricketer that we considered above earlier scored the following runs in seven innings: 35, 56, 45, 43, 67, 70, and 48. Now, to find his average, we take an arbitrary figure, say 50, and first find the deviations of each of the scores from this figure. The deviations of the scores from 50 are –15, +6, –5, –7, +17, +20 and –2. The sum of these deviations is +14.

Hence, the average of the cricketer's scores is

$$50 + \frac{14}{7} = 52$$

Please note that the number P (= 50 above) can be any value. Let us work out the same example taking a different value for P. Let us take P equal to 45. The deviations of the scores from P are –10, +11, 0, –2, +22, +25 and +3. The sum of these deviations is 49. Hence, the average is 45 + 49/7 = 45 + 7 = 52.

Weighted Average

When two groups of items are combined together, then we can talk of the average of the entire group. However, if we know only the average of the two groups individually, we cannot find out the average of the combined group of items.

For example, there are two sections A and B of a class where the average height of section A is 150 cm and that of section B is 160 cm. On the basis of this information alone, we cannot find the average of the entire class (of the two sections). As discussed earlier, the average height of the entire class is

$$\frac{\text{Total height of the entire class}}{\text{Total number of students in the entire class}}$$

Since we do not have any information regarding the number of students in the two sections, we cannot find the average of the entire class. Now, suppose that we are given that there are 60 students in the section A and 40 students in section B, then we can calculate the average height of the entire class which, in this case will be equal to $\frac{60\times150+40\times160}{60+40} = 154$ cm.

This average height 154 cm of the entire class is called '**weighted average**' of the class.

The aforementioned step in calculating the weighted average of the class can be rewritten:

$$\frac{60\times150+40\times160}{60+40} = \frac{60}{100}150 + \frac{40}{100}160 = \frac{3}{5}150 + \frac{2}{5}160$$

It is clear from the aforementioned step that we would have been able to calculate the average height of the entire class even if we had not been given the number of students in the individual sections but only the **ratio** of the number of students in the two sections (which in this case is 3 : 2).

Even if there are more than two groups of items to be combined, then also the weighted average can be calculated by the same method. For example, if three sections in a class have their average marks as 75, 76, and 79, respectively, and their respective strengths are 30, 35, and 35, then the average mark of the entire class is given by

$$\frac{30\times75+35\times76+35\times79}{30+35+35} = 76.75$$

The method of deviations we used for calculating averages can be applied to calculate weighted average also. Here, that method will involve finding out deviations from the arbitrarily chosen number and calculating the weighted average of these deviations. In the aforementioned example, if we take 70 as the arbitrary figure, then the deviations of the three observed values given from 70 are +5, +6, and +9. The weighted average of these deviations is

$$\frac{30\times5+35\times6+35\times9}{30+35+35} = \frac{675}{100} = 6.75$$

Hence, the weighted average will be 70 + 6.75 = 76.75.

The arbitrary figure chosen can be any figure, and if it is selected, as in the previous case, between the smallest and largest observed figures, some of the deviations will be positive and some negative making the final division relatively simpler. For example, in the aforementioned case, if we take with 76 as the arbitrary figure, the deviations are –1, 0, and +3. Then, the weighted average will be

$$\frac{30-(-1)+35-0+35-(+3)}{30+35+35} = \frac{75}{100} = 0.75$$

Hence, the weighted average will be 76 + 0.75 = 76.75.

'Weighted average' can be defined or calculated for any MIXTURE.

Mixtures

Mixing of two or more qualities of things produces a mixture. When two items of different qualities are thus mixed, the quality of the resultant mixture lies in between the qualities of the original constituent items, i.e. it will be higher than the lowest quality and lower than the highest quality of the items being mixed.

In the aforementioned example that we took, the 'quality' that we looked at was the height of the students. We could also have taken their weights or the marks scored by them or any other 'quality' or 'parameter' and calculated the 'weighted average' value of that particular 'quality' for the entire group.

Similarly, if two types of a product of different prices per unit are mixed, the unit price of the resultant mixture will lie between the prices of the two types that form the mixture.

Here, the average quality is essentially the **weighted average** of the two constituent items.

If q_1 is the quantity (or number of items) of one particular item of quality p_1, and q_2 be the quantity (or number of items) of the second item of quality p_2 are mixed together to give a new mixture, then the **weighted average** value (p) of the quality of the mixture is given by

$$\boxed{p = \frac{p_1\, q_1 + p_2\, q_2}{q_1 + q_2}}$$

Even if there are more than two groups of items mixed, the weighted average rule can be applied. We will only have to take figures (as shown in the formula for the two groups) for all the groups in the numerator as well as the denominator and calculate the weighted average. For example, if there are four groups of quantities q_1, q_2, q_3, and q_4 whose respective qualities are p_1, p_2, p_3, and p_4, then the weighted average quality of the group can be written as

$$\boxed{p = \frac{p_1\, q_1 + p_2\, q_2 + p_3\, q_3 + p_4\, q_4}{q_1 + q_2 + q_3 + q_4}}$$

A mixture can also be a solution—that is, a liquid mixed with another liquid which is normally water. The concentration of the solution is expressed as the proportion (or percentage) of the liquid in the total solution.

For example, if 10 litres of pure alcohol is mixed with 40 litres of water, then in a total solution of 50 litres, there is 10 litres of alcohol. Hence, the concentration of this solution is 0.2 (= 10/50) or 20%.

Similarly, if 30 litres of pure milk is mixed with 10 litres of water, the concentration of this solution can be expressed as 75% (= 30/40) milk or 25% water.

We can also have two solutions mixed together to give a new solution. Such problems can also be handled in the same manner as other mixtures. In the weighted average rule, the quality of the constituents (p_1, p_2, etc.) will then be the concentrations of various solutions mixed together.

Solved Examples

Example 1

Rajiv purchased three dozen mangoes at ₹10 per dozen, two dozen mangoes at ₹15 per dozen, and five dozen mangoes at ₹16 per dozen. Find the average cost per dozen of the mangoes that he purchased.

Solution

The cost of first three dozen mangoes = (3) (10) = ₹30

The cost of next two dozen mangoes = (2) (15) = ₹30

The cost of next five dozen mangoes = (5) (16) = ₹80

Total cost of the mangoes purchased = ₹140.

Average cost per dozen

$$= \frac{\text{Total cost of mangoes}}{\text{Number of dozens}} = \frac{140}{10} = ₹14$$

Example 2

The average age of 5 men is 20 years. Their average age increased by 1 year when a new man joined them. Find the age of the new man.

Solution

Total age of 5 men = (5) (20) = 100 years

Total age of 6 men = (6) (21) = 126 years

The age of the new man = 126 – 100

i.e. 26 years

Example 3

Six kilograms of wheat costing of ₹18 per kg is mixed with nine kilograms of wheat costing of ₹12 per kg. Find the price per kg of the mixture.

Solution

Total cost of 6 kg = (6) (18) = ₹108

Total cost of 9 kg = (9) (12) = ₹108

$$\text{Average cost of the mixture} = \frac{\text{Total cost}}{\text{Total quantity}}$$

$$= \frac{108\,(2)}{15} = ₹\,14.40 \text{ per kg}$$

Example 4

The average marks of three sections in the tenth class were 90, 120, and 150. If the number of students in these sections are 30, 40, and 50, respectively, find the average mark of the tenth class.

Solution

Total mark of the first section = (90) (30) = 2700

Total mark of the second section

= (120) (40) = 4800

Total mark of the third section = (150) (50) = 7500

$$\text{Average mark of class } X = \frac{\text{Total mark}}{\text{Number of students}}$$

$$= \frac{2700+4800+7500}{30+40+50} = \frac{15000}{120} = 125$$

Example 5

Tarun earned an average of ₹1500 per month from January to April in a year. He earned an average of ₹1600 per month from May to October in that year. His earning in the month of December of that year was ₹300 more than his earning in

the month of November of that year. His average monthly earnings for that year was ₹1675. Find his earnings in the month of November.

Solution

Total earnings of Tarun from January to April = (1500) (4) = ₹6000

Total earnings of Tarun from May to October = (1600) (6) = ₹9600

Let his earnings in November be ₹x

His earnings in December = ₹ $(x + 300)$

$$\frac{6000+9600+x+x+300}{12} = 1675$$

$$x = 2100$$

Example 6

The average age of a group of friends is 37 years. If 6 new friends whose average age is 35 years join them, the average age of the entire group becomes 36 years. How many people were there in the group initially?

Solution

Let the initial number of people in the group be n.

The total age of the initial group of friends = $37n$ years

The total age of the six friends who joined the group = 35×6 i.e., 210 years.

Given that, $37n + 210 = 36\ (n + 6)$

$\Rightarrow \quad 37n - 36n = 216 - 210$

$\therefore \quad n = 6$

Alligations

We will take the weighted average rule discussed in the previous section and rewrite the formula such that the quantity terms come on one side and the price terms come on the other side. If we do this we get the rule $\frac{q_1}{q_2} = \frac{p - p_2}{p_1 - p}$

This is called the RULE OF ALLIGATION. This rule connects quantities and prices in mixtures. This can also be written as $\frac{q_1}{q_2} = \frac{p_2 - p}{p - p_1}$

In a descriptive manner, the Rule of Alligation can be written as $\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}}$

$$= \frac{\text{Rate of Dearer} - \text{Average Rate}}{\text{Average Rate} - \text{Rate of Cheaper}}$$

This rule is a very powerful rule and is useful in problems on weighted averages and mixtures. This rule is also useful in a number of problems which can be treated as mixtures and applied to parameters other than price also. We will take examples where alligation rule can be applied.

In actual practice, to apply alligation rule, we do not need to remember the aforementioned formula at all. It can be made very simple by representing the rule pictorially. The formula can be represented as follows:

Dearer Price ↘ ↙ Cheaper Price

Average Price

(Average Price – Cheaper Price) ↙ ↘ (Dearer Price – Average Price)

{We write the dearer and cheaper prices in one line with some gap in between them. Then, we write the average price in between these two but in the line below the line in which dearer and cheaper prices are written. Then, take the differences of quantities as shown along the arrows and write along the same direction of the arrows continued, i.e. in a diagonally opposite corner. The difference between the top left hand quantity (dearer price) and average price has to be written at the bottom right hand corner. Similarly, the difference between the top right hand corner (cheaper price) and the average price has to be written at the bottom left hand corner. Now the ratio of the two quantities in the bottommost line will give us the ratio of the quantities of dearer and cheaper varieties. Please note that since we took dearer price on the top left corner, the ratio of the bottom left figure to that of the bottom right figure will give the ratio of dearer quantity to cheaper quantity.}

Example 7

A bag contains a total of 120 coins in the denominations of 50 p and ₹1. Find the number of 50 p coins in the bag if the total value of the coins is ₹100.

Solution

Let the number of 50 p coins be x.

Total value of the coins = ₹$\left[\frac{50x}{100} + 1\,(120 - x)\right]$

$$\frac{50x}{100} + 120 - x = 100$$

$\Rightarrow \quad x = 40$

Alternative method:

Average value per coin = $\frac{(100)(100)}{120}$

$= \frac{500}{6}$ paise

Using allegation rule, $\frac{x}{120 - x}$

$$= \frac{100 - \frac{500}{6}}{\frac{500}{6} - 50} = \frac{1}{2}$$

$\Rightarrow \quad 2x = 120 - x$

$\Rightarrow \quad x = 40$

Example 8

A vessel has 60 litres of solution of milk and water having 80% milk. How much water should be added to it to make it a solution in which milk forms 60%?

Solution

Let the quantity of water to be added be x litres.

Quantity of milk in the vessel

$$= \left(\frac{80}{100}\right)(60) = 48 \text{ litres}$$

$\Rightarrow \qquad 48 = 0.6\,(60 + x)$

$\therefore \qquad x = 20$

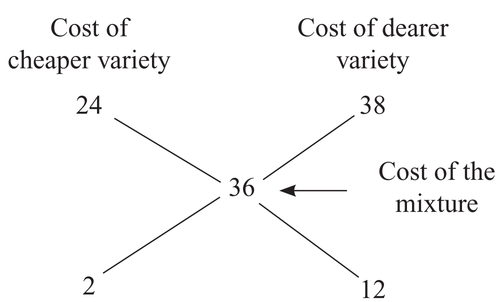
Example 9

In what ratio must Anand mix two varieties of barley costing ₹24 per kg and ₹38 per kg so that by selling the mixture at ₹40 per kg he would make $11\frac{1}{9}\%$ profit?

Solution

Cost price of the mixture $= \dfrac{40(100)}{100 + 11\frac{1}{9}}$

$= ₹36$ per kg



$\therefore$ The ratio of the costs of the cheaper to the dearer variety $= 2 : 12 = 1 : 6$.

If there is P volume of pure liquid initially and, in each operation, Q volume is taken out and replaced by Q volume of water, then at the end of n such operations, the concentration (k) of the liquid in the solution is given by $\left\{\frac{P-Q}{P}\right\}^n = k$

This gives the concentration (k) of the liquid as a Proportion of the total volume of the solution.

If the concentration has to be expressed as a percentage, then it will be equal to 100 k.

If the volume of the liquid is to be found out at the end of n operations, it is given by kP, i.e. the concentration k multiplied by the total volume P of the solution.

Example 10

A vessel has 400 litres of pure milk. 40 litres of milk is removed from the vessel and replaced by water. 40 litres of the mixture thus formed is replaced by water. This procedure is repeated once again. Find the percentage of milk in the resultant solution.

Solution

Let v litre be volume of milk with a concentration of $c_1\%$.

If x litres of the solution is removed and replaced with water, the new concentration is $\left(\frac{v-x}{v} \times c_1\right)\%$

Given that initial concentration is 100%. (pure milk), $v = 400$, $x = 40$ and the replacement is done thrice.

$\therefore$ Concentration of milk in the resultant solution

$$= \left(\frac{400-40}{400}\right)^3 \times 100 = 72.9\%$$

Exercises

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. Find the average of all the two digit numbers divisible by 10.
   (A) 40 (B) 50 (C) 45 (D) 60
2. Find the average of all odd numbers less than 50.
   (A) 26.5 (B) 25.5 (C) 26 (D) 25
3. Find the average of all the multiples of 12 less than 100.
   (A) 48 (B) 54 (C) 60 (D) 66
4. The average salary per month of a man for the first four months, next four months, and the last four months of a year are ₹6000, ₹8000, and ₹13000, respectively. Find his average salary per month in that year (in ₹).
   (A) 7500 (B) 9000
   (C) 10500 (D) 6600
5. In an office there are 20 employees. The average heights of the male employees is 180 cm. The average height of the female employees is 170 cm. Find the average height of all the employees (in cm).
   (A) 172 (B) 174
   (C) 176 (D) Cannot be determined
6. The average age of the boys in a class is ten years. The average age of the girls in the class is eight years. There are 50% more boys than girls in the class. Find the average age of the class (in years).
   (A) 8.4 (B) 8.8 (C) 9.2 (D) 9.6

7. A vessel has 20 litres of a mixture of milk and water having 60% milk. Five litres of pure milk is added to the vessel. Find the percentage of milk in the new solution.
(A) 34% (B) 51%
(C) 68% (D) None of these

8. In what ratio must two kinds of coffee which cost ₹80 per kg and ₹108 per kg be mixed such that the resultant mixture costs ₹96 per kg?
(A) 1 : 2 (B) 2 : 3
(C) 3 : 4 (D) 2 : 1

9. Vessel *A* has 20 litres of a mixture of milk and water having 75% milk. Vessel *B* has *x* litres of a mixture of milk and water having 60% milk. The contents of the vessels are mixed to form a mixture having 66% milk. Find *x*.
(A) 25 (B) 30 (C) 20 (D) 40

10. A milkman has 15 litres of pure milk. How many litres of water have to be added to it so that he gets a 60% profit by selling at cost price?
(A) 9 (B) 10 (C) 8 (D) 12

11. From 90 litres of pure milk, 9 litres is withdrawn and replaced by water. 9 litres of the mixture is then withdrawn and replaced by water. Find the ratio of milk and water in the present mixture.
(A) 19 : 81 (B) 19 : 100
(C) 81 : 19 (D) 81 : 100

12. Just before the last match in a season, the total number of runs scored by Sachin Tendulkar added up to 2100. In his last match, he scored 101 runs. As a result his average score for the season went up by one run. Find the total number of matches he played in that season if he got out in every match.
(A) 31 (B) 5
(C) 71 (D) Either 31 or 71

13. The average weight of all the students of classes I and II equals the average of the average weight of the students of the two classes. There are twice as many students in class II as in class I. The sum of twice the average weight of the students of class I and the average weight of the students of class II is 60 kg. Find the average weight of class I (in kg).
(A) 10 (B) 15 (C) 20 (D) 25

14. Two varieties of wheat are mixed in the proportion of 3 : 4 and the mixture is sold at ₹28 per kg at a profit of 40%. If the second variety of wheat costs ₹3 more than the first variety of wheat, find the cost price of the first variety of wheat.
(A) ₹128/7 per kg (B) ₹120/7 per kg
(C) ₹141/7 per kg (D) ₹149/7 per kg

15. A man buys milk at ₹4 per litre, mixes it with water and sells the mixture at the same price. If his profit is 25%, find the amount of water mixed with each litre of milk.
(A) 0.25 litres (B) 0.5 litres
(C) 0.75 litres (D) 0.6 litres

16. In what proportion can three varieties of sugar priced at ₹10 per kg, ₹12 per kg, and ₹18 per kg, be mixed so that the price of the mixture is ₹14 per kg?
(A) 2 : 2 : 5 (B) 2 : 3 : 4
(C) 1 : 3 : 4 (D) 3 : 4 : 5

17. The ratio of alcohol and water in three mixtures of alcohol and water is 3 : 2, 4 : 1, and 7 : 3. If equal quantities of the mixtures are drawn and mixed, the concentration of alcohol in the resulting mixture will be______.
(A) 65% (B) 70% (C) 75% (D) 80%

18. In what proportion should milk and water be mixed to reduce the cost of litre of milk from ₹18 per litre to ₹16?
(A) 8 : 1 (B) 6 : 1 (C) 10 : 1 (D) 7 : 1

19. *A*'s weight equals the average weight of *B*, *C,* and *D*. *B*'s weight equals the average weight of *A*, *C,* and *D*. The average weight of *C* and *D* is 30 kg. Find the average weight of *A* and *B*.
(A) 15 kg (B) 30 kg (C) 60 kg (D) 45 kg

20. Of five numbers, the first number is thrice the third, the fourth number is two less than the first, the fifth number is one-seventh of the second and the second number is three less than thrice the first. Find the fifth number, if the average of the numbers is 16.2.
(A) 3 (B) 4 (C) 5 (D) 6

21. There are nine two-digit numbers with distinct tens digits. The units digit of each number is one less than its tens digit. Find the average of the units digits.
(A) 3 (B) 4 (C) 5 (D) 6

22. A sum of ₹7.75 is made up of 100 coins, which are in the denominations of 5 paise and 10 paise. Find the number of 5 paise coins.
(A) 50 (B) 55 (C) 75 (D) 45

23. A businessman lends ₹1800 in two parts, one at 10% and the other at 12% interest. At the end of the year, the average interest he obtained worked out to be 10.5%. Find the interest earned by the businessman from the part which was lent at 10%.
(A) ₹135 (B) ₹150 (C) ₹200 (D) ₹250

24. A vessel is full of a mixture of milk and water, with 9% milk. Nine litres are withdrawn and then replaced with pure water. If the milk is now 6%, how much does the vessel hold?
(A) 27 litres (B) 18 litres
(C) 36 litres (D) 40 litres

25. Three varieties of rice, *A*, *B,* and *C* costing ₹6/kg, ₹9/kg and ₹12/kg are mixed together in a certain ratio. The mixture is sold at $66\frac{2}{3}$% profit for ₹15 / kg. Of the total of 100 kg of the mixture, 50 kg is variety *B*. Find the quantity of variety *A* (in kgs)
(A) 15 (B) 25 (C) 20 (D) 10

Answer Keys

1. B	**2.** D	**3.** B	**4.** B	**5.** D	**6.** C	**7.** C	**8.** C	**9.** B	**10.** A
11. C	**12.** D	**13.** C	**14.** A	**15.** A	**16.** D	**17.** B	**18.** A	**19.** B	**20.** D
21. B	**22.** D	**23.** A	**24.** A	**25.** B					

Chapter 7

Time and Work

CHAPTER HIGHLIGHTS

- *Work and Time*
- *Unitary Method*
- *Pipes and Cisterns*

Work

Work to be done is usually considered as one unit. It may be constructing a wall or a road, filling up or emptying a tank or cistern, or eating certain amount of food.

There are some basic assumptions that are made in the problems on Time and Work. These are taken for granted and are not specified in every problem.

1. If a person (or one member of the workforce) does some work in a certain number of days, then we assume (unless otherwise explicitly stated in the problem) that he does the work uniformly, i.e. he does the SAME amount of work everyday.
   For example, if a person can do some work in 15 days, he does 1/15th of the work in one day.
   If a person completes the work in 4 days, he does 1/4th of the work on each day, and conversely, if a person can complete 1/4th of the work in one day, he can complete the work in 4 days.
   If a tap can fill a tank in 20 minutes, then in one minute, it can fill 1/20th part of the tank.
2. If there is more than one person (or members of 'workforce') carrying out the work, it is assumed that each person (or members of the workforce), unless otherwise specified, does the same amount of work each day. This means they share the work equally.

If two people together can do the work in 8, days, it means that one man can do it in 16 days. This, in turn means, each person can do 1/16th of the work per day.

If a man works three times as fast as a boy does, the man takes one-third of the time the boy takes to complete the work. If the boy takes 12 days to complete the work, then the man takes 4 days to complete the work.

This method is known as '**UNITARY METHOD**', i.e. the time taken per '**Unit Work**' or number of persons required to complete 'Unit Work' or work completed by 'Unit Person' in 'Unit Time', etc. is what is first calculated.

We should recollect the fundamentals on variation (direct and inverse) here.

1. Time remaining constant, Work and Men are directly proportional to each other, i.e. if the work increases the number of men required to complete the work in the same number of days increases proportionately and viceversa.
2. Work remaining constant, Men and Days are inversely proportional, i.e. if the number of men increases, the number of days required to complete the same work decreases in inverse proportion, and viceversa.
3. The number of workingmen remaining constant, Work and Days are directly proportional, i.e. if the work increases, the number of days required to complete the work with the same number of working men also proportionately increases and viceversa.

The concept of MANDAYS is very important and useful here. The number of men multiplied by the number of days that they take to complete the work will give the number of mandays required to do the work. The total number of mandays required to complete a specific task will remain

a constant. So, if we change one of the variables—men or days—the other will change accordingly so that their product will remain constant (remember from our knowledge of VARIATION, two variables whose product is a constant are said to be inversely proportional to each other). The two variables—men and days—are inversely proportional to each other, when work is constant.

Solved Examples

Example 1

If 15 men take 60 days to complete a job, find the time taken by 45 men to complete it.

Solution

Number of mandays required to complete the job = 900 mandays. Time taken by 45 men to complete it = $\frac{900}{45}$, i.e. 20 days.

Example 2

18 men take 20 days to complete a job working 12 hours a day. Find the number of days that 15 men will take to complete it if they work 9 hours a day.

Solution

Total time for which 18 men work = 240 hours. Number of man hours required to complete the job = (18) (240) man hours.

Number of days taken by 15 men working 9 hours a day to complete it = $\frac{(18)(240)}{(15)(9)} = 32$.

Hence, in general we can say that

If M_1 men can do W_1 work in D_1 days working H_1 hours per day and M_2 men can do W_2 work in D_2 days working H_2 hours per day (where all men work at the same rate), then

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$

Example 3

20 men take 10 days to complete a job working 12 hours a day. Find the number of men required to complete a job, twice as large, in 30 days working 8 hours a day.

Solution

Number of man hours required to complete the job = (20) (10) (12) = 2400

Number of men required to complete a job twice as large by 240 hours = $\frac{2400 \times 2}{240}$, i.e. 20 days.

Alternative method:

$$M_1 = 20, D_1 = 10, H_1 = 12$$

$$D_2 = 30, H_2 = 8$$

$$D_2 = 2W_1$$

$$M_2 = \frac{M_1 D_1 H_1 W_2}{W_1 D_2 H_2} = \frac{(20)(10)(12)(2W_1)}{W_1(30)(8)} = 20.$$

If two persons A and B can individually do some work in p and q days, respectively, we can find out how much work can be done by them together in one day. Since A can do $1/p$th part of the work in one day and B can do $1/q$th part of the work in one day, the two of them together do $(1/p + 1/q)$th part of the work in one day.

From this, we can find out the number of days that they take to complete the work.

If A can do a piece of work in p days and B can do it in q days then A and B together can complete the same in $\frac{pq}{p+q}$ *days.*

Example 4

A and B can complete a job in 10 days and 12 days, respectively. Find the time taken to complete it, if both A and B work together.

Solution

Time taken by them to complete it

$$= \frac{(10)(12)}{10+12} = \frac{60}{11} \text{ days.}$$

Example 5

A and B together can complete a job in 12 days. A alone can complete it in 24 days. Find the time taken by B to complete it.

Solution

Part of the job that A and B can complete in a day = $\frac{₹1320}{6}$

Part of the job that A can complete in a day = $\frac{1}{24}$

Part of the job that B can complete in a day

$$= \frac{1}{12} - \frac{1}{24} = \frac{1}{24}$$

∴ B can complete it in 24 days.

Example 6

Ajay and Bala working together can complete a job in 16 days. Ajay alone can complete it in 18 days. Both work together for 4 days and then Bala leaves. Find the time taken by Ajay to complete the remaining work.

Solution

Part of the job that can be done by both in a day $= \frac{1}{16}$

Part of the job that can be done by them in 4 days $= 4 \times \frac{1}{16} = \frac{1}{4}$

Remaining part of the job $= \frac{3}{4}$

Time taken by Ajay to complete it

$$= \frac{3}{4}(18) = 13.5 \text{ days.}$$

Example 7

A can complete a job in 16 days. He started the work and after 4 days, *B* joined him. They completed the job in 4 more days. Find the number of days in which *B* alone can complete it.

Solution

Part of the job done by A in a day $= \frac{1}{16}$

A worked for a total of 8 days

$\therefore$ A completed $(8)\left(\frac{1}{16}\right) = \frac{1}{2}$ of the job.

Hence, B can complete the remaining $\frac{1}{2}$ of the job in 4 days.

$\therefore$ B alone can complete the entire job in 8 days.

Example 8

P and Q together can complete a job in $14\frac{2}{5}$ days. Q and R together can complete it in $20\frac{4}{7}$ days. P and R together can complete it in 16 days. Find the time taken by each of them to complete the job.

Solution

Part of the job that P and Q can do in a day $= \frac{5}{72}$

Part of the job that Q and R can do in a day $= \frac{7}{144}$

Part of the job P and R can do in a day $= \frac{1}{16}$

Let the time taken by P, Q, and R to complete the job be p days, q days, and r days, respectively.

$$\frac{1}{p} + \frac{1}{q} = \frac{5}{72} \quad (5)$$

$$\frac{1}{q} + \frac{1}{r} = \frac{7}{144} \quad (6)$$

$$\frac{1}{p} + \frac{1}{r} = \frac{1}{16} \quad (7)$$

Adding (5) and (6) and subtracting (7),

$$\Rightarrow \quad \frac{2}{q} = \frac{5}{72} + \frac{7}{144} - \frac{1}{16} = \frac{8}{144}$$

$$q = 36$$

substitute $q = 36$, in (5),

we get $p = 24$

substi $q = 36$ in (6),

we get $r = 48$.

Example 9

A contractor decided to complete a job in 30 days for which he employed 20 men in the beginning. After 10 days, he released that the job could not be completed on time. Hence, he employed 15 more men and thus completed the job on time. Find the number of extra days, it would have taken to complete the job if the additional men were not employed.

Solution

Number of mandays required to complete the job = (20)(10) + (20 + 15) (20) = 900 mandays.

If the additional men were not employed, number of extra days $= \frac{900}{20} - 30 = 15$ days.

Example 10

A and B together can complete a job in 18 days and 36 days, respectively. They work on alternate days with A starting the job. In how many days will the job be completed?

Solution

Part of the job completed in the first 2 days

$$= \frac{1}{18} + \frac{1}{36} = \frac{1}{12}$$

$\therefore$ To complete the job, 12 cycles of 2 days, i.e. a total of 24 days will be required.

Example 11

P and Q together can complete a job in 8 days and 16 days respectively. They work on alternate days with Q starting the job. In how many days will the job be completed?

Solution

Part of the job completed by P and Q in the first 2 days $= \frac{1}{8} + \frac{1}{16} = \frac{3}{16}$

After 5 cycles of 2 days, i.e. after 10 days, $\frac{15}{16}$th of the job will be completed.

Remaining part $= \frac{1}{16}$th. Q will work on the 11th day and he takes exactly one day to complete the remaining part.

$\therefore$ The job will be completed in 11 days.

In general, money earned should be shared by people doing the work together in the ratio of the **SHARE OF WORK** done by each of them.

For example, if A does two-fifth of the work, then he should get two-fifth of the total earnings for the work. If the remaining three-fifth of the work is done by B and C in the ratio of 1 : 2, then the remaining three-fifth of the earnings (after paying A) should be shared by B, and C in the ratio of 1 : 2. Suppose ₹500 is paid to A, B, and C together for doing the work, then A will get ₹200 (which is 2/5 of ₹500), B will get ₹100, and C, ₹200 (because the remaining ₹300 after paying A is to be divided in the ratio 1 : 2 between B and C).

When people work for the same number of days each, then the ratio of the total work done will be the same as the work done by each of them PER DAY. Hence, if all the people involved work for the same number of days, then the earnings can directly be divided in the ratio of work done per day by each of them.

Example 12

P, Q, and R can together earn ₹3100 in 10 days. Q and R together can earn ₹1320 in 6 days. P and R together can earn ₹1050 in 5 days. Find R's daily earning.

Solution

Total daily wage of P, Q, and $R = \dfrac{₹3100}{10} = ₹310$

Total daily wage of Q and $R = \dfrac{₹1320}{6} = ₹220$

Total daily wage of P and $R = \dfrac{₹1050}{5} = ₹210$

Total daily wage of P, Q, and $2R = ₹430$

$\therefore$ R's daily wage $= ₹120$.

Example 13

Two men under take a job for ₹960. They can complete it in 16 days and 24 days. They work along with a third man and take 8 days to complete it. Find the share that the third man should get.

Solution

The amount payable should be proportional to the fraction of work done.

Part of the job done by the third man

$$= 1 - \left(\frac{8}{16} + \frac{8}{24}\right) = \frac{1}{6}.$$

$\therefore$ The third man should get $\dfrac{₹960}{6} = ₹160$.

Pipes and Cisterns

There can be pipes (or taps) filling (or emptying) tanks with water. The time taken by different taps (to fill or empty the tank) may be different. Problems related to these can also be dealt with in the same manner as the foregoing problems on Work have been dealt with.

There is only one difference between the problems on regular Work (of the type seen earlier on in the chapter) and those in Pipes and Cisterns. In Pipes and Cisterns, a filling pipe or tap does positive work and an emptying pipe or a leak does negative work.

Example 14

Pipes P and Q take 24 minutes and 36 minutes respectively to fill an empty tank. If both take 18 minutes to fill a tank along with an outlet pipe R, find the time R would take to empty the full tank.

Solution

Let the time taken by R to empty the tank be r minutes.

$$\frac{1}{24} + \frac{1}{36} - \frac{1}{r} = \frac{1}{18}; \quad r = 72.$$

Example 15

Pipes X and Y can fill a tank in 30 minutes and 60 minutes, respectively. Both pipes are opened simultaneously. After how much time should X be closed so that the tank is filled in 30 minutes?

Solution

Let us say pipe X should be closed after n minutes.

i.e. pipe X is in operation for n minutes and pipe Y for all the 30 minutes.

So, $\dfrac{n}{30} + \dfrac{30}{60} = 1$

$\Rightarrow$ $n = 15$.

Example 16

Pipes P, Q, and R together can empty a full tank in 6 hours. All the three pipes are opened simultaneously and after 2 hours, P is closed. The tank is emptied in another 6 hours. Find the time in which P can empty the tank.

Solution

Part of the tank that can be emptied by P, Q, and R per hour $= \dfrac{1}{6}$.

Part of the tank that was emptied by P, Q, and R in 2 hours $= \dfrac{1}{3}$

Part of the tank which was emptied by Q and R per hour

$$= \frac{1 - \frac{1}{3}}{6} = \frac{1}{9}$$

Time in which P can empty the tank

$$= \frac{1}{\frac{1}{6} - \frac{1}{9}}, \text{ i.e. 18 hours.}$$

Example 17

A tank has a leak at its bottom which empties it at 6 litres/minutes. It also has a filling tap which can fill the tank in 6 hours. The tank takes 18 hours to become full. Find the capacity of the tank.

Solution

Let the time that would be taken by the leak to empty the full tank be x hours.

$$\therefore \quad \frac{1}{6} - \frac{1}{x} = \frac{1}{18}; x = 9$$

$\therefore$ Capacity of the tank = (6) (9) (60)

= 3240 litres.

EXERCISES

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. X men can complete a work in 120 days. If there were 10 men more, the work would be completed in 20 days less. Find the value of X.
   (A) 75 (B) 50 (C) 90 (D) 60

2. Nine men can complete a job in 15 days. If a man works thrice as fast as a woman, find the number of days taken by 15 women to complete the job.
   (A) 20 (B) 24 (C) 27 (D) 36

3. The ratio of the time taken by A, B, and C to complete a job is 3 : 4 : 6. Find the ratio of the work they can complete in an hour.
   (A) 6 : 4 : 3 (B) 4 : 3 : 2
   (C) 2 : 3 : 4 (D) 3 : 4 : 6

4. Amar, Bharat, and Charu can complete a job in 12, 24, and 24 days, respectively. If they all work together, how long will they take to complete the same work?
   (A) 18 days (B) 6 days
   (C) 20 days (D) 16 days

5. Adam can complete a job in 25 days. Adam and Chris together can complete it in $9\frac{3}{8}$ days. In how many days can Chris alone complete the job?
   (A) $12^5/_8$ (B) 10 (C) 25 (D) 15

6. P and Q can complete a job in 10 days. Q and R can complete it in 12 days. P and R can complete it in 20 days. Who is the slowest of the three workers?
   (A) P
   (B) Q
   (C) R
   (D) Cannot be determined

7. Ten men can do a piece of work in 15 days. How many men are needed to complete a work which is five times as large as the first one, in 10 days?
   (A) 60 (B) 75 (C) 70 (D) 85

8. Tap X can fill a tank in 10 hours. Tap Y can fill it in 15 hours. If the two taps fill the tank together, what fraction of the tank is filled by X?
   (A) 1/10 (B) 1/6 (C) 2/3 (D) 3/5

9. Pipe A can fill an empty tank in 9 hours. Pipe B can empty a full thank in 18 hours. If both pipes are opened simultaneously when the tank is empty, find the time taken to fill the tank (in hours).
   (A) 24 (B) 27 (C) 18 (D) 36

10. Raj can build a wall in 18 days and Kiran can do the same in 30 days. After Raj had built half the wall, Kiran joins him. What is the total number of days taken to build the wall?
    (A) 24 (B) $14\frac{5}{8}$ (C) $15\frac{1}{2}$ (D) $16\frac{1}{2}$

11. Kaushik is one and a half times more efficient than Ravi. Kaushik can do a piece of work in 20 days. What portion of the total work can both of them together complete in 10 days?
    (A) 3/10 (B) 4/5 (C) 9/10 (D) 7/10

12. Had then been one men less, then the number of days required to do a piece of work would have been one more. If the number of mandays required to complete the work is 56, how many workers were there?
    (A) 6 (B) 8 (C) 9 (D) 14

13. In 8 days, Peter can do as much work as Pan can do in 12 days. To do a certain job both together take 36 days. In how many days can Pan, working alone, complete the job?
    (A) 60 days (B) 80 days
    (C) 108 days (D) 90 days

14. X can complete a job in 36 days and Y can complete it in 45 days. Z can complete the job in z days. Z started the job. After 28 days, X and Y joined. The job was completed in 4 more days. Find z.
    (A) 40 (B) 35 (C) 30 (D) 50

15. Working in pairs, PQ, QR, and RP can complete a job in 24 days, 20 days, and 30 days, respectively. Find the respective times taken by P, Q, and R individually to complete the same job (in days).
    (A) 48, 80, $\frac{240}{7}$ (B) 80, 48, $\frac{240}{7}$
    (C) 80, $\frac{240}{7}$, 48 (D) 48, $\frac{240}{7}$, 80

16. A frog was at the bottom of a 80 m deep well. It attempted to come out of it by jumping. In each jump, it covered 1.15 m but slipped down by 0.75 m. Find the number of jumps after which it would out of the well.
(A) 198 (B) 201 (C) 200 (D) 199

17. A man, a woman and a boy can do a piece of work in 2, 4, and 8 days, respectively. How many boys must work together with one man and one woman to complete the work in one day?
(A) 5 (B) 4 (C) 2 (D) 1

18. A machine of type *A* which has to produce a set of 1500 bolts, can do so in 30 days. The machine breaks down after 10 days. A machine of type *B* completes the remaining work in 10 days. In 30 days how many bolts can both of them together produce?
(A) 3000 (B) 4500 (C) 6000 (D) 2500

19. In a farm, each cow eats twice as much grass as each sheep. The cost of grass for 10 cows and 40 sheep for 20 days is ₹ 900. Find the cost of grass for 20 cows and 10 sheep for 18 days (in ₹).
(A) 600 (B) 675 (C) 750 (D) 800

20. The cost of grass for 20 cows and 30 sheep for 30 days is ₹720. If the 30 sheep eat double the grass eaten by the 20 cows, then what is the cost of grass eaten by 20 sheep in 15 days?
(A) ₹200 (B) ₹160 (C) ₹240 (D) ₹100

21. George and Gagan together repair a bridge in 45 days and receive ₹13500. If Gagan is three times as efficient as George, what is the amount of money he earns in 10 days?
(A) ₹2000 (B) ₹2250 (C) ₹2500 (D) ₹2750

22. Two pipes *A* and *B* which can fill a tank in 20 and 30 hours, respectively, were opened simultaneously. But there was a leak and it took 3 hours more to fill the tank. In how many hours can the leak empty the tank?
(A) 60 (B) 50 (C) 30 (D) 40

23. Gokul, Govardhan, and Ganesh can do a piece of work in 10, 20, and 30 days, respectively. They begin a new job of similar nature and each of them works on it for one third of the total period of work. If they get ₹6600 for the new job, how much should Govardhan get, given that the amounts distributed are in proportion to the work done by them?
(A) ₹1800 (B) ₹2200 (C) ₹3300 (D) ₹2400

24. Rakesh and Ramesh take 30 days and 60 days, respectively to complete a job. They work on alternate days to complete it with Rakesh starting the job. Find the time in which the job is completed (in days).
(A) 60 (B) 80 (C) 40 (D) 90

25. If Rakesh and Ramesh had instead taken 10 days and 12 days, respectively, to complete the job, find the time in which the job would have been completed (in days).
(A) $10\frac{1}{3}$ (B) $10\frac{5}{6}$
(C) 11 (D) $10\frac{1}{2}$

Answer Keys

1. B	**2.** C	**3.** B	**4.** B	**5.** D	**6.** C	**7.** B	**8.** D	**9.** C	**10.** B
11. D	**12.** B	**13.** D	**14.** A	**15.** C	**16.** D	**17.** C	**18.** B	**19.** B	**20.** B
21. B	**22.** A	**23.** A	**24.** C	**25.** B					

Chapter 8

Time and Distance

CHAPTER HIGHLIGHTS

- *General problems on Time, Speed, and Distance*
- *Speed*
- *Relative Speed*
- *Average Speed*
- *Boats and Streams*
- *Races and Circular Tracks*

Introduction

In this chapter, we will look at problems in the following different areas:

1. General problems on Time, Speed, and Distance
2. Relative Speed
3. Boats and Streams
4. Races and Circular Tracks

Before we look at problems in various areas, let us first look at some basic concepts pertaining to speed, time and distance.

Speed

Distance covered per unit time is called speed.

i.e. Speed = Distance/Time

The above relationship between the three variables distance, speed, and time can also be expressed as follows:

Distance = Speed × Time or Time = Distance/Speed

1. If two bodies travel with the same speed, distance covered ∝ time (Direct Variation).
2. If two bodies travel for the same period of time, distance covered ∝ speed (Direct Variation).
3. If two bodies travel the same distance,

$$\text{Time} \propto \frac{1}{\text{Speed}} \text{ (Inverse Variation).}$$

Distance is normally measured in kilometres, metres, or miles; time in hours or seconds and speed in km/hr (also denoted by kmph), miles/hr (also denoted by mph), or metres/second (denoted by m/s).

To convert speed in kmph to m/sec, multiply it with 5/18.

To convert speed in m/sec to kmph, multiply it with 18/5.

In the case of moving trains, three different situations need to be considered.

When a train passes a stationary point, the distance covered (in the passing) is the length of the train. If the train is crossing a platform (or a bridge), the distance covered by the train (in the crossing) is equal to the length of the train plus the length of the platform (or bridge). If two trains pass each other (travelling in the same direction or in opposite directions), the total distance covered (in the crossing or the overtaking, as the case may be) is equal to the sum of the lengths of the two trains.

Average Speed

Average speed of a body travelling at different speeds is defined as follows:

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Please note that the **Average speed** of a moving body is **Not equal** to the **Average of the speeds.**

If a body travels from point A to point B with a speed of p and back to point A (from point B) with a speed of q, then the

average speed of the body can be calculated as $\mathbf{2pq/(p+q)}$. Please note that this does not depend on the distance between A and B.

If a body covers part of the journey at speed p and the remaining part of the journey at speed q and the distances of the two parts of the journey are in the ratio $m : n$, then the average speed for the entire journey is $\mathbf{(m+n)pq/(mq+np)}$.

Solved Examples

Example 1

Ashok covered a certain distance at a certain speed. If his speed was 20% more, he would take 10 minutes less to cover the same distance. Find the time he takes to cover the distance.

Solution

If his speed was 20% more, it would be 120%, i.e. $\frac{6}{5}$ times his actual speed.

$\therefore$ Time taken would be $\frac{5}{6}$ times his actual time.

$$\text{Reduction in time} = \frac{1}{6}(\text{actual time}) = 10 \text{ minutes}$$

$\therefore$ Actual time = 60 minutes.

Example 2

A car covered a certain distance at 90 kmph and returned back at 60 kmph. Find its average speed for the entire journey.

Solution

Let x km be the distance to be covered, each way.

Total time of travel (in hours)

$$= \frac{x}{90} + \frac{x}{60} = \frac{5x}{180} = \frac{x}{36}$$

Average speed (in km/hr)

$$= \frac{\text{Total disance travelled}}{\text{Total time taken}} = \frac{2x}{\frac{x}{36}} = 72.$$

Example 3

Find the time taken by a train, 100 m long, travelling at a speed of 63 kmph to cross a platform of length 250 m.

Solution

Speed of the train $= (63)\left(\frac{5}{18}\right) = \frac{35}{2}$ m/sec

Distance to be travelled by the train to cross the platform = length of the train + length of the platform.

Time taken to cross the platform

$$= \frac{100+250}{\frac{35}{2}} = 20 \text{ sec.}$$

Example 4

Ashok would reach his office 15 minutes early if he walked at 4 kmph from his house. He would reach it 45 minutes late if he walked at 3 kmph from his house. Find the distance between his house and office.

Solution

Let the distance be x km. Time taken by Ashok if he walked at 4 kmph $= \frac{x}{4}$ hours.

Time taken by Ashok if he walked at 3 kmph $= \frac{x}{3}$ hours.

In this case, he would take one hour more to reach his office compared to the time taken if he had walked at 4 kmph.

$$\therefore \quad \frac{x}{3} - \frac{x}{4} = 1$$

$$\Rightarrow \quad x = 12.$$

In general, if a person travelling between two points reaches p hours late travelling at a speed of u kmph and reaches q hours early travelling at v kmph, the distance between the two points is given by $\frac{vu}{v-u}(p+q)$.

Relative Speed

The speed of one (moving) body in relation to another moving body is called the relative speed of these two bodies, i.e. it is the speed of one moving body as observed, from the second moving body.

If two bodies are moving in the same direction, the relative speed is equal to the difference of the speeds of the two bodies.

If two bodies are moving in opposite directions, the relative speed is equal to the sum of the speeds of the two bodies.

Example 5

Find the time taken by a train 175 m long running at a speed of 54 kmph to overtake another train 75 m long running at a speed of 36 kmph.

Solution

Relative speed = 18 kmph = 5 m/sec

Time taken for the faster train to overtake the slower train

$$= \frac{(\text{Length of the faster train}) + (\text{Length of the slower train})}{\text{Their relative speed}}$$

$$= \frac{175+75}{5} = 50 \text{ sec}$$

Example 6

A train overtakes two persons, cyling at 9 kmph and 18 kmph in 40 seconds and 48 seconds, respectively. Find its length and speed.

Solution

Let the length and the speed of the train be ℓ *m* and *s* kmph, respectively.

$$\ell = 40(s-9)\frac{5}{18} = 48(s-18)\frac{5}{18}$$

$$\Rightarrow \quad \frac{s-9}{s-18} = \frac{48}{40}$$

$$\Rightarrow \quad s = 63$$

$$\therefore \quad \ell = 40\ (63-9) \times \frac{5}{18} = 600\ m.$$

Boats and Streams

Problems related to boats and streams are different in the computation of relative speed from those of trains/cars.

When a boat is moving in the same direction as the stream or water current, the boat is said to be moving **WITH THE STREAM OR CURRENT.**

When a boat is moving in a direction opposite to that of the stream or water current, it is said to be moving **AGAINST THE STREAM OR CURRENT.**

If the boat is moving with a certain speed in water that is not moving, the speed of the boat is then called the **SPEED OF THE BOAT IN STILL WATER.**

When the boat is moving upstream, the speed of the water opposes (and hence reduces) the speed of the boat.

When the boat is moving downstream, the speed of the water aids (and thus adds to) the speed of the boat. Thus, we have

Speed of the boat against stream = Speed of the boat in still water – Speed of the stream

Speed of the boat with the stream = Speed of the boat in still water Speed of the stream

These two speeds, the speed of the boat against the stream and the speed of the boat with the stream, are RELATIVE SPEEDS.

If u is the speed of the boat down the stream and v is the speed of the boat up the stream, then we have the following two relationships.

Speed of the boat in still water = $(u + v)/2$

Speed of the water current = $(u - v)/2$

In problems, instead of a boat, it may be a swimmer but the approach is exactly the same. Instead of boats/swimmers in water, it could also be a cyclist cycling against or along the wind. The approach to solving the problems still remains the same.

Example 7

A boat travels 30 km upstream in 5 hours and 100 km downstream in 10 hours. Find the speed of the boat in still water and the speed of the stream.

Solution

Upstream speed $= \frac{30}{5} = 6$ kmph

Downstream speed $= \frac{100}{10} = 10$ kmph

Speed in still water $= \frac{6+10}{2} = 8$ kmph

Speed of the stream $= \frac{10-6}{2} = 2$ kmph .

Example 8

Anand can row 20 km in 2 hours in still water. If the speed of the stream is 6 kmph, he would take 3.75 hours to cover a round trip journey. Find the distance that he would then cover each way.

Solution

Speed of the boat in still water $= \frac{20}{2} = 10$ kmph

Let the total distance covered be $2x$ km.

Given that, $$\frac{x}{10+6} + \frac{x}{10-6} = 3.75$$

$$x = 12$$

Races and Circular Tracks

When two persons *P* and *Q* are running a race, they can start the race at the same time or one of them may start a little later than the other. In the second case, suppose *P* starts the race and after 5 seconds, *Q* starts. Then we say *P* has a 'start' of 5 seconds. Alternatively, in a race between *P* and *Q*, *P* starts first and then when *P* has covered a distance of 10 metres, *Q* starts. Then we say that *P* has a 'start' of 10 metres.

In a race between *P* and *Q* where *Q* is the winner, by the time *Q* reaches the winning post, if *P* still has another 15 metres to reach the winning post, then we say that *Q* has won the race by 15 metres. Similarly, if P reaches the winning post 10 seconds after *Q* reaches it, then we say that *Q* has won the race by 10 seconds.

In problems on RACES, we normally consider a 100 m race or a 1 km race. The length of the track.

NEED NOT necessarily be one of the two figures mentioned above but can be as given in the problem.

When two or more persons running around a circular track (starting at the same point and at the same time), then we will be interested in two main issues:

1. When they will meet for the first time and
2. When they will meet for the first time at the starting point

To solve the problems on circular tracks, you should keep the following points in mind.

When two persons are running around a circular track in **OPPOSITE** directions

1. The relative speed is equal to the sum of the speeds of the two individuals and
2. From one meeting point to the next meeting point, the two of them TOGETHER cover a distance equal to the length of the track.

When two persons are running around a circular track in the **SAME** direction

1. The relative speed is equal to the difference of the speeds of the two individuals and
2. From one meeting point to the next meeting point, the faster person covers one COMPLETE ROUND more than the slower person.

We can now tabulate the time taken by the persons to meet for the first time ever or for the first time at the starting point in various cases.

When TWO people are running around a circular track

Let the two people A and B with respective speeds of a and $b(a > b)$ be running around a circular track (of length L) starting at the same point and at the same time. Then,

	When the two persons are running in the SAME direction	When the two persons are running in OPPOSITE directions
Time taken to meet for the FIRST TIME EVER	$\frac{L}{(a-b)}$	$\frac{L}{(a+b)}$
Time taken to meet for the first time at the STARTING POINT	LCM of $\left\{\frac{L}{a}, \frac{L}{b}\right\}$	LCM of $\left\{\frac{L}{a}, \frac{L}{b}\right\}$

Please note that when we have to find out the time taken by the two persons to meet for the first time at the starting point, what we have to do is to find out the time taken by each of them to complete one full round and then take the LCM of these two timings (L/a and L/b are the timings taken by the two of them respectively to complete on full round).

Example 9

In a 200 m race, A beats B by 10 m or 2 seconds. Find B's speed and A's speed.

Solution

A beat B by 10 m or 2 seconds.

$\Rightarrow$ When A reached the finishing line B was 10 m behind the finishing line and took 2 seconds to cover it.

$$\therefore \quad B\text{'s speed} = \frac{10}{2} = 5 \text{ m/sec}$$

Time taken by B to complete the race

$$= \frac{200 \text{ m}}{5 \text{ m/s}} = 40 \text{ seconds}$$

$\therefore$ Time taken by A to complete the race

$$= 38 \text{ seconds}$$

$$A\text{'s speed} = \frac{200}{38} = \frac{100}{19} \text{ m/sec}$$

Example 10

Ramu is 50% faster than Somu. In a race, Ramu gave Somu a head start of 200 m. Both finished the race simultaneously. Find the length of the race.

Solution

Let the length of the race be x m.

$$\frac{x}{x-200} = \frac{150}{100}$$

$$\Rightarrow \quad x = 600 \text{ m.}$$

Example 11

In a 1200 m race, Ram beats Shyam by 300 m. In the same race, Shyam beats Tarun by 400 m. Find the distance by which Ram beats Tarun.

Solution

Let the speeds of Ram, Shyam, and Tarun be r m/sec, s m/sec, and t m/sec, respectively

$$\frac{r}{s} = \frac{1200}{1200-300} = \frac{4}{3}$$

$$\frac{s}{t} = \frac{1200}{1200-400} = \frac{3}{2}$$

$$\frac{r}{t} = \left(\frac{r}{s}\right)\left(\frac{s}{t}\right) = 2$$

$\Rightarrow$ By the time Ram covers 1200 $\overline{m}$, Tarun covers 600 $\overline{m}$.
$\therefore$ Ram beats Tarun by (1200 – 600), i.e. by 600 $\overline{m}$.

Exercises

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. Convert the following speeds into meters per second

(a) 36 km/hr
(A) 10 (B) 12 (C) 15 (D) 20

(b) 12.6 km/hr
(A) 3.5 (B) 4 (C) 0.35 (D) 6

(c) 252/35 km/hr
(A) 2.2 (B) 2.4 (C) 2 (D) 2.6

2. If a man runs at 6 metres per second, what distance (in km) will he cover in 3 hours and 45 minutes?
(A) 81 (B) 96 (C) 91 (D) 27

3. Travelling at 5/6th of his usual speed a man is 10 minutes late. What is the usual time he takes to cover the same distance?
(A) 50 minutes (B) 70 minutes
(C) 1 hour (D) 75 minutes

4. *X* and *Y* are 270 km apart. At 9:00 a.m, buses *A* and *B* left *X* and *Y* for *Y* and *X*, respectively. If the speeds of *A* and *B* are 50 kmph and 40 kmph, respectively, find their meeting time.
(A) 11:00 a.m (B) 12:00 p.m
(C) 1: 00 p.m (D) 2:00 p.m

5. Car A left *X* for *Y* at 9:00 a.m. Car *B* left *Y* for *X* at 10:00 a.m. *XY* = 180 km. Speeds of *A* and *B* are 30 kmph and 20 kmph, respectively. Find their meeting time.
(A) 12:36 p.m. (B) 1:36 p.m.
(C) 1:00 p.m (D) 2:00 p.m

6. Ashok left *X* and reached *Y* in 4 hours. His average speed for the journey was 90 kmph. Find the distance between *X* and *Y* (in km).
(A) 180 (B) 360 (C) 720 (D) 900

7. Alok travelled from Hyderabad to Tirupati at 60 kmph and returned at 90 kmph. Find his average speed for the journey (in kmph).
(A) 72 (B) 75 (C) 66 (D) 78

8. What is the time taken by a train 650 $\overline{m}$ long travelling at 72 km/hr to cross a 750 $\overline{m}$ long platform?
(A) 60 sec (B) 65 sec
(C) 70 sec (D) 75 sec

9. What is the time taken by a 750 $\overline{m}$ long train travelling at 99 km/hr to cross a boy running at 9 km/hr towards the train?
(A) 30 sec (B) 33 sec
(C) 36 sec (D) 25 sec

10. In a 200 $\overline{m}$ race, Eswar gives Girish a start of 10 $\overline{m}$ and beats him by 10 $\overline{m}$. Find the ratio of their speeds.
(A) 1 : 1 (B) 9 : 10 (C) 10 : 9 (D) 19 : 20

11. In a 100 $\overline{m}$ race, Ganesh beats Harish by 10 $\overline{m}$ or 2 seconds. Find Harish's speed (in m/sec).
(A) 5 (B) $5\frac{5}{9}$
(C) $4\frac{1}{2}$ (D) 6

12. In a 100 $\overline{m}$ race, Akbar gives Birbal a start of 2 seconds. Birlbal covers 10 $\overline{m}$ by the time Akbar starts. If both of them finish together, find Akbar's speed. (in m/sec)
(A) 5 (B) $5\frac{5}{9}$
(C) $4\frac{1}{2}$ (D) 4

13. In a race, *P* beats *Q* by 20 seconds. *Q* beats *R* by 30 seconds. By how many seconds did *P* beat *R*?
(A) 44 (B) 25 (C) 50 (D) 36

14. In a 100 $\overline{m}$ race, *A* beats *B* by 10 $\overline{m}$ and *B* beats *C* by 20 m. Find the distance by which *A* beats *C* (in m).
(A) 30 (B) 28 (C) 32 (D) 36

15. Anand can row a boat in still water at a speed of 5 kmph. The speed of the stream is 3 kmph. Find the time taken by him to row 40 km downstream (in hours).
(A) 5 (B) 20 (C) 8 (D) 10

16. Ram, Shyam, and Tarun started cycling from a point on a circular track 600 $\overline{m}$ long with speeds of 10 m/sec, 15 m/sec, and 20 m/sec, respectively. Find the time taken by them to meet at the starting point for the first time (in seconds).
(A) 120 (B) 60 (C) 240 (D) 600

17. Ashwin and Bhaskar started running simultaneously from a point on a 300 $\overline{m}$ long circular track. They ran in opposite directions with speeds of 6 m/sec and 4 m/sec, respectively. After meeting for the first time, they exchange their speeds. Who will reach the starting point first?
(A) Ashwin
(B) Bhaskar
(C) Both reach simultaneously
(D) Cannot be determined

18. A man reaches his destination which is 16 km away, 9 min late, if he travels at 8 kmph. What should his speed be if he wishes to reach 15 minutes ahead of the right time?
(A) 10 kmph (B) 3 m/sec
(C) 20/9 m/sec (D) 12 kmph

19. The distance between two points *P* and *Q* is 84 km. Two persons start at the same time but one travelling from *P* towards *Q* and the other travelling from *Q* towards *P*. If their respective speeds are 36 kmph and 27 kmph, where do they meet each other?
(A) 48 km from *Q* (B) 24 km from *P*
(C) 36 km from *P* (D) 48 km from *P*

20. Towns P and Q are 80 km apart. Cars A and B are stationed at towns P and Q respectively. If they start simultaneously towards each other, they would meet in an hour. If both start simultaneously in the same direction, the faster car would overtake the slower car in 4 hours. Find the speed of the faster car (in kmph).
(A) 50 (B) 55 (C) 60 (D) 65

21. A cat on seeing a dog 100 *m* away turns around and starts running away at 24 kmph. The dog spots him one minute later and starts chasing the cat at a speed of 33 kmph. After how much time, from the start of the cat's run, will the chase end?
(A) 160 s (B) 220 s
(C) 260 s (D) 280 s

22. Train A starts at 6 a.m. from city P towards city Q at a speed of 54 kmph. Another train 'B' starts at 9 a.m. from P towards Q at 72 kmph. If the distance between P and Q is 1440 km, find at what distance from Q would the two trains meet each other?
(A) 648 km (B) 792 km
(C) 486 km (D) 954 km

23. Mahesh travelled from Hyderabad to Tirupati at a certain speed and returned at a certain speed. His average speed for the entire trip was the average of his onward and return speeds. He travelled a total distance of 1200 km in 12 hours. Find his onward speed (in kmph).
(A) 100 (B) 80 (C) 60 (D) 40

24. Two cars left simultaneously from two places P and Q, and headed for Q and P, respectively. They crossed each other after x hours. After that, one of the cars took y hours to reach its destination while the other took z hours to reach its destination. Which of the following always holds true?
(A) $x = \dfrac{y+z}{2}$ (B) $x = \dfrac{2yz}{y+z}$
(C) $x = \sqrt{yz}$ (D) $x = \dfrac{y^2+z^2}{y+z}$

25. A boat travels 30 km upstream in 5 hours and 24 km downstream in 3 hours. Find the speed of the boat in still water and the speed of the water current
(A) 7 kmph, 2 kmph (B) 14 kmph, 1 kmph
(C) 7 kmph, 1 kmph (D) 8 kmph, 2 kmph

26. Amar, Akbar, and Anthony start running in the same direction and from the same point, around a circular track with speeds 7 m/sec, 11 m/sec, and 22 m/sec, respectively. If Akbar can complete 5 revolutions around the track in 40 sec, when will they meet for the first time after they start?
(A) 56 s (B) 88 s (C) 118 s (D) 79 s

27. If Ashok travelled at $4/5^{\text{th}}$ of his usual speed, he would reach his destination 15 minutes late. By how many minutes would he be early if he travelled at $6/5^{\text{th}}$ of his usual speed?
(A) 12 (B) 10 (C) 15 (D) 20

28. In a 500 ft race, Habib beats Akram by 60 ft. If Habib takes 5 paces for every 4 paces taken by Akram, what is the ratio of the length of Habib's pace to that of Akram?
(A) 10 : 11 (B) 11 : 10
(C) 25 : 22 (D) 22 : 25

29. Girish takes 1 minute to complete a round around a circular track. Harish is twice as fast as Girish, Suresh is thrice as fast as Harish. All three start at the same point. Find the time taken by them to meet at the starting point for the first time (in minutes).
(A) 1 (B) 2
(C) 6 (D) 12

30. Two cars C and D start from a junction along two perpendicular roads at 8:00 a.m. and 9:00 a.m., respectively. If at 12 noon, the cars, which travel at the same speed, are 150 km apart, then, find the speed of each car.
(A) 15 kmph
(B) 45 kmph
(C) 60 kmph
(D) 30 kmph

Answer Keys

1. (a) A	**(b)** A	**(c)** C	**2.** A	**3.** A	**4.** B	**5.** C	**6.** B	**7.** A	**8.** C
9. D	**10.** C	**11.** A	**12.** B	**13.** C	**14.** B	**15.** A	**16.** A	**17.** C	**18.** A
19. D	**20.** A	**21.** C	**22.** B	**23.** A	**24.** C	**25.** C	**26.** B	**27.** B	**28.** A
29. A	**30.** D								

Chapter 9

Indices, Surds, and Logarithms

CHAPTER HIGHLIGHTS

- *Indices*
- *Surds*
- *Rationalization of a Surd*
- *Square Root of a Surd*
- *Comparison of Surds*
- *Logarithms*

Indices

If a number 'a' is added three times to itself, then we write it as $3a$. Instead of adding, if we multiply 'a' three times with itself, we write it as a^3.

We say that 'a' is expressed as an exponent. Here, 'a' is called the 'base' and 3 is called the 'power' or 'index' or 'exponent'.

Similarly, 'a' can be expressed to any exponent 'n' and accordingly written as a^n. This is read as 'a to the power n' or 'a raised to the power n.'

$$a^n = a \times a \times a \times a \times \cdots n \text{ times}$$

For example,

$$2^3 = 2 \times 2 \times 2 = 8 \text{ and } 3^4 = 3 \times 3 \times 3 \times 3 = 81$$

While the example taken is for a positive integer value of n, the powers can also be negative integers or positive or negative fractions. In the sections that follow, we will also see how to deal with numbers where the powers are fractions or negative integers.

If a number raised to a certain power is inside brackets and quantity is then raised to a power again {i.e. a number of the type $(a^m)^n$—read as 'a raised to the power m whole raised to the power n' or 'a raised to power m whole to the power n'}, then the number inside the brackets is evaluated first and then this number is raised to the power which is outside the brackets.

For example, to evaluate $(2^3)^2$, we first find out the value of the number inside the bracket (2^3) as 8 and now raise this to the power 2. This gives 8^2 which is equal to 64. Thus, $(2^3)^2$ is equal to 64.

If we have powers in the manner of 'steps', then such a number is evaluated by starting at the topmost of the 'steps' and coming down one 'step' in each operation.

For example, 2^{4^3} is evaluated by starting at the topmost level '3'. Thus, we first calculate 4^3 as equal to 64. Since 2 is raised to the power 4^3, we now have 2^{64}.

Similarly, 2^{3^2} is equal to '2 raised to the power 3^2' or '2 raised to the power 9' or 2^9, which is equal to 512.

There are certain basic rules/formulae for dealing with numbers having powers. These are called Laws of Indices. The important ones are listed below but you are not required to learn the proof for any of these formulae/rules. The students have to know these rules and be able to apply any of them in solving problems. Most of the problems in indices will require one or more of these formulae. These formulae should be internalized by the students to the extent that after some practice, application of these rules should come naturally and the student should not feel that he is applying some specific formula.

Table 1 Table of Rules/Laws of Indices

	Rule/Law	Example
(1)	$a^m \times a^n = a^{m+n}$	$5^2 \times 5^7 = 5^9$
(2)	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{7^5}{7^3} = 7^2 = 49$
(3)	$(a^m)^n = a^{mn}$	$(4^2)^3 = 4^6$
(4)	$a^{-m} = \frac{1}{a^m}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$
(5)	$\sqrt[m]{a} = a^{1/m}$	$\sqrt[3]{64} = 64^{1/3} = 4$
(6)	$(ab)^m = a^m \cdot b^m$	$(2 \times 3)^4 = 2^4 \cdot 3^4$
(7)	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$
(8)	$a^0 = 1$ (where $a \neq 0$)	$3^0 = 1$
(9)	$a^1 = a$	$4^1 = 4$

These rules/laws will help you in solving a number of problems. In addition, the student should also remember the following rules:

Rule 1: When the bases of two EQUAL numbers are equal, then their powers also will be equal. (If the bases are neither zero nor ±1.)

For example: If $2^n = 2^3$, then it means $n = 3$

Rule 2: When the powers of two equal numbers are equal (and not equal to zero), two cases arise:

1. If the power is an odd number, then the bases are equal. For example, if $a^3 = 4^3$ then $a = 4$.
2. If the powers are even numbers, then the bases are numerically equal but can have different signs. For example, if $a^4 = 3^4$ then $a = +3$ or -3.

The problems associated with indices are normally of THREE types:

Simplification: Here, the problem involves terms with different bases and powers, which have to be simplified using the rules/formulae discussed in the table earlier.

Solving for the value of an unknown: Here, the problem will have an equation where an unknown (like x or y) will appear in the base or in the power, and using Rule 1 and Rule 2 discussed, values of unknown are to be determined.

Comparison of numbers: Here, two or more quantities will be given—each being a number raised to a certain power. These numbers have to be compared in magnitude—either to find the largest or smallest of the quantities or to arrange the given quantities in ascending or descending order.

The following examples will make clear the different types of problems that you may be asked.

Solved Examples

Example 1

Simplify: $\left(\frac{729}{1728}\right)^{\frac{-2}{3}} \times \left(\frac{1024}{9}\right)^{\frac{1}{2}} \div \left(\frac{24}{324}\right)$

Solution

$$\left(\frac{729}{1728}\right)^{\frac{-2}{3}} \times \left(\frac{1024}{9}\right)^{\frac{1}{2}} \div \left(\frac{24}{324}\right)$$

$$= \left(\frac{9^2}{12^2}\right)^{-1} \times \left(\frac{32^2}{3^2}\right)^{\frac{1}{2}} \times \left(\frac{324}{24}\right)$$

$$\left(\frac{9^3}{12^3}\right)^{\frac{-2}{3}} \times \frac{32}{3} \times \frac{324}{24} = \frac{144}{81} \times \frac{32}{3} \times \frac{324}{24} = 256$$

Example 2

In the equation given below, solve for x

$$\sqrt[3]{\left(\frac{5}{7}\right)^{x+1}} = \frac{125}{343}$$

Solution

Given, $\left(\frac{5}{7}\right)^{x+1} = \left[\left(\frac{5}{7}\right)^3\right]^3 = \left(\frac{5}{7}\right)^9$.

By equating their indices.

$$x + 1 = 9$$

$$x = 8.$$

Example 3

If $\left(\frac{49}{2401}\right)^{4-x} = 49^{2x-6}$, find x.

Solution

$$\left(\frac{49}{2401}\right)^{4-x} = \left(49^{-1}\right)^{4-x} = 49^{x-4}$$

Given, $49^{x-4} = 49^{2x-6}$

$$x - 4 = 2x - 6$$

$$x = 2$$

Example 4

Arrange the following in ascending order 625^6, 125^7 and 25^{10}

Solution

$$625^6 = (5^4)^6 = 5^{24}$$
$$125^7 = (5^3)^7 = 5^{21}$$
$$25^{10} = (5^2)^{10} = 5^{20}$$
$$25^{10} < 125^7 < 625^6$$

Surds

Any number of the form p/q, where p and q are integers and $q \neq 0$ is called a rational number. Any real number which is not a rational number is an irrational number. Amongst irrational numbers, of particular interest to us are SURDS. Amongst surds, we will specifically be looking at 'quadratic surds'—surds of the type $a + \sqrt{b}$ and $a + \sqrt{b} + \sqrt{c}$, where the terms involve only square roots and not any higher roots. We do not need to go very deep into the area of surds—what is required is a basic understanding of some of the operations on surds.

If there is a surd of the form $(a + \sqrt{b})$, then a surd of the form $\pm(a - \sqrt{b})$ is called the conjugate of the surd $(a + \sqrt{b})$. The product of a surd and its conjugate will always be a rational number.

Rationalization of a Surd

When there is a surd of the form $\dfrac{1}{a + \sqrt{b}}$, it is difficult to perform arithmetic operations on it. Hence, the denominator is converted into a rational number, thereby facilitating ease of handling the surd. This process of converting the denominator into a rational number without changing the value of the surd is called rationalization.

To convert the denominator of a surd into a rational number, multiply the denominator and the numerator simultaneously with the conjugate of the surd in the denominator so that the denominator gets converted to a rational number without changing the value of the fraction. That is, if there is a surd of the type $a + \sqrt{b}$ in the denominator, then both the numerator and the denominator have to be multiplied with a surd of the form $a - \sqrt{b}$ or a surd of the type $(-a + \sqrt{b})$ to convert the denominator into a rational number.

If there is a surd of the form $(a + \sqrt{b} + \sqrt{c})$ in the denominator, then the process of multiplying the denominator with its conjugate surd has to be carried out TWICE to rationalize the denominator.

Square Root of a Surd

If there exists a square root of a surd of the type $a + \sqrt{b}$, then it will be of the form $\sqrt{x} + \sqrt{y}$. We can equate the square of $\sqrt{x} + \sqrt{y}$ to $a + \sqrt{b}$ and thus solve for x and y. Here, one point should be noted—when there is an equation with rational and irrational terms, the rational part on the left-hand side is equal to the rational part on the right-hand side and, the irrational part on the left-hand side is equal to the irrational part on the right-hand side of the equation.

However, for the problems which are expected in the entrance exams, there is no need of solving for the square root in such an elaborate manner. We will look at finding the square root of the surd in a much simpler manner. Here, first, the given surd is written in the form of $(\sqrt{x} + \sqrt{y})^2$ or $(\sqrt{x} - \sqrt{y})^2$. Then, the square root of the surd will be $(\sqrt{x} + \sqrt{y})$ or $(\sqrt{x} - \sqrt{y})$, respectively.

Comparison of Surds

Sometimes, we will need to compare two or more surds either to identify the largest one or to arrange the given surds in ascending/descending order. The surds given in such cases will be such that they will be close to each other, and, hence, we will not be able to identify the largest one by taking the approximate square root of each of the terms. In such a case, the surds can both be squared and the common rational part be subtracted. At this stage, normally,` one will be able to make out the order of the surds. If even at this stage, it is not possible to identify the larger of the two, then the numbers should be squared once more.

Example 5

Rationalize the denominator: $\dfrac{1}{1+\sqrt{6}-\sqrt{7}}$

Solution

The rationalizing factor of

$1+\sqrt{6}-\sqrt{7}$ is $1 + \sqrt{6} + \sqrt{7}$

$$\frac{1}{1+\sqrt{6}-\sqrt{7}} = \frac{(1+\sqrt{6}+\sqrt{7})}{(1+\sqrt{6}-\sqrt{7})\,(1+\sqrt{6}+\sqrt{7})}$$

$$= \frac{1+\sqrt{6}+\sqrt{7}}{(1+\sqrt{6})^2-(\sqrt{7})^2} = \frac{1+\sqrt{6}+\sqrt{7}}{2\sqrt{6}}$$

The rationalizing factor of $\sqrt{6}$ is $\sqrt{6}$

$$= \frac{\sqrt{6}+6+\sqrt{42}}{12}$$

Example 6

Find the value of $\sqrt{62+\sqrt{480}}$

Solution

Let $\sqrt{62+\sqrt{480}} = \sqrt{a}+\sqrt{b}$

Squaring both sides, $62+\sqrt{480} = a+b+2\sqrt{ab}$

$$62+\sqrt{480} = a+b+\sqrt{4ab}$$

Equating the corresponding rational and irrational parts on both sides, $a+b = 62$

$$4ab = 480 \quad \Rightarrow \quad ab = 120$$

As $a + b = 60 + 2$ and $ab = (60)$ (B) it follows that $a = 60$ and $b = 2$ or vice versa.

$\therefore \qquad \sqrt{a}+\sqrt{b}=\sqrt{60}+\sqrt{2}$

Example 7

Which of the surds given below is greater?

$$\sqrt{3}+\sqrt{23} \text{ and } \sqrt{6}+\sqrt{19}$$

Solution

$$(\sqrt{3}+\sqrt{23})^2 = 26+2\sqrt{69}$$

$$\sqrt{69} \text{ lies between } \sqrt{64} \text{ and } \sqrt{81}$$

$\therefore\ 26+2\sqrt{69}$ lies between $26 + 2(8)$ and $26 + 2(9)$ i.e., 42 and 44.

Similarly $(\sqrt{6}+\sqrt{19})^2$ lies between 45 and 47.

$\therefore \qquad (\sqrt{3}+\sqrt{23})^2 < (\sqrt{6}+\sqrt{19})^2$

$\therefore \qquad \sqrt{6}+\sqrt{19} > \sqrt{3}+\sqrt{23}$

Logarithms

In the equation $a^x = N$, we are expressing N in terms of a and x. The same equation can be re-written as, $a = N^{1/x}$. Here, we are expressing a in terms of N and x. But, among a, x, and N, by normal algebraic methods known to us, we cannot express x in terms of the other two parameters a and N. This is where logarithms come into the picture. When $a^x = N$, then we say x = logarithm of N to the base a and write it as $x = \log_a N$. The definition of logarithm is given as: 'the logarithm of any number to a given base is the index or the power to which the base must be raised in order to equal the given number'.

Thus,

$$\boxed{\textit{if } a^x = N \textit{ then } x = \log_a N}$$

This is read as 'log N to the base a'.

In the above equation, N is a **POSITIVE NUMBER** and a is a **POSITIVE NUMBER OTHER THAN 1**.

This basic definition of logarithm is very useful in solving a number of problems on logarithms.

Example of a logarithm: $216 = 6^3$ can be expressed as $\log_6 216 = 3$.

Since logarithm of a number is a value, it will have an 'integral' part and a 'decimal' part. The integral part of the logarithm of a number is called the CHARACTERISTIC and the decimal part of the logarithm is called the MANTISSA.

Logarithms can be expressed to any base (positive number other than 1). Logarithms from one base can be converted to logarithms to any other base. (One of the formulae given below will help do this conversion.) However, there are two types of logarithms that are commonly used.

(i) **Natural Logarithms or Napierian Logarithms:** These are logarithms expressed to the base of a number called 'e'.

(ii) **Common Logarithms:** These are logarithms expressed to the base 10. For most of the problems under LOGARITHMS, it is common logarithms that we deal with. In examinations also, if logarithms are given without mentioning any base, it can normally be taken to be logarithms to the base 10.

Given below are some **important rules/formulae** in logarithms:

(i) $\log_a a = 1$ (logarithm of any number to the same base is 1)

(ii) $\log_a 1 = 0$ (log of 1 to any base other than 1 is 0)

(iii) $\log_a (mn) = \log_a m + \log_a n$

(iv) $\log_a (m/n) = \log_a m - \log_a n$

(v) $\log_a m^p = p \times \log_a m$

(vi) $\log_a b = \dfrac{1}{\log_b a}$

(vii) $\log_a m = \dfrac{\log_b m}{\log_b a}$

(viii) $\log_{a^q} m^p = \dfrac{p}{q} \log_a m$

(ix) $a^{\log_a N} = N$

(x) $a^{\log b} = b^{\log a}$

You should memorize these rules/formulae because they are very helpful in solving problems.

Like in the chapter on INDICES, in LOGARITHMS also there will be problems on

(i) Simplification using the above-listed formulae/rules and

(ii) Solving for the value of an unknown given in an equation.

In solving problems of the second type earlier, in most of the cases, we take recourse to the basic definition of logarithms (which is very important and should be memorized).

The following examples will give problems of both the above types and some problems on common logarithms.

The following rules also should be remembered while solving problems on logarithms:

Given an equation $\log_a M = \log_b N$,

(i) if $M = N$, then a will be equal to b; if $M \neq 1$ and $N \neq 1$.

(ii) if $a = b$, then M will be equal to N.

The examples that follow will explain all the above types of problems. Please note that unless otherwise specified, all the logarithms are taken to the base 10).

Example 8

Solve for x : $\log_{10} 20x = 4$

Solution

Given that $\log_{10} 20x = 4$

$\Rightarrow \quad 20x = 10^4 = 10000$

$\therefore \quad x = 500$

Example 9

Solve for x : $\log (x + 3) + \log (x - 3) = \log 72$

Solution

$\log (x + 3) + \log (x - 3) = \log 72$

$$\log (x + 3)(x - 3) = \log 72$$
$$(x + 3)(x - 3) = 72$$
$$x^2 = 81$$

$x = 9$ (If $x = -9$, $\log (x - 3)$ would be undefined)

Example 10

If $\log 2 = 0.301$, find the value of $\log 1250$, $\log 0.001250$, and $\log 125000$.

Solution

$$\log 1250 = \log \frac{10000}{8}$$
$$= 4 \log 10 - 3 \log 2 = 4 - 3 (0.3010) = 3.097$$

$$\log 0.001250 = \log \frac{1250}{10^6}$$
$$= 3.097 - 6 = -2.903$$

$$\log 125000 = \log (1250)(100)$$
$$= \log 1250 + 2 = 5.097$$

Example 11

Find the number of digits in 294^{20} given that $\log 6 = 0.778$ and $\log 7 = 0.845$

Solution

$\log 294^{20} = 20 \log (7^2\, 6)$

$= 20 (2 \log 7 + \log 6)$

$= 20 (2 (0.845) + 0.778) = 20 (1.69 + 0.778)\ 49.36$

Characteristic = 49.

$\therefore\ 294^{20}$ has 50 digits

Example 12

Find the value of $\log_{\sqrt[3]{2}} 32\sqrt[3]{16}$.

Solution

$$\log_{\sqrt[3]{2}} 32\sqrt[3]{16} = \log_{2^{1/3}} 2^5 \left(2^4\right)^{\frac{1}{3}}$$
$$= \log_{2^{1/3}} \left(2^{\frac{1}{3}}\right)^{19} = 19$$

Example 13

Find the number of zeros after the decimal point in $\left(\frac{3}{4}\right)^{500}$, given that $\log 3 = 0.4771$ and $\log 2 = 0.3010$.

Solution

$$\log \left(\frac{3}{4}\right)^{500} = 500 \left(\log \frac{3}{4}\right)$$

$= 500 (\log 3 - 2 \log 2)$

$= 500 (0.4771 - 2 (0.3010) = -62.4500$

$\therefore$ Number of zeros after the decimal point is 62.

Exercises

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. Simplify the following:

$$\left(\frac{243}{1024}\right)^{-2/5} \times \left(\frac{144}{49}\right)^{-1/2} \div \left(\frac{8}{343}\right)^{-2/3}$$

(A) $2^5 \times 3^{-1} \times 7$ (B) $2^5 \times 3^{-3} \times 7^{-2}$
(C) $2^4 \times 3^{-3} \times 7^{-1}$ (D) $2^4 \times 3^{-1} \times 7^{-2}$

2. Simplify the following:

$$\left(\frac{x^2 \cdot y^{-3}}{z^4}\right)^{-2} \times \left(\frac{x^2 \cdot y}{z^{-2}}\right)^3 \div \left(\frac{x^{-12} \cdot y^7}{z^{-8}}\right)^{-1}$$

(A) $x^{10} . y^{16} . z^{-22}$ (B) $x^7 . y^{-16} . z^{-22}$
(C) $x^{-7} . y^{16} . z^{-22}$ (D) $x^{-10} . y^{16} . z^{22}$

3. Simplify the following: $\dfrac{1-[1-\{1-(1+y)^{-1}\}]}{(1-y)}$

(A) $\dfrac{y}{(1-y^2)}$ (B) $\dfrac{y}{(1-y)^2}$
(C) $\dfrac{1+y}{(1-y)^2}$ (D) $\dfrac{1+y^2}{1-y^2}$

4. $(x^{a-b})^{(a^2+ab+b^2)} \times (x^{b-c})^{(b^2+bc c^2)} \times (x^{c-a})^{(c^2+ac+a^2)}$

(A) 0
(B) 1
(C) $x^{a^3+b^3+c^3}$
(D) $x^{3(a^2+b^2+c^2+ac+bc+ca)}$

5. $343^{0.12} \times 2401^{0.08} \times 49^{0.01} \times 7^{0.1} =$
(A) 7 (B) $7^{4/5}$ (C) 7^8 (D) $7^{3/5}$

6. Solve for x: $9^{2x+1} = 27^{5x-3}$
(A) 1 (B) 2 (C) –1 (D) –2

7. If $\frac{p}{q} = \frac{r}{s}$ and $p^a = q^b = r^c = s^d$, then $\frac{1}{a} - \frac{1}{b} =$
(A) $\frac{1}{d} - \frac{1}{c}$ (B) $\frac{1}{c} + \frac{1}{d}$
(C) $\frac{1}{c} - \frac{1}{d}$ (D) $-\left(\frac{1}{c} + \frac{1}{d}\right)$

8. Which of the following is the largest in value?
(A) $6^{1/2}$ (B) $7^{1/3}$ (C) $8^{1/4}$ (D) $9^{1/5}$

9. $2\sqrt{\frac{5}{2}} - 5\sqrt{\frac{2}{5}} + \sqrt{10} + \sqrt{1000} =$
(A) $9\sqrt{10}$ (B) $8\sqrt{10}$
(C) $8\sqrt{10}$ (D) $11\sqrt{10}$

10. $\left(\frac{\sqrt{p} - \sqrt[4]{pq}}{\sqrt[4]{pq} - \sqrt{q}}\right)^{-4} =$
(A) $\frac{p}{q}$ (B) $\sqrt{\frac{p}{q}}$ (C) $\sqrt{\frac{q}{p}}$ (D) $\frac{q}{p}$

11. If $y = 12 + 2\sqrt{35}$, then $\sqrt{y} - \frac{1}{\sqrt{y}} =$
(A) $\frac{\sqrt{7}+\sqrt{5}}{2}$ (B) $\frac{3\sqrt{5}-\sqrt{7}}{2}$
(C) $\frac{2\sqrt{5}+\sqrt{7}}{2}$ (D) $\frac{3\sqrt{5}+\sqrt{7}}{2}$

12. Arrange the following in ascending order.
$a = \sqrt{2} + \sqrt{11}$, $b = \sqrt{6} + \sqrt{7}$,
$c = \sqrt{3} + \sqrt{10}$ and $d = \sqrt{5} + \sqrt{8}$
(A) *abcd* (B) *abdc*
(C) *acdb* (D) *acbd*

13. Arrange the following in descending order.
$a = \sqrt{13} + \sqrt{11}$, $b = \sqrt{15} + \sqrt{9}$, $c = \sqrt{18} + \sqrt{6}$,
$d = \sqrt{7} + \sqrt{17}$.
(A) *abdc* (B) *dcab*
(C) *adcb* (D) *acdb*

14. Solve for x and y:
$3.5^x + 2^{y+2} = 107$, $5^{x+1} + 8.2^y = 189$
(A) 3, 2 (B) 5, 7 (C) 7, 5 (D) 2, 3

15. Solve for x, if $\left(5\sqrt{7}\right)^{5x-4} = (35)^3 (25)^{3/2}$.
(A) 2 (B) 5/4 (C) 7/2 (D) 3

16. If $5^{x+3} - 5^{x-3} = 78120$, find x.
(A) 4 (B) 3 (C) 5 (D) 6

17. If $a^a \cdot b^b \cdot c^c = a^b \cdot b^c \cdot c^a = a^c \cdot b^a \cdot c^b$ and a, b, c are positive integers greater than 1, then which of the following can NOT be true for any of the possible values of a, b, c?
(A) $abc = 8$ (B) $a + b + c = 8$
(C) $abc = 27$ (D) $a + b + c = 27$

18. The ascending order of $16^{\frac{7}{12}}$, $81^{\frac{3}{8}}$, $625^{\frac{2}{3}}$ is ______.
(A) $16^{\frac{7}{12}}$, $81^{\frac{3}{8}}$, $625^{\frac{2}{3}}$ (B) $16^{\frac{7}{12}}$, $625^{\frac{2}{3}}$, $81^{\frac{3}{8}}$,
(C) $625^{\frac{2}{3}}$, $16^{\frac{7}{12}}$, $81^{\frac{3}{8}}$ (D) $81^{\frac{3}{8}}$, $16^{\frac{7}{12}}$, $625^{\frac{2}{3}}$

19. If $2\sqrt{2} + \sqrt{3} = x$, what is the value of $\frac{11+4\sqrt{6}}{2\sqrt{2}-\sqrt{3}}$ in terms of x?
(A) $\frac{x^2}{\sqrt{2}}$ (B) x^3 (C) $\frac{x^3}{8}$ (D) $\frac{x^3}{5}$

20. Simplify: $\sqrt{(a+b+c) + 2\sqrt{ac+bc}}$.
(A) $\sqrt{a} + \sqrt{b} + \sqrt{c}$ (B) $\sqrt{a+b} + \sqrt{c}$
(C) $\sqrt{ab+bc}$ (D) $\sqrt{abc}$

21. Find the value of $x^2 - y^2$, if $\log_y(x-1) + \log_y(x+1) = 2$.
(A) 2 (B) $2y$ (C) 1 (D) $2xy$

22. If $a > 1$, $\log_a a + \log_{a^{\frac{1}{2}}} a + \log_{a^{\frac{1}{3}}} a + \cdots + \log_{a^{\frac{1}{20}}} a =$
(A) 420 (B) 210 (C) 380 (D) 190

23. If $\log_7(x-7) + \log_7(x^2 + 7x + 49) = 4$, then $x =$
(A) 196 (B) 7 (C) 49 (D) 14

24. If $\frac{\log a}{5} = \frac{\log b}{6} = \frac{\log c}{7}$, then $b^2 =$
(A) ac (B) a^2 (C) bc (D) ab

25. What is the value of $\log_{(1/5)} 0.0000128$?
(A) –7 (B) –5 (C) 5 (D) 7

26. If $(\log\tan 5°)(\log\tan 10°)(\log\tan 15°)\ldots(\log\tan 60°) = x$, what is the value of x?
(A) $\log(\sin 5°)^{12}$ (B) 1
(C) 0 (D) $\log(\cos 60°)$

27. Solve for x, if $\log_x[\log_5(\sqrt{x+5} + \sqrt{x})] = 0$.
(A) 1 (B) 9 (C) 12 (D) 4

28. If a, b, c are distinct values, what is the value of abc if $(\log_b a)(\log_c a) + (\log_a b)(\log_c b) + (\log_a c)(\log_b c) - 3 = 0$?
(A) 2 (B) 1
(C) $1 - \log a - \log b - \text{loc } c$ (D) 0

29. If $\log_6 161 = a$, $\log_6 23 = b$, what is the value of $\log_7 6$ in terms of a and b?
(A) a/b (B) $a + b$
(C) $1/(a - b)$ (D) b/a

30. $x = y^2 = z^3 = w^4 = u^5$, then find the value of $\log_x xyzwu$.
(A) $1\frac{47}{60}$ (B) $\frac{111}{120}$ (C) $2\frac{17}{60}$ (D) $2\frac{13}{60}$

Answer Keys

1. C	**2.** D	**3.** A	**4.** B	**5.** B	**6.** A	**7.** C	**8.** A	**9.** D	**10.** D
11. D	**12.** C	**13.** A	**14.** D	**15.** A	**16.** A	**17.** B	**18.** A	**19.** D	**20.** B
21. C	**22.** B	**23.** D	**24.** A	**25.** D	**26.** C	**27.** D	**28.** B	**29.** C	**30.** C

Chapter 10

Quadratic Equations

CHAPTER HIGHLIGHTS

- *Quadratic Equations*
- *Finding the Roots by Factorization*
- *Finding the Roots by Using the Formula*
- *Sum and Product of Roots of a Quadratic Equation*
- *Nature of the Roots*
- *Signs of the Roots*
- *Constructing a Quadratic Equation*
- *Maximum or Minimum Value of a Quadratic Expression*

Quadratic Equations

'If a variable occurs in an equation with all positive integer powers and the highest power is two, then it is called a Quadratic Equation (in that variable)'.

In other words, a second degree polynomial in x equated to zero will be a quadratic equation. For such an equation to be a quadratic equation, the co-efficient of x^2 should not be zero.

The most general form of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ (and a, b, c are real).

Some examples of quadratic equations are

$$x^2 - 5x + 6 = 0 \quad (1)$$

$$x^2 - x - 6 = 0 \quad (2)$$

$$2x^2 + 3x - 2 = 0 \quad (3)$$

$$2x^2 + x - 3 = 0 \quad (4)$$

Like a first degree equation in x has one value of x satisfying the equation, a quadratic equation in x will have TWO values of x that satisfy the equation. The values of x that satisfy the equation are called the ROOTS of the equation. These roots may be real or imaginary.

For the above-given four quadratic equations, the roots are as given below:

Equation (1) : $x = 2$ and $x = 3$
Equation (2) : $x = -2$ and $x = 3$
Equation (3) : $x = 1/2$ and $x = -2$
Equation (4) : $x = 1$ and $x = -3/2$

In general, the roots of a quadratic equation can be found out in two ways.

1. By factorizing the expression on the left-hand side of the quadratic equation.
2. By using the standard formula.

All the expressions may not be easy to factorize whereas applying the formula is simple and straightforward.

Finding the Roots by Factorisation

If the quadratic equation $ax^2 + bx + c = 0$ can be written in the form $(x - \alpha)(x - \beta) = 0$, then the roots of the equation are α and β.

To find the roots of a quadratic equation, we should first write it in the form of $(x - \alpha)(x - \beta) = 0$, i.e. the left-hand side $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$ should be factorized into two factors.

For this purpose, we should go through the following steps. We will understand these steps with the help of the equation $x^2 - 5x + 6 = 0$, which is the first of the four quadratic equations we looked at as examples earlier.

1. First write down b (the co-efficient of x) as the sum of two quantities whose product is equal to ac.

In this case, –5 has to be written as the sum of two quantities whose product is 6. We can write –5 as (–3) + (–2) so that the product of (–3) and (–2) is equal to 6.

2. Now rewrite the equation with the 'bx' term split in the aforementioned manner.
   In this case, the given equation can be written as $x^2 - 3x - 2x + 6 = 0$.
3. Take the first two terms and rewrite them together after taking out the common factor between the two of them. Similarly, the third and fourth terms should be rewritten after taking out the common factor between the two of them. In other words, you should ensure that what is left from the first and the second terms (after removing the common factor) is the same as that left from the third and the fourth terms (after removing their common factor).
   In this case, the equation can be rewritten as $x(x - 3) - 2(x - 3) = 0$; Between the first and second terms as well as the third and fourth terms, we are left with $(x - 3)$ is a common factor.
4. Rewrite the entire left-hand side to get the form $(x - \alpha)(x - \beta)$.
   In this case, if we take out $(x - 3)$ as the common factor, we can rewrite the given equation as $(x - 3)(x - 2) = 0$.
5. Now, α and β are the roots of the given quadratic equation.
   ∴ For $x^2 - 5x + 6 = 0$, the roots of the equation are 3 and 2.

For the other three quadratic equations given earlier as examples, let us see how to factorize the expression and get the roots.

For equation (2), i.e. $x^2 - x - 6 = 0$, the co-efficient of x, which is –1 can be rewritten as (–3) + (+2) so that their product is –6, which is equal to ac (1 multiplied by –6). Then, we can rewrite the equation as $(x - 3)(x + 2) = 0$ giving us the roots as 3 and –2.

For equation (3), i.e. $2x^2 + 3x - 2 = 0$, the co-efficient of x, which is 3 can be rewritten as (+4) + (–1) so that their product is –4, which is the value of ac (–2 multiplied by 2). Then, we can rewrite the equation as $(2x - 1)(x + 2) = 0$, giving the roots as 1/2 and –2. For equation (4), i.e. $2x^2 + x - 3 = 0$, the co-efficient of x which is 1 can be rewritten as (+3) + (–2) so that their product is –6 which is equal to ac (2 multiplied by –3). Then, we can rewrite the given equation as $(x - 1)(2x + 3) = 0$, giving us the roots as 1 and –3/2.

Finding the Roots by Using the Formula

If the quadratic equation is $ax^2 + bx + c = 0$, then we can use the standard formula given below to find out the roots of the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The roots of the four quadratic equations we took as examples can be taken and their roots found out by using the aforementioned formula. The student is advised to check it out for himself/herself that the roots can be obtained by using this formula also.

Sum and Product of Roots of a Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$, the sum of the roots and the product of the roots can be given by the following:

Sum of the roots = $-b/a$

Product of the roots = c/a

These two rules will be very helpful in solving problems on quadratic equation.

Nature of the Roots

We mentioned already that the roots of a quadratic equation with real co-efficients can be real or complex. When the roots are real, they can be equal or unequal. All this will depend on the expression $b^2 - 4ac$. Since $b^2 - 4ac$ determines the nature of the roots of the quadratic equation, it is called the 'DISCRIMINANT' of the quadratic equation.

If $b^2 - 4ac > 0$, then the roots of the quadratic equation will be real and distinct.

If $b^2 - 4ac = 0$, the roots are real and equal.

If $b^2 - 4ac < 0$, then the roots of the quadratic equation will be complex conjugates.

Thus, we can write down the following about the nature of the roots of a quadratic equation when a, b, and c are all *rational*.

when $b^2 - 4ac < 0$	the roots are complex and unequal
when $b^2 - 4ac = 0$	the roots are rational and equal
when $b^2 - 4ac > 0$ and a perfect square	the roots are rational and unequal
when $b^2 - 4ac > 0$ but not a perfect square	the roots are irrational and unequal

Whenever the roots of the quadratic equation are irrational, (a, b, c being rational), they will be of the form $a + \sqrt{b}$ and $a - \sqrt{b}$, i.e. whenever $a + \sqrt{b}$ is one root of a quadratic equation, then $a - \sqrt{b}$ will be the second root of the quadratic equation and vice versa.

Signs of the Roots

We can comment on the signs of the roots, i.e. whether the roots are positive or negative, based on the sign of the sum of the roots and the product of the roots of the quadratic equation. The following table will make clear the relationship between the sum and the product of the roots and the signs of the roots themselves.

Sign of product of the roots	Sign of sum of the roots	Sign of the roots
+ ve	+ ve	Both the roots are positive.
+ ve	– ve	Both the roots are negative.
– ve	+ ve	The numerically larger root is positive and the other root is negative.
– ve	– ve	The numerically larger root is negative and the other root is positive.

Constructing a Quadratic Equation

We can build a quadratic equation in the following three cases:

1. When the roots of the quadratic equation are given.
2. When the sum of the roots and the product of the roots of the quadratic equation are given.
3. When the relation between the roots of the equation to be framed and the roots of another equation is given.

If the roots of the quadratic equation are given as α and β, the equation can be written as

$$(x - \alpha)(x - \beta) = 0 \quad \text{i.e., } x^2 - x(\alpha + \beta) + \alpha\beta = 0$$

If p is the sum of the roots of the quadratic equation and q is the product of the roots of the quadratic equation, then the equation can be written as $x^2 - px + q = 0$.

Maximum or Minimum Value of a Quadratic Expression

An equation of the type $ax^2 + bx + c = 0$ is called a quadratic equation. An expression of the type $ax^2 + bx + c$ is called a 'quadratic expression'. The quadratic expression $ax^2 + bx + c$ takes different values as x takes different values.

As x varies from $-\infty$ to $+\infty$, (i.e. when x is real), the quadratic expression $ax^2 + bx + c$

1. Has a minimum value whenever $a > 0$ (i.e., a is positive). The minimum value of the quadratic expression is $(4ac - b^2)/4a$ and it occurs at $x = -b/2a$.
2. Has a maximum value whenever $a < 0$ (i.e. a is negative). The maximum value of the quadratic expression is $(4ac - b^2)/4a$ and it occurs at $x = -b/2a$.

Solved Examples

Example 1

Find the roots of the equation $11x^2 - 37x + 30 = 0$.

Solution

We have to write –37 as the sum of two parts whose product should be equal to $(11) \times (30)$

$$(-22) + (-15) = -37 \text{ and } (-22)(-15) = 11 \times 30$$

Therefore, $11x^2 - 37x + 30 = 0$

$\Rightarrow \quad 11x^2 - 22x - 15x + 30 = 0$

$\Rightarrow \quad 11x(x - 2) - 15(x - 2) = 0$

$\Rightarrow \quad (11x - 15)(x - 2) = 0 \; x = \dfrac{15}{11}$ or 2.

Example 2

Discuss the nature of the roots of the equation $8x^2 - 2x - 4 = 0$.

Solution

For the quadratic equation $ax^2 + bx + c = 0$, the nature of the roots is given by the discriminant $b^2 - 4ac$.

Discriminant of $8x^2 - 2x - 4 = 0$ is

$$(-2)^2 - 4(8)(-4) = 132.$$

Since the discriminant is positive but not a perfect square, the roots of the equation are irrational and unequal.

Example 3

If the sum of the roots of the equation $Rx^2 + 5x - 24 = 0$ is 5/11, then find the product of the roots of that equation.

Solution

For a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is $(-b/a)$ and the product of the roots is (c/a).

The sum of the roots of the equation

$Rx^2 + 5x - 24 = 0$ is $\left(\dfrac{-5}{R}\right)$, which is given as $\dfrac{5}{11}$

$\therefore \; R = -11$

In the given equation, product of the roots $= \dfrac{-24}{R} = \dfrac{-24}{-11} = +\dfrac{24}{11}$.

Example 4

Find the value of k, so that the roots of $6x^2 - 12x - k = 0$ are reciprocals of each other.

Solution

If the roots of the equation are reciprocals of each other, then the product of the roots should be equal to 1.

$\Rightarrow \quad \dfrac{-k}{6} = 1.$

Therefore $k = -6$.

Example 5

If $4 + \sqrt{7}$ is one root of a quadratic equation with rational co-efficients, then find the other root of the equation.

Solution

When the co-efficients of a quadratic equation are rational and the roots are irrational, they occur only in pairs like

$p \pm \sqrt{q}$ i.e., if $p+\sqrt{q}$ is one root, then the other root of the equation will be $p-\sqrt{q}$. So, in this case, the other root of the equation will be $4-\sqrt{7}$.

Example 6

Form a quadratic equation with rational co-efficients, one of whose roots is $5+\sqrt{6}$.

Solution

If $5+\sqrt{6}$ is one root, then the other root is $5-\sqrt{6}$ (because the co-efficients are rational).

The sum of the roots $= 5+\sqrt{6}+5-\sqrt{6} = 10$.

The product of the roots $= (5+\sqrt{6})(5-\sqrt{6}) = 25-6 = 19$.

Thus the required equation is $x^2 - 10x + 19 = 0$.

Example 7

If the price of each book goes up by ₹5, then Atul can buy 20, books less for ₹1200. Find the original price and the number of books Atul could buy at the original price.

Solution

Let the original price of each book be x.

Then the new price of each book will be $x + 5$.

The number of books that can be bought at the original price $= \dfrac{1200}{x}$

The number of books that can be bought at the new price $= \dfrac{1200}{x+5}$

Given that Atul gets 20 books less at new price, i.e. $\dfrac{1200}{x} - \dfrac{1200}{x+5} = 20$

$$\Rightarrow \quad \frac{60}{x} - \frac{60}{x+5} = 1$$

$$\Rightarrow \quad \frac{60(x+5-x)}{x^2+5x} = 1$$

$$\Rightarrow \quad 300 = x^2 + 5x$$

$$\Rightarrow \quad x^2 + 5x - 300 = 0$$

$$\Rightarrow \quad (x+20)(x-15) = 0$$

$$\Rightarrow \quad x = -20 \quad \text{or} \quad 15$$

As the price cannot be negative, the original price is ₹15.

Example 8

If α and β are the roots of the equation $x^2 - 3x - 180 = 0$ such that $\alpha < \beta$, then find the values of

(i) $\alpha^2 + \beta^2$ (ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ (iii) $\alpha - \beta$

Solution

From the given equation, we get $\alpha + \beta = 3$ and $\alpha\beta = -180$

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (3)^2 - 2(-180) = 369$

(ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = \dfrac{+3}{-180} = \dfrac{-1}{60}$

(iii) $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$$\Rightarrow \quad \alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

$$= \pm\sqrt{(+3)^2 - 4(-180)} = \pm\sqrt{9+720}$$

$$= \pm\sqrt{729} = \pm 27\text{; as } \alpha < \beta,\ \alpha - \beta = -27.$$

Example 9

If $\sqrt{x+4} + \sqrt{x+8} = 7$, then find the value of x.

Solution

Given $\sqrt{x+4} + \sqrt{x+8} = 7$

Squaring on both sides, we get

$$x + 4 + x + 8 + 2\left(\sqrt{x+4}\sqrt{x+8}\right) = 49$$

$$\Rightarrow \quad 2x + 12 + 2\sqrt{x^2+12x+32} = 49$$

$$\Rightarrow \quad 2x - 37 = -2\sqrt{x^2+12x+32}$$

Squaring again on both sides, we have

$$(2x-37)^2 = 4(x^2 + 12x + 32)$$

$$\Rightarrow \quad 4x^2 - 148x + 1369 = 4x^2 + 48x + 128$$

$$\Rightarrow \quad 1241 = 196x$$

$$\Rightarrow \quad x = \frac{1241}{196}.$$

Example 10

If $4^{2x+1} + 4^{x+1} = 80$, then find the value of x.

Solution

Given $4^{2x+1} + 4^{x+1} = 80$

$$\Rightarrow \quad 4^{2x} \times 4 + 4^x \times 4 = 80$$

$$4^{2x} + 4^x = 20$$

Substituting $4^x = a$,

we get $a^2 + a = 20$

$$\Rightarrow \quad a^2 + a - 20 = 0$$

$$\Rightarrow \quad (a+5)(a-4) = 0$$

$$\Rightarrow \quad a = -5 \quad \text{or} \quad 4$$

If $4^x = -5$, there is no possible value for x as no power of 4 gives negative value.

If $4^x = 4$, then $x = 1$.

Exercises

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. The roots of the quadratic equation $2x^2 - 7x + 2 = 0$ are
(A) Rational and unequal
(B) Real and equal
(C) Imaginary
(D) Irrational

2. Find the nature of the roots of the quadratic equation $2x^2 + 6x - 5 = 0$.
(A) Complex conjugates
(B) Real and equal
(C) Conjugate surds
(D) Unequal and rational

3. Construct a quadratic equation whose roots are one third of the roots of $x^2 + 6x + 10 = 0$.
(A) $x^2 + 18x + 90 = 0$ (B) $x^2 + 16x + 80 = 0$
(C) $9x^2 + 18x + 10 = 0$ (D) $x^2 + 17x + 90 = 0$

4. A quadratic equation in x has its roots as reciprocals of each other. The co-efficient of x is twice the co-efficient of x^2. Find the sum of the squares of its roots.
(A) 5 (B) 4 (C) 3 (D) 2

5. If one root of the quadratic equation $4x^2 - 8x + k = 0$, is three times the other root, find the value of k.
(A) 3 (B) 9 (C) –3 (D) –6

6. The roots of the quadratic equation $(m - k + \ell)\, x^2 - 2mx + (m - \ell + k) = 0$ are
(A) $1, \dfrac{\ell + m - k}{k + m - \ell}$ (B) $1, \dfrac{2m}{\ell + m - k}$
(C) $1, \dfrac{k + m - \ell}{\ell + m - k}$ (D) $1, \dfrac{2k}{k - m + \ell}$

7. The expression $\dfrac{4ac - b^2}{4a}$ represents the maximum/minimum value of the quadratic expression $ax^2 + bx + c$. Which of the following is true?
(A) It represents the maximum value when $a > 0$.
(B) It represents the minimum value when $a < 0$.
(C) Both (A) and (B)
(D) Neither (A) nor (B)

8. Find the signs of the roots of the equation $x^2 + x - 420 = 0$.
(A) Both are positive.
(B) Both are negative.
(C) The roots are of opposite signs with the numerically larger root being positive.
(D) The roots are of opposite signs with the numerically larger root being negative.

9. If k is a natural number and $(k^2 - 3k + 2)\,(k^2 - 7k + 12) = 120$, find k.
(A) 7 (B) 6 (C) 5 (D) 9

10. Both A and B were trying to solve a quadratic equation. A copied the co-efficient of x wrongly and got the roots of the equation as 12 and 6. B copied the constant term wrongly and got the roots as 1 and 26. Find the roots of the correct equation.
(A) 6, 16 (B) –6, –16
(C) 24, 3 (D) –3, –24

11. If the roots of the equation $(x - k_1)\,(x - k_2) + 1 = 0$, k_1 and k_2 are integers, then which of the following must be true?
(A) k_1, k_2 are two consecutive integers
(B) $k_2 - k_1 = 2$
(C) $k_1 - k_2 = 2$
(D) Either (B) or (C)

12. The roots of the equation $ax^2 + bx + c = 0$ are k less than those of the equation $px^2 + qx + r = 0$. Find the equation whose roots are k more than those of $px^2 + qx + r = 0$.
(A) $ax^2 + bx + c = 0$
(B) $a(x - 2k)^2 + b(x - 2k) + c = 0$
(C) $a(x + 2k)^2 + b(x + 2k) + c = 0$
(D) $a(x - k)^2 + b(x - k) + c = 0$

13. If one root of the equation $x^2 - 10x + 16 = 0$ is half of one of the roots of $x^2 - 4Rx + 8 = 0$. Find R such that both the equations have integral roots.
(A) 1 (B) 2/3 (C) 3/2 (D) 4

14. If $x + y = 4$, find the maximum/minimum possible value of $x^2 + y^2$.
(A) Minimum, 8 (B) Maximum, 8
(C) Maximum, 16 (D) Minimum, 16

15. Find positive integral value(s) of p such that the equation $2x^2 + 8x + p = 0$ has rational roots.
(A) 8 (B) 4
(C) 6 (D) (A) or (C)

16. Two equations have a common root which is positive. The other roots of the equations satisfy $x^2 - 9x + 18 = 0$. The product of the sums of the roots of the two equations is 40. Find the common root.
(A) 1 (B) 2 (C) 3 (D) 4

17. If one root of the equation $x^3 - 11x^2 + 37x - 35 = 0$ is $3 - \sqrt{2}$, then find the other two roots.
(A) $5, 3 - \sqrt{2}$ (B) $-5, 3 + \sqrt{2}$
(C) $5, 3 + \sqrt{2}$ (D) $-5, 3 - \sqrt{2}$

18. The roots [the values of x (and not $|x|$)] of the equation $|x|^2 + 6|x| - 55 = 0$ are α and β. One of the roots of $py^2 + qy + r = 0$ is $\alpha\beta$ times the other root. Which of the following can be concluded?
(A) $25q^2 = -576pr$ (B) $25pr = -576q^2$
(C) $25q^2 = 576pr$ (D) $25pr = 576q^2$

19. The sides of a right-angled triangle are such that the sum of the lengths of the longest and that of the shortest side is twice the length of the remaining side. Find the longest side of the triangle if the longer of the sides containing the right angle is 9 cm more than half the hypotenuse.

(A) 30 cm (B) 25 cm (C) 20 cm (D) 15 cm

20. Solve for x: $2\{3^{2(1+x)}\} - 4(3^{2+x}) + 10 = 0$

(A) $-1, \log_3\left(\frac{5}{3}\right)$ (B) $-1, \log_3 2$

(C) $-1, \frac{5}{3}$ (D) $-1, \log_3\left(\frac{3}{5}\right)$

21. If $\sqrt{x^2-2x-3}+\sqrt{x^2+5x-24}=\sqrt{x^2+7x-30}$, then find x.

(A) 2 (B) 3 (C) 4 (D) 6

22. Two software professionals Ranjan and Raman had 108 floppies between them. They sell them at different prices, but each receives the same sum. If Raman had sold his at Ranjan's price, he would have received ₹722 and if Ranjan had sold his at Raman's price, he would have received ₹578. How many floppies did Ranjan have?

(A) 51 (B) 57 (C) 68 (D) 40

23. The sum and product of the roots of a quadratic equation E are a and b, respectively. Find the equation whose roots are the product of first root of E and the square of the second root of E, and the product of the second root of E and the square of the first root of E.

(A) $x^2 - abx + b^3 = 0$ (B) $x^2 + abx + b^3 = 0$

(C) $x^2 + abx - b^3 = 0$ (D) $x^2 - abx - b^3 = 0$

24. Which of the following options represent(s) a condition for the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ to have exactly one common root, given that the roots of both the equations are real?

(A) $a - b = 1$ (B) $b - a = 1$

(C) $1 + a + b = 0$ (D) Either (A) or (B)

25. If the roots of $2x^2 + (4m + 1)x + 2(2m - 1) = 0$ are reciprocals of each other, find m.

(A) -1 (B) 0 (C) 1 (D) 3/4

Answer Keys

1. D	**2.** C	**3.** C	**4.** D	**5.** A	**6.** C	**7.** D	**8.** D	**9.** B	**10.** C
11. D	**12.** B	**13.** C	**14.** A	**15.** D	**16.** B	**17.** C	**18.** A	**19.** A	**20.** A
21. B	**22.** A	**23.** A	**24.** C	**25.** C					

Chapter 11

Inequalities

CHAPTER HIGHLIGHTS

- *Inequalities and Modulus*
- *Symbols and Notations*
- *Absolute Value*
- *Properties of Modulus*

Inequalities and Modulus

If 'a' is any real number, then 'a' is either positive or negative or zero. When 'a' is positive, we write $a > 0$, which is read 'a is greater than zero'. When 'a' is negative, we write $a < 0$, which is read 'a is less than zero'. If 'a' is zero, we write $a = 0$ and in this case, 'a' is neither positive nor negative.

Symbols and Notations

'>' means 'greater than'
'<' means 'less than'
'≥' means 'greater than or equal to'
'≤' means 'less than or equal to'

For any two non-zero real numbers a and b,

1. a is said to be greater than b when $a - b$ is positive.
2. a is said to be less than b when $a - b$ is negative.

These two statements are written as

1. $a > b$ when $a - b > 0$ and
2. $a < b$ when $a - b < 0$.

For example,
3 is greater than 2 because $3 - 2 = 1$ and 1 is greater than zero. –3 is less than –2 because $-3 - (-2) = -1$ and –1 is less than zero.

Certain properties and useful results pertaining to inequalities are given below. A thorough understanding of these properties results is very essential for being able to solve the problems pertaining to inequalities.

[In the following list of properties and results, numbers like a, b, c, d, etc. are real numbers.]

1. For any two real numbers a and b, either $a > b$ or $a < b$ or $a = b$.
2. If $a > b$, then $b < a$.
3. If $a \nless b$, then $a \geq b$ and if $a > b$, then $a \leq b$.
4. If $a > b$ and $b > c$, then $a > c$.
5. If $a < b$ and $b < c$, then $a < c$.
6. If $a > b$, then $a \pm c > b \pm$ c.
7. If $a > b$ and $c > 0$, then $ac > bc$.
8. If $a < b$ and $c > 0$, then $ac < bc$.
9. If $a > b$ and $c < 0$, then $ac < bc$.
10. If $a < b$ and $c < 0$, then $ac > bc$.
11. If $a > b$ and $c > d$, then $a + c > b + d$.
12. If $a < b$ and $c < d$, then $a + c < b + d$.
13. Let A, G and H be the Arithmetic mean, Geometric mean and Harmonic mean of n positive real numbers. Then $A \geq G \geq H$, the equality occurring only when the numbers are all equal.

14. If the sum of two positive quantities is given, their product is the greatest when they are equal; if the product of two positive quantities is given, their sum is the least when they are equal.
15. If $a > b$ and $c > d$, then we cannot say anything conclusively about the relationship between $(a - b)$ and $(c - d)$; depending on the values of a, b, c, and d, it is possible to have $(a - b) > (c - d)$, $(a - b) = (c - d)$ or $(a - b) < (c - d)$.

Absolute Value

(written as $|x|$ and read as 'modulus of x')

For any real number x, the absolute value is defined as follows:

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \text{ and} \\ -x, & \text{if } x < 0 \end{cases}$$

Properties of Modulus

For any real number x and y,

1. $x = 0 \Leftrightarrow |x| = 0$
2. $|x| \geq 0$ and $-|x| \leq 0$
3. $|x + y| \leq |x| + |y|$
4. $||x| - |y|| \leq |x - y|$
5. $-|x| \leq x \leq |x|$
6. $|x \cdot y| = |x| \cdot |y|$
7. $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$; $(y \neq 0)$
8. $|x|^2 = x^2$

In inequalities, the variables generally take a range of values unlike in the case of equations where the variables in general, take one value or a discrete set of values. (In some specific cases, the variables may take only one value.)

Solved Examples

Example 1

If $13x - 19 \leq 4x + 26$, find the range of x.

Solution

$13x - 4x \leq 26 + 19$

$$x \leq 5.$$

$(-\infty, 5]$ in the interval notation.

Example 2

Solve the following inequalities:

$$5x + 21 < 46 \text{ and } 4x + 18 < 54.$$

Solution

$$5x + 21 < 46 \Rightarrow x < 5 \quad (1)$$

$$4x + 18 < 54 \Rightarrow x < 9 \quad (2)$$

The common inequality satisfying (1) and (2) is $x < 5$ or $(-\infty, 5)$ in the interval notation.

Example 3

Which of the numbers 50^{51} and 51^{50} is greater?

Solution

Let $a = 50^{51}$ and $b = 51^{50}$.

$$\frac{b}{a} = \frac{51^{50}}{50^{51}} = \left(\frac{51}{50}\right)^{50}\left(\frac{1}{50}\right) = \left(1 + \frac{1}{50}\right)^{50}\left(\frac{1}{50}\right)$$

$\left(1 + \frac{1}{x}\right)^x$ where $x > 0$ always lies between 2 and 2.8.

$\therefore$ $\frac{b}{a}$ lies between

$$\frac{2}{50} = 0.04 \text{ and } \frac{2.8}{50} = 0.056$$

$\therefore \quad \frac{b}{a} < 1$

$\therefore \quad a > b.$

Example 4

Solve for x if $4x^2 - 21x + 20 > 0$

Solution

$$4x^2 - 21x + 20 > 0 \Rightarrow (4x - 5)(x - 4) > 0$$

Both factors are positive (i.e. the smaller is positive) or both are negative (i.e. the greater is negative), i.e. $x > 4$ or $x < \frac{5}{4}$ or it can be expressed in the interval notation as $(4, \infty) \cup \left(-\infty, \frac{5}{4}\right)$

Example 5

Solve for x, if $\frac{x^2 + 5x - 24}{2x^2 - 5x - 3} < 0$

Solution

$$x^2 + 5x - 24 = (x + 8)(x - 3)$$

Similarly $2x^2 - 5x - 3 = (2x + 1)(x - 3)$

Given: $\frac{x^2 + 5x - 24}{2x^2 - 5x - 3} < 0$

$$\Rightarrow \quad \frac{(x+8)(x-3)}{(2x+1)(x-3)} < 0$$

$$\Rightarrow \quad \frac{x+8}{2x+1} < 0$$

$$\frac{(x+8)(2x+1)}{(2x+1)^2} < 0$$

$$\Rightarrow \quad (x+8)\left(x+\frac{1}{2}\right)<0$$

$$\therefore \quad -8<x<-\frac{1}{2}.$$

Example 6

Solve the inequality $|3x + 6| > -12$.

Solution

The modulus of any number is always non-negative.

$\therefore \quad |3x + 6| \geq 0$.

$\therefore$ The given inequality is always satisfied.

$\therefore \quad -\infty < x < \infty$

Example 7

Solve the inequality $|2x + 4| < -6$.

Solution

The modulus of any number is always non- negative.

$\therefore \quad |2x + 4| \geq 0$

$\therefore$ The given inequality will never satisfy.
The solution is null set.

Example 8

Solve for x: $|2x - 3| = 5$

Solution

$2x - 3 = 5$ or $2x - 3 = -5$

$$(\text{If } |y| = a, y = \pm a) \quad \Rightarrow \quad x = 4 \text{ or } x = -1.$$

Example 9

Find the maximum value of

$$g(x) = 16 - |-x - 6|; x \in R.$$

Solution

$g(x)$ is maximum when $|-x - 6|$ is minimum.
The minimum value of the modulus of all numbers is 0.
$\therefore$ The maximum value of $g(x) = 16 - 0 = 16$.

Exercises

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. If $a < b$ and $c < 0$, then which of the following is true?
   (A) $ac < bc$ (B) $\frac{a}{c} < \frac{b}{c}$
   (C) $ac > bc$ (D) None of these
2. If p and q are two real numbers, then which of the following statements is always true?
   (A) $\frac{p}{q} < 1 \Rightarrow p < q$
   (B) $p > 0, q > 0$ and $\frac{p}{q} > 1 \Rightarrow p > q$
   (C) $\frac{p}{q} > 1 \Rightarrow p > q$
   (D) All the above
3. If $5x - 8 < 2x + 9$ and $4x + 7 > 7x - 8$, then the range of the values of x that satisfies the inequalities is
   (A) $(5, \infty)$ (B) $(-\infty, 5)$
   (C) $\left(5, \frac{17}{3}\right)$ (D) $\left(-\infty, \frac{17}{3}\right)$
4. Solve for real values of x; $5x^2 - 3x - 2 \geq 0$.
   (A) $\left[\frac{-2}{5}, 1\right]$ (B) $R - \left(\frac{-2}{5}, 1\right)$
   (C) $[1, \infty)$ (D) $R - (0, 1)$
5. If $x^2 - 9x - 36$ is negative, then find the range of x.
   (A) $(-3, 12)$ (B) $[-3, 12]$
   (C) $(-12, 3)$ (D) $[-12, 3]$
6. Which of the following is true?
   (A) $|x+y| \leq |x| + |y|$ (B) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}, y \neq 0$
   (C) $|x-y| \geq ||x| - |y||$ (D) All the above
7. If $6x + 8 > 7x - 9$ and $4x - 7 < 6x - 3$, then the values of x is
   (A) $(-17, 2)$ (B) $(2, 17)$
   (C) $(-2, 17)$ (D) $(-\infty, 17)$
8. The solution set of the inequality $|x - 5| < 9$ is
   (A) $(0, 14)$ (B) $(-4, 14)$ (C) $(-4, 0)$ (D) $(9, 14)$
9. The number of integral values of x that do not satisfy the inequation $\frac{x+5}{x-2} \geq 0$ is
   (A) 7 (B) 5 (C) 6 (D) 4
10. If $(x + 5)(x + 9)(x + 3)^2 < 0$, then the solution set for the inequality is
    (A) $(-9, -3)$ (B) $(-9, -5)$
    (C) $(-3, \infty)$ (D) $(-9, \infty)$
11. Find the range of the real values of x satisfying $8 - 3x \leq 5$ and $4x + 5 \leq -7$.
    (A) $[-3, 1]$ (B) $(-\infty, -3] \cup [1, \infty)$
    (C) $(-3, 1)$ (D) ϕ
12. Which of the following is true?
    (A) $30^{31} < 31^{30}$ (B) $71^{69} > 70^{70}$
    (C) $(155)^{29} < (150)^{30}$ (D) Both (B) and (C)

13. At what value of x is $-|x-3|+\frac{21}{2}$ maximum?

(A) -3 (B) $\frac{21}{2}$ (C) 0 (D) 3

14. Find the range of all real values of x if $|3x+5|<5x-11$.

(A) $(8, \infty)$ (B) $(-\infty, -5/3) \cup (8, \infty)$
(C) $(-5/3, 8)$ (D) $(-5/3, \infty)$

15. If $ac = bd = 2$, then the minimum value of $a^2 + b^2 + c^2 + d^2$ is

(A) 4 (B) 6 (C) 8 (D) 16

16. If $x, y > 0$ and $x + y = 3$ then

(A) $xy \le 0.72$ (B) $xy \le 1.8$
(C) $xy \le 2.25$ (D) $xy \le 1.25$

17. Find the complete range of values of x that satisfies $|x-16| > x^2 - 7x + 24$.

(A) $(0, 2)$ (B) $\left(\frac{3}{2}, \frac{5}{2}\right)$
(C) $(1, 3)$ (D) $(2, 4)$

18. For which of the following range of values of x is $x^2 + x$ less than $x^3 + 1$?

(A) $(-\infty, -1)$ (B) $(1, \infty)$
(C) $(-1, 1) \cup (1, \infty)$ (D) $[-1, 1]$

19. If x, y, z are positive, then the value of

$A=\frac{(4x^2+x+4)(5y^2+y+5)(7z^2+z+7)}{xyz}$ can be

(A) 400 (B) 500 (C) 1000 (D) 1500

20. The range of x for which $2x^2 - 5x - 8 \le |2x^2 + x|$ is

(A) $\left[-\frac{4}{3}, \infty\right)$ (B) $\left(-\frac{4}{3}, -1\right)$
(C) $[-1, \infty)$ (D) $[-1, 2]$

21. For how many integral values of x, is the inequation $\frac{x-5}{x+7} > 4$ satisfied?

(A) 5 (B) 4 (C) 2 (D) 3

22. If $1 \le x \le 3$ and $2 \le y \le 5$, then the minimum value of $\frac{x+y}{y}$ is

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{6}{5}$ (D) $\frac{5}{6}$

23. If $|b| \ge 5$ and $x = |a|\, b$, which of the following is true?

(A) $a - xb > 0$ (B) $a + xb < 0$
(C) $a + xb > 0$ (D) $a - xb \le 0$

24. Find the number of solutions of the equation $\left|x-|x-2|\right| = 6$.

(A) 2 (B) 1 (C) 3 (D) 4

25. If x, y and z are positive real numbers, then the minimum value of

$\frac{x^2y+y^2z+z^2x+xy^2+yz^2+zx^2}{xyz}$ is

(A) 6 (B) 9 (C) 12 (D) 14

Answer Keys

1. C	**2.** B	**3.** B	**4.** B	**5.** A	**6.** D	**7.** C	**8.** B	**9.** A	**10.** B
11. D	**12.** C	**13.** D	**14.** A	**15.** C	**16.** C	**17.** D	**18.** C	**19.** D	**20.** A
21. D	**22.** C	**23.** D	**24.** B	**25.** A					

Chapter 12

Progressions

CHAPTER HIGHLIGHTS

- *Progressions*
- *Arithmetic Progression (AP)*
- *Geometric Progression (GP)*
- *Infinite Geometric Progression*
- *Some Important Results*

PROGRESSIONS

In this chapter, we will look at the problems on sequences or progressions of numbers, where the terms of the sequence follow a particular pattern either addition of a constant (arithmetic sequence or arithmetic progression) or multiplication by a constant (geometric sequence or geometric progression). A third type of progression—harmonic progression—has also been defined later.

Arithmetic Progression (AP)

An arithmetic progression is a sequence of numbers in which any number (other than the first) is more (or less) than the immediately preceding number by a constant value. This constant value is called the common difference. In other words, any term of an arithmetic progression can be obtained by adding the common difference to the preceding term.

Let a be the first term of an arithmetic progression, d the common difference, and n the number of terms in the progression.

The n^{th} term is normally represented by T_n, and the sum to n terms of an arithmetic progression is denoted by S_n

$$T_n = n^{\text{th}} \text{ term} = a + (n - 1)\, d$$

$S_n = \text{Sum of } n \text{ terms} = \frac{n}{2} \times [2a + (n - 1)\, d]$, then the progression can be represented as a, $a + d$, $a + 2d$, …, $[a + (n - 1)\, d]$. Here, quantity d is to be added to any chosen term to get the next term of the progression.

The sum to n terms of an arithmetic progression can also be written in a different manner.

$$\text{Sum of first } n \text{ terms} = \frac{n}{2} \times [2a + (n - 1)d]$$

$$= \frac{n}{2} \times [a + \{a + (n - 1)d\}]$$

But, when there are n terms in an arithmetic progression, a is the first term and $\{a + (n - 1)\,d\}$ is the last term. Hence, $S_n = \frac{n}{2} \times [\text{first term} + \text{last term}]$.

The average of all the terms in an arithmetic progression is called their arithmetic mean (AM). Since average is equal to {sum of all the quantities/number of quantities}, arithmetic progression must be equal to the sum of the terms of the arithmetic progression divided by the number of terms in the arithmetic progression.

Arithmetic mean of n terms in arithmetic progression

$$= \frac{S_n}{n} = \frac{1}{2}\ \{2a + (n - 1)d\}$$

$$= \frac{1}{2} \times (\text{First Term} + \text{Last Term})$$

$$= \frac{(\text{First Term} + \text{Last Term})}{2}$$

i.e. AM is the average of the first and the last terms of the AP.

Arithmetic mean can also be obtained by taking the average of any two terms that are EQUIDISTANT from the two ends of the AP, i.e.

1. The average of the second term from the beginning and the second term from the end will be equal to the AM.
2. The average of the third term from the beginning and the third term from the end will also be equal to the AM and so on.

In general, the average of the k^{th} term from the beginning and the k^{th} term from the end will be equal to the AM.

Conversely, if the AM of an AP is known, the sum to n terms of the series (S_n) can be expressed as

$$S_n = n \times \text{AM}.$$

If three numbers are in arithmetical progression, the middle number is called the arithmetic mean, i.e. if a, b, c are in AP, then b is the AM of the three terms and $b = \frac{a+c}{2}$.

If a and b are in arithmetic progression (AP), then their $\text{AM} = \frac{(a+b)}{2}$.

If three numbers are in AP, we can represent the three numbers as $(a - d)$, a, and $(a + d)$.

If four numbers are in AP, we can represent the four numbers as $(a - 3d)$, $(a - d)$, $(a + d)$, and $(a + 3d)$; (in this case, $2d$ is the common difference).

If five numbers are in AP, we can represent the five numbers as $(a - 2d)$, $(a - d)$, a, $(a + d)$, and $(a + 2d)$.

Solved Examples

Example 1

The sixth and the tenth terms of an arithmetic progression are 22 and 38, respectively. Find the first term and the common difference.

Solution

Let the first term and the common difference be a and d, respectively.

$$a + 5d = 22 \tag{1}$$

$$a + 9d = 38 \tag{2}$$

Subtracting (1) from (2),

$$4d = 16,\ d = 4$$

Substituting d in (1) or (2),

we get $\quad a = 2$

Example 2

The 12^{th} term, the 14^{th} term, and the last term of an arithmetic progression are 25, 31, and 37, respectively. Find the first term, common difference, and the number of terms.

Solution

Let the first term, the common difference and the number of terms be a, d, and n, respectively.

Given that

$$a + 11d = 25 \tag{3}$$

$$a + 13d = 31 \tag{4}$$

Subtracting (3) from (4),

$$2d = 6$$

$$d = 3$$

Substituting $d = 3$ in (3) or (4),

$$a = -8$$

given, $t_n = -8 + (n - 1)3 = 37$

$$n = 16$$

Example 3

Three terms in arithmetic progression have a sum of 45 and a product of 3240. Find them.

Solution

Let the terms be $a - d$, a and $a + d$.

$$a - d + a + a + d = 45$$

$$a = 15$$

$$(a - d)\ a\ (a + d) = 3240$$

$$15^2 - d^2 = 216$$

$$d = \pm 3$$

If $d = 3$ the terms are 12, 15, and 18. If $d = -3$, the terms are same but in the descending order.

Example 4

The first term and the last term of an arithmetic progression are 9 and 69, respectively. If the sum of all the terms is 468, find the number of terms and the common difference.

Solution

Let the number of terms and the common difference be n and d, respectively,

$$S_n = \frac{n}{2}[9 + 69] = 468$$

$$\Rightarrow \quad 39n = 468$$

$$n = 12$$

$$t_n = 9 + 11d$$

$$\Rightarrow \quad 11d = 60$$

$$\Rightarrow \quad d = \frac{60}{11}$$

Example 5

The sum of three numbers which are in arithmetic progression is 24. The sum of their square is 200. Find the numbers.

Solution

Let the numbers be $a - d$, a and $a + d$.

Given, $a - d + a + a + d = 24$

$\therefore \quad a = 8$

$$(a - d)^2 + a^2 + (a + d)^2 = 200$$

$$3a^2 + 2d^2 = 200$$

$\Rightarrow \quad d^2 = 4$

$\therefore \quad d = \pm 2$

If $d = 2$, the numbers are 6, 8 and 10. If $d = -2$, the numbers are same, but in the descending order.

Geometric Progression (GP)

Numbers taken in a certain order are said to be in geometrical progression, if the ratio of any (other than the first number) to the preceding one is the same. This ratio is called the common ratio. In other words, any term of a geometric progression can be obtained by multiplying the preceding number by the common ratio.

The common ratio is normally represented by r. The first term of a geometric progression is denoted by a.

A geometric progression can be represented as a, ar, $1ar^2$, …, where a is the first term and r is the common ratio of the geometric progression.

n^{th} term of the geometric progression is ar^{n-1}.

Sum to n terms:

$$\frac{a\left(1-r^n\right)}{1-r} \text{ or } \frac{a\left(r^n-1\right)}{r-1}$$

$$= \frac{xar^{n-1}-a}{r-1} = \frac{r \times \text{Last term} - \text{First term}}{r-1}$$

Thus, the sum to n terms of a geometric progression can also be written as

$$S_n = \frac{r \times \text{Last term} - \text{First term}}{r-1}$$

If n terms $a_1, a_2, a_3, \ldots, a_n$ are in GP, then the geometric mean (GM) of these n terms is given by $= \sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n}$

If three terms are in geometric progression then the middle term is a geometric mean of the other two terms, i.e. if a, b, and c are in GP, then b is the geometric mean of the three terms and $b^2 = ac$.

If there are two terms a and b, their geometric mean (GM) is given by $\sqrt{ab}$.

For any two unequal positive numbers a and b, their arithmetic mean is always greater than their geometric mean, i.e.

For any two unequal positive numbers a and b, $\frac{a+b}{2} > \sqrt{ab}$; $(a + b) > 2\sqrt{ab}$ $b) >$

When there are three terms in geometric progression, we can represent the three terms to be a/r, a, and ar

When there are four terms in geometric progression, we can represent the four terms as $\frac{a}{r^3}$, $\frac{a}{r}$, ar. and ar^3.

(In this case r^2 is the common ratio.)

Infinite Geometric Progression

If $-1 < r < +1$ or $|r| < 1$, then the sum of a geometric progression does not increase infinitely; it 'converges' to a particular value. Such a GP is referred to as an infinite geometric progression. The sum of an infinite geometric progression is represented by S_∞ and is given by the formula $S_\infty = \frac{a}{1-r}$.

Harmonic progression: If the reciprocals of the terms of a sequence are in arithmetic progression, the sequence is said to be a harmonic progression, For example, $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ is a harmonic progression. In general, the sequence $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$ is a harmonic progression.

If a, b, c are in harmonic progression, b is said to be the harmonic mean of a and c. In general, if $x_1, x_2, \ldots, x_n$ are in harmonic progression, $x_2, x_3, \ldots, x_{n-1}$ are the $n - 2$ harmonic means between x_1 and x_n.

Some Important Results

The results of the sums to n terms of the following series are quite useful and, hence, should be remembered by students.

Sum of the first n natural numbers $= \sum n = \frac{n(n+1)}{2}$

Sum of squares of the first n natural numbers

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of cubes of first n natural numbers

$$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4} = \left[\sum n\right]^2$$

Example 6

Find the 7^{th} term of the geometric progression whose first term is 6 and common ratio is 2.

Solution

n^{th} term of a GP $= ar^{n-1}$

$$7^{\text{th}} \text{ term} = 6\,(2^6) = 384$$

Example 7

A geometric progression has its first term as 64 and its common ratio as $\frac{1}{2}$. Find the sum of its first five terms.

Solution

Sum of the first n terms of a GP $= \dfrac{a(1-r^n)}{1-r}$

$$\text{Sum of its first five terms} = \frac{64\left(1-\left(\frac{1}{2}\right)^5\right)}{1-\frac{1}{2}} = 124$$

Example 8

Find the common ratio of the geometric progression whose first and last terms are 5 and $\dfrac{1}{25}$, respectively, and the sum of its terms is $\dfrac{624}{100}$.

Solution

Sum of the terms of a geometric progression whose common ratio is r is given by $\dfrac{r(\text{last term})-(\text{first term})}{r-1}$

$$\frac{r\left(\frac{1}{25}\right)-5}{r-1} = \frac{624}{100}.$$

$$\Rightarrow \quad 4r - 500 = 624r - 624$$

$$\Rightarrow \quad r = \frac{1}{5}.$$

Example 9

Three numbers in geometric progression have a sum of 42 and a product of 512. Find the numbers.

Solution

Let the numbers be $\dfrac{a}{r}$, a and ar.

$$\frac{a}{r} + a + ar = 42$$

$$\left(\frac{a}{r}\right)(a)(ar) = 512$$

$$a = 8$$

$$\frac{8}{r} + 8 + 8r = 42$$

$$8r^2 - 34r + 8 = 0$$

$$8r^2 - 32r - 2r + 8 = 0$$

$$(r-4)(4r-1) = 0$$

$$r = 4 \quad \text{or} \quad \frac{1}{4}$$

If $r = 4$, the numbers are 2, 8 and 32. If $r = \dfrac{1}{4}$, the numbers are same, but in the descending order.

Example 10

The sum of the terms of an infinite geometric progression is 27. The sum of their squares is 364.5. Find the common ratio.

Solution

Let the first term and the common ratio be a and r, respectively.

$$\text{Given that} \quad \frac{a}{1-r} = 27 \Rightarrow \left(\frac{a}{1-r}\right)^2 = 729$$

$$\text{And} \quad \frac{a^2}{1-r^2} = 364.5$$

$$\Rightarrow \quad a^2 = 729\,(1-r)^2$$

$$= 364.5\,(1-r^2)$$

$$729\,(1-r)^2 - \frac{729}{2}(1-r)(1+r) = 0$$

$$\frac{729}{2}(1-r)[2(1-r)-(1+r)] = 0$$

$$\Rightarrow \quad (1-r)(1-3r) = 0$$

$$r \neq 1 \quad (\because |r| < 1)$$

$$\therefore \quad r = \frac{1}{3}$$

Example 11

If $|x| < 1$, find the value of $3 + 6x + 9x^2 + 12x^3 + \cdots$

Solution

Let $S = 3 + 6x + 9x^2 + 12x^3 + \cdots$ (7)

$xS = 3x + 6x^2 + 9x^3 +$ (8)

Subtracting (8) from (7)

$$S(1-x) = 3\,(1 + x + x^2 + \cdots)$$

$$\text{As} \quad |x| < 1,\; S = \frac{3\left(\frac{1}{1-x}\right)}{1-x} = \frac{3}{(1-x)^2}$$

Example 12

Evaluate $\dfrac{1}{1(2)} + \dfrac{1}{2(3)} + \dfrac{1}{3(4)} + \cdots + \dfrac{1}{99(100)}$.

Solution

$$\frac{1}{1(2)} = \frac{1}{1} - \frac{1}{2}$$

$$\frac{1}{2(3)} = \frac{1}{2} - \frac{1}{3}$$

$$\text{Finally} \quad \frac{1}{99(100)} = \frac{1}{99} - \frac{1}{100}$$

The given expression is $1 - \dfrac{1}{100} = \dfrac{99}{100}$

EXERCISES

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. The sixth term and the eleventh term of an arithmetic progression are 30 and 55, respectively. Find the twenty-first term of the series.
(A) $88\frac{1}{3}$ (B) 105 (C) 110 (D) $92\frac{1}{2}$

2. What is the 15th term of an arithmetic progression whose first term is equal to its common difference and whose 3rd term is 9?
(A) 15 (B) 30 (C) 45 (D) 60

3. If $x + 4$, $6x - 2$, and $9x - 4$ are three consecutive terms of an arithmetic progression, then find x.
(A) 2 (B) 4 (C) 6 (D) 8

4. Find the number of terms and the sum of the terms of the arithmetic progression 32, 28, … 4.
(A) 8; 144 (B) 7; 126
(C) 14; 252 (D) 15; 270

5. Find the sum of the first 31 terms of the arithmetic progression whose first term is 6 and whose common difference is $\frac{8}{3}$.
(A) 1410 (B) 1418 (C) 1426 (D) 1434

6. The sum of five terms of an arithmetic progression is 70. The product of the extreme terms is 132. Find the five terms.
(A) 4, 8, 12, 16, 20 (B) 10, 12, 14, 16, 18
(C) 6, 10, 14, 18, 22 (D) 8, 12, 16, 20, 24

7. The sum to n terms of an arithmetic progression is $5n^2 + 2n$. Find the n^{th} term of the series.
(A) $10n + 5$ (B) $10n - 3$
(C) $5n - 1$ (D) $5n - 2$

8. Which term of the geometric progression 4, $4\sqrt{2}$, 8 … is $64\sqrt{2}$?
(A) 8 (B) 9 (C) 10 (D) 12

9. Find the sixth term of the geometric progression whose first term is 2 and common ratio is 3.
(A) 96 (B) 486 (C) 1458 (D) 162

10. Find the sum of the first 4 terms of a geometric progression whose first term is 6 and whose common ratio is 2.
(A) 90 (B) 84 (C) 96 (D) 102

11. What is the sum of the first 7 terms of a geometric progression whose first term is 1 and 4th term is 8?
(A) 129 (B) 128
(C) 127 (D) None of these

12. If the sum to 37 terms of an arithmetic progression is 703, then find the middle term of the arithmetic progression.
(A) 34 (B) 17 (C) 38 (D) 19

13. Find the sum of the 20 terms of the series 1, (1 + 2), (1 + 2 + 3), (1 + 2 + 3 + 4), (1 + 2 + 3 + 4 + 5), …
(A) 1540 (B) 1435 (C) 1450 (D) 1345

14. If the real numbers a, c and b as well as $a^2 + b^2$, $a^2 + c^2$, and $b^2 + c^2$ are in geometric progression, then which of the following is necessary true?
(A) $a = b$ (B) $b = c$
(C) $a = c$ (D) $a = b = c$

15. How many numbers between 450 and 950 are divisible by both 3 and 7?
(A) 20 (B) 24 (C) 30 (D) 35

16. $S = 2 + 4x + 6x^2 + 8x^3$ … where $|x| < 1$. Which of the following is the value of S?
(A) $\frac{4}{(1-x)^2}$ (B) $\frac{3}{(1-x)^2}$
(C) $\frac{2}{(1-x)^2}$ (D) $\frac{1}{(1-x)^2}$

17. The sum of the first eight terms of a geometric progression. is 510 and the sum of the first four terms of the geometric progression. is 30. Find the first term of the geometric progression, given that it is positive.
(A) 2 (B) 4 (C) 6 (D) 8

18. Find the integer value of y, if $-x$, $2y$, and $2(y + 3)$ are in arithmetic progression and $(x + 2)$, $2(y + 1)$, and $(5y - 1)$ are in geometric progression.
(A) 2 (B) 3 (C) 4 (D) 5

19. Find the number of terms common to the progressions 2, 8, 14, 20, …, 98 and 6, 10, 14, 18, …, 102.
(A) 7 (B) 6 (C) 8 (D) 9

20. Find the sum of the series $2 + 3x + 4x^2 + 5x^3 + \cdots$ to infinity, if $|x| < 1$.
(A) $\frac{2-x}{(1-x)^2}$ (B) $\frac{2+x}{(1+x)^2}$
(C) $\frac{2-x}{(1+x)^2}$ (D) $\frac{2+x}{(1-x)^2}$

21. The mean of the sequence 3, 8, 17, 30, …, 1227 is _____.
(A) 531 (B) 431 (C) 314 (D) 315

22. Find the value of $-1^2 + 2^2 - 3^2 + 4^2 - 5^2 + 6^2 + \cdots -19^2 + 20^2$
(A) 210 (B) 420 (C) 630 (D) 720

23. Find the sum of the given terms in the following series: $\frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \cdots + \frac{1}{\sqrt{119}+\sqrt{121}}$
(A) $2\sqrt{3}+1$ (B) 5
(C) $11 - 2\sqrt{3}$ (D) 10

24. If $\log_3 x + \log_{\sqrt[3]{3}} x + \log_{\sqrt[5]{3}} x + \cdots + \log_{\sqrt[23]{3}} x = 432$, then find x.
(A) 9 (B) 27 (C) $3\sqrt{3}$ (D) 81

25. The sum of the first n terms of two arithmetic progressions S_1 and S_2 are in the ratio $11n - 17 : 5n - 21$. Find the ratio of the 16th terms of S_1 and S_2.
(A) 3 : 2 (B) 162 : 67
(C) 9 : 4 (D) 27 : 8

Answer Keys

1. B	**2.** C	**3.** A	**4.** A	**5.** C	**6.** C	**7.** B	**8.** C	**9.** B	**10.** A
11. C	**12.** D	**13.** A	**14.** A	**15.** B	**16.** C	**17.** A	**18.** A	**19.** C	**20.** A
21. B	**22.** A	**23.** B	**24.** B	**25.** B					

Chapter 13

Permutations and Combinations

CHAPTER HIGHLIGHTS

- *Permutations*
- *Combinations*
- *Total Number of Combinations*
- *Dividing Given Items into Groups*
- *Circular Permutations*
- *Rank of a Word*
- *Arrangements*

Introduction

Permutations and combinations is one of the important areas in many exams because of two reasons. The first is that solving questions in this area is a measure of students' reasoning ability. Secondly, solving problems in areas like probability requires thorough knowledge of permutations and combinations.

Before discussing permutations and combinations, let us look at what is called as the 'fundamental rule'.

'If one operation can be performed in 'm' ways and (when, it has been performed in any one of these ways), a second operation then can be performed in 'n' ways, the number of ways of performing the two operations will be $m \times n$'.

This can be extended to any number of operations.

If there are three cities A, B, and C such that there are 3 roads connecting A and B and 4 roads connecting B and C, then the number of ways one can travel from A to C is 3×4, i.e. 12.

This is a very important principle, and we will be using it extensively in permutations and combinations. Because we use it very extensively, we do not explicitly state every time that the result is obtained by the fundamental rule but directly write down the result.

Permutations

Each of the arrangements which can be made by taking some or all of a number of items is called a permutation. Permutation implies 'arrangement' or that 'order of the items' is important.

The permutations of three items a, b, and c, taken two at a time are ab, ba, ac, ca, cb, and bc. Since the order in which the items are taken is important, ab and ba are counted as two different permutations. The words 'permutation' and 'arrangement' are synonymous and can be used interchangeably.

The number of permutations of n things taking r at time is denoted by ${}^nP_r$ (and read as '${}^nP_r$').

Combinations

Each of the groups or selections which can be made by taking some or all of a number of items is called a combination. In combinations, the order in which the items are taken is not considered as long as the specific things are included.

The combination of three items a, b, and c taken two at a time are ab, bc, and ca. Here, ab and ba are not considered separately because the order in which a and b are taken is not important but it is only required that a combination including a and b is what is to be counted. The words 'combination' and 'selection' are synonymous.

The number of combinations of n things taking r at time is denoted by ${}^nC_r$ (and read as '${}^nC_r$').

Number of linear permutations of 'n' dissimilar items taken 'r' at a time without repetition (${}^nP_r$)

Consider r boxes each of which can hold one item. When all the r boxes are filled, what we have is an arrangement of r items taken from the given n items. So, each time we fill up the r boxes with items taken from the given n items, we

have an arrangement of r items taken from the given n items without repetition. Hence, the number of ways in which we can fill up the r boxes by taking things from the given n things is equal to the number of permutations of n things taking r at a time.

Boxes □ □ □ □ .... □
1 2 3 4 r

The first box can be filled in n ways (because any one of the n items can be used to fill this box). Having filled the first box, to fill the second box, we now have only $(n - 1)$ items; any one of these items can be used to fill the second box, and, hence. the second box can be filled in $(n - 1)$ ways; similarly, the third box in $(n - 2)$ ways and so on the r^{th} box can be filled in $\{n - (r - 1)\}$ ways, i.e. $[n - r + 1]$ ways. Hence, from the Fundamental Rule, all the r boxes together be filled up in

$$n \times (n - 1) \times (n - 2) \ldots (n - r + 1) \text{ ways}$$

So, $\quad {}^nP_r = n \times (n - 1) \times (n - 2) \ldots (n - r + 1)$

This can be simplified by multiplying and dividing the right hand side by $(n - r)\ (n - r - 1) \ldots 3.2.1$ giving us ${}^nP_r = n(n - 1)\ (n - 2) \ldots [n - (r - 1)]$

$$= \frac{(n-1)(n-2)\ldots[n-(r-1).(n-r)\ldots 3.2.1]}{(n-r)\ldots\ldots\ldots 3.2.1} = \frac{n!}{(n-r)!}$$

The number of permutations of n distinct items taking r items at a time is

$$\boxed{{}^n p_r = \frac{n!}{(n-r)!}}$$

If we take n items at a time, then we get ${}^nP_n$. From a discussion similar to that we had for filling the r boxes above, we can find that ${}^nP_n$ is equal to $n!$

The first box can be filled in n ways, the second one in $(n - 1)$ ways, the third one in $(n - 2)$ ways, and so on, then the n^{th} box in 1 way; hence, all the n boxes can be filled in $n(n - 1)\ (n - 2) \ldots 3.2.1$ ways, i.e., $n!$ ways. Hence,

$$\boxed{{}^nP_n = n!}$$

But if we substitute $r = n$ in the formula for ${}^nP_r$, then we get ${}^nP_n = \frac{n!}{0!}$; since we already found that ${}^nP_n = n!$, we can conclude that $0! = 1$

Number of combinations of n dissimilar things taken r at a time.

Let the number of combinations ${}^nC_r$ be x. Consider one of these x combinations. Since this is a combination, the order of the r items is not important. If we now impose the condition that order is required for these r items, we can get $r!$ arrangements from this one combination. So, each combination can give rise to $r!$ permutations. x combinations will thus give rise to $x\,.\,r!$ permutations. But, since these are all permutations of n things taken r at a time, this must be equal to ${}^nP_r$. So,

$$x.r! = {}^nP_r = \frac{n!}{(n-r)!}$$

$$\Rightarrow \quad {}^nC_r = \frac{n!}{r!.(n-r)!}$$

The number of combinations of n dissimilar things taken all at a time is 1.

Out of n things lying on a table, if we select r things and remove them from the table, we are left with $(n - r)$ things on the table — that is, whenever r things are selected out of n things, we automatically have another selection of the $(n - r)$ things. Hence, the number of ways of making combinations taking r out of n things is the same as selecting $(n - r)$ things out of n given things, i.e.

$$\boldsymbol{{}^nC_r = {}^nC_{n-r}}$$

When we looked at ${}^nP_r$, we imposed two constraints which we will now release one by one and see how to find out the number of permutations.

Number of arrangements of n items of which p are of one type, q are of a second type, and the rest are distinct

When the items are all not distinct, then we **cannot** talk of a general formula for $\boldsymbol{{}^nP_r}$ for any r but we can talk of only ${}^nP_n$ (which is given below). If we want to find out ${}^nP_r$ for a specific value of r in a given problem, we have to work on a case to case basis (this has been explained in one of the solved examples).

The number of ways in which n things may be arranged taking them all at a time, when p of the things are exactly alike of one kind, q of them exactly alike of another kind, r of them exactly alike of a third kind, and the rest all distinct is $\frac{n!}{p!\,q!\,r!}$.

Number of arrangements of n distinct items where each item can be used any number of times (i.e. repetition allowed)

You are advised to apply the basic reasoning given while deriving the formula for ${}^nP_r$ to arrive at this result also. The first box can be filled up in n ways; the second box can be filled in again n ways (even though the first box is filled with one item, the same item can be used for filling the second box also because repetition is allowed); the third box can also be filled in n ways, and so on; the r^{th} box can be filled in n ways. Now, all the r boxes together can be filled in $\{n.n.n.n \ldots r \text{ times}\}$ ways, i.e. $\boldsymbol{n^r}$ ways.

The number of permutations of n things, taken r at a time when each item may be repeated once, twice, up to r times in any arrangement is $\boldsymbol{n^r}$.

What is important is not this formula by itself but the reasoning involved. So, even while solving problems of this type, you will be better off if you go from the basic reasoning and not just apply this formula.

Total number of combinations: Out of n given things, the number of ways of selecting **one or more** things is where

we can select 1 or 2 or 3 and so on n things at a time; hence the number of ways is ${}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n$

This is called 'the total number of combinations' and is equal to $2^n - 1$ where n is the number of things.

The same can be reasoned out in the following manner also.

There are n items to select from. Let each of these be represented by a box.

	1	2	3	4		n
No. of ways of dealing	☐	☐	☐	☐	....	☐
with the boxes	2	2	2	2		2

The first box can be dealt with in two ways. In any combination that we consider, this box is **either** included **or** not included. These are the two ways of dealing with the first box. Similarly, the second box can be dealt with in two ways, the third one in two ways, and so on; the n^{th} box in two ways. By the Fundamental Rule, the number of ways of dealing with all the boxes together in $2 \times 2 \times 2 \times \cdots n$ times ways, i.e. in 2^n ways. But out of these, there is one combination where we 'do not include the first box, do not include the second box, do not include the third box and so on, do not include the n th box'. That means, no box is included. But this is not allowed because we have to select **one or more** of the items (i.e. at least one item). Hence, this combination of no box being included is to be subtracted from the 2^n ways to give the result of

Number of ways of selecting one or more items from n given items is $2^n - 1$

Dividing given items into groups: Dividing $(p + q)$ items into two groups of p and q items, respectively.

Out of $(p + q)$ items, if we select p items (which can be done in ${}^{p+q}C_p$ ways), then we will be left with q items, i.e. we have two groups of p and q items, respectively. So, the number of ways of dividing $(p + q)$ items into two groups of p and q items, respectively, is equal to ${}^{p+q}C_p$ which is equal to $\dfrac{(p+q)!}{p!\,q!}$

The number of ways of dividing $(p + q)$ items into two groups of p and q items respectively is $\dfrac{(p+q)!}{p!\cdot q!}$.

If $p = q$, i.e. if we have to divide the given items into two EQUAL groups, then two cases arise

1. When the two groups have distinct identity and
2. When the two groups do not have distinct identity.

In the first case, we just have to substitute $p = q$ in the aforementioned formula which then becomes

The number of ways of dividing 2p items into two equal groups of p each is $\dfrac{(2p)!}{(p!)^2}$ where the two groups have distinct identity.

In the second case, where the two groups do not have distinct identity, we have to divide the above result by 2!, i.e. it then becomes

The number of ways of dividing 2p items into two equal groups of p each is $\dfrac{(2p)!}{2!(p!)^2}$ where the two groups do not have distinct identity.

Dividing $(p + q + r)$ items into three groups consisting of p, q, and r items, respectively

The number of ways in which $(p + q + r)$ things can be divided into three groups containing p, q, and r things, respectively, is $\dfrac{(p+q+r)!}{p!\,q!\,r!}$.

If $p = q = r$, i.e. if we have to divide the given items into three EQUAL groups, then we have two cases where the three groups are distinct and where the groups are not distinct.

When the three groups are distinct, the number of ways is $\dfrac{(3p)!}{(p!)^3}$.

When the three groups are not distinct, then the number of ways is $\dfrac{(3p)!}{3!(p!)^3}$.

Circular Permutations

When n distinct things are arranged in a straight line taking all the n items, we get $n!$ permutations. However, if these n items are arranged in a circular manner, then the number of arrangements will not be $n!$ but it will be less than that. This is because in a straight line manner, if we have an arrangement ABCDE and if we move every item one place to the right (in cyclic order), the new arrangement that we get EABCD is not the same as ABCDE and this also is counted in the $n!$ permutations that we talked of. However, if we have an arrangement ABCDE in a circular fashion, by shifting every item by one place in the clockwise direction, we still get the same arrangement ABCDE. So, if we now take $n!$ as the number of permutations, we will be counting the same arrangement more than once.

The number of arrangements in circular fashion can be found out by first fixing the position of one item. Then the remaining $(n - 1)$ items can be arranged in $(n - 1)!$ ways. Now, even if we move these $(n - 1)$ items by one place in the clockwise direction, then the arrangement that we get will not be the same as the initial arrangement because one item is fixed and it does not move.

Hence, the number of ways in which n distinct things can be arranged in a circular arrangement is $(n - 1)!$

> *The number of **circular arrangements** of **n distinct items** is **(n – 1)!** if there is **DIFFERENCE** between clockwise and anticlockwise arrangements and **(n – 1)!/2** if there is **NO DIFFERENCE** between clockwise and anticlockwise arrangements.*

The number of diagonals in an n-sided regular polygon

An n-sided regular polygon has n vertices. Joining any two vertices, we get a line of the polygon that are ${}^nC_2$ in number. Of these ${}^nC_2$ lines, n of them are sides. Hence, diagonals are ${}^nC_2 - n = \dfrac{n(n-3)}{2}$.

Number of integral solution of the equation

$$x_1 + x_2 + \cdots + x_n = s$$

Consider the equation $x_1 + x_2 + x_3 = 10$.

If we consider all possible integral solutions of this equation, there are infinitely many. But, the number of positive (or non-negative) integral solutions is finite.

We would like the number of positive integral solutions of this equation, i.e. values of (x_1, x_2, x_3) such that each $x_i > 0$.

We imagine 10 identical objects arranged on a line. There are 9 gaps between these 10 objects. If we choose any two of these gaps, we are effectively splitting the 10 identical objects into 3 parts of distinct identity. Conversely, every split of these 10 objects corresponds to a selection of 2 gaps out of the 9 gaps.

Therefore, the number of positive integral solutions is ${}^9C_2$. In general, if $x_1 + x_2 + \cdots + x_n = s$ where $s \geq n$, the number of positive integral solutions is ${}^{s-1}C_{n-1}$.

If we need the number of non negative integral solutions, we proceed as follows. Let $a_1, a_2, \ldots$ be a non-negative integral solution. Then, $a_1 + 1$, $a_2 + 1$, …, $a_n + 1$ is a positive integral solution of the equation $x_1 + x_2 + \cdots + x_n = s + n$. Therefore, the number of non-negative integral solutions of the given equation is equal to the number of positive integral solutions of $x_1 + x_2 + \cdots + x_n = s + n$, which is ${}^{s+n-1}C_{n-1}$.

> *For $x_1 + x_2 + x_3 + \cdots + x_n = s$ where $s \geq 0$, the number of **positive integral solutions** (when $s \geq n$) is ${}^{s-1}\boldsymbol{C_{n-1}}$ and the number of **non-negative integral solutions** is ${}^{n+s-1}\boldsymbol{C_{n-1}}$*

Some additional points

1. Suppose there are n letters and n corresponding addressed envelopes. The numbers of ways of placing these letters into the envelopes such that no letter is placed in its corresponding envelope is often referred as derangements. The number of derangements of n objects is given by

$$D(n) = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^n \frac{1}{n!}\right]$$

For example, when $n = 3$, the number of derangements is $D(3) = 3!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right] = 2$ and when $n = 4$,

$$D(4) = 4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 9$$

2. The total number of ways in which a selection can be made by taking some or all out of $p + q + r + \cdots$ things where p are alike of one kind, q alike of a second kind, r alike of a third kind, and so on is

$$[\{(p + 1)(q + 1)(r + 1) \ldots\} - 1].$$

3. ${}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$ and ${}^nP_r = r.{}^{n-1}P_{r-1} + {}^{n-1}P_r$

Solved Examples

Example 1

Consider the word PRECIPITATION. Find the number of ways in which

(i) a selection
(ii) an arrangement

of 4 letters can be made from the letters of this word.

Solution

The word PRECIPITATION has 13 letters I, I, I, P, P, T, T, E, R, C, A, O, N of 9 different sorts.

In taking 4 letters, the following are the possibilities to be considered.

(a) all 4 distinct.
(b) 3 alike, 1 distinct.
(c) 2 alike of one kind, 2 alike of other kind.
(d) 2 alike, 2 other distinct.

Selections

(a) 4 distinct letters can be selected from 9 distinct letters (I, P, T, E, R, C, A, O, N) in ${}^9C_4 = 126$ ways.
(b) As 3 letters have to be alike, the only possibility is selecting all the I's. Now the 4th letter can be selected from any of the remaining 8 distinct letters in ${}^8C_1 =$ 8 ways.
(c) Two pairs of two alike letters can be selected from I's, Q's, and T's in ${}^3C_2 = 3$ ways.
(d) The two alike letters can be selected in ${}^3C_1 = 3$ ways and the two distinct letters can now be selected from the 8 distinct letters in ${}^8C_2 = 28$ ways. Hence, required number of ways are $3 \times 28 = 84$.

Hence, the total selections are $126 + 8 + 3 + 84 = 221$.

Arrangements: For arrangements, we find the arrangements for each of the aforementioned selections and add them up.

(a) As the 4 letters are distinct, there are 4! arrangements for each selection. Hence required arrangements are $126 \times 4! = 3024$.

(b) Since 3 of the 4 letters are alike, there are $\frac{4!}{3!}$ arrangements for each of the selection. Hence, required arrangements are $8 \times \frac{4!}{3!} = 32$.

(c) The required arrangements here are $3 \times \frac{4!}{2!\,2!} = 18$

(d) The required arrangements are $84 \times \frac{4!}{2!} = 1008$.

Total number of arrangements are 3024 + 32 + 18 + 1008 = 4082.

Example 2

How many four letter words can be formed using the letters of the word 'ROAMING'?

Solution

None of the letters in the word are repeated.
∴ The number of four letter words that can be formed = ${}^7P_4$
$= \frac{7!}{3!} = (7)\,(6)\,(5)\,(4) = 840.$

Example 3

In a party, each person shook hands with every other person present. The total number of hand shakes was 28. Find the number of people present in the party.

Solution

Let the number of people present in the party be n.

Method 1: The first people shakes hands with a total of $(n - 1)$ persons, the second with $(n - 2)$ other people, and so on.

The total number of hand shakes is

$$(n - 1) + (n - 2) + \cdots + 2 + 1$$

$$\frac{n(n-1)}{2} = 28 \quad \text{(given)}$$

$$\Rightarrow \quad n = 8$$

Method 2: Number of hand shakes = Number of ways of selecting 2 people out of $n = {}^nC_2$.

$${}^nC_2 = 28$$

$$\frac{n(n-1)}{2!} = 28$$

$$\Rightarrow \quad n = 8$$

Direction for examples 3 to 7: The following examples are based on the data below.

The letters of NESTLE are permuted in all possible ways.

Example 4

How many of these words begin with T?

Solution

NESTLE has 6 letters of which the letter E occurs two times. Therefore, the required number of words = Number of ways of filling N, E, S, E, and L in the second to sixth positions $= \frac{5!}{2!} = 60$.

Example 5

How many of these words begin and end with E?

Solution

The required number of words = The number of ways of filling N, S, T, and L in the second to fifth positions = 4! = 24.

Example 6

How many of these words begin with S and end with L?

Solution

The required number of words = The number of ways of filling N, E, T, and E in the second to fifth positions $= \frac{4!}{2!} = 12$.

Example 7

How many of these words neither begin with S nor end with L?

Solution

The required number of words = The total number of words which can be formed using the letters N, E, S, T, and E – (Number of words which begin with S or end with L) $= \frac{6!}{2!}$ – (Number of words beginning with S + Number of words ending with L – Number of words beginning with S and ending with L)

$$= \frac{6!}{2!} - \left(\frac{5!}{2!} + \frac{5!}{2!} - \frac{4!}{2!}\right)$$

$$= 360 - (60 + 60 - 12) = 252.$$

Example 8

How many of these words begin with T and do not end with N?

Solution

The required number of words = The number of words beginning with T – The number of words beginning with T and ending with $N = \frac{5!}{2!} - \frac{4!}{2!} = 48$.

Direction for examples 8 to 11: The following examples are based on the data below.

The letters of FAMINE are permuted in all possible ways.

Example 9

How many of these words have all the vowels occupying odd places?

Solution

FAMINE has 3 vowels and 3 consonants.

The vowels can be arranged in the odd places in 3! or 6 ways.

The consonants would have to be arranged in even places. This is possible in 3! or 6 ways as well.

$\therefore$ The required number of words = $6^2 = 36$.

Example 10

How many of these words have all the vowels together?

Solution

If all the vowels are together, the vowels can be arranged in 3! ways among themselves.

Considering the vowels as separate a unit and each of the other letters as a unit, we have a total of 4 units that can be arranged in 4! ways.

$\therefore$ The required number of words = 4! 3! = 144

Example 11

How many of these words have at least two of the vowels separated?

Solution

The required number of words = The total number of words which can be formed using the letters *F, A, M, I, N,* and *E* – The number of words with all the vowels together = 6! – 4! 3! = 576.

Example 12

How many of these words have no two vowels next to each other?

Solution

To ensure that no two vowels are together, we first arrange the 3 consonants say $-c_1 - c_2 - c_3 -$ and place the vowels in the gaps between the consonants or the initial or final position. For each arrangement of the consonants, there are 4 places where the vowels can go. The vowels can be dealt with in 4 (3) (2) ways.

$\therefore$ The total number of words is 3! 4! = 144.

Direction for examples 17 and 18: The following examples are based on the data below.

A committee of 5 is to be formed from 4 women and 6 men.

Example 18

In how many ways can it be formed if it consists of exactly 2 women?

Solution

The committee must have 2 women and 3 men.

$\therefore$ The required number of ways = ${}^4C_2\,{}^6C_3 = 120$.

Example 19

In how many ways can it be formed if it consists of more women than men?

Solution

The committee must have either 4 women and 1 man or 3 women and 2 men.

$\therefore$ The required number of ways

$$= {}^4C_4\,{}^6C_1 + {}^4C_3\,{}^6C_2 = 6 + 60 = 66.$$

Example 20

Find the number of four-digit numbers that can be formed using four of the digits 0, 1, 2, 3, and 4 without repetition.

Solution

The first digit has 4 possibilities (1, 2, 3, and 4).

The second digit has 4 possibilities (0 and any of the three digits not used as the first digit).

The third digit has 3 possibilities.

The last digit has 2 possibilities.

$\therefore$ The required number of numbers = (4) (4) 3 (2) = 96.

Example 21

The number of diagonals of a regular polygon is four times the number of its sides. How many sides does it have?

Solution

Let the number of sides in the polygon be n.

$$\frac{n(n-3)}{2} = 4n$$

$$n(n-11) = 0;\ n > 0$$

$$\therefore \quad n - 11 = 0;$$

$$\therefore \quad n = 11.$$

Exercises

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. A man has 12 blazers, 10 shirts, and 5 ties. Find the number of different possible combinations in which he can wear the blazers, shirts, and ties.
(A) 27 (B) 300 (C) 240 (D) 600

2. How many different words can be formed by using all the letters of the word INSTITUTE?
(A) $\frac{9!}{2!}$ (B) 9 ! (C) $\frac{9!}{3!}$ (D) $\frac{9!}{3!2!}$

3. In how many ways can a cricket team of 11 members be selected from 15 players, so that a particular player is included and another particular player is left out?
(A) 216 (B) 826 (C) 286 (D) 386

4. A group contains n persons. If the number of ways of selecting 6 persons is equal to the number of ways of selecting 9 persons, then the number of ways of selecting four persons from the group is
(A) 1365 (B) 273 (C) 455 (D) 285

5. The number of ways of arranging 10 books on a shelf such that two particular books are always together is
(A) 9! 2! (B) 9! (C) 10! (D) 8

6. Find the number of ways of inviting at least one among 6 people to a party.
(A) 2^6 (B) $2^6 - 1$ (C) 6^2 (D) $6^2 - 1$

7. An eight-letter word is formed by using all the letters of the word ‘EQUATION’. How many of these words begin with a consonant and end with a vowel?
(A) 3600 (B) 10800
(C) 2160 (D) 720

8. A committee of 5 members is to be formed from a group of 6 men and 4 women. In how many ways can the committee be formed such that it contains more men than women?
(A) 180 (B) 186 (C) 126 (D) 66

9. In how many ways can 10 boys and 10 girls be arranged in a row so that all the girls sit together?
(A) 10! (B) 11!
(C) 20! (D) 10! 11!

10. In how many ways can 6 boys and 5 girls be arranged in a row so that boys and girls sit alternately?
(A) $(6!)^2$ (B) $(5!)^2$
(C) 6! 5! (D) 2.5! 6!

11. There are seven letters and corresponding seven addressed envelopes. All the letters are placed randomly into the envelopes—one in each envelope. In how many ways can exactly two letters be placed into their corresponding envelopes?
(A) 44 (B) 924 (C) 308 (D) 189

12. We are given 3 different green dyes, 4 different red dyes, and 2 different yellow dyes. The number of ways in which the dyes can be chosen so that at least one green dye and one yellow dye is selected is
(A) 336 (B) 335 (C) 60 (D) 59

13. Prahaas attempts a question paper that has 3 sections with 6 questions in each section. If Prahaas has to attempt any 8 questions, choosing at least two questions from each section, then in how many ways can he attempt the paper?
(A) 18000 (B) 10125
(C) 28125 (D) 9375

14. Find the number of selections that can be made by taking 4 letters from the word INKLING.
(A) 48 (B) 38 (C) 28 (D) 18

15. A man has $(2n + 1)$ friends. The number of ways in which he can invite at least $n + 1$ friends for a dinner is 4096. Find the number of friends of the man.
(A) 11 (B) 15 (C) 17 (D) 13

16. How many four-digit numbers are there between 3200 and 7300, in which 6, 8, and 9 together or separately do not appear?
(A) 1421 (B) 1420
(C) 1422 (D) 1077

17. Raju has forgotten his six-digit ID number. He remembers the following: the first two digits are either 1, 5 or 2, 6, the number is even and 6 appears twice. If Raju uses a trial and error process to find his ID number at the most, how many trials does he need to succeed?
(A) 972 (B) 2052
(C) 729 (D) 2051

18. A matrix with four rows and three columns is to be formed with entries 0, 1, or 2. How many such distinct matrices are possible?
(A) 12 (B) 36 (C) 3^{12} (D) 2^{12}

19. In how many ways can 4 postcards be dropped into 8 letter boxes?
(A) ${}^8P_4$ (B) 4^8 (C) 8^4 (D) 24

20. In how many ways can 12 distinct pens be divided equally among 3 children?
(A) $\frac{12!}{(3!)^4}$ (B) $\frac{12!}{(4!)^3\,3!}$
(C) $\frac{12!}{3!4!}$ (D) $\frac{12!}{(4!)^3}$

21. If all possible five-digit numbers that can be formed using the digits 4, 3, 8, 6, and 9 without repetition are arranged in the ascending order, then the position of the number 89634 is
(A) 91 (B) 93 (C) 95 (D) 98

22. Manavseva, a voluntary organization, has 50 members who plan to visit 3 slums in an area. They decide to divide themselves into 3 groups of 25, 15, and 10. In how many ways can the group division be made?

(A) 25! 15! 10!
(B) $\frac{50!}{25!\,15!\,10!}$
(C) 50!
(D) 25! + 15! + 10!

23. In how many ways is it possible to choose two white squares so that they lie in the same row or same column on an 8×8 chessboard?

(A) 12 (B) 48 (C) 96 (D) 60

24. The number of four digit telephone numbers that have at least one of their digits repeated is

(A) 9000 (B) 4464 (C) 4000 (D) 3986

25. There are 4 identical oranges, 3 identical mangoes, and 2 identical apples in the basket. The number of ways in which we can select one or more fruits from the basket is

(A) 60 (B) 59 (C) 57 (D) 55

Answer Keys

1. D	**2.** D	**3.** C	**4.** A	**5.** A	**6.** B	**7.** B	**8.** B	**9.** D	**10.** C
11. B	**12.** A	**13.** C	**14.** D	**15.** D	**16.** D	**17.** B	**18.** C	**19.** C	**20.** D
21. C	**22.** B	**23.** C	**24.** B	**25.** B					

Chapter 14

Data Interpretation

CHAPTER HIGHLIGHTS

- ☞ *Methods of Presenting Data*
- ☞ *Data table*
- ☞ *Pie Charts*
- ☞ *Two-Dimensional Graphs*

Introduction

Not a day passes without our coming across figures and statistics. Study and manipulation of such data leads us to an important area: Data Interpretation. Data can be organized in a number of ways so that larger volume of data can be presented in a more compact and precise form. Data thus presented have to be deciphered correctly by the user of the data. This process of deciphering the data from its compactly presented form is called Data Interpretation.

Methods of Presenting Data

Numerical data can be presented in one or more of the following ways:

1. Data Tables
2. Pie Charts
3. 2-Dimensional Graphs
4. Bar Charts
5. 3-Dimensional Graphs
6. Venn Diagrams
7. Geometrical Diagrams
8. Pert Charts
9. Others

The 'Others' category covers miscellaneous forms like descriptive case format customized for the situation. Data can also be presented by using a combination of two or more of the aforementioned forms.

While some data can be presented in many different forms, some other data may be amenable to be presented only in a few ways. In real life situations, the style of data presentation is based on the end-objective. In certain situations, data have to be presented as a combination of two or more forms of data presentation.

Let us understand each of the above forms of data presentation with an example.

Data Table

Here, data are presented in the form of simple table. While any type of data can be presented in tabular form, that too in a very accurate manner, interpreting the data in tabular form is more difficult and time consuming than the other modes, all of which are basically pictorial or graphical in presentation.

Data tables can be of a number of types. They can be of a single-table variety or combination of tables. Some examples of tables are given below.

Table 1 Movement of goods by different modes of transport

(in 000's of metric-ton-kms)

Year	Road	Rail	Air	Water	Total
1985	1000	1500	120	20	2640
1986	1600	2000	129	24	3753
1987	2907	3090	139	28	6164
1988	4625	5200	152	27	10004
1989	6346	7540	174	33	14093
1990	7920	10250	212	40	18422
1991	9540	13780	266	50	23636

From the table, we can deduce the following:

1. Rate of growth by each mode of transport in successive years as well as cumulative annual growth.
2. Rate of growth of total haulage by all modes of transport together in any year.
3. Contribution by each mode of transport to the total haulage in any given year.
4. Trends of growth over time for various modes of transport.
5. Given the cost of transportation for each mode, we can calculate the total annual cost of transportation over the years for various modes of transport as well make a cost comparison.
6. Finding out the mode of transportation in any given year that forms the largest percentage of total haulage.
7. For a given mode of transport, finding out the year in which the percentage increase in haulage over the previous year was the highest.

Pie Charts

This is probably the simplest of all pictorial forms of data presentation. Here, total quantity to be shown is distributed over one complete circle or 360 degrees. In pie-charts, data is essentially presented with respect to only one parameter (unlike in 2- and 3-dimensional graphs described later). This form essentially presents shares of various elements as proportion or percentage of the total quantity. Each element or group in the pie-chart is represented in terms of quantity (or value, as the case may be) or as the angle made by the sector representing the elements or as a proportion of the total or as a percentage of the total.

Figure 1 gives distribution of the population in different geographical zones.

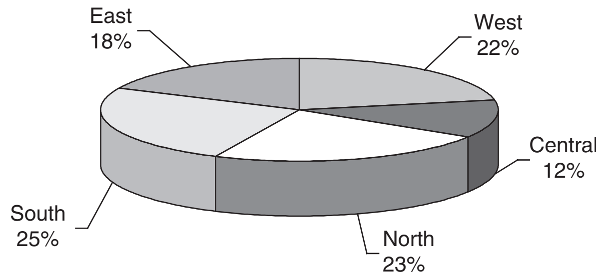


Figure 1 Distribution of population in geographical zones

From the above-mentioned pie chart, we can calculate the following:

1. Population in any zone given the total population.
2. Population of any zone as a percentage of that of another zone.
3. Percentage increase in the total population given the percentage increase in the population of one or more zones.

Pie Charts are also very frequently used in combination with other forms of data or along with other pie charts.

Two-Dimensional Graphs

This is essentially used for continuous data but can also be used for depicting discrete data provided we understand the limitation. Also known as Cartesian Graphs, they represent variation of one parameter with respect to another parameter each shown on a different axis. These types of graphs are useful in studying the rate of change or understanding the trends through extrapolations.

These graphs can be of various types, and a few of them are shown below (Figures 2 and 3):

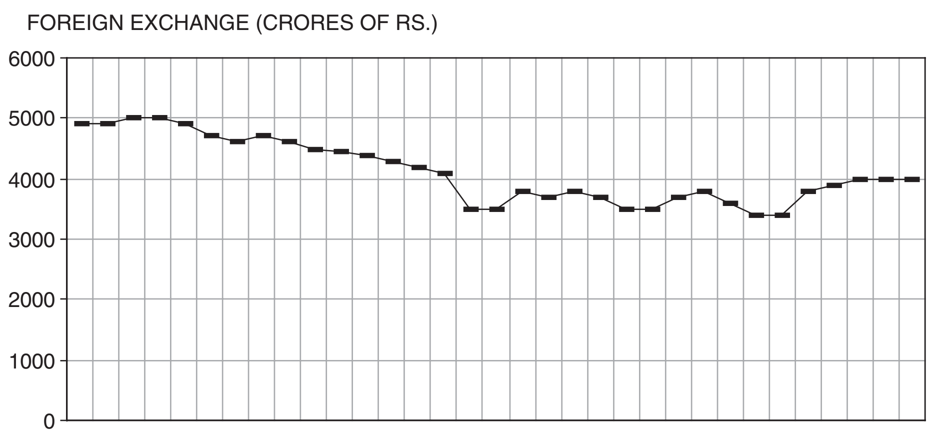


Figure 2 Foreign exchange reserves of India

The graph in Figure 2 shows the changes in the foreign exchange reserves of our country during a period of time. One can find out trends and the growth rates of foreign exchange reserves.

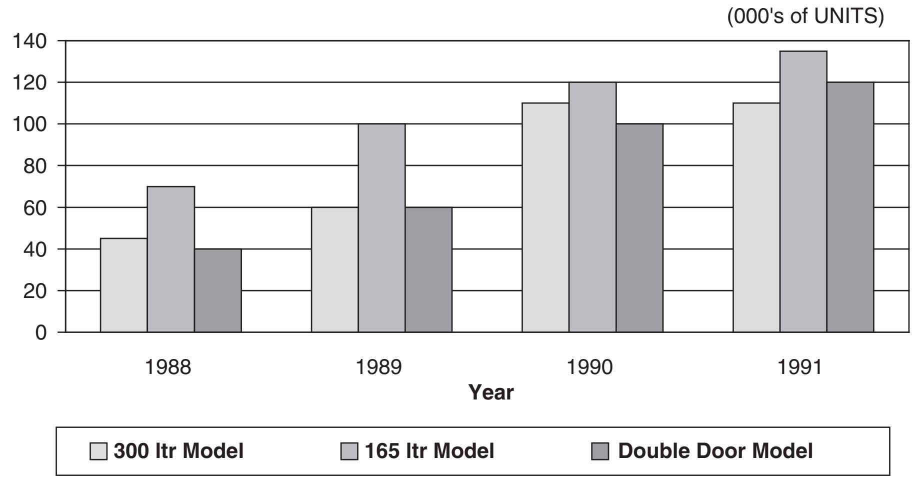


Figure 3 Refrigerator sales of company ABC

Chart 3 shows model wise sales of refrigerators during four years. From this graph, we can obtain the following:

1. Percentage contribution of each model to the company's total sales for four years.
2. Relative increase or decrease in the share of each model.
3. Sales trend of various models.

Using this bar chart, one can carry out a detailed performance evaluation of the company with respect to the sales of the four year period 1988 to 1991 for any given model. These bar charts can also be depicted horizontally. Another variation could be showing each product at one place (rather than each year at one place).

Example: These questions are based on the following table, which gives the details of the sports which students in all the classes of a school like.

The table gives the number of students in each class and the percentage of students in it who like cricket, volleyball, basketball, and football.

Class	Number of students	Cricket	Volleyball	Basketball	Football
6	120	60%	70%	50%	60%
7	140	50%	60%	60%	50%
8	160	40%	65%	55%	45%
9	180	65%	75%	65%	55%
10	240	70%	80%	75%	45%

Solved Examples

Example 1

How many students in the school like cricket?

(A) 436 (B) 432 (C) 491 (D) 511

Solution

Number of students who like cricket

$$= \frac{60}{100}(120) + \frac{50}{100}(140) + \frac{40}{100}(160) + \frac{65}{100}(180) + \frac{70}{100}(240) = 491$$

Example 2

By what percentage is the number of students who like volleyball in class 6 more/less than those who like basketball in class 10?

(A) 40% (B) 50%
(C) 53.33% (D) 56.67%

Solution

Number of students who like volleyball in class 6 $= \frac{70}{100}(120) = 84$

Number of students who like basketball in class 10 $= \frac{75}{100}(240) = 180$

84 is less than 180 by $\frac{180-84}{180}(100) = 53.33\%$.

Example 3

The number of students who like cricket in class 7 is what percentage of the number of students who like football in class 8?

(A) 88% (B) 93.5%
(C) 95.6% (D) 97.2%

Solution

Number of students who like cricket in class 7 = $\frac{50}{100}(140) = 70$

Number of students who like football in class 8 = $\frac{45}{100}(160) = 72$

Required percentage = $\frac{70}{72} \times 100 = 97.2\%$

Example 4

In how many of the given classes can more than 90 students like all the three games?

(A) 2 (B) 3 (C) 1 (D) 0

Solution

In any class, the maximum value of the number of students who like all the three games would be the number of students who like the game liked by the least number of students.

In class 6, the percentage of students who like a game is the least for basketball. Number of those who like basketball = $\frac{50}{100}(120) = 60 < 90$.

In class 7, the percentage of students who like a game is the least for cricket and football. Number of students who like cricket

$$= \frac{50}{100}(140) = 70 < 90$$

In class 8, the number of students who like a game is the least for cricket. Number of students who like cricket = $\frac{40}{100}(160) = 64 < 90$.

In class 9, the percentage of students who like a game is the least for football. Number of those who like football

$$= \frac{55}{100}(180) = 99 > 90$$

In class 10, the percentage of students who like a game is the least for football. Number of students who like football

$$= \frac{45}{100}(240) = 108 > 90$$

∴ In two classes, more than 90 students can like all the games.

Example 5

What can be the maximum percentage of students in class 6 who do not like any of the given games?

(A) 40% (B) 10% (C) 50% (D) 30%

Solution

In class 6, the maximum percentage of students who like a game = Percentage of students who like volleyball, i.e. 70%. Percentage of students who like at least one game would be minimum when all students who like other games are the ones who like volleyball.

∴ Maximum percentage required

= 100 – 70 = 30%.

Exercises

Direction for question 1: Select the correct alternative from the given choices.

1. The table shows the total marks of four students P, Q, R, and S in all their subjects for the two years 2012 and 2013.

Students	2012	2013
P	997	1295
Q	664	876
R	585	732
S	480	689

How many students had a percentage Increase in their total marks of more than 35% from 2012 to 2013?

(A) 1 (B) 2 (C) 3 (D) 4

Direction for questions 2 to 4: These questions are based on the following data which give some details of new states joining the United States of America across time.

State	Capital	Joined the union	Union rank	Population	Number of representatives in the house of representatives
Washington	Olympia	Nov 11, 1889	42	62,87,759	9
Texas	Austin	Dec 29, 1845	28	2,28,59,968	32
Delaware	Dover	Dec 7, 1781	1	8,43,524	1
Virginia	Raleigh	Nov 21, 1789	12	86,83,242	13

(Continued)

State	Capital	Joined the union	Union rank	Population	Number of representatives in the house of representatives
Minnesota	St. Paul	May 11, 1832	32	51,32,799	8
Kansas	Topeka	Jan 29, 1861	34	27,44,687	4
Illinois	Springfield	Dec 3, 1818	21	1,27,63,371	19
New Hampshire	Concord	June 21, 1788	9	13,09,940	2
Arizona	Phoenix	Feb 14, 1912	48	59,39,292	8
Hawai	Honolulu	Aug 21, 1959	50	12,75,194	2
Indiana	Indianapolis	Dec 11, 1816	19	62,71,973	9
Vermont	Montpelier	March 14, 1791	14	6,23,050	1
Nebraska	Lincoln	March 1, 1867	37	17,58,787	3
Georgia	Atlanta	Jan 2, 1788	4	9,07,256	13

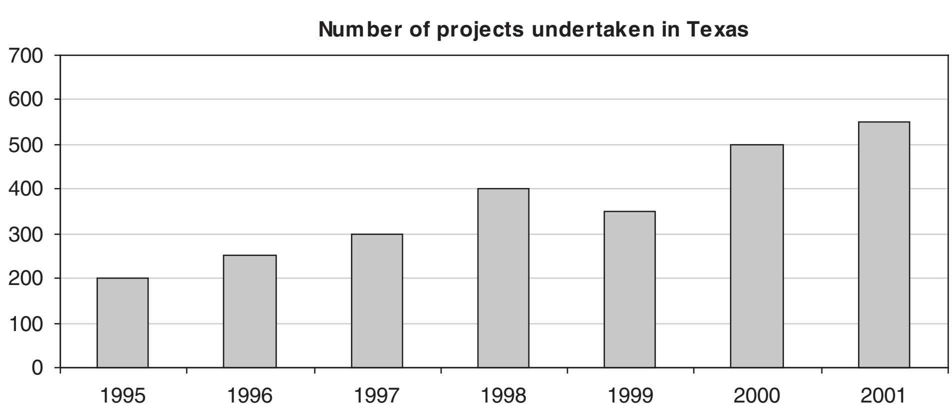
Union rank is the chronological order in which the states joined the Union.

2. How many states joined the Union from March 1, 1867, to Feb 14, 1912?
(A) 11 (B) 12
(C) 13 (D) 14

3. If it is known that the House of Representatives of USA has a strength of 535 members, then the number of representatives in the House of Representatives of the given states will form what approximate percentage of the total strength of the House of Representatives?
(A) 16 (B) 19 (C) 21 (D) 23

4. In how many of the given states is the population less than 15 million but the number of representatives is not less than six?
(A) 4 (B) 5 (C) 6 (D) 7

Direction for questions 5 to 7: These questions are based on the following graphs.



5. In how many years was the number of projects undertaken in Mexico greater than that in Texas?
(A) 3 (B) 6 (C) 5 (D) 4

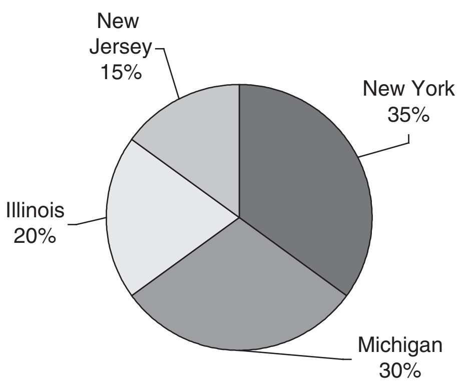
6. How many projects were undertaken in the year 1998 in both places together?
(A) 1000 (B) 900 (C) 750 (D) 500

7. In which of the following years was the average (arithmetic mean) number of projects undertaken in both places the highest?
(A) 2000 (B) 1996 (C) 1998 (D) 1999

Direction for questions 14 to 16: The table below shows the percentages of colleges offering the courses mentioned in medicine in four states, New York, New Jersey, Illinois, and Michigan, in a certain year.

Sl. No.	Course	New York	New Jersey	Illinois	Michigan
1.	Biochemistry	86	80	74	68
2.	Biophysics	74	92	88	64
3.	Biomechanics	59	82	84	68
4.	Biostatistics	56	84	86	70

The total number of colleges offering courses in medicine in the four states is 2000. The percentage-wise distribution of the number of colleges in the four states is as shown below.



8. The number of colleges offering the Biochemistry course is more than 230 in
(A) All the four states. (B) Exactly three states.
(C) Exactly two states. (D) Exactly one state.

9. What percentage of the colleges in the four states do not offer Biophysics as well as Biochemistry?
(A) 41%
(B) 36%
(C) 34%
(D) Cannot the determined

10. What is the total number of colleges offering Biostatistics in all the four states?
(A) 1392 (B) 1408 (C) 1432 (D) 1476

Direction for questions 11 and 12: Study the given table and answer the questions that follow.

Percentage break-up of the workers working in six different factories *J, K, L, M, N*, and *P*.

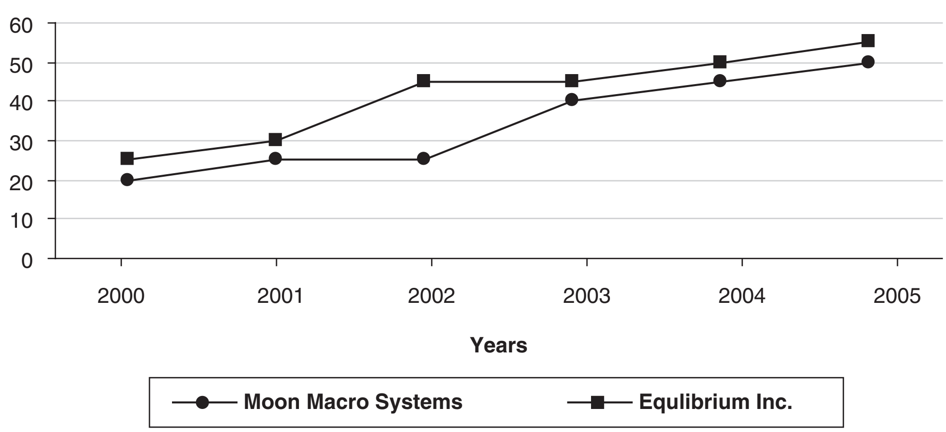
Factory	Total no. of workers	Percentage		
		Men	Women	Boys
J	4800	50	37.5	12.5
K	8750	40	36	24
L	5250	24	56	20
M	12000	35	25	40
N	8500	38	30	32
P	2700	45	40	15

11. By what percent is the number of women working in factory *P* is more than the number of boys working in factory *J*?
(A) 20% (B) $44\frac{4}{9}$%
(C) 80% (D) 180%

12. What is the ratio of the number of men working in factory *M* to the number of women working in factory *L*?
(A) 7 : 10 (B) 10 : 7 (C) 7 : 5 (D) 5 : 7

Direction for questions 13 to 15: These questions are based on the following line graph.

Numbers of employees of two companies each years over the period 2000 to 2005 (in thousands)

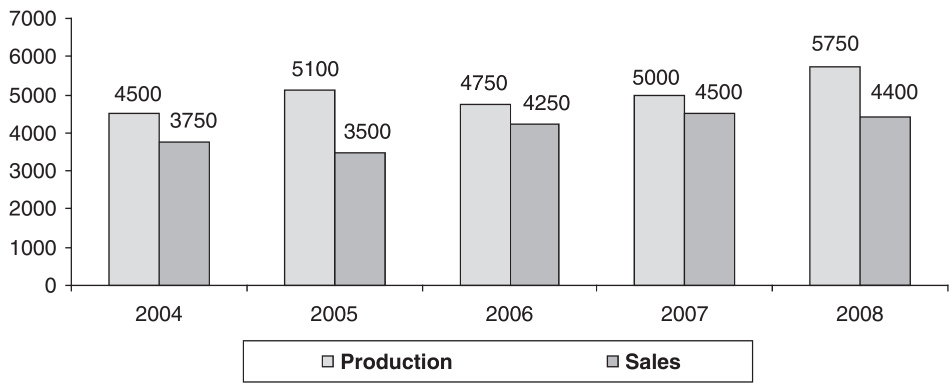


13. What is the ratio of the average number of employees of company Moon Macro Systems for the period 2001, 2002 and 2003 and the average number of employees of company Equilibrium Inc. for the same period?
(A) 2 : 5 (B) 3 : 5 (C) 3 : 4 (D) 4 : 3

14. During which of the following years was the percentage increase in the number of employees of company Equilibrium Inc. over that in the previous year the highest?
(A) 2000
(B) 2001
(C) 2002
(D) 2004

15. What is the approximate percentage increase in the total number of employees of the two companies from 2004 to 2005?
(A) 8.5% (B) 9%
(C) 9.5% (D) 10.5%

Direction for questions 16 and 17: These questions are based on the following bar graph that gives the production and sales of a company across five years from 2004 to 2008.



16. In the given period, what percentage was the average production more than the average sales?
(A) 20% (B) 23%
(C) 25% (D) 28%

17. The percentage increase/decrease in the total sales of the company in a given year with respect to that in the previous year was highest in which of the following years?
(A) 2005 (B) 2006 (C) 2007 (D) 2008

Previous Years' Questions

1. If $\left(z+\frac{1}{z}\right)^2 = 98$, compute $\left(z^2+\frac{1}{z^2}\right)$. **[GATE, 2014]**

2. The roots of $ax^2 + bx + c = 0$ are real and positive. a, b and c are real. Then $ax^2 + b\,|x| + c = 0$ has **[GATE, 2014]**
(A) no roots (B) 2 real roots
(C) 3 real roots (D) 4 real roots

3. Round–trip tickets to a tourist destination are eligible for a discount of 10% on the total fare. In addition, groups of 4 or more get a discount of 5% on the total fare. If the one way signle person fare is ₹100, a group of 5 tourists purchasing round-trip tickets will be charged ₹_______. **[GATE, 2014]**

4. In a survey, 300 respondents were asked whether they own a vehicle or not. If yes, they were further asked to mention whether they own a car or scooter or both. Their responses are tabulated below. What percent of respondents do not own a scooter? **[GATE, 2014]**

		Men	Women
	Car	40	34
Own vehicle	Scooter	30	20
	Both	60	46
Do not own vehicle		20	50

5. When a point inside of a tetrahedron (a solid with four triangular surfaces) is connected by straight lines to its corners, how many (new) internal planes are created with these lines? _______. **[GATE, 2014]**

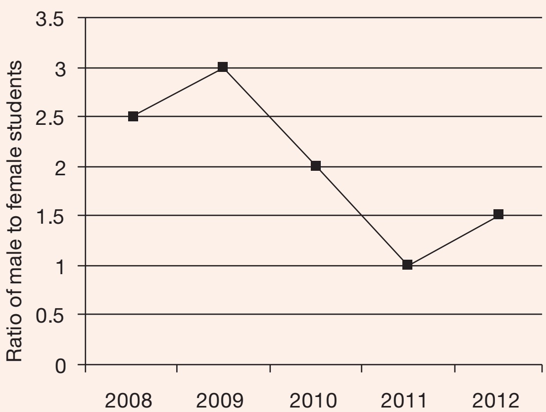
6. What is the average of all multiples of 10 from 2 to 198? **[GATE, 2014]**
(A) 90 (B) 100
(C) 110 (D) 120

7. The value of $\sqrt{12+\sqrt{12+\sqrt{12+\cdots}}}$ is **[GATE, 2014]**
(A) 3.464 (B) 3.932
(C) 4.000 (D) 4.444

8. If x is real and $|x^2 - 2x + 3| = 11$, then possible values of $|-x^3 + x^2 - x|$ include **[GATE, 2014]**

(A) 2, 4 (B) 2, 14
(C) 4, 52 (D) 14, 52

9. The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what percent did the number of male students increase in 2009? **[GATE, 2014]**



(A) 1 : 1 (B) 2 : 1
(C) 1.5 : 1 (D) 2.5 : 1

10. The table below has question-wise data on the performance of students in an examination. The marks for each questions are also listed. There is no negative or partial marking in the examination.

Q.No.	Marks	Answered correctly	Answered wrongly	Not Attempted
1	2	21	17	6
2	3	15	27	2
3	2	23	18	3

What is the average of the marks obtained by the class in the examination? **[GATE, 2014]**

(A) 1.34 (B) 1.74
(C) 3.02 (D) 3.91

11. The Gross Domestic Product (GDP) in Rupees grew at 7% during 2012–2013. For international comparison, the GDP is compared in US Dollars (USD) after conversion based on the market exchange rate. During the period 2012–2013 the exchange rate for the USD increased from ₹50/USD to ₹60/USD. India's GDP in USD during the period 2012–2013 **[GATE, 2014]**

(A) Increased by 5%
(B) Decreased by 13%
(C) Decreased by 20%
(D) Decreased by 11%

12. Consider the equation: $(7256)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y. **[GATE, 2014]**

(A) 1634 (B) 1737
(C) 3142 (D) 3162

13. What will be the maximum sum of 44, 42, 40, ...? **[GATE, 2013]**

(A) 502 (B) 504
(C) 506 (D) 500

14. A tourist covers half of his journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. The average speed of the tourist in km/h during his entire journey is **[GATE, 2013]**

(A) 36 (B) 30
(C) 24 (D) 18

15. The current erection cost of a structure is ₹13,200. If the labour wages per day increase by $\frac{1}{5}$ of the current wages and the working hours decrease by $\frac{1}{24}$ of the current period, then the new cost of erection in ₹ is **[GATE, 2013]**

(A) 16,500 (B) 15,180
(C) 11,000 (D) 10,120

16. Out of all the 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7? **[GATE, 2013]**

(A) $\frac{13}{90}$ (B) $\frac{12}{90}$
(C) $\frac{78}{90}$ (D) $\frac{77}{90}$

17. Find the sum of the expression

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

[GATE, 2013]

(A) 7 (B) 8
(C) 9 (D) 10

18. The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of ₹50 per unit. The number of units to be produced by the firm such that the profit is maximized is **[GATE, 2012]**

(A) 5 (B) 10
(C) 15 (D) 25

19. Which of the following assertions are **CORRECT**?

P: Adding 7 to each entry in a list adds 7 to the mean of the list.

Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list.

R: Doubling each entry in a list doubles the mean of the list.

S: Doubling each entry in a list leaves the standard deviation of the list unchanged. **[GATE, 2012]**

(A) P, Q (B) Q, R
(C) P, R (D) R, S

20. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is **[GATE, 2012]**

(A) 8 meters
(B) 10 meters
(C) 12 meters
(D) 14 meters

21. An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.

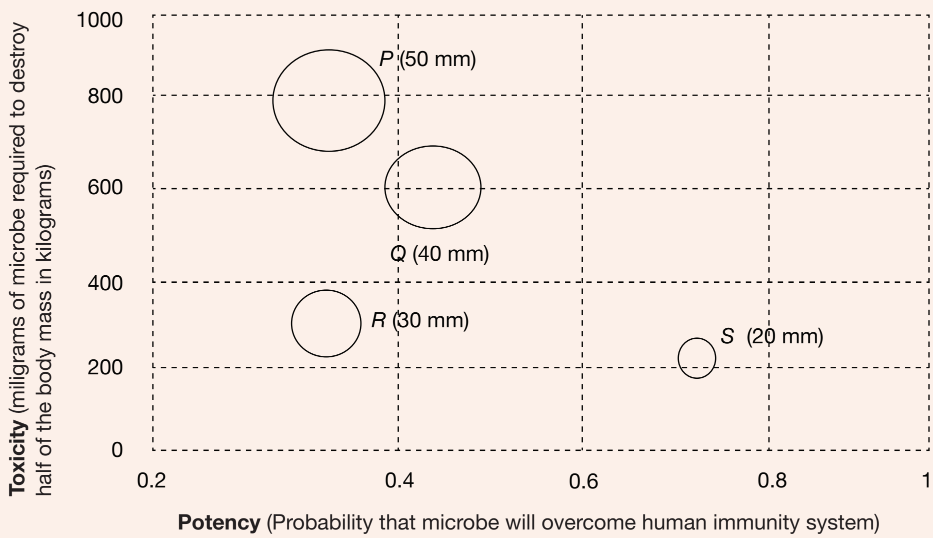
The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is **[GATE, 2012]**

(A) 0.288 (B) 0.334
(C) 0.667 (D) 0.720

22. If $\text{Log}(P) = \left(\frac{1}{2}\right)\text{Log}(Q) - \left(\frac{1}{3}\right)\text{Log}(R)$ then which of the following options is TRUE? **[GATE, 2011]**

(A) $P^2 = Q^3R^2$ (B) $Q^2 = PR$
(C) $Q^2 = R^3P$ (D) $R = P^3Q^2$

23. P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below:



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt? **[GATE, 2011]**

(A) P (B) Q
(C) R (D) S

24. The variable cost (V) of manufacturing a product varies according to the equation $V = 4q$, where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation $F = \frac{100}{q}$. How many units should be produced to minimize the total cost ($V + F$)? **[GATE, 2011]**

(A) 5 (B) 4
(C) 7 (D) 6

25. A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks required so that there will be no pending order at the end of the 5th day? **[GATE, 2011]**

(A) 4 (B) 5
(C) 6 (D) 7

26. A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1

litre of water. Subsequently 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much spirit is now left in the container? **[GATE, 2011]**

(A) 7.58 litres (B) 7.84 litres
(C) 7 litres (D) 7.29 litres

27. If 137 + 276 = 435, how much is 731+672? **[GATE, 2010]**

(A) 534 (B) 1403
(C) 1623 (D) 1513

28. 5 skilled workers can build a wall in 20 days, 8 semi-skilled workers can build the wall in 25 days, 10 unskilled workers can build the wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall? **[GATE, 2010]**

(A) 20 (B) 18
(C) 16 (D) 15

29. From the digits 2, 2, 3, 3, 3, 4, 4, 4, 4, how many distinct 4-digit numbers greater than 3000 can be formed? **[GATE, 2010]**

(A) 50 (B) 51
(C) 52 (D) 54

Answer Keys

Exercises

1. A	**2.** B	**3.** D	**4.** D	**5.** A	**6.** C	**7.** A	**8.** A	**9.** D	**10.** B
11. C	**12.** B	**13.** 0	**14.** 0	**15.** D	**16.** B	**17.** B			

Previous Years' Questions

1. 96	**2.** D	**3.** 850	**4.** 48%	**5.** 6	**6.** B	**7.** C	**8.** D	**9.** C	**10.** C
11. D	**12.** C	**13.** C	**14.** C	**15.** B	**16.** D	**17.** B	**18.** A	**19.** C	**20.** B
21. B	**22.** B	**23.** D	**24.** A	**25.** C	**26.** D	**27.** C	**28.** D	**29.** B	

Test

Quantitative Ability

Time: 30 Minutes

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. P, Q, R, and S have a total amount of ₹220 with them. P has ₹30 more than Q. S has half the amount with Q. R has ₹10 more than S. Find the amount with S (in ₹).
   (A) 20 (B) 40
   (C) 30 (D) 50

2. In a room, there are some girls and some benches. If 5 girls sit on each bench, three girls will have no bench to sit on. If there is one bench less, 6 girls can sit on each bench. Find the number of benches.
   (A) 6 (B) 8
   (C) 7 (D) 9

3. A test has 60 questions. Each correct answer fetches 1 mark. For each wrong answer and each unanswered question 1 mark is deducted. A candidate who wrote this test scored 20 marks. Find the number of questions he correctly answered.
   (A) 50 (B) 45
   (C) 35 (D) 40

4. If $\frac{a+b-c}{x} = \frac{a-b+c}{y} = \frac{c+b-a}{z}$ then $x(b-a) + y(a-c) + z(c-b) =$
   (A) 0 (B) 2
   (C) 3 (D) 1

5. The value of a diamond varies directly with the cube of its weight. It broke into two pieces whose weights are in the ratio 3 : 4. The loss due to breakage is ₹504000. Find its initial value (in ₹).
   (A) 1029000 (B) 686000
   (C) 1372000 (D) 1715000

6. The average of 25 observations is 120. By mistake one of the observations, 144, is taken as 169. Find the average of the 25 observations, after the mistake is corrected.
   (A) 120 (B) 119
   (C) 125 (D) 132

7. The average of 13 observations is 50. The average of first seven observations is 45 and the average of last seven observations is 52. Find the value of seventh observation.
   A) 41 (B) 30
   (C) 29 (D) 62

8. A container contains 100 litres of milk. 10 litres of milk is replaced by 10 litres of water. From the solution formed, 10 litres of solution is replaced by 10 litres of water, and this process is repeated one more time. Find the percentage of water in the resulting solution.
   (A) 33.3% (B) 67%
   (C) 36.5% (D) 27.1%

9. In a 729 ml of solution, the ratio of acid to water is 7 : 2. How much more water should be mixed so that the resulting mixture contains acid and water in the ratio 7 : 3 (in ml)?
   (A) 100 (B) 40
   (C) 37 (D) 81

10. A merchant buys sulphuric acid at a certain rate per gallon and after mixing it with water, sells it at the same rate. If the merchant makes a profit of 20%, how many gallons of water are there per gallon of acid?
    (A) 0.2 (B) 0.5
    (C) 0.7 (D) 0.25

11. If A travelled a certain distance at 6 km/h, he would have reached his destination 10 minutes early. If he travelled it at 4 km/h, he would have reached his destination 10 minutes late. Find the speed at which he must travel to reach his destination on time (in km/h).
    (A) 5 (B) 5.4
    (C) 4.8 (D) 4.5

12. A car travelled the first hour of its journey at 30 km/h, the next 5 hours of its journey at 50 km/h, and the remaining 4 hours of its journey at 75 km/h. Find its average speed for its journey (in km/h).
    (A) 56 (B) 60
    (C) 58 (D) 62

13. Without stoppages, a train can cover 54 km in an hour. With stoppages it can cover 36 km in an hour. Find its stoppage time per hour in a journey it covers with stoppages (in minutes).
    (A) 15 (B) 18
    (C) 20 (D) 12

14. A and B can complete a job in 40 days. B and C can complete it in 30 days. A and C can complete it in 20 days. Find the time taken by A to complete it (in days).
    (A) $\frac{180}{7}$ (B) 30
    (C) 48 (D) $\frac{240}{7}$

15. 3 men and 4 women can complete a job in 10 days. 24 men and 2 women can complete it in 2 days. Find the time taken by 5 men and 10 women to complete it (in days).
    (A) 4 (B) 5
    (C) 3 (D) 6

16. Abhilash spends 25% of his income towards rent, 20% of the remaining income towards food, 8% of the remaining towards medical expenses, and 25% of the remaining towards miscellaneous expenses. If he saves ₹82,800, what is his income?

(A) ₹2,00,000 (B) ₹2,25,000
(C) ₹2,40,000 (D) ₹2,50,000

17. By selling 30 articles, a shopkeeper gained the selling price of 10 articles. Find the profit percent.

(A) 20% (B) 30%
(C) 50% (D) 40%

18. When 1036 is divided by N, the remainder is 12 and when 1545 is divided by N, the remainder is 9. Find the greatest possible value of N.

(A) 128 (B) 512
(C) 250 (D) 64

19. Five bells toll at regular intervals of 10, 15, 20, 25, and 30 seconds, respectively. If they toll together at 8:00 am, then at what time will they toll together for the first time after 8:00 am?

(A) 8:04 am (B) 8:06 am
(C) 8:05 am (D) 8:07 am

20. Find the value of

$$\frac{3}{\sqrt{2}+\sqrt{11}}+\frac{3}{\sqrt{5}+\sqrt{8}}+\frac{3}{\sqrt{8}+\sqrt{11}}+\cdots+\frac{3}{\sqrt{26}+\sqrt{29}}.$$

(A) $\sqrt{29}-\sqrt{2}$ (B) $\sqrt{26}+\sqrt{5}$
(C) $\sqrt{29}+\sqrt{26}$ (D) $\sqrt{26}-\sqrt{8}$

21. If $a+b+c=0$, find the value of $(3^{a^8})^{\frac{1}{a^6bc}}(3^{b^8})^{\frac{1}{ab^6c}}(3c^8)^{\frac{1}{abc^6}}$

(A) 2 (B) 6
(C) 27 (D) 81

22. If $3^{x+3} - 3^{x+2} = 486$, then find x.

(A) 3 (B) 5
(C) 6 (D) 2

23. $\log_p q = \frac{5}{4}, \log_r q = \frac{5}{6}$ and $\log_r p = 3x$, then find x.

(A) $\frac{1}{9}$ (B) $\frac{2}{3}$
(C) $\frac{1}{18}$ (D) $\frac{2}{9}$

24. $\frac{5}{1+\log_p qr}+\frac{5}{1+\log_q pr}+\frac{5}{1+\log_r pq}=$

(A) 0 (B) 1
(C) 5 (D) 10

25. If $\log_{10}2 = 0.3010$ then find the number of digits in 2^{55}.

(A) 17 (B) 11
(C) 18 (D) 16

26. The maximum sum of the arithmetic progression 45, 41, 37, ... is

(A) 256 (B) 274
(C) 276 (D) 264

27. The greatest value of n such that $1 + 3 + 3^2 + 3^3 + \cdots + 3^n$, which is less than 3000 will be:

(A) 6 (B) 8
(C) 7 (D) 9

28. $3 + 33 + 333 + \cdots +$ upto n terms =

(A) $\frac{(10^n-1)}{3}-n$ (B) $\frac{10(10^n-1)}{27}-n$

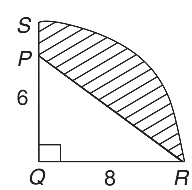
(C) $\frac{(10^n-1)}{3}-\frac{n}{9}$ (D) $\frac{10(10^n-1)}{27}-\frac{n}{3}$

29. What is the minimum value of the function $f(x) = x^2 - 15x + 9$?

(A) $\frac{289}{4}$ (B) $\frac{-200}{9}$

(C) $\frac{295}{2}$ (D) $\frac{-189}{4}$

30. Find the area of the shaded region, where PQR is a triangle and QRS is a quadrant. $PQ = 6$ cm and $QR = 8$ cm.



(A) $4\pi - 8$ sq. cm. (B) $2\pi - 3$ sq. cm.
(C) $8\pi - 16$ sq. cm. (D) $16\pi - 24$ sq. cm.

Answer Keys

1. C	**2.** D	**3.** D	**4.** A	**5.** B	**6.** B	**7.** C	**8.** D	**9.** D	**10.** A
11. C	**12.** C	**13.** C	**14.** C	**15.** B	**16.** A	**17.** C	**18.** B	**19.** C	**20.** A
21. C	**22.** A	**23.** D	**24.** C	**25.** A	**26.** C	**27.** C	**28.** D	**29.** D	**30.** D

Quantitative Ability Test I

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. What is the percentage change in the volume of a cylinder if its height increases by 20% and radius remains the same?
   (A) No change
   (B) 10% increase
   (C) 20% decrease
   (D) 20% increase

2. 30% of a number when subtracted from twice the number equals to 33 less than five times the same number. What is the number?
   (A) 15 (B) 10
   (C) 20 (D) 5

3. In the second and the third match of a series Dravid's score increases by 10% and $21^7/_{33}$% respectively over the previous match. By what percentage did his score increase in the third match as compared to the first match?
   (A) $33^1/_3$% (B) $31^7/_{33}$%
   (C) $35^{15}/_{33}$% (D) $37^9/_{33}$%

4. Two tests are given with maximum marks of 300 and 200 respectively. A student scores an overall percentage of 60%. If he has scored 40% in the second test, then how many marks did he score in the first test?
   (A) 200 (B) 180
   (C) 220 (D) 240

5. If a kerosene dealer sells kerosene at ₹8 per litre, he loses ₹400, but if he sells at ₹10 per litre he makes a profit of ₹400. Then, how many litres of kerosene did the dealer sell?
   (A) 250 (B) 200
   (C) 350 (D) 400

6. By selling 50 metres of cloth, a merchant gains the cost of 10 metres. Find the gain percentage.
   (A) 25% (B) $33^1/_3$%
   (C) 20% (D) 30%

7. By selling 60 metres of cloth, a merchant gains the selling price of 10 metres. The gain percentage is
   (A) 25% (B) 20%
   (C) 15% (D) $16^2/_3$%

8. If 3/5th of *B*'s income is 25% less than *A*'s income, then what percentage of *B*'s income is *A*'s income?
   (A) 60% (B) 80%
   (C) 120% (D) 125%

9. If a dealer gets a commission of 10% on the list price from the company, then the profit made by the company is 50% of its manufacturing cost. If the dealer's commission is increased to 25%, then what will be the profit percentage on the cost of manufacturing for the company?
   (A) 10% (B) 25%
   (C) 30% (D) 20%

10. Chetan started a cable TV service with an investment of ₹2,00,000. After a few months David joined him with an investment of ₹1,50,000. If at the end of the year David's share was ₹3,00,000 out of a total profit of ₹7,80,000 after how many months did David join Chetan?
    (A) 2 (B) 4
    (C) 8 (D) 10

11. A man sells an article at a profit of 25%. Had he bought it at 25% less and sold for ₹25 less, he would still have gained 25%. Find the cost of the article.
    (A) ₹80 (B) ₹40
    (C) ₹60 (D) ₹50

12. A man sold a Doberman and a German Shepherd for ₹5,750 each. On the Doberman he made a profit of 25%, and on the German Shepherd he made a profit of 15%. Find the approximate profit percentage on the whole transaction.
    (A) 23% (B) 16%
    (C) 18% (D) 20%

13. *A*, *B*, and *C* invested capitals in the ratio 4 : 5 : 6. At the end of the year, they received the profits in the ratio 6 : 5 : 4. Find the ratio of time periods for which they invested their capitals.
    (A) 9 : 6 : 2 (B) 9 : 8 : 6
    (C) 9 : 3 : 4 (D) 9 : 6 : 4

14. A man purchases a certain number of chocolates at 2 per rupee and the same number of pepper-mints at 5 per rupee. He mixes them together and sells them at 3 per rupee. What is his approximate profit or loss percentage?
    (A) 5% profit (B) 5% loss
    (C) 4% profit (D) 4% loss

15. A shopkeeper bought a table marked at ₹600 at successive discounts of 10% and 20% respectively. He spent ₹8 on transportation and sold the table for ₹450. Find his profit percentage.
    (A) $2^7/_{11}$% (B) $2^3/_{11}$%
    (C) $2^5/_{11}$% (D) $2^8/_{11}$%

16. A sum of money is invested at a certain rate of simple interest. Find the annual rate of interest if the sum becomes 84% more in 6 years.
    (A) 12% (B) 14%
    (C) 16% (D) 18%

17. A sum of money becomes $1^{91}/_{125}$ times itself, when invested at compound interest at 20% p.a. Find the duration of the investment.
(A) 1 year (B) 2 years
(C) 3 years (D) 4 years

18. Two sums of money in the ratio 3:4 are lent for a period of 5 years. The rates of interest on the two sums are in the ratio 1:2. The difference in the simple interest on the two sums is ₹1000. Find the total simple interest on the two sums.
(A) ₹2000 (B) ₹2200
(C) ₹2400 (D) ₹2500

19. Two litres of 20% H_2SO_4 solution, three litres of water and five litres of 10% H_2SO_4 are mixed together. How many litres of the resulting solution must be mixed with thirty litres of 18% H_2SO_4 solution so as to get a 15% H_2SO_4 solution?
(A) 30 (B) 10
(C) 15 (D) 20

20. In which of the following ratios by weight should three alloys of gold of purity 18, 20 and 22 carats be mixed to form a fourth alloy whose purity is $20^1/_3$ carats?
(A) 3 : 4 : 5 (B) 4 : 3 : 5
(C) 4 : 5 : 6 (D) 6 : 5 : 4

21. A family consists of a grandfather, a grandmother, father, mother and three children. The average age of father, mother and the 3 children is 25 years. The average age of the three children is 15 years. The average age of parents and grandparents is 50 years, then find the average age of the grand parents (in years).
(A) 70 (B) 68
(C) 58 (D) 60

22. Groundnuts contain 70% oil by weight. Oil is partially extracted and what is left behind is groundnut cake which contains 17% oil by weight. What is the weight of the groundnuts which yield 1 kilogram of cake?
(A) 3.77 kg (B) 2.67 kg
(C) 3.58 kg (D) 2.77 kg

23. One-third of a bottle full of pure listerene is replaced with water. Find the ratio of water and listerene if the above process is carried out for a total of four times.
(A) 16 : 65 (B) 16 : 89
(C) 89 : 16 (D) 65 : 16

24. The number of matches played by a cricketer in the year 2003 is one-fourth of the total number of matches played by him upto the end of 2002. His average score (*i.e.*, runs scored per match played) upto the end of the year 2003 is four-thirds of his average upto the end of the year 2002. What is the ratio of the runs scored in the year 2003 to that of the total scored upto the end of the year 2002?
(A) 2 : 3 (B) 3 : 2
(C) 3 : 4 (D) 15 : 16

25. The weighted average of the scores of all the students of three sections *X*, *Y* and *Z* of a class is $33^1/_3$% more than the average of the section *X*. The weighted average of sections *Y* and *Z* is $65^5/_{11}$. If section *X* has a strength of 40 and an average score of 45, what is the combined strength of the section *Y* and Z?
(A) 60 (B) 75
(C) 110 (D) 85

26. Find the expression for the sum of *n* terms of an arithmetic progression, if the tenth term is 40 and the 12th term is 44.
(A) $10n + 25n^2$ (B) $20n + 20n^2$
(C) $25n + 15n^2$ (D) $n^2 + 21n$

27. The first term of an arithmetic progression consisting of 30 terms is 10 and the common difference is 5. Find the ratio of the sum of the 30 terms of the arithmetic progression. to the sum of the last 20 terms of the A.P.
(A) 99 : 13 (B) 96 : 17
(C) 99 : 86 (D) 99 : 68

28. If the sum of the fifth, thirteenth and eighteenth terms of an A.P is zero, find the 12th term of the arithmetic progression.
(A) –2 (B) –1
(C) 0 (D) 1

29. Find the sum of the first 10 terms of the series: $3 (2^2) + 4 (3^2) + 5 (4^2) + \ldots\ldots..$
(A) 3009 (B) 4860
(C) 3408 (D) 3608

30. The sum of the first eight terms of a geometric progression. is 510 and the sum of the first four terms of the geometric progression. is 30. Find the first term of the geometric progression, given that it is positive.
(A) 2 (B) 4
(C) 6 (D) 8

31. The terms of an arithmetic progression are all positive. The square of fourth term equals the sum of the squares of the previous two terms. The sum of the first four terms is 14. Find the common difference.
(A) 1
(B) 2
(C) 2
(D) Cannot be determined

32. The first, second and third terms of a geometric progression are equal to the first, seventh and twelfth terms of an arithmetic progression. If the first term and common difference have opposite signs, find the 37th term of the arithmetic progression.
(A) 1 (B) 0
(C) 1 (D) 2

33. Find the sum of the terms of the series
(1) × (20), (2) × (19), (3) × (18), …(20) × (1).
(A) 1750 (B) 1645
(C) 1540 (D) 1435

34. In a geometric progression, each term is the sum of all the terms following it. The sum to infinity of the terms is 32. If all the terms are positive, then find the first term.

(A) 16 (B) $16\sqrt{2}$

(C) 64 (D) $8\sqrt{2}$

35. The sum of five numbers in geometric progression is 62. The sum of their reciprocals is $\frac{31}{32}$. Find the square of the third number.

(A) 100 (B) 121

(C) 36 (D) 64

Answers Key

1. D	**2.** B	**3.** A	**4.** C	**5.** D	**6.** C	**7.** B	**8.** B	**9.** B	**10.** A
11. A	**12.** D	**13.** D	**14.** B	**15.** B	**16.** B	**17.** C	**18.** B	**19.** C	**20.** A
21. D	**22.** D	**23.** D	**24.** A	**25.** C	**26.** D	**27.** C	**28.** C	**29.** B	**30.** A
31. A	**32.** B	**33.** C	**34.** A	**35.** D					

Hints and Explanations

1. Volume of a cylinder = $\pi r^2 h$
As π is constant and there is no change in radius, volume varies only with height. New height = (1.2) h
$\therefore$ New volume = 1.2 $\pi r^2 h$ i.e., the volume increases by 20%. Choice (D)

2. Let the number be = x
According to the problem, $2x - 0.3x = 5x - 33$
$\Rightarrow$ $3.3x = 33 \Rightarrow x = 10$ Choice (B)

3. Let Dravid's score in the first match be x.
It increases by 10% in the second to $\frac{110}{100}x = \frac{11x}{10}$.
In the third match it increases by
$$21\frac{7}{33}\% = \frac{700}{33}\% \text{ to } \left(100 + \frac{700}{33}\right)\% = \frac{4000}{33}\%$$
i.e., score in the third match is
$$\left(\frac{1}{100}\right)\left(\frac{4000}{33}\right)\left(\frac{11x}{10}\right) = \frac{4x}{3}$$
Over the two year it increases by $\frac{4x}{3} - x = \frac{x}{3}$.
Fractional increase $= \frac{\frac{x}{3}}{x} = \frac{1}{3} = 33\frac{1}{3}\%$. Choice (A)

4. The two tests together had 300 + 200 = 500 marks
The student scored an overall percentage of 60%
$\therefore$ Total marks scored by the student = 500 × 0.6 = 300
Marks scored by the student in the second test
= 200 × 0.4 = 80
Hence marks scored in the first test = 300 – 80 = 220. Choice (C)

5. Let the quantity of kerosene sold by the dealer be x litres
$\therefore$ CP – $8x$ = 400 and $10x$ – CP = 400
$\Rightarrow$ $8x + 400 = 10x - 400 \Rightarrow 5x = 400$ Choice (D)

6. Let the cost price of 1m = CP
Let the sale price of 1m = SP
Given 50SP = 50CP + 10 CP i.e., 60CP = 50SP
$\therefore$ % Profit = (60 – 50) × 100/50 = 20%. Choice (C)

7. Let the cost price of 1m = CP
Let the sale price of 1m = SP
Given 60SP = 60CP + 10SP i.e., 60CP = 50SP
$\therefore$ %Profit = (60 – 50) × 100/50 = 20%. Choice (B)

8. Let A's and B's increase be a and b
$$\frac{3b}{5} = a - \frac{25a}{100} = a\left(\frac{3}{4}\right)$$
$$\therefore a = \frac{4}{5}b = \frac{80}{100}b$$
a in 80% of b. Choice (B)

9. Let the list price be = LP
If 10% of LP is given as commission to the dealer the profit for the company is 50% $\Rightarrow$ 0.9 LP = 1.5 CP
If 25% commission is given then
0.75 LP = 0.75 × (1.5 CP / 0.9) = 1.25 CP
% of profit = 25% Choice (B)

10. Chetan's total investment = 2,00,000 X 12
David's total investment = 1,50,000 X x
(where x is the period of investment of David)
Chetan's share in total profit
= 7,80,000 – 3,00,000 = Rs.4,80,000
Now, $\frac{2,00,000 \times 12}{1,50,000 \times x} = \frac{4,80,000}{3,00,000} \Rightarrow x = 10.$
$\therefore$ David joined after 2 months. Choice (A)

11. SP = 1.25 CP
Also given SP - 25 = 1.25 × 0.75 CP
Solving the above equations CP = 80 Choice (A)

12. Total selling price = 5750 × 2 = ₹11,500
Total cost price =
$$\frac{5750}{1.25} + \frac{5750}{1.15} = 4600 + 5000 = ₹9600$$
$$\therefore \text{Overall profit percentage} = \frac{1900}{9600} \times 100 \cong 20$$
Choice (D)

13. Ratio of time is 6/4 : 5/5 : 4/6

$\frac{6 \times 60}{4} : \frac{5 \times 60}{5} : \frac{4 \times 60}{6} = 9 : 6 : 4$ Choice (D)

14. Let the number of éclairs be x

There CP = $x/2$

CP of peppermint = $x/5$

Therefore total CP = $\frac{x}{2} + \frac{x}{5} = \frac{7x}{10}$

Total SP = $\frac{2x}{3}$

P% = $\frac{\frac{2x}{3} - \frac{7x}{10}}{\frac{7x}{10}} \times 100$. Therefore loss = 5% Choice (B)

15. CP = 600 (.9) (.8) = 432

Total CP = 432 + 8 = 440

SP = 450

P% = $\frac{10}{440}$ x 100 = $2^{3}/_{11}$% Choice (B)

16. If a sum of money invested under simple interest at R% p.a. becomes N times after t years, R is given by $\frac{100(N-1)}{t}$.

As $N = 1 \times 84$ and $t = 6$ years, $R = \frac{100(1 \cdot 84 - 1)}{6} = 14$

∴ the rate is 14% p.a. Choice (B)

17. Let the sum of money be ₹ p and duration of the investment be n years.

$1\frac{91}{125}p = \frac{216}{125}p = p\left(1+\frac{20}{100}\right)^n$

$\left(\frac{6}{5}\right)^3 = \left(\frac{6}{5}\right)^n \Rightarrow n = 3.$ Choice (C)

18. Let the two sums of money be $3x$ and $4x$ and the rates of interest on the two sums be R% p.a. and $2R$% p.a. respectively. Difference in the simple interest on the two sums = $\frac{(4x)(2R)(5)}{100} - \frac{(3x)(R)(5)}{100} = 1000$

$\Rightarrow \frac{xR}{4} = 1000$

$\Rightarrow$ $xR = 4000$

Total simple interest on the two sums

$= \frac{(4x)(2R)(5)}{100} + \frac{(3x)(R)(5)}{100} = \frac{55xR}{100} = ₹2200.$

($\because$ xR = 4000) Choice (B)

19. Concentration of H_2SO_4 in the resulting solution

$= \frac{2(0.2) + 3(0) + 5(0.1)}{2 + 3 + 5} = 9\%$

Let the amount of the resulting solution that is being mixed with 30 lt of 0.18 H_2SO_4 be x.

$(0.09)x + 0.18(30) = 0.15\,(x + 30)$

$0.9 = 0.06x \Rightarrow x = 15$ Choice (C)

20. Let the weight of 18, 20 and 22 carat gold be x, y and z respectively.

$\frac{18x + 20y + 22z}{x+y+z} = \frac{61}{3}$

Only option (1) satisfies this condition. Choice (A)

21. Average age of children and parents 5 members = 25

∴ Sum of ages = 125 years.

Average age of 3 children = 15

Sum of ages of 3 children = 45

∴ Sum of ages of parents = 80

Sum of the ages of parents and grand parents = 50(4) = 200.

Sum of the ages of grand parents = 120

Average = 120/2 = 60 Choice (D)

22. Since groundnut contains 70% of oil, it means in 1kg of groundnut there is 30% of (or 0.3kg) solid material. From this 1kg of groundnut, 0.3kg is solid which translates to 83% of cake.

∴ The cake in 1kg of groundnut is $\frac{0.3}{0.83}$kg.

Groundnut cake	Groundnut
$\frac{0.3}{0.83}$kg –	1 kg
1kg –	?

$\Rightarrow \frac{1(1\text{kg})}{\left(\frac{0.3}{0.83}\right)} = 2.77\text{kg}$ Choice (D)

23. Let the capacity of the bottle be 1 litre.

The fraction of listerine left after repeating the process 4 times = $\left(\frac{3-1}{3}\right)^4 = \left(\frac{2}{3}\right)^4 = \left(\frac{16}{81}\right)$

$\Rightarrow$ For every 81 lt of mixture, listerine is 16 lt and water is 81 – 16 = 65 lt.

∴ The ratio of water and listerine is 65 : 16.

Choice (D)

24. The number of matches and average score are tabulated below.

	Upto end of 2002	In 2003	Upto end of 2003
Average	3x	—	4x
No. of matches	4n	n	5n

Score upto end of 2002 $(3x)(4n) = 12\,xn$.

Score upto end of 2003 $(4x)(5n) = 20\,xn$.

Score in 2003 = $8\,xn$.

Required ratio = $8\,xn : 12\,xn = 2 : 3$ Choice (A)

25. Let $A_{(x+y+z)}$ be the average score of the three section combined. It is given that $A_{(x+y+z)}$

= (1 + 33 1/3%) of Average of x.

= (4/3)45 = 60….. (1).

Let n be the combined strength of sections Y and Z.
Average of the two sections Y and Z together = $65^5/_{11}$
Total strength of X + Total strength of (Y and Z)
= Total score of $(X + Y + Z)$

$\Rightarrow$ $(45)(40) + n(65\frac{5}{11}) = (n + 40)(60)$

$\Rightarrow$ $1800 + \frac{720n}{11} = 60n + 2400 \Rightarrow \frac{60n}{11} = 600$

$\Rightarrow$ $n = 110.$

$\therefore$ The combined strength of section Y and Z = 110.
Choice (C)

26. If the first term of the A.P is a and the common difference is d, we have the tenth term as
$a + 9d = 40$ ….. (1)
and the twelfth term as a + 11d = 44 …. (2)
Subtracting (1) from (2), we have $2d = 44 - 40$,
$d = 2$
Substituting the value of d in (1) we get, $a = 22$.
The sum of n terms of the A.P.
$= \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2(22) + (n-1)2]$
$= \frac{n}{2}[44 + 2n - 2] = \frac{n}{2}[42 + 2n] = 21n + n^2$ Choice (D)

27. Sum of the first 30 terms of the A.P $= \frac{30}{2}[2(10) + 29(5)]$
= 15 [20 + 145] = 15 [165] = 2475.
Sum of the first 10 terms of the A.P $= \frac{10}{2}[2(10) + 9(5)]$
= 5[20 + 45] = 5 [65] = 325. Ratio of the sum of the first 30 terms of the A.P. to the sum of the last 20 terms of the A.P = (2475) : (2475 – 325) = 2475 : 2150
= 5[495] : 5[430] = 495 : 430 = 99 : 86. Choice (C)

28. If the first term is a and the common difference is d, we have the sum of the fifth, thirteenth and eighteenth terms as
$(a + 4d) + (a + 12d) + (a + 17d) \Rightarrow 3a + 33d = 0.$
Dividing by 3, we have $a + 11d = 0$. Hence the 12th term of the A.P is 0. Choice (C)

29. $S = 3(2)^2 + 4(3)^2 + 5(4)^2 + \ldots\ldots$ 10 terms.
$= (2 + 1)2^2 + (3 + 1)3^2 + (4 + 1)4^2 + \ldots$ 10 terms
$= (2^3 + 2^2) + (3^3 + 3^2) + (4^3 + 4^2) + \ldots$ 10 terms
$= (2^3 + 3^3 + 4^3 + \ldots + 11^3) + (2^2 + 3^2 + 4^2 \ldots + 11^2)$
$= (1^3 + 2^3 + 3^3 \ldots + 11^3) + (1^2 + 2^2 + 3^2 + \ldots + 11^2) - 1^3 - 1^2$
= 4355 + 505 = 4860. Choice (B)

30. If the first term of the G.P is a and the common ratio is r, we have $\frac{a(r^8-1)}{r-1} = 510$ … (1)
and $\frac{a(r^4-1)}{r-1} = 30$ …. (2)
Dividing (1) by (2) we have $\dfrac{\frac{a(r^8-1)}{r-1}}{\frac{a(r^4-1)}{r-1}}$
$= r^4 + 1 = \frac{510}{30} = 17.$
$r^4 = 17 - 1 = 16$
$r = \pm\sqrt[4]{16} = \pm 2$
First term of the G. P, a $= \frac{510(r-1)}{r^8-1}$
As first term is positive, $r = 2$ is taken.
$a = \frac{510(2-1)}{2^8-1} = \frac{510(1)}{255} = 2.$ Choice (A)

31. Let the first term and the common difference be a and d respectively.
$(a + 3d)^2 = (a + 2d)^2 + (a + d)^2$
$a^2 + 6ad + 9d^2 = 2a^2 + 6ad + 5d^2 \Rightarrow a = \pm 2d$
As all the terms are positive, $a = 2d$
$a + a + d + a + 2d + a + 3d = 14 \Rightarrow d = 1.$
Choice(A)

32. Let the first term of either progression be a.
Let the common difference of the arithmetic progression be $d.d\,(a + 36d) = 0$
as a and d have opposite signs, $d \neq 0$.
$a + 36d = 0$
37th term = 0 Choice (B)

33. The terms of the series are in the form $x(21 - x)$
Required sum $= \sum_{x=1}^{20} x(21 - x)$
$= \frac{(21)(20)(21)}{2} - \frac{1}{6}(20)(21)(41) = 1540$ Choice (C)

34. Let the first term and the common ratio be a and r respectively
First term = sum of all the terms following it
$a = \frac{a}{1-r} - a$
$a(1 - 2r) = 0$
As all the terms are positive, $a \neq 0$.
$1 - 2r = 0$
$r = 1/2$
$\frac{a}{1-r} = 16$ Choice (A)

35. Let the first number and the common ratio be $\frac{a}{r^2}$ and r respectively.
$\frac{a}{r^2} + \frac{a}{r} + a + ar + ar^2 = 62$ …. (1)
$\frac{r^2}{a} + \frac{r}{a} + \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} = \frac{31}{32}$
$\Rightarrow \frac{1}{a^2}\left(ar^2 + ar + a + \frac{a}{r} + \frac{a}{r^2}\right) = \frac{31}{32}$
(1) ÷ (2) $\Rightarrow a^2 = \dfrac{62}{\frac{31}{32}} = 64$ Choice (D)

Quantitative Ability Test 2

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 16: Select the correct alternative from the given choices.

1. Two men, two women and six part-time workers take 12 days to complete a job. The same job can be completed by 10 men and 18 part-timers in 4 days. If two men and three women take 16 days to complete that job, find the time taken by one woman to complete that job (in days).
(A) 96 (B) 60
(C) 75 (D) 100

2. *A* can complete a job in 20 days. *B* works twice as fast as *A*. They both work together for 5 days. On the 6th day, they complete the job with the help of *C*. Find the time taken by *C* alone to complete the job (in days).
(A) 5 (B) 6
(C) 10 (D) 12

3. Four men take ten days to complete one-third of a work. How many more men are required to complete the remaining work in five days?
(A) 16 (B) 14
(C) 15 (D) 12

4. *A* and *B* working separately can do a piece of work in 5 days and 10 days respectively. They work on alternate days starting with *B* on the first day. In how many days will the work be completed?
(A) 6 (B) 7
(C) 8 (D) 9

5. A tank is fitted with three pipes *A*, *B* and *C*. The three pipes can be used as inlet or outlet pipes with the same flow rates. When one among *A*, *B* and *C* in turns works as outlet pipe and the other two as inlet pipes, it takes 30, 40 and 24 minutes to fill the tank respectively. Find the time taken by *A* to fill the empty tank (in minutes).
(A) 30 (B) 40
(C) 25 (D) 20

6. *A* and *B* take respectively 12 days and 27 days more time to complete a piece of work, working alone, than when they work together. Find the time taken by them to complete the work working together.
(A) 15 days (B) 20 days
(C) 24 days (D) 18 days

7. *A*, *B* and *C* work together to complete a job. *A* gets ₹600 out of the total share of ₹2400. If *A* works twice as fast as *B*, find the share of *C*.
(A) ₹1200 (B) ₹1500
(C) ₹1000 (D) ₹1600

8. *A*, *B* and *C* work at the same rate. *A* starts the job and after 25% of the work is completed, he leaves. *B* and *C* take over and complete the remaining work together in 18 more days. Find the time for which *A* worked (in days).
(A) 6 (B) 20
(C) 12 (D) 24

9. Six taps working together take 12 minutes to fill a tank. Find the time taken (in minutes) by 24 taps working together to fill a tank twice as big.
(A) 6 (B) 8
(C) 12 (D) 24

10. *P* can do a piece of work in 12 days working 6 hrs a day. *Q* can do the same work in 18 days working 5 hrs a day. If *P* and *Q* work together 4 hrs a day, then in how many days can they complete the work?
(A) 10 (B) 11
(C) 12 (D) 14

11. *A*, *B* and *C* can complete a piece of work in 20, 30 and 20 days respectively. They start the work together but *A* leaves after 5 days. After some more days *C* leaves. *B* completes the remaining work in 5/3 more days. For how many days does *B* work?
(A) $\frac{10}{3}$ (B) 10
(C) $\frac{14}{3}$ (D) 14

12. *A*, *B* and *C* take 20, 30 and 60 days to complete a job. *A* works along with *B* on the 1st day and with *C* on the 2nd day. If they continue in this manner, then find the time taken (in days) to complete the work.
(A) $12\frac{5}{6}$ (B) $18\frac{4}{5}$
(C) $14\frac{1}{4}$ (D) $13\frac{1}{4}$

13. Amar can complete a job in 15 days, while Bhavan can complete it in 10 days. They start working together and two days before the work was expected to be completed, Bhavan left. Find the time taken by Amar to complete the remaining work (in days).
(A) 4 (B) 5
(C) 6 (D) 8

14. A pipe can fill a 1000 litre tank in 10 minutes while another pipe can empty a 600 litre tank in 8 minutes. If they work together, then how long will they take (in minutes) to fill a 500 litre tank?
(A) 10 (B) 15
(C) 20 (D) 25

15. *A* and *B* can complete a job in 25 days and 20 days respectively, working alone. With the help of *C*, they can complete the job in $6^2/_3$ days. Find the

percentage of work completed by the fastest worker of the three.

(A) $20\frac{1}{3}\%$ (B) 25%

(C) 40% (D) $16\frac{2}{3}\%$

16. A tank has three inlet pipes I, II and III fitted to it whose flow rates are in the ratio 2 : 5 : 6. Pipe III takes 1 hour less than pipe II to fill the tank. Find the time (in hours) taken by pipe I to fill the tank.

(A) 6 (B) 10
(C) 5 (D) 15

Directions for questions 17 and 18: These questions are based on the following data.

In a city there are 5 major traffic junctions – *A*, *B*, *C*, *D* and *E*. There are no direct roads connecting *AC*,*BE* or *CE* but for every other pair of junctions, there are direct connecting roads, which all happen to be of equal length. Traffic moves at recommended uniform speeds on each road – at 20 km/hr on *BD* and *AD*, at 30 km/hr on *AE*, at 40 km/hr on *BC* and *CD*, and at 60 km /hr on *AB* and *DE* (Assume any direct connecting road is straight line).

17. A traffic inspector wants to visit any three traffic junctions in the shortest possible time, starting from *A*. What are the three points that he can visit (other than *A*) in order if he drives at the recommended speed on each road?

(A) *BCD* (B) *EDB*
(C) *BDC* (D) *BDE*

18. A new direct connecting road is constructed joining *A* and *C* with its recommended speed fixed at 50 km/hr. If *AB* = 10 km, find the time (in minutes) in which the traffic inspector can complete the round trip *ABCA*.

(A) $12 + 25\sqrt{3}$
(B) $25 + 12\sqrt{3}$
(C) $24 + 12.5\sqrt{3}$
(D) Cannot be determined

Directions for questions 19 and 20: These questions are based on the following data.

Cities *P*, *Q* and *A* are in different time zones. *P* and *Q* are located at 4500 km, east and west of *A* respectively. The table below describes the schedule of an airline operating non-stop flights between *A* and *P*, *A* and *Q*. All the times indicated are local and on the same day.

Departure		Arrival	
City	**Time**	**City**	**Time**
A	7 : 00 am	P	3 : 00 pm
A	9 : 00 am	Q	12 : 00 noon

Planes cruise at the same speed to both the cities but effective speed is influenced by a steady wind blowing from east to west at 75 kmph.

19. What is the plane's cruising speed (in kmph)?

(A) 825
(B) 900
(C) 875
(D) Cannot be determined

20. What is the time difference between cities *A* and *Q*?

(A) 1 hour
(B) $2\frac{1}{2}$ hours
(C) 2 hours
(D) Cannot be determined

Directions for questions 21 to 35: Select the correct alternative from the given choices.

21. Amar covered the first one-fourth of a certain distance at 2 km/hr, half of the remaining distance at 3 km/hr and the remaining distance at 4 km/hr. Find his average speed (in km/hr) for the entire journey.

(A) $2\frac{7}{11}$ (B) $2\frac{8}{11}$

(C) $2\frac{9}{11}$ (D) $2\frac{10}{11}$

22. A man starts from *P* at 8 a.m. and reaches *Q* by 9 : 30 a.m. At what time should he start from *Q* to reach *R* at 11 : 30 a.m., where *PQ* : *QR* = 10 : 11?

(A) 10 : 01 a.m. (B) 9 : 59 a.m.
(C) 9 : 50 a.m. (D) 9 : 51 a.m.

23. A boat covered a certain distance upstream and returned to the starting point. If the speed of the boat in still water is doubled and the speed of the stream is tripled, it would have taken the same time for the round trip. Find the ratio of the speed of the boat in still water to the speed of the stream.

(A) $\sqrt{5} : \sqrt{2}$ (B) $\sqrt{3} : \sqrt{2}$
(C) $\sqrt{7} : \sqrt{2}$ (D) 3 : 2

24. A boat started travelling downstream from a point *A* on a river. After it had travelled 12 km, a log started floating from *A*. The boat travels for 2 more hours in the same direction and then turns around and meets the log at a point 12 km from *A*. If the speed of the boat in still water is thrice the speed of the stream, find the speed of the stream (in km/hr).

(A) 1 km/hr (B) 2 km/hr
(C) $\frac{3}{2}$ km/hr (D) 4 km/hr

25. In a race, *A* gives *B* a start of 25 m and *C* a start of 50 m. If *B* runs 50% faster than *C* and all the three reach the finishing point simultaneously, then find the ratio of the speeds of *A* and *C*.

(A) 2 : 1 (B) 4 : 3
(C) 5 : 4 (D) 3 : 1

26. In a 200 m race, A gives B a start of 10 m and beats him by 10 m or 2 seconds. Find the speed of A (in m/s).

(A) $\frac{200}{17}$ (B) $\frac{150}{17}$

(C) $\frac{125}{17}$ (D) $\frac{50}{9}$

27. On a 900 m long circular track, A, B and C start running from the same point simultaneously. A runs in the clockwise direction at 2 m/s while B and C run in the anti-clockwise direction at 3 m/s and 4 m/s respectively. Find the time interval (in seconds) between A and C meeting for the first time at the starting point and B and C meeting for the first time.

(A) 600 (B) 450

(C) 750 (D) 800

28. A, B and C run along a circular track with speeds in the ratio 1 : 2 : 4 starting from the same point simultaneously. If A takes 3 minutes to complete one round of the track, find the time taken (in minutes) by the three to meet at the starting point for the first time.

(A) 6 (B) $\frac{3}{2}$

(C) 3 (D) Cannot be determined

29. Two men A and B start from two points P and Q simultaneously towards each other. They meet after two hours of their starting, B takes 3 hours less to reach P than A takes to reach Q. Find the ratio of the speeds of A and B.

(A) 3 : 2 (B) 2 : 1

(C) 1 : 2 (D) 3 : 1

30. A man travels 51 km in 61 minutes and 30 seconds with an usual speed of 50 km/hr. There are some speed breakers on the road. Each speed breaker reduces his speed to 80% of his usual speed for a distance of 50 m about the speed breaker. Find the number of speed breakers that he crossed.

(A) 20 (B) 25

(C) 30 (D) 35

31. When the speed of a train is increased by 5 m/s, it would take 40 seconds to cross a 200 m long platform. If it crosses a 300 m long platform in 50 seconds, at its original speed, then find the original speed of the train (in m/s).

(A) 35 (B) 15

(C) 20 (D) 30

32. Two trains take 80 seconds to cross each other, when travelling in the same direction. They take 60 seconds to cross each other, when travelling in opposite directions. Find the ratio of the speeds of the faster and the slower train.

(A) 4 : 1 (B) 7 : 1

(C) 3 : 1 (D) 6 : 1

33. There are two cars 80 km apart. When they travel in the same direction, they would take twice the time to meet, compared to the time they would take to meet while travelling towards each other. Find the ratio of their speeds.

(A) 3 : 1 (B) 2 : 1

(C) 4 : 3 (D) 3 : 2

34. By travelling 20% faster than his usual speed, a person reaches his office from home 10 minutes earlier than his usual time. By how many minutes would he be delayed as compared with his usual time, if he travels 25% slower than his usual speed?

(A) 10 (B) 15

(C) 20 (D) 25

35. A frog spots a snake 30 m behind it. It starts moving away from it at 12 m/s. After 5 seconds, it sees that the snake has just begun to move towards it at 20 m/s and increases its speed by 3 m/s. Find the time taken by the snake (in seconds) to catch the frog.

(A) 15 (B) 18

(C) 21 (D) 24

Answer Key

1. A	**2.** C	**3.** D	**4.** B	**5.** A	**6.** D	**7.** B	**8.** C	**9.** A	**10.** A
11. B	**12.** D	**13.** B	**14.** C	**15.** C	**16.** D	**17.** A	**18.** B	**19.** A	**20.** C
21. D	**22.** D	**23.** C	**24.** B	**25.** A	**26.** D	**27.** B	**28.** C	**29.** C	**30.** A
31. D	**32.** B	**33.** A	**34.** C	**35.** B					

Hints and Explanations

1. Let the work done by 1 man, 1 woman and part-timer in a day be m, w and p units respectively.

Given $(2m + 2w + 6p)\ 12 = (10m + 18p)\ 4$

$24w = 16m \Rightarrow m = \frac{3}{2}w$

Work done by 2 men and 3 women in 16 days

$= [[3\ w] + 3\ w]\ 16 = 96w$

Time taken by 1 woman to complete that job

$= \frac{96w}{w} = 96$ days

Choice (A)

2. A can do $\frac{1}{20}th$ of the job in a day. B can do $\frac{1}{10}th$ of the job in a day. In 6 days they will together complete $\frac{9}{10}th$ of the job. The balance $\frac{1}{10}th$ of the job is done by C on 6th day. Hence C would take 10 days to complete the job independently. Choice (C)

3. Four men can do one-third of the work in 10 days. In 5 days, one-third of the work can be completed by 8 men. Two-thirds of the work can be completed by 16 men. As there are 4 men, 12 men are required additionally. Choice (D)

4. A can do the work in 5 days. The part of the work done by A in one day $= \frac{1}{5}$

B can do the work in 10 days.

The part of the work done by B in one day $= \frac{1}{10}$

both A and B in two days $= \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$

The part of the work done in 6 days $= 3 \times \frac{3}{10} = \frac{9}{10}$

The part of the remaining work $= 1 - \frac{9}{10} = \frac{1}{10}$

As B starts the work, $\frac{1}{10}$th of the work can be done by B on 7th day.

$\therefore$ In 7 days, the work will be completed.

Choice (B)

5. Let the time (in minutes) taken by A, B and C to either fill or empty the tank be a, b and c respectively.

$$\frac{1}{b} + \frac{1}{c} - \frac{1}{a} = \frac{1}{30} \quad \text{..... (1)}$$

$$\frac{1}{c} + \frac{1}{a} - \frac{1}{b} = \frac{1}{40} \quad \text{..... (2)}$$

$$\frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{1}{24} \quad \text{..... (3)}$$

Adding the above three equations, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{10}$

--- (4)

Subtracting equation (1) from equation (4),

we get $\frac{2}{a} = \frac{2}{30}$

$\Rightarrow$ $a = 30$. Choice (A)

6. Let the time taken by A and B working together to complete the work be t days. Time taken by A alone and B alone to complete the work is $(t + 12)$ days and $(t + 27)$ days respectively.

Work done by A and B working together in a day

$= \frac{1}{t+12} + \frac{1}{t+27}$ which is equal to $\frac{1}{t}$.

$$\Rightarrow \frac{1}{t+12} + \frac{1}{t+27} = \frac{1}{t}$$

$$\Rightarrow \frac{t+27+t+12}{(t+12)(t+27)} = \frac{1}{t}$$

$\Rightarrow$ $2t^2 + 39t = t^2 + 39t + 324$

$\Rightarrow$ $t^2 = 324 \Rightarrow t = 18$. Choice (D)

7. As A gets $\frac{1}{4}$th of the total share, he completes $\frac{1}{4}$th of the total work. B whose rate is half that of A, completes $\frac{1}{8}$th of the total work, for which he gets ₹300.

$\therefore$ Share of $C = 2400 - 600 - 3000$

$= ₹1500$. Choice (B)

8. The time taken by B and C together to complete $\left(\frac{3}{4}\right)$th of the work is 18 days

$\therefore$ The time taken to complete 1 unit of work

$= 18 \times \left(\frac{4}{3}\right) = 24$ days

$\therefore$ The time taken by each alone to complete the work $= (24)(2) = 48$ days.

$\Rightarrow$ The time taken by A to complete (1/4)th of the work $= \left(\frac{1}{4}\right)(48) = 12$ days Choice (C)

9. Time taken by 24 taps working together to fill the tank would be $\frac{1}{4}th$ of the time taken by 6 taps working together i.e. 3 minutes. To fill a tank twice as big, 24 taps would take 6 minutes working together

Choice (A)

10. Time taken by P to complete the work in 12×6 i.e., 72 man hours.

Time taken by Q to complete the work in 18×5 i.e., 90 man hours

The part of the work done by P and Q in

1 hr $= \frac{1}{72} + \frac{1}{90} = \frac{90+72}{72(90)} = \frac{1}{40}$

$\therefore$ They complete the work in 40 hours.

By working 4 hrs per day, they can complete in $\frac{40}{4}$ i.e., 10 days Choice (A)

11. Let the total work be 1 unit

$$\frac{5}{20}+\frac{x+5}{20}+\frac{x+5+\frac{5}{3}}{30}=1$$

$\Rightarrow$ $x = 10/3$

Time for which B worked = $x+5+\frac{5}{3}=10$ days.

Choice (B)

12. A and B complete $\frac{1}{20}+\frac{1}{30}$ *i.e.*, $\frac{1}{12}$th of the work on 1st day. A and C complete $\frac{1}{20}+\frac{1}{60}$ *i.e.*, $\frac{1}{15}$th of the work on the 2nd day. In 2 days, $\frac{3}{20}$th of the work would be completed. Working in this way, $\frac{9}{10}$th of the work would be completed in 12 days. Of the balance $\frac{1}{10}$th of the work, A and B would complete $\frac{1}{12}$th of the work the next day. A and C would complete the balance $\frac{1}{60}$th of the work in another $\frac{1}{4}$th of a day. Hence a total of $13\frac{1}{4}$ days would be taken to complete the work.

Alternate method:
Assume the work (in units) to be the LCM of the individual time taken by A, B and C to complete the job i.e., 60 units. Capacities of A, B and C would be 6 units a day. A and B would complete 5 units the first day. A and C would complete 4 units the second day. Hence 9 units would be completed in two days. In 12 days, 54 units would be completed. Of the balance 6 units, 5 units would be completed the 13th day. The balance 1 unit would be completed in $\frac{1}{4}$th of the 14th day. Hence a total of $13^1/_4$ days would be taken to complete the work. Choice (D)

13. Amar and Bhavan would have taken 6 days to complete the job working together. Bhavan left after 4 days of the start of the work. In 4 days, Bhavan would have completed $\frac{4}{10}$ *i.e.* $\frac{2}{5}$th of the work. Amar completes the remaining $\frac{3}{5}$th of the work for which he would have taken $\frac{3}{5}(15)$ i.e., 9 days. Hence Amar completes the remaining work in 5 days. Choice (B)

14. Filling rate of first pipe $=\frac{1000\,\text{lit}}{10\,\text{min}}=100\,\text{lit/min}$.

Emptying rate of second pipe $=\frac{600\,\text{lit}}{8\,\text{min}}=75\text{lit/min}$.

Working together they can fill 25 lit/min. Time taken by them working together to fill a 500 litre tank $=\frac{500}{25}$ or 20 minutes. Choice (C)

15. Let us assume that the third person takes c days to complete the work independently. Total work completed by the three in a day, working together $=\frac{1}{25}+\frac{1}{20}+\frac{1}{c}=\frac{1}{6\frac{2}{3}}$

$$\frac{1}{c}=\frac{3}{20}-\frac{1}{20}-\frac{1}{25}\Rightarrow\frac{1}{c}=\frac{3}{50}$$

As the third person completes most of the work in a day, he is the fastest.
Ratio of work completed by the three persons $=\frac{1}{25}:\frac{1}{20}:\frac{3}{50}=4:5:6$

Percentage of work done by the third person $=\frac{6}{(4+5+6)}(100\%)=40\%$. Choice (C)

16. Let the volume of the tank be V litres. Let the filling rates of pipes I, II and III be $2x$, $5x$ and $6x$ respectively (in litres/hour)

$$\frac{V}{6x}=\frac{V}{5x}-1\Rightarrow V=30x$$

Time taken by pipe I to fill the tank $=\frac{30x}{2x}=15\,\text{hours}$.

Choice (D)

If the 10 possible pairs of points, for 7 pairs, the distances between the points equal. This is possible if of the 5 points, 4 are consecutive vertices of a regular hexagon and the 5th is the centre of the circum circle. We can think of the following figure.
Now we have to final which of these 5 points in A, which is B etc., AC, BE, CE have no direct roads connecting them. In the figure 13, 25 and also 35 have no direct roads connecting them (35 is connected through 4 not directly)

$\therefore$ We get the following possibilities. For either questions that follow it does not matter, which of the two figures we use.

17. The routes and recommended speeds are shows in the figure below.
The routes and the time taken are tabulated below.
We can take
$AB = BC = CD = DB = AD = DE = EA = r$ km

Route time Route Time

(1) $ABCD$ $\frac{r}{60}+\frac{r}{40}+\frac{r}{40}$

(2) $AEDC$ $\frac{r}{30}+\frac{r}{60}+\frac{r}{40}$

(3) $AEDB$ $\frac{r}{30}+\frac{r}{60}+\frac{r}{20}$

(4) $ABDE$ $\frac{r}{60}+\frac{r}{20}+\frac{r}{60}$

Multiplying all the time by LCM (60, 40, 30, 20) or 120, we get $8r$, $12r$, $9r$, $10r$ respectively

$\therefore$ For $ABCD$ the time is the least. Choice (A)

18. If AC and BE are also connected, each dist each distance would be $\sqrt{3}\,r$. The time for the round trip $ABCD$ is

$$\frac{r}{60}+\frac{r}{40}+\frac{\sqrt{3}r}{50}=\frac{10r+15r+12\sqrt{3}r}{600}=\frac{\left(25+12\sqrt{3}\right)(10)}{600}$$

($\because$ r = 10 km) = 25 + 12$\sqrt{3}$ min. Choice (B)

Solutions for questions 19 and 20:

Let the speed of the plane be x km

Let the time difference between A and P be t hours (i.e., P is t hours ahead of A).

$\therefore$ The time difference between A and Q is t hours (Q is t hours behind A)

$\therefore$ we have the following equations.

(from A to P) $\frac{4500}{x-75}=8-t$ …… (1)

(from A to Q) $\frac{4500}{x+75}=3+t$ …… (2)

(1) + (2) $\Rightarrow 4500\left[\frac{2x}{x^2-75^2}\right]=11$

$\Rightarrow$ $11x^2 - 11(75)^2 = 9000x$

$$\therefore\quad x=\frac{9000\pm\sqrt{(9000)^2-4(11)(-11)75^2}}{2(11)}$$

$$=\frac{9000\pm(150)\sqrt{60^2+121}}{22}=\frac{9000\pm150(61)}{22}$$

As x is positive, $x=\frac{9000+9150}{22}=825$

From (1), $\frac{4500}{825-75}=8-t$

$\Rightarrow$ $6 = 8 - t \Rightarrow t = 2$

19. Choice (A)

20. Choice (C)

Solutions for questions 21 to 35:

21. Let the total distance covered by Amar be d km. Amar covered $\frac{d}{4}$ km at 2 km/hr, $\frac{3d}{8}$ km at 3 km/hr and $\frac{3d}{8}$ km at 4 km/hr.

Total travel time of Amar $=\frac{\frac{d}{4}}{2}+\frac{\frac{3d}{8}}{3}+\frac{\frac{3d}{8}}{4}=\frac{11d}{32}$ hours

Average speed of Amar

$=\frac{\text{Total distance he travelled}}{\text{His total travel time}}$

$=\frac{d}{\frac{11d}{3}11d/3}=2\frac{10}{11}$ hours

Alternative method:

It can be seen from the normal method, as d cancels finally, any value of d can be taken. Taking d = 8, total travel time of Amar $=\frac{2}{2}+\frac{3}{3}+\frac{3}{4}=\frac{11}{4}$ hours

Average speed of Amar $=\frac{8}{\frac{11}{4}}=2\frac{10}{11}$ km/hr.

Choice (D)

22. From Q to R, he has to cover $\frac{11}{10}$ of the distance from P to Q.

$\therefore$ Time taken by him to reach R from Q will be $\frac{11}{10}$ times the time taken by him to reach R from Q i.e., 90 minutes $\times\frac{11}{10}$.

$\therefore$ He needs 99 minutes.

To reach R by 11 : 30 a.m. he should start from Q at 9 : 51 a.m. Choice (D)

23. Let the distance travelled in each direction (upstream as well as downstream) be d km. Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr. Total travel time of the boat

= upstream travel time + downstream travel time

$=\frac{d}{x+y}+\frac{d}{x-y}$

Now the speed of the boat in still water and the speed of the stream are $2x$ km/hr and $3y$ km/hr respectively,

Total travel time $=\frac{d}{2x+3y}+\frac{d}{2x-3y}$

Given, $\frac{d}{x+y}+\frac{d}{x-y}=\frac{d}{2x+3y}+\frac{d}{2x-3y}$

$\frac{2dx}{x^2-y^2}=\frac{4dx}{4x^2-9y^2}$; $dx\,(2x^2-7y^2)=0$

$dx = 0$ or $x=\pm\sqrt{\frac{7}{2}}y$

As dx cannot be 0 ($\because$ $d > 0$ and $x > 0$) and x and y are both positive,

$x=\frac{\sqrt{7}}{\sqrt{2}}y \Rightarrow \frac{x}{y}=\frac{\sqrt{7}}{\sqrt{2}}$. Choice (C)

24. Let the point 12 km from A and the point where the boat turns back be B and C respectively.
If the speed of the stream is y km/hr, speed of the boat in still water = $3y$ km/hr. It travels for 2 hrs to cover BC.
$\therefore$ $BC = (3y + y)\,2 = 8y$ km
Time taken by the boat to travel from C to B
$=\frac{8y}{3y-y}=4$ hr.
As the boat takes 6 hours to travel from B to C and back,
$\frac{12}{y}=6$ or $y = 2$ Choice (B)

25. Let the length of the race be x m.
By the time A finish the race, B and C would have run $(x - 25)$ and $(x - 50)$ m respectively.
As B is 50% faster than C, $\frac{x-25}{x-50}=\frac{3}{2} \Rightarrow x=100$
Ratio of the speeds of A and $C = x : (x - 50) = 2 : 1$.
Choice (A)

26. Speed of $B = 10/2 = 5$ m/s
Time for which B would have run when A finishes the race $=\frac{200-(10+10)}{5}=36$ seconds.
So, A takes 36 seconds to run the race.
$\therefore$ Speed of $A=\frac{200}{36}=\frac{50}{9}$ m/s. Choice (D)

27. Time taken by A and C to meet for the first time at the starting point = LCM $\left(\frac{900}{2},\frac{900}{4}\right)=450$ sec
Time taken by B and C to meet for the first time
$=\frac{900}{\text{Difference of the speeds of } B \text{ and } C}=900$ seconds
Required time interval is 450 seconds.
Choice (B)

28. Let the speeds (in m/min) of A, B and C be x, $2x$ and $4x$ respectively. Let the length of the track be L m. Given that the time taken by A to complete one round = L/X = 3 minutes. Time taken by all the three to meet for the first time = LCM$\left(\frac{L}{x},\frac{L}{2x},\frac{L}{4x}\right)=\frac{L}{x}=3$ minutes.
Choice (C)

29. Let the speeds of A and B be x km/hr and y km/hr respectively. Distance from P and Q to their first meeting point are $2x$ km and $2y$ km respectively. Times taken by A and B to reach Q and P from their first meeting point are $\frac{2y}{x}$ hours and $\frac{2x}{y}$ hours respectively.

$2\frac{y}{x}=2\frac{x}{y}+3$

Substituting the choices in the above equation, only $\frac{x}{y}=\frac{1}{2}$, satisfies the condition. Choice (C)

30. If the man covers the entire distance at the usual speed, he takes $\frac{51}{50}$ hr or 1.02 hr = 1 hr $\frac{2(60)}{100}$ min = 1 hr / min 12 s.
But he actually takes 1 hr 1 min 30 s, i.e., 18 s more. For one speed breaker, he takes a certain extra time, which is $\left(\frac{0.05}{40}-\frac{0.05}{50}\right)$hr $=\frac{0.05}{10}\left(\frac{1}{20}\right)$hr $=\frac{5(36)}{200}$s
= 0.9s
$\therefore$ He has to cross $\frac{18}{0.9}$ or 20 speed breakers.
Choice (A)

31. Let the length of the train be L m and speed of the train be s m/sec. Time taken by the train to cross a 200 m long platform at increased speed $=\frac{L+200}{s+5}=40$
$\Rightarrow$ $L = 40s$
Time taken by the train to cross a 300 m long platform (in seconds) $=\frac{L+300}{s}=50$
As $L = 40s$,
$40s + 300 = 50s$
$s = 30$. Choice (D)

32. Let the lengths of the two trains be L_1 and L_2
Let the speeds of the faster and slower trains be S_1 and S_2 respectively.

$\frac{L_1+L_2}{S_1-S_2}=80 \Rightarrow L_1+L_2=80\,(S_1-S_2)$

$\frac{L_1+L_2}{S_1-S_2}=60 \Rightarrow L_1+L_2=60\,(S_1-S_2)$

$80\,(S_1-S_2)=60\,(S_1+S_2)$

$20S_1=140S_2 \Rightarrow \frac{S_1}{S_2}=\frac{7}{1}$. Choice (B)

33. Let the speeds (in km/hr) of the faster and slower cars be x and y respectively.Time taken by the cars to meet when they travel in the same direction and in the opposite direction are $\frac{80}{x-y}$ hours and $\frac{80}{x+y}$ hours respectively.

$\frac{80}{x-y} = 2\left(\frac{80}{x+y}\right)$

$80\ (x + y) = 160\ (x - y)\ 240y = 80x$

$\frac{x}{y} = \frac{3}{1}$. Choice (A)

34. Let the usual speed of the man be S km/hr. If he travels 20% faster, he would travel at $S + \frac{20}{100}S = \frac{6}{5}S$ km/hr.

As his speed is $\frac{6}{5}th$ of his usual speed, he would take $\frac{5}{6}th$ of the usual time to travel to office. He saves one sixth of his usual time = 10 minutes

$\Rightarrow$ His usual time = 60 minutes

If he travels 25% slower than his usual speed, he would travel at $S - \frac{25}{100}S = \frac{3}{4}S$

If his speed is $\frac{3}{4}ths$ of his usual speed, he would take $\frac{4}{3}rds$ of the usual time to travel to office. He would be late by $\frac{1}{3}rd$ of the usual time i.e., 20 minutes. Choice (C)

35. In 5 seconds, the frog would move 60 m. When the snake is spotted by the frog, the frog would be 90 m ahead of the snake. Time, the snake would take to catch the frog (in seconds)

$= \frac{90}{\text{DIfference of speeds of snake and frog}}$

$= \frac{90}{20-15} = 18$. Choice (B)

QUANTITATIVE ABILITY TEST 3

(INDICES, SURDS AND LOGARITHMS)

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Find the value of x, if $2^x = 8^y$ and $6^{4y} = 216^{x+y-2}$.

(A) $2\frac{1}{4}$ (B) $2\frac{1}{2}$

(C) $3\frac{1}{2}$ (D) $3\frac{1}{3}$

2. If $x = \sqrt[3]{9\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9\ldots\ldots\ldots\ldots\ldots}}}}$, what is the value of x?

(A) 3 (B) –3

(C) $\sqrt{3}$ (D) 9

3. If $9^{3x-4} = 6561.\sqrt{27^{x-2}}$, then find the value of x.

(A) 1 (B) 2

(C) 3 (D) None of these

4. If $p^{\ell} = q^m = r^n$ and $\frac{1}{\ell} + \frac{1}{n} = \frac{2}{m}$,then which of the following is valid? ($p > 1$, $q > 1$, $r > 1$)

I. $p\ell = q^2$ II. $pq = \ell^2$ III. $p^2 = qr$

(A) Both I and I (B) Both II and III

(C) Both III and I (D) None of these

5. Find the value of x if $(125)^{2x-3} = (25)^{3^{(-1)^{2^{3^4}}}}$.

(A) 4.5 (B) 2.5

(C) 1.5 (D) None of these

6. If $2^{2x^2+3} = 8^{2x+1}$ and x is positive, then what is the value of x?

(A) 2 (B) 3

(C) 1 (D) 4

7. If $t_1 = \sqrt{5}$, $t_2 = \sqrt{5\sqrt{5}}$, $t_3 = \sqrt{5\sqrt{5\sqrt{5}}}$ and so on, then the product of the first ten terms $(t_1)\,(t_2)\,(t_3)\,(t_4)\ldots.\,(t_{10})$ is equal to

(A) $\sqrt[512]{5^{4609}}$ (B) $\sqrt[2048]{5^{18431}}$

(C) $\sqrt[1024]{5^{9217}}$ (D) $\sqrt[512]{5^{4607}}$

8. If $x^2 \neq x$, then

$$\frac{x^{4b} + x^{2(a+b)} + x^{4a}}{\left(x^{2a} + x^{a+b} + x^{2b}\right)\left(x^{2a} - x^{a+b} + x^{2b}\right)}$$ is equal to

(A) $\frac{x^a}{x^b}$ (B) $\frac{x^{2a}}{x^{2b}}$

(C) x^{a+b} (D) None of these

9. If $3^{x+3} - 3^{x-3} = 6552$, then find x^2.

(A) 5 (B) 25

(C) 3 (D) 9

10. If x, y, z are real numbers such that $xyz = 1$, then the expression $\frac{1}{1+x+y^{-1}} + \frac{1}{1+y+z^{-1}} + \frac{1}{1+z+x^{-1}}$ is equal to

(A) 1 (B) $\frac{3}{x+y+z}$

(C) $\frac{3}{x^{-1}+y^{-1}+z^{-1}}$ (D) $\frac{x+y+z}{3}$

11. If $\frac{(81^a)^a\,(81^b)^b\,(81^c)^c}{(6561^b)^{-c}\,(6561^c)^{-a}\,(6561^a)^{-b}} = 3.$

Then $a + b + c$ could be

(A) 2 (B) $\frac{1}{3}$

(C) $-\frac{1}{2}$ (D) $-\frac{1}{3}$

12. If $x = \cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots.\infty}}}}$ find x.

(A) $\sqrt{3}+\frac{3}{2}$ (B) $\sqrt{3}-\frac{3}{2}$

(C) $\sqrt{3}+\frac{1}{2}$ (D) $\sqrt{3}-\frac{1}{2}$

13. If $A = 8^{88^8}$, $B = 8^{8^{88}}$, $C = 8^{888}$ and $D = 8^{8^{8^8}}$, which of the following represents the ascending order of the values of A, B, C, D?

(A) *CDAB* (B) *CABD*

(C) *CBAD* (D) *ACBD*

14. Solve for x: $\sqrt{x} + \sqrt{x - \sqrt{1-x}} = 1$.

(A) 1 (B) $\frac{16}{25}$

(C) $\frac{4}{5}$ (D) 0

15. The arithmetic mean of two surds is $5 + 9\sqrt{2}$, and one of the surds is $1 + 12\sqrt{2}$

What is the square root of the other surd?

(A) $6 - 21\sqrt{2}$ (B) $4 - 3\sqrt{2}$

(C) $\sqrt{3}\left(\sqrt{2}+1\right)$ (D) $\sqrt{2}\left(2-\sqrt{3}\right)$

16. $\dfrac{1}{\sqrt{6}+\sqrt{7}-\sqrt{13}}+\dfrac{1}{\sqrt{6}-\sqrt{7}-\sqrt{13}}=$

(A) $\sqrt{6}$ (B) $\dfrac{1}{\sqrt{6}}$

(C) 6 (D) $\dfrac{1}{6}$

17. Find the square root of

$$\left[1+\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\ldots\ldots\ldots \frac{1}{\sqrt{324}+\sqrt{323}}\right]$$

(A) $3\sqrt{2}$ (B) $\dfrac{1}{\sqrt{2}}$

(C) $2\sqrt{3}$ (D) $\dfrac{\sqrt{3}-1}{2}$

18. If $x^y = y^z = z^x$ and $(x, y, z) > 0$, then

(A) $3\left[\dfrac{xy+yz+zx}{xyz}\right]$ (B) $\dfrac{xy+yz+zx}{xyz}$

(C) $\dfrac{x+y+z}{xyz}$ (D) $\dfrac{xy+yz+zx}{x+y+z}$

19. If $\log_4 3$, $\log_4 (3^m - 2)$ and $\log_4\left(3^m - \dfrac{8}{3}\right)$ are in arithmetic progression, then the number of possible values of m are

(A) 1 (B) 2

(C) 4 (D) 5

20. If $\log_x 162 = m$ and $\log_x 72 = n$, then what is the value of $\log_x 7776$ in terms of m and n?

(A) $\dfrac{m+3n}{m+5n}$ (B) $\dfrac{3m-5n}{m+2n}$

(C) $\dfrac{m+3n}{2}$ (D) $\dfrac{3m-5n}{2}$

21. Which of the following is a possible value of x if $\log_3 x^2 - \log_3 x\sqrt{x} = 8\log_x 3$?

(A) $\dfrac{1}{81}$ (B) $\dfrac{1}{243}$

(C) 243 (D) 9

22. If $a =$ $a = \log_4 31$, then ______ .

(A) $a < 2$ (B) $2 < a < 2.5$

(C) $2.5 < a < 2.8$ (D) $2.8 < a$

23. If $\log_{10}(2x + 3) - 1 = \log_{10} x$, then find x.

(A) $\dfrac{2}{7}$ (B) $\dfrac{3}{4}$

(C) $\dfrac{7}{8}$ (D) $\dfrac{3}{8}$

24. If $a^2 + 4b^2 = 12ab$, what is the value of $\log(a + 2b)$?

(A) $\log\left(\dfrac{a}{2}\right)+\log\left(\dfrac{b}{2}\right)+\log 2$

(B) $(\log a+\log b-\log 2)\dfrac{1}{2}$

(C) $\dfrac{1}{2}(\log a+\log b+4\log 2)$

(D) $\dfrac{1}{2}(\log a-\log b+4\log 2)$

25. Simplify $\dfrac{\log_m p.\log_n p}{\log_m p+\log_n p}$.

(A) 1 (B) $\log_p (m + n)$

(C) $\log_p mn$ (D) $\log_{mn} p$

26. If $a > 1$, $\log_a a + \log_{a^{\frac{1}{2}}} a + \log_{a^{\frac{1}{3}}} a + \ldots..+ \log_{a^{\frac{1}{20}}} a =$

(A) 420 (B) 210

(C) 380 (D) 190

27. If $\dfrac{\log x}{y-z}=\dfrac{\log y}{z-x}=\dfrac{\log z}{x-y}$, then find the value of

$\log\left[\left(x^{y^2+yz+z^2+z}\right)\left(y^{z^2+zx+x^2+z}\right)\left(z^{x^2+xy+y^2+z}\right)\right]$

(A) 2 (B) 0

(C) 3 (D) None of these

28. If $abc = 1$, then find the value of

$\dfrac{1}{\log_{bc} a^3}+\dfrac{1}{\log_{ac} b^3}+\dfrac{1}{\log_{ab} c^3}$

(A) $\dfrac{-1}{3}$ (B) $\dfrac{1}{3}\log abc$

(C) -1 (D) $\log_{a+b+c} abc$

29. If $\log_6 27 = t$, then find the value of $\log_{18} 4$ in terms of t.

(A) $3\left(\dfrac{2-t}{2+t}\right)$ (B) $2\left(\dfrac{3-t}{3+t}\right)$

(C) $\dfrac{6+t}{3+t}$ (D) $\dfrac{4-t}{3+t}$

30. For $a \geq b$, $b > 1$ the value of the expression $\log_a\left(\dfrac{a}{b}\right)+\log_b\left(\dfrac{b}{a}\right)$ can never be

(A) 0 (B) 1

(C) -2 (D) -0.5

31. If $\log_4(x^2+x) - \log_4(x+1) = 2$, then $\sqrt{x} =$

(A) 2 (B) 4

(C) 8 (D) 16

32. If $\log_{10} 3 = 0.4771$, then find the number of digits in $(243)^{50}$.

(A) 200 (B) 205

(C) 120 (D) 210

33. If (log 16) (log 27) = (log x) (log y) and (log 4096) [log x – log 27] = [log16 – log y] (log 512), which of the following can be the value of (x – y)?
(A) –11
(B) 73
(C) –73
(D) More than one options

34. What is the value of?

$$\log_{64}\sqrt{512\sqrt{512\sqrt{512\sqrt{512\ldots\ldots\ldots\ldots\infty}}}}.$$

(A) 2.5 (B) 3
(C) 1.5 (D) 1

35. If $\log_{bc}a = \frac{1}{p}$, $\log_{ca}b = \frac{1}{q}$ and $\log_{ab}c = \frac{1}{r}$, find the value of $\frac{1}{p+1}+\frac{1}{q+1}+\frac{1}{r+1}$
(A) 1 (B) $\frac{3}{2}$
(C) 2 (D) None of these

Answer Keys

1. A	**2.** A	**3.** D	**4.** D	**5.** B	**6.** B	**7.** C	**8.** D	**9.** B	**10.** A
11. C	**12.** B	**13.** B	**14.** B	**15.** C	**16.** B	**17.** A	**18.** A	**19.** B	**20.** C
21. A	**22.** B	**23.** D	**24.** C	**25.** D	**26.** B	**27.** B	**28.** C	**29.** B	**30.** B
31. B	**32.** C	**33.** D	**34.** C	**35.** A					

Hints and Explanations

1. Given $2^x = 8^y \Rightarrow 2^x = (2^3)^y$
$\Rightarrow \quad x = 3y$
Also $6^{4y} = 216^{x+y-2}$
$\Rightarrow \quad 6^{4y} = 6^{3(3y+y-2)}$
$\Rightarrow \quad 6^{4y} = 6^{12y-6}$
$\Rightarrow \quad 4y = 12y - 6$
$\Rightarrow \quad 6 = 8y \Rightarrow y = \frac{3}{4}$
$\therefore \quad x = 3y = \frac{9}{4} = 2\frac{1}{4}$
Choice (A)

2. Given $x = \sqrt[3]{9\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9\ldots\ldots\ldots\ldots}}}}$

$\Rightarrow \quad x^3 = 9\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9\sqrt[3]{9\ldots\ldots\ldots\ldots}}}} = 9x$

$\Rightarrow x = 0, 3, –3$. Since x is positive, $x = 3$. Choice (A)

3. $9^{3x-4} = 3^8.(27)^{\frac{(x-2)}{2}}$

$9^{3x-4} = 3^8.(3^3)^{\frac{(x-2)}{2}}$

$3^{2(3x-4)} = 3^8.3^{3\frac{(x-2)}{2}}$

$\frac{3x}{2} = 8 + 8 - 3$

$6x - 8 = 8 + \frac{(3x-6)}{2}$

$\frac{9x}{2} = 13 \Rightarrow x = \frac{26}{9}$ Choice (D)

4. Given that $p^{\ell} = q^m = r^n$
Let each be equal to k.
Hence $p^{\ell} = k$; $\log_p k = \ell$
$\log_q k = m$; $\log_r k = n$ given that
$\frac{1}{\ell} + \frac{1}{n} = \frac{2}{m}$
$\Rightarrow \quad \frac{1}{\log_p k} + \frac{1}{\log_r k} = \frac{2}{\log_q k}$
$\Rightarrow \quad \log_k p + \log_k r = 2 \cdot \log_k q$
$\Rightarrow \quad \log_k (pr) = \log_k q^2 \; pr = q^2$ Choice (D)

5. 2^{3^4} is always even, as 2 raised to any power is even.
Hence $(-1)^{2^{3^4}} = (-1)^{\text{even number}} = +1$
Hence, $(25)^{3(-1)^{2^{3^4}}} = (25)^{3^1} = 5^6$.
$(125)^{2x-3} = 5^6. \Rightarrow 5^{3(2x-3)} = 5^6. \Rightarrow 2x - 3 = 2; x = 2.5$
Choice (B)

6. By equating the index of 2 on both sides we get
$2x^2 + 3 = 6x + 3 \Rightarrow x = 3$ (as $x > 0$) Choice (B)

7. $t_1 = 5^{\frac{1}{2}} = 5^{1-\frac{1}{2}}$

$t_2 = 5^{\frac{3}{4}} = 5^{1-\frac{1}{2^2}}$

$t_3 = 5^{\frac{7}{8}} = 5^{1-\frac{1}{2^3}}$

$t_{10} = 5^{1-\frac{1}{2^{10}}}$

$= (t_1)(t_2)(t_3)\ldots\ldots(t_{10}) = \left(5^{1-\frac{1}{2}}\right)\left(5^{1-\frac{1}{2^2}}\right)\left(5^{1-\frac{1}{2^3}}\right)$

$\ldots\ldots\left(5^{1-\frac{1}{2^{10}}}\right)$

$$= 5^{10-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+....+\frac{1}{2^{10}}\right)}$$

$$= 5^{10-\frac{1}{2}\left[1+\frac{1}{2}+\frac{1}{2^2}+....+\frac{1}{2^9}\right]}$$

$$= 5^{10-\frac{1}{2}\frac{\left[1-\frac{1}{2^{10}}\right]}{1-\frac{1}{2}}}$$

$$= 5^{10-1+\frac{1}{2^{10}}} = \sqrt[1024]{5^{9217}}$$

Choice (C)

8. Let $E =$ $E = \dfrac{x^{4b}+x^{2(a+b)}+x^{4a}}{\left(x^{2a}+x^{a+b}+x^{2b}\right)\left(x^{2a}-x^{a+b}+x^{2b}\right)}$.

Den (E) = $x^{2b}\left(x^{2a-2b}+x^{a-b}+1\right).\ x^{2b}\left(x^{2a-2b}-x^{a-b}+1\right)$

$$E=\frac{x^{4b}\left(1+x^{2(a\text{-}b)}+x^{4(a\text{-}b)}\right)}{x^{4b}\left[1\text{-}x^{(a\text{-}b)}+x^{2(a\text{-}b)}\right]\left[1+x^{a\text{-}b}+x^{2(a\text{-}b)}\right]}$$

Considering $x^{a\text{-}b} = t$, we get

$$\frac{1+\ t^2+t^4}{\left(1\text{-}t+t^2\right)\left(1+t+t^2\right)}=\frac{1+t^2+t^4}{1+\ t^2+t^4}=1$$

$(1 - t + t^2)\ (1 + t + t^2) = (1 + t^2 - t)\ (1 + t^2 + t)$

$=(\ (1 + t^2)^2 - t^2)$

Note: The condition $x^2 \neq x$ means $x \neq 0$, $x \neq 1$. If $x = 0$, E is not defined. If $x = 1$, $E = 1$.

∴ This condition need not be imposed. But imposing the condition does not make the statement (that $E = 1$) false. Choice (D)

9. $3^{x+3} - 3^{x-3} = 6552$

$$3^x\left[3^3-\frac{1}{3^3}\right]=6552$$

$$3^x\left[\frac{728}{27}\right]=6552$$

$3^x = 243 = 3^5$

$\Rightarrow\ \ x = 5$

∴ $x^2 = 5^2 = 25$ Choice (B)

10. Given $xyz = 1$

$\Rightarrow\ \ qxy = \dfrac{1}{z}, \dfrac{1}{xy} = z$ -- (1)

Given expression,

$$\frac{1}{1+x+y^{-1}}+\frac{1}{1+y+z^{-1}}+\frac{1}{1+z+x^{-1}}$$

$$=\frac{y}{y+xy+1}+\frac{1}{1+y+xy}+\frac{1}{1+\frac{1}{xy}+\frac{1}{x}}\ \text{(from (1))}$$

$$=\frac{y}{y+xy+1}+\frac{1}{1+y+xy}+\frac{xy}{xy+1+y}$$

$$=\frac{y+1+xy}{1+xy+y}=1$$

Choice (A)

11. $\dfrac{81^{a^2+b^2+c^2}}{81^{[-2bc-2ca-2ab]}}=81^{a^2+b^2+c^2+2ab+2bc+2ca}$

$= 81^{(a+b+c)^2} = 3 = 81^{\frac{1}{4}}$

$\Rightarrow\ \ a+b+c=\pm\ \dfrac{1}{2}$

Choice (C)

12. Given $x = \cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+......\infty}}}}$

$\Rightarrow\ \ x=\dfrac{1}{4+\dfrac{1}{3+x}}\ \Rightarrow\ x=\dfrac{3+x}{4\left(3+x\right)+1}$

$\Rightarrow\ \ x=\dfrac{3+x}{4x+13}$

$\Rightarrow\ \ 4x^2 + 12x - 3 = 0$

$\Rightarrow\ \ x=\dfrac{-12\pm\sqrt{144+48}}{8}$

$\Rightarrow\ \ x=\dfrac{4\left(-3\pm\sqrt{12}\right)}{8}$

$\Rightarrow\ \ x=\dfrac{-3\pm2\sqrt{3}}{2}$

Since $x > 0$,

$x =\ x=\dfrac{-3}{2}+\sqrt{3}$ Choice (B)

13. $A=8^{88^8}\quad B=8^{8^{88}}\ \ C=8^{888}\ \ D=8^{8^{8^8}}$

Since the base of all the numbers is 8, the number power with highest index is the greatest number. Clearly 'C' has the lowest value.

Consider $A=8^{88^8}$

and $B=8^{888}$.

Consider the indices is 88^8

and 8^{88}

$(88)^8$ and $(8^{11})^8$

Since $8^{11} > 88$

$8^{88} > 88^8$

∴ $B > A$

Also, among the four powers the greatest power is 8^{8^8}. Hence D is the largest number.

∴ the ascending order is $CABD$. Choice (B)

14. $\sqrt{x}+\sqrt{x-\sqrt{1-x}}=1$

Squaring both sides we get, $x-\sqrt{1-x}=1+x-2\sqrt{x}$

Squaring again, we get, $1 - x = 1 + 4x - 4\sqrt{x}$

$16x = 25x^2$

$x=\dfrac{16}{25}$ Choice (B)

15. Let the other surd be "a".

$a + 1 + 12\sqrt{2}$

$= 10 + 18\sqrt{2}$

$a = 9 + 6\sqrt{2}$

$= 9 + 2\sqrt{18} = \left(\sqrt{6}+\sqrt{3}\right)^2$

$\therefore \quad \sqrt{a} = \left(\sqrt{6}+\sqrt{3}\right)$ Choice (C)

16. $\dfrac{1}{\sqrt{6}+\sqrt{7}-\sqrt{13}} = \dfrac{\left(\sqrt{6}+\sqrt{7}+\sqrt{13}\right)}{\left(\sqrt{6}+\sqrt{7}+\sqrt{13}\right)} \dfrac{(1)}{\left(\sqrt{6}+\sqrt{7}-\sqrt{13}\right)}$

$= \dfrac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{\left(\sqrt{6}+\sqrt{7}\right)^2-\left(\sqrt{13}\right)^2} = \dfrac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{13+2\sqrt{42}-13}$

$= \dfrac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{2\sqrt{42}}$

$\dfrac{1}{\sqrt{6}-\sqrt{7}-\sqrt{13}} = \dfrac{1\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{\left(\sqrt{6}-\sqrt{7}-\sqrt{13}\right)\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}$

$= \dfrac{\sqrt{6}-\sqrt{7}+\sqrt{13}}{\left(\sqrt{6}-\sqrt{7}\right)^2-\left(\sqrt{13}\right)^2} = \dfrac{\sqrt{6}-\sqrt{7}+\sqrt{13}}{13-2\sqrt{42}-13}$

$= \dfrac{-\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{2\sqrt{42}}$

Required value $= \dfrac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{2\sqrt{42}} + \dfrac{-\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{2\sqrt{42}}$

$= \dfrac{2\sqrt{7}}{2\sqrt{42}} = \dfrac{1}{\sqrt{6}}$ Choice (B)

17. The given function is $1 + \dfrac{1}{\sqrt{2}+1} + \dfrac{1}{\sqrt{3}+\sqrt{2}} +$

$= 1 + \dfrac{\sqrt{2}-1}{2-1} + \dfrac{\sqrt{3}-\sqrt{2}}{3-2} + \dfrac{\sqrt{4}-\sqrt{3}}{4-3} + \ldots +$

(on rationalizing the denominator of each term)

$= 1 + \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \ldots + \sqrt{324} - \sqrt{323}$

$= \sqrt{324} = 18$ ($\because$ all terms cancel off except $\sqrt{324}$)

Hence, the square root of the given expression is $\sqrt{18}$

$= 3\sqrt{2}$. Choice (A)

18. Let $x^y = y^z = z^x = k$

$\Rightarrow \quad x = k^{\frac{1}{y}}, y = k^{\frac{1}{z}}, z = k^{\frac{1}{x}}$

consider $\dfrac{1}{x}\log_z xyz$

$= \dfrac{1}{x}\log_{k^{\frac{1}{x}}}\left(k^{\frac{1}{y}}.k^{\frac{1}{z}}.k^{\frac{1}{x}}\right)$

$= \dfrac{1}{x}\log_{k^{\frac{1}{x}}}\left(k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}\right) = \dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}$

Similarly $\dfrac{1}{y}\log_x xyz = \dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}$ and $\dfrac{1}{z}\log_y xyz$

$= \dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}$

Hence the given expression is equal to

$3\left[\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right] = 3\left(\dfrac{xy+yz+zx}{xyz}\right)$ Choice (A)

19. Given $\log_4 3 + \log_4\left(3^m - \dfrac{8}{3}\right) = 2\log_4\left(3^m - 2\right)$

$\Rightarrow \quad 3\left(3^m - \dfrac{8}{3}\right) = \left(3^m - 2\right)^2$

$\Rightarrow \quad 3^{m+1} - 8 = 3^{2m} + 4 - 4(3^m)$

$\Rightarrow \quad 3^{2m} - 7(3^m) + 12 = 0$

Let $3^m = x$

$\Rightarrow \quad x^2 - 7x + 12 = 0$

$x^2 - 4x - 3x + 12 = 0$

$x(x-4) - 3(x-4) = 0$

$\Rightarrow \quad (x-4)(x-3) = 0$

$\Rightarrow \quad x = 4$ or 3

$\Rightarrow \quad 3^m = 4$ or $3^m = 3$

$\Rightarrow \quad m = \log_3 4$ (or) $m = 1$

Hence m can take two values. Choice (B)

20. Given $\log_x 162 = m$

$\Rightarrow \quad \log_x 3^4 \ (2) = m$

$\therefore \quad m = 4\log_x 3 + \log_x 2$

Given $\log_x 72 = n$

$\Rightarrow \quad \log_x 3^2\, 2^3 = n$

$\therefore \quad n = 2\log_x 3 + 3\log_x 2$

Let $\log_x 3 = 1$ and $\log_x 2 = b$

$\Rightarrow m = 4a + b$ --- (1)

$n = 2a + 3b$ --- (2)

2 (2) – (1) gives

$5b = 2n - m \Rightarrow b = \dfrac{2n-m}{5}$

similarly $a = \dfrac{3m-n}{10}$

Now consider $\log_x 7776$

$= \log_x 3^5.2^5$

$= 5[\log_x 3 + \log_x 2]$

$= 5\left[\dfrac{3m-n}{10} + \dfrac{2n-m}{5}\right]$

$= 5\left[\dfrac{m+3n}{10}\right] = \dfrac{m+3n}{2}$ Choice (C)

21. $\log_3 x^2 - \log_3 x\sqrt{x}$
$= 8\log_x 3$

$\Rightarrow \quad \log_3 \dfrac{x^2}{x\sqrt{x}} = 8\log_x 3$

$\Rightarrow \quad \log_3 \sqrt{x} = 8\log_x 3$

$\Rightarrow \quad \dfrac{1}{2}\log_3 x = \dfrac{8}{\log_3 x}$

$\Rightarrow \quad (\log_3 x)^2 = 16$

$\Rightarrow \quad \log_3 x = 4$

$\Rightarrow \quad x = 3^4 = 81$ or $x = 3^{-4} = \dfrac{1}{81}$

Choice (A)

22. $\log_4 31 = \log_{2^2} 31 = \dfrac{1}{2}\log_2 31$

$2^4 < 31 < 2^5$

$\Rightarrow \quad \log_2 2^4 < \log_2 31 < \log_2 2^5$

$\Rightarrow \quad 4\log_2 2 < \log_2 31 < 5\log_2 2$

$\Rightarrow \quad \dfrac{4}{2} < \dfrac{1}{2}\log_2 31 < \dfrac{5}{2}$

$\Rightarrow \quad 2 < \dfrac{1}{2}\log_2 31 < 2.5.$ Choice (B)

23. $\log(2x + 3) - 1 = \log x$

$\log(2x + 3) - \log 10 = \log x$

$\Rightarrow \quad \log\left(\dfrac{2x+3}{10}\right) = \log x$

$\Rightarrow \quad \dfrac{2x+3}{10} = x$

$\Rightarrow \quad 2x + 3 = 10x$

$\Rightarrow \quad x = \dfrac{3}{8}$

Choice (D)

24. $a^2 + 4b^2 = 12ab$; adding $4ab$ to both sides of the equation, we get $(a + 2b)^2 = 16ab$

$2\log(a + 2b) = 4\log 2 + \log a + \log b$

$\log(a + 2b) = \dfrac{1}{2}$

$[\log a + \log b + 4\log 2]$ Choice (C)

25. $\dfrac{\log_m p.\log_n p}{\log_m p + \log_n p}$

$= \dfrac{1}{\dfrac{\log_m p + \log_n p}{\log_m p.\log_n p}} = \dfrac{1}{\dfrac{1}{\log_n p} + \dfrac{1}{\log_m p}}$

$= \dfrac{1}{\log_p n + \log_p m} = \dfrac{1}{\log_p mn}$

$= \log_{mn} p$ Choice (D)

26. $\log_a a + \log_a \frac{1}{2} a + \log_a \frac{1}{3} a + \ldots + \log_a \frac{1}{20}$ a

$= \dfrac{\log a}{\log a} + \dfrac{\log a}{\log a^{\frac{1}{2}}} + \dfrac{\log a}{\log a^{\frac{1}{3}}} + \ldots + \dfrac{\log a}{\log a^{\frac{1}{20}}}$

$= 1 + 2 + 3 + \ldots + 20 = \dfrac{20\times21}{2} = 210$ Choice (B)

27. Let $\dfrac{\log x}{y-z} = \dfrac{\log y}{z-x} = \dfrac{\log z}{x-y} = k$

$\log x = k(y - z)$

$\log y = k(z - x)$

$\log z = k(x - y)$

$\log x^{y^2+yz+z^2+z} = (y^2 + yz + z^2 + z)(\log x)$

$= k(y-z)(y^2 + yz + z^2 + z)$

$= k(y^3 - z^3) + k(y - z)z$ --- (1)

$\log y^{z^2+xz+x^2+z} = k(z-x)(z^2 + xz + x^2 + z)$

$= k(z^3 - x^3) + k(z - x)z$ --- (2)

$\log z^{x^2+xy+y^2+2} = k(x-y)(x^2 + xy + y^2 + z)$

$= k(x^3 - y^3) + k(x\ y)z$ --- (3)

Adding (1), (2) and (3),

$\log x^{y^2-yz+z^2+z} + \log y^{z^2+xz+x^2+z} + \log z^{x^2+xy+y^2+z}$

$= k(y^3 - z^3) + kz(y - z) + k(z^3 - x^3) + kz(z - x) + k(x^3 - y^3) + kz(x - y)$

$= k(y^3 - z^3 + z^3 - x^3 + x^3 - y^3) + kz(y - z + z - x + x - y)$

$= 0 + 0 = 0$ Choice (B)

28. $\dfrac{1}{\log_{bc} a^3} + \dfrac{1}{\log_{ac} b^3} + \dfrac{1}{\log_{ab} c^3}$

$= \dfrac{1}{\log_{1/a} a^3} + \dfrac{1}{\log_{1/b} b^3} + \dfrac{1}{\log_{1/c} c^3}$

$\left[\because bc = \dfrac{1}{a}, ac = \dfrac{1}{b} \text{ and } ab = \dfrac{1}{c}\right]$

$= \dfrac{1}{\log_{a^{-1}} a^3} + \dfrac{1}{\log_{b^{-1}} b^3} + \dfrac{1}{\log_{c^{-1}} c^3}$

$= \dfrac{1}{-3} + \dfrac{1}{-3} + \dfrac{1}{-3} = -1$ Choice (C)

29. $t = \log_6 27 = \dfrac{\log_{10} 27}{\log_{10} 6} = \dfrac{3\log_{10} 3}{\log_{10} 2 + \log_{10} 3}$

Now $3 - t = 3 - \dfrac{3\log 3}{\log 2 + \log 3} = \dfrac{3\log 2}{\log 2 + \log 3}$

$3 + t = 3 + \dfrac{3\log 3}{\log 2 + \log 3} = \dfrac{3\log 2 + 6\log 3}{\log 2 + \log 3}$

$\dfrac{3-t}{3+t} = \dfrac{(3\log 2)/(\log 2 + \log 3)}{(3\log 2 + 6\log 3)/(\log 2 + \log 3)}$

$= \dfrac{\log 2}{\log 2 + 2\log 3} = \log_{18} 2$

$\therefore \quad 2\dfrac{(3-t)}{3+t} = 2\log_{18} 2 = \log_{18} 4$

Choice (B)

30. Given: $\log_a\left(\dfrac{a}{b}\right) + \log_b\left(\dfrac{b}{a}\right)$

$= \log_a a - \log_a b + \log_b b - \log_b a = 2 - (\log_a b + \log_b a)$

But $\log_a b + \log_b a$ is in the form of $x + \dfrac{1}{x}$ which is ≥ 2

$\therefore$ $\log_a b + \log_b a \geq 2$

$\therefore$ The given expression can not be positive

$\therefore$ It can not be 1

Choice (B)

31. $\log_4\left[\dfrac{(x^2+x)}{(x+1)}\right] = 2$

$\dfrac{(x^2+x)}{(x+1)} = 4^2$

$\Rightarrow \quad x^2 + x = 16x + 16$

$= x^2 - 15x - 16 = 0$

$x^2 - 16x + x - 16 = 0$

$x(x - 16) + 1(x - 16) = 0$

$\Rightarrow \quad (x + 1)(x - 16) = 0$

$x = -1$ or 16

$\sqrt{x} = \sqrt{16} = 4$

Choice (B)

32. Let $k = (243)^{50} = 3^{250}$

Taking log on both sides, we get

$\log k = 250 \log 3$

$= 250\,(0.4771)$

$= 25(4.771) = \dfrac{100}{4}(4.771) = 119.4$

The characteristic of $\log k$ is 119.

Number of digits in $(243)^{50}$ are $119 + 1 = 120$

Choice (C)

33. Given $(\log 16)(\log 27) = (\log x)(\log(y))$

let $\log x = X$ and $\log y = Y$ then $(\log 4096)(\log x - \log 27)$

$= \log 512\,(\log 16 - \log y)$

becomes

$12 \log 2\,(X - 3\log 3) = 9\log 2(4\log 2 - Y)$

$4(X - 3\log 3) = 3(4\log 2 - Y)$ ———— (1)

and $(\log 16)(\log 27) = (\log x)(\log y)$ becomes $12 \log 2 \log 3 = XY$ ———— (2)

eliminating Y using (1) and (2) we have

$4(X - 3\log 3) = 3(4\log 2 - 12(\log 2)(\log 3) \div X)$

$4X(X - 3\log 3) = 3(4X\log 2 - 12\log 2.\log 3)$

$4X^2 - 12X\log 3 = 12X\log 2 - 36\log 2.\log 3$

$X^2 - 3X\log 3 - 3X\log 2 + 9\log 2\log 3 = 0$

$(X - 3\log 3)(X - 3\log 2) = 0$

$\Rightarrow \quad X = \log 27$ or $X = \log 8$

i.e., $\log x = \log 27$ or $\log x = \log 8$

$\Rightarrow \quad x = 27$ or 8

when $x = 27$ then $y = 16$ and $x - y = 11$ and when $x = 8$, then $y = 81$ and $x - y = -73$

Choice (D)

34. Let $\sqrt{512\sqrt{512\sqrt{512\sqrt{512\ldots\ldots\ldots\infty}}}} = x$

$x^2 = 512\sqrt{512\sqrt{512\sqrt{512\ldots\ldots\infty}}}$

$x^2 = 512x, \Rightarrow x = 0$ or 512.

As x is clearly not zero, $x = 512$.

Hence the required quantity is $\log_{64} 512$.

$= \log_{8^2} 8^3 = \dfrac{3}{2} = 1.5$

Choice (C)

35. $\log_{bc} a = 1/p \Rightarrow p = \log_a bc \Rightarrow p + 1 = \log_a abc$

Similarly $\log_{ca} b = 1/q \Rightarrow q + 1 = \log_b abc$

and $\log_{ab} c = 1/r \Rightarrow r + 1 = \log_c abc$

$\therefore \quad \dfrac{1}{p+1} + \dfrac{1}{q+1} + \dfrac{1}{r+1} = \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} abc = 1$

Choice (A)

Quantitative Ability Test 4

(Data Interpretation)

Number of Questions: 30 **Section Marks: 30**

Directions for questions 1 to 4: These questions are based on the information given below.

Vinod Melkote, HR Manager of *XYZ* Ltd. received partial information from his subordinates about the employees for annual appraisal.

	Performance			Total
	Below Average	Average	Above Average	
Males				96
Females		30		
Total	48			

Vinod also received the following information.

(1) One-third of the total employees are above average performers.

(2) 25% of the males are below average performers.

(3) The total number of females is twice the number of average performing males.

1. How many people are above average performers?

(A) 48 (B) 60
(C) 66 (D) 54

2. How many females are there?

(A) 54 (B) 72
(C) 90 (D) 84

3. What is the difference between the total number of average performers and that of males?

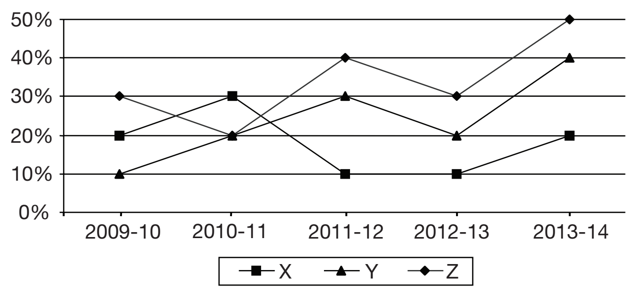
(A) 18 (B) 12
(C) 24 (D) 20

4. What is the ratio of the number of average performers and the number of females?

(A) 6 : 7 (B) 11 : 12
(C) 3 : 4 (D) 2 : 3

***Directions for questions 5 to 8*:** These questions are based on the following line graph.

Profit percentages of companies X, Y and Z from 2009-10 to 2013-14



Note: Profit = Income − Expenditure.; Profit % = $\frac{\text{Profit}}{\text{Expenditure}} \times 100$

5. If the profit of *X* in 2010-11 is ₹120 lakhs, then what is its income in 2010-11? (in ₹lakhs)

(A) 520 (B) 200
(C) 320 (D) 140

6. If the income of *Y* in 2013-14 is thrice that of *X* in 2009-10, then what is the ratio of profits of *X* and *Y* in 2013-14?

(A) 12 : 1 (B) 12 : 13
(C) 1 : 13 (D) None of these

7. If in 2009-10, the ratio of profits of *X*, *Y* and *Z* is 4 : 2 : 3, then what is the ratio of their expenditures in that year?

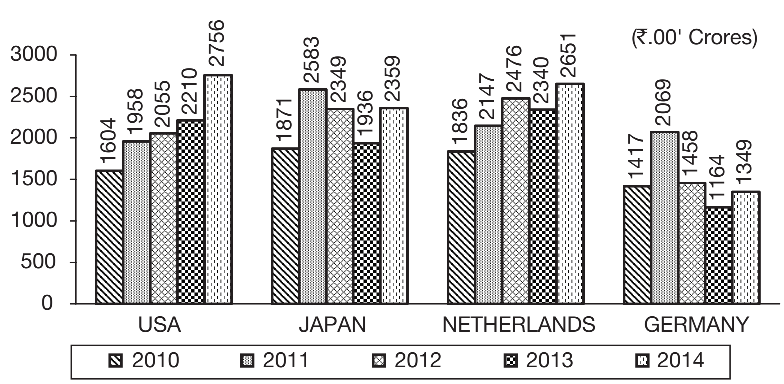
(A) 3 : 6 : 4 (B) 2 : 2 : 1
(C) 3 : 2 : 4 (D) None of these

8. If both *Y* and *Z* had the same profit in 2011-12, then what is the ratio of their incomes in that year?

(A) 4 : 3 (B) 1 : 1
(C) 26 : 21 (D) 2 : 3

Directions for questions 9 to 13: These questions are based on the following bar graph.

Trade volumes with India by four countries from 2010 through 2014



9. From the year 2010 to 2011, the trade with India by which of the following countries showed the highest percentage increase?
(A) USA (B) Japan
(C) U.K (D) Germany

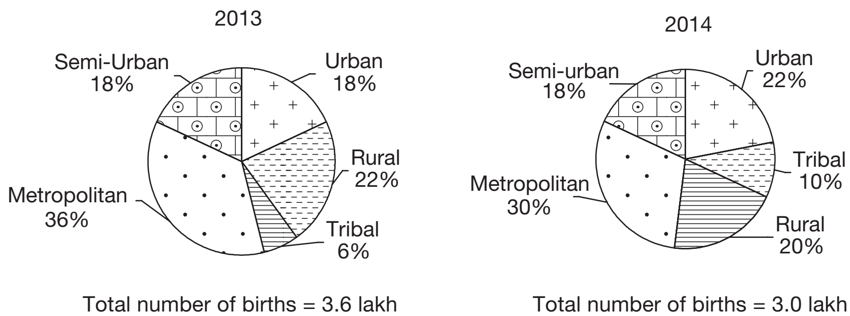
10. In the year 2013, the trade volume by the given countries constitutes 62.5% of the total trade volume of India. What is the trade volume (in ₹'00 crores) by all other countries with India in that year?
(A) 4590 (B) 5430
(C) 6140 (D) 7250

11. During which year is the total trade volume by the given countries with India the highest?
(A) 2010 (B) 2011
(C) 2012 (D) 2014

12. In how many of the given years is the trade volume of Netherlands with India greater than the average trade volume per year by the Netherlands in India in the given period?
(A) 1 (B) 2
(C) 3 (D) 4

13. For which country is the percentage increase in the trade volume with India in any year when compared to that in the previous year, the greatest?
(A) USA (B) Japan
(C) UK (D) Germany

Directions for questions 14 to 18: These questions are based on the following pie-charts which show the percentage distribution of births in different areas, during the two years 2013 and 2014.



14. What is the difference in the number of births in the year 2013 and 2014 in rural areas?
(A) 16,400 (B) 19,200
(C) 18,600 (D) 17,600

15. The percentage increase/decrease in the number of births in semi-urban areas from 2013 to 2014 is ______.
(A) 16.66% decrease
(B) 20% increase
(C) 16.66% increase
(D) 20% decrease

16. What is the ratio of the number of births in the metropolitans in 2013 to that in 2014?
(A) 36 : 25 (B) 15 : 17
(C) 3 : 7 (D) 7 : 3

17. The number of births in tribal areas in the year 2013 is what percentage of that in 2014?
(A) 56% (B) 64%
(C) 72% (D) 84%

18. If 18% of the children born in 2013 and 16% of the children born in 2014 lacked good medical facilities in the same year, then what is the total number of children (in lakhs) who doesn't lacked good medical facilities in these two years?
(A) 6.329 (B) 5.472
(C) 4.289 (D) 5.689

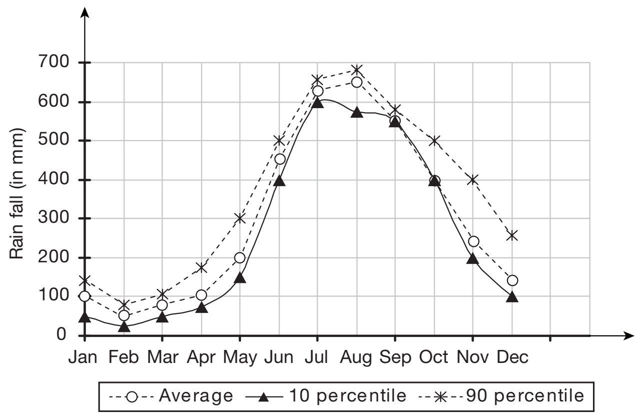
Directions for questions 19 to 20: Select the correct alternative from the given Choices.

19. A total of 600 persons participated in a survey. Each respondent was asked whether he/she owned a vehicle. Any respondent owning a vehicle was asked to mention whether he/she owned a two-wheeler or four wheeler or both.
The results of the survey are tabulated below.

		men	Women
Number of persons having own vehicle	Only Four-wheeler	80	68
	Only Two-wheeler	60	40
	Both	120	92
Number of persons not owning a vehicle		40	100

What percent of the respondents do not own a four wheeler?
(A) 60% (B) 40%
(C) 30% (D) 80%

20. The monthly rainfall chart for a certain city was prepared, based on 40 years of data. The graph below shows the x (x percentile means that for x% of the 40 years, the rainfall was less than the indicated value.



Which of following statements can be concluded?

(i) The average rainfall in December is less than that in June.

(ii) Each year, the amount of rainfall in July is more than that in February.

(iii) In May, there is at least 250mm of rainfall each year.

(iv) The rainfall in August can be estimated with better certainity than the rainfall in November.

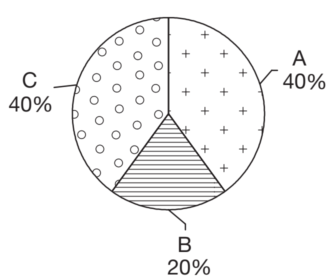
(A) (i) and (iii) (B) (i) and (iv)

(C) (ii) and (iv) (D) (ii) and (iii)

Directions for questions 21 and 22: Study the following table and pie chart carefully to the answer the questions.

Percentage of two wheelers of the total vehicles manufactured in a year by a certain company.

Type	Percentage of total vehicles	Out of which	
		100 cc	150 cc
Scooters (without gear)	25%	70%	30%
Scooter with gear	35%	40%	60%
Bikes	40%	65%	35%



The two wheelers are manufactured in three models as shown in the above pie chart.

Note: The above percentages mentioned in the table are applicable for all models. The total number of vehicles produced in that year is 70,000.

21. What is the total number of 150 cc bikes produced by the company in that year?

(A) 9000 (B) 9600

(C) 9500 (D) 9800

22. Which of the following numbers is the least?

(A) 100 cc bikes

(B) 150 cc bikes

(C) 100 cc scooters with gear

(D) 150 cc scooters without gear

Directions for questions 23 and 24: These questions are based on the following table which gives the percentage by weight of proteins, carbohydrates, minerals and fats in four formulations – *A*, *B*, *C* and *D*.

Formulation	Proteins	Carbohydrates	Minerals	Fats
A	10	20	25	45
B	25	35	20	20
C	30	10	40	20
D	15	50	30	5

	Proteins	Carbohydrates	Minerals	Fats
Cost (in ₹) per 10 g	4	3	2	1

23. What is the cost of a mixture containing 100 g each of *A*, *B*, *C* and *D*?

(A) ₹96.5 (B) ₹98

(C) ₹98.5 (D) ₹99

24. Which of the following would cost the least?

(A) 400 g of *B*

(B) 400 g of *C*

(C) 200 g of *A* and 200 g of *D*

(D) 300 g of *A* and 150 g of *D*

Directions for question 25: Select the correct alternative from the given choices.

25. The table below shows the test batting averages of 5 cricket players from 2010 to 2014. The test batting average of any batsman in any number of matches is his total score in those matches divided by the number of those matches.

Year	P	Q	R	S	T
2010	44	46	41	42	19
2011	55	52	44	48	22
2012	50	55	36	52	28
2013	46	51	40	46	34
2014	48	52	35	40	39

Who among the five players had the least percentage increase in the test batting average from 2010 to 2014?

(A) P (B) Q

(C) T (D) S

Directions for questions 26 and 27: These questions are based on the data given below.

A survey was conducted among 100 students in a hostel to find their favourite breakfast dish. Five students liked cutlet and sandwich only. 21 students liked omlette only. 25 students liked sandwich and 30 liked cutlet. There are 3 students who liked both sandwich and omlette. 48 students liked exactly one dish among the three. 2 students liked all the three dishes.

26. How many students like at least one dish?
(A) 69 (B) 100
(C) 2 (D) 21

27. How many students like cutlet or sandwich but not both?
(A) 48 (B) 41
(C) 31 (D) 62

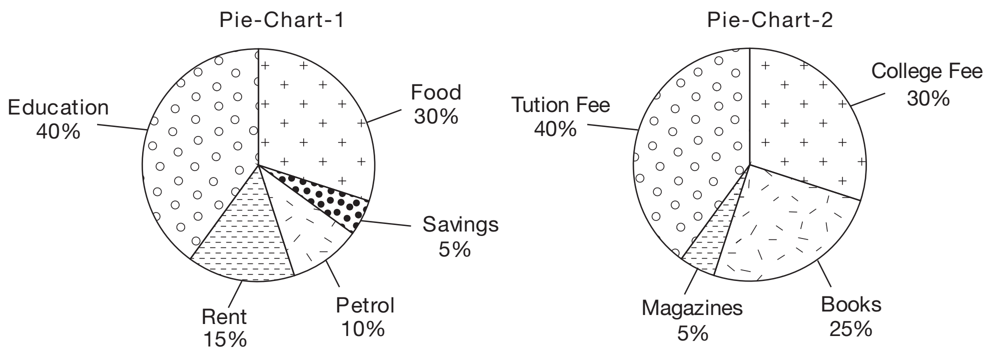
Directions for question 28: Select the correct alternative from the given choices.

28. Ten companies produce the same tool. Each of those companies rejects all the defective units of the tool produced. The table below gives the percentage of accepted units and the number of rejected units among all the units produced by the companies in month *M*.

Company	Percentage of accepted units	Number of rejected units
C1	80%	32
C2	90%	29
C3	85%	27
C4	81%	38
C5	76%	36
C6	77%	46
C7	84%	20
C8	88%	21
C9	93%	28
C10	95%	30

The company which produced the greatest total number of units in month *M* is
(A) *C*10 (B) *C*9
(C) *C*2 (D) *C*4

Directions for questions 29 and 30: These questions are based on the pie charts given below. Pie chart – 1 represents the distribution of income of Manohar and pie chart – 2 represents the split up of expenditure on education.



Note: Total income of Manohar is ₹15000

29. Which of the following statement/s is/are true?
(A) The expenditure on food is ₹2800 more than the expenditure towards college fee.
(B) The expenditure on education is ₹2,250 less than the remaining expenditure.
(C) The tuition fee is ₹2,460.
(D) Both (B) and (C).

30. The expenditure on books is how much more/less than the expenditure on food?
(A) ₹750 more (B) ₹750 less
(C) ₹3,000 more (D) ₹3,000 less

Answer Keys

1. B	**2.** D	**3.** C	**4.** A	**5.** A	**6.** D	**7.** B	**8.** C	**9.** D	**10.** A
11. D	**12.** C	**13.** D	**4.** B	**15.** A	**16.** A	**17.** C	**18.** B	**19.** B	**20.** B
21. D	**22.** D	**23.** C	**24.** C	**25.** A	**26.** A	**27.** B	**28.** A	**29.** B	**30.** D

Hints and Explanations

1. Given that 25% of the males are below average performers. Hence their number is 25% of 96 = 24.
As the total number of below average performers is 48 and 24 of them are males, remaining 24 are females.
Given, number of females = 2 (Number of male average performers)
Let the number of male average performers be x. Then, we get the following table.

	Below Average	Average	Above Average	Total
Males	24	x	72 – x	96
Females	24	30	2x – 54	2x
Total	48	30 + x	x + 18	2x + 96

Given, above average performers = $\frac{1}{3}$ (total number of employees)
i.e., $x + 18 = \frac{1}{3}(2x + 96)$
$\Rightarrow$ $3x + 54 = 2x + 96 \Rightarrow x = 42$
Number of above average performers = x + 18 i.e., 60. Choice (B)

2. Given that 25% of the males are below average performers. Hence their number is 25% of 96 = 24.
As the total number of below average performers is 48 and 24 of them are males, remaining 24 are females.
Given, number of females = 2 (Number of male average performers)
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Total	48	30 + x	x + 18	2x + 96

Given, above average performers = $\frac{1}{3}$ (total number of employees)
i.e., $x + 18 = \frac{1}{3}(2x + 96)$
$\Rightarrow$ $3x + 54 = 2x + 96$
$\Rightarrow$ $x = 42$
Number of females = $2x$
i.e., 2(42) = 84. Choice (D)

3. Given that 25% of the males are below average performers. Hence their number is 25% of 96 = 24.
As the total number of below average performers is 48 and 24 of them are males, remaining 24 are females.
Given, number of females = 2 (Number of male average performers)
Let the number of male average performers be x. Then, we get the following table.

	Below Average	Average	Above Average	Total
Males	24	x	72 – x	96
Females	24	30	2x – 54	2x
Total	48	30 + x	x + 18	2x + 96

Given, above average performers = $\frac{1}{3}$ (total number of employees)
i.e., $x + 18 = \frac{1}{3}(2x + 96)$
$\Rightarrow$ $3x + 54 = 2x + 96$
$\Rightarrow$ $x = 42$
Total number of average performers = x + 30 i.e., 72
Also as the number of males = 96,
the required difference is 96 – 72 = 24. Choice (C)

4. Given that 25% of the males are below average performers. Hence their number is 25% of 96 = 24.
As the total number of below average performers is 48 and 24 of them are males, remaining 24 are females.
Given, number of females = 2 (Number of male average performers)
Let the number of male average performers be x. Then, we get the following table.

	Below Average	Average	Above Average	Total
Males	24	x	72 – x	96
Females	24	30	2x – 54	2x
Total	48	30 + x	x + 18	2x + 96

Given, above average performers = $\frac{1}{3}$ (total number of employees)
i.e., $x + 18 = \frac{1}{3}(2x + 96)$
$\Rightarrow$ $3x + 54 = 2x + 96$
$\Rightarrow$ $x = 42$
Required ratio of above average performers : Number of females = $x + 30 : 2x$ i.e., 72 : 84 = 6 : 7. Choice (A)

5. Given profit of X in 2010-11 = ₹120 lakhs
Profit percentage of X in 2010-11 = 30%
As profit % = $\frac{\text{Profit}}{\text{Expenditure}} \times 100$
$30 = \frac{120\,\text{lakhs}}{\text{expenditure}} \times 100 =$ ₹400 lakhs.
$\therefore$ Expenditure of X in 2010–11 is ₹400 lakhs
Hence, its income in 2010–11 = ₹520 lakhs. Choice (A)

6. As the income of X in 2013–14 is not known, the required ratio cannot be determined. Choice (D)

7. Given, 20% of X; 10% of Y : 30% of Z = 4 : 2 : 3
i.e., $X : Y : Z = 20 : 20 : 10 = 2 : 2 : 1$. Choice (B)

8. Let the profit of Y and R in 2010-11 be ₹x. Then, using the graph, we have $\frac{x}{\exp Q} = 30$ and $\frac{x}{\exp R} = 40$

Let $\exp_Q = 400x$ and $\exp_R = 300x$

∴ $30(\exp_Q) = 40(\exp_R)$

⇒ Expenditures of X and Y are in the ratio 4 : 3.
Incomes ratio = $520x : 420x = 26 : 21$. Choice (C)

9. The percentage increase in the trade volume from 2010 to 2011

for USA = $\frac{354}{1604} \times 100 < 25\%$

for Japan = $\frac{711}{1871} \times 100 < 40\%$

for UK = $\frac{311}{1836} \times 100 < 20\%$

for Germany = $\frac{652}{1417} \times 100 > 40\%$

The percentage increase in trade volume is the greatest for Germany. Choice (D)

10. Total trade volume by the given countries in 2013
= 2210 + 1936 + 2340 + 1164 = 7650
Given that 62.5% of the total trade volume = 7650

Then, 37.5% of the total trade volume = $\frac{37.5}{62.5} \times 7650$

= $\frac{3}{5} \times 7650$ = ₹459,000 Crores. Choice (A)

11. By observation, we find that the total trade volume is the least in 2010 and 2013.
By comparing the total trade volume in 2011 and that in 2012, we find that the trade volume in 2012 is lower. Now, by comparing the total trade volume in 2011 and that in 2014 we find that the total trade volume in 2014 is the highest. Choice (D)

12. We can observe that the trade volume by the Netherlands in India is less than 2000 in one year and more than that in four years. By assuming the average to be 2000, we find the average of the deviations

$= \frac{-164+147+476+340+651}{5} = \frac{1450}{5} = 290$

The average = 2290. Three values are more than the average. Choice (C)

13. The trade volume from Germany registered more than 40% growth. By observation, we find that for no other country there is more than 40% growth in the trade volume in any two successive years. Choice (D)

14. No. of births in rural area in 2013 = 22% of 3.6 lakh = 79,200
No. of births in rural area in 2014 = 20% of 3 lakh = 60,000
Difference = 79,200 − 60,000 = 19,200. Choice (B)

15. The number of births in semi-urban area in 2013 = 18% of 3.6 lakh = 64,800
The number of births in semi-urban area in 2014 = 18% of 3 lakh = 54,000
Percentage decrease

$= \frac{64,800 - 54,000}{64,800} \times 100 = 16.66\%$. Choice (A)

16. The number of births in metropolitan area in 2013 = 36% of 3.6 lakh = 1,29,600
The number of births in metropolitan area in 2014 = 30% of 3 lakh = 90,000
Required ratio = 129600 : 90000 = 36 : 25. Choice (A)

17. The number of births in tribal area in 2013 = 6% of 3.6 lakh = 21,600
The number of births in tribal area in 2014 = 10% of 3 lakh = 30,000

∴ Required percentage $= \frac{21600}{30000} \times 100 = 72\%$. Choice (C)

18. The number of children lacking good medical facilities in 2013 = 82% of 3.6 lakh = 2,95,200
The number of children lacking good medical facilities in 2014 = 84% of 3 lakh = 2,52,000
∴ Total number of children who does not lack good medical facilities = 2,95,200 + 2,52,000 = 5,47,200 = 5.472 lakhs. Choice (B)

19. A total of 80 + 68 + 120 + 92, viz., 360 persons own a four wheeler.
∴ The remaining 60 + 40 + 40 + 100, viz., 240 of the respondents do not own a four wheeler, i.e., 40% of the respondents do not own a four-wheeler. Choice (B)

20. (i) The average rainfall in December is 150mm. The average rainfall in June is between 400mm and 500mm. ∴ (i) is true.
(ii) For each month, we know the average rainfall, the 10 percentile value (*i.e.*, the value below which 10 percent of the data falls) and the 90 percentile value. In a particular year, the rainfall in July may be less than that in February. (ii) cannot be concluded
(iii) This statement does not follow (From the explanation in (ii) above)
(iv) We see a narrow gap between any two of the three curves in August and a significant gap between any two of the three curves in November. ∴ (iv) follows. Only (i) and (iv) follow. Choice (B)

Solutions for questions 21 and 22:
Total number of vehicles produced = 70,000
Scooters without gear = 25% of 70,000.

$= 70,000 \times \frac{25}{100} = 17,500$

100 cc scooters without gear = $17,500 \times \frac{70}{100} = 12,250$

150 cc scooters without gear = 17,500 – 12,250 = 5,250

Number of scooters with gear = 35% of 70,000

i.e., $70,000 \times \frac{35}{100} = 24,500$

100 cc scooters with gear = $24500 \times \frac{40}{100} = 9,800$

150 cc scoters with gear = $24,500 \times \frac{60}{100} = 14,700$

Number of bikes = $70,000 \times \frac{40}{100} = 28,000$

100 cc bikes = $28,000 \times \frac{65}{100} = 18,200$

150 cc bikes = $28,000 \times \frac{35}{100} = 9,800$

21. Number of 150 cc bikes produced in that year = 9,800. Choice (D)

22. Of the four options the number of 150 cc scooters without gear is the least in number i.e., 5,250. Choice (D)

Solutions for questions 23 and 24:

The cost of 100 g of A = 4 (1) + 3 (2) + (in ₹) 2 (2.5) + (4.5) = 19.5

100 g of B (in ₹) = 26.5

100 g of C (in ₹) = 25

100 g of D (in ₹) = 27.5

23. The cost of the required mixture = 19.5 + 26.5 + 25 + 27.5 = ₹98.5. Choice (C)

24. Cost of 400 g of B = ₹106

400 g of C = ₹100

200 g of A and 200 g of D = ₹94

300 g of A and 150 g of D = ₹99.75. Choice (C)

25. Among the five players, only P, Q and T had an increase in the test batting average from 2010 to 2014. Percentage increase in the test batting average from 2010 to 2014 of

$$P = \frac{48-44}{44} \times 100\%$$

$$= \frac{100\%}{11} = 9\frac{1}{11}\%$$

$$Q = \frac{52-46}{46} \times 100\%$$

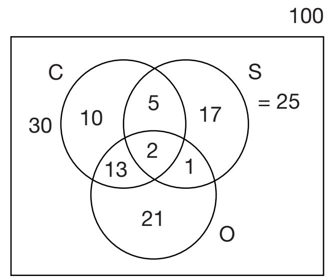
$$= \frac{300}{23}\% = 13\frac{1}{23}\%$$

$T = \frac{39-19}{19} \times 100\% = \frac{20}{19} \times 100\%$ which is > 100%.

P had the least percentage increase in the test batting average. Choice (A)

Solutions for questions 26 and 27:

From the given data we get the following diagram.



(i) Only cutlet = 10

(ii) Only omlette = 21

(iii) Only sandwich = 17

(iv) All the three = 2

(v) Exactly two dishes = 13 + 5 + 1 = 19

(vi) At least two dishes = 19 + 2 = 21

(vii) Cutlet = 10 + 5 + 2 + 13 = 30

(viii)Sandwich = 5 + 2 + 1 + 17 = 25

(ix) Omlette = 13 + 2 + 1 + 21 = 37

(x) None = 100 – (10 + 17 + 21 + 5 + 13 + 1 + 2) = 31

26. The number of students who like at least one dish = sum of the elements = 69. Choice (A)

27. The number of students who like cutlet or sandwich but not both is given by 10 + 13 + 17 + 1 = 41. Choice (B)

Solutions for question 28:

28. The number of units produced by C_1, C_2, C_3 …….C_{10}

Are $\frac{32\times100}{100-80}, \frac{29\times100}{100-90}, \frac{27\times100}{100-85}, \frac{38\times100}{100-81}, \frac{36\times100}{100-76}$

$\frac{46\times100}{100-77}, \frac{20\times100}{100-84}, \frac{21\times100}{100-88}, \frac{28\times100}{100-93}$, and

$\frac{30\times100}{100-95}$

Respectively i.e.160, 290, 180, 200, 150, 200, 125, 175, 400 and 600

∴ $C10$ produced the greatest total number of units in month M. Choice (A)

Solutions for questions 29 and 30:

29. Statement A is not true as (30% – 40% of 40%) of 15000 ≠ 2800

Statement B is true, since the expenditure on education = $\left(\frac{40}{100}\right)(15000) = 6000$.

The remaining expenditure = $\frac{(15+10+30)}{100}(15000)$ = 8250.

∴ Difference is ₹2250. Choice (B)

30. Expenditure on books = $15000\left(\frac{40}{100}\right)\left(\frac{25}{100}\right) = 1500$.

The expenditure on food = $\left(\frac{30}{100}\right)(15000) = 4500$.

Expenditure on books is less than the expenditure on food by 4500 – 1500 = ₹3000. Choice (D)

Quantitative Ability Test 5

(Permutations and Combinations)

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. In how many ways can five boys and five girls be arranged around a circular table such that on either side of every boy, a girl must sit?

(A) 2560 (B) 2880
(C) 120 (D) 1440

2. Twelve boys have to be seated in a row such that two particular boys occupy the middle two positions. In how many ways can they be seated?

(A) $\frac{12!}{2!}$ (B) 10! 2!
(C) 12! (D) $\frac{10!}{2!}$

3. How many five letter words can be formed using the letters of the word "QUESTION" so that the word contains 2 vowels and 3 consonants?

(A) 5670 (B) 120
(C) 2880 (D) 1440

4. There are seven pairs of shoes. In how many ways can one select 4 shoes from them such that no complete pair is included among them?

(A) 840 (B) 420
(C) 13440 (D) 560

5. There are 4 apples, 3 oranges and 6 mangoes in a basket. In how many ways can one select one or more fruits from the basket?

(A) 139 (B) 140
(C) 71 (D) 72

6. There are 10 questions in a paper each with four options, of which only one is correct. In how many ways can a student get exactly 7 questions correct given that he attempted all the questions?

(A) 2400 (B) 2880
(C) 3240 (D) 3600

7. A number is formed using the digits 5, 8, 1, 4 and 3. When we arrange the numbers in ascending order find the rank of the number 58413. (Each digit occurs at most once in each number)

(A) 228 (B) 290
(C) 299 (D) 300

8. A team of 11 is to be selected from two groups *A* and *B*, which consist of 10 and 8 persons respectively. In how many ways can this team be selected such that exactly five members are selected from the first eight persons of group *A*?

(A) 11,760 (B) 10,760
(C) 12,760 (D) 11,670

9. In how many arrangements of the word MATHEMATICS, the two *A*'s are separated?

(A) $\frac{10!}{2!2!2!}$ (B) $\frac{9!}{2!2!2!}$
(C) $9 \times 10!$ (D) $\frac{9 \times 10!}{2!2!2!}$

10. Find the number of ways of arranging the letters of the word CALENDAR in such a way that exactly two letters are present in between *L* and *D*?

(A) 2640 (B) 3600
(C) 2600 (D) 7200

11. There are 12 men and 7 women. In how many ways can a team of six members be formed such that there are atmost two women?

(A) 16,683 (B) 16,386
(C) 16,863 (D) 16,638

12. Three bags *X*, *Y* and *Z* contain six, five and four marbles respectively. A person has to choose 11 marbles at random. In how many ways can this be done such that at least 3 marbles are to be chosen from each bag.

(A) 860 (B) 840
(C) 960 (D) 870

13. In a factory there are three class I employees, two class II employees, three class III employees and four class IV employees. A team of five members is to be formed. In how many ways can this be done if the team must have at least one class I and at most two class IV employees?

(A) 590 (B) 491
(C) 600 (D) 591

14. There are two groups *X* and *Y* in a colony. *X* consists of five boys and four girls, *Y* consists of four boys and five girls. They plan an educational tour of four boys and four girls such that exactly four persons are selected from each of the two groups. In how many ways can this be done?

(A) 5626 (B) 5026
(C) 15876 (D) 43758

15. There are 20 players and 6 of them are from Hyderabad. In how many ways can a team of 12 players be formed so that exactly three persons of the team are from Hyderabad?

(A) 4004 (B) 20020
(C) 40040 (D) None of these

16. In how many ways can five men and five women be seated in a row, so that all men are sitting together and all women are sitting together?

(A) $(5!)^2$ (B) $(5!)^2 2!$

(C) $\frac{(5!)^2}{2!}$ (D) $(5!)^2 4!$

17. For a company board meeting, eight directors and a chairperson have to be seated around a circle. If two particular directors are seated on either sides of the chairperson, in how many ways can they be seated?

(A) 1440 (B) 1200
(C) 10080 (D) 1080

18. In how many ways can seven persons be selected from 14 persons such that two particular persons are selected and three other particular persons are not selected?

(A) 120 (B) 126
(C) 300 (D) 240

19. There are six different consonants and three different vowels of the English alphabet. The number of words that can be formed using them such that each word contains two vowels and three consonants is

(A) 3600 (B) 1800
(C) 2400 (D) 7200

20. Find the number of sides of a regular polygon in which the number of diagonals is equal to one and half times the number of its sides.

(A) 5 (B) 6
(C) 8 (D) 10

21. Find the number of ways of arranging the letters of the word RAINBOW such that the vowels occupy odd places.

(A) 7! (B) 720
(C) 576 (D) 120

22. On Sports Day in a school, some competitions are held. Every student has to play exactly one game with every other student. It was found that in 36 games both players were girls and in 126 games one player was a boy and the other was a girl. Find the number of games played in which both players were boys.

(A) 56 (B) 72
(C) 91 (D) 45

23. A box contains coins of denominations 50 paisa, ₹1, ₹2 and ₹5. Given that there are unlimited coins of each type, find the number of ways of selecting the coins so that any such selection gives a total amount of ₹10.

(A) 49 (B) 12
(C) 48 (D) 46

24. Find the number of ways in which the letters of the word THURSDAY can be arranged such that no word starts with *T* or ends with *Y*.

(A) 9360 (B) 31680
(C) 29520 (D) 30960

25. How many natural numbers are there from 10000 to 1000000 for which the sum of the digits is 3?

(A) 16 (B) 36
(C) 27 (D) 35

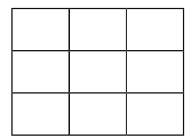
26. The number of ways of posting seven different letters into three post boxes so that at least one letter is posted in each post box is

(A) 1806 (B) 1803
(C) 2184 (D) 2059

27. A palindrome is a word, which spells the same when read from left to right or from right to left. How many palindromes of length 8 can be formed using the operative symbols +, –, ÷ and ×?

(A) 8! (B) 2^{16}
(C) 256 (D) 243

28. Six identical balls have to be placed in the square cells of the given figure such that each row contains at least one ball. In how many ways can this be done? (Given that each square can take at the most one ball)



(A) 84 (B) 76
(C) 77 (D) 81

29. The number of two digit codes that can be formed using the digits 0 to 9 with '0' not taking the tens place and an odd number taking the units place is

(A) 40 (B) 50
(C) 45 (D) 36

30. There are 5 balls of different colours and 5 boxes of colours the same as those of the balls. The number of ways in which the balls, one in each box can be placed such that a ball does not go to a box of its own colour is

(A) 40 (B) 44
(C) 42 (D) 36

31. A question paper consists of 5 problems, each problem having 3 internal choices. In how many ways can a candidate attempt one or more problems?

(A) 63 (B) 511
(C) 1023 (D) 15

32. Six points are marked on a straight line and five points marked on another line which is parallel to the first line. How many straight lines, including the first two, can be formed with these points?

(A) 29 (B) 33
(C) 55 (D) 32

33. Sixteen guests have to be seated around two circular tables, each accommodating 8 members. 3 particular guests desire to sit at one particular table and 4 others at the other table. The number of ways of arranging these guests is

(A) ${}^9C_5$ (B) $\frac{9!(7!)}{4!5!}$

(C) $\frac{9!(7!)^2}{4!5!}$ (D) $(7!)^2$

34. In how many ways can one or more of 5 letters be posted into 4 mail boxes, if any letter can be posted into any of the boxes?
(A) 5^4 (B) 4^5
(C) $5^5 - 1$ (D) $4^5 - 1$

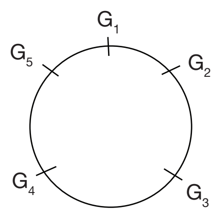
35. The number of non negative integral solutions to the equation $a + b + c = 14$ is
(A) 78 (B) 45
(C) 120 (D) 110

Answer Keys

1. B	**2.** B	**3.** C	**4.** D	**5.** A	**6.** C	**7.** D	**8.** A	**9.** D	**10.** B
11. C	**12.** D	**13.** D	**14.** A	**15.** C	**16.** B	**17.** A	**18.** B	**19.** D	**20.** B
21. C	**22.** C	**23.** A	**24.** D	**25.** B	**26.** A	**27.** C	**28.** D	**29.** C	**30.** B
31. C	**32.** D	**33.** C	**34.** C	**35.** C					

Hints and Explanations

1. First arrange the girls in the circle as shown in the diagram. They can be arranged in (5 – 1)! ways.



Then there are 5 gaps and the 5 boys can be seated in these gaps in 5! Ways.
∴ The total number of arrangements is $4! \times 5! = 2880$. Choice (B)

2. There are two middle positions (6th and 7th). The two particular boys can occupy the middle two positions in 2! ways. The remaining 10 boys can be seated in 10! ways.
∴ The total number of ways = (10!) (2!). Choice (B)

3. The word QUESTION, contains 4 vowels and 4 consonants; 2 vowels and 3 consonants can be selected from 4 vowels and 4 consonants in ${}^4C_2 \cdot {}^4C_3$ ways. $= 6 \times 4 = 24$
Again these 5 letters can be rearranged among themselves in 5! ways.
∴ The total number of 5 letter words formed = $5! \times 24 = 120 \times 24 = 2880$. Choice (C)

4. Since there are 7 pairs of shoes and 4 shoes are to be selected without any complete pair being included in it, the 4 shoes must be from 4 different pairs.
This can be done in ${}^7C_4$ ways (choosing 4 pairs from the 7 pairs)
Now from each of these 4 pairs, one can select a shoe in 2 ways
(*i.e.*, any one of the two shoes present in each pair)
Therefore the total number of ways in which one can select 4 shoes from 7 pairs of shoes such that no complete pair is included among them = ${}^7C_4(2)(2)(2)(2) = 35(16) = 560$ ways. Choice (D)

5. We can choose the apples in 5 ways i.e., either choosing 0 apples or 1 apple or 2 apples or 3 apples or 4 apples.
Similarly the oranges can be chosen in 4 ways and the mangoes can be chosen in 7 ways.
Therefore the total number of ways in which one can select one or more fruits from 4 apples, 3 oranges and 6 mangoes = $5 \times 4 \times 7 - 1 = 139$ ways.
(we subtract 1 for the case where we select 0 apples, 0 oranges and 0 mangoes i.e., no fruit at all). Choice (A)

6. In each question, there are 3 wrong options and exactly one correct option.
The number of ways in which a student gets exactly 7 questions correct = ${}^{10}C_7(3)^3$
${}^{10}C_7$ gives us the number of ways in which the student selects seven questions (in which the student marks correct option) from the given 10 questions.
3^3 gives us the number of ways in which the student answers the 3 questions which he gets wrong.
Therefore the required number of ways = 3240.
Choice (C)

7. The number of single digit numbers = 5.
Given digits are {5, 8, 1, 3, 7}.
The number of two digit numbers formed with the digits is ${}^5P_2 = 20$. The number of three digit numbers formed with the digits is ${}^5P_2 = 60$.
The number of four digit numbers formed with digits is ${}^5P_4 = 120$.
The number of five digit numbers formed which begin with either 1 or 3 or 4 is 4!
The number of five digit numbers formed which begin with 51 or 53 or 54 is 3!
The number of 5 digit numbers which begin with 581 or 583 is 21.
The next number 58413
The rank of the number 58413 is 5 + 20 + 60 + 120 + 3 (24) + 3 (6) + 2(2) + 1 = 300. Choice (D)

8. In group *A*, there are 10 persons.
From first 8 persons 5 can be selected in ${}^8C_5$ ways and remaining 6 must be selected from the remaining 2

persons of group A and 8 persons from group B.
This can be done in $^{10}C_6$.

$\therefore$ The required number of ways = $^8C_5\ ^{10}C_6$ = 11760. Choice (A)

9. The total number of words that can be formed is $\frac{11!}{2!2!2!}$

Number of arrangements in which the 2 A's are together $= \frac{10!}{2!2!}$

Total number of arrangements in which the A's are separated = Total number of words – number of words, in which the two A's together.

$= \frac{10!}{2!2!}\left(\frac{11}{2}-1\right) = \frac{9(10!)}{2!2!2!}$ Choice (D)

10. C A L E N D AR
$\overline{1}\,\overline{2}\,\overline{3}\,\overline{4}\,\overline{5}\,\overline{6}\,\overline{7}\,\overline{8}$

There are 5 positions to fix the L and D i.e., (1, 4), (2, 5), (3, 6), (4, 7) and (5, 8) and L and D can be interchanged. The remaining 6 letters can be arranged in $\frac{6!}{2!}$ ways.

$\therefore$ Required number of ways = $\frac{6!}{2!} \times 5 \times 2 = 360 \times 10$
= 3600. Choice (B)

11. The following table gives the complete possibilities of selecting the team as per the given conditions

Men (12)	Women (7)	Number of ways
4	2	$^{12}C_4 \times {}^7C_2$
5	1	$^{12}C_5 \times {}^7C_1$
6	–	$^{12}C_6$

Total number of ways of selecting the team
$= {}^{12}C_4\ {}^7C_2 + {}^{12}C_5\ {}^7C_1 + {}^{12}C_6$
= 10395 + 5544 + 924 = 16863. Choice (C)

12. The following table will give the number of marbles selected from each bag and the number of ways of selecting them.

X(6)	Y(5)	Z(4)	Number of ways of selecting them
3	4	4	$^6C_3\ ^5C_4\ ^4C_4 = 100$
3	5	5	$^6C_3\ ^5C_5\ ^4C_3 = 80$
4	3	4	$^6C_4\ ^5C_3\ ^4C_4 = 150$
4	4	3	$^6C_4\ ^5C_4\ ^4C_3 = 300$
5	3	3	$^6C_5\ ^5C_3\ ^4C_3 = 240$
			Total number of ways = 870

Choice (D)

13. The following table gives the number of persons of each category and the number of ways of selecting them as per the given conditions

Class I (3) (3)	Class II + Class III (5)	Class IV (4)	Number of ways of selecting them
1	4	0	$^3C_1\ ^5C_4\ ^4C_0 = 15$
1	3	1	$^3C_1\ ^5C_3\ ^4C_1 = 120$
1	2	2	$^3C_1\ ^5C_2\ ^4C_2 = 180$
2	3	0	$^3C_2\ ^5C_3\ ^4C_0 = 30$
2	2	1	$^3C_2\ ^5C_2\ ^4C_1 = 120$
2	1	2	$^3C_2\ ^5C_1\ ^4C_2 = 90$
3	2	0	$^3C_3\ ^5C_2\ ^4C_0 = 10$
3	1	1	$^3C_3\ ^5C_1\ ^4C_1 = 20$
3	0	2	$^3C_3\ ^5C_0\ ^4C_2 = 6$
			Total = 591

Choice (D)

14. The following table shows the number of persons and the number of ways of selecting them as per the given conditions.

Boys		Girls		Number of ways of selecting them
X(5)	Y(4)	X(4)	Y(5)	
0	4	4	0	$^5C_0{}^4C_4\ ^4C_0\ ^5C_0$ (or) = 1
1	3	3	1	$^5C_1\ ^4C_3\ ^4C_3\ ^5C_1$ (or) = 400
2	2	2	2	$^5C_2\ ^4C_2\ ^4C_2\ ^5C_2$ (or) = 3600
3	1	1	3	$^5C_3\ ^4C_1\ ^4C_1\ ^5C_3$ (or) = 1600
4	0	0	4	$^5C_4\ ^4C_0\ ^4C_0\ ^5C_1$ (or) = 25
				Total = 5626

Choice (A)

15. There are 20 players, of them 6 are from Hyderabad. Exactly 3 players can be selected from 6 players in $^6C_3$ ways and the remaining 9 players are to be selected from the remaining 14 players.
This can be done in $^{14}C_9$ ways.

$\therefore$ Total number of ways of selecting the team
$= {}^6C_3\ {}^{14}C_9 = 40040$. Choice (C)

16. There are 5 men and 5 women.
As all men and all women are to sit together, treat all men as one unit and all women as one unit.
The two units can be arranged in 2! ways.
But five men and five women can arrange among themselves in 5! 5! ways.

$\therefore$ Total number of ways they can sit is given by $(5!)^2\ 2!$. Choice (B)

17. As two particular directors are to sit on either sides of the chairperson we treat these three as one unit. The remaining six directors and this one unit of three persons can sit around a circular table in (7 – 1)! = 6! ways.

But the two directors sitting on either side of the chairperson can arrange themselves in 2! ways.

$\therefore$ The required number of arrangements possible
= 6! 2! = 1440 ways. Choice (A)

18. Total there are 14 persons, of them two particular persons are always selected.
We need to select only 5 persons from 14 – 2 = 12 persons. As three other particular persons are not to be selected, keeping them away there are only nine persons left from which we have to select any five persons.
This is possible in ${}^9C_5 = 126$ ways. Choice (B)

19. There are 6 consonants , of them 3 can be selected in ${}^6C_3$ ways. Similarly, 2 vowels can be selected in ${}^3C_2$ ways.
These 3 consonants and 2 vowels can be arranged in 5! Ways.
$\therefore$ Total number of words that can be formed
$= {}^6C_3 \times {}^3C_2 \times 5! = 7200$ Choice (D)

20. The number of diagonals of a regular polygon of n sides $= \dfrac{n(n-3)}{2}$

It is given that, $\dfrac{n(n-3)}{2} = \dfrac{3}{2}n \Rightarrow n = 6.$ Choice (B)

21. There are seven letters in the word RAINBOW, of them three are vowels and four are consonants.
There are four odd places, three vowels can occupy four places in ${}^4P_3$ ways and the remaining 4 places can be occupied by the remaining 4 consonants in 4! ways.
$\therefore$ Total number of arrangements possible in given by ${}^4P_3 \times 4! = 576$ Choice (C)

22. Let m boys and n girls participate in the competitions.
Given the number of games in which both players are girls = 36

$\Rightarrow {}^nC_2 = 36$

$\Rightarrow \dfrac{n(n-1)}{2} = 36 \Rightarrow n = 9$

$\therefore$ Number of girls = n = 9
The number of games in which one boy and one girl participated = 126
${}^mC_1\,{}^nC_1 = 126$
$m(9) = 126 \Rightarrow m = 14$
Number of boys, $m = 14$
Number of games, in which both players were boys

$= {}^mC_2 = {}^{14}C_2 = \dfrac{14(13)}{1(2)} = 91.$ Choice (C)

23. The number of ₹5 coins that can be selected is 0, 1 or 2.
Let x, y and z represent the number of 50 paise, ₹1 and ₹2 coins respectively to be selected.
Case 1 : When the number of ₹5 coins selected is zero, then

$\dfrac{x}{2} + y + 2z = 10$ i.e., $x + 2y + 4z = 20$

If $z = 0$, then $y = 0, 1, 2, \ldots 10$; the number of solutions = 11
$z = 1$, then $y = 0, 1, 2, .. 8$; the number of solutions = 9
- -
- -
$z = 5$, then $y = 0$ and $x = 0$; the number of solutions = 1
$\therefore$ The number of possible selections in this case
= 1 + 3 + 5 + 7 + 9 + 11 = 36.
Similarly, we can show that, the number of selections when one 5 rupee coin is selected = 12
The number of selections when two 5 rupee coins are selected = 1
Hence, the total number of possible selections
= 36 + 12 + 1 = 49. Choice (A)

24. The number of words that start with T and end with Y = 6!
Keeping T and Y fixed in the first and the last places, the remaining 6 letters can be arranged in the remaining 6 places in 6! or 720 ways.
The number of words starting with T = 7!
Keeping T fixed in the first place, the remaining seven letters can be arranged in the remaining seven places in 7! or 5040 ways.
$\therefore$ (5040 – 720) words start with T but do not end with Y. Similarly (5040 – 720) words end with Y but do not start with T. The total number of words that can be formed using all the letters in the given word = 8!
Therefore, the number of words that do not start with T or end with Y = 8! – 4320 – 4320 – 720 = 30960. Choice (D)

25. It could be five–a digit or a six–digit number.
Case I: It is a five digit number.

Digits used (in that order)		No of values
3, 0	3 – – – –	1
2, 1	2 – – – –	${}^4C_1 = 4$
1, 2	1 – – – –	${}^4C_1 = 4$
1, 1, 1	1 – – – –	${}^4C_2 = 6$
	Total	15

Case II: It is a six–digit number

Digits used		No. of values
3, 0	3 – – – – –	1
2, 1	2 – – – – –	${}^5C_1 = 5$
1, 2	1 – – – – –	${}^5C_1 = 5$
1, 1, 1	1 – – – – –	${}^5C_2 = 10$
	Total	21

Therefore the total number of natural numbers from 10000 to 1000000 such that the sum of the digits is 3 = 15 + 21 = 36. Choice (B)

26. The possible combinations, internal arrangements and total number of ways in case of each combination are listed below.

Combinations	Internal arrangements	Number of ways
1, 1, 5	$\frac{3!}{2!}=3$	$3\times\frac{7!}{5!}=126$
1, 2, 4	3! = 6	$6\times\frac{7!}{4!\,2!}=630$
1, 3, 3	$\frac{3!}{2!}=3$	$3\times\frac{7!}{3!\,3!}=420$
2, 2, 3	$\frac{3!}{2!}=3$	$3\times3\times\frac{7!}{2!\,2!\,3!}=630$
	Total	1806

Choice (A)

27. The first and eighth places must be filled with the same symbol. This can be done in 4 ways.
Similarly the second and seventh place must be filled with same symbol. This can be done in 4 ways.

1st	2nd	3rd	4th	5th	6th	7th	8th

Similarly, the other entries can be filled.
$\therefore$ The number of ways for forming a palindrome of length 8 symbols is $4^4 = 256$. Choice (C)

28. There are 9 boxes in total, so the 6 identical balls can be placed in these boxes in ${}^9C_6$ ways. These include 3 ways in which the balls are placed in exactly two rows.
$\therefore$ Required number of ways = ${}^9C_6 - 3$
= 84 – 3 = 81. Choice (D)

29. Units place can be filled with 1, 3, 5, 7 or 9 and the tens can be filled with any of the 9 non-zero digits.
$\therefore$ The number of two digit codes that can be formed = 9 × 5 = 45. Choice (C)

30. $\therefore$ Required number of ways

$$= n!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+----+(-1)^n\,\frac{1}{n!}\right)$$

$$= 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

$$= 5!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

= 60 – 20 + 5 – 1 = 44. Choice (B)

31. Given that, the question paper consists of 5 problems. For each problem, one or two or three or none of the choices can be attempted.
$\therefore$ Hence, the required number of ways = 4^5 - 1.
= 2^{10} - 1= 1024 - 1 = 1023.

Choice (C)

32. We know that, the number of straight lines that can be formed by the '*n*' points in which *r* points are collinear and no other set of three points, except those that can be selected out of these *r* points are collinear) is ${}^nC_2$ - ${}^rC_2$ +1.
$\therefore$ Hence, the required number of straight lines
= ${}^{11}C_2$ - ${}^6C_2$ – ${}^5C_2$ + 1 + 1
= 55 - 15 - 10 + 2 = 32.

Choice (D)

33. After arranging 3 and 4 particular guests, the remaining number of people is 9.
To arrange on first table we require 5 members. They can be selected in ${}^9C_5$ ways.
To arrange on the second table, we require 4 members. They can be selected in ${}^4C_4$ ways.
$\therefore$ Hence, required arrangements is = ${}^9C_5$ (7!) (7!)
= ${}^9C_5$ $(7!)^2$.

Choice (C)

34. Let the letters be L_1, L_2, L_3, L_4 and L_5 and the mail boxes be B_1, B_2, B_3, B_4. Now L_1 can be dealt in 5 ways i.e., either post it into B_1 or B_2 or B_3 or B_4 or do not post it at all (since one or more letters have to be posted, there is a possibility of not posting L_1 at all). Similarly each of L_2, L_3, L_4 and L_5 can be dealt in 5 ways, giving us a total of 5^5 possibilities which includes the case of not posting any of the letters, which has to be ruled out. Hence the required ways are 5^5 –1.

Choice (C)

35. We know that, the number of non negative integral solutions of $a_1 + a_2 + a_3$+-------+ $a_r = n$ is ${}^{n+r-1}C_{r-1}$ here $n = 14$ $r = 3$
$\therefore$ Required answer is (14 + 3 – 1) C_{3-1} = ${}^{16}C_2$ = 120.
Choice (C)

Quantitative Aptitude Test 6

Number of Questions: 35 **Section Marks: 30**

(ERPV, Numbers)

Directions for questions 1 to 6: Select the correct alternative from the given choices.

1. A three-digit number when reversed and subtracted from the original number gives the result as 792. If the sum of the digits of the number is 18, find the tens digit of the number.

(A) 6 (B) 7
(C) 5 (D) 8

2. A sum of ₹209 was intended to be divided among *A*, *B* and *C* in the ratio 2 : 4 : 5. By mistake, it got divided in the reciprocal of the intended ratio. Find the gain of *A* due to this mistake.

(A) ₹48 (B) ₹72
(C) ₹56 (D) ₹60

3. In a fraction, the numerator exceeds the denominator by 7. If twice the numerator exceeds thrice the denominator by 2, find the fraction.

(A) $\frac{8}{15}$ (B) $\frac{19}{12}$
(C) $\frac{20}{3}$ (D) $\frac{17}{24}$

4. If $a : b = 3 : 4$, $c : b = 2 : 7$ and $c : d = 5 : 7$, find $a : b : c : d$.

(A) 105 : 140 : 50 : 70 (B) 105 : 140 : 40 : 49
(C) 105 : 140 : 40 : 56 (D) 105 : 140 : 50 : 49

5. In a parking place, there are a total of 20 vehicles (bikes and cars) are parked. If the total number of tyres of all these vehicles is 70, find the ratio of the number of bikes and cars. (No vehicle has spare tyres).

(A) 3 : 1 (B) 1 : 2
(C) 1 : 3 (D) 2 : 1

6. Ajay, Balu, Chetan and Dinesh have a total of ₹240 with them. The amount with Ajay is half of the total amount with the others. The amount with Balu is one-fourth of the total amount with the others. The amount with Chetan is one-fifth of the total amount with the others. Find the amount with Dinesh.

(A) ₹30 (B) ₹72
(C) ₹40 (D) ₹48

Directions for questions 7 and 8: These questions are based on the following information.

Siddharth has two landlines of a particular telephone service provider. For each line,he has to pay a monthly rent of ₹250. This entitles him to a certain fixed number of free calls per line. Calls over and above this allowance are charged at the rate of ₹1 per call. One month he received bills of ₹450 and ₹350 for the two lines. Had he made all his calls on a single line, his bill for that line would have been ₹700.

7. What was the total number of calls on the first line?

(A) 375 (B) 300
(C) 350 (D) 400

8. What is the number of free calls per line?

(A) 150 (B) 125
(C) 180 (D) 200

Directions for questions 9 to 35: Select the correct alternative from the given choices.

9. The cost of 2 pens, 4 erasers and 5 sharpeners is ₹36. The cost of 3 pens,7 erasers and 9 sharpeners is ₹63. Find the total cost of one pen, one eraser and one sharpener.

(A) ₹9 (B) ₹8
(C) ₹10 (D) ₹11

10. If $a + b - c : b + c - a : a + c - b = 3 : 4 : 5$, find $a : b : c$.

(A) 1 : 2 : 3 (B) 8 : 7 : 9
(C) 5 : 7 : 8 (D) 1 : 3 : 2

11. For which of the following values of *k* does the system of equations $2x + 5y = 1$ and $6x + 15y = k/2$ have infinite solutions?

(A) 6 (B) 3
(C) –6 (D) Any value except 6

12. If $a : b = 3 : 4$, find the value of $\frac{3a^2 + 4b^2}{4a^2 + 3b^2}$.

(A) $\frac{11}{12}$ (B) $\frac{12}{11}$
(C) $\frac{12}{13}$ (D) $\frac{13}{12}$

13. The distance a stone falls under free fall varies directly with the square of the time for which it falls. If a stone falls at a distance of 35 m in the fourth second, find the total distance it falls in the first 5 seconds.

(A) 100 m (B) 140 m
(C) 135 m (D) 125 m

14. A sum of ₹750 is divided among *P*, *Q* and *R*. If ₹30, ₹20 and ₹10 is added to their respective shares then the ratio of amounts of *P*, *Q* and *R* becomes 10 : 8 : 9. What is the share of *P*?

(A) 280 (B) ₹250
(C) ₹260 (D) ₹270

15. Ajay told Bharat, "I am four times as old as you were when I was as old as you are". Bharat told Ajay "Ten

years ago, I was nine years younger to you". Find the sum of present ages of Ajay and Bharat.

(A) 39 years (B) 36 years
(C) 42 years (D) 45 years

16. The ratio of the incomes of A and B is 3 : 2. The ratio of their expenditures is 5 : 4. If the savings of A is twice that of B, find the ratio of the income and expenditure of B.

(A) 3 : 16 (B) 3 : 4
(C) 3 : 2 (D) 5 : 4

17. A bag has a total of 40 coins in denominations of ₹1, ₹2 and ₹5. If the total value of the coins is ₹130, find the maximum number of ₹5 coins.

(A) 24 (B) 21
(C) 23 (D) 22

18. The age of Harish is 8 years more than twice the age of Ganesh. After how many years will the age of Harish be twice the age of Ganesh?

(A) 4
(B) 6
(C) 8
(D) Cannot be determined

19. Rohan made 13 calls from a public booth. Each was either a local call or STD call or ISD call. The average costs of his local calls, STD calls and ISD calls were ₹6, ₹11 and ₹13, respectively. The total amount spent by Ram on the calls was ₹119. Find the number of ISD calls he made

(A) 3 (B) 4
(C) 2 (D) 5

20. Ram has a certain number of notes of the denominations ₹5, ₹10 and ₹20. The total amount he has is ₹540. If he has a total of 24 currency notes in the denominations of ₹5 and ₹20, then what is the greatest number of notes of ₹10 notes he could have?

(A) 42 (B) 40
(C) 39 (D) 38

21. Three vessels are filled to their capacities with mixtures of milk and water. The ratioof their capacities is 2:3:4. The ratio of the quantities of milk and water in the first,the second and the third vessels is 1:3, 5:1 and 3:5 respectively. Find the ratio of the total quantity of milk in the vessels to the total quantity of water in the vessels.

(A) 1 : 1 (B) 2 : 3
(C) 3 : 2 (D) 3 : 4

22. a, b, c are positive integers such that $a : b = 4 : 3$ and $b : c = 4 : 3$. If the sum of the squares of c and b is less than square of the sum of b and a by 2236, then what is the value of the number which is neither the greatest nor the least?

(A) 24 (B) 32
(C) 18 (D) 36

23. What is the least value of x, if the nine-digit number $23x4567y4$ is divisible by 44?

(A) 1 (B) 0
(C) 5 (D) 7

24. A number when divided by 259 leaves a remainder of 161. If one-seventh of the same number is divided by 37, the remainder will be

(A) 14
(B) 23
(C) 7
(D) Cannot be determined

25. $(36^3 + 36^2 - 37)$ is not divisible by, which of the following?

(A) 185
(B) 37
(C) 36
(D) More than one of the above

26. Mohit has 290 toffees with him. He distributes all his toffees among his friends, such that each of his friends gets a different number of toffees, which is at least 5 and at most 29. What could be the least number of friends?

(A) 19 (B) 20
(C) 13 (D) 14

27. When the King of Patiala distributed 33,274 gold coins equally among his subjects, the Maharaja of Mysore also distributed 30,905 gold coins equally to his subjects, each giving the same number of coins to his subjects. Surprisingly, both the kings were left with the same number of gold coins.If the number of coins that each subject received is a 2-digit number, what is the difference in the number of subjects of the two kings?

(A) 309
(B) 103
(C) 23
(D) Cannot be determined

28. The units digit of $(2^{4n})(6^{7n}) + (5^{3n})(7^{9n})$, (where n is a natural number) is

(A) 3
(B) 1
(C) 5
(D) Cannot be determined

29. Three bells X, Y and Z ring at regular intervals and ring simultaneously 24 times in a day. If Y rings less frequently than X but more frequently than Z, what could be the minimum number of times for which Y rings in a day?

(A) 48 (B) 36
(C) 54 (D) 72

30. If a, b and c are three consecutive positive integers, then which of the following is not necessarily true?

I. $(4a + 5b + 3c)$ is an odd number.
II. $(2a + 3b + 4c)$ is an even number.
III. $a^2 b^3 c^4$ is an even number.

IV. $(3a + 2b + 4c)$ is an odd number.
(A) Only I (B) Only II and III
(C) Only III and IV (D) Only II and IV

31. Some students have to be seated in some rows, such that equal number of students is seated in each row. If there are 13 rows 3 students will be left out and if there are 21 rows, 11 students will be left out. How many students will be left out if there are 19 rows and less than 300 students?
(A) 7 (B) 9
(C) 11 (D) 16

32. After the division of a number successively by 4, 5 and 3 the remainders obtained are 1, 2 and 2 respectively. What will be the remainder if the least of such numbers is divided by 37?
(A) 12 (B) 14
(C) 17 (D) 31

33. The sum and the difference of the LCM and the HCF of two numbers is 784 and 756 respectively. What is the least possible difference between the two numbers?
(A) 112 (B) 70
(C) 84 (D) 98

34. A number when divided by 30 and 36 leaves a remainder of 15 and 21 respectively. When such a number is divided by 35, it leaves a remainder of 10. What is such least number?
(A) 885 (B) 895
(C) 985 (D) 15

35. What will be the remainder when $223 \times 226 \times 228$ is divided by 11?
(A) 10 (B) 7
(C) 6 (D) 1

Answer Key

1. D	**2.** B	**3.** B	**4.** C	**5.** C	**6.** B	**7.** C	**8.** A	**9.** A	**10.** B
11. A	**12.** D	**13.** D	**14.** D	**15.** A	**16.** C	**17.** D	**18.** C	**19.** A	**20.** C
21. A	**22.** A	**23.** A	**24.** B	**25.** C	**26.** C	**27.** B	**28.** B	**29.** A	**30.** D
31. D	**32.** A	**33.** C	**34.** A	**35.** D					

Hints and Explanations

1. Let the three-digit number be *abc*.
Its value is $1000a + 10b + c$.
On reversing it becomes *cba* whose values are
$100c + 10b + a$
$abc - cba = 99(a - c) = 792$
$\Rightarrow$ $a - c = 8$
(a, c) can be (9, 1) or (8, 0)
$a + b + c = 18$
when $a = 9$, $c = 1$, then $b = 8$
when $a = 8$, $c = 0$, then
$b = 10$, which is not possible.
Hence $b = 8$
Choice (D)

2. A should have got 2 out of the total 11 parts i.e., 2/11 (209) = ₹38
Ratio of actual division of ₹209 among A, B and C is 1/2 : 1/4 : 1/5 = 10 : 5 : 4
A actually got 10 out of 19 parts i.e.,
10/19(209) = ₹110
Gain of A is 110 – 38 = ₹72. Choice (B)

3. Let the numerator and denominator of the fraction be n and d.
$2n - 3d = 2$ ….. (1)
$n - d = 7$ ….. (2)
Solving the two equations we get $n = 19$ and $d = 12$.
Choice (B)

4. $a = \frac{3}{4}b, b = \frac{7}{2}c, c = \frac{5}{7}d$

Hence $a = \frac{3}{4}\left(\frac{7}{2}\left(\frac{5}{7}d\right)\right) = \frac{15}{8}d$

$b = \frac{7}{2}\left(\frac{5}{7}\right)d = \frac{5}{2}d$

Hence $a : b : c : d = \frac{15}{8}d : \frac{5}{2}d : \frac{5}{7}d : d$

Multiplying each term of the ratio by LCM of denominators of the coefficients of d i.e. 56,
We got, $a : b : c : d = 105 : 140 : 40 : 56$
Alternate method:
Going by the choices, $a : b = 3 : 4$, $b : c = 7 : 2$ and $c : d = 5 : 7$ is satisfied only in Choice (C).
Choice (C)

5. Let the number of bikes sand cars parked be S and C respectively.
$S + C = 20$ …… (1)
Each bike has 2 tyres and each car has 4 tyres.
Total number of tyres = $2S + 4C = 70$
$\Rightarrow$ $S + 2C = 35$ ……. (2)
Subtracting the second equation from the first equation, we get $C = 15$,
so $S = 20 - C = 5$ $S : C = 1 : 3$ Choice (C)

6. Let the amounts with Ajay, Balu, Chetan and Dinesh be a, b, c and d respectively.

$a + b + c + d = 240$ $a = \frac{1}{2}$ $(b + c + d) = \frac{1}{2}$ $(240 – a)$

$\Rightarrow$ $2a = 240 – a$

$\Rightarrow$ $3a = 240 \Rightarrow a = \frac{240}{3}$ $= ₹80$

It can be seen above that half of the total amount with Balu, Chetan and Dinesh becomes one-third of the total amount.

Similarly amounts with Balu and Chetan which are one-fourth and one-fifth of the total amount with the others become one-fifth and one-sixth of the total amounts.

Hence $b = \frac{240}{5}$ $= 48$

$c = \frac{240}{6}$ $= 40$

$d = 240 – (a + b + c) = 72$ Choice (B)

Solutions for questions 7 and 8:

On line 1, ₹250 was rent and ₹200 was call charges. On line 2, ₹250 was rent and ₹100 was call charges. The total chargeable calls was 300 ….. (1)

On a single line, ₹250 is the rent and ₹450 is the call charges, i.e. there are 450 chargeable calls ……. (2)

Comparing (1), (2) we conclude that the number of free calls is 150. The total number of calls (free + chargeable) on the first line is 150 + 200 = 350

7. 350 Choice (C)

8. 150 Choice (A)

Solutions for questions 9 to 35:

9. Let the cost of each pen, eraser and sharpener be p, e and s respectively.

$2p + 4e + 5s = 36$

$3p + 7e + 9s = 63$

Multiplying the first equation by 2 and subtracting the second equation from it, $p + e + s = 9$ Choice (A)

10. Let $a + b – c = 3x$ $\rightarrow$ (1)

$b + c – a = 4x$ $\rightarrow$ (2)

$a + c – b = 5x$ $\rightarrow$ (3)

Adding these three equations, $a + b + c = 12x$ $\rightarrow$ (4)

$2c = 9x \rightarrow (4) – (1)$

so $c = 4.5x$

$2a = 8x \rightarrow$ (4) – (2)

so $a = 4x$

$2b = 7x \rightarrow (4) – (3)$

so $b = 3.5x$

$a : b : c = 4x : 3.5x : 4.5x = 8 : 7 : 9$ Choice (B)

11. If two equations should be in the form

$a_1x + b_1y = k_1$ and

$a_2x + b_2y = k_2$ to have infinite solutions,

$a_1/a_2 = b_1/b_2 = k_1/k_2$ must be satisfied.

Hence for the given equations, $2/6 = 1/k/2$ so $k = 6$

Choice (A)

12. Given that $a : b = 3 : 4$

so $\frac{a}{b} = \frac{3}{4}$

Dividing both numerator and denominator of $\frac{3a^2 + 4b^2}{4a^2 + 3b^2}$

by b^2, it becomes

$$\frac{3\left(\frac{a}{b}\right)^2 + 4}{4\left(\frac{a}{b}\right)^2 + 3} = \frac{3\left(\frac{9}{16}\right) + 4}{4\left(\frac{9}{16}\right) + 3} = \frac{13}{12}$$

Choice (D)

13. Let the distance travelled by the stone and the time of travel of the stone be denoted by d and t respectively. $d = ct^2$ where c is a constant.

Distance travelled in the fourth second by the stone = Total distance travelled in first four seconds – the total distance it travelled in first three seconds

$= c(4^2 – 3^2) = 35.$

$7c = 35 \Rightarrow c = 5$

Total distance it falls in the first 5 seconds

$= c(5^2)$.= 125 m. Choice (D)

14. The shares of P, Q and R be ₹x, ₹y and ₹z respectively.

So $x + y + z = 750$

$(x + 30) : (y + 20) : (z + 10) = 10 : 8 : 9$

$x + y + z + 30 + 20 + 10 = 750 + 60 = 810.$

So $x + 30 = \frac{810 \times 10}{(10 + 8 + 9)} = 300$. So $x = 270$

Share of p = ₹270 Choice (D)

15. Let the present age of Ajay be x years.

Some time in the past, Bharat was $x/4$ years.

The age of Ajay at that time is Bharat's present age.

Bharat's present age is $(x – 9)$ years.

As their difference of ages is constant,

$x – (x – 9) = (x – 9) – x/4$. $x = 24$

Sum of their present ages = $2x – 9 = 39$ years

Choice (A)

16. Let the incomes of A and B be $3x$ and $2x$, respectively. Let the expenditures of A and B be $5y$ and $4y$, respectively. Savings of A and B are $3x – 5y$ and $2x – 4y$ respectively. Given that, $3x – 5y = 2(2x – 4y) \Rightarrow x = 3y$

Ratio of income and expenditure of $B = 2x : 4y$

$= 2(3y) : 4y = 3 : 2$ Choice (C)

17. Let the number of ₹1, ₹2 and ₹5 coins be x, y and z respectively.

$x + y + z = 40$ ……. (1)

$x + 2y + 5z = 130$ …… (2)

Subtracting the equation (1) from (2), $y + 4z = 90$

In order to satisfy the above equation, y must be divisible by 2.

As z must be maximum, y must be minimum.
Hence when y has the least value, then z is obtained as 22 from the above equation. Choice (D)

18. Let the present age of Ganesh be G.
Present age of Harish = $2G + 8$
Let Harish become twice the age of Ganesh k years from now. Hence, $2G + 8 + k = 2(G + k) \Rightarrow k = 8$. Choice (C)

19. Let the number of local calls, STD calls and ISD calls made by Rohan be l, s and i respectively.
$l + s + i = 13$ and $6l + 11s + 13i = 119$
$6l + 11s + 13i - 6(l + s + i) = 119 - 6(13)$
$5s + 7i = 41$
$5s$ ends with 0 or 5. Also, R.H.S ends with 1. $\therefore 7i$ must end with 1 or 6. Also $7i < 41$.
$\therefore$ $7i = 21$ ($\therefore$No value of $7i$ ends with 6)
$i = 3$ Choice (A)

20. Let Ram have f, t and w notes of denomination ₹5, ₹10 and ₹20 respectively. Given that
$5f + 10t + 20w = 540$ ------ (1) (f, t, w are integers)
and $f + w = 24$ -------------- (2)
(1) – 5 × (2) gives $10t + 15w = 420$
$\Rightarrow$ $t = \dfrac{420-15w}{10}$

As $w > 0$, $t_{max} = \dfrac{420-15\times2}{10} = 39$

Choice (C)

21. The vessels are filled to their capacities
Let the capacities of the first, the second and the third vessels be $2x$, $3x$ and $4x$ respectively
Required ratio = $\dfrac{1}{1+3}(2x) + \dfrac{5}{6}(3x) + \dfrac{3}{8}(4x) : 3/4(2x) + \dfrac{1}{6}(3x) + \dfrac{5}{8}(4x) = \dfrac{x}{2} + \dfrac{5x}{2} + \dfrac{3x}{2} : \dfrac{3x}{2} + \dfrac{x}{2} + \dfrac{5x}{2}$
= 1 : 1. Choice (A)

22. Given that $a : b = 4 : 3$ and $b : c = 4 : 3$
$\Rightarrow$ $a : b = 16 : 12$ and $b : c = 12 : 9$
$\therefore$ $a : b : c = 16 : 12 : 9$
Let $a = 16k$, $b = 12k$ and $c = 9k$
Also given $(a + b)^2 - (b^2 + c^2) = 2236$
$(28k)^2 + (144k^2 + 81k^2) = 784k^2 - 225k^2 = 2236$
$\Rightarrow$ $k^2 = 4 \Rightarrow k = 2$
$\therefore$ $b = 12k = 12 \times 2 = 24$. Choice (A)

23. The number 23x 456 7y 4 is divisible by 4 and by 11.
$\therefore$ $y = 0, 2, 4, 6$ or 8 and $(2 + x + 5 + 7 + 4) - (3 + 4 + 6 + y)$,
i.e., $(18 + x) - (13 + y)$ or $(5 + x - y)$ is also a multiple of 11.
As the least value of $5 + x - y$ is $5 + 0 - 9$ or -4 and the greatest value is $5 + 9 - 0$ or 14, it could be 0 or 11.
$\therefore$ $x = y - 5$ or $x + y + 6$. The least value of x is $6 - 5$ or 1. Choice (A)

24. Let the number be N
$\therefore$ $N = 259K + 161$.
$\Rightarrow$ $N/7 = 37K + 23$
$\therefore$ $N/7$ leaves a remainder of 23, when divided by 37. Choice (B)

25. The given number N is $36^2(36 + 1) - 37$
$= 36^2(37) - 37 = 37(36^2 - 1) = (37)(35)(37)$
$\therefore$ N is not divisible by 36. Choice (C)

26. Since the number of friends has to be the least, the number of toffees should be as great as possible. Let the toffees distributed be 29, 28, 27, 26 …..
Given that, $29 + 28 + 27 + 26 + \ldots$ up to n terms ≤ 290.
If $n = 12$, the number of toffees that can be distributed is $29 + 28 + \ldots\ldots + 19 + 18 = 6(29 + 18) = 282$
The last person gets 8 toffees. The least number of friends is 12 + 1 = 13. Choice (C)

27. Let there be m subjects in Patiala and n in Mysore. Let each subject get p coins and say the number of coins left with either king is r.
$\therefore$ $mp + r = 33, 274$ and $np + r = 30, 905$
$\therefore$ $(m - n)p = 2369 = (23)(103)$
As p is a 2–digit number $p = 23$ and $m - n = 103$. Choice (B)

28. $2^{4n} \times 6^{7n} + 5^{3n} \times 7^{9n}$ can be written as $16^n \times 6 + 5 \times 7^{9n}$.
As 16^n can be written as 6^n, the unit's digit of the sum becomes $6 \times 6 + 5 \times$ (odd number) = 6 + 5 =1. Choice (B)

29. In a day, the three bells toll together = 24 times, or once in every hour or 60 minutes. Let X, Y, Z ring once in every x, y, z minutes respectively. The maximum value of z is 60 and $y < 60$. Since y is a factor of 60, its greatest value is 30. Hence if Y rings once every 30 minutes, the number of times it tolls in a day will be the minimum, which is = $\dfrac{(24)(60)}{30} = 48$ Choice (A)

30. The numbers a, b, c are consecutive. Therefore there are only 2 possibilities for the types (odd / even) of a, b, c. Either a, b, c are o, e, o or they are e, o, e. We tabulate the expressions, and the truth values (definitely true, could be false, definitely false, i.e., dt, cf, df).

				dt	cf	df
I	4a + 5b + 3c	4(o) + 5(e) + 3(o) = o	4(e) + 5(o) + 3(e) = o	✓		
II	2a + 3b + 4c	2(o) + 5(e) + 4(o) = e	2(e) + 5(o) + 4(e) = o		✓	
III	a2 b3 c4	(o) (e) (o) = e	(e) (o) (e) = e	✓		
IV	3a + 2b + 4c	3(o) + 2(e) + 4(o) = o	3(e) + 2(o) + 4(e) = e		✓	

We see that II, IV are not definitely true

Choice (D)

31. When the students are arranged in 13 rows, let the number of students in each row be x. The total number of students is $(13x + 3)$. Similarly in the second case, the total number of students is $21y + 11$.
This is LCM model II
Number of students: = (LCM of 13 and 21) – 10 = 263
Required remainder 263 ÷ 19 = 16. Choice (D)

32. Divisors are 4, 5, 3. Remainders are 1, 2, 2.
The least number is [(2) (5) + 2] (4) + 1 = 49.
When 49 is divided by 37, the remainder is 12.
Choice (A)

33. Let the two numbers be ha and hb respectively,where a and b are co-primes. Then LCM of the two numbers is hab.
Given $hab + h = 784$ and $hab - h = 756$
$\Rightarrow$ $hab = 770$ and $h = 14$ Þ $ab = 55$.
The possible values of a, b are (1, 55) and (5, 11). When the numbers are close, the difference will bethe least, i.e., $\{a, b\} = \{5, 11\}$.
The least possible difference is 14 (6) = 84.
Choice (C)

34. The given conditions represent the problem as LCM model-2. The general form of such numbers is (LCM of 30, 36)K – [common difference i.e., 15 in this case] = $180K - 15$.
Now, when $180K - 15$ is divided by 35, ($175K$ is a already divisible by 35), the remainder is given to be 10. Hence $180K - 25$ i.e., $5K - 25$ is exactly divisible by 35. This will be possible when $K = 5$.
Hence the required number is 180(5) – 15 = 885.
Choice (A)

35. Remainder required

$$= \text{Rem}\left(\frac{(220+3)(220+6)(220+8)}{11}\right)$$

(220 + 3) (220 + 6) (220 + 8) = (A multiple of 220 + 3.6) (220 + 8) = A multiple of 220 + (3) (6) (8). As 220 is divisible by 11, reminder required = Rem$\left(\frac{(3)(6)(8)}{11}\right)$

$= \text{Rem}\left(\frac{144}{1}\right) = 1.$ Choice (D)

QUANTITATIVE APTITUDE TEST 7

Number of Questions: 35 **Section Marks: 30**

(QUADRATIC EQUATIONS AND INEQUALITIES)

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Two students independently attempted to solve a quadratic equation in x. One of them copied the constant term wrong and obtained roots as –15 and 16. The other student copied the coefficient of x wrong and obtained his roots as –10 and 21. Find the correct roots of the quadratic equation?
(A) (–15, 14) (B) (–14, 15)
(C) (–25, 7) (D) (–7, 25)

2. $\sqrt{132+\sqrt{132+\sqrt{132+.......}}} =$ ______
(A) 11.11 (B) 12.32
(C) 11 (D) 12

3. The range of k for which the sum as wells as the product of the roots of $6x^2 - kx + 9 - k^2 = 0$ are negative is __________
(A) (–3, 3) (B) $(-\infty, 3)$
(C) $(-\infty, -3)$ (D) $(3, \infty)$

4. Find the range of k, for which $-x^2 + 4kx + 3k - 1$, is always negative.
(A) $\left(-\frac{1}{4}, 1\right)$ (B) $(1, \infty)$
(C) $\left(-\infty, -\frac{1}{4}\right)$ (D) $\left(-1, \frac{1}{4}\right)$

5. a and b are the roots of the equation $2x^2 - 15x + k = 0$. Find the value of k if $a^2 - b^2 = 45$.
(A) $5\frac{1}{16}$ (B) $10\frac{1}{8}$
(C) $13\frac{1}{2}$ (D) $12\frac{1}{2}$

6. If the sum of the roots of the quadratic equation $3x^2 + (2k + 1)x - k - 5 = 0$ is equal to the product of the roots, which of the following is true?
(A) $k^2 - 4 = 0$ (B) $k^2 - 9 = 0$
(C) $k^2 - 16 = 0$ (D) $k^2 - 25 = 0$

7. If the roots of the equation $x^2 - 7x - 12 = 0$ are diminished by one and then multiplied by two, which of the following equations is formed with those values as the roots?
(A) $x^2 - 10x + 24 = 0$ (B) $x^2 - 6x - 76 = 0$
(C) $x^2 - 2x - 48 = 0$ (D) $x^2 - 10x - 72 = 0$

8. Which of the following statements is true about the roots of the equation $k^2x^2 - kx + (1 + 2x^2) = 0$, where k is a real number?
I. Roots are equal
II. Roots are complex
III. Roots are rational
IV. Roots are real
(A) I and IV (B) III only
(C) I and II (D) II only

9. All the roots of two quadratic equations are positive integers. The sum of the squares of the roots of the first quadratic equation is equal to that of the second quadratic equation. If the sum of the roots of the two equations are 10 and 8 respectively, then what is the greatest possible root of these quadratic equations?
(A) 7 (B) 6
(C) 8 (D) 5

10. If a and b are positive numbers, what is the nature of the roots of the equation $(a + b)x^2 + 2abx + \frac{(a+b)^3}{16} = 0$?
(A) Real and distinct. (B) Real and equal.
(C) Non-real and distinct. (D) Either (B) or (C)

11. If a positive number is increased by three and then squared, the result is 23 more than the original number. Find the original number.
(A) 1 (B) 2
(C) 3 (D) 4

12. Find the value of R, so that one of the roots of $x^2 + 6Rx + 64 = 0$ is the square of the other root.
(A) $\frac{-10}{3}$ (B) $\frac{8}{3}$
(C) $\frac{5}{3}$ (D) $\frac{7}{3}$

13. If the value of p in the equation $x^2 + 2(p + 1)x + 2p = 0$, is real, the roots of the equation are
(A) rational and unequal.
(B) irrational and unequal.
(C) real and unequal.
(D) real and equal.

14. Find the equation whose roots are twice the roots of the equation $3x^2 - 7x + 4 = 0$.
(A) $3x^2 - 14x + 8 = 0$ (B) $3x^2 + 14x + 16 = 0$
(C) $3x^2 + 14x - 16 = 0$ (D) $3x^2 - 14x + 16 = 0$

15. The length of a rectangle is 1 cm more than its breadth. If its diagonal is 29 cm, what is the measure of its breadth? (in cm)
(A) 18 (B) 20
(C) 17 (D) 21

16. A is any single-digit prime number and B is any natural number. How many equations of the form $x^2 - 4\sqrt{A}\,x + 3B = 0$ have both real roots?
(A) 15 (B) 18
(C) 21 (D) 24

17. In a class, eight students play basketball. The remaining students, who represent 7 times the square root of the strength of the class, play football. Find the strength of the class.
(A) 36 (B) 16
(C) 64 (D) 100

18. If the price of a book goes down by ₹20 per dozen, a person can purchase 50 dozen books more for ₹30,000. Find the original price of each book.
(A) ₹10 (B) ₹12
(C) ₹9 (D) ₹8

19. If $-9 \le p \le -5$ and $-17 \le q \le -12$ then which of the following can be concluded?
(A) $\frac{5}{12} \le \frac{p}{q} \le \frac{9}{17}$ (B) $\frac{17}{9} \le \frac{p}{q} \le \frac{12}{5}$
(C) $\frac{5}{17} \le \frac{p}{q} \le \frac{3}{4}$ (D) $\frac{12}{9} \le \frac{p}{q} \le \frac{17}{5}$

20. If $|3x-4| = |5x-12|$, then the sum of the possible values of x is ______.
(A) 4 (B) 6
(C) –4 (D) –6

21. If $|4x-9| = 7$, then the values of $4|x| - |-x^3|$ is ______.
(A) $48, \frac{15}{8}$ (B) $-48, \frac{-15}{8}$
(C) $48, \frac{-15}{8}$ (D) $-48, \frac{15}{8}$

22. Find the range of values of x that satisfy the relation $|2x - 1| - 1 < |x - 2| + 3$.
(A) $-4 < x < 4$ (B) $-6 < x < \frac{1}{2}$
(C) $-6 < x < 4$ (D) $-5 < x < 3$

23. If $E = |x + 4| + |x + 7| + |x - 1|$, then how many integral values of x satisfy the inequality $E \le 14$?
(A) 8 (B) 10
(C) 11 (D) More than 11

24. Which of the following inequalities gives a finite range of values for x?
(A) $6x^3 - x^2 - x < 0$
(B) $x^4 + x^3 - 3x^2 - x + 2 < 0$
(C) $x^3 - x^2 - 5x - 3 < 0$
(D) $x^4 + 3x^3 + 2x^2 > 0$

25. If $\frac{x}{x+1} - \frac{x+2}{x-1} < 0$, then find the range of x.
(A) $\left(-1, -\frac{1}{2}\right) \cup (1, \infty)$ (B) $(-2, -1) \cup (0, 1)$
(C) $(-\infty, -1) \cup \left(-\frac{1}{2}, 1\right)$ (D) $\left(-\frac{1}{2}, \infty\right)$

26. Find the range of x, for which $|x + 2| - 3\,|x - 1| + 4 \ge 0$.
(A) $-2 \le x \le 1$ (B) $-2 \le x \le \frac{9}{2}$
(C) $-\frac{3}{4} \le x \le 4$ (D) None of these

27. a, b, c and d are four positive real numbers whose sum is equal to 4. If $p = \frac{abcd}{(abc + bcd + acd + abd)}$, then find the maximum value of p.
(A) 16 (B) 4
(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

28. If $2 < x < 5$ and $10 < y < 30$, then $\frac{y}{x}$ lies between
(A) 5 and 6 (B) 2 and 6
(C) 2 and 15 (D) 6 and 15

29. If $|x| > 6$ and $y > -4$, then which of the following is necessarily true?
(A) $|xy| > 24$ (B) $|xy| < 24$
(C) $|x|\,|y| > 0$ (D) None of these

30. Let $f(x) = \max(3x + 5, 7 - 2x)$, where x is any real number. Then the minimum possible value of $f(x)$ is
(A) $\frac{31}{5}$ (B) $\frac{27}{5}$
(C) $\frac{21}{5}$ (D) $\frac{29}{5}$

31. If $20 \le x \le 35$ and $3y - 2x = 5$, then the minimum value of $\frac{x}{x+y}$ is
(A) 1 (B) $\frac{1}{3}$
(C) $\frac{4}{9}$ (D) $\frac{4}{7}$

32. If a, b and c are positive real numbers. Find the minimum value of $\left(1+a+\frac{1}{a}\right)\left(1+b+\frac{1}{b}\right)\left(1+c+\frac{1}{c}\right)$.
(A) 9 (B) 12
(C) 27 (D) 81

33. If $a \le 25$ and $a + b \ge 10$, then which of the following is always true?

(A) $a - b \ge 40$ (B) $b - a \ge -40$
(C) $a + b \ge 40$ (D) $ab \le 250$

34. If $1 \le x \le 3$, $4 \le y \le 10$ and $2 \le z \le 5$, what is the maximum possible value of $\frac{y}{x+y+z}$?

(A) 5 (B) $\frac{10}{3}$
(C) $\frac{10}{13}$ (D) $\frac{10}{7}$

35. Find the range of values of x for which $\left|\frac{18-2x}{4}\right| < 3$.

(A) $6 < x < 30$ (B) $-3 < x < 15$
(C) $-15 < x < 3$ (D) $3 < x < 15$

Answer Key

1. B	**2.** D	**3.** C	**4.** D	**5.** B	**6.** C	**7.** D	**8.** D	**9.** A	**10.** D
11. B	**12.** A	**13.** C	**14.** D	**15.** B	**16.** C	**17.** C	**18.** A	**19.** C	**20.** B
21. D	**22.** D	**23.** B	**24.** B	**25.** A	**26.** D	**27.** D	**28.** C	**29.** D	**30.** A
31. D	**32.** C	**33.** B	**34.** C	**35.** D					

Hints and Explanations

1. The quadratic equation which has α and β are the roots is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Quadratic equation taken by the first student is

$x^2 - (-15 + 16)x + (-15 \times 16) = 0$

$x^2 - x - 240 = 0$ ——————(1)

Quadratic equation taken by the second student is

$x^2 - (-10 + 21)x + (-10 \times 21) = 0$

$x^2 - 11x - 210 = 0$ ——————(2)

∴ Required correct quadratic equation is $x^2 - x - 210 = 0$

⇒ $x^2 - 15x + 14x - 210 = 0$

⇒ $x(x - 15) + 14(x - 15) = 0$

⇒ $(x - 15)(x + 14) = 0$

∴ Required roots are -14 and 15. Choice (B)

2. Let $X = \sqrt{132+\sqrt{132+\sqrt{132+......}}}$

⇒ $x = \sqrt{132+x}$

Squaring on both sides

⇒ $x^2 = 132 + x \Rightarrow x^2 - x - 132 = 0$

⇒ $x^2 - 12x + 11x - 132 = 0$

⇒ $(x - 12)(x + 11) = 0$

∴ $x = 12$ (∴ x is always positive) Choice (D)

3. Given $6x^2 - kx + 9 - k^2 = 0$

The sum of the roots = $\frac{k}{6}$

The product of the roots = $\frac{9-k^2}{6}$

$\frac{k}{6}$ and $\frac{9-k^2}{6}$ are both negative

⇒ $\frac{k}{6} < 0$ and $\frac{9-k^2}{6} < 0$

⇒ $k < 0$ and $9 < k^2$

⇒ $k < 0$ and $k > 3$ or $k < -3$

∴ $k < -3 \Rightarrow k \in (-\infty, -3)$. Choice (C)

4. It is given that $-x^2 + 4kx + 3k - 1 < 0$

⇒ $-(x^2 - 4kx) + 3k - 1 < 0$

⇒ $-[(x - 2k)^2 - 4k^2] + 3k - 1 < 0$

⇒ $-[(x - 2k)^2] + 4k^2 + 3k - 1 < 0$

Now, for the above expression to be always negative $4k^2 + 3k - 1 < 0 \Rightarrow (4k - 1)(k + 1) < 0$

This is true when $-1 < k < \frac{1}{4}$. Choice (D)

5. The given equation is $2x^2 - 15x + k = 0$

The sum of the roots, $a + b = \frac{15}{2}$ and

the product $ab = \frac{k}{2}$

It is given that $a^2 - b^2 = 45 \Rightarrow a - b = 6$

$a + b = \frac{15}{2}$, $a - b = 6$, $\Rightarrow a = \frac{27}{4}$, $b = \frac{3}{4}$

∴ The product of the roots $ab = \left(\frac{27}{4}\right)\left(\frac{3}{4}\right) = \frac{81}{16}$

Now $\frac{k}{2} = \frac{81}{16} \Rightarrow k = \frac{81}{8} = 10\frac{1}{8}$ Choice (B)

6. Sum of the roots = $\frac{-(2k+1)}{3}$

Product of the roots = $\frac{-(k+5)}{3}$

$\frac{-(2k+1)}{3} = \frac{-(k+5)}{3}$

⇒ $2k + 1 = k + 5 \Rightarrow k = 4$

Choice (C)

7. Roots are to be diminished by one and then multiplied by two. i.e., if A, B are roots of given equation, then $2(A-1) = A_1$ and $2(B-1) = B_1$, where A_1 and B_1 are the roots of the new equation. i.e.,

$\Rightarrow \quad A = 1 + \frac{A_1}{2} = \frac{A_1+2}{2}$ and $B = \frac{B_1+2}{2}$

i.e., x of the given equation is to be replaced by $\frac{x+2}{2}$, to obtain the required equation. Given equation is: $x^2 - 7x - 12 = 0$.

Required equation is $\left(\frac{x+2}{2}\right)^2 - \frac{7(x+2)}{2} - 12 = 0$

$(x+2)^2 - 7(2)(x+2) - 4(12) = 0$.

$\Rightarrow \quad x^2 + 4x + 4 - 14x - 28 - 48 = 0$.

$\Rightarrow \quad x^2 - 10x - 72 = 0$. Choice (D)

8. When rewritten, the equation becomes:
$(k^2+2)x^2 - kx + 1 = 0$
Discriminant, $D = (k)^2 - 4(1)(k^2+2)$
$= -3k^2 - 8 = -(3k^2+8)$
$3k^2$ is positive for all real values of k, and hence $(3k^2+8)$ is positive; and so $-(3k^2+8)$ is negative.
As the discriminant is negative, roots are complex.
Choice (D)

9. Let the roots of the first quadratic equation be α and β and those of the second equation be γ and δ respectively.
Given $\alpha^2 + \beta^2 = \gamma^2 + \delta^2$
Also $\alpha + \beta = 10$ and $\gamma + \delta = 8$.
The possible values of $\alpha^2 + \beta^2$ are 50, 52, 58, 68 and 82 while the possible values $\gamma^2 + \delta^2$ as are 32, 34, 40 and 50. As only 50 is a common value, $\alpha = 5$, $\beta = 5$, $\gamma = 7$ and $\delta = 1$

$\therefore$ The greatest possible root is 7. Choice (A)

10. Dividing both sides of the given equation by $a + b$,

$x^2 + \frac{2abx}{a+b} + \frac{(a+b)^2}{16} = 0$

Discriminant

$= \left(\frac{2ab}{a+b}\right)^2 - \frac{4(a+b)^2}{16} = \left(\frac{2ab}{a+b}\right)^2 - \left(\frac{a+b}{2}\right)^2$

Shown below is the proof that this is always non-positive provided a and b are positive.
$(a-b)^2 \geq 0 \Rightarrow a^2 + b^2 + 2ab \geq 4ab$

dividing both sides by $2(a+b)$ $\frac{a+b}{2} \geq \frac{2ab}{a+b}$

As the expressions on both sides of the inequality are positive, $(\frac{a+b}{2})^2 \geq (\frac{2ab}{a+b})^2$

$\therefore \quad \Delta < 0$ or $\Delta = 0$
If $\Delta = 0$, the roots are real and equal.
If $\Delta < 0$, the roots are non-real and distinct.
Choice (D)

11. Let the required original number be x.
$(x+3)^2 = 23 + x$.
Hence $x^2 + 6x + 9 = 23 + x$
$\Rightarrow \quad x^2 + 5x - 14 = 0$.
$(x+7)(x-2) = 0$
$\Rightarrow \quad x = -7$ or $x = 2$.
Since the original number is positive, $x = 2$.
Choice (B)

12. If one of the roots is α, the other root is α^2.
Hence the product of the roots $= \alpha(\alpha^2)$.
$\alpha^3 = 64 \Rightarrow \alpha = \sqrt[3]{64} = 4$ and $\alpha^2 = 4^2 = 16$

The sum of roots $= -\left(\frac{6R}{1}\right) = -6R = 4 + 16 = 20$

$R = \left(\frac{20}{-6}\right) = -\frac{10}{3}$ Choice (A)

13. For the equation $x^2 + 2(p+1)x + 2p = 0$
$b^2 - 4ac = [2(p+1)]^2 - 4(2p)] = 4p^2 + 8p + 4 - 8p$
$= 4p^2 + 4$ which is always positive.
Hence the roots of the equation are always real and unequal. Choice (C)

14. For the equation, whose roots are twice the roots of the equation A : $3x^2 - 7x + 4 = 0$, the sum of the roots is twice the sum of the roots of A and the product of the roots is 4 times the product of the roots of A.The required equation is $x^2 - \left(2\left(\frac{7}{3}\right)\right)x + 4\left(\frac{4}{3}\right) = 0$

i.e., $3x^2 - 14x + 16 = 0$ Choice (D)

15. Let ℓ and b be the length and breadth in cm.
Given that $\ell = b + 1$
Also given that diagonal = 29 cm
$\Rightarrow \quad \sqrt{\ell^2 + b^2} = 29$
By squaring on both sides, $(b+1)^2 + b^2 = 29^2$
$\Rightarrow \quad 2b^2 + 2b - 840 = 0$
$\Rightarrow \quad b^2 + b - 420 = 0$
$\Rightarrow \quad (b+21)(b-20) = 0$
$\therefore \quad b = 20$ Choice (B)

16. $(4\sqrt{A})^2 - 4(3B) \geq 0$

$\frac{4}{3}A \geq B$

As A is a single digit prime number, A can be 2, 3, 5 or if $A = 2$, B has 2 possibilities. If $A = 3$, B has 4 possibilities. If $A = 5$, B has 6 possibilities. If $A = 7$, B has 9 possibilities. A total of 21 equations are possible.
Choice (C)

17. Let the strength be x. The number of students who play basketball = 8
The number of students who play football
$= x - 8 = 7\sqrt{x}$
Substituting the choices in place of x in the equation above, only choice (C) satisfies it. Choice (C)

18. Let the initial number of books in dozens = b

Let initial price (in ₹) of books per dozen be p.

$pb = 30{,}000$. →(I)

$(50 + b)(p - 20) = 30{,}000$

$50p - 1000 + pb - 20b = 30{,}000$

or, $50p - 20b = 1000$

$5p - 2b = 100$. →(II)

From (I) and (II) $5p - \dfrac{60{,}000}{p} = 100$

$5p^2 - 100p - 60{,}000 = 0$

$5p^2 - 600p + 500p - 60{,}000 = 0$

$5p(p - 120) + 500(p - 120) = 0 \Rightarrow p = 120$

The price of each book $= \dfrac{120}{12} = 10$ Choice (A)

19. $\dfrac{p}{q} = \dfrac{-p}{-q}$

$5 \le -p \le 9$ and $12 \le -q \le 17$

$\dfrac{-p}{-q}$ is maximum when p is maximum and q is minimum

$\therefore$ $\text{Max}\left(\dfrac{-p}{-q}\right) = \dfrac{9}{12} = 3/4$

$\dfrac{-p}{-q}$ is minimum when p is minimum and q is maximum.

$\therefore$ $\text{Min}\left(\dfrac{-p}{-q}\right) = \dfrac{5}{17}$

$\dfrac{5}{17} \le \dfrac{p}{q} \le \dfrac{3}{4}$ Choice (C)

20. $|3x - 4| = |5x - 12|$

When ever $|p| = |q|$ it follows that $p = \pm q$

$3x - 4 = \pm(5x - 12)$

$\Rightarrow$ $3x - 4 = 5x - 12$ or $3x - 4 = -5x + 12$

$\Rightarrow$ $2x = 8$ or $8x = 16$

$\Rightarrow$ $x = 4$ or $x = 2$

$\therefore$ Required sum of the possible values of x is 6.

Choice (B)

21. $|a| = b \Rightarrow a = \pm b$

$|4x - 9| = 7$

$\Rightarrow$ $4x - 9 = 7$ or $4x - 9 = -7$

$\Rightarrow$ $x = 4$ or $x = 1/2$

$4|x| - |-x|^3$

$= 4(4) - (4)^3$ or $4\left(\dfrac{1}{2}\right) - \left(\dfrac{1}{2}\right)^3$

$= -48$ or $15/8$ Choice (D)

22. $|2x - 1| - 1 < |x - 2| + 3$

$\Rightarrow$ $|2x - 1| - |x - 2| < 4$

We need to consider 3 cases

(1) $x < \dfrac{1}{2}$ (2) $\dfrac{1}{2} \le x < 2$ and (3) $2 \le x$.

For $x < \dfrac{1}{2}$, we get

$-(2x - 1) + (x - 2) < 4$

$\Rightarrow$ $-x < 5 \Rightarrow x > -5$

$\therefore$ $-5 < x < \dfrac{1}{2}$

For $\dfrac{1}{2} \le x < 2$, $2x - 1 + x - 2 < 4$

$\Rightarrow$ $x < \dfrac{7}{3}$ $\therefore \dfrac{1}{2} < x < 2$

For $x \ge 2$, $(2x - 1) - (x - 2) < 4$

$\Rightarrow$ $x < 3$ $\therefore 2 \le x < 3$

Thus the range of x such that the given relation is satisfied is $-5 < x < 3$ Choice (D)

23. For $x = -9$, $E = |-9 + 4| + |-9 + 7| + |-9 - 1| = 17$

For $x = -8$, $E = |-8 + 4| + |-8 + 7| + |-8 - 1| = 14$

For $x = 1$, $E = |1 + 4| + |1 + 7| + |1 - 1| = 13$

For $x = 2$, $E = |2 + 4| + |2 + 7| + |2 - 1| = 16$

Therefore the integral values of x for which the given inequality is satisfied are $-8, -7, -6, -5, -4, -3, -2, -1$, 0 and 1 i.e. a total of 10 values. Choice (B)

24. We need to considering each option separately,

Option A:

$6x^3 - x^2 - x < 0$

$x(2x - 1)(3x + 1) < 0$

The above inequality is satisfied for

$x < -\dfrac{1}{2}$ or $0 < x < \dfrac{1}{2}$

which does not give a finite range of values for x

Option B:

$x^4 + x^3 - 3x^2 - x + 2 < 0$

$(x - 1)^2(x + 1)(x + 2) < 0$ which gives the same solution set as $(x + 1)(x + 2) < 0$ $((x - 1)^2 \ge 0]$

The above inequality is satisfied for $-2 < x < -1$ this is a finite range of values for x.

Option C:

$x^3 - x^2 - 5x - 3 < 0$

$(x + 1)^2(x - 3) < 0$ which gives the same solution set as $x - 3 < 0 \Rightarrow x < 3$

It does not give a finite range of values for x.

Option D:

$x^4 + 3x^3 + 2x^2 > 0$

$x^2(x^2 + 3x + 2) > 0$

$x^2(x + 2)(x + 1) > 0$

The above inequality gives the same solution set as $(x + 2)(x + 1) > 0$ $[x^2 \ge 0]$

The inequality is satisfied for $x < -2$ or $x > -1$ which does not give a finite range of values for x.

Note: A polynomial of odd degree can take values from $-\infty$ to ∞ but a polynomial of even degree has a finite range of values for which it has values of a particular sign. If the coefficient of the leading term (say a) is positive, $f(x) < 0$ for a finite range if $a < 0$, $f(x) > 0$ for a finite range. Choice (B)

25. $\frac{x}{x+1} - \frac{x+2}{x-1} < 0$

$\frac{x(x-1)-(x+2)(x+1)}{(x+1)(x-1)} < 0$

$\frac{x^2 - x - x^2 - 3x - 2}{(x+1)(x-1)} < 0$

$\frac{-4x-2}{(x+1)(x-1)} < 0$

$\frac{-2(2x+1)}{(x+1)(x-1)} < 0$

$\frac{(2x+1)}{(x+1)(x-1)} > 0$

Multiplying both Nr & Dr by $(x + 1)(x - 1)$ we get

$\frac{(2x+1)(x+1)(x-1)}{(x+1)^2(x-1)^2} > 0$

The solution set for the above inequality is the same as that for $(2x + 1)(x + 1)(x - 1) > 0$

Therefore the inequality holds true for $-1 < x < -\frac{1}{2}$ or $x > 1$, i.e., $x \in \left(-1, -\frac{1}{2}\right) \cup (1, \infty)$ Choice (A)

26. $|x + 2| - 3\,|x - 1| + 4 \geq 0$

For $x < -2$, $|x + 2| - 3\,|x - 1| + 4 = -(x + 2) + 3\,(x - 1) + 4$

$\therefore$ $-x - 2 + 3x - 3 + 4 \geq 0 \Rightarrow 2x \geq 1 \Rightarrow x \geq \frac{1}{2}$

But we have taken $x < -2$, thus no solution exists in this range.

For $-2 \leq x < 1$,

$|x + 2| - 3\,|x - 1| + 4 = (x + 2) + 3\,(x - 1) + 4$

$\Rightarrow$ $x + 2 - 3x + 3 + 4 \geq 0$

$\Rightarrow$ $9 - 2x \geq 0$

$\Rightarrow$ $9 \geq 2x$ $\Rightarrow$ $2x \leq 9$

$\therefore$ $x + 2 + 3x - 3 + 4 \geq 0 \Rightarrow 4x \geq -3 \Rightarrow x \geq -\frac{3}{4}$

Therefore, the range of x satisfying the given condition is $-\frac{3}{4} \leq x < 1$

For $x \geq 1$,

$|x + 2| - 3\,|x - 1| + 4 = (x + 2) - 3\,(x - 1) + 4 - 2x \leq -9$

$\therefore$ $x + 2 - 3x + 3 + 4 \geq 0$

$\Rightarrow$ $9 - 2x \geq 0$

$\Rightarrow$ $2x \leq 9$

$\Rightarrow$ $x \leq \frac{9}{2}$

$\therefore$ $1 \leq x \leq \frac{9}{2}$

Thus, the range of x satisfying the given inequality is $-\frac{3}{4} \leq x \leq \frac{9}{2}$ Choice (D)

27. $AM\,(a, b, c, d) \geq HM\,(a, b, c, d)$

$\frac{a+b+c+d}{4} \geq \frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d} \geq \frac{16}{a+b+c+d}$

$\frac{bcd+acd+abd+abc}{abcd} \geq \frac{16}{a+b+c+d}$

$\frac{abcd}{abc+bcd+acd+abd} \leq \frac{a+b+c+d}{16}$

$(\because a + b + c + d = 4)$

$\therefore$ $p \leq \frac{1}{4}$. Choice (D)

28. Given $2 < x < 5$ and $10 < y < 30$.

The value of y/x is minimum, for the minimum value of y and the maximum value of x.

$\therefore$ $y/x > \frac{10}{5}$ or $y/x > 2$

The value of y/x is maximum, for the maximum value of y and the minimum value of x.

$\therefore$ $y/x < \frac{30}{2}$ or $y/x < 15$

$\Rightarrow$ $2 < y/x < 15$ Choice (C)

29. Given $|x| > 6$, $y > -4$.

Consider $x = 7$ and $y = 2$; $xy = 14$

$\Rightarrow$ $|xy| = 14|xy| > 24$ is not necessarily true.

Consider $x = 8$ and $y = 5$; $xy = 40$; $|xy| > 40 > 24$

$\therefore$ The second option is not necessarily true.

For $y = 0$; $|x|\,|y| = 0$, hence none of the given options is necessarily true. Choice (D)

30. Given $f(x) = \max(3x + 5, 7 - 2x)$

$f(x)$ has the minimum value when the two expressions are equal.

$\therefore$ $3x + 5 = 7 - 2x$

$5x = 2 \Rightarrow x = \frac{2}{5}$

$\therefore$ The minimum value of $f(x)$ is

$$f\left(\frac{2}{5}\right) = \max\left(\frac{3(2)}{5} + 5, 7 - \frac{2(2)}{5}\right)$$

$$= \max\left(\frac{31}{5}, \frac{31}{5}\right) = \frac{31}{5}$$ Choice (A)

31. $20 \le x \le 35$

$$y = \frac{2x+5}{3}$$

$$\therefore \quad \frac{x}{x+y} = \frac{x}{x + \frac{2x+5}{3}}$$

$$= \frac{3x}{5x+5} = \frac{3}{5 + \frac{5}{x}}$$

This expression is positive for the given range of values of x and it has its minimum value when $5/x$ has its maximum value, i.e. when $x = 20$.

The corresponding value is $\frac{3(4)}{21} = \frac{4}{7}$.

Choice (D)

32. If x is a positive number, the minimum value of $x + \frac{1}{x}$ is 2.

Hence, the minimum value of $1 + x + \frac{1}{x}$ is 3 and for the given expression, it is $3(3)(3) = 27$. Choice (C)

33. Given $a \le 25$ and $a + b \ge 10$

$\Rightarrow$ $a \le 25$ and $b \ge 10 - a$

$\Rightarrow$ $a \le 25$ and $b \ge 10 - 25$

$\Rightarrow$ $a \le 25$ and $-b \ge 15$

$\Rightarrow$ $a - b \le 40$

$\Rightarrow$ $b - a \ge -40$ Choice (B)

34. $\frac{y}{x+y+z} = \frac{1}{\frac{x}{y} + 1 + \frac{z}{y}}$; to maximize the given expression, $x + z$ should take minimum and y should take maximum possible value.

$\therefore$ Maximum value $= \frac{1}{\frac{1}{10} + 1 + \frac{2}{10}} = \frac{10}{13}$ Choice (C)

35. Given $\left|\frac{18-2x}{4}\right| < 3$

$\Rightarrow$ $|18 - 2x| < 12$

$\Rightarrow$ $|x - 9| < 6$

The expression $|x - a|$ denotes the distance of the point x from the point a on the number line.

$|x - 9| < 6$ Þ x lies within a distance 6 units from the point 9. i.e., $3 < x < 15$.

Choice (D)

PART - II

Engineering Mathematics

Chapter 1

Calculus

CHAPTER HIGHLIGHTS

- *Limit of a function*
- *Derivatives*
- *Mean value theorem*

Limit of a Function

Let $y = f(x)$ be a function of x and let 'a' be any real number.

We must first understand what a 'limit' is. A limit is the value, function approaches, as the variable within that function (usually 'x') gets nearer and nearer to a particular value. In other words, when x is very close to a certain number, what is $f(x)$ very close to?

Meaning of '$x \to a$'

Let x be a variable and 'a' be a constant. If x assumes values nearer and nearer to 'a', then we say that 'x tends to a' or 'x approaches a' and is written as '$x \to a$'. By $x \to a$, we mean that $x \neq a$ and x may approach 'a' from left or right, which is explained in the example given below.

Let us look at an example of a limit: What is the limit of the function $f(x) = x^3$ as x approaches 2? The expression 'the limit as x approaches to 2' is written as: $\lim\limits_{x\to 2}$ Let us check out some values of $\lim\limits_{x\to 2}$ as x increases and gets closer to 2, without even exactly getting there.

When $x = 1.9$, $f(x) = 6.859$
When $x = 1.99$, $f(x) = 7.88$
When $x = 1.999$, $f(x) = 7.988$
When $x = 1.9999$, $f(x) = 7.9988$

As x increases and approaches 2, $f(x)$ gets closer and closer to 8 and since x tends to 2 from left this is called 'left-hand limit' and is written as $\lim\limits_{x\to 2^-}$.

Now, let us see what happens when x is greater than 2.

When $x = 2.1$, $f(x) = 9.261$
When $x = 2.01$, $f(x) = 8.12$
When $x = 2.001$, $f(x) = 8.01$
When $x = 2.0001$, $f(x) = 8.001$

As x decreases and approaches 2, $f(x)$ still approaches 8. This is called 'right-hand limit' and is written as $\lim\limits_{x\to 2^+}$

$\overrightarrow{x}\ \underline{\qquad 2}\quad \underline{2 \qquad} \overleftarrow{x}$

We get the same answer while finding both, left and right hand limits. Hence we write that $\lim\limits_{x\to 2} x^3 = 8$.

Meaning of the Symbol: $\lim\limits_{x\to a} f(x) = l$

Let $f(x)$ be a function of x where x takes values closer and closer to 'a' ($\neq a$), then $f(x)$ will assume values nearer and nearer to l. Hence we say, $f(x)$ tends to the limit 'l' as x tends to a.

The following are some of the simple algebraic rules of limits.

1. $\lim\limits_{x\to a} kf(x) = k \lim\limits_{x\to a} f(x)$
2. $\lim\limits_{x\to a}[f(x) \pm g(x)] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x)$
3. $\lim\limits_{x\to a}[f(x)\cdot g(x)] = \lim\limits_{x\to a} f(x) \cdot \lim\limits_{x\to a} g(x)$
4. $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{x \to a}{\lim\limits_{x\to a} g(x)} (\lim\limits_{x\to a} g(x) \neq 0)$

NOTES

1. If the left hand limit of a function is not equal to the right hand limit of the function, then the limit does not exist.
2. A limit equal to infinity is not the same as a limit that does not exist.

Continuous Functions

Let $f: A \to B$ be any given function and let $c \in A$. We say f is continuous at c, if given $\in > 0$, there exists $\delta > 0$ such that $|f(x) - f(c)| < \in$ whenever $|x - c| < \delta$

In words, this means that, if x is very close to c in domain, then $f(x)$ is very close to $f(c)$ in range.

Equivalently f is continuous at c. If $\lim_{x\to c} f(x) = f(c)$

We observe

1. $c \in A$, i.e., $f(c)$ must exist
2. $\lim_{x\to c} f(x)$ exists
3. $f(c)$ and $\lim_{x\to c} f(x)$ are equal.

If any of these three conditions fail, then f is discontinuous at $x = c$.

Algebra of Continuous Functions

If f, g be two continuous functions at c, then $f + g$, $f - g$, fg are also continuous at $x = c$.

To solve a problem of continuous functions at a point a, you can take the following approach.

1. Find the value $f(x)$ at $x = a$. If a is in the domain of f, $f(a)$ must exist. If a is not in the domain, then $f(a)$ does not exist. In such a case, f is not continuous at $x = a$.
2. Find $\lim_{x\to a} f(x)$. For this you have to first find $\lim_{x\to\infty} f(x) = l_1$(say) and $\lim_{x\to a^+} f(x) = l_2$(say). If $l_1 \neq l_2$ then $\lim_{x\to a} f(x)$ does not exist and so f is not continuous at $x = a$. If $l_1 = l_2$, then $\lim_{x\to a} f(x)$ exists.
3. If $\lim_{x\to a} f(x)$ exists and also $f(a)$ exists.

Then verify whether $\lim_{x\to a} f(x) = f(a)$.

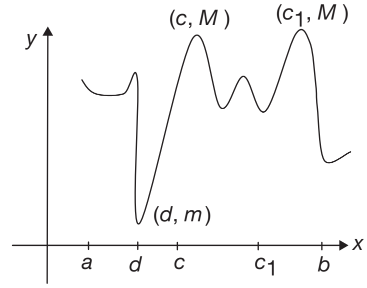
If $\lim_{x\to a} f(x) = f(a)$. Then f is continuous, otherwise it is not continuous at $x = a$.

Problems on continuous functions can be grouped into the following categories.

1. Using $\in$, δ notation.
2. Using existence of right and left hand limits.
3. To find the value of the unknown in $f(x)$ when f is given to be continuous at a point.
4. To find $f(a)$ when f is given to be continuous at $x = a$.

For functions that are continuous on (a, b) the following holds:

f is bounded and attains its bounds at least once on $[a, b]$, i.e., for some $c, d \in [a, b]$,
M = supremum of $f = f(c)$ and m = Infimum of $f = f(d)$



NOTE

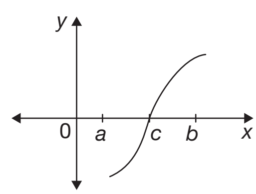
The converse may not be true as $f(x) = \begin{cases} 1; & 0 < x \le 1 \\ -1; & 1 < x \le 2 \end{cases}$ is bounded on [1, 2] but it is not continuous at $x = 1$.

Intermediate-value Theorem

If f is continuous on $[a, b]$ and $f(a) \neq f(b)$ then f takes every value between $f(a)$ and $f(b)$.

Equivalently, if f is continuous on $[a, b]$ and $f(a) < k < f(b)$ or $f(b) < k < f(a)$, then there exists $c \in (a, b)$ such that $f(c) = k$.

Equivalently, If $f(a)$ and $f(b)$ are of opposite signs then there exists $c \in (a, b)$ such that $f(c) = 0$.



$f(a) < 0$ and $f(b) > 0$, clearly $f(c) = 0$.

NOTES

1. If $f(x)$ is continuous in $[a, b]$ then f takes all values between m and M at least once as x moves from a to b, where M = Supremum of f on $[a, b]$ and m = infimum of f on $[a, b]$.
2. If $f(x)$ is continuous in $[a, b]$, then $|f|$ is also continuous on $[a, b]$, where $|f|(x) = |f(x)|\ x \in [a, b]$.
3. Converse may not be true

For instance, $f(x) = \begin{cases} 1; & 0 < x \le 3 \\ -1; & 3 < x \le 5 \end{cases}$

is not continuous at $x = 3$, but $|f|(x) = 1x \in [0, 5]$, being a constant function is continuous [0, 5].

Inverse-function Theorem

If f is a continuous one-to-one function on $[a, b]$ then f^{-1} is also continuous on $[a, b]$.

Uniform Continuity A function f defined on an interval I is said to be uniformly continuous on I if given $\in > 0$ there exists a $\delta > 0$ such that if x, y are in I and $|x - y| < \delta$ then $|f(x) - f(y)| < \in$.

NOTE

Continuity on $[a, b]$ implies uniform continuity whereas continuity on (a, b) does not mean uniform continuity.

Types of Discontinuity If f is a function defined on an interval I, it is said to have

(TD1) **a removable discontinuity** at $p \in I$, if $\lim_{x\to p} f(x)$ exists, but is not equal to $f(p)$.

(TD2) **a discontinuity of first kind from the left** at p if $\lim_{x\to p^-} f(x)$ exists but is not equal to $f(p)$.

(TD3) **a discontinuity of first kind from the right** at p if $\lim_{x\to p^+} f(x)$ exists but is not equal to $f(p)$.

(TD4) **a discontinuity of first kind** at p if $\lim_{x\to\infty} f(x)$ and $\lim_{x\to p^+} f(x)$ exists but they are unequal.

(TD5) **a discontinuity of second kind from the left** at p if $\lim_{x\to p-} f(x)$ does not exist.

(TD6) **a discontinuity of second kind from the right** at p if $\lim_{x\to p^+} f(x)$ does not exist.

(TD7) **a discontinuity of second kind** at p if neither $\lim_{x\to p^-} f(x)$ nor $\lim_{x\to p^+} f(x)$ exist.

Examples for each type are presented in the following table:

Type	Example	Point of Discontinuity
TD1	$f(x) = \dfrac{x^2-1}{x-1}, x \neq$ $f(1) = 3$	$x = $ i
TD2	$f(x) = x + 3$ for $0 < x < 1$ $f(x) = 5$ for $x \geq 1$	$x = 1$
TD3	$f(x) = x + 3$, for $x > 2$ $f(x) = 8$ for $x \leq 2$	$x = 2$
TD4	$f(x) = \begin{cases} x+3; & x > 2 \\ 7; & x = 2 \\ x-3; & x < 2 \end{cases}$	$x = 2$
TD5	$f(x) = \tan x$ for $x < \pi/2$ $f(x) = 1$, for $x \geq \pi/2$	$x = \dfrac{\pi}{2}$
TD6	$f(x) = 1$, for $x \leq \pi/2$ $f(x) = \tan x$ for $x > \pi/2$	$x = \dfrac{\pi}{2}$
TD7	$f(x) = 1/x$ at $x \neq 0$ $f(0)$ $= 3$ at $x = 0$	$x = 0$

NOTES

1. Every differentiable function is continuous, but the converse is not true.
   The example of a function which is continuous but not differentiable at a point $f(x) = |x - 3|$ for $x \in R$ is continuous at $x = 3$, but it is not differentiable at $x = 3$.
2. The function may have a derivative at a point, but the derivative may not be continuous.
   For example the function
   $$f(x) = \begin{cases} x^3 \sin\dfrac{1}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$$ has the derivative function
   as
   $$f'(x) = \begin{cases} 3x^2 \sin\dfrac{1}{x} - x\cos\dfrac{1}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$$
   However $\lim_{x\to 0} f'(x)$ doesn't exist.

SOLVED EXAMPLES

Example 1

Discuss the continuity of the function at $x = 1$ where $f(x)$ is defined by

$$f(x) = \frac{3x-2}{x} \text{ for } 0 < x \leq 1$$
$$= \frac{\sin(x-1)}{(x-1)} \text{ for } x > 1$$

Solution

Consider the left and right handed limits

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} \frac{3x-2}{x} = 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} \frac{\sin(x-1)}{x-1}$$

$$= \lim_{(x-1)\to 0} \frac{\sin(x-1)}{(x-1)} = 1 \text{ and } f(1)$$

$$= \frac{3(1)-2}{1} = 1$$

$$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$

$\therefore f$ is continuous at $x = 1$.

Example 2

If $f(x) = \dfrac{(2^x-1)^2}{(\sin 2x)\log(1+x)}$ for $x \neq 0$ and $f(x) = \log 2$ for $x = 0$, discuss the continuity at $x = 0$.

Solution

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{(2^x-1)}{(\sin 2x)\log(1+x)}$$

$$= \lim_{x\to 0}\frac{\left(\frac{2^x-1}{x}\right)^2}{\frac{\sin 2x}{2x}(2)\frac{\log(1+x)}{x}}$$

$$= \lim_{x\to 0}\frac{\left(\frac{2^x-1}{x}\right)^2}{2\left(\frac{\sin 2x}{2x}\right)\log(1+x)^{\frac{1}{x}}}$$

$$= \frac{1}{2}\frac{\lim\limits_{x\to 0}\left(\frac{2^x-1}{x}\right)^2}{\left(\lim\limits_{x\to 0}\frac{\sin 2x}{2x}\right)\left(\log\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}\right)}$$

$$= \frac{1}{2}(\log 2)^2.$$

But given $f(x) = 2\log 2$ at $x = 0$

$\therefore \lim\limits_{x\to 0} f(x) \neq f(0)$

$\therefore f(x)$ is not continuous at $x = 0$.

Example 3

Find the value of k if

$$f(x) = \frac{2x^3-5x^2+4x+11}{x+1},\ \text{for } x \neq -1$$

And $f(-1) = k$ is continuous at $x = -1$.

Solution

Given $f(x)$ is continuous at $x = -1$

$$\Rightarrow \lim_{x\to -1} f(x) = f(-1) = k.$$

$$\Rightarrow \lim_{x\to -1} f(x) \lim_{x\to -1}\left[\frac{2x^3-5x^2+4x+11}{x+1}\right]$$

$$= \lim_{x\to -1}\frac{(x+1)(2x^2-7x+11)}{x+1}$$

$$= 2(-1)^2 - 7(-1) + 11$$

$$= 2 + 7 + 11 = 20$$

$$\therefore k = 20$$

Example 4

If $f(x) = \frac{x-4}{|x-4|}+a,$ for $x < 4$, $= a+b$ for $x = 4$, $= \frac{x-4}{|x-4|}+b,$ for $x > 4$

And $f(x)$ is continuous at $x = 4$, then find the values of a and b.

Solution

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-}\frac{x-4}{|x-4|}+a$$

$$= \lim_{x\to 4^-}\frac{(x-4)}{-(x-4)}+a = -1+a$$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}\frac{x-4}{|x-4|}+b$$

$$= \lim_{x\to 4^+}\frac{x-4}{(x-4)}+b = 1+b$$

Since given $f(x)$ is continuous at $x = 4$

$$\lim_{x\to 4^-} f(x) = f(4) = \lim_{x\to 4^+} f(x)$$

$\Rightarrow -1 + a = a + b = 1 + b \Rightarrow a = 1, b = -1$

Example 5

Examine the continuity of the given function at origin where,

$$f(x) = \begin{cases} \dfrac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Solution

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} = 0$$

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0^+}\frac{x}{e^{-1/x}+1} = 0$$

Then,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 0$$

Thus the function is continuous at the origin.

Derivatives

In this section we will look at the simplistic form of the definition of a derivative, the derivatives of certain standard functions and application of derivatives.

For a function $f(x)$, the ratio $\frac{[f(a+h)-f(a)]}{h}$ is the rate of change of $f(x)$ in the interval $[a, (a + h)]$.

The limit of this ratio as h tends to zero is called the derivative of $f(x)$. This is represented as $f'(x)$, i.e.,

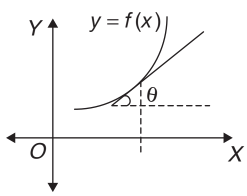
$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = f'(x)$$

The derivative $f'(x)$ is also represented as $\frac{d\{f(x)\}}{dx}$ or $\frac{d}{dx}\{f(x)\}$

Hence, if $y = f(x)$, i.e., y is a function of x, then $\frac{dy}{dx}$ is the derivative of y with respect to x.

NOTES

1. $\frac{dy}{dx}$ is the rate of change of y with respect to x.
2. If the function y can be represented as a general curve, and a tangent is drawn at any point where the tangent makes an angle θ with the horizontal (as shown in the figure), then $\frac{dy}{dx} = \tan\theta$, In other words, derivative of a function at a given point is the slope of the curve at that point, i.e., tans of the angle, the tangent drawn to the curve at that point, makes with the horizontal.



Standard Results

If $f(x)$ and $g(x)$ are two functions of x and k is a constant, then

1. $\frac{d}{dx}(c) = 0$ (c is a constant)

2. $\frac{d}{dx}k \cdot f(x) = k\frac{d}{dx}f(x)$ (k is a constant)

3. $\frac{d}{dx}(f(x) \pm g(x))$

$$= \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$

Product Rule

4. $\frac{d}{dx}\{f(x)\cdot g(x)\} = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

Quotient Rule

5. $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}$

Chain Rule

6. If $y = f(u)$ and $u = g(x)$ be two functions, then $\frac{dy}{dx} = \left(\frac{dy}{du}\right)\times\left(\frac{du}{dx}\right)$

Derivatives of Some Important Functions

1. (a) $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

   (b) $\frac{d}{dx}\left[\frac{1}{x^n}\right] = \frac{-n}{x^{n+1}}$

   (c) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$; $x \neq 0$

2. $\frac{d}{dx}[ax^n + b] = an \cdot x^{n-1}$

3. $\frac{d}{dx}[ax + b]^n = n\,a\,(ax + b)^{n-1}$

4. $\frac{d}{dx}[e^{ax}] = a \cdot e^{ax}$

5. $\frac{d}{dx}[\log x] = \frac{1}{x}$; $x > 0$

6. $\frac{d}{dx}[a^x] = a^x \log a$; $a > 0$

7. (a) $\frac{d}{dx}[\sin x] = \cos x$

   (b) $\frac{d}{dx}[\cos x] = -\sin x$

   (c) $\frac{d}{dx}[\tan x] = \sec^2 x$

   (d) $\frac{d}{dx}[\cot x] = -\text{cosec}^2 x$

   (e) $\frac{d}{dx}[\sec x] = \sec x \cdot \tan x$

   (f) $\frac{d}{dx}[\text{cosec } x] = -\text{cosec } x \cdot \cot x$

Inverse Rule

If $y = f(x)$ and its inverse $x = f^{-1}(y)$ is also defined, then $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$.

Second Derivative

If $y = f(x)$, then the derivative of derivative of y is called as second derivative of y and is represented by $\frac{d^2y}{dx^2}$.

$\frac{d^2y}{dx^2} = f''(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ where $\frac{dy}{dx}$ is the first derivative of y.

8. (a) $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

   (b) $\frac{d}{dx}\text{cosec}^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$

   (c) $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

   (d) $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

(e) $\dfrac{d}{dx}\tan^{-1}x = \dfrac{1}{1+x^2}$

(f) $\dfrac{d}{dx}\cot^{-1}x = \dfrac{-1}{1+x^2}$

9. (a) $\dfrac{d}{dx}\sinh x = \cosh x$

(b) $\dfrac{d}{dx}\cosh x = \sinh x$

(c) $\dfrac{d}{dx}\tanh x = \text{sech}^2\, x$

(d) $\dfrac{d}{dx}\coth x = -\text{cosech}^2 x$

(e) $\dfrac{d}{dx}\text{sech}\, x = -\text{sech}\, x \tanh x$

(f) $\dfrac{d}{dx}\text{cosech}x = -\text{cosech}x \coth x$

10. (a) $\dfrac{d}{dx}\sinh^{-1}x = \dfrac{1}{\sqrt{1+x^2}}$

(b) $\dfrac{d}{dx}\cosh^{-1}x = \dfrac{1}{\sqrt{x^2-1}}$

(c) $\dfrac{d}{dx}\tanh^{-1}x = \dfrac{1}{1-x^2}$

(d) $\dfrac{d}{dx}\coth^{-1}x = \dfrac{-1}{x^2-1}$

(e) $\dfrac{d}{dx}\text{sech}^{-1}x = \dfrac{-1}{x\sqrt{1-x^2}}$

(f) $\dfrac{d}{dx}\text{cosech}^{-1}x = \dfrac{-1}{x\sqrt{x^2+1}}$

Successive Differentiation

If f is differentiable function of x and the derivative f' is also a differentiable function of x, then f'' is called the second derivative of f. Similarly 3rd, 4th ... nth derivative of f may be defined and are denoted by f''', f'''', ... f^n or $y_3, y_4 \dots y_n$.

11. The n^{th} derivatives of some special functions:

(a) $\dfrac{d^n}{dx^n}x^n = n\,!$

(b) $\dfrac{d^n}{dx^n}x^m = \dfrac{m!}{(m-n)}x^{m-n}$ s(m being a positive integer more than n)

(c) $\dfrac{d^n}{dx^n}e^{ax} = a^n\, e^{ax}$

(d) $\dfrac{d^n}{dx^n}\left(\dfrac{1}{x-a}\right) = \dfrac{(-1)^n n!}{(x+a)^{n+1}}; x \neq -a$

(e) $\dfrac{d^n}{dx^n}\log(x+a) = \dfrac{(-1)^{n-1}(n-1)!}{(x+a)}$; $(x+a) > 0$

(f) $\dfrac{d^n}{dx^n}\sin(ax+b) = a^n \sin\left(\dfrac{n\pi}{2}+ax+b\right)$

(g) $\dfrac{d^n}{dx^n}\cos(ax+b) = a^n \cos\left(\dfrac{n\pi}{2}+ax+b\right)$

(h) $\dfrac{d^n}{dx^n}(e^{ax}\sin bx)$

$= (a^2+b^2)^{n/2}\, e^{ax} \sin\left(bx + n\tan^{-1}\dfrac{b}{a}\right)$

(i) $\dfrac{d^n}{dx^n}(e^{ax}\cos bx)$

$= (a^2+b^2)^{n/2}\, e^{ax} \cos\left(bx + n\tan^{-1}\dfrac{b}{a}\right)$

(j) $\dfrac{d^n}{dx^n}\left(\dfrac{1}{x^2+a^2}\right) = \dfrac{(-1)^n n}{a^{n+2}}\sin^{n+1}\theta \sin(n+1)\theta$

where $\theta = \tan^{-1}\left(\dfrac{x}{a}\right)$

(k) $\dfrac{d^n}{dx^n}(\tan^{-1}x) = (-1)^{n-1}(n-1)!\, \sin^n\theta \cdot \sin n\theta$

where $\theta = \cot^{-1}x$.

Application of Derivatives

Errors in Measurement

Problems relating to errors in measurement can be solved using the concept of derivatives. For example, if we know the error in measurement of the radius of a sphere, we can find out the consequent error in the measurement of the volume of the sphere. Without going into further details of theory, we can say dx = error in measurement of x and dy = consequent error in measurement of y, Where $y = f(x)$. Hence, we can rewrite $\dfrac{dy}{dx} = f'(x)$ as $dy = f'(x) \cdot dx$.

Thus, if we know the function $y = f(x)$ and dx, error in measurement of x, we can find out dy, the error in measurement of y.

NOTES

1. An error is taken to be positive when the measured value is greater than the actual value and negative when it is less.
2. Percentage error in y is given by $\left(\dfrac{dy}{y}\right)\times 100$.

Rate of Change

While defining the derivative, we have seen that derivative is the 'rate of change'. This can be applied to motion of bodies to determine their velocity and acceleration.

Velocity If we have s, the distance covered by a body expressed as a function of t, i.e., $s = f(t)$, then rate of change of s is called velocity (v). $v = \dfrac{ds}{dt} = f'(t)$.

Acceleration Rate of change of velocity is defined as acceleration. Since $v = f'(t)$ itself is a function of t, we can write $v = f'(t)$.

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$, i.e., acceleration is the second derivative of the function $s = f(t)$.

Maxima and Minima

A function takes a maximum value or a minimum value when the slope of the tangent of the curve at that point is zero, i.e., when the first derivative of the function is zero. If $y = f(x)$, then y is maximum or minimum at the point $x = x_1$ if $\left(\frac{dy}{dx}\right)_{x=x_1} = 0$.

Thus we can find the value of x_1 by equating $\frac{dy}{dx} = 0$.

As mentioned above that y can have a maximum or a minimum value at $x = x_1$. Whether y is a maximum value or minimum is governed by the sign of the second derivative. The function y has a minimum value if the second derivative is positive. In other words, y is maximum at $x = x_1$ if $\frac{d^2y}{dx^2} < 0$ at $x = x_1 \cdot y$ is minimum at $x = x_1$ if $\frac{d^2y}{dx^2} > 0$ at $x = x_1 \cdot \left(\frac{dy}{dx}\right)_{x=x_1} = 0.$ in both the cases discussed above.

The above discussion can be summerized as follows:

1. If $f'(c) = 0$ and $f''(c)$ is negative, then $f(x)$ is maximum for $x = c$
2. If $f'(c) = 0$ and $f''(c)$ is positive, then $f(x)$ is minimum for $x = c$
3. If $f'(c) = f''(c) = \cdots = f^{r-1}(c) = 0$ and $f^r(c) \neq 0$, then
   (a) If r is even, then $f(x)$ is maximum or minimum for $x = c$ according as $f^r(c)$ is negative or positive.
   (b) If r is odd, then there is neither maximum nor a minimum for $f(x)$ at $x = c$.

Mean Value Theorems

Rolle's Theorem Let f be a function defined on $[a, b]$ such that

1. f is continuous on $[a, b]$;
2. f is differentiable on (a, b) and
3. $f(a) = f(b)$, then there exists $c \in (a, b)$ such that $f'(c) = 0$

Lagrange's Mean Value Theorem Let f be a function defined on $[a, b]$ such that

1. f is continuous on $[a, b]$,
2. f is differentiable on (a, b) then there exists $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Another Form If f is defined on $[a, a + h]$ such that

1. f is continuous on $[a, a + h]$.
2. f is differentiable on $(a, a + h)$ then there exists atleast one $\theta \in (0, 1)$ such that $f(a + h) = f(a) + hf'(a + \theta h)$.

Meaning of the sign of the derivative

SIGN OF $f'(x)$ on [a, b]	Meaning
$f'(x) \geq 0$	f is non-decreasing
$f'(x) > 0$	f is increasing
$f'(x) < 0$	f is non-increasing
$f'(x) < 0$	f is decreasing
$f'(x) = 0$	f is constant

Example: The function f, defined on R by $f(x) = x^3 - 15x^2 + 75x - 125$ is non-decreasing in every interval as $f'(x) = 3(x^2 - 10x + 15) = 3(x - 5)^2 \geq 0$

Thus f is non-decreasing on R.

Cauchy's Mean Value Theorem Let f and g be two functions defined on $[a, b]$ such that

1. f and g are continuous on $[a, b]$
2. f and g are differentiable on (a, b)
3. $g'(x) \neq 0$ for any $x \in (a, b)$ then there exists at least one real number $c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}.$$

Taylor's Theorem

Let f be a real-valued function defined on $[a, a + h]$ such that

1. f^{n-1} is continuous on $[a, a + h]$
2. f^{n-1} is derivable on $(a, a + h)$, then there exists a number $\theta \in (0, 1)$ such that

$$f(a + h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \cdots$$

$$+ \frac{h^{n-1}}{(n-1)} f^{n-1}(a) + R_n.$$

Where

$$R_n = \frac{h^n f^n(a + \theta h)}{n!}$$

(Lagranges' form of remainder)

$$R_n = \frac{h^n (1 - \theta)^{n-1} f^n(a + \theta h)}{(n-1)!}$$

(Cauchy's form of remainder)

Maclaurin's Theorem Let $f \cdot [0, x] \to R$ such that

1. f^{n-1} is continuous on $[0, x]$,
2. f^{n-1} is derivable on $(0, x)$

Then there exists a real number $\theta \in (0, 1)$ such that

$$f(x) = f(0) + xf'(0) + \frac{x^2}{n!} f''(0) + \cdots$$

$$+ \frac{x^{n-1}}{(n-1)!} f^{(n-1)}(0) + R_n.$$

Where

$$R_n = \frac{x^n}{n!} f^n(\theta x)$$

(Lagranges form of remainder)

$$R_n = \frac{x^n (1-\theta)^{n-1} f^n(\theta x)}{(n-1)!}$$

(Cauchy's form of remainder)

Maclaurin's Series Let $f(x)$ be a function which posses derivatives of all orders in the interval $[0, x]$, then

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \cdots + \frac{x^{n-1}}{(n-1)} f^{(n-1)}(0)$$

$$+ \frac{x^n}{n!} f^n(0) + \cdots \text{ is known as}$$

Maclaurin's infinite series.

Series expansions of some standard functions

1. $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots + \dfrac{x^n}{n!} + \cdots$
2. $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \cdots + \dfrac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$
3. $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \cdots + \dfrac{(-1)^n x^{2n}}{(2n)!} + \cdots$
4. $\sinh x = x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \cdots + \dfrac{x^{2n+1}}{(2n+1)!} + \cdots$
5. $\cosh x = 1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \cdots + \dfrac{x^{2n}}{(2n)!} + \cdots$
6. $\log(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} - \cdots + \dfrac{(-1)^{n-1} x^n}{n}$
7. $(1 + x)^{-1} = 1 - x + x^2 - x^3 + \cdots$
8. $(1 - x)^{-1} = 1 + x + x^2 + x^3 + \cdots$
9. $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$
10. $(1-x)^{-\frac{1}{2}} = 1 + \dfrac{x}{2} + \dfrac{1 \cdot 3}{2 \cdot 3} x^2 + \dfrac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot x^3 + \cdots$
11. $\tan^{-1} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \cdots + \dfrac{(-1)^{n-1}}{(2n-1)} x^{2n-1} + \cdots$
12. $\sin^{-1} x = x + \dfrac{1}{2} \cdot \dfrac{x^3}{3} + \dfrac{1 \cdot 3}{2 \cdot 4} \cdot \dfrac{x^5}{5} + \cdots$

Example 6

For the function $f(x) = x(x^2 - 1)$ test for the applicability of Rolle's theorem in the interval $[-1, 1]$ and hence find c such that $-1 < c < 1$.

Solution

Given $f(x) = x(x^2 - 1)$

1. f is continuous in $[-1, 1]$
2. f is differentiable in $(-1, 1)$
3. $f(-1) = f(1) = 0$

$\therefore$ $f(x)$ satisfies the hypothesis of Rolle's theorems
$\therefore$ We can find a number c such that $f'(c) = 0$, i.e., $f'(x) = 3x^2 - 1$

$$f'(c) = 0 \quad \Rightarrow \quad 3c^2 - 1 = 0 \quad \Rightarrow \quad c = \pm\sqrt{\frac{1}{3}}$$

$$\Rightarrow \quad c = \sqrt{\frac{1}{3}}$$

Example 7

If $f(x) = 2x^2 + 3x + 4$, then find the value of θ in the mean value theorem.

Solution

$f(a) = 2a^2 + 3a + 4$

$f(a + h) = 2(a^2 + 2ah + h^2) + 3a + 3h + 4$

$f(a + h) - f(a) = 4ah + 2h^2 + 3h = 2(2ah + h^2) + 3h$

$$\frac{f(a+h) - f(a)}{h} = 2(2a + h) + 3$$

$$= 4\left(a + \frac{h}{2}\right) + 3 \qquad (1)$$

Now $f'(x) = 4x + 3$, $f^1(a + \theta h)$

$= 4a + 4h\theta + 3$ (2)

Comparing Eqs. (1) and (2) we have $4\left(a + \dfrac{h}{2}\right) + 3$

$$= 4a + 4h\theta + 3 \quad \Rightarrow \quad a + h\theta = a + \frac{h}{2}$$

$$\Rightarrow \quad \theta = \frac{1}{2}$$

Partial Differentiation

Let u be a function of two variables x and y. Let us assume the functional relation as $u = f(x, y)$. Here x alone or y alone

or both x and y simultaneously may be varied and in each case a change in the value of u will result. Generally the change in the value of u will be different in each of these three cases. Since x and y are independent, x may be supposed to vary when y remains constant or the reverse.

The derivative of u wrt x when x varies and y remains constant is called the partial derivative of u wrt x and is denoted by $\frac{\partial u}{\partial x}$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right), \quad \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right).$$

Total Differential Co-efficient

If u be a continuous function of x and y and if x and y receive small increments Δx and Δy, u will receive, in turn, a small increment Δu. This Δu is called total increment of u.

$\Delta u = f(x + \Delta x, y + \Delta y) - f(x, y)$

In the differential form, this can be written as

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy.$$

du is called the total differential of u. If $u = f(x, y, z)$ then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \cdot\frac{\partial u}{\partial z}\cdot\frac{dz}{dt}$$

Implicit Function

If the relation between x and y be given in the form $f(x, y) = c$ where c is a constant, then the total differential co-efficient wrt x is zero.

Homogeneous Functions

Let us consider the function $f(x, y) = a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \cdots + a_ny^n$. In this expression the sum of the indices of the variable x and y in each term is n. Such an expression is called a homogeneous function of degree n.

Euler's Theorem

If $f(x, y)$ is a homogeneous function of degree n, then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf.$

This is known as Euler's theorem on homogeneous function.

Maxima and Minima for Function of Two Variables

A function $f(x, y)$ is said to have a local maximum at a point (a, b), if $f(a + h, b + k) \le f(a, b)$ for all small values of h and k, i.e., $f(x, y)$ has a local maximum at (a, b), if $f(a, b)$ has a highest value in a neighbourhood of (a, b).

Similarly, $f(x, y)$ is said to have a local minimum at a point (a, b), if $f(x, y)$ has least value at (a, b) in a neighbourhood of (a, b).

Procedure to Obtain Maxima and Minima

Let $f(x, y)$ be a function of two variables for which we need to find maxima and minima.

1. Find $f_x = \frac{\partial f}{\partial x}$ and $f_y = \frac{\partial f}{\partial y}$

2. Take $f_x = 0$ and $f_y = 0$ and solve them as simultaneous equations to get pairs of values for x and y, which are called stationary points.

3. Find $r = f_{xx} = \frac{\partial^2 f}{\partial x^2}$, $\quad s = f_{xy} = \frac{\partial^2 f}{\partial x \partial y}$ and

$t = f_{yy} = \frac{\partial^2 f}{\partial y^2}$ and find $rt - s^2$.

4. At a stationary point, say (a, b)

(a) If $rt - s^2 > 0$, then (a, b) is called an extreme point of $f(x, y)$ at which $f(x, y)$ has either maximum or minimum which can be found as follows.

Case 1: If $r < 0$, then $f(x, y)$ has a local maximum at $a, b)$

Case 2: If $r > 0$, then $f(x, y)$ has a local minimum at (a, b).

(b) If $rt - s^2 < 0$, then (a, b) is called as saddle point of $f(x, y)$ where $f(x, y)$ has neither maximum nor minimum at (a, b).

Example 8

Find the stationary points of the function $f(x, y) = x^2y + 3xy - 7$ and classify them into extreme and saddle points.

Solution

Given $f(x, y) = x^2y + 3xy - 7$

$\therefore f_x = \frac{\partial f}{\partial x} = 2xy + 3y$ and $f_y = \frac{\partial f}{\partial y} = x^2 + 3x$

Now $f_x = 0 \Rightarrow 2xy + 3y = 0$ and $f_y = 0$

$\Rightarrow x^2 + 3x = 0$

$\Rightarrow \quad y = 0$ and $x = \frac{-3}{2}$; $x(x + 3)x = 0$ and $x = -3$

But for $x = \frac{3}{2}$, $f_y \ne 0$

$\therefore$ The stationary points of $f(x, y)$ are $(0, 0)$ and $(-3, 0)$

Now $r = f_{xx} = 2y$; $s = f_{xy} = 2x + 3$ and $t = f_{yy} = 0$

And $rt - s^2 = 2y \times 0 - (2x + 3)^2 = -(2x + 3)^2$

$\therefore rt - s^2 < 0$ at $(0, 0)$ as well as $(-3, 0)$

Hence the two stationary points $(0, 0)$ and $(-3, 0)$ are saddle points where $f(x, y)$ has neither maximum nor minimum.

Example 9

Find the maximum value of the function $f(x, y, z) = z - 2x^2 - 3y^2$ where $3xy - z + 7 = 0$.

Solution

Given $f(x, y, z) = z - 2x^2 - 3y^2$ (1)

Where $3xy - z + 7 = 0$ (2)

$\Rightarrow z = 3xy + 7$ (3)

Substituting the value of z in (1), we have $f = 3xy + 7 - 2x^2 - 3y^2$

$$\therefore f_x = \frac{\partial f}{\partial x} = 3y - 4x \text{ and } f_y = \frac{\partial f}{\partial y} = 3x - 6y$$

$f_x = 0 \Rightarrow 3y - 4x = 0$ and $f_y = 0 \Rightarrow 3x - 6y = 0$

$f_x = 0$ and $f_y = 0$ only when $x = 0$ and $y = 0$

$\therefore$ The stationary point is (0, 0)

Now $r = f_{xx} = \dfrac{\partial^2 f}{\partial x^2} = -4;\ s = f_{xy} = \dfrac{\partial^2 f}{\partial x \partial y} = 3$ and

$t = f_{yy} = \dfrac{\partial^2 f}{\partial y^2} = -6$

$\therefore rt - s^2 = (-4)(-6) - 3^2 = 24 - 9 = 15 > 0$ and $r = -4 < 0$

$\therefore f$ has a maximum value at (0, 0)

For $x = 0, y = 0$, from (3), $z = 3 \times 0 \times 0 + 7 \Rightarrow z = 7$

$\therefore$ The maximum value exists for $f(x, y, z)$ at (0, 0, 7) and that maximum value is $f(x, y, z)_{at\ (0,0,7)} = 7 - 2 \times 0^2 - 3 \times 0^2 = 7$.

Indefinite Integrals

If $f(x)$ and $g(x)$ are two functions of x such that $g'(x) = f(x)$, then the integral of $f(x)$ is $g(x)$. Further, $g(x)$ is called the antiderivative of $f(x)$.

The process of computing an integral of a function is called Integration and the function to be integrated is called integrand.

An integral of a function is not unique. If $g(x)$ is any one integral of $f(x)$, then $g(x) + c$ is also its integral, where C is any constant termed as constant of integration.

Some Standard Formulae

1. $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c\ (n \neq -1)$
2. $\int (ax+b)^n dx = \dfrac{(ax+b)^{n+1}}{(n+1)a} + c\ (n \neq -1)$
3. $\int \dfrac{1}{x} dx = \log x + c$
4. $\int \dfrac{1}{ax+b} dx = \dfrac{\log(ax+b)}{a} + c$
5. $\int a^x dx = \dfrac{a^x}{\log a} + c$
6. $\int e^x dx = e^x + c$
7. $\int \sin x\, dx = -\cos x + c$
8. $\int \cos x\, dx = \sin x + c$
9. $\int \sec^2 x\, dx = \tan x + c$
10. $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$
11. $\int \sec x \tan x\, dx = \sec x + c$
12. $\int \operatorname{cosec} x \cot x\, dx = \operatorname{cosec} x + c$
13. $\int \tan x\, dx = \log(\sec x) + c$
14. $\int \cot x\, dx = \log(\sin x) + c$
15. $\int \sec x\, dx = \log(\sec x + \tan x) + c$

    $= \log\tan\left[\dfrac{\pi}{4} + \dfrac{x}{2}\right] + c$
16. $\int \operatorname{cosec} x\, dx = \log(\operatorname{cosec} x + \cot x) + c$

    $= \log\tan\dfrac{x}{2} + c$
17. $\int \dfrac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$ or $-\cos^{-1} x + c$
18. $\int \dfrac{1}{1+x^2} dx = \tan^{-1} x + c$ or $-\cot^{-1} x + c$
19. $\int \dfrac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$ or $-\operatorname{cosec}^{-1} x + c$
20. $\int \sinh x\, dx = \cosh x + c$
21. $\int \cosh x\, dx = \sinh x + c$
22. $\int \operatorname{sech}^2 x\, dx = \tanh x + c$
23. $\int \operatorname{cosech}^2 x\, dx = -\coth x + c$
24. $\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x + c$
25. $\int \operatorname{sech} x \coth x\, dx = -\operatorname{cosech} x + c$
26. $\int K f(x) dx = K \int f(x) dx + c$
27. $\int (f(x) \pm g(x))\, dx = \int f(x)\, dx \pm \int g(x)\, dx + c$
28. $\int \dfrac{f'(x)}{f(x)} dx = \log[f(x)] + c$
29. $\int f(x)^n \cdot f'(x) dx = \dfrac{[f(x)]^{n+1}}{n+1} + c$

30. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+c$

31. $\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\frac{x}{a}+c$ or

$\log|x+\sqrt{a^2+x^2}|+c$

32. $\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\frac{x}{a}+c$ or

$\log|x+\sqrt{x^2+a^2}|+c$

33. $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

34. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$

35. $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

36. $\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2+x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$

37. $\int \sqrt{a^2+x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin h^{-1}\frac{x}{a}+c$

38. $\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\cos h^{-1}\frac{x}{a}+c$

39. $\int \log x\,dx = x(\log x-1) = x\log\left(\frac{x}{e}\right)+c$

40. $\int e^x[f(x)+f'(x)]\,dx = e^x f(x)+c$

Definite Integrals

The difference in the values of an integral of a function $f(x)$ for two assigned values say a, b of the independent variable x, is called the Definite Integral of $f(x)$ over the interval $[a, b]$ and is denoted by $\int_a^b f(x)dx$.

The number 'a' is called the lower limit and the number 'b' is the upper limit of integration.

Fundamental Theorem of Integral Calculus

If $f(x)$ is a function of x continuous in $[a, b]$, then $\int_a^b f(x)dx = g(b)-g(a)$ where $g(x)$ is a function such that $\frac{d}{dx}g(x) = f(x)$.

Properties of definite integrals

1. If $f(x)$ is a continuous function of x over $[a, b]$, and c belongs to $[a,b]$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$.
2. If $f(x)$ is continuous function of x over $[a, b]$, then $\int_a^b Kf(x)dx = K\int_a^b f(x)dx$.
3. If $f(x)$ is continuous function of x over $[a, b]$, then $\int_b^a f(x)dx = -\int_a^b f(x)dx$.
4. If $f(x)$ is continuous in some neighbourhood of a, then $\int_a^a f(x)dx = 0$.
5. If $f(x)$ and $g(x)$ are continuous in $[a, b]$, then $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.
6. $\int_a^b f(x)dx = \int_a^b f(z)dz = \int_a^b f(t)dt$
7. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
8. $\int_{-a}^a f(x) = 0$, if $f(x)$ is odd
9. $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ if $f(x)$ is even
10. $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, if $f(2a-x) = f(x)$

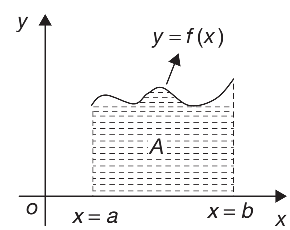
    $= 0$ if $f(2a-x) = -f(x)$
11. $\int_0^{na} f(x)dx = n\int_0^a f(x)dx$, if $f(a+x) = f(x)$

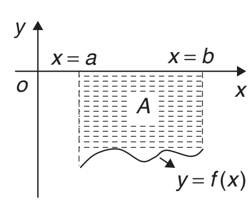
Applications of Integration

Area as a Definite Integral

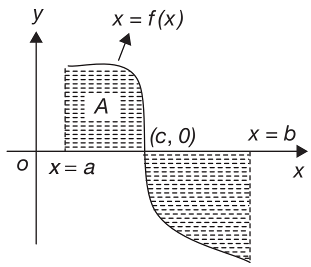
1. The area enclosed by a curve $y = f(x)$, the lines $x = a$ and $x = b$ and the x-axis is given by:

$$A = \int_a^b |f(x)|dx = \begin{cases} \int_a^b f(x)dx, & \text{if } f(x) \geq 0,\ a \leq x \leq b \\ -\int_a^b f(x)dx, & \text{if } f(x) \leq 0,\ a \leq x \leq b \end{cases}$$

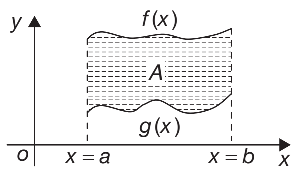


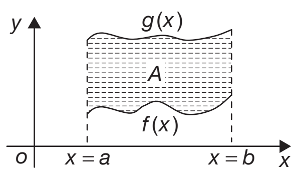


2. Similarly, the area enclosed by the curve $x = g(y)$, the lines $y = c$ and $y = d$ and the y-axis is $A = \int_c^d |g(y)|dy$
3. When $f(x) \geq 0$ for $a \leq x \leq c$ and $f(x) \leq 0$ for $c \leq x \leq b$, then the area enclosed by the curve $y = f(x)$, the lines $x = a$ and $x = b$ and the x-axis is $A = \int_a^c f(x)\,dx - \int_c^b f(x)\,dx$



4. The area enclosed by the curves $y = f(x)$ and $y = g(x)$ and the lines $x = a$ and $x = b$ is given by,





$$A = \int_a^b |f(x) - g(x)|dx = \begin{cases} \int_a^b (f(x) - g(x))dx, \ \text{if } f(x) \geq g(x), \\ \quad a \leq x \leq b \\ \int_a^b (g(x) - f(x))dx, \ \text{if } f(x) \leq g(x); \\ \quad a \leq x \leq b \end{cases}$$

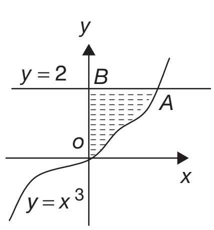
Example 10

Find the area enclosed by the curve $y = x^3$, the line $y = 2$ and the y-axis in first quadrant?

Solution

The area bounded by $y = x^3$, $y = 2$ and the y-axis is the area OAB as shown in the figure.

So, the region OAB is bounded by the curve $x = y^{\frac{1}{3}}$, the lines $y = 0$ and $y = 2$ and the y-axis and $x = y^{\frac{1}{3}} \geq 0,\ y \in [0,\ 2]$



$\therefore$ The required area

$$= \left[\int_{y=0}^{2} y^{\frac{1}{3}} dy = \frac{3}{4} y^{\frac{4}{3}} \right]_0^2$$

$$= \frac{3}{4} \times 2^{\frac{4}{3}}$$

$$= \frac{3}{2^{\frac{2}{3}}}$$

$$= \frac{3}{\sqrt[3]{4}}$$

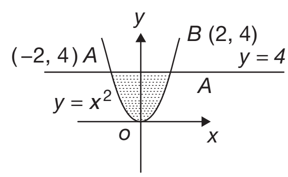
Example 11

Find the area enclosed by the curve $y = x^2$ and line $y = 4$?

Solution

The area enclosed by the curve $y = x^2$ and the line $y = 4$ is the region OAB.

$\therefore$ The region OAB is bounded by line $y = 4$ and the curve $y = x^2$ from $x = -2$ to $x = 2$ and $4 \geq x^2$ for all $x \in [-2, 2]$



$\therefore$ The required area $= \int_{x=-2}^{2} (4 - x^2)\,dx$

$$= 2\int_0^2 (4 - x^2)dx \ (\because \quad 4 - x^2 \text{as even})$$

$$= 2\left[4x - \frac{x^3}{3}\right]_0^2 = \frac{32}{3}$$

Rectification

The process of determining the length of arcs of plane curves is called Rectification. The length of the arc can be calculated by any one of the methods given below.

Cartesian Equations Let $y = f(x)$ be a function of x. The length of arc between the points with x-coordinates 'a' and 'b' is given by $S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$, provided $\frac{dy}{dx}$ is continuous on $[a, b]$.

NOTE

If the equation of the curve is given in the form $x = f(y)$, then the length of the arc between the points with y-coordinates 'c' and 'd' is given by

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy \text{ provided } \frac{dx}{dy} \text{ is continuous on } [c, d]$$

Parametric Equations Let $x = f(t)$ and $y = g(t)$ be parametric functions of 't'.The length of the arc between the points $\{f(t_1), g(t_1)\}$ and $\{f(t_2), g(t_2)\}$ is given by

$\int_{t_1}^{t_2} \sqrt{\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right]}\, dt$ provided $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are both continuous on $[t_1, t_2]$.

Polar Equations Let $r = f(\theta)$ be a function of θ, the length of the arc between the points $\{f(\theta_1), \theta_1\}$ and $\{f(\theta_2), \theta_2\}$ is given by $S = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ provided $\frac{dr}{d\theta}$ is continuous along the arc.

If the equation of the curve is given in the form $\theta = f(r)$, then the length of the arc between the points $(r_1, f(r_1))$, $(r_2, f(r_2))$ is given by

$S = \int_{r_1}^{r_2} \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\, dr$ provided $\frac{d\theta}{dr}$ is continuous along the arc.

Theorems on Integration

1. If f is a continuous function on $[a, b]$ then there exists $c \in (a, b)$ such that $\int_a^b f(x)\,dx = f(c)(b-a)$
2. If $f, g \in R\,[a, b]$ and g keeps the same sign on $[a, b]$ then there exists $\mu \in R$ lying between the infimum and the suprimum of f such that $\int_a^b f(x)g(x) = \mu \int_a^b g(x)dx$

NOTE

This is called the first mean value theorem.

3. If $f, g \in R\,[a, b]$, g is positive and decreasing on $[a, b]$ Then there exists $\mu \in [a, b]$ such that

$$\int_a^b f(x)g(x)\,dx = g(a)\int_a^\mu f(x)dx$$

NOTE

This is known as Bonnet mean value theorem.

4. If $f, g \in R\,[a, b]$ and is monotonic on $[a, b]$ then there exists $\mu \in (a, b)$ such that $\int_a^b f(x)g(x)\,dx = g(x)\int_a^\mu f(x)\,dx + g(x)\int_\mu^b f(x)\,dx$

NOTE

This is known as second mean value theorem or weierstrars theorem.

Example 12

Prove that these exists $\mu \in \left(0, \frac{\pi}{2}\right)$ such that

$$\int_0^{\frac{\pi}{2}} x\cos x\,dx = \mu$$

Solution

Take $f(x) = x$ and $g(x) = \cos x$

$\therefore$ f is continuous on $\left[0, \frac{\pi}{2}\right]$ and g is integrable on $\left[0, \frac{\pi}{2}\right]$ also

$$g(x) \geq 0 \text{ in } \left[0, \frac{\pi}{2}\right]$$

$\therefore$ By first mean value theorem,

$$\int_0^{\frac{\pi}{2}} x\cos dx = \mu\int_0^{\pi} \cos x\,dx = \mu$$

$\therefore$ There exists is $\mu \in \left(0, \frac{\pi}{2}\right)$ such that $\int_0^{\frac{\pi}{2}} \cos x\,dx = \mu$

Example 13

Verify second mean value theorem for $f(x) = x^2$ and $g(x) = x^2$ on $[-1, 1]$.

Solution

Given $f(x) = x^2$ and $g(x) = x^2$ on $[-1, 1]$ both f and g are continuous and integrable on $[-1, 1]$ but g is a decreasing function on $[-1, 0]$ and increasing function on $[0, 1]$ $\therefore$ g is not monotonic.

$$\therefore \int_{-1}^1 f(x)g(x)dx = \int_{-1}^1 x^2 \cdot x^2 dx$$

$$= \left[\frac{x^5}{5}\right]_{-1}^1 = \frac{1}{5} + \frac{1}{5} = \frac{2}{5} \quad (1)$$

But by second mean value theorem,

$$\int_a^b f(x)g(x)\,dx = g(a)\int_a^\mu f(x)dx + g(b)\int_\mu^b f(x)\,dx$$

$$\therefore \int_{-1}^1 x^4 dx = g(-1)\int_{-1}^\mu x^2 dx + g(1)\int_\mu^1 x^2 dx$$

$$= \int_{-1}^\mu x^2 dx + \int_\mu^1 x^2 dx = \int x^2 dx = \frac{2}{3} \quad (2)$$

Since (1) and (2) are not equal the mean value theorem does not hold.

Improper Integrals

Consider definite integral $\int_a^b f(x)dx$ (1)

If $f(x)$ is a function defined in a finite interval $[a, b]$ and $f(x)$ is continuous for all x which belongs to $[a, b]$

Then (1) is called proper integral.

If $f(x)$ is violated, at least one of these conditions then the integral is known as improper integral. These improper integrals are classified into three kinds.

Improper Integral of the First Kind In a definite integral if one or both limits of integration are infinite then it is an improper integral of first kind.

1. $\int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx.$

   (Singularity at upper limit)

2. $\int_{-\infty}^b f(x)dx = \lim_{a\to-\infty}\int_a^b f(x)dx.$

3. $\int_{-\infty}^\infty f(x)dx = \lim_{\substack{b\to\infty \\ a\to-\infty}} \int_a^b f(x)dx.$ Or

4. $\int_{-\infty}^{\infty} f(x)dx = \lim_{a\to-\infty}\int_a^0 f(x)dx + \lim_{b\to\infty}\int_0^b f(x)dx.$

Or $= \lim_{r\to\infty}\int_{-r}^{r} f(x)dx.$

Convergent: If the limits of the above integral exists or finite then the integral is said to be converge.

Divergent: If the limits do not exist then they are said to be Divergent.

NOTES

1. Geometrically for $f(x) \ge 0$, the improper integral $\int_a^{\infty} f(x)dx$ denotes the area of an unbounded region lying between the curve $y = f(x)$ the ordinate $x = a$ and x-axis.
2. Let $f(x)$ and $g(x)$ be non-negative functions and $0 \le f(x) \le g(x)$ for $x \ge a$. If $\int_a^{\infty} g(x)dx$ converges then $\int_a^{\infty} f(x)dx$ is also converges and $\int_a^{\infty} f(x)dx \le \int_a^{\infty} g(x)dx.$

   Similarly let $0 \le g(x) \le f(x)$. If $\int_a^{\infty} g(x)dx$ diverges then $\int_a^{\infty} f(x)dx$ also diverges.

   That is the convergent or divergent of an improper integral by comparing it with a simple integral.

Improper Integral of the Second Kind

Consider $$\int_a^b f(x)dx \qquad (1)$$

If both the limits of Eq. (1) are finite and $f(x)$ is undefined or discontinuous at a point in between a and b, then Eq. (1) is known as Improper integral of second kind.

This can be evaluated as follows.

Let $f(x)$ be undefined at a point c which belongs to (a, b) then

$$\int_a^b f(x)dx = \lim_{\epsilon\to 0}\int_a^{c-\epsilon} f(x)dx + \lim_{\epsilon\to 0}\int_{c+\epsilon}^{b} f(x)dx.$$

If these limits exist then it is convergent otherwise it is divergent.

Improper Integral of Third Kind If the limits of the integral are infinite or $f(x)$ may be discontinuous or both then the improper integral is known as third kind.

NOTES

1. $\int_1^{\infty} \frac{1}{x^p}dx$ is convergent when $p > 1$ and it is divergent when $p \le 1$. This result is used in comparison test for testing the convergence or divergence of the integral of first kind.
2. $\int_a^{c} \frac{1}{(x-c)^p}dx$ is convergent for $p < 1$ and is divergent for $p \le 1$. This is used for convergence or divergence of an improper integral of second kind.

Example 14

Examine $\int_1^{\infty} \frac{dx}{x^p}$ for convergence/divergence.

Solution

Consider $\int_1^k \frac{dx}{x^p} = \int_1^k x^{-p}dx = \left[\frac{x^{-p+1}}{-p+1}\right]_1^k$ if $p \ne 1$

And $\Rightarrow$ $[\log x]_1^k$ if $p = 1$

Case 1: If $p = 1$, $\int_1^k \frac{dx}{x} = \log k - \log 1 = \log k \to \infty$ when $k \to \infty$ it does not tend to a finite limit.

$\therefore$ It is divergent.

Case 2: If $p \ne 1 \int_1^k \frac{dx}{x^p} = \frac{1}{1-p}[k^{1-p}]$ it converges

If $p > 1$ and diverges if $p \le 1$.

Multiple integrals

Double Integrals: Integration of $f(x, y)$ over a region R in xy-plane is called a double integral.

$$\iint_R f(x, y)dR = \int_{x=x_1}^{x_2}\int_{y=y_1}^{y_2} f(x, y)dxdy$$

Order of Integration in a Double Integral Order of integration depends on the nature of limits of the variables.

Case 1: If the limits of y are function of x, say $y_1 = f_1(x)$ and $y_2 = f_2(x)$ and the limits of x are constants, say $x_1 = a$ and $x_2 = b$, where a and b are constants, then integrate wrt y first treating x as constant and then integrate wrt x.

$$\text{That is, } \iint_R f(x, y)dR = \int_{x_1=a}^{x_2=b}\left(\int_{y_1=f_1(x)}^{y_2=f_2(x)} f(x, y)dy\right)dx$$

Case 2: If the limits of x are function of y, say $x_1 = g_1(y)$ and $x_2 = g_2(y)$ and the limits of y are constants, say $y_1 = c$ and $y_2 = d$, then integrate wrt x first treating y as constant and then integrate wrt y.

$$\text{That is, } \iint_R f(x, y)dR = \int_{y_1=c}^{y_2=d}\left(\int_{x_1=g_1(y)}^{x_2=g_2(y)} f(x, y)dx\right)dy$$

Case 3: If both the variables x and y have constant limits, then one can follow any order of integration.

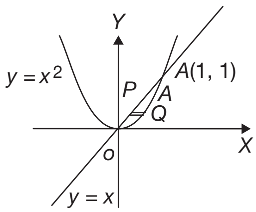
Change of Order of Integration Evaluation of some of the double integrals can be made simple by changing the order of integration. In change of order of integration, we take the limits of the variables for the given region of integration in such a way that the order of integration reverses.

Example 15

Evaluate $\int_{x=0}^{1}\int_{y=x^2}^{x}\frac{x}{y}e^{-\frac{x^2}{y}}\,dy\,dx$

Solution

Let $I = \int_{x=0}^{1}\int_{y=x^2}^{x}\frac{x}{y}e^{-\frac{x^2}{y}}\,dy\,dx$ (1)



Evaluation of this integral can be made simple by changing the order of integration.

From the limits of x and y given, the region of integration is the region bounded by the line $y = x$ and the parabola $y = x^2$ as shown in figure.

Now by changing the order of integration, we first integrate wrt x, along the horizontal strip PQ from $P(x = y)$ to $Q(x = \sqrt{y})$ and then

We integrate wrt y from $0(y = 0)$ to $A(y = 1)$

$$\therefore\ I = \int_{y=0}^{1}\left(\int_{x=y}^{\sqrt{y}}\frac{x}{y}e^{-\frac{x^2}{y}}\,dx\right)dy \qquad (2)$$

Put $\frac{x^2}{y} = t \quad\Rightarrow\quad \frac{2x}{y}dx = dt$

$$\Rightarrow\quad \frac{x}{y}dx = \frac{1}{2}dt$$

$$x = y \quad\Rightarrow\quad t = \frac{y^2}{y} = y \text{ and } x = \sqrt{y} \quad\Rightarrow\quad t\frac{(\sqrt{y})^2}{y} = 1$$

$\therefore$ Eq. (2) Becomes

$$I = \int_{y=0}^{1}\left(\int_{t=y}^{1}e^{-1}\frac{1}{2}dt\right)dy$$

$$= \int_{y=0}^{1}(-e^{-t})\Big]_{t=y}^{1}dy = \int_{y=0}^{1}[-e^{-1}+e^{-y}]dy$$

$$= -ye^{-1} - e^{-y}\Big]_0^1$$

$$= (-e^{-1} - e^{-1}) - (0 - e^{-0})$$

$$= 1 - 2e^{-1} = \frac{e-2}{e}$$

Triple Integrals Integration of a function $f(x, y, z)$ over a 3-dimensional region V is called the triple integral.

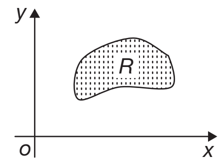
$$\iint_V\int f(x, yx, z)dv = \int_{x=x_1}^{x_2}\int_{y=y_1}^{y_2}\int_{z=z_1}^{z_2} f(x, y, z)dxdydz$$

Like double integrals, in triple integrals also the order of integration depends on the nature of the limits of the variables.

Applications of Double and Triple Integrals

1. Area of the region R in xy-plane is given by

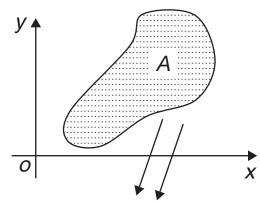
Area of $R = \int_R\int dxdy$



2. Volume of the solid of revolution:

(a) The volume of the solid of revolution obtained by revolving the area A about x-axis is

$$\text{Volume} = V = \int_A\int 2\pi y\,dx\,dy$$

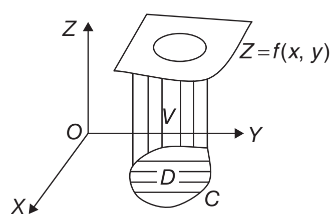


(b) The volume of the solid of revolution obtained by revolving the area A about y-axis is

$$\text{Volume} = V = \iint_A 2\pi x\,dx\,dy$$

(c) Volume under the surface as a double integral: The volume V of the solid under the surface $z = f(x, y)$ and above the xy-plane with the projection of $z = f(x, y)$ on xy plane as its base is

$$\text{Volume} = \int\int_D f(x, y)dx\,dy$$



(d) Volumes as a triple integral: The volume of the 3-dimensional region V is given by $\int_V\iint dx\,dy\,dz$

Example 16

Find the volume under the surface $x + 2y + z = 4$ and above the circle $x^2 + y^2 = 4$ in the xy-plane.

Solution

Given surface is $x + 2y + z = 4$

$\Rightarrow z = 4 - x - 2y$ (1)

Let D be the region bounded by the circle $x^2 + y^2 = 4$ in xy-plane

$\therefore$ In D, y varies from $y = -\sqrt{4-x^2}$ to $y = \sqrt{4-x^2}$ and x varies from $x = -2$ to $x = +2$.

$\therefore$ The volume under the surface $x + 2y + z = 4$ and above the circle $x^2 + y^2 = 4$ in xy-plane is

$$V = \int_D \int z dxdy = \int_{x=-2}^{2} \int_{y=-2\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4-x-2y)dxdy \qquad (2)$$

Evaluation of this double integral can be made simple by changing it into polar coordinates.

In polar coordinates, $x = r\cos\theta$, $y = r\sin\theta$ and

$$J = \frac{\partial(x, y)}{\partial(r, \theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$\therefore$ $J = r$. Also, in the circle $x^2 + y^2 = 4$, r varies from $r = 0$ to $r = 2$ and θ varies from $\theta = 0$ to $\theta = 2\pi$

$\therefore$ From (2),

$$V = \iint_D (4-x-2y)dx\,dy$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{2} (4 - r\cos\theta - 2r\sin\theta)\,|J|\,drd\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{2} (4 - r\cos\theta - 2r\sin\theta) rdr\,d\theta$$

$$= \int_{\theta=0}^{2\pi} \left(\int_{r=0}^{2} (4r - r^2\cos\theta - 2r^2\sin\theta)dr \right) d\theta$$

$$= \int_{\theta=0}^{2\pi} \left[2r^2 - \frac{r^3}{3}\cos\theta - \frac{2r^3}{3}\sin\theta \right]_{r=0}^{2} d\theta$$

$$= \int_{\theta=0}^{2\pi} \left[8 - \frac{8}{3}\cos\theta - \frac{16}{3}\sin\theta \right] d\theta$$

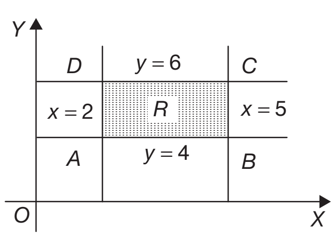
$$= 8\theta - \frac{8}{3}\sin\theta + \frac{16}{3}\cos\theta \Big]_{\theta=0}^{2\pi} = 16\pi$$

Example 17

Find the volume generated by the revolution of the rectangle formed by the lines $x = 2$, $x = 5$, $y = 4$ and $y = 6$ about x-axis.

Solution

The volume of the solid generated by revolving the rectangle $ABCD$ about x-axis $= V = \int_R \int 2\pi\, ydxdy$



$$= \int_{x=2}^{5} \int_{y=4}^{6} 2\pi\, ydxdy = \left(\int_{x=2}^{5} dx \right) \left(\int_{y=4}^{6} 2\pi\, y\, dy \right)$$

$$= \left(x \right]_{x=2}^{5} \left(\pi y^2 \right]_{y=4}^{6} = 3 \times 20\pi = 60\pi$$

Change of Variables Evaluation of some of the double (or) triple integrals can be made simple by changing the variables.

1. **In a double integral:** Let a double integral $\int_{R_{xy}} \iint f(x, y)dxdy$ in x and y is to be converted into the variables u and ϑ where $x = \phi(u, \vartheta)$ and $y = \Psi(u, \vartheta)$. Then

$$\int_{R_{xy}} \int f(x, y)dxdy = \int_{R^1_{u\vartheta}} \int f(\phi(u, \vartheta), \psi(u, \vartheta))\,|J|\,dud\vartheta$$

Where $J = \frac{\partial(x, y)}{\partial(u, \vartheta)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial \vartheta} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial \vartheta} \end{vmatrix}$

Is the Jacobian of x and y wrt u and ϑ and $R'_{u\vartheta}$ is the region of integration in u, ϑ-plane corresponding to R_{xy} in xy-plane.

2. **In a triple integral:** Let a triple integral $\int_{R_{xyz}} \iint f(x, y, z)\, dxdydz$ in x, y and z is to be converted into the variables u, ϑ and w, where $x = \phi(u, \vartheta, \omega)$, $y = \Psi(u, \vartheta, \omega)$ and $z = h(u, \vartheta, \omega)$

Then $\int_{R_{xyz}} \iint f(x, y, z)dxdydz = \int_{R^1_{u\vartheta\omega}} \iint f(\phi(u, \vartheta, \omega),$

$$\psi(u, \vartheta, \omega),\ h(u, \vartheta, \omega))\,|J|\,dud\vartheta d\omega$$

where $J = \frac{\partial(x, y, z)}{\partial(u, \vartheta, \omega)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial \vartheta} & \frac{\partial x}{\partial \omega} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial \vartheta} & \frac{\partial y}{\partial \omega} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial \vartheta} & \frac{\partial z}{\partial \omega} \end{vmatrix}$ is the jacobian of x, y and z wrt u, ϑ and ω and $R'_{u\vartheta\omega}$ is the region of integration in u, ϑ, ω, coordinate system corresponding to the region R_{xyz} in xyz co-ordinate system.

Vector Calculus

If $\bar{r}$ is the position vector of a point P, having co-ordinates (x, y, z), then $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, where $\bar{i}, \bar{j}, \bar{k}$ are unit vectors along OX, OY, OZ respectively, and

$$|\bar{r}| = |x\bar{i} + y\bar{j} + z\bar{k}| = \sqrt{x^2 + y^2 + z^2}.$$

Given any vector $\bar{v} = a\bar{i} + b\bar{j} + c\bar{k}$ its direction ratios are a, b, c and its direction cosines are given by:

$$l = \frac{a}{|\bar{v}|},\ m = \frac{b}{|\bar{v}|},\ n = \frac{c}{|\bar{v}|} \text{ and } l^2 + m^2 + n^2 = 1$$

Linear Combinations

A vector $\bar{r}$ is said to be a linear combination of the vectors $\bar{a}, \bar{b}, \bar{c}\ldots$ etc. if there exist scalars $x, y, z, \ldots$ such that

$\bar{r} = x\bar{a} + y\bar{b} + z\bar{c} + \cdots$

Test of Collinearity

Three points A, B, C with position vectors $\bar{a}, \bar{b}, \bar{c}$ respectively are collinear if *f* there exist scalars x, y, z not all zero such that $x\bar{a} + y\bar{b} + z\bar{c} = 0$, where $x + y + z = 0$

Test of Coplanarity

Four points A, B, C and D with position vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ are coplanar if there exist scalars x, y, z and u (not all zero) such that $x\bar{a} + y\bar{b} + z\bar{c} + u\bar{d} = 0$, where $x + y + z + u = 0$

Linear Dependence and Independence

A system of vectors $\bar{a}, \bar{b}, \bar{c}\ldots$ is said to be linearly independent (L.I.) if $x\bar{a} + y\bar{b} + z\bar{c} + \cdots = 0$

$\Rightarrow x = y = z \cdots = 0$

If $\bar{a}, \bar{b}, \bar{c}\ldots$ is a system of vectors which is not LI, then they are linearly dependent (L.D) and for such system of vectors there exist scalars $x, y, z \cdots$ (not all zeros) such that

$x\bar{a} + y\bar{b} + z\bar{c} + \cdots = 0$

NOTE

Every non-zero vector is LI.
Every pair of non-zero non-collinear vectors is LI.
Every pair of collinear vectors is LD.
Three non-coplanar vectors are LI.
Three coplanar vectors are LD.

Multiplication of Vectors

Scalar or Dot Product If $\bar{a}$ and $\bar{b}$ are two non-zero vectors and θ is the angle between them $(0 \le \theta \le \pi)$, then their dot or scalar product is given by $\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta \cdot \bar{a}\cdot\bar{b}$ is a scalar.

NOTES

1. If one or both of $\bar{a}, \bar{b}$, are $\bar{0}$, then $\bar{a}\cdot\bar{b} = 0$
2. $\bar{a}\cdot\bar{b} = |\bar{a}|\cdot$(scalar component of $\bar{b}$ along $\bar{a}$)
   $= |\bar{b}|$ (scalar component of $\bar{a}$ along $\bar{b}$),
3. $\bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{a}$
4. If $\bar{a}, \bar{b}, \bar{c}$ are any three vectors, then $\bar{a}\cdot(\bar{b}+\bar{c}) = \bar{a}\cdot\bar{b} + \bar{a}\cdot\bar{c}$
5. Two non-zero vectors $\bar{a}$ and $\bar{b}$ are perpendicular if $\bar{a}\cdot\bar{b} = 0$
6. $\bar{i}\cdot\bar{j} = \bar{j}\cdot\bar{i} = \bar{j}\cdot\bar{k} = \bar{k}\cdot\bar{j} = \bar{j}\cdot\bar{k} = \bar{k}\cdot\bar{i} = 0$
7. $\bar{a}\cdot\bar{b}$ is positive, negative or zero according as $0 \le \theta < 90°$, $90° < \theta \le 180°$ or $\theta = 90°$
8. The square of a vector is the square of its modulus, i.e., $(\bar{a})^2 = |\bar{a}|^2$
   $\bar{i}^2 = \bar{k}^2 = \bar{j}^2 = 1$
9. m is a scalar, then
   $m(\bar{a}\cdot\bar{b}) = (m\bar{a})\cdot\bar{b} = \bar{a}\cdot(m\bar{b})$
10. If $\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}$ and $\bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k}$, then $\bar{a}\cdot\bar{b} = a_1b_1 + a_2b_2 + a_3b_3$ and angle between the vectors is
    $$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||b|} = \frac{a_1\cdot b_1 + a_2\cdot b_2 + a_3\cdot b_3}{\sqrt{a_1^2 + a_2^2 + a_3^3}\cdot\sqrt{b_1^2 + b_2^2 + b_3^2}}$$
11. Work done $= \bar{F}\cdot\bar{S}$

Vector or Cross Product

$\bar{a}\times\bar{b} = |\bar{a}||\bar{b}|\sin\theta\cdot\hat{n}$ where θ $(0 \le \theta \le 180)$ is the angle between $\bar{a}$ and $\bar{b}$, and $\hat{n}$ is a unit vector such that it is perpendicular to both $\bar{a}$ and $\bar{b}$.

$\bar{a}, \bar{b}$ and $\hat{n}$ (in the same order) are in the right handed orientation (i.e., the rotation of a right handed screw from $\bar{a}$ to $\bar{b}$ advances it in the direction of $\hat{n}$).

NOTES

1. $\bar{a}\times\bar{b} \ne \bar{b}\times\bar{a}$ but $\bar{a}\times\bar{b} = -\bar{b}\times\bar{a}$
2. If $\bar{a}$ and $\bar{b}$ are parallel, then $\bar{a}\times\bar{b} = \bar{0}$
3. $\bar{i}\times\bar{j} = \bar{k}, \bar{j}\times\bar{k} = \bar{i}, \bar{k}\times\bar{i} = \bar{j}$ and
   $\bar{j}\times\bar{i} = -\bar{k}, \bar{k}\times\bar{j} = -\bar{i}, \bar{i}\times\bar{k} = -\bar{j}$
   $\bar{i}\times\bar{i} = \bar{j}\times\bar{j} = \bar{k}\times\bar{k} = 0$? [In particular $\bar{a}\times\bar{a} = 0$]
4. The angle between two vectors: $\sin\theta = \dfrac{|\bar{a}\times\bar{b}|}{|\bar{a}||b|}$
5. A unit vector perpendicular to the plane of $\bar{a}$ and $\bar{b}$ is given by $\hat{n}$ where $\hat{n} = \dfrac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}$
6. Area of parallelogram whose adjacent sides are $\bar{a}$ and $\bar{b}$ is given by $|\bar{a}\times\bar{b}|$
7. When the diagonals are given, the vector area of parallelogram $ABCD$ is $\dfrac{1}{2}(\overline{AB}\times\overline{AC})$
8. The vector area of the triangle $ABC = \dfrac{1}{2}(\overline{AB}\times\overline{AC})$
9. If $\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}$ and $\bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k}$,
   $$\text{Then } \bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
10. Vector product is distributive with respect to vector addition $\bar{a}\times(\bar{b}+\bar{c}) = \bar{a}\times\bar{b} + \bar{a}\times\bar{c}$

Triple Products

Scalar Triple Product The Scalar triple product of three vectors $\bar{a}$, $\bar{b}$, $\bar{c}$ is $(\bar{a}\times\bar{b})\cdot\bar{c}$ denoted by $[\bar{a}\bar{b}\bar{c}]$

The Scalar triple product of orthonormal right handed vector triad $\bar{i}$, $\bar{j}$, $\bar{k}$ is equal to unity

That is, $[\bar{i}\ \bar{j}\ \bar{k}]=[\bar{j}\ \bar{k}\ \bar{i}]=[\bar{k}\ \bar{i}\ \bar{j}]=1.$

1. The volume of a parallelepiped having $\bar{a}$, $\bar{b}$, $\bar{c}$ as co-terminus edges $=[\bar{a}\bar{b}\bar{c}]$.
2. If three vectors are coplanar then $[\bar{a}\bar{b}\bar{c}]=0$
3. If two of the three vectors are equal, then their scalar triple product is zero, i.e., $[\bar{a}\ \bar{b}\ \bar{c}]=0$
4. If $\bar{a}=a_1\bar{i}+a_2\bar{j}+a_3\bar{k}$, $\bar{b}=b_1\bar{i}+b_2\bar{j}+b_3\bar{k}$,
   $\bar{c}=c_1\bar{i}+c_2\bar{j}+c_3\bar{k}$, then $\lfloor \bar{a}\bar{b}\bar{c}\rfloor=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
5. The volume of a tetrahedron with co-terminus edges $\bar{a}$, $\bar{b}$, $\bar{c}$ is $\frac{1}{6}[\bar{a}\bar{b}\bar{c}]$ cubic units.
6. $[\bar{a}\bar{b}\bar{c}]=(\bar{a}\times\bar{b})\cdot\bar{c}=\bar{a}\cdot(\bar{b}\times\bar{c})$

Vector Triple Product If $\bar{a}$, $\bar{b}$, $\bar{c}$ are three vectors, then the triple product $\bar{a}\times(\bar{b}\times\bar{c})$ is called the vector triple product.

If $\bar{a}$, $\bar{b}$, $\bar{c}$ are any three vectors, then $\bar{a}\times(\bar{b}\times\bar{c}) =(\bar{a}\cdot\bar{c})\bar{b}-(\bar{a}\cdot\bar{b})\bar{c}$

Vector Variable

A variable of the form $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$ is called a vector variable and x, y, z are scalar variables.

Scalar Function If t is a scalar variable on a range $a\le t\le b$ and a function f defined as $f=f(t)$ for $t\in[a, b]$ is called a scalar function of t.

Example: $f(t)=9t^3+4t^2+7$,
$f(t)=\sin t+5\cos t+e^t$, etc.

Vector Function If t is a scalar variable defined on a domain $[a, b]$, and a function $\bar{F}(t)=x(t)\hat{i}+y(t)\hat{j}+z(t)\hat{k}$ is called a vector function of the scalar variable t.

NOTE

t is generally taken as 'time'.

Differentiation If $\bar{F}(t)$ is a continuous single valued vector function of the variable t, then the derivative of $\bar{F}(t)$ is defined as $\frac{d\bar{F}}{dt}=\lim_{\Delta t\to 0}\frac{F(t+\Delta t)-F(t)}{\Delta t}$ where Δt is a small increment in t.

One can also look at second and higher order derivatives in a similar way.

Differentiation Formula

1. The derivative of a constant vector with respect to any scalar variable is 0.
2. $\frac{d}{dt}[\bar{F}(t)\pm\bar{G}(t)]=\frac{d\bar{F}}{dt}\pm\frac{d\bar{G}}{dt}$.
3. $\frac{d}{dt}[s(t)\bar{F}(t)]=s(t)\cdot\frac{d\bar{F}}{dt}+\frac{ds}{dt}\cdot\bar{F}$
4. Chain rule: $\frac{d\bar{F}}{dt}=\frac{d\bar{F}}{du}\times\frac{du}{dt}$, where $\bar{F}=\bar{F}(u)$ and u is a function of t.
5. Dot and cross products:
   $$\frac{d}{dt}(\bar{F}\cdot\bar{G})=\bar{F}\cdot\frac{d\bar{G}}{dt}+\frac{d\bar{F}}{dt}\cdot\bar{G},$$
   $$\frac{d}{dt}(\bar{F}\times\bar{G})=\bar{F}\times\frac{d\bar{G}}{dt}+\frac{d\bar{F}}{dt}\times\bar{G}.$$
6. Partial derivatives: If $\bar{F}$ is vector function dependent on x, y and z, say $\bar{F}=\bar{F}(x, y, z)$, then partial derivative of $\bar{F}$ with respect to x is defined as $\frac{\partial\bar{F}}{\partial x}=\lim_{\Delta x\to 0}\frac{\bar{F}(x+\Delta x, y, z)-\bar{F}(x, y, z)}{\Delta x}$.

Likewise, one can also define $\frac{\partial\bar{F}}{\partial y}$ and $\frac{\partial\bar{F}}{\partial z}$.

It is also possible to define higher order partial derivatives as:

$$\frac{\partial^2\bar{F}}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial\bar{F}}{\partial x}\right),\ \frac{\partial^2\bar{F}}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial\bar{F}}{\partial y}\right).$$

$$\frac{\partial^2\bar{F}}{\partial x\partial z}=\frac{\partial}{\partial x}\left(\frac{\partial\bar{F}}{\partial z}\right),\ \text{etc}$$

Differential Vectors

1. If $\bar{G}=\bar{G}(x, y, z)$ then
   $$d\bar{G}=\frac{\partial\bar{G}}{\partial x}dx+\frac{\partial\bar{G}}{\partial y}dy+\frac{\partial\bar{G}}{\partial z}dz$$
2. If $\bar{F}=F_1\hat{i}+F_2\hat{j}+F_3\hat{k}$, then
   $$d\bar{F}=dF_1\hat{i}+dF_2\hat{j}+dF_3\hat{k}$$
3. $d(\bar{F}\cdot\bar{G})=\bar{F}\cdot d\bar{G}+d\bar{F}\cdot\bar{G}$
4. $d(\bar{F}\times\bar{G})=\bar{F}\times d\bar{G}+d\bar{F}\times\bar{G}$

Vector Differential Operators ∇ is to be read as del or nabla

$$\nabla=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$$

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\ \text{is called Laplacian.}$$

Gradient of a Scalar Function

If $\phi\,(x, y, z)$ is a scalar function, then $\hat{i}\dfrac{\partial\phi}{\partial x}+\hat{j}\dfrac{\partial\phi}{\partial y}+\hat{k}\dfrac{\partial\phi}{\partial z}$ is known as the gradient of ϕ and is denoted by grad ϕ. One can also write the gradient of ϕ using the ∇ operator as grad

$$\phi=\hat{i}\frac{\partial\phi}{\partial x}+\hat{j}\frac{\partial\phi}{\partial y}+\hat{k}\frac{\partial\phi}{\partial z}=\nabla\phi$$

Now $\nabla\,\phi$ denotes a vector field.

NOTES

1. If ϕ is a constant, then $\nabla\,\phi=\bar{0}$
2. If a vector $\bar{G}(x, y, z)$ is defined at all points in a region we say $\bar{G}$ is a vector field. A vector field is said to be irrotational if $\bar{G}=$ grad ϕ for some scalar function ϕ.
3. Gradient can be used in finding directional derivative. (An example is discussed in worked examples section)
4. $\nabla\,\phi$ also gives the normal to the surface $\phi\,(x, y, z)=C$.
5. If $\nabla^2\phi = 0$, the function is called the harmonic function.
6. The directional derivative of $\phi\,(x, y, z)$ in the direction of a vector $\bar{a}$ is $\nabla\phi\cdot\hat{n}$, where $\hat{n}=\dfrac{\bar{a}}{|\bar{a}|}$.

Divergence of Vector

$\bar{F}(x, y, z)$ be a vector field which is differentiable at each point (x, y, z) in some region of space, i.e., $\bar{F}$ is differentiable vector field. The scalar product of the vector operator ∇ and $\bar{F}$ gives a scalar which is termed as divergence.

$$\nabla\cdot\bar{F}=\hat{i}\cdot\frac{\partial\bar{F}}{\partial x}+\hat{j}\cdot\frac{\partial\bar{F}}{\partial y}+k\cdot\frac{\partial\bar{F}}{\partial z}$$

NOTE

If div $(\bar{F})$ or $\nabla\cdot\bar{F}=0$, then $\bar{F}$ is called 'solenoidal'

Curl of a Vector

Let $\bar{F}(x, y, z)$ is a vector field defined for all (x, y, z) in a certain region of space and is differentiable, i.e., $\bar{F}$ is a differentiable vector field. The cross product of the vector operator ∇ with the vector $\bar{F}$ is termed as curl $\bar{F}$.

$$\text{curl } F=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ F_1 & F_2 & F_3\end{vmatrix};\ \bar{F}=F_1\hat{i}+F_2\hat{j}+F_3\hat{k}$$

NOTE

If curl $\bar{F}=\bar{0}$, then $\bar{F}$ is said to be irrotational.

Standard Results

1. div $(\phi\bar{F})=\phi$ div $\bar{F}+\bar{F}\cdot$grad ϕ or $\nabla\cdot\phi\bar{F}=\phi\,\nabla\cdot\bar{F}+\bar{F}\cdot\nabla\,\phi$
2. curl $(\phi\bar{F})=\nabla\phi\times\bar{F}+\phi$ curl $\bar{F}$
3. div $(\bar{F}\times\bar{G})=\bar{F}\cdot$ curl $\bar{G}-\bar{G}\cdot$ curl $\bar{F}$
4. $\nabla\cdot\nabla\phi=$ div (grad ϕ) or $\nabla\cdot\nabla\phi=\nabla^2\phi$
5. curl (grad ϕ) $=\bar{0}$ or $\nabla\times(\nabla\,\phi)=\bar{0}$, i.e., curl of a gradient equals $\bar{0}$.
6. div (curl $\bar{F}$) $=0$ or $\nabla\cdot(\nabla\times\bar{F})=0$
7. curl (curl $\bar{F}$) $=$ grad (div $\bar{F}$) $-\nabla^2\bar{F}$(or) $\nabla\times(\nabla\times\bar{F})=\nabla(\nabla\cdot\bar{F})-\nabla^2\bar{F}$

Integration

Line Integral

Let $\bar{F}(x, y, z)$ be a vector function defined on a region of space and let C be curve in that region, then the integral $\int_c\bar{F}\cdot d\bar{r}$ is called the line integral.

For Riemann Integration,

$\int_{x=a}^{x=b}fdx$ the limits of integration are along the line segment joining $(a, 0)$, $(b, 0)$, where $a<b$.

Here instead of line, we integrate along the curve C.

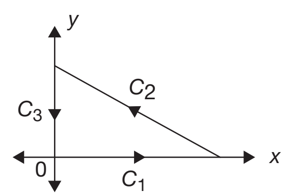
Circulation

The line integral around a closed curve C denoted by $\oint F\cdot dr$ is called circulation of F around C.

Example 18

Evaluate $\int_C\bar{F}\cdot d\hat{r}$, where $\bar{F}=xy\hat{i}+y^2\hat{j}$ along the triangle $x=0$, $y=0$ and $x+y=1$ in the first quadrant.

Solution



$$\int_c F\cdot dr=\int_{C_1}(xydx+y^2dy)+\int_{C_2}(xydx+y^2dy)+\int_{C_3}(xydx+y^2dy)$$

C_1	C_2	C_3
$y=0$	$y=1-x$	$x=0$
$0<x<1$	$1<x<0$	$dx=0$
$dy=0$	$dy=-dx$	$1<y<0$

$$=\int_{x=0}^{1}[x(0)dx+0+\int_{x=1}^{0}x(1-x)dx+(1-x)^2(-dx)+\int_1^0 y^2dy$$

$$=\int_1^0(x-x^2-1-x^2+2x)dx+\int_1^0 y^2dy$$

$$=\int_1^0(-2x^2+3x-1)dx-\int_0^1 y^2dy$$

$$=\left(\frac{2}{3}-\frac{3}{2}+1\right)-\frac{1}{3}=\frac{-1}{6}$$

Surface Integral Let S be a closed surface, then the normal surface integral $\int_s FN\,ds$ is called the flux of F over S.

Cartesian Form Let $F(r) = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$, where, F_1, F_2, F_3, are continuous and differentiable functions of x, y, z. If $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ be the direction cosines of the unit normal N, then

$$N = \hat{i}\cos\alpha + \hat{j}\cos\beta + \hat{k}\cos\gamma.$$

$$\therefore \int_S F\cdot N\,ds = \int_S (F_1\cos\alpha + F_2\cos\beta + F_3\cos\gamma)ds$$

But then $ds\cos\alpha$, $ds\cos\beta$ and $ds\cos\gamma$ are the projections of ds on yz, zx and xy planes. If dx, dy, dz are the differentials along the areas then

$ds\cos\alpha = dy\,dz$; $ds\cos\beta = dz\,dx$; $ds\cos\gamma = dx\,dy$.

$$\therefore \int_S F\cdot N\,ds = \iint_S (F_1 dy\,dz + F_2 dz\,dx + F_3 dx\,dy)$$

NOTE

If R_1 is the projection of S on xy-plane, then

$$\int_S F\cdot N ds = \int_{R_1}\int F\cdot N\frac{dxdy}{\cos\gamma}$$

$$= \iint_S F\cdot N\cdot\frac{dxdy}{|N\cdot\hat{k}|}(|N\cdot\hat{k}| = \cos\gamma)$$

Equivalently,

$$\iint_S F\cdot N ds = \iint_{R_2} F\cdot N\frac{dydz}{|N\cdot\hat{i}|} = \iint_{R_3} F\cdot N\frac{dzdx}{|N\cdot\hat{j}|}$$

Volume Integral

$$\int_{x_1}^{x^2}\int_{y_1}^{y^2}\int_{z_1}^{z^2} f(x,\ y,\ z)dz\,dy\,dx$$

$$= \boxed{\int_{x_1}^{x_2}\boxed{\int_{y_1}^{y_2}\boxed{\int_{z_1}^{z_2} f(x, y, z)\,dz}\,dy}\,dx}$$

Gauss' Divergence Theorem

If $\bar{F}$ is continuously differentiable vector function in the region bounded by a surface S, then $\iint_S \bar{F}\cdot\bar{N}\,ds = \iiint_V \text{div }\bar{F}dv$ where $\bar{N}$ is the unit normal to the surface.

Green's Theorem If P and Q are scalar point functions, possessing continuous derivatives of the first order, in a region S of the xy plane bounded by a closed curve C then $\int_C Pdx + Qdy = \iint_S\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy.$

Stoke's Theorem If S is an open surface bounded by a closed curve C and $\bar{F}$ is a continuously differentiable vector point function, then $\int_C \bar{F}\cdot d\bar{r} = \int_S \text{curl }\bar{F}\cdot Nds$, where N is unit outward drawn normal at any point on the surface.

Example 19

If $A = x^3\hat{i} + x^2\hat{j} + x\hat{k}$ and $B = -x\hat{i} + x^2\hat{j} + x^3\hat{k}$, then find the values of

(i) $\frac{d}{dx}(A\cdot B)$ and (ii) $\frac{d}{dx}(A\times B)$.

Solution

(i) $$\frac{d}{dx}(A\cdot B) = A\cdot\frac{d}{dx}(B) + B\cdot\frac{d}{dx}(A)$$

$$= (x^3\hat{i} + x^2\hat{j} + x\hat{k})\cdot\frac{d}{dx}(-x\hat{i} + x^2\hat{j} + x^3\hat{k})$$
$$+(-x\hat{i} + x^2\hat{j} + x^3\hat{k})\cdot\frac{d}{dx}(x^3\hat{i} + x^2\hat{j} + x\hat{k})$$
$$= (x^3\hat{i} + x^2\hat{j} + x\hat{k})\cdot(-\hat{i} + 2x\hat{j} + 3x^2\hat{k})$$
$$+(-x\hat{i} + x^2\hat{j} + x^3\hat{k})\cdot(3x^2\hat{i} + 2x\hat{j} + \hat{k})$$
$$= -x^3 + 2x^3 + 3x^3 - 3x^3 + 2x^3 + x^3 = 4x^3.$$

(ii) $$\frac{d}{dx}(A\times B) = A\times\frac{dB}{dx} + \frac{dA}{dx}\times B$$

$$= (x^3\hat{i} + x^2\hat{j} + x\hat{k})\times\frac{d}{dx}(-x\hat{i} + x^2\hat{j} + x^3\hat{k})$$
$$+\frac{d}{dx}(x^3\hat{i} + x^2\hat{j} + x\hat{k})\times(-x\hat{i} + x^2\hat{j} + x^3\hat{k})$$
$$= (x^3\hat{i} + x^2\hat{j} + x\hat{k})\times(-\hat{i} + 2x\hat{j} + 3x^2\hat{k})$$
$$+(3x^2\hat{i} + 2x\hat{j} + \hat{k})\times(-x\hat{i} + x^2\hat{j} + x^3\hat{k})$$
$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x^3 & x^2 & x\\ -1 & 2x & 3x^2\end{vmatrix} + \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3x^2 & 2x & 1\\ -x & x^2 & x^3\end{vmatrix}$$
$$= \hat{i}(5x^4 - 3x^2) - \hat{j}(6x^5 + 2x) + \hat{k}(5x^4 + 3x^2)$$

Example 20

If $f = x^3 - 6xy^2 - 9xyz$ is a scalar function, then find $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial x\partial y}$.

Solution

$f = x^3 - 6xy^2 - 9xyz$

$$\therefore \frac{\partial f}{\partial x} = 3x^2 - 6y^2 - 9yz$$

$$\therefore \frac{\partial^2 f}{\partial x^2} = 6x - 0 = 6x$$

$$\frac{\partial f}{\partial y} = -12xy - 9xz$$

$$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(-12xy - 9xz)$$
$$= -12y - 9z.$$

Example 21

If $\phi \equiv x^3 + y^3 + z^3 - 3xyz$, then find the value of grad ϕ at (2, 1, 1).

Solution

$$\text{Grad } \phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z} \quad \text{(by definition)}$$

$$= \hat{i}\frac{\partial}{\partial x}(x^3+y^3+z^3-3xyz) + \hat{j}\frac{\partial}{\partial y}(x^3+y^3+z^3-3xyz)$$

$$+\hat{k}\frac{\partial}{\partial z}(x^3+y^3+z^3-3xyz).$$

$$= 3[\hat{i}(x^2-yz)+\hat{j}(y^2-xz)+\hat{k}(z^2-xy)]$$

$\therefore$ grad ϕ at (2, 1, 1)

$$= 3[\hat{i}(4-1)+\hat{j}(1-2)+\hat{k}(1-2)]$$

$$= 9\hat{i}-3\hat{j}-3\hat{k}\,.$$

Example 22

If $\bar{P} = x^2y\hat{i} - x^3\hat{j} + xyz^2\hat{k}$, then find div p and curl p.

Solution

(i) div $\bar{P} = \nabla \cdot \bar{P}$

$$= \frac{\partial}{\partial x}(x^2y) - \frac{\partial}{\partial y}(-x^3) + \frac{\partial}{\partial z}(xyz^2)$$

$$= 2xy - 0 + 2xyz = 2xy\,(1 + z)$$

(ii) curl $\bar{P} = \nabla \times \bar{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & -x^3 & xyz^2 \end{vmatrix}$

$$= \hat{i}\left(\frac{\partial}{\partial y}(xyz)^2 - \frac{\partial}{\partial z}(-x^3)\right) - \left(\hat{j}\frac{\partial}{\partial x}(xyz^2) - \frac{\partial}{\partial Z}(x^2y)\right)$$

$$+\hat{k}\left(\frac{\partial}{\partial x}(-x^3) - \frac{\partial}{\partial y}(x^2y)\right)$$

$$= xz^2\hat{i} - yz^2\hat{j} + \hat{k}(-3x^2 - x^2)$$

$$= xz^2\hat{i} - yz^2\hat{j} - 4x^2\hat{k}.$$

Example 23

Find the value of r if,

$p = xy^2\hat{i} + xyz^2\hat{j} + (r-2)xyz^3\hat{k}$ is solenoidal at (1, – 1, 1).

Solution

p is solenoidal $\Rightarrow$ div $p = 0 \Rightarrow \nabla \cdot p = 0$

$$\Rightarrow \quad \frac{\partial p_1}{\partial x} + \frac{\partial p_2}{\partial y} + \frac{\partial p_3}{\partial z} = 0$$

$$\Rightarrow \quad \frac{\partial}{\partial x}xy^2 + \frac{\partial}{\partial y}(xyz^2) + \frac{\partial}{\partial z}[(r-2)xyz^3] = 0$$

$\Rightarrow y^2 + xz^2 + (r-2)\,3xyz^2 = 0$ at $(1, -1, 1)$, div $p = 0$

$\Rightarrow (-1)^2 + (1)^2 + (r-2)\,3\,(1)\,(-1)\,(1)^2 = 0$

$$\Rightarrow \quad 1+1-3r+6=0 \quad \Rightarrow \quad r = \frac{8}{3}.$$

Example 24

Find the value of a, if $P = (y^2+2xz)\hat{i} + (z^2+2xy)\hat{j} + (x^2+ayz)\hat{k}$ is irrotational.

Solution

The vector P is irrotational

$\Rightarrow$ curl $P = \bar{0} \Rightarrow \nabla \times P = \bar{0}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2+2xz & z^2+2xy & ayz+x^2 \end{vmatrix} = \bar{0}$$

$$\Rightarrow \hat{i}\left(\frac{\partial}{\partial y}(x^2+ayz) - \frac{\partial}{\partial z}(2xy+z^2)\right)$$

$$-\hat{j}\left(\frac{\partial}{\partial x}(x^2+ayz) - \frac{\partial}{\partial z}(y^2+2xz)\right)$$

$$+\hat{k}\left(\frac{\partial}{\partial x}(z^2+2xy) - \frac{\partial}{\partial y}(y^2+2xz)\right) = \bar{0}$$

$$\Rightarrow \quad \hat{i}(az-2z) + \hat{j}(2x-2x) + \hat{k}(2y-2y) = \bar{0}$$

$$\Rightarrow \quad \hat{i}\,z(a-2) = 0 = 0\,\hat{i} \quad \Rightarrow \quad z(a-2) = 0$$

$$\Rightarrow \quad a - 2 = 0 \Rightarrow a = 2$$

Example 25

Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at $(1, -2, 1)$.

Solution

Let $f = xy^2z - 3x - z^2 = 0$ and

$g = 3x^2 - y^2 + 2z - 1 = 0.$

$\therefore$ grad $f = \hat{i}(y^2z-3) + \hat{j}(2xyz) + \hat{k}(xy^2-2z)$

grad $g = \hat{i}(6x) + \hat{j}(-2y) + \hat{k}(2)$

But, angle between two surfaces at a point is equal to angle between the normals to the surfaces at that point.

$\therefore$ Let $\bar{n}_1 =$ grad f at $(1, -2, 1)$ and $\bar{n}_2 =$ grad g at $(1, -2, 1)$ respectively

$\therefore \quad \bar{n}_1 = (\text{grad} f)$ at $(1, -2, 1)$

$$= \hat{i}[(-2)^2 1 - 3] + \hat{j}[2(1)(-2)1]$$

$$+\hat{k}[1(-2)^2 - 2(1)] = \hat{i} - 4\hat{j} + 2\hat{k}$$

$\bar{n}_2 = (\text{grad } g)$ at $(1, -2, 1)$

$$= \hat{i}[6\ (A)] + \hat{j}[-2(-2)] + \hat{k}(2) = 6\hat{i} + 4\hat{j} + 2\hat{k}$$

Let the angle between the normals $\bar{n}_1$ and $\bar{n}_2$ be θ.

So, $\bar{n}_1 \cdot \bar{n}_2 = |n_1||n_2|\cos\theta \;\Rightarrow\; 6-16+4$

$$= (\sqrt{1+16+4})(\sqrt{36+16+4})\cos\theta$$

$$\therefore \cos\theta = \left|\frac{-6}{\sqrt{21}\sqrt{56}}\right| = \left|\frac{-3}{7\sqrt{6}}\right| = \frac{3}{7\sqrt{6}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{3}{7\sqrt{6}}\right)$$

Example 26

If $\bar{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$ evaluate $\oint \bar{F}\cdot d\bar{r}$ along the straight line C from (0, 0, 0) to (1, 2, 3).

Solution

The equation of the line joining (0, 0, 0) and (1, 2, 3) is $\frac{x}{1} = \frac{y}{2} = \frac{z}{3} = (t)$.

Then along the line C, $x = t$, $y = 2t$, $z = 3t$.

$$\therefore \quad \bar{r} = x\hat{i} + y\hat{j} + z\hat{k} = t\,\hat{i} + 2t\,\hat{j} + 3t\,\hat{k}$$

$$d\bar{r} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Given $\bar{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$

And along C, $\bar{F} = [t^2 + (2t)^2]\hat{i} - 2t(2t)\hat{j} = 5t^2\hat{i} - 4t^2\hat{j}$

$\therefore \; \bar{F}\cdot d\bar{r} = (5t^2 - 8t^2 + 0)\,dt = -3t^2\,dt$

at (0, 0, 0), $t = 0$ and at (1, 2, 3), $t = 1$.

$$\therefore \int_c \bar{F}\cdot d\bar{r} = \int_{t=0}^{1} -3t^2 dt = \left(\frac{-3t^3}{3}\right)_0^1 = -1.$$

Example 27

If $F = 3x\hat{i} - z^2\hat{k}$, , evaluate $\oint F\cdot dr$, where the curve C is the rectangle in the xz bounded by $z = 0$, $z = 2$, $x = 0$, $x = 3$.

Solution

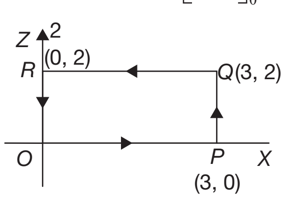
Since the integration takes place in xz-plane ($y = 0$)

$$\therefore \int_0 F\cdot dr = \int_0 f_1 dx + f_2 dz = \int_0 3x\,dx - z^2 dz$$

$$\oint_C \bar{F}\cdot d\bar{r} = \int_{OP} \bar{F}\cdot d\bar{r} + \int_{PQ} \bar{F}\cdot d\bar{r} + \int_{QR} \bar{F}\cdot d\bar{r} + \int_{RO} \bar{F}\cdot d\bar{r}$$

(i) Along OP:
z = 0, $dz = 0$ and x varies from 0 to 3

$$\int F\cdot dr = \int_0^3 3x\,dx = \left[\frac{3x^2}{2}\right]_0^3 = \frac{27}{2}$$



(ii) Along PQ:
$x = 3$, $dx = 0$ and z changes from 0 to 2.

$$\therefore \int_{PQ} \bar{F}\cdot dr = \int_0^2 -z^2 dz = \left[\frac{-Z^3}{3}\right]_0^2 = \frac{-8}{3}$$

(3) Along QR:
$y = 2$, $dy = 0$ and x changes from 3 to 0

$$\therefore \int_{QR} F\cdot dr = \int_3^0 3x\,dx = \left(3\frac{x^2}{2}\right)_3^0 = \frac{-27}{2}$$

(4) Along RO:
$x = 0$,
$dx = 0$ and y varies from 2 to 0.

$$\therefore \int_{RO} F\cdot dr = \int_2^0 -z^2 dz = -\left(\frac{z^3}{3}\right)_2^0 = \frac{8}{3}$$

$$\text{Thus } \oint_C F\cdot dr = \frac{27}{2} - \frac{8}{3} - \frac{27}{2} + \frac{8}{3} = 0$$

Example 28

Evaluate by Green's theorem $(\oint)$ $(xy + y^2)\,dx + x^2\,dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

Solution

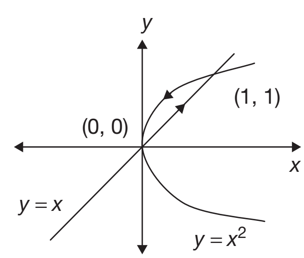
$$\text{Here } P = xy + y^2 \quad \therefore \quad \frac{\partial P}{\partial y} = x + 2y$$

$$Q = x^2 \quad \therefore \quad \frac{\partial Q}{\partial x} = 2x$$

Hence by Green's theorem,

$$\int_C (xy+y^2)dx + x^2 dy = \iint_S (2x - x - 2y)dx\,dy$$

$$= \iint_S (x-2y)dx\,dy = \int_{x-0}^{1}\left[\int_{y-x}^{x}(x-2y)dy\right]dx$$



$$= \int_{x=0}^{1}[xy - y^2]_{y=x}^{x^2} dx = \int_{x=0}^{1}(x^3 - x^4)dx$$

$$= \left(\frac{x^4}{4} - \frac{x^5}{5}\right)_0^1 = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$$

Example 29

By applying Gauss theorem, evaluate $\iint_S (x^3 dy\,dz + x^2 y dz\,dx + x^2 z dx\,dz)$, where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular discs $z = 0$ and $z = b$.

Solution

We have

$F_1 = x^3; F_2 = x^2y; F^3 = x^2z$

$$\therefore \frac{\partial F_1}{\partial x} = 3x^2, \frac{\partial F_2}{\partial y} = x^2, \frac{\partial F_3}{\partial z} = x^2$$

$$\therefore \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 3x^2 + x^2 + x^2 = 5x^2$$

$\therefore$ Using Gauss theorem,

$$\iint_S F_1 dy\, dz + F_2 dz\, dx + F_3 dx\, dy$$

$$= \iint_V \int \left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right) dx\, dy\, dz$$

$$\therefore \iint_S x^3 dy\, dz + x^2 ydz\, dx + x^2 zdx\, dy$$

$$= \iint_V \int 5x^2 dx\, dy\, dz = 20 \int_{x=0}^{a} \int_{y=0}^{\sqrt{a^2-x^2}} \int_{z=0}^{b} x^2 dx\, dy\, dz$$

$$= 20 \int_{x=0}^{a} \int_{y=0}^{\sqrt{a^2-x^2}} x^2 b\, dx\, dy.$$

$$= 20b \int_0^a x^2 \sqrt{a^2 - x^2}\, dx$$

[Let $x = a \sin \theta$; $dx = a \cos \theta\, d\theta$

Upper limit: $x = a \Rightarrow a \sin \theta = a \Rightarrow \theta = \dfrac{\pi}{2}$

Lower limit: $x = 0 \Rightarrow a \sin\theta = 0 \Rightarrow \theta = 0$]

$$= 20b \int_0^{\frac{\pi}{2}} a^2 \sin^2 \theta \sqrt{a^2(1 - \sin^2 \theta)}\, a \cos\theta d\theta$$

$$= 20a^4 b \int_0^{\frac{\pi}{2}} \sin^2 \theta \cos^2 \theta d\theta$$

$$= 20a^4 b \int_0^{\frac{\pi}{2}} \frac{1}{4} \sin^2 2\theta\, d\theta$$

$$= 5a^4 b \int_0^{\frac{\pi}{2}} \left(\frac{1 - \cos 4\theta}{2}\right) d\theta$$

$$= \frac{5a^4 b}{2}\left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{5a^4 b}{2}\left[\frac{\pi}{2} - 0\right] = \frac{5\pi}{4} a^4 b$$

Example 30

Evaluate $\int_c F \cdot dr$ by Stokes theorem,

If $F = (x^2 + y^2)\hat{i} - 2xy\hat{j}$, where c is the rectangle formed by the lines $x = \pm a$, $y = 0$ and $y = b$.

Solution

$\bar{F} = (x^2 + y^2)\hat{i} - 2xyj$

By Stoke's theorem,

$$\int (\nabla \times \bar{F}) \cdot \bar{N}\, ds = \int_c \bar{F} \cdot d\bar{r}$$

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2 + y^2 & -2xy & 0 \end{vmatrix} = -4y\hat{k}$$

$$\therefore \int (\nabla \times \bar{F}) \cdot (N \cdot k) ds$$

$$= \int (-4ky) \cdot N\, ds = \int -4y\, (N \cdot k)\, ds \int_R \int -4y\, dx\, dy$$

Since N $\cdot$ $k\, ds = dx\, dy$

And R is the region bounded by the rectangle.

$$= \int_{x=-a}^{a} \int_{y=0}^{b} (-4y) dy\, dx = \int_{-a}^{a} \left[\frac{-4y^2}{2}\right]_0^b dx$$

$$= -2\int_{-a}^{a} (b^2 - 0) dx = -2b^2 [x]_{-a}^{a} = -4ab^2.$$

EXERCISES

1. $\lim\limits_{x \to \infty} \{3x - \sqrt{9x^2 - x}\} =$ ______.

(A) $\dfrac{1}{6}$ (B) 3

(C) 6 (D) None of these

2. $\lim\limits_{x \to 0} \left[\dfrac{24 \cos x - 24 + 12x^2 - x^4}{24x^6}\right] =$

(A) $\dfrac{1}{720}$ (B) $\dfrac{-1}{120}$

(C) $\dfrac{1}{120}$ (D) $\dfrac{-1}{720}$

3. Evaluate $\lim\limits_{x \to 2.7} (x - [x])$, where $[x]$ is the greatest integer less than equal to x.

(A) -0.3 (B) 0.7

(C) 4.7 (D) 2

4. Evaluate $\lim\limits_{x \to 0} \dfrac{1}{x^{189}}$.

(A) 0 (B) ∞

(C) $-\infty$ (D) None of these

5. $\lim\limits_{x \to 0} \left(\dfrac{2^x + 3^x}{2}\right)^{1/x} =$

(A) 1 (B) $\sqrt{3}$

(C) $\sqrt{6}$ (D) $\sqrt{2}$

6. $\lim_{x\to 2} |x-2| + [x-2] =$
(A) 0.
(B) only left limit exists.
(C) only right limit exists.
(D) limit does not exist.

7. Let the function $f(x) = [x]$. Where $[x]$ is the greatest integer less than or equal to x. Which of the following is/are true?
(A) $f(x)$ has jump discontinuity at all $x \in Z$.
(B) $f(x)$ has removable discontinuity at all $x \in Z$.
(C) $f(x)$ is continuous at all irrational values.
(D) both (A) and (C).

8. $f(x) = \begin{cases} 5x-4 & 0 < x \leq \\ 4x^2 - 3x & 1 < x < 2 \end{cases}$ at $x = 1$
(A) Left hand continuous at $x = 1$.
(B) Right hand continuous at $x = 1$.
(C) continuous at $x = 1$.
(D) None of these

9. The function $f(x) = \dfrac{x \sin x}{(x^2+2)}$ is
(A) continuous for all x.
(B) discontinuous for all x.
(C) constant function.
(D) discontinuous only at $x = \pm 2$.

10. Check the continuity of the following function

$$f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & \text{when } x \neq 0 \\ a^2 & \text{when } x=0 \end{cases} \quad \text{at } x = 0$$

(A) continuous at $x = 0$
(B) discontinuous at $x = 0$
(C) discontinuous of first kind
(D) None of these

11. If $f(x) = \begin{cases} 7 & x < 5, \\ ax+b & 5 < x < 7, \\ 11 & x > 7 \text{ is } c \text{ continuous on } R \end{cases}$
then the values of a and b are
(A) $a = 2, b = 3$ (B) $a = -2, b = 3$
(C) $a = 3, b = -2$ (D) $a = 2, b = -3$

12. Let $f(x) = \max(1-x, x^2-1)$. Then f is
(A) not continuous at $x = 1, -2$.
(B) continuous and differentiable everywhere.
(C) not differentiable at $x = -2, 1$.
(D) continuous but not differentiable at $x = 1, -1$.

13. Consider the function $f(x) = \dfrac{1}{x-1} + \dfrac{1}{3-x}$ defined in the interval [1, 3]
P. f is continuous on [1, 3]
Q. f is differentiable on (1, 3)
R. there exists $c \in (1, 3)$ such that $f'(c) = 0$ which of the above statements are true?
(A) P, Q only (B) Q, R only
(C) P, R only (D) P, Q, R

14. A function $f: R \to R$ is such that $f(x+y) = f(x) \cdot f(y)$ for all x, y in R and $f(x) \neq 0$ for any x in R. If $f(x)$ is differentiable and $f'(0) = 2$, then
(A) $f'(x) = 2f(x)$ (B) $f(x) = 2f'(x)$
(C) $f(x) = f'(x)$ (D) $f'(x) = -f(x)$

15. Which of the following statement(s) is/are true?
(A) $y = x^2$ has a minimum value at $x = 0$
(B) $y = |x-3|$ has a minimum value at $x = 3$
(C) The maximum value of the function $y = \dfrac{1}{1+x^2}$ is 1
(D) All of these

16. The maximum and minimum values of $f(x) = 3\sin^2 x + 4\cos^2 x$ is
(A) $\{-4, -3\}$ (B) $\{7, 3\}$
(C) $\{4, -3\}$ (D) $\{4, 3\}$

17. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at $x = p$ and $x = q$ respectively such that $p^2 = q$, then the value of 'a' is
(A) 2 (B) $\dfrac{1}{4}$
(C) $\dfrac{1}{8}$ (D) 4

Direction for questions 18 and 19:

The sum of the hypotenuse and one side of a right angled triangle is given as a units.

18. When the area is maximum the ratio of the side and the hypotenuse is ______.
(A) 2 : 1 (B) 1 : 3
(C) 1 : 2 (D) 2 : 3

19. When the area is maximum, find the angle between the hypotenuse and the other side is ______.
(A) 60° (B) 30°
(C) 45° (D) None of these

20. Consider $f(x) = |x^2 - 3|$, $0 \leq x \leq \sqrt{6}$ and $g(x) = \begin{cases} 3^x, & 0 \leq x \leq 1 \\ 4-x, & 1 < x \leq 3 \end{cases}$. Then Rolle's theorem can be applied in the respective intervals
(A) to both $f(x)$ and $g(x)$.
(B) only to $f(x)$.
(C) only to $g(x)$.
(D) neither to $f(x)$ nor to $g(x)$.

21. If the function $f(x) = px^2 + qx^2 + rx + s$ on [0, 1], satisfies the mean value theorem, then the value of c in the interval (0, 1) is
(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$
(C) $\dfrac{2}{3}$ (D) $\dfrac{2}{3\sqrt{3}}$

22. $f(x)=\dfrac{x^2}{x+1}$ increases in

(A) $(-2, 0)$

(B) $[-4, -2]$

(C) $(-\infty, -2] \cup [0, \infty)$

(D) $(-\infty, -2) \cup (0, \infty)$

23. Let $f(x) = e^{ax}$ and $g(x) = e^{-ax}$ be two functions defined in $[p, q]$, If the functions satisfies Cauchy mean value theorem then the value of 'c' is _______.

(A) $p+q$ (B) $\dfrac{p+q}{2}$

(C) $2(p+q)$ (D) None of these

24. If $x = \cos(z + y^2)$, then $\dfrac{\partial z}{\partial y} =$

(A) 1 (B) y

(C) $2y$ (D) $-2y$

25. If $u=\left[\dfrac{\sqrt[4]{x}+\sqrt[4]{y}}{\sqrt[6]{x}+\sqrt[6]{y}}\right]^6$, then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial x}=$

(A) $\dfrac{u}{2}$ (B) $\dfrac{4}{u}$

(C) $4u$ (D) $6u$

26. The stationary points of the function $f(x, y) = x^3 + y^4 - 27x + 32y + 100$ is/are

(A) $(3, 2), (3, -2)$

(B) $(-3, 2), (-3, -2)$

(C) $(3, 2), (-3, -2)$

(D) $(3, -2), (-3, -2)$

27. For the function $f(x, y) = 2x^2 + 4y^2 + 4xy + 2x + 10y + 7$.

(A) Local maximum exists, but no local minimum.

(B) Local minimum exists, but no local maximum.

(C) Neither local minimum nor local maximum exists.

(D) Both local minimum and local maximum exists.

28. For the function xyz, if $x + y + z = 3$, then the local maximum occurs for xyz at the point _______.

(A) $\left(4,\dfrac{1}{2},\dfrac{1}{2}\right)$

(B) $(5, -1, -1)$

(C) $(1, 1, 1)$

(D) $(7, -3, -1)$

29. The ratio of the dimensions of a rectangular box of volume 64 cubic units and open at the top that requires least material for its construction is

(A) 2 : 2 : 1 (B) 2 : 4 : 5

(C) 2 : 3 : 4 (D) 1 : 2 : 3

30. Which of the following function/s is/are integrable but not continuous on (0, 10)?

(A) $f(x) = [x]$ (greatest integer function)

(B) $f(x) = |x - 3|$

(C) $f(x) = |x - 5| + |x - 2|$

(D) $f(x) = x^2 + 5x + 9$

31. $\int \sec^3 x \; dx =$ _______.

(A) $\dfrac{\sec x\tan x}{3}+\log\left(\sec x+\text{an } x\right)$

(B) $\dfrac{\sec^2 x\tan x}{3}+\dfrac{1}{3}\log\tan\left(\dfrac{\pi}{4}+x\right)$

(C) $\dfrac{\sec x\tan x}{2}+\dfrac{1}{2}\log\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$

(D) None of these

32. $\int_0^{\pi/2} \sin^4 x\cos^6 x dx =$ _______.

(A) $\dfrac{3\pi}{128}$ (B) $\dfrac{2\pi}{425}$

(C) $\dfrac{3\pi}{2560}$ (D) $\dfrac{3\pi}{512}$

33. Area bounded by the curve $y = -3x^2$, $x = 2$ and the two coordinate axes is _______ sq units

(A) 2 (B) 3

(C) 6 (D) 8

34. The volume of the solid obtained by revolving the area bounded by the parabola $y^2 = x - 4$, x-axis and the lines $x = 4$ and $x = 7$, about x-axis is _______ cubic unit

(A) $\dfrac{9}{2}\pi$ (B) $\dfrac{11}{2}\pi$

(C) $\dfrac{13}{2}\pi$ (D) $\dfrac{15}{2}\pi$

35. The length of arc of the curve $y = \ln(\cos x)$ from $x = 0$ to $x=\dfrac{\pi}{4}$ is _____.

(A) $\ln(1+\sqrt{2})$ (B) $\ln(\sqrt{2}-1)$

(C) $\ln(2+\sqrt{3})$ (D) $\ln(2-\sqrt{3})$

36. Evaluate $\int_0^{\pi/4}\int_0^{\pi/4}(3\cos\theta+4\sin\theta)d\theta d\phi$ _______.

(A) $\left(\dfrac{\sqrt{2}-1}{\sqrt{2}}\right)\pi$ (B) $\dfrac{(4\sqrt{2}-1)\pi}{4\sqrt{2}}$

(C) $\dfrac{(4\sqrt{2}-1)\pi}{\sqrt{2}}$ (D) $\dfrac{(4\sqrt{2}-1)\pi}{4\sqrt{2}}$

37. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dxdy}{\sqrt{1-x^2-y^2}}$

(A) $\frac{\pi}{4}$ (B) 0

(C) $\frac{\pi}{2}$ (D) 1

38. By changing the order of integration, the integral $\int_2^{\infty}\int_0^{x-2} f(x,y)dxdy$ becomes ______.

(A) $\int_0^{\infty}\int_2^{y+2} f(x,y)dxdy$ (B) $\int_0^{\infty}\int_{y+2}^{\infty} f(x,y)\,dxdy$

(C) $\int_0^{\infty}\int_0^{\infty} f(x,y)dxdy$ (D) $\int_1^{\infty}\int_{y+2}^{1} f(x,y)dxdy$

39. By changing the variables in the double integral $\iint_R \frac{dxdy}{xy}$, where $x = e^{u+v}$ and $y = uv$, it changes to $\iint_R \phi(uv)dudv$ then $\phi(u, v)$ is

(A) $\frac{R}{(e^{u+v})(uv)}$ (B) $\frac{e^{u+v}}{uv}$

(C) $\frac{1}{v}+\frac{1}{u}$ (D) $\frac{1}{v}-\frac{1}{u}$

40. By changing the variables from x, y to u, v where $x = u + 2v$ and $y = 4u + 3v$, the given integral $\iint_R f(x,y)dxdy$ changes to $\iint_R f(u+2v, 4u+3v)\psi(u,v)dud\,v$ then $\Psi(u, v)$ is ______.

(A) 5 (B) −5

(C) $\frac{1}{5}$ (D) $-\frac{1}{5}$

41. The area bounded by the circle $x^2 + y^2 = 6$ and the parabola $y = x^2$ is given by:

(A) $\int_{x=-2}^{2}\int_{y=\sqrt{x}}^{\sqrt{x^2-6}} dydx$

(B) $\int_{x=-\sqrt{2}}^{\sqrt{2}}\int_{y=\sqrt{x}}^{\sqrt{6-x^2}} dydx$

(C) $\int_{x=-2}^{2}\int_{y=\sqrt{x}}^{\sqrt{x^2-6}} (x^2+y^2)dydx$

(D) $\int_{x=-\sqrt{2}}^{\sqrt{2}}\int_{y=\sqrt{x}}^{\sqrt{6-x^2}} (y-x^2)dxdy$

42. The volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 4$ is ______ cubic units.

(A) $\frac{32}{3}$ (B) $\frac{64}{3}$

(C) 32 (D) 64

43. The acute angle between the vectors $3i + j + 2k$ and $i - j + k$ is θ, then the value of $\cos\theta$ is

(A) $\frac{8}{21}$ (B) $\frac{8}{\sqrt{21}}$

(C) $21\sqrt{8}$ (D) $\sqrt{\frac{8}{21}}$

44. If $\bar{r}$ is the position vector of a particle which passes along the curve $x = 3 \sin 4t$, $y = 3 \cos 4t$, and $z = 5t$ $(t > 0)$. The magnitude of its velocity and acceleration respectively are

(A) 13, 45 (B) 12, 48

(C) 13, 48 (D) 12, 45

45. $\bar{f}(t)$ be a vector function and $\bar{f}\times\frac{d\bar{f}}{dt}=0$ implies

(A) f is a vector function with constant magnitude.

(B) f is a vector function both in direction and magnitude.

(C) f is a vector function of constant direction.

(D) Either A or C.

46. The directional derivative of $f = x^3 y + y^3 z + z^3 x$ in the direction of $i + 2j + 2k$ at (0, 1, −1) is

(A) $\frac{5}{3}$ (B) $\frac{4}{3}$

(C) $\frac{-4}{3}$ (D) $\frac{-5}{3}$

47. If $\bar{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ and $|r| = r$, then $\nabla r^n =$

(A) $n(n-1)r^{n-1}\,\bar{r}$ (B) $n(n-2)r^{n-2}\,\bar{r}$

(C) $n \cdot r^{n-2} \times \bar{r}$ (D) $n(n-1)\bar{r}$

Direction for questions 48 and 49:

Two equations $f = x\,y^2\,z - 2y + z^2$ and $g = x^2 + yz - x - 2$ represents two surfaces

48. Find normal vector to 'g' at (1, −1, 2)

(A) $i + 2j + 2k$ (B) $i + 2j - k$

(C) $2\,i - j - k$ (D) $i - j - 2k$

49. The acute angle between the surfaces f and g at (1, −1, 2) is

(A) $\cos^{-1}\left(\frac{15}{\sqrt{390}}\right)$ (B) $\cos^{-1}\left(\sqrt{\frac{15}{390}}\right)$

(C) 60° (D) 30°

50. The magnitude of maximum directional derivative of $\phi = 2xy^2 - xyz + y^2 z$ in the direction from the point $(1, -1, 1)$ is

(A) 62 (B) $\sqrt{52}$

(C) $\sqrt{62}$ (D) $\sqrt{56}$

51. The directional derivative of a scalar point function is a function of

(A) only direction (B) only position

(C) either A or B (D) both A and B

52. The values of div $\bar{r}$ and curl $\bar{r}$ respectively when $\bar{r} = 2x\hat{i} - y\hat{j} + 3z\bar{k}$ is

(A) $4; \hat{i}$ (B) $0, \bar{0}$

(C) $4, 4\bar{k}$ (D) $4, \bar{0}$

53. The necessary and sufficient condition that the force field $\bar{F}(x, y, z)$ is conservative is

(A) (curl $\bar{F}$) $= -\bar{F}$ (B) div $\bar{F} = \bar{0}$

(C) curl $\bar{F} = \bar{F}$ (D) curl $\bar{F} = \bar{0}$

54. Which of the following is/are true?

(A) $\nabla(\bar{r} \times \bar{a}) = 0$

(B) Grad $(\bar{r} \cdot \bar{a}) = \bar{a}$

(C) $\nabla \times (\bar{r} \times \bar{a}) = -2\bar{a}$

(D) All of these

55. Compute the value of div$(\nabla\phi \times \nabla f)$.

(A) ∇f curl $(\nabla\phi)$ (B) $\nabla\phi$ curl (∇f)

(C) curl $(\nabla\phi \times \nabla f)$ (D) 0

56. For what value of p the vector $f = (2x + 3y)i + (z + 2y)j + (x - pz)k$ is solenoidal?

(A) 4 (B) -4

(C) 2 (D) 0

57. For what values of p, q and r the vector $\bar{f} = (x + ry - z)i + (3x - y + qz)j + (px + y - z)k$ is irrotational?

(A) $p = 1, q = -1, r = 3$

(B) $p = -1, q = 1, r = 3$

(C) $p = -1, q = 1, r = -3$

(D) $p = 1, q = 1, r = -3$

58. If $\nabla\phi = yz\hat{i} + zx\hat{j} + xy\hat{k}$, then $\phi(x, y, z) =$

(A) $xyz + f(y, z)$; $f \neq$ constant

(B) $xyz + g(x, z)$; $g \neq$ constant

(C) $xyz + h(x, y)$; $h \neq$ constant

(D) $xyz + k$; k is a constant

59. If $\bar{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$, compute the line integral $\int_C \bar{F}\,d\bar{r}$ where $C \equiv y = x^3$ in the xy-plane joining $(1, 1)$ and $(2, 8)$.

(A) 35 (B) -32

(C) 12 (D) 18

60. Compute $\int_S x^2 y^2 ds$ around the circle $x = \cos t$ and $y = \sin t$.

(A) $\frac{\pi}{4}$ (B) 0

(C) $\frac{\pi}{2}$ (D) π

61. If $\bar{F} = y^2\hat{i} - 2xy\hat{j}$, compute the circulation $\int_C \bar{F}\,d\bar{r}$ where C is the rectangle bounded by $y = 0, y = 1, x = 0$ and $x = 2$.

(A) 3 (B) 4

(C) -4 (5) -3

62. A particle in the force field $\bar{F} = 2x^2 i + (y - 3xz)j + 2z\,k$ is moving along a space curve defined by $x = 2t, y = t^2, z = 3t^2 - 2$. Find the work done by $\bar{F}$ in moving a particle along the straight line from $A(0, 0, 0)$ to $B(2, 1, 1)$.

(A) $\frac{107}{30}$ (B) $\frac{121}{30}$

(C) $\frac{113}{30}$ (D) $\frac{109}{30}$

63. Evaluate $\oint_C (x^2 y dx + xy^2 dy)$ using greens theorem where C is the triangle with vertices (0, 0), (2, 0) and (2, 1).

(A) $\frac{11}{24}$ (B) $\frac{11}{12}$

(C) $\frac{-11}{6}$ (D) $\frac{11}{4}$

64. Find the area of the region in the first quadrant bounded by the curves $y = 4x$, $y = \frac{1}{x}$ and $y = \frac{x}{4}$ using green's theorem.

(A) log 2 (B) $\frac{1}{2}$ log 2

(C) log 4 (D) log 16

65. Evaluate $\iint_s F\, nds$ where $F = 2xz\hat{i} - yz\hat{j} + yx\bar{k}$ where S is the cube bounded by $x = 0, x = 3, y = 0, y = 3$ and $z = 0, z = 3$.

(A) $\frac{27}{2}$ (B) $\frac{81}{4}$

(C) $\frac{27}{4}$ (D) $\frac{81}{2}$

66. For the force field $\bar{F} = x^2 i + xyj$ in the square region in the xy-plane bounded by the lines $x = 0, y = 0, x = 2, y = 2$. Using stokes theorem, find the value of $\int_C F, dr$.

(A) 4 (B) 6

(C) 8 (D) 2

67. Evaluate the volume integral $\int_V div\,\bar{N}\,dv$, where N is the outward drawn normal to the surface described by $x^2 + (y-5)^2 + (z-8)^2 = 12$.

(A) 8π (B) 12π
(C) 48π (D) 24π

68. If S is a closed surface and n is unit normal to the surface 'S' then $\iint_S \bar{r}\, nds =$ ______.

(A) 4V (B) 3V
(C) 2V (D) V

69. $\int_{1x}^{\infty} \frac{1}{1.0001} dx =$ ______.

(A) 1000 (B) 100000
(C) 10000 (D) 1000000

70. $\int_0^3 \frac{1}{(x-2)^{4/5}} dx =$ ______.

(A) $5 - 2^{1/5}$ (B) $5 + 2^{1/5}$
(C) $5(1-2)^{1/5}$ (D) $5\left[1+2^{1/2}\right]$

Previous Years' Questions

1. Evaluate $\int_0^{\infty} \frac{\sin t}{t} dt$ **[GATE, 2007]**

(A) π (B) $\frac{\pi}{2}$
(B) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

2. A velocity vector is given as $\bar{v} = 5xy\bar{i} + 2y^2\bar{j} + 3yz^2\bar{k}$. The divergence of the this velocity vector at (1, 1, 1) is **[GATE, 2007]**

(A) 9 (B) 10
(C) 14 (D) 15

3. The value of $\int_0^3\int_0^x (6-x-y)dx\,dy$ is **[GATE, 2008]**

(A) 13.5 (B) 27.0
(C) 40.5 (D) 54.0

4. The inner (dot) product of two vectors $\bar{P}$ and $\bar{Q}$ is zero. The angle (degrees) between the two vectors is **[GATE, 2008]**

(A) 0 (B) 30
(C) 90 (D) 120

5. For a scalar function $f(x, y, z) = x^2 + 3y^2 + 2z^2$, the gradient at the point $P(1, 2, -1)$ is **[GATE, 2009]**

(A) $2\vec{i} + 6\vec{j} + 4\vec{k}$ (B) $2\vec{i} + 12\vec{j} - 4\vec{k}$
(C) $2\vec{i} + 12\vec{j} + 4\vec{k}$ (D) $\sqrt{56}$

6. The $\lim\limits_{x\to 0} \frac{\sin\left[\frac{2}{3}\right]x}{x}$ is **[GATE, 2010]**

(A) $\frac{2}{3}$ (B) 1
(C) $\frac{3}{2}$ (D) ∞

7. Given function
$F(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$. The optimal value of $f(x, y)$ **[GATE, 2010]**

(A) is a minimum equal to $\frac{10}{3}$

(B) is a maximum equal to $\frac{10}{3}$

(C) is a minimum equal to $\frac{8}{3}$

(D) is a maximum equal to $\frac{8}{3}$

8. What is the value of the definite integral, $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$? **[GATE, 2011]**

(A) 0 (B) $\frac{a}{2}$
(C) a (D) $2a$

9. If $\bar{a}$ and $\bar{b}$ are two arbitrary vectors with magnitudes a and b, respectively, $\left|\bar{a}\times\bar{b}\right|^2$ will be equal to **[GATE, 2011]**

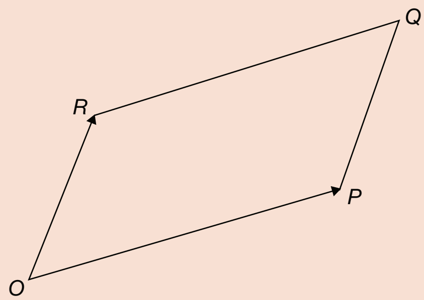
(A) $a^2b^2 - (\vec{a}\cdot\vec{b})^2$

(B) $ab - \vec{a}\cdot\vec{b}$

(C) $a^2b^2 + (\vec{a}\cdot\vec{b})^2$

(D) $ab + \vec{a}\cdot\vec{b}$

10. For the parallelogram $OPQR$ shown in the sketch, $\overrightarrow{OP} = a\bar{i} + bj$ and $\overrightarrow{OR} = ci + dj$. The area of the parallelogram is



[GATE, 2012]

(A) $ad - bc$
(B) $ac + bd$
(C) $ad + bc$
(D) $ab - cd$

11. There is no value of x that can simultaneously satisfy both the given equations. Therefore, find the least square error solution to the two equations, i.e., find the value of x that minimizes the sum of squares of the errors in the two equations
$2x = 3$
$4x = 1$ **[GATE, 2013]**

12. The solution for $\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta \, d\theta$ is **[GATE, 2013]**

(A) 0 (B) $\frac{1}{15}$
(C) 1 (D) $\frac{8}{3}$

13. $\lim_{x\to\infty}\left(\frac{x+\sin x}{x}\right)$ equals is **[GATE, 2014]**

(A) $-\infty$ (B) 0
(C) 1 (D) ∞

14. The expression $\lim_{x\to 0}\frac{x^\alpha - 1}{\alpha}$ is equal to **[GATE, 2014]**

(A) $\ln x$ (B) 0
(C) $x\ln x$ (D) ∞

15. With reference to the conventional cartesian (x, y) coordinate system, the vertices of a triangles have the following coordinates: $(x_1, y_1) = (1, 0)$: $(x_2, y_2) = (2, 2)$: and (x_3, y_3), $= (4, 3)$. The area of the triangle is equal to **[GATE, 2014]**

(A) $\frac{3}{2}$ (B) $\frac{3}{4}$
(C) $\frac{4}{5}$ (D) $\frac{5}{2}$

16. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}$ is equal to **[GATE, 2015]**

(A) e^{-2} (B) e
(C) 1 (D) e^2

17. While minimizing the function $f(x)$, necessary and sufficient conditions for a point, x_0 to be a minima are: **[GATE, 2015]**

(A) $f'(x_0) > 0$ and $f''(x_0) = 0$
(B) $f'(x_0) < 0$ and $f''(x_0) = 0$
(C) $f'(x_0) = 0$ and $f''(x_0) < 0$
(D) $f'(x_0) = 0$ and $f''(x_0) > 0$

18. The directional derivative of the field $u(x, y, z) = x^2 - 3yz$ in the direction for the vector $(\hat{i} + \hat{j} - 2\hat{k})$ at point $(2, -1, 4)$ is ________. **[GATE, 2015]**

19. The optimum value of the function $f(x) = x^2 - 4x + 2$ is **[GATE, 2016]**

(A) 2 (maximum) (B) 2 (minimum)
(C) -2 (maximum) (D) -2 (minimum)

20. The quadratic approximation of $f(x) = x^3 - 3x^2 - 5$ at the point $x = 0$ is **[GATE, 2016]**

(A) $3x^2 - 6x - 5$ (B) $-3x^2 - 5$
(C) $-3x^2 + 6x - 5$ (D) $3x^2 - 5$

21. What is the value of $\lim_{\substack{x\to 0\\ y\to 0}}\frac{xy}{x^2+y^2}$? **[GATE, 2016]**

(A) 1 (B) -1
(C) 0 (D) Limit does not exist

22. The area between the parabola $x^2 = 8y$ and the straight line $y = 8$ is ________. **[GATE, 2016]**

23. The area of the region bounded by the parabola $y = x^2 + 1$ and the straight line $x + y = 3$ is **[GATE, 2016]**

(A) $\frac{59}{6}$ (B) $\frac{9}{2}$
(C) $\frac{10}{3}$ (D) $\frac{7}{6}$

24. The angle of intersection of the curves $x^2 = 4y$ and $y^2 = 4x$ at point $(0, 0)$ is **[GATE, 2016]**

(A) 0° (B) 30°
(C) 45° (D) 90°

25. The value of $\int_0^\infty \frac{1}{1+x^2}dx + \int_0^\infty \frac{\sin x}{x}dx$ is **[GATE, 2016]**

(A) $\frac{\pi}{2}$ (B) π
(C) $\frac{3\pi}{2}$ (D) 1

Answer Keys

Exercises

1. A	**2.** D	**3.** B	**4.** D	**5.** C	**6.** D	**7.** D	**8.** C	**9.** A	**10.** A
11. D	**12.** C	**13.** B	**14.** A	**15.** D	**16.** D	**17.** A	**18.** C	**19.** B	**20.** D
21. A	**22.** D	**23.** B	**24.** D	**25.** A	**26.** D	**27.** B	**28.** C	**29.** A	**30.** A
31. C	**32.** D	**33.** D	**34.** A	**35.** A	**36.** B	**37.** C	**38.** B	**39.** D	**40.** A
41. B	**42.** A	**43.** D	**44.** C	**45.** C	**46.** D	**47.** C	**48.** B	**49.** A	**50.** C
51. D	**52.** D	**53.** D	**54.** D	**55.** D	**56.** A	**57.** B	**58.** D	**59.** A	**60.** A
61. C	**62.** D	**63.** C	**64.** C	**65.** D	**66.** A	**67.** C	**68.** B	**69.** C	**70.** D

Previous Years' Questions

1. B	**2.** D	**3.** A	**4.** C	**5.** B	**6.** A	**7.** A	**8.** B	**9.** A	**10.** A
11. 0.875	**12.** B	**13.** C	**14.** A	**15.** A	**16.** D	**17.** D	**18.** −5.72 to −5.70		**19.** D
20. B	**21.** D	**22.** 85.33	**23.** B	**24.** D	**25.** B				

Chapter 2

Ordinary Differential Equations

CHAPTER HIGHLIGHTS

- *Introduction*
- *Differential equations*
- *Laplace transforms*

Introduction

Familiarity with various methods used in evaluating indefinite integrals or finding anti-derivatives of functions [or, in other words, evaluating $\int f(x)\,dx$] is a pre-requisite.

Differential Equations

An equation involving derivatives of a dependent variable with respect to one or more independent variables is called a differential equation. The equation may also contain the variables and/or their functions and constants. If there is only one independent variable, the corresponding equation is called an ordinary differential equation. If the number of independent variables is more than one, the corresponding equation is called a partial differential equation.

Examples:

1. $\dfrac{dy}{dx} = x^4 + e^{-x} + y$

2. $x^2 \dfrac{d^2y}{dx^2} + 3\left(\dfrac{dy}{dx}\right)^2 + 3y^4x = \sin x + 6$

3. $\dfrac{dy}{dx} + 5y = x^3 - \tan x$

4. $\dfrac{d^2y}{dx^2} + 4y = 0$

5. $\left(\dfrac{d^3y}{dx^3}\right)^2 + 5\left(\dfrac{dy}{dx}\right)^4 + e^{2xy} = 6$

6. $\dfrac{d^3y}{dx^3} + 8\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + 9y = 16x^2$

7. $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 8u$

8. $\dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial x^2} = 10$

9. $\dfrac{\partial^2 u}{\partial y^2} = 25\dfrac{\partial^2 u}{\partial x^2}$

10. $\dfrac{\partial^4 u}{\partial x^4} + 6\dfrac{\partial^2 u}{\partial x^2 \partial y^2} + \dfrac{\partial^4 u}{\partial y^4} = e^{3xy}$

We note that in the given examples, Eqs. (1) to (6) are ordinary differential equations while Eqs. (7) to (10) are partial differential equations. We refer to these examples later on in next chapter.

Certain Geometrical Results may also be Expressed as Differential Equations

Illustration 1 Consider a family of parallel lines. All these lines have the same slope. If k represents the slope, we may interpret the family of parallel lines as curves having the same slope. As $\dfrac{dy}{dx}$ represents the slope of the tangent to a curve at any point (x, y), we may say that the differential equation $\dfrac{dy}{dx} = k$ represents a family of parallel lines.

Illustration 2 The differential equation $y\frac{dy}{dx} = k$ (a constant) may be said to represent the family of curves having the length of subnormal equal k at every point (x, y) on the curve. (We may note that the family of curves is the family of parabolas). Our study is confined to ordinary differential equations. In what follows, differential equation means ordinary differential equations.

Order of a Differential Equation

It is defined as the order of the highest derivative present in the equation. Examples (1), (3) are of first order; (2), (4) are of second order and (5), (6) are of third.

Degree of a Differential Equation

The degree of a differential equation is defined as the degree of the highest order derivative present in the equation. (It is assumed that the various order differential co-efficients or derivatives present in the equation are made free from fractional powers).

Examples (1), (2), (3), (4), (6) are of first degree while Example (5) is of second degree.

Consider the differential equation,

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{5/2} = 4\frac{d^3y}{dx^3}.$$

Taking the square on both sides (to free it from fractional powers), the differential equation is

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{5} = 16\left(\frac{d^3y}{dx^3}\right)^2.$$

This is a third order second degree differential equation.

Linear Differential Equation

If, in a differential equation, the dependent variable and the derivatives appear only in the first degree and there is no term involving products of the above or containing functions of the dependent variable, it is called **linear differential equation.**

1. $\frac{dy}{dx} + Py = Q$ (where P and Q are functions of only x) is an example of a first order linear differential equation.
2. $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$, where P, Q, R are functions of only x; $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2y = f(x)$, where a_1, a_2 are constants and $f(x)$ is a function of x are examples of second order linear differential equations.

Similarly, we can have nth order linear differential equation.

$$P_0\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + \cdots + P_{n-1}\frac{dy}{dx} + P_ny = Q$$

where $P_0, P_1, P_2, \ldots, P_n, Q$ are functions of x or constants. If an equation is not linear, it is called a non-linear differential equation. In examples, 1, 3, 4, 6 are linear differential equations, while examples 2 and 5 are non-linear differential equations.

Solution of a Differential Equation

A function $y = f(x)$ or $F(x, y) = 0$ is called a solution of a given differential equation if it is defined and differentiable (as many times as the order of the given differential equation) throughout the interval where the equation is valid, and is such that the equation becomes an identity when $y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots$ are replaced by $f(x)$, $f'(x)$, $f''(x), \ldots$ respectively.

[In the case of $F(x, y) = 0$ one has to get $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots$ by successive differentiation of $F(x, y) = 0$ with respect to x].

Examples:

1. $y = e^{7x}$ is a solution of $\frac{dy}{dx} = 7y$, since on substitution of $y = e^{7x}$, both left and right sides of the differential equation become identical. We find that $y = e^{7x}, 3e^{7x}, \frac{-1}{2}e^{7x}$ or, in general, $y = Ce^{7x}$, where C is an arbitrary constant represents solutions of $\frac{dy}{dx} = 7y$.
2. $y^2 - x^2 = 4$ is a solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$. Also, $y^2 - x^2 = 5$, $y^2 - x^2 = -10, \ldots$ or, in general, $y^2 - x^2 = C$ where C is an arbitrary constant represents solutions of $\frac{dy}{dx} = \frac{x}{y}$.

In both the above examples, we could represent the solutions of the differential equations which involve an arbitrary constant denoted by C. We now define the general solution of a first order differential equation.

The ***general solution*** of a first order differential equation is a relation between x and y involving one arbitrary constant such that the differential equation is satisfied by this relation or, the general solution of a first order differential equation is a one parameter family of curves where the parameter is the arbitrary constant. By assigning particular values to the arbitrary constant, we generate ***particular solutions*** of the equation.

In Example (1) $y = Ce^{7x}$ represents the general solution of the differential equation $\frac{dy}{dx} = 7y$ and the solutions

$y = e^{7x}, y = 3e^{7x}, \ldots$ are its particular solutions. The general solution represents a family of exponential curves.

In Example (2) $y^2 - x^2 = C$ represents the general solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$ and the solutions $y^2 - x^2 = 4$, $y^2 - x^2 = 5$, … are its particular solutions. The general solution in this case represents a family of rectangular hyperbolas.

3. $y = 2e^{-3x} + 5e^{6x}$ is a solution of the second order differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 18y = 0$. (which can be verified by actual substitution). Also, $y = 4e^{-3x} - 10e^{6x}$, $e^{-3x} + e^{6x}, \ldots$ or, in general, $y = Ae^{-3x} + Be^{6x}$ where A and B are arbitrary constants represents solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 18y = 0$.
4. $y = 2 \cos 4x + 3 \sin 4x$ or, in general, $y = A \cos 4x + B \sin 4x$ where A and B are arbitrary constants represents solutions of $\frac{d^2y}{dx^2} + 16y = 0$.

In Example (3), the general solution is $y = Ae^{-3x} + Be^{6x}$ and in Example (4), the general solution is $y = A \cos 4x + B \sin 4x$.

By assigning particular values to the arbitrary constants one can generate particular solutions.

From Examples (3) and (4), we infer that the general solution of a second order differential equation is a relation between x and y involving two arbitrary constants such that the differential equation is satisfied by this relation or the general solution of a second order differential equation is a two-parameter family of curves where the parameters are the arbitrary constants.

To sum up, the general solution of an nth order differential equation is a relation between x and y involving n arbitrary constants, such that the differential equation is satisfied by this relation or the general solution of an nth order differential equation is an n-parameter family of curves where the parameters are the arbitrary constants. For the first and second order differential equations, we have

First Order Equation

One parameter family of curves:

Representation: Relation between x and y involving one arbitrary constant, say C.

Eliminate: Eliminate C to obtain a DE representing the given curve.

Second Order Equation

Two-parameter family of curves:

Representation: Relation between x and y involving two arbitrary constants, say A and B

Elimination: Eliminate A and B to obtain a DE representing the two-parameter family of curves.

We shall work out a few examples to illustrate the formation of differential equations.

SOLVED EXAMPLES

Example 1

Form the differential equation representing the one-parameter family of curves

$$x^3 - Ay = 0.$$

Solution

Given, $x^3 - Ay = 0$ (1)

$$Ay = x^3$$

$$A\frac{dy}{dx} = 3x^2 \quad \Rightarrow \quad A = \frac{3x^2}{\frac{dy}{dx}} \tag{2}$$

Substituting A in the Eq. (1),
we have

$$x^3 - \frac{3x^2}{\frac{dy}{dx}} \cdot y = 0 \quad \Rightarrow \quad x\frac{dy}{dx} - 3y = 0.$$

Example 2

Obtain the differential equation of all the circles in the first quadrant, which touch the co-ordinate axes.

Solution

The equation of any circle in the first quadrant, which touches the co-ordinate axes may be represented as $(x - h)^2 + (y - h)^2 = h^2$.

Differentiating with respect to x,

$$2(x-h) + 2(y-h)\frac{dy}{dx} = 0$$

or

$$h = \frac{x + y\frac{dy}{dx}}{\left(1 + \frac{dy}{dx}\right)}$$

Substituting the above expression for h in the equation of the circle

$$\left(x - \frac{x + y\frac{dy}{dx}}{1 + \frac{dy}{dx}}\right)^2 + \left(y - \frac{x + y\frac{dy}{dx}}{1 + \frac{dy}{dx}}\right)^2 = \left(\frac{x + y\frac{dy}{dx}}{1 + \frac{dy}{dx}}\right)^2$$

or

$$(x-y)^2\left(\frac{dy}{dx}\right)^2 + (x-y)^2 = \left(x + y\frac{dy}{dx}\right)^2$$

or

$$(x-y)^2\left[1 + \left(\frac{dy}{dx}\right)^2\right] = \left[x + y\left(\frac{dy}{dx}\right)\right]^2.$$

Initial Value Problems A first order differential equation with a condition that $y = y_0$ when $x = x_0$ [written as $y(x_0) = y_0$] is known as an initial value problem. For example,

1. $\dfrac{dy}{dx} = \dfrac{x}{y}$; $y(0) = 1$

2. $\dfrac{dy}{dx} + 2xy = x^3$; $y(1) = 6$

3. $\dfrac{dy}{dx} + \dfrac{3y}{x} = e^x$; $y(0) = 4$

To solve such problems, we first obtain the general solution and find that particular value of the arbitrary constant in the general solution which satisfies the condition $y(x_0) = y_0$. This means that the solution of an initial value problem is a particular solution of the given differential equation.

First Order First Degree Equations The general form of the equation will be $\dfrac{dy}{dx} = f(x, y)$.

Separable Equations (or Variables Separable Type) Here, the given differential equation can be reduced to the form $f(y)dy = g(x)dx$. [Recall that $\dfrac{dy}{dx}$ may be thought as the ratio of the differential of y to the differential of x]. Direct integration of the relation with respect to the variable on each side gives general solution or, in other words, the general solution of the differential equation above may be written as $\int f(y)\,dy = \int g(x)\,dx + C$, where C is an arbitrary constant.

Example 3

Solve: $\dfrac{dy}{dx} = \sqrt{\dfrac{1+y^2}{1+x^2}}$.

Solution

$$\frac{dy}{dx} = \sqrt{\frac{1+y^2}{1+x^2}}$$

$$\frac{1}{\sqrt{1+y^2}}dy = \frac{1}{\sqrt{1+x^2}}dx$$

Integrating on both sides,

$$\int \frac{1}{\sqrt{1+y^2}}dy = \int \frac{1}{\sqrt{1+x^2}}dx.$$

$$\sinh^{-1} y = \sinh^{-1} x + c.$$

Example 4

Solve: $(x - xy^2)\dfrac{dy}{dx} + (y + x^2 y) = 0$.

Solution

$$(x - xy^2)\frac{dy}{dx} + (y + x^2 y) = 0$$

$$(x - xy^2)\,dy + (y + x^2y)\,dx = 0$$

$$x(1 - y^2)\,dy + y(1 + x^2)\,dx = 0$$

$$\frac{1-y^2}{y}dy + \frac{1+x^2}{x}dx = 0$$

Integrating on both sides,

$$\int\left(\frac{1}{y} - y\right)dy + \int\left(\frac{1}{x} + x\right)dx = 0$$

$$\log y - \frac{y^2}{2} + \log x + \frac{x^2}{2} = \log C$$

$$\log_e \frac{xy}{C} = \frac{y^2 - x^2}{2} \quad \Rightarrow \quad \frac{xy}{C} = e^{\left(\frac{y^2-x^2}{2}\right)}$$

$$\Rightarrow \quad xy = Ce^{\left(\frac{y^2-x^2}{2}\right)}$$

Example 5

Solve the initial value problem

$$y^2\frac{dy}{dx} = x^2 e^{y^3}, \quad y(1) = (0)$$

Solution

Given: $y^2 \dfrac{dy}{dx} = x^2 e^{y^3}$

$$y^2 e^{-y^3}\,dy = x^2 dx.$$

$$\int y^2 e^{-y^3}\,dy = \int x^2 dx$$

Let $e^{-y^3} = t \quad \Rightarrow \quad e^{-y^3} \cdot -3y^2 dy = dt$

$$-\frac{1}{3}\int dt = \int x^2 dx$$

$$\frac{-1}{3}t = \frac{x^3}{3} + c$$

$$-\frac{1}{3}e^{-y^3} = \frac{x^3}{3} + c.$$

Given: When $x = 1$, $y = 0$;

$$-\frac{1}{3}e^{\circ} = \frac{1}{3} + c$$

$$c = -\frac{2}{3}$$

$\therefore$ The solution is $-\dfrac{1}{3}e^{-y^3} = \dfrac{x^3}{3} - \dfrac{2}{3}$.

$$x^3 + e^{-y3} - 2 = 0.$$

Homogeneous Differential Equations

Homogeneous differential equation will be of the form $f(x, y)dy = g(x, y)dx$, where $f(x, y)$ and $g(x, y)$ are homogeneous functions in x and y of the same degree.

Definition

A function $F(x, y)$ in x and y is a homogeneous function in x and y of degree n(n, a rational number), if $F(x, y)$ can be expressed as $x^n\phi\left(\dfrac{y}{x}\right)$ or $y^n\psi\left(\dfrac{x}{y}\right)$.

1. $x^3+4x^2y-y^3=x^3\left(1+\frac{4y}{x}-\frac{y^3}{x^3}\right)$ is a homogeneous function in x and y of degree 3.
2. $x^3\tan\left(\frac{y}{x}\right)$ is a homogeneous function in x and y of degree 3.
3. $\frac{x+y}{2x-3y}$ is a homogeneous function in x and y of degree 0. We change the dependent variable y to v by the substitution $y=vx$. Then, $\frac{dy}{dx}=v+x\frac{dv}{dx}$.

On substitution y and $\frac{dy}{dx}$ in the given homogeneous equation, it reduces to the variables separable form.

Example 6

Solve: $x^2\frac{dy}{dx}=x^2+7xy+9y^2$.

Solution

$$x^2\frac{dy}{dx}=x^2+7xy+9y^2$$

$$\frac{dy}{dx}=1+\frac{7y}{x}+9\left(\frac{y}{x}\right)^2$$

Put $y=xv \quad\Rightarrow\quad \frac{dy}{dx}=v+\frac{dv}{dx}$

$$v+\frac{dv}{dx}=1+7v+9v^2$$

$$x\frac{dv}{dx}=9v^2+6v+1$$

$$\frac{1}{9v^2+6v+1}dv=\frac{1}{x}dx$$

Integrating on both sides,

$$\int\frac{1}{9v^2+6v+1}dv=\int\frac{1}{x}dx$$

$$\int\frac{1}{(3v+1)^2}dv=\int\frac{1}{x}dx \quad -\frac{1}{3(3v+1)}=\log x+\log c$$

$$=-\frac{1}{3\left(\frac{3y}{x}+1\right)}=\log_e cx=\frac{-x}{9y+3x}=\log_e cx$$

where C is an arbitrary constant.

Example 7

Solve $x\frac{dy}{dx}=y+x\sin\left(\frac{y}{x}\right)$

Solution

Given: $x\frac{dy}{dx}=y+x\sin\left(\frac{y}{x}\right)$

$$\frac{dy}{dx}=\frac{y}{x}+\sin\left(\frac{y}{x}\right) \tag{1}$$

Put $y=vx,\ \frac{dy}{dx}=v+x\frac{dv}{dx}$.

Substituting in (1) we get,

$$v+x\frac{dv}{dx}=v+\sin v$$

$$\Rightarrow\quad \frac{xdv}{dx}=\sin v \quad\Rightarrow\quad \frac{1}{\sin v}dv=\frac{1}{x}dx$$

$$=\int\operatorname{cosec} v\,dv=\int\frac{1}{x}dx$$

$\Rightarrow\quad \log(\operatorname{cosec} v-\cot v)=\log x+\log c$

$\Rightarrow\quad \operatorname{cosec} v-\cot v=cx$

$$\operatorname{cosec}\left(\frac{y}{x}\right)-\cot\left(\frac{y}{x}\right)=cx.$$

Example 8

Solve $3y^2\,dx+(2xy+3x^2)\,dy=0$.

Solution

$3y^2\,dx+(2xy+3x^2)\,dy=0$.

$$\frac{dy}{dx}=\frac{-3y^2}{2xy+3x^2}$$

Put $y=vx \quad\Rightarrow\quad \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$v+x\frac{dv}{dx}=\frac{-3v^2}{2v+3}$$

$$x\frac{dv}{dx}=\frac{-3v^2}{2v+3}-v$$

$$x\frac{dv}{dx}=\frac{-3v^2-2v^2-3v}{2v+3}$$

$$\frac{2v+3}{-5v^2-3v}dv=\frac{1}{x}dx$$

$$\Rightarrow\quad \frac{2v+3}{v(5v+3)}dv+\frac{1}{x}dx=0$$

Integrating on both sides,

$$\Rightarrow\quad \int\left[\frac{1}{v}-\frac{3}{5v+3}\right]dv+\int\frac{1}{x}dx=0$$

$$\Rightarrow\quad \log v-\frac{3}{5}\log(5v+3)+\log x=\log c.$$

$$\Rightarrow\quad 5\log v-3\log(5v+3)+5\log x=5\log c.$$

$$\Rightarrow \quad \log \frac{v^5}{(5v+3)^3} x^5 = \log c^5$$

$$\Rightarrow \quad \frac{y^5}{\left(5\frac{y}{x}+3\right)^3} = c_1, \text{ where } c_1 = c^5$$

$$\Rightarrow \quad \frac{y^5x^3}{(5y+3x)^3} = c_1 \quad \Rightarrow \quad x^3y^5 = c_1(5y+3x)^3$$

Exact Differential Equations

If M, as well as N, is a function in x and y, then the equation $Mdx + Ndy = 0$ is said to be an exact differential equation if there exists a function $f(x, y)$ such that

$$d(f(x, y)) = Mdx + Ndy.$$

That is, $$\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = Mdx + Ndy$$

Example: $3x^2ydx + x^3dy = 0$ is an exact differential equation since there exists a function x^3y such that

$$d(x^3 y) = 3x^2ydx + x^3dy$$

The necessary and sufficient condition for an equation of the form $Mdx + Ndy = 0$ to be an exact equation is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

The solution of the exact differential equation

$$Mdx + Ndy = 0 \text{ is } U + \int \phi(y)\, dy = C$$

where $U = \int Mdx$ and $\phi(y) = N - \frac{\partial u}{\partial y}$

Or $\int^x Mdx + \int (\text{terms of } N \text{ not containing } x)dy = C$

Here $\int^x Mdx$ denotes integration of M with respect to x treating y as a constant.

Example 9

Find the solution of

$$(3x - 2y + 5)\, dx + (3y - 2x + 7)dy = 0.$$

Solution

$M = 3x - 2y + 5$, $N = 3y - 2x + 7$

$$\frac{\partial M}{\partial y} = -2\frac{\partial N}{\partial x} = -2.\ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ The given equation is exact.

The solution is

$$\int^x Mdx + \int (\text{the terms of } N \text{ not containing } x)\, dy = C$$

$$\int^x (3x-2y+5)\, dx + \int (3y+7)\, dy = C$$

$$\frac{3x^2}{2} - 2yx + 5x + \frac{3y^2}{2} + 7y = C$$

Example 10

Find the solution of $(e^y + 1)\cot x\, dx + e^y \log(\sin x)\, dy = 0$.

Solution

Given $(e^y + 1)\cot x\, dx + e^y \log(\sin x)dy = 0$

Let $M = (e^y + 1)\cot x$ and $N = e^y \log(\sin x)$

$$\frac{\partial M}{\partial y} = e^y \cot x \text{ and } \frac{\partial N}{\partial x} = e^y \cot x$$

$$\therefore \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ The given equation is exact.

The solution is

$$\int^x Mdx + \int (\text{the terms of } N \text{ not containing } x)dy = C$$

$$\therefore \quad \int^x (e^y+1)\cot x\, dx + \int 0\, dy = C$$

$$(e^y + 1)\log(\sin x) = C$$

Integrating factors: Let us say $M(x, y)dx + N(x, y)dy = 0$ be a non-exact differential equation. If it can be made exact by multiplying it by a suitable function $\mu(x, y)$, then $\mu(x, y)$ is called an integrating factor.

Methods to Find the Integrating Factors

Method I

If $Mdx + Ndy = 0$ is a homogeneous differential equation and $Mx + Ny \neq 0$, then $\frac{1}{Mx+Ny}$ is an integrating factor of $Mdx + Ndy = 0$

Example 11

Find the solution of $(x + 2y)dx + (y - 2x)dy = 0$.

Solution

Here $M = x + 2y$ and $N = y - 2x$

$$\frac{\partial M}{\partial y} = 2\ \frac{\partial N}{\partial x} = -2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

The above equation is not an exact equation.

But M and N are homogeneous functions

$\therefore$ The integrating factor $= \frac{1}{Mx+Ny}$

$$(x + 2y)x + (y - 2x)y = x^2 + y^2 \tag{1}$$

Now by multiplying Eq. (1) by $\frac{1}{x^2+y^2}$, it become an exact equation.

$$\left(\frac{x+2y}{x^2+y^2}\right)dx+\left(\frac{y-2x}{x^2+y^2}\right)dy=0$$

The solution is $U+\int\phi(y)\,dy=C$

$$U=\int^x M_1dx,\ \text{where } M_1=\frac{x+2y}{x^2+y^2}$$

$$=\int^x\frac{x+2y}{x^2+y^2}\,dx$$

$$=\int\frac{x}{x^2+y^2}dx+2y\int\frac{1}{x^2+y^2}dx$$

$$=\frac{1}{2}\log(x^2+y^2)+2y\frac{1}{y}\tan^{-1}\left(\frac{x}{y}\right)$$

$$=\frac{1}{2}\log(x^2+y^2)+2\tan^{-1}\left(\frac{x}{y}\right)$$

Since in $N_1=\dfrac{y-2x}{x^2+y^2}$ there is no term independent of x, the solution is

$$\frac{1}{2}\log(x^2+y^2)+2\tan^{-1}\left(\frac{x}{y}\right)=C$$

Method 2

If the differential equation $Mdx+Ndy=0$ is of the form $y,f(xy)dx+x\,g(xy)dy=0$ and $Mx-Ny\neq 0$, then $\dfrac{1}{Mx-Ny}$ is an integrating factor of $Mdx+Ndy=0$.

Method 3

In the equation $Mdx+Ndy=0$,

if $\dfrac{1}{N}\left[\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}\right]=f(x)$, then $e^{\int f(x)\,dx}$ is an integrating factor of the given equation.

Similarly if $\dfrac{1}{M}\left[\dfrac{\partial N}{\partial x}-\dfrac{\partial M}{\partial y}\right]=g(y)$ then $e^{\int g(y)\,dy}$ is an integrating factor of the given equation.

Example 12

Find the solution $(x^2-y^2)dx+2xy\,dy=0$.

Solution

Given $(x^2-y^2)dx+2xy\,dy=0$ (1)

$$M=x^2-y^2 \text{ and } N=2xy$$

$$\frac{\partial M}{\partial y}=-2y \text{ and } \frac{\partial N}{\partial x}=2y$$

$$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$$

$$\frac{1}{N}\left[\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right]=\frac{1}{2xy}[-2y-2y]$$

$$=\frac{-2}{x}=f(x)$$

Integrating factor (IF)

$$=e^{\int f(x)\,dx}$$

$$=e^{\int\frac{-2}{x}dx}=e^{\int-2\log x}=e^{\log\frac{1}{x^2}}=\frac{1}{x^2}$$

∴ Multiplying the given equation with $\dfrac{1}{x^2}$, we get

$$\left(\frac{x^2-y^2}{x^2}\right)dx+\frac{2xy}{x^2}dy=0$$

$$\left(\frac{x^2-y^2}{x^2}\right)dx+2\frac{y}{x}dy=0 \qquad (2)$$

$$M_1=\frac{x^2-y^2}{x^2} \text{ and } N_1=\frac{2y}{x}$$

$$\frac{\partial M_1}{\partial y}=\frac{-2y}{x^2},\ \text{and } \frac{\partial N_1}{\partial x}=\frac{-2y}{x^2}$$

$$\frac{\partial M_1}{\partial y}=\frac{\partial N_1}{\partial x}$$

∴ Eq. (2) is an exact equation and its solution is

$$\int^x M_1dx+\int(\text{the terms of } N_1 \text{ not containing } x)dy=C$$

$$\int^x\frac{x^2-y^2}{x^2}dx+\int 0dy=C$$

$$\Rightarrow\int^x 1-\frac{y^2}{x^2}dx=C\ \Rightarrow\ x+\frac{y^2}{x}=C.$$

Example 13

Find the solution of $xy^2dx+(y+y^2)dy=0$.

Solution

Given $xy^2dx+(y+y^2)dy=0$ (1)

$$Mdx+Ndy=0$$

$$M=xy^2;\quad N=y+y^2$$

$$\frac{\partial M}{\partial y}=2xy \text{ and } \frac{\partial N}{\partial x}=0$$

$$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$$

$$\frac{1}{M}\left[\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right]=\frac{1}{xy^2}[-2xy]$$

$$=\frac{-2}{y}=g(y)$$

Integrating factor is $e^{\int g(y)dy}$

$$=e^{\int\frac{-2}{y}dy}=e^{-2\log y\,dy}=e^{\log\frac{1}{y^2}}=\frac{1}{y^2}$$

Multiplying Eq. (1) by $\dfrac{1}{y^2}$, we get $\dfrac{xy^2dx}{y^2}+\left(\dfrac{y+y^2}{y^2}\right)dy=0$

$$xdx + \left(\frac{1}{y} + 1\right)dy = 0$$

Integrating on both sides we get $\dfrac{x^2}{2} + \log y + y = C$

Linear Equations

Consider the linear differential equation $\dfrac{dy}{dx} + Py = Q$ (1)

where P and Q are functions of only x. We explain below, how such equations can be solved. Consider the equation

$$\frac{dy}{dx} + Py = 0 \tag{2}$$

The Eq. (2) is called the homogeneous linear equation corresponding to Eq. (1). We find the general solution of Eq. (2).

Eq. (2) is a variables separable type. We write it as

$$\frac{dy}{y} = -Pdx.$$

Integrating the above equation given.

$$\log y = -\int Pdx + \log C \text{ or } y = Ce^{-\int Pdx} \tag{3}$$

This represents the general solution of Eq. (2).

Eq. (3) may also be written as $ye^{\int Pdx} = c$.

Now, $\dfrac{d}{dx}(ye^{\int Pdx}) = 0$

That is, $e^{\int Pdx}\dfrac{dy}{dx} + ye^{\int Pdx} \times P = 0$ or $e^{\int Pdx}\left[\dfrac{dy}{dx} + Py\right] = 0.$

This means that if we multiply both sides of Eq. (2) by $e^{\int Pdx}$, the product

$e^{\int Pdx}\left[\dfrac{dy}{dx} + Py\right]$ is $\dfrac{d}{dx}\{ye^{\int Pdx}\}$. The factor $e^{\int Pdx}$ is called an integrating factor of Eq. (2).

Suppose we multiply both sides of Eq. (1) by $e^{\int Pdx}$, it is reduced to $\dfrac{d}{dx}(ye^{\int Pdx}) = \dfrac{d}{dx}\left(\int Qe^{\int Pdx}dx\right)$, $\left[\text{since } \dfrac{d}{dx}\left(\int Qe^{\int Pdx} dx\right) = Qe^{\int Pdx}\right]$. Hence, we get the general solution of Eq. (1) as

$$ye^{\int Pdx} = C + \int Qe^{\int Pdx} dx.$$

Example 14

Solve $\sin x \dfrac{dy}{dx} + y \cos x = 1$.

Solution

$$\sin x \frac{dy}{dx} + y\cos x = 1$$

$$\frac{dy}{dx} + (\cot x)y = \operatorname{cosec} x.$$

This is a linear equation in y

Here, $P = \cot x$, $Q = \operatorname{cosec} x$.

$$\int Pdx = \int \cot x\, dx = \log(\sin x)$$

$$\text{IF} = e^{\int Pdx} = e^{\log \sin x} = \sin x.$$

$\therefore$ The general solution is $y \cdot \text{IF} = \int \text{QIF}\, dx + c$

$$y \sin x = \int \operatorname{cosec} x \cdot \sin x\, dx + c$$

$$y \sin x = \int dx + c$$

$$y \sin x = x + c.$$

Example 15

Solve $(1 + x^4)\dfrac{dy}{dx} + 4x^3 y = \sin^3 x$.

Solution

Given: $(1 + x^4)\dfrac{dy}{dx} + 4x^3 y = \sin^3 x$

$$\frac{dy}{dx} + \frac{4x^3}{1+x^4}y = \frac{\sin^3 x}{1+x^4}$$

It is a linear differential equation in y.

Here, $P = \dfrac{4x^3}{1+x^4}$ and $Q = \dfrac{\sin^3 x}{1+x^4}$

$$\int Pdx = \int \frac{4x^3}{1+x^4}dx = \log(1+x^4)$$

$$\text{IF} = e^{\int pdx} = e^{\log(1+x^4)} = 1 + x$$

General solution

$y \cdot \text{IF} = \int Q \cdot \text{IF}\, dx + c.$

$$y(1+x^4) = \int \frac{\sin^3 x}{1+x^4}(1+x^4)\, dx + c$$

$$= \int \sin^3 x\, dx + C = \int \frac{3\sin x - \sin 3x}{4} dx + c$$

$$y(1+x^4) = \frac{\cos 3x}{12} - \frac{3}{4}\cos x + c$$

$$12y(1 + x^4) = \cos 3x - 9 \cos x + c$$

Example 16

Solve $x^2\left(\dfrac{dy}{dx} + y\right) = 4x^2 + 8 - 2y$.

Solution

Given: $x^2\left(\dfrac{dy}{dx} + y\right) = 4x^2 + 8 - 2y$

$$\frac{dy}{dx} + y = 4 + \frac{8}{x^2} - \frac{2y}{x^2}$$

$$\frac{dy}{dx} + y\left(1 + \frac{2}{x^2}\right) = 4 + \frac{8}{x^2}$$

Here, $P = 1 + \dfrac{2}{x^2}$ and $Q = 4 + \dfrac{8}{x^2}$

$$\int Pdx = \int 1 + \frac{2}{x^2}\, dx = x - \frac{2}{x}$$

$$\text{IF} = e^{\int P dx} = e^{\left(x-\frac{2}{x}\right)}$$

General solution is $y \cdot \text{IF} = \int Q \cdot \text{IF} dx + c$

$$ye^{x-\frac{2}{x}} = \int\left(4+\frac{8}{x^2}\right)e^{x-\frac{2}{x}}dx + c$$

$$= 4\int\left(1+\frac{2}{x^2}\right)e^{x-\frac{2}{x}}dx + c$$

(Put $e^{x-\frac{2}{x}} = t$

$$\Rightarrow \quad e^{x-\frac{2}{x}}\left(1+\frac{2}{x^2}\right)dx = dt)$$

$$= 4\int dt + c = 4t + c$$

The general solution is

$$ye^{x-\frac{2}{x}} = 4e^{x-\frac{2}{x}} + c$$

Bernoulli's Linear Equations

An equation of the form $\frac{dy}{dx} + Py = Qy^n$ is called Bernoulli's linear equation, where P, Q are continuous functions in x.

Example 17

Solve $\frac{dy}{dx} + xy = -(3xy^2)$.

Solution

Given $\frac{dy}{dx} + xy = -(3xy^2)$

Throughout the equation dividing with y^2 we get

$$y^{-2}\frac{dy}{dx} + xy^{-1} = -3x \tag{1}$$

$$\text{Let } y^{-1} = u \quad \Rightarrow \quad -y^{-2}\frac{dy}{dx} = \frac{du}{dx}$$

The Eq. (1) becomes $\frac{-du}{dx} + xu = -3x$

$$\frac{du}{dx} - xu = 3x$$

The above equation is a linear differential equation in u.

$$\therefore \quad \text{IF} = e^{\int P dx} = e^{-\int x dx} = e^{\frac{-x^2}{2}}$$

$\therefore$ Solution is $u \cdot \text{IF} = \int \text{QIF}\, dx$

$$u \cdot e^{\frac{-x^2}{2}} = \int 3xe^{\frac{-x^2}{2}}dx.$$

$$= -\int 3e^{-t}dt \text{ when } t = \frac{-x^2}{2}$$

$$= \frac{-3e^{-t}}{-1} = 3e^{-t}$$

$$u \cdot e^{\frac{-x^2}{2}} = 3e^{\frac{-x^2}{2}} + C$$

$$\frac{1}{y} = 3 + Ce^{\frac{x^2}{2}}$$

$$y = \frac{1}{3 + Ce^{\frac{x^2}{2}}}$$

Example 18

Solve $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^3}(\log y)^2$.

Solution

Given $\frac{dy}{dx} + \frac{y\log y}{x} = \frac{y(\log y)^2}{x^3}$

$$\Rightarrow \quad \frac{1}{y(\log y)^2}\frac{dy}{dx} + \frac{1}{x}\cdot\frac{1}{(\log y)} = \frac{1}{x^3} \tag{1}$$

Let $\frac{1}{\log y} = u$,

Differenting wrt x

$$\frac{-1}{(\log y)^2}\cdot\frac{1}{y}\frac{dy}{dx} = \frac{du}{dx}$$

$\therefore$ Eq. (1) becomes

$$\Rightarrow \quad \frac{-du}{dx} + \frac{1}{x}u = \frac{1}{x^3} \quad \Rightarrow \quad \frac{du}{dx} - \frac{1}{x}u = \frac{-1}{x^3}$$

It is a linear equation in u.

Here $P = \frac{-1}{x}$ and $Q = \frac{-1}{x^3}$

$$\text{IF} = e^{\int P dx} = e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$$

$\therefore$ Solution is $u \cdot \text{IF} = \int \text{QIF} dx + c$

$$\frac{1}{x}u = \int \frac{-1}{x^3}\cdot\frac{1}{x}dx + c$$

$$\frac{1}{x}u = -\int x^{-4}dx + c$$

$$\frac{1}{(\log y)x} = \frac{1}{3x^3} + c$$

Second Order Linear Differential Equations with Constant Co-efficients

The standard form of a second order linear differential equation with constant co-efficients is

$$a_0\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = F(x) \tag{1}$$

where a_0, a_1, a_2 are real constants and $F(x)$ is a function of only x. The second order equation,

$$a_0\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0 \tag{2}$$

represents the corresponding homogeneous equation.

Let $y = u(x)$ represent the general solution of Eq. (2) [$u(x)$ will contain two arbitrary constants]. This means that

$$a_0 \frac{d^2u}{dx^2} + a_1 \frac{du}{dx} + a_2 u = 0 \qquad (3)$$

Let $y = v(x)$ represent a particular solution of the given equation of Eq. (1). We have, then,

$$a_0 \frac{d^2v}{dx^2} + a_1 \frac{dv}{dx} + a_2 v = F(x) \qquad (4)$$

Substituting $y = u(x) + v(x)$ in Eq. (1),

$$a_0 \frac{d^2}{dx^2}(u+v) + a_1 \frac{d}{dx}(u+v) + a_2(u+v)$$

$$= \left(a_0 \frac{d^2u}{dx^2} + a_1 \frac{du}{dx} + a_2 u\right) + \left(a_0 \frac{d^2v}{dx^2} + a_1 \frac{dv}{dx} + a_2 v\right)$$

$= 0 + F(x)$ (by Eqs. (3) and (4))

$= F(x)$.

We infer that $y = u(x) + v(x)$ is the general solution of the Eq. (1). Thus, the general solution of Eq. (2) is the sum of the general solution of the corresponding homogeneous equation (2) and a particular solution of the given equation (1). $y = u(x)$ is called the *complementary function* of Eq. (2) and $y = v(x)$ is called a particular integral of Eq. (1). The general solution of Eq. (1) is given by $y = u(x) + v(x)$.

= [Complementary function] + [Particular integral]

= CF + PI (in short).

To find the complementary function of Eq. (1) or to obtain the general solution of the homogeneous equation (2): As $y = e^{mx}$ is a solution of $\frac{dy}{dx} - my = 0$, we assume $y = e^{mx}$ (for some value of m) to be a solution of Eq. (2).

Then, $a_0 \frac{d^2}{dx^2}(e^{mx}) + a_1 \frac{d}{dx}(e^{mx}) + a_2 e^{mx}$ must be equal to zero (or) $e^{mx}\{a_0 m^2 + a_1 m + a_2\} = 0$.

Since e^{mx} cannot be equal to zero, $a_0 m^2 + a_1 m + a_2 = 0$ (5)

Eq. (5) is called the auxiliary equation corresponding to (1) [or (2)]. Eq. (5) is quadratic in m and gives two values for m, which may be real or complex.

Case 1: Let the roots of Eq. (5) be real and distinct, say m_1 and m_2 ($m_1 \neq m_2$). Then, $y = e^{m_1 x}$ and $y = e^{m_2 x}$ are two distinct solutions of (2) or $y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$ (6)

(C_1 and C_2 are arbitrary constants) is the general solution of (2) or the complementary function of (1).

Case 2: Let the roots of (5) be real and equal and each equals to m_1.

$$\text{Let } \frac{d}{dx} \equiv D, \quad \frac{d^2}{dx^2} \equiv D^2.$$

Then Eq. (2) may be expressed as $(a_0 D^2 + a_1 D + a_2)y = 0$.

Since the roots of the auxiliary equation are equal and each equal to m_1, this reduces to

$$a_0(D - m_1)^2 y = 0 \text{ or } (D - m_1)^2 y = 0 \qquad (7)$$

(since $a_0 \neq 0$)

Let $(D - m_1)y = Y_1$ (8)

Then, Eq. (7) becomes $(D - m_1)Y_1 = 0$. (9)

Now, Eq. (9) is reduced to $\frac{dY_1}{dx} - m_1 Y_1 = 0$, giving $Y_1 = C_1 e^{m_1 x}$ as the solution.

Substituting in Eq. (8), $\frac{dy}{dx} - m_1 y = c_1 e^{m_1 x}$ is a linear equation. The general solution is given by $ye^{-m_1 x} = c_2 + \int c_1 e^{m_1 x} \times e^{-m_1 x} dx = c_2 + c_1 x$

or $$y = c_2 e^{m_1 x} + c_1 x e^{m_1 x} = e^{m_1 x}(c_2 + c_1 x)$$

where c_1 and c_2 are arbitrary constants.

Case 3: Let the roots of (V) be complex. Let us assume the roots as the conjugate pairs $\alpha \pm i\beta$. (The co-efficients a_0, a_1, a_2 being real, roots occur in conjugate pairs).

The general solution is $y = c_1 e^{(\alpha + i\beta)x} + c_2 e^{(\alpha - i\beta)x}$

$$= c_1 e^{\alpha x}(\cos \beta x + i \sin \beta x) + c_2 e^{\alpha x}(\cos \beta x - i \sin \beta x)$$

$$= e^{\alpha x}\{(c_1 + c_2) \cos \beta x + i(c_1 - c_2) \sin \beta x\}$$

$$= e^{\alpha x}\{A_1 \cos \beta x + A_2 \sin \beta x).$$

where A and B are arbitrary constants. We may now summarize the nature of the complementary function of Eq. (1) as follows:

Roots of the Auxiliary Equation $a_0 m^2 + a_1 m + a_2 = 0$	Complementary Function of (1), or General Solution of (2)
Roots, real and distinct, say m_1, m_2	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
Roots, real and equal, say each equals m_1	$y = (c_1 + c_2 x)e^{m_1 x}$
Roots, complex, say $\alpha \pm i\beta$	$y = e^{\alpha x}\{c_1 \cos \beta x + c_2 \sin \beta x\}$
Roots, complex and repeated, say $m_1 = m_2 = \alpha + i\beta$ and $m_3 = m_4 = \alpha - i\beta$	$y = e^{\alpha x}[(c_1 + c_2 x) \cos \beta x + (c_3 + c_4 x) \sin \beta x$

Example 23

Obtain the complementary function of the equation $\frac{d^2y}{dx^2} - \frac{7dy}{dx} + 6y = x^4$.

Solution

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = x^4$$

$\Rightarrow \quad (D^2 - 7D + 6)y = x^4$

Auxiliary equation is $m^2 - 7m + 6 = 0$

$$m = 1, 6.$$

$\therefore$ The complementary function of the given equation.

$$y = c_1 e^x + c_2 e^{6x}.$$

Example 24

Obtain the general solution of the equation $\frac{d^2y}{dx^2}-10\cdot\frac{dy}{dx}+25y=0.$

Solution

Given: $\frac{d^2y}{dx^2}-10\frac{dy}{dx}+25y=0$

$$\Rightarrow \quad (D^2-10D+25)y=0$$

Auxiliary equation is $m^2-10m+25=0$

The roots are $(m)=5, 5$

$\therefore$ The general solution of the equation is $(c_1+c_2x)e^{5x}$.

Example 25

Obtain the complementary function of the equation $\frac{d^2y}{dx^2}-6\cdot\frac{dy}{dx}+10y=e^{3x}.$

Solution

Given: $\frac{d^2y}{dx^2}-6\cdot\frac{dy}{dx}+10y=e^{3x}$

$\Rightarrow \quad (D^2-6D+10)y=e^{3x}$

Auxiliary equation is $m^2-6m+10=0$

$$m=\frac{6\pm\sqrt{36-40}}{2}=\frac{6\pm 2i}{2}=3\pm i$$

$\therefore$ The complementary function is given by $y_c=e^{3x}(c_1\cos x+c_2\sin x)$.

To find a particular integral of Eq. (1) or to find a particular solution of the Eq. (1):

$$a_0\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=F(x)$$

We may write the above as $(a_0D^2+a_1D+a_2)y=F(x)$ or $f(D)\,y=F(x)$ where $f(D)$ stands for $(a_0D^2+a_1D+a_2)$. Particular integral y is that function of x independent of arbitrary constants such that $f(D)$ on y or $f(D)\,y$ yields $F(x)$.

This is symbolically represented as $y=\frac{1}{f(D)}\{F(x)\}$.

Case 1: $F(x)=e^{kx}$ where k is a constant.

We have $D(e^{kx})=ke^{kx}$, $D^2(e^{kx})=k^2e^{kx}$ … or, in general, $g(D)\,(e^{kx})=g(k)\,e^{kx}$ where $g(D)$ is a polynomial in D, in particular, $f(D)\,\{e^{kx}\}=f(k)\,e^{kx}$.

Since $\frac{1}{f(D)}e^{kx}$ is that function of x which when operated by $f(D)$ gives e^{kx}, it is clear that $\frac{1}{f(D)}e^{kx}=\frac{1}{f(k)}e^{kx}$ provided $f(k)\neq 0$. $f(k)$ reduces to zero when one or both the roots of the auxiliary equation $a_0m^2+a_1m+a_2=0$, is k.

1. Suppose one of the roots is k. Then, $f(D)=a_0(D-k)(D-m_0)$, where $m_0\neq k$. Particular integral

$$=\frac{1}{a_0(D-k)(D-m_0)}e^{kx}$$

$$=\frac{1}{D-k}\left\{\frac{1}{a_0(D-m_0)}e^{kx}\right\}$$

$$=\frac{1}{a_0(k-m_0)}\frac{1}{(D-k)}e^{kx}$$

Let $\frac{1}{(D-k)}e^{kx}=X_1$

Then $(D-k)X_1=e^{kx}$ or $\frac{dX_1}{dx}-kX_1=e^{kx}$

This is a linear equation and the particular solution of the above equation is xe^{kx}. Therefore, particular integral $=\frac{1}{a_0(k-m_0)}xe^{kx}$.

2. Suppose both the roots of the auxiliary equation are k. Then, particular integral

$$=\frac{1}{a_0(D-k)^2}[e^{kx}]$$

$$=\frac{1}{a_0(D-k)}\left[\frac{1}{(D-k)}e^{kx}\right]$$

$$=\frac{1}{a_0(D-k)}[xe^{kx}],$$

Use the result in (1). Now, let $\frac{1}{D-k}(xe^{kx})=X_2$

We have, therefore, $(D-k)X_2=xe^{kx}$ or $\frac{dX_2}{dx}-kX_2=xe^{kx}$ which is a linear equation.

Particular solution is $X_2=\frac{x^2}{2}e^{kx}$ or, particular integral in this case is given by $y=\frac{x^2}{2}e^{kx}$.

Example 26

Solve the differential equation:

$$(D^2+5D+6)y=e^{-4x}$$

Solution

$(D^2+5D+6)y=e^{-4x}$

Auxiliary equation is $m^2+5m+6=0$.

$$(m+3)(m+2)=0.$$

$\therefore$ Roots are $m=-3,-2$.

Complementary function is $c_1e^{-3x} + c_2e^{-2x}$.

$$\text{Particular integral } = \frac{1}{D^2+5D+6}\cdot e^{-4x}$$

$$= \frac{1}{(-4)^2+5(-4)+6}e^{-4x} = \frac{e^{-4x}}{2}$$

∴ General solution is

$$y = c_1e^{-3x} + c_2e^{-2x} + \frac{e^{-4x}}{2}.$$

Example 27

Solve $(3D^2 - D - 10)y = 6e^{2x}$

Solution

Given $(3D^2 - D - 10)y = 6e^{2x}$

Auxiliary equation $3m^2 - m - 10 = 0$

$$m = 2, \frac{-5}{3}.$$

∴ Complementary function is

$$\text{CF} = c_1e^{2x} + c_2e^{\frac{-5}{3}x}$$

$$\text{PI} = \frac{1}{3D^2-D-10}6e^{2x}$$

$$= \frac{1}{(D-2)(3D+5)}6e^{2x}$$

$$= 6\frac{1}{D-2}\left[\frac{1}{3D+5}e^{2x}\right] = 6\frac{1}{(D-2)}\frac{1}{11}e^{2x}1$$

$$= \frac{6}{11}\frac{1}{(D-2)}e^{2x} = \frac{6}{11}xe^{2x}$$

∴ General solution is

$$y = c_1e^{2x} + c_2e^{-\frac{5}{3}x} + \frac{6}{11}xe^{2x}.$$

Example 28

Solve $(D^2 - 12D + 36)y = e^{6x}$

Solution

Given: $(D^2 - 12D + 36)y = e^{6x}$

Auxiliary equation is $m^2 - 12m + 36 = 0$.

$$m^2 - 12m + 36 = 0.$$

$$m = 6, 6$$

Complementary function (CF) $= (c_1 + c_2x)e^{6x}$

$$\text{PI} = \frac{1}{D^2-12D+36}e^{6x} = \frac{1}{(D-6)^2}e^{6x}$$

$$= \frac{x^2}{2!}e^{6x}$$

∴ General solution is $y = \text{CF} + \text{PI}$

$$= (c_1+c_2x)e^{6x} + \frac{x^2}{2!}e^{6x0}$$

Case 2: $F(x) = \sin kx$ or $\cos kx$ where k is a constant.

We have $D\{\sin kx\} = k\cos kx$

$$D^2\{\sin kx\} = -k^2 \sin kx$$

Similarly, $D^2\{\cos kx\} = -k^2\cos kx$

If $g(D^2)$ is a polynomial in D^2,

$$g(D^2)\ \{\sin kx \text{ or } \cos kx\} = g(-k^2)\sin kx \text{ or } g(-k^2)\cos kx.$$

Hence, $\frac{1}{g(D^2)}\sin kx = \frac{1}{g(-k^2)}\sin kx$ and $\frac{1}{g(D^2)}\cos kx$

$= \frac{1}{g(-k^2)}\cos kx$, provided $g(-k^2) \neq 0$.

We shall illustrate the above technique by considering two examples.

Example 29

Find the particular integral of the equation $(D^2 + 16)y = \cos 3x$.

Solution

$$\text{PI} = \frac{1}{D^2+16}\cos 3x = \frac{1}{-(3)^2+16}\cos 3x = \frac{\cos 3x}{7}$$

Example 30

Find the particular integral of the equation $(D^2 - 5D + 6)y = \sin 3x$.

Solution

$$\text{PI} = \frac{1}{D^2-5D+6}\sin 3x$$

$$= \frac{1}{-3^2-5D+6}\sin 3x$$

$$\frac{1}{-5D-3}\sin 3x$$

$$= -\frac{5D-3}{(5D+3)(5D-3)}\sin 3x$$

$$= \frac{(5D-3)}{-(25D^2-9)}\sin 3x = \frac{3-5D}{25\times(-9)-9}\sin 3x$$

$$= \frac{1}{-234}[(3-5D)\sin 3x]$$

$$= \frac{-1}{234}[3\sin 3x - 5D(\sin 3x)]$$

$$= \frac{-1}{234}[3\sin 3x - 15\cos 3x]$$

$$\text{PI} = \frac{15\cos 3x}{234} - \frac{3\sin 3x}{234}$$

NOTE

Suppose $g(-k^2) = 0$.

Let us discuss the technique of finding particular integral in this case.

Suppose we have to find $\frac{1}{D^2+k^2}[\sin kx]$.

By Euler's formula, $e^{ikx} = \cos kx + i \sin kx$ or $\sin kx$ = imaginary part of e^{ikx}.

$$\text{Particular integral} = \frac{1}{D^2+k^2}[\sin kx].$$

$$= \text{Imaginary part of } \frac{1}{D^2+k^2}(e^{ikx})$$

$$= \text{Imaginary part of } \frac{1}{(D-ik)(D+ik)}e^{ikx}$$

$$= \text{Imaginary part of } \frac{1}{D-ik}\left[\frac{e^{ikx}}{2ik}\right]$$

$$= \text{Imaginary part of } \frac{xe^{ikx}}{2ik}$$

$$= \text{Imaginary part of } \frac{x}{2ki}(\cos kx + i\sin kx)$$

$$= \text{Imaginary part of } \frac{x}{2k}(-i\cos kx + \sin kx)$$

$$= \frac{-x\cos kx}{2k}.$$

Similarly, if we have to find $\frac{1}{D^2+k^2}[\cos kx]$.

We write it as the real part of $\frac{1}{D^2+k^2}(e^{ikx})$

$$= \text{Real part of } \frac{1}{(D-ik)(D+ik)}(e^{ikx})$$

$$= \text{Real part of } \frac{x}{2k}(-i\cos kx + \sin kx)$$

$$= \frac{x\sin kx}{2k}.$$

$$\boxed{\begin{aligned} \frac{1}{D^2+k^2}\sin kx &= \frac{-x}{2k}\cos kx \\ \frac{1}{D^2+k^2}\cos kx &= \frac{x}{2k}\sin kx \end{aligned}}$$

Example 31

Solve the equation $(D^2 + 16)y = \sin 4x$.

Solution

Given: $(D^2 + 16)y = \sin 4x$

Auxiliary equation is $m^2 + 16 = 0$

$$m = \pm 4i$$

$\therefore$ CF $= c_1 \cos 4x + c_2 \sin 4x$

$$PI = \frac{1}{D^2+16}\sin 4x.$$

$$= -\frac{x}{2 \cdot 4}\cos 4x$$

$$\left(\because \frac{1}{D^2+k^2}\sin kx = -\frac{x}{2kx}\cos kx\right)$$

$$= -\frac{x}{8}\cos 4x$$

General solution is $y =$ CF + PI

$$= c_1\cos 4x + c_2\sin 4x - \frac{x}{8}\cos 4x.$$

Cauchy's Homogeneous Linear Equations

An equation of the form

$$x^n\frac{d^ny}{dx^n} + p_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \cdots p_ny = Q(x) \qquad (1)$$

where $p_1, p_2, \ldots, p_n$ are constants is called Cauchy's linear equation. To convert the above equation into linear differential equation with constant co-efficients, we substitute $x = e^z$ or $z = \log x$.

$\therefore \quad z = \log x,$

$$\Rightarrow \quad \frac{dz}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{1}{x}$$

$$\frac{dy}{dz} = x\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{x}\cdot\frac{dy}{dz}\right)$$

$$= \frac{-1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{-1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{dz}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{x^2}\frac{dy}{dz} + \frac{1}{x^2}\frac{d^2y}{dz^2}$$

$$x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz} = \frac{d}{dz}\left[\frac{dy}{dz} - y\right]$$

$$\text{Let } \frac{dy}{dz} = \theta y \quad \Rightarrow \quad x\frac{dy}{dx} = \theta y,\ x^2\frac{d^2y}{dx^2} = \theta(\theta-1)y$$

Similarly $x^3 \dfrac{d^3y}{dx^3} = \theta(\theta-1)(\theta-2)y$, and so on.

Then Eq. (1) is changed into a linear differential equation.

We solve this by methods discussed earlier.

Example 32

Solve $x^2 \dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} - 3y = 0$

Solution

Let $x = e^z$ or $z = \log x$

Then $x\dfrac{dy}{dx} = \theta y;\ x^2\dfrac{d^2y}{dx^2} = \theta(\theta-1)y$

The above equation becomes

$$[\theta(\theta-1)+3\theta-3]\,y = 0$$

$$\left\lfloor \theta^2+2\theta-3 \right\rfloor y = 0$$

Auxiliary equation is $m^2 + 2m - 3 = 0$

$\Rightarrow \quad (m+3)(m-1) = 0$

$\Rightarrow \quad m = -3, 1$

$\therefore \quad y = c_1e^{-3z} + c_2e^z$

$\quad\quad = c_1x^{-3} + c_2x.$

Example 33

Solve $x^3\dfrac{d^3y}{dx^3} + 6x^2\dfrac{d^2y}{dx^2} + 8\dfrac{dy}{dx} + 2y = x^2\log x.$

Solution

Put $x = e^z$ or $z = \log x$. Then

$$x\frac{dy}{dx} = \theta y,\ x^2\frac{d^2y}{dx^2} = \theta(\theta-1)y,$$

$$x^3\frac{d^3y}{dx^3} = \theta(\theta-1)(\theta-2)y$$

The given equation becomes

$$[\theta(\theta-1)(\theta-2) + 6\theta(\theta-1) + 8\theta + 2]y = e^{2z}\cdot z$$

$$(\theta^3 + 3\theta^2 + 4\theta + 2)y = e^{2z}\cdot z$$

$$\text{AE} = m^3 + 3m^2 + 4m + 2 = 0$$

$$(m+1)(m^2+2m+2) = 0$$

$$m = -1 \text{ or } m = -1 \pm i$$

$$\text{CF} = C_1e^{-z} + e^{-z}(C_2\cos z + C_3\sin z)$$

$$\text{PI} = \frac{1}{\theta^3+3\theta^2+4\theta+2}\cdot e^{2z}z$$

$$= e^{2z}\frac{1}{(\theta+2)^3+3(\theta+2)^2+4(\theta+2)+2}z$$

$$= e^{2z}\frac{1}{\theta^3+9\theta^2+28\theta+30}\cdot z$$

$$\frac{e^{2z}}{30}\left[1+\frac{\theta^3+9\theta^2+28\theta}{30}\right]^{-1}z$$

$$= \frac{e^{2z}}{30}\left[1-\frac{\theta^3+9\theta^2+28\theta}{30}\right]z$$

$$= \frac{e^{2z}}{30}z - \frac{28}{(30)^2}e^{2z}$$

$$y = \text{CF} + \text{PI}$$

$$= C_1e^{-z} + e^{-z}(C_2\cos z + C_3\sin z) + \frac{e^{2z}}{30}z - \frac{28}{(30)^2}e^{2z}$$

$$= \frac{C_1}{x} + \frac{1}{x}(C_2\cos(\log x) + C_3\sin(\log x)) + \frac{x^2\log x}{30} - \frac{28}{900}x^2$$

Example 34

Solve $(2x-1)\dfrac{d^2y}{dx^2} + 2(2x-1)\dfrac{dy}{dx} - 100y = 32(2x-1)^2$

Solution

Let $2x - 1 = u$

$$2 = \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = 2\frac{dy}{du}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(2\frac{dy}{du}\right)$$

$$= 2\frac{d}{du}\left(\frac{dy}{du}\right)\cdot\frac{du}{dx} = 2^2\frac{d^2y}{du^2}$$

$\therefore$ The given equation becomes

$$2^2u^2\frac{d^2y}{du^2} + 2\cdot 2u\frac{dy}{du} - 100y = 32u^2$$

$$u^2\frac{d^2y}{du^2} + u\frac{dy}{du} - 25y = 8u^2$$

Let $u = e^z,\ u\dfrac{dy}{dx} = 0;\ x^2\dfrac{d^2y}{dx^2} = \theta(\theta-1)$

$$[\theta(\theta-1)+\theta-25]y = 8e^{2z}$$

$$\left\lfloor \theta^2-25 \right\rfloor y = 8e^{2z}$$

$$\text{AE} = m^2 - 25 = 0 \quad \Rightarrow \quad m = \pm 5$$

$$\text{CF} = C_1e^{-5z} + C_2e^{5z}$$

$$\text{PI} = \frac{1}{\theta^2-25}\cdot 8e^{2z} = 8.e^{2z}\frac{1}{2^2-25} = \frac{-8}{21}e^{2z}$$

$$y = \text{CF} + \text{PI} = C_1e^{5z} + C_2e^{5z} - \frac{8}{21}e^{2z}$$

$$= C_1u^{-5} + C_2u^5 - \frac{8}{21}u \text{ where } u = (2x-1).$$

Method of Variation of Parameters

An equation of the form $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=R(x)$, where $P(x)$, $Q(x)$ and $R(x)$ are real valued functions of x, is called the linear equation of the second order with variable co-efficients.

The above equation is solved by the method of variation of parameters.

The method is explained below:

1. Find the solution of $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0$ and let the solution be $y_c = C_1U(x) + C_2V(x)$
2. Write particular solution as follows:

   $y_p = AU(x) + BV(x)$

   where $A=\int\frac{-VR}{W}dx$

   and $B=\int\frac{UR}{W}dx$

   where $W=\begin{vmatrix} U & V \\ \frac{dU}{dx} & \frac{dV}{dx}\end{vmatrix}=U\frac{dV}{dx}-V\frac{dU}{dx}$ is called the Wronskian of U and V.
3. Then the solution is $y_c + y_p$

   i.e., $y = C_1U(x) + C_2V(x) + AU(x) + BV(x)$

Example 35

Solve the differential equation $(D^2 + 4)y = \sec 2x$ by variation of parameters.

Solution

Given $(D^2 + 4)y = \sec 2x$

$$\text{AE} = m^2 + 4 = 0 \quad \Rightarrow \quad m = \pm 2i$$

$$\text{CF} = y_c = C_1\cos 2x + C_2\sin 2x$$

$$\therefore\ U(x) = \cos 2x;\ V(x) = \sin 2x$$

$$y_p = AU(x) + BV(x)$$

$$W=U\frac{dV}{dx}-V\frac{dU}{dx}$$

$$=\cos 2x\frac{d}{dx}(\sin 2x)-\sin 2x\cdot\frac{d}{dx}(\cos 2x)$$

$$= 2\cos^2 2x + 2\sin^2 2x = 2$$

$$A=-\int\frac{VR}{W}dx=-\int\frac{\sin 2x\cdot\sec 2x}{2}dx$$

$$=-\int\frac{\tan 2x}{2}dx=\frac{1}{4}\log(\cos 2x)$$

$$B=\int\frac{UR}{W}dx=\int\frac{\cos 2x\cdot\sec 2x}{2}dx=\frac{1}{2}x$$

$$\therefore\quad y_p=\frac{1}{4}[\log(\cos 2x)]\cdot\cos 2x+\frac{1}{2}x\sin 2x$$

$$\therefore\quad y=y_c+y_p=C_1\cos 2x+C_2\sin 2x+\frac{1}{4}[\log(\cos 2x)]\cos 2x+\frac{1}{2}x\sin 2x.$$

Example 36

Solve the differential equation $y'' + 4y' + 4y = x^3e^{2x}$

Solution

Given equation

$$(D^2 + 4D + 4)y = x^3 e^{2x}$$

The auxiliary equation is

$$m^2 + 4m + 4 = 0$$

$$(m + 2)^2 = 0 \quad \Rightarrow \quad m = -2$$

$$y_C = C_1 e^{-2x} + C_2xe^{-2x}$$

Let $U(x) = e^{-2x}$ and $V(x) = xe^{-2x}$

$$y_p = AU(x) + BV(x)$$

$$A=-\int\frac{VR}{W}dx,\ B=\int\frac{UR}{W}dx$$

$$W=u\frac{dv}{dx}-v\frac{du}{dx}=e^{-2x}\frac{d}{dx}(xe^{-2x})-xe^{-2x}\frac{d}{dx}(e^{-2x})$$

$$=e^{-2x}\left[e^{-2x}-2xe^{-2x}\right]+2xe^{-2x}e^{-2x}=e^{-4x}$$

$$A=-\int\frac{UR}{\frac{udv}{dx}-\frac{vdu}{dx}}dx=-\int\frac{xe^{-2x}x^3\cdot e^{2x}}{e^{-4x}}dx$$

$$=-\int x^4e^{4x}dx$$

$$=-x^4\frac{e^{4x}}{4}+\frac{x^3e^{4x}}{4}-3\frac{x^2e^{4x}}{16}+6\frac{xe^{4x}}{16\times 4}-6\frac{e^{4x}}{16\times 16}$$

$$B=\int\frac{UR}{W}dx=\int\frac{e^{-2x}x^3e^{2x}}{e^{-4x}}dx=\int x^3e^{4x}dx$$

$$=x^3\frac{e^{4x}}{4}-\frac{3}{4}\left[x^2\frac{e^{4x}}{4}-2\frac{xe^{4x}}{16}+\frac{e^{4x}}{32}\right]$$

$$y = y_c + y_p = AU(X) + BV(x) + C_1e^{-2x} + C_2xe^{-2x}$$

$$=C_1e^{-2x}+C_2xe^{-2x}-x^4\frac{e^{2x}}{4}+\frac{x^3e^{2x}}{4}-\frac{3x^2e^{2x}}{16}$$

$$+\frac{6xe^{2x}}{64}-\frac{3e^{2x}}{3128}+\frac{x^4e^{2x}}{4}-\frac{3x^3e^{2x}}{16}+\frac{3}{32}x^2e^{2x}-\frac{3}{128}xe^{2x}$$

$$=C_1e^{-2x}+C_2xe^{-2x}-\frac{1}{16}x^3e^{2x}-\frac{3}{32}x^2e^{2x}\frac{9}{128}-\frac{xe^{2x}}{128}+\frac{9}{128}e^{2x}$$

Laplace Transforms

Let $f(t)$ be a given function defined for all $t \geq 0$. The Laplace transform of $F(t)$ is denoted by $L\{f(t)\}$ or $L\{f\}$ and is defined as $L\{f(t)\}=\int_0^\infty e^{-st}f(t)dt=F(s)$.

Here L is Laplace transform operator. $f(t)$ is the determining function depends on it. $F(s)$ is the function to be

determined called generating function. e^{-st} is called kernel of the transform.

Some standard results of Laplace transforms are given below.

1. $L\left\{e^{at}\right\}=\dfrac{1}{s-a},\ s>a$

2. $L\left\{e^{-at}\right\}=\dfrac{1}{s+a},$

3. (a) Let k be a constant $L\{k\}=\dfrac{k}{s}$

   (b) $L\{1\}=\dfrac{1}{s},\ s>0$

4. $L\left\{t^n\right\}=\dfrac{n!}{s^{n+1}},\ s>0$

5. $L\{\cos at\}=\dfrac{s}{s^2+a^2},\ s>0$

6. $L\{\sin at\}=\dfrac{a}{s^2+a^2},\ s>0$

7. $L\{\cosh at\}=\dfrac{s}{s^2-a^2},\ s>|a|$

8. $L\{\sinh at\}=\dfrac{a}{s^2-a^2},\ s>|a|$

9. $L\{t^n\cdot e^{at}\}=\dfrac{n!}{(s-a)^{n+1}},\ n\in Z^+$

10. $L\left\{\dfrac{1}{t}f(t)\right\}=\displaystyle\int_s^\infty F(s)\,ds$

Example 37

Find the Laplace transform of the function

$$f(x)=5e^{2x}+7e^{-3x}$$

Solution

$$L\{f(x)\}=L(5e^{2x}+7e^{-3x})$$
$$=5L(e^{2x})+7L(e^{-3x})$$
$$L\{f(t)\}=5\cdot\frac{1}{s-2}+7\cdot\frac{1}{s+3}$$
$$=\frac{5}{s-2}+\frac{7}{s+3}.$$

Example 37

Find $L\{f(t)\}$ where

$$f(t)=0,\ 0<t<1$$
$$=1,\ 1<t<2$$
$$=t,\ t>2.$$

Solution

As the given function is not defined at $t=0$, 1 and 2

$$L\{f(t)\}=\int_0^\infty e^{-st}\cdot F(t)dt$$
$$=\int_0^1 e^{-st}\cdot 0dt+\int_1^2 e^{-st}\cdot 1dt+\int_2^\infty e^{-st}\cdot tdt$$
$$=\int_1^2 e^{-st}dt+\int_2^\infty e^{-st}\cdot tdt$$
$$=\frac{e^{-st}}{-s}]\int_1^2+t\cdot\frac{e^{-st}}{-s}]\int_2^\infty-\int_2^\infty\frac{e^{-st}}{-s}\cdot dt$$
$$=-\frac{e^{-2s}}{s}+\frac{e^{-s}}{s}+\frac{2e^{-2s}}{s}+\frac{1}{s}\frac{e^{-st}}{-s}\Bigg]_2^\infty$$
$$=\frac{-e^{-2s}}{s}+\frac{e^{-s}}{s}+2\frac{e^{-2s}}{s}+\frac{1}{s^2}e^{-2s}$$
$$=\frac{e^{-2s}}{s}\left(1+\frac{1}{s}\right)+\frac{e^{-s}}{s}.$$

Example 39

Find the Laplace transform of the function

$$f(t)=\sin 2t,\ 0<t<\pi\ =\frac{e^{-st}}{-s}\Bigg]_1^2\int+t\cdot\frac{e^{-st}}{-s}\Bigg]_2^\infty\int-\int_2^\infty\frac{e^{-st}}{-s}\cdot dt$$
$$=0,\ t>\pi$$

Solution

$$L\{f(t)\}=\int_0^\infty e^{-st}f(t)dt$$
$$=\int_0^\pi e^{-st}\cdot\sin 2tdt+\int_\pi^\infty e^{-st}\cdot 0dt$$
$$=\int_0^\pi e^{-st}\sin 2tdt$$
$$=\frac{e^{-st}}{s^2+4}[-s\sin 2t-2\cos 2t]]_0^\pi$$
$$=\frac{2(1-e^{-\pi s})}{s^2+4}.$$

Example 40

Find the Laplace transform of the function $f(t)=(\sin t+\cos t)^2$

Solution

$L\{(\sin t+\cos t)^2\}=L\{1+\sin 2t\}=L\{1\}+L\{\sin 2t\}=$
$\dfrac{1}{s}+\dfrac{2}{s^2+4}$

Some important (theorems) properties of Laplace transforms:

1. **Linear property:** Let f and g be any two functions of t and a_1, a_2 are constants, then $L\{a_1f(t) + a_2g(t)\} = a_1L\{f(t)\} + a_2L\{g(t)\}$
2. **First shifting property:** If $L\{f(t)\} = F(s)$ then $L\{e^{at}f(t) = F(s-a)$

   Example: $L\{e^{at}\cos ct\} = \dfrac{s-a}{(s-a)^2+c^2}$
3. **Change of scale property:** If $L\{f(t)\} = F(s)$ then $L\{f(at)\} = \left|\dfrac{1}{a}\right| F\left(\dfrac{s}{a}\right)$

   Example: We know

   $$L\{e^{at}\} = \frac{1}{s-a} = F(s)$$

   Then $L\{be^{at}\} = \dfrac{1}{|b|}F\left(\dfrac{s}{b}\right) = \dfrac{1}{|b|}\dfrac{1}{\frac{s}{b}-a} = \dfrac{1}{|b|}\cdot\dfrac{b}{s-ab}$
4. **Differentiation theorem:** If derivatives of $f(t)$ are continuous and $L\{f(t)\} = F(s)$ then $L\{f'(t)\} = sF(s) - f(0)$ and

   $L\{f^n(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-3}f''(0)\ldots . f^{n-1}(0) = s^nF(s) - \sum_{r=0}^{n-1} s^{n-1-r}\cdot f^r(0)$ (f^r represents rth derivative of f)
5. **Multiplication theorem:** If $L\{f(t)\} = F(s)$ then $L\{t\cdot f(t)\} = -F'(s)$

   and $L\{t^n\cdot f(t)\} = (-1)^n \dfrac{d^n}{ds^n}[F(s)]$
6. **Division theorem:** If $L\{f(t)\} = F(s)$, then $L\left\{\dfrac{1}{t}f(t)\right\} = \displaystyle\int_s^\infty F(s)ds$
7. **Transforms of integrals (theorem)**

   If $L\{f(t)\} = F(s)$, then $L\{\displaystyle\int_0^t f(u)du\} = \dfrac{1}{s}F(s)$

Example 41

Find the Laplace transform of $te^{-2t}\sin^2 t$.

Solution

$$L\{\sin^2 t\} = \frac{1}{2}L\{1-\cos 2t\} = \frac{1}{2}\left\{\frac{1}{s} - \frac{s}{s^2+4}\right\}$$

$$\therefore \quad L\{t\cdot\sin^2 t\} = (-1)\frac{d}{ds}\left(\frac{1}{2}\left\{\frac{1}{s} - \frac{s}{s^2+4}\right\}\right)$$

(using multiplication theorem)

$$L\{t\sin^2 t\} = \frac{-1}{2}\left(\frac{-1}{s^2} - \frac{(s^2+4)-s(2s)}{(s^2+4)}\right)$$

$$= \frac{1}{2s^2} + \frac{4-s^2}{2(s^2+4)^2}$$

$$L\{e^{-2t}\cdot t\sin^2 t\} = \frac{1}{2(s+2)^2} + \frac{4-(s+2)^2}{2[(s+2)^2+4]^2}$$

(using shifting property)

$$= \frac{1}{2(s+2)^2} - \frac{4s+s^2}{2(s^2+4s+8)^2}.$$

Example 42

Find the Laplace transform of $\dfrac{\sin 2t - \cos 2t}{t}$.

Solution

$$L\{\sin 2t - \cos 2t\} = \frac{2}{s^2+4} - \frac{s}{s^2+4}$$

$$L\{\frac{\sin 2t - \cos 2t}{t}\} = \int_s^\infty \left(\frac{2}{s^2+4} - \frac{s}{s^2+4}\right)ds$$

(using division property)

$$= \frac{2}{2}\left(\tan^{-1}\frac{s}{2}\right)_s^\infty - \frac{1}{2}[\log(s^2+4)]_s^\infty$$

$$= \frac{\pi}{2} - \tan^{-1}\frac{s}{2} + \frac{1}{2}\log(s^2+4)$$

$$= \cot^{-1}\frac{s}{2} + \frac{1}{2}\log(s^2+4).$$

Example 43

Find the Laplace transform of $\displaystyle\int_0^t \frac{\sin 2u}{u}du$.

Solution

$$L\{\sin 2u\} = \frac{2}{s^2+4}$$

and $\left\{\dfrac{\sin 2u}{u}\right\} = \displaystyle\int_s^\infty \frac{2}{s^2+4}ds$

(using division theorem)

$$\frac{2}{2}\tan^{-1}\frac{s}{2}\Big]_s^\infty = \frac{\pi}{2} - \tan^{-1}\frac{s}{2} = \cot^{-1}\frac{s}{2}$$

$$\therefore \quad L\left\{\int_0^t \frac{\sin 2u}{u}du\right\} = \frac{1}{s}\cot^{-1}\frac{s}{2}$$

(using transform of integral theorem).

Inverse Laplace Transforms

If $F(s)$ is the Laplace transform of the function $f(t)$ i.e., $L\{f(t)\} = F(s)$ then $f(t)$ is called the inverse Laplace transform of the function $F(s)$ and is written as $f(t) = L^{-1}\{F(s)\}$. Here L^{-1} is called inverse Laplace transformation operator.

Some important standard results for inverse Laplace transform.

1. $L^{-1}\left(\frac{1}{s}\right) = 1$
2. $L^{-1}\left(\frac{1}{s^{n+1}}\right) = \frac{t^n}{n!}$ where n is a positive integer

   or $L^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{(n-1)!}$
3. $L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$
4. $L^{-1}\left(\frac{1}{(s-a)^n}\right) = \frac{e^{at}t^{n-1}}{(n-1)!}$
5. $L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{1}{a}\sin at$
6. $L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$
7. $L^{-1}\left(\frac{s}{s^2-a^2}\right) = \cos hat$
8. $L^{-1}\left(\frac{1}{s^2-a^2}\right) = \frac{1}{a}\sin hat$
9. $L^{-1}\left[\frac{1}{(s-a)^2+b^2}\right] = \frac{1}{b}e^{at}\sin bt$
10. $L^{-1}\left\{\frac{s-a}{(s-a)^2+b^2}\right\} = e^{at}\cos bt$
11. $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\} = \frac{1}{2a^3}$ (sin $at - at$ cos at)
12. $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{1}{2a}t\sin at$

To find the inverse Laplace transform we use the following methods.

1. Using the following properties

   (a) If $L^{-1}\{F(s)\} = f(t)$, than $L^{-1}\{F(s-a)\} = e^{at}f(t)$

   (b) If $L^{-1}\{F(s)\} = f(t)$ and $f(0) = 0$; then

   (i) $L^{-1}\{sF(s)\} = \frac{d}{dt}(f(t))$

   (ii) $L^{-1}\{s^nF(s)\} = \frac{d^n}{dt^n}(f(t))$ if $f(0) = f^1(0) = f^{(n-1)}(0) = 0$

   (c) If $L^{-1}\{F(s)\} = f(t)$, then

   (i) $L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(t)dt$

   (ii) $L^{-1}\left\{\frac{F(S)}{s^2}\right\} = \int_0^t\left\{\int_0^t f(t)dt\right\}dt$

2. **Convolution theorem:** Let $f(t)$ and $g(t)$ be two functions and

   $L^{-1}\{F(s)\} = f(t)$ and $L^{-1}\{G(s)\} = f(t)$, then

   $$L^{-1}\{F(s)\cdot G(s)\} = \int_0^t f(x)g(t-x)dx$$

   It is denoted by $f(t) * g(t)$ here $*$ represents convolution.

3. **Unit step function:** This function is defined as $u(t-a) = H(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases}$ the Laplace transform of $H(t-a) = L\{H(t-a)\}$

   $$= \int_0^\infty e^{-st}u(t-a)dt = \frac{e^{-as}}{s}$$

   NOTE

   This is also called as Heavisides unit function

4. **Periodic function:** If $f(t)$ is a periodic function with period a i.e., $f(t+a) = f(t)$, then

   $$L\{f(t)\} = \frac{\int_0^a e^{-st}f(t)dt}{1-e^{-sa}}$$

5. **Using partial fractions:** If $F(s)$ is of the from $\frac{G(s)}{H(s)}$ where G and H are polynomials in S then break $F(s)$ into partial fractions and manipulate term by term.

6. **Heavisides expansion formula:** Let $F(s)$ and $G(s)$ be two polynomials in 's' where $F(s)$ has degree less than that of $G(s)$. If $G(s)$ has n distinct zeros a_r, $r = 1, 2, 3, \ldots, n$

   i.e., $G(s) = (s-\alpha_1)(s-\alpha_2)\ldots(s-\alpha_n)$, then

   $$L^{-1}\left[\frac{F(S)}{G(S)}\right] = \sum_{r=1}^{n}\frac{F(\alpha_r)}{G'(\alpha_r)}e^{\alpha_r t}$$

Transform of Special Functions

7. **Bessel function:**

   $$J_0(x) = 1 - \frac{x^2}{2} + \frac{x^4}{2^2\cdot 4^2} - \frac{x^6}{2^2\cdot 4^2\cdot 6^2} + \cdots$$

   then $L\{J_0(x)\} = \frac{1}{\sqrt{s^2+1}}$

8. Error function: Error function is denoted as $erf(t)$

$$erf(\sqrt{x}) = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{x}} e^{-t^2}dt,$$

then $L\{erf(\sqrt{x}) = \frac{1}{s\sqrt{s+1}}$

9. Complex inversion (theorem) formula: If $f(t)$ has a continuous derivative and is of exponential order and $L\{f(t)\} = F(s)$ then $L^{-1}\{F(s)\}$ is given by

$$f(t) = \frac{1}{2\pi i}\int_{r-i\infty}^{r+i\infty} e^{st}F(s)ds, \quad t>0 \text{ and } f(t)=0 \text{ for } t<0$$

NOTES

1. The above result is also known as Bromwich's integral formula
2. The integration is to be performed along a line $s=r$ in the complex plane where $s=x+iy$. The real number r is chosen so that $p=r$ lies to the right of all the singularities.

10. The Gamma function: If $n > 0$, then the gamma function is defined by $\Gamma(n) = \int_0^{\infty} u^{n-1}e^{u}du$

11. Exponential Integral: The exponential integral is denoted by

$$E_i(t) = = \int_t^{\infty}\frac{e^{-u}}{u}du$$

Example 44

Evaluate $L^{-1}\left\{\frac{e^{2-3s}}{(s+2)^{5/2}}\right\}$

Solution

We have

$$L^{-1}\left\{\frac{1}{(s+2)^{5/2}}\right\} = e^{-2t}L^{-1}\left\{\frac{1}{s^{5/2}}\right\}$$

$$= e^{-2t}\frac{t^{\frac{5}{2}-1}}{\Gamma\left(\frac{5}{2}\right)} = \frac{4t^{\frac{3}{2}}e^{-2t}}{3\sqrt{\pi}}$$

$$\therefore \quad L^{-1}\left\{\frac{e^{2-3s}}{(s+2)^{5/2}}\right\} = e^2L^{-1}\left\{\frac{e^{-3s}}{(s+2)^{5/2}}\right\}$$

$$= \frac{4}{3\sqrt{\pi}}(t-3)^{3/2}e^{-2(t-4)}\cdot H(t-3)$$

(when expressed in terms of Heaviside's unit step function)

Example 45

Evaluate $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$

Solution

$$L^{-1}\left\{\frac{3(s-1)+10}{(s-1)^2-4}\right\}$$

$$= L^{-1}\left\{\frac{3(s-1)}{(s-1)^2-4} + \frac{10}{(s-1)^2-4}\right\}$$

$$= 3L^{-1}\left(\frac{s-1}{(s-1)^2-4}\right) + 10L^{-1}\left\{\frac{1}{(s-1)^2-4}\right\}$$

$$= 3e^tL^{-1}\left\{\frac{s}{s^2-2^2}\right\} + 10e^tL^{-1}\left\{\frac{1}{s^2-2^2}\right\}.$$

$$= 3e^t\cosh 2t + 5e^t\sinh 2t = 4e^{3t} - e^{-t}$$

Example 46

Evaluate $L^{-1}\left\{\frac{1}{s(s^2+4)^2}\right\}$

Solution

$$L^{-1}\left\{\frac{1}{s^2}\cdot\frac{s}{(s^2+4)^2}\right\}$$

Let $F_1(s) = \frac{1}{s^2}$ and $F_2(s) = \frac{s}{(s^2+4)^2}$ so that

$$L^{-1}\{F_1(s)\} = L^{-1}\left\{\frac{1}{s^2}\right\} = t = f_1(t)$$

and $$L^{-1}\{F_2(s)\} = L^{-1}\left\{\frac{s}{\left(s^2+4\right)^2}\right\}$$

$$= = \frac{t\cdot\sin 2t}{4} = f_2(t) \text{ (say)}$$

$\therefore$ By convolution theorem, we have

$$L^{-1}\left\{\frac{1}{s^2}\cdot\frac{s}{(s^2+4)^2}\right\} = L^{-1}\{F_1(s)\cdot F_2(s)\}$$

$$= \int_0^t f_2(x)f_1(t-x)dx = \int_0^t\left(\frac{x}{4}\sin 2x\right)(t-x)dx$$

$$= \frac{t}{4}\int_0^t x\sin 2x dx - \frac{1}{4}\int_0^t x^2\sin 2x dx$$

$$= \frac{t}{4}\left(-\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x\right)_0^t$$

$$-\frac{1}{4}\left(-\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x\right)_0^t$$

$$= \frac{1}{16}(1 - t\sin 2t - \cos 2t)$$

Application of Laplace transforms to solutions of differential equations: Solution of ordinary differential equations with constant co-efficients:

Consider a linear differential equation with constant co-efficients

$(D^n + C_1D^{n-1} + C_2D^{n-2} + \cdots + (C_n)y = F(t)$ (1)

where $F(t)$ is a function of the independednt variable t

Let $y(0) = A_1, y^1(0) = A_2, \ldots, y^{n-1}(0) = A_{n-1}$ (2)

be the given initial or boundary conditions where $A_1, A_2 \ldots A_{n-1}$ are constants.

By taking the Lapalce transform on both sides of (1) and using the conditions (2), we obtain an algebraic equation known as subsidiary equation from which $y(s) = L\{y(t)\}$ is determined. The required solution is obtained by finding the inverse Laplace transform of $y(s)$.

Example 47

Solve $(D + 3)^2 y = 9e^{-3t}$, $y(0) = -1$ and $y'(0) = 9$.

Solution

The given equation can be written as

$$(D^2 + 6D + 9)y = 9e^{-3t}$$

applying Laplace transform we get

$\therefore L\{y''\} + 6L\{y'\} + 9L\{y\} = 9L\{e^{-3t}\}$

or $s^2L\{y\} - sy(0) - y'(0) + 6[sL\{y\} - y(0)] + 9L\{y\} = \dfrac{9}{s+3}$

or $s^2L\{y\} + s - 9 + 6s\,L\{y\} + 6 + 9L\{y\} = \dfrac{9}{s+3}$

$\Rightarrow \quad (s^2 + 6s + 9)\,L\{y\} = \dfrac{9}{s+3} - s + 3$

$$(s+3)^2 L\{y\} = \frac{18-s^2}{s+3}$$

$$L\{y\} = \frac{18-s^2}{(s+3)^3}$$

$$\therefore \quad y = L^{-1}\left\{\frac{9-(s+3)^2+6(s+3)}{(s+3)^3}\right\}$$

$$= e^{-3t}L^{-1}\left\{\frac{9-s^2+6s}{s^3}\right\}$$

$$= e^{-t}\left[L^{-1}\left\{\frac{9}{s^3}\right\} - L^{-1}\left\{\frac{1}{s}\right\} + 6L^{-1}\left\{\frac{1}{s^2}\right\}\right]$$

$$y = e^{-3t}\left(9\cdot\frac{t^2}{2!} - 1 + 6t\right)$$

$\therefore$ The required solution is

$$y = \frac{e^{-3t}}{2}(9t^2 + 12t - 2).$$

Exercises

1. The order and degree of the DE $\dfrac{d^2y}{dx^2} = n^2y$ respectively are

(A) 1, 2 (B) 1, 1
(C) 2, 2 (D) 2, 1

2. The differential equation whose solution is $y = mx + \dfrac{4}{m}$, where 'm' is parameter is

(A) $x\left(\dfrac{dy}{dx}\right)^2 - y\dfrac{dy}{dx} + 4\dfrac{dy}{dx} = 0.$

(B) $\left(\dfrac{dy}{dx}\right)^2 - \dfrac{dy}{dx} + 4 = 0.$

(C) $x\dfrac{dy}{dx} - y + 4 = 0.$

(D) $x\left(\dfrac{dy}{dx}\right)^2 + \dfrac{dy}{dx} + 4 = 0.$

3. If $y = c_1 \log x + c_2 \log c_3 + c_4 e^x + c_5$ is the general solution of a homogeneous linear differential equation, then the order of the equation is

(A) 2 (B) 3
(C) 4 (D) 5

4. Find the solution of $\tan y \sec^2 x\, dx + \tan x \sec^2 y dy = 0$ when $x = y = \dfrac{\pi}{4}$.

(A) $\tan x \tan y = 1$
(B) $\cot x \tan y = 1$
(C) $\tan x \cot y = 1$
(D) $\cot x \cot y = 1$

5. The general solution of the DE, $(e^x + 1)ydy = (y + 1)e^x dx$ is

(A) $\log(e^x + 1) - \log(y + 1) + c = 0$
(B) $\log(e^x + 1) = y - \log(y + 1) + c$
(C) $\log(e^x - 1) + \log(y + 1) + c = 0$
(D) $\log\left(\dfrac{e^x}{y+1}\right) = c$

6. Solve $\dfrac{dy}{dx} = |x|$

(A) $y = \dfrac{x^2}{2} + c$ (B) $y = \dfrac{x^2}{2} + x + c$

(C) $y = \dfrac{-x|x|}{2} + c$ (D) $y = \dfrac{x|x|}{2} + c$

7. Solve $(x+y)^2 \dfrac{dy}{dx} = k^2$.

(A) $y = \tan^{-1}(x+y)$

(B) $y = \sin^{-1}\left(\dfrac{x+y}{k}\right) + c$

(C) $y = k\tan^{-1}\left(\dfrac{x+y}{k}\right) + c$

(D) $y = \cot^{-1}\left(\dfrac{x+y}{k}\right) + c$

8. The general solution of the DE, $\dfrac{dy}{dx} = (3x+y+1)^2$ is

(A) $\sec^{-1}(3x+y+1) = x+c$

(B) $\dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{3x+y+1}{\sqrt{3}}\right) = x+c$

(C) $\tan^{-1}(3x+y+1) = x+c$

(D) $\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{2x-y+1}{\sqrt{3}}\right) = x+c$

9. The general solution of $\dfrac{dy}{dx} = \dfrac{x-y}{x+y}$ is

(A) $x^2 + xy + y^2 = k$ (B) $x^2 - y^2 = k$

(C) $x^2 - 2xy - y^2 = k$ (D) $x^2y^2 = k$

10. The general solution of $\dfrac{dy}{dx} = \dfrac{x-2y+1}{2x-4y+3}$ is

(A) $x^2 - 4xy - 6y = c$

(B) $x^2 - 4xy + 4y^2 + 2x - 6y = c$

(C) $x^2 + 4xy + 4y^2 + 2x - 6y = c$

(D) $x^2 + 4xy - x + 6y = c$

11. The solution of the differential equation $2xy\,dy + (x^2 + y^2 + 1)dx = 0$ is

(A) $x^3 + xy^2 + 3x = c$

(B) $x^3 + 3xy^2 + x = c$

(C) $\dfrac{x^3}{3} + xy^2 + x = c$

(D) $3x^2 + y^2 + 2x = c$

12. The general solution of $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$ is

(A) $e^x + y^2 = c$ (B) $e^{xy} + y^2 = c$

(C) $e^{y^2} + xy = c$ (D) $e^y + xy = c$

13. The solution of the differential equation $(3xy + 2y^2)dx + (x^2 + 2xy)dy = 0$ is

(A) $x^3y + x^2y = c$ (B) $x^3y + x^2y^2 = c$

(C) $x^2y + xy^2 = c$ (D) $2xy(x+y) = c$

14. The integrating factor of the equation $(x^2 + xy - y^2)dx + (xy - x^2)dy = 0$ is

(A) $\dfrac{1}{x^2}$ (B) $\dfrac{1}{x^3}$

(C) x^2 (D) x^3

15. The solution of $(1+x)\dfrac{dy}{dx} - xy = 1 - x$ satisfying the initial conditions at $x = 0$ and $y = 1$ is

(A) $1 + x = y + e^x$ (B) $y(1+x) = x + e^x$

(C) $x + y = e^x$ (D) $x(1+y) = ce^x$

Direction for questions 16 to 17:

Consider the differential equation $\dfrac{dy}{dx} + y\cot x = y^2 \sin x$

16. The integrating factor of the above equation is

(A) $\operatorname{cosec} x$ (B) $\sin x$

(C) $\cos x$ (D) $\sec x$

17. The solution of the above equation when $x = \dfrac{\pi}{2}$, $y = 1$ is

(A) $y\operatorname{cosec} x - x = \dfrac{\pi+2}{2}$

(B) $\dfrac{\operatorname{cosec} x}{y} + x = \dfrac{\pi+2}{2}$

(C) $y\operatorname{cosec} x + x = \dfrac{\pi-2}{2}$

(D) $\dfrac{\operatorname{cosec} x}{y} - x = \dfrac{\pi+2}{2}$

18. The general solution of $x\dfrac{dy}{dx} + y = y^2\log x$ is

(A) $y = \log x + cx$ (B) $y = x + c\log x$

(C) $\dfrac{1}{y} = 1 + cx$ (D) $\dfrac{1}{y} = 1 + cx + \log x$

19. Consider the differential equation $\cos y\dfrac{dy}{dx} + 3x^2 \sin y = x^2$.

To convert the above equation into linear form the substituted variable is

(A) $z = \cos y$ (B) $z = \operatorname{cosec} y$

(C) $z = \sin y$ (D) $z = \sec y$

20. The solution of $(aD^2 + bD + c)\,y = 0$ whose auxiliary equation has its discriminant as zero and has 5 as one of its roots is

(A) $y = c_1e^{5x} + c_2e^{5x}$ (B) $y = c_1e^x + c_2e^x$

(C) $y = (c_1 + c_2x)e^{5x}$ (D) $y = c_1 + c_2x$

21. Find the general solution of $\dfrac{d^3y}{dx^3} + 3\dfrac{d^2y}{dx^2} - 4y = 0$.

(A) $y = (c_1 + c_2x)e^x + c_3e^{-2x}$

(B) $y = (c_1 + c_2x)e^{-2x} + c_3e^x$

(C) $y = (c_1 + c_2x)e^{2x} + c_3e^{-x}$

(D) $y = (c_1 + c_2x)e^{-x} + c_3e^{2x}$

22. The general solution of the differential equation $\frac{d^4x}{dt^4}+13\frac{d^2x}{dt^2}+36x=0$ is ______.

(A) $x=(c_1+c_2t)\cos 2t+(c_3+c_4t)\sin 3t$
(B) $x=c_1e^{2t}+c_2e^{-2t}+c_3e^{3t}+c_4e^{-3t}$
(C) $x=(c_1+c_2t)e^{2t}+(c_3+c_4t)e^{3t}$
(D) $x=c_1\cos 2t+c_2\sin 2t+c_3\cos 3t+c_4\sin 3t$

23. The particular integral of $(D^2-4D+3)y=e^{3x}$ is

(A) $\frac{xe^{3x}}{2}$ (B) e^{3x}
(C) $\frac{1}{2}e^{3x}$ (D) xe^{2x}

24. The particular integral of $(D^3-4D^2)y=6$ is

(A) x^2 (B) $\frac{3}{4}x^2$
(C) $-\frac{3}{4}x^2$ (D) $\frac{-x^2}{4}$

25. The particular of integral of $(D^2+3D+2)y=\cos 2x$ is

(A) $3\sin 2x-\cos 2x$ (B) $\frac{3\sin 2x-\cos 2x}{20}$
(C) $\frac{\cos 2x-3\sin 2x}{10}$ (D) $\frac{\cos x-\sin 2x}{40}$

26. The particular integral of $(D^2-D)\,y=x^2-2x+4$ is

(A) x^3-8x+4 (B) $-x^3+4x-4$
(C) $\frac{x^3}{3}+8x-4$ (D) $\frac{-x^3}{3}-4x-4$

27. If $y_1=e^{2x}$ and $y_2=xe^{2x}$ are two solutions of a second order linear differential equation, then the Wronskian W of y_1 and y_2 is ______.

(A) e^{4x} (B) xe^{4x}
(C) $2e^{4x}$ (D) $2xe^{4x}$

28. The complementary function of the differential equation $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=5e^{3x}$ is $y_c=c_1e^{-2x}+c_2e^{-3x}$ using the method of variation of parameters, its particular is found to be $y_p=A(x)\,e^{-2x}+B(x)\,e^{-3x}$. Then $A(x)=$

(A) $5e^{5x}$ (B) e^{5x}
(C) $\frac{1}{5}e^{-5x}$ (D) e^{-5x}

29. The solution of the DE $(D^2+1)y=0$ given $x=0$, $y=2$ and $x=\frac{\pi}{2}$, $y=-2$ is

(A) $y=\sin x-\cos x$ (B) $y=2(\cos x-\sin x)$
(C) $y=2\cos x\sin x$ (D) $y=2(e^x+e^{-x})$

30. Solve the equation

$$3x^2\frac{d^2}{dx^2}+x\frac{dy}{dx}-y=x^2.$$

(A) $y=C_1x^{-3}+C_2x^{-1}+x^3/7$
(B) $y=C_1x^3+C_2x+x^2/7$
(C) $y=C_1x^{1/3}+C_2x^{-1}+x/7$
(D) $y=C_1x^{-1/3}+C_2x+x^2/7$

31. Laplace transform of $2\sin^2 2t=$ ______.

(A) $\frac{1}{s}+\frac{1}{s^2+16}$ (B) $\frac{s}{s^2+16}$
(C) $\frac{1}{s}-\frac{1}{s^2+16}$ (D) $\frac{1}{s}+\frac{1}{s^2+16}$

32. The Laplace transform of $(t+1)^3$ is ______.

(A) $\frac{6-6s+3s^2-s^3}{s^3}$ (B) $\frac{6+6s+3s^2+s^3}{s}$
(C) $\frac{6(1+s+s^2+s^3)}{s^4}$ (D) $\frac{6+6s+3s^2+s^3}{s^4}$

33. The value of $L\,\{\sinh 3t\cos 3t\}$ ______.

(A) $\frac{s^2+18}{s^4+81}$ (B) $\frac{s^2+18}{s^4+324}$
(C) $\frac{3(s^2-18)}{s^4+324}$ (D) $\frac{3(s^2+18)}{s^4-324}$

34. The value of $L\{t^2\cos 3t\}$ is ______.

(A) $\frac{s^2-27}{(s^2+9)^4}$ (B) $\frac{2s(s^2-27)}{(s^2+9)^3}$
(C) $\frac{s^3-27}{(s^2+9)^4}$ (D) $\frac{s(s^3-27)}{(s^2+9)^3}$

35. Laplace transform of $\frac{\cos 4t}{t}$ ______.

(A) $\frac{64}{s^2+16}$ (B) $\frac{16}{(s^2+16)^2}$
(C) $\frac{8}{(s^2+16)^2}$ (D) Does not exist

36. The Laplace transform of the function defined by $f(t)=\begin{matrix}2, & 0<t<1\\ 1, & t>1\end{matrix}$ is ______.

(A) $\frac{2-e^{-s}}{s}$ (B) $\frac{2-e^{-s}}{2}$
(C) $\frac{2+e^{-s}}{s}$ (D) $\frac{2+e^{-s}}{2}$

37. If $f(t) = t;\ 0 < t < 3$ and $f(t+3) = f(t)$, then $L\{f(t)\}$ is

(A) $\dfrac{1}{s^2(1-e^{-3s})}[1+e^{3s}+e^{-3s}]$

(B) $\dfrac{1}{s(1-e^{-3s})}[1-e^{-3s}+se^{-3s}]$

(C) $\dfrac{1}{s^2(1-e^{-3s})}[1-e^{-3s}-3se^{-3s}]$

(D) $\dfrac{1}{s(1-e^{-3s})}[1-e^{-3s}-se^{-3s}]$

38. The value of $\int_0^\infty \dfrac{e^{-4t}-e^{-8t}}{t}\,dts$ is _______.

(A) log 2 (B) log 4
(C) log 8 (D) log 6

39. $\int_0^\infty t\cdot e^{-2t}\sin 3t dt$ = _______.

(A) $\dfrac{5}{169}$ (B) $\dfrac{10}{169}$

(C) $\dfrac{6}{169}$ (D) $\dfrac{12}{169}$

40. The inverse Laplace transform of $\left(\dfrac{1}{S^{9/2}}\right)$ is _______.

(A) $\dfrac{16}{105}\sqrt{\dfrac{t^7}{\pi}}$ (B) $\dfrac{8}{15}\sqrt{\dfrac{t^5}{\pi}}$

(C) $\dfrac{16}{35}\sqrt{\dfrac{t}{\pi}}$ (D) $\dfrac{8}{105}\sqrt{\dfrac{t^7}{\pi}}$

41. The value of $L^{-1}\left\{\dfrac{8}{3s-2}-\dfrac{4+2s}{16s^2-25}\right\}$ is _______.

(A) $\dfrac{8}{3}\sin h\dfrac{5t}{4}-\cos h\dfrac{5t}{4}$

(B) $\dfrac{8}{3}e^{2/3t}-\sin h\dfrac{5t}{4}-\cos h\dfrac{5t}{4}$

(C) $\dfrac{8}{3}e^{2/3t}-\dfrac{1}{5}\sin h\dfrac{5t}{4}-\dfrac{1}{8}\cos h\dfrac{5t}{4}$

(D) None of these

42. The inverse Laplace transform of $\dfrac{1}{s^2-8s+20}$ is _______.

(A) $\dfrac{e^{2t}}{2}\sin 2t$ (B) $\dfrac{e^{4t}}{2}\sin 2t$

(C) $e^{4t}\sin 2t$ (D) $e^{4t}\sin 4t$

43. The inverse Laplace transform of $\dfrac{1}{s^3(s^2+4)}$ is

(A) $\dfrac{1}{16}(2t^2+\cos 2t-1)$ (B) $2t^2-\cos 2t-1$

(C) $\dfrac{1}{16}(1-\cos 2t-4t^2)$ (D) $\dfrac{1}{8}(2+\cos 2t-4t^2)$

44. The inverse Laplace transform of $\dfrac{e^{-3s}}{(s-4)^5}$ when expressed in terms of Heaviside unit step function is _______.

(A) $\dfrac{1}{16}t^4e^{4(t-3)}H(t-3)$

(B) $\dfrac{1}{24}(t-3)^4e^{4t}H(t-3)$

(C) $\dfrac{1}{24}(t-3)^4e^{4(t-3)}H(t-3)$

(D) $\dfrac{1}{24}t^4e^{4t}H(t-3)$

45. The value of $L^{-1}\left\{\log\dfrac{s-4}{s+3}\right\}$ is

(A) $e^{4t}-e^{-3t}$ (B) $\dfrac{1}{t}(e^{4t}-e^{-3t})$

(C) $\dfrac{1}{t}(e^{-3t}-e^{4t})$ (D) $t(e^{-3t}-e^{4t})$

46. Using convolution theorem, the value of $\int_0^t \sin x\cos(t-x)dx$ is _______.

(A) $\dfrac{1}{2}\cos t$ (B) $\dfrac{t}{2}\sin t$

(C) $t\sin\dfrac{t}{2}$ (D) $t\cos\dfrac{t}{2}$

47. Solve $(D^4-16)y = 1,\ y = y' = y'' = y''' = 0$.

(A) $y=\dfrac{-1}{16}-[\cos h\,2t+\sin h\,2t]$

(B) $y=\dfrac{1}{32}(1-\cos h\,2t+\cos 2t)$

(C) $y=\dfrac{-1}{16}+\dfrac{1}{32}(\cos h\,2t-\sin t)$

(D) $y=\dfrac{-1}{16}+\dfrac{1}{32}(\cos h\,2t+\cos 2t)$

48. Solve $(D^2 - 5D + 6)y = 1 - e^{-2t}$, $y = 1$, $y' = 0$ when $t = 0$.

(A) $y = \frac{1}{20}e^{-2t} + \frac{11}{4}e^{2t} - \frac{59}{30}e^{3t}$

(B) $y = \frac{1}{6} - \frac{1}{20}e^{-2t} + \frac{11}{4}e^{2t} - \frac{28}{15}e^{3t}$

(C) $y = \frac{1}{6} - \frac{1}{20}e^{2t} + \frac{11}{4}e^{-2t} + \frac{59}{30}e^{3t}$

(D) $y = \frac{1}{6} - \frac{1}{20}e^{2t} - \frac{11}{4}e^{-2t} + \frac{59}{30}e^{3t}$

Previous Years' Questions

1. The solution for the differential equation $\frac{dy}{dx} = x^2 y$ with the condition that $y = 1$ at $x = 0$ is **[GATE, 2007]**

(A) $y = e^{\frac{1}{2x}}$ (B) $\ln(y) = \frac{x^3}{3} + 4$

(C) $\ln(y) = \frac{x^2}{2}$ (D) $y = e^{\frac{x^3}{3}}$

2. The general solution of $\frac{d^2y}{dx^2} + y = 0$ is **[GATE, 2008]**

(A) $y = P\cos x + Q\sin x$

(B) $y = P\cos x$

(C) $y = P\sin x$

(D) $y = P\sin^2 x$

3. Solution of $\frac{dy}{dx} = -\frac{x}{y}$ at $x = 1$ and $y = \sqrt{3}$ is **[GATE, 2008]**

(A) $x - y^2 = -2$ (B) $x + y^2 = 4$

(C) $x^2 - y^2 = -2$ (D) $x^2 + y^2 = 4$

4. Solution of the differential equation $3y\frac{dy}{dx} + 2x = 0$ represents a family of **[GATE, 2009]**

(A) ellipses (B) circles

(C) parabolas (D) hyperbolas

5. Laplace transform for the function $f(x) = \cosh(ax)$ is **[GATE, 2009]**

(A) $\frac{a}{s^2 - a^2}$ (B) $\frac{s}{s^2 - a^2}$

(C) $\frac{a}{s^2 + a^2}$ (D) $\frac{s}{s^2 + a^2}$

6. The order and degree of the differential equation $\frac{d^3y}{dx^3} + 4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2} = 0$ are respectively **[GATE, 2010]**

(A) 3 and 2 (B) 2 and 3

(C) 3 and 3 (D) 3 and 1

7. The solution to the ordinary differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ is **[GATE, 2010]**

(A) $y = c_1e^{x} + c_2e^{-2x}$

(B) $y = c_1e^{3x} + c_2e^{2x}$

(C) $y = c_1e^{-3x} + c_2e^{2x}$

(D) $y = c_1e^{-3x} + c_2e^{-2x}$

8. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x$, with the condition that $y = 1$ at $x = 1$, is **[GATE, 2011]**

(A) $y = \frac{2}{3x^2} + \frac{x}{3}$ (B) $y = \frac{x}{2} + \frac{1}{2x}$

(C) $y = \frac{2}{3} + \frac{x}{3}$ (D) $y = \frac{2}{3x} + \frac{x^2}{3}$

9. The solution of the ordinary differential equation $\frac{dy}{dx} + 2y = 0$ for the boundary condition, $y = 5$ at $x = 1$ is **[GATE, 2012]**

(A) $y = e^{-2x}$

(B) $y = 2e^{-2x}$

(C) $y = 10.95e^{-2x}$

(D) $y = 36.95e^{-2x}$

10. The integrating factor for the differential equation $\frac{dp}{dt} + k_2P = k_1L_0e^{kt}$ is **[GATE, 2014]**

(A) e^{-k_1t} (B) e^{-k_2t}

(C) e^{-k_1t} (D) e^{k_2t}

11. Consider the following differential equation:

$$x(ydx + xdy)\cos\frac{y}{x} = y(xdy - ydx)\sin\frac{y}{x}$$

Which of the following is the solution of the above equation (c is an arbitrary constant)? **[GATE, 2015]**

(A) $\frac{x}{y}\cos\frac{y}{x} = C$ (B) $\frac{x}{y}\sin\frac{y}{x} = C$

(C) $xy\cos\frac{y}{x} = C$ (D) $xy\sin\frac{y}{x} = C$

12. Consider the following second order linear differential equation $\frac{d^2y}{dx^2} = -12x^2 + 24x - 20$.

The boundary conditions are: at $x = 0$, $y = 5$ and at $x = 2$, $y = 21$

The value of y at $x = 1$ is ________. **[GATE, 2015]**

13. The respective expressions for complimentary function and particular integral part of the solution of the differential equation are **[GATE, 2016]**

(A) $\left[c_1 + c_2x + c_3\sin\sqrt{3x} + c_4\cos\sqrt{3x}\right]$ and $\left[3x^4 - 12x^2 + c\right]$

(B) $\left[c_2x + c_3\sin\sqrt{3x} + c_4\cos\sqrt{3x}\right]$ and $\left[5x^4 - 12x^2 + c\right]$

(C) $\left[c_1 + c_3\sin\sqrt{3x} + c_4\cos\sqrt{3x}\right]$ and $\left[3x^4 - 12x^2 + c\right]$

(D) $\left[c_1 + c_2x + c_3\sin\sqrt{3x} + c_4\cos\sqrt{3x}\right]$ and $\left[5x^4 - 12x^2 + c\right]$

Answer Keys

Exercises

1. D	**2.** A	**3.** B	**4.** A	**5.** B	**6.** D	**7.** C	**8.** B	**9.** C	**10.** B
11. C	**12.** B	**13.** B	**14.** B	**15.** B	**16.** A	**17.** B	**18.** D	**19.** C	**20.** C
21. B	**22.** D	**23.** A	**24.** C	**25.** B	**26.** D	**27.** A	**28.** B	**29.** B	**30.** D
31. C	**32.** D	**33.** C	**34.** B	**35.** D	**36.** A	**37.** C	**38.** A	**39.** D	**40.** A
41. C	**42.** B	**43.** A	**44.** C	**45.** C	**46.** B	**47.** D	**48.** B		

Previous Years' Questions

1. D	**2.** A	**3.** D	**4.** A	**5.** B	**6.** A	**7.** C	**8.** D	**9.** D	**10.** D
11. C	**12.** 18	**13.** A							

Chapter 3

Partial Differential Equations

CHAPTER HIGHLIGHTS

- *Fourier series*
- *Heat equation*

Fourier Series

Periodic Function A function $f(x)$ is said to be periodic if $f(x + a) = f(x)$ for all x. The least value of a is called the period of $f(x)$.

Example: $\sin x$, $\cos x$ are periodic functions with period 2π.

NOTES

1. $f(x)$ and $g(x)$ are periodic functions with period k then $af(x) + bg(x)$ is also a periodic function with period k.
2. If $f(x)$ is a periodic function with period k, then the period of $f(ax)$ is $\dfrac{k}{a}$.
3. If the periods of functions $f(x)$, $g(x)$ and $h(x)$ are a, b, c, respectively, then the period of $f(x) + g(x) + h(x)$ is the lcm of a, b and c.

Euler's Formula for the Fourier Coefficients

Let $f(x)$ is a periodic function whose period is 2π and is integrable over a period. Then $f(x)$ can be represented by trigonometric series.

$$f(x) = \frac{a_0}{2} + \int_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where a_0 a_n, b_n are called Fourier coefficients and these are obtained by

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx,$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx \text{ for } n = 1, 2, 3, \ldots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx \text{ for } n = 1, 2, 3, \ldots$$

SOLVED EXAMPLES

Example 1

Obtain the Fourier series expansion of $f(x) = e^x$ in $(0, 2\pi)$.

Solution

$$a_0 = \frac{1}{\pi}\int_{0}^{2\pi} f(x)dx$$

$$= \frac{1}{\pi}\int_{0}^{2\pi} e^x dx$$

$$= \frac{1}{\pi} e^x \Big]_0^{2\pi} = \frac{1}{\pi}(e^{2\pi} - 1) \quad (1)$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx dx$$
$$= \frac{1}{\pi}\int_0^{2\pi} e^x \cos nx dx$$

we know that $\int e^{ax}\cos bx dx$

$$= \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$$

$$\therefore a_n = \frac{1}{\pi}\left[\frac{e^x}{1+n^2}(\cos nx + n\sin nx)\right]_0^{2\pi}$$
$$= \frac{1}{\pi}\left[\frac{e^{2\pi}}{1+n^2}(\cos 2\pi n) - \frac{1}{1+n^2}\right]$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx dx = \frac{1}{\pi}\int_0^{2\pi} e^x \sin nx dx$$
$$= \frac{1}{\pi}\frac{e^x(\sin nx - n\cos nx]_0^{2\pi}}{1+n^2}$$
$$\left(\because \int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}\right)(a\sin bx - b\cos bx))$$
$$= \frac{1}{\pi}\frac{1}{1+n^2}(n - e^{2\pi} n\cos 2\pi n)$$
$$= \frac{n}{\pi(1+n^2)}(1-e^{2\pi}\cos 2\pi n)$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n \sin nx)$$
$$= \frac{1}{2\pi}(e^{2\pi}-1) + \sum_{n=1}^{\infty}$$
$$\left[\frac{1}{\pi}\frac{1}{1+n^2}(e^{2\pi}\cos 2\pi n - 1) + \frac{n}{\pi(1+n^2)}(1-e^{2\pi}\cos 2\pi n)\right]$$

Even and Odd Functions

Even function: A function $f(x)$ is said to be even if $f(-x) = f(x)$ for all x.

Example: x^2, $\cos x$

Odd function: A function $f(x)$ is said to be odd if $f(-x) = -f(x)$ for all x

Example: x^3, $\sin x$

NOTES

1. The sum of two odd functions is odd.
2. The product of an odd function and an even function is odd.
3. Product of two odd functions is even.

Fourier Series for Odd Function and Even Function

Case 1: Let $f(x)$ is an even function in $(-\pi, \pi)$. Then the Fourier series of the even function contains only cosine terms and is known as Fourier cosine series and it is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx, \quad \text{where}$$
$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx, a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$$

Case 2: If $f(x)$ is an odd function, then the Fourier series of an odd function contains only sine terms, and is known as Fourier sine series.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx,$$

where $$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx dx$$

Example 2

Expand the function $f(x) = \frac{\pi^2}{24} - \frac{x^2}{8}$ in Fourier series in the interval $(-\pi, \pi)$.

Solution

$$f(x) = \frac{\pi^2}{24} - \frac{x^2}{8}$$
$$f(-x) = \frac{\pi^2}{24} - \frac{(-x)^2}{8} = \frac{\pi^2}{24} - \frac{x^2}{8} = f(x)$$

$\therefore f(x)$ is an even function.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx$$
$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx = \frac{2}{2\pi}\int_0^{\pi}\frac{\pi^2}{24} - \frac{x^2}{8}dx$$
$$= \frac{1}{\pi}\left(\frac{\pi^2 x}{24} - \frac{x^3}{24}\right)\Big]_0^{\pi} = 0$$
$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx\, s$$
$$= \frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi^2}{24} - \frac{x^2}{8}\right)\cos nx dx$$
$$= \frac{2}{\pi}\left\{\begin{array}{l}\left(\frac{\pi^2}{24} - \frac{x^2}{8}\right) - \left(\frac{\sin nx}{n}\right)\Big]_0^{\pi} \\ -\frac{1}{8}\int_0^{\pi}\left(\frac{-(-2x)\sin nx}{n}\right)dx\end{array}\right\}$$

$$= -\frac{2}{\pi}\int_0^{\pi}\frac{2x}{8}\frac{\sin nx}{n}dx$$

$$= \frac{2}{\pi}\left[\frac{2}{8n}\left\{\left[\frac{-x\cos nx}{n}\right]_0^{\pi} - \int_0^{\pi}\frac{\cos nx}{n}dx\right\}\right]$$

$$= \frac{-4}{8\pi n^2}(\pi\cos n\pi)$$

$$= \frac{-1}{2n^2}(\text{con}\, n\pi), n = 1, 2, 3, \ldots$$

$$= \frac{(-1)^{n+1}}{2n^2}$$

$$\therefore f(x) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n^2}\cos nx$$

$$= \frac{1}{2}\left[\cos x - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \cdots\right].$$

Function of Any Period (P = 2L)

If the function $f(x)$ is of period $P = 2L$ has a Fourier series, then $f(x)$ can be expressed as,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi}{L}x + b_n\sin\frac{n\pi}{L}x\right)$$

where the Fourier coefficients are as follows:

$$a_0 = \frac{1}{L}\int_{-L}^{L}f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi}{L}xdx$$

$$b_n = \frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi}{L}xdx$$

Fourier Series of Even and Odd Functions Let $f(x)$ be an even function in (–L, L), then the Fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}$$

Where $$a_0 = \frac{1}{L}\int_{-L}^{L}f(x)dx = \frac{2}{L}\int_0^{L}f(x)dx$$

$$a_n = \frac{2}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx$$

Let $f(x)$ be an odd function in (–L, L) then Fourier series is

$$f(x) = \sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$$

where $b_n = \frac{2}{L}\int_0^{L}f(x)\sin\frac{n\pi}{L}dx.$

Half Range Expansion

In the pervious examples we define the function $f(x)$ with the period $2L$.

Suppose $f(x)$ is not periodic function and defined in half the interval say (0, L) of lengths L. such expansions are known as half range expansions or half range Fourier series. In particular a half range expansion containing only cosine series of $f(x)$ in the interval (0, L) in a similar way half range Fourier sine series contains only sine terms. To find the Fourier series of $f(x)$ which is neither periodic nor even nor odd we obtain Fourier cosine series and Fourier sine series of $f(x)$ as follows. We define a function $g(x)$ such that $g(x) = f(x)$ in the interval from (0, L) and $g(x)$ is an even function in (–L, L) and is periodic with period $2L$ and g(x) is obtained by previous methods which are discussed earlier. Similarly we can obtain a fourier sine series as follows. Assume $f(x) = h(x)$ in (0, L) and $h(x)$ is an odd function in the interval (–L, L) with period $2L$ and evaluate $h(x)$ by pervious methods which are discussed earlier.

Example 3

If $f(x) = 1 - x$ in $0 < x < 1$ find Fourier cosine series and Fourier sine series.

Solution

Given $f(x) = 1 - x$ in $0 < x < 1$ since $f(x)$ is neither periodic nor even nor odd function.

Let us assume $g(x) = f(x) = 1 - x$ in $0 < x < 1$

$= 1 + x$ in $-1 < x < 0$

$\therefore g(x)$ is even function in (–1, 1)

$$\therefore g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}$$

$$a_0 = \frac{2}{L}\int_0^{L}f(x)dx = 2\int_0^{1}f(x)dx \text{ (here } L = 1)$$

$$= 2\int_0^{1}(1-x)dx = 2\left[x - \frac{x^2}{2}\right]_0^1 = \frac{1}{2}\times 2 = 1$$

$$a_n = \frac{2}{L}\int_0^{L}f(x)\cos\frac{n\pi x}{L}dx$$

$$= 2\int_0^{1}(1-x)\cos n\pi x dx$$

$$= 2\left[(1-x)\frac{\sin n\pi x}{n\pi}\bigg|_0^1 - \int_0^{1}(-1)\frac{\sin n\pi xx}{n\pi}\right]$$

$$= 2\left[(1-x)\frac{\sin n\pi x}{n\pi}\Big|_0^1 - \int_0^1 (-1)\frac{\sin n\pi x}{n\pi}\,dx \right]$$

$$= -2\frac{\cos n\pi x}{n^2\pi^2}\Big]_0^1 = 2\left(\frac{1}{n^2\pi^2} - \frac{\cos n\pi}{n^2\pi^2}\right)$$

$$= \frac{2}{n^2\pi^2}(1-(-1)^n).$$

∴ Fourier cosine series is

$$g(x) = \frac{1}{2} + \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1-(-1)^2}{n^2}\cos n\pi x$$

Fourier sine series in (0, 1)

$$\therefore\ h(x) = f(x) = 1 - x;\ 0 < x < 1$$
$$= -(1 + x);\ -1 < x < 0$$

$h(x)$ is an odd function

$$\therefore h(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$

$$= 2\int_0^1 (1-x)\sin n\pi x dx$$

$$= 2(1-x)\frac{\cos n\pi x}{-n\pi}\Big]_0^1 - 2\int_0^1 \frac{\cos n\pi x}{n\pi}\,dx = 2/n\pi$$

$$\therefore h(x) = 2/\pi \int_{n=1}^{\infty}\frac{1}{n}\sin(n\pi x)$$

Partial Differential Equations (PDE)

An equation involving two or more independent variables and a dependent variable and its partial derivatives is called a partial differential equation.

$$\therefore\quad f\left(x, y, z, \frac{\partial z}{\partial z}, \frac{\partial z}{\partial y}\ldots\right) = 0.$$

Standard Notation

$$\frac{\partial z}{\partial x} = p = z_x,\quad \frac{\partial z}{\partial y} = q = z_y$$

$$\frac{\partial^2 z}{\partial x^2} = r = z_{xx},\quad \frac{\partial^2 z}{\partial y^2} = t = z_{yy}$$

$$\frac{\partial^2 z}{\partial x \partial y} = z_{xy} = s$$

Formation of Partial Differential Equations

Partial differential equation can be formed by two ways.

1. By eliminating arbitrary constants.
2. By eliminating arbitrary functions.

Formation of PDE by Eliminating Arbitrary Constants

Consider a function $f(x, y, z, a, b) = 0$
where a, b, are arbitrary constants.

Differentiating this partially wrt, x and y eliminate a, b from these equations we get an equation $f(x, y, z, p, q) = 0$, which is partial differential equation of first order.

Example 4

$z = ax^2 - by^2$, a, b are arbitrary constants.

Solution

Given

$$z = ax^2 - by^2 \tag{1}$$

Differentiating z partially wrt x,

$$\frac{\partial z}{\partial x} = 2ax \quad\Rightarrow\quad p = 2ax \quad\Rightarrow\quad a = \frac{p}{2x}$$

Differentiate z partially wrt y,

$$\frac{\partial z}{\partial y} = 2by\text{, i.e., } q = -2by$$

$$\Rightarrow\quad b = \frac{-q}{2y}$$

Substituting the values of a and b in Eq. (1), we get

$$z = \frac{p}{2x}x^2 + \frac{q}{2y},\ y^2$$

$2z = px + qy$ which is a partial differential equation of order 1.

Formation of PDE by Eliminating Arbitrary Function

Consider $z = f(u)$ (1)

f is an arbitrary function in u and u is function in x, y, z.

Now differentiate Eq. (1) wrt x, y partially by chain rule we get

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x} \tag{2}$$

$$\frac{\partial z}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial y} \tag{3}$$

by eliminating the arbitrary functions from Eqs. (1), (2), (3) we get a PDE of first order.

Formation of PDE when Two Arbitrary Functions are Involved

When two arbitrary functions are involved, we differentiate the given equation two times and eliminate the two arbitrary functions from the equation obtained.

Example 5

Form the partial differential equation of

$$z = \frac{f(x)}{g(y)}$$

Solution

Given $$z = \frac{f(x)}{g(y)}$$

$$p = z_x = \frac{f'(x)}{g(y)} \tag{1}$$

$$q = z_y = \frac{-f(x)}{[g(y)]^2} \cdot g'(y) \tag{2}$$

$$s = \frac{\partial^2 z}{\partial x \partial y} = \frac{-f'(x)}{[g(y)]^2} \cdot g'(y) \tag{3}$$

$$\text{Eq.(1)} \times \text{Eq.(2)} = pq = \frac{f'(x)}{g(y)} \cdot \left(\frac{-f(x) \cdot g'(y)}{[g(y)]^2} \right) = -s \cdot z$$

$$\because \quad pq + sz = 0$$

Forming PDE by the Elimination of Arbitrary Function of Specific Functions

Consider $f(u, v) = 0$

Where u, v are the functions in x, y, z.

Differentiate the above equation wrt x and y by chain rule and eliminate the $\frac{\partial F}{\partial u}$, $\frac{\partial F}{\partial v}$ and convert them in the form $Pp + Qq = R$, which is a first order linear PDE where P, Q, R are functions of x, y, z.

Linear Equation of First Order

Linear equation of first order is $Pp + Qq = R$. This is also called Lagrange's equation, where P, Q, R are the functions in x, y, and z.

Procedure For solving Lagrange's Equations

Take the auxiliary equation as

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

Solve any two equations and take the solutions as u and v. The complete solution is $\phi(u, v) = 0$ or $u = f(v)$.

Example 6

Solve $(z - y)p + (x - z)q = y - x$.

Solution

Auxiliary equation is

$$\frac{dx}{z-y} = \frac{dy}{x-z} = \frac{dz}{y-x}.$$

Using the multipliers as x, y, z we get

$$\frac{x\,dx + y\,dy + z\,dz}{x(z-y) + y(x-z) + z(y-x)}$$

$$= xdx + ydy + zdz = 0$$

$$\therefore \quad x^2 + y^2 + z^2 = 0$$

and also $\frac{dx + dy + dz}{z - y + x - z + y - z} = 0$

$$dx + dy + dz = 0, x + y + z = 0.$$

$\therefore$ The required solution is $x^2 + y^2 + z^2 = f(x + y + z)$.

Non-linear Equations of First Order

There are four types of non linear equations of first order.

Type 1:

$$f(p, q) = 0.$$

If the given equation contains only p and q then the solution is taken as $z = ax + by + c$. Where a, b and c are arbitrary, such that $f(a, b) = 0$.

Example 7

Solve $2p + 3q = 5$

Solution

Given $2p + 3q = 5$

$z = ax + by + c$.

Where $$2a + 3b = 5,$$

$$b = \frac{5 - 2a}{3}$$

$\therefore$ The solution is $z = ax + \left(\frac{5-2a}{3}\right) y + c$.

Type 2:

$$f(z, p, q) = 0$$

When the equation is not containing x and y then to solve the equation assume $u = x + ay$ and substitute $p = \frac{dz}{du}$, $q = a\frac{dz}{du}$.

Solve the resulting equation and replace u by $x + ay$.

Type 3:

$$f(x, p) = g(y, q).$$

The equation is not containing z.

Assume $f(x, p) = a$ and $g(y, q) = a$.

Solve the equations for p and q and then write the solution.

Example 8

Solve $p^2 - q^2 = x^2 - y^2$.

Solution

$$p^2 - q^2 = x^2 - y^2$$

$$p^2 - x^2 = -y^2 + q^2$$

Let $$p^2 - x^2 = a^2 = -y^2 + q^2$$

$$p^2 = a^2 + x^2 \qquad q^2 = y^2 + a^2$$

$$p = \sqrt{a^2 + x^2} \qquad q = \sqrt{a^2 + y^2}$$

$\therefore$ Take $dz = pdx + qdy$

Integrating on both sides, $\int dz = \int pdx + \int qdy$

$$z = \int\sqrt{a^2 + x^2 dx} + \int\sqrt{a^2 + y^2 dy}$$

$$= \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a} + \frac{y}{2}\sqrt{a^2 + y^2}$$

$$+ \frac{a^2}{2}\sinh^{-1}\frac{y}{a} + b.$$

Type 4:

$$z = px + qy + f(p, q)$$

The equation in the above form is Clairaut's equation. The solution is $z = ax + by + f(a, b)$.

Classification of Second Order Homogeneous Linear Equations

A second order linear homogeneous PDE of the form

$$A\frac{\partial^2\phi}{\partial x^2} + B\frac{\partial^2\phi}{\partial x\partial y} + C\frac{\partial^2\phi}{\partial y^2} + D\frac{\partial\phi}{\partial x} + E\frac{\partial\phi}{\partial y} + F\phi(x, y) = 0 \quad (1)$$

Where A, B, C, D, E and F are either functions of x and y only or constants, is called

1. a parabolic equation, if $B^2 - 4AC = 0$
2. an elliptic equation, if $B^2 - 4AC < 0$
3. a hyperbolic equation, if $B^2 - 4AC > 0$

Examples:

1. Consider the one-dimensional heat equation:

$$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$$

$$\Rightarrow \quad c^2\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$$

Comparing it with Eq. (1), we have

$A = c^2$, $B = 0$ and $C = 0$

$\therefore$ $B^2 - 4AC = 0^2 - 4 \times c^2 \times 0 = 0$

$\therefore$ One dimensional heat equation is parabolic.

Similarly, it can be easily observed that

2. One-dimensional wave equation:

$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ is hyperbolic ($B^2 - 4AC > 0$) and

3. The Laplace equation:

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is elliptic ($B^2 - 4AC < 0$)

Method of Separation of Variables

Consider a PDE involving a dependent variable u and two independent variables x and y. In the method of separation of variables, we find a solution of the PDE in the form of a product of a function of x and a function of y, i.e., we write

$$u(x, y) = X(x) \cdot Y(y) \qquad (1)$$

Then $\quad \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(XY) = X'Y; \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(XY) = XY'$

$\frac{\partial^2 u}{\partial x^2} = X''Y, \frac{\partial^2 u}{\partial x\partial y} = X'Y', \frac{\partial^2 u}{\partial y^2} = XY''$ and so on

Here $\quad X' = \frac{dX}{dx}; Y' = \frac{dY}{dy}; X'' = \frac{d^2X}{dx^2}; Y'' = \frac{d^2Y}{dy^2}.$

Substituting these in the given PDE, separating X and its derivatives from Y and its derivatives, finding solutions for x and y and substituting them in Eq. (1), we get the solution of the given PDE

This is best explained through the examples given below:

Example 9

Solve $xp + yq = 0$ by the method of separation of variables.

Solution

For the PDE,

$$xp + yq = 0 \qquad (1)$$

Let $\quad z = X(x) \cdot Y(y) \qquad (2)$

be the solution

$$\therefore \quad p = \frac{\partial z}{\partial x} = X'Y \text{ and } q = \frac{\partial z}{\partial y} = XY'$$

Substituting these in Eq. (1)

$$x X' Y + y XY' = 0$$

$$\Rightarrow xX'Y = -y XY'$$

$$\Rightarrow \quad x\frac{X'}{X} = -y\frac{Y'}{Y} \qquad (3)$$

In Eq. (3), as LHS is a function of x alone and RHS is a function of y alone, they are equal only if each of them is equal to some constant

$$\therefore \quad x\frac{X'}{X} = -y\frac{Y'}{Y} = k\,(\text{say}) \qquad (4)$$

Where k is a constant

From Eq. (4), $x\frac{X'}{X} = k \quad \Rightarrow \quad xX' = kX$

$$\Rightarrow \quad x\frac{dX}{dx} = kx$$

$$\Rightarrow \quad \frac{dX}{X} = k\cdot\frac{dx}{x}$$

Integrating on both sides we have,

$$\int \frac{dx}{X} = k \int \frac{dx}{x}$$

$$\Rightarrow \quad \log X = k \log x + \log C_1$$

$$\Rightarrow \quad \log X = \log x^k C$$

$$\Rightarrow \quad X = C_1 x^k \qquad (5)$$

Again from Eq. (4), $-y\dfrac{Y'}{Y} = k$

$$\Rightarrow \quad -yY' = kY$$

$$\Rightarrow \quad y\frac{dY}{dy} = -kY$$

$$\Rightarrow \quad \frac{dY}{Y} = -k\frac{dy}{y}$$

Integrating on both sides,

$$\int \frac{dY}{Y} = -k \int \frac{dy}{y}$$

$$\Rightarrow \quad \log Y = -k \log y + \log C_2$$

$$\Rightarrow \quad \log Y = \log y^{-k} C_2$$

$$\Rightarrow \quad Y = C_2 y^{-k} \qquad (6)$$

Substituting Eqs. (5) and (6) in Eq. (2), we get the solution of Eq. (1) as,

$$z = (C_1 x^k)(C_2 y^{-k})$$

$$= C_1 C_2 x^k y^{-k}$$

$$\therefore \quad z = C\left(\frac{x}{y}\right)^k \text{ where } C = C_1 C_2.$$

Example 10

Solve the PDE $u_x + u_t = 3u$; $u(0, t) = 4e^t$ by the method of separation of variables.

Solution

Let $u = X(x).\ T(t)$ (1)

be the solution of the PDE

$$u_x + u_t = 3u \qquad (2)$$

$$u = XT \quad \Rightarrow \quad u_x = \frac{\partial u}{\partial x} = X'T \text{ and } u_t$$

$$= \frac{\partial u}{\partial t} = XT'$$

Substituting these in Eq. (2), we get

$$X'T + XT' = 3XT$$

Dividing throughout by XT, we have

$$\frac{X'}{X} + \frac{T'}{T} = 3$$

$$\Rightarrow \quad \frac{X'}{X} = \frac{-T'}{T} + 3 = k \qquad (3), \text{(say)}$$

From Eq. (3), $\dfrac{X'}{X} = k \quad \Rightarrow \quad X' = kX$

$$\Rightarrow \quad X' - kX = 0 \qquad (4)$$

Which is a linear equation with its auxiliary equation being

$$m - k = 0 \quad \Rightarrow \quad m = k$$

Hence its solution is $X = C_1 e^{kx}$ (5)

Again from Eq. (3), $\dfrac{-T'}{T} + 3 = k$

$$\Rightarrow \quad \frac{T'}{T} = 3 - k$$

$$\Rightarrow \quad T' = (3 - k)T$$

$$\Rightarrow \quad T' - (3 - k)\,T = 0 \qquad (6)$$

Which is a linear equation with its auxiliary equation being $m - (3 - k) = 0$

$$\Rightarrow \quad m = 3 - k$$

$\therefore$ The solution of Eq. (6) is $T = C_2 e^{(3-k)t}$ (7)

Substituting Eqs. (5) and (7) in Eq. (1), we get the general solution of given PDE (2) as

$$u = X \cdot T = (C_1 e^{kx})(C_2 e^{(3-k)t})$$

$$= C_1 C_2 e^{kx+(3-k)t}$$

$$\therefore\ u = ce^{kx+(3-k)t}; \text{ where } c = c_1 c_2$$

$$\therefore \quad u(x, t) = ce^{kx+(3-k)t} \qquad (8)$$

Given $u(0, t) = 4e^t$

$\therefore$ From Eq. (8), $u(0, t) = ce^{(3-k)t} = 4e^t$

Comparing on both sides, we get

$$C = 4,\ 3 - k = 1$$

$$\Rightarrow \quad C = 4;\ k = 2$$

Substituting these in Eq. (8), we get the required solution of Eq. (2) as

$$u(x, t) = 4e^{2x+t}$$

One Dimensional Diffusion Equation The diffusion equation is a partial differential equation that describes density fluctuations in a material undergoing diffusion. The partial differential equation representing the one dimensional diffusion equation is

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$$

where $u(x, t)$ is the density of the diffusing material at time t and D is diffusion coefficient

Example 11

Find the solution of the one dimensional diffusion equation $\dfrac{\partial u}{\partial t} = D\dfrac{\partial^2 u}{\partial x^2}$ on the interval $x \in [0, L]$ with initial condition

$$u\,(x, 0) = f(x), \forall x \in [0, L]$$

and Dirichlet's boundary conditions

$$u(0, t) = u(L, t) = 0 \quad \forall t > 0$$

Solution

We will solve the one-dimensional diffusion equation

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} \qquad (1)$$

by the method of separation of variables.

Let $u(x, t) = X(x)\,T(t)$ (2)

be the solution of Eq. (1)

$$\therefore \frac{\partial u}{\partial x} = X'T \quad \text{and} \quad \frac{\partial^2 u}{\partial x^2} = X''T \quad \text{and} \quad \frac{\partial u}{\partial t} = XT'$$

Substituting these in Eq. (1),

$$XT' = DX''\,T$$

$$\Rightarrow \frac{1}{D}\frac{T'}{T} = \frac{X''}{X} \tag{3}$$

As the left hand side depends only on the variable t and the right hand side depends only on the variable x, both sides are equal to some constant say $-\lambda$

(Negative sign is taken for convenience reason)

From Eq. (3), $\dfrac{1}{D}\dfrac{T'}{T} = \dfrac{X''}{X} = -\lambda$

$$\Rightarrow \frac{1}{D}\frac{T'}{T} = -\lambda \text{ and } \frac{X''}{X} = -\lambda$$

$$\Rightarrow T' + DT = 0 \tag{4}$$

and
$$X'' + \lambda X = 0 \tag{5}$$

Clearly Eqs. (4) and (5) are linear ordinary differential equations involving the variables t and x respectively.

Solving (4), we get

$$T(t) = Ce^{-\lambda Dt} \tag{6}$$

Solving Eq. (5), we get different possible solutions depending on the value of λ as given below.

$$X(x) = \begin{cases} A\sin(\sqrt{\lambda}x) + B\cos(\sqrt{\lambda}x);\ \text{for } \lambda > 0 \\ A'e^{\sqrt{-\lambda}x} + B'e^{\sqrt{-\lambda}x};\ \text{for } \lambda < 0 \\ A''x + B'';\ \text{for } \lambda = 0 \end{cases}$$

Given boundary conditions are

$$u(0, t) = 0 \text{ and } u(L, t) = 0$$

From Eq. (2), $u\,(0, t) = X(0)\,T(t) = 0$

$$\Rightarrow X(0) = 0$$

and
$$u(L, t) = X(L)\,T(t) = 0 \Rightarrow X(L) = 0$$

Taking into account, the boundary conditions $X(0) = 0$ and $X(L) = 0$, the values of $X(x)$ for $\lambda = 0$ and $\lambda < 0$ leads to only the trivial solutions and hence we take the value $X(x)$ given for $\lambda > 0$, which on application of the boundary conditions becomes,

$$X(x) = C_n \sin\left(\frac{n\pi x}{L}\right); \quad n = 1, 2, 3\ldots$$

and Eq. (6) becomes,

$$T(t) = B_n \exp\left(-D\left(\frac{n\pi}{L}\right)^2 t\right), \quad n = 1, 2, 3\ldots$$

where B_n is a constant

$\therefore$ Substituting the values of $X(x)$ and $T(t)$ in Eq. (2),

We get

$$u(x, t)$$

$$= \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}x\right)\exp\left(-D\left(\frac{n\pi}{L}\right)^2 t\right) \tag{7}$$

where $A_n = \text{Constant } (= B_n C_n)$

Given initial condition is

$$u(x, 0) = f(x)$$

i.e.,
$$\sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}x\right) = f(x) \text{ (From Eq. (7))} \tag{8}$$

By writing $f(x)$ as a half range Fourier sine series in $[0, L]$ we have

$$f(x) = \sum_{n=1}^{\infty} F_n \sin\left(\frac{n\pi}{L}x\right)$$

where $F_n = \dfrac{2}{L}\displaystyle\int_0^L f(\xi)\sin\left(\frac{n\pi}{L}\xi\right)d\xi$

$\therefore$ Eq. (8) becomes,

$$\sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}x\right) = \sum_{n=1}^{\infty} F_n \sin\left(\frac{n\pi}{L}x\right)$$

$$\Rightarrow A_n = F_n = \frac{2}{L}\int_0^L f(\xi)\sin\left(\frac{n\pi}{L}\xi\right)d\xi$$

Substituting the value of A_n in Eq. (7), we get the solution of Eq. (1) as

$$u\,(x, t)$$

$$= \sum_{n=1}^{\infty}\left(\left(\frac{2}{L}\int_0^L f(\xi)\sin\left(\frac{n\pi}{L}\xi\right)d\xi\right)\sin\left(\frac{n\pi x}{L}\right)\right)$$

$$\exp\left(-D\left(\frac{n\pi}{L}\right)^2 t\right)$$

Heat Equation

The heat flow in a body of homogeneous material is governed by the heat equation $\dfrac{\partial u}{\partial t} = c^2\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}\right)$ where $c^2 = \dfrac{k}{\sigma\rho}$ and $u\ (x, y, z, t)$ is the temperature in a body, k is the thermal conductivity, σ is specific heat of the body, ρ is the density of the material and c^2 the constant is called the diffusivity of the body. If the heat flow is in x-direction only then u depends on x and t, then the heat equation becomes $\dfrac{\partial u}{\partial t} = c^2\left(\dfrac{\partial^2 u}{\partial x^2}\right)$, which is known as one-dimensional heat equation.

Wave Equation

The one-dimensional wave equation of a vibrating elastic string is given by,

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} \text{ where } c^2 = \frac{T}{\rho}$$

Laplace Equation

When the temperature in a homogeneous material is in steady state and the temperature does not vary with time then the heat conduction equation becomes $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$ and this is known as

Laplace's equation in cartesian system While solving the boundary value problems the following results may be used

If $u(x, t)$ is a function of x and t

1. $L\left\{\frac{\partial u}{\partial t}\right\}=s\bar{u}(x,s)-u(x,0)$
2. $L\left\{\frac{\partial^2 u}{\partial t^2}\right\}=s^2\bar{u}(x,s)-su(x,0)-u_t(x,0)$
3. $L\left\{\frac{\partial u}{\partial x}\right\}=\frac{d\bar{u}}{dx}$
4. $L\frac{\partial^2 u}{\partial x^2}=\frac{d^2\bar{u}}{dx^2}$ where $L\{u(x,t)\}=\bar{u}\ (x,s)$

Example 49

Solve the one dimensional heat equation $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$ satisfying the boundary conditions $u(0,t)=0=u(4,t)$ and $u(x,0)=8\sin 2\pi x$.

Solution

Taking Laplace transform on both sides of the equation $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$

$$L\left\{\frac{\partial u}{\partial t}\right\}=2L\left\{\frac{\partial^2 u}{\partial x^2}\right\}$$

$$s\bar{u}-u(x,0)=2\cdot\frac{d^2\bar{u}}{dx^2}$$

or $\frac{\partial^2\bar{u}}{\partial x^2}-\frac{s}{2}\bar{u}=-4\sin 2\pi x$ as $u(x,0)=8\sin 2\pi x$

The general solution of the above equation is $\bar{u}$

$$=Ae^{(\sqrt{S/2})X}+Be^{-(\sqrt{S/2})X}-\frac{4\sin 2\pi x}{-(2\pi)^2-\frac{s}{2}}$$

or

$$\bar{u}=Ae^{\sqrt{S/2}X}+Be^{-\sqrt{S/2}X}+\frac{8\sin 2\pi x}{8\pi^2+s} \qquad (1)$$

But $u(0,t)=0=u(4,t)$

$$\therefore \qquad \bar{u}(0,s)=0,\ \bar{u}(4,s)=0$$

$\therefore$ From Eq. (1), we have $A+B=0$

$$\text{and } 0=Ae^{4\sqrt{s/2}}+Be^{-4\sqrt{s/2}}+\frac{8\sin 8\pi}{8\pi^2+s}$$

$$\Rightarrow Ae^{4\sqrt{s/2}}+Be^{-4\sqrt{s/2}}=0$$

Solving we get $A=B=0$

$$\therefore \text{ From (1) we have } \bar{u}=\frac{8\sin 2\pi x}{8\pi^2+s}$$

$$\therefore y=L^{-1}\left\{\frac{8}{8\pi^2+s}\sin 2\pi x\right\}$$

i.e.,
$$y=8e^{-8\pi^2 t}\sin 2\pi x.$$

Example 50

Solve the wave equation of a stretched string given by $\frac{\partial^2 u}{\partial t^2}=9\frac{\partial^2 u}{\partial x^2}$ satisfying the boundary conditions $u(x,0)=0$, $u_t(x,0)=0, x>0$ and $\bar{u}(0,t)=F(t)$, $\lim_{x\to\infty}u(x,t)=0, t\geq 0$.

Solution

Given $\frac{\partial^2 u}{\partial t^2}=9\frac{\partial^2 u}{\partial x^2}$.

Taking Laplace transform on both sides of the equation with the boundary conditions we have

$$L\left\{\frac{\partial^2 u}{\partial t^2}\right\}=9L\left\{\frac{\partial^2 u}{\partial x^2}\right\}$$

$$\text{or } s^{2\bar{u}}(x,s)-su(x,0)-u_t(x,0)=9\cdot\frac{d^2\bar{u}}{dx^2} \text{ or } \frac{d^2\bar{u}}{dx^2}-\frac{s^2}{9}\bar{u}=0 \qquad (1)$$

Also $\bar{u}(0,s)=\int_0^\infty F(t)e^{-st}dt=\bar{F}$ and $u(x,s)=0$ as $x\to\infty$

$\therefore$ The general solution Eq. of (1) is $\bar{u}\ (x,s)$

$$=c_1e^{\frac{s}{3}x}+c_2e^{\frac{-s}{3}x}$$

and $\bar{u}(x,s)=0$ as $x\to\infty\Rightarrow c_1=0$

and $\bar{u}(0,s)=\bar{F}(s)=c_2$

Hence, $\bar{u}(x,s)=\bar{F}(s)\ e^{-\frac{sx}{3}}$

$$\therefore u(x,t)=L^{-1}\left\{e^{-\frac{sx}{3}}\bar{F}(s)\right\}$$

$$=\begin{cases}F\left(t-\frac{x}{3}\right), t>\frac{x}{3} \\ 0, t<\frac{x}{3}\end{cases} \text{ as } L^{-}\{\bar{F}(s)=F(t)\},$$

when expressed in terms of Heaviside's unit step function. $u(x,t)$

$$=F\left(t-\frac{x}{3}\right)\cdot H\left(t-\frac{x}{3}\right).$$

Exercises

1. Let $f(x) = \begin{cases} 0 & \text{if} \quad -\pi \le x \le 0 \\ x^3 & \text{if} \quad 0 < x \le \pi \end{cases}$

Which is a periodic function with period $2p$, then $a_0 =$ ______.

(A) $\frac{\pi^3}{4}$ (B) $\frac{\pi^3}{8}$

(C) $\frac{\pi^3}{12}$ (D) $\frac{\pi^3}{16}$

2. The value of the fourier coefficient a_n for $n \ge 2$ for $f(x) = x \sin x$ in $(-\pi, \pi)$ is ______.

(A) $\cos \frac{(n-1)\pi}{n-1} \cos nx$

(B) $\cos \frac{2(n-1)\pi}{n-1} \cos nx$

(C) $\frac{1}{2} + \sum_{n=1}^{\infty} \cos nx$

(D) $\left[\cos \frac{(n-1)\pi}{n-1} - \cos \frac{(n+1)}{n+1}\right]$

3. If $f(x) = x^3$ in $(-\pi, \pi)$ then the value of b_n is ______.

(A) $\frac{2}{n^3}[6 + n^2\pi^2]$

(B) $\frac{2\cos n\pi}{n^3}[6 - n^2\pi^2]$

(C) $\frac{2}{n^3}[6 - n^2\pi^2]$

(D) $\frac{\cos n\pi}{n^3}[6 + n^2\pi^2]$

4. Find the Fourier series of $f(x)$ which is defined as follows:

$f(x) = 2 \qquad 0 < x < 1$

$\quad = 3 \qquad 1 < x < 2$

(A) $\sum_{n=1}^{\infty} \sin \frac{\pi x}{2}$

(B) $\frac{5}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin \frac{(2n-1)\pi x}{2}$

(C) $\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin \frac{\pi x}{2}$

(D) None of these

Direction for questions 5 and 6:

Let $f(x) = \frac{\pi - x}{2}$ in the interval $(0, 2\pi)$

5. The Fourier series of $f(x)$ is ______.

(A) $\cos x + \frac{\cos 2x}{2} + \frac{\cos 3x}{2} + \cdots$

(B) $\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} +$

(C) $\sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots$

(D) None of these

6. The value of $\frac{\pi}{4}$ is ______.

(A) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots$

(B) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{6} + \cdots$

(C) $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9}$

(D) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \cdots$

Direction for questions 7 and 8:

Let $f(x) = x^2$ in the interval $(-\pi, \pi)$.

7. The Fourier series of $f(x)$ is ______.

(A) $-\frac{\pi^2}{3}$

(B) $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} \cos nx$

(C) $\sum_{n=1}^{\infty} \cos nx$

(D) $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} 4\frac{(-1)^n}{n^2} \cos nx$

8. The value of $\frac{\pi^2}{12}$ is

(A) $1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

(B) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

(C) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

(D) $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

9. The value of $\dfrac{\pi^2}{6}$ is

(A) $1-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\dfrac{1}{4^2}$

(B) $1+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\dfrac{1}{4^2}\cdots$

(C) $1+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\dfrac{1}{7^2}$

(D) None of these

10. The half-range sine series of $f(x) = e^x$ in $0 < x < l$ is _____.

(A) $\sum_{n=1}^{\infty}\dfrac{1}{1+n^2\pi^2}\,(1 - e(-1)^n)\sin n\pi x$

(B) $2\pi\sum_{n=1}^{\infty}\dfrac{n}{1+n^2\pi^2}\,(1 - e(-1)^n)\sin n\pi x$

(C) $\sum_{n=1}^{\infty}\dfrac{1}{1+n^2\pi^2}\,(1 - e(-1)^n)\sin n\pi x$

(D) None of these

11. The order and degree of the $\dfrac{\partial^2 z}{\partial x^2}+3xy\left(\dfrac{\partial z}{\partial x}\right)^2+5\dfrac{\partial z}{\partial y}$ $= 8$ are

(A) 1, 1 (B) 1, 2
(C) 2, 1 (D) 2, 2

12. The differential equation whose solution is $z = (x - a)(y - b)$ is _______.

(A) $pq = 2z$ (B) $pq = z$
(C) $p = 2zq$ (D) $p = zq$

13. Form a PDE of $z = (x - y)\,\phi\,(x^2 - y^2)$

(A) $py - xq = z$ (B) $py + xq = z$
(C) $px + yq = z$ (D) $px - yq = z$

14. The solution of $x^2p + y^2q = (x + y)z$ is ________.

(A) $f(xy, x - y) = 0.$ (B) $f\left(\dfrac{xy}{z},\dfrac{x-y}{z}\right)=0$
(C) $f(zx, z - x) = 0$ (D) None of these

15. Solve $(2p + 1)\,q = pz$

(A) $a\log(z - a) = x - ay + b$
(B) $2a\log(z + a) = ay + b$
(C) $2a\log(z - a) = x + ay + b$
(D) $a\log(z + a) = 3x + ay + b$

16. The solution of $q^2x\,(1 + y^2) = py^2$ is _______.

(A) $z = a\,(1 + y^2)$

(B) $z=\dfrac{ax^2}{2} - a\,(1 + y^2) + b$

(C) $z=\dfrac{ax^2}{2}+\sqrt{a(1+y^2)}+b$

(D) $z=\dfrac{ax}{2}+\sqrt{a(1+y^2)}+b$

17. Solve $pqz = q^2(yp + q^2) + p^2(xq + p^2)$.

(A) $z = ax + by + \dfrac{a^3}{b} + \dfrac{b^3}{a}$

(B) $z = ax - by$

(C) $z = ax + by + \dfrac{a}{a^3} + \dfrac{b}{b^3}$

(D) None of these

18. In the process of solving the partial differential equation $\dfrac{\partial^2 u}{\partial x^2}+5\dfrac{\partial^2 u}{dy^2}=0$ by the method of separation of variables, the linear differential equation involving the independent variable 'X' is _____. (Here k is a constant)

(A) $\dfrac{d^2X}{dx^2} + kX(x) = 0$

(B) $\dfrac{d^2X}{dx^2} - kX(x) = 0$

(C) $\dfrac{d^2X}{dx^2}+k\dfrac{dx}{dx}+k^2X(x)=0$

(D) $\dfrac{d^2X}{dx^2}-k\dfrac{dx}{dx}+2k\,X(x)=0$

19. The second order partial differential equation $3x^2\dfrac{\partial^2 u}{\partial x^2}$ $-6xy\dfrac{\partial^2 u}{\partial x\partial y}+3y^2\dfrac{\partial^2 u}{\partial y^2}-5\dfrac{\partial u}{\partial x}+7\dfrac{\partial u}{\partial y}=6x^2y$ is _______.

(A) elliptic equation
(B) parabolic equation
(C) hyperbolic equation
(D) depends on the value of x and y

20. Which of the following partial differential equations represents the one-dimensional diffusion equation?

(A) $\dfrac{\partial^2 u}{\partial t^2}=c^2\dfrac{\partial^2 u}{\partial x^2}$

(B) $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$

(C) $\dfrac{\partial u}{\partial t}=D\dfrac{\partial^2 u}{\partial x^2}$

(D) $\dfrac{\partial^2 u}{\partial x^2}=c^2\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial u}{\partial x}$

21. In the one-dimensional diffusion equation, $\dfrac{\partial u}{\partial t}=D\dfrac{\partial^2 u}{\partial x^2}$, $u(x, t)$ and D represent respectively

(A) density of and diffusion coefficient.
(B) diffusion and density coefficient.
(C) viscosity and diffusion coefficient.
(D) diffusion and viscosity coefficient.

22. Which of the following pair can be represented by the same partial differential equation? (Except possibly a change in the constant multiplying the partial derivatives)
(A) The one-dimensional wave equation and the one -dimensional heat equation.
(B) The one-dimensional wave equation and the two -dimensional Laplace equation.
(C) The one-dimensional heat equation and the two-dimensional Laplace equation.
(D) The one-dimensional heat equation and the one-dimensional diffusion equation.

23. Solution of the one dimensional heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $x > 0$, $t > 0$ satisfying the boundary condition $u(0, t) = 1$, $u(x, 0) = 0$ is ______.

(A) $erf\left(\frac{x}{2\sqrt{t}}\right)$ (B) $erf\left(\frac{\sqrt{x}}{2\sqrt{t}}\right)$

(C) $erf\left(\frac{1}{2\sqrt{t}}\right)$ (D) $erf\left(\frac{x}{\sqrt{t}}\right)$

24. A string is stretched between two fixed points follows the equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ $(t > 0, x > 0)$ satisfying the boundary conditions $y(x, 0) = 0$, $x > 0$ and $y(0, t) = t$ $\underset{x\to\infty}{\text{Lt}}\ y(x, t) = 0, t \geq 0$, Find $y(x, t)$ in terms of Heaviside's unit step function.

(A) $(t - x)\, H(t - x)$

(B) $\left(t - \frac{x}{a}\right) H\left(t - \frac{x}{a}\right)$

(C) $(t - xa)\, H(t - xa)$

(D) None of these

25. The one dimensional wave equation is ______.

(A) $\frac{\partial u}{\partial t} = c\frac{\partial u}{\partial x}$

(B) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

(C) $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$

(D) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial u}{\partial x}$

Previous Years' Questions

1. The equation $K_x \frac{\partial^2 h}{\partial x^2} + K_z \frac{\partial^2 h}{\partial z^2} = 0$ can be transformed to $\frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0$ by substituting **[GATE, 2008]**

(A) $x_t = x\frac{K_z}{K_x}$ (B) $x_t = x\frac{K_x}{K_z}$

(C) $x_t = x\sqrt{\frac{K_x}{K_z}}$ (D) $x_t = x\sqrt{\frac{K_z}{K_x}}$

2. The partial differential equation that can be formed from $z = ax + by + ab$ has the form (with $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$) **[GATE, 2010]**

(A) $z = px + qy$
(B) $z = px + pq$
(C) $z = px + qy + pq$
(D) $z = qy + pq$

3. The Fourier series of the function,
$$f(x) = 0, \quad -\pi < x \leq 0$$
$$= \pi - x. \quad 0 < x < \pi$$
In the interval $[-\pi, \pi]$ is
$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \cdots\right] +$$
$$\left[\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots\right]$$
The convergence of the above Fourier series at $x = 0$ gives **[GATE, 2016]**

(A) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ (B) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

(C) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ (D) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi^2}{4}$

4. The type of partial differential equation $\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + 3\frac{\partial^2 p}{\partial x \partial y} + 2\frac{\partial p}{\partial x} - \frac{\partial p}{\partial y} = 0$ is **[GATE, 2016]**

(A) elliptic
(B) parabolic
(C) hyperbolic
(D) None of these

5. The solution of the partial differential equation $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ is of the form **[GATE, 2016]**

(A) $C\cos(kt)[C_1 e^{(\sqrt{k/\alpha})x} + C_2 e^{-(\sqrt{k/\alpha})x}]$

(B) $Ce^{kt}[C_1 e^{(\sqrt{k/\alpha})x} + C_2 e^{-(\sqrt{k/\alpha})x}]$

(C) $Ce^{kt}\left[C_1 \cos\left(\sqrt{\frac{k}{\alpha}}\right)x + C_2 \cos\left(-\sqrt{\frac{k}{\alpha}}\right)x\right]$

(D) $C\sin(kt)\left[C_1 \cos\left(\sqrt{\frac{k}{\alpha}}\right)x + C_2 \cos\left(-\sqrt{\frac{k}{\alpha}}\right)x\right]$

Answer Keys

Exercises

1. A	**2.** D	**3.** B	**4.** B	**5.** C	**6.** D	**7.** D	**8.** D	**9.** B	**10.** B
11. C	**12.** B	**13.** B	**14.** B	**15.** C	**16.** C	**17.** A	**18.** B	**19.** B	**20.** C
21. A	**22.** D	**23.** A	**24.** B	**25.** B					

Previous Years' Questions

1. D	**2.** C	**3.** C	**4.** C	**5.** B

Chapter 4

Linear Algebra

CHAPTER HIGHLIGHTS

- *Introduction*
- *Determinants*
- *Systems of linear equations*

Introduction

A set of 'mn' elements arranged in the form of rectangular array having 'm' rows and 'n' columns is called an $m \times n$ matrix (read as 'm by n matrix') and is denoted by $A = [a_{ij}]$ where $1 \le i \le m$; $1 \le j \le n$

or
$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots & a_{2n} \\ \vdots & \vdots & \vdots \cdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} \cdots & a_{mn} \end{pmatrix}$$

The element a_{ij} lies in the ith row and jth column.

Type of Matrices

Square Matrix A matrix $A = [a_{ij}]_{m \times n}$ is said to be a square matrix, if $m = n$ (i.e., Number of rows of A = Number of columns of A)

The elements $a_{11}, a_{22}, a_{33}, \ldots, a_{nn}$ are called 'DIAGONAL ELEMENTS'.

The line containing the diagonal elements is the 'PRINCIPAL DIAGONAL'.

The sum of the diagonal elements of 'A' is the 'TRACE' of A.

Row Matrix A matrix $A = [a_{ij}]_{m \times n}$ is said to be row matrix, if $m = 1$ (i.e., the matrix has only one row)

General form is $A = [a_1, a_2, \ldots, a_n]$ or $[a_{ij}]_{1 \times n}$

Column Matrix A matrix which has only one column

$$A = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \text{ or } [a_{ij}]_{n \times 1}$$

Diagonal Matrix A square matrix is said to be a diagonal matrix if all its elements except those in the principal diagonal are zeros. That is, if

1. $m = n$ (A is a square matrix) and
2. $a_{ij} = 0$ if $i \ne j$ (The non-diagonal elements are zeros)

A diagonal matrix of order 'n' with diagonal elements d_1, $d_2, \ldots, d_n$ is denoted by Diag $[d_1\ d_2 \ldots d_n]$.

Scalar Matrix A diagonal matrix whose diagonal elements are all equal is called a scalar matrix. That is, if

1. $m = n$
2. $a_{ij} = 0$ if $i \ne j$
3. $a_{ij} = k$ if $i = j$ for some constant 'k'.

Unit or Identity Matrix A scalar matrix of order 'n' in which each diagonal element is '1' (unity) is called a unit matrix or identity matrix of order 'n' and is denoted by I_n. That is,

1. $m = n$
2. $a_{ij} = 0$ if $i \neq j$
3. $a_{ij} = 1$ if $i = j$

Example: $I_1 = [1],\ I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Null Matrix or Zero Matrix A matrix is a 'null matrix' or zero matrix if all its elements are zeros.

Upper Triangular Matrix A square matrix is said to be an upper triangular matrix, if each element below the principal diagonal is zero. That is,

1. $m = n$
2. $a_{ij} = 0$ if $i > j$

For example, $\begin{pmatrix} 1 & 4 & 3 & 2 \\ 0 & -1 & 6 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 9 \end{pmatrix}_{4\times 4}$

Lower Triangular Matrix A square matrix is said to be a lower triangular matrix, if each element above the principal diagonal is zero, i.e., if

1. $m = n$
2. $a_{ij} = 0$ if $i < j$

For example, $\begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 7 & 8 & 0 \\ 5 & 4 & 2 & 1 \end{pmatrix}$

Horizontal Matrix If the number of rows of a matrix is less than the number of columns, i.e., $m < n$, then the matrix is called horizontal matrix.

Vertical Matrix If the number of columns in a matrix is less than the number of rows, i.e., if $m > n$, then the matrix is called a vertical matrix.

Comparable Matrices Two matrices $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{p\times q}$ are said to be comparable, if they are of same order, i.e., $m = p$; $n = q$.

Equal Matrices Two comparable matrices are said to be 'equal', if the corresponding elements are equal, i.e., $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{p\times q}$ are equal if

1. $m = p$; $n = q$ (i.e., they are of the same order)
2. $a_{ij} = b_{ij}\ \forall\ i, j$ (i.e., the corresponding elements are equal)

Transpose of a Matrix

The matrix obtained by interchanging the rows and the columns of a given matrix 'A' is called the 'transpose' of A and is denoted by A^T or A'. If A is an $(m \times n)$ matrix, A^T will be an $(n \times m)$ matrix. Thus if $A = [a_{ij}]_{m\times n}$ then $A^T = [u_{ij}]_{n\times m}$, where $u_{ij} = a_{ji}$.

Properties of Transpose

$T-1$: $(A')' = A$, for any matrix A

$T-2$: $(A + B)' = A' + B'$, for any two matrices A, B of same order

$T-3$: $(KA)' = KA'$, for any matrix A

$T-4$: $(AB)' = B'A'$, for any matrices A, B such that number of columns of A = number of rows of B (REVERSAL LAW)

$T-5$: $(A^n)' = (A')^n$, for any square matrix A

Trace of a Matrix

Let 'A' be a square matrix. The trace of A is defined as the sum of elements of 'A' lying in the principal diagonal.

Thus if $A = [a_{ij}]_{n\times n}$ then trace of 'A' denoted by $t_r A = a_{11} + a_{22} + \cdots + a_{nn}$.

Properties of Trace of a Matrix Let A and B be any two square matrices and K any scalar then,

1. $t_r(A + B) = t_r A + t_r B$
2. $t_r(KA) = Kt_r A$
3. $t_r(AB) = t_r(BA)$

Conjugate of a Matrix

A matrix obtained by replacing each element of a matrix 'A' by its complex conjugate is called the 'conjugate matrix' of A and is denoted by $\bar{A}$. If $A = [a_{ij}]_{m\times n,}$ then $\bar{A} = \left[\overline{a_{ij}}\right]$ where $\bar{a}_{ij}$ is the conjugate of 'a_{ij}'.

Properties of Conjugate of a Matrix

$C-1$: $\overline{(\overline{(A)})} = A$for any matrix '$A$'

$C-2$: $\overline{(A+B)} = \bar{A} + \bar{B}$ for any matrices A, B of same order.

$C-3$: $\overline{(KA)} = \bar{K}\ \bar{A}$ for any matrix 'A' and any Scalar K.

$C-4$: $\overline{(AB)} = (\bar{A}) \cdot \bar{I}$for any matrices A and B with the condition that number of columns of A = number of rows of B.

$C-5$: $(\bar{A})^n = \left(\overline{A^n}\right)$for any square matrix '$A$'.

Tranjugate or Transposed Conjugate of a Matrix

Tranjugate of a matrix 'A' is obtained by transposing the conjugate of A and is denoted by A^θ. Thus $A^\theta = (\bar{A})^T$.

Properties of Tranjugate of a Matrix

TC – 1: $(A^\theta)^\theta = A$ for any matrix A

TC – 2: $(A + B)^\theta = A^\theta + B^\theta$ for any matrices A, B of the same order.

TC – 3: $(KA)^\theta = KA^\theta$ for any matrix A and any scalar K.

TC – 4: $(BA)^\theta = B^\theta A^\theta$ for any matrix A, B with the condition that number of columns of A = number of rows of B.

TC – 5: $(A^n)^\theta = (A^\theta)^n$ for any square matrix 'A'.

Symmetric Matrix A matrix A is said to be symmetric, if $A^T = A$ (i.e., transpose of $A = A$).

NOTE

A symmetric matrix must be a square matrix.

Skew-symmetric Matrix A matrix 'A' is said to be skew-symmetric matrix, if $A^T = (-A)$, i.e., $A = [a_{ij}]_{m\times n}$ is skew symmetric if

1. $m = n$
2. $a_{jI} = -a_{ij} \forall i, j$

NOTE

In a skew-symmetric matrix, all the elements of the principal diagonal are zero.

Orthogonal Matrix A square matrix 'A' of order $n \times n$ is said to be an orthogonal matrix, if $AA^T = A^TA = I_n$.

Involutory Matrix A square matrix 'A' is said to be involutory matrix, if $A^2 = I$ (where I is identity matrix).

Idempotent Matrix A square matrix 'A' is said to be an idempotent matrix, if $A^2 = A$.

Nilpotent Matrix A square matrix 'A' is said to be nilpotent matrix, if there exists a natural number 'n' such that $A^n = O$. If 'n' is the least natural number such that $A^n = O$, then 'n' is called the index of the nilpotent matrix 'A'. (Where O is the null matrix).

Unitary Matrix A square matrix 'A' is said to be a unitary matrix if, $AA^\theta = A^\theta A = I$. (Where A^θ is the transposed conjugate of A.)

Hermitian Matrix A matrix 'A' is said to be a hermitian matrix, if $A^\theta = A$, i.e., $A = [a_{ij}]_{m\times n}$ is hermitian if

1. $m = n$
2. $a_{ij} = \bar{a}_{ij} \forall i, j$

NOTE

The diagonal elements in a hermitian matrix are real numbers.

Skew-hermitian Matrix A matrix 'A' is said to be a skew-hermitian matrix, if $A^\theta = -A$.

Operations on Matrices

Scalar Multiplication of Matrices

If A is a matrix of order $m \times n$ and 'K' be any scalar (a real or complex number), then KA is defined to be a $m \times n$ matrix whose elements are obtained by multiplying each element of 'A' by K, i.e., if $A = [a_{ij}]_{m\times n}$ then $KA = [Ka_{ij}]_{m\times n}$ in particular if $K = -1$; then $KA = -A$ is called the negative of A and is such that,

$A + (-A) = [a_{ij}] + [-a_{ij}] = [a_{ij} - a_{ij}] = [0] = O$ (zero matrix)
$(-A) + A = [-a_{ij}] + [a_{ij}] = [-a_{ij} + a_{ij}] = [0] = O$
That is, $A + (-A) = (-A) + A = O$.

Properties of Scalar Multiplication

Let A, B are two matrices of same order and α, β are any scalars, then

$S-1$: $\alpha(A + B) = \alpha A + \alpha B$
$S-2$: $(\alpha + \beta)A = \alpha A + \beta A$
$S-3$: $\alpha(\beta A) = (\alpha\beta)A$
$S-4$: $1A = A$

Addition of Matrices

If A and B are two matrices of the same order, then they are 'conformable' for addition and their sum '$A + B$' is obtained by adding the corresponding elements of A and B, i.e., if $A = [a_{ij}]_{m\times n}$; $B = [b_{ij}]_{m\times n}$, then $A + B = [a_{ij} + b_{ij}]_{m\times n}$.

Properties of Addition Let A, B and C be three matrices of same order say $m \times n$, then

$A-1$: $A + B$ is also a $m \times n$ matrix (CLOSURE)
$A-2$: $(A + B) + C = A + (B + C)$ (ASSOCIATIVITY)
$A-3$: If 'O' is the $m \times n$ zero (null) matrix, then $A + O = O + A = A$ ('O' is the ADDITIVE IDENTITY)
$A-4$: $A + (-A) = (-A) + A = O$ ($-A$ is the ADDITIVE INVERSE)
$A-5$: $A + B = B + A$ (COMMUTATIVITY)

NOTE

The set of matrices of same order form an 'Abelian Group' under addition.

Multiplication of Matrices

Let A and B be two matrices. A and B are conformable for multiplication, only if the number of columns of A is equal to the number of rows of B.

Let $A = [a_{ij}]$ be an $m \times n$ matrix, $B = [b_{jk}]$ be an $n \times p$ matrix. Then the product 'AB' is defined as the matrix $C = [c_{ik}]$ of order $m \times p$ where $c_{ik} = a_{i1}b_{1k} + a_{i2}b_{2k} + \cdots + a_{in}b_{nk}$

$$= \sum_{j=1}^{n} a_{ij}b_{jk}.$$

c_{ij} calculated for $i = 1, 2, \ldots m$ and $k = 1, 2, \ldots, p$ will give all the elements of the matrix C.

Properties of Multiplication

$M-1$: If A, B, C be $m \times n$, $n \times p$, $p \times q$ matrices respectively, then $(AB)C = A(BC)$ (ASSOCIATIVITY).
$M-2$: If A is a $m \times n$ matrix, then $A\,I_n = A$ and $I_m\,A = A$ and if A is a square matrix, i.e., $m = n$, then $AI = IA = A$ (I is the MULTIPLICATIVE IDENTITY).
$M-3$: If A, B, C be $m \times n, n \times p, p \times q$ matrices respectively, then $A(B + C) = AB + AC$ (DISTRIBUTIVE LAW).
$M-4$: Matrix multiplication is NOT COMMUTATIVE in general.
$M-5$: The INVERSE of a given matrix may not always exist.

Determinants

Let $A = [a_{ij}]$ be a square matrix of order 'n'. Then the determinant of order 'n' associated with 'A' is denoted by $|A|$ or $|a_{ij}|$ or Det(A) or Δ.

NOTES

1. Determinant of a matrix exists, only if it is a square matrix.
2. The value of a determinant is a single number.

Determinant of Order 1 (or First Order Determinant)

If 'a' be any number, then determinant of 'a' is of order '1' and is denoted by $|a|$. The value of $|a| = a$.

Determinant of Order 2 (or Second Order Determinant)

If 'A' is a square matrix of order 2 given by

$A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ then $\text{Det}(A) = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is determinant of order 2 and its value is $\Delta = a_1b_2 - a_2b_1$

Minor and Cofactor of a Matrix

Let $A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ be a 3×3 matrix

Then the minor of an element a_{ij} of 'A' is the determinant of the 2×2 matrix obtained after deleting the i-th row and j-th column of A and is denoted by M_{ij}.

The cofactor of a_{ij} is denoted by A_{ij} and is defined as $(-1)^{i+j} M_{ij}$, i.e., $A_{ij} = (-1)^{i+j} M_{ij}$

Determinant of Order 3 (Third Order Determinant)

If A is a square matrix of order '3', given by $A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$. Then the determinant of 'A' is given by

$\Delta = \text{Det } A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is a determinant of order 3 and the value is obtained by taking the sum of the products of the elements of any row (or column) by their corresponding cofactors.

Thus for A, $\Delta = a_1A_1 + b_1B_1 + c_1C_1$

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

or also $\Delta = a_1A_1 + a_2A_2 + a_3A_3$

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

(This is by expanding by C_1) and so on.

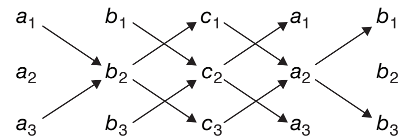
The sign to be used before a particular element can be judged by using the following rule:

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

The value of the determinants of order 3 can also be evaluated by using 'Sarrus' method given as follows:

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Enter the first column and then the second column after the third column and take the product of numbers as shown by the arrows, taking care of signs indicated



Then

$$\Delta = a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - a_3b_2c_1 - b_3c_2a_1 - c_3a_2b_1$$

We can now define the cofactor of an element a_{ij} in a 4×4 matrix as $(-1)^{i+j} \times$ (Determinant of the 3×3 matrix obtained by deleting the i-th row and j-th column) and determinant of a 4×4 matrix to be the sum of products of elements of any row (or column) by their corresponding cofactors. We can similarly define determinant of a square matrix of any order.

Properties of Determinant

1. If two rows (or columns) of a determinant are interchanged, the value of the determinant is multiplied by (-1).
2. If the rows and columns of a determinant are interchanged, the value of the determinant remains unchanged, i.e., $\text{Det}(A) = \text{Det}(A^T)$.
3. If all the elements of a row (or column) of a determinant are multiplied by a scalar (say 'K'), the value of the new determinant is equal to 'K' times the value of the original determinant.
4. If two rows (or columns) of a determinant are identical, then the value of the determinant is zero.
5. If the elements of a row (or a column) in a determinant are proportional to the elements of any other row (or column), then the determinant is '0'.
6. If every element of any row (or column) is zero, then determinant is '0'.
7. If each element in a row (or column) of a determinant is the sum of two terms, then its determinant can be expressed as the sum of two determinants of the same order.
8. (The theorem of 'false cofactor') The sum of products of elements of a row (or column) with the cofactors of any other row (or column) is zero.

   Thus in $A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$

   $a_1A_2 + b_1B_2 + c_1C_2 = 0$

   $a_2A_1 + b_2B_1 + c_2C_1 = 0$ and so on in general

   $a_rA_s + b_rB_s + c_rC_s = 0$ if $r \neq s$

9. If the elements of a determinant are polynomials in x and the determinant vanishes for $x = a$, then $x - a$ is a factor of the determinant.

Singular and Non-singular Matrices

A square matrix 'A' is said to be singular, if $\text{Det}(A) = 0$ and is non-singular, if $\text{Det}(A) \neq 0$.

NOTES

1. A unit matrix is non-singular (since its Det = 1)
2. If A and B are non-singular matrices of the same 'type', then AB is non-singular of the same 'type'.

Inverse of a Matrix

Let 'A' be a square matrix. A matrix 'B' is said to be an inverse of 'A', if $AB = BA = I$.

NOTE

If B is the inverse of 'A', then 'A' is the inverse of 'B'.

Some Results of Inverse

1. Inverse of a square matrix, when it exists, is unique.
2. The inverse of a square matrix exists, if and only if it is non-singular.
3. If 'A' and 'B' are square matrices of the same order, then 'AB' is invertible (i.e., inverse of AB exists) if 'A' and 'B' are both invertible.
4. If 'A' and 'B' are invertible matrices of the same order, then $(AB)^{-1} = B^{-1} A^{-1}$.
5. If A is invertible, then so is A^T and $(A^T)^{-1} = (A^{-1})^T$.
6. If A is invertible, then so is A^θ and $(A^\theta)^{-1} = (A^{-1})^\theta$.

Adjoint of a Matrix

The adjoint of a square matrix 'A' is the transpose of the matrix obtained by replacing the elements of 'A' by their corresponding cofactors.

NOTE

The adjoint is defined only for square matrices and the adjoint of a matrix 'A' is denoted by Adj(A). If

$$A = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \\ b_1 & b_2 & \cdots & b_n \\ \vdots & \vdots & \vdots & \vdots \\ l_1 & l_2 & \cdots & l_n \end{pmatrix}$$

$$\text{Adj } A = \begin{pmatrix} A_1 & A_2 & \cdots & A_n \\ B_1 & B_2 & \cdots & B_n \\ \vdots & \vdots & \vdots & \vdots \\ L_1 & L_2 & \cdots & L_n \end{pmatrix}^T = \begin{pmatrix} A_1 & B_1 & \cdots & L_1 \\ A_2 & B_2 & \cdots & L_2 \\ \vdots & \vdots & \vdots & \vdots \\ A_n & B_n & \cdots & L_n \end{pmatrix}$$

Results

1. If 'A' is of order 3×3 and K is any number, then $\text{Adj}(KA) = K^2(\text{Adj } A)$.
2. $A(\text{Adj } A) = (\text{Adj } A)A = |A|\, I$ for any square matrix 'A'.
3. Adj $I = I$; Adj $O = O$ where I is the identity matrix and O is the null matrix.
4. $\text{Adj}(AB) = (\text{Adj } B)\,(\text{Adj } A)$ if A, B are non-singular and are of same type.
5. If $A = A_{n\times n}$, then
   $\det(\text{Adj } A) = (\det A)^{n-1}$.
   $\text{Adj}(\text{Adj } A) = (\det A)^{n-2}(A)$.
   $|\text{Adj}(\text{Adj } A)| = (\det A)^{(n-1)^2}$

Evaluating Inverse of a Square Matrix

If A is a square matrix, then $A^{-1} = \dfrac{1}{|A|}(\text{Adj } A)$

NOTES

1. The inverse of an identity matrix is itself.
2. $(\text{Adj } A)^{-1} = \dfrac{1}{|A|} A$
3. If A is a non-singular square matrix (say of order 3) and K is any non-zero number, then
$$(KA)^{-1} = \frac{1}{K} A^{-1}$$

Rank and Nullity of a Matrix

Rank of a Matrix The Matrix 'A' is said to be of rank 'r', if and only if it has at least one non-singular square sub-matrix of order 'r' and all square sub-matrices of order $(r + 1)$ and higher orders are singular. The rank of a matrix A is denoted by rank (A) or $\rho(A)$.

Nullity of a Matrix If A is a square matrix of order 'n', then $n - \rho(A)$, i.e., n – rank (A) is defined as nullity of matrix 'A' and is denoted by $N(A)$.

Remark 1: If there is a non-singular square sub-matrix of order 'K', then $\rho(A) \geq K$.

Remark 2: If there is no non-singular square sub-matrix of order 'K', then $\rho(A) < K$.

Remark 3: If A' is the transpose of A, then $\rho(A) = \rho(A')$.

Remark 4: The rank of a null matrix is '0'.

Remark 5: The rank of a non-singular square matrix of order 'n' is 'n' and its nullity is '0'.

Remark 6: Elementary operations do not change the rank of a matrix.

Remark 7: If the product of two matrices A and B is defined, then $\rho(AB) \leq \rho(A)$ and $\rho(AB) \leq \rho(B)$. That is, the rank of product of two matrices cannot exceed the rank of either of them.

Elementary Operations or Elementary Transformations

1. Elementary row operations
   (a) $R_i \leftrightarrow R_j$: Interchanging of ith and jth rows
   (b) $R_i \to KR_i$: Multiplication of every element of ith row with a non-zero scalar K
   (c) $R_i \to R_i + kR_j$: Addition of k times the elements of jth row to the corresponding elements of ith row.
2. Elementary column operations
   (a) $C_i \leftrightarrow C_j$: Interchanging of ith and jth columns
   (b) $C_i \to KC_i$: Multiplication of every element of ith column with a non-zero scalar K.
   (c) $C_i \to C_i + KC_j$: Addition of K times the elements of jth column to the corresponding elements of ith column.

Example: Consider the matrix $A = \begin{bmatrix} 2 & 3 & -4 & 1 \\ 3 & 0 & 1 & 5 \\ 4 & 7 & 1 & 2 \end{bmatrix}$

$$R_2 \to 2R_2 \sim \begin{bmatrix} 2 & 3 & -4 & 1 \\ 6 & 0 & 2 & 10 \\ 4 & 7 & 1 & 2 \end{bmatrix}$$

$$C_2 \leftrightarrow C_3 \sim \begin{bmatrix} 2 & -4 & 3 & 1 \\ 3 & 1 & 0 & 5 \\ 4 & 1 & 7 & 2 \end{bmatrix}$$

$$C_1 \to C_1 - 2C_4 \sim \begin{bmatrix} 0 & -4 & 3 & 1 \\ -7 & 1 & 0 & 5 \\ 0 & 1 & 7 & 2 \end{bmatrix}$$

NOTE

The rank of a matrix is invariant under elementary operations

Row and Column Equivalence Matrices

Row Equivalence Matrix If B is a matrix obtained by applying a finite number of elementary row operations successively on matrix A, then matrix B is said to be row equivalent to A (or a row equivalent matrix of A).

Column Equivalence Matrix If B is obtained by applying a finite number of elementary column operations successively on matrix A, then matrix B is said to be column equivalent to A (or a column equivalent matrix of A).

Example: $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & -2 \\ 1 & 4 & -3 \end{bmatrix}$

$$R_2 - 2R_1, R_3 - R_1 \sim \begin{bmatrix} 1 & 3 & 4 \\ 0 & -1 & -10 \\ 0 & 1 & -7 \end{bmatrix} = B \text{ (say)}$$

B is a row equivalent matrix of A.

Example: $B = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 4 & -4 \\ 1 & 1 & 6 \end{bmatrix}$

$$C_2 - 3C_1, \frac{1}{2}C_3 \sim \begin{bmatrix} 1 & 0 & 1 \\ 3 & -5 & -2 \\ 1 & -2 & 6 \end{bmatrix} = C \text{ (say)}$$

C is a column equivalent to B.

Row Reduced Matrix A matrix A of order $m \times n$ is said to be row reduced if,

1. The first non-zero element of a non-zero row is 1.
2. Every other element in the column in which such 1's occur is 0.

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix} \text{ is a row reduced matrix}$$

$$B = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ is not a row reduced matrix.}$$

Row Reduced Echelon Matrix A matrix 'X' is said to be row reduced echelon matrix if,

1. X is row reduced.
2. There exists integer $\underline{P}(0 \le p \le m)$ such that first 'p' rows of X are non-zero and all the remaining rows are zero rows.
3. For the ith non-zero row, if the first non-zero element of the row (i.e., 1) occurs in the jth column then, $j_1 < j_2 < j_3 < \cdots < j_p$.

Example: $P = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}; Q = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

are echelon matrices. The number of non-zero rows (i.e., value of P and Q) are 3 and 2 respectively. The value of i and j are tabulated below

P:	i	1	2	3
	j	1	2	3

Q:	i	1	2
	j	2	4

Normal form of a Matrix

By means of elementary transformations, every matrix 'A' of order $m \times n$ and rank r (> 0) can be reduced to one of the following forms.

1. $\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$ **2.** $[I_r/0]$ **3.** $[\underline{I_r}]$ **4.** $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$

and these are called the normal forms. I_r is the unit matrix of order 'r'.

NOTE

If a $m \times n$ matrix 'A' has been reduced to the normal form say $\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$ then 'r' is the rank of A.

Systems of Linear Equations

Let
$$\left.\begin{matrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}X_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}X_n = b_2 \\ \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \\ a_{n1}x_1 + a_{n2}x_2 + \cdots \ a_{nn}x_n = b_n \end{matrix}\right\} \tag{1}$$

be a system of 'n' linear equations in 'n' variables $x_1, x_2, \ldots, x_n$. The above system of equations can be written as

$$\begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \quad \text{or} \quad AX = B$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} \cdots & a_{nn} \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

A is called the co-efficient matrix.

Any set of values of $x_1, x_2, x_3, \ldots$ which simultaneously satisfy these equations is called a solution of the system. When the system of equations has one or more solutions, the equations are said to be CONSISTENT and the system of equations are said to be INCONSISTENT if it does not admit any solution. The system of equations (1) is said to be

HOMOGENEOUS, if $B = 0$

NON-HOMOGENEOUS, if $B \neq 0$

Let the system of equations be

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{12}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

This is a system of 'm' equations in 'n' variables $x_1, x_2, \ldots, x_n$. The system of equations can be written as $AX = B$ where

$$A = \begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{mn} \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

The matrix $\begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \\ a_{m1} & a_{m2} \cdots & a_{mn} & b_m \end{pmatrix}$ is called the augmented matrix of the system of equations and is denoted by $[A : B]$.

Let $AX = B$ represents 'm' linear equations with 'n' variables. Let rank of $A = r$ and rank $(A, B) = r_1$ [where (A, B) is an augmented matrix]. If $r_1 \neq r$, then the system of equations are inconsistent.

If $r_1 = r$, the table follows:

	$m = n$		$m > n$		$m < n$	
	$r = n$	$r < n$	$r = n$	$r < n$	$r = m$	$r < m$
Homo-geneous	Only trivial solution	Infinite solutions	Only trivial solution	Infinite solutions	Infinite solutions	Infinite solutions
Non-homo geneous	Unique solution	Infinite solutions	Unique solution	Infinite solutions	Infinite solutions	Infinite solutions

Solving System of Linear Equations

The following methods of solving system of linear equations (1) is applicable only when the co-efficient matrix 'A' is non singular, i.e., $|A| \neq 0$.

Cramers Method

Let $AX = B$ represent the system of equations (1) where A, X and B are as defined earlier.

Let Δ be $|A|$ and $\Delta_1, \Delta_2, \ldots, \Delta_n$ be the determinants obtained by replacing the elements of 1st, 2nd, $\ldots$, nth column of A by the elements of B. Then if $\Delta \neq 0$, we have

$$x_1 = \Delta_1/\Delta;\ x_2 = \Delta_2/\Delta;\ x_3 = \Delta_3/\Delta;\ \ldots;\ x_n = \Delta_n/\Delta.$$

Inverse Method

Let the system of linear equations be $AX = B$, where A, X, B are as defined earlier.

If $|A| \neq 0$ then pre-multiplying with A^{-1}, we get $A^{-1}(AX) = A^{-1}B$.

$\Rightarrow \quad X = A^{-1}B$ which gives the values of the variables.

Gauss–Jordan Method

Consider the augmented matrix $[A : B]$ of the system of 'n' non-homogeneous equations (1) in n-variables

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} & b_n \end{bmatrix}$$

Reduce this augmented matrix to the standard form

$$\begin{bmatrix} 1 & 0 & \cdots & 0 & d_1 \\ 0 & 1 & \cdots & 0 & d_2 \\ \vdots & \vdots & & \vdots & \\ 0 & 0 & \cdots & 1 & d_n \end{bmatrix}$$

By applying the elementary operations, the solution of the equations is $x_1 = d_1, x_2 = d_2, \ldots, x_n = d_n$.

Gauss Elimination Method

Let the system of linear equations given by

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= c_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= c_2 \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n &= c_3 \\ \cdot \quad \cdot \quad \cdot \quad & \cdot \\ \cdot \quad \cdot \quad \cdot \quad & \cdot \\ \cdot \quad \cdot \quad \cdot \quad & \cdot \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= c_n \end{aligned}\right\} \quad (1)$$

Let $a_{11} \neq 0$ write the above equations in the matrix form $AX = B$

Write the augmented matrix $[A\ B]$.

Using elementary row operations, eliminate the unknown x_1 from all the equations except the first. Eliminate the unknown x_2 from all the equations except from first and second rows, continuing in this way we finally get the following equivalent system of equations at the $(n-1)$th step.

$$\begin{aligned} a'_{11}x_1 + a'_{12}x_2 + a'_{13}x_3 + \cdots + a'_{1n}x_n &= c'_1 \\ a'_{22}x_2 + \cdots + a'_{2n}x_n &= c'_2 \\ a'_{33}x_3 + \cdots + a'_{3n}x_n &= c'_3 \\ a'_{nn}x_n &= c'_n \end{aligned}$$

From the above system of equations we can find the values of the unknowns.

Linear Dependence

A set of vectors of n dimensions is said to be linearly dependent if one of these vectors can be expressed as a linear combination of some other vectors in the set.

If no vector can be expressed as a linear combination of the others, then the set of vectors is said to be linearly independent.

NOTE

The maximum number of linearly independent rows or columns of a matrix is called the rank of the matrix.

LU Decomposition Method of Factorisation or Method of Triangularization

Consider the system of equations

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}\right\} \quad (1)$$

In matrix notation, Eq. (1) can be written as $AX = B$ (2)

where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

Step 1: Write $A = LU$, where $L \to$ Lower triangular matrix with principal diagonal elements being equal to 1 and $U \to$ Upper triangular matrix.

That is, $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

Step 2: Now Eq. (2) becomes $LUX = B$ (3)

Step 3: Let $UX = Y$ (4)

where $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

Step 4: Combining Eqs. (3) and (4), we get $LY = B$ (5)
On solving Eq. (5) we get y_1, y_2, y_3.

Step 5: Substituting Y in Eq. (4), we get $UX = Y$
On solving, we get X, i.e., x_1, x_2, x_3.

The Characteristic Equation of a Matrix

Characteristic Matrix If A is any square matrix, the matrix $A - \lambda I$ where λ is a scalar, is called the characteristic matrix of A.

Characteristic Polynomial If A is any square matrix of order n, then the determinant $|A - \lambda I|$ yields a polynomial $\phi(\lambda)$ of degree n in λ which is known as the characteristic polynomial of the matrix A.

Characteristic Equation If $\phi(\lambda)$ is the characteristic polynomial of a matrix A, then $\phi(\lambda) = 0$, is called the characteristic equation of A.

And the roots of this equation, say $\lambda_1, \lambda_2, \ldots, \lambda_n$ are called the characteristic roots or latent roots or **eigen values**. If λ is a characteristic root of ordert, then t is called the algebraic multiplicity of λ.

Characteristic Vectors Corresponding to each characteristic root λ, there is a non-zero vector which satisfies the characteristic equation $|A - \lambda I| = 0$. These non-zero vectors are called the characteristic vectors or **eigen vectors** or latent vectors.

NOTES

1. The characteristic roots of a matrix and its transpose are the same.
2. 0 is a characteristic roots of a matrix, if the matrix is singular.
3. The characteristic roots of a triangular matrix are just the diagonal elements of the matrix.
4. If K is any scalar, the characteristic roots of matrix KA are K times the characteristic roots of matrix A.
5. If $a_1, a_2, a_3, \ldots, a_n$ are characteristic roots of matrix A and K is a scalar, then the characteristic roots of matrix $A - KI$ are $a_1 - K, a_2 - K, \ldots, a_n - K$.
6. If λ is a characteristic root of a non-singular matrix, then λ^{-1} is a characteristic root of A^{-1}.
7. If the eigen values of A are $\lambda_1, \lambda_2, \ldots, \lambda_n$ then the eigen values of A^2 are $\lambda_1^2, \lambda_2^2, \ldots, \lambda_n^2$.

Cayley–Hamilton Theorem

Every square matrix satisfies its characteristic equation.

Inverse by Cayley–Hamilton Theorem

Let A be non-singular square matrix of order n

Let the characteristic equation of A be

$$|A - \lambda I| = (-1)^n \lambda^n + C_1\lambda^{n-1} + C_2\lambda^{n-2} + \cdots + C_{n-1}\lambda + C_n = 0$$

Where $C_1, C_2, \ldots, C_n$ are all scalar constants

Then by Cayley–Hamilton theorem

$$(-1)^n A^n + C_1 A^{n-1} + C_2 A^{n-2} + \cdots + C_{n-1}A + C_n I = O \qquad (1)$$

Multiplying Eq. (1) throughout by A^{-1}, we have

$$A^{-1}[(-1)^n A^{n-1} + C_1 A^{n-1} + C_2 A^{n-2} + \cdots + C_{n-1} A + C_n I] = A^{-1} \cdot 0$$

$$\Rightarrow \quad (-1)^n A^{n-1} + C_1 A^{n-2} + C_2 A^{n-3} + \cdots + C_{n-1} I + C_n A^{-1}$$

$$\Rightarrow \quad A^{-1} = \frac{-1}{C_n}[(-1)^n A^{n-1} + C_1 A^{n-2} + C_2 A^{n-3} + \cdots + C_{n-1} I]$$

NOTE

Similarly, we can find $A^{-2}, A^{-3}, \ldots$ for the matrix A_x provided A is non-singular.

Power of a Matrix by Cayley–Hamilton Theorem

Cayley–Hamilton theorem is also helpful in finding higher powers of a square matrix with least possible number of matrix multiplications. This is explained in Examples 11 and 12.

Reduction to Diagonal Form

If A is a square matrix of order n with n linearly independent eigen vectors, then A can be reduced to a diagonal matrix, called diagonal form of A.

Procedure to Reduce a Square Matrix into Diagonal Form

Let A be a square matrix of order n that can be reduced to diagonal form

1. Find the eigen values and their corresponding eigen vectors of A. Let $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ be the eigen values and let $X_1, X_2, X_3, \ldots, X_n$ be their corresponding eigen vectors that are linearly independent.
2. Form the matrix P with $X_1, X_2, X_3, \ldots, X_n$ as its columns i.e., $P = [X_1\, X_2\, X_3 \ldots X_n]$ it can be easily observed that P is invertible.
3. Find the inverse of P (i.e., find P^{-1})
4. The diagonal form of A is given by $D = P^{-1}$ AP.

Where $D = \begin{bmatrix} \lambda_1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_n \end{bmatrix}$ is a diagonal matrix

with eigen values of A as its principal diagonal elements.

NOTE

Here P is called the modal matrix and D is the spectral matrix of the matrix A

Power of a Matrix by Using its Diagonal Form

If D is the diagonal form of a square matrix A, then for any positive integer n, we have $A^n = P D^n P^{-1}$.

Where P is the modal matrix of A.

SOLVED EXAMPLES

Example 1

Find the value of

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}.$$

Solution

$c_1 \to c_1 + c_2 + c_3$

$$\begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$R_2 \to R_2 - R_1 \qquad R_3 \to R_3 - R_1$

$$2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix}$$

$$= 2(a+b+c)^3\begin{vmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 2(a+b+c)^3.$$

Example 2

Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -2 \\ 2 & 0 & -1 \\ 1 & 4 & 1 \end{bmatrix}$.

Solution

Given

$$\begin{bmatrix} 3 & 1 & -2 \\ 2 & 0 & -1 \\ 1 & 4 & 1 \end{bmatrix} R_1 \leftrightarrow R_3 \begin{bmatrix} 1 & 4 & 1 \\ 2 & 0 & -1 \\ 3 & 1 & -2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$$\sim \begin{bmatrix} 1 & 4 & 1 \\ 0 & -8 & -3 \\ 0 & -11 & -5 \end{bmatrix}$$

$$R_3 \to R_3 + \frac{-11}{8} R_2 \sim \begin{bmatrix} 1 & 4 & 1 \\ 0 & -8 & -3 \\ 0 & 0 & \frac{-7}{8} \end{bmatrix}$$

which is a row echelon form. The number of non zero rows = 3.

The rank of the matrix = The number of non-zero rows in it = 3

∴ Rank of the matrix = 3.

Example 3

Find whether the vectors given below are linearly dependent or independent {(1, 3, 2), (1, –4, 1), (–1, 2, 5)}.

Solution

Let $x, y, z \in R$ such that $x(1, 3, 2) + y(1, -4, 1) + z(-1, 2, 5) = (0, 0, 0)$

$$\Rightarrow \left.\begin{aligned} x + y - z &= 0 \\ 3x - 4y + 2z &= 0 \\ 2x + y + 5z &= 0 \end{aligned}\right\} \qquad (1)$$

The above system of equations when expressed in determinant form, we have

$$\begin{vmatrix} 1 & 1 & -1 \\ 3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix} \xrightarrow{R_2 - 3R_1,\ R_3 - 2R_1} \begin{vmatrix} 1 & 1 & -1 \\ 0 & -7 & 5 \\ 0 & -1 & 7 \end{vmatrix}$$

$$\xrightarrow{R_3 - \frac{1}{7}R_2} \begin{vmatrix} 1 & 1 & -1 \\ 0 & -7 & 5 \\ 0 & 0 & \frac{44}{7} \end{vmatrix}$$

∴ Rank = 3 = number of unknowns

∴ There exists a unique solution $x = 0, y = 0$ and $z = 0$

$\Rightarrow \quad x(1, 3, 2) + y(1, -4, 1) + z(-1, 2, 5)$
$= (0, 0, 0)$ only when $x = 0, y = 0, z = 0$.

∴ The set of vectors are linearly independent.

Example 4

Show that the set of vectors {(2, 3, 9), (3, –2, –6), (–1, 5, 15)} are linearly dependent.

Solution

Let $x, y, z \,\varepsilon\, R$ such that

$x(2, 3, 9) + y(3, -2, -6) + z(-1, 5, 15) = (0, 0, 0)$

$$\begin{aligned} 2x + 3y - z &= 0 \\ \Rightarrow \quad 3x - 2y + 5z &= 0 \\ 9x - 6y + 15z &= 0 \end{aligned}$$

The above system when expressed in matrix form we have the coefficient matrix

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & -2 & 5 \\ 9 & -6 & 15 \end{bmatrix}$$

$$\begin{vmatrix} 2 & 3 & -1 \\ 3 & -2 & 5 \\ 9 & -6 & 15 \end{vmatrix} = 0$$

as $\quad R_3 = 3R_2$ and $\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} \neq 0$

∴ Rank of $A = 2 <$ the number of variables which is 3.

∴ The system will possess a non-zero solution, i.e.,

$$\begin{aligned} 2x + 3y - z &= 0 \\ 3x - 2y + 5z &= 0 \end{aligned}$$

$$\frac{x}{15-2} = \frac{y}{-3-10} = \frac{z}{-4-9} = k \text{ (say)}$$

$\Rightarrow \quad x = 13k, y = -13k$ and $z = -13k$

Let $k = 1 \Rightarrow x = 13, y = -13, z = -13$

∴ There exists a non-zero solution such that $x, y, z \,\varepsilon\, R$

$x(2, 3, 9) + y(3, -2, -6) + z(-1, 5, 15) = (0, 0, 0)$

∴ The set of given vectors are linearly dependent.

Example 5

How many solutions are there for the system of linear equations $x + 2y + z = 0$, $3x + 2y - z = 0$ and $4x + y - 3z = 0$?

Solution

Determinant of the co-efficient matrix of the given equations

is $\begin{vmatrix} 1 & 2 & 1 \\ 3 & 2 & -1 \\ 4 & 1 & -3 \end{vmatrix}$

$$= 1(-6 + 1) - 2(-9 + 4) + 1(3 - 8) = 0$$

∴ The system has infinite number of solutions.

Example 6

Solve the system of equations

$x_1 + x_2 + x_3 = 1$, $3x_1 + x_2 - 3x_3 = 5$ and $x_1 - 2x_2 - 5x_3 = 10$ by LU decomposition method.

Solution

$$AX = B \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -3 \\ 1 & -2 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 10 \end{bmatrix}$$

Step 1: $LU = A$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -3 \\ 1 & -2 & -5 \end{bmatrix}$$

Expanding and on solving we get, $u_{11} = 1$, $u_{12} = 1$, $u_{13} = 1$,

$u_{22} = -2$, $u_{23} = -6$, $u_{33} = 3$, $l_{21} = 3$, $l_{31} = 1$, $l_{32} = \dfrac{3}{2}$

Step 2: Now $LUX = B$

Step 3: Let $UX = Y$

Step 4: $\therefore$ $LY = B$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & \frac{3}{2} & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 10 \end{bmatrix}$$

On solving, $y_1 = 1$, $y_2 = 2$ and $y_3 = 6$.

Step 5: $UX = Y$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -6 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$$

On solving we get $x_1 = 6$, $x_2 = -7$ and $x_3 = 2$

$\therefore$ The solution is (6, –7, 2).

Example 7

Solve: $x + y + z = 6$, $3x - 2y - z = -4$ and $2x + 3y - 2z = 2$.

Solution

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & -1 \\ 2 & 3 & -2 \end{vmatrix} = 1(7) - 1(-4) + 1(9+4) \neq 0$$

$\therefore$ The set of given equations are non-homogeneous and the number of equations is equal to the number of variables.

$\therefore$ The given system of equations is consistent and has a unique solution.

Augmented matrix,

$$[AB] \text{ is } \begin{bmatrix} 1 & 1 & 1 & 6 \\ 3 & -2 & -1 & -4 \\ 2 & 3 & -2 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$, and $R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -5 & -4 & -22 \\ 0 & 1 & -4 & -10 \end{bmatrix}$$

$R_1 \to R_1 + \dfrac{1}{5}R_2$ and $R_3 \to R_3 + \dfrac{1}{5}R_2$

$$\sim \begin{bmatrix} 1 & 0 & \frac{1}{5} & \frac{8}{5} \\ 0 & -5 & -4 & -22 \\ 0 & 0 & -\frac{24}{5} & -\frac{72}{5} \end{bmatrix}$$

$$R_2 \to \frac{-5}{6}R_3 + R_2;\ R_3 \to \frac{-5}{24}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{1}{5} & \frac{8}{5} \\ 0 & -5 & 0 & -10 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$R_1 \to -\frac{1}{5}R_3 \text{ and } R_2 \to \frac{-1}{5}R_2$$

$$R_2 \to -\frac{1}{5}R_2 \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$\therefore$ Solution is $x = 1$, $y = 2$ and $z = 3$.

Example 8

Solve $3x + 2y - z = 0$, $4x + y + 2z = 0$ and $x - 5y + 7z = 0$.

Solution

Determinant of the co-efficient matrix of the equations when written in matrix form is $\begin{vmatrix} 3 & 2 & -1 \\ 4 & 1 & 2 \\ 1 & -5 & 7 \end{vmatrix}$

$$= 3(7 + 10) - 2(28 - 2) - 1(-20 - 1)$$
$$= 51 - 52 + 21 = 20$$

$\therefore$ The given system of equations have only one solution, i.e., $x = y = z = 0$.

Example 9

Determine the eigen values and eigen vectors of $A = \begin{pmatrix} 2 & 4 \\ 3 & 3 \end{pmatrix}$.

Solution

Characteristic equation of the given matrix is $|A - \lambda| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 4 \\ 3 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda^2 - 5\lambda - 6 = 0$$
$$(\lambda - 6)(\lambda + 1) = 0$$

$\Rightarrow$ $\lambda = -1$ and $\lambda = 6$ are the eigen values. Eigen vector corresponding to $\lambda = -1$ is obtained as follows:

$$\left[\begin{pmatrix} 2 & 4 \\ 3 & 3 \end{pmatrix} + 1\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3 & 4 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad 3x_1 + 4x_2 = 0$$

$$3x_1 + 4x_2 = 0 \Rightarrow x_1 = -\frac{4}{3}x_2$$

$\therefore$ Eigen vector corresponding to $\lambda = -1$ is,

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \dfrac{-4}{3}x_2 \\ x_2 \end{pmatrix} = x_2\begin{pmatrix} \dfrac{-4}{3} \\ 1 \end{pmatrix}$$

Similarly eigen vector corresponding to $\lambda = 6$ is obtained as follows:

$$\left[\begin{pmatrix} 2 & 4 \\ 3 & 3 \end{pmatrix} - 6\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -4 & 4 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad -4x_1 + 4x_2 = 0 \text{ and } 3x_1 - 3x_2 = 0$$
$$\Rightarrow \quad x_1 = x_2$$

Eigen vector corresponding to $\lambda = 6$ is,

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_2 \end{pmatrix} = x_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Example 10

Find the eigen values of the matrix

$$A = \begin{pmatrix} 6 & 2 & 2 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix}.$$

Solution

Characteristic equation of the given matrix is $|A - \lambda| = 0$

$$\Rightarrow \begin{vmatrix} 6-\lambda & 2 & 2 \\ 2 & 3-\lambda & 1 \\ 2 & 1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$$
$$\lambda = 2, 2, 8$$

$\therefore$ Eigen values are 2, 2, 8.

Example 11

If $A = \begin{bmatrix} 4 & 2 \\ -7 & -4 \end{bmatrix}$, then find A^{16} by using Cayley–Hamilton theorem.

Solution

The characteristic equation of

$$A = \begin{bmatrix} 4 & 2 \\ -7 & -4 \end{bmatrix} \text{ is } |A - \lambda I| = 0$$

$$\Rightarrow \begin{bmatrix} 4-\lambda & 2 \\ -7 & -4-\lambda \end{bmatrix} = 0$$

$$\Rightarrow \quad (4 - \lambda)(-4 - \lambda) + 14 = 0$$
$$\Rightarrow \quad -16 - 4\lambda + 4\lambda + \lambda^2 + 14 = 0$$
$$\Rightarrow \quad \lambda^2 - 2 = 0 \tag{1}$$

By Cayley–Hamilton theorem, the matrix A satisfies its characteristic equation (1).

$\therefore A^2 - 2I = O$

where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\Rightarrow \quad A^2 = 2I \tag{2}$$

Now $A^{16} = (A^2)^8 = (2I)^8$ (From Eq. (2))

$$= 2^8 I^8 = 256I = 256\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad A^{16} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$$

Example 12

If $A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 4 & -5 \\ 0 & 1 & 0 \end{bmatrix}$; then find the value of the matrix polynomial $3A^9 - 18A^8 + 39A^7 - 32A^6 + 12A^5 - 26A^4 + 16A^3 + 24A^2 - 50A + 40I$.

Solution

The characteristic equation of

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 4 & -5 \\ 0 & 1 & 0 \end{bmatrix} \text{ is } |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 0 & 3 \\ 0 & 4-\lambda & -5 \\ 0 & 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (2-\lambda)\{(4-\lambda)(-\lambda)+5\}=0$

$\Rightarrow \quad (2-\lambda)\{(\lambda^2-4\lambda+5)\}=0$

$\Rightarrow 2\lambda^2-8\lambda+10-\lambda^3+4\lambda^2-5\lambda=0.$

$\Rightarrow \quad -\lambda^3+6\lambda^2-13\lambda+10=0$

$\Rightarrow \quad \lambda^3-6\lambda^2+13\lambda-10=0 \qquad (1)$

By Cayley–Hamilton theorem, the matrix A will satisfy its characteristic Eq. (1)

$\therefore A^3-6A^2+13A-10I=O,$

where $I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ and $O=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

$\therefore \quad A^3-6A^2+13A-10I=0 \qquad (2)$

Now consider the given matrix polynomial

$3A^9-18A^8+39A^7-32A^6+12A^5-26A^4+16A^3+24A^2-50A+40I$

$=3A^9-18A^8+39A^7-30A^6-2A^6+12A^5-26A^4+20A^3-4A^3+24A^2-52A+2A+40I$

$=3A^6(A^3-6A^2+13A-10I)-2A^3(A^3-6A^2+13A-10I)-4(A^3-6A^2+13A-10I)+2A$

$=3A^6\times 0-2A^3\times 0-4\times 0+2A$

(From Eq. (2))

$$=2A=2\begin{bmatrix}2&0&3\\0&4&-5\\0&1&0\end{bmatrix}=\begin{bmatrix}4&0&6\\0&8&-10\\0&2&0\end{bmatrix}.$$

Exercises

1. Which of the following is false?
(A) Every diagonal matrix is a square matrix.
(B) Every unit matrix is a scalar matrix.
(C) Every square matrix is a diagonal matrix.
(D) Every scalar matrix is a diagonal matrix.

2. If the trace of the matrix $\begin{pmatrix}1&a_{12}&\cdots&a_{1n}\\a_{21}&2&\cdots&a_{2n}\\\vdots&\vdots&\vdots&\vdots\\a_{n1}&a_{n2}&\cdots&n\end{pmatrix}$ is 55 then the value of n is
(A) 10 (B) 11
(C) 9 (D) Cannot be determined

3. Which of the following statement is/are false?
(A) $A^T\cdot B^T$ always defined for square matrices of same order.
(B) $A^T\cdot B$ is defined for matrices of the same order.
(C) $t_r(A^T)+t_r(B^T)$ is always defined for matrices A, B of same order.
(D) A^T+B^T is always defined for matrices A, B of same order.

4. Consider the following statements about two square matrices A and B of the same order:
P: $(A+B)^2=A^2+2AB+B^2$
Q: $(A+B)(A-B)=A^2-B^2$
Then,
(A) both P and Q are true.
(B) both P and Q are false
(C) both P and Q are true if A and B commute
(D) P is true but Q is false.

5. If $\begin{pmatrix}2&1&2\\1&0&1\\2&2&1\end{pmatrix}\begin{pmatrix}-2x&3&x\\x&-2&0\\2&-2x&-x\end{pmatrix}=I_{3\times3}$, then $x=$
(A) -1 (B) 1
(C) $\frac{1}{2}$ (D) 2

6. If $\Delta=\begin{vmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{vmatrix}$, then which of the following is true? (Here, A_{ij} is the cofactor of the element a_{ij})
(A) $a_{11}A_{11}+a_{21}A_{12}+a_{23}A_{32}=\Delta$
(B) $a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}=\Delta$
(C) $a_{21}A_{12}+a_{23}A_{32}+a_{12}A_{21}=\Delta$
(D) $a_{12}A_{21}+a_{21}A_{12}+a_{31}A_{13}=\Delta$

7. The determinant value of $\begin{pmatrix}2&3&-3\\1&-2&2\\7&4&-4\end{pmatrix}$ is
(A) 0 (B) 10
(C) -10 (D) 15

8. The value of $\begin{vmatrix}n!&(n+1)!&(n+2)!\\(n+1)!&(n+2)!&(n+3)!\\(n+2)!&(n+3)!&(n+4)!\end{vmatrix}$ is
(A) $2n!\,(n+1)!$
(B) $2n!\,(n+1)!\,(n+2)!$
(C) $(2n)!\,(n+1)!\,(n+2)!$
(D) $2n!\,(n+3)!$

9. If $f(x)=\begin{vmatrix}{}^xC_0&{}^xC_1&{}^{x+1}C_1\\2{}^xC_1&2{}^xC_2&2{}^{(x+1)}C_2\\6{}^xC_2&6{}^xC_3&6{}^{(x+1)}C_3\end{vmatrix}$, then $f(200)$ is
(A) 200 (B) -200
(C) 0 (D) -2001

10. The determinant $\begin{vmatrix}2&3+i&-1\\3-i&0&-1+i\\-1&-1-i&1\end{vmatrix}$ is
(A) purely imaginary (B) zero
(C) real (D) 10

11. If $A = \begin{pmatrix} x & y & z \\ 2x & y & 3z \\ \frac{x}{2} & \frac{y}{2} & \frac{z}{2} \end{pmatrix}$, then $|A| =$ ______.

(A) $10xyz$ (B) 1

(C) 0 (D) $\frac{1}{2}(x^3 + y^3 + z^3 - 3xyz)$

12. If the elements of a row or column of a given square matrix is multiplied by 2, then the value of determinant is ______ times the original determinant.

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) 4

13. If A is a square matrix of order k and det(kA) = 27 det(A), then $k =$ ______.

(A) 9 (B) 1

(C) 2 (D) 3

14. If A and B are two square matrices of order 4 such that $|A| = -2$ and $|B| = 5$, then $|4AB|$ is

(A) -80 (B) -160

(C) -2560 (D) -256

15. I. $(a - b)$, $(b - c)$, $(c - a)$ are factors of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$.

II. If the elements of a determinant are functions of x and its two rows or columns become identical (i.e., determinant equals zero,) when we substitute $x = k$, then $(x - k)$ is a factor of the determinant.

Which of the following is correct?

(A) Both I and II true. (B) Both I and II false.

(C) I is true, II is false (D) I is false, II is true

16. A lower triangular matrix $A = (a_{ij})_{n \times n}$ is singular if and only if

(A) $a_{ii} = 0$ for all $i = 1, 2, \ldots n$

(B) $a_{ii} = 0$ for atleast one $i = 1, 2, \ldots n$

(C) $a_{ii} \neq 0$ for all $i = 1, 2, \ldots n$

(D) $a_{ii} \neq 0$ for atleast one i, $i = 1, 2, \ldots n$

17. Inverse of the matrix $\begin{pmatrix} 2 & -1 & 0 \\ 1 & 2 & 3 \\ -4 & 1 & -1 \end{pmatrix}$ is

(A) $\begin{pmatrix} -5 & -11 & 9 \\ -1 & -2 & 2 \\ -3 & -6 & 5 \end{pmatrix}$ (B) $\begin{pmatrix} -5 & -1 & -3 \\ -11 & -2 & -6 \\ 9 & 2 & 5 \end{pmatrix}$

(C) $\begin{pmatrix} 5 & 11 & 9 \\ 1 & 2 & -2 \\ 3 & 6 & -5 \end{pmatrix}$ (D) $\begin{pmatrix} 5 & 1 & 3 \\ 11 & 2 & 6 \\ 9 & -2 & -5 \end{pmatrix}$

18. I. If $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$, then A^{-1} is symmetric.

II. If a non-singular matrix A is symmetric, then A^{-1} is also symmetric.

Which of the following is correct?

(A) Both I and II true. (B) Both I and II false.

(C) I is true, II is false. (D) I is false, II is true.

19. A is a third order matrix. If the value of the square of the determinant of the matrix of co-factors of A is 28561, then $|A|$ equals

(A) 25 (B) ± 13

(C) 120 (D) ± 169

20. If A is a square matrix of order 3, then the product of A and its transpose is

(A) unit matrix. (B) zero matrix.

(C) identity matrix. (D) symmetric matrix.

21. If A and B are two skew symmetric matrices of the same order then AB is skew symmetric if and only if

(A) $AB + BA = O$ (B) $AB - BA = O$

(C) $AB + BA = I$ (D) $AB - BA = I$

22. Rank of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ is

(A) 1 (B) 2

(C) 3 (D) 4

23. The rank of the matrix $\begin{pmatrix} 2 & -1 & -3 \\ -4 & 2 & 6 \\ -10 & 5 & 15 \end{pmatrix}$ is

(A) 0 (B) 1

(C) 2 (D) 3

24. If $A = (1 \;\; 2 \;\; 3)$ and $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ then $\rho(AB)$ is

(A) 0 (B) 1

(C) 2 (D) 4

25. Which of the following matrix is row echelon form?

(A) $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$

26. Which of the following set of vectors are linearly dependent?

(A) (2, 3, 3), (3, −1, 3), (4, −2, 5)
(B) (3, 4, −1), (−1, 3, 1), (−2, −7, −2)
(C) (2, 1, 4), (1, −2, 2), (−3, 1, −6)
(D) (1, 3, −5), (−5, −1, 3), (4, −2, −2)

27. The system of equations $2x - y + 3z = 9$, $x + y + z = 0$ and $x - y + z = 0$ has/is

(A) unique solution.
(B) infinite solutions.
(C) only zero solution.
(D) inconsistent.

28. The system of equations $6x + 7y + 8z = 1$, $13x + 14y + 15z = 2$ and $x + 2y + 3z = 2$ is

(A) consistent with unique solution.
(B) consistent with infinite solutions.
(C) inconsistent.
(D) None of these

29. The value of λ for which the following system of equation does not have a solution is

$x + y + z = 6$
$4x + \lambda y - \lambda z = 0$
$3x + 2y - 4z = -8$

(A) 3 (B) −3
(C) 0 (D) 1

30. If the number of variables in the linear homogeneous system $AX = O$ is n, then the system will have exactly one solution $X = O$, if the rank of the matrix A is

(A) 1 (B) $< n$
(C) $\leq n$ (D) n

31. If the equations $2x - y - z = 0$, $kx - 3y + 2z = 0$ and $-3x + 2y + kz = 0$ have a non-zero solution, then the value of k is

(A) 2 (B) 1
(C) 7 (D) Both 1 and 7

32. The system of equations $\alpha + 3y + 5z = 0$, $2x - 4\alpha y + \alpha z = 0$, $-4x + 18y + 7z = 0$ has only trivial solution if α is

(A) −1 or −3 (B) 1 or −3
(C) not equal to 1, −3 (D) not equal to −1 and 3

33. The eigen values of $\begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$ is

(A) 0, 0, 0 (B) 0, 1, 0
(C) 2, 1, 3 (D) −2, −1, −3

34. The characteristic roots of the inverse of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are

(A) −1, −1, 5 (B) 1, 1, 5
(C) $1, 1, \frac{1}{5}$ (D) $-1, -1, \frac{1}{5}$

35. The sum and product of the eigen values of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 4 & -2 \\ 1 & 3 & -5 \end{bmatrix}$ is respectively

(A) 0, 24 (B) 1, −24
(C) 2, 20 (D) 4, −24

36. The eigen values of a matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & p \\ 1 & 0 & q \end{bmatrix}$ are 1, 2, and 3. Then the values of p and q are _______.

(A) $p = 0, q = 0$
(B) $p =$ any real number, $q = 2$
(C) $p = 2, q = 0$
(D) $p = 2, q = 2$

37. The eigen values of the matrix $\begin{bmatrix} 0 & -1 & 2 & -3 \\ 1 & 0 & 4 & 6 \\ -2 & -4 & 0 & 5 \\ 3 & -6 & -5 & 0 \end{bmatrix}$ is

(A) real only (B) imaginary
(C) zero only (D) imaginary or zero

38. The number of linearly independent eigen vectors of $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$ is _______.

(A) 0 (B) 1 (C) 2 (D) infinite

39. Which of the following is an eigen vector for the matrix $\begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix}$?

(A) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (B) $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

(C) $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ (D) $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$

40. For a matrix $A = \begin{bmatrix} 6 & -6 & 2 \\ -6 & 5 & -4 \\ 2 & -4 & 1 \end{bmatrix}$, $X = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$ is an eigen vector. The corresponding eigen value is _______.

(A) −2 (B) 1
(C) 2 (D) 13

41. Let A be a 2×2 square matrix with $\lambda_1 = -2$ and $\lambda_2 = -3$ as its eigen values and $x_1 = \begin{bmatrix} -4 \\ -4 \end{bmatrix}$, $x^2 = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$ as its eigen vectors then A is given by

(A) $\begin{bmatrix} 0 & 2 \\ 4 & -5 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & -6 \\ 7 & -9 \end{bmatrix}$

(C) $\begin{bmatrix} -2 & 6 \\ 7 & -3 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 6 \\ -4 & 3 \end{bmatrix}$

42. Consider the matrix $A = \begin{bmatrix} 2 & 5 & 4 \\ 0 & 1 & 0 \\ 0 & -3 & -2 \end{bmatrix}$ let $B = A^{-1}$, then $B =$ ______

(A) $\frac{-1}{4}[A^2 - A - 4I]$ (B) $\frac{1}{4}[A^2 - A - 4I]$

(C) $\frac{1}{4}[A^2 + A - 4I]$ (D) $\frac{-1}{4}[A^2 - A + 4I]$

43. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$, then $A^{15} =$

(A) $8^{14}A$ (B) $8^{15}A$

(C) $8^{16}A$ (D) $15A$

44. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 6 & 7 \\ 9 & 0 & 1 \end{bmatrix}$, then the value of the matrix polynomial $2A^{10} - 18A^9 + 40A^8 - 25A^7 + 9A^6 - 20A^5 + 13A^4 - 9A^3 + 20A^2 - 10A$ is ______.

(A) $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 6 & 7 \\ 9 & 7 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 0 & 0 \\ 6 & 12 & 14 \\ 18 & 0 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

45. For the matrix $A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ 1 & 1 & -1 \end{bmatrix}$, consider the following statements

(P) The characteristic equation of A is $\lambda^3 - 5\lambda^2 + 4\lambda = 0$

(Q) A^{-1} exists

(R) The matrix A is diagonalizable

Which of the above statements are TRUE?

(A) P, Q and R

(B) P and R but not Q

(C) P and Q but not R

(D) Q and R but not P

46. If P is a modal matrix and D is a spectral matrix of a diagonalizable matrix A, then which of the following relations is NOT TRUE among A, P and D?

(A) $PD = AP$ (B) $DP^{-1} = P^{-1}A$

(C) $A^2P = PD^2$ (D) $DP = PA$

47. If A is a 3×3 square matrix with eigen values 0, 2, 3 with P as its modal matrix, then the eigen values of the matrix $P^{-1}AP$ are ______.

(A) 0, 2, 3

(B) 0, 4, 6

(C) $0, \frac{1}{2}, \frac{1}{3}$

(D) $1, \frac{1}{2}, \frac{1}{3}$

Previous Years' Questions

1. For what value of α and β, the following simultaneous equations have an infinite number of solutions?
$x + y + z = 5$; $x + 3y + 3z = 9$; $x + zy + az = b$
[GATE, 2007]

(A) 2, 7 (B) 3, 8

(C) 8, 3 (D) 7, 2

2. The product of matrices $(PQ)^{-1}P$ is **[GATE, 2008]**

(A) P^{-1} (B) Q^{-1}

(C) $P^{-1}Q^{-1}P$ (D) PQP^{-1}

3. The following simultaneous equations

$x + y + z = 3$

$x + 2y + 3z = 4$

$x + 4y + kz = 6$

will NOT have a unique solution for k equal to
[GATE, 2008]

(A) 0 (B) 5

(C) 6 (D) 7

4. A square matrix B is skew-symmetric if
[GATE, 2009]

(A) $B^T = -B$ (B) $B^T = B$

(C) $B^{-1} = B$ (D) $B^{-1} = BT$

5. The inverse of the matrix $\begin{bmatrix} 3+2i & i \\ -i & 3-2i \end{bmatrix}$ is
[GATE, 2010]

(A) $\frac{1}{12}\begin{bmatrix} 3+2i & -i \\ -i & 3-2i \end{bmatrix}$

(B) $\frac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

(C) $\frac{1}{14}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$

(D) $\frac{1}{14}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

6. $[A]$ is a square matrix which is neither symmetric nor skew-symmetric and $[A]^T$ is its transpose. The sum and difference of these matrices are defined as $[S] = [A] + [A]^T$ and $[D] = [A] - [A]^T$, respectively. Which of the following statements is TRUE? **[GATE, 2011]**

(A) Both [S] and [D] are symmetric.
(B) Both [S] and [D] are skew-symmetric.
(C) [S] is skew-symmetric and [D] is symmetric.
(D) [S] is symmetric and [D] is skew-symmetric.

7. The eigen vales of matrix $\begin{bmatrix} 9 & 5 \\ 5 & 8 \end{bmatrix}$ are **[GATE, 2012]**

(A) –2.42 and 6.86 (B) 3.48 and 13.53
(C) 4.70 and 6.86 (D) 6.86 and 9.50

8. What is the minimum number of multiplications involved in computing the matrix product PQR? Matrix P has 4 rows and 2 columns, matrix Q has 2 rows and 4 columns, a matrix R has 4 rows and 1 column? **[GATE, 2013]**

9. Given the matrices $J = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}$ and $K = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ the product $K^T JK$ is. **[GATE, 2014]**

10. The determinant of matrix $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$ is. **[GATE, 2014]**

11. The rank of the matrix $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$ is. **[GATE, 2014]**

12. The sum of Eigen values of the matrix [M] is.

Where $[M] = \begin{bmatrix} 215 & 650 & 795 \\ 655 & 150 & 835 \\ 485 & 355 & 550 \end{bmatrix}$ **[GATE, 2014]**

(A) 915 (B) 1355
(C) 1640 (D) 2180

13. Let $A = [a_{ij}]$, $1 \le i, j \le n$ with $n \ge 3$ and $a_{ij} = i \cdot j$. The rank of A is **[GATE, 2015]**

(A) 0 (B) 1
(C) $n - 1$ (D) n

14. For what value of p the following set of equations will have no solution?
$2x + 3y = 5$
$3x + py = 10$
[GATE, 2015]

15. The smallest and largest eigen values of the following matrix are: $\begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix}$ **[GATE, 2015]**

(A) 1.5 and 2.5 (B) 0.5 and 2.5
(C) 1.0 and 3.0 (D) 1.0 and 2.0

16. The two eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & p \end{bmatrix}$ have a ratio of 3 : 1 for $p = 2$. What is another value of p for which the eigen values have the same ratio of 3 : 1? **[GATE, 2015]**

(A) –2 (B) 1
(C) $\frac{7}{3}$ (D) $\frac{14}{3}$

17. Consider the following linear systems:
$x + 2y - 3z = a$
$2x + 3y + 3z = b$
$5x + 9y - 6z = c$
This system is consistent if a, b and c satisfy the equation **[GATE, 2016]**

(A) $7a - b - c = 0$ (B) $3a + b - c = 0$
(C) $3a - b + c = 0$ (D) $7a - b + c = 0$

18. If the entries in each column of a square matrix M add up to 1, then an eigen value of M is **[GATE, 2016]**

(A) 4 (B) 3
(C) 2 (D) 1

Answer Keys

Exercises

1. C	**2.** A	**3.** C	**4.** C	**5.** B	**6.** B	**7.** A	**8.** B	**9.** C	**10.** C
11. C	**12.** C	**13.** D	**14.** C	**15.** A	**16.** B	**17.** B	**18.** D	**19.** B	**20.** D
21. A	**22.** B	**23.** B	**24.** B	**25.** A	**26.** C	**27.** A	**28.** C	**29.** A	**30.** D
31. D	**32.** C	**33.** C	**34.** C	**35.** B	**36.** B	**37.** B	**38.** B	**39.** B	**40.** D
41. B	**42.** A	**43.** A	**44.** B	**45.** B	**46.** D	**47.** A			

Previous Years' Questions

1. A	**2.** B	**3.** D	**4.** A	**5.** B	**6.** D	**7.** B	**8.** 16	**9.** 23	**10.** 88
11. 2	**12.** A	**13.** B	**14.** 4.49 to 4.51		**15.** D	**16.** D	**17.** B	**18.** D	

Chapter 5

Probability and Statistics

CHAPTER HIGHLIGHTS

- *Probability*
- *Some special continuous distributions*
- *Statistics*
- *Hypothesis testing*

Probability

The word PROBABILITY is used, in a general sense, to indicate a vague possibility that something might happen. It is also used synonymously with chance.

Random Experiment

If the result of an experiment conducted any number of times under essentially identical conditions, is not certain but is any one of the several possible outcomes, the experiment is called a trial or a random experiment. Each of the out comes is known as an event.

Examples:

1. Drawing 3 cards from a well shuffled pack is a random experiment while getting an Ace or a King are events.
2. Throwing a fair die is a random experiment while getting the score as '2' or an odd number' are events.

Mutually Exclusive Events If the happening of any one of the events in a trial excludes or prevents the happening of all others, then such events are said to be mutually exclusive.

Example: The events of getting a head and that of getting a tail when a fair coin is tossed are mutually exclusive.

Equally Likely Events Two events are said to be equally likely when chance of occurrence of one event is equal to that of the other.

Example: When a die is thrown, any number from 1 to 6 may be got. In this trial, getting any one of these events are equally likely.

Independent Events Two events E_1 and E_2 are said to be independent, if the occurrence of the event E_2 is not affected by the occurrence or non-occurrence of the event E_1.

Example: Two drawings of one ball each time are made from a bag containing balls.

Here, we have two events drawing a ball first time (E_1) and drawing a ball second time (E_2). If the ball of the first draw is replaced in the bag before the second draw is made, then the outcome of E_2 does not depend on the outcome of E_1. In this case E_1 and E_2 are Independent events.

If the ball of the first draw is not replaced in the bag before the second draw is made, then the outcome of E_2 depends on the outcome of E_1. In this case, events E_1 and E_2 are Dependent events.

Compound Events When two or more events are in relation with each other, they are known as compound events.

Example: When a die is thrown two times, the event of getting 3 in the first throw and 5 in the second throw is a compound event.

Definition of Probability

If an event E can happen in m ways and fail in k ways out of a total of n ways and each of them is equally likely, then the probability of happening E is $\dfrac{m}{(m+k)} = \dfrac{m}{n}$ where $n = (m+k)$.

In other words, if a random experiment is conducted n times and m of them are favourable to event E, then the

probability of happening of E is $P(E) = \frac{m}{n}$. Since the event does not occur $(n - m)$ times, the probability of non-occurrence of E is $P(\bar{E})$.

$$P(\bar{E}) = \frac{n-m}{n} = -\frac{m}{n}n = 1 - P(E)$$

Therefore, $P(E) + P(\bar{E}) = 1$.

NOTES

1. Probability $[P(E)]$ of the happening of an event E is known as the probability of success and the probability $[P(\bar{E})]$ of the non-happening of the event is the probability of failure.
2. If $P(E) = 1$, the event is called a certain event and if $P(E) = 0$ the event is called an impossible event.
3. Instead of saying that the chance of happening of an event is $\frac{m}{n}$, we can also say that the odds in favour of the event are m to $(n - m)$ or the odds against the event are $(n - m)$ to m.

Addition Theorem of Probability

If A and B are two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap \mathrm{B})$$

This result follows from the corresponding result in set theory. If $n(X)$ represents the number of elements in set X, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$.

Example: If a die is rolled, what is the probability that the number that comes up is either even or prime?

A = The event of getting an even number = $\{2, 4, 6\}$

B = The event of getting a prime = $\{2, 3, 5\}$

$A \cup B = \{2, 3, 4, 5, 6\}$

$A \cap B = \{2\}$

$P(A) = \frac{3}{6}$, $P(B) = \frac{3}{6}$, $P(A \cup B) = \frac{5}{6}$ and $P(A \cap B) = \frac{1}{6}$. We can verify that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

SOLVED EXAMPLES

Example 1

When a cubical dice is rolled, find the probability of getting an even integer.

Solution

When a dice is rolled, the number of possible out comes is 6. The number of favourable outcomes of getting an even integer is 3.

The required probability $= \frac{3}{6} = \frac{1}{2}$.

Example 2

If a card is drawn from a pack of cards, find the probability of getting a queen.

Solution

When a card is drawn, the number of possible outcomes is 52. The number of favourable outcomes of getting a queen card is 4.

The required probability $= \frac{4}{52} = \frac{1}{13}$.

Example 3

A bag contains 5 green balls and 4 red balls. If 3 balls are picked from it at random, then find the odds against the three balls being red.

Solution

The total number of balls in the bag = 9. Three balls can be selected from 9 balls in ${}^9C_3$ ways.

Three red balls can be selected from 4 red balls in ${}^4C_3$ ways.

Probability of picking three red balls

$$= \frac{{}^4C_3}{{}^9C_3} = \frac{4}{84} = \frac{1}{21};\ P(\bar{E}) = \frac{20}{21}$$

Odds against the three balls being red are $= P(\bar{E}) : P(E) = \frac{20}{21} : \frac{1}{21} = 20:1$.

Example 4

When two dice are rolled together, find the probability of getting at least one 4.

Solution

Let E be the event that at least one dice shows 4. $\bar{E}$ be the event that no dice shows 4. The number of favourable outcomes of $\bar{E}$ is $5 \times 5 = 25 \cdot P(\bar{E}) = \frac{25}{36}$

$$\therefore\ P(E) = 1 - P(\bar{E}) = 1 - \frac{25}{36} = \frac{11}{36}.$$

Example 5

When two dice are rolled together find the probability that total score on the two dice will be 8 or 9.

Solution

When two dice are rolled, the total number of outcomes $= 6 \times 6 = 36$.

Favourable outcomes for getting the sum 8 or 9 are {(2, 6), (6, 2), (3, 5), (5, 3), (4, 4), (3, 6), (6, 3), (4, 5), (5, 4)}, i.e., the total number of favourable outcomes = 9.

The required probability $= \frac{9}{36} = \frac{1}{4}$.

Example 6

If two cards are drawn simultaneously from a pack of cards, what is the probability that both will be jacks or both are queens?

Solution

Here two events are mutually exclusive, $P(J \cup Q) = P(J) + P(Q)$. Probability of drawing two jacks is $P(J) = \frac{{}^4C_2}{{}^{52}C_2}$

Probability of drawing two queens is $P(Q) = \frac{{}^4C_2}{{}^{52}C_2}$

$$P(J \cup Q) = P(J) + P(Q)$$

$$= \frac{{}^4C_2}{{}^{52}C_2} + \frac{{}^4C_2}{{}^{52}C_2} = 2 \cdot \frac{{}^4C_2}{{}^{52}C_2} = \frac{2}{221}.$$

Example 7

When two cards are drawn from a pack of cards, find the probability that the two cards will be kings or blacks.

Solution

The probability of drawing two kings $= \frac{{}^4C_2}{{}^{52}C_2}$

The probability of drawing two black cards is $= \frac{{}^{26}C_2}{{}^{52}C_2}$

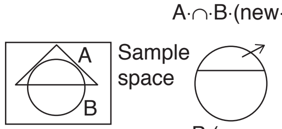
The probability of drawing two black kings is $\frac{{}^2C_2}{{}^{52}C_2}$

$\therefore$ The required probability

$$= \frac{{}^4C_2}{{}^{52}C_2} + \frac{{}^{26}C_2}{{}^{52}C_2} - \frac{{}^2C_2}{{}^{52}C_2} = \frac{55}{221}.$$

Conditional Probability

Let S be a finite sample space of a random experiment and A, B are events, such that $P(A) > 0$, $P(B) > 0$. If it is known that the event B has occurred, in light of this we wish to compute the probability of A, we mean conditional probability of A given B. The occurrence of event B would reduce the sample space to B, and the favourable cases would now be $A \cap B$.



Notation The conditional probability of A given B is denoted by $P\left(\frac{A}{B}\right)$.

$$\therefore \quad P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{P(A \cap B)}{P(B)}.$$

NOTES

1. This definition is also valid for infinite sample spaces.
2. The conditional probability of B given A is denoted by $P\left(\frac{B}{A}\right)$ and $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$.

Multiplication Theorem

Let A and B be two events of certain random experiment such that A occurs only when B has already occurred. Then, for the conditional event $\frac{A}{B}$, the total possible outcomes are the outcomes favourable to the event B and its favourable outcomes are the outcomes favourable to both A and B.

So, $P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$

$$= \frac{n(A \cap B)}{n(S)} \times \frac{n(S)}{n(B)} = P(A \cap B) \times \frac{1}{P(B)}$$

That is, $P\left(\frac{A}{B}\right) \cdot P(B) = P(A \cap B)$

This is called the multiplication theorem on probability.

Example 8

A letter is selected at random from the set of English alphabet and it is found to be a vowel. What is the probability that it is 'e'?

Solution

Let A be the event that the letter selected is 'e' and B be the event that the letter is a vowel. Then, $A \cap B = \{e\}$ and $B = \{a, e, i\ o, u\}$

So, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\left(\frac{1}{26}\right)}{\left(\frac{5}{26}\right)} = \frac{1}{5}.$

Independent Events In a random experiment, if A, B are events such that $P(A) > 0$, $P(B) > 0$ and if $P\left(\frac{A}{B}\right) = P(A)$ or $P\left(\frac{B}{A}\right) = P(B)$ (conditional probability equals to unconditional probability) then we say A, B are independent events. If A, B are independent, $P(A \cap B) = P(A)\,P(B)$.

Example 9

Two coins are tossed one after the other and let A be the event of getting tail on second coin and B be the event of getting head on first coin, then find $P\left(\frac{A}{B}\right)$.

Solution

Sample space = {HH, HT, TH, TT}, A = {HT, TT} and B = {HH, HT}, $(A \cap B)$ = {HT}

$$\therefore\ P(A) = \frac{2}{4} = \frac{1}{2} \quad \text{and} \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Thus $P\left(\frac{A}{B}\right) = P(A)$

$\therefore$ Logically too we understand that occurrence or non-occurrence of tail in 2nd coin.

Baye's Rule

Suppose $A_1, A_2, \ldots, A_n$ are n mutually exclusive and exhaustive events such that $P(A_i) \neq 0$. Then for $i = 1, 2, 3, \ldots, n$,

$$P\left(\frac{A_i}{A}\right) = \frac{P(A_i) \cdot P\left(\frac{A}{A_i}\right)}{\sum_{k=1}^{n} P(A_K) P\left(\frac{A}{A_K}\right)}$$

Where A is an arbitrary event of S.

Example 10

Akshay speaks the truth in 45% of the cases, In a rainy season, on each day there is a 75% chance of raining. On a certain day in the rainy season, Akshay tells his mother that it is raining outside. What is the probability that it is actually raining?

Solution

Let E denote the event that it is raining and A denote the event that Akshay tells his mother that it is raining outside.

Then, $P(E) = \frac{3}{4}$, $P(\bar{E}) = \frac{1}{4}$

$$P\left(\frac{A}{E}\right) = \frac{45}{100} = \frac{9}{20} \text{ and } P\left(\frac{A}{\bar{E}}\right) = \frac{11}{20}$$

By Baye's Rule, we have

$$P\left(\frac{E}{A}\right) = \frac{P(E)P\left(\frac{A}{E}\right)}{P(E)P\left(\frac{A}{E}\right) + P(\bar{E})P\left(\frac{A}{\bar{E}}\right)}$$

$$= \frac{\frac{3}{4} \times \frac{9}{20}}{\frac{3}{4} \times \frac{9}{20} + \frac{1}{4} \times \frac{11}{20}} = \frac{27}{38}.$$

Advanced Probability

Random Variable

A random variable is a real valued function defined over the sample space (discrete or continuous).

A **discrete random variable** takes the values that are finite or countable. For example when we consider the experiment of tossing of 3 coins, the number of heads can be appreciated as a discrete random variable (X). X would take 0, 1, 2 and 3 as possible values.

A continuous random variable takes values in the form of intervals. Also, in the case of a **continuous random variable** $P(X = c) = 0$, where c is a specified point. Heights and weights of people, area of land held by individuals, etc., are examples of continuous random variables.

Probability Mass Function (PMF)

If X is a discrete random variable, which can take the values $x_1, x_2, \ldots$ and $f(x)$ denote the probability that X takes the value x_i, then $p(x)$ is called the Probability Man Function (pmf) of X.

$p(x_i) = P(x = x_i)$. The values that X can take and the corresponding probabilities determine the probability distribution of X. We also have

1. $p(x) \geq 0$;
2. $\sum p(x) = 1$.

Probability Density Function (PDF)

If X is a continuous random variable then a function $f(x)$, $x \in I$ (interval) is called a Probability Density Function. The probability statements are made as $P(x \in I) = \int_I f(x)dx$.

We also have,

1. $f(x) \geq 0$
2. $\int_{-\infty}^{\infty} f(x)dx = 1$

The probability $P(X \leq x)$ is called the cumulative distribution function (CDF) of X and is denoted by $F(X)$. It is a point function. It is defined for discrete and continuous random variables.

The following are the properties of probability distribution function $F(x)$,

1. $F(x) \geq 0$
2. $F(x)$ is non-decreasing i.e., for $x > y$, $F(x) \geq F(y)$
3. $F(x)$ is right continuous
4. $F(-\infty) = 0$ and $F(+\infty) = 1$

Also,

5. $P(a < x \leq b) = F(b) - F(a)$.

For a continuous random variable:

6. $Pr\{x < X \leq x + dx\} = F(x + dx) - F(x) = f(x)\, dx$; where dx is very small
7. $f(x) = \frac{d}{dx}[F(x)]$ where;

   (a) $f(x) \geq 0\ \forall\ x \in R$.

   (b) $\int_R f(x)dx = 1$.

Mathematical Expectation [$E(X)$]

Mathematical Expectation is the weighted mean of values of a variable.

If X is a random variable which can assume any one of the values $x_1, x_2, \ldots, x_n$ with the respective probabilities $p_1, p_2, \ldots, p_n$, then the mathematical expectation of X is given by $E(X) = p_1x_1 + p_2x_2 + \cdots + p_n x_n$

For a continuous random variable,

$$E(X) = \int_{-\infty}^{+\infty} xf(x)dx \text{ where } f(x) \text{ is the PDF of } X.$$

Some Special Discrete Distributions

Discrete Uniform Distribution

A discrete random variable defined for values of x from 1 to n is said to have a uniform distribution if its probability mass function is given by

$$f(x) = \begin{cases} \frac{1}{n}; \text{ for } x = 1, 2, 3, \ldots, n \\ 0, \text{ otherwise} \end{cases}$$

- The cumulative distribution function $F(x)$ of the discrete uniform random variable x is given by
$$F(x) = \begin{cases} 0, \text{ for } x < 1 \\ \frac{x}{n}; \text{ for } 1 \le x \le n \\ 1; \text{ for } x > 1 \end{cases}$$
- Mean of $X = \mu = \dfrac{n+1}{2}$
- Variance of $X = \sigma^2 = \dfrac{n^2-1}{12}$

Binomial Distribution

An experiment which is made of n independent trials, each of which resulting in either 'success' with probability 'p' or 'failure' with probability 'q' ($q = 1 - p$), then the probability distribution for the random variable X when represents the number of success is called a binomial distribution. The probability mass function,

$$p(x) = b(x; n, p) = {}^nC_x\, p^x q^{n-x}; x = 0, 1, 2, \ldots, n$$

Example: Hitting a target in 5 trials. Here the random variable (X) represents the number of trials made for hitting the target, i.e., $x = 0$ or 1 or 2 or 3 or 4 or 5.

We have a set of 5 trials $n = 5$

Each trial may hit the target termed to be success (p) or not termed to be failure (q), which are independent.

$\therefore$ This is an example for Binomial distribution.

Properties of Binomial Distribution

1. $E(X) = np$ (mean)
2. $V(X) = E(X^2) - (E(X))^2 = npq$; (variance)
   (mean > variance)
3. SD $(X) = \sqrt{npq}$
4. Mode of a binomial distribution lies between $(n + 1)p - 1 \le x \le (n + 1)p$
5. If $X_1 \sim b(n_1, p)$ and $X_2 \sim b(n_2, p)$ and if X_1 and X_2 are independent, then . $X_1 + X_2 \sim b(n_1 + n_2, p)$ where (n, p) is the pmf of binomial distribution.

Poisson Distribution A random variable X is said to follow a **Poisson distribution** with parameter λ, $\lambda > 0$, if it assumes only non-negative values and its probability mass function is given by

$$p(x) = p(x; \lambda) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & : x = 0, 1, 2, \ldots \\ & \lambda > 0 \\ 0 & \text{otherwise} \end{cases}$$

In a binomial distribution if n is large compared to p, then np approaches a fixed constant say λ. Such a distribution is called poisson distribution (limiting case of binomial distribution)

Properties of Poisson Distribution

1. $E(X) = \sum_x x \cdot \dfrac{e^{-\lambda}\lambda^x}{x!} = \lambda$
2. $V(X) = E(X^2) - (E(X))^2 = \lambda$
   SD $(X) = \sqrt{\lambda}$
   $\therefore$ Mean $= \lambda =$ Variance
3. Mode of a Poisson distribution lies between $\lambda - 1$ and λ
4. If $X_1 \sim P(\lambda_1)$ and $X_2 \sim P(\lambda_2)$, and X_1, X_2 independent then $X_1 + X_2 \sim P(\lambda_1 + \lambda_2)$.

Some Special Continuous Distributions

Continuous Uniform Distribution or Rectangular Distribution

A continuous random variable x defined on $[a, b]$ is said to have a uniform distribution, if its probability density function is given by

$$F(x) = \begin{cases} \dfrac{1}{b-a}; \text{ for } x \in [a, b] \\ 0; \text{ otherwise} \end{cases}$$

- The cumulative distribution function of the continuous uniform random variable X is given by

$$F(x) = \begin{cases} 0; \text{ if } x \le a \\ \dfrac{x-a}{b-a}; \text{ if } a < x < b \\ 1; \text{ if } x \ge b \end{cases}$$

- Mean of $X = \mu = \dfrac{a+b}{2}$
- Variance of $X = \sigma^2 = \dfrac{(b-a)^2}{12}$.

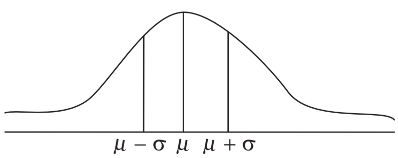
Normal Distribution

A continuous random variable X is said to have **a normal distribution** with parameters μ and σ^2 if its density function is given by the probability density function,

$$f(x) = \begin{cases} \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} & \begin{matrix} -\infty < x < \infty \\ -\infty < \mu < \infty \\ \sigma > 0 \end{matrix} \\ 0 & \text{otherwise} \end{cases}$$

It is denoted as $X \sim N(\mu, \sigma^2)$.

The graphical representation of normal distribution is as given below.



Properties of Normal Distribution

1. The function is symmetrical about the value μ.
2. It has a maximum at $x = \mu$
3. The area under the curve within the interval $(\mu \pm \sigma)$ is 68%.

   That is, $P(\mu - \sigma \le X \le \mu + \sigma) = 0.68$.
4. A fairly large number of samples taken from a 'Normal' population will have average, median and mode nearly the same, and within the limits of average $\pm 2 \times$ SD, there will be 95% of the values.
5. $E(X) = \int_{-\infty}^{+\infty} x \cdot f(x)dx = \mu$.
6. $V(X) = \sigma^2$; S.D $(X) = \sigma$
7. For a normal distribution,

   Mean = Median = Mode
8. All odd order moments about mean vanish for a normal distribution.

   That is, $\mu_{2n+1} = 0 \forall = n = 0, 1, 2, \ldots$
9. If $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$, X_1, X_2 independent, then,

   $X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

   Also, $X_1 - X_2 \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$
10. If $\mu = 0$ and $\sigma^2 = 1$, we call it as standard normal distribution. The standardization can be obtained by the transformation,

$$z = \frac{x-\mu}{\sigma}. \quad \text{Also, } \frac{X-\mu}{\sigma} \sim N(0, 1).$$

Exponential Distribution

A continuous random variable X is said to have an exponential distribution if its probability density function $f(x)$ is given by,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}; \text{ for } x > 0 \\ 0; \text{ otherwise} \end{cases}$$

Here λ is the parameter of the exponential distribution and $\lambda > 0$.

The cumulative distribution function $F(x)$ of an exponential distribution with λ as parameter is

$$F(x) = \begin{cases} 1 - e^{-\lambda x}; \text{ if } x > 0 \\ 0, \text{ otherwise} \end{cases}$$

Mean $= \mu = \dfrac{1}{\lambda}$,

Variance $= \sigma^2 = \dfrac{1}{\lambda^2}$.

Example 11

An unbiased die is thrown at random. What is the expectation of the number on it?

Solution

Let X denotes the number on the die, which can take the values 1, 2, 3, 4, 5 or 6;

Probability of each will be equal to $\dfrac{1}{6}$

X	1	2	3	4	5	6
***P*(*X* = *x*)**	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(X) = \sum_x xP(X = x)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

$$= \frac{1}{6}(1+2+3+4+5+6) = \frac{6 \times 7}{6 \times 2} = \frac{7}{2}$$

$$= 3.5.$$

Example 12

In a city 5 accidents take place in a span of 25 days. Assuming that the number of accidents follows the Poisson distribution, what is the probability that there will be 3 or more accidents in a day? (Given $e^{-0.2} = 0.8187$)

Solution

Average number of accidents per day $= \frac{5}{25} = 0.2$; $\therefore\ \lambda = 0.2$.

Probability (3 or more accidents per day)

$= 1 - P$ (2 or less accidents)

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - [e^{-0.2} + 0.2e^{-0.2} + 0.02e^{-0.2}]$

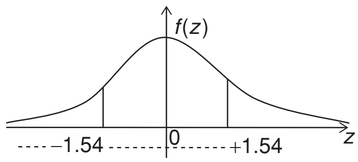
$= 1 - e^{-0.2}[1.22] = 1 - 0.99814 = 0.001186.$

Example 13

What is the area under the normal curve to the left of $Z = -1.54$ (given area between 0 and $-1.54 = 0.4382$)?

Solution

Required area $= 0.5 - 0.4382 = 0.0618$



Example 14

A family consists of five children. If the random variable (X) represents the number of boys in that family then,

1. Find the expected value $E(X)$ of X.
2. Find the variance of X.

Solution

This situation can be modelled as binomial distribution.

$$X \sim b\left(5, \frac{1}{2}\right);\ E(X) = np = 5\times\frac{1}{2} = 2.5$$

$$V(X) = npq = 5\times\frac{1}{2}\times\frac{1}{2} = 1.25$$

Example 15

Ram and Shyam play a game in which their chances of winning are in the ratio 2 : 3. Find Shyam's chance of winning at least 3 games out of five games played.

Solution

$$P(\text{Shyam wins}) = \frac{3}{5};$$

$$P(\text{Shyam loses}) = \frac{2}{5}$$

Let X denote the number of games won by Shyam.

P(Shyam wins at least 3 games) $= P(X \geq 3)$

$$= \sum_{x=3}^{5} {}^5C_x \left(\frac{3}{5}\right)^x \left(\frac{2}{5}\right)^{5-x} = \sum_{x=3}^{5} {}^5C_x \frac{3^x 2^{5-x}}{5^5}$$

$$= \frac{3^3}{5^5}[{}^5C_3 2^2 + {}^5C_4 \times 3\times 2 + 1\times 3^2\times 1]$$

$$= \frac{27\times 79}{3125} = 0.68.$$

Example 16

The PDF of a random variable X is

$$f(x) = \begin{cases} \left(\frac{1}{10}\right)e^{\left(-\frac{x}{10}\right)}; & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

What is $P(X \leq 10)$? (given $e^{-1} = 0.3679$)

Solution

$$P(X \leq 10) = \int_0^{10} f(x)\,dx = \int_0^{10} \frac{1}{10} e^{-\frac{x}{10}}\,dx$$

$$= \frac{1}{10}\left(\frac{e^{-\frac{x}{10}}}{-\frac{1}{10}}\right)_0^{10} = 1 - e^{-1} = 0.6321.$$

Joint Distribution of Random Variables Joint Probability Mass Function

Let X and Y be two discrete random variables on the same sample space S with the range space of X as $R_x = \{x_1, x_2, \ldots, x_m\}$ and the range space of y as, $R_y = \{y_1, y_2, \ldots, y_n\}$ and $P_X(x)$ and $P_y(y)$ as the probability mass functions of x and y. Then the joint probability mass function $P_{xy}(x, y)$ of the two dimensional random variable (x, y) on the range space $R_x \times R_y$ is defined as,

$$P_{XY}(x_i, y_j) = \begin{cases} P(X = x_i,\ Y = y_j), \text{ for } (x_i, y_j) \in R_X \times R_Y \\ 0, \text{ otherwise} \end{cases}$$

This joint probability mass function can be represented in the form of a table as follows:

X \ Y	y_1	y_2	y_3	…	y_n	$\sum_{j=1}^{n} P_{xy}(x_i, y_j)$
x_1	$P_{xy}(x_1, y_1)$	$P_{xy}(x_1, y_2)$	$P_{xy}(x_1, y_3)$	…	$P_{xy}(x_1, y_n)$	$P_x(x_1)$
x_2	$P_{xy}(x_2, y_1)$	$P_{xy}(x_2, y_2)$	$P_{xy}(x_3, y_3)$	…	$P_{xy}(x_3, y_n)$	$P_x(x_2)$
x_3	$P_{xy}(x_3, y_1)$	$P_{xy}(x_3, y_2)$	$P_{xy}(x_3, y_3)$	…	$P_{xy}(x_3, y_n)$	$P_x(x_3)$
⋮	⋮	⋮	⋮		⋮	⋮
x_m	$P_{xy}(x_m, y_1)$	$P_{xy}(x_m, y_2)$	$P_{xy}(x_m, y_3)$	…	$P_{xy}(x_m, y_n)$	$P_x(x_m)$
$\sum_{i=1}^{m} P_{xy}(x_i, y_j)$	$P_y(y_1)$	$P_y(y_2)$	$P_y(y_3)$	…	$P_y(y_n)$	

From the above table, it can be easily observed that the marginal probability mass functions of X and Y namely $P_x(x)$ and $P_Y(y)$ respectively can be obtained from the joint probability mass function $P_{xy}(x, y)$ as

$$P_x(x_i) = \sum_{j=1}^{n} P_{xy}(x_i, y_j), \text{ for } i = 1, 2, \ldots, m$$

And

$$P_y(y_j) = \sum_{j=1}^{m} P_{xy}(x_i, y_j) \text{ for } j = 1, 2, 3, \ldots, n$$

- $P_{xy}(x_i, y_j) \geq 0 \ \forall \ i, j$
- $\sum_{i=1}^{m} \sum_{j=1}^{n} P_{xy}(x_i, y_j) = 1$
- The cumulative joint distribution function of the two dimensional random variable (X, Y) is given by $F_{xy}(x, y) = P(X \leq x, Y \leq y)$.

Joint Probability Density Function

Let X and Y are two continuous random variables on the same sample space S with $f_x(x)$ and $f_y(y)$ as the probability density functions respectively. Then a function $f_{xy}(x, y)$ is called the joint probability density function of the two dimensional random variable (X, Y) if the probability that the point (x, y) will lie in the infinitesimal rectangular region of area $dx\,dy$ is $f_{xy}(x, y)\,dx\,dy$,

That is,

$$P\left(x - \frac{1}{2}dx \leq X \leq x + \frac{1}{2}dx,\ y - \frac{1}{2}dy \leq Y \leq y + \frac{1}{2}dy\right)$$

$= f_{XY}(x, y)\,dx\,dy$

- $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x, y)dxdy = 1$
- The marginal probability density functions $f_X(x)$ and $f_Y(y)$ of the two continuous random variables X and Y are given by,

  $$f_x(x) = \int_{-\infty}^{\infty} f_{XY}(x, y)dy \text{ and } f_y(y) = \int_{-\infty}^{\infty} f_{XY}(x, y)dx$$

- The cumulative joint distribution function $F_{XY}(x, y)$ of the two-dimensional random variable (X, Y) (where X and Y are any two continuous random variables defined on the same sample space) is given by,

  $$F_{XY}(x, y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{XY}(x, y)dxdy.$$

Conditional Probability Functions of Random Variables

Let X and Y be two discrete (continuous) random variables defined on the same sample space with joint probability mass (density) function $f_{XY}(x, y)$, then

1. The conditional probability mass (density) function $f_{\frac{X}{Y}}\left(\frac{x}{y}\right)$ of X, given $Y = y$ is defined as

   $$F_{\frac{X}{Y}}\left(\frac{x}{y}\right) = \frac{f_{XY}(x, y)}{f_Y(y)}, \text{ where } f_Y(y) \neq 0 \text{ and}$$

2. The conditional probability mass (density) function $f_{\frac{Y}{X}}\left(\frac{y}{x}\right)$ of Y, given $X = x$ is defined as $f_{\frac{Y}{X}}\left(\frac{y}{x}\right)$

   $$= \frac{f_{XY}(x, y)}{f_X(x)} \text{ where } f_X(x) \neq 0.$$

Independent Random Variables

Two discrete (continuous) random variables X and Y defined on the same sample space with joint probability mass (density) function $P_{XY}(x, y)$ are said to be independent, if and only if,

$$P_{XY}(x, y) = P_X(x)\,P_Y(y)$$

Where $P_X(x)$ and $P_Y(y)$ are the marginal probability mass (density) functions of the random variables X and Y respectively.

NOTE

If the random variables X and Y are independent then

$$P_{xy}(a \leq X \leq b, c \leq Y \leq d) = P_x(a \leq X \leq b)\,P_y(c \leq Y \leq d)$$

Statistics

Statistics is basically the study of numeric data. It includes methods of collection, classification, presentation, analysis and inference of data. Data as such is qualitative or quantitative in nature. If one speaks of honesty, beauty, colour, etc., the data is qualitative while height, weight, distance, marks, etc are quantitative.

The present course aims to systematically study statistics of quantitative data. The quantitative data can be divided into three categories

1. Individual series
2. Discrete series and
3. Continuous series

Individual Series

Examples:

1. Heights of 8 students

   5.0, 4.9, 4.5, 5.1, 5.3, 4.8, 5.1, 5.3 (in feet)

2. The weight of 10 students

   46, 48, 52, 53.4, 47, 56.8, 52, 59, 55, 52 (in kgs)

Discrete Series

Example:

x : Number of children in a family

f : Number of families

Total number of families = 50

x	0	1	2	3	4
f	8	10	19	8	5

Continuous Series

Example: Total number of students = 50

Class Interval (CI)	Frequency (*f*)
0–10	8
10–20	12
20–30	13
30–40	10
40–50	7

In order to analyze and get insight into the data some mathematical constants are devised. These constants concisely describe any given series of data. Basically we deal with two of these constants,

1. Averages or measures of central tendencies
2. Measures of spread or dispersion

Measures of Central Tendencies These tell us about how the data is clustered or concentrated. They give the central idea about the data. The measures are

1. Arithmetic mean or mean
2. Geometric mean
3. Harmonic mean
4. Median
5. Mode

The first three are mathematical averages and the last two are averages of position.

Measures of Dispersion It is possible that two sets of data may have the same central value, yet they may differ in spread. So there is a need to study about the spread of the data.

The measures we deal with are,

1. Range
2. Quartile deviation or semi inter-quartile range
3. Mean deviation
4. Standard deviation (including variance)

The formulae for each of the above mentioned measures is listed for each of the series in what follows.

Measures of Central Tendencies

Arithmetic Mean (AM or $\bar{x}$)

1. Individual series:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\Sigma x_i}{n}$$

2. Discrete series:

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \cdots + f_n x_n}{f_1 + f_2 + \cdots + f_n} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

where $x_1, x_2, \ldots, x_n$ are n distinct values with frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively.

3. Continuous series:

$$\bar{x} = \frac{f_1 m_1 + f_2 m_2 + \cdots + f_n m_n}{f_1 + f_2 + \cdots + f_n} = \frac{\Sigma f_i m_i}{\Sigma f_i}$$

where $f_1, f_2, f_3, \ldots, f_n$ are the frequencies of the classes whose mid-values are $m_1, m_2, \ldots, m_n$ respectively.

Some Important Results Based on AM

1. The algebraic sum of deviations taken about mean is zero.
2. Its value is based on all items.
3. Mean of first n natural numbers is $\frac{(n+1)}{2}$.
4. Arithmetic mean of two numbers a and b is $\frac{(a+b)}{2}$.
5. If b is AM of a and c then a, b, c are in arithmetic progression.

Combined Mean If x_1 and x_2 are the arithmetic means of two series with n_1 and n_2 observations respectively, the combined mean, $\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$.

Median

If for a value the total frequency above (or below) it is half of the overall total frequency the value is termed as median. Median is the middle-most item.

Individual Series If $x_1, x_2, \ldots, x_n$ are arranged in ascending order of magnitude then the median is the size of $\left(\frac{n+1}{2}\right)$th item.

Some Results Based on Median

1. Median does not take into consideration all the items.
2. The sum of absolute deviations taken about median is least.
3. Median is the abscissa of the point of intersection of the cumulative frequency curves.
4. Median is the best suited measure for open end classes.

Mode The most frequently found item is called mode. Being so, it is easy and straight forward to find for individual and discrete series.

Empirical Formula

- For moderately symmetrical distribution,
- Mode = 3 median – 2 mean
- For a symmetric distribution, Mode = Mean = Median. This formula is to be applied in the absence of sufficient data. Given any two, of the mean, median or mode the third can be found.

Measures of Dispersion

Range

The range of a distribution is the difference between the greatest and the least values observed.

Some Important Results Based on Range

1. Range is a crude measure of dispersion as it is based only on the value of extreme observations.
2. It is also very easy to calculate.
3. It does not depend on the frequency of items.

Quartile Deviation (QD)

$$\text{QD} = \frac{Q_3 - Q_1}{2}$$

Individual Series The numbers are first arranged in ascending or descending order, then we find the quartiles Q_1 and Q_3 as

$Q_1 \rightarrow$ size of $[(n + 1)/4]$th item

$Q_3 \rightarrow$ size of $[3(n + 1)/4]$th item

The first quartile (or the lower quartile) Q_1 is that value of the variable, which is such that one-quarter of the observations lies below it. The third quartile Q_3 is that value of the variable, which is such that three-quarters of the observations lie below it.

Mean Deviation (MD)

It is defined as the arithmetic mean of the deviation from origin, which may be either mean or median or mode.

Individual Series

$$\text{MD} = \frac{|x_1 - A| + |x_2 - A| + \cdots + |x_n - A|}{n}$$

where $x_1, x_2, \ldots, x_n$ are the n observations and A is the mean or median or mode.

Some Results Based on MD

1. Mean deviation depends on all items.
2. By default, mean deviation is to be computed about mean.
3. Mean deviation about the median is the least.
4. Mean deviation of two numbers a and b is $\dfrac{|a-b|}{2}$.

Standard Deviation (SD)

Standard deviation is referred to as root mean squared deviation about the mean.

Individual Series

$$SD(\sigma) = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

where $x_1, x_2, \ldots, x_n$ are n observations with mean as $\bar{x}$.

Alternatively $\sigma = \sqrt{\dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2}$ is a useful formula for computational purpose.

Some Results Based on SD

1. The square of standard deviation is termed as variance.
2. SD is the least mean square deviation.
3. If each item is increased by a fixed constant the SD does not alter or SD is independent of change of origin.
4. Standard deviation depends on each and every data item.
5. For a discrete series in the form $a, a+d, a+2d, \ldots$ (AP), the standard deviation is given by $\text{SD} = d\sqrt{\dfrac{n^2-1}{12}}$, where n is number of terms in the series.

Co-efficient of Variation (CV)

Co-efficient of variation (CV) is defined as, $\text{CV} = \dfrac{\text{SD}}{\text{AM}} \times 100$.

This is a relative measure, which helps in measuring the consistency. Smaller the co-efficient of variation, greater is the consistency.

Example 17

For the individual series, compute the mean, median and mode 8, 11, 14, 17, 20, 23, 26, 29.

Solution

$$\text{Mean } = \bar{x} = \frac{\Sigma x_i}{n} = \frac{8 + 11 + \cdots + 29}{8} = 18.5$$

Median: As the numbers are in ascending order and the number 17 and 20 being middle terms.

$$\text{Median } = \frac{17+20}{2} = \frac{37}{2} = 18.5$$

Mode: As no term can be regarded as 'most often found', mode is not-defined. However using empirical formula,

Mode = 3 median − 2 mean

= 3(18.5) − 2(18.5) = 18.5.

Example 18

The arithmetic mean of 8, 14, x, 20 and 24 is 16; then find x.

Solution

$$\bar{x} = \frac{8 + 14 + x + 20 + 24}{5} = 16$$

$$\Rightarrow \quad \bar{x} = 80 - 66 = 14.$$

Example 19

Calculate standard deviation of first five prime numbers.

Solution

Given set of observations {2, 3, 5, 7, 11}

$$\frac{\Sigma x^2}{n} = \frac{208}{5}$$

$$\frac{\Sigma x}{n} = \frac{28}{5}$$

$$\therefore \text{ SD} = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{208}{5} - \left(\frac{28}{5}\right)^2} = 3.2.$$

Example 19

In a series of observations, co-efficient of variation is 25 and mean is 50. Find the variance.

Solution

Co-efficient of variation: $\text{CV} = \frac{\text{SD}}{\bar{x}} \times 100$

$$\Rightarrow \quad \text{SD} = \frac{\text{CV}}{100} \cdot \bar{x}$$

$$= 50 \times \frac{25}{100} = 12.5$$

Variance = $(12.5)^2$ = 156.25.

Hypothesis Testing

Introduction

In probability theory, we set up mathematical models of processes and systems that are affected by 'chance'. In statistics, we check these models against the reality, to determine whether they are faithful and accurate enough for practical purposes. The process of checking models is called statistical inference.

Methods of statistical inference are based on drawing samples (or sampling). One of the most important methods of statistical inference is 'Hypothesis Testing'.

Some Basic Definitions

Population

Population is the set of individuals or objects, animate or inanimate, actual or hypothetical under study.

Size of the Population The number of individuals or objects or observations in the population.

- The size of the population is denoted by N.
- N can be finite or infinite (i.e., population can be finite or infinite)

Sample: Any subset of the population is called as sample.

- The size (i.e., the number of elements) of the sample is denoted by n.
- n is always finite.

Examples:

1. All the GATE applicants—Population

   GATE applicants form a city—Sample
2. Cars manufactured by Tata Motors—Population

   Nano cars manufactured by Tata Motors—Sample
3. All possible outcomes of 10 roles of a die—Population

   12 possible outcomes of 10 roles of a die—Sample
4. Number of units of electricity consumed by the residents of a colony in a city—Population

   Number of units of electricity consumed by the residents of 5 houses of that colony—Sample
5. Diameters of screws produced by a company—Population

   Diameters of screws produced on one machine of that company—Sample

Sampling

The process of drawing samples from the population is called sampling.

Random Sampling A sampling in which each member of the population has the same chance of being included in the sample is called random sampling.

Simple Sampling A random sampling in which the chance of being included in the sample for different members of the population is independent of whether included or not in the previous trails is called simple sampling.

Large and Small Samples If the size of the sample is greater than or equal to 30 (i.e., $n \geq 30$), then the sample is called a large sample. Otherwise it is called a small sample.

Parameter A statistical measure or constant of the population is called a parameter.

Examples:

1. Population mean (denoted by μ)
2. Population standard deviation (denoted by σ)

Statistic A statistical measure or constant of the sample drawn from the population is called a statistic. (statistic–singular, statistics–plural)

Examples:

1. Sample mean (denoted by $\bar{x}$)
2. Sample standard deviation (denoted by s)

NOTE

In general, the population parameters are not known and their estimates given by the corresponding sample statistics are used.

Sampling Distribution Consider samples of size n drawn from a given population. Compute some statistic S, say mean ($\bar{x}$) or variance (s^2) for each of the samples. The values of the statistics can be given in the form of a frequency table. The frequency table so formed is known as a sampling distribution of the statistic.

Example: Consider the set of numbers {1, 2, 3, 4, 5, 6} as population.

Consider the following 15 samples each of size 3 drawn from the above population.
(1, 2, 3), (3, 5, 5), (2, 4, 6), (5, 5, 5), (1, 2, 6)
(1, 3, 5), (6, 6, 6), (4, 4, 5), (2, 3, 4), (1, 1, 4)
(2, 5, 5), (2, 2, 5), (3, 4, 6), (2, 4, 5), (4, 5, 6)

Then the sampling distribution of means for these samples is

Sample Mean ($\bar{x}$)	2	3	3.67	4	4.33	5	6
Frequency	2	4	1	2	3	2	1

Standard Error The standard deviation of the sampling distribution of a statistic is called the standard error (SE) of that statistic.

- The standard deviation of the sampling distribution of means is called the standard error of means where as the standard deviation of the sampling distribution of variances is called the standard error of variances.

Precision: The reciprocal of the standard error is called precision.

NOTE

If the sample size n is large, (i.e., $n \geq 30$), then the sampling distribution of a statistic is approximately normal. (Irrespective of the population distribution being normal or not)

Testing of Hypothesis

We have some information about a characteristic of the population which may or may not be true. This information is called statistical hypothesis or briefly hypothesis. We wish to know, whether this information can be accepted or to be rejected. We choose a random sample and obtain information about this characteristic. Based on this information, a process that decides whether the hypothesis to be accepted or rejected is called testing of hypothesis. i.e., In brief, the test of hypothesis or the test of significance is a procedure to determine whether observed samples differ significantly from expected results.

Null Hypothesis and Alternative Hypothesis

Null Hypothesis A statistical hypothesis which is to be actually tested for acceptance or rejection is called a null hypothesis.

(According to RA Fisher, Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true)

- Null hypothesis is denoted by H_0

Alternative Hypothesis Any hypothesis other than the null hypothesis is called an alternative hypothesis.

- Alternative hypothesis is denoted by H_1
- Let θ be a population parameter and θ_0 be the specified value of θ. Then we define null and alternative hypotheses as follows.

Null hypothesis $H_0 : \theta = \theta_0$
Alternative Hypothesis $H_1 : \theta \neq \theta_0$ (two tailed alternative)
(OR) $H_1 : \theta > \theta_0$ (right tailed alternative)
(OR) $H_1 : \theta < \theta_0$ (left tailed alternative)

Type I and Type II Errors

Type I Error Rejecting the null hypothesis (H_0), when it should be accepted is called type I error.

Type II Error Accepting the null hypothesis (H_0) when it should be rejected is called type II error.

	Accept H_0	Reject H_0
H_0 is true	Correct decision	Type I error
H_0 is false	Type II error	Correct decision

Level of Significance

The probability level, below which, we reject the null hypothesis is called the level of significance.

(OR)

The probability of committing type I error is known as the level of significance.

- The level of significance is denoted by 'α'.
- It is customary to fix α, before sample information is collected.
- In most of the cases, we choose α as 0.05 or 0.01.
- $\alpha = 0.05$ is used for moderate precision and $\alpha = 0.01$ is used for high precision.
- Level of significance can also be expressed as percentage. $\alpha = 5\%$ means there are 5 chances in 100 that the null hypothesis H_0 is rejected when it is true or one is 95% confident that a right decision is made.
- The probability of committing type II error is denoted by β.
  $\therefore\ \beta = P$ (accept H_0 when H_0 is false)
- Level of significance (α) is also known as the size of the test.
- $1 - \beta$ is known as the power of the test.

Critical Region and Critical Value

Consider the area under the probability curve of the sampling distribution of the test statistic which follows some known distribution. The area under the probability curve is divided into two regions, namely the region of rejection where null hypothesis is rejected and the region of acceptance where null hypothesis is accepted.

Critical Region (or) the Region of Rejection (or) the Significant Region

The region under the probability curve of the sampling distribution of the test statistic, where the null hypothesis (H_0) is rejected is called the critical region.

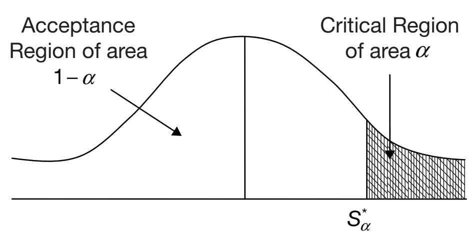
- The area of the critical region is equal to the level of significance α.

Critical Value (OR) Significant Value

The value of the test statistic (for given level of significance α) which separates the area under the probability curve into critical and non-critical regions.

One Tailed and Two Tailed Tests

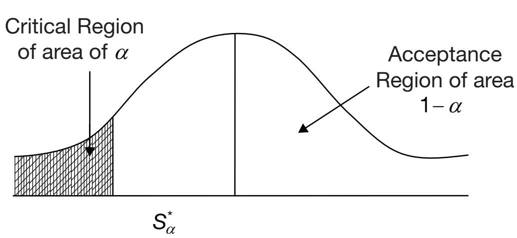
1. **Right one-tailed test:** If the alternative hypothesis H_1 is of greater than type (For example, $H_1 : \mu > \mu_0$ or $H_1 : \sigma_1^2 > \sigma_2^2$) then the entire critical region of area α lies on the right side tail of the probability curve of the test statistic S^* as shown in the figure. In this case, the test of hypothesis is known as right one-tailed test.



Right one-tailed test

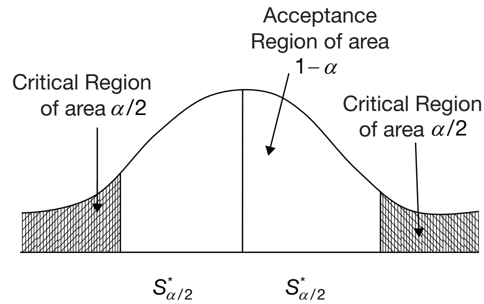
2. **Left one-tailed test:** If the alternative hypothesis H_1 is of less than type (For example, $H_1 : \mu_1 - \mu_2 < 0$ or $H_1 : \sigma^2 < \sigma_1^2$) then the entire critical region of area α lies on the left side tail of the probability curve of the test statistic S^* as shown in the figure.

   In this case, the test of hypothesis is known as left one-tailed test.



Left one-tailed test

3. **Two tailed test:** If the alternative hypothesis H_1 is of not equal to type (For example, $H_1 : \mu \neq \mu_0$ or $H_1 : \sigma_1^2 \neq \sigma_2^2$), then the critical region lies on both sides (right and left tails) of the probability curve of the test statistic S^* such that the critical region of area $\frac{\alpha}{2}$ lies on the right tail and the critical region of area $\frac{\alpha}{2}$ lies on the left tail as shown in the figure.



Two-tailed test

In this case, the test of hypothesis is known as two-tailed test.

Procedure for Test of Hypothesis

Step 1: Formulate null hypothesis H_0

Step 2: Formulate alternative hypothesis H_1

Step 3: Choose the level of significance α

Step 4: Identify the critical region based on the critical value S_α^* and the alternative hypothesis.

Step 5: Compute the test statistic S^* using the sample data (Formulae for finding the values of the test statistics under different tests of hypothesis were given while describing those tests).

Step 6: If the value of S^* comes under the critical region, then reject the null hypothesis H_0 and if the value of S^* comes under the non-critical (acceptance) region, then accept the null hypothesis H_0.

Central Limit Theorem If $\overline{x}$ is the mean of a sample of size n drawn from a population with mean μ and finite variance σ^2, then the limiting distribution of $Z = \dfrac{\overline{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$ as $n \to \infty$ is the standard normal distribution (i.e., mean zero and standard deviation 1)

NOTES

1. If the sample size is large, then whether the population is normally distributed or not, the sampling distribution of means always follow a normal distribution.
2. If the sample size is small and the population from which the samples are drawn follows a normal distribution, then the sampling distribution of means also follows a normal distribution.

Tests of Hypothesis for Large Samples

If the sample size is large, then the standard error (SE) forms the basis for the testing of hypothesis. Also, we know that if sample size is large, then the sampling distribution of any statistic S is normal. So, in large sampling, we can relate the value of the test statistic S^* with the standard normal random variable Z as:

$$Z = \frac{S^* - E(S^*)}{SE(S^*)}$$

Where

S^* = Value of the test statistic

$E(S^*)$ = Expected Value (value of the corresponding population parameter)

$SE(S^*)$ = Standard error of the test statistic.

Following table gives the information about the standard errors and test statistics for various cases in testing of hypothesis for large samples.

Test of Hypothesis (significance)		Standard Error = SE = σ^*	Test Statistic = Z	Expansions for Notations
Difference between	Sample mean ($\bar{x}$) and population mean (μ)	$\frac{\sigma}{\sqrt{n}}$	$\frac{\bar{x}-\mu}{\sigma^*}$	μ = Population mean $\bar{x}$ = Sample mean
	Means of two samples ($\bar{x}_1$ and $\bar{x}_2$)	$\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$	$\frac{\overline{x_1}-\overline{x_2}}{\sigma^*}$	σ = Population standard deviation
	Sample standard deviation (s) and population standard deviation (σ)	$\frac{\sigma}{\sqrt{2n}}$	$\frac{s-\sigma}{\sigma^*}$	n = Sample size s = Sample standard deviation
	Sample standard deviations (s_1 and s_2)	$\sqrt{\frac{\sigma_1^2}{2n_1}+\frac{\sigma_2^2}{2n_2}}$	$\frac{s_1-s_2}{\sigma^*}$	P = Population proportion $Q = 1 - P$
	Sample proportion (p) and population proportion (P)	$\sqrt{\frac{PQ}{n}}$	$\frac{p-P}{\sigma^*}$	p = Sample proportion
	Two sample proportions (P_1, P_2)	$\sqrt{\frac{P_1Q_1}{n_1}+\frac{P_2Q_2}{n_2}}$	$\frac{P_1-P_2}{\sigma^*}$	

Example 21

The dean of an engineering college claims that the average attendance of students in the final semester of B.Tech is 72.5% with a standard deviation of 5.7%. To test this claim, the attendance of a random sample of 49 students of final semester of B.Tech were examined, which showed the average as 74.4%. Can the claim be accepted or not at 1% level of significance?

Solution

Here population mean = $\mu_0 = 72.5$

Simple mean = $\bar{x} = 74.4$

Population standard deviation = σ

= 5.7

Level of significance $= \alpha = \frac{1}{100} = 0.01$

Sample size = $n = 49$

Null hypothesis H_0: $\mu = \mu_0 = 72.5$

Alternative hypothesis H_1: $\mu \neq \mu_0$ (= 72.5)

Level of significance, $\alpha = 0.01$

Test statistic,

$$Z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{74.4-72.5}{\frac{5.7}{\sqrt{49}}}$$

$$= \frac{1.9}{5.7} \times 7 = 2.333$$

Critical region: As the alternative hypothesis is of ≠ type, the test should be a two tailed test, where the critical region lies on both sides of the curve as shown in the figure.

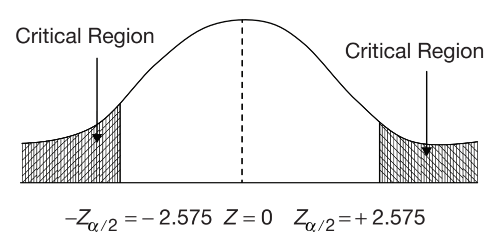
$\alpha = 0.01$

$\Rightarrow \frac{\alpha}{2} = 0.005$

$\therefore P(Z \leq -Z_{\alpha/2}) = 0.05$

$\Rightarrow -Z_{\alpha/2} = -2.575$

$\Rightarrow Z_{\alpha/2} = 2.575$



Decision: It can be easily observed that the value of the test statistic lies between $-Z_{\alpha/2}$ and $Z_{\alpha/2}$ i.e., The test statistic is not in the critical region.

Hence we accept the null hypothesis H_0

$\therefore$ The claim of the dean can be accepted.

Example 22

Can it be concluded that the average life span of an electric bulb is more than 200 hours, if a random sample of 100 electric bulbs has an average life span of 202 hours with a standard deviation of 8 hours with level of significance 0.05

Solution

Here population mean $\mu_0 = 200$

Sample mean $= \bar{x} = 202$

Sample standard deviation $= s = 8$

Sample size $= n = 100$

Null hypothesis H_0: $\mu = \mu_0 = 200$

Alternative hypothesis H_1: $\mu > \mu_0$ (= 202)

Level of significance, $\alpha = 0.05$

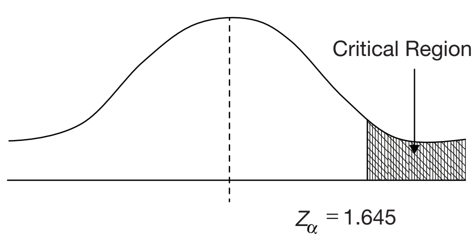
Test statistic, $Z = \dfrac{\bar{x} - \mu_0}{\dfrac{\sigma}{\sqrt{n}}} = \dfrac{\bar{x} - \mu_0}{\dfrac{s}{\sqrt{n}}}$

($\because$ As the population standard deviation σ is not given and the sample size is large, we can consider the sample standard deviation s as the population standard deviation)

$$\therefore Z = \frac{202 - 200}{\dfrac{8}{\sqrt{100}}}$$

$$Z = 2.5$$

Critical region: As the alternative hypothesis H_1 is of > type, the test should be a right one tailed test, where the critical region lies on the right tail of the standard normal curve as shown in the figure.



Here $\alpha = 0.05$

$\therefore P(Z \geq Z_\alpha) = 0.05$

$\Rightarrow P(Z \leq -Z_\alpha) = 0.05$

$\Rightarrow -Z_\alpha = -1.645$

$\Rightarrow Z_\alpha = 1.645$

$\therefore$ The critical region is to the right of Z_α

(i.e., to the right of $Z = 1.645$) under the standard normal curve.

Decision: As the value of the test statistic $Z = 2.5$ is greater than that of $Z_\alpha = 1.645$, the test statistic lies in the critical region.

$\therefore$ Reject the null hypothesis $H_0 : \mu = 200$

Hence accept the alternative hypothesis $H_1 : \mu > 200$

$\therefore$ We can conclude that the average life span of an electric bulb is more than 200 hours.

NOTE

In the process of testing of hypothesis for large samples, if the population standard deviation σ is not given, then the sample standard deviation s can be assumed as the population standard deviation.

Example 23

In a city, a random sample of 36 men has an average life span of 71 years with a standard deviation of 9 years, while a random sample of 49 women has an average life span of 76 years with a standard deviation of 14 years. Does this substantiate the claim that the life span of men is less than that of women in that city with 1% level of significance?

Solution

Let $\bar{x}_1$ = Average life span of men = 71 years

And $\bar{x}_2$ = Average life span of women

= 76 years

$s_1 = 9$ and $s_2 = 14$

$n_1 = 36$ and $n_2 = 49$

Level of significance $= \alpha = 0.01$

Null hypothesis H_0: $\mu_1 = \mu_2$ (i.e., The average life span of men and women in the city is same)

Alternative hypothesis H_1: $\mu_1 < \mu_2$ (i.e., The average life span of men is less than that of women in the city)

Level of significance: $\alpha = 0.01$

Test statistic,

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

i.e., $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

($\because$ The standard deviations s_1 and s_2 are given and the standard deviations of populations are unknown)

$$= \frac{(71 - 76)}{\sqrt{\dfrac{9^2}{36} + \dfrac{14^2}{49}}} = \frac{-5}{\sqrt{\left(\dfrac{9}{4} + 4\right)}}$$

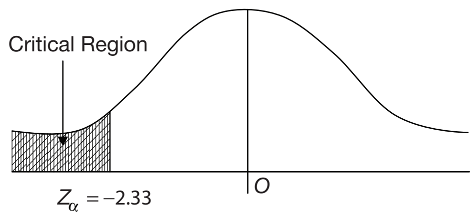
$Z = -2$

Critical region: As the alternative hypothesis H_1 is of < type, the critical region should be in the left tail of the standard normal curve as shown in the figure.

Here $\alpha = 0.01$

$\therefore P(Z \le Z_\alpha) = \alpha = 0.01$

$\Rightarrow Z_\alpha = -2.33$



$\therefore$ The critical region is to the left of $Z_\alpha = -2.33$ under the standard normal curve.

Decision: As the value of the test statistic $Z = -2$ is greater than that of $Z_\alpha = -2.33$, the test statistic does not lie in the critical region.

$\therefore$ Accept the null hypothesis H_0

Hence reject the alternative hypothesis H_1

$\therefore$ The given information does not substantiate the claim that the life span of men is less than that of women in that city.

Example 24

The manufacturer of electronic weighing machines finds that in a random sample of 120 machines, 15 machines are defective. Find the standard error of the proportion of defective machines in the sample.

Solution

Total number of machines = Sample size = $n = 120$

$\therefore$ The number of defective machines = 15 = x (say)

$\therefore$ Proportion of defective machines

$$= p = \frac{x}{n} = \frac{15}{120} = \frac{1}{8}$$

$$\therefore q = 1 - p = 1 - \frac{1}{8} = \frac{7}{8}$$

As the population proportion P (and hence $Q = 1 - P$) is unknown, we take p and q as P and Q respectively.

$\therefore$ Standard error of he proportion of defective machines

$$= SE = \sqrt{\frac{PQ}{n}}$$

$$= \sqrt{\frac{pq}{n}} = \sqrt{\frac{\frac{1}{8} \times \frac{7}{8}}{120}}$$

$\therefore SE = 0.0302$

Example 25

A home appliances company claims that the life of its geysers has a standard deviation of 16 hours. The life of a sample of 98 geysers of that company was found to have a standard deviation of 18 hours. Find the test statistic Z that is used in the process of testing whether the claim of the company be accepted or not.

Solution

Population standard deviation = $\sigma = 16$

Sample standard deviation = $s = 18$

Sample size = $n = 98$

As the situation is a testing of hypothesis of difference between population and sample standard deviations, we have

$$\text{Test statistic } = Z = \frac{(s - \sigma)}{\frac{\sigma}{\sqrt{2n}}}$$

$$= \frac{(18 - 16)}{\frac{16}{\sqrt{2 \times 98}}} = \frac{2 \times 14}{16}$$

$\therefore Z = 1.75.$

Tests of Hypothesis for Small Samples

In case of large samples, we often made use of the fact that the sampling distribution of many statistics are approximately normal and values of sample statistics are considered best estimates of the parameters of a population. However, in case of small samples, the sampling distributions of many statistics are not normal and the approximations of population parameters by the corresponding sample statistics are not valid. So, we shall discuss different tests of hypothesis which are applicable to small sampling. Note that these tests of hypothesis for small samples can also be applied to the cases of large samples. First we will discuss three important distributions that are used in testing of hypothesis for small samples namely, t-distribution, F-distribution and χ^2-distribution. These distributions require the knowledge of the concept of 'Degrees of freedom'.

Degrees of Freedom

The number of degrees of freedom is defined as the number of values in a set, which may be assigned arbitrarily.

For example, if $x_1 + x_2 + x_3 + x_4 + x_5 = 18$, then assign any values for four of the five variables arbitrarily (say, x_1, x_2, x_3 and x_4 were given arbitrary values). Then the value of the fifth variable (x_5) has to be taken based on the values of x_1, x_2, x_3 and x_4. So, in this case, the degrees of freedom is 4.

With reference to statistics, if n is the number of observations in the small sample and k is the number of constraints

on them (or k values are already available), then the number of degrees of freedom can be obtained by $n - k$.

The number of degrees of freedom is denoted by ν.

Example: If $x_1, x_2, \ldots x_n$ are the observations given, then the number of degrees of freedom for the mean $\bar{x}$ is n

($\because$ we use all values to find $\bar{x}$)

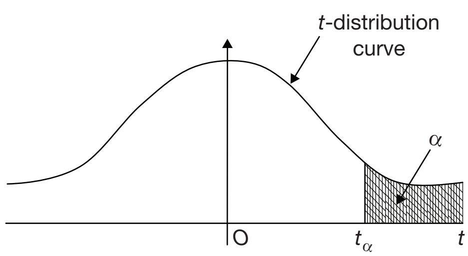
The number of degrees of freedom for the variance is $n - 1$ ($\because$ The variance depends on the mean)

Student's t-Distribution (or) t-Distribution

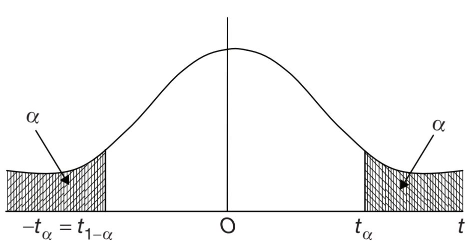
A random variable t is said to follow the t-distribution with $\nu = n - 1$ degrees of freedom, n being the sample size, if its probability density function is given by

$$f(t) = \frac{1}{\sqrt{\upsilon\beta\left(\frac{1}{2},\frac{\upsilon}{2}\right)}}\left(1+\frac{t^2}{\upsilon}\right)^{-(\upsilon+1)/2} \quad ; -\infty < t < \infty$$

The t-distribution curve is as shown in the figure, which is symmetric about the mean 0 and bell shaped. The total area under the t-distribution curve is unity.



- The t-distribution curve is similar to normal curve.
- The variance of t-distribution is greater than 1 and depends on the degrees of freedom ν.
- As the sample size n (i.e., the degrees of freedom $n - 1$) becomes large, the variance of corresponding t-distribution approaches 1 and hence for large samples, t-distribution can be approximated by the standard normal distribution.
- Critical values of t-distribution (see the t-distribution tables) are denoted by t_α, which is such that the area under the curve to the right of t_α equals to α.
- As the t-distribution is symmetric, it follows that $t_{1-\alpha} = -t_\alpha$.

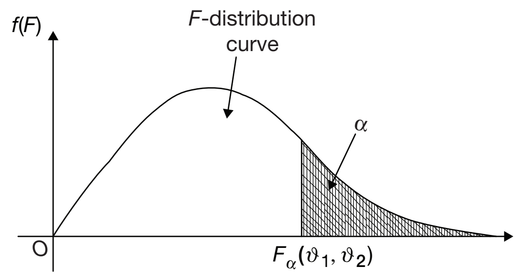


F-Distribution

A random variable F is said to follow the F-distribution with (ν_1, ν_2) degrees of freedom, if its probability density function $f(F)$ is given by

$$f(F) = \frac{\left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}} F^{\left(\frac{\nu_1}{2}\right)-1}}{\beta\left(\frac{\nu_1}{2},\frac{\nu_2}{2}\right)\left[1+\left(\frac{\nu_1}{\nu_2}\right)F\right]^{(\nu_1+\nu_2)/2}}, F > 0$$

The graph of F-distribution is given below.



- The F-distribution curve entirely lies in the first quadrant.
- F-distribution is not symmetric.
- $F_\alpha(\nu_1, \nu_2)$ is the value of F with ν_1 and ν_2 degrees of freedom such that the area under the F-distribution curve to the right of $F_\alpha(\nu_1, \nu_2)$ is α.
- The value of $F_\alpha(\nu_1, \nu_2)$ not only depends on the values of the degrees of freedom ν_1 and ν_2, but also the order in which they were taken.
- $F_{1-\alpha}(\nu_1, \nu_2) = \dfrac{1}{F_\alpha(\nu_2, \nu_1)}$
- F-distribution is also known as variance ratio distribution.
- The values of F_a for $\alpha = 0.05$ and $\alpha = 0.01$ for various combinations of the degrees of freedom ν_1 and ν_2 were presented in the tables.
- For large values of ϑ_1 and ϑ_2, F-distribution can be approximated by a normal distribution $N\left[1, 2\left(\frac{1}{\partial_1}+\frac{1}{\partial_2}\right)\right]$ with mean 1 and variance $2\left(\frac{1}{\partial_1}+\frac{1}{\partial_2}\right)$.
- F-distribution and t-distribution can be related as follows. If a statistic t follows t-distribution with ν degrees of freedom, then t^2 follows F-distribution with degrees of freedom $\nu_1 = 1$ and $\nu_2 = \nu$.

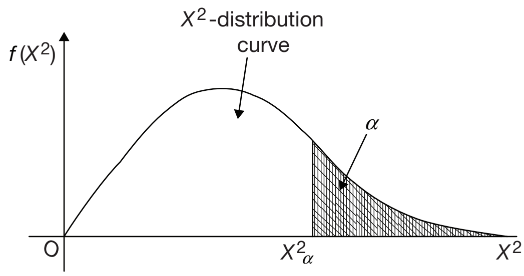
Chi-square Distribution

If a random variable X follows chi-square distribution (denoted as χ^2-distribution or $\chi^2(\nu)$), then the probability density function of X is given by

$$f(\chi^2)=\frac{1}{2\frac{v}{2\Gamma}\left(\frac{v}{2}\right)}e^{-\chi^2/2}(\chi^2)^{\left(\frac{v1}{2}\right)^{-1}},0\le\chi^2<\infty$$

where v is the degrees of freedom

The graph of chi-square distribution is given below.



- The chi-square distribution curve entirely lies in the first quadrant.
- Chi-square distribution is not symmetric.
- χ^2-distribution depends only on ϑ, the degrees of freedom.
- If χ_1^2 and χ_2^2 are two independent distributions with v_1 and v_2 degrees of freedom respectively, then $\chi_1^2+\chi_2^2$ will follow chi-square distribution with (v_1+v_2) degrees of freedom.
- χ_α^2 represents the value of χ^2 such that the area under the chi-square curve to the right of χ_α^2 is α
- The value of χ_α^2 for various combinations of α and ϑ were presented in the table.
- As the number of degrees of freedom $\vartheta\to\infty$, the χ^2-distribution tends to the normal distribution.

Identifying the Values of t_α, F_α and χ_α^2 from the Tables

1. The value of t_α for a given degrees of freedom ϑ is the value in the t-table at the intersection point of the column headed by α and the row headed by the degrees of freedom ϑ.

Example 26

Find the values of

(i) $t_{0.1}$ with degrees of freedom $\vartheta = 12$

(ii) $t_{0.05}$ with degrees of freedom $\vartheta = 17$

(iii) $t_{0.98}$ with degrees of freedom $\vartheta = 23$

Solution

(i) $t_{0.1}$ with degrees of freedom $\vartheta = 12$ = The value in the t-table at the intersection point of the column headed by $\alpha = 0.1$ and the row headed by $\vartheta = 12$ $= 1.356$

(ii) $t_{0.05}$ with degrees of freedom $\vartheta = 17$ = The value in the t-table at the intersection point of the column headed by $\alpha = 0.05$ and the row headed by $\vartheta = 17$ $= 1.740$.

(iii) $t_{0.98}$ with degrees of freedom $\vartheta = 23$. In the given t-table, there is no column corresponding to $\alpha = 0.98$

But we know that

$t_{1-\alpha} = -t_\alpha$

i.e., $t_\alpha = -t_{1-\alpha}$

$\therefore\ t_{0.98} = -t_{1-0.98}$

$\Rightarrow t_{0.98} = -t_{0.02}$ (1)

Now $t_{0.02}$ with degrees of freedom $\vartheta = 23$

$= 2.177$

$\therefore$ From (1), $t_{0.98}$ with degrees of freedom $\vartheta = 23$

$= -2.177$.

2. Two tables were given for F-distribution, one each for the values of $\alpha = 0.05$ and $\alpha = 0.01$ for various combinations of degrees of freedom ϑ_1 and ϑ_2.
   - The value of F_α for a given pair of values of degrees of freedom ϑ_1 and ϑ_2 is the value in the respective F_α-table at the intersection point of column headed by ϑ_1 and the row headed by ϑ_2.
3. The value of χ_α^2 for a given degrees of freedom ϑ is the value in the χ^2-table at the intersection point of the column headed by α and the row headed by the degrees of freedom ϑ.

Example 27

Find the values of

(i) $F_{0.05}$ (6, 13);

(ii) $F_{0.01}$(12, 17) and

(iii) $F_{0.95}$(15, 24)

Solution

(i) $F_{0.05}$(6, 13) = The value in the F-table corresponding to $\alpha = 0.05$ at the intersection point of the column headed by $\vartheta_1 = 6$ and the row headed by $\vartheta_2 = 13$ $= 2.92$

(ii) $F_{0.01}$(12, 17) = The value in the F-table corresponding to $\alpha = 0.01$ at the intersection point of the column headed by $\vartheta_1 = 12$ and the row headed by $\vartheta_2 = 17$ $= 3.46$

(iii) $F_{0.95}$(15, 24)

e know that $F_\alpha(\vartheta_1, \vartheta_2)$

$$= \frac{1}{F_{1-\alpha}(\vartheta_2,\vartheta_1)}$$

$\therefore\ F_{0.95}$(15, 24)

$$= \frac{1}{F_{1-0.95}(24,15)}$$

$$= \frac{1}{F_{0.05}(24,15)}$$

$$= \frac{1}{2.29} = 0.4367.$$

Example 28

Find the values of

(i) $\chi^2_{0.05}$ with degrees of freedom $\vartheta = 16$

(ii) $\chi^2_{0.01}$ with degrees of freedom $\vartheta = 21$

(iii) $\chi^2_{0.10}$ with degrees of freedom $\vartheta = 4$

Solution

(i) $\chi^2_{0.05}$ with degrees of freedom $\vartheta = 16$

= The value in the χ^2 – table at the intersection point of the column headed by $\alpha = 0.05$ and the row headed by $\vartheta = 16$

= 26.296

(ii) $\chi^2_{0.01}$ with degrees of freedom $\vartheta = 21$

= The value in the χ^2 – table at the intersection point of the column headed by $\alpha = 0.01$ and the row headed by $\vartheta = 21$

= 38.932

Similarly, we have

(iii) $\chi^2_{0.10}$ with degrees of freedom $\vartheta = 4$

= 7.779.

Test of Hypothesis	Test Statistic	Distribution with Degrees of Freedom	Expansions for Notations
Difference between means of population and sample (σ unknown)	$t = \dfrac{\bar{x} - \mu_0}{\dfrac{s}{\sqrt{n-1}}}$	t-distribution with $v = n - 1$	$\bar{x}$ = Sample mean
Difference between means of two samples (σ unknown) (a) If $n_1 \neq n_2$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\left(\dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$	t-distribution with $v = n_1 + n_2 - 2$	μ_0 = Population mean
(b) If $n_1 = n_2 = n$	$t = \dfrac{(\overline{x_1} - \overline{x_2})}{\sqrt{\dfrac{(s_1^2 + s_2^2)}{(n-1)}}}$	t-distribution with $v = 2n - 2$	s = Sample standard deviation n = Sample size ϑ = Degrees of freedom
(c) $n_1 = n_2 = n$ and the two samples are not independent i.e., they are related in some way (This implies that the pairs of observations (x_i, y_i) belong to same sample unit	$t = \dfrac{\bar{d}}{\dfrac{s}{\sqrt{n-1}}}$ where $\bar{d} = \overline{x_i} - \overline{y_i}$ $d_i = x_i - y_i$ and $s_2 = \dfrac{1}{n}\sum_i (d_i - \bar{d})^2$	t-distribution with $v = n - 1$	
Equality of the population variances **Note:** Take the larger of the estimates of variances of the samples as $\hat{\sigma}_1^2$ and the corresponding degrees of freedom as ϑ_1	$F = \dfrac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2}$ where $\hat{\sigma}_1^2 = \left(\dfrac{n_1}{n_1 - 1}\right)s_1^2$ and $\hat{\sigma}_2^2 = \left(\dfrac{n_2}{n_2 - 1}\right)s_2^2$	F-distribution with $v_1 = n_1 - 1$ $v_2 = n_2 - 1$	
Population variance	$\chi^2 = \dfrac{ns^2}{\sigma^2}$ 0 where $s^2 = \sum_{i=1}^{n} \dfrac{(x_i - \bar{x})^2}{n}$	χ^2-distribution with $v = n - 1$	

Example 29

The highest temperature in the month of June at a certain place is normally distributed with mean 40°C. The highest temperatures in June during the last five years are 43°C, 37°C, 35°C, 39°C and 38°C. From this data, can we conclude that the average highest temperature in June during the last five years is less than the normal highest temperature? (Test at 0.05 level of significance)

Solution

Population mean = $\mu_0 = 40$

Sample mean $= \bar{x} = \dfrac{43+37+35+39+38}{5}$

$\therefore \bar{x} = \dfrac{192}{5} = 38.4$

Sample size v $n = 5$

$\therefore$ Number of degrees of freedom $= \vartheta = n - 1 = 5 - 1 = 4$

Sample variance $= s^2 = \left(\frac{1}{n}\sum_i x_i^2\right) - \overline{x^2}$

$= \frac{43^2 + 37^2 + 35^2 + 39^2 + 38^2}{5} - (38.4)^2$

$= 1,481.6 - 1,474.56$

$\therefore s^2 = 7.04$

$\Rightarrow s = 2.6533$

Null hypothesis H_0: $\mu = \mu_0 = 40$

Alternative hypothesis: H_1: $\mu < \mu_0$ (=40)

Level of significance: $\alpha = 0.05$

Test statistic:

$$t = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n-1}}}$$

$$= \frac{38.4 - 40}{\frac{2.6533}{\sqrt{5-1}}} = \frac{-1.6}{\frac{2.6533}{2}}$$

$\therefore t = -1.2060$

Critical region: As the alternative hypothesis is < type, the critical region is in the left tail of the *t*-distribution curve as shown in the figure.

As $\alpha = 0.05$ with $\vartheta = 4$, we have

t_α with $\vartheta = 4$

= The area under the *t*-distribution curve to the right of $\alpha = 0.05$ with $\vartheta = 4$

= 2.132

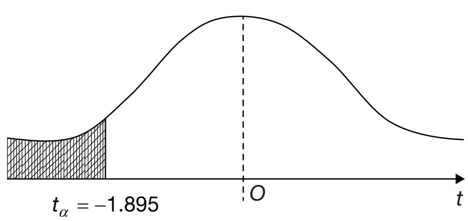
$\therefore$ The critical region is the region (area = 0.05) in the left tail of the *t*-distribution curve.

$= -t_\alpha$ with $\vartheta = 4$

($\because$ *t*-distribution is symmetric)

$\therefore$ The critical value is

$\therefore -t_\alpha = -2.132$



Decision: As the test statistic $t = -1.2060$ is greater than the critical value -2.132, it lies in the acceptance region.

$\therefore$ Accept the null hypothesis.

Hence there is no significant difference between the normal temperature and the average temperature of the last five years in the month of June.

Example 30

In a CBSE school marks scored in Mathematics by 10 students of section-A of *X* standard has a mean of 68 and a variance of 109 where as that of 8 students of Section-B has a mean of 57 with a variance of 128. Test of 2% level of significance whether there is a significant difference between the means of marks scored by the students of sections-A and B or not. Assume that the marks of the students of sections-A and B follow normal distribution with same variance.

Solution

Let μ_1 and μ_2 be the means of the marks of students of the sections A and B respectively.

Mean of first sample $= \overline{x}_1 = 68$

Mean of second sample $= \overline{x}_2 = 57$

Variance of first sample $= s_1^2 = 109$

Variance of second sample

$= s_2^2 = 128$

$n_1 = 10$ and $n_2 = 8$

Null hypothesis: H_0: $\mu_1 = \mu_2$

Alternative hypothesis: H_1: $\mu_1 \neq \mu_2$

Level of significance: $\alpha = 0.02$

Test statistic:

$$t = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{\left(\frac{n_1 s_1{}^2 + n_2 s_2{}^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

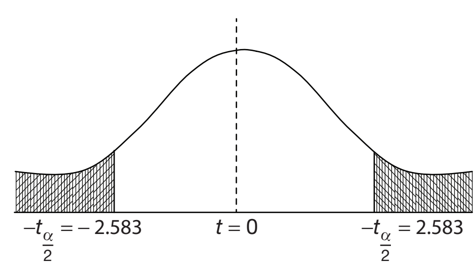
$$= \frac{(68 - 57)}{\sqrt{\left(\frac{10 \times 109 + 8 \times 128}{10 + 8 - 2}\right)\left(\frac{1}{10} + \frac{1}{8}\right)}}$$

$$= \frac{11}{5.4523}$$

$= 2.0175$

$\therefore$ The test statistic is $t = 2.0175$

Critical region: As the alternative hypothesis is of the ≠ type, the critical region lies on both sides of the *t*-distribution curve as shown in the figure.



Here $\alpha = 0.02$ and υ = degrees of freedom $= n_1 + n_2 - 2 = 10 + 8 - 2$

$\upsilon = 16$

$\frac{t_\alpha}{2}$ with $\upsilon = 16$

$= t_{0.01}$ at $\upsilon = 16$

$= 2.583$

∴ The critical region is to the left of $-\frac{t_\alpha}{2} = -2.583$ and to the right of $\frac{t_\alpha}{2} = 2.583$.

Decision: As the test statistic $t = 2.0175$ lies between $-\frac{t_\alpha}{2}$ and $\frac{t_\alpha}{2}$, we accept the null hypothesis.

∴ There is no significant difference between the means of marks scored by the students of sections A and B.

Example 31

The variances of two samples of sizes 9 and 13 are 15.778 and 19.175 respectively. Test whether the two samples be regarded as drawn from normal populations with the same variance at 5% level of significance.

Solution

Always the sample having higher variance will be taken as the first sample in this test of hypothesis.

∴ Variance of first sample $= s_1^2$

$= 19.175$

Variance of second sample $= s_2^2$

$= 15.778$

Sample size of first sample $= n_1 = 13$

Sample size of second sample $= n_2 = 9$

$$\hat{\sigma}_1^2 = \left(\frac{n_1}{n_1 - 1}\right) s_1^2 = \frac{13}{(13-1)} \times 19.175$$

$$\hat{\sigma}_1^2 = 20.7729$$

$$\hat{\sigma}_2^2 = \left(\frac{n_2}{n_2 - 1}\right) s_2^2 = \frac{9}{(9-1)} \times 15.778$$

$$\hat{\sigma}_2^2 = 17.7520$$

Null hypothesis: H_0: $\hat{\sigma}_1^2 = \hat{\sigma}_2^2$

Alternative hypothesis: H_1: $\hat{\sigma}_1^2 \neq \hat{\sigma}_2^2$

Level of significance: $\propto = 0.05$

Test statistic:

The test statistic is

$$F = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_{21}^2}$$

$$= \frac{20.7729}{17.7502}$$

$F = 1.1703$

Critical region:

Here $\upsilon_1 = n_1 - 1 = 13 - 1 = 12$

And $\upsilon_2 = n_2 - 1 = 9 - 1 = 8$

∴ The critical value is

$F_\alpha(\upsilon_1, \upsilon_2) = F_{0.05}(12, 8) = 3.28$

And the critical region is to the right of 3.28

As $F = 1.1703 < F_\alpha(\upsilon_1, \upsilon_2) = 3.28$,

we conclude that the two random samples are drawn from two normal populations with the same variance.

Non-Parametric Tests

Goodness of Fit Test

To determine, if a population follows a specified theoretical distribution such as binomial, Poisson or normal distribution, χ^2 test can be used. χ^2 test, which is based on how good a fit is there between the observed frequencies (O_i from the sample) and the expected frequencies (E_i from the theoretical distribution) is known as 'goodness of fit test'.

Let a distribution be given. Let O_i and E_i ($I = 1, 2, 3... n$) be the observed and expected frequencies of the ith class (or cell) such that $\sum_{i=1}^{n} O_i = \sum_{i=1}^{n} E_i = N =$ Total Frequency.

Test statistic (OR) Statistic for test of 'goodness of fit'

$$= \chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where n is the number of class intervals or cells, in the given frequency distribution and χ^2 is a random variable which is very closely approximated with χ^2-distribution with degrees of freedom v.

Degrees of Freedom for Goodness of Fit Test Based on the theoretical distribution given, the degrees of freedom are as given below.

1. For uniform distribution, $v = n - 1$
2. For binomial and Poison distribution, $v = n - 2$
3. For normal distribution, $v = n - 3$

NOTES

Given data should satisfy the following conditions.

1. Sample size (OR) the number of sample observations, N should be more than 50($N \geq 50$)
2. If individual frequencies (O_i and/or E_i) is/are small say less than 10, then combine neighbouring frequencies in such a way that, they will be ≥ 10.
3. The number of class or cells n should be neither too small nor too large. Generally, $4 \leq n \leq 16$.

Example 32

Fitting a Poisson distribution to the following data:

x_i	0	1	2	3	4
Observed Frequencies (O_i)	30	62	46	10	2

The following respective expected frequencies are obtained.

Expected Frequencies (E_i): 42 54 34 15 5

Test the goodness of fit of a Poisson distribution to the above data with 1% level of significance.

Solution

We have

Observed Frequencies (O_i)	30	62	46	10	2
Expected Frequencies (E_i)	42	54	34	15	5

Grouping the classes so that each class frequency is ≥10,

We have

O_i	30	62	46	12
E_i	42	54	34	20
$O_i - E_i$	−12	8	12	−8

Null hypothesis: H_0: Good fit exists between the theoretical (Poisson) distribution and given data (observed frequencies)

Alternative hypothesis: H_1: No good fit exists between the theoretical (Poisson) distribution and given data (observed frequencies).

Level of Significance: $-\alpha = 0.01$

Test Statistic:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(-12)^2}{42} + \frac{8^2}{54} + \frac{12^2}{34} + \frac{(-8)^2}{20}$$

$= 3.4286 + 1.1852 + 4.2353 + 3.2$

$\therefore \chi^2 = 12.0491$

Critical region: As we are testing the fitting of Poisson distribution,

Degrees of Freedom = $\upsilon = n - 2$
$= 4 - 2 = 2$

$\therefore \chi^2\ \alpha = \chi^2_{0.01}$ With $\upsilon = 2$

$= 9.210$

$\therefore$ The critical region is to the right of 9.210 under χ^2 distribution curve.

Decision: As the value of the test statistic

$\chi^2 = 12.0491 > \chi^2_\alpha (= 9.210)$

We reject the null hypothesis.

$\therefore$ Accept the alternative hypothesis.

Hence no good fit exists between the theoretical (Poisson) distribution and given data (observed frequencies).

Analysis of rXc Contingency Tables

Consider two attributes A and B of the given population. Let each of these attributes are classified into different classes (categories), say the attribute A is divided into r classes $A_1, A_2, \ldots, A_r$ and the attribute B is divided into c classes $B_1, B_2, \ldots, B_c$. Let a table (matrix) be formed with the classed of attribute A as heading rows and the classes of attribute B as heading columns as shown below. In the table, the values $O_{i*}(i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots,$ (C) are known as observed frequencies which denote the number of items belonging to both A_i and B_j; O_{i*} denote the number of items belonging to the class A_i and O_{*j} denote the number of items belonging to the class B_j. This table is known as *rxc* contingency table.

***rXc* Contingency Table**

A \ B	B_1	B_2	…	B_j	…	B_c	Row Total
A_1	O_{11}	O_{12}	…	O_{ij}	…	O_{1c}	O_{1*}
A_2	O_{21}	O_{22}	…	O_{2j}	…	O_{2c}	O_{2*}
⋮	⋮	⋮		⋮		⋮	⋮
A_i	O_{i1}	O_{i2}	…	O_{ij}		O_{ic}	O_{i*}
⋮	⋮	⋮		⋮		⋮	⋮
A_r	O_{r1}	O_{r2}	…	O_{rj}	…	O_{rc}	O_{r*}
Column Total	O_{*1}	O_{*2}	…	O_{*j}	…	O_{*c}	$\sum_{i=i}^{r} O_{i*} = \sum_{j=1}^{c} O_{*j} = N$

In general, these tables arise in two kinds of problems.

1. Test for Independence.
2. Test for Homogeneity.

Various requirements for testing of hypothesis in these two types of problems were described in the following table.

	Test for Independence	Test for Homogeneity
(A) Description	To test whether the given two attributes of the population are independent or not.	To test whether different classes of the attributes are homogeneous or not
(B) Expected frequency (E_{ij})	$\frac{(\text{Total observed frequency in } i\text{th row}) \times (\text{Total observed frequency in } j\text{th column})}{\text{Total frequency}}$ i.e., $\frac{(O_{i*})(O_{*j})}{N}$	
(C) Test Statistic	$\chi^2 = \sum_{i-1}^{r}\sum_{j=1}^{c}\left[\frac{(O_{ij}-E_{ij})^2}{E_{ij}}\right]$	
(D) Degrees of Freedom (υ)	$v = (r-1)\times(c-1)$	
(E) Decision	1. If $\chi^2 < \chi^2_\alpha$ with v degrees of freedom, then accept the null hypothesis H_0. 2. If $\chi^2 > \chi^2_\alpha$ with v degrees of freedom, then accept the alternative hypothesis H_1.	

Example 33

Test the hypothesis with 1% level of significance that the heart problem is independent of drinking (alcoholic drinks) habits from the following experimental data on 200 persons.

	Non Drinkers	Moderate Drinkers	Heavy Drinkers
Heart problem	25	40	35
No Heart problem	50	30	20

Solution

Given contingency table is

	Non Drinkers	Moderate Drinkers	Heavy Drinkers	Row Total
Heart Problem	25	40	35	100
No Heart problem	50	30	20	100
Column Total	75	70	55	

Total number of persons $= N = 200$

The expected frequency E_{ij} is given by

$$E_{ij} = \frac{(i\text{th row total})\times(j\text{th column total})}{\text{Total number of persons }(N)}$$

$$E_{11} = \frac{100\times 75}{200} = 37.5,$$

$$E_{12} = \frac{100\times 70}{200} = 35,$$

$$E_{13} = \frac{100\times 55}{200} = 27.5,$$

$$E_{21} = \frac{100\times 75}{200} = 37.5,$$

$$E_{22} = \frac{100\times 70}{200} = 35,$$

$$E_{23} = \frac{100\times 55}{200} = 27.5$$

Null hypothesis: H_0: Heart problem and the drinking habits are independent.

Alternative hypothesis: H_1: Heart problem and the drinking habits are not independent.

Level of significance: $\alpha = 0.01$

Test statistic:

$$X^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\left[\frac{(O_{ij}-E_{ij})^2}{E_{ij}}\right]$$

$$= \sum_{i=1}^{2}\sum_{j=1}^{3}\left[\frac{(O_{ij}-E_{ij})^2}{E_{ij}}\right]$$

$$= \frac{(25-37.5)^2}{37.5} + \frac{(40-35)^2}{35} + \frac{(35-27.5)^2}{27.5}$$

$$= \frac{(50-37.5)^2}{37.5} + \frac{(30-35)^2}{35} + \frac{(20-27.5)^2}{27.5}$$

$= 4.1667 + 0.7143 + 2.0454 + 4.1667 + 0.7143 + 2.0454$

$\therefore \chi^2 = 13.8528$

Critical region:

Here $\alpha = 0.01$

r = Number of rows in the table = 2

c = Number of columns in the table = 3

$\therefore$ Degrees of freedom $= v = (r-1)(c-1)$

$= (2-1)(3-1) = 2$

$v = 2$

$\therefore \chi^2_\alpha$ with υ degrees of freedom.

$= \chi^2_{0.01}$ with 2 degrees of freedom

$= 9.210.$

Decision: As the test statistic $= \chi^2 = 13.8528 > \chi^2_\alpha$ $(= 9.210)$, reject the null hypothesis.

i.e., accept the alternative hypothesis.

$\therefore$ Heart problem and the drinking habits are not independent.

Example 34

Following table shows the opinions of 300 persons about 'Love Marriages'.

	Married Persons	Unmarried Persons
Good	40	50
Not good	20	60
Undecided	60	70

Test whether the opinions of married and unmarried persons are homogeneous (same) with respect to 'Love Marriages' at 0.01 level of significance.

Solution

Given contingency table is

	Married Persons	Unmarried Persons	Row Total
Good	40	50	90
Not Good	20	60	80
Undecided	60	70	130
Column total	120	180	

The expected frequencies are given by,

$$E_{11} = \frac{90\times 120}{300} = 36,$$

$$E_{12} = \frac{90\times 180}{300} = 54,$$

$$E_{21} = \frac{80\times 120}{300} = 32,$$

$$E_{22} = \frac{80\times 180}{300} = 48,$$

$$E_{31} = \frac{30\times 120}{300} = 52,$$

$$E_{32} = \frac{130\times 180}{300} = 78$$

Null hypothesis: H_0: The opinion of married and unmarried persons about 'Love Marriages' is homogeneous (same).

Alternative hypothesis: H_1: The opinion of married and unmarried persons about 'Love Marriages' is not homogeneous.

Level of significance: $\alpha = 0.01$

Test statistic:

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\left[\frac{(O_{ij}-E_{ij})^2}{E_{ij}}\right]$$

$$\sum_{i=1}^{3}\sum_{j=1}^{2}\left[\frac{(O_{ij}-E_{ij})^2}{E_{ij}}\right]$$

$$= \frac{(40-36)^2}{36} + \frac{(50-54)^2}{54} + \frac{(20-32)^2}{32} + \frac{(60-48)^2}{48}$$

$$+ \frac{(60-52)^2}{52} + \frac{(70-78)^2}{78}$$

$\therefore\ \chi^2 = 10.292$

Critical region:

Here $\alpha = 0.01$

r = Number of rows in the table = 3

c = Number of columns in the table = 2

$\therefore$ Degrees of freedom $= \upsilon = (r-1)(c-1)$

$= (3-1)(2-1) = 2$

$\upsilon = 2$

$\therefore\ \chi^2_\alpha$ with υ degrees of freedom.

$= \chi^2_{0.01}$ with 2 degrees of freedom

$= 9.210$

Decision:

As the test statistic $= \chi^2$

$= 10.292 > \chi^2_\alpha$ $(= 9.210)$, reject the null hypothesis.

i.e., Accept the alternative hypothesis.

$\therefore$ The opinion of married and unmarried persons about 'Love Marriages' is not homogeneous (not same).

Exercises

1. If eight unbiased coins are tossed together, then the probability that the number of heads exceeds the number of tails is

(A) $\frac{31}{128}$ (B) $\frac{1}{2}$

(C) $\frac{93}{256}$ (D) $\frac{57}{256}$

2. If A and B are two mutually exclusive and exhaustive events and the probability that the non-occurrence of A is $\frac{3}{4}$, then the probability of occurrence of B is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{3}{4}$ (D) $\frac{1}{16}$

3. A bag contains five red balls, three black balls and a white ball. If three balls are drawn from the bag, the probability that the three balls are of different colours is

(A) $\frac{23}{28}$ (B) $\frac{5}{28}$

(C) $\frac{3}{28}$ (D) None of these

4. From a box containing 18 bulbs, of which exactly $\frac{1}{3}$ rd are defective, five bulbs are chosen at random to fit into the five bulb holders in a room. The probability that the room gets lighted is

(A) $1-\frac{^6C_5}{^{18}C_5}$ (B) $\frac{^6C_5}{^{18}C_5}$

(C) $\frac{^{12}C_5}{^{18}C_5}$ (D) $1-\frac{^{12}C_5}{^{18}C_5}$

5. On a biased dice, any even number appears four times as frequently as any odd number. If the dice is rolled thrice what is the probability that the sum of the scores on them is more than 16?

(A) $\frac{26}{375}$ (B) $\frac{112}{375}$

(C) $\frac{26}{3375}$ (D) $\frac{112}{3375}$

6. A five digit number is formed using the digits 0, 1, 2, 3, 4 and 5 at random but without repetition. The probability that the number so formed is divisible by 5 is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{4}{25}$ (D) $\frac{9}{25}$

7. If six people sit around a circular table, the probability that two specified persons always sit side by side is

(A) $\frac{14}{15}$ (B) $\frac{11}{15}$

(C) $\frac{2}{5}$ (D) $\frac{4}{15}$

8. Eight letters are to be placed in eight addressed envelopes. If the letters are placed at random into the envelopes, the probability that exactly one letter is placed in a wrong addressed envelopes is

(A) $\frac{1}{6}$ (B) $\frac{1}{8!}$

(C) $\frac{1}{7!}$ (D) None of these

9. A puzzle in logic was given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. The probability that the problem being solved is

(A) $\frac{29}{32}$ (B) $\frac{31}{32}$

(C) $\frac{1}{8}$ (D) $\frac{7}{8}$

10. If A and B are two events of an experiment such that $P(A \cup B) = \frac{3}{4}$, $P(A) = \frac{7}{20}$, then find $P(B)$ given that

(i) A and B are mutually exclusive

(A) $\frac{1}{4}$ (B) $\frac{1}{5}$

(C) $\frac{3}{5}$ (D) $\frac{2}{5}$

(ii) A and B are equally likely

(A) $\frac{7}{20}$ (B) $\frac{3}{4}$

(C) $\frac{2}{5}$ (D) $\frac{13}{20}$

(iii) A and B are independent events

(A) $\frac{7}{13}$ (B) $\frac{8}{13}$

(C) $\frac{6}{13}$ (D) $\frac{2}{5}$

11. The probability that a square selected at random from *a* 8×8 chessboard is of size 3×3 is

(A) $\frac{8}{51}$ (B) $\frac{14}{17}$

(C) $\frac{3}{17}$ (D) $\frac{25}{204}$

12. A dice has two of its sides painted pink, two blue and two green. If the dice is rolled twice the probability that same colour appears both the times is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{7}{9}$ (D) $\frac{8}{9}$

13. X and Y are independent events. The probability that both X and Y occur is $\frac{1}{8}$ and the probability that neither of these occur is $\frac{3}{8}$. The probability of occurrence of X can be

(A) $\frac{2}{3}$ (B) $\frac{1}{4}$

(C) $\frac{1}{3}$ (D) $\frac{3}{4}$

14. A bag contains 12 cards. 5 of these cards have the letter 'M' printed on them, 3 cards have the letter 'A' printed

on them and the remaining cards have the letter 'N' printed on them. If three cards are picked up one after the other at random, and placed on a table in that order, then what is the probability that the word formed will be 'MAN'?

(A) $\frac{5}{44}$ (B) $\frac{1}{22}$

(C) $\frac{3}{22}$ (D) $\frac{3}{44}$

15. A and B pick a card at random from a well shuffled pack of cards, one after the other replacing it every time till one of them gets a spade. The person who picks a spade is declared the winner. If A begins the game, then the probability that B wins the game is

(A) $\frac{5}{9}$ (B) $\frac{4}{9}$

(C) $\frac{3}{7}$ (D) $\frac{4}{7}$

16. A number is randomly chosen from the numbers 10 to 99. It is observed that the sum of the digits of the number is ten. Find the probability that it is divisible by five.

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) $\frac{2}{9}$

17. An unbiased coin is tossed a person gets ₹30 if the coin shows head, and he loses ₹15 if the coin shows tail. If three coins are tossed, the probability that the person gets ₹45 is

(A) $\frac{3}{8}$ (B) $\frac{1}{2}$

(C) $\frac{1}{10}$ (D) $\frac{1}{25}$

18. What is the probability of getting at least 6 heads when a coin is tossed 7 times if it is known that there are at least 5 heads?

(A) $\frac{5}{29}$ (B) $\frac{8}{29}$

(C) $\frac{9}{29}$ (D) None of these

19. If $P(A) = \frac{3}{5}$, $P(B^C) = \frac{6}{7}$ and $P(A \cap B) = \frac{1}{4}$, then find $P\left(\frac{A^c}{B^c}\right)$.

(A) $\frac{17}{60}$ (B) $\frac{71}{120}$

(C) $\frac{19}{60}$ (D) $\frac{29}{60}$

20. If two events A and B are such that $P(\bar{A}) = 0.4$, $P(B) = 0.7$ and $P(A \cap B) = 0.2$, then $P\left(\frac{B}{A \cup \bar{B}}\right)$ is

(A) $\frac{3}{5}$ (B) $\frac{2}{5}$

(C) $\frac{1}{4}$ (D) $\frac{4}{5}$

21. A cinema historian noted that for a brief period, all movies released were either directed by Nolan or starred Bale. Also no movie directed by Nolan starred Bale. The probability that a movie was directed by Nolan is 0.5, and the probability that a movie starred Bale is 0.5. The probability that a movie is a hit if directed by Nolan is 0.6, while the probability that a movie is a hit given that Bale acted in it is 0.4. Given that a movie is a hit, find the probability that it is directed by Nolan.

(A) 0.4 (B) 0.5

(C) 0.6 (D) 0.7

22. Probability mass function of a variate x is as follows:

x	0	1	2	3	4
$P(X = x)$	k	$2k$	$3k$	$4k$	$5k$

then $P(x \geq 3) =$

(A) $\frac{1}{3}$ (B) $\frac{4}{15}$

(C) $\frac{3}{5}$ (D) $\frac{5}{7}$

23. The expected number of trials required to open a door using a bunch of n keys of which only one is the correct key is

(A) $\frac{n}{2}$ (B) $\frac{n-1}{2}$

(C) $\frac{n+1}{2}$ (D) n

Direction for questions 24 and 25:

A variate x has the probability distribution as

x	4	8	12
$P(X = x)$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{1}{15}$

24. Values of $E(x)$ and $E(x^2)$ respectively are

(A) $\frac{104}{15}, \frac{160}{3}$ (B) $\frac{102}{15}, \frac{150}{3}$

(C) $\frac{21}{3}, \frac{160}{5}$ (D) $\frac{104}{15}, \frac{151}{3}$

25. The value of $E[(3x + 2)^2]$ is ________.

(A) 675.2 (B) 560.2

(C) 134.56 (D) 567.2

26. In the random experiment of drawing a card from 15 cards numbered 1 to 15, if x is the random variable defined by the number appeared on the card, then the expectation of x is

(A) 8 (B) 7
(C) 6 (D) 5

27. For a binominal distribution, mean is 6 and variance is 2. The number of Bernouli trials is

(A) 8 (B) 9
(C) 10 (D) 11

28. If $X(n, p)$ follows a binominal distribution with $n = 6$ such that $9P[X = 4] = P[X = 2]$, then $p =$

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$
(C) 1 (D) $\frac{1}{4}$

29. The variance of a Poisson variate is given to be 1. Then, $P(X = 3)$ is

(A) $\frac{1}{e}$ (B) $\frac{1}{2e}$
(C) $\frac{1}{3e}$ (D) $\frac{1}{6e}$

30. A random variable X follows a Poisson distribution such that $P[X = 1] = P[X = 2]$. Its mean and variance are, respectively,

(A) 1, 1 (B) 2, 2
(C) $\sqrt{3}, 2$ (D) $\sqrt{2}, \sqrt{2}$

31. The probability that a person hits a target is 0.003. What is the probability of hitting the target with 2 or more bullets if the number of shots is 2000?

(A) $1 - e^{-6}$ (B) $1 - e^{6}$
(C) $1 - 7e^{6}$ (D) $1 - 7e^{-6}$

32. The expected value of a random variable with uniform distribution over the interval (2, 5) is

(A) 2 (B) $2\frac{1}{2}$
(C) $3\frac{1}{2}$ (D) $4\frac{1}{2}$

33. If X is a continuous random variable with PDF $f(x) = \frac{1}{4}$ if $-2 \leq x \leq 2$ and $f(x) = 0$ elsewhere, the mean of X is ______.

(A) 1 (B) 1.5
(C) 2 (D) 0

34. If X is a uniformly distributed random variable in [1, 4] then $P\left(x > \frac{3}{2}\right)$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$
(C) $\frac{5}{6}$ (D) $\frac{1}{4}$

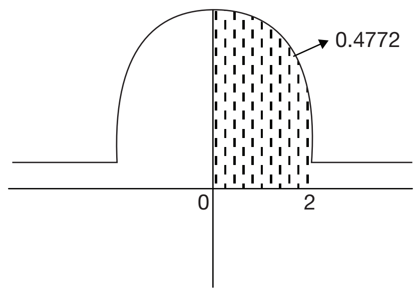
35. If X is a uniformly distributed random variable in [2, 5] then $E(X^2)$ is

(A) 2 (B) 8
(C) 13 (D) 15

36. If the life time of bulbs (in months) is exponential with mean 5 months, then the probability that the bulb lasts for atleast 7 months is

(A) 0.2466 (B) 0.7534
(C) 0.4932 (D) 0.5068

37. x is a normal variate with mean 35 and variance 25 probability of $31 \leq x < 45$ is $(-0.8 \leq z < 0 = 0.2881)$



(A) 0.6735 (B) 0.7563
(C) 0.7653 (D) 0.5736

38. Let X_1 and Y_1 be two discrete random variables with joint probability mass function as given below

X_1 \ Y_1	2	3	$P(X_1 = x_i)$
1	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$
4	$\frac{4}{15}$	$\frac{8}{15}$	$\frac{4}{5}$
$P(Y_1 = y_j)$	$\frac{1}{3}$	$\frac{2}{3}$	

Let X_2 and Y_2 be two discrete random variables with joint probability mass function given as follows:

X_2 \ Y_2	0	4	7	$P(X_2 = x_i)$
–1	$\frac{1}{7}$	$\frac{3}{14}$	$\frac{1}{14}$	$\frac{3}{7}$
3	$\frac{4}{21}$	$\frac{2}{7}$	$\frac{2}{21}$	$\frac{4}{7}$
$P(Y_2 = y_j)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$	

Which of the following statements is TRUE about the random variables X_1, X_2, Y_1 and Y_2?

(A) Only X_1 and Y_1 are independent.
(B) Only X_2 and Y_2 are independent.
(C) X_1 and Y_1 are independent as well as X_2 and Y_2 are independent.
(D) Neither X_1 and Y_1 are independent nor X_2 and Y_2 are independent.

39. If X and Y are two independent random variables with expectations 3 and 4 respectively. Then the expectation of X Y is
(A) 1 (B) 7
(C) 12 (D) 16

40. If X and Y are two independent random variables that are uniformly distributed over the same interval [2, 5] then $P\left(X \le \frac{11}{4}, Y \ge \frac{11}{3}\right)$ is
(A) $\frac{1}{9}$ (B) $\frac{2}{9}$
(C) $\frac{1}{3}$ (D) $\frac{4}{7}$

41. The mean of cubes of first 10 natural numbers is
(A) 305 (B) 300
(C) 302.5 (D) 310

42. The mean of 25 observations was found to be 38. It was later discovered that 23 and 38 were misread as 25 and 36, then the mean is
(A) 32 (B) 36
(C) 38 (D) None of these

43. If 3, 2 and 9 occur with frequencies 2, 5 and 3 respectively, then their arithmetic mean is
(A) 4.3 (B) 5
(C) 6 (D) 4.8

44. The median of first ten prime numbers is
(A) 11 (B) 13
(C) 12 (D) 10

45. If the mean of a set of 12 observations is 10 and another set of 8 observations is 12, then the mean of combined set is
(A) 12.6 (B) 10.8
(C) 12.8 (D) 10.6

46. The mode of a distribution of 13 and its mean is 4 then its median is
(A) 7 (B) 9
(C) 8 (D) 11

47. Consider the non-decreasing series of the numbers, 1, 8, 8, 13, 14, 14, x, y, 18, 20, 31, 34, 38 and 40. If the median of the series is 15, then the mode of the series is
(A) 14 (B) 16
(C) 18 (D) Cannot be determined

48. The standard deviation of 5, 5, 5, 5, 5, 5, 5, 13 is
(A) $2\sqrt{2}$ (B) $\sqrt{6}$
(C) 5 (D) $\sqrt{7}$

49. If the standard deviation of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 is M, then the standard deviation of 101, 102, 103, 104, … and 111 is
(A) M (B) $100 + M$
(C) $100 - M$ (D) $M - 100$

50. If the standard deviation of 10, 20, 30, 40 and 50 is S, then the standard deviation of 20, 30, 40, 50 and 60 is
(A) S (B) $S + 10$
(C) $S - 10$ (D) $10S$

51. The arithmetic mean of five observations is 6.4 and the variance is 8.24. If three of the observations are 3, 4, 8, then find the other two observations.
(A) 6, 11 (B) 10, 7
(C) 8, 9 (D) 5, 12

52. The director of a sports academy claims that the average height of sports persons in their academy is more than 170 cms. A random sample of 40 sports persons of that academy has an average height of 174 cm with a standard deviation of 15 cm. Then the claim of the director can be accepted with ______.
(A) both 1% as well as 5% levels of significance
(B) 5% level of significance but not with 1% level of significance
(C) neither 5% nor 1% levels of significance
(D) no level of significance

53. In a survey conducted on the increase in pay packages of male and female managers across IT industry by taking a random sample of 32 male managers and another random sample of 36 female managers, the following information was derived.

	Sample Size	Average Increases in Pay Package/ Annum	Standard Deviation of Increase in Pay Package/Annum
Male Managers	32	20%	4%
Female Managers	36	17%	3%

The standard error of the difference between the average increase in pay packages is ______
(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3}{\sqrt{2}}$
(C) $\frac{3}{2}$ (D) $\frac{3}{4}$

54. Match the following:

	List I		List II
P.	Standard error	**1.**	Level of significance
Q.	Type I error	**2.**	Standard deviation of the sampling distribution of a statistic
R.	Type II error	**3.**	Accepting the null hypothesis when it should be rejected
S.	Size of a test	**4.**	Rejecting the null hypothesis when it should be accepted

Codes:

(A) P – 1, Q – 2, R – 3, S – 4
(B) P – 2, Q – 4, R – 3, S – 1
(C) P – 2, Q – 3, R – 4, S – 1
(D) P – 1, Q – 3, R – 2, S – 4

55. If the population distribution is not normal, then for which of the following sample sizes, the sampling distribution of a statistic will always be normal?

(A) 4 (B) 8
(C) 16 (D) 32

56. One can reduce both type I and type II errors by

(A) reducing the sample size.
(B) increasing the sample size.
(C) changing the null hypothesis.
(D) changing the alternative hypothesis.

57. Assume that the marks obtained in GATE by the students of Civil Engineering follow normal distribution. The mean and standard deviation of marks of two groups with 10 students in each group are as given below.

	Mean	Standard Deviation
Group 1	57	3.36
Group 2	53	5.44

Then the test statistic that is to be used to test whether the difference in means of marks is significant or not is _______.

(A) 1.8768 (B) 2.3541
(C) 3.6172 (D) 4.5376

58. To test whether there is any significant difference in the marks scored by 13 students in a test before and after a yoga course using the t-distribution, the degrees of freedom to be taken is _______.

(A) 12 (B) 13
(C) 24 (D) 26

59. If $F_{0.01}$ with the degrees of freedom $\upsilon_1 = 8$ and $\upsilon_2 = 24$, is 3.36, then the value of $F_{0.99}$ with the degrees of freedom $\upsilon_1 = 24$ and $\upsilon_2 = 8$ is _______.

(A) 0.64 (B) 6.33
(C) 4.95 (D) 0.2976

60. Fitting a normal distribution to the following data:

Class	5–9	10–14	15–19	20–24	25–29	30–34	35–39
Observed frequencies (O_i)	1	10	37	36	13	2	1

The respective expected frequencies when the data is fitted to normal distribution are:

Expected Frequencies (E_i): 2, 12, 32, 36, 15, 3 and 0

The critical value to test the goodness of fit of normal distribution to the above data with 5% level of significance is _______.

(A) 14.067 (B) 9.488
(C) 3.841 (D) 2.706

61. For the contingency table given below, the test stastic (chi-square value) is _______.

A \ B		
	10	20
	30	40

(A) 0.7936 (B) 7.8361
(C) 4.8312 (D) 3.8142

PREVIOUS YEARS' QUESTIONS

1. If the standard deviation of the spot speed of vehicles in a highway is 8.8 km/h and the mean speed of the vehicles is 33 km/h, the coefficient of variation in speed is **[GATE, 2007]**

(A) 0.1517 (B) 0.1867
(C) 0.2666 (D) 0.3646

2. A person on a trip has a choice between private car and public transport. The probability of using a private car is 0.45. while using the public transport, further choices available are bus and metro, out of which the probability of commuting by a bus is 0.55. In such a situation, the probability (rounded up to two decimals) of using a car, bus and metro, respectively would be **[GATE, 2008]**

(A) 0.45, 0.30 and 0.25
(B) 0.45, 0.25 and 0.30
(C) 0.45, 0.55, and 0.00
(D) 0.45, 0.35 and 0.20

3. If probability density function of a random variable x is $F(x) = x^2$ for $-1 \le x \le 1$ and $= 0$ for other value of x then, the percentage probability $P\left(-\frac{1}{3} \le x \le \frac{1}{3}\right)$ is **[GATE, 2008]**

(A) 0.247 (B) 2.47
(C) 24.7 (D) 247

4. Two coins are simultaneously tossed. The probability of two heads simultaneously appearing is **[GATE, 2010]**

(A) $\frac{1}{8}$ (B) $\frac{1}{6}$

(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

5. There are two containers, with one containing 4 Red and 3 Green balls and the other containing 3 Blue and 4 Green balls. One ball is drawn at random from

each container. The probability that one of the ball is Red and the other is Blue will be **[GATE, 2011]**

(A) $\frac{1}{7}$ (B) $\frac{9}{49}$

(C) $\frac{12}{49}$ (D) $\frac{3}{7}$

6. In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is **[GATE, 2012]**

(A) $\frac{1}{32}$ (B) $\frac{2}{32}$

(C) $\frac{3}{32}$ (D) $\frac{6}{32}$

7. The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is **[GATE, 2012]**

(A) <50% (B) 50%
(C) 75% (D) 100%

8. Find the value of λ such that the function $f(x)$ is a valid probability density function

$F(x) = \lambda(x - 1)(2 - x)$ for $1 \le x \le 2$

$= 0$ otherwise **[GATE, 2013]**

9. A fair (unbiased) coin was tossed four times in succession and resulted in the following outcomes; (i) Head, (ii) Head (iii) Head (iv) Head. The probability of obtaining a 'Tail' when the coin is tossed again is **[GATE, 2014]**

(A) 0 (B) $\frac{1}{2}$

(C) $\frac{4}{5}$ (D) $\frac{1}{5}$

10. The probability density function of evaporation E on any day during a year in a watershed is given by

$$F(E) = \begin{cases} \frac{1}{5} & 0 \le E \le 5 \text{ mm/day} \\ 0, & \text{otherwis} \end{cases}$$

The probability the E lies in between 2 and 4 mm/day in a day in the watershed is (in decimal). **[GATE, 2014]**

11. A traffic office imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days is independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is ______. **[GATE, 2014]**

12. If $\{x\}$ is a continuous, real valued random variable defined over the interval $(-\infty, +\infty)$ and its occurrence is defined by the density function given as $f(x) = \frac{1}{\sqrt{2\pi} * b} e^{\frac{1}{2}\left(\frac{x-a}{b}\right)^2}$ where 'a' and 'b' are the statistical attributes of the random variable $\{x\}$. The value of the integral $\int_{-\infty}^{a} \frac{1}{\sqrt{2\pi} * b} e^{\frac{1}{2}\left(\frac{x-a}{b}\right)^2} dx$ is **[GATE, 2014]**

(A) 1 (B) 0.5

(C) π (D) $\frac{\pi}{2}$

13. Consider the following probability mass function (pmf) of a random variable X:

$$p(x, q) = \begin{cases} q & \text{if } X = 0 \\ 1 - q & \text{if } X = 1 \\ 0 & \text{othewise} \end{cases}$$

If $q = 0.4$, the variance of X is ________. **[GATE, 2015]**

14. The probability density function of a random variable, x is

$f(x) = \frac{x}{4}(4 - x^2)$ for $0 \le x \le 2$

$= 0$ otherwise

The mean, μ_x of the random variable is ______. **[GATE, 2015]**

15. X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^C) = 0.7$. Which one of the following is the value of $P(X \cup Y)$? **[GATE, 2016]**

(A) 0.7 (B) 0.5
(C) 0.4 (D) 0.3

16. Probability density function of a random variable X is given below

$$f(x) = \begin{cases} 0.25 \text{ if } 1 \le x \le 5 \\ 0 \quad \text{otherwise} \end{cases}$$

$P(x \le 4)$ is **[GATE, 2016]**

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{1}{8}$

17. If $f(x)$ and $g(x)$ are two probability density functions,

$$f(x) = \begin{cases} \frac{x}{a}+1 & : -a \le x < 0 \\ -\frac{x}{a}+1 & : 0 \le x \le a \\ 0 & : \text{otherwise} \end{cases}$$

$$g(x) = \begin{cases} -\frac{x}{a} & : -a \le x < 0 \\ \frac{x}{a} & : 0 \le x \le a \\ 0 & : \text{otherwise} \end{cases}$$

Which one of the following statement is true?

[GATE, 2016]

(A) Mean of $f(x)$ and $g(x)$ are same; Variance of $f(x)$ and $g(x)$ are same.
(B) Mean of $f(x)$ and $g(x)$ are same; Variance of $f(x)$ and $g(x)$ are different.
(C) Mean of $f(x)$ and $g(x)$ are different; Variance of $f(x)$ and $g(x)$ are same.
(D) Mean of $f(x)$ and $g(x)$ are different; Variance of $f(x)$ and $g(x)$ are different.

18. The spot speeds (expressed in km/h) observed at a road section are 66, 62, 45, 79, 32, 51, 56, 60, 53 and 49. The median speed (expressed in km/h) is ______. (Note: answer with one decimal accuracy)

[GATE, 2016]

19. Type II error in hypothesis testing is **[GATE, 2016]**
(A) acceptance of the null hypothesis when it is false and should be rejected.
(B) rejection of the null hypothesis when it is true and should be accepted.
(C) rejection of the null hypothesis when it is false and should be rejected.
(D) acceptance of the null hypothesis when it is true and should be accepted.

Answer Keys

Exercises

1. C	**2.** C	**3.** B	**4.** A	**5.** D	**6.** D	**7.** C	**8.** D	**9.** A	
10. (i) D	(ii) A	(iii) B	**11.** C	**12.** A	**13.** B	**14.** B	**15.** C	**16.** A	**17.** A
18. B	**19.** B	**20.** B	**21.** C	**22.** C	**23.** C	**24.** A	**25.** D	**26.** A	**27.** B
28. D	**29.** D	**30.** B	**31.** D	**32.** C	**33.** D	**34.** C	**35.** C	**36.** A	**37.** C
38. C	**39.** C	**40.** A	**41.** C	**42.** C	**43.** A	**44.** C	**45.** B	**46.** A	**47.** D
48. D	**49.** A	**50.** A	**51.** A	**52.** B	**53.** A	**54.** B	**55.** D	**56.** B	**57.** A
58. A	**59.** D	**60.** C	**61.** A						

Previous Years' Questions

1. C	**2.** A	**3.** B	**4.** C	**5.** C	**6.** D	**7.** A	**8.** 6	**9.** B	**10.** 0.4
11. 0.26 to 0.27		**12.** B	**13.** 0.23 to 0.25		**14.** 1.06 to 1.07		**15.** A	**16.** A	**17.** B
18. 54.5	**19.** A								

Chapter 6

Numerical Methods

CHAPTER HIGHLIGHTS

- *Numerical methods*
- *Accuracy and precision*
- *Curve fitting*
- *Numerical integration*
- *Numerical solutions of ordinary differential equations*
- *Multi-step methods*
- *Runge–Kutta methods*
- *Predictor-corrector methods*

Numerical Methods

We encounter problems in Engineering mathematics for which analytical methods are not available to find solutions. Further, it may be sufficient in engineering applications to find approximate solutions. The numerical methods offer us approximate solutions.

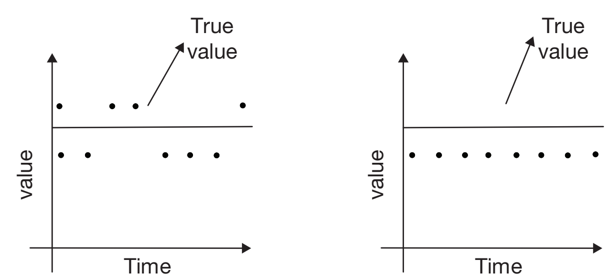
1. Methods for finding roots of algebraic or transdental equations
2. Solutions to system of linear equation
3. Numerical Integration
4. Numerical solutions of ordinary differential equation.

Accuracy and Precision

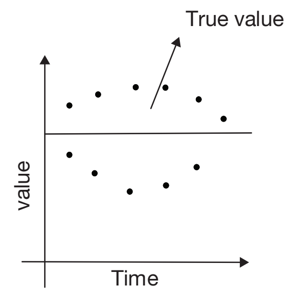
Solutions of problems computed by numerical methods are approximate. Errors associated with calculations can be characterized with reference to accuracy and precision.

Accuracy: Accuracy refers to how closely a computed value agrees with the true value.

Precision: Precision refers to how closely computed values agree with each other after repeated iterations. The following figures illustrate the difference between accuracy and precision where the horizontal line denote the true value (or) actual value of the solution where as the dots denote the values computed by a numerical method.



Accurate but not precise **Precise but not accurate**



Neither accurate nor precise

Errors in the solutions obtained by numerical methods: As the numerical methods give approximate solutions, these solutions contain errors.

Let x denote the actual value (or) true value and let $\bar{x}$ denote an approximate value of the solution obtained by a numerical method.

Error $= \in = x - \bar{x}$

Absolute error $= |\in| = |x - \bar{x}|$

Relative error $= \in_r = \dfrac{|\in|}{|x|} = \dfrac{|x - \bar{x}|}{|x|}$

Percentage error $= \in_p = \in_r \times 100 = \dfrac{|x - \bar{x}|}{x} \times 100.$

Types of Errors

Inherent Error

The error which is already present in the statement of the problem before its solution, is called the inherent error.

This type of errors arise due to any one or more of the following reasons.

- Wrong formulation of the problem
- Unsuitable solution procedure
- Invalid assumptions in the formulation
- Inaccurate data

Round off Error

Real numbers such as $\dfrac{5}{6}, \sqrt{2}, \pi$, etc., contain an infinite number of digits when expressed in decimal form. In general, in scientific and engineering computations, a real number x is represented as $x = \pm\, 0.\, d_1 d_2 d_3 \ldots d_n \ldots \times 10^k$, known as floating point form of x.

(where $d_1, d_2, \ldots d_n \ldots$ are all digits from 0 to 9 and k is a non zero integer). Each digit $d_1, d_2 \ldots$ other than the leading zeros (the zeros that occur before the first non-zero digit) is called a significant digit. As its not possible to retain infinite number of digits in a number, we round off the number to, say n significant digits.

To round off a number to n significant digits, proceed as follows:

Possibility	Procedure to Follow
The $(n + 1)$th digit is less than 5 (OR) The $(n + 1)$th digit is equal to 5 and the nth digit is even	Discard all the digits to the right of nth digit and leave the nth digit as it is
The $(n + 1)$th digit is greater than 5 (OR) The $(n + 1)$th digit is equal to 5 and the nth digit is odd.	Discard all the digits to the right of nth digit and increase the nth digit by 1.

For example,

Consider the number 25.31465.

When written in floating point form

$25.31465 = 0.2531465 \times 10^2$

$\approx 0.253146 \times 10^2$ (Rounded off to six significant digits)

$\approx 0.25315 \times 10^2$ (Rounded off to five significant digits)

$\approx 0.2532 \times 10^2$ (Rounded off to four significant digits)

$\approx 0.253 \times 10^2$ (Rounded off to three significant digits)

Definition The difference between the true value and its rounded off value is called the rounded off error.

- If x is the true value and x^* is its rounded off value such that $|x - x^*| \le 0.5 \times 10^{-m}$ (OR) $|x - x^*| \le 5 \times 10^{-(m+1)}$ then x^* is said to denote x correct to m significant digits.

Truncation Error The error in a method, which occurred because some series (finite or infinite) is truncated to a fewer (and finite) number of terms is called the truncation error.

For instance,

$$\text{Let } f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) + \cdots + \frac{(x - x_0)^{m-1}}{(m-1)} f(m-1)(x_0) + \frac{(x - x_0)^m}{m} f^{(m)}(x_0) + \cdots \infty \tag{1}$$

denote the Taylor's series expansion of $f(x)$ about $x = x_0$.

If we retain the first m terms, we get

$$f(x) \approx f(x_0) + (x - x_0) f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) + \cdots + \frac{(x - x_0)^{m-1}}{(m-1)!} f^{(m-1)}(x_0) \tag{2}$$

where the series of infinite terms $\dfrac{(x - x_0)^m}{m!}$

$$f^{(m)}(x_0) + \frac{(x - x_0)^{m-1}}{(m+1)!} f^{(m+1)}(x_0) + \cdots \infty \tag{3}$$

is neglected.

The first term in this neglected part of the series is called the principal part of the truncation error or simply the truncation error.

$$\therefore \text{Truncation error} = TE = \frac{(x - x_0)^m}{m!} f^{(m)}(\xi);\ x_0 < \xi < x$$

As ξ is an unknown function of x, we have

$$|TE| \le \frac{1}{m!} \max[|(x - x_0)^m| \, M_m]$$

where $M_m = [a, b] \max|f^{(m)}(x)|$

SOLVED EXAMPLES

Example 1

If the number $\frac{\pi}{4} = 0.785398163$ is approximated by $\frac{11}{14}$, then

(i) Find the number of digits upto which, this approximation is accurate.

(ii) Find the absolute and the percentage errors.

Solution

Given $\frac{\pi}{4} = 0.785398163$

Let $x = \frac{\pi}{4} = 0.785398163$ (Exact value) and $\bar{x} = \frac{11}{14}$

$= 0.785714285 \left(\text{Approximate value of } \frac{\pi}{4}\right)$

(i) $|x - \bar{x}| = \left|\frac{\pi}{4} - \frac{11}{14}\right|$

$$= |0.785398163 - 0.785714285|$$
$$= 3.16122 \times 10^{-4}$$
$$\leq 5 \times 10^{-4}$$

$\therefore$ The approximation $\frac{11}{14}$ to $\frac{\pi}{4}$ is accurate upto three significant digits

(ii) Absolute error $= |x - \bar{x}|$

$$= \frac{11}{14} - \frac{\pi}{4} = 3.16122 \times 10^{-4}$$

Percentage error $= \frac{|x - \bar{x}|}{x} \times 100 = 0.04\%$.

Example 2

Using the Taylor's series expansion about $x = 0$, find a second degree polynomial approximation to $f(x) = \sqrt{1+3x}$. Also find the maximum error for this approximation when $x \in [0, 1]$.

Solution

We know that the Taylor's series expansion of $f(x)$ about $x = 0$ is

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots + \cdots \infty \quad (1)$$

$\therefore$ Considering the terms upto second degree, we have

$$f(x) \approx f(0) + x f'(0) + \frac{x^2}{2!} f''(0) \quad (2)$$

Here $f(x) = \sqrt{1+3x} \Rightarrow f(0) = 1$

$$f'(x) = \frac{3}{2\sqrt{1+3x}} \Rightarrow f'(0) = \frac{3}{2}$$

$$f''(x) = \frac{-9}{4(1-3x)^{\frac{3}{2}}} \Rightarrow f''(0) = \frac{-9}{4}$$

and $f'''(x) = \frac{27}{8} \times \frac{1}{(1+3x)^{\frac{5}{2}}}$

$\therefore$ Substituting these in (2), we get

$$f(x) = \sqrt{1+3x} \approx 1 + \frac{3}{2} \times - \frac{9}{4}\frac{x^2}{2!}$$

$$= 1 + \frac{3}{2} \times - \frac{9}{8} x^2$$

Truncation error $= \frac{x^3}{3!} f'''(0)$

$$\leq \frac{1}{3!}\left(\underset{0 \leq x \leq 1}{\text{Max}} \quad x^3\right)$$

$$\left(\underset{0\leq x\leq 1}{\text{Max}} \frac{27}{8(1+3x)^{5/2}}\right)$$

$$= \frac{1}{3!}(1)\left(\frac{27}{8}\right)$$

$= 0.5625$.

Methods for Finding the Real Roots (Zeros) of $f(x) = 0$

The equation of the form $f(x) = 0$ is called an Algebraic or Transcendental according as $f(x)$ is purely a polynomial in x or contains some other functions such as exponential, logarithmic and trigonometric functions etc.

Examples:

1. $x^9 + 8x^5 - 4x^3 - 11x + 3 = 0 \rightarrow$ Algebraic equation
2. $10x^4 - \log(x^2 - 3) + e^{-x}\sin x + \tan^2 x = 0 \rightarrow$ Transcendental equation

In this chapter, we obtain the solution of an equation $f(x) = 0$, i.e., we mean to find the zeros of $f(x)$.

We shall now discuss few methods to find the real roots of both algebraic (with numerical coefficients) and transcendental equations.

We first find an approximate value of the root of the given equation and then successively improve it to some desired degree of accuracy.

We start with an initial approximate value, say x_0, and then find the better approximations successively $x_1, x_2, x_3 \ldots, x_n$ by repeating the same method.

If the successive approximations at each step of a method approach the root more and more closely, we say that the method converges.

The Intermediate Value Theorem

If a function $f(x)$ is continuous between a and b and $f(a)$ and $f(b)$ are of opposite signs, then there exists at least one root say α between a and b of the equation

$f(x) = 0$, i.e., $f(\alpha) = 0$

NOTE

Root 'α' of $f(x) = 0$, will be unique in (a, b) if $f'(x)$ has the same sign in (a, b) (i.e., $f'(x) > 0$ or $f'(x) < 0$ in $a < x < b$)

Relations between Roots and Coefficients

An nth order equation has n roots. Corresponding to every root, there is a factor. If α is a root of $f(x) = 0$, then $x - \alpha$ is a factor of $f(x)$. Sometimes $(x - \alpha)^2$ may also be a factor. In such a case, α is said to be a double root. Similarly equations can have triple roots, quadruple roots and roots of multiplicity m. If m is the greatest value of k, for which $(x - \alpha)^k$ is a factor of $f(a)$, then α is said to be a root of multiplicity m. If all the roots are counted by taking their multiplicity into account, the number of roots is equal to n, the degree of the equation.

If $\alpha_1, \alpha_2, \ldots, \alpha_n$ (not necessarily distinct) are the roots of $f(x) = 0$, then

$$\begin{aligned} f(x) &= a_n(x - \alpha_1)(x - \alpha_2) \ldots (x - \alpha_n) \\ &= a_n [x^n - S_1 x^{n-1} + S_2 x^{n-2} + \cdots + (-1)^n S_n] \end{aligned}$$

Where

S_1 = The sum of the roots

S_2 = The sum of the products of the roots taken 2 at a time

S_3 = The sum of the product of the roots taken 3 at a time and so on.

S_n = The 'sum' of the product of the roots taken n (or all) at a time. Thus, S_n is a single term.

$S_n = \alpha_1 \alpha_2 \ldots \alpha_n$

Let us write down the polynomial $f(x)$ in two forms:

The standard form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0$$

In terms of the roots of the corresponding equation.

$$f(x) = a_n [x_n - S_1 x^{n-1} + S_2 x^{n-2} + \cdots + (-1)^{x-1} S_{n-1} x + (-1)^{nS_n}]$$

These polynomials are identically equal, i.e., equal for all values of x. Therefore the corresponding coefficients are equal. The sum of the roots $S_1 = -\dfrac{a_{n-1}}{a_n}$.

The sum of the products of the roots, taken two at a time, $S_2 = \dfrac{a_{n-2}}{a_n}$.

The sum of the products of the roots, taken three at a time, $S_3 = -\dfrac{a_{n-3}}{a_n}$ and so on.

The 'sum' of the 'products' of the roots taken $m (m \le n)$ at a time $S_m = \Sigma \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_m = (-1)^m \dfrac{a_{n-m}}{a_n}$.

$$\therefore \quad S_n = \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_n = (-1)^n \frac{a_0}{a_n}$$

For example, consider the polynomial equation

$$(x - 1)(x - 2)(x - 3) = x^3 - 6x^2 + 11x - 6 = 0$$

(We can see immediately that the roots are 1, 2, 3)

The sum of roots $= (1 + 2 + 3) = -\dfrac{(-6)}{1}$

The sum of the products of the roots, taken two at a time

$S_2 = 1(2) + 1(3) + 2(3) = 11 = \dfrac{11}{1}$

We can drop the word 'sum' and 'products' for the last relation, because there is only one term (only one product).

The product $= 1(2)(3) = 6 = -\dfrac{(-6)}{1}$.

Roots of Equations and Descartes' Rule

If the coefficients are all real and the complex number z_1, is a root of $f(x) = 0$, then the conjugate of z_1, viz, $\bar{z}_1$ is also a root of $f(x) = 0$. Thus, for equations with real, coefficients, complex roots occur in pairs.

A consequence of this is that any equation of an odd degree must have at least one real root.

The number of roots is related to very simple properties of the equation as illustrated below.

Let α_1 be a positive root, i.e., $x - \alpha_1$, is a factor.

Let α_2 be another positive root, i.e., $x^2 - (\alpha_1 + \alpha_2)x + \alpha_1 \alpha_2$ is a factor.

Let α_3 be another positive root i.e., $x^3 - (\alpha_1 + \alpha_2 + \alpha_3)x_2 + (\alpha_1 \alpha_2 + \alpha_2 \alpha_3 + \alpha_3 \alpha_1)x - \alpha_1 \alpha_2 \alpha_3$ is a factor.

We note that every positive root introduces a sign change in the polynomial. For 1 root, there is 1 sign change (the coefficient of x is positive and $-\alpha_1$ is negative)

The second root results in a second sign change [$x^2 - (\alpha_1 + \alpha_2)x + \alpha_1 \alpha_2$ has 2 sign changes] and so on.

But every sign change need not correspond to a real positive root. (For example, $x^2 - 2x + 4$ has two sign changes but the corresponding equation $x^2 - 2x + 4 = 0$ has no real roots.

The number of positive roots is at the most equal to the number of sign changes. It could also be less than that by 2, 4…, i.e., if there are k sign changes in $f(x)$, the number of positive roots could be $k, k - 2, k - 4, \ldots$

This is called **Descartes' Rule of Signs**. This rule can be extended to negative roots as follows. The number of negative roots of $f(x) = 0$ is equal to the number of positive roots of $g(x) = f(-x) = 0$

For example, consider $f(x) = x^5 - 3x^3 + 6x^2 - 28x + 24$. There are 4 sign changes in $f(x)$

$\therefore$ The number of positive roots could be 4, 2 or 0.

$$\begin{aligned} \text{Consider } g(x) &= f(-x)(-x)^5 - 3(-x)^3 + 6(-x)^2 - 28(-x) + 24 \\ &= -x^5 + 3x^3 + 6x^2 + 28x + 24 \end{aligned}$$

There is only one sign change in $f(-x)$.

$\therefore$ The number of negative roots of $f(x) = 0$ is 1. (It can't be –1, –3, …).

The following table shows the various possibilities for the roots.

Negative	Positive	Complex
1	4	0
1	2	2
1	0	4

We have considered one specific equation and this specific equation has 5 specific roots. We can use more advanced techniques to find the actual roots. But even without that, using only Descartes rule, we expect exactly one of the 3 situations shown in the table above.

Example 3

Find the nature of roots of the equation, $f(x) = x^3 + x - 2 = 0$.

Solution

There is only 1 change of sign in $f(x)$.

We know that when $f(x)$ has r changes of sign then $f(x)$ has $r, r-2, r-4, \ldots$ positive roots.

$\therefore$ $f(x) = 0$ has one positive root.

Now $f(-x) \equiv -x^3 - x - 2 = 0$. $q = 0$

Since there is no change of sign in $f(-x)$, $f(x)$ has no negative roots. The number of complex roots is even.

$\therefore$ The equation has one positive root, and two complex roots.

Hence $f(x) = 0$ has 1 real root and two complex roots.

Example 4

How many non real-roots does the equation $x^4 - 2x^2 + 3x - 2 = 0$ have?

Solution

Let $f(x) = x^4 - 2x^2 + 3x - 2$

$f(x) = 0$ has 3 sign changes

$\therefore$ $f(x)$ has 3 or 1 positive roots.

$f(-x) = x^4 - 2x^2 - 3x - 2$

$\therefore$ $f(-x)$ has one sign change

$\therefore$ $f(x)$ has exactly one negative root.

As the sum of the co-efficient of $f(x)$ is zero,

$x = 1$ is a root of $f(x) = 0$

$\therefore$ $f(x) = (x-1)(x^3 + x^2 - x + 2) = (x-1)\,f_1(x)$. By trial, $f_1(-2) = 0$

$\therefore$ $f_1(x) = (x+2)(x^2 - x + 1)$

We can see that $x^2 - x + 1 = 0$ has two non-real roots.

$\therefore$ $f(x)$ has one positive, one negative and two non-real roots.

Example 3

If $p-q, p, p+q$ are the roots of the equation $x^3 - 18x^2 + 99x - 162 = 0$, then find the values of p and q.

Solution

Given $p-q, p, p+q$ are the roots of the equation.

$\therefore$ The sum of the roots is $(p-q) + p + (p+q) = 18$

$\Rightarrow \quad 3p = 18 \quad \Rightarrow \quad p = 6$

and the product of the roots is $(p-q)\,p(p+q) = 162$

$$p^2 - q^2 = \frac{162}{6} = 27 \quad \Rightarrow \quad 36 - q^2 = 27$$

$\Rightarrow q = \pm 3 \quad \therefore \quad p = 6$ and $q = \pm 3$.

Bisection Method (Bolzano Method) or (Halving Method)

Consider the equation $f(x) = 0$ (1)

If $f(x)$ is continuous between a and b and $f(a)\,f(b) > 0$, then there exists one root between a and b. Let $f(a)$ be negative and $f(b)$ be positive. The bisection method isolates the root in $[a, b]$ by halving process, approximately dividing the given interval $[a, b]$ into two, four, eight, etc. equal parts.

Thus, the first approximation to the root is given by:

$$x_0 = \frac{a+b}{2}$$

$$a \text{ — — —}\underset{\frac{a+b}{2}}{\bullet}\text{— — — } b$$

If $f(x_0) = 0$, then x_0 is a root, otherwise the root lies either between a and x_0 or x_0 and b depending on whether $f(x_0)$ is positive or negative. We again bisect the interval and repeat the process until the root is obtained to desired accuracy.

Example 4

Find a real root of the equation $f(x) = x^3 - 2x^2 + 3x - 1$ on the interval (0, 1) using bisection method with four iterations.

Solution

We have $f(0) = -1 < 0$ and

$$f(1) = 1 - 2 + 3 - 1 = 1 > 0$$

$\therefore$ A root lies between 0 and 1

$\therefore$ The first approximation to the root is $\dfrac{0+1}{2} = 0.5$. Now $f(0.5) = (0.5)^3 - 2(0.5)^2 + 3(0.5) - 1 = 0.125 > 0$ and $f(0) < 0$

$\therefore$ The root lies between 0 and 0.5. The second approximation to the root is $\dfrac{0+0.5}{2} = 0.25$.

Now $f(0.25) = (0.25)^3 - 2(0.25)^2 + 3(0.25) - 1$
$= -0.359 < 0$ and $f(0.5) > 0$

$\therefore$ The root lies between 0.25 and 0.5.

$\therefore$ The third approximation to the root is $\dfrac{0.25+0.5}{2} = \dfrac{0.75}{2} = 0.375$.

Now $f(0.375) = (0.375)^3 - 2(0.375)^2 + 3(0.375) - 1$

$= -0.103 < 0$ and $f(0.5) > 0$

∴ The root lies between 0.375 and 0.5.

∴ The fourth approximation to the root is $\frac{0.375+0.5}{2}$

$$= \frac{0.875}{2} = 0.4375.$$

Convergence of Bisection Method

If $x_1, x_2, x_3, \ldots, x_n$ is the sequence of midpoints obtained by bisection method, then $|c - x_n| \le \frac{b-a}{2^n}$, $n = 1, 2, 3 \ldots$ where 'c' is between a and b.

NOTE

In bisection method, the convergence is very slow but definite. The order of convergence is linear or of first order.

Regula Falsi Method or (The Method of False Position)

In this method, to find the real root of the equation $f(x) = 0$, we consider a sufficiently small interval (a, b), $a < b$ such that $f(a)$ and $f(b)$ will have opposite signs. This implies a root lies between a and b according to intermediate value theorem. Also the curve $y = f(x)$ will meet the X-axis at some point between $A\,[a, f(a)]$ and $B\,[b, f(b)]$. The equation of the chord joining $A\,[a, f(a)]$ and $B\,[b, f(b)]$ is given by:

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a) \qquad (1)$$

The point of intersection of the chord (1) with X-axis is given by $y = 0$ in Eq. (1)

$$-f(a) = \frac{f(b) - f(a)}{b - a}(x - a),$$

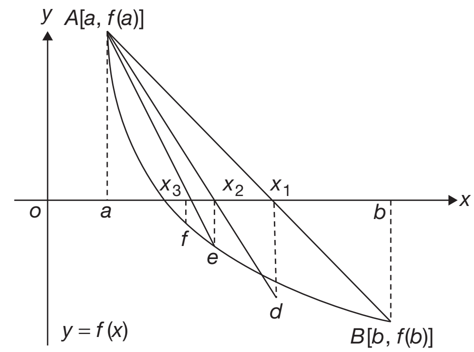
$$\Rightarrow \quad x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

∴ The first approximation $x_1 = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$ (2)

If $f(x_1) = 0$, then x_1 is the root. If $f(x_1) \ne 0$ and if $f(x_1)$ and $f(a)$ have opposite signs, the second approximation

$$x_2 = \frac{af(x_1) - x_1 f(a)}{f(x_1) - f(a)} \qquad (3)$$

Proceeding in the same way, we get x_3, x_4 and so on. Geometrically, the required root is shown in the figure below.



NOTE

This method is faster than the first order fixed point iteration.

Convergence of Regula Falsi Method

The order of convergence of the method of false position is greater than 1.

The Secant Method

This method is quite similar to that of Regula–Falsi method except for the condition $f(a)\,f(b) < 0$. The interval at each iteration may not contain the root. Let the initial limits of the interval be 'a' and 'b'.

The formula for successive approximation general form is

$$x_{n+1} = x_n + \frac{(x_n - x_{n-1})f(x_n)}{f(x_{n-1}) - f(x_n)}$$

In case at any stage $f(x_n) = f(x_{n-1})$ the method fails.

NOTES

1. This method does not converge always, but Regula–Falsi method always converges.
2. If it converges, it converges with order 1.62 approximately, which is more rapidly than the Regula–Falsi method.

Example 7

Find a root for $2e^x \sin x = 3$ using Regula–Falsi method and correct to three decimal places with three iterations.

Solution

Let $f(x) = 2e^x \sin x - 3$

$f(0) = -3 < 0$, $f(1) = 2e' \sin 1 - 3$

$= 1.5747 > 0$

∴ A root lies between 0 and 1.

Here $a = 0$ and $b = 1$

∴ The first approximation

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0(1.5747) - 1(-3)}{1.5747 - (-3)}$$

$$= \frac{3}{4.5747} = 0.6557.$$

Now $f(0.6557) = 2e^{0.6557}\sin(0.6557) - 3$
$= -0.6507 < 0$ and $f(1) > 0$

∴ The root lies between 0.6557 and 1.

The second approximation x_2

$$= \frac{(0.6557)(1.5747) - 1(-0.6507)}{1.5747 - (-0.6507)}$$

$$= \frac{1.0325 + 0.6507}{2.2254} = \frac{1.6832}{2.2254} = 0.7563$$

Now $f(0.7563) = -0.0761 < 0$ and $f(1) > 0$

∴ The root lies between 0.7563 and 1

∴ The third approximation to the root x_3

$$= \frac{(0.7653)(1.5747) - 1(-0.0761)}{1.5747 - (-0.0761)}$$

$$= \frac{1.1909 + 0.0761}{1.6508} = 0.7675$$

∴ The best approximation to the root upto three decimal places is 0.768

Newton–Raphson Method

Let x_0 be the approximate root of $f(x) = 0$ and let $x_1 = x_0 + h$ be the correct root. Then $f(x_1) = 0$

$\Rightarrow \quad f(x_0 + h) = 0 \qquad (1)$

Expanding Eq. (1) using Taylor's theorem,
We get $f(x_0) + hf'(x_0) + \cdots = 0$

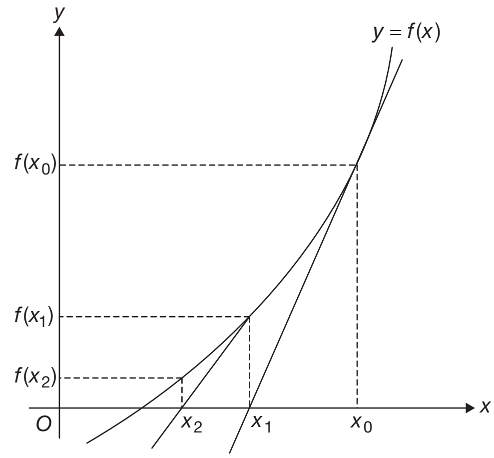
$$\Rightarrow \quad h = \frac{-f(x_0)}{f'(x_0)},$$

$$\therefore \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Now x_1 is the better approximation than x_0. Proceeding this way, the successive approximations $x_2, x_3, \ldots, x_{n+1}$ are given by $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

This is called Newton–Raphson formula.

Geometrical Interpretation of Newton–Raphson Formula



The geometrical meaning of Newton–Raphson method is a tangent is drawn at the point $[x_0, f(x_0)]$ to the curve $y = f(x)$. It cuts the x-axis at x_1 which will be a better approximation of the root. Now drawing another tangent at $[x_1, f(x_1)]$ which cuts the x-axis at x_2 which is a still better approximation and the process can be continued till the desired accuracy has been achieved.

Convergence of Newton–Raphson Method

The order of convergence of Newton–Raphson method is 2 or the convergence is quadratic. It converges if $|f(x). f''(x)| < |f'(x)|^2$. Also this method fails if $f'(x) = 0$

Newton's Iterative Formula to Find bth Root of a Positive Real Number a

The iterative formula is given by x_{n+1}

$$= \frac{1}{b}\left\{(b-1)x_n + \frac{a}{x_n^{\,b-1}}\right\}$$

Newton's Iterative Formula to Find a Reciprocal of a Number N

The iterative formula is given by

$$x_{i+1} = x_i\,(2 - x_i N)$$

Example 8

Find a real root of the equation $-4x + \cos x + 2 = 0$, by Newton–Raphson method upto four decimal places assuming $x_0 = 0.5$

Solution

Let $f(x) = -4x + \cos x + 2$ and

$$f'(x) = -4 - \sin x$$

Also $f(0) = 1 + 2 = 3 > 0$ and

$f(1) = -4 + \cos 1 + 2 = -1.4596 < 0$

So, a root lies between 0 and 1.

Given $x_0 = 0.5$

∴ The first approximation

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.5 - \frac{[-4(0.5) + \cos(0.5) + 2}{-4 - \sin(0.5)}$$

$$= (0.5) - \frac{[-2 + 2 + \cos(0.5)}{-4 - \sin 0.5}$$

$$= 0.5 - \frac{0.8775}{-4.4794}$$

$$= 0.5 + 0.1958 = 0.6958.$$

Example 9

Obtain the cube root of 14 using Newton–Raphson method, with the initial approximation as 2.5.

Solution

We know that, the iterative formula to find $\sqrt[b]{a}$ is

$$x_{n+1} = \frac{1}{b}\left\{(b-1)x_n + \frac{a}{x_n^{b-1}}\right\}$$

Here $b = 3$ and $a = 14$ and let $x_0 = 2.5$

$$\therefore \quad x_1 = \frac{1}{3}\left\{2x_0 + \frac{14}{x_0^{\ 2}}\right\}$$

$$x_1 = \frac{1}{3}\left\{2(2.5) + \frac{14}{(2.5)^2}\right\}$$

$$= \frac{1}{3}\left\{5 + \frac{14}{6.25}\right\} = \frac{1}{3}\{5 + 2.24\} = 2.413$$

$$x_2 = \frac{1}{3}\left\{2(2.413) + \frac{14}{(2.413)^2}\right\}$$

$$= \frac{1}{3}\left\{4.826 + \frac{14}{5.822569}\right\}.$$

$$= \frac{1}{3}\{4.826 + 2.4044\} = 2.410$$

∴ The approximate cube root of 14 is 2.41.

Example 10

Find the reciprocal of 24 using Newton–Raphson method with the initial approximation as 0.041.

Solution

The iterative formula to find $\frac{1}{N}$ is,

$$x_{n+1} = x_n(2 - x_n N)$$

Let $x_0 = 0.041$. Then $x_1 = x_0(2 - x_0\,(24))$

$\Rightarrow \quad x_1 = (0.041)\,(2 - (24)\,(0.041))$

$= 0.04165$

$x_2 = (0.0416)\ \{2 - (24)\ (0.04165)\} = 0.04161$, similarly proceeding we get $x_3 = 0.041666$

∴ The reciprocal of 24 is 0.04166.

Curve Fitting

In engineering applications, many a times, we need to find a suitable relation or law that may exist between the variables x and y from a given set of observed values (x_1, y_1), (x_2, y_2), …, (x_n, y_n), The relation connecting x and y is called as empirical law.

The process of finding the equation of the curve of best fit which may be most suitable for predicting the value of y for a given value of x is known as curve fitting.

Least Squares Approximation

Least squares approximation method is one of the best methods available for curve fitting.

Let (x_1, y_1), (x_2, y_2), …, (x_n, y_n) be the pairs of observed set of values of x and y. Let $y = f(x)$ be the functional relationship sought between the variables x and y where $f(x)$ consists of some unknown parameters. We need to find the relationship $y = f(x)$ by using the observed values.

Procedure

1. Find the residual $d_i = y_i - f(x_i)$ $(i = 1, 2, \ldots, n)$ for every pair of observed value y_i and $f(x_i)$, the value of the functional relation $f(x)$ at $x = x_i$
2. Find the sum of the squares of residuals corresponding to all pairs of values of y_i and $f(x_i)$ and let it be S

   i.e., $S = \sum_{i=1}^{n}(y_i - f(x))^2.$
3. Find the values of the parameters in $f(x)$ such that S is minimum.

Fitting a Straight Line Let $y = a + bx$ be a straight line to be fitted to the data $(x_1\ y_1)$, $(x_2\ y_2)$, …, $(x_n\ y_n)$.

∴ Residual $= d_i = y_i - (a + bx_i)$, $i = 1, 2, \ldots, n$

Sum of the squares of the residuals $= S = \Sigma(y_i - (a + bx_i\,))^2$

Now we have to find the parameters a and b such that S is minimum

$$\frac{\partial S}{\partial a} = \sum 2[y_i - (a + bx_i)(-1)] \text{ and}$$

$$\frac{\partial S}{\partial b} = \sum [2(y_i - (a + bx_i))(-x_i)]$$

For S to be minimum,

$$\frac{\partial S}{\partial a} = 0 \text{ and } \frac{\partial S}{\partial a} = 0$$

$$\Rightarrow \sum[-2(y_i-(a+bx_i))]=0 \text{ and}$$
$$\sum[2(y_i-(a+bx_i))(-x_i)]=0$$
$$\Rightarrow \sum y_i = na + b\sum x_i \quad (1)$$
$$\text{and } \sum x_i y_i = a\sum x_i + b\sum x_i^2 \quad (2)$$

Eqs. (1) and (2) are known as normal equations.

By solving these equations, we get the values of 'a' and b

Fitting a Parabola (Quadratic Equation)

To fit a parabola of the type $y = a + bx + cx^2$ to the set of data points (x_1, y_1), (x_2, y_2), …, (x_n, y_n), by proceeding as above, we get the normal equations as

$$\sum y_i = na + b\sum x_i + c\sum x_i^2 \quad (1)$$
$$\sum x_i y_i = a\sum x_i + b\sum x_i^2 + c\sum x_i^3 \quad (2)$$
$$\sum x_i^2 y_i = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4 \quad (3)$$

By solving Eqs. (1), (2) and (3), we can get the values of the parameters a, b and c

Fitting of various exponential curves that can be brought into the form of a straight line: Exponential curves of the type $y = ax^b$, $y = ab^x$ and $y = ae^{bx}$ can be fitted to the given data by transforming it into the form of a straight line by applying logarithm as follows.

Equation of the curve to be fitted	Equation obtained after applying $\log_e(= ln)$	Transformed equation into the form of a straight line
$y = ax^b$	$\ln y = \ln a + b\ln x$	$Y = A + bX$ where $Y = \ln y$; $A = \ln a$ and $X = \ln x$
$y = ab^x$	$\ln y = \ln a + x\ln b$	$Y = A + Bx$ where $Y = \ln y$; $A = \ln a$ and $B = \ln b$
$y = ae^{bx}$	$\ln y = \ln a + bx$	$Y = A + bx$ where $Y = \ln y$; $A = \ln a$

Example 11

Using the method of least squares, fit a straight line $y = a + bx$ to the following data.

x	1	2	3	4
y	4	11	35	100

Hence find the value of y at $x = 5$.

Solution

We have to fit the line $y = a + bx$ to the given data.

The normal equations are

$$\sum y_i = na + b\sum x_i \quad (1)$$
$$\sum x_i y_i = a\sum x_i + b\sum x_i^2 \quad (2)$$

The required values in the normal equations can be found using the following table

x_i	y_i	$x_i y_i$	x_i^2
1	4	4	1
2	11	22	4
3	35	105	9
4	100	400	16

$$\sum x_i = 10; \sum y_i = 150; \sum x_i y_i = 531; \sum x_i^2 = 30$$

Substituting these values in Eqn. (1) and (2), we get

$150 = 4a + 10b$ and $531 = 10a + 30b$

$\Rightarrow 4a + 10b = 150$

$10a + 30b = 531$

Solving these linear equations, we get

$a = -40.5$ and $b = 31.2$

$\therefore$ The straight line that fits to the given data is $y = a + bx$

$\Rightarrow y = -40.5 + 31.2x$

The value of y at $x = 5$ is

$y = -40.5 + 31.2 \times 5 \Rightarrow y = 115.5.$

Example 12

Fit a quadratic equation $y = a + bx + cx^2$ to the following data by the method of least squares.

x	–2	–1	0	1	2
y	1	5	10	22	38

Solution

We have to fit the curve $y = a + bx + cx^2$ to the given data.

Here the normal equations are

$$\sum y_i = na + b\sum x_i + c\sum x_i^2 \quad (1)$$
$$\sum x_i y_i = a\sum x_i + b\sum x_i^2 + c\sum x_i^3 \quad (2)$$
$$\sum x_i^2 y_i = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4 \quad (3)$$

The values required in the normal equations can be obtained by the following table:

x_i	y_i	$x_i y_i$	x_i^2	x_i^3	x_i^4	$x_i^2 y_i$
–2	1	–2	4	–8	16	4
–1	5	–5	1	–1	1	5
0	10	0	0	0	0	0
1	22	22	1	1	1	22
2	38	76	4	8	16	152

$\sum x_i = 0$; $\sum y_i = 76$; $\sum x_i y_i = 91$; $\sum x_i^2 = 10$; $\sum x_i^3 = 0$; $\sum x_i^4 = 34$; $\sum x_i^2 y_i = 183$

Substituting these values in the normal equation we have

$$76 = 5a + b \times 0 + c \times 10$$
$$91 = a \times 0 + b \times 10 + c \times 0$$

$$183 = a \times 10 + b \times 0 + c \times 34$$
$$\Rightarrow 5a + 10c = 76$$
$$10b = 91$$
$$10a + 34c = 183$$

Solving these equations for a, b and c we get

$a = 10.77$, $b = 9.1$ and $c = 2.21$

Substituting these in $y = a + bx + cx^2$, we get the required parabola as

$y = 10.77 + 9.1x + 2.21x^2$.

Interpolation

The process of finding the most appropriate estimate for the unknown values of a function $y = f(x)$ at some values of x by using the given pairs of values $(x, f(x))$ is called interpolation.

Assumptions in Interpolation

1. The frequency distribution is normal and not marked by sudden ups and downs.
2. The changes in the series are uniform within a period.

   Before looking into interpolation, let us get familiarity with the finite differences which we use in interpolation.

Finite Differences

1. **Forward differences:** Consider a function $y = f(x)$. Let we were given the following table representing the values of $y = f(x)$ corresponding to the values x_1, x_2, …, x_n of x that are equally spaced (i.e., $x_i = x_0 + ih$; $i = 1, 2, \ldots, n$).

$x = x_i$	x_1	x_2	x_3	…	x_n
$y = f(x_i)$	y_1	y_2	y_3		y_n

The forward difference of $f(x)$ denoted by $\Delta y = \Delta[f(x)]$ can be defined as

$$\Delta y = \Delta[f(x)] = f(x + h) - f(x)$$
$$\therefore \Delta y_0 = f(x_0 + h) - f(x_0) = y_1 - y_0$$
$$\Delta y_1 = y_2 - y_1$$
$$\Delta y_2 = y_3 - y_2$$

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$$\Delta y_{n-1} = y_n - y_{n-1}$$

where Δ is called the forward difference operator and $\Delta y_0, \Delta y_1, \ldots, \Delta y_{n-1}$ are called the first order forward differences of $y = f(x)$

Similarly, $\Delta^2 y = \Delta[f(x + h)] - \Delta[f(x)]$

$$\therefore \Delta^2 y_0 = \Delta y_1 - \Delta y_0$$
$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$$
$$\Delta^2 y_{n-2} = \Delta y_{n-1} - \Delta y_{n-2}$$

where $\Delta^2 y_0, \Delta^2 y_1, \ldots, \Delta^2 y_{n-2}$ are called the second order forward differences.

And in general, the nth order forward differences are given by

$$\Delta^n y = \Delta^n[f(x)] = \Delta^{n-1}[f(x + h)] - \Delta^{n-1}[f(x)]$$
$$\therefore \Delta^n y_0 = \Delta^{n-1} y_1 - \Delta^{n-1} y_0$$
$$\Delta^n y_1 = \Delta^{n-1} y_2 - \Delta^{n-1} y_1$$

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These forward differences of various orders can be found and represented in a table called the forward difference table as shown below

x	**y = f(x)**	**Δy**	**Δ²y**	**Δ³y**	**Δ⁴y**	**Δ⁵y**
x_0	y_0					
		Δy_0				
$x_1 = x_0 + h$	y_1		$\Delta^2 y_0$			
		Δy_1		$\Delta^3 y_0$		
$x_2 = x_0 + 2h$	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$
$x_3 = x_0 + 3h$	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
		Δy_3		$\Delta^3 y_2$		
$x_4 = x_0 + 4h$	y_4		$\Delta^2 y_3$			
		Δy_4				
$x_5 = x_0 + 5h$	y_5					

2. **Backward differences:** Consider a function $y = f(x)$. Let we were given the following table representing the values of $y = f(x)$ corresponding to the values x_1, x_2, …, x_n of x that are equally spaced (i.e; $x_i = x_0 + ih$, $i = 1, 2, \ldots n$)

$x = x_i$	x_1	x_2	x_3	…	x_n
$y_i = f(x_i)$	y_1	y_2	y_3	…	y_n

The backward difference of $f(x)$ denoted by ∇y (or) $\nabla[f(x)]$ can be defined as

$$\nabla y = \nabla[f(x)] = f(x) - f(x - h)$$
$$\therefore \nabla y_1 = y_1 - y_0$$
$$\nabla y_2 = y_2 - y_1$$
$$\nabla y_3 = y_3 - y_2$$

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$$\nabla y_n = y_n - y_{n-1}$$

where ∇ is called the backward difference operator and $\nabla y_1, \nabla y_2, \ldots, \nabla y_n$ are called the first order backward differences of $y = f(x)$

Similarly, $\nabla^2 y = \nabla^2[f(x)] = \nabla[f(x) - \nabla[f(x - h)]$

$\therefore\ \nabla^2 y_2 = \nabla y_2 - \nabla y_1$

$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$

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$\nabla^2 y_n = \nabla y_n - \nabla y_{n-1}$

where $\nabla^2 y_2, \nabla^2 y_3, \ldots, \nabla^2 y_n$ are called the second order backward differences.

And in general, the nth order backward differences are given by

$\nabla^n y = \nabla^n[f(x)] = \nabla^{n-1}[f(x)] - \nabla^{n-1}[f(x-h)]$

$\therefore\ \nabla^n y_n = \nabla^{n-1} y_n - \nabla^{n-1} y_{n-1}$

These backward differences of various orders can be found and represented in a table called the backward difference table as shown below

x	$y = f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
x_0	y_0					
		∇y_1				
$x_1 = x_0 + h$	y_1		$\nabla^2 y_2$			
		∇y_2		$\nabla^3 y_3$		
$x_2 = x_0 + 2h$	y_2		$\nabla^2 y_3$			
		∇y_3		$\nabla^3 y_4$	$\nabla^4 y_4$	
$x_3 = x_0 + 3h$	y_3		$\nabla^2 y_4$			
		∇y_4		$\nabla^3 y_5$	$\nabla^4 y_5$	$\nabla^5 y_5$
$x_4 = x_0 + 4h$	y_4		$\nabla^2 y_5$			
		∇y_5				
$x_5 = x_0 + 5h$	Y_5					

Relation between forward and backward differences: First order: $\nabla[f(x+h)] = \Delta[f(x)]$

Second order: $\nabla^2[f(x+2h)] = \Delta^2[f(x)]$

Third order: $\nabla^3[f(x+3h)] = \Delta^3[f(x)]$

In general, the nth order forward and backward differences are connected by the relation.

$\nabla^n[f(x+nh)] = \Delta^n[f(x)]$

3. **Divided differences:** Consider a function $y = f(x)$. Let we were given a table of values of $y = f(x)$ at x_1, $x_2, x_3, \ldots, x_n$, (need not be equally spaced) as shown below.

$x = x_i$	x_1	x_2	x_3	…	x_n
$y_i = f(x_i)$	y_1	y_2	y_3	…	y_n

Then the first order divided differences are given by:

$$[x_0 x_x] = \frac{y_1 - y_0}{x_1 - x_0}$$

$$[x_1 x_2] = \frac{y_2 - y_1}{x_2 - x_1}$$

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$$[x_{n-1}, x_n] = \frac{y_n - y_{n-1}}{x_n - x_{n-1}}$$

The second order divided differences are given by:

$$[x_0, x_1, x_2] = \frac{|x_1, x_2| - |x_0, x_1|}{x_3 - x_0}$$

$$[x_1, x_2, x_3] = \frac{[x_2, x_3] - [x_1, x_2]}{x_3 - x_1}$$

Similarly, the third order divided differences are given by

$$[x_0, x_2, x_3, x_4] = \frac{|x_1, x_2, x_3| - |x_0, x_1, x_2|}{x_3 - x_0}$$

$$[x_1, x_2, x_3, x_4] = \frac{|x_2, x_3, x_4| - |x_1, x_2, \ldots}{x_4 - x_1}$$

Note that $[x_0, x_1] = [x_1, x_0]$

And $[x_0, x_1, x_2] = [x_1, x_2, x_0] = [x_2, x_0, x_1]$

Interpolation Formulae

1. **Newton's forward interpolation formula:** If the function $y = f(x)$ takes the values $y_0, y_1, y_2, \ldots, y_n$ respectively at the equally spaced points $x_0, x_1, x_2, \ldots, x_n$ (i.e., $x_{i+1} = x_0 + ih$ (OR) $x_{i+1} = x_i + h$, then the Newton's forward interpolation formula is given by:

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}$$

$$\Delta^3 y_0 + \cdots + \frac{p(p-1)(p-2)\cdots(p-(n-1))}{n!}\Delta^n y_0$$

where $p = \dfrac{x - x_0}{h}$ (OR) $x = x_0 + ph$.

2. **Newton's backward interpolation formula:** If the function $y = f(x)$ takes the values $y_0, y_1, y_2, \ldots, y_n$ respectively at the equally spaced points $x_0, x_1, x_2, \ldots, x_n$ (i.e., $x_{i+1} = x_0 + ih$ (OR) $x_{i+1} = x_i + h$), then the Newton's backward interpolation formula is given by

$$Y_p = \nabla y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}$$

$$\nabla^3 y_n + \cdots + \frac{p(p+1)(p+2)\ldots(p+(n-1))}{n!}\nabla^n y_n$$

where $p = \dfrac{x - x_n}{h}$ (OR) $x = x_n + ph$.

NOTES

1. Newton's forward interpolation formula is used to interpolate (estimate) the values $y = f(x)$ near the beginning of the set of tabulated values given or for estimating the value of $y = f(x)$ to the left of the beginning.
2. Newton's backward interpolation formula is used to interpolate (estimate) the values $y = f(x)$ near the end of the set of tabulated values or for estimating the values of $y = f(x)$ to the right of the last tabulated value y_n.

When the values $x_0, x_1, x_2, \ldots, x_n$ of x are not equally spaced, then we can't make use of Newton's forward as well as backward interpolation formulae. In such situations, the following two interpolation formulae will be helpful.

3. **Newton's divided difference formula:** If the function $y = f(x)$ takes the values $y_0, y_1, y_2, \ldots, y_n$ corresponding to the values $x_0, x_1, x_2, \ldots, x_n$ (need not be equally spaced) of x, then the Newton's divided difference interpolation polynomial is given by
$y(x) = f(x) = y_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] + \cdots + (x - x_0)(x - x_1)\cdots(x - x_{n-1})[x_0, x_1, \ldots x_n]$
4. **Lagrange's interpolation formula:** If the function $y = f(x)$ takes the values $y_0, y_1, \ldots, y_n$ respectively at the points $x_0, x_1, x_2, \ldots, x_n$ (need not be equally spaced) of x, then the Lagrange's interpolation polynomial is given by $y(x) = f(x) =$

$$\frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}y_1 + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}y_n.$$

Example 13

If Δ denotes the forward difference operator, then show that $\Delta^4 y_0 = y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$

Hence find the general expression for $\Delta^n y_0$ in terms of $y_0, y_1, \ldots, y_n$ that does not involve the difference operators.

Solution

We know that $\Delta y_0 = y_1 - y_0$

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$

$= (y_2 - y_1) - (y_1 - y_0)$

$= y_2 - 2y_1 + y_0$

$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$

$= (y_3 - 2y_2 + y_1) - (y_2 - 2y_1 + y_0) = y_3 - 3y_2 + 3y_1 - y_0$

and $\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$

$= (y_4 - 3y_3 + 3y_2 - y_1) - (y_3 - 3y_2 + 3y_1 - y_0)$

$\therefore\ \Delta^4 y_0 = y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$

From the above discussion, one can observe that the coefficients of y_i s are nothing but the binomial coefficients with positive and negative signs alternatively.

$\therefore\ \Delta^n y_0 = y_n - {}^nC_1\, y_{n-1} + {}^nC_2\, y_{n-2} + \cdots + (-1)^n y_0.$

Example 14

A function $y = f(x)$ is given by the following table

x	5	10	15	20	25
$y = f(x)$	31	42	51	62	76

Using Newton's forward interpolation formula, find the value of y at $x = 7$.

Solution

First let us form the forward difference table:

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
5	31				
		11			
10	42		–2		
		9		4	
15	51		2		–3
		11		–1	
20	62		3		
		14			
25	76				

By Newton's forward difference formula,

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \cdots \quad (1)$$

Here $x_0 = 5$; $h = 5$ and $x = 7$

$$\therefore\ p = \frac{x - x_0}{h} = \frac{7-5}{5} = 0.4$$

Substituting these in Eq. (1), we have

$$y(7) = 31 + (0.4)\times 11 + \frac{(0.4)(0.4-1)}{2!}\times(-2) + \frac{(0.4)(0.4-1)(0.4-2)}{3!}\times 4 + \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)}{4!}\times(-3)$$

$= 31 + 4.4 + 0.24 + 0.256 + 0.1248 = 36.0208$

$\therefore$ The value of y at $x = 7$ is 36.0208.

Example 15

Following table shows the values of a function $y = f(x)$ at 0, 2, 5 and 9

x	0	2	5	9
$y = f(x)$	6	15	27	40

Using the Lagrange's interpolation formula, find $y(6)$.

Solution

Given values of $y = f(x)$ are

x	0	2	5	9
Y = f(x)	6	15	27	40

By Lagrange's interpolation formula, we have

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0$$

$$+ \frac{(x-x_0(x-x_2)(x-c_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$\therefore f(x) = \frac{(x-2)(x-5)(x-9)}{(0-2)(0-5)(0-9)} \times 6 + \frac{(x-0)(x-5)(x-9)}{(2-0)(2-5)(2-9)} \times$$

$$15 + \frac{(x-0)(x-2)(x-9)}{(5-0)(5-2)(5-9)} \times 27 + \frac{(x-0)(x-2)(x-5)}{(9-0)(9-2)(9-5)} \times 40$$

Now, $y(6) = f(6) = \dfrac{(6-2)(6-5)(6-9)}{(-2)\times(-5)\times(-9)}$

$$\times 6 + \frac{(6-0)(6-5)(6-9)}{2x(-3)\times(-6)} \times 15 + \frac{(6-0)(6-2)(6-9)}{-5x-3x(-4)} \times$$

$$2 + \frac{(6-0)(6-2)(6-5)}{9\times7\times4} \times 40 = \frac{4}{5} - \frac{45}{7} + \frac{162}{5} + \frac{80}{21} = 30.5807.$$

Numerical Differentiation

In numerical differentiation, we find the derivatives by using the interpolation formulae.

1. **Derivatives using newton's forward difference interpolation formula:** We know that the Newton's forward difference interpolation formula is

$$y = y_0 + p\Delta\, y_0 + \frac{p(p-1)}{2!}\ \Delta^2 y_0\ + \frac{p(p-1)(p-2)}{3!}$$

$\Delta^3 y_0 + \ldots$

Differentiating both sides wrt p,

$$\frac{dy}{dp} = \Delta y_0 + \frac{2p-1}{2!}\Delta^2 y_0 + \frac{3p^2-6p+2}{3!}\Delta^3 y_0 + \cdots$$

As $p = \dfrac{x-x_0}{h} = \dfrac{dp}{dx} = \dfrac{1}{h}$

$$\text{Now } \frac{dy}{dx} = \frac{dy}{dp}\frac{dp}{dx} = [\Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0$$

$$+ \frac{3p^2-6p+2}{3!}\Delta y_0 + \cdots]\frac{1}{h}$$

$$\therefore \frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 \right.$$

$$\left. + \frac{(3p^2-6p+2)}{3!}\Delta^3 y_0 + \cdots\right]$$

$\therefore \left(\dfrac{dy}{dx}\right)$ at $x = x_0$

$$= \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5{}_0 \ldots\right]$$

$(\because \text{At } i = x_0;\ p = 0)$

$$\text{And } \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dp}\left(\frac{dy}{dx}\right)\frac{dp}{dx} = \frac{1}{h}$$

$$\left[\Delta^2 y_0 + \frac{6p-6}{3!}\Delta^3 y_0 + \frac{12p^2-36p+22}{4!}\Delta^4 y_0 \cdots\right]\frac{1}{h}$$

$$\left(\frac{d^2y}{dx^2}\right) = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{6p-6}{3!}\Delta^3 y_0 + \frac{12p^2-36p+22}{1!}\right.$$

$$\left.\Delta^4 y_0 \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{\text{at } x=x_0}$$

$$= \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \cdots\right]$$

$\therefore$ By using Newton's forward interpolation formula, the first and second derivatives of $y = f(x)$ at $x = x_0$ are given by $\left(\dfrac{dy}{dx}\right)_{x=x_0}$

$$= \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 \ldots\right]$$

and

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \ldots\right]$$

2. **Derivatives using newton's backward difference interpolation formula:** We know that the Newton's backward difference interpolation formula is $y = y_n +$

$$p\nabla\, y_n + \frac{p(p+1)}{2!}\ \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\ + \nabla^3 y_n \ldots$$

Differentiating on both sides wrt p,

$$\frac{dy}{dx} = \nabla y_n + \frac{2p+1}{2}\nabla^2 y_n + \frac{3p^2+6p+2}{3!}\nabla^3 y_n + \cdots$$

As $p = \dfrac{x-x_n}{h}\dfrac{dp}{dx} = \dfrac{1}{h}$

$$\therefore \frac{dy}{dx} = \frac{dy}{dp}\frac{dp}{dx}$$

$$= \left[\nabla y_n + \frac{2p+1}{2!}\nabla^2 y_n + \frac{3p^2+6p+2}{3!}\nabla^3 y_n + \cdots\right]\frac{1}{h}$$

$$\therefore \frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2p+1}{2!}\nabla^2 y_n + \frac{3p^2+6p+2}{3!}\nabla^3 y_n + \cdots\right]$$

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \frac{1}{5}\nabla^5 y_n + \ldots\right]$$

$(\because \text{At } x = x_n;\ p = 0)$

$$\text{And } \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dp}\left(\frac{dy}{dx}\right)\frac{dp}{dx}$$

$$= \frac{1}{h}\left[\nabla^2 y_n + \frac{6p+6}{3!}\nabla^3 y_n + \frac{12p^2+36p+22}{4!}\nabla^4 y_n + \cdots\right]\frac{1}{h}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{6p+6}{3!}\nabla^3 y_n + \frac{12p^2+36p+22}{4!}\nabla^4 y_n + \cdots\right]$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n\right]$$

$\therefore$ By using Newton's backward difference interpolation formula, the first and second derivatives of $y = f(x)$ at $x = x_n$ are given by $\left(\frac{dy}{dx}\right)_{x=x_n}$

$$= \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \frac{1}{5}\nabla^5 y_n + \cdots\right]$$

and $\left(\frac{d^2y}{dx^2}\right)_{x=x_n}$

$$= \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \cdots\right]$$

Example 16

Using the values of a function $y = f(x)$ given in the following table, find the first two derivatives of $f(x)$ at $x = 7$.

x	2	3	4	5	6	7
y = f(x)	4	8	15	27	36	42

Solution

As we have to find the first two derivatives of $y = f(x)$ at $x = 7$, (end point of the given data), we use the derivatives' formulae obtained from Newton's backward interpolation formula.

The backward difference table for the given data is

X	y = f(x)	Δy	Δ²y	Δ³y	Δ⁴y	Δ⁵y
2	4					
		4				
3	8		3			
		7		–2		
4	15		5		6	
		12		–8		2
5	27		–3		8	
		9		0		
6	36		–3			
		6				
7	42					

By Newton's backward interpolation formula, we have $\left(\frac{dy}{dx}\right)_{x=x_n=7}$

$$= \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \nabla^4 y_n + \frac{1}{5}\nabla^5 y_n\right]$$

$$= \frac{1}{1}\left[6 + \frac{1}{2}\times(-3) + \frac{1}{3}\times 0 + \frac{1}{4}\times 8 + \frac{1}{5}\times 2\right] = \frac{69}{10} = 6.9$$

And $\left(\frac{d^2y}{dx^2}\right)_{x=x_n=7}$

$$= \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} + \nabla^4 y_n + \frac{5}{6}\nabla^5 y_n\right]$$

$$= \frac{1}{1^2}\left[-3 + 0 + \frac{11}{12}\times 8 + \frac{5}{6}\times 2\right] = 6.$$

Example 17

Find the first derivative at $x = 6$ for a function $y = f(x)$ with the following data.

x	5	7	10	11	13
y = f(x)	100	294	900	1210	2028

Solution

As the given values of x are not equally spaced, to find the first derivative of $f(x)$, we will make use of the Newton's divided difference interpolation formula, which is given by

$$y = f(x) = f(x_0) + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] + (x - x_0)(x - x_1)(x - x_2)[x_0, x_1, x_2, x_3] + (x - x_0)(x - x_1)(x - x_2)(x - x_3)][x_0, x_1, x_2, x_3, x_4] + \cdots \quad (1)$$

The divided differences of various orders for the given data can be represented as shown below.

x	$y = f(x)$	First divided differences	Second divided differences	Third divided differences	Fourth divided differences
5	100				
		$\frac{294-100}{7-5} = 97$			
			$\frac{202-97}{10-5} = 21$		
7	294				
		$\frac{900-294}{10-7} = 202$		$\frac{27-21}{11-5} = 1$	
			$\frac{310-202}{11-7} = 27$		$\frac{1-1}{13-5} = 0$
10	900				
		$\frac{1210-900}{11-10} = 310$		$\frac{33-27}{13-7} = 1$	
			$\frac{409-310}{13-10} = 33$		
11	1,210				
		$\frac{2028-1210}{13-11} = 409$			
13	2,028				

Substituting these in Eq. (1), we get

$y = f(x) = 100 + (x-5) \times 97 + (x-5)(x-7)\, 21 + (x-5)(x-7)(x-10) \times 1 + (x-5)(x-7)(x-10)(x-11) \times 0$

$\therefore f(x) = 100 + 97(x-5) + 21(x^2 - 12x + 35) + (x^3 - 22x^2 + 155x - 350)$

$$\therefore \frac{dy}{dx} = 97 + 21(2x - 12) + (3x^2 - 44x + 155)$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=6} = 97 + 21(2 \times 6 - 12) + (3 \times 6^2 - 44 \times 6 + 155)$$

$= 97 + 0 + (-1) = 96.$

Numerical Integration

The numerical integration can be stated as follows: Given a set of $(n+1)$ data points (x_i, y_i), $i = 0, 1, 2, 3, \ldots, n$ of the function $y = f(x)$, where $f(x)$ is not known explicitly, it is required to find $\int_{x_0}^{x_n} f(x)dx$.

NOTE

Numerical integration is also known as Numerical quadrature.

Newton–Cote's Quadrature Formula

[General Quadrature formula]

Consider the integral $I = \int_a^b f(x)dx$

Let the interval $[a, b]$ be divided into 'n' equal subintervals of width h so that $a = x_0, x_1 = x_0 + h, x_2 = x_0 + 2h \ldots b = x_0 + nh$

$$\therefore I = \int_{x_0}^{x_0+nh} f(x)dx$$

Put $x = x_0 + mh \Rightarrow dx = h \cdot dm$ as $x \to x_0$, $m \to 0$ and $x \to x_0 + nh$, $m \to n$

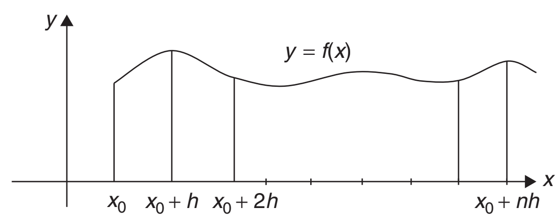
$$I = h\int_0^n f(x_0 + mh)dm$$

Applying Newton's forward interpolation formula

$$I = h\int_0^n \left(y_0 + m\Delta y_0 + \frac{m(m-1)}{2!}\Delta^2 y_0 + \cdots\right)dm$$

Integrating term by term and applying the limits, we get

$$\int_{x_0}^{x_0+nh} f(x)dx = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \cdots\right]$$ (Newton–Cotes quadrature formula)



On substituting $n = 1, 2, 3, \ldots$ in Newton-Cote's quadrature formula, we get various quadrature formulae.

Trapezoidal Rule [Two-point Quadrature]

Substituting $n = 1$ in the Newton–Cotes formula and taking the curve $y = f(x)$ through (x_0, y_0) and (x_1, y_1) as a straight line so that differences of order higher than one becomes zero, we get

$$\int_{x_0}^{x_n} f(x)dx = \int_{x_0}^{x_0+h} f(x)dx = h\left[y_0 + \frac{1}{2}\Delta y_0\right]$$

$$= h\left[y_0 + \frac{1}{2}(y_1 - y_0)\right] = \frac{h}{2}[y_0 + y_1]$$

Similarly,

$$\int_{x_1}^{x_2} f(x)dx = \int_{x_0+h}^{x_0+2h} f(x)dx = h\left[y_1 + \frac{1}{2}\Delta y_1\right] = \frac{h}{2}[y_1 + y_2]$$

$$\int_{x_2}^{x_3} f(x)dx = \int_{x_0+2h}^{x_0+3h} f(x)dx = \frac{h}{2}(y_2 + y_3$$

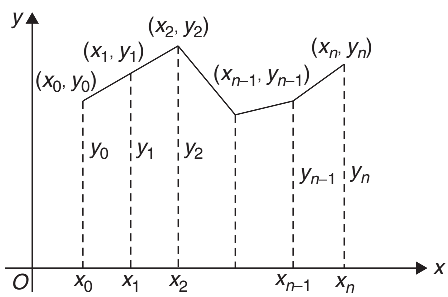
Proceeding, $$\int_{x_0+(n-1)h}^{(x_0+nh)} f(x)dx = \frac{h}{2}(y_{n-1} + y_n)$$

Hence, $$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$$

Thus, $$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}\left[\begin{array}{l}\text{(sum of the first and last ordinates)} \\ +2\text{ (sum of remaining ordinates)}\end{array}\right]$$

The above rule is known as Trapezoidal rule.

Geometrical Interpretation of Trapezoidal Rule



Geometrically, the curve $y = f(x)$ is replaced by n straight line segments joining the points (x_0, y_0) and (x_1, y_1); (x_1, y_1) and (x_2, y_2); …, (x_{n-1}, y_{n-1}) and (x_n, y_n). The area bounded by the curve $y = f(x)$ is then approximately equal to the sum of the areas of n trapeziums as shown in the figure.

Simpson's One-third Rule [Three-point Quadrature]

Substituting $n = 2$ in the Newton–Cotes quadrature formula taking the curve through (x_0, y_0), (x_1, y_1) and (x_2, y_2) as a parabola, so that the differences of order higher than 2 becomes zero, we get

$$\int_{x_0}^{x_0+2h} f(x)dx = 2h\left(y_0 + \Delta y_0 + \frac{1}{6}\Delta^2 y_0\right)$$

$$= \frac{h}{3}(y_0 + 4y_1 + y_2)$$

Similarly,

$$\int_{x_0+2h}^{x_0+4h} f(x)dx = \frac{h}{3}(y_2 + 4y_3 + y_4)$$

$$\int_{x_0+(n-2)h}^{x_0+nh} f(x)dx = \frac{h}{3}(y_{n-2} + 4y_{n-1} + y_n)$$

Therefore adding all these we get when 'n' is even,

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}\left[\begin{array}{l}(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1}) \\ + 2(y_2 + y_4 + \cdots + y_{n-2})\end{array}\right]$$

$= \frac{h}{3}$ [(sum of the first and last ordinates + 4 (sum of the odd ordinates) + 2 (sum of the even ordinates)]

This is known as Simpson's $\frac{1}{3}$ rule.

Simpson's Three-eighth Rule

Substituting $n = 3$ in the Newton Cotes quadrature formula and taking curve through (x_0, y_0), (x_1, y_1), (x_2, y_2) and (x_3, y_3) so that the differences of order higher than three becomes zero, we get

$$\int_{x_0}^{x_3} f(x)dx = 3h\left[y_0 + \frac{3}{2}\Delta y_0 + \frac{3}{2}\Delta^2 y_0 + \frac{1}{8}\Delta^3 y_0\right]$$

$$= \frac{3h}{8}(y_0 + 3y_1 + 3y_2 + y_3)$$

Similarly,

$$\int_{x_3}^{x_6} f(x)dx = \frac{3h}{8}(y_3 + 3y_4 + 3y_5 + y_6) \text{ and so on.}$$

Adding all these integrals from x_0 to x_n where 'n' is a multiple of 3, we get

$$\int_{x_0}^{x_n} f(x)dx = \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-2})$$
$$+ 2(y_3 + y_6 + y_9 + \cdots + y_{n-3})]$$

The above rule is called Simpson's $\frac{3}{8}$ rule which is applicable only when 'n' is a multiple of 3.

Example 18

Evaluate: $\int_0^2 \sqrt{1+x^2}\,dx$ taking $h = 0.2$ using

(i) Trapezoidal rule and

(ii) Simpson's $\frac{1}{3}$rd rule

Solution

Here, $a = 0$, $b = 2$, $h = 0.2$

So, $n = \frac{b-a}{h} = \frac{2-0}{0.2} = 10$

The values of x and y are tabulated as follows:

x	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
$y=\sqrt{1+x^2}$	1 y_0	1.0198 y_1	1.077 y_2	1.1661 y_3	1.2806 y_4	1.414 y_5	1.562 y_6	1.7204 y_7	1.8867 y_8	2.059 y_9	2.236 y_{10}

(i) By Trapezoidal rule,

$$\int_0^2 \sqrt{1+x^2}\,dx = \frac{h}{2}\left[(y_0+y_{10})+2(y_1+y_2+\cdots+y_9)\right]$$

$$= \frac{0.2}{2}[(1+2.236)+2(1.0198+1.077+1.1661+1.2806+1.414+1.562+1.7204+1.8867+2.059)]$$

$$= 0.1\,[(3.236)+2(13.1856)]$$

$$= 0.1\,[29.6072] = 2.96072.$$

(ii) By Simpson's $\frac{1}{3}$ rule,

$$\int_0^2 \sqrt{1+x^2}\,dx$$

$$= \frac{h}{3}\left[\begin{array}{l}(y_0+y_{10})+4(y_1+y_3+y_5+y_7+y_9)\\ \quad +2(y_2+y_4+y_6+y_8)\end{array}\right]$$

$$= \frac{0.2}{3}[(1+2.236)+4(1.0198+1.1661+1.414+1.7204+2.059)+2(1.077+1.2806+1.562+1.8867)]$$

$$= \frac{0.2}{3}[(3.236)+29.5172+11.6126]$$

$$= 2.95772.$$

Example 19

Evaluate $\int_0^{\pi/2} e^{\cos x}\,dx$ by Simpson's three-eighth rule.

Solution

Taking $h=\frac{\pi}{6}$, the range can be divided into three equal, sub intervals with the division points. The values of x and y are tabulated as below.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y=e^{\cos x}$	2.718(y_0)	2.3774(y_1)	1.6487(y_2)	1(y_3)

By Simpon's three-eighth rule,

$$\int_0^{\pi/2} e^{\cos x}\,dx = \frac{3h}{8}\left[(y_0+y_3)+3(y_1+y_2)\right]$$

$$= \frac{3}{8}\times\frac{\pi}{6}\left[(2.718+1)+3(2.3774+1.6487)\right]$$

$$= \frac{\pi}{16}\left[(3.718)+(12.0783)\right] = 3.10159.$$

Numerical Solutions of Ordinary Differential Equations

The following methods are discussed on the numerical solutions of ordinary differential equations.

Single-step Methods

1. Taylor's series method
2. Picard's method of successive approximation

Multi-step Methods

1. Euler's method
2. Modified euler's method
3. Runge–Kutta method
4. Predictor–Corrector methods [Milne's and Adam's]

Taylor's Series Method

Consider the differential equation

$$\frac{dy}{dx} = f(x,y) \text{ with initial condition } y(x_0)=y_0 \qquad (1)$$

Let $y=f(x)$ be the solution of Eq. (1)

Writing the Taylor's series expansion of $f(x)$ at x_0

$$f(x) = f(x_0)+(x-x_0)f'(x_0)+\frac{(x-x_0)^2}{2!}f''(x_0)+\cdots$$

$$\Rightarrow y = y_0+(x-x_0)y_0'+\frac{(x-x_0)^2}{2!}y_0''+\cdots$$

Put $x=x_1$, we get

$$y_1 = y_0+(x_1-x_0)y_0'+\frac{(x_1-x_0)^2}{2!}y_0''+\cdots$$

If we take $h=x_1-x_0$

$$\Rightarrow y_1 = y_0+hy_0'+\frac{h^2}{2!}y_0''+\cdots$$

∴ In general,

$y_{n+1} = y_n + hy_n' + \frac{h^2}{2!} y_n'' + \ldots$ will be the iterative formula.

Example 22

Given $\frac{dy}{dx} = x - y^2$ with the initial condition $y(0) = 1$

Find $y(0.1)$ using Taylor series method with step size 0.1.

Solution

$$f(x, y) = x - y^2$$
$$x = 0.1, x_0 = 0, y_0 = 1, h = 0.1$$
$$y' = x - y^2 \Rightarrow y'(0) = x_0 - y_0^{\,2} = -1;$$
$$y'' = 1 - 2yy' \Rightarrow y''(0) = 1 - 2y_0y_0'$$
$$= 1 - 2\,(1)\,(-1) = 3$$
$$y''' = -2yy'' - 2(y')^2$$
$$\Rightarrow y'''\,(0) = -2\,(1)\,(3) - 2\,(-1)^2$$
$$= -6 - 2 = -8$$

By Taylor's formula,

$$y\,(0.1) = y_1 = y_0 + hy'\,(0) + \frac{h^2}{2!} y''\,(0) + \frac{h^3}{3!} y'''\,(0) + \cdots$$

$$\Rightarrow y_1 = 1 + (0.1)\,(-1) + \frac{(0.1)^2}{2!}(3) + \frac{(0.1)^3}{3!}(-8)\ + \cdots$$

$= 1 - 0.1 + 0.015 - 0.0013 + \cdots$

$y_1 = 0.9137.$

Picard's Method of Successive Approximation

Given the differential equation $\frac{dy}{dx} = f(x, y)$ (1)

Integrate Eq. (1) from x_0 to x, we get

$$\int_{x_0}^{x} dy = \int_{x_0}^{x} f(x, y)dx$$

$$\Rightarrow y(x) - y(x_0) = \int_{x_0}^{x} f(x, y)dx$$

$$\Rightarrow y(x) = y(x_0) + \int_{x_0}^{x} f(x, y)dx \qquad (2)$$

Put $y = y_0$, we get the first approximation,

$$y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})dx.$$

Example 23

Given $\frac{dy}{dx} = 1 + xy$ and $y\,(0) = 1$. Evaluate $y\,(0.1)$ by Picard's method upto three approximations.

Solution

$$f\,(x, y) = 1 + xy$$
$$x_0 = 0, y_0 = 1$$

The first approximation $y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)dx$

$$1 + \int_{x_0}^{x} 1 + xy_0 dx\ = 1 + \int_{0}^{x} 1 + xdx$$

$$y_1 = 1 + x + \frac{x^2}{2}$$

At $x = 0.1$, $y_1 = 1 + (0.1) + \frac{(0.1)^2}{2} = 1.105$

The second approximation y_2,

$$= y_0 + \int_{x_0}^{x} f(x, y_1)dx$$

$$\Rightarrow y_2 = 1 + \int_{0}^{x} 1 + xy_1 dx$$

$$\Rightarrow y_2 = 1 + \int_{0}^{x}\left[1 + x\left(1 + x + \frac{x^2}{2}\right)\right]dx$$

$$= 1 + \int_{0}^{x}\left(1 + x + x^2 + \frac{x^3}{2}\right)dx$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}$$

At $x = 0.1$, $y_2 = 1 + (0.1) + \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} + \frac{(0.1)^4}{8}$

$y(0.1) = 1.10534$

The third approximation $y_3 = y_0 + \int_{x_0}^{x} f(x, y_2)dx$

$$\Rightarrow y_3 = 1 + \int_{0}^{x}(1 + xy_2)dx.$$

$$1 + \int_{0}^{x}\left(1 + x\left[1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}\right]\right)dx$$

$$1 + \int_{0}^{x}\left(1 + x + x^2 + \frac{x^3}{2} + \frac{x^4}{3} + \frac{x^5}{8}\right)dx$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} + \frac{x^5}{15} + \frac{x^6}{48}$$

At $x = 0.1$,

$$y_3 = 1 + (0.1) + \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} + \frac{(0.1)^4}{8} + \frac{(0.1)^5}{15} + \frac{(0.1)^6}{48}$$

$= 1 + 0.1 + 0.005 + 0.0003 + 0.0000125 + 0.0000006 + 0.00000002$

$y_3 = 1.105313$

Multi-step Methods

Euler's Method

For the differential equation $\frac{dy}{dx} = f(x, y)$ with initial condition $y(x_0) = y_0$, the Euler's iteration formula is

$$y_n = y_{n-1} + h f(x_{n-1}, y_{n-1}),\ n = 1, 2, 3,\ldots$$

NOTE

The process is very slow and to obtain accuracy, h must be very small, i.e., we have to divide $[x_0, x_n]$ into a more number of subintervals of length 'h'.

Example 24

Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$, find $y(0.5)$ by Euler's method choosing $h = 0.25$.

Solution

$$f(x, y) = \frac{y-x}{y+x}$$

$$x_0 = 0, y_0 = 1, h = 0.25$$

Euler's iteration formula,

$$y_n = y_{n-1} + h f(x_{n-1}, y_{n-1})$$

Put $n = 1$,

$$x_1 = 0.25 \Rightarrow y_1 = y(0.25) = y_0 + h f(x_0, y_0)$$

$$= 1 + (0.25)\left(\frac{y_0 - x_0}{y_0 + x_0}\right)$$

$$= 1 + (0.25)\frac{1-0}{1+0} = 1.25$$

Put $n = 2$

$$x_2 = 0.5 \Rightarrow y_2 = y(0.5) = y_1 + h f(x_1, y_1)$$

$$= (1.25) + (0.25)\left[\frac{y_1 - x_1}{y_1 + x_1}\right]$$

$$= 1.25 + (0.25)\left[\frac{1.25 - 0.25}{1.25 + 0.25}\right]$$

$$= 1.25 + 0.166666 = 1.4166$$

$\therefore\ y(0.5) = 1.4166$

Modified Euler's Method

For the differential equation $\frac{dy}{dx} = f(x, y)$ with initial condition $y(x_0) = y_0$, the Modified Euler's iteration formula is

$$y_r^{(n)} = y_{r-1} + \frac{h}{2}\left[f(x_{r-1}, y_{r-1}) + f(x_r, y_r^{n-1})\right]$$

NOTE

To find y_n, we proceed to find the approximations $y_n^{(0)}$, $y_n^{(1)}$, $y_n^{(2)}$ … until the two successive approximations are approximately equal.

$y_n^{(0)}$ is found by Euler's method, i.e., $y_n^{(0)} = y_{n-1} + hf(x_{n-1}, y_{n-1})$

Example 25

Find y for $x = 0.1$ using modified Euler's method for the differential equation $\frac{dy}{dx} = \log(x + y)$ with $y(0) = 1$.

Solution

Given $f(x, y) = \log(x + y)$

$$x_0 = 0, y_0 = 1, h = 0.1$$

To find y_1, $x_1 = 0.1$

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

$$= 1 + (0.1)\log(0 + 1) = 1$$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= y_0 + \frac{h}{2}\left[\log(x_0 + y_0) + \log(x_1 + y_1^{(0)})\right]$$

$$= 1 + \frac{0.1}{2}\left[\log(0 + 1) + \log(0.1 + 1)\right]$$

$$= 1 + \frac{0.1}{2}\left[\log 1 + \log 1.1\right] = 1.0047$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= y_0 + \frac{h}{2}\left[\log(0 + 1) + \log(x_1 + y_1^{(1)})\right]$$

$$= 1 + \frac{0.1}{2}\left[\log(0 + 1) + \log(0.1 + 1.0047)\right]$$

$$= 1.0049$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1 + \frac{0.1}{2}\left[\log(0 + 1) + \log(0.1 + 1.0049)\right]$$

$$= 1.0049$$

$\therefore\quad y_1 = 1.0049.$

Runge–Kutta Methods

First Order Runge–Kutta Method

$$y_1 = y_0 + hy_0^1 \text{ [same as Euler's method]}$$

Second Order Runge–Kutta Method

The formula is $y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$

where $k_1 = h f(x_0, y_0)$ and $k_2 = h f(x_0 + h, y_0 + k_1)$

Third Order Runge-Kutta Method

The formula is $y_1 = y_0 + \frac{1}{6}(k_1 + 4k_2 + k_3)$

where $k_1 = h f(x_0, y_0)$

$$k_2 = hf\left(x_0 + \frac{1}{2}h,\ y_0 + \frac{1}{2}k_1\right) \text{ and}$$

$k_3 = hf(x_0 + h, y_0 + k')$ where $k' = hf(x_0 + h, y_0 + k_1)$.

Fourth Order Runge–Kutta Method

The formula is $y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_0, y_0)$

$$k_2 = hf\left(x_0 + \frac{1}{2}h,\ y_0 + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(x_0 + \frac{1}{2}h,\ y_0 + \frac{1}{2}k_2\right)$$

and $k_4 = hf(x_0 + h, y_0 + k_3)$

Example 26

Given $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.2$. Find $y(1.05)$ applying fourth order Runge–Kutta method, with $h = 0.05$.

Solution

$$f(x, y) = x^2 + y^2,\ x_0 = 1,\ y_0 = 1.2,\ h = 0.05$$

$$\therefore k_1 = hf(x_0, y_0) = (0.05)\,[x_0{}^2 + y_0{}^2]$$

$$= (0.05)\,[1^2 + (1.2)^2] = 0.122$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= (0.05)\,[f(x_0 + 0.025, y_0 + 0.061]$$

$$= (0.05)\,[f(1.025, 1.261)]$$

$$= (0.05)\,[(1.025)^2 + (1.261)^2] = 0.1320$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= (0.05)\,f(1 + 0.025,\ 1.2 + 0.066)$$

$$= (0.05)\,f(1.025,\ 1.266)$$

$= (0.05)\,[(1.025)^2 + (1.266)^2] = 0.1326$ and $k_4 = h\,f(x_0 + h, y_0 + k_3)$

$$= (0.05)\,f(1 + 0.05,\ 1.2 + 0.1326)$$

$$= (0.05)\,f(1.05,\ 1.3326)$$

$$= (0.05)\,[(1.05)^2 + (1.3326)^2] = 0.1439$$

$$\therefore y_1 = y(1.05) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.2 + \frac{1}{6}[0.122 + 2(0.1320) + 2(0.1326) + 0.1439]$$

$$= 1.2 + \frac{1}{6}[0.7951] = 1.3325$$

Predictor–Corrector Methods

Milne's Predictor Formula

$$y_{n+1}^p = y_{n-3} + \frac{4h}{3}\,(2y_{n-2} - y_{n-1} + 2y_n)$$

Milne's Corrector Formula

$$y_{n+1}^c = y_{n-1} + \frac{h}{3}\,[y_{n-1} + 4y_n + y_{n+1}{}^p]$$

Adams–Bashforth Predictor Formula

$$y_{n+1}^p = y_n + \frac{h}{24}[55y_n - 59y_{n-1} + 37y_{n-2} - 9y_{n-3}]$$

Adams–Moulton Corrector Formula

$$y_{n+1}^c = y_n + \frac{h}{24}\,[9y_{n+1}^p + 19y_n - 5y_{n-1} + y_{n-2}]$$

Exercises

1. Three of the roots of the equation $x^4 + lx^3 + mx^2 + nx + 24 = 0$ are 3, 1 and –2. Which of the following could be the value of $l + m - n$?

(A) 0 (B) 1
(C) 2 (D) 3

2. If one of the roots of the equation $x^3 + 5x^2 - 12x - 36 = 0$ is thrice another root, then the third root is

(A) –6 (B) 3
(C) –2 (D) $-\frac{89}{13}$

3. If the equation $x^6 + 5x^5 + 11x^4 + 34x^2 + 20x + 24 = 0$ has exactly four non-real roots, then the number of negative roots is

(A) 1 (B) 0
(C) 3 (D) 2

4. A student finds, by trial, two negative and one positive root(s) of the equation $x^5 + 5x^4 + 2802x + 3024 = 103x^3 + 329x^2$. How many non–real roots does the equation have?

(A) 0 (B) 1
(C) 2 (D) 4

5. If the equation $3x^4 - 13x^3 + 7x^2 + 17x + a - 10 = 0$ has exactly three positive roots, then a can be

(A) 11 (B) 4
(C) 13 (D) 12

6. If two of the roots of the equation $x^3 + 3x^2 - 10x - 24 = 0$ are such that one is twice the other, then the third root is

(A) –4 (B) –3
(C) –2 (D) 3

7. If 2.236146 is an approximation to $\sqrt{5}$, then the relative error is

(A) 3.4883×10^{-5} (B) 4.8383×10^{-5}
(C) 8.3483×10^{-4} (D) 5.8438×10^{-4}

8. The least number of terms required to be considered in the Taylor's series approximation of $f(x) = \dfrac{1}{(2+x)}$ about $x = 0$ such that the truncation error is at most 5×10^{-4} for $x \in [0, 1]$ is

(A) 3 (B) 5
(C) 6 (D) 8

9. Let $f(x) = x^3 - x - 5 = 0$. By bisection method first two approximations x_0 and x_1 are 1.5 and 2.25 respectively, then x_2 is

(A) 1.625 (B) 1.875
(C) 1.999 (D) None of these

10. Find the fourth approximation of the root of the equation $x^3 + x - 11 = 0$, between 2 and 3, using Bisection method.

(A) 1.925 (B) 2.832
(C) 2.5215 (D) 2.0625

11. The absolute error bisection method is

(A) 2^n (B) $\dfrac{1}{2^n}|b - a|$
(C) $\dfrac{1}{|b-a|}$ (D) $|b - a|2^n$

12. If the first two approximations x_0 and x_1 to a root of $x^3 - x - 4 = 0$ are 1.666 and 1.780 respectively, then find x_2 by Regula–Falsi method.

(A) 1.974 (B) 1.794
(C) 1.896 (D) None of these

13. Find the second approximation to the root of the equation $2x - 5 = 3\sin x$ between (2, 3) using the method of false position.

(A) 2.2523 (B) 2.012
(C) 2.8804 (D) None of these

14. For $N = 28$ and $x_0 = 5.5$, the first approximation to $\sqrt{N}$ by Newton's iteration formula is

(A) 5.295 (B) 5.582
(C) 5.396 (D) None of these

15. The Newton's iterative formula to find the value of $\sqrt[3]{N}$ is

(A) $x_{i+1} = \left(2x_i - \dfrac{N}{x_i^2}\right)$

(B) $x_{i+1} = \dfrac{1}{3}\left(x_i - \dfrac{N}{x_i^2}\right)$

(C) $x_{i+1} = \dfrac{1}{3}\left(2x_i + \dfrac{N}{x_i^2}\right)$

(D) $x_{i+1} = \dfrac{1}{3}\left(2x_i - \dfrac{N}{x_i^2}\right)$

16. Find the second approximation to the cube root of 24 correct to three decimal places using Newton's iterative formula.

(A) 2.695 (B) 2.885
(C) 3.001 (D) None of these

17. The Newton's iterative formula to find the value of $\dfrac{1}{N}$ is

(A) $x_{i+1} = x_i(2 + x_i N)$
(B) $x_{i+1} = x_i(2 - x_i N)$
(C) $x_{i+1} = x_i^2(2 + x_i N)$
(D) None of these

18. Find the reciprocal of 22 using Newton–Raphson method.

(A) 0.0454545 (B) 0.4504504
(C) 0.54054 (D) None of these

19. If the first approximation of the root of $x^3 - 3x - 5 = 0$ is $(x_0 =)$ 2, then find x_1 by Newton–Raphson method.

(A) 2.2806 (B) 2.2790
(C) 2.3333 (D) None of these

20. Find the first approximation of the real root by Newton–Raphson method for $x^4 + x^3 - 7x^2 - x + 5 = 0$ by taking $x_0 = 2$.

(A) 2.066 (B) 2.981
(C) 2.819 (D) None of these

21. If $y = 2.6 + 0.7x$ is a line that fits the data:

x	–2	–1	0	1	2
y	1	2	3	*K*	4

Then the value of K is

(A) 3 (B) 5
(C) 6 (D) 7

22. If a curve $y = ab^x$ is fitted to the following data, then the value of 'b' is

x	–2	–1	0	1	2
y	11	13	20	25	34

(A) 0.2911 (B) 0.9845
(C) 1.3379 (D) 2.0034

23. For a set of 5 pairs of values (x_0, y_0), (x_1, y_1), (x_2, y_2) (x_3, y_3) and (x_4, y_4) of (x, y), if $\Delta^3 y_0 = 4$ and $\Delta^3 y_1 = 10$, then the value of $\nabla^4 y_4$ is

(A) 4 (B) 6
(C) 10 (D) 14

24. The value of $y(1.5)$ computed from the following data using Newton's forward interpolation formula is

x	1	2	3	4	5
y	6	7	12	21	34

(A) 6 (B) 6.5
(C) 7 (D) 7.5

25. The Lagrange's interpolation polynomial corresponding to the pairs of values of x and y given in the following table is

x	1	3	4	7
y	36	16	9	72

(A) $x^3 - 6x^2 + 9x + 36$
(B) $x^3 - 6x^2 + 18x - 45$
(C) $3x^3 + 4x^2 - 5x + 27$
(D) $x^3 - 7x^2 + 5x + 37$

26. The values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 5$ from the following table respectively are

x	0	1	2	3	4	5
y = f(x)	1	4	9	16	21	28

(A) 15.00 and 26.45
(B) 13.73 and 23.33
(C) 17.13 and 31.42
(D) 21.64 and 43.00

27. The value of $\frac{dy}{dx}$ at $x = 12$ from the table given below is

x	10	15	20	25
y = f(x)	354	332	291	260

(A) –3.9427 (B) 4.6125
(C) –0.4652 (D) 1.3549

28. The magnitude of error when $\frac{dy}{dx}$ at $x = 2$ is found by Newton's forward interpolation formula for $y =$ $y = \frac{1}{x}$ using the following data is

x	2	4	6	8
y = 1/x	0.5000	0.2500	0.1667	0.1250

(A) 0.0005 (B) 0.0025
(C) 0.0125 (D) 0.0625

29. Find the value of $\int_2^3 \frac{1}{1+x^2}dx$ taking four intervals by trapezoidal rule and also find the error when compared to its exact value.

(A) 0.1759, 0.000004 (B) 0.1826, 0.04
(C) 0.1953, 0.004 (D) 0.1423, –0.0004

30. Find $\int_0^1 \frac{x^2}{1+8x^3}dx$ using Trapezoidal rule by taking 4 strips.

(A) 0.0911 (B) 0.9011
(C) 0.1901 (D) None of these

31. The estimate of $\int_{0.5}^{1.5} \frac{dx}{x}$ obtained using Simpson's rule with three point function evaluation exceeds the exact value by

(A) 0.235 (B) 0.068
(C) 0.024 (D) 0.012

32. Find the value of $\int_2^6 x \log x dx$ taking 4 strips by Simpson's $\frac{1}{3}$ rd rule upto four decimals.

(A) 21.8901 (B) 22.8661
(C) 23.6581 (D) None of these

33. Evaluate $\int_0^{\pi/2} \sin x \, dx$ by Simpson's $\frac{1}{3}$ rule using six intervals.

(A) 0.97768 (B) 0.98869
(C) 0.99968 (D) None of these

34. Find the maximum error in evaluating the above when compared to its exact value.

(A) 0.000032 (B) 0.00032
(C) 0.00000032 (D) 0.0032

35. Evaluate $\int_0^3 \frac{1}{2+x^2}dx$ by using Simpson's $\frac{3}{8}$ rule by taking 3 strips.

(A) 0.507 (B) 0.5007
(C) 0.7839 (D) None of these

36. If $\frac{dy}{dx} = 1 - 3xy^2$, y (0) = 0, then by Taylor's method y (0.1) =

(A) 0.02 (B) 0.001
(C) 0.05 (D) 0.1

37. If $\frac{dy}{dx} = 2x + y$, $y(0) = 1$, the Picard's approximate of y upto second degree terms is

(A) $1 + x + x^2$ (B) $1 + x + \frac{x^2}{2}$
(C) $1 - x - \frac{x^2}{2}$ (D) None of these

38. If $y_0 = 1, f(x_0, y_0) = 1.2, f(x_1, y_1^{(0)}) = 1.9312, h = 0.3$, by modified Euler's formula $y_1^{(1)} =$

(A) 1.4696 (B) 1.2015
(C) 1.325 (D) 1.525

39. Using Euler's modified method, find a solution of the equation $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$ with $y(0) = 1$ at $y(0.2)$.

(A) 1.3902 (B) 1.2309
(C) 1.3092 (D) None of these

40. Find k_1, by Runge–Kutta method of fourth order if $\frac{dy}{dx} = 2x + 3y^2$ and $y(0.1) = 1.1165$, $h = 0.1$.

(A) 0.3993 (B) 0.9393
(C) 0.3939 (D) None of these

Direction for questions 41 and 42:

41. Find $y(0.8)$ by Milne's predictor formula, given $\frac{dy}{dx} = x - y^2$, $y_2 = 0.0795$, $y\,(0.6) = 0.1762$, $y_0 = 0.0000$, $y_1^1 = 0.1996$, $y_2^1 = 0.3937$, $y_3^1 = 0.5689$, $h = 0.2$

(A) 0.9304 (B) 0.4930
(C) 0.3049 (D) None of these

42. For the above problem find $y(0.8)$ using Milne's corrector formula.

(A) 0.3046 (B) 0.4036
(C) 0.436 (D) None of these

Direction for questions 43 and 44:

43. Find using the Adams–Bashforth corrector formula y (0.4), for the differential equation $\frac{dy}{dx} = \frac{1}{2}xy$, given $y(0.1) = 1.01, y(0.2) = 1.022, y\,(0.3) = 1.023, y_0^1 = 0, y_1^1 = 0.0505, y_2^1 = 0.1022, y_3^1 = 0.1535$.

(A) 1.5418 (B) 1.0410
(C) 1.4100 (D) None of these

44. For the above differential equation find $y(0.5)$ using Adams–Bashforth predictor formula.

(A) 1.00463 (B) 1.06463
(C) 1.00599 (D) None of these

45. The Runge–Kutta methods has the error of order ______.

(A) 1 (B) 3
(C) 5 (D) 2

Previous Years' Questions

1. Given that one root of the equation $x^3 - 10x^2 + 31x - 30 = 0$, is 5 the other two roots are **[GATE, 2007]**

(A) 2 and 3 (B) 2 and 4
(C) 3 and 4 (D) −2 and −3

2. The following equation needs to be numerically solved using the Newton–Raphson method $x^3 + 4x - 9 = 0$

The iterative equation for this purpose is (k indicates the iteration level) **[GATE, 2007]**

(A) $x_{k+1} = \dfrac{2x_k^3 + 9}{3x_k^2 + 4}$ (B) $x_{k+1} = \dfrac{3x_k^2 + 4}{2x_k^2 + 9}$

(C) $x_{k+1} = x_k - 3x_k^2 + 4$ (D) $x_{k+1} = \dfrac{4x_k^2 + 3}{9x_k^2 + 2}$

3. Three values of x and y are to be fitted in a straight line in the form $y = a + bx$ by the method of least squares. Given : $\Sigma x = 6$, $\Sigma y = 21$, $\Sigma x^2 = 14$, and $\Sigma xy = 46$ the values of a and b are respectively. **[GATE, 2008]**

(A) 2 and 3 (B) 1 and 2
(C) 2 and 1 (D) 3 and 2

4. The table below gives values of a function $F(x)$ obtained for values of x at intervals of 0.25.

X	0	0.25	0.5	0.75	1.0
***F*(*x*)**	1	0.9412	0.8	0.64	0.50

The value of the integral of the function between the limits 0 to 1 using Simpon's rule is **[GATE, 2010]**

(A) 0.7854 (B) 2.3562
(C) 3.1416 (D) 7.5000

5. The square root of a number N is to be obtained by applying the Newton–Raphson iterations to the equation $x^2 - N = 0$. If i denotes the iteration index, the correct iterative scheme will be **[GATE, 2011]**

(A) $x_{i+1} = \dfrac{1}{2}\left(x_i + \dfrac{N}{x_i}\right)$

(B) $x_{i+1} = \dfrac{1}{2}\left(x_i^2 + \dfrac{N}{x_i^2}\right)$

(C) $x_{i+1} = \dfrac{1}{2}\left(x_i + \dfrac{N^2}{x_i}\right)$

(D) $x_{i+1} = \dfrac{1}{2}\left(x_i - \dfrac{N}{x_i}\right)$

6. Find the magnitude of the error (correct to two decimal places) in the estimation of following integral using Simpson's $\frac{1}{3}$ rule. (Take the step length as 1)

$$\int_0^4 (x^4 + 10)dx$$ **[GATE, 2013]**

7. In Newton–Raphson iterative method, the initial guess value (x_{inj}) is considered as zero while finding the roots of the equation: $f(x) = -2 + 6x - 4x^2 + 0.5x^3$. The correction, Δx, to be added to x_{inj} in the first iteration is ______. **[GATE, 2015]**

8. The quadratic equation $x^2 - 4x + 4 = 0$ is to be solved numerically, starting with the initial guess $x_0 = 3$. The Newton–Raphson method is applied once to get a new estimate and then the Secant method is applied once using the initial guess and this new estimate. The estimated value of the root after the application of the Secant method is ______. **[GATE, 2015]**

9. For step-size, $\Delta x = 0.4$, the value of following integral using Simpson's $\frac{1}{3}$ rule is ________.

$$\int_0^{0.8} (0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5)dx$$

[GATE, 2015]

10. Newton–Raphson method is to be used to find root of equation $3x - e^x + \sin x = 0$. If the initial trial value for the root is taken as 0.333, the next approximation for the root would be ______. (note: answer up to three decimal) **[GATE, 2016]**

Answer Keys

Exercises

1. D	**2.** B	**3.** D	**4.** A	**5.** B	**6.** D	**7.** A	**8.** C	**9.** B	**10.** D
11. B	**12.** B	**13.** C	**14.** A	**15.** C	**16.** B	**17.** B	**18.** A	**19.** C	**20.** A
21. A	**22.** C	**23.** B	**24.** A	**25.** D	**26.** B	**27.** A	**28.** D	**29.** D	**30.** A
31. D	**32.** B	**33.** C	**34.** B	**35.** C	**36.** D	**37.** A	**38.** A	**39.** B	**40.** C
41. C	**42.** A	**43.** B	**44.** B	**45.** C					

Previous Years' Questions

1. A	**2.** A	**3.** D	**4.** A	**5.** A	**6.** 0.52 to 0.55	**7.** 0.3 to 0.4
8. 2.32 to 2.34		**9.** 1.36 to 1.37		**10.** 0.360		

TEST

Engineering Mathematics

Time: 60 Minutes

1. If λ is an eigen value of an orthogonal matrix A then which of the following is always TRUE?

P: $\frac{1}{\lambda}$ is also an eigen value of A

Q: λ is a non-zero eigen value.

(A) P only (B) Q only
(C) Both P and Q (D) Neither P nor Q

2. A fair die is rolled independently four times. The probability that a non-composite number turns up for atleast 3 times is ______.

(A) $\frac{16}{27}$ (B) $\frac{11}{27}$
(C) $\frac{1}{27}$ (D) $\frac{5}{27}$

3. The value of $I = \int_1^2 x^3 \ln x dx$ is ______.

(A) $4\ln 16 - 15$ (B) $4\ln 16 + 15$
(C) $\ln 16 + \frac{15}{16}$ (D) $\ln 16 - \frac{15}{16}$

4. If $x = \ln \frac{y}{x}$, then y has

(A) a local maximum at $x = -1$
(B) a local minimum at $x = -1$
(C) a local maximum at $x = 1$
(D) a local minimum at $x = 1$

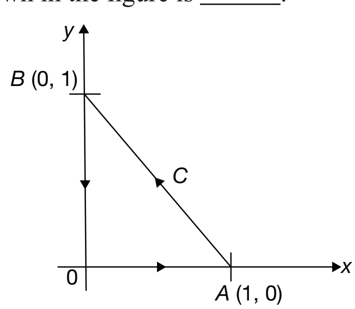
5. The complete solution of the initial value problem $\frac{d^2y}{dx^2} + 4y = 0$; $y(0) = 4$ and $y'(0) = 8$ is ______.

(A) $y = 4(\cos 2x + 2\sin 2x)$
(B) $y = 4\cos 2x$
(C) $y = 4\sin 2x$
(D) None of these

6. The value of $\lim_{n\to 0}\left(\frac{1+2n}{1+3n}\right)^{1/n}$ is ______

(A) e (B) e^{-1}
(C) e^{-2} (D) 1

7. The value of $\oint_c \bar{F}\cdot\overline{dr}$, where $\bar{F} = xy^2\bar{i} - x^2y\bar{j}$ over the path shown in the figure is ______.



(A) $\frac{1}{3}$ (B) $\frac{-1}{3}$
(C) $\frac{1}{6}$ (D) $\frac{-1}{6}$

8. If the eigen values of a matrix $A = \begin{bmatrix} 7 & a \\ 5 & b \end{bmatrix}$ are 10 and 2, then the values of 'a' and 'b' respectively are ______.

(A) $a = 3, b = -5$ (B) $a = -3, b = -5$
(C) $a = 3, b = 5$ (D) $a = 3, b = -5$

9. The Taylor's series expansion of $\frac{\cos x}{\frac{3\pi}{2} - x}$ at $x = \frac{3\pi}{2}$ is given by

(A) $1 - \frac{\left(x - \frac{3\pi}{2}\right)^2}{2!} + \frac{\left(x - \frac{3\pi}{2}\right)^4}{4!} \cdots \infty$

(B) $\left(x - \frac{3\pi}{2}\right) - \frac{\left(x - \frac{3\pi}{2}\right)^3}{3!} + \frac{\left(x - \frac{3\pi}{2}\right)^5}{5!} \cdots \infty$

(C) $1 - \frac{\left(x - \frac{3\pi}{2}\right)^2}{3!} + \frac{\left(x - \frac{3\pi}{2}\right)^4}{5!} \cdots \infty$

(D) $-1 + \frac{\left(x - \frac{3\pi}{2}\right)^2}{3!} - \frac{\left(x - \frac{3\pi}{2}\right)^4}{5!} + \cdots \infty$

10. If a scalar field f and a vector field $\bar{V}$ are related by $f = \text{Div}\,\bar{V}$, which of the following is TRUE?
Here S is a closed surface, enclosing a volume V_S.

(A) $\iint_S \text{curl}\,\bar{V}\cdot d\bar{s} = \iiint_{V_s} f\, dv$

(B) $\iint_S \bar{V}\cdot d\bar{s} = \iiint_{V_s} f\, dv$

(C) $\iiint_S f\, dv = \iiint_{V_s} \text{Div}\,\bar{V}\, d\bar{s}$

(D) $\iint_S \text{curl}\,\bar{V} \times d\bar{s} = \iiint_{V_s} f\, dv$

11. The curl of the vector field $2xy\bar{i} + xy^2z\bar{j} - 6xz^3\bar{k}$ at the point (0, 2, 3) is ______

(A) $2\bar{i} + 5\bar{j} - 6\bar{k}$ (B) $5\bar{i} + 2\bar{j}$
(C) $162\bar{j} + 12\bar{k}$ (D) None of these

12. Let S denote the set of all possible arrangements of letters of the word 'AUTHORISED' without repetitions. If an element in S is selected at random, then what is

the probability that, the letter 'I' appears at a later position, than all other vowels?

(A) $\frac{1}{5}$ (B) $\frac{1}{5!}$

(C) $\frac{4!}{5!}\times 2$ (D) $\frac{4!\times 5!}{10!}$

13. In the process of finding a solution to the equation $f(x) = 4x^2 = 4x - 15 = 0$ by Newton–Raphson method, with initial solution as $x_0 = 1.6$, the method converges to actual solution after _______ interaction, when the calculator is fixed to four decimal places.

(A) 2nd (B) 4th

(C) 6th (D) 8th

14. In the process of finding a root for $x^3 - 3x^2 - 5x + 6 = 0$ in between $a = 3$ and $b = 4$, by Regula Falsi method, the value of the root in the first iteration is _______.

(A) 3.8182 (B) 3.5

(C) 3.6235 (D) 3.3218

15. For the system of linear equations

$x + 2y + 3z = 4$

$2x + 3y + (a - 4)z = b$

$4x + 7y - z = 5$

has infinite number of solutions, then the values of 'a' and 'b' are _______

(A) $a = b = 3$ (B) $a = -b = 3$

(C) $a = b = -3$ (D) $a = -b = -3$

16. Evaluate $\int_0^{\pi/3} f(x)dx$ by Simpson's $\frac{3}{8}$ Rule using the following table

x	0	$\frac{\pi}{18}$	$\frac{\pi}{9}$	$\frac{\pi}{6}$	$\frac{2\pi}{9}$	$\frac{5\pi}{18}$	$\frac{\pi}{3}$
y = f(x)	0	0.1762	0.3638	0.5770	0.8385	1.1907	1.7299

(A) 0.5862 (B) 0.6929

(C) 0.5928 (D) 0.7234

17. If 'r' is the standard deviation of $a_1, a_2, a_3 \ldots a_n$, then the standard deviation of $ka_1 + 1, ka_2 + 1, ka_3 + 1, \ldots, ka_n + 1$ is

(A) kr (B) $kr + 1$

(C) $\frac{r}{k} - 1$ (D) $\frac{r}{k}$

18. If $\bar{a} = \bar{i} + \bar{j} - 2\bar{k}$ and $\bar{b} = \bar{i} - 2\bar{j} + \bar{k}$, then determinant of the matrix $\begin{bmatrix} \bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} \\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} \end{bmatrix}$, where $\bar{a}\cdot\bar{b}$ denotes the dot product of the vector $\bar{a}$ and $\bar{b}$ is _______.

(A) 9 (B) 27

(C) 18 (D) 54

19. If x and y are two random variables, a and b are any two constants and $E(x)$ and var(x) denote the expectation and the variance of the random variable x, then which of the following is INCORRECT?

(A) var $(ax + b) = a^2$ var(x)

(B) $E(ax + b) = aE(x) + b$

(C) $E(x + y) = E(x) + E(y)$

(D) $E(x^2) = \text{var}(x)$

20. Which of the following pairs of vectors are orthonormal?

(A) $\begin{bmatrix}1\\1\\1\end{bmatrix};\begin{bmatrix}-1\\2\\-1\end{bmatrix}$ (B) $\begin{bmatrix}\frac{1}{\sqrt{4}}\\\frac{1}{\sqrt{4}}\\\frac{1}{\sqrt{2}}\end{bmatrix};\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{4}}\\\frac{-1}{\sqrt{4}}\end{bmatrix}$

(C) $\begin{bmatrix}1\\2\\3\end{bmatrix};\begin{bmatrix}3\\2\\1\end{bmatrix}$ (D) $\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\\0\end{bmatrix};\begin{bmatrix}\frac{1}{\sqrt{3}}\\\frac{1}{\sqrt{3}}\\\frac{-1}{\sqrt{3}}\end{bmatrix}$

21. An integrating factor of the non-exact differential equation $(x^2 + 2xy - 2y^2)dx + (y^2 + 2xy - 2x^2)dy = 0$ is

(A) $x^3 + y^3$

(B) $\frac{1}{x^3 + y^3}$

(C) $x^3 - y^3$

(D) $\frac{1}{x^3 - y^3}$

22. The solution of the differential equation $x\frac{dy}{dx} + y = 3x^2$, $y(1) = 2$ is

(A) $y = x + \frac{1}{x}$

(B) $y = x + \frac{1}{x^2}$

(C) $y = x^3$

(D) $y = x^2 + \frac{1}{x}$

23. If $x = \sqrt{y + \sqrt{y + \sqrt{y + \cdots\infty}}}$, then the value of $\frac{dy}{dx}$ at $x = 1$ is _______.

(A) 0 (B) 1

(C) 4 (D) Undefined

24. Let $f(x) = \begin{Bmatrix} 2x-3\,;\ \text{for } x \geq \dfrac{3}{2} \\ 3-2x\,;\ \text{for } x < \dfrac{3}{2} \end{Bmatrix}$

Then which of the following is true?

(A) $f(x)$ is continuous and differentiable for all real values of x.

(B) $f(x)$ is not continuous at $x = \dfrac{3}{2}$.

(C) $f(x)$ is continuous for real values of x, except $x = \dfrac{3}{2}$.

(D) $f(x)$ is continuous for every x and differentiable for all values of x, except $x = \dfrac{3}{2}$.

25. If $L[f(t)] = \dfrac{2s+3}{s^2+5s+6}$, then the initial value of $f(t)$ is ______.

(A) 1 (B) 2

(C) 3 (D) 6

Answer Keys

1. C	**2.** A	**3.** D	**4.** B	**5.** D	**6.** B	**7.** D	**8.** C	**9.** D	**10.** B
11. C	**12.** A	**13.** B	**14.** A	**15.** C	**16.** B	**17.** A	**18.** B	**19.** D	**20.** D
21. B	**22.** D	**23.** B	**24.** D	**25.** B					

Engineering Mathematics Test I

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. $\underset{x\to 0}{Lt}\dfrac{5^x-1}{\sqrt{x^2-x+1}-1}=$

(A) $\log_e 5$ (B) $2\log_e 5$

(C) $-2\log_e 5$ (D) 1

2. If $f(x) = x^2 + 5x - 13$, if $x < 1$,

$x - 8$ if $x \geq 1$, then $\underset{x\to 0}{Lt} f(x) =$

(A) –8 (B) –7

(C) 7 (D) does not exist

3. $f(x) = \dfrac{e^{\frac{1}{x}}}{1-e^{\frac{1}{x}}}$. for $x \neq 0$ is

$= 0$ for $x = 0$

(A) continuous at $x = 0$.

(B) not continuous at $x = 0$.

(C) continuous everywhere.

(D) none of these.

4. If $f(x) = a[x - 5] + b[x + 5]$ is continuous at $x = 5$, then the value of $a + b$ ($[x]$ is the greatest integer less than or equal to x) is

(A) 0 (B) 1

(C) 5 (D) 2

5. If $f(x) = \begin{cases}\dfrac{\sqrt{x}-3}{x-9}; \text{for } x\neq 9\\ 3k;\ \text{for } x=9\end{cases}$ is continuous everywhere,

then k is equal to ———.

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$

(C) $\dfrac{1}{9}$ (D) $\dfrac{1}{18}$

6. If $y = ax^2 + 3\log x + bx$ has its extreme value at $x = 1$ and $x = -1$, then the value of $2a - b$ is

(A) 3 (B) 0

(C) –1 (D) –3

7. If $xy = 6$, then find the minimum value of $2x + 3y$, $x, y \in R^+$.

(A) 12 (B) 15

(C) 9 (D) 6

8. Which of the following functions have neither maximum nor minimum?

(A) $\dfrac{1}{2x}$ (B) $4x + 7$

(C) e^{5x+1} (D) All the above

9. Find the maximum value of the function $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$ in the interval $[-1, 2]$.

(A) $\dfrac{39}{16}$ (B) 2

(C) 21 (D) 18

10. Let $f(x)$ and $g(x)$ be two continuous functions in $[a, b]$ and differentiable in (a, b) and $g^1(x) \neq 0$ for any $x \in (a, b)$ then there exists at least one value $c \in (a, b)$, by Cauchy mean value theorem which of the following is true?

(A) $\dfrac{f^2(c)}{g^1(c)}=\dfrac{f(b)-g(a)}{g(b)+f(a)}$

(B) $\dfrac{g^1(c)}{f^1(c)}=\dfrac{f(b)+g(b)}{f(a)+f(b)}$

(C) $\dfrac{f^1(c)}{g^1(c)}=\dfrac{f^1(b)-f^1(a)}{g^1(b)-g^1(a)}$

(D) $\dfrac{f'(c)}{g'(c)}=\dfrac{f(b)-f(a)}{g(b)-g(a)}$

11. If the function $f(x) = (\sin 2x)\, e^{-2x}$ satisfies Rolle's theorem in the interval $\left(0,\dfrac{\pi}{2}\right)$, then the value of $c \in \left(0,\dfrac{\pi}{2}\right)$ such that $f'(c) = 0$ is

(A) $\dfrac{\pi}{8}$ (B) $\dfrac{\pi}{4}$

(C) $\dfrac{\pi}{6}$ (D) $\dfrac{\pi}{3}$

12. By using Lagrange's mean value theorem for $f(x) = x(x + 2)(x - 1)$, find c for $c \in (-1, 0)$.

(A) $\dfrac{-1}{3}$ (B) $\dfrac{-3}{4}$

(C) $\dfrac{-2}{3}$ (D) $\dfrac{-4}{5}$

13. If $u = \tan^{-1}\left(\dfrac{x^3-y^3}{x+y}\right)$, then the value of $x\dfrac{\vartheta u}{\vartheta x}+y\dfrac{\vartheta u}{\vartheta y}$ is

(A) $2\sin u$ (B) $2\tan u$

(C) $\sin 2u$ (D) $2\cos 2u$

14. If $f(x, y)$ is a homogeneous function of degree n, then the value of

$y\dfrac{\vartheta^2 f}{\vartheta y^2}+x\dfrac{\vartheta^2 f}{\vartheta x\vartheta y}=$

(A) $n.\frac{\partial f}{\partial x}$ (B) $(n-1)\frac{\partial f}{\partial y}$

(C) $(n-1)\frac{\partial f}{\partial x}$ (D) $n.\frac{\partial f}{\partial y}$

15. If $u = \cos^{-1}\left(\frac{x^3+2y^3-4z^3}{x^7-y^7-z^7}\right)$, then the value of $\sum_{x,y,z} x\frac{\partial u}{\partial x}$ is

(A) $4\cot u$ (B) $-4\tan u$
(C) $-4\cos u$ (D) $-4\sin u$

16. If $z = \ln(x^2 + y^3)$, then

(A) $3x\frac{\partial z}{\partial x} = 2y\frac{\partial z}{\partial y}$

(B) $3x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y} = 6$

(C) $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$

(D) $2y\frac{\partial z}{\partial x} + 3x\frac{\partial z}{\partial y} = 6$

17. The value of the definite integral $\int_0^1 (x^2-2x+3)e^{-xdx}$ is ______.

(A) $3 + 4e$ (B) $3 - 4e$

(C) $3 + \frac{4}{e}$ (D) $3 - \frac{4}{e}$

18. $\int_0^{\frac{\pi}{2}} \frac{3\sec x + 4\operatorname{cosec} x}{\sec x + \operatorname{cosec} x}\,dx =$

(A) $\frac{7\pi}{4}$ (B) $\frac{7}{2}$

(C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{2}$

19. $\int_0^{\infty} \frac{x^2}{(1+x^2)^{\frac{5}{2}}}dx =$

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

20. Evaluate $\int_1^4 \frac{dx}{(x-2)(x-3)}$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) diverges

21. $\int_0^{\infty} \frac{dx}{9+x^2} =$

(A) $\frac{\pi}{2}$ (B) –

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

22. The area bounded by the cure $y^2 = x + 2$ and $y = x - 4$ (in sq units) is

(A) $20\frac{5}{6}$ (B) $15\frac{5}{6}$

(C) $12\frac{2}{3}$ (D) $18\frac{3}{4}$

23. The area bounded by the curves $y^2 = 9ax$ and $x^2 = 9ay$ is

(A) $54a^2$
(B) $48a^2$
(C) $36a^2$
(D) $27a^2$

24. The volume of the solid formed by the revolution of the area A about y – axis is

(A) $\iint_A 2\pi dxdy$ (B) $\iint_A 2\pi ydxdy$

(C) $\iint_A \pi x^2 dxdy$ (D) $\iint_A \pi y^2 dxdy$

25. The taylor's series expansion of $2e^x + 3\sin x$ about $x = 0$ is

(A) $2 + 5x + x^2 + \frac{x^3}{6} + \frac{x^4}{12} + \frac{x^5}{24} + \ldots\ldots.$

(B) $2 + 5x + x^2 - \frac{x^3}{6} + \frac{x^4}{12} + \frac{x^5}{24} + \ldots\ldots\ldots..$

(C) $2 + 5x + x^2 + \frac{x^3}{6} - \frac{x^4}{12} + \frac{x^5}{24} + \ldots\ldots\ldots$

(D) $2 + 5x + x^2 + \frac{x^3}{+} + \frac{x^4}{12} + \frac{x^5}{24} + \ldots\ldots\ldots\ldots$

Answer Keys

1. C	**2.** B	**3.** B	**4.** A	**5.** D	**6.** D	**7.** A	**8.** D	**9.** C	**10.** D
11. A	**12.** C	**13.** C	**14.** B	**15.** A	**16.** B	**17.** D	**18.** A	**19.** B	**20.** D
21. C	**22.** A	**23.** D	**24.** A	**25.** B					

Hints and Explanations

1. $\underset{x\to 0}{Lt}\dfrac{5^x-1}{\sqrt{x^2-x+1}-1}\times\dfrac{\sqrt{x^2-x+1}+1}{\sqrt{x^2-x+1}+1}$

$= \underset{x\to 0}{Lt}\dfrac{(5^x-1)\left(\sqrt{x^2-x+1}+1\right)}{x^2-x+1-1}$

$= \underset{x\to 0}{Lt}\dfrac{(5^x-1)\sqrt{x^2-x+1}+1}{x(x-1)}$

$= \underset{x\to 0}{Lt}\dfrac{5^x-1}{x}\underset{x\to 0}{Lt}\dfrac{\sqrt{x^2-x+1}+1}{x-1}.$

$=\log_e 5\times\dfrac{2}{-1}=2\log_e 5$ Choice (C)

2. we know that $\underset{x\to 1}{Lt}\ f(x)=\underset{x\to 1^-}{Lt}\ f(x)=\underset{x\to 1^+}{Lt}\ f(x)$

$\underset{x\to 1^-}{Lt}\ f(x)=\underset{x\to 1^-}{Lt}\ (x^2+5x-13)=-7$

$\underset{x\to 1^+}{Lt}\ f(x)=\underset{x\to 1^+}{Lt}\ (x-8)=-7$

$\therefore\ \underset{x\to 1^-}{Lt}\ f(x)=\underset{x\to 1^+}{Lt}\ f(x)=-7$

$\Rightarrow\ \underset{x\to 1}{Lt}\ f(x)=-7.$ Choice (B)

3. $\underset{x\to 0^-}{Lt}\dfrac{e^{\frac{1}{x}}}{1-e^{\frac{1}{x}}}=\dfrac{0}{1-0}=0$ ----------- (1)

$\underset{x\to 0^+}{Lt}\dfrac{e^{\frac{1}{x}}}{1-e^{\frac{1}{x}}}=\underset{x\to 0^+}{Lt}\dfrac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}\left(e^{\frac{-1}{x}}-1\right)}$

$=\underset{x\to 0^+}{Lt}\dfrac{1}{e^{\frac{-1}{x}}-1}=\dfrac{1}{0-1}=-1$ ------------- (2)

From (1) and (2) $\underset{x\to 0^-}{Lt}\ f(x)\neq\underset{x\to 0^+}{Lt}\ f(x)$

$\therefore\ f(x)$ is not continuous at $x=0$. Choice (B)

4. $f(x)=a[x-5]+b\,[x+5]$

$\therefore\ f(x)=a\,([x]-5)+b([x]+5)$

$\underset{x\to 5^-}{Lt}\ f(x)=\underset{x\to 5^{-1}}{Lt}\ \{a([x]-5)+b\,([x]+5)$

$=-a+9b$ ------ (1)

$\underset{x\to 5^+}{Lt}\ f(x)=\underset{x\to 5^+}{Lt}\ \{a([x]-5)+b\,([x]+5)\}$

$=10\,b$ ---- (2)

As $f(x)$ is continuous at $x=5$

$\Rightarrow\ \underset{x\to 5^-}{Lt}\ f(x)=\underset{x\to 5^+}{Lt}\ f(x)$

$\Rightarrow\ -a+9b=10b$

$\Rightarrow\ a+b=0.$ Choice (A)

5. Given $f(x)=\begin{cases}\dfrac{\sqrt{x}-3}{x-9}, & \text{for } x\neq 9\\ 3k, & \text{for } x=9\end{cases}$

As $f(x)$ is continuous everywhere

$f(x)$ is continuous at $x=9$

$\therefore\ \underset{x\to 9}{Lt}\ f(x)=f(9)$

$\Rightarrow\ \underset{x\to 9}{Lt}\left(\dfrac{\sqrt{x}-3}{x-9}\right)=3k$

$\Rightarrow\ \underset{x\to 9}{Lt}\left(\dfrac{\frac{1}{2\sqrt{x}}}{1}\right)=3k$ (By L'Hospital's Rule)

$\Rightarrow\ \dfrac{1}{2\sqrt{9}}=3k$

$\Rightarrow\ \dfrac{1}{6}=3k$

$\Rightarrow\ k=\dfrac{1}{18}.$ Choice (D)

6. Let $f(x)=ax^2+3\log x+bx$

$f^1(x)=2ax+\dfrac{3}{x}+b$

Given at $x=-1$ and 1 $f(x)$ has extreme values

$\Rightarrow\ f^1(-1)=0$ and $f^1(1)=0$

$f^1(-1)=2a\,(-1)+\dfrac{3}{-1}+b=0$

$\Rightarrow\ -2a+b=3$

$\Rightarrow\ 2a-b=-3.$ Choice (D)

7. Given $xy=6\Rightarrow y=\dfrac{6}{x}$

$2x+3y=2x+\dfrac{3\times 6}{x}$

Let $f(x)=2x+\dfrac{18}{x}$

$f^1(x)=2-\dfrac{18}{x^2}$

$\Rightarrow\ f^{11}(x)=\dfrac{36}{x^3}$ which is always positive as x, y and R^+

For maximum or minimum $f^1(x)=0$

$=2-\dfrac{18}{x^2}=0$

$\Rightarrow\ x=3$

$\therefore\ y=\dfrac{6}{x}$

$\Rightarrow\ y=2$

$\therefore$ At $x=3$ and $y=2$, $2x+3y$ has minimum value

The minimum value is $2(3)+3(2)=12.$ Choice (A)

8. Choice A: Let $f(x) = \dfrac{1}{2x} \Rightarrow f^1(x) \neq 0$ for any real value of x

Choice B: $f_1(x) = 4x + 7 \Rightarrow f_1^1(x) \neq 0$ for any real value of x

Choice C: $f_2(x) = e^{5x+1} \Rightarrow f_2^1(x) \neq 0$ for any real value of x

$\therefore$ None of the options have either maximum or minimum values. Choice (D)

9. Let $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$

$f^1(x) = 12x^3 - 6x^2 - 12x + 6$

$f^1(x) = 0 \Rightarrow 2x^3 - x^2 - 2x + 1 = 0$

$(x - 1)(x + 1)(2x - 1) = 0$

$\Rightarrow x = \pm 1, \dfrac{1}{2}$

Max of $f(x)$ in $[-1, 2]$ = Max $\{f(-1), f\left(\dfrac{1}{2}\right), f(1), f(2)\}$

$= \text{Max}\left\{-6, \dfrac{39}{16} 2,\ 21\right\} = 21$. Choice (C)

10. Standard result of Cauchy mean value theorem. Choice (D)

11. Given $f(x) = e^{-2x} \sin 2x$

Clearly $f(x)$ is continuous on $\left[0, \dfrac{\pi}{2}\right]$ and differentiable in $\left(0, \dfrac{\pi}{2}\right)$ and $f(0) = f\left(\dfrac{\pi}{2}\right)$ then there exists $c \in \left(0, \dfrac{\pi}{2}\right)$ such that $f^1(c) = 0$

$f^1(x) = \sin 2x\,(-2e^{-2x}) + 2\cos 2x e^{-2x}$

$f^1(c) = 2e^{-2c}(\cos 2c - \sin 2c) = 0$

$\Rightarrow \sin 2c = \cos 2c$ or $\tan 2c = 1$

$\Rightarrow c = \dfrac{\pi}{8} \in \left(0, \dfrac{\pi}{2}\right)$

The required value of $c = \dfrac{\pi}{8}$. Choice (A)

12. Given $f(x) = x(x + 2)(x - 1)$

Clearly $f(x)$ is continuous on $[-1, 0]$ and differentiable on $(-1, 0)$

$a = -1, b = 0;$

$f(b) = 0, f(a) = (-1)(1)(-2) = 2$

$f^1(x) = 3x^2 + 2x - 2$ and $f^1(c) = 3c^2 + 2c - 2$

By Lagrange's mean value theorem

$$f^1(c) = \frac{f(b) - f(b)}{b - a}$$

$\Rightarrow 3c^2 + 2c - 2 = \dfrac{0 - 2}{1}$

$\Rightarrow 3c^2 + 2c = 0$

$c(3c + 2) = 0 \Rightarrow c = 0$ or $c = \dfrac{-2}{3}$ As $c \neq 0$,

$c = \dfrac{-2}{3} \in (-1, 0)$. Choice (C)

13. $U = \tan^{-1}\left(\dfrac{x^3 - y^3}{x + y}\right) \Rightarrow \tan u = \dfrac{x^3 - y^3}{x + y}$

Let $x = kx$ and $y = ky$ then

$\text{tanu} = \dfrac{k^3(x^3 - y^3)}{k(x + y)} = k^2\left(\dfrac{x^3 - y^3}{x + y}\right) = f(x)$ say

$\therefore$ f is a homogeneous function of degree 2

$\therefore$ By Euler's theorem $x \dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} = = n(f)$ (n is degree of f)

$= x.\dfrac{\partial(\tan u)}{\partial x} + y\dfrac{\partial(\tan u)}{\partial y} = 2\tan u$

$= x\sec^2 u \cdot \dfrac{\partial u}{\partial x} + y\ \sec^2 u\ \dfrac{\partial u}{\partial y}\ \dfrac{\partial u}{\partial y} = 2\tan u$

$\therefore\ x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$. Choice (C)

14. Given $f(x, y)$ is a homogeneous function of degree n then by Euler's theorem we know that x $\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}$

$= nf$ ------- (1)

Differentiating (1) w. r. to y partially we have

$$x.\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f}{\partial y} + y.\frac{\partial^2 f}{\partial y^2} = n.\frac{\partial f}{\partial y}$$

$\Rightarrow y\dfrac{\partial^2 f}{\partial y^2} + x\dfrac{\partial^2 f}{\partial x \partial y} = (n-1)\dfrac{\partial f}{\partial y}$ Choice (B)

15. $U = \cos^{-1}\left(\dfrac{x^3 + 2y^3 - 4z^3}{x^7 - y^7 - z^7}\right)$

$\Rightarrow \cos u = \dfrac{x^3 + 2y^3 - 4z^3}{x^7 - y^7 - z^7} = f(\text{say})$

Clearly f is a homogeneous function of degree – 4

$\therefore$ By Euler's theorem $x.\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} + z\dfrac{\partial f}{\partial z} = n.f$

$\therefore\ x\dfrac{\partial(\cos u)}{\partial x} + y\dfrac{\partial(\cos u)}{\partial y} + z\dfrac{\partial(\cos u)}{\partial z}s = -4.\cos u$

$= -\left(x.\dfrac{\partial u}{\partial x}.(\sin u) + y.\dfrac{\partial u}{\partial y}(\sin u) + z\dfrac{\partial u}{\partial z}(\sin u)\right)$

$= -4\cos u$

$\therefore\ x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 4\cot u$. Choice (A)

16. Given $z = \text{in}\,(x^2 + y^3)$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{2x}{x^2+y^3} \text{ and } \frac{\partial z}{\partial y} = \frac{3y^2}{x^2+y^3}$$

Consider $3x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y} = \frac{6x^2}{x^2+y^3} + \frac{6y^3}{x^2+y^3}$

$$= \frac{6(x^2+y^3)}{x^2+y^3} = 6.$$

Choice (B)

17. We have $\int_0^1 (x^2-2x+3)e^{-x}dx$

$= (x^2-2x+3)(-e^{-x}) - (2x-2)(e^{-x}) + 2(-e^{-x})_0^1$

$= (-2e^{-1} - 0 - 2e^{-1}) - (-3 + 2 - 2)$

$= 3 - 4e^{-1} = 3 - \frac{4}{e}$

Choice (D)

18. We know that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Let $I = \int_0^{\frac{\pi}{2}} \frac{3\sec x + 4\text{cosec}\, x}{\sec x + \text{c osec}\, x}$

$$= \int_0^{\frac{\pi}{2}} \frac{3\sec\left(\frac{\pi}{2}-x\right) + 4\,\text{c osec}\left(\frac{\pi}{2}-x\right)}{\sec\left(\frac{\pi}{2}-x\right) + \text{cosec}\left(\frac{\pi}{2}-x\right)}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{3\text{cosec}\, x + 4\sec x}{\text{cosec}\, x + \sec x}dx$$

$I + I$

$$= \int_0^{\frac{\pi}{2}} \frac{3\sec x + 4\,\text{cosec}\, x}{\text{cosec}\, x + \text{cosec}\, x}dx + \int_0^{\frac{\pi}{2}} \frac{3\text{cosec}\, x + 4\sec x}{\sec x + \text{cosec}\, x}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{3(\text{cosec}x + \sec x) + 4(\sec x + \text{cosec}x)}{\sec x + \text{cosec}x}dx$$

$$= \int_0^{\frac{\pi}{2}} 7dx = 7(x)_0^{\frac{\pi}{2}} = 7\frac{\pi}{2}$$

$\therefore\quad 2I = 7\frac{\pi}{2}$

$\Rightarrow\quad I = 7\frac{\pi}{4}$

Choice (A)

19. $\int_0^{\infty} \frac{x^2}{(1+x^2)^{\frac{5}{2}}}dx$ Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$ $L.L$

When $x = 0$ $\theta = 0$ and $U.L$ when $x = \infty$, $\theta = \frac{\pi}{2}$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\tan^2\theta.\sec^2\theta}{(1+\tan^2\theta)^{\frac{5}{2}}}d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^2\theta.\sec^2\theta}{\cos^2 0.\sec^5\theta}d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^2\theta.\cos\theta d\theta = \left.\frac{\sin^3\theta}{3}\right]_0^{\frac{\pi}{2}} = \frac{1}{3}$$

Choice (B)

20. $\int_1^4 \frac{dx}{(x-2)(x-3)}$

$$= \int_1^2 \frac{dx}{(x-2)(x-3)} + \int_2^3 \frac{dx}{(x-2)(x-3)} + \int_3^4 \frac{dx}{(x-2)(x-3)}$$

$I = I_1 + I_2 + I_3$ (say)

Clearly as I has discontinuity at $x = 2$, I_1 diverges

As I_1 diverges I also diverges

$\therefore \int_1^4 \frac{dx}{(x-2)(x-3)}$ diverges.

Choice (D)

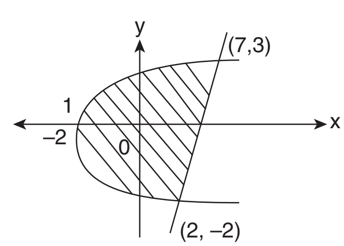
21. $\int_0^{\infty} \frac{dx}{9+x^2} = \underset{a\to\infty}{Lt} \int_0^a \frac{dx}{3^2+x^2} = \underset{a\to\infty}{Lt} \frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^a$

$\underset{a\to\infty}{Lt} \frac{1}{3}\tan^{-1}\left(\frac{a}{3}\right) = \frac{\pi}{6}$

The given improper integral converges to $\frac{\pi}{6}$

Choice (C)

22.



$y^2 = x + 2$ and $y = x - 4$

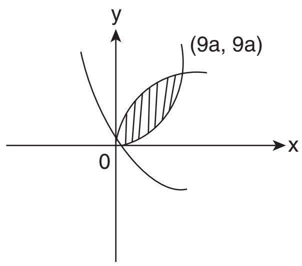
The above two intersect at (2, –2) and (7, 3).

The area bounded $= \int_{-2}^{+3} \left[(y+4) - (y^2-2)\right]dy$

$$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{3} + 2y\right]_{-2}^{3}$$

$= \frac{125}{6} = 20\frac{5}{6}$ sq units. Choice (A)

23.



Given $y^2 = 9ax$ and $x^2 = 9ay$

The above two curves intersect at (0, 0) and ($9a$, $9a$).

The required area = $\int_0^{9a}\left(\sqrt{9ax} - \frac{x^2}{9a}\right)dx$

$= 3\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{9a} - \frac{1}{9a}\left(\frac{x^3}{3}\right)_0^{9a}$

$= 2\sqrt{a}\left((9a)^{\frac{3}{2}}\right) - \frac{1}{27a}(9a)^3$

$= 54a^2 - 27a^2 = 27a^2$. Choice (D)

24. Standard Result. Choice (A)

25. We have $2e^x + 3\sin x$

$= 2\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}.....\right) + 3\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}.....\right)$

$= \left(2 + 2x + 2\frac{x^2}{2!} + 2.\frac{x^3}{3!} + 2\frac{x^4}{4!} + 2\frac{x^5}{5!} + ...\right)$

$+ \left(3x - 3\frac{x^3}{3!} + 3.\frac{x^5}{5!} - 3\frac{x^7}{7!} + ...\right)$

$= 2 + 5x + x^2 - \frac{x^3}{3!} + \frac{x^4}{12} + \frac{x^5}{24} + ...$ Choice (B)

Engineering Mathematics Test 2

Number of Questions: 25 **Time: 60 min.**

(Ordinary Differential Equations, Calculus (Vector Calculus))

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The directional derivative of $f(x, y) = x^3y^2 + 3xy$ at (1, 2) in the direction of unit vector which makes an angle of $\frac{\pi}{4}$ with the x-axis is ________.

(A) $16\sqrt{2}$ (B) $8\sqrt{2}$

(C) $\frac{15}{\sqrt{2}}$ (D) $\frac{25}{\sqrt{2}}$

2. If the vector $e^{px-y-z}\left(\bar{i}+\bar{j}+\bar{k}\right)$ is solenoidal, then the value of p is ______.

(A) –3 (B) 3

(C) –2 (D) 2

3. Evaluate $\int_c \left(x^2+3yz\right)ds$ where c is the curve defined by $x = 6y$; $z = 4$ from $\left(3, \frac{1}{2}, 4\right)$ to (6, 1, 4).

(A) $15\sqrt{37}$ (B) $2\sqrt{37}$

(C) 199 (D) $37\sqrt{33}$

4. The unit normal vector to the surface $xy^3 + 3yz = 3$ at the point (3, –1, –2) is __________

(A) $\frac{-\bar{i}-3\bar{j}-3\bar{k}}{\sqrt{19}}$ (B) $\frac{-\bar{i}+3\bar{j}-3\bar{k}}{\sqrt{19}}$

(C) $\frac{\bar{i}+3\bar{j}-3\bar{k}}{\sqrt{19}}$ (D) $\frac{\bar{i}-3\bar{j}-3\bar{k}}{\sqrt{19}}$

5. Evaluate the integral $\int_c \left(yz+z+z^2\right)dx + (xz-1)\,dy + (xy+x+2xz)\,dz$ from (2, 3, 3) to (3, 4, 5).

(A) 146 (B) 107

(C) 39 (D) 185

6. Evaluate the integral $\oint_c (x+2y)dx + x^2y\,dy$, C is the triangle with vertices at (0, 0), (3, 0) and (3, 3) taken in that order.

(A) 18 (B) $\frac{45}{4}$

(C) $\frac{37}{3}$ (D) 15

7. Evaluate the surface integral $\iint_S \bar{F}.\bar{n}\,dA$, where $\bar{F} = z^2\,\bar{i} + xy\,\bar{j} + y^2\,\bar{k}$ and S is the portion of the surface of the cylinder $x^2 + y^2 = 49$; $0 \le z \le 5$ included in first octant.

(A) 518 (B) 2590

(C) $\frac{2590}{3}$ (D) 624

8. If $\bar{V} = (2x-y) - yz^2\,j - y^2z\,\bar{k}$, then evaluate $\oint_c \bar{V}.d\bar{r}$ where C is the curve bounding the projection on the semi sphere $x^2 + y^2 + z^2 = 9$, $z > 0$; in the xy plane.

(A) 18 π (B) 6 π

(C) 8 π (D) 9 π

9. If $\bar{r} = 2x\bar{i} + 3y\,\bar{j} + z\bar{k}$ and V is the volume of the sphere $x^2 + y^2 + z^2 = 16$ then $\iint_S \bar{r}.\bar{n}\,dA =$ _______.

(A) 38 π (B) 512 π

(C) 438 π (D) 348 π

10. The work done by the force $\bar{F} = 2xy\bar{i} + y^2\bar{j} + z\bar{k}$ in moving a particle over the circular path $x^2 + y^2 = 9$; $z = 0$ from (3, 0, 0) to (0, 3, 0) is ________.

(A) 17 (B) –12

(C) –9 (D) 6

11. The solution of $3(xdy + ydx) = 2xydy$ when $x = 1$, $y = 1$ is

(A) $3 \log(xy) - 2y + 2 = 0$

(B) $\log(xy) + y - 2 = 0$

(C) $\log x + \log y + 2 = 0$

(D) $\log(xy) - 5y + 2 = 0$

12. Solve $dy = (9x + y - 1)^2\,dx$ when $x = 0$, $y = 1$.

(A) $3 \tan x = 3x + y - 1$

(B) $3 \tan(3x) = 9x + y - 1$

(C) $\tan x = 9x + y - 1$

(D) $3 \tan 3x = 9x - y + 1$

13. The solution of the differential equation $\frac{x\,dy - ydx}{x} = \cos^2\left(\frac{y}{x}\right)dx$ is

(A) $\tan\left(\frac{y}{x}\right) = \log(cx)$ (B) $\tan\left(\frac{y}{x}\right) = x + c$

(C) $\tan^{-1}\left(\frac{y}{x}\right) = \log(cx)$ (D) $\tan\left(\frac{x}{y}\right) = \log x + c$

14. The integrating factor of $\frac{dy}{dx} - y\tan x - \cos x = 0$ is

(A) $\cos x$ (B) $\sin x$
(C) $\sec x$ (D) $\operatorname{cosec} x$

15. Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ with the boundary conditions $x = 1, y = 1$.

(A) $x^2 + y^2 = 3$ (B) $x^2 - y^2 = 0$
(C) $x^2 - y^2 = 2$ (D) $x^2 + y^2 = 2$

16. The solution of the differential equation $ydx = (x + 3y^3)dy$ when $x = 1, y = 1$ is

(A) $x = 3y^2 - 1$ (B) $3x = y(2y^2 - 1)$
(C) $2x = y(3y^2 + 1)$ (D) $2x = y(3y^2 - 1)$

17. The particular integral solution of the differential equation

$\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 3y = e^{4x}\sinh x$ is

(A) $\frac{1}{8}\left[e^{5x} - 2e^{3x}\right]$

(B) $\frac{1}{16}\left[e^{3x} - xe^{-5x}\right]$

(C) $\frac{1}{64}\left[e^{5x} - 8x\,e^{3x}\right]$

(D) $\frac{1}{4}\left[x\,e^{5x} - 2e^{3x}\right]$

18. The solution of the differential equation $(D^3 + 5D^2)y = 4$ is

(A) $y = C_1 + C_2e^{-5x} + \frac{1}{5}x^2$

(B) $y = C_1 + C_2x + C_3e^{-5x} + \frac{2}{5}x^2$

(C) $y = (C_1 + C_2x)\,e^{5x} + \frac{2}{5}x^2$

(D) $y = (C_1 + C_2x)\,e^{x} + \frac{2}{5}x^2 + e^{5x}$

19. The particular integral of the differential equation given by $(D^2 - 2D + 4)y = x^2 e^x$ is

(A) $\frac{1}{9}e^x(3x^2 - 2)$ (B) $\frac{1}{6}e^x(2x^2 - 3)$

(C) $\frac{1}{8}e^x(3x^2 - 1)$ (D) $\frac{1}{3}e^x(2x^2 - 3)$

20. The solution of the differential equation $(D^4 + D^2 + 36D + 52)y = 0$ is

(A) $y = (C_1 + C_2x)e^{-2x} + (C_3 + C_4x)e^{2x}$
(B) $y = (C_1 + C_2x + C_3\cos 3x + C_4\sin 3x)e^{2x}$
(C) $y = (C_1 + C_2x + (C_3\cos 3x + C_4\sin 3x)e^{-2x}$
(D) $y = (C_1 + C_2x)e^{-2x} + (C_3\cos 3x + C_4\sin 3x)e^{2x}$

21. The Laplace Transform of the function $f(t) = t^2\sin 3t$, $t > 0$ is ______.

(A) $\frac{(s^2 - 2s + 9)}{(s^2 + 9)^3}$ (B) $\frac{18(s^2 - 3)}{(s^2 + 9)^3}$

(C) $\frac{-(s^2 - 2s + 9)}{(s^2 + 9)^3}$ (D) $\frac{6(s^2 - 2s + 9)}{(s^2 + 9)^3}$

22. If $u(t - a)$ denotes the unit step function, then the Laplace Transform of $(t^2 + 3)\,u(t - 2)$ is ________.

(A) $\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{7}{s}\right)e^{-2s}$ (B) $\left(\frac{2}{s^3} + \frac{3}{s}\right)e^{-2s}$

(C) $\left(\frac{2}{s^2} + 3\right)e^{-2s}$ (D) $\left(\frac{1}{s^3} + \frac{4}{s^2} + \frac{7}{s}\right)e^{-2s}$

23. The Laplace Transform of solution of the initial value problem $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0$, $y(0) = 1$ and $y^1(0) = -2$ will be ____

(A) $\frac{1}{(s-4)(s+2)}$ (B) $\frac{1}{s-4}$

(C) $\frac{-1}{(s-4)(s+2)}$ (D) $\frac{1}{s+2}$

24. The inverse Laplace Transform of $\frac{3}{2}\left[\frac{1}{\sqrt{s^5}} - \frac{1}{\sqrt{s^3}}\right]$ is ______.

(A) $\sqrt{\frac{t}{\pi}}\,(2t - 3)$ (B) $\sqrt{\frac{t}{\pi}}\,(4t - 3)$

(C) $\sqrt{\frac{\pi}{t}}\,(2t + 3)$ (D) $\sqrt{\frac{\pi}{t}}\,(4t + 3)$

25. The inverses Laplace Tranform of $\frac{4(2s + 3)}{(s^2 + 4s + 20)}$ is

(A) $e^{2t}[2\cos 4t - \sin 4t]$
(B) $e^{2t}\{2\sin 4t - \cos 4t]$
(C) $e^{-2t}[2\cos 4t - \sin 4t]$
(D) $e^{-2t}[2\sin 4t - \cos 4t]$

Answer Keys

1. D	**2.** D	**3.** A	**4.** B	**5.** B	**6.** B	**7.** C	**8.** D	**9.** B	**10.** C
11. A	**12.** B	**13.** A	**14.** A	**15.** B	**16.** D	**17.** C	**18.** B	**19.** A	**20.** D
21. B	**22.** A	**23.** D	**24.** A	**25.** C					

Hints and Explanations

1. Given $f(x, y, z) = x^3 y^2 + 3xy$

$$\nabla f = \frac{\partial f}{\partial x}\bar{i} + \frac{\partial f}{\partial y}\bar{j} = (3x^2 y^2 + 3y)\bar{i} + (2x^3 y + 3x)\bar{j}$$

$$(\nabla f)_{(1,2)} = (12 + 6)\bar{i} + (4 + 3)\bar{j} = 18\bar{i} + 7\bar{j}$$

Given that unit vector makes an angle $\frac{\pi}{4}$ with x-axis

$\therefore$ unit vector must be $\bar{b} = \cos\frac{\pi}{4}\bar{i} + \sin\frac{\pi}{4}\bar{j} = \frac{\bar{i}+\bar{j}}{\sqrt{2}}$

The directional derivative of f in the direction $\bar{b}$ is $\nabla f.\bar{b}$

$$(18\bar{i}+7\bar{j}).\frac{(\bar{i}+\bar{j})}{\sqrt{2}} = \frac{18+7}{\sqrt{2}} = \frac{25}{\sqrt{2}}$$ Choice (D)

2. Given $\bar{V} = e^{px-y-z}(\bar{i}+\bar{j}+\bar{k})$ is solenoidal

We know that if $\bar{V}$ is solenoidal, div $\bar{V} = 0$

$$\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z} = 0$$

$$= e^{px-y-z}.p + e^{px-y-z}.(-1) + e^{px-y-z}(-1) = 0$$

$\therefore p - 1 - 1 = 0 \Rightarrow p = 2$ Choice (D)

3. Let $x = t$; $y = \frac{t}{6}$; $z = 4$ and $3 \le t \le 6$.

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,.\,dt = \sqrt{1+\frac{1}{36}}\,dt = \frac{\sqrt{37}}{6}dt$$

$$\int_C (x^2+3yz)\,ds = \int_3^6 \left(t^2+3\frac{t}{6}.4\right)\frac{\sqrt{37}}{6}dt = d\left(\tan\left(\frac{y}{x}\right)\right)$$

$$= \tan\left(\frac{y}{x}\right) = \tan\left(\frac{y}{x}\right)$$ Choice (A)

4. Let $f(x, y, z) = xy^3 + 3yz - 3$

The normal vector to the surface $f(x, y, z)$ is

$$\frac{dy}{dx} - y\tan x - \cos x$$

$$\nabla f = y^3\,\bar{i} + (3xy^2 + 3z)\bar{j} + 3y\bar{k}$$

The normal vector at (3, –1, –2)

$$(\nabla f)_{(3,-1,-2)} = -\bar{i} + (9-6)\bar{j} - 3\bar{k} = -\bar{i} + 3\bar{j} - 3\bar{k}$$

$\therefore$ The unit normal vector to the surface f is $\frac{\nabla f}{|\nabla f|}$

$$= \frac{-\bar{i}+3\bar{j}-3\bar{k}}{\sqrt{19}}$$ Choice (B)

5. $\int (yz+z+z^2)dx + (xz-1)dy + (xy+x+2xz)dz$

Let $f(x, y, z) = yz + z + z^2$

$g(x, y, z) = xz - 1$

$h(x, y, z) = xy + x + 2xz$

$$\frac{\partial f}{\partial y} = z = \frac{\partial g}{\partial x}$$

$$\frac{\partial f}{\partial z} = y + 1 + 2z = \frac{\partial h}{\partial x}$$

$$\frac{\partial g}{\partial z} = x = \frac{\partial h}{\partial y}$$

The integral is independent of the path C.

The integral is exact differential

So there exists a function Φ

Such that $\frac{\partial \phi}{\partial x} = yz + z + z^2$ $\rightarrow$ (1)

$\frac{\partial \phi}{\partial y} = xz - 1$ $\rightarrow$ (2)

$\frac{x}{y} = \int 3y^2 .\frac{1}{y}dy + c = xy + x + 2xz$ $\rightarrow$ (3)

$$\frac{\partial \phi}{\partial x} = yz + z + z^2$$

Integrate wrt x.

$\phi = xyz + xz + xz^2 + Q(y, z)$ $\rightarrow$ (4)

Diff wrt y

$\frac{\partial \phi}{\partial y} = xz + \phi^1(y, z)$ $\rightarrow$ (5)

Comparing (2) and (5)

$\phi^1(y, z) = -1 \Rightarrow \phi(y, z) = -y + R(z)$

$\therefore$ $\phi = xyz + xz + xz^2 - y + R(z)$ $\rightarrow$ (6)

$$\frac{ydx - xdy}{y^2} = 3y\,dy = xy + x + 2xz + R^1(z)$$

Comparing (3) and (6)

$R^1(z) = 0$

$\Rightarrow$ $R(z) = k$

$\therefore$ $\phi = xyz + xz + xz^2 - y + k$

$$\therefore \int_{(3,4,5)} (yz+z+z^2)dx + (xz-1)dy + (xy+x+2xz)dz$$

$$= \int_{(2,3,3)} d(xyz + xz + xz^2 - y)$$

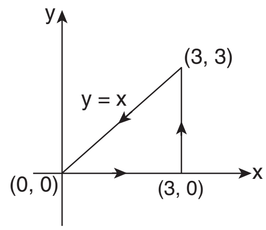
$$= xyz + xz + xz^2 - y\Big]_{(2,3,3)}^{(3,4,5)}$$

$= 146 - 39 = 107$ Choice (B)

6. $\oint_C (x+2y)dx + x^2 y\,dy$

$f(x, y) = x + 2y$ $\quad g(x, y) = x^2 y$

$$\frac{\partial f}{\partial y} = 2 \text{ and } \frac{\partial g}{\partial x} = 2xy$$



By green's theorem

$$\oint_C (x+2y)dx + x^2 y dy = \iint_R (2xy-2)dxdy$$

$$= \int_0^3 \int_0^x (2xy-2)dydx$$

$$= \int_0^3 \left. xy^2 - 2y\right]_0^x dx$$

$$= \int_0^3 (x^3 - 2x)dx$$

$$= \left.\frac{x^4}{4} - x^2\right]_0^3 = \frac{81}{4} - 9 = \frac{45}{4}$$

Choice (B)

7. Let $f(x, y, z) = x^2 + y^2 - 49$
Surface then
grad $f = 2x\bar{i} + 2y\,j$

$$\overset{\Delta}{n} = \frac{\text{grad } f}{|\text{grad } f|} = \frac{2x\bar{i}+2y\,j}{2\sqrt{x^2+y^2}} = \frac{1}{7}\left(x\bar{i}+y\bar{j}\right)$$

Consider the projection of S on the yz plane. It is a rectangle with sides 7 and 5.

$$dA = \frac{dy\,dz}{\bar{n}.\bar{i}} = \frac{dy\,dz}{\frac{x}{7}}$$

$$\bar{F}.\overset{\Delta}{n} = \left(z^2\,\bar{i} + xy\,\bar{j} + y^2\,\bar{k}\right).\left(\frac{x\bar{i}+y\bar{j}}{7}\right) = \frac{xz^2+xy^2}{7}$$

$$\therefore \quad \iint_S \bar{F}.\overset{\Delta}{n}\,dA = \iint_s \frac{xz^2+xy^2}{7}\,\frac{dy\,dz}{\frac{x}{7}}$$

$$= \int_{z=0}^{5}\int_{y=0}^{7} \left(z^2+y^2\right)dy\,dz = \int_{z=0}^{5} \left. z^2 y + \frac{y^3}{3}\right]_0^7 dz$$

$$= \int_{z=0}^{5} (7z^2 + \frac{343}{3})dz$$

$$\left.\frac{7z^3}{3} + \frac{343}{3}z\right]_0^5 = \frac{875}{3} + \frac{1715}{3} = \frac{2590}{3}$$

Choice (C)

8. Given $\bar{V} = (2x-y)\bar{i} - yz^2\,\bar{j} - y^2 z\,\bar{k}$

$$\oint \bar{V}.d\bar{r} = \iint_S \nabla\times V.ndA$$

(By stoke's theorem)

$$\nabla\times V = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x-y & -yz^2 & -y^2 z \end{vmatrix}$$

$$= i(-2yz + 2yz) - j(0) + \bar{k}\,(0 - (-1)) = \bar{k}$$

$f(x, y, z) = x^2 + y^2 + z^2 - 9$ be the surface

grad $f = 2x\bar{i} + 2y\,\bar{j} + 2z\bar{k}$

$$\hat{n} = \frac{\text{grad } f}{|\text{grad } f|} = \frac{2x\bar{i}+2y\bar{j}+2z\bar{k}}{2\sqrt{x^2+y^2+z^2}}$$

$$\hat{n} = \frac{x\bar{i}+y\bar{j}+z\bar{k}}{3}$$

$$\therefore \quad \nabla\times\bar{V}.\overset{\Delta}{n} = \bar{k}.\frac{\left(x\bar{i}+y\bar{j}+z\bar{k}\right)}{3} = \frac{z}{3}$$

$$\iint_S \nabla\times V.\overset{\Delta}{n}\,dA = \iint_R \frac{z}{3}\frac{dx\,dy}{\bar{n}.\bar{k}}$$

$$= \iint_R \frac{z}{3}\frac{dx\,dy}{\frac{z}{3}} = \iint_R dx\,dy$$

Area of circular region in x-y plane $= 9\pi$ Choice (D)

9. By using divergence theorem

$$\iint_S \bar{r}.\bar{n}\,dA = \iiint_V div\,\bar{r}\,dV$$

$$\bar{r} = 2x\,\bar{i} + 3y\,\bar{j} + z\bar{k}$$

$$\text{Div } \bar{r} = 2+3+1 = 6 = \iiint_V div\,\bar{r}\,dv = \iiint_V 6dV = 6V$$

V is the volume of the sphere

$$= 6 \times \frac{4}{3} \times \pi(4)^3 = 512\,\pi$$

Choice (B)

10. Given $\bar{F} = 2xy\bar{i} + y^2\,\bar{j} + z\bar{k}$

Work done by the force is $\int_C F.d\bar{r}$

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k} \Rightarrow d\bar{r} = dxi + dy\,\bar{j} + dz\bar{k}$$

$$\bar{F}.d\bar{r} = \left(2xy\bar{i} + y^2\,\bar{j} + z\bar{k}\right).\left(dx\bar{i} + dy\,\bar{j} + dz\bar{k}\right)$$

$$\bar{F}.d\bar{r} = 2xy\,dx + y^2\,dy + z\,dz$$

$$\int_C F.d\bar{r} = \int (2xy\,dx + y^2\,dy + z\,dz$$

Convert x, y, z in parametric form
$x = 3\cos t, y = 3\sin t, z = 0$
$dx = -3\sin t\, dt, dy = 3\cos t\, dt$
The limit of t is 0 to $\pi/2$

$$\int_C F.d\bar{r} = \int_0^{\frac{\pi}{2}} 2.3\cos t.3\sin t.(-3\sin t)\,dt + 9\sin^2 t\, 3\cos t\, dt$$

$$= -\int_0^{\frac{\pi}{2}} 27\sin^2 t\cos t\, dt$$

$$= -27\left.\frac{\mathrm{Sin}^3 t}{3}\right]_0^{\frac{\pi}{2}} = \frac{-27}{3} = -9$$ Choice (C)

11. $3(x\,dy + y\,dx) = 2xy\,dy$
$x(3-2y)\,dy + 3y\,dx = 0$

$$\frac{(3-2y)dy}{y} + \frac{3dx}{x} = 0$$

$$\int\left(\frac{3}{y} - 2\right)dy + 3\int\frac{dx}{x} = C_1$$

$3\log y - 2y + 3\log x = \log C$
$3\log(xy) - 2y = \log C$
Given when $x = 1, y = 1$
$\Rightarrow\ -2 = \log C$
$\therefore$ The required solution is $3\log(xy) - 2y + 2 = 0$.
Choice (A)

12. $dy = (9x + y - 1)^2 dx$ or $\frac{dy}{dx} = (9x + y - 1)^2$ ……. (1)

Let $9x + y - 1 = u$

$$9 + \frac{dy}{dx} = \frac{du}{dx}$$

$$\Rightarrow \frac{du}{dx} - 9 = u^2 \text{ (from 1)}$$

$$\frac{du}{dx} = u^2 + 9 \text{ or } \frac{du}{u^2+9} = dx$$

$$\int\frac{du}{u^2+9} = \int dx + c \Rightarrow \frac{1}{3}\mathrm{Tan}^{-1}\left(\frac{u}{3}\right) = x + c$$

i.e., $\frac{1}{3}\tan^{-1}\left(\frac{9x+y-1}{3}\right) = x + c$ or

$3\tan(3x + 3c) = 9x + y - 1$, when $x = 0, y = 1$

$$\frac{1}{3}\tan^{-1}(0) = C \Rightarrow C = 0$$

$\therefore$ The required solution is $3\tan(3x) = 9x + y - 1$.
Choice (B)

13. $\frac{x\,dy - y\,dx}{x} = \cos^2\left(\frac{y}{x}\right)dx$

Dividing both sides by $x\cos^2\left(\frac{y}{x}\right)$, we have

$$\left(\frac{x\,dy - y\,dx}{x^2}\right)\sec^2\left(\frac{y}{x}\right) = \frac{dx}{x}$$

$$d\left(\tan\left(\frac{y}{x}\right)\right) = d(\log x)$$

Integrating on both sides, we have

$$\tan\left(\frac{y}{x}\right) = \log x + \log c$$

$$\tan\left(\frac{y}{x}\right) = \log(cx).$$ Choice (A)

14. $\frac{dy}{dx} - y\tan x - \cos x = 0$

$$\Rightarrow \frac{dy}{dx} - y\tan x = \cos x$$

which is in the form of $\frac{dy}{dx} + Py = Q$

The integrating factor of above equation $e^{\int p dx} = e^{-\int \tan x\, dx}$
$= e^{\log\cos x} = \cos x$. Choice (A)

15. $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ --------- (1)

$(x^2 + y^2)dx - 2xy\,dy = 0$
which is in the form of $Mdx + Ndy = 0$
Here $M = x^2 + y^2$, $N = -2xy$

$$\frac{\partial M}{\partial y} = 2y, \frac{\partial N}{\partial x} = -2y$$

$$\Rightarrow \text{ Here } \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{-1}{2xy}(2y + 2y) = \frac{-2}{x}$$

which is a function of x alone say $f(x)$ then the Integrating factor ($I.F$) is

$$e^{\int f(x)dx} = e^{\int\frac{-2}{x}dx} = e^{-2\log x} = e^{\log x^{-2}} = x^{-2}$$

Multiplying (1) by $I.F$, we have

$\left(1 + \frac{y^2}{x^2}\right)dx - \frac{2y}{x}dy = 0$ which is in the form of $M\,dx + N\,dy = 0$

$\therefore$ Solution is

$$\int m\,dx(\text{taking } y \text{ constant}) + \int(\text{terms of } N \text{ not containing } x)dy = C$$

$$\Rightarrow \int\left(1 + \frac{y^2}{x^2}\right)dx + \int 0\,dy$$

$$x - \frac{y^2}{x} = C$$

Given when $x = 1, y = 1 \Rightarrow C = 0$

$\therefore$ The required solution is $x - \frac{y^2}{x} = 0$ or $x^2 - y^2 = 0$.

Choice (B)

16. $y\,dx = (x + 3y^3)\,dy$

or $y\frac{dx}{dy} = x + 3y^3$

or $\frac{dx}{dy} - \frac{x}{y} = 3y^2$, which is a linear equation in y of the form $\frac{dx}{dy} + px = Q$ here $P = -\frac{1}{y}$ and $Q = 3y^2$

$\therefore$ $I.F = e^{\int pdy} = e^{-\int \frac{1}{y}dy} = e^{\log y^{-1}} = \frac{1}{y}$

$\therefore$ The solution is $x \,.\, e^{I.F} = \int Q.e.^{I.F}.dy$

$$\Rightarrow \frac{x}{y} = \int 3y^2.\frac{1}{y}dy + c$$

$$\frac{x}{y} = \frac{3y^2}{2} + c$$

Given when $x = 1, y = 1$

$\Rightarrow$ $1 = \frac{3}{2} + c$ or $c = \frac{-1}{2}$

$\therefore$ The required solutions is $\frac{x}{y} = \frac{3y^2}{2} - \frac{1}{2}$

i.e., $2x = y(3y^2 - 1)$

Alternate solution:

Given

$$ydx = (x + 3y^3)dy$$

$$ydx - xdy = 3y^3\,dy$$

$$\frac{ydx - xdy}{y^2} = 3y\,dy$$

$$d\left(\frac{x}{y}\right) = 3y\,dy$$

Integrating on both sides

$$\left(\frac{x}{y}\right) = \left(\frac{3y^2}{2}\right) + C$$

$2x = y(3y^2 + C_1)$

Given $x = 1; y = 1$

$2 = 3 + C_1 \Rightarrow C_1 = -1$

$\therefore$ required solution is $2x = y\,(3y^2 - 1)$. Choice (D)

17. $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 3y = e^{4x}\sin hx$

i.e., $(D^3 - 5D^2 + 7D - 3)y$

$$= e^{4x}\left[\frac{e^x - e^{-x}}{2}\right] = \frac{e^{5x} - e^{3x}}{2}$$

i.e., $(D - 1)^2\,(D - 3)y = \frac{e^{5x} - e^{3x}}{2}$

Particular integral is $\frac{e^{5x} - e^{3x}}{(D-1)^2(D-3)}.\frac{1}{2}$

$$\frac{1}{2}\frac{e^{5x}}{(D-1)^2(D-3)} - \frac{1}{2(D-1)^2(D-3)}.e^{3x}$$

$$= \frac{1}{2}.\frac{e^{5x}}{(5-1)^2(5-3)} - \frac{1}{2}\frac{1}{(3-1)^2}.\frac{e^{3x}}{(D-3)}$$

$$= \frac{e^{5x}}{64} - .\frac{1}{8}.x.e^{3x} = \frac{1}{64}\left[e^{5x} - 8x\,e^{3x}\right]$$

Choice (C)

18. $(D^3 + 5D^2)\,y = 4$

Auxiliary equation $m^3 + 5m^2 = 0$

$m^2(m + 5) = 0$

$\Rightarrow$ $m = 0, 0, -5$

$\therefore$ $C.F$ is $(C_1 + C_2x)e^{0.x} + C_3e^{-5x} = (C_1 + C_2x) + C_3e^{-5x}$

$$P.I = \frac{1}{D^2(D+5)}4.e^{0.x}$$

$$= \frac{1}{5}.\frac{1}{D^2}4 = \frac{1}{5}\left[\frac{1}{D^2}.4\right] = \frac{2x^2}{5}.$$

Complementary Solution $y = C.F + P.I$

$$y = C_1 + C_2x + C_3\,e^{-5x} + \frac{2x^2}{5}$$

Choice (B)

19. $(D^2 - 2D + 4)\,y = x^2\,e^x$

$$P.I = \frac{e^x\,x^2}{(D^2 - 2D + 4)} = e^x\frac{1}{(D+1)^2 - 2(D+1) + 4}.x^2$$

$$= e^x\frac{1}{D^2 + 3}x^2$$

$$\Rightarrow e^x\frac{1}{3\left(1 + \frac{D^2}{3}\right)}x^2 = e^x\frac{1}{3}\left(1 + \frac{D^2}{3}\right)^{-1}x^2$$

$$= e^x\frac{1}{3}\left(1 - \frac{D^2}{3}\right)x^2$$

$$= e^x\frac{1}{3}\left(x^2 - \frac{2}{3}\right) = e^x\frac{1}{9}(3x^2 - 2)$$

Choice (A)

20. $(D^4 + D^2 + 36D + 52)\,y = 0$

Auxiliary equation of the above is $m^4 + m^2 + 36m + 52 = 0$

By trail and error we notice m = –2, –2, are the roots of the above

$\therefore$ $(m + 2)^2\,(m^2 - 4m + 13) = 0$

The roots are $m = -2, -2$, and $2 \pm 3i$

$\therefore$ The solution is

$y = (C_1 + C_2x)e^{-2x} + e^{2x}\,(C_3\cos 3x + C_4\sin 3x)$

Choice (D)

21. Let $g(t) = \sin 3t \Rightarrow f(t) = t^2 g(t)$

Now $L[g(t)] = L:[\sin 3t] = \dfrac{3}{s^2+9}$

$$L[f(t)] = L[t^2 g(t)] = \frac{d^2}{ds^2}\ (L[g(t)])$$

$$= \frac{d}{ds}\left(\frac{d}{ds}\left(\frac{3}{s^2+9}\right)\right)$$

$$= \frac{d}{ds}\left(\frac{-6s}{\left(s^2+9\right)^2}\right)$$

$$= \frac{-6\left[\left(s^2+9\right)^2.1 - s.2\left(s^2+9\right).2s\right]}{\left(s^2+9\right)^4}$$

$$= \frac{-6\left[s^2+9-4s^2\right]}{\left(s^2+9\right)^3}$$

$$= \frac{18(s^2-3)}{\left(s^2+9\right)^3}$$ Choice (B)

22. Let $f(t) = (t^2 + 3)\ u\ (t-2)$

$L\ [f(t)] = L\ [(t^2 + 3)\ u\ (t-2)]$

$L\ [((t-2) + 2)^2 + 3)\ u\ (t-2)]$

$\therefore\ \ L\ [f(t)] = L\ [(t-2)^2 + 4(t-2) + 7)\ u\ (t-2)]$ ----- (1)

Let $g\ (t) = t^2 + 4t + 7$

$\therefore\ \ L\ [g\ (t)] = L\ [t^2 + 4t + 7] = L\ [t^2] + 4L[t] + 7L\ [1]$

$\therefore\ \ L\ [g\ (t)] = \dfrac{2}{s^3} + \dfrac{4}{s^2} + \dfrac{7}{s}$

Now from (1)

$L\ [f(t)] = L\ [(t-2)^2 + 4(t-2) + 7)\ u\ (t-2)]$

$= L\ [g\ (t-2).\ U\ (t-2)]$

$= L\ [g\ (t)]\ e^{-2s}$ (By second shifting theorem)

$= \left(\dfrac{2}{s^3} + \dfrac{4}{s^2} + \dfrac{7}{s}\right)e^{-2s}.$ Choice (A)

23. Given initial value problem is

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0 \qquad \text{----------- (1)}$$

Where $y(0) = 1$ and $y^1(0) = -2$

Applying laplace transform on both sides of (1)

$$L\left[\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y\right] = L[0]$$

$$\Rightarrow\ L\left[\frac{d^2y}{dt^2}\right] - 2L\left[\frac{dy}{dt}\right] - 8L\ [y] = 0$$

$$\Rightarrow\ \left(s^2\bar{y} - sy(0) - y^1(0)\right) - 2\ \left(s\bar{y} - y(0)\right) - 8\ \bar{y} = 0$$

Where $\bar{y} = L[y]$

$$\Rightarrow\ s^2\ \bar{y} - s \times 1 - (-2) - 2\ s\ \bar{y} + 2 \times 1 - 8\ \bar{y} = 0$$

$$\Rightarrow\ \left(s^2 - 2s - 8\right)\ \bar{y} - s + 4 = 0$$

$$\Rightarrow\ \left(s^2 - 2s - 8\right)\ \bar{y} = s - 4$$

$$\Rightarrow\ \bar{y} = \frac{(s-4)}{\left(s^2-2s-8\right)} = \frac{(s-4)}{(s-4)(s+2)}$$

$$\bar{y} = \frac{1}{s+2}$$

The laplace transform of the solution of (1) is

$\bar{y} = L[y] = \dfrac{1}{s+2}.$ Choice (D)

24. We have to find $L^{-1}\left[\dfrac{3}{2}\left(\dfrac{1}{\sqrt{s^5}} - \dfrac{1}{\sqrt{s^3}}\right)\right]$

$$= \frac{3}{2}\left(L^{-1}\left[\frac{1}{s^{\frac{5}{2}}}\right] - L^{-1}\left[\frac{1}{s^{\frac{3}{2}}}\right]\right)$$

$$= \frac{3}{2}\left[\frac{t^{\frac{3}{2}}}{\Gamma\left(\frac{5}{2}\right)} - \frac{t^{\frac{1}{2}}}{\Gamma\left(\frac{3}{2}\right)}\right]$$

$$= \frac{3}{2}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}\times\frac{1}{2}\left(\sqrt{\pi}\right)} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}\left(\sqrt{\pi}\right)}\right]$$

$$= \frac{3}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{4}\sqrt{\pi}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}\sqrt{\pi}}\right) = -\times\frac{\sqrt{t}}{\sqrt{\pi}}\left[\frac{4t}{3} - 2\right]$$

$$= \sqrt{\frac{t}{\pi}} \times 3\left[\frac{2t}{3} - 1\right]$$

$$= \sqrt{\frac{t}{\pi}}\ (2t - 3).$$ Choice (A)

25. We have $L^{-1}\left[\dfrac{4(2s+3)}{s^2+4s+20}\right]$

$$= 4\ L^{-1}\left[\frac{2s+3}{\left(s^2+4s+4\right)+16}\right]$$

$$= 4L^{-1}\left[\frac{2(s+2-2)+3}{(s+2)^2+4^2}\right] = 4L^{-1}\left[\frac{2(s+2)-1}{(s+2)^2+4^2}\right]$$

$$= 4L^{-1}\left[\frac{2(s+2)}{(s+2)^2+4^2}\right] - 4L^{-1}\left[\frac{1}{(s+2)^2+4^2}\right]$$

$$= 8 \times \frac{1}{4}\ e^{-2t}\cos 4t - 4 \times \frac{1}{4}\ e^{-2t}\sin 4t$$

$= e^{-2t}\ [2\cos 4t - \sin 4t].$ Choice (C)

Engineering Mathematics Test 3

(Linear Algebra, Partial Differential Equations)

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. If A is a square matrix of order 5 with $A^{-1} = A^T$ and non-negative determinant, then the determinant of A is _______.
   (A) 0 (B) 1
   (C) 2 (D) 5
2. For two matrices A and B, if $AB = A$ and $BA = B$, then which of the following statements is/are correct?
   I. A is an idempotent matrix.
   II. B is an idempotent matrix.
   (A) I only
   (B) II only
   (C) Both I and II
   (D) Neither I nor II
3. Consider the matrix $A = \begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$. Which of the following is NOT equal to the determinant of A?
   (A) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ (B) $\begin{vmatrix} a+bc & a & 1+a \\ b+ca & b & 1+b \\ c+ab & c & 1+c \end{vmatrix}$
   (C) $\begin{vmatrix} 0 & a-b & bc-ac \\ 0 & b-c & ca-ab \\ 1 & c & ab \end{vmatrix}$ (D) $\begin{vmatrix} 1 & a+1 & a^2+a \\ 1 & b+1 & b^2+b \\ 1 & c+1 & c^2+c \end{vmatrix}$
4. For a non-singular square matrix A, if $A^3 = A$, then A must be _______.
   (A) a nilpotent matrix
   (B) an idempotent matrix
   (C) an involutory matrix
   (D) None of these
5. If A is a matrix of order 6×9 with rank 5, then which of the following is true?
   (A) All the rows of A are linearly independent.
   (B) 5 columns of A are linearly independent.
   (C) AA^T is invertible.
   (D) A^TA is invertible.
6. The rank of the matrix $P = \begin{bmatrix} 1 & 2 & 4 & -3 \\ 2 & -3 & 5 & -4 \\ 4 & 1 & 13 & -10 \\ 3 & -8 & 6 & -5 \end{bmatrix}$ is _____.
   (A) 1 (B) 2
   (C) 3 (D) 4
7. The value of x_3 in the solution of the system of linear equations $x_1 + 2x_2 + 2x_3 = 4$, $2x_1 - 2x_2 - x_3 = -3$, $4x_1 + x_2 + 2x_3 = 3$ is —
   (A) 1 (B) −1
   (C) 2 (D) −2
8. For a homogeneous system of linear equations $AX = O$ with four equations in four unknowns, if the number of linearly independent solutions is one, then the rank of A is _______.
   (A) 1 (B) 2
   (C) 3 (D) 4
9. If $A = \begin{bmatrix} 1 & 10 & 16 & -20 \\ 0 & -1 & 159 & 237 \\ 0 & 0 & 1 & -431 \\ 0 & 0 & 0 & 1 \end{bmatrix}$; then the determinant of $A^9 - 7A^5 + 4A$ is _____.
   (A) 4 (B) −4
   (C) 16 (D) −16
10. If the characteristic equation of a 2×2 matrix A is $\lambda^2 - 4\lambda + 1 = 0$, then the trace and determinant of A respectively are _______.
    (A) −1 and 4 (B) 1 and −4
    (C) 4 and 1 (D) 4 and −1
11. If λ_1 and λ_2 are the eigenvalues of a 2×2 non-singular matrix A, then the eigenvalues of ad joint of A are _______.
    (A) λ_1 and λ_2 (B) λ_1^2 and λ_2^2
    (C) $\lambda_1 + \lambda_2$ and $\lambda_1 - \lambda_2$ (D) $\lambda_1 \times \lambda_2$ and $\dfrac{\lambda_1}{\lambda_2}$
12. If A is a 3×3 matrix with the characteristic equation $\lambda^3 - 5\lambda^2 + 2\lambda - 3 = 0$, then $3A^9 - 15A^8 + 6A^7 - 11A^6 + 10A^5 - 4A^4 + 10A^3 - 20A^2 + 8A - 9I$ is equal to ________.
    (A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
    (C) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$
13. Which of the following is NOT an eigenvector of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & -1 & 4 \end{bmatrix}$?

(A) $\begin{bmatrix} 3 \\ -3 \\ -4 \end{bmatrix}$ B) $\begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix}$

(C) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

14. If the system of linear equations
$2x + 3y + 4z = 1$
$5x - y + z = 4$
$3x + ay - 3z = 3$
has a unique solution, then the value of $a + 4$ _______
(A) must be equal to 0,
(B) should not be equal to 0
(C) can be any real number,
(D) can be any rational number

15. If a_1, b_1, c_1, d_1, a_2, b_2, c_2 and d_2 are any non zero real numbers, then which of the following types of solution is NOT possible for the system of linear equations.
$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
(A) Unique solution
(B) No solution
(C) Infinitely many solution
(D) None of these

16. The partial differential equation of $z = f(x + at) - g(x - at)$ is ______.

(A) $\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$ (B) $\frac{\partial z}{\partial t} = a\frac{\partial z}{\partial x}$

(C) $\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2}$ (D) $\frac{\partial^2 z}{\partial t^2} + \frac{\partial^2 z}{\partial x^2}$

17. The first order partial differential equation by eliminating the arbitary function from $z = f(x^3 - y^3)$ is
(A) $p + q = 0$ (B) $yp + xq = 0$
(C) $y^2p + x^2q = 0$ (D) $2y^2p + 3x^2q = 0$

18. The general solution of the partial differential equation $x^3 (y - z) p + y^3(z - x)q = z^3(x - y)$ is

(A) $\phi\left(xyz, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$

(B) $\phi\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$

(C) $\phi\left(\frac{1}{x} - \frac{1}{y} - \frac{1}{z}, \frac{1}{x^2} + \frac{1}{y^2} - \frac{1}{z^2}\right) = 0$

(D) $\phi\left(\frac{1}{x^2} - \frac{1}{y^2} - \frac{1}{z^2}, \frac{1}{x} - \frac{1}{y} + \frac{1}{z}\right) = 0$

19. The solution of the partial differential equation $xy^2z^2p + x^2yz^2q = x^2y^2z$ is
(A) $x^2 + y^2 = \phi\,(x^2 - y^2)$
(B) $x^2 + z^2 = \phi\,(x^2 - z^2)$
(C) $x^2 - y^2 = \phi\,(y^2 - z^2)$
(D) $y^2 - z^2 = \phi\,(x^2 - y^2 - z^2)$

20. The solution of $(p - q)\,(z - x\,p - y\,q) = 1$ is __________

(A) $z = ax - by + \frac{1}{a+b}$ (B) $z = ax - by$

(C) $z = ax + by$ (D) $z = ax + by + \frac{1}{a-b}$

21. If $u(x, y) = X(x).Y(y)$ be the solution of the partial differential equation $4\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} = 0$, which is obtained by solving it by the method of separation of variables, then $X(x)$ (the function of x only in $u(x, y)$) is _______
[Note: Here c and k are arbitrary constants]

(A) $X(x) = ce^{(kx)}\,x^2$ (B) $X(x) = ce^{\left(\frac{4k}{x}\right)}$

(C) $X(x) = ce^{\left(\frac{k}{4}\right)x}$ (D) $X(x) = ce^{(-5k)}x^2$

22. Which of the following second order partial differential equations is an elliptic equation?

(A) $3\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} - 5\frac{\partial^2 u}{\partial y^2} + 7\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 7x^2$

(B) $3\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 6x^2 y$

(C) $-3\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial^2 u}{\partial y^2} + 4x\frac{\partial u}{\partial x} - 7y\frac{\partial u}{\partial y} = 0$

(D) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} = 6xy^2$

23. The Fourier cosine series of the function $f(x) = \frac{1}{2}$, $0 \le x \le 1$ is
(A) 1 (B)
(C) 0 (D)

24. The Fourier series of $f(x) = e^{2x}$ in the $\frac{1}{4}\frac{a_0}{2}$ interval $(0, 2\pi)$ is

$f(x) = + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$, then the value of $\frac{a_0}{2}$ is

(A) $\frac{e^{4\pi} - 1}{4\pi}$ (B) $\frac{e^{2\pi} - 1}{2\pi}$

(C) $\frac{e^{4\pi} - 1}{2\pi}$ (D) $\frac{e^{2\pi} - 1}{4\pi}$

25. The Fourier series of the function

$$f(x) = \begin{cases} -2 \text{ for} -\pi < x < \frac{-\pi}{2} \\ 0 \text{ for} \frac{-\pi}{2} < x < \frac{\pi}{2} \\ 2 \text{ for} \frac{\pi}{2} < x < \pi \end{cases}$$

is $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$, then $b_n =$

(A) $\frac{1}{n\pi}\left[\sin\frac{n\pi}{2} - \cos n\pi\right]$

(B) $\frac{4}{n\pi}\left[\cos\frac{n\pi}{2} - \cos n\pi\right]$

(C) $\frac{1}{4n\pi}\left[\cos n\pi - \cos\frac{n\pi}{2}\right]$

(D) $\frac{4}{n\pi}\left[\sin\frac{n\pi}{2} - \sin n\pi\right]$

Answer Keys

1. B	**2.** C	**3.** B	**4.** C	**5.** B	**6.** B	**7.** A	**8.** C	**9.** D	**10.** C
11. A	**12.** C	**13.** D	**14.** B	**15.** A	**16.** A	**17.** C	**18.** B	**19.** C	**20.** D
21. C	**22.** B	**23.** B	**24.** A	**25.** B					

Hints and Explanations

1. Given $A^{-1} = A^T$ and det of A is non-negative

$\therefore$ $A.A^T = A^T.A = I_5$

$\Rightarrow$ Det of $(A.A^T)$ = Det of I_5

$\Rightarrow$ $|A.A^T| = |I_5|$

$\Rightarrow$ $|A|\,|A^T| = 1$

$\Rightarrow$ $|A|\,.\,|A| = 1$ ($\because$ $|A| = |A^T|$)

$\Rightarrow$ $|A|^2 = 1 \Rightarrow |A| = \pm 1$

$\therefore$ The determinant of $A = 1$.

($\because$ $|A|$ is non-negative) Choice (B)

2. Given $AB = A$ and $BA = B$

Consider $BA = B$

$\Rightarrow$ $A(BA) = AB$

$\Rightarrow$ $(AB)A = A$ ($\because$ $AB = A$)

$AA = A \Rightarrow A^2 = A$

$\Rightarrow$ A is an idempotent matrix $\rightarrow$ (1)

Consider $AB = A$

$\Rightarrow$ $B(AB) = BA$

$\Rightarrow$ $(BA)B = B$ ($\because$ $BA = B$)

$\Rightarrow$ $BB = B \Rightarrow B^2 = B$

$\Rightarrow$ B is an idempotent matrix $\rightarrow$ (2)

$\therefore$ From (1) and (2), both I and II are correct.

Choice (C)

3. Given $A = \begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$

Consider the determinant given in option (B)

$$\begin{vmatrix} a+bc & a & 1+a \\ b+ca & b & 1+b \\ c+ab & c & 1+c \end{vmatrix} = \begin{vmatrix} a & a & 1+a \\ b & b & 1+b \\ c & c & 1+c \end{vmatrix} + \begin{vmatrix} bc & a & 1+a \\ ca & b & 1+b \\ ab & c & 1+c \end{vmatrix}$$

$$= 0 + \begin{vmatrix} bc & a & 1 \\ ca & b & 1 \\ ab & c & 1 \end{vmatrix} + \begin{vmatrix} bc & a & a \\ ca & b & b \\ ab & c & c \end{vmatrix}$$

$$= \begin{vmatrix} bc & a & 1 \\ ca & b & 1 \\ ab & c & 1 \end{vmatrix} + 0$$

$$= -1 \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

= –Det of $A \neq$ Det of A

$\therefore$ The determinant given in option B is NOT equal to det of A. Choice (B)

4. Given A is non-singular and $A^3 = A$

$\Rightarrow$ $AA^2 = A$

$\Rightarrow$ $A^{-1}(AA^2) = A^{-1}A$

$\Rightarrow$ $(A^{-1}A)A^2 = A^{-1}A$

$\Rightarrow$ $A^2 = I$

$\Rightarrow$ A must be an involutory matrix. Choice (C)

5. Given A is a matrix of order 6×9.

Rank of $A = 5$

$\therefore$ Maximum number rows/columns of A that are linearly independent = 5.

$\therefore$ Option (B) is TRUE. Choice (B)

6. Given matrix is $P = \begin{bmatrix} 1 & 2 & 4 & -3 \\ 2 & -3 & 5 & -4 \\ 4 & 1 & 13 & -10 \\ 3 & -8 & 6 & -5 \end{bmatrix}$

$R_2 \rightarrow R_2 - 2R_1$, $R_3 - 4R_1$ and $R_4 \rightarrow R_4 - 3R_1$

$$\sim \begin{bmatrix} 1 & 2 & 4 & -3 \\ 0 & -7 & -3 & 2 \\ 0 & -7 & -3 & 2 \\ 0 & -14 & -6 & 4 \end{bmatrix}$$

$R_3 \to R_3 - R_2$ and $R_4 \to R_4 - 2R_2$

$$\sim \begin{bmatrix} 1 & 2 & 4 & -3 \\ 0 & -7 & -3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to \frac{-1}{7} R_2$

$$\therefore \quad P \sim \begin{bmatrix} 1 & 2 & 4 & -3 \\ 0 & 1 & 3/7 & -2/7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which is in Row Echelon form.

$\therefore$ The rank of P = the number of non-zero rows in its Row Echelon form = 2. Choice (B)

7. Given system of equations is

$x_1 + 2x_2 + 2x_3 = 4$

$2x_1 - 2x_2 - x_3 = -3 \to (1)$

$4x_1 + x_2 + 2x_3 = 3$

It can be written in matrix form as

$AX = B \to (2)$

Where $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & -2 & -1 \\ 4 & 1 & 2 \end{bmatrix}$; $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -3 \\ 3 \end{bmatrix}$

Consider the augmented matrix

$$[A/B] = \begin{vmatrix} 1 & 2 & 2 & 4 \\ 2 & -2 & -1 & -3 \\ 4 & 1 & 2 & 3 \end{vmatrix}$$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 4R_1$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 4 \\ 0 & -6 & -5 & -11 \\ 0 & -7 & -6 & -13 \end{bmatrix}$$

$R_3 \to 6R_3 - 7R_2$

$$\therefore [A/B] \sim \begin{bmatrix} 1 & 2 & 2 & 4 \\ 0 & -6 & -5 & -11 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

Hence the system of equations that has same solution as that of $AX = B$ is

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & -6 & -5 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -11 \\ -1 \end{bmatrix}$$

$\Rightarrow \quad x_1 + 2x_2 + 2x_3 = 4$

$-6x_2 - 5x_3 = -11$

$-x_3 = -1 \Rightarrow x_3 = 1.$ Choice (A)

8. Given the system of equations $AX = O$ has

Number of unknowns = $n = 4$

Number of equations = 4

$\therefore$ A is a 4×4 matrix

Also given the number of linearly independent solutions = 1. We know that the number of linearly independent solutions of a system of homogeneous linear equations.

$AX = O$ is $= n - r$

Where n = the number of unknowns and r = the rank of A.

$\therefore \quad n - r = 1$

$\Rightarrow \quad 4 - r = 1$

$\Rightarrow \quad r = 4 - 1 = 3$

$\therefore$ The rank of $A = 3$ Choice (C)

9. Given A = $\begin{bmatrix} 1 & 10 & 16 & -20 \\ 0 & -1 & 159 & 237 \\ 0 & 0 & 1 & -431 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The eigenvalues of A are 1, –1, 1 and 1.

If $\lambda = -1$ is an eigenvalue of A, then $(-1)^9 - 7(-1)^5 + 4(-1) = 2$ is an eigenvalue of $A^9 - 7A^5 + 4A$.

Also if $\lambda = 1$ is an eigenvalue of A then

$(1)^9 - 7(1)^5 + 4(1) = -2$ is an eigenvalue of $A^9 - 7A + 4A$

$\therefore$ The eigenvalues of $A^9 - 7A^5 + 4A$ are –2, 2, –2 and –2.

$\therefore$ The determinant of $A^9 - 7A^5 + 4A$ = Product of the eigenvalues of $A^9 - 7A^5 + 4A = (-2)(2)(-2)(-2) = -16.$ Choice (D)

10. Given the characteristic equation of a 2×2 matrix A is $\lambda^2 - 4\lambda + 1 = 0$

Let λ_1 and λ_2 be the eigenvalues of A.

$\therefore$ Trace of A = sum of the eigenvalues of A

$= \lambda_1 + \lambda_2 = (-(-4)) = 4$

Determinant of A = Product of the eigenvalues of $A = \lambda_1 . \lambda_2 = 1.$ Choice (C)

11. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2×2 matrix with λ_1 and λ_2 as its eigenvalues.

The characteristic equation of A is

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (a - \lambda)(d - \lambda) - bc = 0$ ---- (1)

The adjoint of A is adj$(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\therefore$ The characteristic equation of adj(A) is

$$\begin{vmatrix} d-\lambda & -b \\ -c & a-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (d-\lambda)(a-\lambda) - bc = 0$ ---- (2)

As (1) and (2) are one and the same and λ_1 and λ_2 being the roots of (1), λ_1 and λ_2 will be the roots of (2).

$\therefore$ The eigenvalues of adj(A) are λ_1 and λ_2.

Choice (A)

12. The characteristic equation of A is $\lambda^3 - 5\lambda^2 + 2\lambda - 3 = 0$

$\therefore$ By Cayley Hamilton theorem, we have

$A^3 - 5A^2 + 2A - 3I = 0$ ----- (1)

Consider

$3A^9 - 15A^8 + 6A^7 - 11A^6 + 10A^5 - 4A^4 + 10A^3 - 20A^2 + 8A - 9I$

$= 3A^6(A^3 - 5A^2 + 2A - 3I) - 2A^6 + 10A^5 - 4A^4 + 10A^3 - 20A^2 + 8A - 9I$

$= 3A^6 \times 0 - 2A^3(A^3 - 5A^2 + 2A - 3I) + 4A^3 - 20A^2 + 8A - 9I$

(From (1)) $= 0 - 2A^3 \times 0 + 4(A^3 - 5A^2 + 2A - 3I) + 3I$

(From (1)) $= 4 \times 0 + 3I$ (From (1))

$$= 3I = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$ Choice (C)

13. Given $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & -1 & 4 \end{bmatrix}$

The eigenvalues of A are 1, 3 and 4

If $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is an eigenvector of A, then x should satisfy any one of the three conditions. $AX = x$, $AX = 3X$ and $AX = 4X$

From the options given, it can be easily observed that the vectors given in options (A), (B) and (C), will satisfy one of these three conditions.

Consider the vector given in option (D),

$$AX = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & -1 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ -2 \end{bmatrix} \neq \lambda \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

For, $\lambda = 1, 2$ or 3

$\therefore$ Its not an eigenvector of A Choice (D)

14. Given system of linear equations is

$2x + 3y + 4z = 1$

$5x - y + z = 4$

$3x + ay - 3z = 3$ --------- (1)

It can be written in matrix form as $AX = B$

Where $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & -1 & 1 \\ 3 & a & -3 \end{bmatrix}$;

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ consider the augmented matrix

$$[A|B] = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 5 & -1 & 1 & 4 \\ 3 & a & -3 & 3 \end{bmatrix}$$

$R_2 \to 2R_2 - 5R_1$, $R_3 \to 2R_3 - 3R_1$

$$\sim \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & -17 & -18 & 3 \\ 0 & 2a-9 & -18 & 3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$[A|B] \sim \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & -17 & -18 & 3 \\ 0 & 2a+8 & 0 & 0 \end{bmatrix}$$

The given system of equations has a unique solution, if $P(A) = p([A/B]) = 3$ (= The no. of unknowns)

This is possible only if $2a + 8 \neq 0$

$\Rightarrow \quad a + 4 \neq 0.$ Choice (B)

15. Given system of equations is

$a_1x + b_1y + c_1z = d_1$ ______(1)

$a_2x + b_2y + c_2z = d_2$

It can be written in matrix form as

$AX = B$

Where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$; $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$

Here two possibilities arise

(i) $P(A) \neq P(A/B])$

In this case, (1) has no solution

(ii) $P(A) = P([A/B]) < 3$(= The no. of unknowns)

In this case, (1) has infinitely many solutions.

So, the given system (1) do not have a unique solution. Choice (A)

16. $z = f(x + at) - g(x - at)$

$$\frac{\partial z}{\partial x} = f^1(x+at) - g^1(x-at)$$

$$\frac{\partial^2 z}{\partial x^2} = f^{11}(x+at) - g^{11}(x-at)$$

$$\frac{\partial z}{\partial t} = af^1(x+at) + ag^1(x-at)$$

$$\frac{\partial^2 z}{\partial t^2} = a^2f^{11}(x+at) - a^2g^{11}(x-at)$$

$$\frac{\partial^2 z}{\partial t^2} = a^2\frac{\partial^2 z}{\partial x^2}$$ Choice (A)

17. Given $z = f(x^3 - y^3)$

Let $x^3 - y^3 = u$

$z = f(u)$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}.\frac{\partial u}{\partial x} = f\ (u).3x^2$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}.\frac{\partial u}{\partial y} = -f\ (u)3y^2$$

$$y^2\frac{\partial z}{\partial x} + x^2\frac{\partial z}{\partial y} = f\ (u)3x^2y^2\ - f^1(u)3x^2y^2 = 0$$

$\therefore$ The first order partial dE is $y^2p + x^2q = 0$

Choice (C)

18. $x^3(y - z)\, p + y^3\, (z - x)q = z^3(x - y)$

The subsidiary equation of the given differential equation is

$$\frac{dx}{x^3(y-z)} = \frac{dy}{y^3(z-x)} = \frac{dz}{z^3(x-y)} \quad \ldots\ldots\ldots\ldots (1)$$

using the multipliers $\frac{1}{x^2}, \frac{1}{y^2}$ and $\frac{1}{z^2}$ each fraction of

(1) is equal to $\dfrac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{0}$

$$\Rightarrow \frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz = 0$$

On integrating the above, we get

$$-\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = C \text{ or } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = C_1 \quad \ldots\ldots\ldots\ldots (2)$$

using the multipliers $\frac{1}{x^3}, \frac{1}{y^3}$ and $\frac{1}{z^3}$ each of the fraction (1) equal to $\dfrac{\frac{1}{x^3}dx + \frac{1}{y^3}dy + \frac{1}{z^3}dz}{0}$

On integrating both the sides, we get

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = C_2 \quad \ldots\ldots\ldots\ldots (3)$$

From (2) and (3) the general solutions is

$$\varphi\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right) = 0.$$

Choice (B)

19. $xy^2z^2p + x^2yz^2q = x^2\, y^2\, z \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$

The subsidiary equations of (1) are

$$\frac{dx}{xy^2z^2} = \frac{dy}{x^2yz^2} = \frac{dz}{x^2y^2z} \quad \ldots\ldots\ldots\ldots (2)$$

Considering first two fractions of (2) we have $x\, dx = y\, dy$.

On integrating we get $x^2 = y^2$ or $x^2 - y^2 = C_1 \ldots\ldots\ldots (3)$

Considering the last two fractions of (2), we have $y\, dy = z\, dz$. On integrating, we get

$y^2 - z^2 = C_2 \quad \ldots\ldots\ldots\ldots\ldots (4)$

From (3) and (4) the general solution of (1) is

$x^2 - y^2 = \phi\, (y^2 - z^2)$.

Choice (C)

20. Given $(p - q)\, (z - x\, p - yq) = 1$

$$z = x\,p + yq + \frac{1}{p-q}$$

This is a clairaut equation and its solution is

$$z = ax + by + \frac{1}{a-b}$$

Choice (D)

21. Given $u = x(x), y(y)$ ------ (1) is the solution of the PDE

$$4\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} = 0 \text{ ----------- } (2)$$

Obtained by solving (2) by the method of separation of variables

$$\therefore \frac{\partial u}{\partial x} = x^1y \text{ and } \frac{\partial u}{\partial y} = xy^1$$

where $x^1 = \frac{dx}{dx}$ and $y^1 = \frac{dY}{dy}$

$\therefore$ (2) becomes

$4x^1y + 5xy^1 = 0$

$$\Rightarrow \frac{4x^1}{x} + 5\frac{y^1}{y} = 0$$

$$\Rightarrow \frac{4x^1}{x} = -5\frac{y^1}{y} = k \text{ (say) where } k \text{ is a constant}$$

$$\Rightarrow \frac{4x^1}{x} = k \text{ and } \frac{-5y^1}{y} = k$$

$$\Rightarrow x^1 = \frac{kx}{4} \qquad y^1 = \frac{-ky}{5}$$

$$\Rightarrow x^1 - \frac{kx}{4} = 0 \Rightarrow y^1 + \frac{ky}{5} = 0$$

$$\Rightarrow \frac{dX}{dx} = \frac{kx}{4} \Rightarrow \frac{dx}{x} = \frac{k}{4}\ dx$$

$$\Rightarrow \int\frac{dx}{x} = \frac{k}{4}\int dx \Rightarrow = \frac{k}{4}x + c^1$$

$$\Rightarrow x = e^{\frac{kx}{4}} + c^1 = e^{\frac{kx}{4}}c^1$$

$$\Rightarrow x = ce^{\left(\frac{kx}{4}\right)}; \text{ where } c = e^{c^1}$$

$$\therefore x(x) = ce^{\left(\frac{kx}{4}\right)}.$$

Choice (C)

22. A PDE is of the form

$Au_{xx} + Bu_{xy} + Cu_{yy} + F(x, y, u, u_x, u_y) = 0 \quad$ ----- (1)

is elliptic, if $B^2 - 4AC < 0$

From the PDE in the options, consider the PDE in option (B)

Comparing it with (1), we have

$A = 3$, $B = -4$ and $C = 5$

$\therefore$ $B^2 - 4AC = (-4)^2 - 4 \times 3 \times 5 = -44 < 0$

$\Rightarrow$ $B^2 - 4AC < 0$

Hence the PDE given in option (B) is elliptic

Also, it can be easily observed that the PDE given in options (A), (C) and (D) do not satisfy the property, $B^2 - 4AC < 0$. Choice (B)

23. The coefficients of Fourier cosine series are given by

$$a_0 = \frac{2}{1}\int_0^1 f(x)dx \text{ and}$$

$$a_n = \frac{2}{1}\int_0^1 f(x)\cos\left(\frac{n\pi x}{1}\right)dx$$

$$\therefore \quad a_0 = \frac{2}{1}\int_0^1 \frac{1}{2}dx = x\Big]_0^1 = 1$$

$$a_n = \frac{2}{1}\int_0^1 \frac{1}{2}\cos(n\pi x)dx = \int_0^1 \cos(n\pi x)dx$$

$$= \frac{\sin(n\pi x)}{n\pi}\Bigg]_0^1 = 0$$

$\therefore$ The required series is $f(x) = \frac{1}{2}$. Choice (B)

24. In the Fourier series of e^{2x}; a_0 is given by $\frac{1}{\pi}\int_0^{2\pi} f(x)dx$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} e^{2x}dx = \frac{1}{2\pi}\left[e^{2x}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[e^{4\pi} - 1\right] \; \frac{a_0}{2} = \frac{e^{4\pi}-1}{2\times 2\pi} = \frac{e^{4\pi}-1}{4\pi}.$$ Choice (A)

25. In the Fourier series of $f(x)$, b_n is given by $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\frac{-\pi}{2}} (-2)\sin nx dx + \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} 0.\sin nx dx + \int_{\frac{\pi}{2}}^{\pi} 2\sin nx dx\right]$$

$$= \frac{2}{\pi}\left[\frac{\cos nx}{n}\right]_{-\pi}^{\frac{-\pi}{2}} + \frac{2}{\pi}\left[\frac{-\cos nx}{n}\right]_{\frac{\pi}{2}}^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\cos n\frac{\pi}{2}}{n} - \frac{\cos n\pi}{n} - \frac{\cos n\pi}{n} + \frac{\cos n\frac{\pi}{2}}{n}\right]$$

$$= \frac{4}{n\pi}\left[\cos\frac{n\pi}{2} - \cos n\pi\right].$$ Choice (B)

ENGINEERING MATHEMATICS TEST 4

Number of Questions: 25 **Time: 60 min.**

(PROBABILITY AND STATISTICS)

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. If A, B and C are mutually exclusive such that $5P(B) = 8\ P(A)$, $4P(C) = 3P(B)$ and $19P(A \cup B) = 13$ then $P(A \cup B \cup C)$ is ________.
(A) 0.75
(B) 1
(C) 0.625
(D) Cannot be determined

2. 240 passengers travelling in a plane from Hyderabad to Sharjah like one or more of the three meals among sandwich, burger and pizza as given below. 140 passengers like sandwich,110 passengers like burger and 90 passengers like pizza. 40 of them like both sandwich and burger, 50 of them like both Sandwich and Pizza, 40 of them like both burger and pizza where as 30 of them like all the three meals. What is the probability that a randomly selected passenger likes pizza only?
(A) 0.125 (B) 0.250
(C) 0.375 (D) 0.500

3. Let S be the set of all 4 digit numbers that can be formed using the digits 2, 3, 5, 7, 8 and 9. Probability that a randomly selected number of S has all digits distinct is _______.
(A) $\frac{5}{12}$ (B) $\frac{5}{18}$
(C) $\frac{5}{24}$ (D) $\frac{5}{36}$

4. An unbiased coin is tossed until it shows up the same face in two consecutive throws. What is the probability that the number of tosses is not more than 4?
(A) $\frac{3}{4}$ (B) $\frac{1}{8}$
(C) $\frac{7}{8}$ (D) $\frac{1}{4}$

5. What is the probability that a quadratic equation $ax^2 + bx + c = 0$ has equal roots if a, b and c are distinct and are taken from {1, 2, 3, 4, 6, 8, 9}?
(A) $\frac{1}{35}$ (B) $\frac{2}{35}$
(C) $\frac{1}{105}$ (D) $\frac{2}{105}$

6. A bag contains 4 five rupee coins, 3 two rupee coins and 3 one rupee coins. If 6 coins are drawn from the bag at random, what are the odds in favour of the draw yielding maximum amount?
(A) 1 : 70 (B) 1 : 69
(C) 69 : 70 (D) 70 : 1

7. Kids and Toys factory is transporting balls of 5 different colours – yellow, blue, red, green and white. Mr. Bholeram, a worker in the factory has to separate these balls as per their colours into different boxes and label them with the corresponding coloured labels.
Mr. Bholeram, after separating the balls, sealed the boxes and then labelled the boxes at random. What is the probability that all the boxes are incorrectly labelled?
(A) 1 (B) 0
(C) $\frac{11}{120}$ (D) $\frac{11}{30}$

8. While shuffling a pack of cards, 4 cards are accidentally dropped. The probability that all of them are numbered cards (2 to 10) of the same suit is
(A) $\frac{4 \times {}^{9}C_{4}}{{}^{52}C_{4}}$ (B) $\frac{\left({}^{9}C_{4}\right)^{4}}{{}^{52}C_{4}}$
(C) $\frac{4 \times {}^{9}C_{1}}{{}^{52}C_{4}}$ (D) $\frac{\left({}^{9}C_{1}\right)^{4}}{{}^{52}C_{4}}$

9. Arpit and Bipin pick up a ball at random from a bag containing 5 violet, 2 red and 3 orange balls one after the other, replacing it every time till one of them gets an orange ball and the one who first gets an orange ball is declared a winner. If Arpit begins the game, then the probability of Bipin winning the game is
(A) $\frac{10}{17}$ (B) $\frac{7}{17}$
(C) $\frac{7}{10}$ (D) $\frac{3}{10}$

10. An urn A contains 6 white balls and 7 black balls. And urn B contains 8 white balls and 6 black balls. A person draws a ball at random from one of the two urns. It turns out to be black. What is the probability that the ball was drawn from urn A?
(A) $\frac{7}{14}$ (B) $\frac{49}{88}$
(C) $\frac{39}{88}$ (D) None of the above

11. The bivariate probability distribution of X and Y is as follows.

X \ Y	0	1	2
0	$\frac{1}{40}$	$\frac{2}{40}$	$\frac{3}{40}$
1	$\frac{2}{40}$	$\frac{3}{40}$	$\frac{1}{40}$
2	$\frac{3}{40}$	$\frac{1}{40}$	$\frac{7}{40}$
3	$\frac{4}{40}$	$\frac{5}{40}$	$\frac{8}{40}$

Find $P(X \le 1, Y = 2)$.

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$
(C) $\frac{3}{10}$ (D) $\frac{1}{10}$

12. In a book of 500 pages, there are 50 typing errors. Assuming that the number of errors per page follows poission distribution, find the probability that randomly chosen 5 pages will contain no error.

(A) 0.6065 (B) 0.6078
(C) 0.6538 (D) 0.3935

13. The continuous random variable X is uniformly distributed with mean 2 and variance 12. Find $P(X > 0)$.

(A) $\frac{1}{3}$ (B) $\frac{4}{5}$
(C) $\frac{2}{3}$ (D) $\frac{1}{5}$

14. X and Y are two independent normal variates with means 3, 6 and variances, 1, 9 respectively. Find the value of k such that $P(X + Y \le k) = P(9X - Y \ge 2k)$.

(A) 9.3 (B) 9.6
(C) 8.6 (D) 10.3

15. Bag A contains 9 white balls and 5 green balls. Bag B contains 6 white balls and 7 green balls. One ball is drawn from bag A and is placed in bag B. Now one ball is drawn at random from bag B. It is found that the ball is green. Find the probability that white ball is transferred from bag A.

(A) $\frac{20}{103}$ (B) $\frac{63}{103}$
(C) $\frac{80}{103}$ (D) $\frac{75}{103}$

16. A dice is rolled twice the sum of the numbers appearing is 7, what is the probability that at least one dice shows 3?

(A) $\frac{3}{7}$ (B) $\frac{2}{3}$
(C) $\frac{1}{3}$ (D) $\frac{4}{7}$

17. A random variable X has the following probability distribution.

X = xi	0	1	2	3	4
P(x = xi)	K	2K	3K	5K	4K

Then find $P(X \ge 2)$.

(A) $\frac{4}{5}$ (B) $\frac{1}{5}$
(C) $\frac{2}{5}$ (D) $\frac{1}{15}$

18. The standard error is ______.
(A) accepting the null hypothesis when it is false.
(B) rejecting the null hypothesis when it is true.
(C) the standard deviation of the sampling distribution of a statistic.
(D) the probability that the test statistic does not lie in the critical region.

19. In large sampling, the sampling distribution of means follows ______
(A) Normal distribution
(B) t – distribution
(C) F – distribution
(D) x^2 – distribution

20. Which of the following distributions is used to test the equality of variances of two populations from which two small random samples are drawn?
(A) Normal distribution
(B) t – distribution
(C) F – distribution
(D) x^2 – distribution

21. If a statistic s follows t – distribution with $\nu = 10$ degrees of freedom, then s^2 follows F – distribution with degrees of freedom (ν_1, ν_2) = ______.

(A) (1,9) (B) (1,10)
(C) (1,11) (D) (9,1)

22. In testing of hypothesis, if the test statistic is outside the critical region, then we will
P : Accept the null hypothesis
Q : Reject the null hypothesis
R : Accept the alternative hypothesis
S : Reject the alternative hypothesis
Which of the following is true?
(A) P only
(B) R only
(C) P and S only
(D) Q and R only

23. Three letters are placed into three addressed envelopes randomly. A random variable X denotes the number of letters placed into corresponding envelopes. Find the variance of X.

(A) $\frac{5}{6}$ (B) 2

(C) 1 (D) 3

24. The variance of the data $x, x + 3, x + 5, x + 7, x + 10$ is

(A) 11.2 (B) 11.6

(C) $11.6 + x$ (D) $11.2 + x$

25. The median of the following data can be 3, 8, 12, 28, 16, 15, x

(A) 13 (B) 14

(C) 15 (D) Any of the above

Answer Keys

1. B	**2.** A	**3.** B	**4.** C	**5.** C	**6.** B	**7.** D	**8.** A	**9.** B	**10.** B
11. D	**12.** A	**13.** C	**14.** B	**15.** B	**16.** C	**17.** A	**18.** C	**19.** A	**20.** C
21. B	**22.** C	**23.** C	**24.** B	**25.** D					

Hints and Explanations

1. Given $5P(B) = 8P(A)$ and $4P(C) = 3P(B)$

$\Rightarrow P(A) = \frac{5}{8}P(B)$ and $P(C) = \frac{3}{4}P(B)$ ----- (1)

Now $19\,P(A \cup B) = 13 \Rightarrow P(A \cup B) = \frac{13}{19}$

$P(A) + P(B) = \frac{13}{19}$ ($\because$ A and B mutually exclusive)

$\Rightarrow \frac{5}{8}P(B) + P(B) = \frac{13}{19}$

$\Rightarrow \frac{13}{8}P(B) = \frac{13}{19}$

$\Rightarrow P(B) = \frac{8}{19}$

Now $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$= \frac{5}{8}P(B) + P(B) + \frac{3}{4}P(B)$

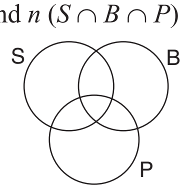
$= \left(\frac{5}{8} + 1 + \frac{3}{4}\right)P(B) = \frac{19}{8}P(B) = \frac{19}{8} \times \frac{8}{19} = 1.$

Choice (B)

2. The total number of passengers = 240

Let S, B and P denote the sets of passengers who like sandwich, burger and pizza respectively.

$\therefore$ $n(S) = 140$, $n(B) = 110$, $n(P) = 90$, $n(S \cap B) = 40$, $n(B \cap P) = 40$, $n(P \cap S)$ $= 50$ and $n(S \cap B \cap P) = 30$



$\therefore$ Probability that a randomly selected passenger likes only

pizza

$= \frac{(\text{The number of passengers who like only pizza})}{(\text{The total number of passengers})}$

$= \frac{n(P) - n(P \cap S) - n(B \cap P) + n(S \cap B \cap P)}{240}$

$= \frac{90 - 40 - 50 + 30}{240} = \frac{1}{8} = 0.125.$ Choice (A)

3. The number of 4 digit numbers that can be formed using the digits 2, 3, 5, 7, 8 and 9

= The number of elements of $S = 6^4$

The number of 4 digit numbers of S that have all digits distinct = The number of 4 digit numbers that can be formed using the digits 2, 3, 5, 7, 8 and 9 = ${}^6P_4$

$\therefore$ Probability that a randomly selected number of S has all digits distinct $= \frac{{}^6P_4}{6^4} = \frac{5}{18}$ Choice (B)

4. The number of tosses may be 2 or 3 or 4.

The possible cases and their corresponding probabilities:

Case 1 : HH OR TT $\rightarrow 2\left(\frac{1}{2}\right)^2$

Case 2 : *HTT* OR *THH* $\rightarrow 2\left(\frac{1}{2}\right)^3$

Case 3 : *HTHH* OR *THTT* $\rightarrow 2\left(\frac{1}{2}\right)^4$

Hence, the required probability is

$2\left[\frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right] = \frac{7}{8}$ Choice (C)

5. Considering different values of a, b and c from the set {1, 2, 3, 4, 6, 8, 9}, we get different quadratic equations.

As a, b and c are distinct, ${}^7P_3 = 210$ different quadratic equations can be formed.

$\therefore$ Total ways are 210

For the quadratic equation $ax^2 + bx + c = 0$ to have equal roots, $b^2 = 4ac$.

The possible combinations of a, b and c respectively are 1, 6, 9 and 9, 6, 1.

Hence favourable cases are 2

$\therefore$ Required probability $=\frac{2}{210}=\frac{1}{105}$. Choice (C)

6. We have 4 five rupee coins, 3 two rupee coins and 3 one rupee coins.
For the draw to yield a maximum amount, of the 6 coins drawn 4 should be five rupee coins and 2 should be two rupee coins. The required probability is

$$\frac{{}^4C_4 \times {}^3C_2}{{}^{10}C_6}=\frac{3}{210}=\frac{1}{70}$$

Hence, odds in favour are favourable ways : unfavourable ways = 1 : 69. Choice (B)

7. There are 5 boxes and 5 labels. Hence the boxes can be labelled in 5! i.e. 120 different ways

$$P\text{ (all labelled incorrectly)}=\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}$$

$$=\frac{44}{120}=\frac{11}{30}$$ Choice (D)

8. There are 9 numbered cards in each suit.
P(all the 4 cards are numbered cards of same suit)

$$=\frac{{}^9C_4+{}^9C_4+{}^9C_4+{}^9C_4}{{}^{52}C_4}=\frac{4\times{}^9C_4}{{}^{52}C_4}$$ Choice (A)

9. The probability of picking up an orange ball is $\frac{3}{10}$ while not picking up an orange ball is $\frac{7}{10}$.

We compute the probability of Arpit (the beginner) winning the game.
Let A and B be the events of Arpit and Bipin picking up an orange ball respectively
The winning sequence of Arpit can be
$A,\ \bar{A}\,\bar{B}\,A,\quad \bar{A}\,\bar{B}\,\bar{A}\,\bar{B}\,A, \ldots\ldots$

As the above sequence indicates, Arpit may pick an orange ball right in the 1st trial with a probability of $\frac{3}{10}$ (or) in the third trial (as the 2nd trial is made by Bipin, and for Arpit to win, Bipin should not be getting an orange ball). The probability here being $\left(\frac{7}{10}\right)^2\times\frac{3}{10}$ (or) in the fifth trial with a probability of $\left(\frac{7}{10}\right)^4\times\frac{3}{10}$ and so on.

$$\therefore\ P(A)=\frac{3}{10}+\left(\frac{7}{10}\right)^2\times\frac{3}{10}+\left(\frac{7}{10}\right)^4\times\frac{3}{10}+\ldots..$$

$$=\frac{\frac{3}{10}}{1-\left(\frac{7}{10}\right)^2}=\frac{30}{51}=\frac{10}{17}$$

Probability of Bipin winning is the same as probability of Arpit losing i.e.,

$$\therefore\ P(B)=P(\bar{A})=1-\frac{10}{17}=\frac{7}{17}$$ Choice (B)

Note: If 'p' is the probability of success (in this case picking up an orange ball), the probability that the beginner wins the game $=\frac{1}{2-p}$

10. Probability of selecting urn A is $P(A)=\frac{1}{2}$.

and that of selecting urn B is $P(B)=\frac{1}{2}$

Probability of drawing a black ball (event E) when urn A is selected $P\left(\frac{E}{A}\right)=\frac{{}^7C_1}{{}^{13}C_1}$ and probability of E when urn B is selected $P\left(\frac{E}{B}\right)=\frac{{}^6C_1}{{}^{14}C_1}$

Probability of selecting black ball

$$=P(A).P\left(\frac{E}{A}\right)+P(B).P\left(\frac{E}{B}\right)$$

$$\frac{1}{2}.\frac{{}^7C_1}{{}^{13}C_1}+\frac{1}{2}.\frac{{}^6C_1}{{}^{14}C_1}$$

$$\text{Required Probability}=\frac{\frac{1}{2}.\frac{{}^7C_1}{{}^{13}C_1}}{\frac{1}{2}.\frac{{}^7C_1}{{}^{13}C_1}+\frac{1}{2}.\frac{{}^6C_1}{{}^{14}C_1}}$$

$$=\frac{\frac{7}{13}}{\frac{7}{13}+\frac{6}{14}}=\frac{\frac{7}{13}}{\frac{98+75}{13\times14}}=\frac{7\times14}{176}=\frac{49}{88}$$ Choice (B)

11. The marginal distributions are given below.

X \ Y	0	1	2	$P_x(x)$
0	$\frac{1}{40}$	$\frac{2}{40}$	$\frac{3}{40}$	$\frac{6}{40}$
1	$\frac{2}{40}$	$\frac{3}{40}$	$\frac{1}{40}$	$\frac{6}{40}$
2	$\frac{3}{40}$	$\frac{1}{40}$	$\frac{7}{40}$	$\frac{11}{40}$
3	$\frac{4}{40}$	$\frac{5}{40}$	$\frac{8}{40}$	$\frac{17}{40}$
$P_y(y)$	$\frac{10}{40}$	$\frac{11}{40}$	$\frac{19}{40}$	1

$P(X \le 1, Y = 2) = P(X = 0, Y = 2) + P(X = 1, Y = 2)$

$= \frac{3}{40} + \frac{1}{40} = \frac{4}{40} = \frac{1}{10}$ Choice (D)

12. Average number of errors per page $\lambda = \frac{50}{500} = \frac{1}{10}$

Average number of errors per 5 pages $= 5 \times \frac{1}{10} = \frac{1}{2}$

Probability of k errors per page is $P(x = k) = \frac{\lambda^k}{k!} e^{-\lambda}$.

$\therefore$ here $k = 0$

$\therefore$ $P(k = 0) = \frac{\lambda^k}{k!} e^{-\lambda} = e^{-\frac{1}{2}}$

$\therefore$ Probability that a random sample of 5 pages has no error $= e^{-0.5} = 0.6065$ Choice (A)

13. We know that X is uniform random variable in the interval $[a, b]$ then $p(x) = \frac{1}{b-a}$, $a < x < b$ and mean $= \frac{a+b}{2}$, variance $= \frac{(b-a)^2}{12}$

Given mean $= 2 \Rightarrow \frac{a+b}{2} = 2$

$\Rightarrow a + b = 4$ ---- (1)

Variance $= 12 \Rightarrow \frac{(b-a)^2}{12} = 12$

$\Rightarrow (b - a)^2 = 144 \Rightarrow b - a = 12$ ------ (2)

Solving (1) and (2) $a = -4$, $b = 8$

$\therefore$ $P(x) = \frac{1}{b-a} = \frac{1}{8-(-4)} = \frac{1}{12}$

$$P(x > 0) = \int_0^b p(x)dx = \int_0^8 \frac{1}{12} dx = \frac{1}{12} x \Big]_0^8 = \frac{8}{12} = \frac{2}{3}$$

Choice (C)

14. Given mean of $X = 3$
Variance of $X = 1$
Mean of $Y = 6$
Variance of $Y = 9$
$X = N(3, 1)$ $Y = N(6, 3)$ and X and Y are independent.
Let $u = x + y$; and $v = 9x - y$
Then u, v are also normal variates
$U = x + y = N(3 + 6, 1 + 9) = N(9, 10)$
$V = 9x - y = N(9(3) - 6, 81(1) + 9) = N(21, 90)$
By definition

$Z = \frac{u-9}{\sqrt{10}}$ and for $u = k \Rightarrow Z = \frac{k-9}{\sqrt{10}} = z$

Again $z = \frac{v-21}{\sqrt{90}}$ and for $v = 2k \Rightarrow Z = \frac{2k-21}{\sqrt{90}} = z_1$

Given $P(x + y \le k) = p(9x - y \ge 2k)$
$P(z \le z_1) = p(z \ge z_2)$
$P(z \le z_1) = p(z \le -2z)$

$= \frac{k-9}{\sqrt{10}} = \frac{-(2k-21)}{\sqrt{90}} = k - 9$

$= \frac{-2k+21}{3}$

$3k - 27 = -2k + 21$

$5k = 48 \Rightarrow k = \frac{48}{5} = 9.6$ Choice (B)

15. Let B_1: transfer of white ball to bag B.
B_2: transfer of green ball to bag B.

$P(B_1) = \frac{9}{14}; P(B_2) = \frac{5}{14}$

Let E be the event of drawing a green ball from bag B after transfer.

$P\left(\frac{E}{B_1}\right)$ = probability of drawing green ball if white ball is transferred to bag $B = \frac{7}{14}$

$P\left(\frac{E}{B_2}\right)$ = probability of drawing a green ball if green ball is transferred to bag $B = \frac{8}{14}$.

$\therefore$ $P(E) = P(B_1) \,.\, P\left(\frac{E}{B_1}\right) + P(B_2) \,.\, P\left(\frac{E}{B_2}\right)$

$= \frac{9}{14} . \frac{7}{14} + \frac{5}{14} . \frac{8}{14} = \frac{63}{196} + \frac{40}{196} = \frac{103}{196}$

$\therefore$ The required probability $P\left(\frac{B_1}{E}\right)$

$= \frac{P(B_1).P\left(\frac{E}{B_1}\right)}{P(E)} = \frac{\frac{9}{14}.\frac{7}{14}}{\frac{103}{196}} = \frac{63}{103}$ Choice (B)

16. Let A be the event that the number 3 appears atleast once.
B be the event that sum of the numbers appearing is 7.
$A \cap B$ be the event that the sum is 7 and 3 appear atleast once $P(B) = \frac{6}{36} \Rightarrow P(A \cap B) = \frac{2}{36}$

A/B denotes atleast one number show 3 while the sum of the numbers is 7.

$P\left(\frac{A}{B}\right) = \frac{P(B \cap A)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$ Choice (C)

17. We know $\sum P(X = x_1) = 1$

$\therefore\ k + 2k + 3k + 4k + 5k = 1$

$15k = 1 \Rightarrow k = \dfrac{1}{15}$

$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$

$= 3k + 5k + 4k = 12k = \dfrac{12}{15} = \dfrac{4}{5}$ Choice (A)

18. By definition. Choice (C)

19. Standard Result. Choice (A)

20. Standard Result. Choice (C)

21. We know that, if a statistic 's' follows t-distribution with degrees of freedom = ν, then 's^2' follows F–distribution with degrees of freedom $(1, \nu)$

Here ν = 10

$\therefore$'s^2' follows F-distribution with degrees of freedom $= (1, \nu) = (1, 10)$. Choice (B)

22. When the test statistic is outside the critical region, it lies in the acceptance region. So, we will accept the null hypothesis and reject the alternative hypothesis.

$\therefore$ Both P and S are true. Choice (C)

23. Three letters are placed into 3 addressed envelopes randomly in 3! = 6 ways.

X denotes the number of letters placed into corresponding addressed envelopes. The probability distribution table is as follows.

$X = x_i$	0	1	2	3
$P(x = x_i)$	$\frac{2}{6}$	$\frac{3}{6}$	0	$\frac{1}{6}$

$\therefore$ mean $(M) = \sum X_i P(x = x_i)$

$$= 0 \times \frac{2}{6} + 1 \times \frac{3}{6} + 2 \times \frac{0}{6} + 3 \times \frac{1}{6}$$

$$= \frac{0+3+0+3}{6} = \frac{6}{6} = 1$$

Variance $= \sum x_i^2 P(x = x_i) - \mu^2$

$$= 0 \times \frac{2}{6} + 1 \times \frac{3}{6} + 4 \times 0 + 9 \times \frac{1}{6} - 1$$

$$= \frac{3+9}{6} - 1 = 2 - 1 = 1.$$ Choice (C)

24. We know that variance $(x, x + 3, x + 5, x + 7, x + 10)$ = variance (0, 3, 5, 7, 10)

$$AM(0, 3, 5, 7, 10) = \frac{0+3+5+7+10}{5} = \frac{25}{5} = 5$$

$$\text{Variance } (0, 3, 5, 7, 10) = \frac{\sum (x_1 - A)^2}{n}$$

$$= \frac{(0-5)^2 + (3-5)^2 + (5-5)^2 + (7-5)^2 + (10-5)^2}{5}$$

$$= \frac{25+4+0+4+25}{5} = \frac{58}{5}$$

Variance $= \dfrac{58}{5} = 11.6$. Choice (B)

25. The ascending order of the given data except x is 3, 8, 12, 15, 16, 28

If $x < 12$, the fourth observation is 12 hence median is 12 if $x > 15$, the fourth observation is 15, hence median is 15. If $12 < x < 15$, the fourth observation is x, hence median is x.

Median is always lies between [12, 15]. Choice (D)

Engineering Mathematics Test 5

Number of Questions: 25 **Time: 60 min.**

Numerical Methods

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. In the process of finding an approximate root of $f(x) = 0$ in $[a, b]$ (where $f(a)$ and $f(b)$ are of opposite signs) by Regula – Falsi method, we assume that the curve $f(x) = 0$ in between $x = a$ and $x = b$ can be approximated to ________.
   (A) a parabola
   (B) a straight line
   (C) a hyperbola
   (D) a rectangular hyperbola

2. The iterative formula to find a root of the equation $f(x) = x^3 - 5x + 7 = 0$ by Newton Raphson method is _______.
   (A) $x_{k+1} = \dfrac{x_k^3 + 5x - 7}{3x_k^2 + 5}$ (B) $x_{k+1} = \dfrac{2x_k^3 + 5x}{3x_k^2 + 7}$
   (C) $x_{k+1} = \dfrac{2x_k^3 - 7}{3x_k^2 - 5}$ (D) $x_{k+1} = \dfrac{x_k^3 - 5x}{3x_k^2 + 7}$

3. With $x_0 = 0.5$ as the initial approximation, the value of the root of $f(x) = x + \sin x - 1 = 0$, after first iteration by Newton Raphson method is _______.
   (A) 0.7456 (B) 0.5110
   (C) 0.4998 (D) 0.2644

4. Applying the secant method, the first approximation to the root of $f(x) = xe^x - 2 = 0$, starting with function value at $x = 0.5$ and $x = 1$ is _______.
   (A) 1.1756 (B) 0.4035
   (C) 0.8104 (D) 0.5473

5. The extreme (minimum or maximum) point of a function $f(x)$ is to be determined by solving $\dfrac{df(x)}{dx} = 0$ using the Newton Raphson method. Let $f(x) = x^3 - 4x^2 + 5$ and $x_0 = 3$ be the initial guess of x. The value of x after first iteration (x_1) is ______.
   (A) 2.70 (B) 4.33
   (C) 3.30 (D) 1.77

6. In the process of evaluating $\int_0^{\frac{\pi}{2}} \left(x^3 + \sin 2x + 5\right) dx$ using Simpson's Rule with $h = \dfrac{\pi}{8}$, the absolute value of the error does not exceed ______.
   (A) 0.12351×10^{-4} (B) 1.03503×10^{-4}
   (C) 3.01243×10^{-4} (D) 6.2475×10^{-4}

7. The following table gives the velocity v of a particle at time t.

T(in seconds)	0	2	4	6	8	10	12
n(in m/sec)	6	10	16	20	22	30	40

   The distance moved by the particle in 12 seconds, when calculated by the Trapeziodal rule with $h = 2$ is ________.
   (A) 200 meters (B) 210 meters
   (C) 242 meters (D) 262 meters

8. A curve is drawn to pass through the points given by the following table

X	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Y	1.0	1.7	2.5	3.4	4.1	3.7	2.9

   The area bounded by the curve, the x – axis and the lines $x = 1$ and $x = 4$, when calculated by the Simpson's $\dfrac{3}{8}th$ Rule is _____ square units.
   (A) 8.7562 (B) 5.7435
   (C) 6.7134 (D) 8.4296

9. The absolute error (correct up to 4 decimal places) in calculating the value $\log_e 2$ by trapezoidal rule, with 4 intervals using the formulae $\log_e 2 = \int_1^2 \dfrac{dx}{x}$ is _______.
   (A) 0.1314 (B) 0.0039
   (C) 0.0000 (D) 0.0004

10. With reference to finding solution of a differential equation by numerical methods, which of the following methods is NOT a predictor correct method?
   (A) Picard's method
   (B) Modified Euler's method
   (C) Adams – Bash forth method
   (D) Milne's method

11. The differential equation $\dfrac{dy}{dx} - x^2 = y$; $y(0) = 1$ is to be solved by the modified Euler's method. With $h = 0.1$, the value of y_1 correct to four decimal places is ______.
   (A) 1.2046 (B) 1.1058
   (C) 0.9954 (D) 0.8764

12. Using Taylor's series method, the solution of the differential equation $\dfrac{dy}{dx} - xy = 1$ with $y(0) = 3$ at $x = 0.1$ with $h = 0.1$ is correct upto three decimal places is _______.
   (A) 3.1153 (B) 2.9847
   (C) 4.1572 (D) 3.7893

13. The solution of the differential equation $\frac{dy}{dx} = x + y$; $y(0) = 0$ at $x = 0.2$ by Runge Kutta method of fourth order with $h = 0.2$ is ______.

(A) 1.0034 (B) 0.0456
(C) 0.9984 (D) 0.0214

14. Consider an equation $f(x) = 0$ for which $x = 4.50$ is an exact root. In the process of finding a root of $f(x) = 0$ by a numerical method, the approximations obtained in four successive iterations are 4.45, 4.54, 4.47 and 4.52 respectively. Then these approximate values of the root of $f(x) = 0$ are ______.

(A) precise but not accurate
(B) not precise but accurate
(C) both precise and accurate
(D) neither precise nor accurate

15. For an equation $f(x) = 0$, if x_e is the exact root and x_a is the approximate root, then the percentage error is ______.

(A) $(x_e - x_a)\times 100$ (B) $|x_e - x_a|\times 100$
(C) $\frac{(x_e - x_a)}{x_e}\times 100$ (D) $\frac{|x_e - x_a|}{|x_e|}\times 100$

16. Consider the following two statements

P: Truncation error in numerical analysis arise when approximations are used to estimate some quantity.

Q: Round off error in numerical analysis occurs because of the computing devices inability to deal with certain number.

Then

(A) Both *P* and *Q* are true
(B) *P* is true but *Q* is false
(C) *P* is false but *Q* is true
(D) Both *P* and *Q* are false

17. In the process of fitting a quadratic equation of the form $y = a + bx + cx^2$ to a set of n points (x_1, y_1), (x_2, y_2), ….., (x_n, y_n) by the method of least squares, which of the following is not a normal equation?

(A) $\sum y_i = na + b\sum x_i + c\sum x_i^2$
(B) $\sum x_i y_i = a\sum x_i + b\sum x_i^2 + c\sum x_i^3$
(C) $\sum x_i y_i^2 = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4$
(D) $\sum x_i^2 y_i = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4$

18. If $y = 3x + 7$ is the best fit for 6 pairs of values of x and y by the method of least squares and $\sum y = 150$, then $\sum x$ is ______.

(A) 144 (B) 102
(C) 46 (D) 36

19. In the process of fitting a curve $y = \frac{x^2}{ax+b}$ to a given set of n pairs of values of x and y by converting it into a linear form $Y = a + bX$, X and Y respectively stand for ______.

(A) $\frac{1}{x^2}$ and $\frac{x^2}{y}$ (B) $\frac{1}{x}$ and $\frac{x}{y}$
(C) x^2 and $\frac{y}{x^2}$ (D) x and $\frac{y}{x}$

20. In the process of fitting a curve $\exp(y) = ab^x$ to a given set of n pairs of values of x and y by converting it into a linear form $y = A + Bx$, A and B respectively stand for ______

(A) $\ln a$ and $\ln b$ (B) $\ln a$ and $\log_{10} b$
(C) $\log_{10} a$ and $\ln b$ (D) $\log_{10} a$ and $\log_{10} b$

21. If Δ denotes the forward difference operator then the value of $\Delta^{18}\left[(1+2x^3)(1-3x^4)(1+4x^5)(1-5x^6)\right]$ is ______.

(A) $5! \times 18!$
(B) $6! \times 18!$
(C) $5! \times 17!$
(D) $6! \times 17!$

22. The central difference operator δ is defined as

$y_r - y_{r-1} = \delta y_{r-\frac{1}{2}}$

Then which of the following is an identity? (Note that Δ and ∇ denote the forward and the backward difference operators respectively)

(A) $\Delta y_5 = \nabla y_4 = \delta y_{\frac{7}{2}}$
(B) $\Delta y_5 = \nabla y_4 = \delta y_4$
(C) $\Delta y_4 = \nabla y_5 = \delta y_{\frac{9}{2}}$
(D) $\Delta y_4 = \nabla y_5 = \delta y_4$

23. Match the following

	Group – I		Group – II
P.	To extrapolate the values of y to the left of y_0 when x values are equally spaced	1.	Newton's divided difference formula
Q.	To interpolate the values of y near the end value y_n when x values are equally spaced	2.	Lagrange's interpolation formula
R.	To split the given function into partial fractions	3.	Newton's forward interpolation formula
S.	To interpolate the values of y when x values are unequally spaced.	4.	Newton's backward interpolation formula

(A) $P-(1), Q-(2), R-(3), S-(4)$
(B) $P-(3), Q-(2), R-(4), S-(1)$
(C) $P-(3), Q-(4), R-(2), S-(1)$
(D) $P-(2), Q-(1), R-(4), S-(3)$

24. The 9th divided difference of a polynomial of degree 8 is ______.
(A) zero
(B) a non – zero constant
(C) a linear polynomial
(D) a quadratic polynomial

25. If $f(0) = -12$, $f(3) = 6$ and $f(4) = 12$, then the value of $f(6)$ obtained by the Lagrange's interpolation formula is ______.
(A) 18 (B) 24
(C) 20 (D) 26

Answer Keys

1. B	**2.** C	**3.** B	**4.** C	**5.** A	**6.** B	**7.** C	**8.** A	**9.** B	**10.** A
11. B	**12.** A	**13.** D	**14.** B	**15.** D	**16.** A	**17.** C	**18.** D	**19.** B	**20.** A
21. A	**22.** C	**23.** C	**24.** A	**25.** B					

Hints and Explanations

1. Standard Result. Choice (B)

2. Given, $f(x) = x^3 - 5x + 7 = 0$
$\Rightarrow f^1(x) = 3x^2 - 5$.
By Newton Raphson's method, the interative formulae to find a root is

$$x_{k+1} = x_k - \frac{f(x_k)}{f^1(x_k)} = x_k - \frac{(x_k^3 - 5x_k + 7)}{(3x_k^2 - 5)}$$

$$\therefore \quad x_{k+1} = \frac{2x_k^3 - 7}{3x_k^2 - 5}.$$ Choice (C)

3. Given, $f(x) = x + \sin x - 1 = 0$
$\Rightarrow f^1(x) = 1 + \cos x$ and $x_0 = 0.5$
By Newton Raphson's method

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)}$$

$$= (0.5) - \frac{(0.5 + \sin(0.5) - 1)}{(1 + \cos(0.5))}$$

$= 0.5110$. Choice (B)

4. Here, $f(x) = xe^x - 2 = 0$
By the secant method, the approximate root of $f(x) = 0$ after first iteration is given by

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \qquad \text{---------- (1)}$$

Here, $x_0 = 0.5$ and $x_1 = 1$
$\therefore$ $f(x_0) = f(0.5) = -1.1756$ and $f(x_1) = f(1) = 0.7183$
Substituting these in (1) we have

$$x_2 = \frac{(0.5)(0.7183) - (1)(-1.1756)}{(0.7183) - (-1.1756)}$$

$\therefore \quad x_2 = 0.8104$. Choice (C)

5. Given, $f(x) = x^3 - 4x^2 + 5$

$$\frac{df(x)}{dx} = 0 \Rightarrow 3x^2 - 8x = 0$$

Let, $g(x) = 3x^2 - 8x = 0$.

$\therefore$ We have to find the approximate root of $g(x) = 0$ after first interaction by the Newton Raphson method with $x_0 = 3$.
$\therefore$ $g^1(x) = 6x - 8$
By Newton Raphson method

$$x_1 = x_0 - \frac{g(x_0)}{g^1(x_0)}$$

$$= 3 - \frac{(3(3)^2 - 8(3))}{6(3) - 8} = 2.7.$$ Choice (A)

6. We have $\int_0^{\frac{\pi}{2}} (x^3 + \sin 2x + 5)dx$.

Let $y = f(x) = x^3 + \sin 2x + 5$.

The absolute value of the maximum error in Simpson's Rule is

$$|E|_{\max} = \frac{(b-a)h^4}{180} m \qquad \text{----------- (1)}$$

Where $m = \max - \{y_0^{(iv)}, y_2^{(iv)}, y_4^{(iv)}\}$

Here, $h = \frac{\pi}{8}$ and $y^{(iv)} = 16 \sin 2x$, $a = 0$, $b = \frac{\pi}{2}$

$\therefore \quad m = \max [16\sin 0, 16\sin\left(2 \times \frac{\pi}{4}\right),$

$16 \sin\left(2 \times \frac{\pi}{2}\right)] = 16$

From (1)

$$|E_{\max}| = \frac{\left(\frac{\pi}{2}\right)\left(\frac{\pi}{8}\right)}{180} \times 16 = 1.0350 \times 10^{-4}$$

The absolute value of the maximum error cannot exceed 1.03503×10^{-4}.

Choice (B)

7. Given velocity of the particle at various times is

T	0	2	4	6	8	10	12
n	6	10	16	20	22	30	40

Distance traveled in 12 seconds = $\int_0^{12} \nu dt$.

By trapezoidal rule

$$\int_0^{12} \nu dt = \frac{h}{2}[(\nu_0 + \nu_6) + 2(\nu_1 + \nu_2 + \nu_3 + \nu_4 + \nu_5)]$$

$$= \frac{2}{2}[(6 + 40) + 2(10 + 16 + 20 + 22 + 30)]$$

$= 242$ meters. Choice (C)

8. Let $y = f(x)$ be the curve, that pass through the points

X	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Y	1.0	1.7	2.5	3.4	4.1	3.7	2.9

$\therefore$ The area bounded by the curve $y = f(x)$, x – axis andthe lines $x = 1$ and $x = 4$ is $\int_1^4 f(x)dx$.

By Simpson's $\frac{3}{8}th$ Rule, we have

$$\int_1^4 f(x)dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3\times(0.5)}{8}[(1.0 + 2.9) + 3(1.7 + 2.5 + 4.1 + 3.7) + 2 \times 3.4] = 8.7562.$$

Choice (A)

9. Let $y = f(x) = \frac{1}{x}$

Here, $a = 1$, $b = 2$ and $n = 4$

$\therefore \quad h = \frac{b-a}{n} = 0.25$

X	1	1.25	1.5	1.75	2
F(x)	1	0.8	0.667	0.5714	0.5

We have $\log_e 2 = \int_1^2 \frac{dx}{x}$

By the trapezoidal rule, we have

$$\int_1^2 \frac{dx}{x} = \int_1^2 f(x)dx = \frac{h}{2}[(y_0 + y_4) + 2(y_1 + y_2 + y_3)]$$

$$\frac{0.25}{2}[(1+ 0.5) + 2(0.8 + 0.6667 + 0.5714)]$$

$$\therefore \quad \log_e 2 = \int_1^2 \frac{dx}{x} = 0.6970 \qquad \text{---------- (1)}$$

The exact value of $\log_e 2 = 0.6931$ -------- (2)

The absolute error in calculating $\log_e 2$ by the trapezoidal rule = 0.6970 – 0.6931 = 0.0039. Choice (B)

10. A predictor corrector method is one in which we predict the solution first and then we improve it for accuracy Picard's method is not a predictor corrector method and all other methods are predictor corrector methods. Choice (A)

11. Given different equation is $\frac{dy}{dx} - x^2 = y$ and $y(0) = 1$

$$\Rightarrow \frac{dy}{dx} = x^2 + y$$

$\therefore \quad f(x, y) = x^2 + y$, $x_0 = 0$, $y_0 = y(x_0) = 1$ and $h = 0.1$

By Euler's method

$$y_1^{(0)} = y_0 + h\,(f(x_0, y_0) = y_0 + h\left(x_0^2 + y_0\right)$$

$$= 1 + (0.1)\,(0 + 1)$$

$$y_1^{(0)} = 1.1$$

By modified Euler's method

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f\left(x_0, y_0\right) f\left(x_1 y_1^{(0)}\right)\right]$$

$$= y_0 + \frac{h}{2}\left[\left(x_0^2 + y_0\right) + \left(x_1^2 + y_1^{(0)}\right)\right]$$

$$= 1 + \frac{(0.1)}{2}[(0 + 1) + (10.1)^2 + 1.1)] = 1.1055$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(1)}\right)\right]$$

$$= y_0 + \frac{h}{2}\left[\left(x_0^2 + y_0\right) + \left(x_1^2 + y_1^{(1)}\right)\right]$$

$$= 1 + \frac{(0.1)}{2}[(0 + 1) + ((0.1)^2 + 1.1055)]$$

$$= 1.1058$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(2)}\right)\right]$$

$$= y_0 + \frac{h}{2}\left[\left(x_0^2 + y_0\right) + \left(x_1^2 + y_1^{(2)}\right)\right]$$

$$= 1 + \frac{(0.1)}{2}[(0 + 1) + ((0.1)^2 + 1.1058)]$$

$$= 1.1058$$

The solution of a given different equation at $x_1 = 0.1$ is $y_1 = 1.1058$. Choice (B)

12. Given differential equation is $\frac{du}{dx} - xy = 1$ and $y(0) = 3$

$$\Rightarrow \frac{dy}{dx} = 1 + xy$$

Here $f(x, y) = 1 + xy$, $x_0 = 0$, $y_0 = y(x_0) = 3$ and $h = 0.1$

By Taylar's series we have

$$y_1 = y(x_1) = y_0 + h_0\, y_0^1 + \frac{h^2}{2!}\, y_0^{11} + \frac{h^3}{3!}\, y_0^{111} + \ldots\ldots \infty \qquad \text{----(1)}$$

$$y_0^1 = \left(\frac{dy}{dx}\right)_{x=0} = f(x_0, y_0) = 1 + x_0\, y_0 = 1 + (0)(3) = 1$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\,(1 + xy) = xy^1 + y$$

$$\therefore \quad y_0^{11} = \left(\frac{d^2y}{dx^2}\right)_{atx=x_0} = x_0\; y_0^1 + y_0 = 0 \times 1 + 3 = 3$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}\,(xy^1 + y) = xy^{11} + 2y^1$$

$$\therefore \quad y_0^{111} = \frac{d^3y}{dx^3}_{atx=x_0} = x_0\; y_0^{11} + 2\, y_0^1 = 0 \times 3 + 2 \times 1 = 2$$

$$\frac{d^4y}{dx^4} = \frac{d}{dx}\left(\frac{d^3y}{dx^3}\right) = \frac{d}{dx}\,(xy^{11} + 2y^1) = xy^{111} + 3y^{11}$$

$$y_0^{(iv)} = \frac{d^4y}{dx^4}_{atx=x_0} = x_0\; y_0^{111} + 3\, y_0^{11} = 0 \times 2 + 3 \times 3 = 9$$

Substituting these in (1), we have

$$y_1 = 3 + (0.1) \times 1 + \frac{(0.1)^2}{2!} \times 3 + \frac{(0.1)^3}{3!} \times 2 + \frac{(0.1)^4}{4!} \times 9 + \ldots = 3.1153.$$

Choice (A)

13. Given differential equation is $\frac{dy}{dx} = x + y, y(0) = 0$

Here $f(x, y) = x + y$, $x_0 = 0$; $y_0 = 0$ and $h = 0.2$

$\therefore \quad x_1 = x_0 + h = 0.2$

By R – K method of fourth order we have

$Y_{\text{at } x = 0.1} = y_1 = y_0 + \Delta y$ ---------- (1)

Where $\Delta y = \frac{1}{6}\,[k_1 + 2k_2 + 2k_3 + k_4]$ ------- (2)

Here $k_1 = h\, f(x_0, y_0) = h(x_0 + y_0)$

$= (0.2)\,(0 + 0)$

$\therefore \quad k_1 = 0$

$$k_2 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= h\left(\left(x_0 + \frac{h}{2}\right) + \left(y_0 + \frac{k_1}{2}\right)\right)$$

$$= (0.2)\left[\left(0 + \frac{(0.2)}{2}\right) + \left(0 + \frac{0}{2}\right)\right]$$

$\therefore \quad k_2 = 0.02$

$K_3 = 0.022$ and $k_4 = h\, f(x_0 + h\,, y_0 + k_3)$

$= h\,[(x_0 + h) + (y_0 + k_3)]$

$= (0.2)\,[(0 + 0.2) + (0 + 0.022)]$

$\therefore \quad k_4 = 0.0444$

$\therefore$ From (2),

$$\Delta y = \frac{1}{6}\,[0 + 2 \times 0.02 + 2 \times 0.022 + 0.0444]$$

$\Delta y = 0.0214$

$\therefore$ From (1), $y_1 = y_0 + \Delta y = 0 + 0.0214 = 0.0214.$

Choice (D)

14. The four approximations given are not relatively close to each other. So, they are not precise. All the four approximations are close to the exact root $x = 4.50$. So, they are accurate. Choice (B)

15. By definition. Choice (D)

16. By definitions of the round off and the truncation errors. Choice (A)

17. Given set of n points are $(x_1, y_1,)$, $(x_2, y_2), \ldots, (x_n, y_n)$

We have to fit the quadratic equation

$y = a + bx + cx^2$ $\rightarrow$ (1)

to the given set of n points

Here a, b and c are constants to be determined such that

$$S = \sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]^2 \quad \rightarrow (2)$$

is minimum

S is minimum for those values of a, b and c at which

$$\frac{\partial S}{\partial a} = 0, \frac{\partial S}{\partial b} = 0 \text{ and } \frac{\partial S}{\partial c} = 0.$$

i.e., $-2\left(\sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]\right) = 0$

$$-2x_i\left(\sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]\right) = 0$$

and $-2x_i^2\left(\sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]\right) = 0$

$\therefore$ the normal equations are

$$\Rightarrow \quad \sum y_i = na + b\sum x_i + c\sum x_i^2$$

$$\sum x_i y_i = a\sum x_i + b\sum x_i^2 + c\sum x_i^3$$

And $\sum x_i^2 y_i = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4$

$\therefore$ the equation given in option (C) is not a normal equation. Choice (C)

18. Given that $y = 3x + 7$ is the best fit for 6 pairs of values of x and y also given $\sum y = 150$

$\therefore$ We know that

$\sum y = 3\sum x + n7$

(Here n = number of points = 6)

$\Rightarrow$ $150 = 3\sum x + 6 \text{ x } 7$

$\Rightarrow$ $3\sum x = 108$

$\Rightarrow$ $\sum x = 36.$ Choice (D)

19. Given curve is $y = \frac{x^2}{ax + b}$

$$\Rightarrow \quad \frac{1}{y} = \frac{ax + b}{x^2}$$

$$\Rightarrow \quad \frac{}{y} = \frac{a}{x} + \frac{b}{x}$$

$$\Rightarrow \quad \frac{x}{y} = a + \frac{b}{x}$$

$$\Rightarrow \quad \frac{x}{y} = a + \frac{b}{x}$$

Which is of the linear from $Y = a + bX$

Where $X = \frac{1}{x}$ and $Y = \frac{x}{y}$ Choice (B)

20. Given curve is $\exp(y) = ab^x$

i.e., $e^y = ab^x$

Applying logarithm (ln) on both sides,

We have

$\ln(e^y) = \ln(ab^x)$

$\Rightarrow \quad y = \ln a + \ln b^x$

$\Rightarrow \quad y = \ln a + x\ln b$

Which is of the form

$y = A + Bx$

Where $A = \ln a$ and $B = \ln b$. Choice (A)

21. We have $\Delta^{18}\left[\left(1+2x^3\right)\left(1-3x^4\right)\left(1+4x^5\right)\left(1-5x^6\right)\right]$

$= \Delta^{18}\left[2x(-3)x4x(-5)x^{18} + k_1x^{17} + k_2x^{16} + \ldots + k_{15}x^3 + 1\right]$

$= 5!\ \Delta^{18}\ [x^{18}]$

$(\because\ \Delta^{18}[x^n] = 0$ for $n < 18)$

$= 5! \times 18!$. Choice (A)

22. We know that $\Delta y_{r-1} = y_r - y_{r-1}$ $\rightarrow (1)$

$\nabla y_r = y_r - y_{r-1}$ $\rightarrow (2)$

and given that $\delta y_{r-\frac{1}{2}} = y_r - y_{r-1}$ $\rightarrow (3)$

From (1), (2) and (3), we have

$\Delta y_{r-1} = \nabla y_r = \delta y_{r-\frac{1}{2}}$ $\rightarrow (4)$

Among the options given, we can get option (C) by taking $r = 5$ in (4),

$\therefore \quad \Delta y_4 = \nabla y_5 = \delta y_{\frac{9}{2}}$ Choice (C)

23. Standard result Choice (C)

24. We know that the n^{th} divided difference of any polynomial of degree less than n is always zero. Choice (A)

25. Given pairs of values of x and $f(x)$ are

X	0	3	4
f(x)	–12	6	12

By Lagrange's interpolation formula, we have

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0)$$

$$+ \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$= \frac{(x-3)(x-4)}{(0-3)(0-4)} \times (-12) + \frac{(x-0)(x-4)}{(3-0)(3-4)} \times 6$$

$$+ \frac{(x-0)(x-3)}{(4-0)(4-3)} \times 12$$

By taking $x = 6$ on both sides,

We have

$$f(6) = \frac{(6-3)(6-4)}{3\times 4} \times (-12) + \frac{(6-0)(6-4)}{3\times(-1)} \times 6$$

$$+ \frac{(6-0)(6-3)}{4\times 1} \times 12$$

$\therefore \quad f(6) = 24$ Choice (B)

PART - III

- ↪ Digital Logic Design
- ↪ C programming
- ↪ Data structure
- ↪ Design and analysis of algorithm
- ↪ Computer Organisation and Architecture
- ↪ Database Management System
- ↪ Automata theory
- ↪ Compiler design
- ↪ Software Engineering
- ↪ Computer Network
- ↪ Web Technology
- ↪ Information System

Chapter 1

Number Systems

LEARNING OBJECTIVES

- *Digital circuits*
- *Number system with different base*
- *Conversion of number systems*
- *Complements*
- *Subtraction with complement*
- *Numeric codes*
- *Weighted and non-weighted codes*
- *Error detection and correction code*
- *Sequential, reflective and cyclic codes*
- *Self complementing code*

Digital Circuits

Computers work with binary numbers, which use only the digits '0' and '1'. Since all the digital components are based on binary operations, it is convenient to use binary numbers when analyzing or designing digital circuits.

Number Systems with Different Base

Decimal number system

Decimal numbers are usual numbers which we use in our day-to-day life. The base of the decimal number system is 10. There are ten numbers 0 to 9.

The value of the nth digit of the number from the right side = nth digit $\times$ (base)$^{n-1}$

Example 1: $(99)_{10} \rightarrow 9 \times 10^1 + 9 \times 10^0$
$= 90 + 9 = 99$

Example 2: $(332)_{10} \rightarrow 3 \times 10^2 + 3 \times 10^1 + 2 \times 10^0$
$= 300 + 30 + 2$

Example 3: $(1024)_{10} \rightarrow 1 \times 10^3 + 0 \times 10^2 + 2 \times 10^1 + \times 10^0$
$= 1000 + 0 + 20 + 4 = 1024$

Binary number system

In binary number system, there are only two digits '0' and '1'. Since there are only two numbers, its base is 2.

Example 4: $(1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 8 + 4 + 1 = (13)_{10}$

Octal number system

Octal number system has eight numbers 0 to 7. The base of the number system is 8. The number $(8)_{10}$ is represented by $(10)_8$.

Example 5: $(658)_8 = 6 \times 8^2 + 5 \times 8^1 + 8 \times 8^0$
$= 384 + 40 + 8 = (432)_{10}$

Hexadecimal number system

In hexadecimal number system, there are 16 numbers 0 to 9, and digits from 10 to 15 are represented by A to F, respectively. The base of hexadecimal number system is 16.

Example 6: $(1A5C)_{16} = 1 \times 16^3 + A \times 16^2 + 5 \times 16^1 + C \times 16^0$
$= 1 \times 4096 + 10 \times 256 + 5 \times 16 + 12 \times 1$
$= 4096 + 2560 + 80 + 12 = (6748)_{10}$.

Table 1 *Different number systems*

Decimal	Binary	Octal	Hexadecimal
0	000	0	0
1	001	1	1
2	010	2	2
3	011	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

(*Continued*)

Table 1 (*Continued*)

Decimal	Binary	Octal	Hexadecimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14

1. For a number system with base n, the number of different symbols in the number system will be n. Example: octal number system will have total of 8 numbers from 0 to 7.
2. The number 'n' in the number system with base 'n' is represented as $(10)_n$.
3. The equivalent of number $(a_3a_2a_1a_0 \cdot a_{-1}a_{-2})_n$ in decimal is $a_3 \times n^3 + a_2 \times n^2 + a_1 \times n^1 + a_0 \times n^0 + a_{-1} \times n^{-1} + a_{-2} \times n^{-2}$.

Conversion of Number Systems

The conversion of decimal to any other number system involves successive division by the radix until the dividend reaches 0. At each division, the remainder gives a digit of converted number; and the last one is most significant digit, the remainder of the first division is least significant digit.

The conversion of other number system to decimal involves multiplying each digit of number system with the weight of the position (in the power of radix) and sum the products calculated, the total is the equivalent value in decimal.

Decimal to binary conversion

Example 7: $(66)_{10}$

$$\begin{array}{r|ll} 2 & 66 & \\ 2 & 33 & 0 \\ 2 & 16 & 1 \\ 2 & 8 & 0 \\ 2 & 4 & 0 \\ & 2 & 0 \\ & 1 & 0 \end{array}$$

Reading remainders from bottom to top

$= (1000010)_2$

Example 8: $(928)_{10}$

$$\begin{array}{r|ll} 2 & 928 & \\ 2 & 464 & 0 \\ 2 & 232 & 0 \\ 2 & 116 & 0 \\ 2 & 58 & 0 \\ 2 & 29 & 0 \\ 2 & 14 & 1 \\ 2 & 7 & 0 \\ 2 & 3 & 1 \\ & 1 & 1 \end{array}$$

$= (1110100000)_2$

Example 9: $(105.75)_{10}$

$$\begin{array}{r|ll} 2 & 105 & \\ 2 & 52 & 1 \\ 2 & 26 & 0 \\ 2 & 13 & 0 \\ 2 & 6 & 1 \\ 2 & 3 & 0 \\ & 1 & 1 \end{array}$$

$(105)_{10} = (1101001)_2$

$(0.75)_{10}$

Multiply 0.75 by 2 = 1.50

Multiply 0.50 by 2 = 1.00

Reading integers from top to bottom $0.75 = (0.11)_2$

$\therefore (105.75)_{10} = (1101001.11)_2$

Binary to decimal conversion

Example 10: $(10100011)_2$

$= 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 128 + 0 + 32 + 0 + 0 + 0 + 2 + 1$

$= (163)_{10}$

Example 11: $(11010011.101)_2$

$= 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3})$

$= 128 + 64 + 0 + 16 + 0 + 0 + 2 + 1 + 0.5 + 0 + 0.125$

$= (211.625)_{10}$

Decimal to octal conversion

Example 12: $(16)_{10}$

$$\begin{array}{r|ll} 8 & 16 & 0 \\ & 2 & \end{array}$$

Remainder from bottom to top $= (20)_8$

Example 13: $(347.93)_{10}$

$(.93)_{10}$

$0.93 \times 8 =$ 7.44

$0.44 \times 8 =$ 3.52

$0.52 \times 8 =$ 4.16

$0.16 \times 8 =$ 1.28

……..

Read the integers of octal point from top to bottom.

$\therefore (0.93)_{10} = (0.7341)_8$

$(347)_{10}$

$$\begin{array}{r|ll} 8 & 347 & 3 \\ 8 & 43 & 3 \\ & 5 & \end{array}$$

$\therefore (347)_{10} = (533)_8$

Ans: $(533.7341)_8$

Octal to decimal conversion

Example 14: $(33)_8$

$3 \times 8^1 + 3 \times 8^0 = 24 + 3$

$(27)_{10}$

Example 15: $(1023.06)_8$

$1 \times 8^3 + 0 \times 8^2 + 2 \times 8^1 + 3 \times 8^0 + 0 \times 8^{-1} + 6 \times 8^{-2}$

$= 512 + 0 + 16 + 3 + 0 + 0.0937 = (2095.0937)_{10}$

Octal to binary conversion

To convert octal to binary, replace each octal digit with their equivalent 3-bit binary representation.

Example 16: $(7777)_8$

Convert each octal digit to binary

$= \frac{7}{111}\ \frac{7}{111}\ \frac{7}{111}\ \frac{7}{111}$

$= (111\ \ 111\ \ 111\ \ 111)_2$

Example 17: $(567.62)_8$

5	6	7	.	6	2
101	110	111	.	110	010

$= (101110111.110010)_2$

Binary to octal conversion

To convert a binary number to an octal number, starting from the binary point, make groups of 3-bits each on either side of the binary point, and replace each 3-bit binary group by the equivalent octal digit.

Example 18: $(010011101)_2$

$\frac{010}{2}\ \frac{011}{3}\ \frac{101}{5} = (235)_8$

Example 19: $(10010111011.1011)_2$

$\frac{010}{2}\ \frac{010}{2}\ \frac{111}{7}\ \frac{011}{3} \cdot \frac{101}{5}\ \frac{100}{4} = (2273.54)_8$

Decimal to hexadecimal conversion

Example 20: $(527)_{10}$

$$\begin{array}{r|ll} 16 & 527 & \\ 16 & 32 & 15 \\ & 2 & 0 \end{array}$$

Decimal		Hexa
2	→	2
0	→	0
15	→	F

$= (20F)_{16}$

Example 21: $(18.675)_{10}$

$(18)_{10}$

$$\begin{array}{r|ll} 16 & 18 & \\ & 1 & 2 \end{array}$$

Decimal		Hexa
1	–	1
2	–	2

$(18)_{10} = (12)_{16}$

$(0.675)_{10}$

$$\begin{array}{l|l} 0.675 \times 16 & 10.8 \\ 0.800 \times 16 & 12.8 \\ 0.800 \times 16 & 12.8 \\ 0.800 \times 16 & 12.8 \end{array}$$

Decimal	Hexa
10	A
12	C
12	C
12	C

$= (0.ACCC)_{16}$

∴ Hexadecimal equivalent is

$= (12.AC\ CC)_{16}$

Hexadecimal to decimal conversion

Example 22: $(A3F)_{16}$

Decimal		Hexa
A	–	10
3	–	3
F	–	15

$\rightarrow 10 \times 16^2 + 3 \times 16^1 + 15 \times 16^0$

$\rightarrow 2560 + 48 + 15 \quad \rightarrow (2623)_{10}$

Example 23: $(1F63.0EB)_{16}$

1	1
F	15
6	6
3	3
0	0
E	14
B	11

$\rightarrow 1 \times 16^3 + 15 \times 16^2 + 6 \times 16^1 + 3 \times 16^0 \times (0 \times 16^{-1})$
$+ (14 \times 16^{-2}) + (11 \times 16^{-3})$

$\rightarrow 4096 + 3840 + 96 + 3 + 0 + 0.0546 + 0.0026$

$\rightarrow (8035.0572)_{10}$

Hexadecimal to binary number system

To represent hexadecimal in binary, represent each HEX number with its 4-bit binary equivalent.

Example 24: $(34F)_{16}$

Hexa	Decimal	Binary
3	3	0011
4	4	0100
F	15	1111

$= (001101001111)_2$

Example 25: $(AFBC \cdot BED)_{16}$

Hexa	Decimal	Binary
A	10	1010
F	15	1111
B	11	1011
C	12	1100
B	11	1011
E	14	1110
D	13	1101

$= (1010111110111100.101111101101)_2$

Binary to hexadecimal number system

To convert binary number to a hexadecimal number, starting from the binary point, make groups of 4-bits each on either side of the binary point and replace each 4-bit group by the equivalent hexadecimal digit.

Example 26: $(11001001)_2$

$\rightarrow \underline{1100}\ \underline{1001}$
$\quad 12 \quad 9$

$\rightarrow (C9)_{16}$

Example 27: $(1011011011.01111)_2$

$$\frac{0010}{2}\ \frac{1101}{D}\ \frac{1011}{B} \cdot \frac{0111}{7}\ \frac{1000}{8} = (2\,DB.78)_{16}$$

Hexadecimal to octal number system

The simplest way to convert hexadecimal to octal is, first convert the given hexadecimal number to binary and the Binary number to Octal.

Example 28: $(C3AF)_{16}$

$\rightarrow 001100001110101111$

$\rightarrow (141657)_8$

Example 29: $(C6.AE)_{16}$

$\rightarrow 0011000110.10101110$

$\rightarrow (306.534)_8$

Octal to hexadecimal number system

The simplest way to convert octal to hexadecimal is first convert the given octal number to binary and then the binary number to hexadecimal.

Example 30: $(775)_8$

$\rightarrow (000111111101)_2$

$\rightarrow (1FD)_{16}$

Example 31: $(34.7)_8$

$\rightarrow (00011100.1110)_2$

$\rightarrow (1C.E)_{16}$

Complements

Complements are used in digital computers to simplify the subtraction operation and for logical manipulation.

There are two types of complements for each base - r-system.

1. Radix complement (or) r's complement: the r's complement of an m digit number N in base r is $r^m - N$ for $N \neq 0$.
   For example, $N = 0$, r's complement is 0.
2. Diminished radix complement: (or) $(r-1)$'s complement: Given a number N in base r having m digits, then $(r-1)$'s complement is $(r^m - 1) - N$.
   For example, decimal number system will have 10's complement and 9's complement.
   Similarly, binary number system will have 2's complement and 1's complement.

Example 32: 10's complement of $(2657)_{10}$ is $(10)^4 - 2657$

10000
<u>2657</u>
<u>7343</u>

Example 33: 9's complement of $(2657)_{10}$ is $(10^4 - 1) - 2657$

10000
<u>– 1</u>
9999
<u>2657</u>
<u>7342</u>

- r's complement can be obtained by adding 1 to $(r-1)$'s complement.

$$r^m - N = \{(r^m - 1) - N\} + 1$$

Example 34: 2's complement of $(101101)_2$ is

$= (2)^6 - 101101$

$(2^6)_{10} = (100000)_2$

2's complement is 100000
<u>–101101</u>
010011

Example 35: 1's complement of $(101101)_2$ is

$2^6 - 1 = 1000000$
<u>– 1</u>
111111
<u>101101</u>

1's complement –010010

The one's complement of a binary number is formed by changing 1's to 0's and 0's to 1's, The 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged, and replacing 1's with zeros and zeros with 1's in all other bits.

If the number M contains radix point, the point should be removed temporarily in order to form r's/$(r-1)$'s complement.

The radix point is then restored to the complemented number in the same relative position.

Example 36: What is 1's complement of $(1001.011)_2$?

$\rightarrow$ Consider without radix point 1001011

Take 1's complement 0110100

Place radix point again $(0110.100)_2$

Example 37: What is 2's complement of $(1001.011)_2$?

Consider without radix point 1001011

Take 2's complement 0110101

Place radix point again $(0110.101)_2$

Complement of a complement is equal to the original number $r^m - (r^m - M) = M$

Subtraction with Complements

Subtraction of two n digit unsigned numbers $A - B$ in base r can be done as follows by r's complement method.

Add A to the r's complement of B. Mathematically $A + (r^n - B) = A - B + r^n$

If $A \geq B$ the sum will produce an end carry r^n; which can be discarded. (Discarding carry is equivalent to subtracting r^n from result). What is left is the result $A - B$?

$$
\begin{array}{llr}
A = 1100 & \rightarrow & 1100 \\
B = 1010 & \text{(2's complement)} & +\ \underline{0110} \\
 & \text{Sum:} & 10010 \\
\text{discard carry } (-r^n) & & -\ \underline{10000} \\
 & A - B: & \underline{0010}
\end{array}
$$

If $A < B$, the sum does not produce an end carry and result is $r^n - (B - A)$. Then take r's complement of the sum, and place a negative sign in front.

If $A = 1010$
$B = 1100$
$A - B$ can be done as

$$
\begin{array}{llr}
A \rightarrow & & 1010 \\
B \rightarrow \text{2's complaint} & & +\ \underline{0100} \\
 & \text{Sum:} & 1110
\end{array}
$$

Here, no carry generated, so result is a negative number.
2's complement of result $\rightarrow 0010 = 2$
result $= -2$

Subtraction of unsigned numbers by using $(r - 1)$'s complement can be done in similar way. However, $(r - 1)$'s complement is one less than the r's complement. Because of this, the sum produced is one less than the correct difference when an end carry occurs. So end carry will be added to the sum. Removing the end carry and adding 1 to the sum is referred to as an end-around-carry.

Consider $A = 1100$, $B = 1010$
For $A - B$

$$
\begin{array}{llr}
A \rightarrow & & 1100 \\
B \rightarrow \text{(1's complement)} & & +\ \underline{0101} \\
 & \text{Sum:} & \underline{10001} \\
\text{End around carry} & & +\ \underline{1} \\
 & & A - B = 0010
\end{array}
$$

For $B - A$

$$
\begin{array}{llr}
B \rightarrow & & 1010 \\
A \rightarrow \text{(1's complement)} & & +\ \underline{0011} \\
 & \text{Sum:} & \underline{1101}
\end{array}
$$

There is no end carry, for there result is
$-(B - A) = -(\text{1's complement of } 1101)$
$= -0010 = -2$

Signed Binary Numbers

Positive integers can be represented as unsigned numbers; but to represent negative integer, we need a notation for negative values in binary.

It is customary to represent the sign with a bit placed in the left most position of the number. The convention is to make the sign bit 0 for positive and 1 for negative. This representation of signed numbers is referred to as sign-magnitude convention

S Magnitude

+24 is $\underbrace{0}_{\text{sign}}\ \underbrace{11000}_{\text{magnitude}}$

–24 is $\underbrace{1}_{\text{sign}}\ \underbrace{11000}_{\text{magnitude}}$

Other notation for representation of signed numbers is signed complement system. This is convenient to use in a computer for arithmetic operations. In this system, a negative number is indicated by its complement (i.e., complement of corresponding positive number) whereas the sign-magnitude system negates a number by changing its sign bit, the signed-complement system negates a number by taking its complement. Positive numbers use same notation in sign-magnitude as well as sign-complement systems.

The signed-complement system can be used either as the 1's complement or the 2's complement.

But 2's complement is the most common.
+24 in 1's/2's complement representation is 011000
–24 in 1's complement representation 100111
–24 in 2's complement representation 101000

Table 2 *Signed binary numbers – (4-bits)*

Decimal	Signed-Magnitude	Signed 1's Complement	Signed 2's Complement
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
–0	1000	1111	–
–1	1001	1110	1111
–2	1010	1101	1110
–3	1011	1100	1101
–4	1100	1011	1100
–5	1101	1010	1011
–6	1110	1001	1010
–7	1111	1000	1001
–8	–	–	1000

The ranges of signed binary numbers with n-bits

Signed-magnitude: $-2^{n-1} + 1$ to $+2^{n-1} - 1$
1's complement representation: $-2^{n-1} + 1$ to $+2^{n-1} - 1$
2's complement representation: -2^{n-1} to $+2^{n-1} - 1$

Signed 2's complement representation can be directly used for arithmetic operations. The carryout of the sign bit position is discarded.

In order to obtain a correct answer, we must ensure that the result has a sufficient number of bits to accommodate the sum/product.

Example 38: $X = 00110$, $Y = 11100$ are represented in 5-bit signed 2's complement system

Then their sum $X + Y$ in 6-bit signed 2's complemented representation is?

Solution: $X = 00110$
$Y = 11100$
are 5-bit numbers
But result needs to be in 6-bit format.
Operands X and Y also should be in 6-bit format

$$X = 000110$$
$$Y = \underline{111100}$$
$$X + Y = (1)\ 000010$$

The carry out of sign bit position is discarded result is 000010.

Example 39: $(36x\ 70)_{10}$ is 10's complement of $(yzyz0)_{10}$ Then values of x, y, z are?
(A) 4, 5, 2
(B) 4, 6, 3
(C) 3, 6, 3
(D) 3, 5, 4

Solution: (C)
$(36x70)_{10}$ is 10's complement of $(yzyz0)_{10}$.
10's complement of $(yzyz0)_{10}$ is

$$10^5 - yzyz0 = 36 \times 70$$

So $36x70 + yzyz0 = 100000$

$$\begin{array}{r} 36x70 \\ +\underline{yzyz0} \\ 100000 \end{array}$$

so $7 + z = 10$,
$1 + x + y = 10 \quad z = 3$
$1 + 6 + z = 10 \quad y = 6$
$1 + 3 + y = 10$,
$\rightarrow x = 3$

Example 40: The 10's complement of $(843)_{11}$ is?
(A) $(157)_{11}$
(B) $(267)_{11}$
(C) $(156)_{11}$
(D) $(268)_{11}$

Solution: (B)
Given $(843)_{11}$ is base 11 number system and the number in the number system range from 0 to 9 & A ($A = 10$)
10's complement for $(843)_{11}$ means $(r - 1)$'s complement.
So $(r^n - 1) - N = [(11)^n - 1] - N$

$$\begin{array}{r} (11)^n - 1 \Rightarrow 1000 \\ \underline{-\quad 1} \\ AAA \\ \underline{-843} \\ 267 \end{array}$$

10's complement is $(267)_{11}$

Example 41: Consider the signed binary number to be 10111011. What is the decimal equivalent of this number if it is in Sign-Magnitude form, or 1's complement form, or 2's complement form?

Solution: Given binary number = 10111011. As sign bit is 1, it is a negative number. If it is in sign-magnitude format, then MSB is sign bit, and remaining bits represent the magnitude,

$(0111011)_2 = 32 + 16 + 8 + 2 + 1 = 59$. So if the given number is in sign-magnitude format, then the number is –59.

If it is in 1's complement/2's complement form, then the magnitude of negative number can be obtained by taking 1's complement/2's complement for the number, respectively.

10111011 ⇒ 1's complement ⇒ 01000100 = $(68)_{10}$.
In 1's complement format, the number is –68.
10111011 ⇒ 2's complement ⇒ 01000101 = $(69)_{10}$.
In 2's complement format, the number is –69.

Example: Find $(-9.625)_{10}$ in signed 2's complement representation.

Signed binary fraction can be represented in the same way of signed integer.

$$\begin{array}{r|l} 2 & 9 \\ 2 & 4-1 \\ 2 & 2-0 \\ & 1-0 \end{array}$$

$0.625 \times 2 = 1.25$
$0.25 \times 2 = 0.5$
$0.5 \times 2 = 1.0$
$= 0.101$
$+(9.625) = 01001.101$
$-9.625 = 10110.011$ (by taking 2's complement)

Binary Multipliers

Multiplication of binary number is done in the same way as multiplication of decimal.

The multiplicand (m) is multiplied by each bit of the multiplier (N), starting from the LSB.

Let
$M = B_3\ B_2\ B_1\ B_0$
$N = A_3\ A_2\ A_1\ A_0$
If $M \times N = P$

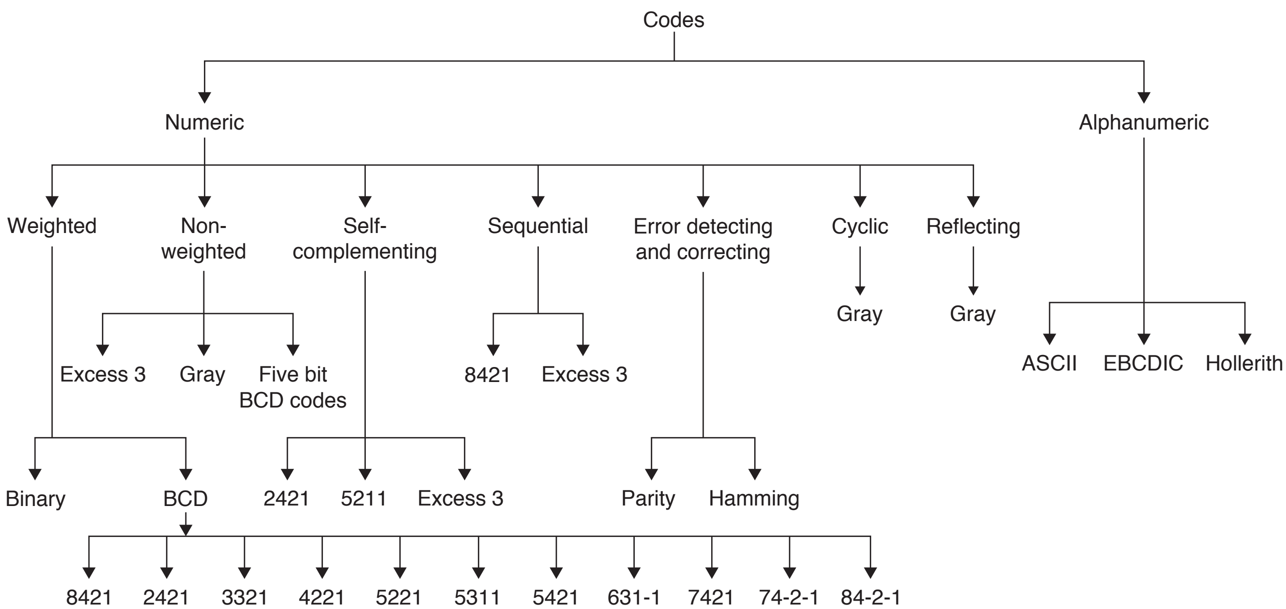
			A_0B_3	A_0B_2	A_0B_1	A_0B_0	
		A_1B_3	A_1B_2	A_1B_1	A_1B_0		
	A_2B_3	A_2B_2	A_2B_1	A_2B_0			
A_3B_3	A_3B_2	A_3B_1	A_3B_0				
P_7P_6	P_5	P_4	P_3	P_2	P_1	P_0	$=P$

Example: Let $M = 1\ 0\ 1\ 1$
$N = 1\ 1\ 0\ 0$
$M \times N = P$

$$\begin{array}{r} 1\,0\,1\,1 \times \\ \underline{1\,1\,0\,0} \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \\ 1\,0\,1\,1 \\ 1\,0\,1\,1 \\ \underline{1\,1\,1} \\ 1\,0\,0\,0\,0\,1\,0\,0 = P \end{array}$$

Binary Codes

Binary codes can be classified as numeric codes and alphanumeric codes. The figure below shows the classification of codes.



Numeric Codes

Numeric Codes are the codes which represent numericals in binary, i.e., only numbers as a series of 0s and 1s.

Weighted and non-weighted codes

- The weighted codes are those which obey the position-weighting principle. Each position of a number represents a specific weight.
  Example: BCD, Binary, 84-2-1, 2421,
- Non-weighted codes are codes which are not assigned fixed values.
  Example: Excess-3, Gray code
  2421, 5211, 84 – 2 – 1 are examples of weighted codes, in which weight is assigned to each position in the number.
  $(27)_{10}$ in 2421 code $\rightarrow$ 0010 1101
  $(45)_{10}$ in 5211 code $\rightarrow$ 0111 1000
  $(36)_{10}$ in 84 – 2 – 1 code $\rightarrow$ 0101 1010
  Any digit in decimal will be represented by the weights represented by the code.

Error-detecting and correcting codes

Codes which allow only error detection are error-detecting codes.

Example: Parity

Codes which allow error detection as well as correction are called error correcting codes.

Example: Hamming codes

Sequential codes

A sequential code is one in which each succeeding code word is one binary number greater than the preceding code word.
Example: XS–3, BCD

Cyclic codes (unit distance codes)

Cyclic codes are those in which each successive code word differs from the preceding one in only one bit position.
Example: Gray code

Reflective codes

Binary code in which the n least significant bits for code words 2^n through $2^{n+1} - 1$ are the mirror images of than for 0 through $2^n - 1$ is called reflective codes.
Example: Gray Code

Self-complementing codes

A code is said to be self-complementing, if the code word of the 9's complement of number 'N', i.e., of "9-N" can be obtained from the code word of 'N' by interchanging all the zeros and ones, i.e., by taking 1's complement. In other words, logical complement of number code is equivalent to representation of its arithmetic complement.

Example: 84-2-1, 2421, XS -3.
All weighted BCD codes are self-complementing codes.

Binary-coded decimal (BCD)

In BCD, each decimal digit 0 to 9 is coded by a 4-bit binary number. BCD codes are convenient to convert to/or from decimal number system.

Decimal	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Example 42: $(628)_{10} = (0110\ 0010\ 1000)_{BCD}$

BCD addition

- BCD addition is performed by individually adding the corresponding digits of the decimal number expressed in 4-bit binary groups starting from the LSB.
- If there is no carry and the sum term is not an illegal code, no correction is needed.
- If there is a carry out of one group to the next group or if the sum term is an illegal code, the $(6)_{10}$ is added to the sum term of that group, and the resulting carry is added to the next group.

Example 43: 44 + 12

0100	0100	(44 in BCD)
0001	0010	(12 in BCD)
0101	0110	(56 in BCD)

Example 44: 76.9+ 56.6

	0111	0110 . 1001		
	0101	0110 . 0110		
	1100	1100 . 1111		(all are illegal codes)
	0110	0110 . 0110		
	0010	0010 . 0101		
	+1	+1	+1	(propagate carry)
0001	0011	0011 . 0101		
1	3	3 . 5		

BCD subtraction BCD subtraction is performed by subtracting the digits of each 4-bit group of the subtrahend from the corresponding 4-bit group of the minuend in binary starting from the LSB.

Example 45:

42	0100	0010	(42 in BCD)
–12	–0001	0010	(12 IN BCD)
30	0011	0000	(No borrow, so this is the correct difference)

Example 46:

247.7	0010	0100	0111 .	0111	(Borrow are present, subtract 0110)
–156.9	0001	0101	0110 .	1001	
90.8	0000	0111	·0000 .	1110	
		–01001	–0110		
		1001	000 ·	1000	Corrected difference (90.8)

Excess-3 (XS-3) code

Excess-3 code is a non-weighted BCD code, where each digit binary code word is the corresponding 8421 code word plus 0011.

Find the XS-3 code of

Example 47: $(3)_{10} \rightarrow (0011)_{BCD} = (0110)_{xS3}$

Example 48: $(16)_{10} \rightarrow (0001\ 0110)_{BCD}$
$\rightarrow (0100\ 1001)_{xS3}$

Gray code

Each gray code number differs from the preceding number by a single bit.

Decimal	Gray Code
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111

Binary to gray conversion

Step I: Shift the binary number one position to the right, LSB of the shifted number is discarded.

Step II: Exclusive or the bits of the binary number with those of the binary number shifted.

Example 49: Convert $(1001)_2$ to gray code

Binary $\rightarrow$ 1010
Shifted Binary $\rightarrow$ 101 $\oplus$
Gray $\rightarrow$ 1111

Gray to binary conversion

(a) Take the MSB of the binary number is same as MSB of gray code number.
(b) X-OR the MSB of the binary to the next significant bit of the gray code.
(c) X-OR the 2nd bit of binary to the 3rd bit of Gray code to get 3rd bit binary and so on.
(d) Continue this till all the gray bits are exhausted.

Example 50: Convert, gray code 1010 to binary

Gray	1		0		1		0
1010	$\Downarrow$	$\oplus$	$\parallel$	$\oplus$	$\parallel$	$\oplus$	$\parallel$
1100	1		1		0		0

$= (1100)_2$

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Assuming all the numbers are in 2's complement representation, which of the following is divisible by 11110110?
(A) 11101010 (B) 11100010
(C) 11111010 (D) 11100111

2. If $(84)_x$ (in base x number system) is equal to $(64)_y$ (in base y number system), then possible values of x and y are
(A) 12, 9 (B) 6, 8
(C) 9, 12 (D) 12, 18

3. Let A = 1111 1011 and B = 0000 1011 be two 8-bit signed 2's complement numbers. Their product in 2's complement representation is

(A) 11001001 (B) 10011100
(C) 11010101 (D) 10101101

4. Let r denotes number system's radix. The only value(s) of r that satisfy the equation $\sqrt[3]{(1331)_r} = (11)_r$ is/are
(A) 10 (B) 11
(C) 10 and 11 (D) any $r > 3$

5. X is 16-bit signed number. The 2's complement representation of X is $(F76A)_{16}$. The 2's complement representation of $8 \times X$ is
(A) $(1460)_{16}$ (B) $(D643)_{16}$
(C) $(4460)_{16}$ (D) $(BB50)_{16}$

6. The HEX number $(CD.EF)_{16}$ in octal number system is
(A) $(315.736)_8$ (B) $(513.637)_8$
(C) $(135.673)_8$ (D) $(531.367)_8$

7. 8-bit 2's complement representation a decimal number is 10000000. The number in decimal is
(A) +256 (B) 0
(C) −128 (D) −256

8. The range of signed decimal numbers that can be represented by 7-bit 1's complement representation is
(A) −64 to + 63 (B) −63 to + 63
(C) −127 to + 128 (D) −128 to +127

9. Decimal 54 in hexadecimal and BCD number system is respectively
(A) 63, 10000111 (B) 36,01010100
(C) 66, 01010100 (D) 36, 00110110

10. A new binary-coded hextary (BCH) number system is proposed in which every digit of a base −6 number system is represented by its corresponding 3-bit binary code. For example, the base −6 number 35 will be represented by its BCH code 011101.

In this numbering system, the BCH code 001001101011 corresponds to the following number in base −6 system.
(A) 4651 (B) 4562
(C) 1153 (D) 1353

11. The signed 2's complement representation of $(-589)_{10}$ in Hexadecimal number system is
(A) $(F24D)_{16}$ (B) $(FDB3)_{16}$
(C) $(F42D)_{16}$ (D) $(F3BD)_{16}$

12. The base of the number system for which the following operation is to be correct $\frac{66}{5} = 13$
(A) 6 (B) 7
(C) 8 (D) 9

13. The solution to the quadratic equation $x^2 - 11x + 13 = 0$ (in number system with radix r) are $x = 2$ and $x = 4$. Then base of the number system is (r) =
(A) 7 (B) 6
(C) 5 (D) 4

14. The 16's complement of BADA is
(A) 4525 (B) 4526
(C) ADAB (D) 2141

15. $(11A1B)_8 = (12CD)_{16}$, in the above expression A and B represent positive digits in octal number system and C and D have their original meaning in Hexadecimal, the values of A and B are?
(A) 2, 5 (B) 2, 3
(C) 3, 2 (D) 3, 5

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. The hexadecimal representation of $(567)_8$ is
(A) 1AF (B) D77
(C) 177 (D) 133

2. $(2326)_8$ is equivalent to
(A) $(14D6)_{16}$ (B) $(103112)_4$
(C) $(1283)_{10}$ (D) $(09AC)_{16}$

3. $(0.46)_8$ equivalent in decimal is?
(A) 0.59375 (B) 0.3534
(C) 0.57395 (D) 0.3435

4. The 15's complement of $(CAFA)_{16}$ is
(A) $(2051)_{16}$ (B) $(2050)_{16}$
(C) $(3506)_{16}$ (D) $(3505)_{16}$

5. 53 in 2's complement from is?
(A) 1001011 (B) 001010
(C) 0110101 (D) 001011

6. Signed 2's complement representation of $(-15)_{10}$ is
(A) 11111 (B) 10001
(C) 01111 (D) 10000

7. $(0.25)_{10}$ in binary number system is?
(A) (0.01) (B) (0.11)
(C) 0.001 (D) 0.101

8. The equivalent of $(25)_6$ in number system with base 7 is?
(A) 22 (B) 23
(C) 24 (D) 26

9. The operation 35 + 26 = 63 is true in number system with radix
(A) 7 (B) 8
(C) 9 (D) 11

10. The hexadecimal equivalent of largest binary number with 14-bits is?
(A) 2FFF (B) 3FFFF
(C) FFFF (D) 1FFFF

11. If x is radix of number system, $(211)_x = (152)_8$, then x is
(A) 6 (B) 7
(C) 9 (D) 5

12. The value of r for which $\sqrt{(224)_r} = (13)_r$ is valid expression, in number system with radix r is?
(A) 5 (B) 6
(C) 7 (D) 8

13. Which of the representation in binary arithmetic has a unique zero?
(A) sign-magnitude (B) 1's compliment
(C) 2's complement (D) All of these

14. For the binary number 101101111 the equivalent hexadecimal number is
(A) 14E (B) 9E
(C) B78 (D) 16F

15. Subtract 1001 from 1110
(A) 0010 (B) 0101
(C) 1011 (D) 1010

16. Which of the following is a positively weighted code?
(A) 8421 (B) 84-2-1
(C) EXS-3 (D) 74-2-1

17. Match the items correctly

Column 1	Column 2
(P) 8421	(1) Cyclic code
(Q) 2421	(2) self-complementing
(R) 5212	(3) sequential code
(S) Gray code	(4) non-sequential code

(A) P–2, Q–4, R–3, S–1
(B) P–1, Q–4, R–3, S–2
(C) P–3, Q–2, R–4, S–1
(D) P–2, Q–4, R–1, S–2

18. Perform the subtraction in XS-3 code 57.6 – 27.8
(A) 0101 1100.1011 (B) 0010 1001.1100
(C) 00011101.1100 (D) 1010 1110.1011

19. The 2's complement representation of –17 is
(A) 101110 (B) 111110
(C) 101111 (D) 110001

20. The decimal 398 is represented in 2421 code by
(A) 110000001000 (B) 001110011000
(C) 001111111110 (D) 010110110010

Previous Years' Questions

1. $(1217)_8$ is equivalent to **[2009]**
(A) $(1217)_{16}$ (B) $(028F)_{16}$
(C) $(2297)_{10}$ (D) $(0B17)_{16}$

2. P is a 16-bit signed integer. The 2's complement representation of P is $(F87B)_{16}$. The 2's complement representation of $8*P$ is **[2010]**
(A) $(C3D8)_{16}$ (B) $(187B)_{16}$
(C) $(F878)_{16}$ (D) $(987B)_{16}$

3. The smallest integer that can be represented by an 8-bit number in 2's complement form is **[2013]**
(A) –256 (B) –128
(C) –127 (D) 0

4. The base (or radix) of the number system such that the following equation holds is ______ $\frac{312}{20} = 13.1$ **[2014]**

5. Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is ______. **[2014]**

6. Consider the equation $(43)_x = (y3)_8$ where x and y are unknown. The number of possible solutions is ______ **[2015]**

7. Suppose X_i for i = 1, 2, 3 are independent and identically distributed random variables whose probability mass functions are $Pr[X_i = 0] = Pr[X_i = 1] = ½$ for i = 1, 2, 3. Define another random variable $Y = X_1 X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then
$Pr[Y = 0|X_3 = 0]$ = ______ **[2015]**

8. The 16-bit 2's complement representation of an integer is 1111 1111 1111 0101; its decimal representation is ____ . **[2016]**

9. Consider an eight - bit ripple - carry adder for computing the sum of A and B, where A and B are integers represented in 2's complement form. If the decimal value of A is one, the decimal value of B that leads to the longest latency for the sum to stabilize is _____ . **[2016]**

10. Let $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$ where x_1, x_2, x_3, x_4 are Boolean variables, and $\oplus$ is the XOR operator. Which one of the following must always be **TRUE**? **[2016]**
(A) $x_1 x_2 x_3 x_4 = 0$
(B) $x_1 x_3 + x_2 = 0$
(C) $\bar{x}_1 \oplus \bar{x}_3 = \bar{x}_3 \oplus \bar{x}_4$
(D) $x_1 + x_2 + x_3 + x_4 = 0$

11. Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base b. The solutions of this equation in the same base b are $x = 5$ and $x = 6$. Then b = ______. **[2017]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** C	**3.** A	**4.** D	**5.** D	**6.** A	**7.** C	**8.** B	**9.** B	**10.** C
11. B	**12.** D	**13.** C	**14.** B	**15.** D					

Practice Problems 2

1. C	**2.** B	**3.** A	**4.** D	**5.** D	**6.** B	**7.** A	**8.** B	**9.** B	**10.** B
11. B	**12.** A	**13.** C	**14.** D	**15.** B	**16.** A	**17.** C	**18.** A	**19.** C	**20.** C

Previous Years' Questions

1. B	**2.** A	**3.** B	**4.** 5	**5.** 3	**6.** 5	**7.** 0.75	**8.** –11	**9.** –1.0	**10.** C
11. 8.0 to 8.0									

Chapter 2

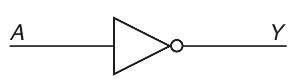
Boolean Algebra and Minimization of Functions

LEARNING OBJECTIVES

- *Logic gates*
- *Boolean algebra*
- *AXIOMS and Laws of Boolean algebra*
- *Properties of Boolean algebra*
- *Conversion from Min term to Max term*
- *Minimization of Boolean function*
- *K-map method*
- *Prime implicant*
- *Implementation of function by using NAND-NOR Gates*
- *EX-OR, EX-NOR GATE*

Logic Gates

1. **Inverter or NOT gate (7404 IC):** The inverter performs a basic logic operation called inversion or complementation. The purpose of the inverter is to change one logic level to the opposite level. In terms of digital circuits, it converts 1 to 0 and 0 to 1.



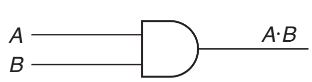
Symbol

$Y = \overline{A}$

Table 1 *Truth Table*

Input	Output
A	***Y***
0	1
1	0

2. **AND gate (logical multiplier 7408 IC):** The AND gate performs logical multiplication more commonly know as AND function. The AND gate is composed of 2 or more inputs and a single output



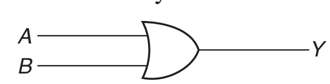
$Y = A \cdot B$

Figure 1 2 input AND gate

Table 2 *Truth Table*

Input		Output
A	***B***	***Y***
0	0	0
0	1	0
1	0	0
1	1	1

3. **OR gate (logical adder 7432 IC):** The OR gate performs logical, addition commonly known as OR function.



Symbol

$Y = A + B$

Figure 2 2 input OR gate

Table 3 *Truth Table*

Input		Output
A	***B***	***Y***
0	0	0
0	1	1
1	0	1
1	1	1

4. **NAND gate (7400 IC):** The NAND gate's function is basically AND + NOT function.

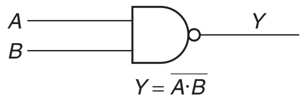


Figure 3 2 input NAND gate

Table 4 *Truth Table*

Input			Output
A	B	$A \cdot B$	$\overline{A+B}$ (Y)
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

5. **NOR gate (7402 IC):** The NOR gate is basically OR + NOT function.

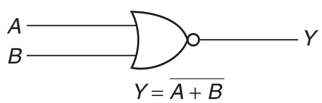


Figure 4 2 input NOR gate

Table 5 *Truth Table*

Input			Output
A	B	$A + B$	$\overline{A+B}$ (Y)
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

6. **Exclusive OR gate X-OR (7486 IC):** X-OR is a gate in which unequal inputs create a high logic level output and if both inputs are equal, the output will be low. Other name for EX-OR gate is unequivalent gate.
2 input X-OR Gate

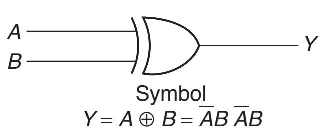


Figure 5 2 input X-OR Gate

Table 6 *Truth Table*

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

7. **Exclusive NOR gate (X-NOR):** X-NOR is a gate in which equal inputs create a high logic level output; and if both inputs are unequal, then the output will be low. Other name for X-NOR gate is equivalent gate.

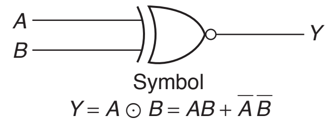


Figure 6 2 input X-NOR Gate

Table 7 *Truth Table*

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

X-NOR Gate is complement of X-OR Gate.

Boolean Algebra

Boolean algebra is a system of mathematical logic. It is an algebraic system consisting of the set of elements (0, 1), two binary operators OR and AND and one unary operator NOT. The Boolean algebra is governed by certain well-developed rules and laws.

AXIOMS and Laws of Boolean Algebra

1. **AXIOMS**
   - **(a) AND operation**
     - (1) $0 \cdot 0 = 0$
     - (2) $0 \cdot 1 = 0$
     - (3) $1 \cdot 0 = 0$
     - (4) $1 \cdot 1 = 1$
   - **(b) OR operation**
     - (5) $0 + 0 = 0$
     - (6) $0 + 1 = 1$
     - (7) $1 + 0 = 1$
     - (8) $1 + 1 = 1$
   - **(c) NOT operation**
     - (9) $\overline{1} = 0$
     - (10) $\overline{0} = 1$
2. **Laws**
   - **(a) Complementation law**
     - (1) $\overline{0} = 1$
     - (2) $\overline{1} = 0$
     - (3) If $A = 0$, then $\overline{A} = 1$
     - (4) If $A = 1$, then $\overline{A} = 0$
     - (5) $\overline{\overline{A}} = A$
   - **(b) AND laws**
     - (1) $A \cdot 0 = 0$ (NULL Law)
     - (4) $A \cdot 1 = A$ (Identity Law)
     - (3) $A \cdot A = A$
     - (4) $A \cdot \overline{A} = 0$

(c) OR laws

(1) $A + 0 = A$ (NULL Law)

(2) $A + 1 = 1$ (Identity Law)

(3) $A + A = A$

(4) $A + \overline{A} = 1$

(d) Commutative laws

(1) $A + B = B + A$

(2) $A \cdot B = B \cdot A$

(e) Associative laws

(1) $(A + B) + C = A + (B + C)$

(2) $(A \cdot B)C = A(B \cdot C)$

(f) Distributive laws

(1) $A(B + C) = AB + AC$

(2) $A + BC = (A + B)(A + C)$

(g) Redundant literal rule (RLR)

(1) $A + \overline{A}B = A + B$

(2) $A(\overline{A} + B) = AB$

(h) Idempotence laws

(1) $A \cdot A = A$

(2) $A + A = A$

(i) Absorption laws

(1) $A + A \cdot B = A$

(2) $A(A + B) = A$

3. Theorems

(a) Consensus theorem

Theorem 1:

$$AB + \overline{A}C + BC = AB + \overline{A}C$$

Proof:

$$\begin{aligned} \text{LHS} &= AB + \overline{A}C + BC \\ &= AB + \overline{A}C + BC(A + \overline{A}) \\ &= AB + \overline{A}C + BCA + BC\overline{A} \\ &= AB(1 + C) + \overline{A}C(1 + B) \\ &= AB(1) + \overline{A}C(1) \\ &= AB + \overline{A}C \\ &= \text{RHS.} \end{aligned}$$

Theorem 2:

$$(A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C)$$

Proof:

$$\begin{aligned} \text{LHS} &= (A + B)(\overline{A} + C)(B + C) \\ &= (A\overline{A} + AC + B\overline{A} + BC)(B + C) \\ &= (AC + BC + \overline{A}B)(B + C) \\ &= ABC + BC + \overline{A}B + AC + BC + \overline{A}BC \\ &= AC + BC + \overline{A}B \end{aligned}$$

$$\begin{aligned} \text{RHS} &= (A + B)(\overline{A} + C) \\ &= A\overline{A} + AC + BC + \overline{A}B \\ &= AC + BC + \overline{A}B \\ &= \text{LHS.} \end{aligned}$$

(b) Transposition theorem

$$AB + \overline{A}C = (A + C)(\overline{A} + B)$$

Proof:

$$\begin{aligned} \text{RHS} &= (A + C)(\overline{A} + B) \\ &= A\overline{A} + C\overline{A} + AB + CB \\ &= 0 + \overline{A}C + AB + BC \\ &= \overline{A}C + AB + BC(A + \overline{A}) \\ &= AB + ABC + \overline{A}C + \overline{A}BC \\ &= AB + \overline{A}C \\ &= \text{LHS} \end{aligned}$$

(c) De Morgan's theorem

Law 1: $\overline{A + B} = \overline{A} \cdot \overline{B}$

This law states that the complement of a sum of variable is equal to the product of their individual complements.

Law 2: $\overline{AB} = \overline{A} + \overline{B}$

This law states that the complement of the product of variables is equal to the sum of their individual complements.

Example 1: Simplify the Boolean function $Y = A(A + \overline{B})$

$$Y = A \cdot A + A \cdot \overline{B}$$

Solution:

$$\begin{aligned} Y &= A + A\overline{B} \\ &= A(1 + \overline{B}) \\ &= A \end{aligned}$$

Example 2: Simplify the Boolean function $Y = A + \overline{A}B$

Solution:

$$\begin{aligned} Y &= A \cdot (B + 1) + \overline{A} \cdot B \\ &= A \cdot B + A + \overline{A}B \\ &= B(A + \overline{A}) + A \\ &= A + B \end{aligned}$$

Example 3: Simplify the Boolean function

$$Y = A(A + B) + B(\overline{A} + B)$$

Solution:

$$\begin{aligned} Y &= A \cdot A + A \cdot B + B \cdot \overline{A} + B \cdot B \\ &= A + B(A + \overline{A}) + B \\ &= A + B \cdot 1 + B \\ &= A + B + B \\ &= A + B \end{aligned}$$

Example 4: Simplify the Boolean function

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + AB\overline{C} + A\overline{B}\,\overline{C}$$

Solution:

$$\begin{aligned} Y &= \overline{A}\,\overline{C}(\overline{B} + B) + A\overline{C}(B + \overline{B}) \\ &= \overline{A}\,\overline{C} + A\overline{C} \\ &= \overline{C}(\overline{A} + A) \\ &= \overline{C} \end{aligned}$$

Example 5: Simplify the Boolean function

$$Y = \overline{\overline{A}BC + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C}$$

Solution:

$$\begin{aligned} &= \overline{\overline{A}C(B + \overline{B}) + \overline{A}B\overline{C}} \\ &= \overline{\overline{A}C + \overline{A}B\overline{C}} \\ &= \overline{\overline{A}(C + B\overline{C})} \\ &= \overline{\overline{A}(C + B)} \\ &= A + \overline{C}\,\overline{B} \end{aligned}$$

Example 6: Simplify the Boolean function

$$Y = AB + C\overline{B} + CA + ABD$$

Solution: $Y = AB(1+D) + C\overline{B} + CA$

$$= AB + C\overline{B} + CA$$
$$= AB + C\overline{B}$$

Properties of Boolean Algebra

With n variables, maximum possible distinct functions $= 2^{2^n}$.

Duality consider the distributive law

1. $x(y+z) = xy + xz$
2. $x + yz = (x+y)(x+z)$

Second one can be obtained from the first law if the binary operators and the identity elements are interchanged. This important property of Boolean algebra is called the duality principle.

The dual of an algebraic expression can be written by interchanging OR and AND operators, 1s by 0, and 0s by 1s.

Example 7: $x + x' = 1 \xleftrightarrow{\text{Dual}} x \cdot x' = 0$

Solution: $xy + xy' = x \xleftrightarrow{\text{Dual}} (x+y)(x+y') = x$

$$x + x'y = x + y \xleftrightarrow{\text{Dual}} x(x'+y) = xy$$

Example 8: The dual of $F = xy + xz + yz$ is?

Solution: Dual of $F = (x+y)(x+z)(y+z)$

$$= (x + xz + xy + yz)(y+z) = xy + yz + xz$$

So dual of $xy + xz + yz$ is same as the function itself; For N variables maximum possible self-dual functions $= 2^{2^{n-1}} = 2^{(2^n/2)}$

Example 9: Which of the following statement/*s* is/are true

S_1: The dual of NAND function is NOR

S_2: The dual of X-OR function is X-NOR

(A) S_1 and S_2 are true
(B) S_1 is true
(C) S_2 is true
(D) None of these

Solution: (A)

NAND $= (xy)' = x' + y'$

Dual of NAND $= (x+y)' = x'y'$

X-OR $= xy' + x'y$

Dual of X-OR $= (x+y')(x'+y) = xy + x'y' =$ X-NOR

Both S_1 and S_2 are true

Operator precedence The operator precedence for evaluating Boolean expression is

1. Parentheses
2. NOT
3. AND
4. OR

So the expression inside the parentheses must be evaluated before all the operations. The next operation to be performed is the complement and then follows AND and finally the OR.

Complement of function The complement of a function F is F' is obtained from an interchange of 0s for 1s and 1s for 0s in the value of F. The complements of a function may be derived algebraically through De Morgan's theorems.

$$(x_1 \,.\, x_2 \,.\, x_3 \ldots x_n)' = x_1' + x_2' + x_3' + \cdots + x_n'$$
$$(x_1 + x_2 + x_3 + \cdots + x_n)' = x'_1 \,.\, x'_2 \,.\, x'_3 \,.\, x'_4 \ldots x'_n$$

Example 10: The complement of function $F = a(b'c + bc')$ is?

Solution: $(F)' = [a(b'c + bc')]'$

$$= a' + (b'c + bc')'$$
$$= a' + (b'c)' \cdot (bc')'$$
$$= a' + (b + c)'(b' + c)$$
$$F' = a' + bc + b'c'$$

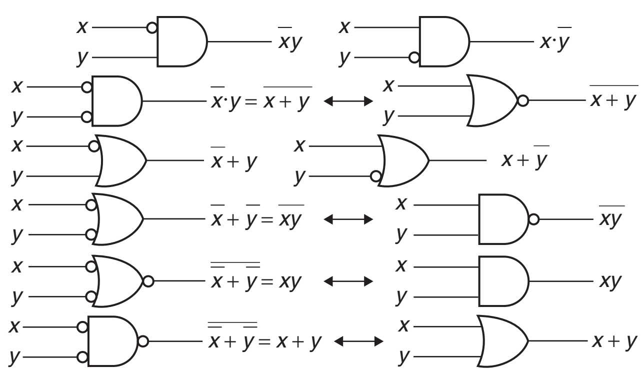


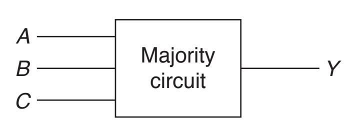
Figure 7 Gates with inverted inputs

Boolean Functions, Min Terms and Max Terms

The starting point for designing most logic circuits is the truth table, which can be derived from the statement of problem. The truth table is then converted into a Boolean expression and finally create the assembly of logic gates accordingly.

Let us consider the example of majority circuit. This circuit takes three inputs (A, B, C) and have one output (Y) which will give the majority of the inputs, i.e., if A, B, C are having more number of zeros. $Y = 0$ else if A, B, C are having more number of 1s, $Y = 1$.

So from the statements we can derive the truth table as follows:



As we are using three Boolean variables A, B, C, total number of combinations in truth table are $2^3 = 8$.

Similarly for n variables, the truth table will have total of 2^n combinations, for a Boolean function.

	Input			Output	
Sl. no.	*A*	*B*	*C*	*Y*	
1	0	0	0	0	$\longrightarrow$ $Y = 0$, If inputs are having more zeros.
2	0	0	1	0	
3	0	1	0	0	
4	0	1	1	1	$\longrightarrow$ $Y = 1$, If inputs are having more 1's
5	1	0	0	0	
6	1	0	1	1	
7	1	1	0	1	
8	1	1	1	1	

For some combinations, output $Y = 1$, and for others $Y = 0$. The input combinations for which output $Y = 1$ are called as min terms.

Similarly the input combinations for which output $Y = 0$ are called as max terms.

Min terms are expressed as product terms, Similarly, max terms are expressed as sum terms.

The output $Y = 1$, only in rows 4, 6, 7, 8.

So the min terms combinations are 011, 101, 110, 111 in Boolean Algebra, 1 input will be written as A, B, C and 0 input will be written as $\overline{A}, \overline{B}, \overline{C}$ in complement form, we express these min terms as product terms, $\overline{A}BC, A\overline{B}C, AB\overline{C}, ABC$.

To express Y as Boolean expression, we can write it as sum of the min terms.

$$Y = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

We know that AND operation is a product while OR is sum. So the above equation is a sum of the products (SOP), (or) min terms expression.

The other way of expressing Y is $Y = \Sigma m$ (3, 5, 6, 7).

$$Y = m_3 + m_5 + m_6 + m_7.$$

The min term numbers are the decimal equivalent of input binary combinations.

Similar to SOP we can have product of sums (POS) Boolean expression.

The output $Y = 0$ for the input combinations 000, 001, 010, 100. For max terms 1 input will be indicated as $\overline{A}, \overline{B}, \overline{C}$ in complement form, 0 input will be indicated as A, B, C and max terms are expressed as sum terms.

$$A + B + C,\ A + B + \overline{C},\ A + \overline{B} + C,\ \overline{A} + B + C$$

Any function can be expressed as product of max terms.

So $Y = (A + B + C)\,(A + B + \overline{C})\,(A + \overline{B} + C)\,(\overline{A} + B + C)$

The above equation is a product of sum expression (POS) or max terms expression.

In other way $Y = \pi M\,(0, 1, 2, 4)$

$$= M_0 \cdot M_1 \cdot M_2 \cdot M_4$$

The max term numbers are decimal equivalents of corresponding input binary combinations.

Min Term and Max Term

All the Boolean expressions can be expressed in a standard sum of product (SOP) form or in a standard product of sum (POS) form.

- A standard SOP form is one in which a number of product terms, each contains all the variables of the function either in complement or non-complement form are summed together.
- A standard POS form is one in which a number of sum terms, each one of which contain all the variable of he function either in complemented or non-complemented form are multiplied together.
- Each of the product term in standard SOP form is called a min term.
- Each of the sum term in the standard POS form is called a max term.

Conversion from min terms to max terms representation

$$Y = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$Y' = (\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC)'$$

$$= (\overline{A}BC)'(A\overline{B}C)'(AB\overline{C})'(ABC)'$$

$$(Y')' = [(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})]'$$

$$= [\pi(3, 5, 6, 7)]^1 = \pi(0, 1, 2 ,4)$$

$$Y = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + C)$$

$$\text{or } Y = \Sigma(3, 5, 6, 7) = \pi(0, 1, 2, 4)$$

Conversion from normal SOP/POS form to canonical SOP/POS

Let us consider $f(A, B, C) = A + BC + \overline{A}C$

The above function is in normal (minimized) SOP from, to convert this function to standard SOP(or) canonical SOP form, include missing variable in each and every term, to make it complete. First term A, Missing literals are B, C. Consider $A\ X\ X$, so possible combinations are $\overline{A}BC, A\overline{B}C, AB\overline{C}, ABC$ or we can write

$$A = A = A(B + \overline{B})(C + \overline{C}) = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\overline{C}$$

Second term BC-missing literal is A. Consider $XBC \Rightarrow$ So possible combinations are ABC, $\overline{A}BC$ or we can write

$$BC = (A + \overline{A})BC$$

$$= ABC + \overline{A}BC$$

Third term $\overline{A}C$ = missing literal is B. Consider $\overline{A}XC \rightarrow$ so possible combinations are $\overline{A}BC, \overline{A}, \overline{B}C$ or we can write

$$\overline{A}C = \overline{A}(B + \overline{B})C$$

$$= \overline{A}BC + \overline{A}\overline{B}C$$

Now, $f(A,B,C) = ABC + AB\overline{C} + A\overline{B}C + \overline{A}BC + \overline{A}\overline{B}C$, after removing the redundant terms.

Now consider

$$f(A,B,C) = (A+B)\,(\overline{A}+C)$$

To convert this expression to canonical form or standard POS form we can write

$$f(A,B,C) = (A+B+C\cdot\overline{C})\,(\overline{A}+B\cdot\overline{B}+C)$$

Here the C variables is absent from first term and B from second term. So add $C\cdot\overline{C} = (0)$ to first, and $B\cdot\overline{B}$ to second, and using distributive law arrive at the result.

$$f(A,B,C) = (A+B+C)\,(A+B+\overline{C})\,(\overline{A}+B+C)(\overline{A}+\overline{B}+C)$$

Minimization of Boolean Functions

Simplification Procedure

- Obtain truth table, and write output in canonical form or standard form
- Generate K-map!
- Determine Prime implicants.
- Find minimal set of prime implicants.

Karnaugh Map (K-map) Method

Karnaugh map method is a systematic method of simplifying the Boolean expression. K-map is a chart or a graph composed of an arrangement of adjacent cell, each representing a particular combination of variable in sum or product form. (i.e., min term or max term).

Two-variable K-map

x	y	F
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

m_0	m_1
m_2	m_3

$x \backslash y$	0	1
0	$x'y'$	$x'y$
1	xy'	xy

Three-variable K-map

A three-variable map will have eight min terms (for three variables $2^3 = 8$) represented by 8 squares

x	y	z	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

$x \backslash yz$	00	01	11	10
0	$x'y'z$	$x'y'z$	$x'yz$	$x'yz'$
1	$xy'z'$	$xy'z$	xyz	xyz'

3-variable K-map

Four-variable K-maps

The K-map for four variables is shown here, 16 min terms are assigned to 16 squares.

$wx \backslash yz$	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

The map is considered to lie on a surface with the top and bottom edges as well as the right and left edge touching each other to form adjacent squares.

- One square $\rightarrow$ a min term of four literals
- Two adjacent square $\rightarrow$ a term of three literals
- Four adjacent square $\rightarrow$ a term of two literals
- Eight adjacent square $\rightarrow$ a term of one literal
- Sixteen adjacent square $\rightarrow$ The constant one

Don't-care Combinations

It can often occur that for certain input combinations, the value of the output is unspecified either because the input combination are invalid or because the precise value of the output is of no consequence. The combination for which the values of the expression are not specified are called don't-care combinations. During the process of design using an SOP, K-map, each don't-care is treated as a 1 if it is helpful in Map Reduction, otherwise it is treated as a 0 and left alone. During the process of design using a POS K-map, each Don't-care is treated as a 0 if it is useful in Map Reduction, otherwise it is treated as a 1 and left alone.

Example 11: Find the Minimal expression
$\Sigma m\,(1, 5, 6, 12, 13, 14) + d\,(2, 4)$

Solution:

$AB \backslash CD$	00	01	11	10
00		1		×
01	×	1		1
11	1	1		1
10				

$$\therefore \quad F = B\overline{C} + B\overline{D} + \overline{A}\,\overline{C}D$$

Pairs, Quads and Octets

A \ BC	00	01	11	10
0		1	1	
1				

The map contains a pair of 1s those are horizontally adjacent. Those cells represent $\bar{A}\,\bar{B}C, \bar{A}BC$.

For these two min terms, there is change in the form of variable B. By combining these two cells we can form a pair, which is equal to $\bar{A}\,\bar{B}C + \bar{A}BC = \bar{A}C(\bar{B} + B) = \bar{A}C$.

If more than one pair exists on K-map, OR the simplified products to get the Boolean function.

A \ BC	00	01	11	10
0	1			1
1		1	1	

$F = \bar{A}\,\bar{C} + AC$

AB \ CD	00	01	11	10
00	1	1	1	
01	1			
11				1
10	1	1		1

$F = \bar{A}\,\bar{C}\,\bar{D} + \bar{A}\,\bar{B}D + A\bar{B}\,\bar{C} + AC\bar{D}$

So Pair eliminates one variable by minimization.

Quad

Quad is a group of four 1s those are horizontally or vertically adjacent.

A \ BC	00	01	11	10
0		1	1	
1		1	1	

$F = C$

A \ BC	00	01	11	10
0		1	1	
1		1	1	

$F = \bar{A}C + AC = (\bar{A} + A)C = C$

By considering two pairs also it will be simplified to C.
Quad eliminates two variables from the function

AB \ CD	00	01	11	10
00		1	1	
01	1			1
11	1			1
10		1	1	

$F = B\bar{D} + \bar{B}D$

Corner min terms can from a Quad

PQ \ RS	00	01	11	10
00	1			1
01				
11				
10	1			1

$F = \bar{Q}\,\bar{S}$

Octet

The group of eight cells will form one octet.

XY \ ZW	00	01	11	10
00				
01	1	1	1	1
11	1	1	1	1
10				

$F = Y$

Other variable X, Z, W changes their form in octet. Octet can eliminate three variables and their complements.

AB \ CD	00	01	11	10
00	1			1
01	1			1
11	1			1
10	1			1

$F = \bar{D}$

Other variable A, B, C are vanished.

Eliminating Redundant Groups

A \ BC	00	01	11	10
0		1	1	
1			1	1

$AB + \bar{A}C + BC$

A \ BC	00	01	11	10
0		1	1	
1			1	1

$AB + \bar{A}C$

Here BC is redundant pair, which covers already covered min terms of $AB, \bar{A}C$.

PQ \ RS	00	01	11	10
00		1		
01		1	1	1
11	1	1	1	
10			1	

This K-map gives fourpairs and one quad.

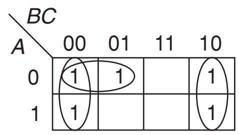
PQ \ RS	00	01	11	10
00		1		
01		1	1	1
11	1	1	1	
10			1	

But only four pairs are enough to cover all the min times, Quad is not necessary.

$\bar{P}\,\bar{R}S + \bar{P}QR + PQ\bar{R} + PRS$ is minimized function.

Prime Implicant

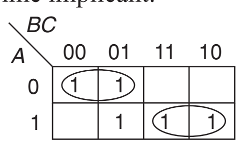
The group of adjacent min terms is called a Prime Implicant, i.e., all possible pairs, quads, octets, etc.



Prime implicants are $\bar{B}\bar{C}$, $B\bar{C}$, $\bar{C}$, $\bar{A}\bar{B}$. Minimized function is $\bar{C}+\bar{A}\bar{B}$

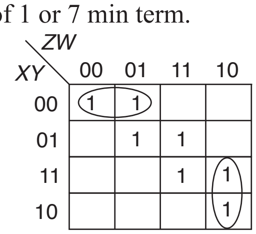
Essential Prime Implicant

The prime implicant which contains at least one min term which cannot be covered by any other prime implicant is called Essential prime implicant.



Min term 0, 6 can be grouped with only one pair each.

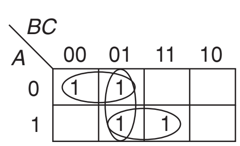
The total possible prime implicants are $\bar{A}\bar{B}$, $\bar{B}C$, AC, AB but min term 0, 6 can be covered with $\bar{A}\bar{B}$, AB. So we call them as essential prime implicants. Min term 5 can be paired with any of 1 or 7 min term.



The essential prime Implicants are $XZ\bar{W}$, $\bar{X}\bar{Y}\bar{Z}$

Redundant Prime Implicant

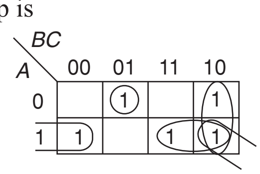
The prime implicant whose min terms are already covered by at least one min term is called redundant prime implicants.



Here prime implicants are $\bar{A}\bar{B}$, AC, $\bar{B}C$. But $\bar{B}C$ is already covered by other min terms So $\bar{B}C$ is redundant prime implicant.

Example 12: Find the minimal expression for $\Sigma m(1, 2, 4, 6, 7)$ and implement it using Basic gates.

Solution: K-map is



$$F = A\bar{C} + AB + B\bar{C} + \bar{B}C + \bar{A}\bar{B}C$$

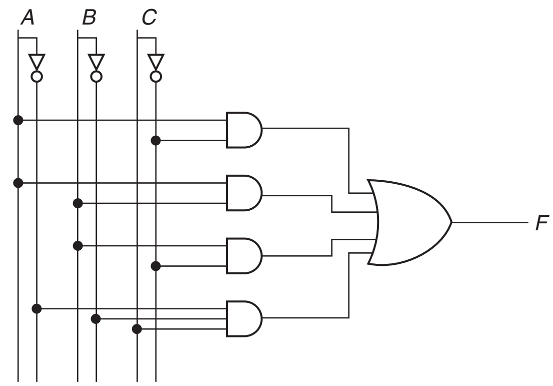
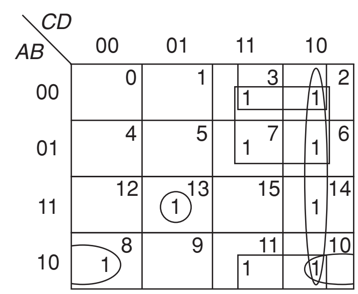


Figure 8 Logic diagram

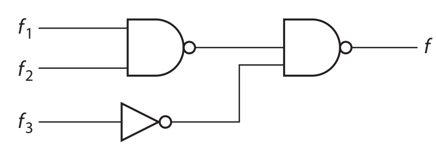
Example 13: Find the minimal expression for Σm (2, 3, 6, 7, 8, 10, 11, 13, 14)

Solution: K-map is:



$$\therefore \quad F(A,B,C,D) = AB\bar{C}D + A\bar{B}\bar{D} + \bar{A}C + \bar{B}C + C\bar{D}$$

Example 14: Three Boolean functions are defined as below $f_1 = \Sigma m(0, 1, 3, 5, 6)$, $f_2 = \Sigma m(4, 6, 7)$ $f_3 = \Sigma m(1, 4, 5, 7)$, then find f.



Solution: When two Boolean functions are ANDed, the resultant will contain the common min terms of both of the functions (like, intersection of min terms). If two Boolean functions are ORed, then resultant is the combination of all the min terms of the functions (like union of min terms)

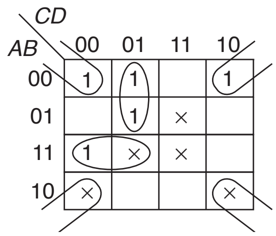
Here $f = \overline{\overline{f_1 f_2} \cdot \overline{f_3}} = f_1 f_2 + f_3$

Here $f_1 \cdot f_2$ = Common min terms in f_1 and $f_2 = \Sigma m(6)$

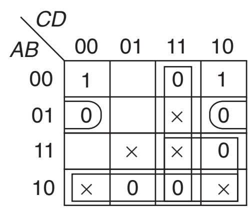
$f_1 \cdot f_2 + f_3$ = Combination of min terms of $f_1 \cdot f_2$ and f_3 $= \Sigma(1, 4, 5, 6, 7)$

Example 15: What is the literal count for the minimized SOP, and minimized POS form for the following function? $f(A, B, C, D) = \Sigma m(0, 1, 2, 5, 12) + \phi d(7, 8, 10, 13, 15)$

Solution: $f(A, B, C, D) = \Sigma m(0, 1, 2, 5, 12) + \phi(7, 8, 10, 13, 15)$



$f = 1$ quad + 2 pairs
Literal count $= 1 \times 2 + 2 \times 3 = 8$
$f(A, B, C, D) = \pi M(3, 4, 6, 9, 11, 14) + \phi(7, 8, 10, 13, 15)$



f will consists of 3 quads + 1pair
$= 3 \times 2 + 1 \times 3 = 6 + 3 = 9$

Implementation of Function by Using NAND–NOR Gates

NAND or NOR gates are called as universal gates, because any function can be implemented by using only NAND gates or only using NOR gates.

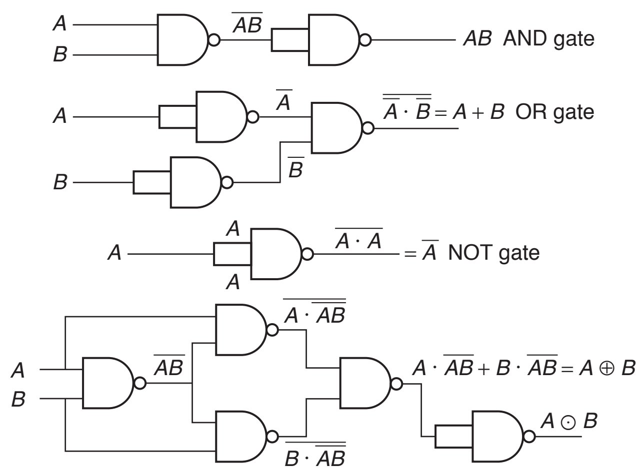


Figure 9 Implement of basics gates by using NAND gates

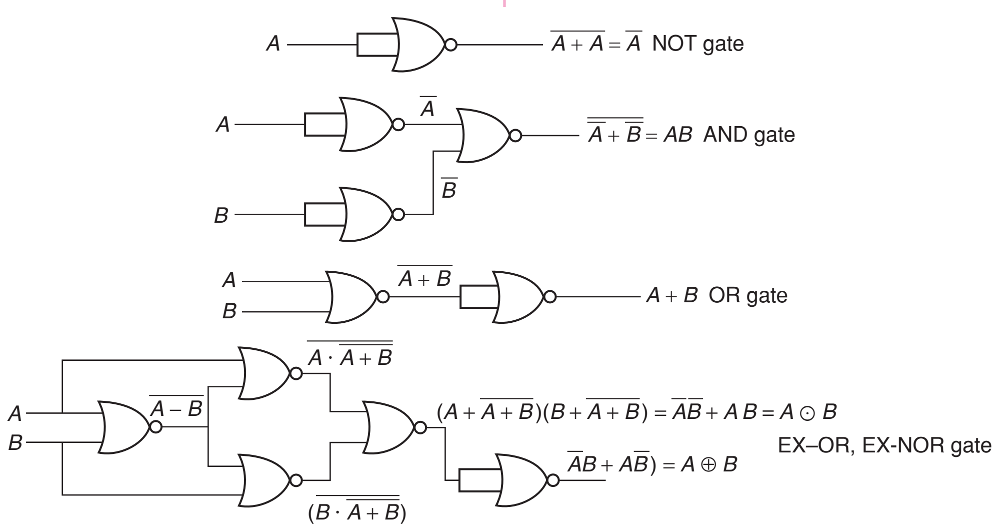
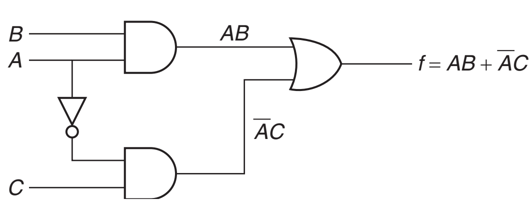


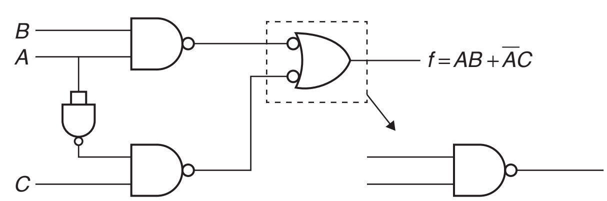
Figure 10 Implementation of basic gates by using NOR gates

Any function which is in the SOP form can be implemented by using AND-OR gates, which is also equivalent to NAND–NAND gates.

$$F = AB + \overline{A}C$$



By considering bubble at AND gate output and OR gate input, and by changing NOT gates to NAND gates the circuit becomes as,



Now the circuit is in completely in NAND – NAND form

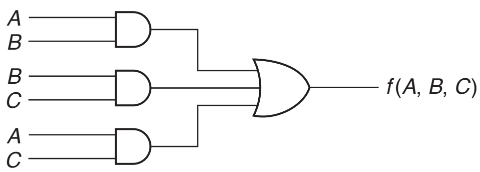
So the functions expressed in SOP form, can be implemented by using AND – OR, (or) NAND–NAND gates.

Any function in POS from, can be implemented by using OR–AND gates, which is similar to NOR–NOR gate.

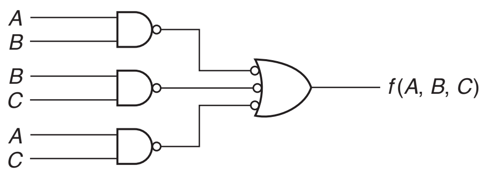
Example 16: How many number of NAND gates are required to implement $f(A, B, C) = AB + BC + AC$

(A) 3 (B) 4 (C) 5 (D) 6

Solution:



By considering bubbles at output of AND gate and input of OR gate.

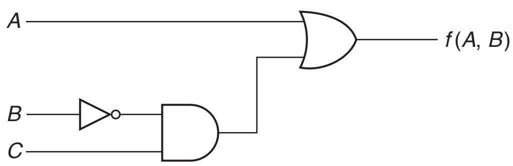


So four NAND gates are required.

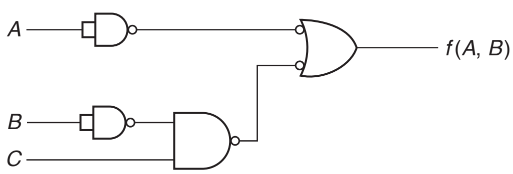
Example 17: Number of NAND gates required for implementation of $f(A,B) = A + \bar{B}C$ is

(A) 3 (B) 4 (C) 5 (D) 6

Solution:



To convert the all gates into NAND gates, place bubble output of AND, and inputs of OR gates. Now, the circuit can be drawn as



Four NAND gates are required.

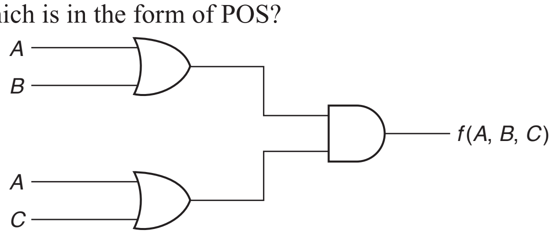
Example 18: $f = A + BC$, the number of NOR gates required to implement f, are?

(A) 3 (B) 4 (C) 5 (D) 2

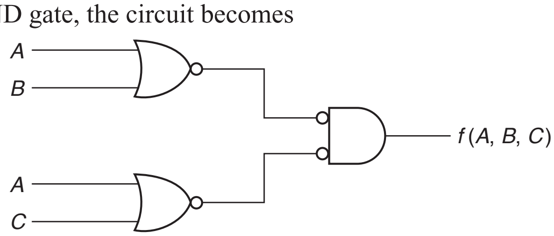
Solution: $A + BC$ is in SOP form.

To implement this function by using NOR gates, we can write $f(A, B, C) = A + BC = (A + B)(A + C)$

Which is in the form of POS?



By including bubbles at output of OR gate, and input of AND gate, the circuit becomes



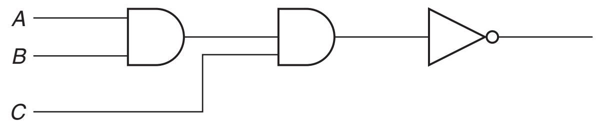
Now the circuit consists of all NOR gates. Three NOR Gates are required.

Example 19: How many number of two-input NAND–NOR gates are required to implement three-input NAND–NOR gates respectively?

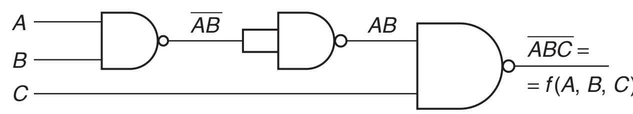
(A) 2, 2 (B) 2, 3
(C) 3, 2 (D) 3, 3

Solution: $f(A,B,C) = \overline{ABC} = \overline{AB} + \overline{C}$

(1) Implement above function by using two-inputs gates

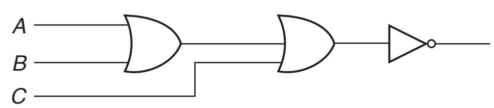


Now convert each gate to NAND gate

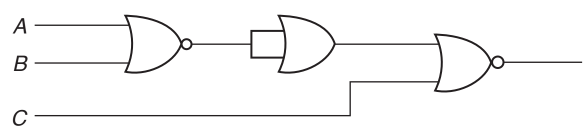


Three two-input NAND gates are required.

(2) $G(A,B,C) = \overline{A+B+C}$ Implement it by using two-input gates



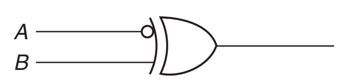
Now convert each gate to NOR gate



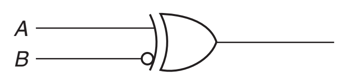
Three two-input, NOR gates are required.

EX-OR, EX-NOR Gates

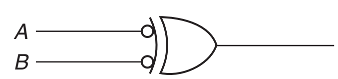
Inverted inputs for EX OR, EX-NOR gates



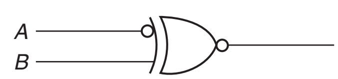
$$\bar{A} \oplus B = \bar{\bar{A}}B + \bar{A}\,\bar{B} = AB + \bar{A}\,\bar{B} = A \odot B$$



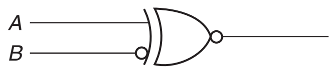
$$A \oplus \bar{B} = A\bar{\bar{B}} + \bar{A}\,\bar{B} = AB + \bar{A}\,\bar{B} = A \odot B$$



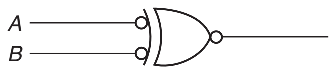
$$\bar{A} \oplus \bar{B} = \bar{A}\,\bar{\bar{B}} + \bar{\bar{A}}\,\bar{B} = \bar{A}B + A\bar{B} = A \oplus B$$



$$\bar{A} \odot B = \bar{A}B + \bar{\bar{A}}\,\bar{B} = \bar{A}B + A\bar{B} = A \oplus B$$



$$A \odot \overline{B} = A\overline{B} + \overline{A}\,\overline{\overline{B}} = A\overline{B} + \overline{A}B = A \oplus B$$

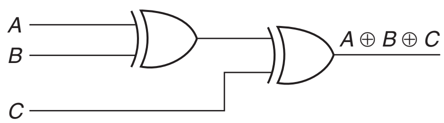


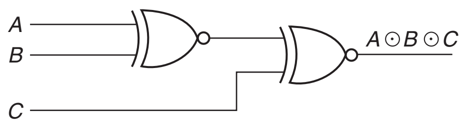
$$\overline{A} \odot \overline{B} = \overline{A}\,\overline{B} + \overline{\overline{A}}\,\overline{\overline{B}} = \overline{A}\,\overline{B} + AB = A \odot B$$

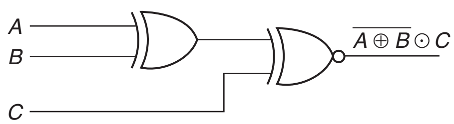
From the above discussions we can conclude that inverted input EXOR gate is EX-NOR gate.

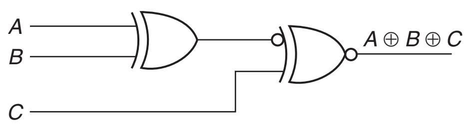
Similarly, inverted input EX-NOR gate is EX-OR gate. If both inputs are inverted the EX-OR / EX-NOR will remain as it is.

Consider a three-inputs X-OR gates by using two-input XOR gates.



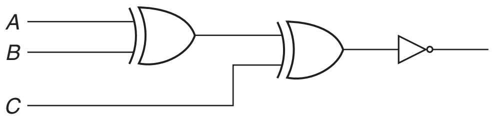


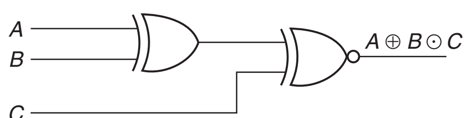




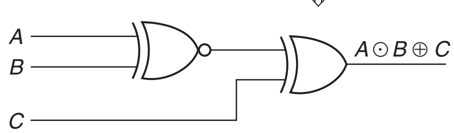
So we can conclude that $A \oplus B \oplus C = A \odot B \odot C$

$$\overline{A \oplus B \oplus C} = \overline{A \odot B \odot C}$$





$\Downarrow A$



$$\overline{A \oplus B \oplus C} = \overline{A \odot B \odot C} = A \oplus B \odot C$$

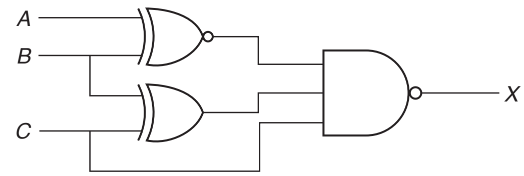
$= A \odot B \oplus C$

$A \oplus B \oplus C \oplus D = A \oplus B \odot C \odot D = A \odot B \odot C \oplus D = A \odot B \oplus C \odot D$

$$A \odot B \odot C \odot D = \overline{A \oplus B \oplus C \oplus D}$$

$A \odot B \odot C \odot D = A \oplus B \oplus C \odot D = A \oplus B \odot C \oplus D$

Example 20: For the logic circuit shown in figure, the required input condition (A, B, C) to make the output $X = 0$ is?



(A) 1, 1, 1 (B) 1, 0, 1
(C) 0, 1, 1 (D) 0, 0, 1

Solution: (D)

To get output $X = 0$, all inputs to the NAND gate should be 1, so $C = 1$.

When $C = 1$, the output of X-OR gate $B \oplus C = 1$ only when $B = 0$.

If $B = 0$ the output of X-NOR gate $A \odot B = 1$.

Only when $A = 0$

So $X = 1$, only when $(A, B, C) = (0, 0, 1)$.

Example 21: The minimized expression of

$(A + \overline{B})\,(A\overline{B} + AC)\,(\overline{A}\,\overline{C} + \overline{B})$ is

Solution: $(A + \overline{B})\,(A\overline{B} + AC)\,(\overline{A}\,\overline{C} + \overline{B})$

$$= (A + \overline{B})\,(A\overline{B}\cdot\overline{A}\,\overline{C} + A\overline{B}\cdot\overline{B} + AC\cdot\overline{A}\,\overline{C} + AC\cdot\overline{B})$$

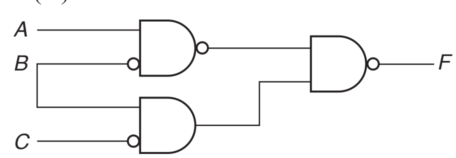
$$= (A + \overline{B})\,(A\overline{B} + A\overline{B}C) = (A + \overline{B})A\overline{B}(1 + C)$$

$$= A\overline{B} + A\overline{B} = A\overline{B}$$

Example 22: The Boolean function f is independent of

(A) a (B) b
(C) c (D) None of these

Solution: (A)

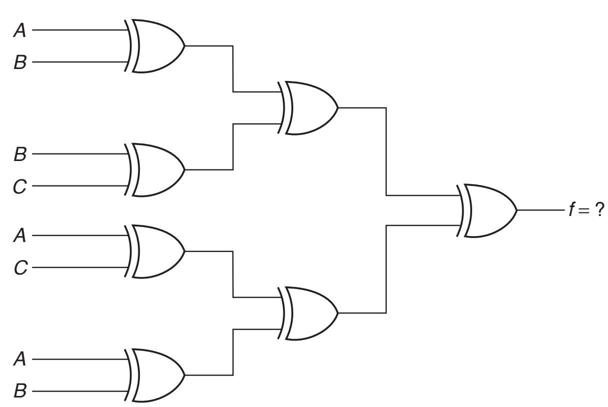


$F = \overline{\overline{ab}\cdot\overline{bc}}$

$= ab + \overline{\overline{b}c} = ab + b + \overline{c}$

$= b + \overline{c}$ is independent of 'a'.

Example 23:



Solution: $f = \{A \oplus B \oplus B \oplus C\} \oplus \{A \oplus C \oplus B \oplus A\}$

$= \{A \oplus 0 \oplus C\} \oplus \{0 \oplus C \oplus B\}$

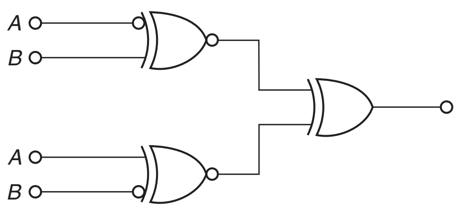
$= A \oplus C \oplus C \oplus B = A \oplus 0 \oplus B = A \oplus B$

Solved Examples

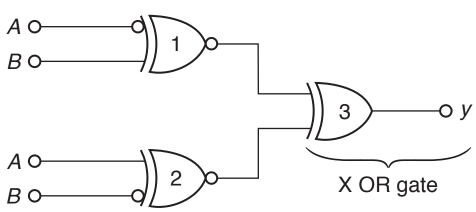
Example 1: Simplify the Boolean function, $x\,y + x'z + y\,z$

Solution: $x\,y + x'\,z + y\,z$
By using consensus property
$xy + x'z + yz = xy + x'z$
$Y = xy + x'z$

Example 2: The output of the given circuit is equal to



Solution: $\overline{A} \odot B = \overline{A}B + A\overline{B}$

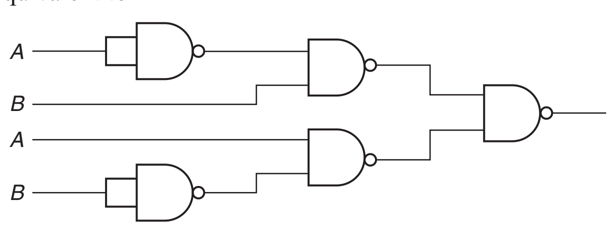


$$A \odot \overline{B} = \overline{A}B + A\overline{B}$$

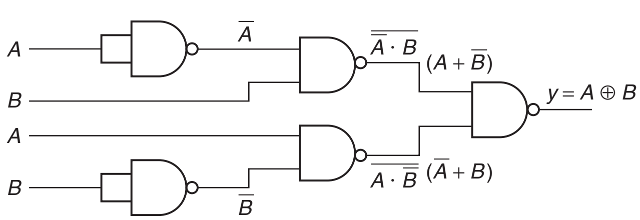
So the output of above circuit is '0'. As two inputs are same at third gate.
Output of XOR gate with two equal inputs is zero.

$$\therefore\ y = 0$$

Example 3: The circuit shown in the figure is functionally equivalent to



Solution:



$Y = \overline{(\overline{\overline{A}\cdot B}\cdot\overline{A\overline{B}})} = \overline{(A+\overline{B})(\overline{A}+B)} \quad \because \quad (\overline{A\cdot B} = \overline{A}+\overline{B})$

$= \overline{(A+\overline{B})} + \overline{(\overline{A}+B)} = \overline{A}\cdot\overline{\overline{B}} + \overline{\overline{A}}\cdot\overline{B}$

$= \overline{A}\cdot B + A\cdot\overline{B} = A \oplus B$

Example 4: Simplify the Boolean function $A \oplus \overline{A}B \oplus \overline{A}$

Solution: $A \oplus \overline{A}B \oplus \overline{A}$

Associativity

$= 1 \oplus \overline{A}B = \overline{\overline{A}B}$

$= A + \overline{B}$ $(\because$ De Morgan's)

Example 5:

CD \ AB	00	01	11	10
00	0	0	1	1
01	0	×	×	1
11	×	×	1	×
10	1	0	1	1

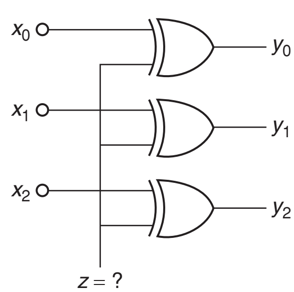
The minimized expression for the given K–map is

Solution:

CD \ AB	00	01	11	10
00	0	0	1	1
01	0	×	×	1
11	×	×	1	×
10	1	0	1	1

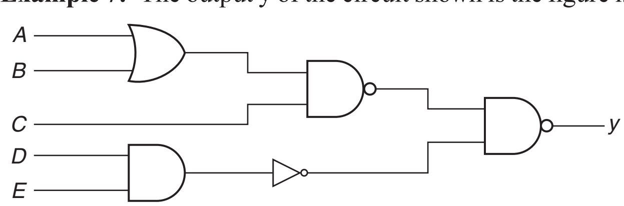
$= A + \overline{B}C$

Example 6: In the figure shown, y_2, y_1, y_0 will be 1s complement of $x_2\,x_1\,x_0$ if $z = ?$

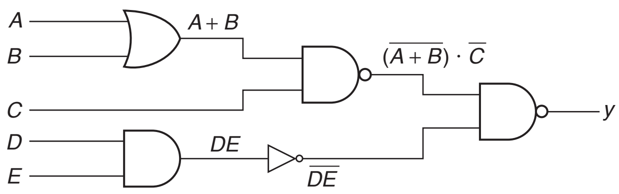


Solution: We are using X-OR gate
$\therefore$ XOR out-put is complement of input only when other input is high.
$\therefore Z = 1$

Example 7: The output y of the circuit shown is the figure is



Solution:



$$y = \overline{\overline{(A+B)\cdot C}\cdot \overline{DE}} = \overline{\overline{(A+B)\cdot C}} + \overline{\overline{DE}}$$

$$= (A+B)C + DE(\overline{x\cdot y} = \bar{x} + \bar{y})$$

Example 8: Simplify the following function

$$f = \overline{\overline{\bar{A}(\overline{AB})}\cdot\overline{\bar{B}(\overline{AB})}}$$

Solution: $\overline{\overline{\overline{\bar{A}(\overline{AB})}}\cdot\overline{\overline{\bar{B}(\overline{A\cdot B})}}}$

$$\overline{[A+(AB)]\cdot[B+(AB)]} = \overline{A+(AB)} + \overline{(B+AB)}$$

$$= \bar{A}\cdot\overline{(AB)} + \bar{B}\cdot\overline{(AB)} = \bar{A}\cdot(\bar{A}+\bar{B}) + \bar{B}(\bar{A}+\bar{B})$$

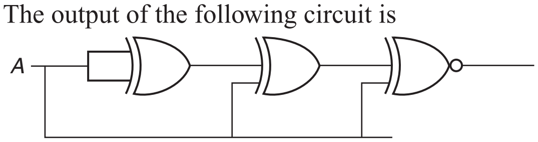
$$= \bar{A}\cdot\bar{A} + \bar{A}\,\bar{B} + \bar{B}\,\bar{A} + \bar{B}\cdot\bar{B} = \bar{A}+\bar{B} = \overline{AB}$$

Exercises

Practice Problems I

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The output of the following circuit is



(A) 0 (B) 1
(C) A (D) A'

2. The circuit which will work as OR gate in positive level will work as ___ gate in negative level logic
(A) NOR gate
(B) NAND gate
(C) Both NAND and NOR gate
(D) AND gate

3. Four logical expressions are given below:

(a) $\bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}\bar{E}\cdot\bar{F}\cdot\bar{G}\cdot\bar{H}$

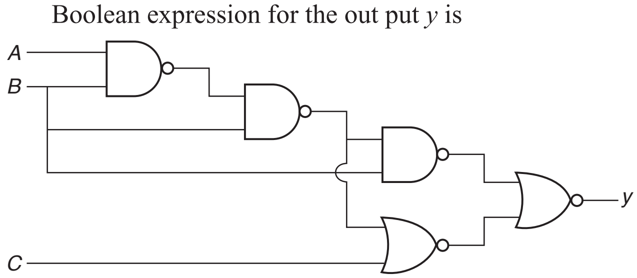
(b) $\overline{AB}\cdot\overline{CD}\cdot\overline{EF}\cdot\overline{GH}$

(c) $\bar{A}+\bar{B}+\bar{C}+\bar{D}+\bar{E}+\bar{F}+\bar{G}+\bar{H}$

(d) $(\bar{A}+\bar{B})\,(\bar{C}+\bar{D})\,(\bar{E}+\bar{F})\,(\bar{G}+\bar{H})$

Two of these expression are equal. They are
(A) c and d (B) b and d
(C) a and b (D) a and c

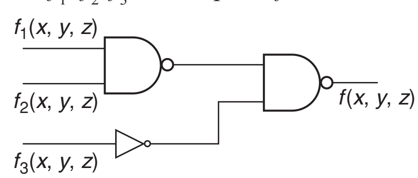
4. For the logic circuit shown in figure, the simplified Boolean expression for the out put y is



(A) $A + B + C$ (B) A
(C) ABC (D) $B\bar{C}$

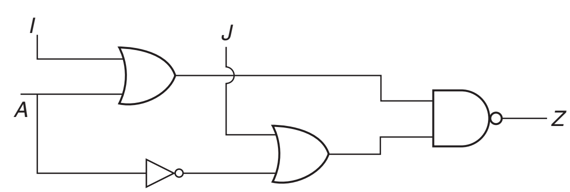
5. In a digital system, there are three inputs A, B and C. The output should be high when at least two inputs are high. The minimized Boolean expression for the out put is
(A) $AB + BC + AC$
(B) $ABC + AB\bar{C} + \bar{A}BC + A\bar{B}C$
(C) $AB\bar{C} + A\bar{B}C + \bar{A}BC$
(D) $A\bar{B} + B\bar{C} + \bar{A}C$

6. Consider the following logic circuit whose inputs are functions f_1, f_2, f_3 and output is f.



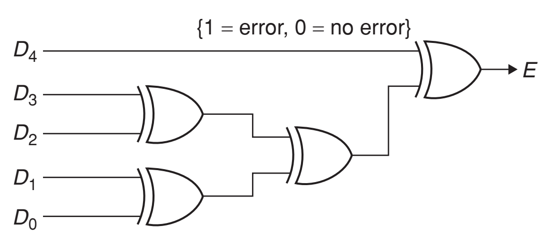
Given that $f_1(x, y, z) = \bullet(0, 1, 3, 5)$ $f_2(x, y, z) = \bullet(6, 7)$ and $f(x, y, z) = \bullet(1, 4, 5)$, then f_3 is
(A) •(1, 4, 5) (B) •(6, 7)
(C) •(0, 1, 3, 5) (D) None of these

7. The circuit shown above is to be used to implement the function $z = f(A, B) = \bar{A} + B$ what values are to be selected for I and J?



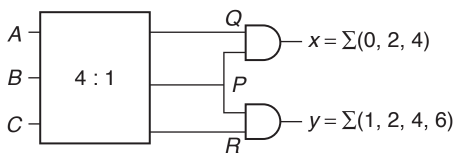
(A) $I = 0, J = B$ (B) $I = 1, J = B$
(C) $I = B, J = 1$ (D) $I = B, J = 0$

8. Parity checker output from the below figure, if input is 11111 ($D_4\,D_3\,D_2\,D_1\,D_0$) and 10000 ($D_4\,D_3\,D_2\,D_1\,D_0$).



(A) error, error
(B) error, no error
(C) no error, error
(D) no error , no error

9. For the given combinational network with three inputs A, B and C, three intermediate outputs P, Q and R, and two final outputs $X = P \cdot Q = \Sigma(0, 2, 4)$ and $Y = P \cdot R = \Sigma(1, 2, 4, 6)$ as shown below. Find the smallest function P(containing minimum number of min terms that can produce the output x and y)



(A) $\Sigma(2, 4)$
(B) $\Sigma(0, 1, 2, 4, 6)$
(C) $\Sigma(3, 5, 7)$
(D) $\Sigma(1, 2, 6)$

10. The standard form of expression $AB + ACD + \bar{A}C$ is

(A) $AB\bar{C}\bar{D} + ABC\bar{D} + AB\bar{C}D + ABCD + A\bar{B}CD + \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}\,\bar{B}CD + \bar{A}\bar{B}C\bar{D}$

(B) $AB + ACD + \bar{A}C$

(C) $AB\bar{C} + ABC + ABCD + \bar{A}CB + \bar{A}C\bar{B}\,\bar{D}$

(D) $\bar{A}\bar{B}\bar{C}\,\bar{D} + ABCD + \bar{A}BC + AB\bar{D} + ABC$

11. Factorize $\bar{A}\,\bar{B}\bar{C}\,\bar{D} + \bar{A}\,\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D$

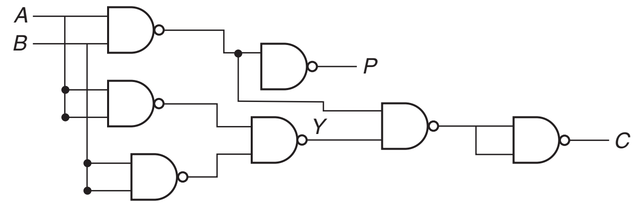
(A) $B + C$
(B) $AB + CD$
(C) $\bar{B}\bar{C}$
(D) AC

12. The K-map of a function is as shown. Find the function.

wx \ yz	00	01	11	10
	1			1
	1	1	1	1
	1			1
	1			1

(A) wx
(B) $\bar{z}$
(C) $\overline{w}(z + \bar{z}) + \bar{z}w$
(D) $\overline{w}x + \bar{z}$

13. The Boolean expression for P is



(A) AB
(B) $\overline{AB}$
(C) $\bar{A} + \bar{B}$
(D) $A + B$

14. The Boolean expression for the truth table is

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

(A) $B(A + C)(\bar{A} + \bar{C})$
(B) $\bar{B}(A + \bar{C})(\bar{A} + \bar{C})$
(C) $B(A + \bar{C})(\bar{A} + C)$
(D) $\bar{B}(A + C)(\bar{A} + \bar{C})$

15. Simplify (d represents don't-care)

C \ AB	00	01	11	10
0	1	d		1
1	1	d		1

(A) $\bar{B}$
(B) $\bar{B} + C$
(C) $\bar{B} + \bar{A}$
(D) $A + \bar{C}$

16. Simplify $\overline{(AB + \bar{C})(\overline{A + B} + C)}$

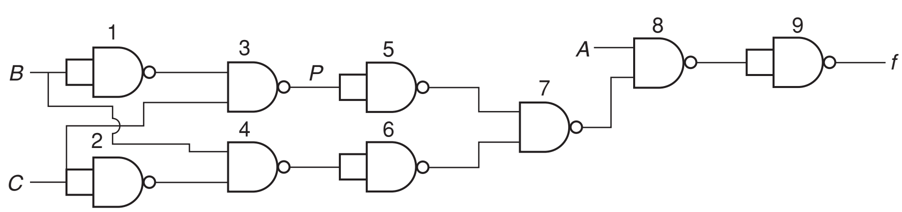
(A) $(\bar{A} + \bar{B} + \bar{C}) \cdot (A + B + C)$

(B) $(\bar{A} + B + C) \cdot (A + \bar{B} + \bar{C})$

(C) $(\bar{A} + \bar{B}) \cdot (A + B + C)$

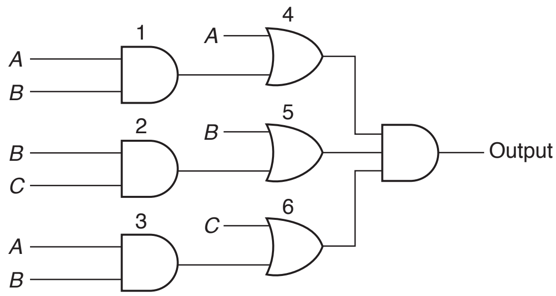
(D) None of these

17. The point P in the figure is stuck at 1. The output f will be



(A) $\overline{ABC}$
(B) $\bar{A}$
(C) $AB\bar{C}$
(D) A

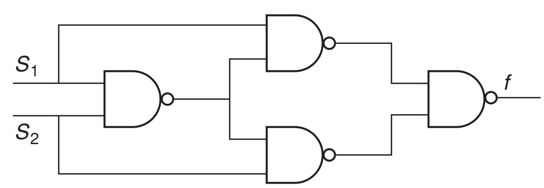
18. Find the function represented by the figure



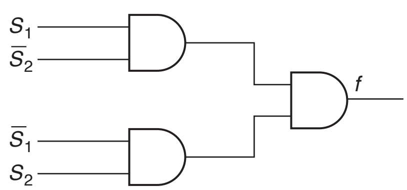
(A) $A + B + C$ (B) AB
(C) $AB + C$ (D) $B + C$

19. A staircase light is controlled by two switches, one is at the top of the stairs and other at the bottom of stairs. Realization of this function using NAND logic results in which of the following circuits? (Assume S_1 and S_2 are the switches)

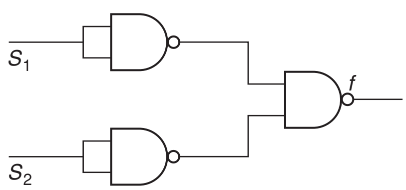
(A)



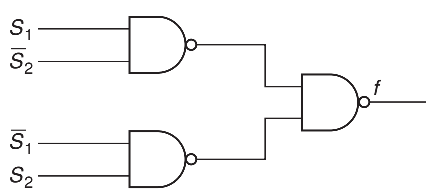
(B)



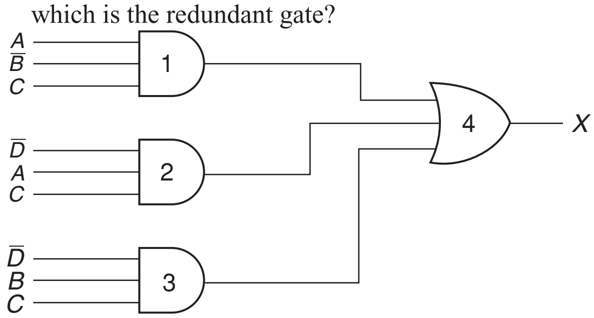
(C)



(D)



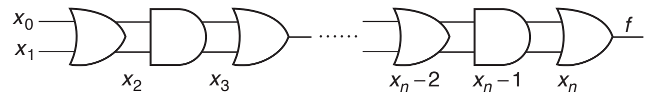
20. For the given figure simplify the expression and find which is the redundant gate?



(A) $ABC + DBC$, 4 (B) $A\bar{B}C + \bar{D}AC$, 3
(C) $\bar{D}AC + \bar{D}BC$, 1 (D) $A\bar{B}C + \bar{D}BC$, 2

21. The function $f = A \oplus B \oplus C \oplus D$ is represented as
(A) $f(A, B, C, D) = \Sigma(2, 6, 10, 11, 12, 13, 14)$
(B) $f(A, B, C, D) = \Sigma(3, 5, 7, 10, 11, 12, 13, 14)$
(C) $f(A, B, C, D) = \Sigma(1, 2, 6, 8, 10, 12, 13, 14)$
(D) $f(A, B, C, D) = \Sigma(1, 2, 4, 7, 8, 11, 13, 14)$

22. Find the function represented

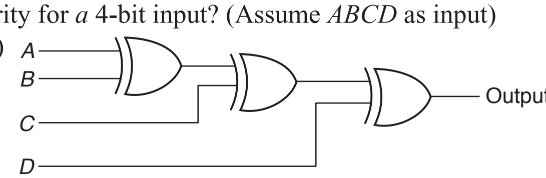


(A) $(x_0 + x_1)(x_2 + x_3)(x_4 + x_5) \ldots (x_{n-1} + x_n)$
(B) $x_0 + x_1 + x_2 + x_3 + \cdots + x_n$
(C) $x_0x_2x_4 \ldots x_n + x_1x_2 \ldots x_n + x_{n-1}x_n$
(D) $x_0x_1 + x_2x_3 + \cdots + x_{n-1}x_n$

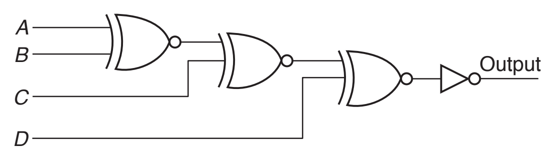
23. The minimum number of NAND gates required to implement $A \oplus B \oplus C$ is
(A) 8 (B) 10
(C) 9 (D) 6

24. Which of the following circuit will generate an odd parity for *a* 4-bit input? (Assume *ABCD* as input)

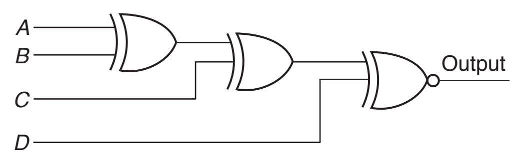
(A)



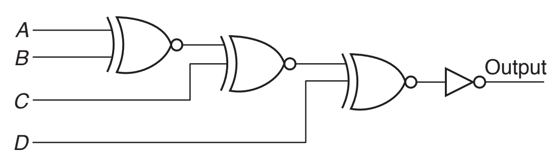
(B)



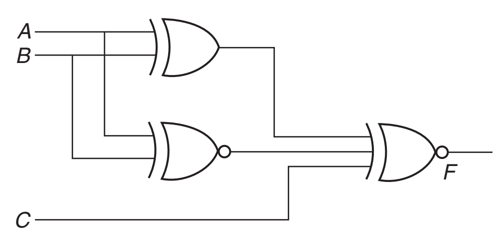
(C)



(D)



25. For the output F to be 1 in the circuit, the input combination should be



(A) $A = 1, B = 1, C = 0$ (B) $A = 1, B = 0, C = 0$
(C) $A = 0, B = 1, C = 0$ (D) $A = 0, B = 0, C = 1$

Practice Problems 2

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. An OR Gate has six inputs. How many input words are there in its truth table?

(A) 6 (B) 36
(C) 32 (D) 64

2. Sum of product form can be implemented by using

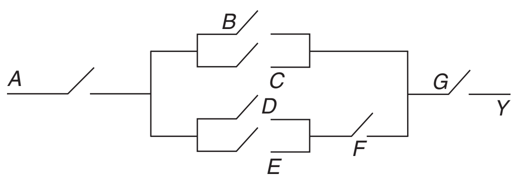
(A) AND – OR
(B) NAND – NAND
(C) NOR – NOR
(D) Both A and B

3. Which one of the following is equivalent to the Boolean expression?

$Y = AB + BC + CA$

(A) $\overline{AB + BC + CA}$
(B) $(\bar{A} + \bar{B})(\bar{B} + \bar{C})(\bar{A} + \bar{C})$
(C) $\overline{(A + B)(B + C)(A + C)}$
(D) $\overline{(\bar{A} + \bar{B})(\bar{B} + \bar{C})(\bar{C} + \bar{A})}$

4. What Boolean function does the following circuit represents?



(A) $A\,[F + (B + C)\cdot(D + E)]\,G$
(B) $A + BC + DEF + G$
(C) $A\,[(B + C) + F\,(D + E)]\,G$
(D) $ABG + ABC + F\,(D + E)$

5. The minimum number of two input NOR gates are required to implement the simplified value of the following equation

$f(w, x, y, z) = \Sigma m(0, 1, 2, 3, 8, 9, 10, 11)$

(A) One (B) Two
(C) Three (D) Four

6. The out put of a logic gate is '1' when all inputs are at logic '0'. Then the gate is either

(1) NAND or X-OR gate
(2) NOR or X-OR gate
(3) NOR or X-NOR gate
(4) NAND or X-NOR gate

(A) 1 and 2 (B) 2 and 3
(C) 3 and 4 (D) 4 and 1

7. If the functions w, x, y, and z are as follows.

$w = R + \bar{P}Q + \bar{R}S$

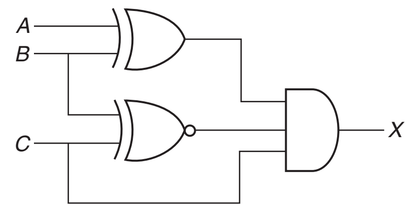
$x = PQ\bar{R}\bar{S} + \bar{P}\bar{Q}\bar{R}\bar{S} + P\bar{Q}\bar{R}\bar{S}$

$y = RS + \overline{PR + P\bar{Q} + \bar{P}\cdot\bar{Q}}$

$z = R + S + \overline{PQ + \bar{P}\cdot\bar{Q}\cdot\bar{R} + P\cdot\bar{Q}\cdot\bar{S}}$

(A) $w = z,\ x = y$ (B) $w = z,\ x = \bar{z}$
(C) $w = y$ (D) $w = y = z$

8. For the logic circuit shown in the figure, the required in put condition (A, B, C) to make the output (x) = 1 is

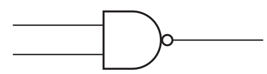


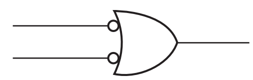
(A) 0, 0, 1 (B) 1, 0, 1
(C) 1, 1, 1 (D) 0, 1, 1

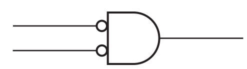
9. Which of the following is a basic gate?

(A) AND (B) X-OR
(C) X-NOR (D) NAND

10. Which of the following represent the NAND gate?

(a) 

(b) 

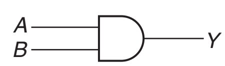
(c) 

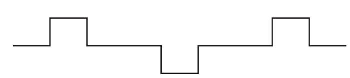
(A) a only (B) a, b, c
(C) b, a (D) a, c

11. The universal gates are

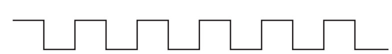
(A) NAND and NOR (B) AND, OR, NOT
(C) X-OR and X-NOR (D) All of these

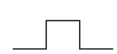
12. In the circuit the value of input A goes from 0 to 1 and part of B goes from 1 to 0. Which of the following represent output under a static hazard condition?



(a) 

(b) 

(c) 

(d) 

(A) Output a (B) Output b
(C) Output c (D) Output d

13. The consensus theorem states that

(A) $A + \bar{A}B = A + B$
(B) $A + AB = A$
(C) $AB + \bar{A}C + BC = AB + \bar{A}C$
(D) $(A + B)\cdot(A + \bar{B}) = A$

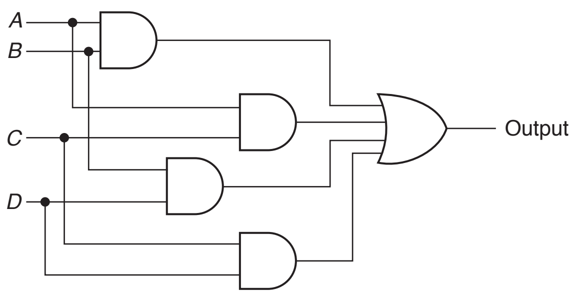
14. The dual form of expression

$AB + \bar{A}C + BC = AB + \bar{A}C$ is

(A) $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$
(B) $(A+B)(\bar{A}+C)(B+C) = (\bar{A}+\bar{B})(A+\bar{C})$
(C) $(\bar{A}+\bar{B})(\bar{A}+\bar{C})(\bar{B}+\bar{C}) = (\bar{A}+\bar{B})(A+\bar{C})$
(D) $\bar{A}\bar{B} + A\bar{C} + \bar{B}\bar{C} = \bar{A}\bar{B} + A\bar{C}$

15. The max term corresponding to decimal 12 is
(A) $\bar{A}+\bar{B}+C+D$ (B) $A+B+\bar{C}+\bar{D}$
(C) $\bar{A}\bar{B}CD$ (D) $ABC\bar{D}$

16. The given circuit is equivalent to



(A) $(A + C)(B + D)$ (B) $AC + BD$
(C) $(A + D)(B + C)$ (D) $(\bar{A} + \bar{B})(\bar{C} + \bar{D})$

17. Minimized expression for Karnaugh map is

C \ AB	00	01	11	10
0	1			1
1	1			1

(A) $AB + C$ (B) $\bar{A}B + C$
(C) $\bar{B}$ (D) $\bar{B} + C$

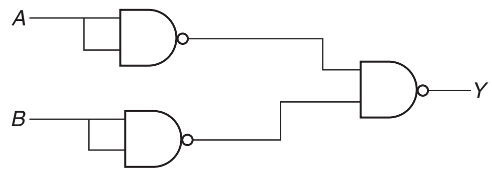
18. An XOR gate will act as ________ when one of its input is one and as ________ when one of its input is zero.
(A) buffer, buffer (B) buffer, inverter
(C) inverter, buffer (D) inverter, inverter

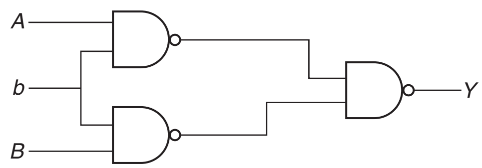
19. The minimum number of two input NAND gates required to implement $A \odot B$ if only A and B are available
(A) 6 (B) 3
(C) 5 (D) 4

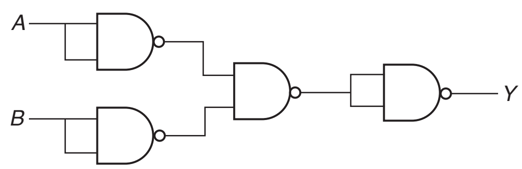
20. Negative logic in a logic circuit is one in which
(A) logic 0 and 1 are represented by GND and positive voltage respectively.
(B) logic 0 and 1 are represented by negative and positive voltage.
(C) logic 0 voltage level is lower than logic 1 voltage level.
(D) logic 0 voltage level is higher than logic 1 voltage level.

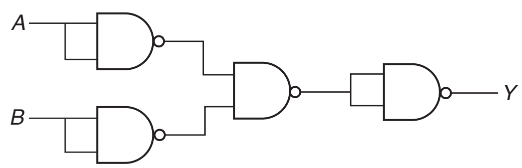
21. If the input to a gate is eight in number, then its truth table contains ________ input words.
(A) 128 (B) 8
(C) 64 (D) 256

22. The X-OR gate implementation using NAND gate is

(A)


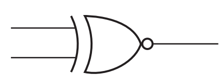
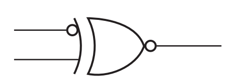
(B)


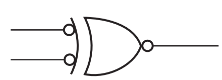
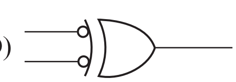
(C)


(D)


23. The equivalent of AND–OR logic circuit is
(A) NAND–NOR (B) NOR–AND
(C) NAND–NAND (D) NAND–OR

24. The X-OR is equivalent to

(A)  (B) 

(C)  (D) 

25. Simplify $A\bar{B}C + B + B\bar{D} + AB\bar{D} + \bar{A}C$
(A) B (B) $B + C$
(C) $C + A$ (D) $\bar{A} + B$

Previous Years' Questions

1. Let $f(w,x,y,z)=\Sigma(0,\ 4,\ 5,\ 7,\ 8,9,\ 13,\ 15)$. Which of the following expressions are NOT equivalent to f? **[2007]**

(P) $x'y'z' + w'xy' + wy'z + xz$
(Q) $w'y'z' + wx'y' + xz$
(R) $w'y'z' + wx'y' + xyz + xy'z$
(S) $x'y'z' + wx'y' + w'y$

(A) P only (B) Q and S
(C) R and S (D) S only

2. Define the connective $*$ for the Boolean variables X and Y as: $X * Y = XY + X'Y'$. Let $Z = X * Y$. Consider the following expressions P, Q and R. **[2007]**

$P : X = Y * Z$ $Q : Y = X * Z$
$R: X * Y * Z = 1$

Which of the following is TRUE?

(A) Only P and Q are valid.
(B) Only Q and R are valid.
(C) Only P and R are valid.
(D) All P, Q, R are valid.

3. In the Karnaugh map shown below, × denotes a don't-care term. What is the minimal form of the function represented by the Karnaugh map? **[2008]**

cd \ ab	00	01	11	10
00	1	1		1
01	×	1		
11	×			
10	1	1		×

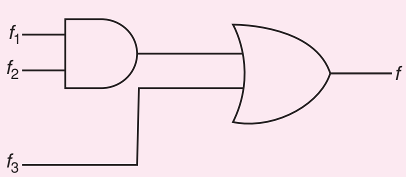
(A) $\bar{b}\cdot\bar{d}+\bar{a}\cdot\bar{d}$

(B) $\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{d}+\bar{a}\cdot b\cdot\bar{d}$

(C) $\bar{b}\cdot\bar{d}+\bar{a}\cdot b\cdot\bar{d}$

(D) $\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{d}+\bar{a}\cdot\bar{d}$

4. Given f_1, f_3 and f in canonical sum of products form (in decimal) for the circuit **[2008]**



$f_1 = \Sigma m(4, 5, 6, 7, 8)$
$f_3 = \Sigma m(1, 6, 15)$
$f = \Sigma m(1, 6, 8, 15)$
then f_2 is

(A) $\Sigma m(4, 6)$ (B) $\Sigma m(4, 8)$
(C) $\Sigma m(6, 8)$ (D) $\Sigma m(4, 6, 8)$

5. If P, Q, R are Boolean variables, then $(P+\bar{Q})(P\cdot\bar{Q}+P\cdot R)(\bar{P}\cdot\bar{R}+\bar{Q})$ simplifies to **[2008]**

(A) $P\cdot\bar{Q}$ (B) $P\cdot\bar{R}$
(C) $P\cdot\bar{Q}+R$ (D) $P\cdot\bar{R}+Q$

6. What is the minimum number of gates required to implement the Boolean function $(AB + C)$, if we have to use only two-input NOR gates? **[2009]**

(A) 2 (B) 3
(C) 4 (D) 5

7. The binary operation □ is defined as follows **[2009]**

P	Q	P□Q
T	T	T
T	F	T
F	T	F
F	F	T

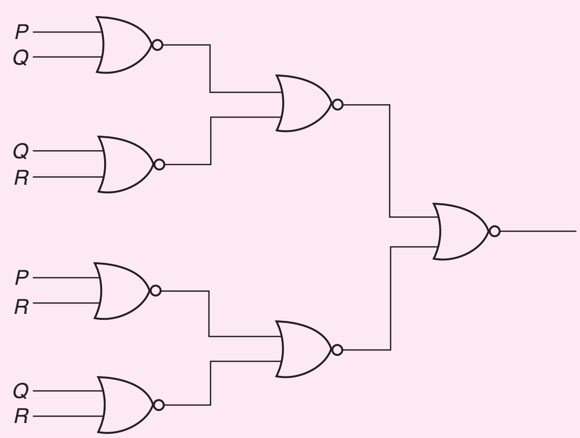
Which one of the following is equivalent to P□Q?

(A) $\neg Q\ \neg P$ (B) P□$\neg Q$
(C) $\neg P$□Q (D) $\neg P$□$\neg Q$

8. The min term expansion of $f(P, Q, Q) = PQ + Q\bar{R} + P\bar{R}$ is **[2010]**

(A) $m_2 + m_4 + m_6 + m_7$
(B) $m_0 + m_1 + m_3 + m_5$
(C) $m_0 + m_1 + m_6 + m_7$
(D) $m_2 + m_3 + m_4 + m_5$

9. What is the Boolean expression for the output f of the combinational logic circuit of NOR gates given below? **[2010]**



(A) $\overline{Q+R}$ (B) $\overline{P+Q}$
(C) $\overline{P+R}$ (D) $\overline{P+Q+R}$

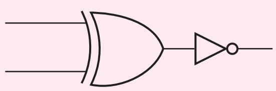
10. The simplified SOP (Sum of Product) form of the Boolean expression. **[2011]**

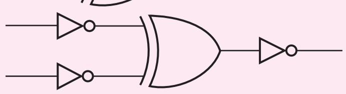
$(P+\overline{Q}+\overline{R})(P+\overline{Q}+R)(P+Q+\overline{R})$ is

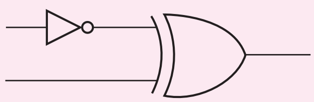
(A) $(PQ+\overline{R})$ (B) $(P+\overline{Q}\,\overline{R})$

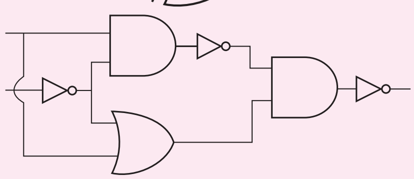
(C) $(\overline{P}Q+R)$ (D) $(PQ+R)$

11. Which one of the following circuits is NOT equivalent to a two-input X-NOR (exclusive NOR) gate? **[2011]**

(A) 

(B) 

(C) 

(D) 

12. The truth table **[2012]**

X	Y	F(X, Y)
0	0	0
0	1	0
1	0	1
1	1	1

represents the Boolean function

(A) X (B) $X+Y$

(C) $X \oplus Y$ (D) Y

13. What is the minimal form of the Karnaugh map shown below? Assume that × denotes a don't-care term. **[2012]**

cd \ ab	00	01	11	10
00	1	×	×	1
01	×			1
11				
10	1			×

(A) $\overline{b}\,\overline{d}$ (B) $\overline{b}\,\overline{d}+\overline{b}\,\overline{c}$

(C) $\overline{b}\,\overline{d}+a\overline{b}\,\overline{c}d$ (D) $\overline{b}\,\overline{d}+\overline{b}\,\overline{c}+\overline{c}\,\overline{d}$

14. Which one of the following expressions does not represent exclusive NOR of x and y? **[2013]**

(A) $xy+x'y'$ (B) $x \oplus y'$

(C) $x' \oplus y$ (D) $x' \oplus y'$

15. Consider the following Boolean expression for F:

$F(P,Q,R,S)=PQ+\overline{P}QR+\overline{P}Q\overline{R}S$

The minimal sum of products form of F is **[2014]**

(A) $PQ+QR+QS$ (B) $P+Q+R+S$

(C) $\overline{P}+\overline{Q}+\overline{R}+\overline{S}$ (D) $\overline{P}R+\overline{P}\,\overline{R}S+P$

16. The dual of a Boolean function $f(x_1, x_2, \ldots x_n, +, ., ')$, written as F^D, is the same expression as that of F with + and . swapped. F is said to be self-dual if $F = F^D$. The number of self-dual functions with n Boolean variables is **[2014]**

(A) 2^n (B) 2^{n-1}

(C) 2^{2^n} (D) $2^{2^{n-1}}$

17. Consider the following min term expression for F:

$F(P, Q, R, S) = \Sigma m(0, 2, 5, 7, 8, 10, 13, 15)$

The min terms 2, 7, 8 and 13 are 'don't-care terms. The minimal sum of products form for F is **[2014]**

(A) $Q\overline{S}+\overline{Q}S$

(B) $\overline{Q}\,\overline{S}+QS$

(C) $\overline{Q}\,\overline{R}\,\overline{S}+\overline{Q}R\overline{S}+Q\overline{R}S+QRS$

(D) $\overline{P}\,\overline{Q}\,\overline{S}+\overline{P}QS+PQS+P\overline{Q}\,\overline{S}$

18. The binary operator ≠ is defined by the following truth table

p	q	p ≠ q
0	0	0
0	1	1
1	0	1
1	1	0

Which one of the following is true about the binary operator ≠? **[2015]**

(A) Both commutative and associative
(B) Commutative but not associative
(C) Not commutative but associative
(D) Neither commutative nor associative

19. Consider the operations **[2015]**

$f(X, Y, Z) = X^1\,YZ + XY^1 + Y^1Z^1$ and

$g(X, Y, Z) = X^1\,YZ + X^1\,YZ^1 + XY.$

Which one of the following is correct?

(A) Both $\{f\}$ and $\{g\}$ are functionally complete
(B) Only $\{f\}$ is functionally complete
(C) Only $\{g\}$ is functionally complete
(D) Neither $\{f\}$ nor $\{g\}$ is functionally complete

20. The number of min-terms after minimizing the following Boolean expression is _______ **[2015]**

$[D^1 + AB^1 + A^1C + AC^1D + A^1C^1D]^1$

21. Let # be a binary operator defined as **[2015]**

$X \# Y = X^1 + Y^1$ where X and Y are Boolean variables.

Consider the following two statements.

(S1) $(P \# Q) \# R = P \# (Q \# R)$

(S2) $Q \# R = R \# Q$

Which of the following is/are true for the Boolean variables P, Q and R?

(A) Only S_1 is true
(B) Only S_2 is true

(C) Both S_1 and S_2 are true
(D) Neither S_1 nor S_2 are true

22. Given the function $F = P^1 + QR$, where F is a function in three Boolean variables P, Q and R and $P^1 = !P$, consider the following statements. **[2015]**

(S_1) $F = \Sigma(4, 5, 6)$

(S_2) $F = \Sigma(0, 1, 2, 3, 7)$

(S_3) $F = \pi(4, 5, 6)$

(S_4) $F = \pi(0, 1, 2, 3, 7)$

Which of the following is true?
(A) (S_1) – False, (S_2) – True, (S_3) – True, (S_4) – False
(B) (S_1) – True, (S_2) – False, (S_3) – False, (S_4) – True
(C) (S_1) – False, (S_2) – False, (S_3) – True, (S_4) – True
(D) (S_1) – True, (S_2) – True, (S_3) – False, (S_4) – False

23. The total number of prime implicants of the function $f(w, x, y, z) = \Sigma(0, 2, 4, 5, 6, 10)$ is ______ **[2015]**

24. Consider the Boolean operator # with the following properties: **[2016]**

$x \# 0 = x$, $x \# 1 = \bar{x}$, $x \# x = 0$ and

$x \# \bar{x} = 1$. Then $x \# y$ is equivalent to

(A) $x\bar{y} + \bar{x}y$
(B) $x\bar{y} + \bar{x}\bar{y}$
(C) $\bar{x}y + xy$
(D) $xy + \bar{x}\bar{y}$

25. Consider the Karnaugh map given below, where X represents *"don't care"* and blank represents 0.

dc \ ba	00	01	11	10
00		X	X	
01	1			X
11	1			1
10		X	X	

Assume for all inputs (a, b, c, d), the respective complements ($\bar{a}, \bar{b}, \bar{c}, \bar{d}$) are also available. The above logic is implemented using 2-input NOR gates only. The minimum number of gates required is ________. **[2017]**

26. If w, x, y, z are Boolean variables, then which one of the following is INCORRECT? **[2017]**
(A) $wx + w(x + y) + x(x + y) = x + wy$
(B) $\overline{w\bar{x}(y+\bar{z})} + \bar{w}x = \bar{w} + x + \bar{y}z$
(C) $\left(w\bar{x}\left(y + x\bar{z}\right) + \bar{w}\bar{x}\right)y = x\bar{y}$
(D) $(w + y)(wxy + wyz) = wxy + wyz$

27. Given $f(w,x,y,z) = \Sigma m(0,1,2,3,7,8,10) + \Sigma d(5,6,11,15)$, where d represents the *don't-care* condition in Karnaugh maps. Which of the following is a minimum product-of-sums (POS) form of $f(w, x, y, z)$? **[2017]**

(A) $f = (\bar{w}+\bar{z})(\bar{x}+z)$
(B) $f = (\bar{w}+z)(x+z)$
(C) $f = (w + z)(\bar{x} + z)$
(D) $f = (w+\bar{z})(\bar{x}+z)$

28. Let ⊕ and ⊙ denote the Exclusive OR and Exclusive NOR operations, respectively. Which one of the following is NOT CORRECT? **[2018]**
(A) $\overline{P \oplus Q} = P \odot Q$
(B) $\bar{P} \oplus Q = P \odot Q$
(C) $\bar{P} \oplus \bar{Q} = P \oplus Q$
(D) $(P \oplus \bar{P}) \oplus Q = (P \odot \bar{P}) \odot \bar{Q}$

29. Consider the minterm list form of a Boolean function F given below.

$$F(P, Q, R, S) = \sum m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$$

Here, m denotes a minterm and d denotes a don't care term. The number of essential prime implicants of the function F is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** D	**3.** B	**4.** C	**5.** A	**6.** A	**7.** B	**8.** A	**9.** B	**10.** A
11. C	**12.** D	**13.** A	**14.** A	**15.** A	**16.** A	**17.** D	**18.** B	**19.** A	**20.** D
21. D	**22.** C	**23.** A	**24.** C	**25.** D					

Practice Problems 2

1. D	**2.** D	**3.** D	**4.** C	**5.** A	**6.** C	**7.** B	**8.** D	**9.** A	**10.** C
11. A	**12.** D	**13.** A	**14.** A	**15.** A	**16.** C	**17.** C	**18.** C	**19.** C	**20.** D
21. D	**22.** C	**23.** C	**24.** B	**25.** B					

Previous Years' Questions

1. D	**2.** D	**3.** A	**4.** C	**5.** A	**6.** B	**7.** B	**8.** A	**9.** A	**10.** B
11. D	**12.** A	**13.** B	**14.** D	**15.** A	**16.** D	**17.** B	**18.** A	**19.** B	**20.** 1
21. B	**22.** A	**23.** 3	**24.** A	**25.** 1	**26.** C	**27.** A	**28.** D	**29.** 3	

Chapter 3

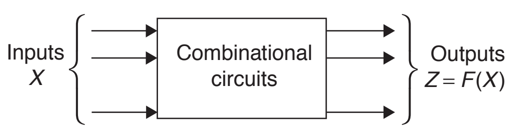
Combinational Circuits

LEARNING OBJECTIVES

- *Combinational logic design*
- *Arithmetic circuit*
- *Half adder*
- *Full adder*
- *Half subtractor*
- *Full subtractor*
- *n-bit comparator*
- *Parity bit generator and parity bit checker*
- *Code converter*
- *Decoder*
- *Designing high order decoders from lower order decoder*
- *Combinational logic implementation*
- *Encoders*
- *Multiplexer*
- *Demultiplexer*

Introduction

Combinational logic is a type of logic circuit whose output is a function of the present input only.



Combinational Logic Design

The design of combinational circuit starts from the problem, statement and ends with a gate level circuit diagram.

The design procedure involves the following steps:

1. Determining the number of input variables and output variables required, from the specifications.
2. Assigning the letter symbols for input and output.
3. Deriving the truth table that defines the required relationship between input and output.
4. Obtaining the simplified Boolean function for each output by using K-map or algebraic relations.
5. Drawing the logic diagram for simplified expressions.

We will discuss combinational circuits under the following categories:

- Arithmetic circuits
- Code converters
- Data processing circuits

Arithmetic Circuits

Arithmetic circuits are the circuits that perform arithmetic operation. The most basic arithmetic operation is addition.

Half Adder

Addition is an arithmetic operation, and here to implement addition in digital circuits we have to implement by logical gates. So the addition of binary numbers will be represented by the logical expressions. Half adder is an arithmetic circuit which performs the addition of two binary bits, and the result is viewed in two output—sum and carry.

The sum 'S' is the X-OR of 'A' and 'B' where A and B are inputs.

$$\therefore \quad S = A\bar{B} + B\bar{A} = A \oplus B$$

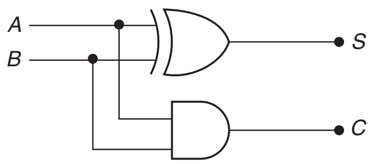
The carry 'C' is the AND of A and B.

$$\therefore C = AB$$

Table 1 *Truth Table*

Inputs		Outputs	
A	**B**	**S**	**C**
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

So, half adder can be realized by using one X-OR gate and one AND gate.

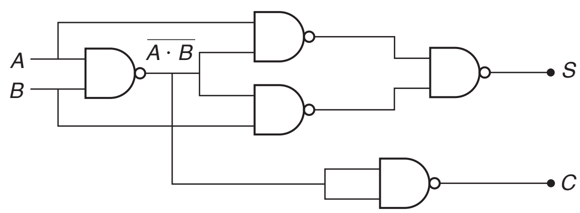


Half adder can also be realized by universal logic such as only NAND gate or only NOR gate as given below.

NAND logic

$$
\begin{aligned}
S &= A\overline{B} + \overline{A}B \\
&= A\overline{B} + A\overline{A} + \overline{A}B + B\overline{B} \\
&= A(\overline{A} + \overline{B}) + B(\overline{A} + \overline{B}) \\
&= \overline{\overline{A\overline{AB}} \cdot \overline{B\overline{AB}}}
\end{aligned}
$$

$$C = AB = \overline{\overline{A \cdot B}}$$

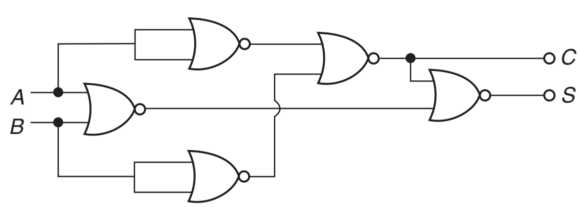


Half adder using NAND logic

NOR logic

$$
\begin{aligned}
S &= A \cdot \overline{B} + \overline{A}B \\
&= A\overline{B} + A\overline{A} + \overline{A}B + B\overline{B} \\
&= A(\overline{A} + \overline{B}) + B(\overline{A} + \overline{B}) \\
&= (A + B)\,(\overline{A} + \overline{B}) \\
&= \overline{\overline{A + B} + \overline{(\overline{A} + \overline{B})}}
\end{aligned}
$$

$$C = A \cdot B = \overline{\overline{A \cdot B}} = \overline{\overline{A} + \overline{B}}$$



Half adder using NOR logic

Full Adder

Full adder is an arithmetic circuit that performs addition of two bits with carry input. The result of full adder is given by two outputs—sum and carry. The full adder circuit is used in parallel adder circuit as well as in serial adder circuit.

For full adder, if total number of 1's is odd at input lines, the sum output is equal to logic 1, and if total number of 1's at input lines are more than or equal to 2, then the carry output is logic 1.

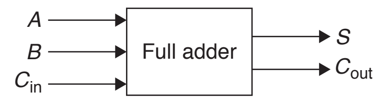


Figure 1 Block diagram

Table 2 *Truth Table*

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$
\begin{aligned}
S &= \overline{A}\,\overline{B}C_{in} + \overline{A}B\overline{C_{in}} + A\overline{B}\,\overline{C_{in}} + ABC_{in} \\
&= A \oplus B \oplus C_{in} \\
C_{out} &= \overline{A}BC_{in} + A\overline{B}C_{in} + AB\overline{C_{in}} + ABC_{in} \\
&= AB + (A \oplus B)\,C_{in} \\
&= AB + A\,C_{in} + B\,C_{in}
\end{aligned}
$$

Full adder can also be realized using universal logic gates, i.e., either only NAND gates or only NOR gates as explained below.

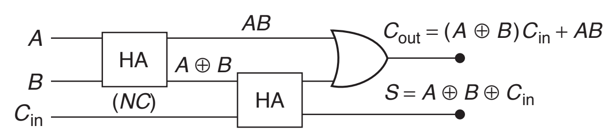


Figure 2 Block diagram of full adder by using Half adder

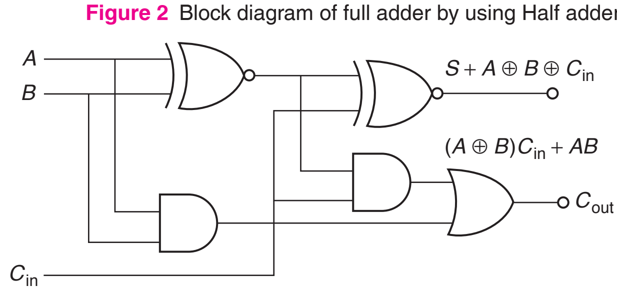


Figure 3 Logic diagram of full adder

NAND logic

$$A \oplus B = \overline{\overline{A\overline{AB}} \quad \overline{B\overline{AB}}}$$

So $A \oplus B \oplus C_{in}$

Let $A \oplus B = x$ then $s = \overline{\overline{X \cdot \overline{XC_{in}}} \quad \overline{C_{in}\overline{X \cdot C_{in}}}}$

$$= X \oplus C_{in}$$

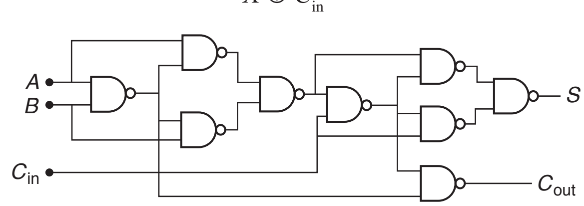


Figure 4 Logic diagram of a full adder using only 2-input NAND gates

NOR logic

Full adder outputs

Sum = $a \oplus b \oplus c$, carry = $ab + bc + ac$ are self dual functions

[∵ A function is called as self dual if its dual is same as the function itself $f^D = f$]

For self dual functions, the number of NAND gates are same as number of NOR gates.

By taking the dual for above NAND gate implementation, all gates will become NOR gates, and the output is dual of the sum and carry, but they are self dual ($f^D = f$).

So, output remain same, and only 9 NOR gates are required for full adder, structure similar to NAND gate circuit.

Half Subtractor

Half subtractor is an arithmetic circuit which performs subtraction of one bit (subtrahend) from other bit (minuend), and the result gives difference and borrow each of one bit. The borrow output is logic 1 only if there is any subtraction of 1 from 0.

When a bit 'B' is subtracted from another bit 'A', a difference bit (d) and a borrow bit (b) result according to the rule given below.

Table 3 *Truth Table*

A	B	d	b
0	0	0	0
1	0	1	0
1	1	0	0
0	1	1	1

$$\begin{aligned} d &= A\overline{B} + B\overline{A} \\ &= A \oplus B \\ b &= \overline{A}B \end{aligned}$$

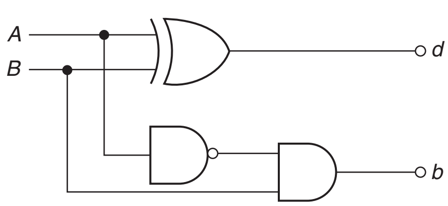


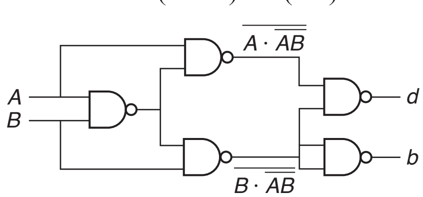
Figure 5 Logic diagram of a half subtractor

A half subtractor can also realized using universal logic either using only NAND gates or only NOR gates as explained below.

NAND logic

$$\begin{aligned} d &= A \oplus B \\ &= \overline{\overline{A\overline{AB}} \cdot \overline{B\overline{AB}}} \end{aligned}$$

$$b = \overline{A}B = B(\overline{A} + \overline{B}) = B(\overline{AB}) = \overline{\overline{B \cdot \overline{AB}}}$$



NOR logic

$$\begin{aligned} d &= A \oplus B \\ &= A\overline{B} + \overline{A}B \\ &= A\overline{B} + B\overline{B} + \overline{A}B + A\overline{A} \\ &= \overline{B}(A+B) + \overline{A}(A+B) \\ &= \overline{B + \overline{A+B}} + \overline{A + \overline{A+B}} \\ b &= \overline{A}B \\ &= \overline{A}(A+B) \\ &= \overline{\overline{\overline{A}(A+B)}} \\ &= \overline{A + (\overline{A+B})} \end{aligned}$$

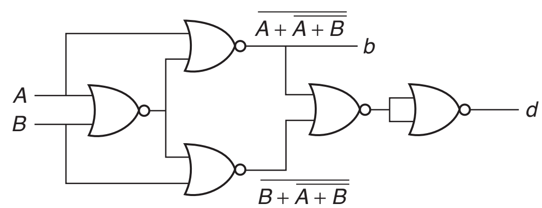


Figure 6 Logic diagram of half subtractor using NOR gate

Full Subtractor

Full subtractor is an arithmetic circuit similar to half subtractor but it performs subtraction with borrow, it involves subtraction of three bits—minuend, subtrahend and borrow-in, and two outputs—difference and borrow. The subtraction of 1 from 0 results in borrow to become logic 1. The presence of odd number of 1's at input lines make difference as logic 1.

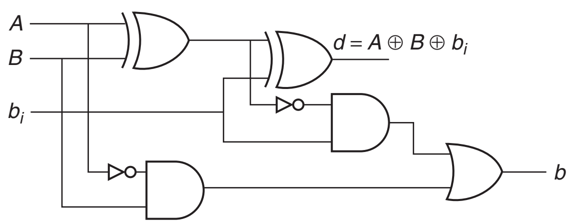
Table 4 *Truth Table*

A	B	b_i	d	b
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$\begin{aligned} d &= \overline{A}\,\overline{B}b_i + \overline{A}B\overline{b_i} + A\overline{B}\,\overline{b_i} + ABb_i \\ &= b_i(AB + \overline{A}\,\overline{B}) + \overline{b_i}(A\overline{B} + \overline{A}B) \\ &= b_i(\overline{A \oplus B}) + \overline{b_i}(A \oplus B) \\ &= A \oplus B \oplus b_i \end{aligned}$$

and

$$b = \bar{A}\bar{B}b_i + \bar{A}B\bar{b}_i + \bar{A}Bb_i$$
$$= \bar{A}B + \overline{(A\oplus B)}b_i$$



NAND logic

$$d = A \oplus B \oplus b_i$$
$$= \overline{\overline{(A\oplus B)\overline{(A\oplus B)b_i}}\;\overline{b_i\overline{(A\oplus B)b_i}}}$$
$$b = \bar{A}B + b_i\overline{(A\oplus B)}$$
$$= \overline{\overline{\bar{A}B + b_i\overline{(A\oplus B)}}}$$
$$= \overline{\overline{\bar{A}B}\;\overline{b_i\,\overline{(A\oplus B)}}}$$
$$= \overline{\overline{B(\bar{A}+\bar{B})}\;\overline{b_i\left[\overline{b_i + (A\oplus B)}\right.}}$$

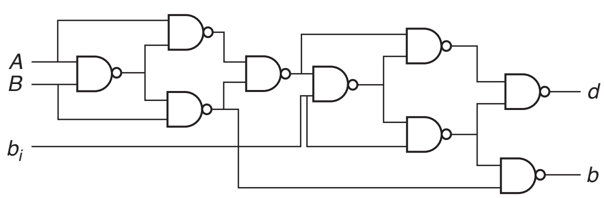


Figure 7 Logic diagram of a full subtractor using NAND logic

NOR logic

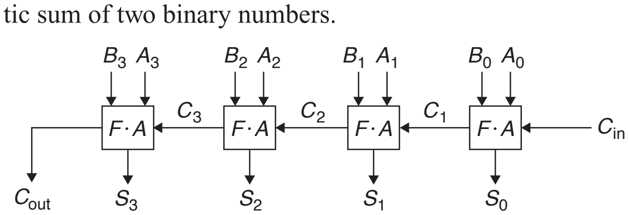
Output of full subtractor is also self dual in nature. So, same circuit, with all NAND gates, replaced by NOR gates gives the NOR gate full subtractor. 9 NOR gates required.

Example 1: How many NAND gates are required for implementation of full adder and full subtractor respectively?
(A) 11, 10 (B) 11, 11 (C) 9, 9 (D) 9, 10

Solution: (C)
From the circuit diagrams in the previous discussion, full adder requires 9 NAND gates and full subtractor requires 9 NAND gates.

Binary Adder

A binary adder is a digital circuit that produces the arithmetic sum of two binary numbers.



Four bit parallel adder the output carry from each full adder is connected to the input carry of next full adder.

The bits are added with full adders, starting from the LSB position to form the sum bit and carry bit.

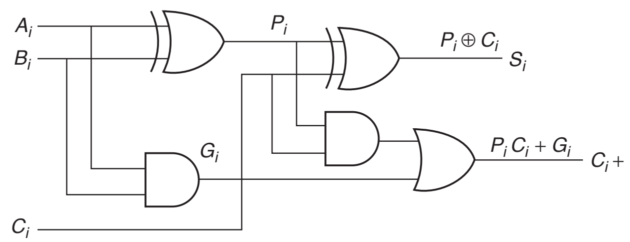
The longest propagation delay time in parallel adder is the time it takes the carry to propagate through the full adders.

For n-bit parallel adders consider t_{pds} is the propagation delay for sum of each full adder and t_{pdc} is the propagation delay of carry.

The total time required to add all n-bits at the nth full adder is

$$T_S = t_{pds} + (n-1)t_{pdc}$$

So propagation delay increases with number of bits. To overcome this difficulty we use look ahead carry adder, which is the fastest carry adder.



Consider the full adder circuit for ith stage, in parallel adder, with two binary variables A_i, B_i, input carry C_i are:

Carry propagate (P_i) and carry generate (G_i)

$$P_i = A_i \oplus B_i$$
$$G_i = A_i \cdot B_i$$

The output sum and carry can be expressed as

$$S_i = P_i \oplus C_i$$
$$C_{i+1} = P_i C_i + G_i$$

Now, the Boolean functions for each stage can be calculated as substitute $i = 0$

C_0 is input carry

$$C_1 = G_0 + P_0 C_0$$

Substitute $i = 1, 2 \ldots$

$$C_2 = G_1 + P_1C_1 = G_1 + P_1(G_0 + P_0C_0)$$
$$= G_1 + P_1 G_0 + P_1 P_0 C_0$$
$$C_3 = G_2 + P_2 C_2 = G_2 + P_2(G_1 + P_1 G_0 + P_1 P_0 C_0)$$
$$= G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0$$

Since the Boolean function for each output carry is expressed in SOP form, each function can be implemented with AND–OR form or two level NAND gates.

From the above equations we can conclude that this circuit can perform addition in less time as C_3 does not have to wait for C_2 and C_1 to propagate. C_3, C_2, C_1 can have equal time delays.

The gain in speed of operation is achieved at the expense of additional complexity (hardware).

n-bit Comparator

The comparison of two numbers is an operation that determines whether one number is greater than, less than, or equal to the other number.

A magnitude comparator is a combinational circuit that compares two input numbers A and B, and specifies the output with three variables, $A > B$, $A = B$, $A < B$:

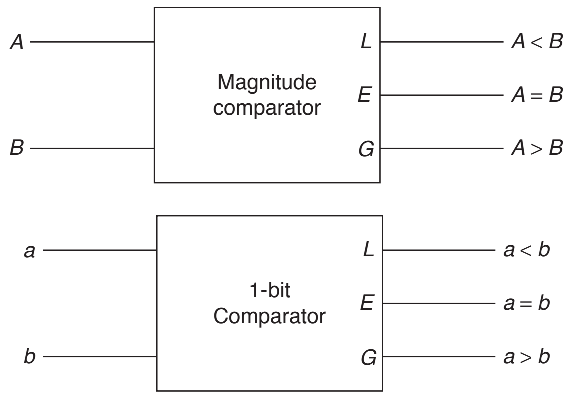


Figure 8 1-bit comparator will have only 1 bit input *a*, *b*.

a	b	$a < b$	$a = b$	$a > b$
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0

By considering minterms for each output.

$$(a < b) = a'b$$
$$(a = b) = a'b' + ab = a \odot b$$
$$(a > b) = ab'$$

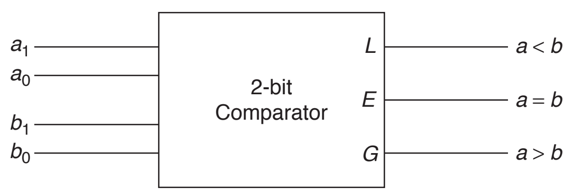


Figure 9 2-bit comparator will have 2-bit inputs a_1 a_0 and b_1 b_0.

a_1	a_0	b_1	b_0	L $a < b$	E $a = b$	G $a > b$
0	0	0	0	0	1	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	1	0

$$(a < b) = \Sigma(1, 2, 3, 6, 7, 11)$$
$$(a > b) = \Sigma(4, 8, 9, 12, 13, 14)$$
$$(a = b) = \Sigma(0, 5, 10, 15)$$

a_1a_0 \ b_1b_0	00	01	11	10
00		1	1	1
01			1	1
11				
00			1	

$$a < b = a_1'a_0'b_0 + a_0'b_1b_0 + a_1'b_1$$
$$L = \overline{a_1}\,b_1 + (a_1 \odot b_1)\,\overline{a_0}\,b_0$$

Similarly, $a > b = a_0b_1'b_0' + a_1a_0b_0' + a_1b_1'$

$$G = a_1\overline{b_1} + (a_1 \odot b_1)\,a_0\overline{b_0}$$

$a = b$ is possible when $a_1 = b_1$, $a_0 = b_0$

So $(a = b) = (a_1 \odot b_1)(a_0 \odot b_0)$

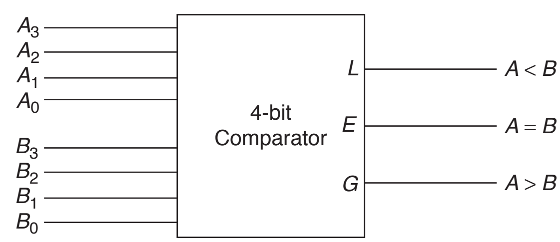


Figure 10 4-bit comparator will compare 2 input numbers each of 4-bits A_3 A_2 A_1 A_0 and B_3 B_2 B_1 B_0 ($A = B$) output will be 1 when each bit of input A is equal to corresponding bit in input B.

So we can write $(A = B) = (A_3 \odot B_3)\,(A_2 \odot B_2)\,(A_1 \odot B_1)\,(A_0 \odot B_0)$.

To determine whether A is greater or less than B, we inspect the relative magnitudes of pairs of significant bits, starting from MSB. If the two bits of a pair are equal, we compare the next lower significant pair of bits. The comparison continues until a pair of unequal bits is reached.

for $A < B$, $A = 0$, $B = 1$

for $A > B$, $A = 1$, $B = 0$

$$A < B = A_3'B_3 + (A_3 \odot B_3)\,A_2'B_2 + (A_3 \odot B_3)(A_2 \odot B_2) \times A_1'B_1 + (A_3 \odot B_3)(A_2 \odot B_2)\,(A_1 \odot B_1)\,A_0'B_0$$

$$A > B = A_3B_3' + (A_3 \odot B_3)\,A_2B_2' + (A_3 \odot B_3)\,(A_2 \odot B_2) \times A_1B_1' + (A_3 \odot B_3)(A_2 \odot B_2)\,(A_1 \odot B_1)\,A_0B_0'$$

4-bit comparator will have total 8 inputs and $2^8 = 256$ input combinations in truth table.

For 16 combinations $(A = B) = 1$, and for 120 combinations $A < B = 1$.

For remaining 120 combination $A > B = 1$

Parity Bit Generator and Parity Bit Checker

When digital information is transmitted, it may not be received correctly by the receiver. To detect one bit error at receiver we can use parity checker.

For detection of error an extra bit, known as parity bit, is attached to each code word to make the number of 1's in the code even (in case of even parity) or odd (in case of odd parity).

For n-bit data, we use n-bit parity generator at the transmitter end. With 1 parity bit and n-bit data, total $n + 1$ bit will be transmitted. At the receiving end $n + 1$ parity checker circuit will be used to check correctness of the data.

For even parity transmission, parity bit will be made 1 or 0 based on the data, so that total $n + 1$ bits will have even number of 1's. For example, if we want to transmit data 1011 by even parity transmission, then we will use parity bit as 1, so data will have even number of 1's, i.e., data transmitted will be $\underline{1}1011$. At the receiving end this data will be received and checked for even number of ones.

To transmit data $B_3B_2B_1B_0$ using even parity, we will transmit sequence $P\,B_3B_2B_1B_0$, where $P = B_3 \oplus B_2 \oplus B_1 \oplus B_0$. (Equation for parity generator)

At the receiving end we will check data received $PB_3B_2B_1B_0$ for error, $E = P \oplus B_3 \oplus B_2 \oplus B_1 \oplus B_0$ (equation for parity checker). If $E = 0$ (no error), or if $E = 1$ (1 bit error).

We use EX-OR gates for even parity generator/checker as EX-OR of bits gives output 1 if there are odd number of 1's else EX-OR output is 0.

Odd parity generator/checker is complement of even parity generator/checker. Odd parity circuits check for presence of odd number of 1's in data.

Code Converters

There are many situations where it is desired to convert from one code to another within a system. For example, the information from output of an analog to digital converter is often in gray code, before it can be processed in arithmetic unit, conversion to binary is required.

Let us consider simple example of 3-bit binary to gray code converter. This will have input lines supplied by binary codes and output lines must generate corresponding bit combination in gray code. The combination circuit code converter performs this transformation by means of logic gates.

The output logic expression derived for code converter can be simplified by using the usual techniques including 'don't-care' if any present. For example, BCD code uses only codes from 0000 to 1001 and remaining combinations are treated as don't-care combinations. Similarly, EXS-3 uses only combinations from 0011 to 1100 and remaining combinations are treated as don't-care.

The relationship between the two codes is shown in the following truth table:

Decimal	B_2	B_1	B_0	G_2	G_1	G_0
0	0	0	0	0	0	0
1	0	0	1	0	0	1
2	0	1	0	0	1	1
3	0	1	1	0	1	0
4	1	0	0	1	1	0
5	1	0	1	1	1	1
6	1	1	0	1	0	1
7	1	1	1	1	0	0

For conversion we have to find out minimized functions of

$$G_2(B_2, B_1, B_0) = \sum m(4, 5, 6, 7)$$
$$G_1(B_2, B_1, B_0) = \sum m(2, 3, 4, 5)$$
$$G_0(B_2, B_1, B_0) = \sum m(1, 2, 5, 6)$$

B_0 \ B_1B_2	00	01	11	10
0		1		1
1		1		1

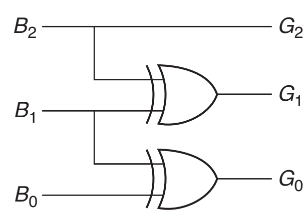
$$G_0(B_2, B_1, B_0) = B'_1B_0 + B_1B'_0 = B_1 \oplus B_0$$

B_0 \ B_1B_2	00	01	11	10
0			1	1
1	1	1		

$$G_1(B_2, B_1, B_0) = B'_1B_2 + B_1B'_2 = B_2 \oplus B_1$$

B_0 \ B_1B_2	00	01	11	10
0				
1	1	1	1	1

$$G_2(B_2, B_1, B_0) = B_2$$



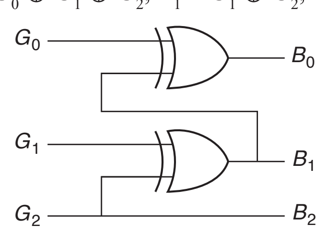
In similar fashion we can derive n-bit binary to gray code conversion as

$$G_n = B_n$$
$$G_{n-1} = B_{n-1} \oplus B_n$$
$$G_{i-1} = B_{i-1} \oplus B_i$$

Thus conversion can be implemented by $n - 1$ X-OR gates for n-bits.

For reverse conversion of gray to binary, by following similar standard principle of conversion, we will get

$$B_0 = G_0 \oplus G_1 \oplus G_2,\ B_1 = G_1 \oplus G_2,\ B_2 = G_2$$



In general for n-bit gray to binary code conversion

$$B_i = G_n \oplus G_{n-1} \oplus G_{n-2} \ldots \oplus G_{i-1} \oplus G_i$$

$B_n = G_n$ (MSB is same in gray and binary). It also requires $n-1$ X-OR gates for n-bits.

Example 2: Design 84-2-1 to XS-3 code converter.

Solution: Both 84-2-1 and *XS*-3 are *BCD* codes, each needs 4-bits to represent. The following table gives the relation between these codes. 84-2-1 is a weighted code, i.e., each position will have weight as specified. *XS*-3 is non-weighted code; the binary code is 3 more than the digit in decimal.

Decimal	84-2-1 $B_3B_2B_1B_0$	*XS*-3 $X_3X_2X_1X_0$
0	0000	0011
1	0111	0100
2	0110	0101
3	0101	0110
4	0100	0111
5	1011	1000
6	1010	1001
7	1001	1010
8	1000	1011
9	1111	1100

We will consider minterm don't-care combinations as 1, 2, 3, 12, 13, 14. For these combinations 84-2-1 code will not exist and the remaining minterms can be found from truth table.

$$X_0(B_3, B_2, B_1, B_0) = \sum m(0, 4, 6, 8, 10) + \sum \Phi(1, 2, 3, 12, 13, 14) = \overline{B_0}$$

$$X_1(B_3, B_2, B_1, B_0) = \sum m(0, 4, 5, 8, 9, 15) + \sum \Phi(1, 2, 3, 12, 13, 14) = \overline{B_1}$$

$$X_2(B_3, B_2, B_1, B_0) = \sum m(4, 5, 6, 7, 15) + \sum \Phi(1, 2, 3, 12, 13, 14) = B_2$$

$$X_3(B_3, B_2, B_1, B_0) = \sum m(8, 9, 10, 11, 15) + \sum \Phi(1, 2, 3, 12, 13, 14) = B_3$$

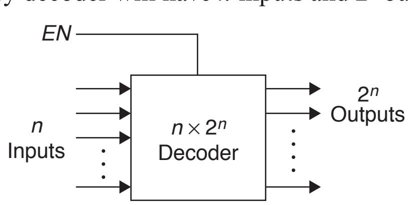
Decoder

A binary code of n-bits is capable of representing up to 2^n elements of distinct elements of coded information.

The three inputs are decoded into eight outputs, each representing one of the minterms of the three input variables.

A decoder is a combinational circuit that converts binary information from n input lines to a maximum 2^n unique output lines.

A binary decoder will have n inputs and 2^n outputs.



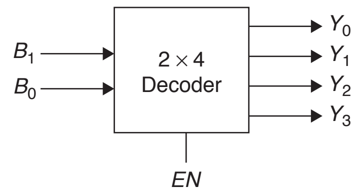


Figure 11 2 × 4 decoder

Table 5 *Truth Table*

EN	B_1	B_0	Y_3	Y_2	Y_1	Y_0
0	X	X	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0

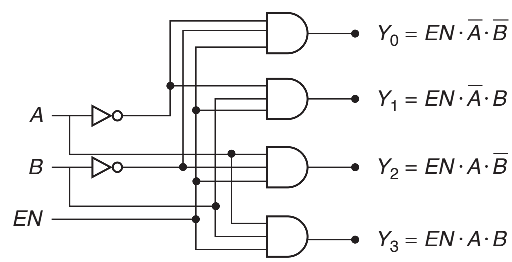


Figure 12 2 × 4 decoder

Decoder outputs are implemented by AND gates, but realization of AND gates at circuit level is done by the NAND gates (universal gates). So, the decoders available in IC form are implemented with NAND gates, i.e., the outputs are in complemented form and outputs are maxterms of the inputs rather than minterms of inputs as in AND gate decoders.

Furthermore, decoders include one or more enable inputs to control the circuit operation. Enable can be either active low/high input.

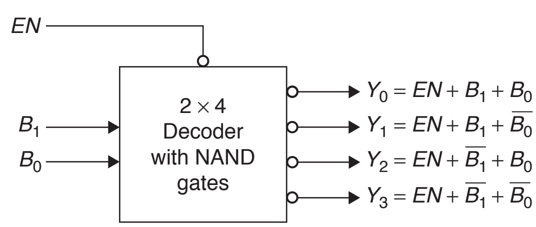


Figure 13 Active low 2 × 4 decoder

Table 6 *Truth Table*

EN	B_1	B_0	Y_3	Y_2	Y_1	Y_0
1	X	X	1	1	1	1
0	0	0	1	1	1	0
0	0	1	1	1	0	1
0	1	0	1	0	1	1
0	1	1	0	1	1	1

The block diagram shown here is 2 × 4 decoder with active low output and active low enable input.

The logic diagram is similar to the previous 2 × 4 decoder, except, all AND gates are replaced by NAND gates and *EN* will have inverter, $\overline{EN}$ is connected to all NAND inputs, as $\overline{EN}$ is active low input for this circuit.

The decoder is enabled when EN is equal to 0.

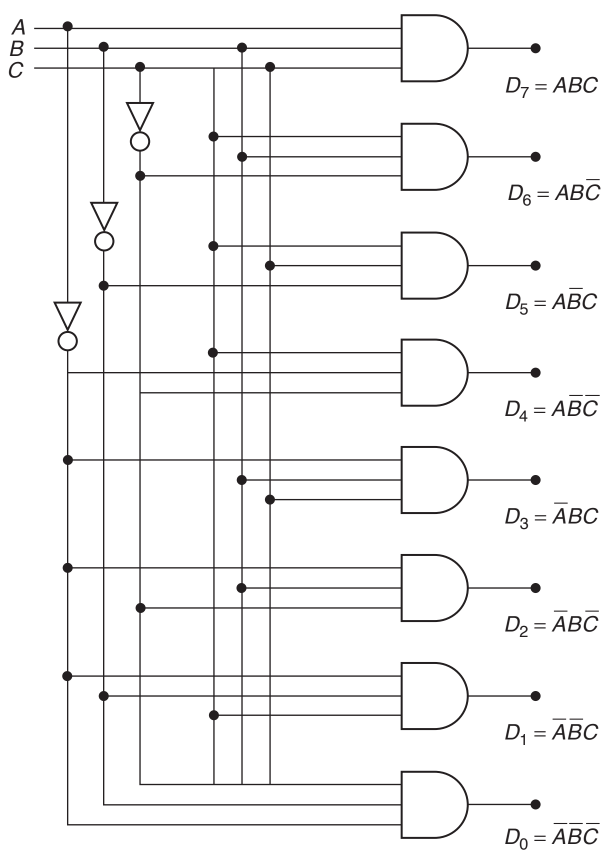
As shown in the truth table, only one output can be equal to 0 at any given time, all other outputs are equal to 1. The output whose value is equal to 0 represents the minterm selected by inputs, enable.

Consider a 3–8 line decoder

Table 7 *Truth Table*

Inputs			Outputs							
A	*B*	*C*	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

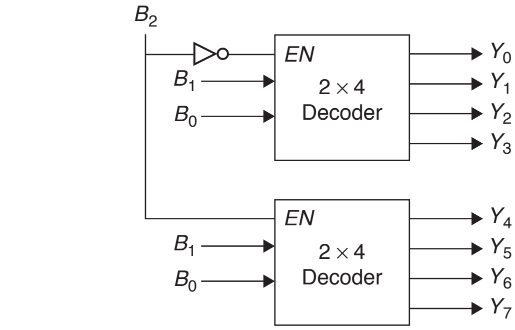
A 3–8 decoder has 3 input lines and 8 output lines, based on the combination of inputs applied for the 3 inputs, one of the 8 output lines will be made logic 1 as shown in the truth table. So, each output will have only one minterm.



Designing High Order Decoders from Lower Order Decoders

Decoder with enable input can be connected together to form larger decoder circuit.

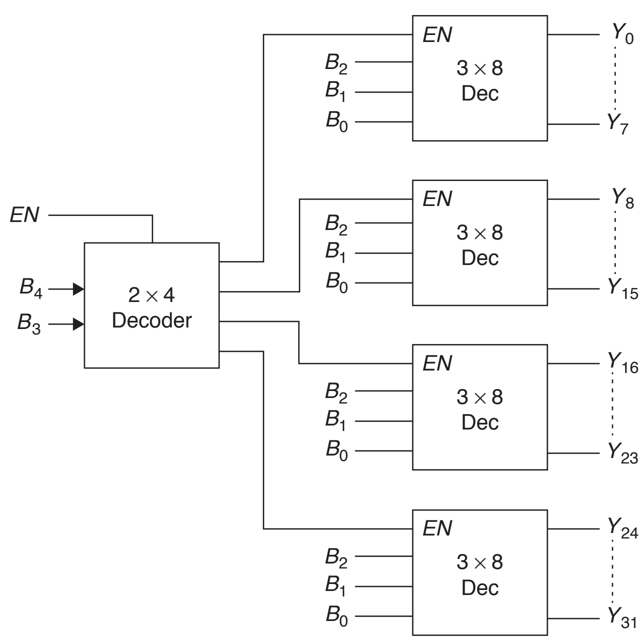
The following configuration shows 3 × 8 decoder with 2 × 4 decoders.



When $B_2 = 0$, top decoder is enabled and other is disabled, for 000–011 inputs, outputs are Y_0–Y_3, respectively, and other outputs are 0.

For $B_2 = 1$, the enable conditions are reversed.

The bottom decoder outputs generates minterms 100–111, while the outputs of top decoder are all 0's. 5 × 32 decoder with 3 × 8 decoders, 2 × 4 decoders



5 × 32 decoder will have 5 inputs $B_4 B_3 B_2 B_1 B_0$. 3 × 8 decoder will have 8 outputs, so 5 × 32 requires four 3 × 8 decoders, and we need one of the 2 × 4 decoders to select one 3 × 8 decoders and the connections are as shown in the circuit above.

Combinational Logic Implementation

An $n \times 2^n$ decoder provides 2^n minterms of *n* input variables. Since any Boolean function can be expressed in sum-of-minterms form, a decoder that generates the minterms of

the function, together with an external OR gate that forms their logical sum, provides a hardware implementation of the function.

Similarly, any function with n inputs and m outputs can be implemented with $n \times 2^n$ decoders and m OR gates.

Example 3: Implement full adder circuit by using 2 × 4 decoder.

Sum = Σ (1, 2, 4, 7), Carry = Σ (3, 5, 6, 7)

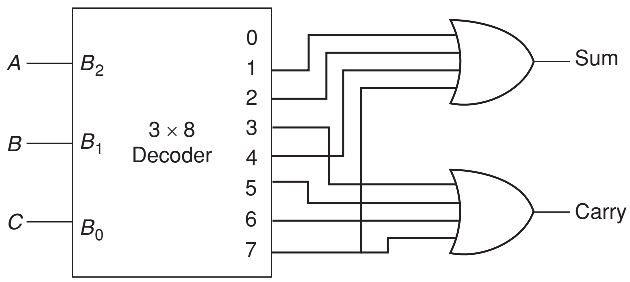
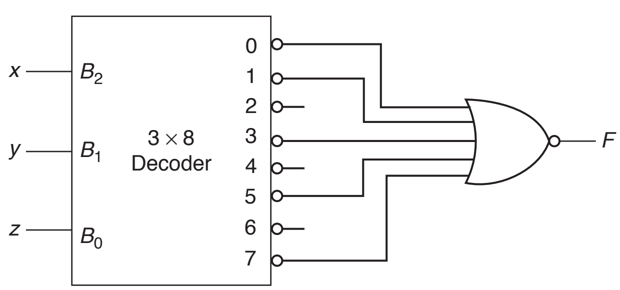


Figure 14 Implementation of full adder circuit with decoder

The 3 × 8 decoder generates the 8 minterms for A, B, and C. The OR gate for output sum forms the logical sum of minterms 1, 2, 4 and 7. The OR gate for output carry forms the logical sum of minterms 3, 5, 6 and 7.

Example 4: The minimized SOP form of output $F(x, y, z)$ is

(A) $x' y + z'$ (B) $x' y' + z'$
(C) $x' y' + z'$ (D) $x' + y' z$



Solution: (C)

The outputs of decoder are in active low state. So, we can express outputs as $\overline{Y_7}, \overline{Y_6} \cdots \overline{Y_0}$

Outputs 0, 1, 3, 5, 7 are connected to NAND gate to form function $F(x, y, z)$

So $$F = \overline{\overline{Y_0} \cdot \overline{Y_1} \cdot \overline{Y_3} \cdot \overline{Y_5} \cdot \overline{Y_7}}$$
$$= Y_0 + Y_1 + Y_3 + Y_5 + Y_7$$
$$= \Sigma(0, 1, 3, 5, 7)$$

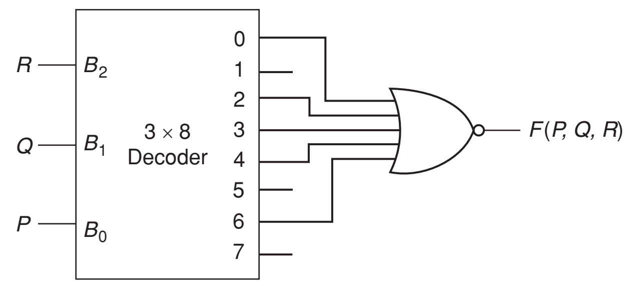
By using K-maps

x \ yz	00	01	11	10
0	1	1	1	
1		1	1	

$F = z + x'y'$

Example 5. The minimal *POS* form of output function $f(P, Q, R)$ is

(A) $P\overline{Q} + PR$ (B) $P + \overline{Q}R$
(C) $P(\overline{Q} + R)$ (D) $Q(\overline{P} + R)$



Solution: (C)

The outputs of decoder are in normal form. 0, 2, 3, 4, 6 outputs are connected to NOR gate to form $F(P, Q, R)$

So $$F = \overline{Y_0 + Y_2 + Y_3 + Y_4 + Y_6}$$
$$= \overline{Y_0} \cdot \overline{Y_2} \cdot \overline{Y_3} \cdot \overline{Y_4} \cdot \overline{Y_6}$$

$Y_0, Y_1, \ldots, Y_7$ indicate minterms, whereas $\overline{Y_0}, \overline{Y_1}, \cdots, \overline{Y_7}$ are maxterms.

So $F = \pi$ (0, 2, 3, 4, 6)

Here, from the decoder circuit MSB is R, LSB is P.

By using K-map

R \ QP	00	01	11	10
0	0		0	0
1	0			0

$$F(P, Q, R) = P(R + \overline{Q})$$

ENCODERS

It is a digital circuit that performs the inverse operation of a decoder.

An encoder has 2^n (or fewer) input lines and n output lines.

It is also known as an octal to binary converter.

Consider an 8–3 line encoder:

Table 8 *Truth Table*

Inputs								Outputs		
D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	A	B	C
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	1	0	0	1	1
0	0	0	0	1	0	0	1	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

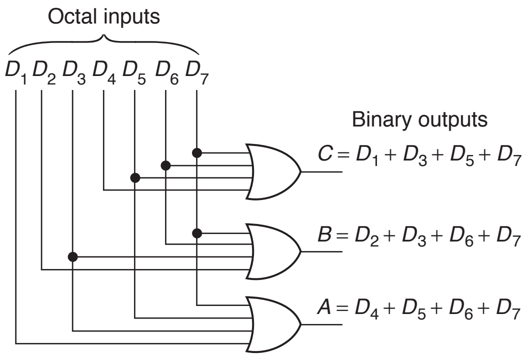


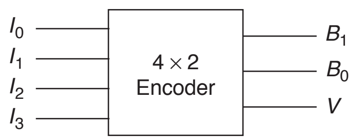
Figure 15 Logic diagram

Priority Encoder

A priority encoder is an encoder circuit that includes the priority function.

When two or more inputs are present, the input with higher priority will be considered.

Consider the 4 × 2 priority encoder.



I_3	I_2	I_1	I_0	B_1	B_0	V
1	X	X	X	1	1	1
0	1	X	X	1	0	1
0	0	1	X	0	1	1
0	0	0	1	0	0	1
0	0	0	0	X	X	0

I_3–I_0 are inputs and B_1 B_0 are binary output bits, valid (V) output is set to 1, when at least one input is present at input ($I_3 - I_0$).

When there is no input present, ($I_3 - I_0 = 0000$) then $V = 0$, for this combination the output B_1B_0 will not be considered.

The higher the subscript number, the higher the priority of the input. Input I_3 has the highest priority, I_2 has the next priority level. Input I_0 has lowest priority level. The Boolean expressions for output B_1 B_0 are

$$B_1 = I_3 + \bar{I}_3 I_2$$
$$= I_3 + I_2$$
$$B_0 = I_3 + \bar{I}_3 \bar{I}_2 I_1$$
$$= I_3 + \bar{I}_2 I_1$$
$$V = I_3 + I_2 + I_1 + I_0$$

Multiplexer

A multiplexer (MUX) is a device that allows digital information from several sources to be converted on to a single line for transmission over that line to a common destination. The MUX has several data input lines and a single output line. It also has data select inputs that permits digital data on any one of the inputs to be switched to the output line.

Depending upon the binary code applied at the selection inputs, one (out of 2^n) input will be gated to single output. It is one of the most widely used standard logic circuits in digital design. The applications of multiplexer include data selection, data routing, operation sequencing, parallel to serial conversion, and logic function generation.

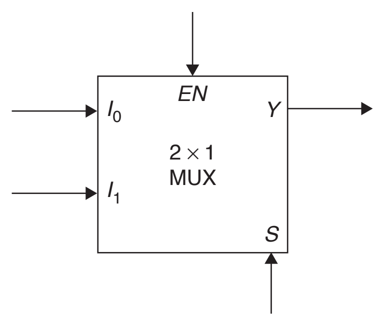
2^n inputs will be controlled by n selection lines and multiplexer will have 1 output, we denote it as $2^n \times 1$ multiplexer (data selector).

In other words, a multiplexer selects 1 out of n input data sources and transmits the selected data to a single output channel, this is called as multiplexing.

Basic 2 × 1 Multiplexer

The figure shows 2 × 1 multiplexer block diagram; it will have 2 inputs—I_0 and I_1, one selection line S, and one output Y. The function table is as shown here.

EN	*S*	*Y*
0	x	0
1	0	I_0
1	1	I_1

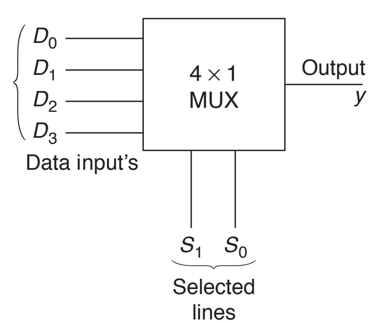


The output equation of 2 × 1 multiplexer is $Y = EN(I_0\bar{S} + I_1 S)$.

When enable is 1, the multiplexer will work in normal mode, else the multiplexer will be disabled.

Sometimes enable input will be active low enable $\overline{EN}$, then $Y = \overline{EN}(I_0\bar{S} + I_1 S)$.

The 4 × 1 Multiplexer



If a binary zero $S_1 = 0$ and $S_0 = 0$ as applied to the data select line the data input D_0 appear on the data output line and so on.

S_1	S_0	y
0	0	D_0
0	1	D_1
1	0	D_2
1	1	D_3

$$y = \overline{S_1}\,\overline{S_0}D_0 + \overline{S_1}S_0D_1 + S_1\overline{S_0}D_2 + S_1S_0D_3$$

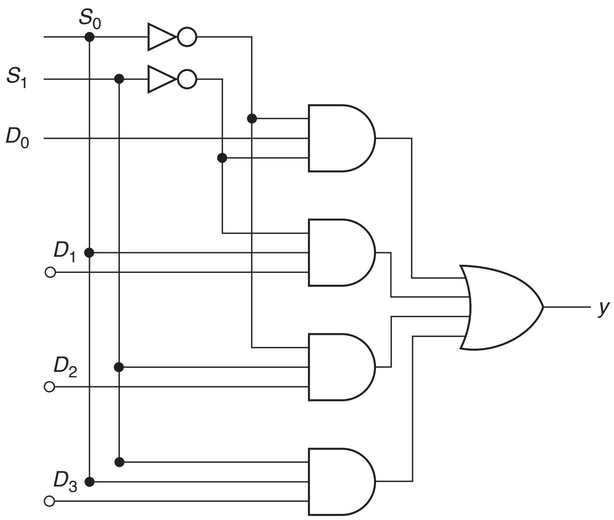


Figure 16 Logic diagram

For 8 × 1 multiplexer with 8 inputs from I_0–I_7 based on selection inputs $S_2 S_1 S_0$, the equation for output

$$Y = I_0\overline{S_2S_1S_0} + I_1\overline{S_2S_1}S_0 + I_2\overline{S_2}S_1\overline{S_0} + I_3\overline{S_2}S_1S_0$$
$$+ I_4S_2\overline{S_1S_0} + I_5S_2\overline{S_1}S_0 + I_6S_2S_1\overline{S_0} + I_7\overline{S_2S_1S_0}$$

From multiplexer equation, we can observe, each input is associated with its minterm (in terms of selection inputs).

Basic Gates by Using MUX

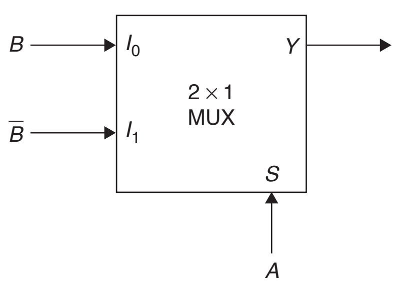


Figure 17 X-OR gate by using 2 × 1 MUX

$Y = \overline{A}B + A\overline{B} =$ X-OR gate, we can interchange inputs A and B also,

By interchanging inputs I_0 and I_1, $Y = \overline{A}\,\overline{B} + AB$, X-NOR gate.

Similarly, we can build all basic gates by using 2 × 1 multiplexer.

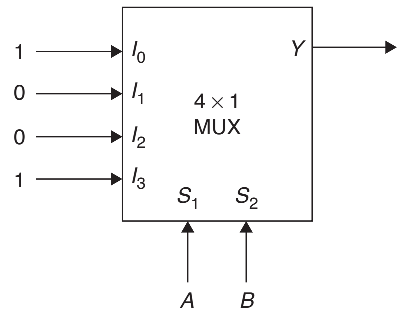
Example 6: If $I_0 = 1$, $I_1 = 0$, $S = A$, then Y is

Solution: $Y = (I_0\overline{S} + I_1S) = \overline{A}$. It Implements NOT gate.

Example 7: What should be the connections to implement NAND gate by using 2 × 1 MUX?

Solution: $Y = \overline{AB} = \overline{A} + \overline{B} = \overline{A} + A\overline{B} = 1\cdot\overline{A} + \overline{B}\cdot A$

By considering $I_0 = 1$, $I_1 = \overline{B}$, $S = A$, we can implement NAND gate, or by interchanging A and B also we can get the same answer.



For the above 4 × 1 multiplexer $Y = \overline{AB} + AB =$ X-NOR gate, similarly to implement 2 input gates by using 4 × 1 multiplexer, the inputs I_0, I_1, I_2, I_3 should be same as the terms in the truth table of that gate.

Logic Function Implementation by Using Multiplexer

Let us consider a full subtractor circuit (borrow) to be implemented by using multiplexer.

Full subtractor borrow (B) is a function of 3 inputs X, Y, Z. The truth table is

X	Y	Z	B	4 × 1 MUX	2 × 1 MUX
0	0	0	0	$B = Z$	$B = Y + Z$
0	0	1	1		
0	1	0	1	$B = 1$	
0	1	1	1		
1	0	0	0	$B = 0$	$B = YZ$
1	0	1	0		
1	1	0	0	$B = Z$	
1	1	1	1		

To implement borrow by using 8 × 1 multiplexer, connect the three variables X, Y, Z directly to selection lines of the multiplexer, and connect the corresponding values of B to inputs, i.e., for $I_0 = 0$, $I_1 = 1$, $I_2 = 1$, etc. as per above truth table.

To implement borrow by using 4 × 1 multiplexer, connect any two variables to selection lines (in this case X, Y) and write output (B) in terms of other variable, for $XY = 00$, output B is same as Z, so connect $I_0 = Z$, similarly 1, 0, Z for remaining inputs.

To implement the function by using 2 × 1 multiplexer, connect 1 variable as selection line (in this case consider X) and write output (B) in terms of other variables, for $X = 0$,

output B is varies as $B = Y + Z$, so connect $I_0 = Y + Z$. For $X = 1$, output B varies as $B = YZ$, connect $I_1 = YZ$.

N–variable function can be implemented by using $2^{N-1} \times 1$ multiplexer without any extra hardware.

Implementation of Higher Order Multiplexer by Using Lower Order Multiplexers

By using lower order multiplexers, we can implement higher order multiplexers, for example by using 4×1 multiplexer, we can implement 8×1 MUX or 16×1 MUX or other higher order multiplexers.

Let us consider implementation of 16×1 MUX by using 4×1 MUX. 16×1 MUX will have inputs I_0–I_{15} and selection lines S_0–S_3, whereas 4×1 MUX will have only 4 input lines, and 2 selection lines, so we require four 4×1 MUX to consider all inputs I_0–I_{15}, and again to select one of the four outputs of these four multiplexers one more 4×1 multiplexer is needed (for which we will connect higher order selection lines S_2 and S_3). So, total of 5, 4×1 multiplexers are required to implement 16×1 MUX.

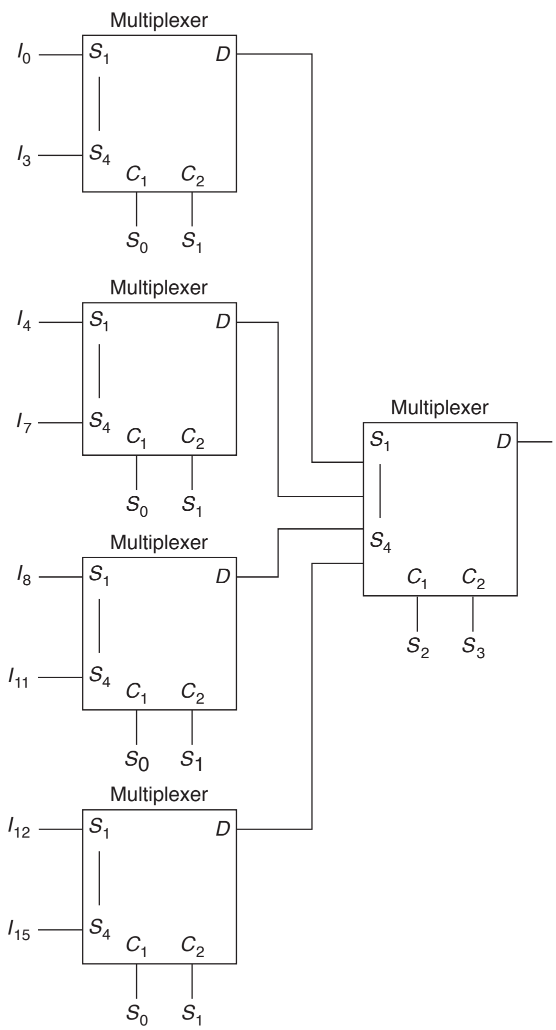


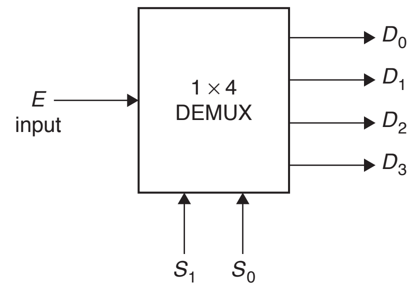
Figure 18 Realization of 16 x 1 multiplexer by using 4 x 1 multiplexers

In a similar fashion, to design 4×1 MUX, we require 3, 2×1 multiplexers, and to design 8×1 multiplexer, we require 7, 2×1 multiplexers.

Demultiplexer

The demultiplexer [DeMUX] basically serves opposite of the multiplexing function. It takes data from one line and distributes them to a given number of output lines.

The other name for demultiplexer is data distributor, as it receives information on a single line and distributes it to a possible 2^n output lines, where n is the number of selection lines, and value of n selects the line.



S_1	S_0	D_3	D_2	D_1	D_0
0	0	0	0	0	E
0	1	0	0	E	0
1	0	0	E	0	0
1	1	E	0	0	0

When $S_1S_0 = 10$; D_2 will be same as input E, and other outputs will be maintained at zero (0).

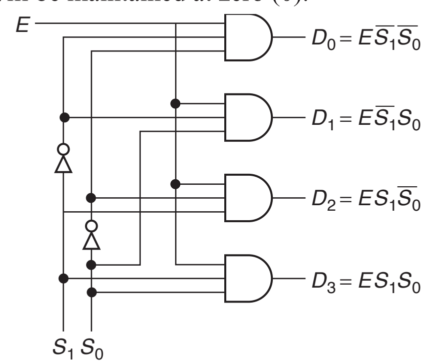
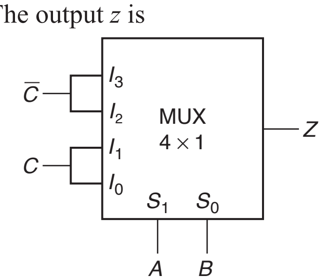


Figure 17 Logic diagram

Solved Examples

Example 1: The multiplexer shown in the figure is a 4 : 1 multiplexer. The output z is

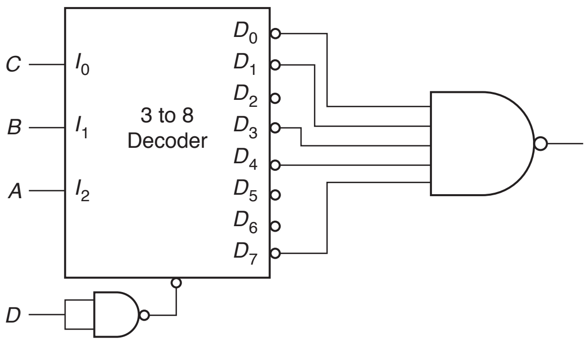


Solution:

A_1	B_0	Z
0	0	C
0	1	C
1	0	$\bar{C}$
1	1	$\bar{C}$

$$\therefore \quad Z = \bar{A}\,\bar{B}C + \bar{A}BC + AB\bar{C} + A\bar{B}\,\bar{C}$$
$$= \bar{B}(\bar{A}C + A\bar{C}) + B(\bar{A}C + A\bar{C})$$
$$= (\bar{A}C + A\bar{C})(B + \bar{B})(x + \bar{x} = 1)$$
$$\therefore \quad = \bar{A}C + A\bar{C} = A \oplus C$$

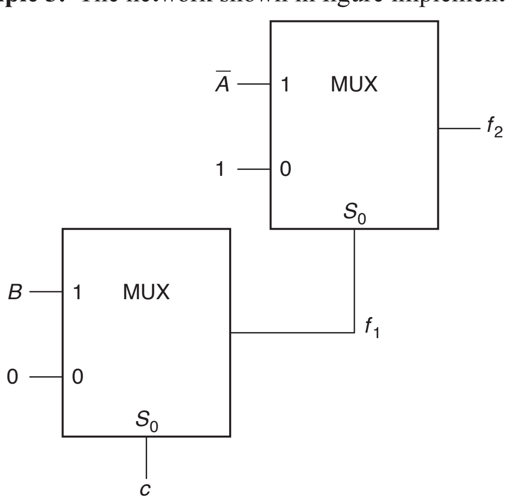
Example 2: The logic circuit shown in figure implements



Solution: $z = D(\bar{A}\,\bar{B}\,\bar{C} + \bar{A}\,\bar{B}C + \bar{A}BC + A\bar{B}\,\bar{C} + ABC)$

$$= D(\bar{A}\,\bar{B}(C + \bar{C}) + BC(\bar{A} + A) + A\bar{B}\,\bar{C})$$
$$\times D(\bar{B}\,\bar{A} + \bar{B}\,\bar{C} + BC)$$
$$= D(B \,\Theta\, C + \bar{A}\bar{B})$$

Example 3. The network shown in figure implements

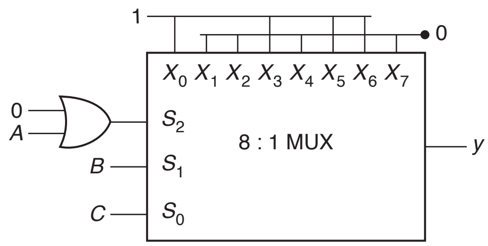


Solution: $f_1 = \bar{C}0 + CB = CB, f_1 = CB$

$$F_2 = \bar{f_1} + f_1\bar{A} = \bar{A} \cdot CB + \overline{CB}$$
$$= \bar{A} + \overline{CB}$$
$$= \bar{A} + \bar{C} + \bar{B} = \overline{ABC}$$

$\therefore$ NAND Gate

Example 4: In the TTL circuit in figure, S_2–S_0 are select lines and x_7–x_0 are input lines. S_0 and X_0 are LSBs. The output Y is

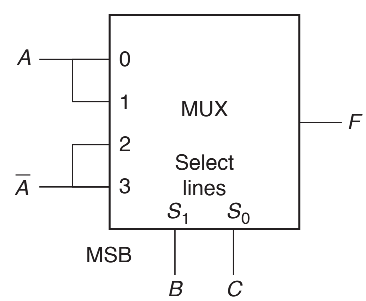


Solution: $S_2 = A, S_1 = B, S_0 = C$

$S_2(A)$	$S_1(B)$	$S_0(C)$	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$Y = \bar{A}\,\bar{B}\,\bar{C} + AB\bar{C} + A\bar{B}C + \bar{A}BC$$
$$= \bar{C}(\bar{A}\,\bar{B} + AB) + C(A\bar{B} + \bar{A}B)$$
$$Y = \bar{C}(\bar{A} \oplus B) + C(A \oplus B) = A \oplus B \odot C$$

Example 5: The logic realized by the adjoining circuit is

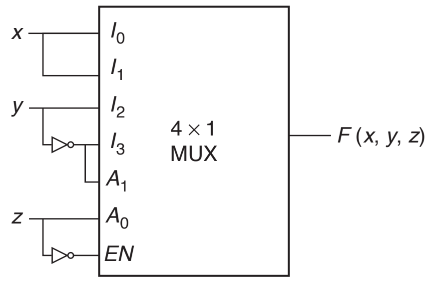


Solution: $F = \bar{B}\,\bar{C}A + \bar{B}CA + B\bar{C}\,\bar{A} + BC\bar{A}$

$$\times \bar{C}(\bar{B}A + B\bar{A}) + C(\bar{B}A + B\bar{A})$$
$$\times A\bar{B} + \bar{A}B(C + \bar{C}) = A \oplus B$$

Example 6: Consider the following multiplexer, where I_0, I_1, I_2, I_3 are four date input lines selected by two address line combinations A_1A_0 = 00, 01, 10, 11, respectively and f

is the output of the multiplexer. EN is the enable input, the function $f(x, y, z)$ implemented by the below circuit is



Solution: $A_1 = \bar{y} \cdot A_0 = z, EN = \bar{z}$

A_1	A_0	S	I
0	0	$(y\bar{z})$	x
0	1	$(y\,z)$	x
1	0	$(\bar{y}\,\bar{z})$	y
1	1	$(\bar{y}z)$	$\bar{y}$

$$f(x, y, z) = \text{S.I} = (xy + 0 + \bar{y}z) \cdot EN$$
$$= xy \cdot \bar{z}$$

Exercises

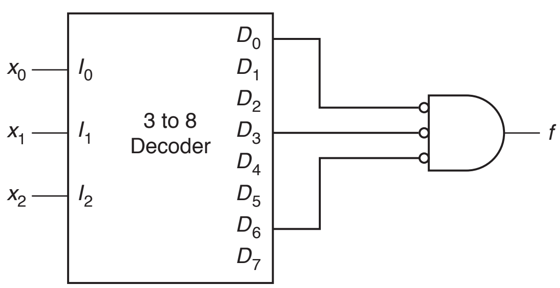
Practice Problems I

Directions for questions 1 to 21: Select the correct alternative from the given choices.

1. The binary number 110011 is to be converted to gray code. The number of gates and type required are
(A) 6, AND (B) 6, X-NOR
(C) 6, X-OR (D) 5, X-OR

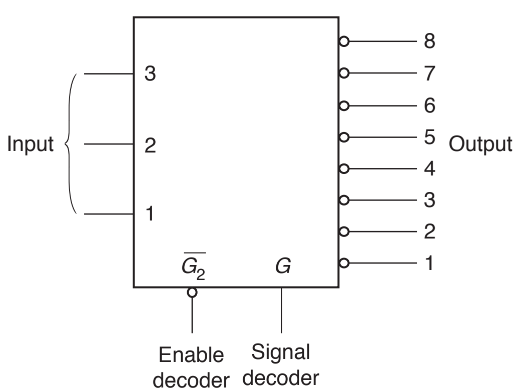
2. The number of 4-to 16-line decoder required to make an 8- to 256-line decoder is
(A) 16 (B) 17
(C) 32 (D) 64

3. $f(x_2, x_1, x_0) = ?$



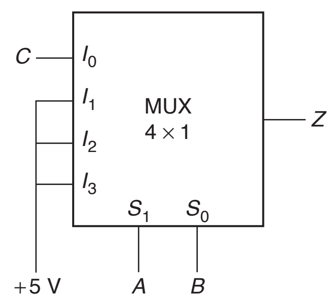
(A) $\pi(1, 2, 4, 5, 7)$ (B) $\Sigma(1, 2, 4, 5, 7)$
(C) $\Sigma(0, 3, 6)$ (D) $\pi(0, 2, 3, 6)$

4. A 3-to-8 decoder is shown below



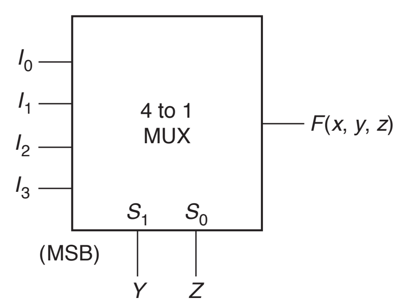
All the output lines of the chip will be high except pin 8, when all the inputs 1, 2, and 3
(A) are high; and G, G_2 are low
(B) are high; and G is low G_2 is high
(C) are high; and G, G_2 are high
(D) are high; and G is high G_2 is low

5. The MUX shown in figure is 4 × 1 multiplexer the output z is



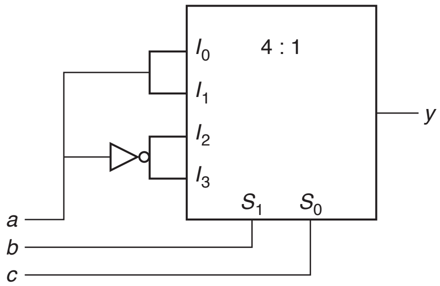
(A) $A\ B\ C$
(B) $A \oplus B \oplus C$
(C) $A \ominus B \ominus C$
(D) $A + B + C$

6. If a 4 to 1 MUX (shown below) realizes a three variable function $f(x, y, z) = xy + x\bar{z}$ then which of the following is correct?



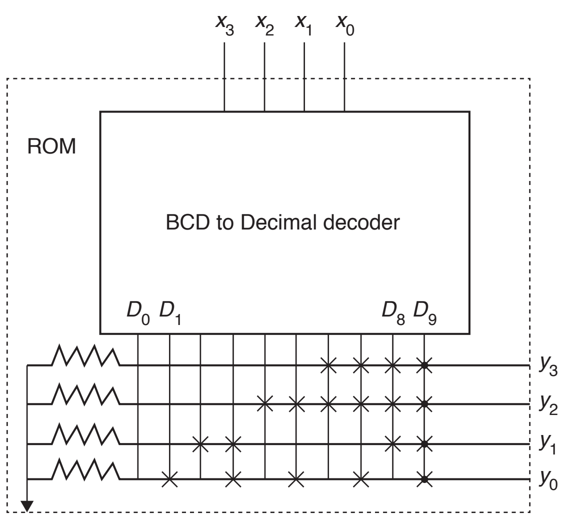
(A) $I_0 = X, I_1 = 0, I_2 = X, I_3 = X$
(B) $I_0 = 0, I_1 = 1, I_2 = Y_1, I_3 = X$
(C) $I_0 = X, I_1 = 1, I_2 = 0, I_3 = X$
(D) $I_0 = X, I_1 = 0, I_2 = X, I_3 = Z$

7. The circuit shown in the figure is same as



(A) two input NAND gate with a and c inputs
(B) two input NOR gate with a and c inputs
(C) two input X-OR gates with a and b inputs
(D) two input X-NOR gate with b and c inputs

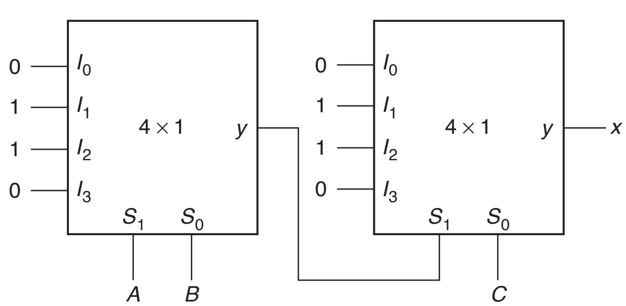
8. If the input x_3, x_2, x_1, x_0 to the ROM in the figure are 8421 BCD numbers, then the outputs y_3, y_2, y_1, y_0 are



(A) gray code numbers (B) 2421 *BCD*
(C) Excess – 3 code numbers (D) 84–2–1

9. A 4-bit parallel full adder without input carry requires
(A) 8 HA, 4 OR gates (B) 8 HA, 3 OR gates
(C) 7 HA, 4 OR gates (D) 7 HA, 3 OR gates

10. In the circuit find X.

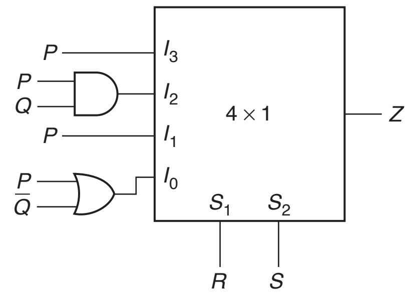


(A) $A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + ABC$

(B) $\bar{A}BC + A\bar{B}C + AB\bar{C} + \bar{A}\bar{B}\bar{C}$

(C) $AB + BC + AC$
(D) $\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}$

11. Find the function implemented.



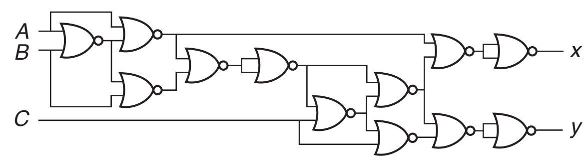
(A) $PQ + PS + \bar{Q}\bar{R}\bar{S}$

(B) $P\bar{Q} + PQ\bar{R} + \bar{P}\bar{Q}\bar{S}$

(C) $P\bar{Q}\bar{R} + \bar{P}QR + PQRS + \bar{Q}\bar{R}\bar{S}$

(D) $PQ\bar{R} + PQR\bar{S} + P\bar{Q}\bar{R}S + \bar{Q}\bar{R}\bar{S}$

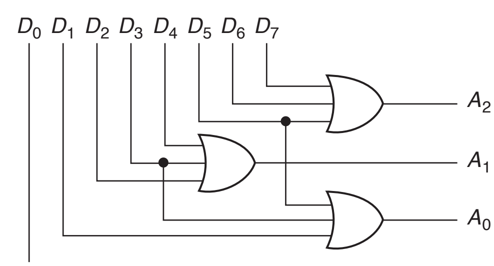
12. Which function is represented by the given circuit?



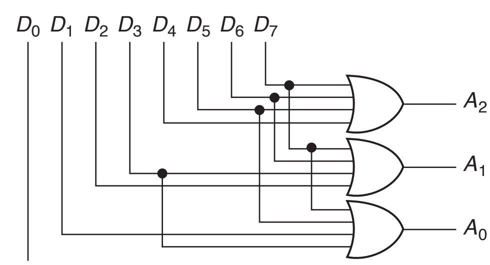
(A) Full adder (B) Full subtractor
(C) Comparator (D) Parity generator

13. Which of the following represents octal to binary encoder?

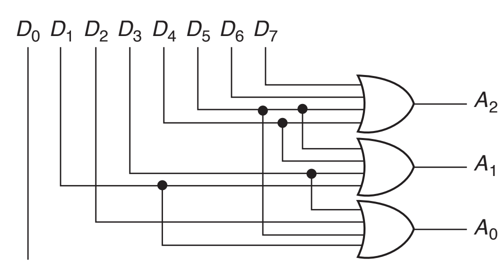
(A)

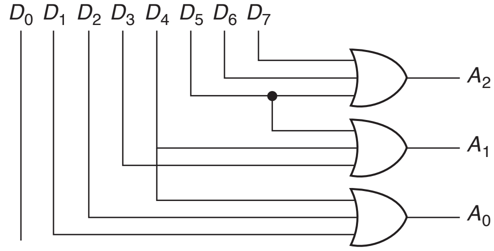


(B)

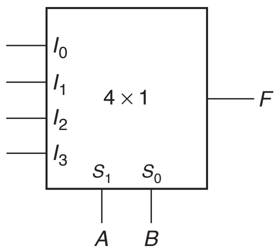


(C)



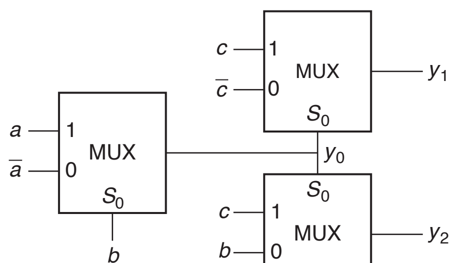
(D)


14. For a MUX to function as a full adder what should be the input provided to the I_0, I_1, I_2, I_3 if the A and B are the select lines?



(A) $I_0 = I_1 = C_{in}$; $I_2 = I_3 = \overline{C}_{in}$
(B) $I_0 = I_1 = \overline{C}_{in}$; $I_2 = I_3 = C_{in}$
(C) $I_0 = I_3 = C_{in}$; $I_1 = I_2 = \overline{C}_{in}$
(D) $I_0 = I_3 = \overline{C}_{in}$; $I_1 = I_2 = C_{in}$

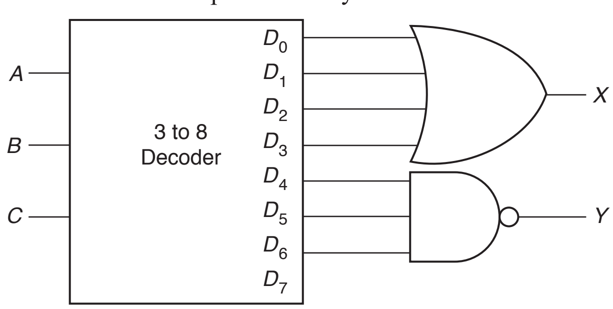
15. The given circuit act as



(A) Full adder (B) Half adder
(C) Full subtractor (D) Half subtractor

16. For a 4 × 16 decoder circuit, the outputs of decoder $(y_0, y_1, y_4 \cdot y_5 \cdot y_{10} \cdot y_{11} \cdot y_{14} \cdot y_{15})$ are connected to 8 input NOR gate, the expression of NOR gate output is
(A) $A \oplus D$ (B) $A \odot D$
(C) $A \odot C$ (D) $A \oplus C$

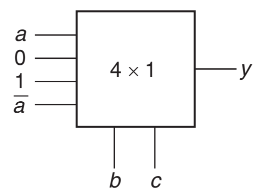
17. The function implemented by decoder is

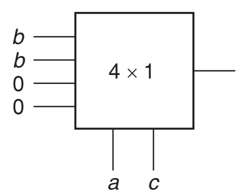


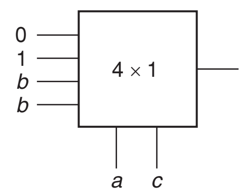
(A) $X = A'BC' + B'C', y = A + B$
(B) $X = A'C' + B'C', y = 1$
(C) $X = \overline{A}, y = 0$
(D) $X = \overline{A}, y = 1$

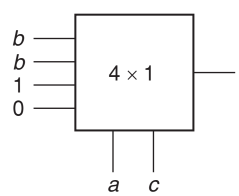
18. A relay is to operate with conditions that it should be on when the input combinations are 0000, 0010, 0101, and 0111. The states 1000, 1001, 1010 don't occur. For rest of the status, relay should be off. The minimized Boolean expression notifying the relationship is
(A) $BC + ACD$
(B) $\overline{B}\overline{D} + \overline{A}BD$
(C) $BD + AC$
(D) $AB + CD$

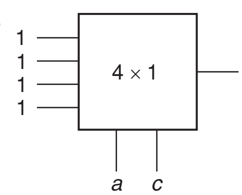
19. If a function has been implemented using MUX as shown, implement the same function with a and c as the select lines



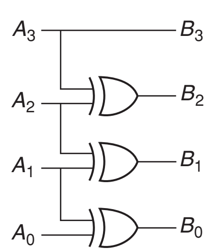
(A)


(B)


(C)


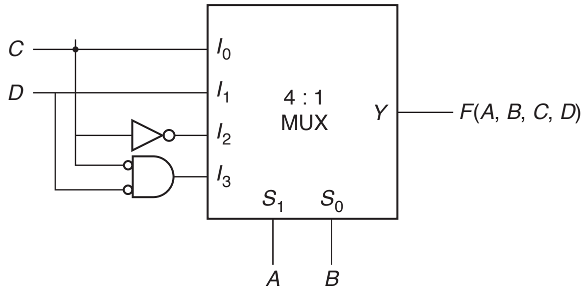
(D)


20. The circuit is used to convert one code to another. Identify it.



(A) Binary to gray
(B) Gray to binary
(C) Gray to XS–3
(D) Gray to 8421

21. The Boolean function realised by logic circuit is



(A) $F = \Sigma m(0, 1, 3, 5, 9, 10, 14)$

(B) $F = \Sigma m(2, 3, 5, 7, 8, 12, 13)$

(C) $F = \Sigma m(1, 2, 4, 5, 11, 14, 15)$

(D) $F = \Sigma m(2, 3, 5, 7, 8, 9, 12)$

Practice Problems 2

Directions for questions 1 to 21: Select the correct alternative from the given choices.

1. For a binary half subtractor having two input A and B, the correct set of logical expression for the outputs D = (A minus B) and X (borrow) are

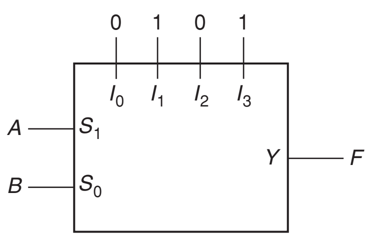
(A) $D = AB + \overline{AB}, X = \bar{A}B$

(B) $D = \overline{AB} + A\bar{B}+, A\bar{B}, X = \bar{A}B$

(C) $D = \bar{A}B + A\bar{B}, X = \bar{A}B$

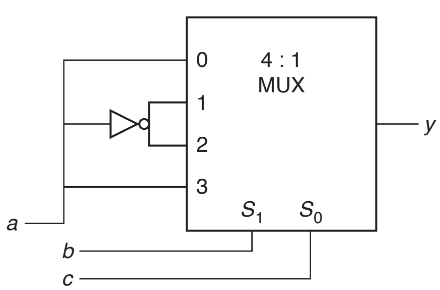
(D) $D = AB + \bar{A}\,\bar{B}, X = \bar{A}B$

2. The function 'F' implemented by the multiplexer chip shown in the figure is



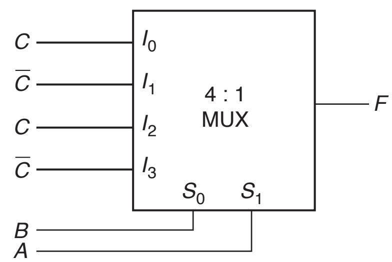
(A) A
(B) B
(C) $\bar{A}B$
(D) $A\bar{B} + \bar{A}B$

3 The following multiplexer circuit is equal to



(A) implementation of sum equation of full adder
(B) implementation of carry equation of full adder
(C) implementation of borrow equation of full substractor
(D) all of the above

4 The output 'F' of the multiplexer circuit shown in the figure will be



(A) $AB + B\bar{C} + \bar{C}A + \bar{B}\bar{C}$
(B) $A \oplus B \oplus C$
(C) $A \oplus B$
(D) $B \oplus C$

5. Full subtractor can be implemented by using
(A) 3-to-8 line decoder only
(B) 3-to-8 line decoder and one OR gate
(C) 3-to-8 line decoder and two OR gates
(D) None

6. What are the difference and borrow equations for the above circuit?
(A) $D = x \,\Theta\, y \,\Theta\, z, B = x'y + yz + zx'$
(B) $D = X \oplus y \oplus z, B = xy + yz + zx$
(C) $D = x \oplus y \oplus z, B = x'y + yz + zx'$
(D) A and C both

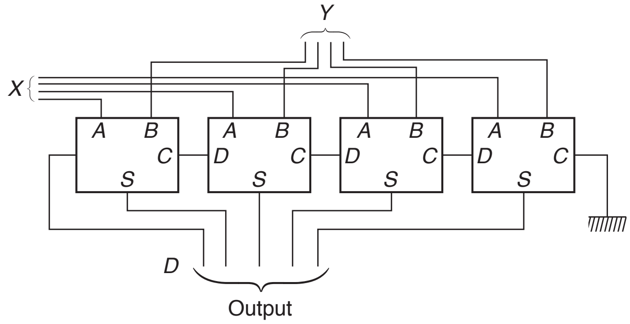
7. Combinational circuits are one in which output depends __________, whereas sequential circuit's output depends __________
(A) only on present input, only on past input
(B) only on present input, only on past and future input
(C) only on present input, only on present input and past output
(D) on present input, on past and present output

8. The sum output of the half adder is given by (assume A and B as inputs)
(A) $S = AB(\overline{A+B})$
(B) $S = (A+B)\overline{AB}$
(C) $S = (A+B)(AB)$
(D) $S = (\bar{A} + \bar{B})(\bar{A}B)$

9. MUX implements which of the following logic?
(A) NAND–XOR
(B) AND–OR
(C) OR–AND
(D) XOR–NOT

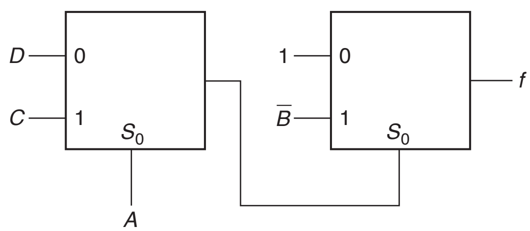
10. A DeMUX can be used as a
(A) Comparator
(B) Encoder
(C) Decoder
(D) Adder

11. If we have inputs as A, B and C and output as S and D. We are given that $S = A \oplus B \oplus C$. $D = BC + \bar{A}B + \bar{A}C$. Which of the circuit is represented by it?



(A) 4-bit adder giving $X + Y$
(B) 4-bit subtractor giving $X - Y$
(C) 4-bit subtractor giving $Y - X$
(D) 4-bit adder giving $X + Y + S$

12. The Boolean function f implemented in the figure using two input multiplexers is

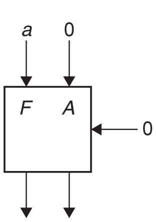


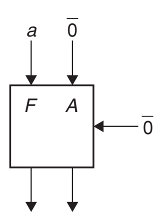
(A) $A\bar{C} + \bar{A}\bar{D} + \bar{D}C + \bar{A}\bar{B}D + A\bar{B}C$
(B) $\bar{A} + A\bar{C} + \bar{A}\bar{D} + \bar{D}\bar{C}$
(C) $\bar{B} + A\bar{C} + \bar{A}\bar{D} + \bar{D}\bar{C}$
(D) $AC + AD + A + B$

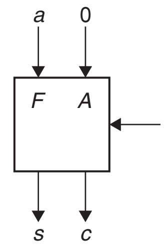
13. The carry generate and carry propagate function of the look ahead carry adder is
(A) $CG = A + B$, $CP = A \oplus B$
(B) $CG = A \oplus B$, $CP = A + B$
(C) $CG = AB$, $CP = A \oplus B$
(D) $CG = AB$, $CP = A + B$

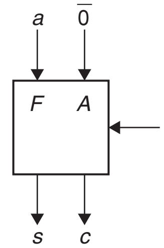
14. If we have a comparator and if E represents the condition for equality i.e., ($A_n \oplus B_n$), if A_n and B_n are to be compared then the expression $A_3\bar{B}_3 + E_3A_2\bar{B}_2 + E_3E_2A_1\bar{B}_1 + E_3E_2E_1A.\bar{B}$. represents which of the condition for a 4-bit number?
(A) $A > B$
(B) $B > A$
(C) $A = B$
(D) None of these

15. When full adder is used to function as a 1-bit incrementor, which of the circuit configurations must be used?

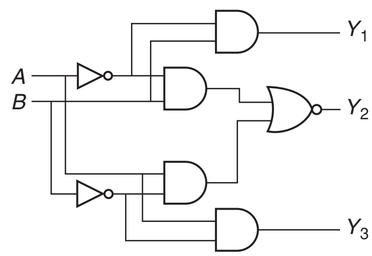
(A)


(B)


(C)


(D)


16. Identify the circuit.



(A) Half adder
(B) Full adder
(C) 1-bit magnitude comparator
(D) Parity generator

17. In order to implement n variable function (without any extra hardware) the minimum order of MUX is
(A) $2^n \times 1$
(B) $2^n \times 1$
(C) $(2^n - 1) \times 1$
(D) $(2^{n-1}) \times 1$

18. A full adder circuit can be changed to full subtractor by adding a
(A) NOR gate
(B) NAND gate
(C) Inverter
(D) AND gate

19. The half adder when implemented in terms of NAND logic is expressed as
(A) $A \oplus B$
(B) $\overline{A\cdot\overline{AB}\cdot B\cdot\overline{AB}}$
(C) $\overline{\overline{A\cdot\overline{AB}}\cdot\overline{B\cdot\overline{AB}}}$
(D) $\overline{\overline{A\cdot\overline{AB}}B\cdot\bar{A}B}$

20. For a DeMUX to act as a decoder, what is the required condition?
(A) Input should be left unconnected and select lines behave as a input to decoder
(B) Input should be always 0 and select line behave as inputs to decoder
(C) Both are same
(D) Input should become enable and select lines behave as inputs to decoder

21. For a full subtractor, which of the combination will give the difference?

(A) $\overline{\overline{(A\oplus B)\overline{(A\oplus B)b_i}}\cdot\overline{b_i\overline{(A\oplus B)b_i}}}$

(B) $\overline{B\cdot\overline{AB}\cdot\overline{b_i(A\oplus B)}}$

(C) $\overline{\bar{A}+\bar{B}}+\overline{\bar{b}_i+\overline{A\oplus B}}$

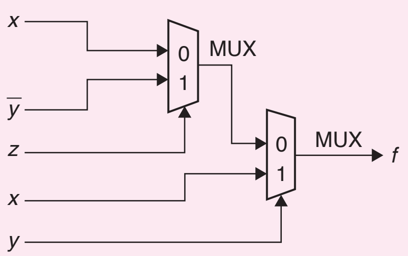
(D) None of these

Previous Years' Questions

1. A 4-bit carry look ahead adder, which adds two 4-bit numbers, is designed using AND, OR, NOT, NAND, NOR gates only. Assuming that all the inputs are available in both complemented and uncomplemented forms and the delay of each gate is one time unit, what is the overall propagation delay of the adder? Assume that the carry network has been implemented using two-level AND–OR logic. **[2004]**

(A) 4 time units (B) 6 time units
(C) 10 time units (D) 12 time units

2.



Consider the circuit above. Which one of the following options correctly represents $f(x, y, z)$? **[2006]**

(A) $x\overline{z} + xy + \overline{y}z$ (B) $x\overline{z} + xy + \overline{yz}$
(C) $xz + xy + \overline{yz}$ (D) $xz + x\overline{y} + \overline{y}z$

3. Given two 3-bit numbers $a_2a_1a_0$ and $b_2b_1b_0$ and c, the carry in, the function that represents the carry generate function when these two numbers are added is **[2006]**

(A) $a_2b_2+a_2a_1b_1+a_2a_1a_0b_0+a_2a_0b_1b_0+a_1b_2b_1+a_1a_0b_2b_0 + a_0b_2b_1b_0$
(B) $a_2b_2+a_2b_1b_0+a_2a_1b_1b_0+a_1a_0b_2b_1+a_1a_0b_2+a_1a_0b_2b_0 + a_2a_0b_1b_0$
(C) $a_2 + b_2 + (a_2 \oplus b_2)(a_1 + b_1 + (a_1 \oplus b_1)(a_0 + b_0))$
(D) $a_2b_2 + \overline{a_2}a_1b_1 + \overline{a_2a_1}a_0b_0 + \overline{a_2}a_0\overline{b_1}b_0 + a_1\overline{b_2}b_1 + \overline{a_1}a_0\overline{b_2}b_0 + a_0\overline{b_2b_1}b_0$

4. We consider the addition of two 2's complement numbers $b_{n-1}b_{n-2} \ldots b_0$ and $a_{n-1}a_{n-2} \ldots a_0$. A binary adder for adding unsigned binary numbers is used to add the two numbers. The sum is denoted by $c_{n-1}c_{n-2} \ldots c_0$ and the carry-out by c_{out}. Which one of the following options correctly identifies the overflow condition? **[2006]**

(A) $c_{out}(\overline{a_{n-1} \oplus b_{n-1}})$
(B) $a_{n-1}b_{n-1}\overline{c_{n-1}} + \overline{a_{n-1}}\,\overline{b_{n-1}}c_{n-1}$
(C) $c_{out} \oplus c_{n-1}$
(D) $a_{n-1} \oplus b_{n-1} \oplus c_{n-1}$

5. Consider numbers represented in 4-bit gray code. Let $h_3h_2h_1h_0$ be the gray code representation of a number n and let $g_3g_2g_1g_0$ be the gray code of $(n + 1)$ modulo 16 value of the number. Which one of the following functions is correct? **[2006]**

(A) $g_0(h_3h_2h_1h_0) = \Sigma(1, 2, 3, 6, 10, 13, 14, 15)$
(B) $g_1(h_3h_2h_1h_0) = \Sigma(4, 9, 10, 11, 12, 13, 14, 15)$
(C) $g_2(h_3h_2h_1h_0) = \Sigma(2, 4, 5, 6, 7, 12, 13, 15)$
(D) $g_3(h_3h_2h_1h_0) = \Sigma(0, 1, 6, 7, 10, 11, 12, 13)$

6. How many 3-to-8 line decoders with an enable input are needed to construct a 6-to-64 line decoder without using any other logic gates? **[2007]**

(A) 7 (B) 8
(C) 9 (D) 10

7. Suppose only one multiplexer and one inverter are allowed to be used to implement any Boolean function of n variables. What is the minimum size of the multiplexer needed? **[2007]**

(A) 2^n line to 1 line (B) 2^{n+1} line to 1 line
(C) 2^{n-1} line to 1 line (D) 2^{n-2} line to 1 line

8. In a look-ahead carry generator, the carry generate function G_i and the carry propagate function P_i for inputs A_i and B_i are given by:

$P_i = A_i \oplus B_i$ and $G_i = A_iB_i$

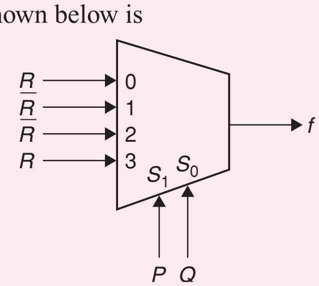
The expressions for the sum bit S_i and the carry bit C_{i+1} of the look-ahead carry adder are given by:

$S_i = P_i \oplus C_i$ and $C_{i+1} = G_i + P_iC_i$, where C_0 is the input carry.

Consider a two-level logic implementation of the look-ahead carry generator. Assume that all P_i and G_i are available for the carry generator circuit and that the AND and OR gates can have any number of inputs. The number of AND gates and OR gates needed to implement the look-ahead carry generator for a 4-bit adder with S_3, S_2, S_1, S_0 and C_4 as its outputs are respectively: **[2007]**

(A) 6, 3 (B) 10, 4
(C) 6, 4 (D) 10, 5

9. The Boolean expression for the output f of the multiplexer shown below is



(A) $\overline{P \oplus Q \oplus R}$
(B) $P \oplus Q \oplus R$
(C) $P + Q + R$
(D) $\overline{P + Q + R}$

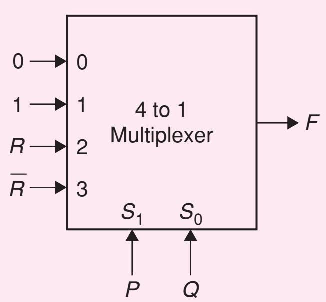
10. The amount of ROM needed to implement a 4-bit multiplier is **[2012]**
(A) 64 bits
(B) 128 bits
(C) 1K bits
(D) 2K bits

11. In the following truth table, $V = 1$ if and only if the input is valid.

Inputs				Outputs		
D_0	D_1	D_2	D_3	X_0	X_1	V
0	0	0	0	x	x	0
1	0	0	0	0	0	1
x	1	0	0	0	1	1
x	x	1	0	1	0	1
x	x	x	1	1	1	1

What function does the truth table represent? **[2013]**
(A) Priority encoder
(B) Decoder
(C) Multiplexer
(D) Demultiplexer

12. Consider the 4-to-1 multiplexer with two select lines S_1 and S_0 given below.



The minimal sum-of-products form of the Boolean expression for the output F of the multiplexer is **[2014]**

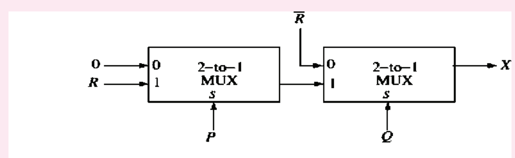
(A) $\bar{P}Q + Q\bar{R} + P\bar{Q}R$
(B) $\bar{P}Q + \bar{P}Q\bar{R} + PQ\bar{R} + P\bar{Q}R$
(C) $\bar{P}QR + \bar{P}Q\bar{R} + Q\bar{R} + P\bar{Q}R$
(D) $PQ\bar{R}\ \bar{P}QR + \bar{P}Q\bar{R} + Q\bar{R} + P\bar{Q}R$

13. Consider the following combinational function block involving four Boolean variables x, y, a, b, where x, a, b are inputs and y is the output. **[2014]**
$f(x, y, a, b)$
{
if (x is 1) $y = a$;
else $y = b$;
}
Which one of the following digital logic blocks is the most suitable for implementing this function?
(A) Full adder (B) Priority encoder
(C) Multiplexer (D) Flip–flop

14. Let ⊕ denote the Exclusive OR (X-OR) operation. Let '1' and '0' denote the binary constants. Consider the following Boolean expression for F over two variables P and Q:
$F(P, Q) = ((1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0))$ **[2014]**
The equivalent expression for F is
(A) $P + Q$ (B) $\overline{P + Q}$
(C) $P \oplus Q$ (D) $\overline{P \oplus Q}$

15. A half adder is implemented with XOR and AND gates. A full adder is implemented with two half adders and one OR gate. The propagation delay of an XOR gate is twice that of an AND/OR gate. The propagation delay of an AND/OR gate is 1.2 microseconds. A 4-bit ripple-carry binary adder is implemented by using four full adders. The total propagation time of this 4-bit binary adder in microseconds is _____ **[2015]**

16. Consider the two cascaded 2-to-1 multiplexers as shown in the figure.



minimal sum of products form of the output x is

The minimal sum of products form of the output X is **[2016]**
(A) $\bar{P}\ \bar{Q}\ + PQR$ (B) $\bar{P}\ Q + QR$
(C) $PQ + \bar{P}\ \bar{Q}\ R$ (D) $\bar{P}\ \bar{Q}\ + PQR$

17. When two 8-bit numbers $A_7 \ldots A_0$ and $B_7 \ldots B_0$ in 2's complement representation (with A_0 and B_0 as the least significant bits) are added using a **ripple-carry adder**, the sum bits obtained are $S_7 \ldots S_0$ and the carry bits are $C_7 \ldots C_0$. An overflow is said to have occurred if **[2017]**
(A) the carry bit C_7 is 1
(B) all the carry bits $(C_7, \ldots, C_0)$ are 1
(C) $(A_7 \cdot B_7 \cdot \overline{S_7} + \overline{A_7} \cdot \overline{B_7} \cdot S_7)$ is 1
(D) $(A_0 \cdot B_0 \cdot \overline{S_0} + \overline{A_0} \cdot \overline{B_0} \cdot S_0)$ is 1

Answer Keys

Exercises

Practice Problems 1

1. D	**2.** B	**3.** B	**4.** D	**5.** D	**6.** A	**7.** C	**8.** B	**9.** D	**10.** A
11. A	**12.** B	**13.** B	**14.** C	**15.** C	**16.** D	**17.** D	**18.** B	**19.** C	**20.** A
21. D									

Practice Problems 2

1. C	**2.** B	**3.** A	**4.** D	**5.** C	**6.** C	**7.** C	**8.** B	**9.** B	**10.** C
11. B	**12.** C	**13.** C	**14.** A	**15.** C	**16.** C	**17.** B	**18.** C	**19.** C	**20.** D
21. A									

Previous Years' Questions

1. B	**2.** A	**3.** A	**4.** B	**5.** C	**6.** C	**7.** C	**8.** B	**9.** B	**10.** D
11. A	**12.** A	**13.** C	**14.** D	**15.** 19.2	**16.** D	**17.** C			

Chapter 4

Sequential Circuits

LEARNING OBJECTIVES

- *Sequential circuit*
- *Basic storage elements*
- *Latches (SR Latch, D Latch, JK Latch)*
- *Flip-flops (JK flip-flop, T flip-flop, D flip-flop)*
- *Counters*
- *Asynchronous counter design*
- *Synchronous counter design*
- *Registers*
- *Various types of registers*
- *Application of shift register*

Sequential Circuits

In sequential circuits the output depends on the input as well as on the previous history of output, i.e., they contain memory elements.

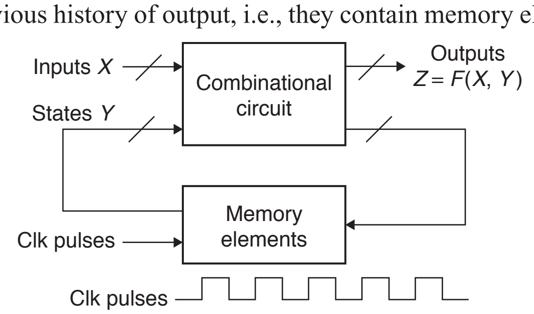


Figure 1 Block diagram of sequential circuit

Table 1 *Comparison between combinational and sequential circuits*

Combinational Circuits	Sequential Circuits
1. Output at any time depends on the combine set of input applied to it simultaneously at that instant of time	Output depends on the present input as well as on the previous history of output
2. Contains no memory element	Contains at least one memory element
3. Easy to design due to absence of memory	Difficult to design
4. Totally described by the set of output values	Its performance is totally described by the set of subsequent values as well as set of output values
5. Faster in speed because all inputs are primary inputs and applied simultaneously	Slower in speed because secondary inputs are also needed which are applied after delay
6. It need more hardware for realization	Less hardware required
7. Expensive in cost	Cheap in cost

Sequential circuits are of two types:

1. Clocked or synchronous
2. Unclocked or asynchronous

In synchronous sequential circuits the logic circuits action is allowed to occur in synchronization with the input clock pulse from a system clock.

In asynchronous sequential circuits the logic sequential action is allowed to occur at any time.

Basic Storage Elements

Latches and flip-flops

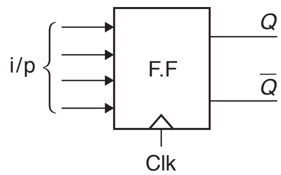
A storage element in digital circuit can maintain a binary state indefinitely until directed by an input signal to switch states. Storage elements that operate with signal levels (i.e., level triggering of signal inputs) are referred to as latches. Those controlled by a clock transition (i.e., edge triggering) are flip-flops.

Latches and filp-flops are related because latches are basic circuits from which all flip-flops are constructed. Latches are useful for storing binary information and for the design of asynchronous sequential circuits. But latches are not practical for use in synchronous sequential circuits, so we use flip-flops.

Flip-flops

They are also known as bistable multivibrators. This is a basic memory element to store 1-bit of information 0 or 1 and is used in storage circuits, counters, shift register, and many other computer applications. It has two stable states: 1 and 0. The high state is called set state and zero as reset.

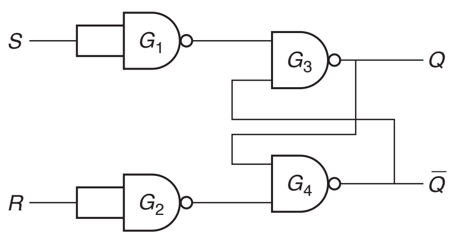
It has two outputs one being the complement of the other usually designated by Q and $\bar{Q}$.



There are different types of flip-flops S-R flip-flop, D-flip-flop, T-flip-flop, J-K flip–flops, etc.

Latches

(i) S-R Latch: The simplest latch is called S-R latch. S-R means Set-Reset. It has two outputs Q and $\bar{Q}$ and two inputs S and R, which represent set or reset signal.



Above figure shows two cross coupled gates G_3 and G_4 and inverters G_1 and G_2. Here output of G_3 is connected to the input of G_4 and output of G_4 is applied to the input of G_3. $S = 1$, $R = 0$ output of $G_1 = 0$ and $G_2 = 1$. Since one of the input of G_3 is 0, so its output will be certainly 1 and consequently both input of G_4 will be 1 and the output $\bar{Q} = 0$.

For $S = 1$, $R = 0$, $Q = 1$, $\bar{Q} = 0$. $S = 0$, $R = 1$ the output will be $Q = 0$ and $\bar{Q} = 1$. The first of the input condition $S = 1$ and $R = 0$ makes $Q = 1$ which referred as the set state and the second condition $S = 0$ and $R = 1$ makes $Q = 0$ which is referred as reset state.

For $S = 0$ and $R = 0$ output of both G_1 and G_2 will be one and hence there will be no change in Q and $\bar{Q}$.

For $S = R = 1$, both the outputs Q and $\bar{Q}$ will try to become one, which produces invalid results and should not be used for the above latch.

Input		Output		
S	R	Q	$\bar{Q}$	State
1	0	1	0	Set
0	1	0	1	Reset
0	0	0	0	No change
1	1	?	?	Invalid

(ii) SR latch by using NAND/NOR gates: The SR latch is a circuit with two cross-coupled NOR gates or two cross-coupled NAND gates. Two inputs labelled S for set and R for reset. Latch will have two outputs:

- Q: output state in normal form and
- Q': output state in complemented form.

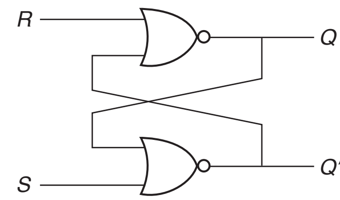


Figure 2 Logic diagram for SR latch

S	R	Q_{n+1}	Q'_{n+1}	
0	0	Q_n	Q'_n	(no change)
0	1	0	1	(Reset)
1	0	1	0	(set)
1	1	0	0	(invalid)

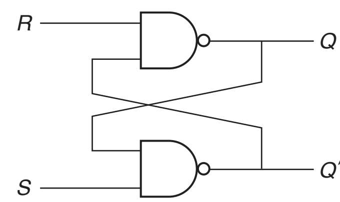
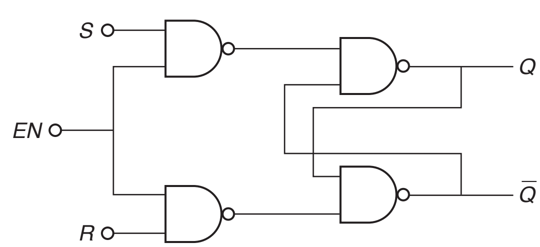


Figure 3 $\bar{S}\,\bar{R}$ Latch

S	R	Q_{n+1}	$\bar{Q}_{n+1}$	
0	0	1	1	(Invalid)
0	1	1	0	(Set)
1	0	0	1	(Reset)
1	1	Q_n	Q'_n	(No change)

$\bar{S}\,\bar{R}$ latch is active low SR latch

(iii) SR latch with control input: The working of gated SR latch is exactly the same as SR latch when the EN pulse is present. When the EN pulse is not present (EN pulse = 0) the gates G_1 and G_2 are inhibited and will not respond to the input.



Characteristic table of SR latch shows the operation of latch in tabular form. Q_t stands as the binary state of the latch before the application of latch pulse and referred to as the present state. The S and R columns give the possible values of the inputs and Q_{t+1} is the state of the latch after the application of a single pulse, referred to as next stage. EN input is not included in the characteristic table.

Characteristic table for SR latch is given below:

Q_t	S	R	Q_{t+1}
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	X
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	X

Characteristic equation of the latch is derived from the K-map.

Q_t \ SR	00	01	11	10
0			×	1
1	1		×	1

$$\therefore Q_{t+1} = S + \bar{R}Q_t$$

This equation specifies the value of the state as a function of the present state and the inputs.

(iv) Preset and clear inputs: For the latch/flip-flop, when the power is switched ON, the state of the circuit is uncertain. It can be either $Q = 0$ (reset) or $Q = 1$ (set) state.

In many applications it is desired to set or reset the circuit, so that initial state of the circuit will be known. This is accomplished by using the asynchronous, inputs referred to as preset (Pr) and clear (Clr), inputs.

These inputs can be applied any time, and are not synchronized with EN input/Clr input.

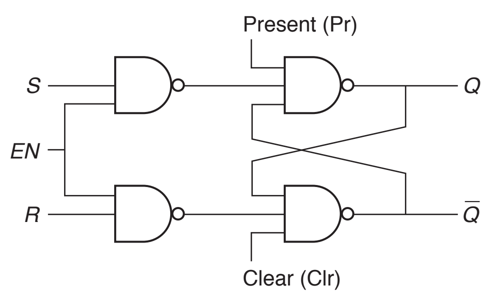
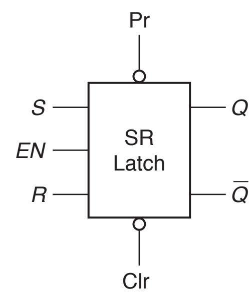


Figure 4 SR latch with Pr and Clr inputs

If Pr = Clr = 1, the circuits operates as of S–R latch explained previously.
If Pr = 0, Clr = 1, the output Q will become 1, which in turn changes $\bar{Q} = 0$.
If Pr = 1, Clr = 0 the output $\bar{Q}$ will become 1, which in turn changes $Q = 0$.
If Pr = Clr = 0, both Q and $\bar{Q}$ will become 1, which is invalid case, so Pr = Clr = 0 condition must not be used.

Pr	Clr	Q_{n+1}
1	1	Q – No change
0	1	1 – Set
1	0	0 – Reset
0	0	X – Invalid



(v) D latch (Transparent latch): One way to eliminate the invalid condition of SR latch (when $S = R = 1$) is to ensure that inputs S and R are never equal to 1 at the same time.

By connecting a NOT gate between S and R inputs. i.e, complement of S will be given to R, we can form D latch as shown in block diagram.

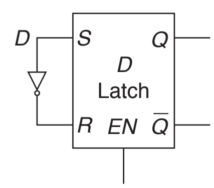


Figure 5 Block diagram for D latch

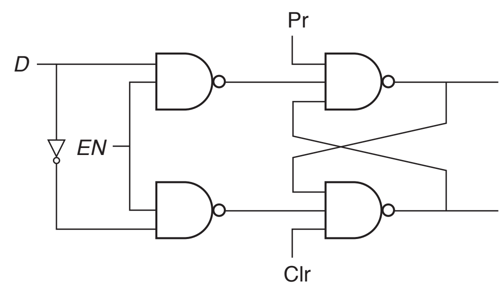


Figure 6 Logic diagram for D latch

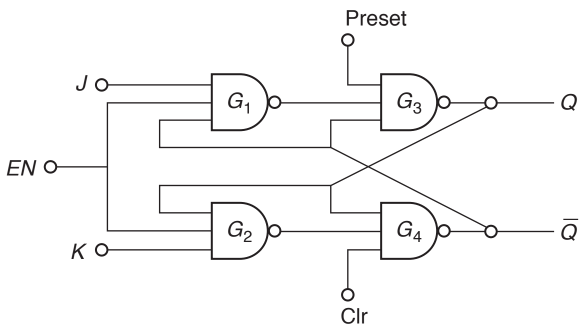
EN	D	Q_{n+1}
0	X	Q_n – No change (Disabled)
1	0	0 – Reset state
1	1	1 – Set state

When EN = 0, the circuit will be disabled and input D will not have only effect on output, and output will be same as previous state.

When EN = 1, $D = 0$, i.e., $S = 0$, $R = 1$ which makes output $Q = 0$ and $\bar{Q} = 1$ (Reset state).

When EN = 1, $D = 1$, i.e., $S = 1$, $R = 0$ which makes output $Q = 1$, and $\bar{Q} = 0$ (Set state).

(vi) JK latch: The function of JK latch is identical to that of SR latch except that it has no invalid state as that of SR latch where $S = R = 1$. In this case the state of the output is changed as complement of previous state.



EN	J	K	Q_{t+}	
1	0	0	Q_t	No change
1	0	1	0	Reset
1	1	0	1	Set
1	1	1	$\bar{Q}_t$	Toggle
0	x	x	Q_t	No change

JK latch by using SR latch: The uncertainty of SR flip-flop (when $S = 1$, $R = 1$) can be eliminated by converting it into JK latch.

The data inputs J and K, which are ANDed with $\bar{Q}$ and Q respectively, to obtain S and R inputs.

$$S = J \cdot \bar{Q}, \quad R = K \cdot Q$$

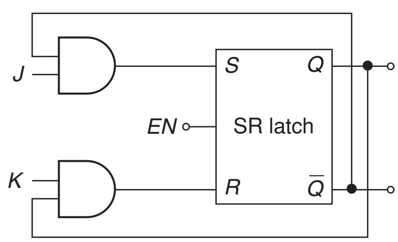
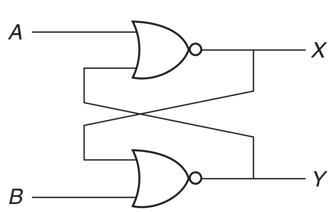


Figure 7 JK latch by using SR latch

J	K	S	R	Q_{n+1}	$\bar{Q}_{n+1}$
0	0	0	0	Q_n	$\bar{Q}_n$-No change
0	1	0	Q_n	0	1-Reset
1	0	$\bar{Q}_n$	0	1	0-Set
1	1	$\bar{Q}_n$	Q_n	$\bar{Q}_n$	Q_n-Toggle

Example 1: The following binary values were applied to A and B inputs of NOR gate latch shown in the figure, in the sequence indicated below. $A = 1, B = 0$; $A = 1, B = 1$; $A = 0$, $B = 0$. The corresponding stable X, Y outputs will be



(A) 10, 01, 10 or 01
(B) 11, 00, 10
(C) 01, 00, 10 or 01
(D) 10, 11, 10 or 01

Solution: (C)

Given circuit is RS latch with NOR gates.

By comparing with RS latch $A = R$, $B = S$, and $X = Q$, $Y = \bar{Q}$, so from truth table of RS latch

S/B	R/A	Q/X	$\bar{Q}$/Y	
0	1	0	1	Reset (Invalid)
1	1	0	0	
0	0	1	0	(Same as previous state)
		0	1	

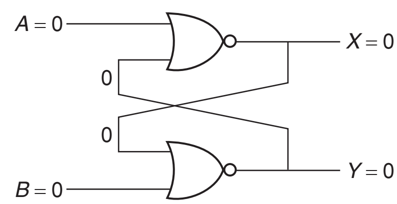
After invalid case $S = 1$, $R = 1$, i.e., $A = B = 1$,

The output $Q = 0, \bar{Q} = 0$, i.e., $X = Y = 0$

By applying $A = 0$, $B = 0$

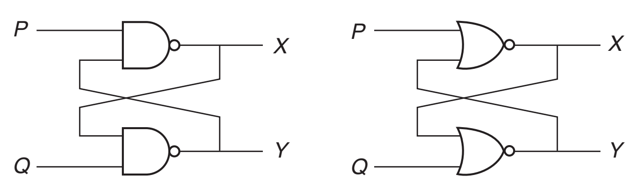
The output X becomes $\overline{(0+0)} = 1$ and which in turn changes

$Y = \overline{(B + X)} = \overline{(0+1)} = 0$



(or) the output Y becomes $\overline{(0+0)} = 1$ and which in turn changes $X = \overline{(Y + A)} = \overline{(0+1)} = 0$. So, output ($X$, Y) cannot be predicted after the invalid condition. So, $X = 0$, $Y = 1$ or $X = 1$, $Y = 0$

Example 2: Refer to the NAND and NOR latches shown in the figure the inputs (P, Q) for both the latches are first made (1, 0) and then after a few seconds, made (0, 0). The corresponding stable outputs (X, Y) are



(A) NAND: first (0, 1) then (0, 1); NOR: first (1, 0) then (1, 0)
(B) NAND: first (0, 1) then (1, 1); NOR: First (0, 1) then (0, 1)
(C) NAND: first (1, 0) then (0, 0); NOR: first (1, 0) then (1, 0)
(D) NAND: first (1, 0), then (1, 0); NOR: first (1, 0) then (1, 1)

Solution: (B)

From the truth table of SR latch and $\bar{S}\,\bar{R}$ latch SR latch with NOR gates:

For $(P, Q) = (1, 0) = (R, S)$ output $(X, Y) = (Q, \bar{Q}) = (0, 1)$

Then (P, Q) are made (0, 0), i.e., (R, S) = (0, 0), which results in no change at output. So, $(X, Y) = (Q, \bar{Q}) = (0,1)$

SR latch with NAND gates:

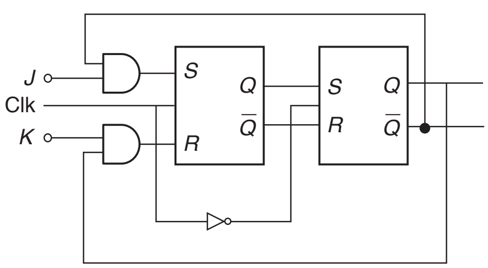
For $(P, Q) = (1, 0) = (S, R)$ output $(X, Y) = (Q, \bar{Q}) = (0,1)$. Then (P, Q) are made (0, 0), i.e., $(S, R) = (0, 0)$ which is invalid conditions for $\bar{S}\,\bar{R}$ latch. So, $(X, Y) = (Q, \bar{Q}) = (1,1)$

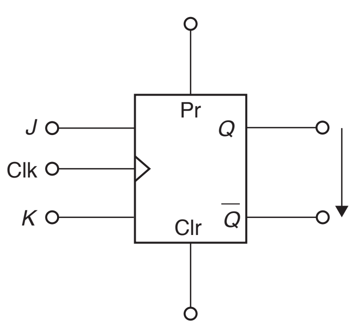
(vii) Race around condition: The difficulties of both the inputs ($S = R = 1$) being not allowed in an SR latch is eliminated in JK latch by using the feedback connection from the output to the input of the gate G_1 and G_2. In a normal JK latch if $J = K = 1$ and $Q = 0$ and enable signal is applied without RC differentiator, after a time interval Δt (the propagation delay through two NAND gate in series) the output will change to $Q = 1$. Now we have $J = K = 1$ and $Q = 1$ and after another time interval of Δt the output will change back to $Q = 0$. Hence for the duration of (t_p) of the enable signal the output will oscillates back and forth between 0 and 1. At the end of the enable signal the values of Q is uncertain. This situation is referred to as race around condition.

The race around condition can be avoided if enable time period $t_p < \Delta t$ but it may be difficult to satisfy this condition, because of very small propagation delays in ICs. To solve this problem the enable signals are converted to narrows spike using RC differentiator circuit having a short time constant. Its output will be high during the high transmission time of the enable. Another method to avoid this problem is master-slave JK flip-flop.

Flip-Flops

(i) Master-slave JK flip-flop: This is a cascade of 2 SR latches with feedback from the output of the second SR latch to the inputs of the first as shown in the figure below.





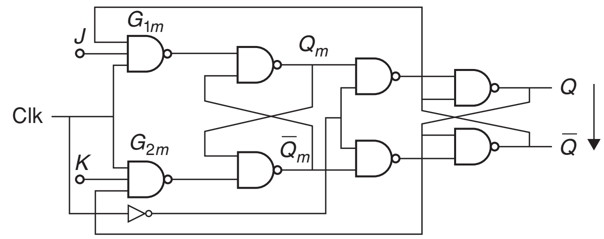


Figure 8 Logic diagram of JK flip-flop

Positive clock pulse is applied to the first latch and the clock pulse will be inverted before its arrival at the second latch. When Clk = 1, the first latch is enabled and the outputs Q_m and $\bar{Q}_m$ responds to the inputs J and K, according to the truth table of JK latch. At this time the 2nd latch is inhibited because its clock is low $(\overline{\text{Clk}} = 0)$. When the clock goes low (Clk = 0), the first latch is inhibited and the second is enabled. Therefore, the outputs Q and $\bar{Q}$ follow the outputs Q_m and $\bar{Q}_m$, respectively. Since the second latch simply follows the first one, it is referred to as slave and the first one as the master. Hence this configuration is known as master-slave JK flip-flop. In this circuit, the input to the gate G_{1m} and G_{2m} do not change, during the clock pulse levels.

The race around condition does not exist.

Table 2 *State/characteristic Table*

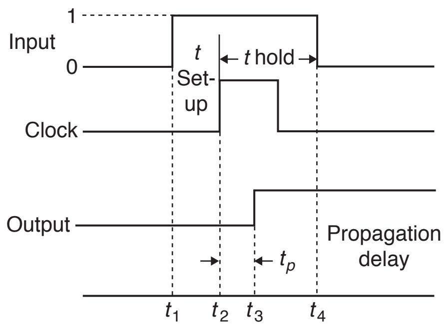
Clk	J	K	Q_t	Q_{t+1}	
↓	0	0	0	0	Q_t
↓	0	0	1	1	
↓	1	0	0	1	1
↓	1	0	1	1	
↓	0	1	0	0	0
↓	0	1	1	0	
↓	1	1	0	1	$\bar{Q}_t$
↓	1	1	1	0	

Q \ JK	00	01	11	10
0			1	1
1	1			1

$$Q_{t+1} = J\bar{Q}_t + Q_t\bar{K}$$

(ii) Flip-flop switching time: In designing circuits with flip-flop the following parameters are important:

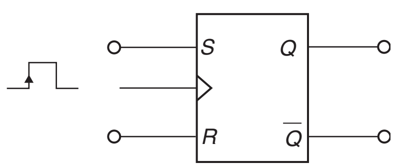
1. **Set-up time:** The minimum amount of time required for the data input to be present before the clock arrived.
2. **Hold time:** The minimum amount of time that the data input to be present after the clock trigger arrived.
3. **Propagation delay:** The amount of time it takes for the output to change states after an input trigger.



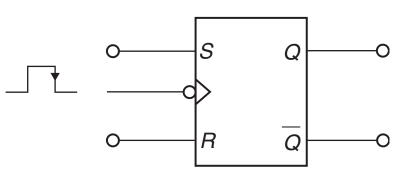
For example, t setup = 50 m sec and t hold = 5 m sec, the data bit has to be the input at least 50 m sec before the clock bit arrives and hold at least 5 m sec after the clock edge.

(iii) Triggering of flip-flop: The flip-flop can be triggered to set or reset either at one of the edges of the clock pulse. There are three types of triggering as described below:

1. **Positive edge triggering flip-flop:** These set or reset at the positive (rising or leading) edge of the clock pulse depending upon the state of i/p signal and o/p remain steady for 1 clock period. Positive edge triggering is indicated by an arrow head at the clock terminal of the flip-flop.



2. **Negative edge triggered flip-flop:** There are flip-flops those in which state transmissions take place only at the negative edge (falling or trailing) of the clock signal. Negative edge triggering is indicated by arrow head with bubble at the clock terminal.



3. **Level triggering:** Level triggering means the specified action occurs based on the steady state value of the input. That is, when a certain level is reached (0 or 1) the output will change states level triggering will be used in latches.

(iv) *D* flip-flop: It receives the designation from its ability to hold data into its internal storage. An SR/JK flip-flop has two inputs. It requires two inputs S/J and R/K to store 1 bit. This is a serious disadvantage in many application to overcome the difficulty D flip-flop has been developed which has only one input line. A D flip-flop can be realized using a SR/JK as show in the figure below.

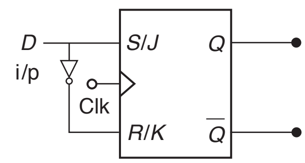


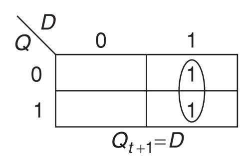
Table 3 *Truth Table*

clk	D	Q_{t+1}
$\not\uparrow$	X	Q_t
↑	0	0
↑	1	1

There is no raising problem with D flip-flop. High or 1 state will set the flip-flop and a low or 0 state will reset the flip-flop. The presence of inverter at the input ensure that S/J and R/K inputs will always be in the opposite state.

Table 4 *Characteristic Table of D Flip-flop*

Q_t	D	Q_{t+1}
0	0	0
0	1	1
1	0	0
1	1	1



From the characteristic table of D.flip-flop, the next state of the flip-flop is independent of the present state since $Q_{t+1} = D$, whether $Q_t = 0$ or 1.

(v) *T* flip-flop: In a JK flip-flop $J = K = 1$ and the resulting flip-flop is referred to as a T flip-flop.

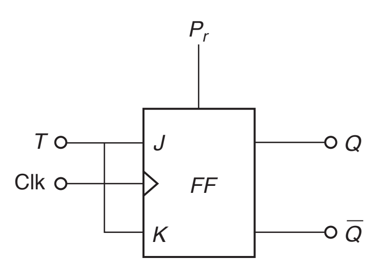


Table 5 *Truth Table*

Clk	T	Q_{n+1}
↑	0	Q_n
↑	1	$\overline{Q}_n$
$\not\uparrow$	x	Q_n

If $T = 1$, it acts as a toggle switch for every Clk pulse with high input, the Q changes to its opposite state.

Table 6 *Characteristic Table*

Q_t	T	Q_{t+1}
0	0	0
0	1	1
1	0	1
1	1	0
1	0	1

Q \ T	0	1
0		1
1	1	

$$Q_{t+1} = T\overline{Q}_t + Q_t\overline{T}$$

(vi) Excitation table of flip-flops: The truth table of flip-flop is also referred to as the characteristic table, which specifies the operational characteristic of flip-flop.

Sometimes we come across situations in which present state and the next state of the circuit are known and we have to find the input conditions that must prevail to cause the desired transition of the state.

Consider initially JK flip-flop output $Q_n = 1, \overline{Q}_n = 0$, after clock pulse it changed to $Q_{n+1} = 0,\ \overline{Q}_{n+1} = 1$,

The input conditions, which made this transition, can be

Toggle – for $J = 1,\ K = 1,\ Q_{n+1} = \overline{Q}_n$

or

Reset – for $J = 0,\ K = 1,\ Q_{n+1} = 0,\ \overline{Q}_{n+1} = 1$

From the above conditions we can conclude that for transition $Q_n = 1$ to $Q_{n+1} = 0$ occurs when $J = 0$ (or) 1 (don't care) and $K = 1$.

Similarly, input conditions can be found out for all possible situations.

Table 7 *Excitation table of flip-flop.*

Present State	Next State	SR Flip-flop		JK Flip-flop		T Flip-flop	D Flip-flop
Q_n	Q_{n+1}	S	R	J	K	T	D
0	0	0	×	0	×	0	0
0	1	1	0	1	×	1	1
1	0	0	1	×	1	1	0
1	1	×	0	×	0	0	1

These excitation tables are useful in the design of synchronous circuits.

(vii) State diagrams of flip-flops: State diagram is a directed graph with nodes connected with directed arcs. State of the circuit is represented by the node, the directed arcs represent the state transitions, from present state (node) to next state (node) at the occurrence of clock pulse.

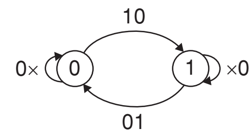


Figure 9 State diagram of SR flip-flop

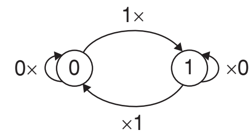


Figure 10 State diagram of JK flip-flop

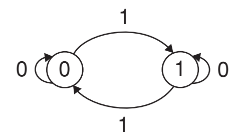


Figure 11 State diagram of T flip-flop

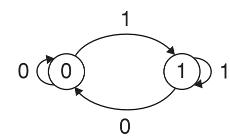


Figure 12 State diagram of D flip-flop

(viii) Conversion of one flip-flop to other flip-flop

Conversion of T flip-flop to JK flip-flop

1. Write the characteristic table of required flip-flop (here JK).
2. Write the excitation table of available or given Flip-flop (here T).
3. Solve for inputs of given flip-flop in terms of required flip-flop inputs and output.

Table 8 *JK flip-flop characteristic and T flip-flop excitation table*

JK Flip-flop Characteristic Table			*T* Flip-flop Excitation Table	
J	K	Q_n	Q_{n+1}	T
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	1

J \ KQ_n	00	01	11	10
0			1	
1	1		1	1

$$T = J\overline{Q}_n + KQ_n$$

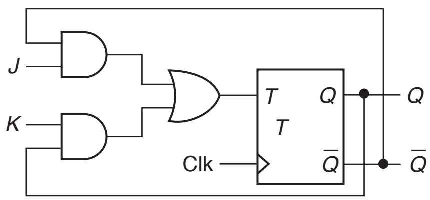


Figure 13 *D* Flip-flop by using other flip-flops

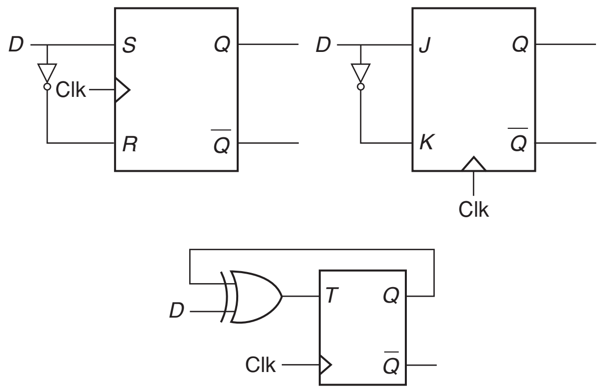
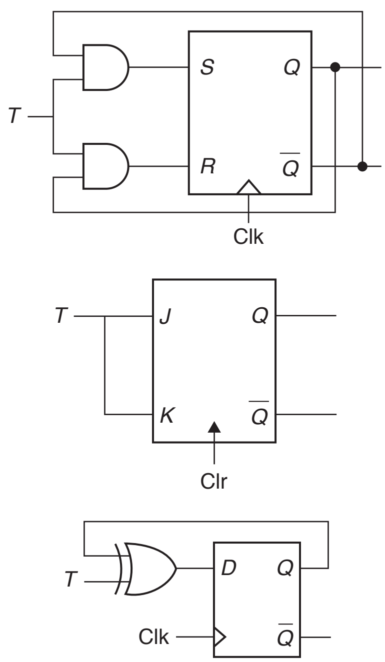
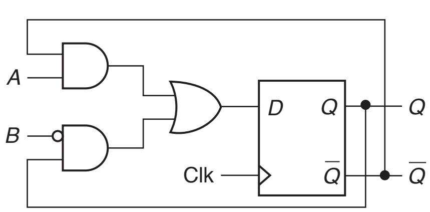


Figure 14 *T* flip-flop by using other flip-flops



Example 3: A sequential circuit using D flip-flop and logic gates is shown in the figure, where A and B are inputs and Q is output.



The circuit is

(A) SR flip-flop with inputs $A = S, B = R$

(B) SR flip-flop with inputs $A = \overline{R}, B = S$

(C) JK flip-flop with inputs $A = J, B = K$

(D) JK flip-flop with inputs $A = K, B = \overline{J}$

Solution: (C)

The characteristic equation of D flip-flop is

$$Q_{n+1} = D$$

Here input $D = A\overline{Q}_n + \overline{B}Q_n$

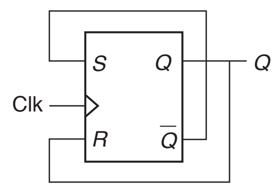
So, output $Q_{n+1} = A\overline{Q}_n + \overline{B}Q_n$

By comparing this equation with characteristic equation of JK

$$Q_{n+1} = J\,\overline{Q}_n + \overline{K}Q_n$$

If $A = J$, $B = K$, then this circuit works like JK flip-flop.

Example 4: The input Clk frequency for the flip-flop given is 10 kHz, then the frequency of Q will be



(A) 10 kHz (B) 5 kHz

(C) 20 kHz (D) 2.5 kHz

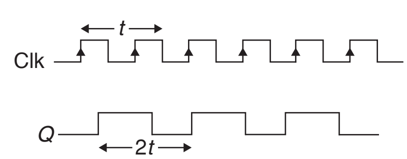
Solution: (B)

Form circuit we can say $S = \overline{Q}_n, R = Q_n$.

If initially $(Q_n, \overline{Q}_n) = (0, 1)$, then inputs $(S, R) = (1, 0)$, by applying clk pulse $(Q_{n+1}\overline{Q}_{n+1})$ becomes $(1, 0)$…

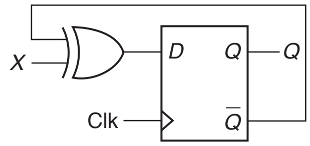
Clk	Q_n	$\overline{Q}_n$	S	R	Q_{n+1}	$\overline{Q}_{n+1}$
1	0	1	1	0	1	0
2	1	0	0	1	0	1
3	0	1	1	0	1	0
4	1	0	0	1	0	1

The output Q_{n+1} toggles for every clock pulse.



So frequency of $Q = \dfrac{1}{2t} = \dfrac{f}{2} = \dfrac{10}{2} = 5$ kHz

Examples 5: For the D flip-flop shown, if initially Q_n is set then what is the output state Q_{n+1} for $X = 0$, and for $X = 1$?



(A) 0, 0 (B) 0, 1
(C) 1, 0 (D) 1, 1

Solution: (B)
The characteristic equation of D is $Q_{n+1} = D$
Here $D = X \oplus \overline{Q_n}$
So $Q_{n+1} = X \oplus \overline{Q_n}$
We have $Q_n = 1$ (Q_n is set) for $X = 0$

$$Q_{n+1} = 0 \oplus 0 = 0$$

We have $Q_n = 1$ (Q_n is set), for $X = 1$

$$Q_{n+1} = 1 \oplus 0 = 1$$

Applications of flip-flops:

1. Data storage: A group of flip-flops connected in series/parallel is called a register, to store a data of N-bits, N-flip-flops are required. Data can be stored in parallel or serial order. Similarly, serial to parallel conversion and parallel to serial conversion can be done by using registers.
2. Counting: A number of flip-flops can be connected in a particular fashion to count the pulses applied (Clk) electronically. One flip-flop can count 2 Clk pulses, two flip-flops can count up to $2^2 = 4$ pulses, similarly n flip-flops can count up to 2^n pulses. Flip-flops may be used to count up/down.
3. Frequency division: Flip-flops may be used to divide input signal frequency by any number. A single flip-flop may be used to divide the input frequency by 2. Similarly n flip-flops may be used to divide the input frequency by 2^n. Output of a MOD-n counter (i.e., which counts n states) will divide input frequency by n.

Counters

Digital counters consist of a number of flip-flops. Their function is to count the number of clock pulses arriving at its clock input.

(i) Counter classification: Counters are classified according to their operational characteristic. Some of these characteristics include:

1. Counter triggering techniques
2. Frequency division characteristic
3. Counter modulus
4. Asynchronous or synchronous

In a synchronous counter all flip-flops are clocked simultaneously. In asynchronous counter the flip-flops are not clocked simultaneously. Each flip-flop is triggered by the previous flip-flop.

(ii) Asynchronous counters (ripple counters): Asynchronous counters do not have a common clock that controls all the flip-flop stages. The control clock is input to the first stage. The clock for each stage subsequent is obtained from the flip-flop of the prior stages. Let us analyze the 3-bit counter and its corresponding wave form diagram shown below.

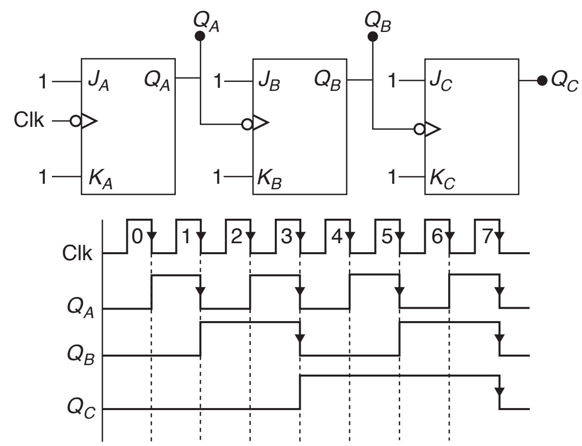


Figure 15 Timing diagrams

- The counter has three flip-flops and three output bits, therefore it is a three stage counter.
- The input clock does not trigger the three flip-flops, therefore it is an asynchronous counter.
- The J and K inputs are tied together as kept high. So they are considered to be toggle flip-flops.
- The flip-flops are negative edge triggered.
- The wave form analysis reveals that Q_A is the LSB and that its frequency is $\frac{1}{2}$ the input clock frequency. Further more, Q_c is the MSB and its frequency is $\frac{1}{8}$ the input clock frequency.
- The count sequence is 000, 001, 010, 011, 100, 101, 110, 111 where the LSB is Q_A. Thus it is MOD-8 binary up counter.
- Asynchronous counters are also known as ripple counters because the effect of the input clock ripples through the counter until it reaches the final stage.

Asynchronous Counter Design

Step I: Write the counting sequence.
Step II: Tabulate the values of reset signals. R for various state of counter.
Step III: Obtain the minimal expression for R and $\overline{R}$ using K-map or any other method.
Step IV: Provide a feedback such that R or $\overline{R}$ resets all the flip-flops after the desired count.

Table 9 *Identification of up/down Counters*

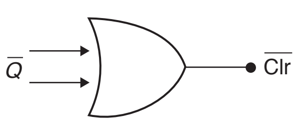
Clock Triggering	Q	Type
+ve edge	$\overline{Q}$	Up
+ve edge	Q	Down
–ve edge	$\overline{Q}$	Down
–ve edge	Q	Up

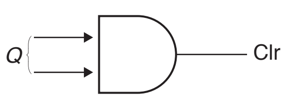
Clock is negative triggering pulse and Q is connected to next level clock, it is acting like a up counter.

Table 10 *Identification of GATE to Clear the Flip-flops*

Input to the Gate	Output of the Gate	Type of Gate
$\overline{Q}$	$\overline{\text{Clr}}$	OR
$\overline{Q}$	Clr	NOR
Q	$\overline{\text{Clr}}$	NAND
Q	Clr	AND

Example:

(i) 

(ii) 

Example 6: Design and Implement a MOD-6 asynchronous counter using T flip-flops.

Solution: Counting sequence is 00, 001, 010, 011, 100, 101

After Pulses	States Q_3, Q_2, Q_1	Reset R
0	000	0
1	001	0
2	010	0
3	011	0
4	100	0
5	101	0
6	110	1
7	↓↓↓	
	000	0
	111	X

From the Truth table $R = Q_3 Q_2$

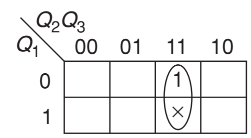
For active Low $\overline{R}$ is used.

$\therefore$ $R = 0$ for 000 to 101

$R = 1$ for 110

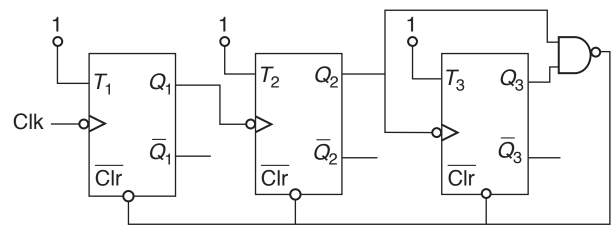
$R = X$ for 111

$\therefore$ K-map is



$\therefore$ $R = Q_2 Q_3$

Logic diagram is



Asynchronous Decode Counter

A ripple counter is an asynchronous sequential circuit, clock is applying only for LSB side. Decade ripple counter it counts from 0 to 9 for up counter.

MOD-10 counter it counts starting from 0000 to 1001. If the NAND gate output is logic '0' at that instant the counter reset to initial state.

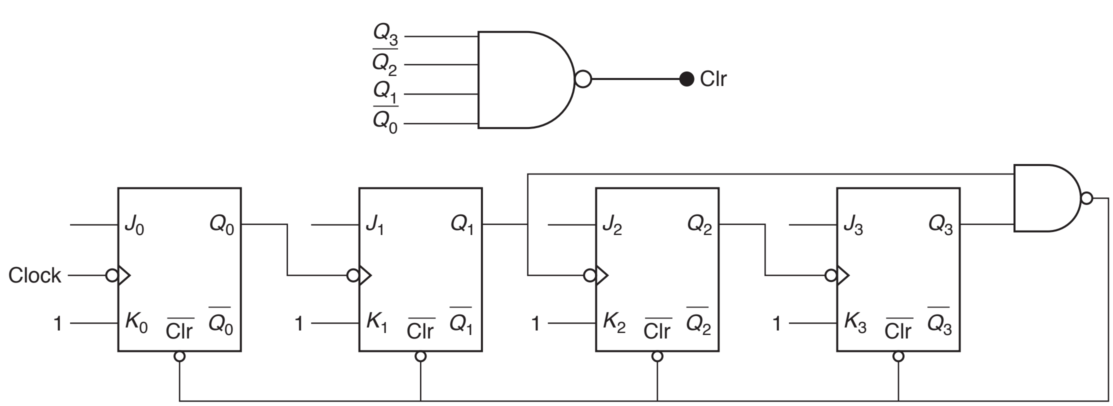


Figure 16 MOD-10 or decade counter

To design a MOD-N counter minimum number of flip-flops required is

$N \le 2^n$

where $N \rightarrow$ MOD

$n \rightarrow$ No. of flip-flops

Example:

MOD-5 counter

$5 \le 2^n$

$\therefore n = 3$

Operating Clock Frequency

(i) Synchronous counter:

$$f_{\text{Clk}} \le \frac{1}{t_{pd}}$$

(ii) Asynchronous counter:

$$f_{\text{Clk}} \le \frac{1}{n t_{pd}}$$

Output frequency of the MOD-N counter is

$$\Rightarrow f_o = \frac{f_{\text{Clk}}}{N}.$$

(iii) Synchronous counter: When counter is clocked such that each flip-flop in the counter is triggered at the same time, the counter is called as synchronous counter.

- Synchronous counters have the advantage of high speed and less severe decoding problems.
- Disadvantage is having more circuiting than that of asynchronous counter.

Synchronous Series Carry Counters

For normal ring counters to count N sequence total N flip-flops are required.

Unused states in ring counter = $2^N - N$.

Unused states in Johnson ring counter = $2^N - 2N$.

Asynchronous counters are slower than the synchronous counters. By using synchronous series carry adders we can design MOD-N counter with n Flip-flops-only.

For non-binary counters $N \le 2^n$

3-bit series carry up counter

It counts from initial state 000 to 111.

$\therefore$ MOD = 2^n = 8 states

$\therefore$ MOD-8

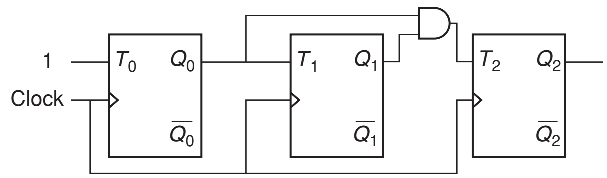


Figure 17 3-bit series carry counter

$$f_{\text{Clk}} \le \frac{1}{t_{pd} + (n-2)t_{pd\ \text{AND}}}$$

where

$t_{pd} \rightarrow$ Propagation delay of each flip-flop.

$t_{pd\ \text{AND}} \rightarrow$ Propagation delay of AND gate.

$n \rightarrow$ Number of flip-flops.

In this, Q_o toggles for every clock pulse.

Q_1 toggles when Q_o is 1.

Q_2 toggles when o/p of AND gate is logic 1.

Note: To design a synchronous series carry down counter. Connect $\overline{Q_o}$ to the next flip-flop input.

Design of Synchronous Counter

Step I: Determine the required number of flip-flop.

Step II: Draw the state diagram showing all possible states.

Step III: Select the type of flip-flop to be used and write the excitation table.

Step IV: Obtain the minimal expressions for the excitations of the FFs using the K-maps.

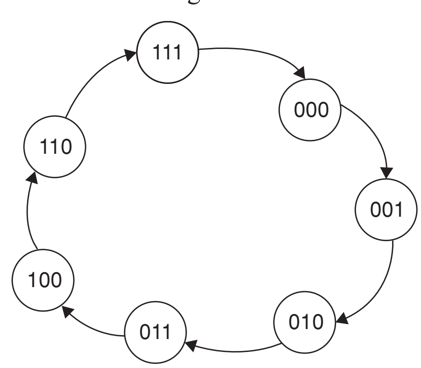
Step V: Draw a logic diagram based on the minimal expression. Let us employ these techniques to design a MOD-8 counter to count in the following.

Example 7: Sequence: 0, 1, 2, 3, 4, 5, 6, and 7. Design a synchronous counter by using JK flip-flops.

Solution:

Step I: Determine the required number of flip-flops. The sequence shows a 3-bit up counter that requires 3 flip-flops.

Step II: Draw the state diagram.

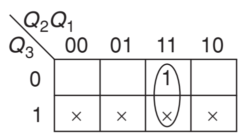


Step III: Select the type of flip-flop to be used and write the excitation table.

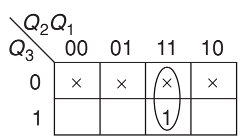
JK flip-flop is selected and excitation table of a 3-bit up counter is

PS	*NS*	Required Excitation
$Q_3\ Q_2\ Q_1$	$Q_3\ Q_2\ Q_1$	$J_3\ K_3\ J_2\ K_2\ J_1\ K_1$
0 0 0	0 0 1	0 x 0 x 1 x
0 0 1	0 1 0	0 x 1 x x 1
0 1 0	0 1 1	0 x x 0 1 x
0 1 1	1 0 0	1 x x 1 x 1
1 0 0	1 0 1	x 0 0 x 1 x
1 0 1	1 1 0	x 0 1 x x 1
1 1 0	1 1 1	x 0 x 0 1 x
1 1 1	0 0 0	x 1 x 1 x 1

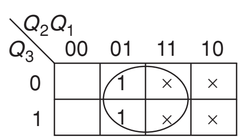
Step IV: Obtain the minimal expression using K-map.



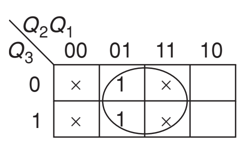
$J_3 = Q_2Q_1$



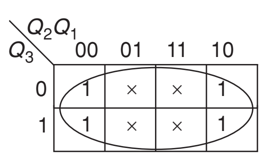
$K_3 = Q_2Q_1$



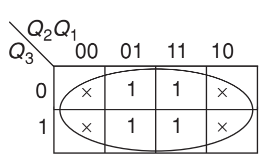
$J_2 = Q_1$



$K_3 = Q_1$



$J_1 = 1$



$K_1 = 1$

Step V: Draw the logic diagram based on the minimal expression.

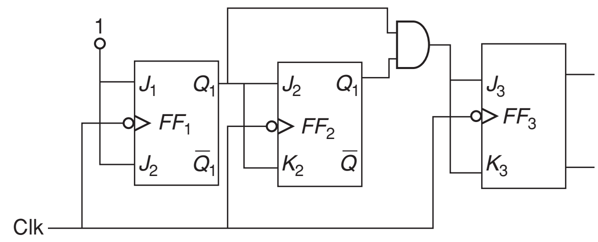


Table 11 *Comparison between asynchronous counter and synchronous counter*

Asynchronous Counter	Synchronous Counter
1. In this type of counter, flip-flops are connected in such a way that output of first flip-flop drives the clock for the next flip-flop	In this type there is no connection between output of first flip-flop and clock input of the next flip-flop
2. All the flip-flops are not clocked simultaneously	All the flip-flops are clocked simultaneously
3. Logic circuit is very simple even for more number of states	Design involves complex logic circuits as number of state increases
4. Main draw back of these counters is their low speed as the clock is propagated through number of flip-flops before it reaches last flip-flop	As clock is simultaneously given to all flip-flops, there is no problem of propagation delay. Hence they are preferred when number of flip-flops increases in the given design.

The main drawback of ripple counters is their high delays, if propagation delay of each flip-flop is assumed as x, then to get output of the first flip-flop it takes x, i.e., after x seconds the second flip-flop will get its clock pulse from previous stage, and output of second flip-flop will be out after another x seconds, similarly the final output of last flip-flop will be after nx seconds, where n is the number of flip-flops. So the propagation delay of ripple counter is nx, which is directly proportionate to the number of flip-flops.

The maximum frequency of operation of ripple counter is inverse of delay, $f_{\max} = \dfrac{1}{nx}$

Maximum operating frequency is the highest frequency at which a sequential circuit can be reliably triggered. If the clock frequency is above this maximum frequency the flip-flops in the circuit cannot respond quickly and the operation will be unreliable.

In case of synchronous counters (synchronous circuits) as clock is applied simultaneously to all the flip-flops, the output of all the flip-flops change by x seconds (delay of one flip-flop) and this delay is independent of number of flip-flops used in circuit.

The maximum frequency of operation of synchronous counter is inverse of delay $f_{\max} = \dfrac{1}{x}$

Example 8: The maximum operation frequency of a MOD-64 ripple counter is 33.33 kHz, the same flip-flops are used to design a MOD-32 synchronous counter, and then the maximum operating frequency of the new counter is

(A) 400 kHz (B) 200 kHz
(C) 40 kHz (D) 500 kHz

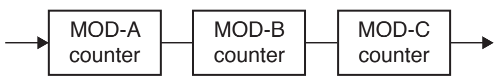
Solution: For ripple counter $f_{\max} = \frac{1}{nx}$, given is a MOD-64 ripple counter, i.e., 2^6 states, so $n = 6$ flip-flops are required.

$$x = \frac{1}{33.33K \times 6} = 5\mu S$$

For synchronous counter

$$f_{\max} = \frac{1}{x} = \frac{1}{5\ \mu S} = 0.2\ \text{MHz} = 200\ \text{kHz}$$

When multiple counters are connected in cascade, then the total number of states of the new counter is $A \times B \times C$, i.e., it will work as MOD-$A \times B \times C$ counter.



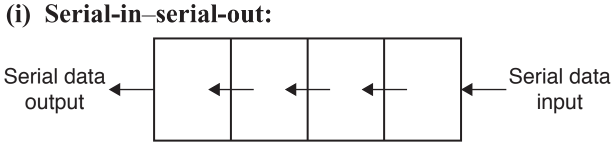
For example, decade counter counts from 0 to 9, 10 states – If two such decade counters are connected in cascade, then the total counting states will be 10 × 10 = 100, it will work as MOD-100 counter, which counts from 00 to 99.

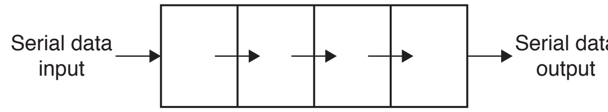
Registers

A number of flip-flops connected together such that data may be shifted into and shifted out of them is called a shift register. There are four basic types of shift register:

1. Serial-in–serial-out
2. Serial-in–parallel-out
3. Parallel-in–serial-out
4. Parallel-in–serial-out

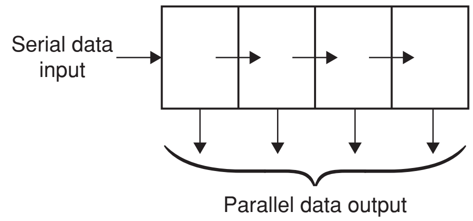
(i) Serial-in–serial-out:



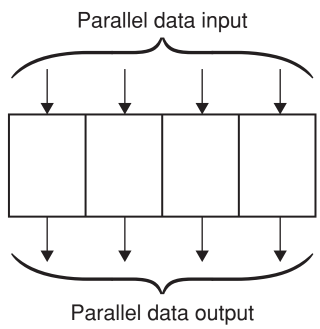


Serial-in–serial-out right-shift register

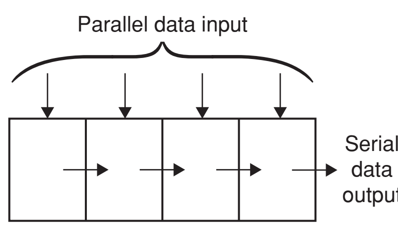
(ii) Serial-in–parallel-out:



(iii) Parallel-in–parallel-out:



(iv) Parallel-in–serial-out:



Serial input and serial output register: This type of shift register accepts data serially, i.e., one bit at a time and also outputs data serially. The logic diagram of 4-bit serial input, serial output, shift-right, shift register is shown in the following figure. With four D flip-flops the register can store up to four bits of data.

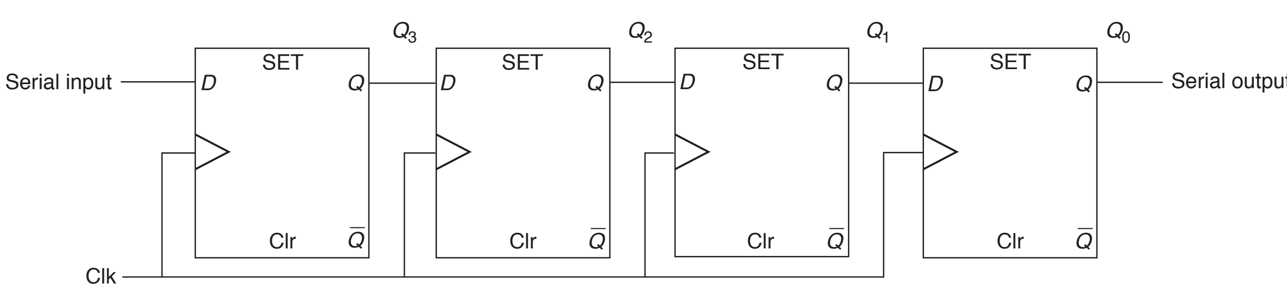


Figure 18 Serial input and serial output register

If initially, all flip-flops are reset, then by applying serial input 1101, the flip-flop states will change as shown in below table.

Clk	S.I	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0	0
1	0	1	0	0	0
2	1	0	1	0	0
3	1	1	0	1	0
4		1	1	0	1

The first data bit 1 will appear at serial output after 4 clock pulses.

Application of Shift Registers

1. Delay line: Serial input and serial output shift register can be used to introduce delay in digital signals.
$$\text{Delay} = \text{no.of flip-flops} \times \frac{1}{\text{Clk frequency}} = \text{No. of}$$
flip-flops × time period of clock pulse
2. Serial to parallel, parallel to serial converter: SIPO, PISO registers used for data conversion.
3. Sequence generator: A circuit, which generates a prescribed sequence of bits, with clock pulses is called as sequence generator
The minimum number of flip-flops 'n' required to generate a sequence of length 'S' bits is given by $S \le 2^n - 1$

Shift register counters

One of the applications of the shift register is that they can be arranged to work as ring counters. Ring counters are constructed by modifying the serial-in, serial-out, shift registers. There are two types of ring counters—basic ring counter and twisted ring counter (Johnson counter). The basic ring counter is obtained from SISO shift register by connecting serial output to serial input.

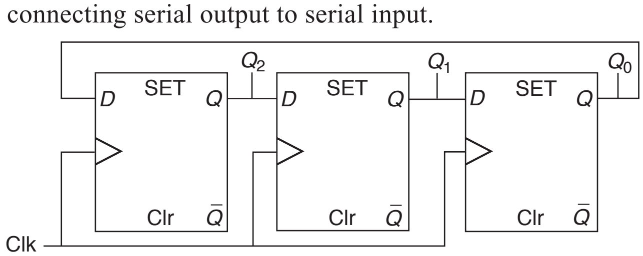
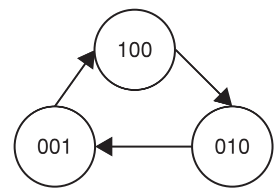


Figure 19 Ring counter

In most instances, only a single 1 or single 0 is in the register and is made to circulate around the register as long as the clock pulses are applied. Consider initially first flip-flop is set, and others are reset. After 3 clock pulses, again we will get initial state of 100. So this is a MOD-3 counter.

Clk	Q_2	Q_1	Q_0
0	1	0	0
1	0	1	0
2	0	0	1
3	1	0	0
4	0	1	0



A ring counter with N flip-flops can count up to N states, i.e., MOD-N counter, whereas, N-bit asynchronous counter can count up to 2^N states. So, ring counter is uneconomical compared to a ripple counter, but has the advantage of requiring no decoder. Since it is entirely synchronous operation and requires no gates for flip-flop inputs, it has further advantage of being very fast.

Twisted ring counter (Johnson counter): This counter is obtained from a SISO shift register by connecting the complement of serial output to serial input as shown in below figure.

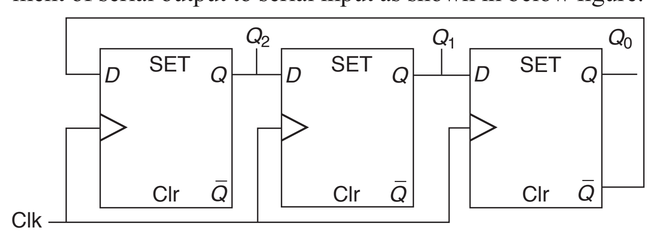
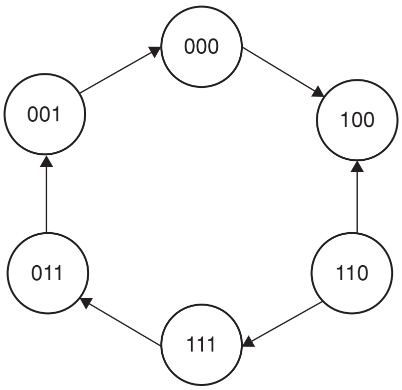


Figure 20 Twisted Ring Counter

Let initially all the FFs be reset, after each clock pulse the complement of last bit will appear as at MSB, and other bits shift right side by 1-bit. After 6 clock pulses the register will come to initial state 000. Similarly, the 3-bit Johnson counter will oscillate between the states 101, 010.

Clk	Q_2	Q_1	Q_0
0	0	0	0
1	1	0	0
2	1	1	0
3	1	1	1
4	0	1	1
5	0	0	1
6	0	0	0



An n-bit Johnson counter can have $2n$ unique states and can count up to $2n$ pulses, so it is a MOD-$2n$ counter. It is more economical than basic ring counter but less economical than ripple counter.

Solved Examples

Example 1: Assume that 4-bit counter is holding the count 0101. What will be the count after 27 clock pulses?

Solution: Total clock pulses: 27 = 16 + 11
0101 + 1011 = 0000

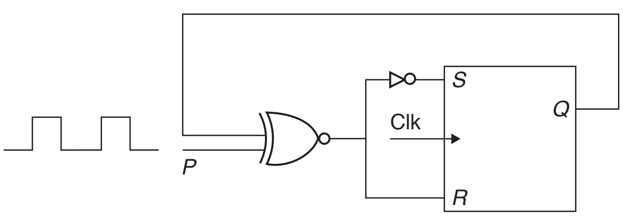
Example 2: A MOD-2 counter followed by MOD-5 counter is

Solution: A decade counter, counts 10 states (5×2).

Example 3: A 4-bit binary ripple counter uses flip-flops with propagation delay time of 25 msec each. The maximum possible time required for change of state will be

Solution: The maximum time = 4×25 ms = 100 ms

Example 4: Consider the circuit, the next state Q^+ is



Solution

P	Q	S	R	Q^+
0	0	0	1	0
0	1	1	0	1
1	0	1	0	1
1	1	0	1	0

So, $Q^+ = P \oplus Q$

Example 5: A certain JK FF has $t_{pd} = 12\ n$ sec what is the largest MOD counter, that can be constructed from these FF and still operate up to 10 MHz?

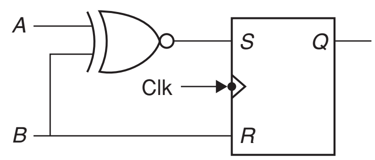
Solution: $N \le \dfrac{1}{f_{\max} \cdot t_{pd}}$

$f_{\max} = 10$ MHz $\;N \le 8$
$t_{pd} = 12$ ns

$$N \le \frac{1}{10 \times 10^6 \times 12 \times 10^{-9}}$$

MOD counter is $= 2^N = 2^8 = 256$

Example 6: An AB flip-flop is constructed from an SR flip-flop as shown below. The expression for next state Q^+ is

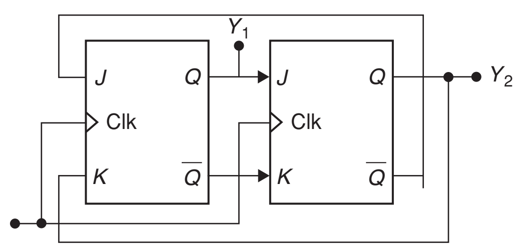


Solution:

A	B	Q	S	R	Q^+
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	0	1	0
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	0	0	1
1	1	0	1	1	×
1	1	1	1	1	×

$\therefore Q^+ = \overline{A}\overline{B} + AQ = \overline{A}\overline{B} + \overline{B}Q$

Example 7: In the circuit shown below, the output y_1 and y_2 for the given initial condition $y_1 = y_2 = 1$ and after four input pulses will be



Solution:
After 1st pulse $y_1 = 0, y_2 = 1$
After 2nd pulse $y_1 = 0, y_2 = 0$
After 3rd pulse $y_1 = 1, y_2 = 0$
After 4th pulse $y_1 = 1, y_2 = 1$

Example 8: A ripple counter is to operate at a frequency of 10 MHz. If the propagation delay time of each flip-flop in the counter is 10 ns and the storbing time is 50 ns, how many maximum stages can the counter have?

Solution: $nt_{pd} + t_s \le \frac{1}{f}$

where, n = number of stages

t_{pd} = propagation delay time

t_s = strobing time

f = frequency of operation = $10 \times 10^{-9}n + 50 \times 10^{-9}$

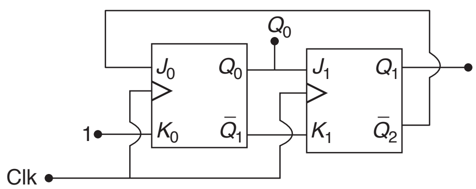
$$\le \frac{1}{10 \times 10^6}$$

(or) $10n + 50 \le 100$

(or) $10n \le 50$

For max stages $n = \frac{50}{10} = 5$

Example 9: In the circuit assuming initially $Q_0 = Q_1 = 0$. Then the states of Q_0 and Q_1 immediately after the 33rd pulse are

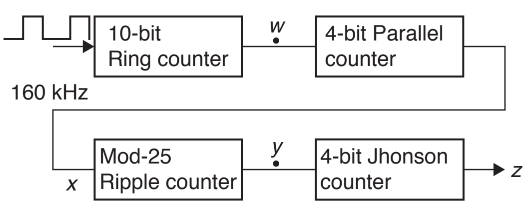


Solution:

J_0	K_0	J_1	K_1	Q_0	Q_1	Count
1	1	0	1	0	0	Initial
1	1	1	0	1	0	1st pulse
0	1	0	1	0	1	2nd
1	1	0	1	0	0	3rd
1	1	1	0	1	0	4th
0	1	0	1	0	1	5th pulse

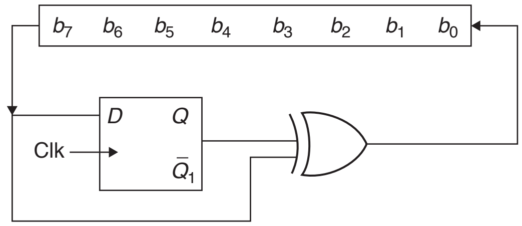
After 4th pulse, output is same as after 1st one, so, sequence gets repeated. So output after 33rd pulse would be same as after 3rd pulse. i.e., (00).

Example 10: The frequency of the pulse at z in the network shown in figure is



Solution: 10-bit ring counter is a MOD-10. So, it divides the 160 kHz input by 10. Therefore, w = 16 kHz. The 4-bit parallel counter is a MOD-16. Thus, the frequency at x = 1 kHz. The MOD-25 ripple counter produces a frequency at y = 40 Hz (1 kHz/25 = 40 Hz). The 4-bit Johnson counter is a MOD-8. The frequency at Z = 5Hz.

Example 11: The 8-bit shift left shift register, and D flip-flop shown in the figure is synchronized with the same clock. The D flip-flop is initially cleared. The circuit acts as



Solution: The output of XOR gate is $Z = b_{i+1} \oplus b_i$ and this output shift the register to left. Initially, $Z = 0$

After 2nd clock $Z = b_7 \oplus 0 = b_7$

After 2nd clock $Z = b_7 \oplus b_6$

3rd clock $Z = b_6 \oplus b_5$

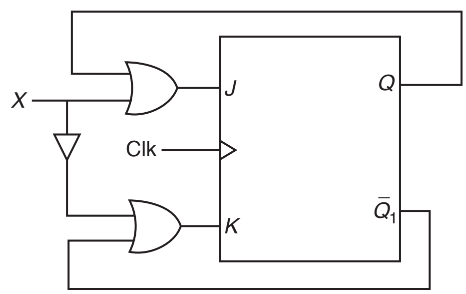
4th clock $Z = b_5 \oplus b_4$

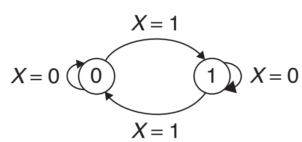
It is a binary to gray code converter.

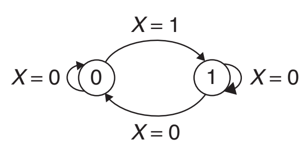
Example 12: A 4-bit MOD-16 ripple counter uses JK flip-flops. If the propagation delay of each flip-flop is 50 ns sec, the maximum clock frequency that can be used is equal to

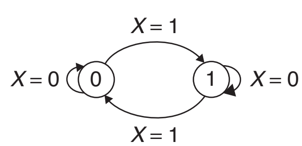
Solution: Max = clock frequency $= \frac{1}{4 \times 50 \times 10^{-9}}$ = 5 MHz

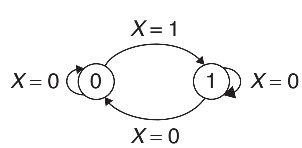
Example 13: What is the state diagram for the sequential circuit shown?



(A) 

(B) 

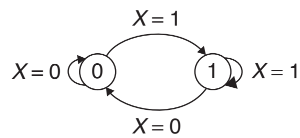
(C) 

(D) 

Solution: (D)

State diagram of a sequential circuit will have states (output) of all the flip-flops.

Present state	Next state	Q_{n+1}
Q_n	For $x = 0$	For $x = 1$
0	0	1
1	0	1



Exercises

Practice Problems I

Directions for questions 1 to 22: Select the correct alternative from the given choices.

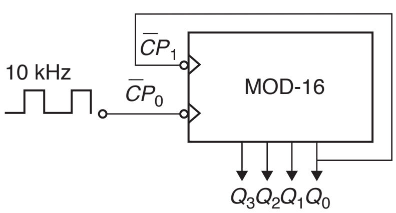
1. How many flip-flops are needed for MOD-16 ring counter and MOD-16 Johnson counter?

(A) 16, 16 (B) 16, 8
(C) 4, 3 (D) 4, 4

2. A 2-bit synchronous counter uses flip-flops with propagation delay time of 25 n sec, each. The maximum possible time required for change of state will be

(A) 25 n sec (B) 50 n sec
(C) 75 n sec D) 100 n sec

3. For given MOD-16 counter with a 10 kHz clock input determine the frequency at Q_3

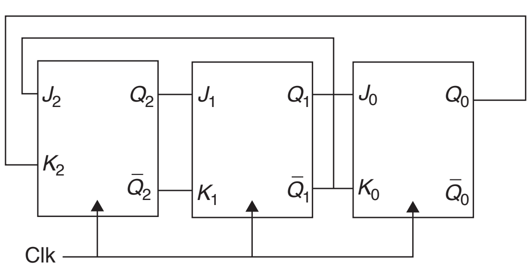


(A) 625 Hz (B) 10 kHz
(C) 2.5 kHz (D) 0 Hz

4. A 4-bit ripple counter and a 4-bit synchronous counter are made using flip-flops having a propagation delay of 10 n sec each. If the worst case delay in the ripple counter and the synchronous counter be R and S, respectively, then

(A) $R = 10$ ns, $S = 40$ ns (B) $R = 40$ ns, $S = 10$ ns
(C) $R = 10$ ns, $S = 30$ ns (D) $R = 30$ ns, $S = 10$ ns

5. The counter shown in the figure has initially $Q_2Q_1Q_0 = 000$. The status of $Q_2Q_1Q_0$ after the first pulse is

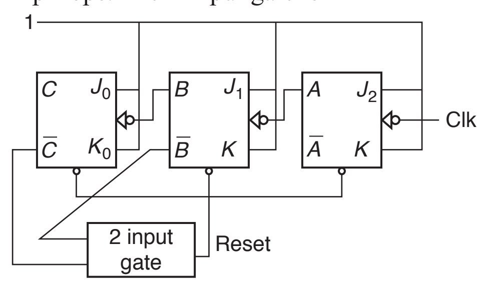


(A) 001 (B) 010
(C) 100 (D) 101

6. 12 MHz clock frequency is applied to a cascaded counter of MOD-3 counter, MOD-4 counter and MOD-5 counter. The lowest output frequency is

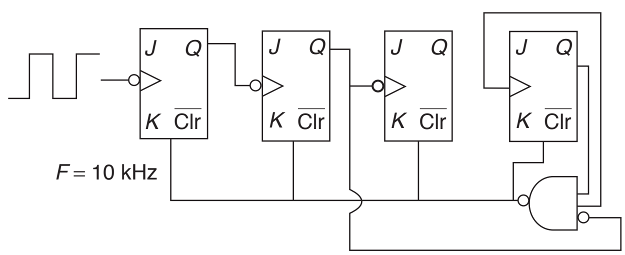
(A) 200 kHz (B) 1 MHz
(C) 3 MHz (D) 4 MHz

7. In the modulo-6 ripple counter shown in the figure below, the output of the 2-input gate is used to clear the JK flip-flops. The 2-input gate is



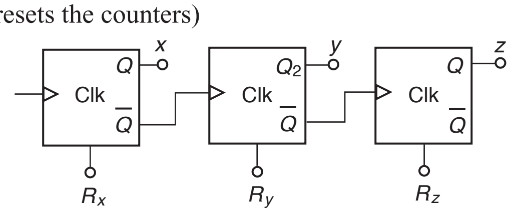
(A) a NAND gate (B) a NOR gate
(C) an OR gate (D) an AND gate

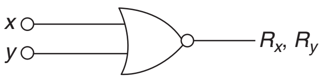
8. In figure, J and K inputs of all the 4 flip-flops are made high, the frequency of the signal at output y is

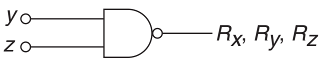


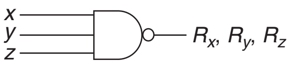
(A) 0.833 kHz (B) 1.0 kHz
(C) 0.91 kHz (D) 0.77 kHz

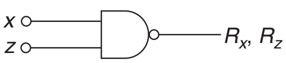
9. In a number system a counter has to recycle to 0 at the sixth count. Which of the connections indicated below will realize this resetting? (a logic '0' at the R inputs resets the counters)



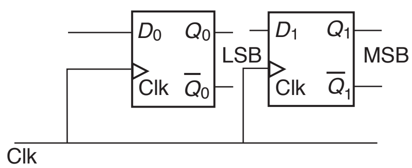
(A) 

(B) 

(C) 

(D) 

10. Two D flip-flops, as shown below, are to be connected as a synchronous counter that goes through the following Q_1Q_0 sequence $00 \rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00$. The inputs D_0 and D_1 respectively should be connected as



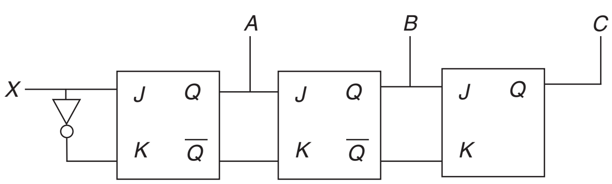
(A) $\overline{Q}_0$ and Q_1 (B) $\overline{Q}_0$ and Q_1

(C) $\overline{Q}_1\overline{Q}_0$ and $\overline{Q}_1Q_0$ (D) $\overline{Q}_1\overline{Q}_0$ and $\overline{Q}_1Q_0$

11. N flip-flops can be used to divide the input clock frequency by

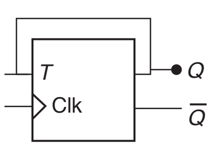
(A) N (B) $2N$
(C) 2^N (D) 2^{N-1}

12. For a shift register as shown, $x = 1011$, with initially FF cleared, ABC will have value of after 3 clock pulses



(A) 101 (B) 011
(C) 001 (D) 111

13. If a FF is connected as shown what will be the output? (initially $Q = 0$)



(A) 11111 (B) 0000
(C) 1010 (D) 0101

14. The excitation table for a FF whose output conditions are if $AB = 00$, no change of state occurs
$AB = 01$, FF becomes 1 with next clock pulse
$AB = 10$, FF becomes 0 with next clock pulse
$AB = 11$, FF changes its state

(A)

Q_n	Q_{n+1}	A	B
0	0	0	x
0	1	1	x
1	0	x	1
1	1	x	0

(B)

Q_n	Q_{n+1}	A	B
0	0	1	x
0	1	0	x
1	0	x	0
1	1	x	1

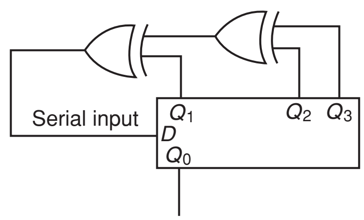
(C)

Q_n	Q_{n+1}	A	B
0	0	x	0
0	1	x	1
1	0	x	x
1	1	0	x

(D)

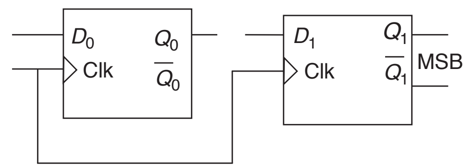
Q_n	Q_{n+1}	A	B
0	0	x	0
0	1	1	x
1	0	x	1
1	1	0	x

15. A shift register that shift the bits 1 position to the right at each clock pulse is initialized to 1100 (Q_0, Q_1, Q_2, Q_3). The outputs are combined using an XOR gate circuit and fed to the D input. After which clock pulse, will the initial pattern reappear at the output?



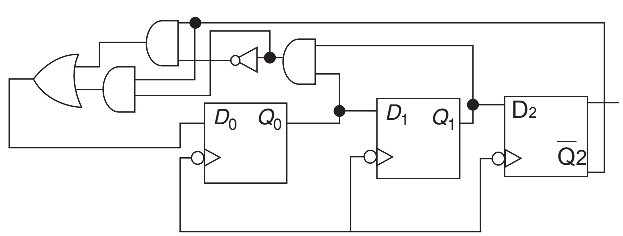
(A) 6th (B) 5th
(C) 4th (D) 7th

16. If we need to design a synchronous counter that goes through the states $00 \rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00$ using D FF, what should be the input to the FF?



(A) $D_0 = Q_0, D_1 = \overline{Q}_1$
(B) $D_0 = \overline{Q}_1, D_1 = Q_0$
(C) $D_0 = \overline{Q}_1 \cdot Q_0,\ D_1 = \overline{Q}_1\overline{Q}_0$
(D) $D_0 = \overline{Q}_0,\ D_1 = Q_1$

17. Find the counter state sequence (Assume Q_0 as MSB).

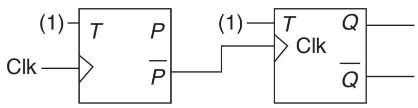


(A) 4, 6, 7, 3, 1, 0, 4 (B) 4, 6, 5, 3, 1, 0, 4
(C) 4, 5, 6, 7, 0, 4, 5 (D) 4, 6, 7, 1, 0, 4

18. If the propagation delay of each FF is 50 ns, and for the AND gate to be 20 ns. What will be the f_{max} for MOD-32 ripple and synchronous counters?

(A) 14.3 MHz, 4 MHz (B) 14.3 MHz, 5 MHz
(C) 5 MHz, 14.3 MHz (D) 3.7 MHz, 14.3 MHz

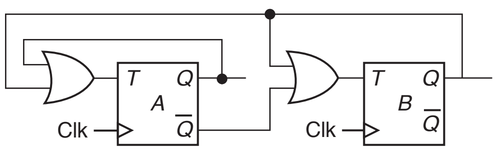
19. For a given counter identify its behavior



The output is taken from *PQ*.

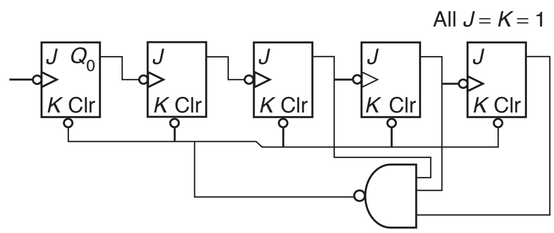
(A) MOD-4 up counter
(B) MOD-2 down counter
(C) MOD-4 down counter
(D) MOD-2 up counter

20. A circuit using *T* FF is given. Identify the circuit.



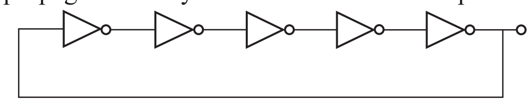
(A) MOD-2 counter
(B) MOD-4 counter
(C) MOD-3 counter
(D) MOD-2 generate 00, 10, 00

21. The MOD number of asynchronous counter shown



(A) 24 (B) 48
(C) 29 (D) 28

22. For the oscillator, find the fundamental frequency if propagation delay of each inverter is 1000 psec.



(A) 100 MHz (B) 10 MHz
(C) 1 GHz (D) 10 GHz

Practice Problems 2

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. Match List 1 (operation) with List 2 (associated device) and select the correct answer using the codes given below:

List 1	List 2
(a) Frequency Ddivision	(1) ROM
(b) Decoding	(2) Multiplexer
(c) Data selection	(3) Demultiplexer
(d) Code conversion	(4) Counter

(A) a–3, b–4, c–2, d–1
(B) a–3, b–4, c–1, d–2
(C) a–4, b–3, c–1, d–2
(D) a–4, b–3, c–2, d–1

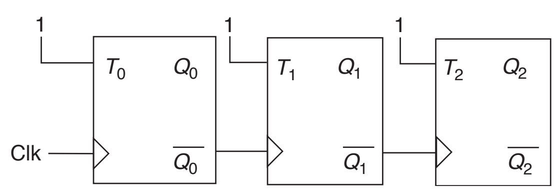
2. A MOD-5 synchronous counter is designed by using JK flip-flop, the number of counts skipped by it will be

(A) 2 (B) 3
(C) 5 (D) 0

3. A counter starts off in the 0000 state, then clock pulses are applied. Some time later the clock pulses are removed and the counter flip-flops read 0011. How many clock pulses have occurred?

(A) 3 (B) 35
(C) 51 (D) Any of these

4. Figure below shown as ripple counter using positive edge triggered flip-flops. If the present state of the counters is $Q_2\,Q_1\,Q_0 = 011$, then its next state ($Q_2Q_1Q_0$) will be

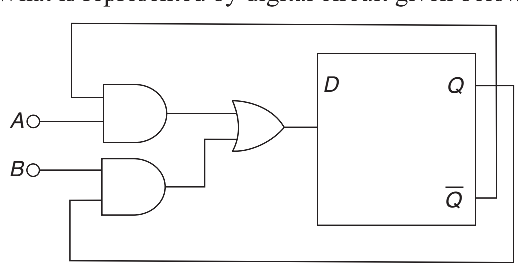


(A) 010 (B) 100
(C) 111 (D) 101

5. A synchronous sequential circuit is designed to detect a bit sequence 0101 (overlapping sequence include). Every time, this sequence is detected, the circuit produces output of 1. What is the minimum number of states the circuit must have?

(A) 4 (B) 5
(C) 6 (D) 7

6. What is represented by digital circuit given below?



(A) An SR flip-flop with $A = S$ and $B = R$
(B) A JK flip-flop with $A = K$ and $B = J$
(C) A JK flip-flop with $A = J$ and $B = \bar{K}$
(D) An SR flip-flop with $A = R$ and $B = S$

7. In a ripple counter, the state whose output has a frequency equal to $\frac{1}{8}$th that of clock signal applied to the first stage, also has an output periodically equal to $\frac{1}{8}$th that of the output signal obtained form the last stage. The counter is
(A) MOD-8 (B) MOD-6
(C) MOD-64 (D) MOD-16

8. A flip-flop is popularly known as
(A) Astable multivibrator
(B) Bistable multivibrator
(C) Monostable multivibrator
(D) None of these

9. Which of the following represents the truth table for JK flip-flop?

(A)

J	K	Output
0	0	Q_0
0	1	0
1	0	1
1	1	$\bar{Q}_0$

(B)

J	K	Output
0	0	$\bar{Q}_0$
0	1	0
1	0	1
1	1	Q_0

(C)

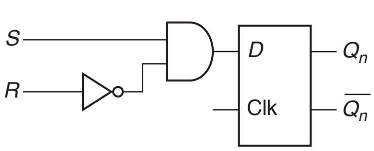
J	K	Output
0	0	Q_0
0	1	0
1	0	1
1	1	Invalid

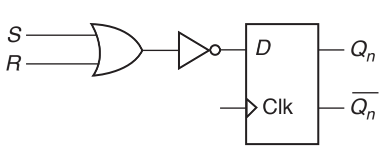
(D)

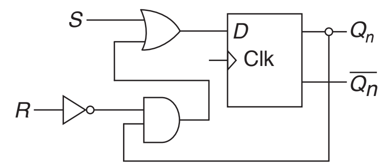
J	K	Output
0	0	1
0	1	0
1	0	1
1	1	0

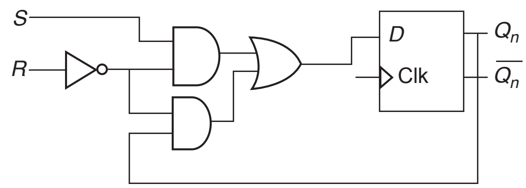
10. One disadvantage of master-slave FF is
(A) setup time becomes longer.
(B) it requires input to be held constant before clock transition.
(C) unpredictable output even if input held constant.
(D) hold time becomes longer.

11. Which of the following converts D FF to SR FF?

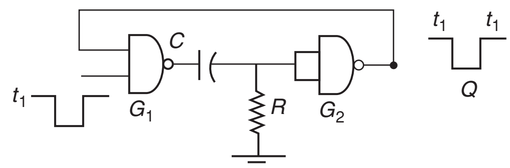
(A)


(B)


(C)


(D)


12. Which of the circuit is being represented by the figure?



(A) NAND gate
(B) Monostable multivibrator
(C) Astable multivibrator
(D) Schmitt trigger

13. Hold time is
(A) time for which output is held constant.
(B) time for which clock is to be held constant on applying input.
(C) time for which input should be maintained constant after the triggering edge of clock pulse.
(D) time for which input should be maintained constant prior to the arrival of triggering edge of clock pulse.

14. Shift registers are made up of
(A) MOS inverters (B) FF
(C) Latches (D) None of these

15. Data from a satellite is received in serial form. If the data is coming at 8 MHz rate, how long will it take to serially load a word in 40-bit shift register?
(A) 1.6 μs (B) 5 μs
(C) 6.4 μs (D) 12.8 μs

16. A JK FF can be converted into T FF by connecting
(A) $\bar{Q}$ to 0
(B) 0 to $\bar{Q}$
(C) 0 to Q
(D) by connecting both J and K inputs to T

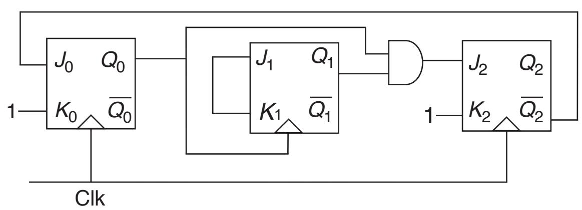
17. The flip-flop that is not affected by race around condition
(A) T FF (B) JK FF
(C) SR FF (D) None of these

18. The characteristic equation of JK FF is
(A) $J'Q(t) + KQ'(t)$ (B) $J'Q(t) + KQ(t)$
(C) $JQ'(t) + K'Q(t)$ (D) None of these

19. For a D.FF input, the s $\overline{Q}$ is connected. What would be the output sequence?
(A) 0000 (B) 1111
(C) 010101 (D) 101010

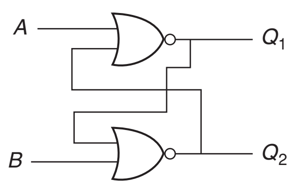
20. In order to implement a MOD-6 synchronous counter we have 3 FF and a combination of 2 input gate(s). Identify the combination circuit.
(A) One AND gate
(B) One OR gate
(C) One AND and one OR gate
(D) Two AND gates

21. Given a MOD-5 counter. The valid states for the counter are (0, 1, 2, 3, 4). The propagation delay of each FF is T_F and that of AND gate is t_A. The maximum rate at which counter will operate satisfactorily



(A) $\dfrac{1}{t_F + t_A}$ (B) $\dfrac{1}{3t_F}$
(C) $\dfrac{1}{2t_F + t_A}$ (D) $\dfrac{1}{3t_F - t_A}$

22. For a NOR latch as shown up A and B are made first (0, 1) and after a few seconds it is made (1, 1). The corresponding output (Q_1, Q_2) are



(A) first (1, 0) then (0, 0)
(B) first (1, 0) then (1, 0)
(C) first (1, 0) then (1, 1)
(D) first (1, 0) then (0, 1)

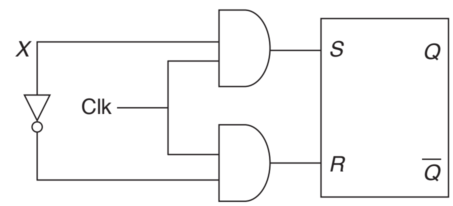
23. In order to design a pulse generator to generate the wave form using a shift register, what is the number of FF required?

(A) 3 (B) 4
(C) 5 (D) 6

24. For what minimum value of propagation delay in each FF will a 10-bit ripple counter skip a count when it is clocked at 5 MHz?
(A) 10 ns (B) 20 ns
(C) 25 ns (D) 15 ns

25. A divide by 50 counter can be realized by using
(A) 5 no. of MOD-10 counter
(B) 10 no. of MOD-5 counter
(C) One MOD-5 counter followed by one MOD-10 counter
(D) 10 no. of MOD-10 counter

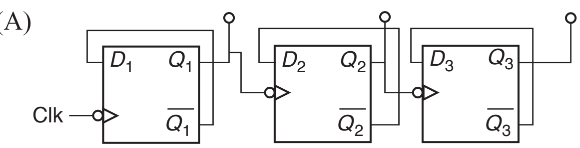
26. The following latch is



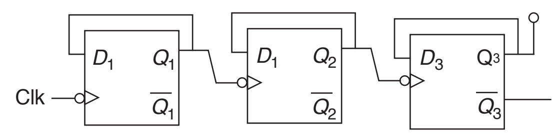
(A) D latch (B) T latch
(C) JK latch (D) RS latch

27. Which of the following represent a 3-bit ripple counter using D FF?

(A)

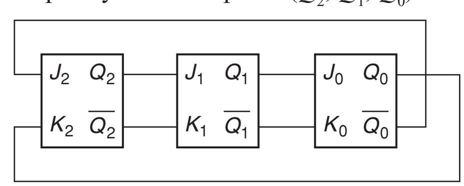


(B)



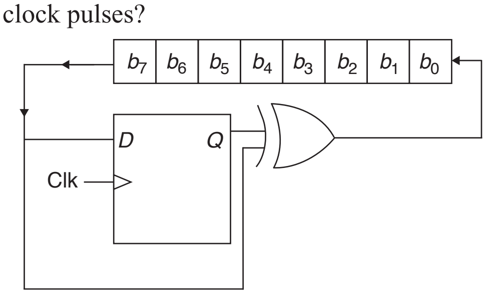
(C) Both (A) and (B)
(D) None of these

28. For the Johnson counter with initial Q_2, Q_1, Q_0 as 101, the frequency of the output is (Q_2, Q_1, Q_0)



(A) $\dfrac{f_c}{8}$ (B) $\dfrac{f_c}{6}$
(C) $\dfrac{f_c}{2}$ (D) $\dfrac{f_c}{4}$

29. For the given circuit the contents of register ($b_7 - b_0$) are 10010101, what will be the register contents after 8 clock pulses?



(A) 10010101 (B) 01101010
(C) 11011111 (D) 01101011

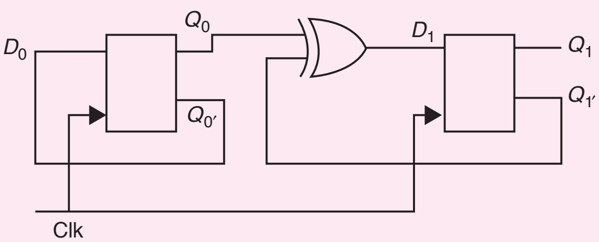
30. A latch is to be build with A and B as input. From the table find the expression for the next state Q^+

A	B	Q	Q+
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

(A) A
(B) B
(C) $A+\overline{B}$
(D) $A\overline{B}+AB$

Previous Years' Questions

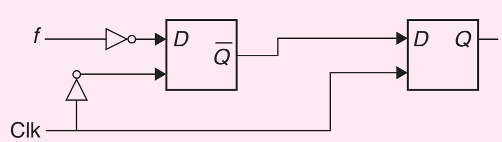
1. Consider the following circuit.

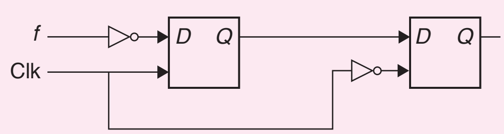


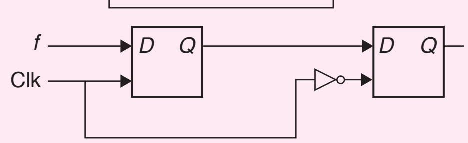
The flip-flops are positive edge triggered D FFs. Each state is designated as a 2-bit string Q_0Q_1. Let the initial state be 00. The state transition sequence is **[2005]**

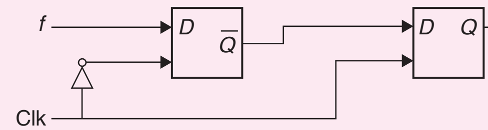
(A) 00 → 11 → 01
(B) 00 → 11
(C) 00 → 11 → 01 → 11
(D) 00 → 11 → 01 → 10

2. You are given a free running clock with a duty cycle of 50% and a digital waveform f which changes only at the negative edge of the clock. Which one of the following circuits (using clocked D flip-flops) will delay the phase of f by 180°? **[2006]**

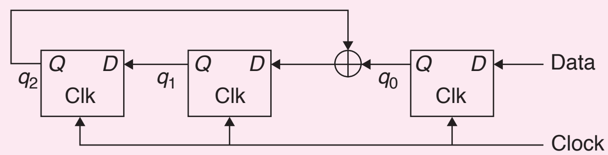
(A)


(B)


(C)


(D)


3. Consider the circuit in the diagram. The ⊕ operator represents Ex-OR. The D flip-flops are initialized to zeros (cleared). **[2006]**



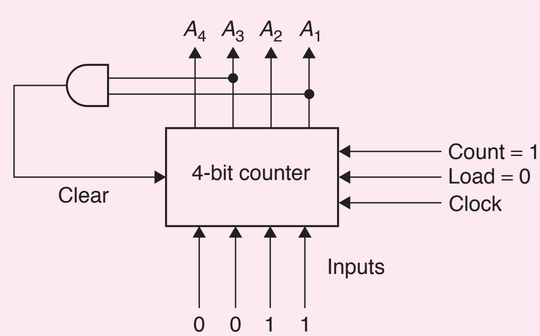
The following data: 100110000 is supplied to the data terminal in 9 clock cycles. After that the values of $q_2q_1q_0$ are

(A) 000 (B) 001
(C) 010 (D) 101

4. The control signal functions of a 4-bit binary counter are given below (where X is 'don't care'):

Clear	Clock	Load	Count	Function
1	X	X	X	Clear to 0
0	X	0	0	No change
0	↑	1	X	Load input
0	↑	0	1	Count next

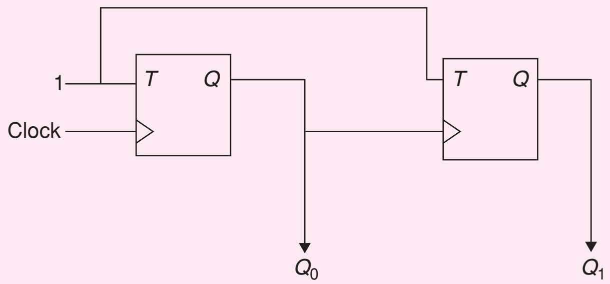
The counter is connected as follows:



Assume that the counter and gate delays are negligible. If the counter starts at 0, then it cycles through the following sequence: **[2007]**

(A) 0, 3, 4 (B) 0, 3, 4, 5
(C) 0, 1, 2, 3, 4 (D) 0, 1, 2, 3, 4, 5

5. In the sequential circuit shown below, if the initial value of the output Q_1Q_0 is 00, what are the next four values of Q_1Q_0? **[2010]**

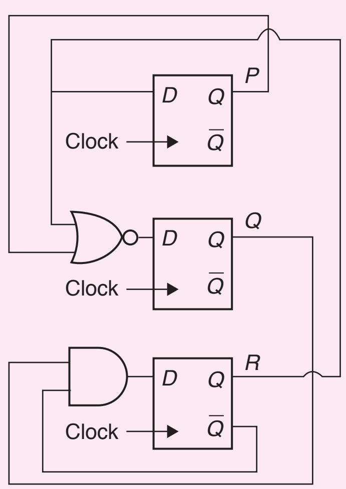


(A) 11, 10, 01, 00 (B) 10, 11, 01, 00
(C) 10, 00, 01, 11 (D) 11, 10, 00, 01

6. The minimum number of *D* flip-flops needed to design a MOD-258 counter is **[2011]**

(A) 9 (B) 8
(C) 512 (D) 258

Common Data for Questions 7 and 8: Consider the following circuit involving three D-type flip-flops used in a certain type of counter configuration.



7. If all the flip-flops were reset to 0 at power on, what is the total number of distinct outputs (states) represented by PQR generated by the counter? **[2011]**

(A) 3 (B) 4
(C) 5 (D) 6

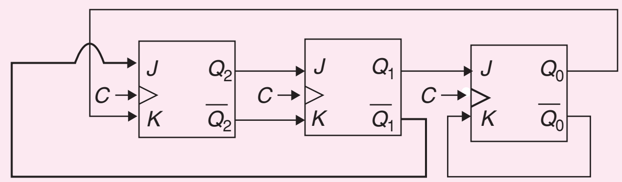
8. If at some instance prior to the occurrence of the clock edge, *P*, *Q*, and *R* have a value 0, 1, and 0, respectively, what shall be the value of PQR after the clock edge? **[2011]**

(A) 000 (B) 001
(C) 010 (D) 011

9. Let $K = 2^n$. A circuit is built by giving the output of an n-bit binary counter as input to an n-to-2^n-bit decoder. This circuit is equivalent to a **[2014)**

(A) K-bit binary up counter
(B) K-bit binary down counter
(C) K-bit ring counter
(D) K-bit Johnson counter

10.



The above synchronous sequential circuit built using *JK* flip-flops is initialized with $Q_2Q_1Q_0 = 000$. The state sequence for this circuit for the next 3 clock cycles is **[2014]**

(A) 001, 010, 011 (B) 111, 110, 101
(C) 100, 110,111 (D) 100, 011, 001

11. Consider a 4-bit Johnson counter with an initial value of 0000. The counting sequence of this counter is **[2015]**

(A) 0, 1, 3, 7, 15, 14, 12, 8, 0
(B) 0, 1, 3, 5, 7, 9, 11, 13, 15, 0
(C) 0, 2, 4, 6, 8, 10, 12, 14, 0
(D) 0, 8, 12, 14, 15, 7, 3, 1, 0

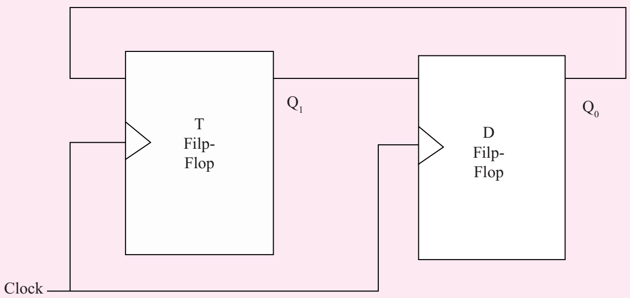
12. A positive edge-triggered D–flip-flop is connected to a positive edge-triggered JK flip-flop as follows. The *Q* output of the D flip-flop is connected to both the *J* and *K* inputs of the JK flip-flop, while the *Q* output of the JK flip-flop is connected to the input of the D flip-flop. Initially, the output of the D flip-flop is set to logic one and the output of the JK flip-flop is cleared. Which one of the following is the bit sequence (including the initial state) generated at the *Q* output of the JK flip-flop when the flip-flops are connected to a free-running common clock? Assume that $J = K = 1$ is the toggle mode and $J = K = 0$ is the state-holding mode of the JK flip-flop. Both the flip-flops have non-zero propagation delays. **[2015]**

(A) 0110110… (B) 0100100…
(C) 011101110… (D) 011001100…

13. The minimum number of JK flip-flops required to construct a synchronous counter with the count sequence (0, 0, 1, 1, 2, 2, 3, 3, 0, 0, …) is _______ **[2015]**

14. We want to design a synchronous counter that counts the sequence 0-1-0-2-0-3 and then repeats. The minimum number of *J-K* flip-flops required to implement this counteris ______ . **[2016]**

15. Consider a combination of T and D flip-flops connected as shown below. The output of the D flip-flop is connected to the input of the T flip-flop and the output of the T flip-flop is connected to the input of the D flip-flop.



Initially, both Q_0 and Q_1 are set to 1 (before the 1st clock cycle). The outputs **[2017]**

(A) $Q_1 Q_0$ after the 3rd cycle are 11 and after the 4th cycle are 00 respectively

(B) $Q_1 Q_0$ after the 3rd cycle are 11 and after the 4th cycle are 01 respectively

(C) $Q_1 Q_0$ after the 3rd cycle are 00 and after the 4th cycle are 11 respectively

(D) $Q_1 Q_0$ after the 3rd cycle are 01 and after the 4th cycle are 01 respectively

16. The next state table of a 2-bit saturating up-counter is given below.

Q_1	Q_0	Q_1^+	Q_0^+
0	0	0	1
0	1	1	0
1	0	1	1
1	1	1	1

The counter is built as a synchronous sequential circuit using T flip-flops. The expressions for T_1 and T_0 are **[2017]**

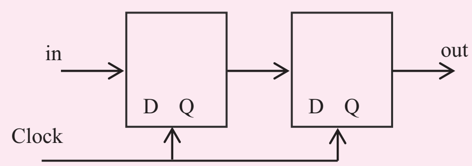
(A) $T_1 = Q_1 Q_0,$ $\quad T_0 = \overline{Q}_1 \overline{Q}_0$

(B) $T_1 = \overline{Q}_1 Q_0,$ $\quad T_0 = \overline{Q}_1 + \overline{Q}_0$

(C) $T_1 = Q_1 + Q_0,$ $\quad T_0 = \overline{Q}_1 + \overline{Q}_0$

(D) $T_1 = \overline{Q}_1 Q_0,$ $\quad T_0 = Q_1 + Q_0$

17. Consider the sequential circuit shown in the figure, where both flip-flops used are positive edge-triggered D flip-flops.



The number of states in the state transition diagram of this circuit that have a transition back to the same state on some value of "in" is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** A	**3.** A	**4.** B	**5.** C	**6.** A	**7.** C	**8.** A	**9.** B	**10.** A
11. C	**12.** B	**13.** B	**14.** C	**15.** D	**16.** B	**17.** A	**18.** D	**19.** A	**20.** C
21. D	**22.** A								

Practice Problems 2

1. D	**2.** B	**3.** D	**4.** B	**5.** A	**6.** C	**7.** C	**8.** B	**9.** A	**10.** B
11. C	**12.** B	**13.** C	**14.** B	**15.** B	**16.** D	**17.** C	**18.** C	**19.** C	**20.** D
21. C	**22.** A	**23.** D	**24.** B	**25.** C	**26.** A	**27.** A	**28.** C	**29.** C	**30.** A

Previous Years' Questions

1. D	**2.** C	**3.** C	**4.** C	**5.** A	**6.** A	**7.** B	**8.** D	**9.** C	**10.** C
11. D	**12.** A	**13.** 3($8 = 2^3$)		**14.** 3	**15.** B	**16.** B	**17.** 2		

TEST

DIGITAL LOGIC

Time: 60 min.

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. What is the range of signed decimal numbers that can be represented by 4-bit 1's complement notation?
(A) –7 to + 7 (B) –16 to +16
(C) –7 to +8 (D) –15 to +16

2. Which of the following signed representation have a unique representation of 0?
(A) Sign-magnitude (B) 1's complement
(C) 0's complement (D) 2's complement

3. Find the odd one out among the following
(A) EBCDIC (B) GRAY
(C) Hamming (D) ASCII

4. Gray code for number 8 is
(A) 1100 (B) 1111
(C) 1000 (D) 1101

5. Find the equivalent logical expression for $z = x + \bar{x}y$
(A) $z = x\underline{y}$ (B) $Z = \bar{x}y$
(C) $Z = \bar{x} + y$ (D) $Z = x + y$

6. The number of distinct Boolean expression of 3 variables is
(A) 256 (B) 16
(C) 1024 (D) 65536

7. The Boolean expression for the truth table shown is

X	*Y*	*Z*	*F*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

(A) $\underline{Y}(X + Z)(\bar{X} + \bar{Z})$ (B) $Y(X + \bar{Z})(\bar{x} + Z)$
(C) $\bar{Y}(X + \bar{Z})(\bar{x} + Z)$ (D) $\bar{Y}(X + Z)(\bar{X} + \bar{Z})$

8. The number of essential prime implicants for the Boolean functions shown in the given K-map.

XY \ WZ	00	01	11	10
00	1	1	0	1
01	1	0	0	1
11	1	0	0	0
10	1	0	0	1

(A) 4 (B) 5
(C) 6 (D) 8

9. The number of product terms in the minimized SOP from is

1	0	0	1
0	*D*	0	0
0	0	*D*	1
1	0	0	1

(A) 2 (B) 4
(C) 5 (D) 3

10. The minimum number of 2 input NAND gates needed to implement $Z = XY + VW$ is
(A) 2 (B) 3
(C) 4 (D) 5

11. The operation $\bar{a} \oplus \bar{b}$ represents
(A) $ab + \bar{a}\,\bar{b}$ (B) $\bar{a}b + \overline{ab}$
(C) $a\bar{b} + \bar{a}b$ (D) $a - \bar{b}$

12. Find the dual of $X + [Y + XZ] + U$
(A) $X + [Y(X + Z)] + U$ (B) $X(Y + XZ)U$
(C) $X + [Y(X + Z)]U$ (D) $X[Y(X + Z)]U$

13. The simplified form of given function $AB + BC + A\bar{C}$ is equal to
(A) $AB + A\bar{C}$ (B) $A\bar{C} + BC$
(C) $\bar{A}C + BC$ (D) $A\bar{B} + A\bar{C}$

14. Simplify the following

WX \ YZ			
1	1	0	1
1	1	0	1
0	0	1	1
0	0	0	0

(A) $\bar{W}\,\bar{Y} + \bar{W}\,\bar{Z} + WXY$
(B) $\bar{W}\,\bar{X} + \bar{W}\,\bar{Z} + WXY$
(C) $WY + WYZ + WXY + XY\bar{Z}$
(D) $\bar{W}\,\bar{X} + \bar{Y}\,\bar{Z} + \bar{W}\,\bar{Z}$

15. Simplify the following
$F = ABCD + A\bar{B}CD + \bar{A}C\bar{B}D + \bar{A}BCD$
(A) CD (B) BC
(C) AB (D) $\bar{C} + \bar{D}$

16. Find the equivalent Boolean expression for $AC + B\bar{C}$
(A) $\bar{A}C + B\bar{C} + AC$
(B) $ABC + A\bar{B}C + AB\bar{C} + \bar{A}B\bar{C}$
(C) $ABC + A\bar{B}C + AB\bar{C} + \bar{A}\,\bar{B}\bar{C}$
(D) $\bar{A}C + B\bar{C} + \bar{A}\bar{C}$

17. Simplify the following expression

$\bar{A}\bar{B}\bar{C}+\bar{A}B\bar{C}+\bar{A}BC+AB\bar{C}$

(A) $\bar{A}\bar{C}+B\bar{C}+\bar{A}B$ (B) $A\bar{C}+B\bar{C}+\bar{A}B$

(C) $\bar{A}\bar{C}+\bar{B}C+\bar{A}B$ (D) $\bar{A}\bar{C}+\bar{B}\bar{C}+\bar{A}B$

18. If $A = 1$ in the logic equation $[A+C\{\bar{B}+(\bar{C}+A\bar{B})\}]$ $[\bar{A}+\bar{C}(A+B)]=1$, then

(A) $B = C$ (B) $B = \bar{C}$

(C) $C = 1$ (D) $C = 0$

19. Which is the odd function with 3 Boolean variables in it

(A) $\Sigma(0, 3, 5, 6)$ (B) $\Sigma(0, 2, 4, 6)$

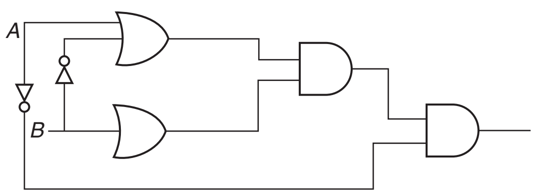
(C) $\Sigma(1, 2, 4, 7)$ (D) $\Sigma(1, 3, 5, 7)$

20. Which of the following expressions is/are incorrect?

(A) $\overline{a+b}=\bar{a}\,\bar{b}$ (B) $\overline{\bar{\bar{a}}+b}=\bar{a}\,\bar{b}$

(C) $\overline{\bar{\bar{a}}\,\bar{\bar{b}}}=\bar{a}+\bar{b}$ (D) $\overline{\bar{a}+\bar{b}}=\bar{a}\,\bar{b}$

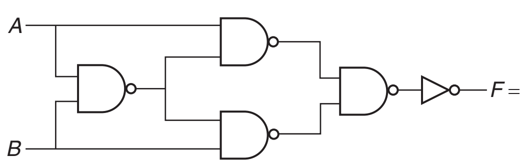
21. The simplified form of logic circuit is



(A) $A + B$ (B) $\overline{AB}$

(C) $\bar{A}+\bar{B}$ (D) $\bar{A}\,\bar{B}$

22. The circuit shown in figure is equivalent to —— gate.



(A) X-OR gate (B) EX-NOR gate

(C) Half adder (D) Half subtractor

23. The truth table of the circuit shown in figure

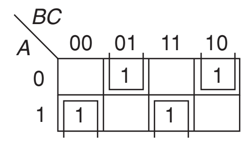
A	B	C	Z
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

The Boolean expression for Z

(A) $\overline{(\overline{\bar{A}+B})(\overline{B+\bar{C}})}$ (B) $\overline{(\overline{A+B})(\overline{B+\bar{C}})}$

(C) $\overline{(\overline{\bar{A}+B})\,(\overline{B+C})}$ (D) All of the above

24. A combinational circuit has input A, B and C and its K-map is as shown in figure. The output of the circuit is given by



(A) $(\bar{A}B+A\bar{B})\bar{C}$ (B) $(AB+\bar{A}\,\bar{B})\bar{C}$

(C) $\bar{A}\,\bar{B}\bar{C}$ (D) $A\oplus B\oplus C$

25. Which of the following two 2-input gates will realize the Boolean expression $X(P, Q, R) = \pi(0, 5)$

(A) AND and OR (B) NAND and OR

(C) AND and X-OR (D) OR and X-OR

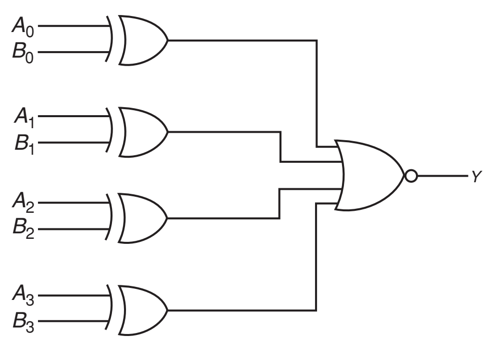
26. Simplify the given function

$f(x, y, z) = \Sigma m(0, 2, 3, 4, 5, 7)$

(A) $\bar{x}y+\bar{y}\,\bar{z}+xz$ (B) $\bar{x}\,\bar{z}+x\bar{y}+yz$

(C) Both (A) and (B) (D) $\bar{x}\,\bar{z}+\bar{x}y+x\bar{y}+xz$

27. Figure below shows a digital circuit, which compares two numbers $A_0\ A_1\ A_2\ A_3$, $B_0\ B_1\ B_2\ B_3$. Choose the pair of correct input number to get output $Y = 0$.



(A) 1100, 1100 (B) 0110, 0110

(C) 1011, 0010 (D) 1011, 1011

28. How many 3 to 8 line decoders with an enable input are required to build 6 of 34 decoder?

(A) 6 (B) 2

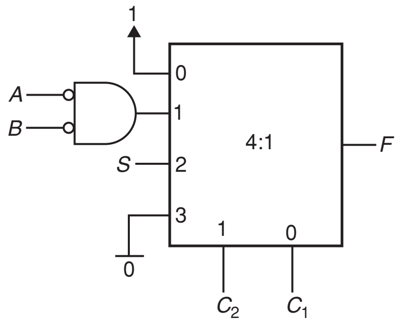
(C) 9 (D) 4

29. It is required to construct a 2^n to 1 multiplexer by using 2-to-1 multiplexer only. How many of 2-to-1 multiplexer are needed?

(A) n (B) 2^{2n}

(C) 2^{n-1} (D) $2^n - 1$

30. Consider the following circuit



Which one of the following give the function implemented by the MUX based digital circuit?

(A) $F = C_2 \cdot \overline{C_1} S + \overline{C_2} C_B (\overline{A} + \overline{A})$

(B) $F = \overline{C_2} \cdot \overline{C_1} + C_2 C_1 + C_2 \overline{C_1} S + \overline{C_2} C_1 \overline{AB}$

(C) $F = \overline{AB} + S$

(D) $F = \overline{C_2} \cdot \overline{C_1} + C_2 \cdot \overline{C_1} S + \overline{C_2} C_1 \overline{A} \cdot \overline{B}$

Answer Keys

1. A	**2.** D	**3.** C	**4.** A	**5.** D	**6.** A	**7.** A	**8.** A	**9.** A	**10.** B
11. C	**12.** D	**13.** B	**14.** A	**15.** A	**16.** B	**17.** A	**18.** D	**19.** C	**20.** D
21. D	**22.** B	**23.** B	**24.** D	**25.** D	**26.** C	**27.** C	**28.** C	**29.** D	**30.** D

Digital Logic Test I

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Assume the propagation delay time of 2 input gates as EXOR-20 ns, AND – 10 ns, OR-10 ns, the propagation delay time for sum and carry output of a full adder circuit are respectively, when all the data inputs are applied simultaneously?

(A) 30 ns, 20 ns (B) 40 ns, 30 ns
(C) 40 ns, 20 ns (D) 20 ns, 20 ns

2. The minimized POS expression of the function $f(A, B, C, D) = AB + A\bar{C} + C + AD + A\bar{B}C + ABC$

(A) $A + \bar{C}$ (B) $\bar{A} + \bar{B}$
(C) AC (D) $A + C$

3. The signed two's complement representation of $(-783)_{10}$ is (in HEX):

(A) 830FH (B) 04F1H
(C) FCF1H (D) F3F1H

4. The two numbers represented in signed 2's complement form are:
P = 11011101 and Q = 11100101, if Q is subtracted from P, the value obtained in signed 2's complement form is?

(A) 11110111 (B) 11000010
(C) 11111000 (D) 00000111

5. The subtraction of a binary number B from another binary number A, done by adding the 2's complement of B to A, results in a binary number without carry, this implies that the result is:

(A) negative and is in normal form
(B) positive and is in normal form
(C) negative and is in 2's complement form
(D) positive and is in 2's complement form

6. $f(a, b, c) = ab + b^1c$ in the canonical POS form is represented as:

(A) $(a + b + c)(a + b + c^1)(a + b^1 + c)(a^1 + b^1 + c)$
(B) $(a + b^1 + c)(a + b^1 + c^1)(a + b + c)(a^1 + b + c)$
(C) $(a + b + c)(a^1 + b^1 + c)(a + b^1 + c)$
(D) $(a^1 + b + c)(a^1 + b^1 + c)(a + b + c)(a + b^1 + c)$

7. The Essential prime Implicants of the function
$f(A, B, C, D) = \bar{A}C + ABD + \bar{A}B + \bar{B}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D}$ are:

(A) $BD, \bar{B}\bar{D}, \bar{A}$ (B) $\bar{A}C, \bar{B}\bar{D}, B$
(C) $BD, \bar{A}C, \bar{B}$ (D) $\bar{A}\bar{B}, \bar{B}\bar{D}, C$

8. A combinational circuit has 3 inputs x, y, z and three outputs A, B, C. When the binary input is 4, 5, 6 and 7, the binary output is 2 less than the binary input. When the binary input is 0, 1, 2 and 3, the output is 4 more than the binary input the Boolean expression for output A and C respectively are:

(A) $x^1 y, z$ (B) $x + y^1, z^1$
(C) x, z (D) $x^1 + y, z$

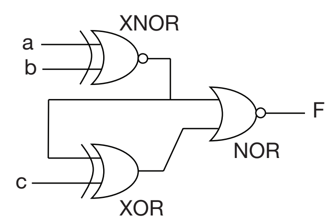
9. In the above problem statement, how many number of NOR are gates required implement output B.

(A) 3 (B) 4
(C) 5 (D) 6

10. A combinational circuit takes 2 inputs and output is the 2's complement of input binary number. Consider the inputs as a and b and output as x and y, the equations of x and y respectively?

(A) $a \odot b, b$ (B) $a^1 b, a \odot b$
(C) $a^1 b, ab^1$ (D) $a \oplus b, b$

11. The output F in the digital logic circuit shown in the figure is:



(A) $a^1 bc + ab^1 c$ (B) $a^1bc^1 + ab^1c^1$
(C) $a^1b^1c + abc$ (D) $a^1b^1c^1 + abc$

12. To construct a 5 to 32 line decoder, how many numbers of 3 to 8 line decoders and 2 to 4 line decoders are required respectively without using any extra hardware?

(A) 3, 2 (B) 4, 1
(C) 2, 4 (D) 2, 2

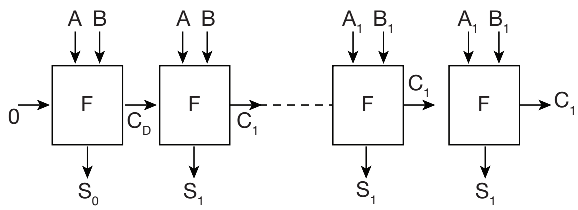
13. Parity is a common error detection mechanism that is often used in data reception or retrieval systems. Consider a parity encoder that is used for data transmission or storage. If a word contains an even number of 1's, the parity bit is 0. If the word has odd number of 1's the parity bit is 1. If the data is w, x, y, z then the min terms for parity bit is?

(A) $\sum m(1, 2, 4, 7, 8, 11, 13, 14)$
(B) $\sum m(0, 3, 5, 6, 9, 10, 12, 15)$
(C) $\sum m(0, 1, 3, 5, 8, 10, 13, 15)$
(D) $\sum m(1, 3, 5, 7, 9, 11, 13, 15)$

14. Consider the Boolean functions
$f_1(A, B, C, D) = AC + BD$
$f_2(A, B, C, D) = \sum m(4, 5, 6, 7, 10, 11, 14, 15)$
Then find $f_1 + f_2$ in minimized POS form

(A) $(\bar{A} + \bar{B})(B + D)(\bar{A} + \bar{B} + C)$
(B) $(A + B)(B + C)(\bar{A} + C + D)$
(C) $(A + \bar{B})(B + D)(\bar{A} + C + D)$
(D) $(A + D)(B + C)(\bar{A} + B + C)$

15. A 16-bit ripple carry adder is realized using 16 identical full address as shown in figure. The carry propagation delay of each FA is 15 ns and the sum propagation delay of each FA is 18 ns.
The worst delay of this 16-bit adder will be:



(A) 243 ns (B) 228 ns
(C) 240 ns (D) 270 ns

16. For an n-variable Boolean function, the maximum number of prime implicants is:

(A) $\frac{n}{2}$ (B) $2^n - 1$
(C) 2^{n-1} (D) 2^n

17. If the Boolean function $f(a, b, c, d) = a + b + c + d$ has to be implemented with only 2 input NAND gates, then how many NAND gates are required?

(A) 6 (B) 7
(C) 8 (D) 9

18. The following expression is valid for the number system with base _______ $\frac{302}{20} = 12.1$.

(A) 6 (B) 5
(C) 4 (D) 8

19. P is a 16 bit signed number integer, the 2's complement representation of P is $(FB8A)_{16}$. The 2's complement representation of $8 \times P$ is:

(A) $(B8\,A0)_{16}$ (B) $(C\,7B4)_{16}$
(C) $(ABCD)_{16}$ (D) $(DC50)_{16}$

20. For a 4 bit magnitude comparator with two inputs each of 4 bit $A(a_3, a_2, a_1, a_0)$ and $B(b_3, b_2, b_1\, b_0)$, the Boolean equation for $A < B$ is:

(A) $a_3^1 b_3 + a_2^1 b_2 + a_1^1 b_1 + a_0^1 b_0$

(B) $a_3 b_3^1 + (a_3 \oplus b_3) a_2 b_2^1 + (a_3 \oplus b_3)(a_2 \oplus b_3) a_2 b_1 + (a_3 \oplus b_3)(a_2 \oplus b_2)(a_1 \oplus b_1) a_0 b_0$

(C) $a_3^1 b_3 + (a_3 \odot b_3) a_2^1 b_2 + (a_3 \odot b_3)(a_2 \odot b_2) a_1^1 b_1 + (a_3 \odot b_3)(a_2 \odot b_2)(a_1 \odot b_1) a_0^1 b_0$

(D) $a_3^1 b_3 + (a_3 \oplus b_3) a_2^1 b_2 + (a_3 \oplus b_3) a_1^1 b_1 + (a_3 \oplus b_3)(a_2 \oplus b_2)(a_1 \oplus b_1) a_0^1 b_0$

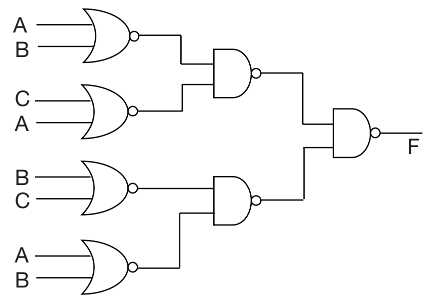
21. The simultaneous equations on the Boolean variables a, b, c and d.

$a + b + c = 1$
$ab = 0$
$ac + d = 1$
$ab + \bar{c}\bar{d} = 0$

have the following solutions for a, b, c and d respectively:

(A) 1011 (B) 1100
(C) 1000 (D) 1101

22. What is Boolean expression for output (F) of the combinational logic circuit of NAND-NOR gate given below?



(A) $\overline{A + B}$ (B) $\overline{A + C}$
(C) $\overline{ABC}$ (D) $\overline{A + B + C}$

23. Let $f(A, B, C, D) = \sum m(0, 2, 3, 4, 5, 7, 9, 13, 15)$
Which of the following expressions is not equivalent of f?

(P) $\bar{A}\bar{B}\bar{D} + \bar{A}B\bar{C} + \bar{A}CD + A\bar{C}DBD$

(Q) $\bar{A}\bar{C}\bar{D} + \bar{A}\bar{B}C + A\bar{C}D + BD$

(R) $\bar{A}\bar{B}\bar{D} + \bar{A}B\bar{C} + BCD + A\bar{C}D$

(S) $\bar{A}\bar{B}\bar{D} + \bar{A}CD + ABD + A\bar{C}D$

(A) only Q (B) only S
(C) P and S (D) P and Q

24. The range of integers that can be represented by an 'n' bit 2's complement signed number system is:

(A) -2^{n-1} to $+(2^{n-1} - 1)$
(B) $-(2^{n-1} - 1)$ to $+(2^{n-1} - 1)$
(C) $-2^{n-1} + 1$ to $+2^{n-1}$
(D) -2^{n-1} to $+2^{n-1}$

25. The minimized POS expression for k-map shown is:

CD \ AB	00	01	11	10
00	0		0	X
01	X		X	X
11	0		0	0
10	0		0	0

(A) $\bar{A} + B$ (B) $\bar{A}B$
(C) $\bar{A} + \bar{B}(\bar{A} + B)(A + B)$ (D) $\bar{A}(A + B)$

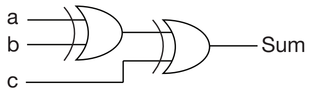
Answer Keys

1. B	**2.** D	**3.** C	**4.** C	**5.** C	**6.** B	**7.** A	**8.** D	**9.** C	**10.** D
11. B	**12.** B	**13.** A	**14.** B	**15.** A	**16.** C	**17.** D	**18.** C	**19.** D	**20.** C
21. A	**22.** D	**23.** D	**24.** A	**25.** B					

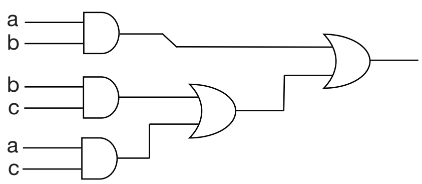
Hints and Explanations

1. Sum $= a \oplus b \oplus c$

Carry $= ab + bc + ac$



$20 + 20 = 40$ ns



$10 + 10 + 10 = 30$ ns

Two level exor gate for sum

So $20 + 20 = 40$ ns

Carry will be implemented with 2 input gates in 3 levels, so $10 + 10 + 10 = 30$ ns Choice (B)

2. $f = AB + A\bar{C} + C + AD + A\bar{B}C + ABC$

$= AB + A + C + AD + AC$

$= A(B + 1 + D + C) + C$

$= A + C$ Choice (D)

3. $783 = 512 + 256 + 8 + 4 + 2 + 1 = 1100001111$

$+783 = 0000\ 0011\ 0000\ 1111$ (add 0's to MSB)

$-783 = 1111\ 1100\ 1111\ 0001$ (2's complement of +783)

In $HEX \Rightarrow FCF1$ Choice (C)

4. $P = 11011101$

00100011 (by taking 2's complement)

$P = -35$

$Q = 11100101$

00011011 (By taking 2's complement)

$Q = -27$

$P - Q = -35 - (-27) = -8 = 1111\ 1000$ (in signed 2's complement form)

(or)

$P = 1101\ 1101$

$Q = 1110\ 0101$ (direct subtraction)

1111 1000 Choice (C)

5. $A - B$ has to be performed

So, the 2's complement of B (which is $2^n - B$, n = no. of bits in B) is added to A

So result is $A + 2^n - B$; and there is no carry

$A + 2^n - B = 2^n - (A - B)$

So, the result is negative and it is in 2's complement form. Choice (C)

6. $f(a, b, c) = ab + b^1 \cdot c$

$= (ab + b^1)(ab + c)$ $[x + yz = (x + y)(x + z)]$

$= (a + b^1)(a + c)(b + c)$

$= (a + b^1 + c \cdot c^1)(a + b \cdot b^1 + c)(a \cdot a^1 + b + c)$

$= (a + b^1 + c)(a + b^1 + c^1)(a + b + c)(a + b^1 + c)(a + b + c)$

$(a^1 + b + c)$

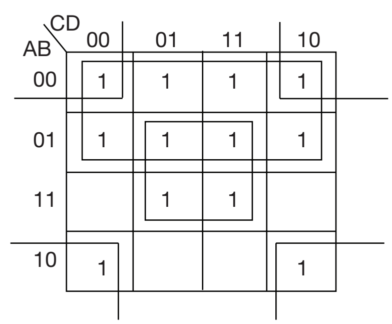
$= (a + b^1 + c)(a + b^1 + c^1)(a + b + c)(a^1 + b + c)$

Choice (B)

7. $f(A, B, C, D) = \bar{A}C + ABD + \bar{A}B + \bar{B}\bar{D} + \bar{A}\bar{B}\bar{C}D$

Product term	Equivalent	Min terms
$\bar{A}C$	0X1X	0010, 0011, 0110, 0111
ABD	11X1	1101, 1111
$\bar{A}B$	01XX	0100, 0101, 0110, 0111
$\bar{B}\bar{D}$	X0X0	0000, 0010, 1000, 1010
$\bar{A}\bar{B}\bar{C}D$	0001	0001

$f(A, B, C, D) = \sum m(0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 13, 15)$



$f(A, B, C, D) = \bar{A} + BD + \bar{B}\bar{D}$

Essential prime implicants are $\bar{A}, BD, \bar{B}\bar{D}$

Choice (A)

8. The truth table is:

x	y	z	A	B	C
0	0	0	1	0	0
0	0	1	1	0	1
0	1	0	1	1	0
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	0	1

output is 4 more than input (rows 000–011)

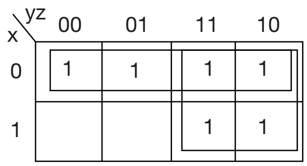
output is 2 less than input (rows 100–111)

$A(x, y, z) = \sum m(0, 1, 2, 3, 6, 7)$

$B(x, y, z) = \sum m(2, 3, 4, 5)$

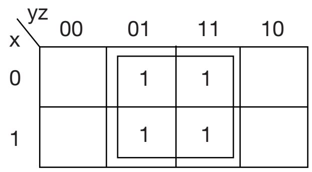
$C(x, y, z) = \sum m(1, 3, 5, 7)$

The k map for A



$A = x^1 + y$

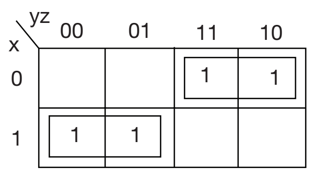
The k map for C



$C = z$

Choice (D)

9. $B(x, y, z) = \sum m(2, 3, 4, 5)$



$B = xy^1 + x^1y = x \oplus y$

2 input XOR required 5 NOR gates. Choice (C)

10. a, b are inputs of 2's complementer, and x, y are the outputs.
So truth table is

a	b	x	y
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	1

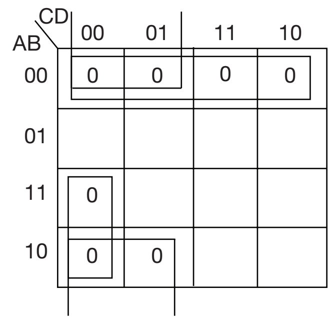
$x = a^1b + ab^1 = a \oplus b$
$y = b$ Choice (D)

11. Output of XNOR $= a \odot b = a^1b^1 + ab$
Output of XOR $= [a \odot b] \oplus c =$
Output of NOR gate $= [a \odot b \oplus c + (a \odot b)]^1$
$(x \oplus y + x) = x^1y + xy^1 + x = x + y$
$[(a \odot b) \oplus c + (a \odot b)]^1 = (a \odot b + c)^1$
$= (a \odot b)^1 \cdot C^1 = (a \oplus b)\, c^1 = a^1bc^1 + ab^1c^1$
Choice (B)

12. 5 to 32 line decoder will have 32 output lines
So 4, 3 to 8 line Decoders are required, these 4 decoders will be selected by one 2 to 4 lines decoder.
So 4, 3 to 8 line decoder and 1, 2 to 4 line decoder
(or)
5 to 32 line decoder will have 32 output lines
So 8, 2 to 4 line Decoders are required, to select one of these 8, one 3 to 8 line Decoder is required.
8, 2 to 4 Decoders, and 1, 3 to 8 Decoder. Choice (B)

13. For even number of 1's parity bit is 0.
So even parity,
Even parity can be implemented by XOR gate
XOR of even 1's given output 0.
XOR of add 1's gives output 1.
So parity bit $P = w \oplus x \oplus y \oplus z$
$P = \sum m(0001, 0010, 0100, 0111, 1000, 1011, 1101, 1110)$
$= \sum m(1, 2, 4, 7, 8, 11, 13, 14)$ Choice (A)

14. $f_1(A, B, C, D) = AC + BD$
$= AC(B + \bar{B})(C + \bar{C}) + (A + \bar{A})(C + \bar{C})BD$
$= \sum m(5, 7, 10, 11, 13, 14, 15)$
$= \Pi M(0, 1, 2, 3, 4, 6, 8, 9, 12)$
$f_2 = \sum m(4, 5, 6, 7, 10, 11, 14, 15)$
$= \Pi M(0, 1, 2, 3, 8, 9, 12, 13)$
$f_1 + f_2 = \Pi M(0, 1, 2, 3, 8, 9, 12)$ [common max terms of f_1 and f_2]

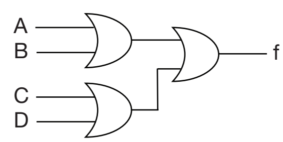


$f_1 + f_2 = (A + B)\,(B + C)\,(A + C + D)$ Choice (B)

15. The final carry we will get after n times of the delay of the carry of each full adder. But sum has more propagation delay. To get the carry of last but one, stage (C_{14}) we required $15 \times t$ carry $= 15 \times 15 = 225$ ns
The next carry C_{15} we get after another 15 ns, but sum S_{15} we get after another 18 ns (= 243 ns)
The worst delay $= (n - 1)t_{\text{carry}} + t_{\text{sum}} = (16 - 1) \times 15 + 18 = 243$ Choice (A)

16. Choice (C)

17.



Each 2 input OR gate required 3–2 input NAND gates
So total 9 NAND gates are required. Choice (D)

18. $\dfrac{(302)_r}{(20)_r} = (12.1)_r$

Convert to decimal number system

$$\frac{3r^2 + 0r^1 + 2r^0}{2r^1 + 0 \cdot r^0} = 1r^1 + 2r^0 + 1\frac{1}{r}$$

$$-\frac{3r^2 + 0 \cdot r^1 + 2 \cdot r^0}{2r^1 + 0 \cdot r^0} = 1 \cdot r^1 + 2 \cdot r^0 + 1 \cdot \frac{1}{r}$$

$$\frac{3r^2 + 2}{2r} = r + 2 + \frac{1}{r} \Rightarrow 3r^2 + 2 = 2r^2 + 4r + 2$$

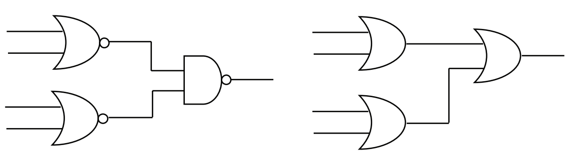
$\Rightarrow r^2 = 4r \Rightarrow r = 0$ or 4
$r = 4$ is valid Choice (C)

19. $P = FB8A = 1111101110001010$
$8P = 2^3\, P \rightarrow P$ shifted to left by 3 bits,
$8P = 1101\ 1100\ 0101\ 0000$
$= (DC50)_{16}$ Choice (D)

20. If $A(a_3\, a_2\, a_1\, a_0)$ and $B(b_3\, b_2\, b_1\, b_0)$ are the two inputs the $A < B$ is possible only when the bits in A are 0 and the bits in B are 1. So we can check MSB by using $a_3^1 b_3$, if the MSB bits are equal, then we check next bits $(a_3 \odot b_3)\ a_2^1\ b_2$ and if the higher order bits are equal then we move to next bits so $(A < B) = a_3^1 b_3 + (a_3 \odot b_3)$ $a_2^1 b_2 + (a_3 \odot b_3)\ (a_2 \odot b_2)\ a_1^1 b_1 + (a_3 \odot b_3)(a_2 \odot b_2)$ $(a_1 \odot b_1)\ a_0^1\, b_0$. Choice (C)

21. $a + b + c = 1$
$ab = 0 \rightarrow$ by substituting in 4th equation
$ab + \overline{c}\overline{d} = 0 \Rightarrow 0 + \overline{c}\overline{d} = 0 \Rightarrow \overline{c + d} = 0$
$\Rightarrow \quad c + d = 1$
$ac + d = 1 \Rightarrow (a + d)\ (c + d) = 1$
already $c + d = 1$ so $a + d = 1$
So we have now $a + b + c = 1$, $ab = 0$, $c + d = 1$, $a + d = 1$ by verification option (A) is valid for all the four equations. Choice (A)

22. NOR–NAND is equivalent to OR-OR



So output of 1st NOR-NAND structure $= A + B + C + A$
$= A + B + C$

Similarly output of 2nd NOR-NAND structure
$= A + B + C$
So output $F = \overline{A + B + C}$ Choice (D)

23. We can verify by writing the min terms

(P) $\overline{A}\overline{B}\overline{D} + \overline{A}B\overline{C} + \overline{A}CD + A\overline{C}D + BD$
00X0 010X 0X11 1X01 X1X1
$\Sigma m(0, 2, 3, 4, 5, 7, 9, 13, 15)$

(Q) $\overline{A}\overline{C}\overline{D} + \overline{A}\overline{B}C + A\overline{C}D + BD$
0X00 001X 1X01 X1X1
$\Sigma m(0, 2, 3, 4, 5, 7, 9, 13, 15)$

(R) $\overline{A}\overline{B}\overline{D} + \overline{A}B\overline{C} + BCD + A\overline{C}D$ 00X0, 010X, X111, 1X01
$\Sigma m(0, 2, 4, 5, 7, 9, 13, 15)$ – min term 3 missing

(S) $\overline{A}\overline{B}\overline{D} + \overline{A}CD + ABD + A\overline{C}D$ 00X0, 0X11, 11X1, 1X01
$\Sigma m(0, 2, 3, 7, 9, 13, 15)$ – min terms 4, 5 missing.
Choice (D)

24. Choice (A)

25. Two octates present so minimized expression is $\overline{A} \cdot B$

CD \ AB	00	01	11	10
00	0		X	X
01	X		X	X
11	0		0	0
10	0		0	0

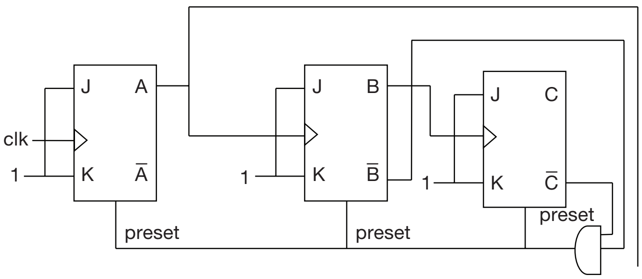
Choice (B)

Digital Logic Test 2

Number of Questions: 25 **Section Marks: 30**

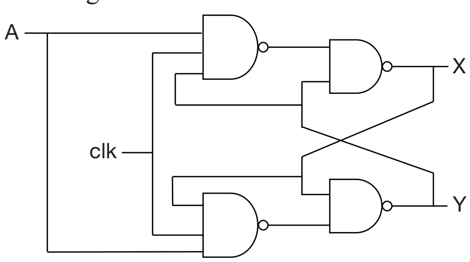
Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The ripple counter shown in figure works as a:



(A) Mod-6 up counter
(B) Mod-5 up counter
(C) Mod-5 down counter
(D) Mod-6 down counter

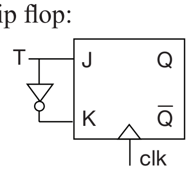
2. Consider the given circuit:

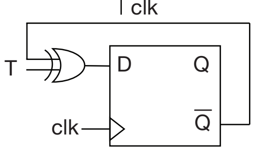


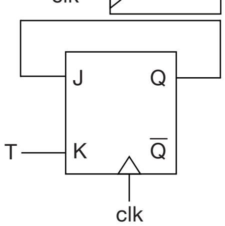
In this circuit, the race around condition
(A) does not occur
(B) occurs when clk = 0
(C) occurs when clk = 0, A = 1 and $X = Y = 1$
(D) occurs when clk = 1, A = 1

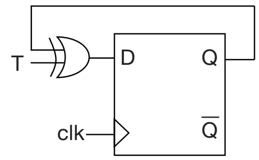
3. Three, modulo-4 counters are cascaded together then the resultant counter modulus is
(A) 3×4 (B) 3^4
(C) $4 \times 4 \times 4$ (D) $3 + 4$

4. Which of the following flip flop configuration works as T flip flop:

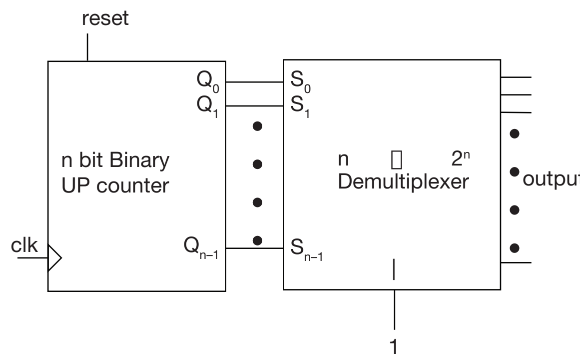
(A)


(B)


(C)


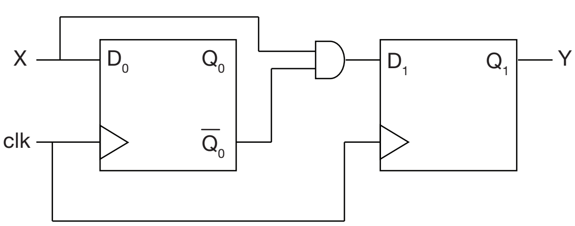
(D)


5.

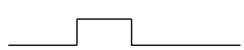


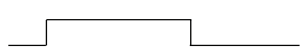
'n' bit Binary UP counter is connected to $n \times 2^n$ Demultiplxer with input I = 1, as shown in the figure.
If the counter starts from reset, the above configuration works as
(A) Modulo – 2^n twisted ring counter
(B) Modulo – n ring counter
(C) Modulo – $2n$ Johnson counter
(D) Modulo – 2^n ring counter

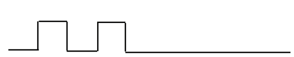
6. Consider the following circuit with initial state $Q_0 = Q_1 = 0$. The D flip flops are positive edge triggered.

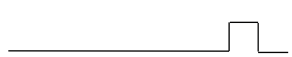


Consider the following timing diagrams of X and clk. Which one is the correct wave form of Y?

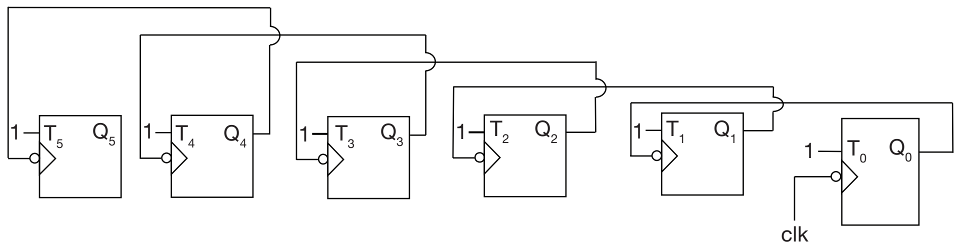
(A) 

(B) 

(C) 

(D) 

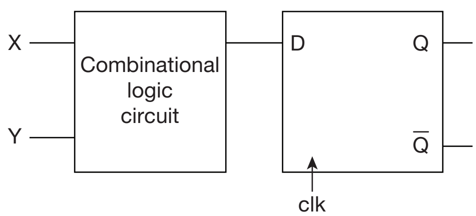
7. Six JK Flip flops are cascaded to form the circuit shown in figure. Clock pulses at a frequency of 128 kHz are applied as shown, The frequency (in kHz) of the wave form at Q_4 is ______.



(A) 32 kHz (B) 16 kHz
(C) 8 kHz (D) 4 kHz

8. The following truth table has to be realized with *D* flip flop.

XY	Q_{n+1}
0 0	Q_n
0 1	1
1 0	0
1 1	$\bar{Q}_n$

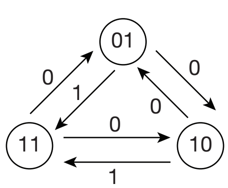


Then what is the equation of combinational logic circuit output in terms of *X*, *Y* and Q_n?

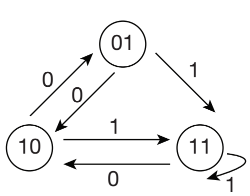
(A) $D = \bar{X}Q_n + Y\bar{Q}_n$ (B) $D = X\bar{Q}_n + \bar{Y}Q_n$
(C) $D = XY + Q_n$ (D) $D = (X + Y) + Q_n$

9. A clocked sequential circuit has 3 states *A*, *B*, *C* and 1 input *X*. As long as input $X = 0$, the circuit alternates between states *A* and *B*, if input $X = 1$ (either in state *A* or *B*), The circuit goes to state *C* and remain in state *C* as long as $X = 1$, from state *C*, circuit returns to state *A* when input $X = 0$, and then a repeats its behavior. Assume $A = 01$, $B = 10$, $C = 11$ the state diagram will be:

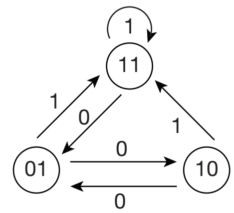
(A)



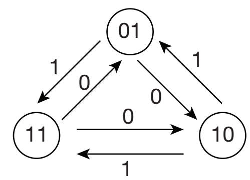
(B)



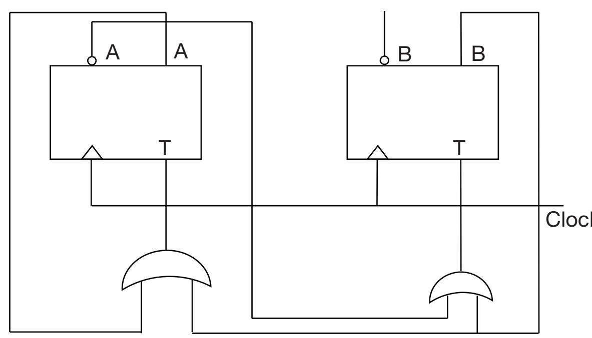
(C)



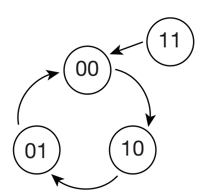
(D)



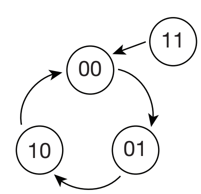
10. Find the state (*AB*) diagram for the following sequential circuit?



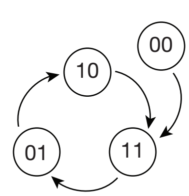
(A)



(B)

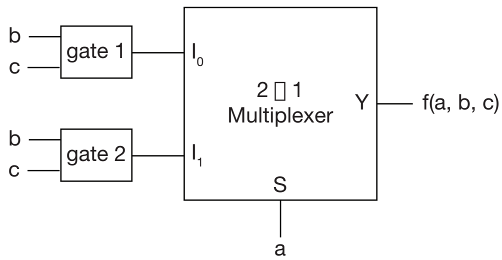


(C)



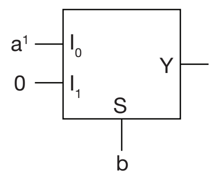
(D) None of these

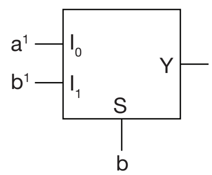
11. The Boolean function $f(a, b, c) = a^1\, b + b^1\, c + ac^1$ has to be implemented by the following 2×1 multiplexer then the gate 1 and gate 2 are respectively?

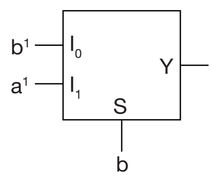


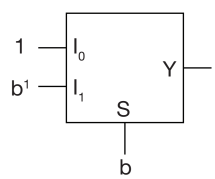
(A) OR, NAND (B) AND, OR
(C) NOR, AND (D) NAND, OR

12. Which of the following multiplexer implements 2 input NAND gate?

(*P*)


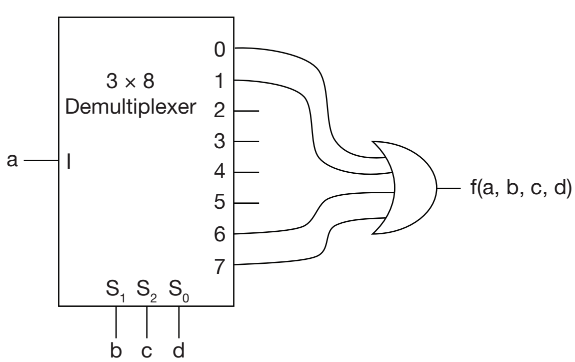
(*Q*)


(*R*)


(*S*)


(A) *Q*, *R* (B) *P*, *R*
(C) *P*, *S* (D) *R*, *S*

13. The output of the following Demultiplexer circuit is:



(A) $a(b \odot c)$ (B) $a(b \odot c)$
(C) $(a \odot b)c$ (D) $(a \odot b)c$

14. An 8 × 1 multiplexer has inputs *A*, *B*, *C* connected to the selection inputs S_2, S_1 and S_0 respectively. The data inputs I_0 through I_7 are as follows. $I_1 = I_2 = 0$; $I_3 = I_5 = I_7 = 1$; $I_0 = I_4 = D$; and $I_6 = D^1$ then the Boolean function that the multiplexer implements is?

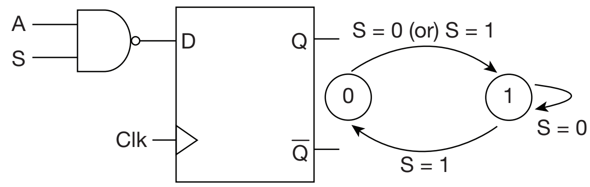
(A) $AB + BC + A\bar{C}D + \bar{B}\bar{C}D$

(B) $AC + BD + A\bar{B}D + B\bar{C}\bar{D}$

(C) $\bar{B}\bar{C}D + AB\bar{D} + BC + AC$

(D) $A\bar{B}D + \bar{B}\bar{C}D + A\bar{C} + BC$

15. The digital circuit shown in the figure satisfies the given state diagram, when *Q*, is connected to input *A* of NAND gate



Suppose the NAND gate is replaced by an OR gate which are of the following options preserves the state diagram.

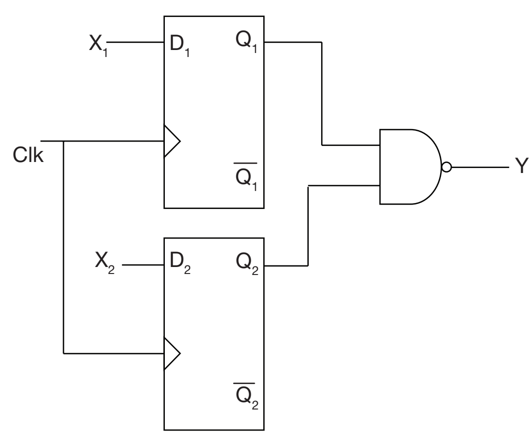
(A) input *A* is connected to $\bar{Q}$

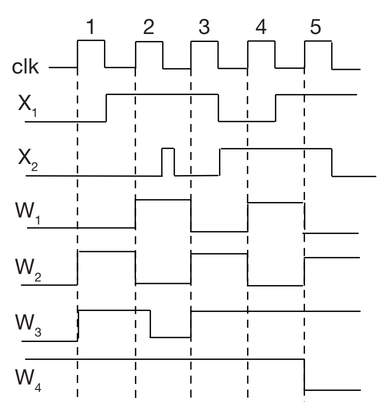
(B) input *A* is connected to 1 and *S* is complemented

(C) input *A* is connected to *S*

(D) input *A* is connected to $\bar{Q}$ and *S* is complemented

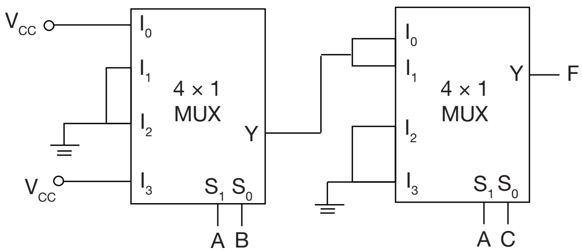
16. In the circuit shown choose the correct timing diagram of output *Y* from the given wave forms W_1, W_2, W_3 and W_4





(A) W_1 (B) W_2
(C) W_3 (D) W_4

17. In the circuit shown the output F is given by:

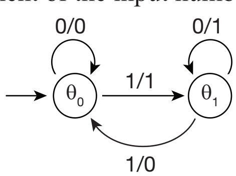


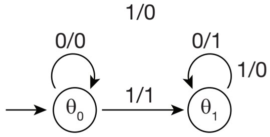
(A) $\overline{A} + \overline{B} + \overline{C}$ (B) ABC
(C) $\overline{A}B$ (D) $\overline{(A + B)}$

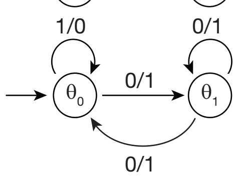
18. Suppose only one multiplexer and one inverter allowed to be used to implement any Boolean function of n variables. What is the minimum size of the multiplexer needed?

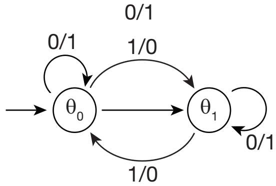
(A) 2^n line to 1 line (B) 2^{n-1} line to 1 line
(C) 2^{n-2} line to 1 line (D) 2^{n+1} line to 1 line

19. Which one of the following state diagram represents finite state machine, which takes as input a binary number from the least significant bit and computes 2's complement of the input number.

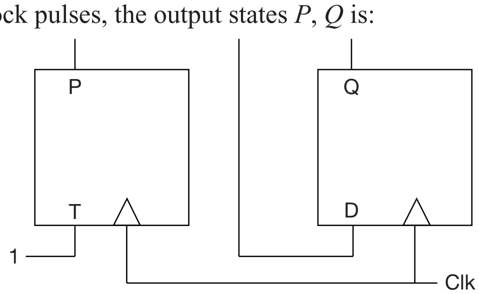
(A)


(B)


(C)


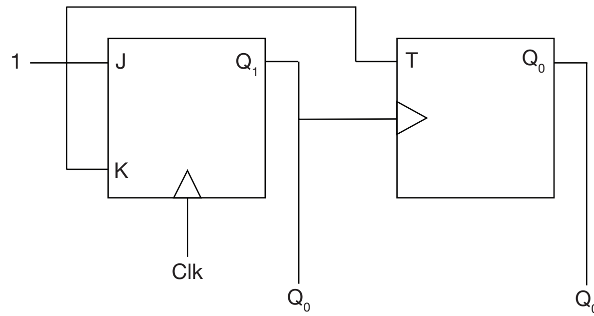
(D)


20. The following arrangement of Master-slave flip flips has the initial state of P, Q as 0, 1 respectively after the 3 clock pulses, the output states P, Q is:



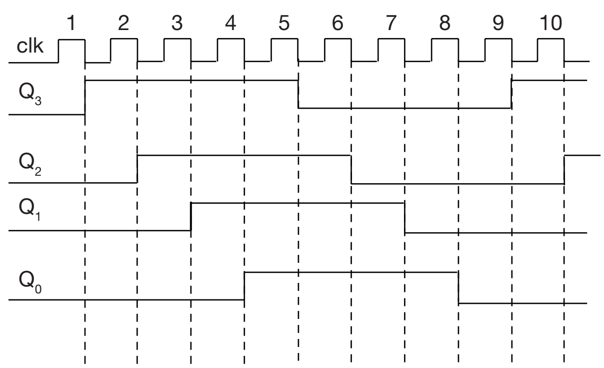
(A) 1, 0 (B) 0, 1
(C) 1, 1 (D) 0, 0

21. In the sequential circuit below, if the initial value of the output Q_1 Q_0 is 00, what are the next four values Q_1, Q_0?



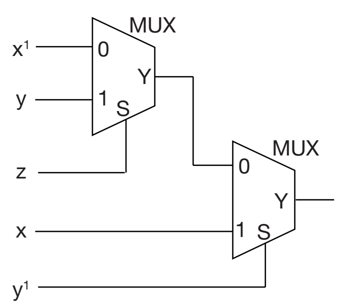
(A) 01, 10, 11, 00
(B) 11, 10, 01, 00
(C) 01, 11, 10, 00
(D) 10, 11, 01, 00

22. The waveforms of a counter are shown below, this is a



(A) synchronous BCD counter
(B) ripple counter
(C) ring counter
(D) twisted ring counter

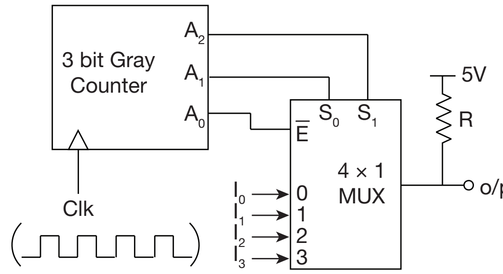
23. Consider the circuit, here, which are the following options correctly represents $f(x, y, z)$



(A) $x^1y^1 + xy + xz$
(B) $xy^1 + x^1y + xz$
(C) $x^1y + xy^1 + y^1z$
(D) $xy + x^1y^1 + xyz$

24. The outputs Q and $\bar{Q}$ of a master slave SR flip flop are connected to its R and S inputs respectively. Its output Q when clock pulses are applied will be:
(A) permanently 0
(B) permanently 1
(C) fixed 0 or 1
(D) complementing with every clock pulse

25. A 3 bit gray counter is used to control the output of the multiplexer as shown in figure, the initial state of the counter is 000. The output is pulled high, the output of the circuit follows the sequence



(A) 1, I_0, I_1, 1, 1, I_3, I_2, 1, 1
(B) I_0, 1, 1, I_1, I_3, 1, 1, I_2, I_0
(C) I_0, 1, I_1, 1, I_2, 1, I_3, 1, I_0,
(D) 1, I_0, 1, I_1, I_2, 1, I_3, 1, I

Answer Keys

1. D	**2.** D	**3.** C	**4.** D	**5.** D	**6.** A	**7.** D	**8.** A	**9.** C	**10.** B
11. A	**12.** D	**13.** B	**14.** C	**15.** D	**16.** D	**17.** D	**18.** B	**19.** B	**20.** A
21. B	**22.** D	**23.** B	**24.** D	**25.** B					

Hints and Explanations

1. All flip flops have $J = K = 1$, so all are toggle switches The output of flip flop is connected to rising edge clk input of next flip flop, so Down counter. clk input is given to flip flop A (LSB).
Preset is active high, so as preset = 1, all flip flops get 111, so AND gate $o/p = 1$, when $A = 1$, $B = 0$, $C = 0$ as $A, \bar{B}, \bar{C}$ are inputs, so the $CBA = 001$,
modulus = 111 – 001 = 6. Choice (D)

2. The given circuit is T latch, The input A is connected to first two NAND gates (J, K inputs connected together to make T latch)
So race around condition occurs when $A = 1$, clk = 1.
Choice (D)

3. When two counters are connected in cascade, the resultant modulus is the multiplication of the individual modulus.
So 3 counters are connected so resultant modulus
= 4 × 4 × 4 = 64. Choice (C)

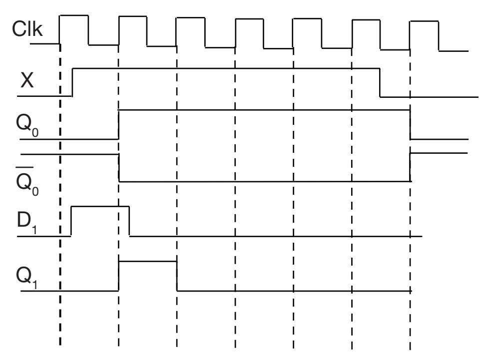
4. JK flip flop works as T flip flop if $J = K = T$
For D flip flop $Q_{n+1} = D$, $Q_{n+1} = T \oplus Q_n$ (T flip flop)
So by taking $D = T \oplus Q_n$ we get T flip flop.
Choice (D)

5. n bit binary UP counter counts in Binary sequence, so same binary sequence is applied to Demultiplexer, so input $I = 1$, will be at output in sequence, for 3 bit example.

$Q_2\ Q_1\ Q_0$	$S_2\ S_1\ S_0$	$Y_0\ Y_1\ Y_2$ ...... Y_7
0 0 0	0 0 0	1 0 0 0
0 0 1	0 0 1	0 1 0 0
0 1 0	0 1 0	0 0 1 0

It works like a module 2^n ring counter. Choice (D)

6. For D flip flop whatever input we apply, same output we get after clk pulse



Choice (A)

7. The frequency of $Q_1 = \dfrac{fclk}{2}$

$$\text{Frequency of } Q_1 = \frac{\frac{fclk}{2}}{2} = \frac{fclk}{4}$$

$$\text{Frequency of } Q_2 = \frac{\frac{fclk}{4}}{2} = \frac{fclk}{8}$$

$$\text{Frequency of } Q_3 = \frac{fclk}{16}$$

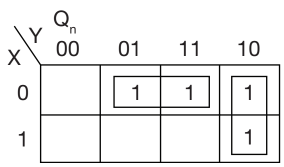
$$\text{Frequency of } Q_4 = \frac{fclk}{32} = \frac{128}{32} = 4 \text{ kHz}$$

$$\text{Frequency of } Q_5 = \frac{fclk}{64}$$

Choice (D)

8. The given XY flip flop truth table is

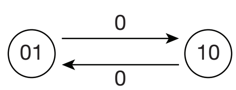
$X\ Y\ Q_n$	Q_{n+1}
0 0 0	0
0 0 1	1
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	1
1 1 1	0

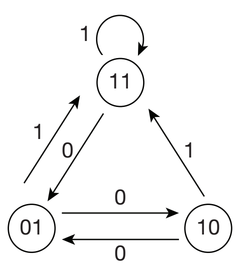


$Q_{n+1} = \bar{X}Q_n + Y\bar{Q}_n$

The characteristic equation of D flip flop is $Q_{n+1} = D$, This has to work like above XY flip flop. By equating $Q_{n+1} = D = \bar{X}Q_n + Y\bar{Q}_n$. Choice (A)

9. $A = 01$, $B = 10$, alternate states when $X = 0$
When $X = 1$, A, B go to state $C = 11$ as long as $X = 1$, C will remain in same state

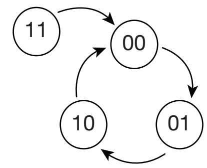




When $X = 0$, C will return to state $A = 01$ Choice (C)

10. Let us consider states as AB if initially state of the circuit is 00 (reset)

Clk	A B	T_A (A + B)	T_B (A^1 + B)	
0 1 2 3	0 0 0 1 1 0 0 0	0 1 1	1 1 0	If initially at 00 state, after 3 Clk pulses it comes back to same state 00.
0 1	1 1 0 0	1	1	If initial state is 11, it goes to 00 state after clock pulse



Choice (B)

11. Given $f(a, b, c) = a^1b + b^1c + ac^1$
$= a^1b + (a + a^1)b^1c + ac^1$
$= a^1b + a^1b^1c + ab^1c + ac^1$
$= a^1[b + b^1c] + a[b^1c + c^1]$
$= a^1[b + c] + a[b^1 + c^1]$
By comparing this equation with output of 2 × 1 multiplexer
$Y = I_0\ \bar{S} + I_1\ S = I_0\bar{a} + I_1 a = [b + c]a^1 + [\overline{bc}]a$
Gate 1 is OR gate, gate 2 is NAND gate.
Choice (A)

12. For multiplexer P, $Y = I_0\ I_0\bar{S} + I_1S = a^1\ b^1 + 0 \cdot b$
For mux, Q $Y = a^1\ b^1 + b^1 \cdot b = a^1\ b^1$
For mux, R, $Y = b^1.b^1 + a^1 \cdot b = a^1 + b^1 = (ab)^1$
For mux, S, $Y = 1 \cdot a^1 + b^1 \cdot a = a^1 + b^1 = (ab)^1$
So, R, S implement NAND gate P, Q implements NOR gate. Choice (D)

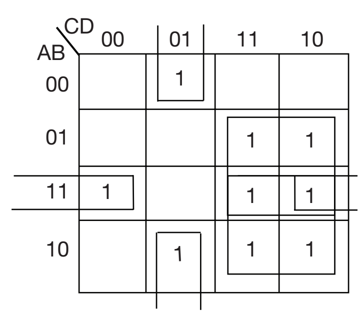
13. The demultiplexer output $Y_0 = I\bar{S}_2\bar{S}_1\bar{S}_0$
$Y_1 = I\bar{S}_2\bar{S}_1S_0$
$Y_2 = I\bar{S}_2S_1\bar{S}_0$
$Y_3 = IS_2\bar{S}_1\bar{S}_0$... ... etc
$f(a, b, c, d) = Y_0 + Y_1 + Y_6 + Y_7$
$= ab^1c^1d^1 + ab^1c^1d + abcd^1 + abcd$
$= ab^1c^1 + abc$
$= a[b \odot c]$ Choice (B)

14. For 8 × 1 multiplexer output

$$Y = l_0\bar{S}_2\bar{S}_1\bar{S}_0 + I_1\bar{S}_2\bar{S}_1S_0 + I_2\bar{S}_2S_1\bar{S}_0 + I_3\bar{S}_2S_1S_0 + I_4S_2\bar{S}_1\bar{S}_0 + I_5S_2\bar{S}_1S_0 + I_6S_2S_1\bar{S}_0 + I_7S_2S_1S_0$$

$$= \bar{A}\bar{B}\bar{C}D + 0 + 0 + \bar{A}BC + A\bar{B}\bar{C}D + A\bar{B}C + AB\bar{C}\bar{D} + ABC$$

$= \sum m\,(1, 6, 7, 9, 10, 11, 12, 14, 15)$



$Y = \bar{B}\bar{C}D + AB\bar{D} + BC + AC$ Choice (C)

15. The input $D = \overline{A \cdot S} = \bar{A} + \bar{S}$
This NAND gate is replaced by OR gate
So A input should be complemented (to $\bar{Q}$) and S input should be complemented. Choice (D)

16. NAND gate output = 0
When two inputs are 1.
Only at last clk pulse $X_1 = X_2 = 1$
So $Y = 0$, remaining cases $Y = 1$, Choice (D)

17. Output of 1st multiplexer is

$Y = I_0\bar{A}\bar{B} + I_1\bar{A}B + I_2A\bar{B} + I_3AB$

$= \bar{A}\bar{B} + AB = A \odot B$

Output of 2nd multiplexer is

$Y = (A \odot B)\,\bar{A}\bar{C} + (A \odot B)\,\bar{A}\,C + 0 \cdot A\bar{C} + 0 \cdot AC$

$= (A \odot B)[\bar{A}\bar{C} + \bar{A}C] = (\bar{A}\bar{B} + AB)\bar{A}$

$= \bar{A}\bar{B} = (\overline{A + B})$ Choice (D)

18. Choice (B)

19. If two 2's complement has to be calculated we require two states.

(Inputs)	0	1
$Q_0(0)$	Q_0, 0	Q_1, 1
$Q_1(1)$	Q_1, 1	Q_1, 0

We can get 2's complement from LSB, by considering all zero's and the first 1 as it is, there after complementing 0's and 1's. Choice (B)

20.

Clk	States	Input
	P Q	T D
0	0 1	1 0
1	1 0	1 1
2	0 1	1 0
3	1 0	1 1

Choice (A)

21. The given circuit is an asynchronous counter (ripple counter) states will change in Binary sequence. Q is connected to rising edge clk pulse so down counter, so sequence is 00, 11, 10, 01, 00 ... Choice (B)

22. By observing sequence 0000 → 1000 → 1100 → 1110 → 1111 → 0111 → 0011 → 0001 → 0000
It is twisted ring counter. Choice (D)

23. Output of 1st multiplexer $Y = I_0\bar{S} + I_1S = x^1z^1 + yz$

Output of 2nd multiplexer $Y = I_0\bar{S} + I_1S$
$= (x^1z^1 + yz)\,(y^1)^1 + xy^1$
$= x^1yz^1 + yz + xy^1 = \sum m\,(2, 3, 4, 5, 7)$

x \ yz	00	01	11	10
0			1	1
1	1	1	1	

f = xy¹ + x¹y + yz

x \ yz	00	01	11	10
0			1	1
1	1	1	1	

f = xy¹ + x¹y + xz

Choice (B)

24. Q is connected to R, $\bar{Q}$ to S input

So $Q_{n+1} = S + \bar{R}Q_n = \bar{Q}_n + \bar{Q}_n \cdot Q_n$

So $Q_{n+1} = \bar{Q}_n$ so next state is complement of present state. Choice (D)

25.

A_2 A_1 A_0	**E** S_1 S_0	**o/p**
0 0 0	0 0 0	I_0
0 0 1	1 0 0	1
0 1 1	1 0 1	1
0 1 0	0 0 1	I_1
1 1 0	0 1 1	I_3
1 1 1	1 1 1	1
1 0 1	1 1 0	1
1 0 0	0 1 0	I_2
0 0 0	0 0 0	I_0

Enable is active low, when $E = 1$, the output is logic 1
When $E = 0$, output is from multiplexer. Choice (B)

Chapter 1

Programming in C

LEARNING OBJECTIVES

- *Basic concepts*
- *Character set*
- *Identifier*
- *Declaring a variable*
- *Visualization of declaration*
- *Constants*
- *Single character constants*
- *String constants*
- *Using const keyword*
- *Precedence decreases as we move from top to bottom*
- *Type conversion*
- *Documentation section*
- *Preprocessing*
- *Global declaration*
- *Control statements*
- *Selection/Decision making statement*
- *Looping statements*
- *Unconditional jump statements*

Basic Concepts

Character Set

A character refers to an alphabet, digit or a special symbol. Alphabets: $A - Z$, $a - z$

Digits: 0 -9
Special symbols:
~ ! # % ∧ and ∗ ( ) − + { } [ ] − < > , . | ? \ | : ; " ' White space

Identifier

Identifier is a user-defined name used for naming a variable or a function.
Rules for naming an identifier

- Consists only letters, digits and underscore
- Starts only with an alphabet or underscore
- Keywords cannot be used.
- Can be as long as you like, first 31 characters are significant.

Example: Valid identifiers: RollNo, Roll_No, _Roll_No
rollno, Name2;
Invalid: 2name, Roll No.

Variable

The name itself represents value, is not constant. Variable is a data name whose value varies/changes during program execution. Variable name is a name given to memory cell (may be one or multiple bytes).

Data Types

Represents type of data and set of operations to perform on data .

Data Type

Primitive/Basic	Derived	User defined	Valueless
– Char	– Array	– Structure	
– float	– pointer	– union	– void
– double		Enumeration	
– integer			

Type	Keyword	Number of Bytes
Integer	int	2
Floating	float	4
Double	double	8
Character	char	1

Declaring a Variable

- Before using a variable, you must give some information to compiler about the variable. i.e., you must declare it.
- Declaration statement includes the type and variable name.

Syntax:
Datatype Var_name;
Example:

```
int roll_no;
char ch;
float age;
```

- When we declare a variable
  - memory space is allocated to hold a value of specified type.
  - space is associated with variable name
  - space is associated with a unique Address.

Table 1 *Visualization of declaration*

	roll no
int roll no;	garbage
	2002
	marks
int marks = 10;	10
	3008
	diameter
float diameter = 5.9	5.9
	4252
	ch → variable name
char ch : 'A'	A → value
	2820 → address

Note: The default value is garbage, i.e., an unknown value is assigned randomly.

Renaming data types with typedef Typedef is a keyword, which can form complex types from the basic type, and will assign some simpler names for such combinations. This is more helpful when some declaration is very tough, confusing or varies from one implementation to another.

For example, the data type unsigned long int is redefined as LONG as follows:

typedef unsigned long int LONG;

Uses of enumerated data types Enumerated data types are most useful when one is working over small, discrete set of values, in which each is having a meaning and it is not a number.

A best example can be given on months jan, feb, mar, …, dec, which are 12 in number, with assigning consecutive numbers for it.

The main advantages are storage efficiency, the *c*-code can become readable

Constants

A constant value is one which does not change during the execution of a program.
C supports several types of constants:

1. Integer constants
2. Real constants
3. Single character constants
4. Strings constants

Integer constants

An integer constant is a sequence of digits. It consists of a set of digits 0 to 9 preceded by an optional + or − sign spaces, commas, and non-digit characters are not permitted between digits.
Examples for valid decimal integer constants are
123
–31
0
562321
+78
Examples for invalid integer constants are
20,000
₹1000

Real constants

Real constants consist of a fractional part in their representation. Integer constants are inadequate to represent quantities that vary continuously.
Examples of real constants are
0.0026
–0.97
435.29
+487.0

Single character constants

A single character constant represents a single character which is enclosed in a pair of quotation symbols.
Examples for character constants are
'5'
'x'
';'

String constants

A string constant is a set of characters enclosed in double quotation marks. The characters in a string constant sequence may be alphabet, number, special character and blank space.
Examples of string constants are
"VISHAL"
"1234"
"C language"
"!….?"

Naming constants

A name given to a constant value. Value of name does not change during program execution.

Using const keyword

When we use 'const' with data type, memory will be allocated to variable and the initialized value does not change.
const int $x = 10$;
const float pi = 3.141;

Using # define

define x 10
define pi 3.141
Where '# define' is instruction to preprocessor so memory is allocated. The preprocessor replace each occurrence of name with value in program before execution.

OPERATOR

An operator is a symbol which performs operations on given data elements.

Table 2 *Precedence and Associativity*

( ) Parenthesis [ ] Index → Member of • Member of	$L - R$
Pre ++, – – (unary) – , &(address of) * (Indirection)	$R - L$
Arithmetic * , /, %	$L - R$
Arithmetic: +, –	$L - R$
Bitwise shift : ≪, ≫	$L - R$
Relational: <, >, =. >, > = = =, ! =	$L - R$
Bitwise ex –OR : ∧	$L - R$
Logical AND : &&	$L - R$
Logical OR : ||	$L - R$
Conditional: ? :	$R - L$
Assignment & compound Assignment =, + =, – =, * =, / = ; % =	$R - L$
Separation operator: , (comma)	$L - R$

Note: For Assignment operator, only a variable is allowed on its left.

Precedence Decreases as We Move from Top to Bottom

Examples:

1. int a,b,c;
   $a = b = c = 0$;
   Assigns '0' to a,b,c;
2. int a, $b = 55$, $c = 10$;
   initializes 'b' with 55 and 'c' with '10'.
   $b + c = a$; // Invalid
   Only variable is allowed on left side of assignment.
3. int $a = 15$, $b = 20$, $c = 2$, $d = 5$, $e = 10$, f, g, h, i;
   $f = a << c$;
   'a' is left shifted for 'c' times and result stored in 'f'
   i.e.,
   $a = 15 = (1\ 1\ 1\ 1)_2$
   ↓ ↓ ↓ ↓
   1 1 1 1 0 (After first shift)
   ↓ ↓ ↓ ↓
   1 1 1 1 0 0 (After second shift)
   One left shift multiplies 15 by 2 = 30
   Again the 2nd left shift multiplies 30 by 2 = 60
   Thus $15 \times 2^2 = 60$, where the power of 2 is the number of times shift is made. Value of 'f' becomes 60.

Note: Left shift multiplies the value by 2. Right shift divides the value by 2.

$g = a$ and b;

a – 0 1 1 1 1
b – 1 0 1 0 0

0 0 1 0 0 = 4

'&' performs bitwise AND. So 'g' value is '4'.

$h = a|b$; a – 0 1 1 1 1
b – 1 0 0 0 0

1 1 1 1 1 = 31

"I" performs bitwise 'OR'. R value is '31'.

$i = a \wedge d$: a – 1 1 1 1
b – 0 1 0 1

1 0 1 0 = 10

'^' performs bit-wise ex – OR. i value is '10'.

4. int $a = 100$, $b = 200$, $c = 300$, x;

```
x = (a>b)?((a>c)?a:c):((b>c)? b:c);
     false                      c
x = c
so, x = 300
```

5. int $i = 10$, $j = 10$, x, y;

```
x = i+++++i+i+++++i+++i
executes as
++ i
++ i  pre-increments
++ i
X = i + i + i + i + i
i ++ ;  post-increments
i ++ ;
so x = 65, i = 15.
y = j - - + - - j + j - -  + - - j +
- - j
```

```
executes as
- - j;
- - j;  pre decrements
- - j;
y = j + j + j + j + j ;
j - -;
j - -;  post decrements.
y = 35; j = 5
```

6. int i = 10;

```
printf("%d%d%d%d%d", i++, ++i, ++i,
i++, ++i);
evaluates the values in printf from
right to left.
So
i++, ++i, ++i, i++, ++i
<------------------------
Prints 14 14 13 11 11
Printf ("%d", i)
Prints 15:
```

Type Conversion

'*C*' allows mixed mode operations, i.e., variables of different type may appear in same expression. To perform the operation, the data need to convert into compatible type.

The conversion takes place in two ways:

Implicit

C automatically converts any intermediate values to proper type so that the expression can be evaluated without losing any significance.

For mixed mode operations, generally the 'lower' type is automatically converted to 'higher' type before the operation proceeds.

Explicit

'*C*' allows programmer to use type conversion operator to convert a data value to the required type.

Syntax:
*V*1 = (type) *V*2;
Type in parenthesis represents the destination type.

Example: int a = 3, b = 2, float x, y;

Case I: $x = a/b$;
results x = 1.000000

Case II: y = (float) a/b;
results y = 1.500000.

Because, in case 1, the integer division is performed and so returns an integer by division operator. While assigning the integer value implicitly converted to 1.000000, then assigns to float variable x where as in case 2, (float)a converts value of 'a' to float, the second variable 'b' is integer. The compiler implicitly converts integer to float. Then it performs float division. So 1.500000 is stored into floating variables.

Notes: '*C*' allows both implicit and explicit type conversion. Type conversion is of two types:

1. Narrowing**:** Conversion of 'higher' type to 'lower' type.
2. Widening**:** Conversion of 'lower' type to 'higher' type.

Widening →

Char – int –– long – float – double – long double

← **Narrowing**

Note: Narrowing causes loss of data.

Input/output Functions

Function	Purpose
printf	prints formatted string
scanf	reads formatted string
getchar	reads character
putchar	displays a character
gets	reads a string
puts	displays a string

Format Specifier	Purpose
%c	single character
%d	decimal integer
%e	floating point
%f	floating point
%h	short int
%o	octal integer
%x	hexa decimal
%s	string
%u	unsigned decimal integer

Note: scanf("%s", string_var); does not read string which contains white space. Hence to read multi word string use gets(string_var);

Example 1: Which of following comment regarding the reading of a string using scanf( ) and gets ( ) is true?

(A) Both can be used interchangeably
(B) scanf is delimited by end of line, gets is delimited by blank space
(C) scanf is delimited by blank, gets is delimited by end of line
(D) None of these

Ans: (C)

Program Structure

```
/* Documentation section */
Preprocessor commands;
Global declaration;
main ()
```

```
{
Body of main;
}
User defined function area;
```

Documentation section/comments Ignored by compiler, provides additional information to user to improve readability.

Preprocessing Tells the compiler to do pre-processing before doing compilation. For example

#include < stdio.h > tells to include stdio header file.

Global declaration It contains variable declarations, these are accessible in more than one function.

Function Functions are main building blocks of 'C' program. Every 'C' program contains one or more functions. A mandatory function called 'main( )' instructs the compiler to start execution from here.

User defined area Here user can define his own functions.

Control Statements

The statement that controls the execution sequence of a program is called "control statement".

The control statements are classified as:

1. Selection statement: if, switch
2. Iterative/looping statement: While, do-while, for
3. Unconditional jump statements: break, continue, return, goto

Selection/Decision-making Statement

Makes a decision to select and execute statement(s) based on the condition. 'C' supports if and switch selection statements.

The if statement "if" is called two-way selection statement.

Syntax:

```
if (expression) // simple-if
       statement(s);
if (expression) // if-else
{
       statement1(s);
}
else
{
       statements(s);
}
if (expression) // ladder else-if.
{
       Statement1(s);
}
else if (expression2)
{
Statement2(s);
}
else
{
Statement3(s);
}
```

Nested if:

```
       if (expression1)
{
       Statement(s)1;
       if (expression(s)2)
 else
       Statement(s)3;
}
else
Statement(s)4;
```

Note: If the expression evaluates to true then the statements of if block gets executed otherwise else block statements will execute.

Example 2: Consider the following program segment:

```
if (a > b) printf ("a > b");
else
printf ("else part");
printf ("a < = b");
a < = b will be printed if
```

(A) $a > b$ (B) $a < b$
(C) $a = b$ (D) all of these

Ans: (D)

Because the statement, printf("$a < = b$"); is not the part of either if block or else block.

The switch statement Switch is a multi-way (n-way) selection statement.

Syntax:

```
switch (var_name/exp)
{
case const1: stmts1;
             break;
case const2: stmts2;
             break;
        .
        .
        .
case constn: stmts n;
             break;
default: statements;
}
```

Notes:

- For switch only the integral (integer/char) type variables or expression evaluates to integral allowed.
- Absence of break after case statements leads continuation execution of all case statements followed by matching case block.

Example 3:

```
main ()
{
int i = 10;
switch(i)
{
case 10 : printf ("case 10");
case 15 : printf ("case 15");
case 20 : printf ("case 20");
default : printf ("default case");
}
}
```

Output: Case 10 case 15 case 20 default case

Reason: Missing break after each case, leads to execution of all the cases from matching case.

Example 4:

```
main ( )
{
int i = 10;
switch (i)
{
case 10 : printf("case 10");
break ;
case 8 + 2 : printf("case 8+2");
             break;
default : printf(" No matching case");
}
}
```

Program raises an error called 'Duplicate case' while compiling because the expression '8 + 2' evaluates to '10'.

Looping Statements

Sometimes, there is a situation to execute statement(s) repeatedly for a number of times or until the condition satisfies. *C*' supports following looping statements: while, do-while, for.

While Statement

Syntax: while (condition)

```
{
Statement(s);
}
```

If the condition is true the block of statements will execute and control returns to condition, i.e., the statement(*s*) executes till the condition becomes false.

Notes:

- 'While' executes the block either '0' or more times.
- 'While' is called entry control loop.

Do-while Statement.

Syntax:

```
do
        {
                Statement(s);
        } while (condition);
```

do-while is same as 'while; except that the statement(s) will execute for at least once.

Notes:

- The condition will not be evaluate to execute the block for first time.
- 'do-while' is called exit-control loop.

Example 5:

```
main ( )
{
int i = 0;
while (i! = 0)
{
        printf("%d", i);
        i++;
}
}
```

No output, because the condition is false for the first time.

```
main ( )
        {
        int  i = 0;
        do
        {
        printf("%d", i);
        i++;
} while (i! = 0);
}
```

Output: Displays 0 to 32767 and–32768 to–1

The for loop 'for' provides more concise loop control structure.

Syntax:

```
for(exp1; exp2; exp3)
{
Statement(s);
}
```

Expression 1: Initialization expression may contain multiple initializations. It executes only once before executing the loop for first time.

Expression 2: Condition expression. Only one condition expression is allowed. That may be single or compound condition, evaluates before every execution.

Expression 3: Modification statement may contain multiple statements. It executes on completion of loop body for every iteration.

Note: All the expressions in parenthesis are optional. Two semi-colons (;) are compulsory even though there are no expressions.

Odd loops In the for loop, while loop, the condition specifies the number of times a loop can be executed. Sometimes a user may not know, about the number of times a loop is to be executed. If we want to execute a loop for unknown number of times, then the concept of odd loops should be implemented, these can be done using the for, while (or) do-while loops. Let us illustrate odd–loop with a program

```
# include <stdio.h>
main()
{
int num, x;
num = 1;
while (num = = 1)
{
printf ("enter a number");
scanf ("%d", & x);
if((x % 2) = = 0)
printf("number is even");
else
printf("number is odd");
printf("do u want to test any num.");
printf("for yes-enter '1', No-enter '0'");
Scanf("%d",& num);
}
}
```

Unconditional Jump Statements

- "*C*" language permits to jump from one statement to another.
- '*C*' supports break, continue, return and goto jump statements.

Break statement Breaks the execution sequence. That is when the break statement executes in a block (loop) it'll come out from block (loop).

Syntax:

```
break;
```

Continue statement Used to skip a part of the loop under certain conditions.

Syntax:

```
continue;
```

Return statement Terminates the execution of a function and returns the control to the calling function.

Syntax:

```
return [exp/value];
```

Goto statement Jumps from one point to another with in a function.

Syntax:

```
label1:              goto label2:
Statement(s);        Statement(s);
goto label1;         label2;
reverse jump         forward jump
```

Reverse jump, executes the statements repeatedly where as in forward jump, the statements are skipped from execution.

Example 6:

```
main( )
{
int i ;
for (i=1; i<=10; i++)
{
        if (i = = 5)
                break;
        printf("%d" , i);
}
}
```

output: 1 2 3 4

if $i = 5$, then the loop will break.

Example 7:

```
main( )
{
        int i ;
        for (i  = 1; i<=10; i++)
        {
                if (i = = 5)
                continue;
                printf("%d" , i);
        }
}
```

o/p: 1 2 3 4 6 7 8 9 10

if $i = 5$, the loop statements skipped for that iteration. So it does not print '5'.

Example 8:

Output for the following program segment

```
for (i = 1, j = 10 ; i < 6; ++i, --j)
printf("\n %d %d", i, j);
```

Output:

1	10
2	9
3	8
4	7
5	6

Note: Since for statement allows multiple initialization and multiple update statements, expression 1 and expression 3, does not raise any error.

Exercises

Practice Problems 1

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. What will be the output of the following program?

```
void main()
{
int i;
char a[ ] =" \0 ";
if (printf("%s\n", a))
printf ("ok \n");
else
printf("program error \n");
}
```

(A) ok (B) progam error
(C) no output (D) compilation error

2. Output of the following will be

```
# define FALSE-1
# define TRUE 1
# define NULL 0
main( )
{
    if(NULL)
    puts("NULL");
    else if(FALSE)
    puts("TRUE");
    else
    puts("FALSE");
    }
```

(A) NULL (B) TRUE
(C) FALSE (D) 1

3. main( )

```
{
    printf("%x",-1 << 4) ;
}
```

For the above program output will be
(A) FFF0 (B) FF00
(C) 00FF (D) 0FFF

4. For the following program
# define sqr (*a*) *a***a*
main( )

```
    {
    int i;
    i = 64 / sqr(4);
    printf( "%d", i);
    }
```

output will be
(A) 4 (B) 16
(C) 64 (D) compilation error

5. #define clrscr ( ) 1000

```
main ( )
{
clrscr();
    printf ( "%d \n", clrscr());
  }
```

Output of the above program will be?
(A) error (B) No output
(C) 1000 (D) 1

6. Output of the following program is

```
main( )
{
    int i = -2;
    +i;
    printf("i = %d, +i = %d\n", i, +i);
```

(A) error (B) –2, +2
(C) –2, –2 (D) –2, 2

7. main( )

```
{
  int n;
  printf("%d", scanf ("%d", & n));
}
```

For the above program if input is given as 20. What will be the output?
(A) 20 (B) 1
(C) 2 (D) 0

8. How many times will the following code be executed?

```
{
    x = 10;
    while (x = 1)
            x ++;
}
```

(A) Never
(B) Once
(C) 15 times
(D) Infinite number of times

9. The following statement

```
printf("%d", 9%5); prints
```

(A) 1.8 (B) 1.0
(C) 4 (D) 2

10. int *a*;

```
printf("%d", a);
```

What is the output of the above code fragment?
(A) 0 (B) 2
(C) Garbage value (D) 3

11. printf("%d", printf("time"));
(A) syntax error
(B) outputs time 4
(C) outputs garbage
(D) prints time and terminates abruptly

12. The following program

```
main( )
{
    int i = 2;
    {
    int i = 4, j = 5;
```

```
        printf ("%d%d",i,j);
        }
        printf ("%d%d",i,j);
    }
```
(A) Compiler error: unrecognised symbol *j*;
(B) Prints 2545
(C) Print 4525
(D) None of the above

13. What is the output of the following program fragment?
```
for (i = 3; i < 15; i + = 3);
printf ("%d", i);
```
(A) a syntax error (B) an execution error
(C) prints 12 (D) prints 15

14. What is the output of the following program segment?
```
int a = 4, b = 6;
printf("%d", a = b);
```
(A) Outputs an error message
(B) Prints 0
(C) Prints 1
(D) None of these

15. The statements:
```
a = 7;
printf("%d", (a++));
prints
```
(A) Value of 8 (B) Value of 7
(C) Value of 0 (D) None of the above

Practice Problems 2

Directions for questions 1 to 12: Select the correct alternative from the given choices.

1. If the condition is missing in a FOR loop of a C program then
(A) It is assumed to be present and taken to be false
(B) It is assumed to be present and taken to be true
(C) It results in syntax error
(D) Execution will be terminated abruptly

2. Which of the following operators in '*C*' does not associate from the right?
(A) = (B) + =
(C) postfix++ (D) >

3. In a C programming language $x - = y + 1$ means
(A) $x = -x - y - 1$ (B) $x = x - y + 1$
(C) $x = x - y - 1$ (D) $x = -x + y + 1$

4. Minimum number of temporary variables needed to swap two variables is
(A) 1 (B) 2
(C) 3 (D) 0

5. A preprocessor command
(A) need not start on a new line
(B) need not start on the first column
(C) has # as the first character
(D) comes after the first executable statement

6. printf ("%d", printf ("%d", printf("time4kids")));
(A) Outputs time (B) Syntax error
(C) Outputs 9 (D) None of the above

7. for (i = 1; i < 5; i++)

if (i!=3)

printf("%d", i);

Outputs:
(A) 12345 (B) Error
(C) 1245 (D) 0000

8. Which operand in '*C*' takes only integer operands?
(A) * (B) /
(C) % (D) +

9. An unrestricted use of 'goto' statement is harmful because
(A) it results in increasing the executing time of the program
(B) it increases the memory of the program
(C) it decreases the readability and testing of program
(D) None of the above

10. What will be the output?
```
main()
{
int i = 0, j = 0;
if(i && j ++)
printf("%d..%d", i++, j);
printf("%d..%d", i, j);
}
```
(A) 1..1 (B) 2..2
(C) 0..0 (D) 1..1, 1..1

11. What is the output?
```
main ()
{
int a = 0;
int b = 20;
char x = 1;
char y = 10;
if(a, b, x, y);
printf("hello");
}
```
(A) logical error (B) Garbage value
(C) hello (D) 20

12. What will be the value of count after executing the below program:
```
main ( ) {
int count = 10, digit = 0;
while (digit < = 9) {
printf ("%d\n", ++count);
++digit;
}
}
```
(A) 10 (B) 11
(C) 20 (D) 21

Previous Years' Questions

1. Which one of the following are essential features of an object-oriented programming language?
(i) Abstraction and encapsulation
(ii) Strictly-typedness
(iii) Type-safe property coupled with sub-type rule
(iv) Polymorphism in the presence of inheritance

[2005]

(A) (i) and (ii) only
(B) (i) and (iv) only
(C) (i), (ii) and (iv) only
(D) (i), (iii) and (iv) only

2. Which of the following are true?

(i) A programming language which does not permit global variables of any kind and has no nesting of procedures/functions, but permits recursion can be implemented with static storage allocation

(ii) Multi-level access link (or display) arrangement is needed to arrange activation records only if the programming language being implemented has nesting of procedures/functions

(iii) Recursion in programming languages cannot be implemented with dynamic storage allocation

(iv) Nesting procedures/functions and recursion require a dynamic heap allocation scheme and cannot be implemented with a stack-based allocation scheme for activation records

(v) Programming languages which permit a function to return a function as its result cannot be implemented with a stack-based storage allocation scheme for activation records

[2008]

(A) (ii) and (v) only (B) (i), (iii) and (iv) only
(C) (i), (ii) and (v) only (D) (ii), (iii) and (v) only

3. What will be the output of the following C program segment?

char inChar = 'A';
switch(inChar) {
case 'A': printf("choice A\n"):
case 'B':
case 'C': printf("choice B");
case 'D'
case 'E':
default: printf("No Choice");} **[2012]**

(A) No choice
(B) Choice A
(C) Choice A
Choice B No choice
(D) Program gives no output as it is erroneous

4. Suppose n and p are unsigned int variables in a C program. We wish to set p to ${}^nC_3$. If n is large, which one of the following statements is most likely to set p correctly?

[2014]

(A) $p = n * (n - 1) * (n - 2)/6;$
(B) $p = n * (n - 1) /2* (n - 2)/3;$
(C) $p = n * (n - 1) /3 * (n - 2)/2;$
(D) $p = n * (n - 1) * (n - 2)/6.0;$

5. The secant method is used to find the root of an equation $f(x) = 0$. It is started from two distinct estimates x_a and x_b for the root. It is an iterative procedure involving linear interpolation to a root. The iteration stops if $f(x_b)$ is very small and then x_b is the solution. The procedure is given below. Observe that there is an expression which is missing and is marked by ?. Which is the suitable expression that is to put in place of ? so that it follows all steps of the secant method?

[2015]

Secant

Initialize: x_a, x_b, ε, N // ε = convergence indicator
// N = maximum no. of iterations
$f_b = f(x_b)$
$i = 0$
while ($i < N$ and $|f_b| > ε$) do
$i = i + 1$ // update counter
$x_t = ?$ // missing expression for
// intermediate value
$x_a = x_b$ // reset x_a
$x_b = x_t$ // reset x_b
$f_b = f(x_b)$ // function value at new x_b
end while
if $|f_b| > ε$ then // loop is terminated with $i = N$
write "Non-convergence"
else
write "return x_b"
end if

(A) $x_b - (f_b - f(x_a))\, f_b / (x_b - x_a)$
(B) $x_a - (f_a - f(x_a))\, f_a / (x_b - x_a)$
(C) $x_b - (x_b - x_a)\, f_b / (f_b - f(x_a))$
(D) $x_a - (x_b - x_a)\, f_a / (f_b - f(x_a))$

6. Consider the following C program:

```
#include<stdio.h>
int main( )
{
int i, j, k = 0;
j = 2 * 3 / 4 + 2.0 / 5 + 8 / 5;
k -= --j;
for (i = 0; i < 5; i ++)
{
switch(i + k)
```

```
{
case 1:
case 2: printf("\n%d", i + k);
case 3: printf("\n%d", i + k);
default: printf(("\n%d", i + k);
}
}
return 0;
}
```

The number of times printf statement is executed is _______. [2015]

7. Consider the C program fragment below which is meant to divide x by y using repeated subtractions. The variables x, y, q and r are all unsigned int.

```
while (r >= y) {
    r = r – y;
    q = q + 1;
}
```

Which of the following conditions on the variables x, y, q and r before the execution of the fragment will ensure that the loop terminates in a state satisfying the condition x == (y*q + r)? [2017]
(A) (q == r) && (r == 0)
(B) (x > 0) && (r == x) &&(y > 0)
(C) (q == 0) && (r == x) && (y > 0)
(D) (q == 0) && (y > 0)

8. Consider the following C Program.

```
#include<stdio.h>
int main () {
        int m = 10;
        int n, nl ;
        n = ++m;
        nl = m++;
        n--;
        --nl;
        n  -= nl;
        printf ("%d",  n) ;
        return 0;
}
```

The output of the program is ___________. [2017]

Answer Keys

Exercises

Practice Problems 1

1. A **2.** B **3.** A **4.** C **5.** C **6.** C **7.** B **8.** D **9.** C **10.** C
11. B **12.** A **13.** D **14.** D **15.** B

Practice Problems 2

1. B **2.** D **3.** C **4.** D **5.** C **6.** D **7.** C **8.** C **9.** C **10.** C
11. C **12.** C

Previous Years' Questions

1. B **2.** D **3.** C **4.** B **5.** C **6.** 10 **7.** C **8.** 0

Chapter 2

Functions

LEARNING OBJECTIVES

- *Functions*
- *Library functions*
- *User defined functions*
- *Defining user defined functions*
- *Recursion*
- *Parameter passing*
- *Pass by value*
- *Pass by address*
- *Scope*
- *Life time*
- *Binding*

Functions

A function is a block of code that performs a specific task. It has a name and is reusable, i.e., it can be executed from as many different parts in a program as required.

Functions make possible top down modular programming. In this style of programming, the high-level logic of overall problem is solved first, whereas the detail of each lower-level function is addressed later. This approach reduces the complexity in writing program.

1. Every C program can be thought of collection of functions.
2. main( ) is also a function.

Types of Functions

Library functions

These are the in-built functions of '*C*' library. These are already defined in header files.

Example 1: printf( ); is a function which is used to print at output. It is defined in '`stdio.h`' file.

User-defined functions

Programmers can create their own function in '*C*' to perform specific tasks.

Example 2:
```
# include <stdio.h>
main( )
{
message( );
}
message( )
{
printf("Hello");
}
```

- A function receives zero (or) more parameters, performs a specific task, and returns zero or one value.
- A function is invoked by its name and parameters.
- No two functions have the same name in a single C program.
- The communication between the function and invoker is through the parameter and the return value.
- A function is independent.
- It is "completely" self-contained.
- It can be called at any place of your code and can be ported to another program.
- Functions make programs reusable and readable.

Example 3: Return the largest of two integers.
```
int maximum (int a, int b)
{
if (a > b)
return a;
else
return b;
}
```

Note: Function calls execute with the help of execution stack. Execution of 'C program' starts with main( ) function. Main( ) is a user-defined function.

Defining User-defined Functions

In order to work with user-defined functions, it requires the following concepts about functions:

- Declaration of a function
- Definition of a function
- Function call

Declaration specifies what

- is the name of the function
- are the parameters to pass (type, order and number of parameters).
- it returns on completion of execution

Example 4: int maximum (int, int); int maximum (int *a*, int *b*);

Syntax:

```
Return_type Function_Name(Parameter_list);
```

- Names of parameters are optional in declaration.
- Default return type for 'C' functions is 'int'.
- A function, whose return type is void returns nothing.
- Empty parenthesis after a function name in declaration says, function does not accept any parameters.

Definition specifies *how*

- to perform the specified task
- to accept passed parameters
- to process parameters or execute instruction to producer equired results (return value).

Function definition is a self-contained block of instructions, will be executed on call:

Syntax:

```
Return _type Function -Name(paralist)
{
Local declaration(s);
Executable statements(s);
}
int maximum (int a, int b)
{
if (a > b)
  return a;
else
  return b;
}
```

Function call specifies

1. where to execute the function
2. when to execute the function

Note: If the function definition provided before use (call), the declaration is optional.

The following example describes control flow during function call:

```
void hello(); // Declaration
void main()
{
printf("\n function");
hello();
printf("\n Main after call to hello")
void hello()//Definition
 {
printf ("\n function Hello");
return;
 }
```

A *return* statement has two important uses:

1. first, it causes an immediate exit from the function.
2. second, it may be used to return a value.

If a function does not return any value, then the return statement is optional.

Recursion

In general, programmers use two approaches to write repetitive algorithms. One approach using *loops*, the other is *recursion*.

Recursive functions typically implement recurrence relations, which are mathematical formula in which the desired expression (function) involving a positive integer, *n*, is described in terms of the function applied to corresponding values for integers less than '*n*'.

1. The function written in terms of itself is a recursive case.
2. The recursive case must call the function with a decreasing '*n*'.
3. Recursion in computer programming is exemplified when a function defined in terms of itself.
4. Recursion is a repetitive process in which a function calls itself.

Note: Recursive function must have an *if* condition to force the function to return without recursive call being executed. If there is no such condition, then the function execution falls into infinite loop.

Rules for designing recursive function
1. Determine base case
2. Determine general case
3. Combine, base and general case into a function

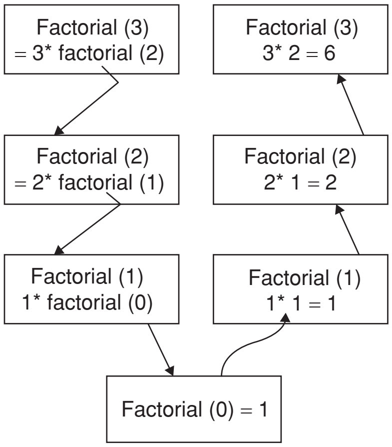
Example 5: Recursive factorial function

```
1. int factorial (int n)
2. {
3. if (n = = 0)
4. return 1;
5. else
6. return (n * factorial (n - 1));
7. }
```

The statement 3 is a base condition, which stops the recursive call of function.

The statement 6 reduces the size of problem by recursively calling the factorial with $(n - 1)$.

Execution sequences for factorial (3):



Disadvantages:

1. Recursive programs increase the execution time of program.
2. Recursive programs typically use a large amount of computer memory and the greater the recursion, the more memory is used.
3. Recursive programs can be confusing to develop and extremely complicated to debug.

Parameter Passing

There are two ways of passing parameters to functions in ‘C’ language.

1. Pass-by-value: When parameters are passed by value, create copies in called function. This mechanism is used when we do not want to change the value of actual parameters.
2. Pass-by-address: In this case, only the addresses of parameters are passed to the called function. Therefore, manipulation of formal parameters affects actual parameters.

Examples 6:

```
void swap1(int, int); /* function – to swap
two numbers by passing values */
void swap2 (int *, int *); /* function to
swap two numbers by passing Address * /
   void main ()
{
int a = 10, b = 15, c = 5, d = 25;
printf("value of a and b before swapping
:%d, %d" a , b );
swap1(a, b);
printf("values of a and b after swapping :
%d, %d", a, b);
printf ("values of c and d before swapping
:%d%d", c,d );
Swap2(&c, &d);
printf("values of c and d after swapping
%d, %d", c, d);
}
void swap1(int x, int y )
{
int temp;
temp = x;
x = y;
y = temp;
}
void swap2 (int *x, int *y)
{
int temp;
temp = *x;
*x = *y:
*y = temp;
}
```

Output:

Value of *a* and *b* before swapping: 10, 15
Value of *a* and *b* after swapping: 10, 15
Value of *c* and *d* before swapping: 5, 25
Value of *c* and *d* after swapping: 25, 5

Solved Examples

Example 1: Consider the program below:

```
#include<stdio.h>
int fun (int n, int *fp)
```

Table 1 *Comparison of pass-by-value and pass-by-address*

	Pass-by-value		Pass-by-address
1.	Also known as call-by-value	1.	Also known as call-by-address or call by-reference
2.	Pass the values of actual parameters	2.	Pass the address of actual parameters
3.	Formal parameters act as duplicates or as a copy to actual parameters	3.	Formal parameters acts as references to the actual parameters
4.	Operations on formal parameter does not affect actual parameters	4.	Operations on formal parameters affect actual parameters
5.	Passing of parameters is time consuming as the data size increases	5.	The size of parameters does not affect the time for transferring references.
6.	Actual parameters are secured	6.	Helps to return multiple parameters

```
{
int t,f;
if (n < =1)
{
*fp=1;
return 1;
 }
t = fun(n-1, fp);
f = t+ *fp;
*fp = t;
return f;
}
int main ()
{
int x = 15;
printf (“%d\n”, fun(5,&x));
return 0;
}
```

What is the output?

(A) 2 (B) 4

(C) 8 (D) 16

Solution: (C)

Execution stack

Function call sequence	Corresponding values of *t*, *f*, *n*, *x*			
	t	*f*	*n*	*x*
main()	–	–	–	15

	t	*f*	*n*	*x*(f_p)
fun (5 &x)	*G*	*G*	5	15
main()	–	–	–	15

	t	*f*	*n*	*x*(f_p)
fun (4 &x)	*G*	*G*	4	15
fun (5 &x)	*G*	*G*	5	15
main()	–	–	–	15

	t	*f*	*n*	*x*(f_p)
fun (3 &x)	*G*	*G*	3	15
fun (4 &x)	*G*	*G*	4	15
fun (5 &x)	*G*	*G*	5	15
main()	–	–	–	15

	t	*f*	*n*	*x*(f_p)
fun (2, &x)	*G*	*G*	2	15
fun (3, &x)	*G*	*G*	3	15
fun (4, &x)	*G*	*G*	4	15
fun (5, &x)	*G*	*G*	5	15
main()	–	–	–	15

Note: ‘–’ indicates *no memory* allocated to variable. ‘*G*’ indicates *garbage value*.

	t	*f*	*n*	*x*(f_p)
fun (1, &x)	*G*	*G*	1	15
fun (2, &x)	*G*	*G*	2	15
fun (3, &x)	*G*	*G*	3	15
fun (4, &x)	*G*	*G*	4	15
fun (5, &x)	*G*	*G*	5	15
main()	–	–	–	15

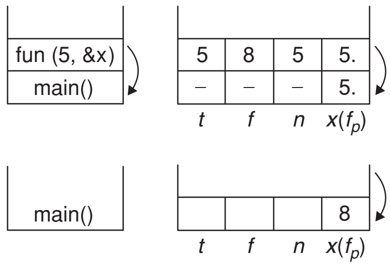
For the function call fun(1, &x) condition (*n*<=1) is true. So Assigns ‘1’ to fp and returns ‘1’.

	t	*f*	*n*	*x*(f_p)
fun (1, &x)	*G*	*G*	1	1.
fun (2, &x)	*G*	*G*	2	1.
fun (3, &x)	*G*	*G*	3	1.
fun (4, &x)	*G*	*G*	4	1.
fun (5, &x)	*G*	*G*	15	1.
main()	–	–	–	1.

	t	*f*	*n*	*x*(f_p)
fun (2, &x)	1	2	2	1.
fun (3, &x)	*G*	*G*	3	1.
fun (4, &x)	*G*	*G*	4	1.
fun (5, &x)	*G*	*G*	15	1.
main()	–	–	–	1.

	t	*f*	*n*	*x*(f_p)
fun (3, &x)	2	3	3	2.
fun (4, &x)	*G*	*G*	4	2.
fun (5, &x)	*G*	*G*	5	2.
main()	–	–	–	1.

	t	*f*	*n*	*x*(f_p)
fun (4, &x)	3	5	4	3.
fun (5, &x)	*G*	*G*	5	3.
main()	–	–	–	1.



Finally, x contains '8', so printf prints '8'.

Example 2: What does the following program prints?

```
#include < stdio.h>
void f (int *p, int *q)
{
p=q;
*p=12;
}
int i = 0, j=1;
int main()
{
f(&i, &j);
printf(" %d%d ", i, j);
return 0 ;
}
```

(A) 2 12 (B) 12 1
(C) 0 1 (D) 0 12

Solution: (D)

```
main( )
f (&i, &j)
address of 'i' is stored in to p.
and address of 'j' is stored into 'q'.
i.e., *p and*q refers i and j.
The statement:
p = q; updates pointer 'p', so that both
pointers refer to parameter 'j'.
*p = 12
Changes value of 'j' to '12' But 'i' does
not effected. So, prints 0 12.
```

Example 3: What is the value printed by the following program?

```
# include <stdio.h>
int f(int *a, int n)
{
if (n<=0) return 0 ;
else if(*a%2 = = 0)
return *a + f(a+1, n-1);
else
return *a - f(a+1, n-1);
}
int main ( )
{
int G [ ] = { 12, 7, 13, 4, 11, 6};
printf("%d", f (a,b));
return 0;
}
```

(a) –9 (b) 12
(c) 15 (d) 20

Solution: (C)

	0	1	2	3	4	5
a	12	7	13	4	11	6

$f(a, 6)$ is the first call to function $f(\,)$.

The array _ name refers to base address of array, i.e., address of first element.

Thus,

$F(a, 6)$

12 % 2 = 0. So,

$$12 + f\left(\overline{(a+1)}, \left(\frac{n-1}{5}\right)\right)[{*a} \text{ is even}]$$
$$\downarrow 7$$

$$12 + \left(7 - \left(\begin{matrix} f(a+1), \left(\dfrac{n-1}{4}\right) \\ \downarrow 13 \end{matrix}\right)\right)[{*a} \text{ is odd}]$$

$$12 + \left(7 - \left(13 - f\left(\begin{matrix}(a+1)\\ \downarrow 4\end{matrix}\right), \left(\frac{n-1}{3}\right)\right)\right)[{*a} \text{ is odd}]$$

$$12 + \left(7 - \left(13 - \left(4 + f\left(\begin{matrix}(a+1), \dfrac{(n-1)}{2} \\ \downarrow 11\end{matrix}\right), \right)\right)\right)[{*a} \text{ is even}]$$

$$12 + \left(7 - \left(13 - \left(4 + \left(11 - f\left(\begin{matrix}(a+1), \left(\dfrac{n-1}{1}\right) \\ \downarrow 6\end{matrix}\right)\right)\right)\right)\right)[{*a} \text{ is odd}]$$

$$12 + \left(7 - \left(13 - \left(4 + \left(11 - \left(6 + \frac{f(a+1), \dfrac{(n-1)}{0}}{\downarrow 0}\right)\right)\right)\right)\right)[{*a} \text{ is even}]$$

$$12 + (7 \quad (13 - (4 + (11 - (6 + 0))))) = 15$$

Scope, Lifetime and Binding

Storage classes specify the scope, lifetime and binding of variables. To fully define a variable, one needs to mention not only its 'type' but also its 'storage class'.

A variable name identifies some physical location within computer memory where a collection of bits are allocated for storing value of variable.

Storage class tells us:

1. Where the variable would be stored (either in memory or CPU registers)?
2. What will be the initial value of a variable, if no value is specifically initialized?
3. What is the scope of a variable (where it can be accessed)?
4. What is the life of a variable?

Scope

The scope defines the visibility of an object. It defines where an object can be referenced/accessed; generally, the scope of variable is local or global.

1. The variables defined within a block have *local scope*. They are *visible only to the block* in which they are defined.
2. The variables defined in global area are visible from their definition until the end of program. It is *visible everywhere* in program.

Lifetime

The lifetime of a variable defines the duration for which the computer allocates memory for it (the duration between allocation and deallocation of memory).

In *C*, variable can have automatic, static or dynamic lifetime.

1. Automatic: Variables with automatic lifetime are created each time their declaration are encountered and are destroyed each time their blocks are exited.
2. Static: A variable is created when the declaration is executed for the first time and destroyed when the execution stops/terminates.
3. Dynamic: The variable's memory is allocated and deallocated through memory management functions.

Binding

Binding finds the corresponding binding occurrence (declaration/definition) for an applied occurrence (usage) of an identifier. For Binding.

1. Scope of variables should be known. What is the block structure? In which block the identifier is variable?
2. What will happen if we use same identifier name again? '*C* forbids use of same identifier name in the same scope'. Same name can be used in different scopes.

Examples:

1.
```
double f,y;
int f(  ) // error
{
.
.
.
}
double y; // error
```

2.
```
double y;
int f( )
{
double f;// legal
int y; //legal
}
```

There are four storage classes in *C*.

Storage class	Storage Area	Default Initial Value	Lifetime	Scope	Keyword
Automatic	Memory	Till the control remains in block	Till the control remains in block	Local	auto
Register	CPU register	An unpredictable value (or) garbage value	Till the control remains in block	Local	register
Static	Memory	Zero	Value of variable persist between function calls	Local	static
External	Memory	Unpredictable or garbage value	Throughout program execution	Global	extern

Note: Default storage class is auto.

Example 4: What will be the output for the program?

```
int i = 33;
main( )
{
   extern int i;
     {
         int i = 22;
          {
               const volatile unsigned i
               = 11;
                   printf (" %d ", i);
              }
                   printf (" %d ", i);
              }
printf ("%d ", i) ;
}
```

(A) error
(B) 11 22 33
(C) 11 22 garbage
(D) 11 11 11

Solution: (B)

'{' introduces new block and thus new scope. In the innermost block, *i* is declared as const volatile unsigned which is a valid declaration. *i* is assumed of type int. So printf prints 11. In the next block, *i* has value 22 and so printf prints 22. In the outermost block, *i* is declared as extern, so no storage space is allocated for it. After compilation is over, the linker resolves it to global variable, *i* since it is the only variable visible there. So it prints its value as 33.

Example 5: Consider the following C program:

```
int f(int n)
{
static int r;
if (n<=0) return 1;
if (n> 3)
{
r=n;
return (f(n-2)+2));
}
return f(n-1) + r;
}
```

What is the value of $f(5)$?

(a) 15 (b) 17
(c) 18 (d) 19

Solution: (C)

Call Sequence	*r*	Return Sequence
$f(5)$ ↓	5	18
$f(3)+2$	5	16+2 ↓
$f(2)+r$	5	11+5 ↓
$f(1)+r$	5	6+5 ↓
$f(0)+r$	5	1+5 ↓

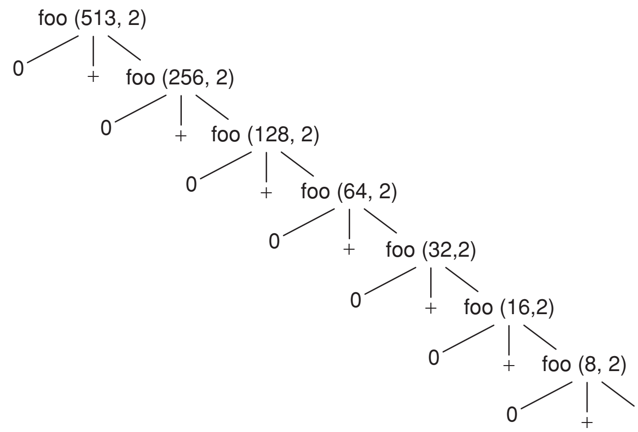
Common data for questions 6 and 7: Consider the following recursive 'C' function that takes two arguments.
unsigned int foo (unsigned int n, unsigned int r)

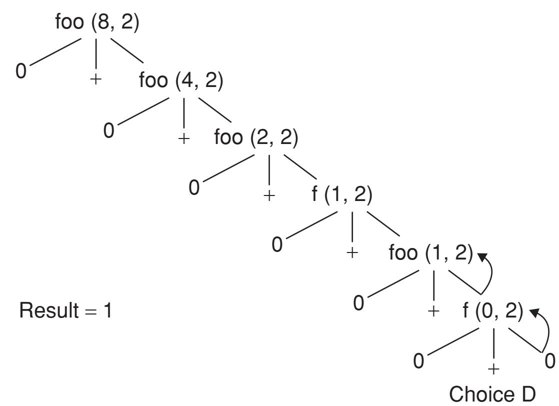
```
{
if (n>0)
return ((n%r)+ foo(n/r,r));
else
return 0;
}
```

Example 6: What is the return value of the function foo when it is called as foo (512,2)?

(A) 9 (B) 8
(C) 2 (D) 1

Solution: (D)



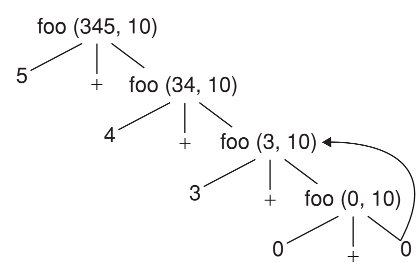


Result = 1

Example 7: What is return value for the function call foo (345, 10)?

(A) 345 (B) 12
(C) 5 (D) 3

Solution: (B)



result 5 + 4 + 3 = 12

Exercises

Practice Problems 1

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. What will be the output of the following program?

```
main( )
{
main( );
}
```

(A) overflow error (B) syntax error
(C) returns 0 (D) returns 1

2. Output of the following program is

```
 main( )
  {
static int var = 6;
printf("%d\t", var--);
if(var)
                   main( );
  }
```

(A) 5 4 3 2 1 0 (B) 6 6 6 6 6 6
(C) 6 5 4 3 2 1 (D) Error

3. Which of the following will be the output of the program?

```
main( )
{
 char str[ ] = "Hello";
 display( str );
}
void display (char *str)
    {
        printf ( "%s", str) ;
    }
```

(A) compilation error (B) hello
(C) print null string (D) no output

4. Consider the following C function

```
int fun (int n)
{
  static int x = 0;
  if (n<=0) return 1;
  if (n>3)
     {
          x = n;
       return fun(n-2)+3;
      }
 return fun(n-1)+ x;
 }
```

What is the value of fun(5)?
(A) 4 (B) 15
(C) 18 (D) 19

5. For the following C function

```
void swap (int a, int b)
{
   int t;
        t = a;
        a = b;
    b = t;
}
```

In order to exchange the values of two variables *w* and *z*,
(A) call swap (*w*, *z*)
(B) call swap (and *w*, and *z*)
(C) swap (*w*, *z*) cannot be used as it does not return any value
(D) swap (*w*, *z*) cannot be used as the parameters are passed by value

6. Choose the correct option to fill? *x* and? *y* so that the program below prints an input string in reverse order. Assume that the input string is terminated by a new line character:

```
void Rev(void) {
  int a;
  if (?x) Rev( );
  ?y
      }
main( ) {
printf("Enter the text");
printf(" \n");
Rev( );
printf("\n");
}
```

(A) `? x is (getchar( )! = '\n')`
`? y is getchar (A);`
(B) `? x is((A = getchar( )) ! = '\n')`
`? y is getchar(A) ;`
(C) `? x is (A! = '\n')`
`? y is putchar (A);`
(D) `? x is (A = getchar ( )) ! = '\n')`
`? y is putchar(A) ;`

7.

```
main ( )
  {
     extern int a;
      a = 30;
       printf ("%d", a);
  }
```

What will be the output of the above program?
(A) 30 (B) Compiler error
(C) Runtime error (D) Linker error

8. Which of the following will be the output of the program?

```
void main ( )
 {
    int n = ret(sizeof(float));
    printf("\n value is %d ", ++n);
}
int ret(int ret)
{
```

```
    ret += 2.5;
   return (ret);
}
```

(A) Value is 6 (B) Value is 6.5
(C) Value is 7 (D) Value is 7.5

9. The following program

```
main( )
{
    pt( ); pt( );pt( );
}
pt( )
{
    static int a;
    printf("%d", ++a) ;
}
prints
```

(A) 0 1 2
(B) 1 2 3
(C) 3 consecutive, but unpredictable numbers
(D) 1 1 1

10. What is the output of the following program?

```
main( ) {
    int i = 0;
    while (i < 4) {
                    sum(i);
                    i++;
                }
          }
void sum(int i) {
    static int k;
printf ("%d", k + i);
k++;
                }
```

(A) 0 2 4 6 (B) 0 1 2 3
(C) 0 2 0 0 (D) 1 3 5 7

11. What will be the output of following code?

```
# include <stdio.h>
aaa() {
printf("hi");
}
bbb() {
printf("hello");
}
ccc()
{
printf("bye");
}
main ()
{
int *ptr[3]( );
ptr[0] = aaa;
ptr[1] = bbb;
ptr[2] = ccc;
ptr[2]();
}
```

(A) hi (B) hello
(C) bye (D) Garbage value

12. What is the output?

```
void main()
{
static int i = 5;
if(--i)
{
main();
printf("%d", i);
}
}
```

(A) 5 (B) 5 5 5 5
(C) 0 0 0 0 (D) 1 1 1 1

13. If the following function gets compiled, what error would be raised?

```
double fun(int x, double y)
{
int x;
x = 100;
return y;
}
```

(A) Function should be defined as int fun(int *x*, double *y*)
(B) Missing parenthesis in return
(C) Redeclaration of *x*
(D) All of these

14. Consider the following function:

```
fun(int x)
{
if ((x/2)! = 0)
return (fun (x/2)  10 + x%2);
else return 1;
}
```

What will happen if the function 'fun' called with value 16 i.e., as fun(16).
(A) Infinite loop
(B) Random value will be returned
(C) 11111
(D) 10000

15. What is the output of the following program?

```
void main( )
{
static int x = 5;
printf("%d", x - - );
if (x ! = 0)
main( );
}
```

(A) error:main( ) cannot be called from main( )
(B) Infinite loop
(C) 5 4 3 2 1
(D) 0

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. An external variable
(A) is globally accessible by all functions
(B) has a declaration "extern" associated with it when declared within a function
(C) will be initialized to 0, if not initialized
(D) all of the above

2. The order in which actual arguments are evaluated in a function call
(A) is from the left (B) is from the right
(C) is unpredictable (D) none of the above

3. In *C* language, it is necessary to declare the type of a function in the calling program if the function
(A) returns an integer (B) Returns a float
(C) both (A) and (B) (D) none of the above

4. What is the output?
```
void main()
{
int k = ret(sizeof(int));
printf("%d", ++k);
}
int ret (int ret)
{
ret + = 2.5;
return (ret);
}
```
(A) 3.5 (B) 5
(C) 4 (D) logical error

5. When a recursive function is called, all its automatic variables are
(A) maintained in stack
(B) retained from last execution
(C) initialized during each call of function
(D) none of these

6. Consider the following program segment:
```
int fun(int x, int y)
{
if(x > 0)
return ((x % y) + fun(x/y, y));
else
return 0;
}
```
What will be the output of the program segment if the function is called as fun(525, 25)?
(A) 25 (B) 12
(C) 21 (D) 42

7. Consider the following C program segment:
```
int fun (int x)
{
static int i = 0;
if (x < = 0)
return 1;
else if (x > 5)
{
i = x;
return fun (x – 3) +2;
}
return fun (x – 2) + i;
}
```
What is the value of fun(7)?
(A) 17 (B) 10
(C) 11 (D) 9

8. Consider the following C program:
```
void rearrange( )
{
char ch;
if (X)
rearrange( );
Y;
}
void main ( )
{
printf("\n enter text to print reverse
order :");
rearrange( ) ;
}
```
Choose the correct option to fill *X* and *Y*, so that the program prints the entered text in reverse order. Assume that input string terminates with new line.
(A) `X: (getchar(ch) = = '\n')`
`Y: putchar(ch);`
(B) `X: (getchar(ch)! = '\n')`
`Y: ch = putchar( );`
(C) `X: ((ch = getchar( ) )! = '\n')`
`Y: putchar(ch);`
(D) `X: ((ch = getchar( )) = = '\n')`
`Y: putchar (ch);`

9. Consider the following C function:
```
int f(int n)
{
static int i = 1;
if (n > = 5) return n;
n = n + i;
i++ ;
return f(n);
}
```
The value returned by *f*(1) is
(A) 5 (B) 6
(C) 7 (D) 8

10. Consider the following C function:
```
int incr (int i)
{
static int count = 0;
count = count + i;
return (count);
```

```
}
main ( )
{
int i, j;
for (i = 0; i < =4; i++)
j = incr (i);
}
```

The *j* value will be

(A) 10
(B) 4
(C) 6
(D) 7

11. The following function

```
int Trial (int a, int b, int c)
{
if ((a > = b) && (c < b))
return b;
else if(a > = b)
return Trail(a, c, b);
else return Trail (b, a, c);
}
```

(A) finds the maximum of *a*, *b*, *c*
(B) finds the middle value of *a*, *b*, *c* after sorting
(C) finds the minimum of *a*, *b*, *c*
(D) none of the above

12. Consider the following pseudo code

```
f(a, b)
{
while(b! = 0)
{
t = b;
b = a % b;
a = t;
}
return a;
}
```

(A) The above code computes HCF of two numbers *a* and *b*
(B) The above code computes LCM of *a* and *b*
(C) The above code computes GCD of *a* and *b*
(D) None of the above

13.
```
1. main ( )
2. {int a = 10, *j;
3. void *k;
4. j = k = &a;
5. j++;
6. k++;
7. printf("\n %u, %u", j, k);
8. }
```

Which of the following is true in reference to the above code?

(A) The above code will compile successfully
(B) Error on line number 6
(C) Error on line number 3
(D) Error on line number 4

14. Aliasing in the context of programming language refers to

(A) multiple variables having the same memory location
(B) multiple variables having the same value
(C) multiple variables having the same identifier
(D) multiple uses of the same variable

15. Match the following:

X:	*m* = malloc (5); *m* = NULL;	1:	Using dangling pointers
Y:	free (*n*); *n* ↓ value = 5;	2:	Using un initialized pointers
Z:	char **p*; **p* = '*a*';	3:	Lost memory

(A) X – 1 Y – 3 Z – 2
(B) X – 3 Y – 1 Z – 2
(C) X – 3 Y – 2 Z – 1
(D) X – 2 Y – 1 Z – 3

Previous Years' Questions

1. In the following C function, let $n \geq m$.

```
int gcd(n,m)
{
if (n%m ==0) return m;
   n = n%m;
return gcd(m,n);
}
```

How many recursive calls are made by this function? **[2007]**

(A) $\Theta(\log_2 n)$ (B) $\Omega(n)$
(C) $\Theta(\log_2 \log_2 n)$ (D) $\Theta(\sqrt{n})$

2. What is the time *complexity* of the following recursive function?

```
int DoSomething (int n) {
if (n <= 2)
     return 1;
 else
return(DoSomething(floor(sqrt(n)))+ n);}
```

[2007]

(A) $\Theta(n^2)$ (B) $\Theta(n \log_2 n)$
(C) $\Theta(\log_2 n)$ (D) $\Theta(\log_2 \log_2 n)$

3. Choose the correct option to fill ? 1 and ? 2 so that the program below prints an input string in reverse order. Assume that the input string is terminated by a newline character.

```
void reverse (void) {
int c;
if (?1) reverse( );
?2
}
main ( ) {
printf ("Enter Text") ; printf ("\ n");
reverse ( ); printf ("\ n") ;
}
```

[2008]

(A) ?1 is (getchar( )! = '\n')
    ?2 is getchar(c);
(B) ?1 is (c = getchar( ))! = '\n')
    ?2 is getchar(c);
(C) ?1 is (c! = '\n')
    ?2 is putchar(c);
(D) ?1 is ((c = getchar( ))! = '\n')
    ?2 is putchar(c);

4. Consider the program below:

```
# include < stdio.h >
   int fun(int n, int * f_p) {
      int t, f;
      if (n <=1) {
            *f_p =1;
      return 1;
      }
      t = fun (n-1, f_p);
      f = t+*f_p;
      *f_p = t;
      return f;
      }
      int main( ) {
      int x = 15;
      printf ("%d\n", fun(5,&x));
      return 0;
      }
```

The value printed is **[2009]**

(A) 6 (B) 8
(C) 14 (D) 15

5. What is the value printed by the following C program?

```
#include <stdio.h>
int f(int *a, int n)
{
if (n <= 0)return 0;
else if(*a % 2 = = 0) return * a + f(a+1,
n-1);
else return *a-f(a+1, n-1);
}
int main( )
{
int a[ ] = {12, 7, 13, 4, 11, 6};
printf("%d", f(a,6));
return 0;
}
```

[2010]

(A) –9 (B) 5
(C) 15 (D) 19

Common data for questions 6 and 7: Consider the following recursive C function that takes two arguments.

unsigned int foo (unsigned int n, unsigned int r)

```
{
if ( n > 0 ) return ((n % r) + foo (n /r,
r));
else return 0;
}
```

6. What is the return value of the function foo when it is called as foo (513, 2)? **[2011]**

(A) 9 (B) 8
(C) 5 (D) 2

7. What is the return value of the function foo when it is called as foo (345, 10)? **[2011]**

(A) 345 (B) 12
(C) 5 (D) 3

Common data for questions 8 and 9: Consider the following C code segment

```
int a, b, c = 0;
void prtFun (void);
main ( )
{ static int a = 1;
prtFun( );
a+ = 1;
prtFun( );
printf("\n %d %d", a, b);
}
void prtFun(void)
{static int a = 2;
int b = 1;
a+ = ++b;
printf("\n %d %d", a, b);
}
```

8. What output will be generated by the given code segment if: **[2012]**

Line 1 is replaced by **auto int** a **= 1;**
Line 2 is replaced by **register int** a **= 2;**

(A)	(B)	(C)	(D)
3 1	4 2	4 2	4 2
4 1	6 1	6 2	4 2
4 2	6 1	2 0	2 0

9. What output will be generated by the given code segment? **[2012]**

(A)	(B)	(C)	(D)
3 1	4 2	4 2	3 1
4 1	6 1	6 2	5 2
4 2	6 1	2 0	5 2

10. What is the return value of $f(p, p)$, if the value of p is initialized to 5 before the call? Note that the first parameter is passed by reference, whereas the second parameter is passed by value.

```
int f(int &x, int c) {
   c = c - 1;
   if (c == 0) return 1;
   x = x + 1;
   return f(x, c) * x;
}
```

[2013]

(A) 3024 (B) 6561
(C) 55440 (D) 161051

11. Consider the following pseudo code. What is the total number of multiplications to be performed? **[2014]**

```
D = 2
for i = 1 to n do
for j = i to n do
for k = j +1 to n do
D = D * 3
```

(A) Half of the product of the three consecutive integers.
(B) One-third of the product of the three consecutive integers.
(C) One-sixth of the product of the three consecutive integers.
(D) None of the above.

12. Consider the function func shown below:

```
int func (int num) {
int count = 0;
while (num) {
count ++;
num>>=1;
}
return (count);
}
```

The value returned by func(435) is ______ **[2014]**

13. Consider the following function

```
double f (double X)
if (abs(X*X – 3) < 0.01)return X;
else return f(X/2 + 1.5/X);
}
```

Give a value q(to two decimals) such that $f(q)$ will return q:________ **[2014]**

14. Consider the following pseudo code, where x and y are positive integers **[2015]**

```
begin
        q := 0
        r := x
        while r ≥ y do
               begin
                      r := r – y
                      q := q + 1
               end
        end
```

The post condition that needs to be satisfied after the program terminates is

(A) $\{r = qx + y \wedge r < y\}$
(B) $\{x = qy + r \wedge r < y\}$
(C) $\{y = qx + r \wedge 0 < r < y\}$
(D) $\{q + 1 < r - y \wedge y > 0\}$

15. Consider the following C function **[2015]**

```
int fun(int n) {
     int x = 1, k;
     if (n == 1) return x;
     for (k = 1; k < n; ++k)
          x = x + fun(k)*fun(n - k);
     return x;
}
```

The return value of fun(5) is ______

16. Consider the following recursive C function

```
void get (int n)
{
     if (n < 1) return;
     get (n - 1);
     get (n - 3);
     printf("%d", n);
}
```

If get (6) function is being called in main ( ) then how many times will the get ( ) function be invoked before returning to the main ( )?

(A) 15 (B) 25
(C) 35 (D) 45

17. Consider the following C program **[2015]**

```
#include<stdio.h>
int f1(void);
int f2(void);
int f3(void);
int x = 10;
int main ( )
{
int x = 1;
x += f1( ) + f2 ( ) + f3 ( ) + f2 ( );
     printf("%d", x);
     return 0;
}
int f1 () { int x = 25; x++; return x;}
int f2 () { static int x = 50; x++; return x;}
int f3 () { x *= 10; return x};
```

The output of the program is ______

18. Suppose $c = < c[0],\ldots,c[k-1]>$ is an array of length k, where all the entries are from the set {0, 1}. For any positive integers a and n, consider the following pseudo code. **[2015]**

DOSOMETHING (c, a, n)
$z \leftarrow 1$
for $i \leftarrow 0$ to $k - 1$
do $z \leftarrow z^2$ mod n
if $c[i] = 1$
then $z \leftarrow (z \times a)$ mod n
return z

If $k = 4$, $c = <1, 0, 1, 1>$, $a = 2$ and $n = 8$, then the output of DOSOMETHING(c, a, n) is ______

19. What will be the output of the following C program? **[2016]**

```
void count (int n) {
static int d = 1;
printf("%d ",n);
printf("%d ",d);
d ++;
if (n > 1) count (n -1);
printf("%d ", d);
}
void main ( ) {
count (3);
}
```

(A) 3 1 2 2 1 3 4 4 4
(B) 3 1 2 1 1 1 2 2 2
(C) 3 1 2 2 1 3 4
(D) 3 1 2 1 1 1 2

20. The following function computes X^Y for positive integers X and Y. **[2016]**

```
int exp (int X, int Y)
{
int res = 1, a = X, b = Y;
while (b! = 0)
{
if (b%2 = = 0) {a = a*a; b = b/2;}
else {res = res *a; b = b -1;}
}
return res;
}
```

Which one of the following conditions is **TRUE** before every iteration of the loop?

(A) $X^Y = a^b$
(B) $(\text{res} * a)^Y = (\text{res} * X)^b$
(C) $X^Y = \text{res} * a^b$
(D) $X^Y = (\text{res} * a)^b$

21. Consider the following two functions.

```
void fun1 (int n) {          void fun2 (int n) {
    if (n == 0) return;      if (n == 0) return;
    printf ("%d", n);        printf ("%d", n);
    fun2 (n - 2) ;           fun1(++n)
    printf ("%d", n);        printf ("%d", n);
}                            }
```

The output printed when fun1 (5) is called is **[2017]**

(A) 53423122233445 (B) 53423120112233
(C) 53423122132435 (D) 53423120213243

22. Consider the C functions foo and bar given below:

```
int foo (int val) {
  int x = 0;
  while (val > 0) {
    x =x + foo (val--);
  }
  return val;
}
int bar (int val) {
  int x = 0;
  while (val > 0) {
    x =x +bar (val - 1);
    }
  return val;
}
```

Invocations of foo (3) and bar (3) will result in: **[2017]**

(A) Return of 6 and 6 respectively.
(B) Infinite loop and abnormal termination respectively.
(C) Abnormal termination and infinite loop respectively.
(D) Both terminating abnormally.

23. The output of executing the following C program is______.

```
# include <stdio.h>
int total (int v)  {
  static int count = 0;
  while (v)  {
    count + = v&1;
    v >> = 1;
  }
   return count;
}
void main ( )  {
    static int x = 0;
    int i = 5;
    for (; i > 0,i--)  {
      x = x + total (i);
    }
    printf ("%d\n", x);
}
```

[2017]

24. Consider the following C program:

```
#include <stdio.h>
int counter = 0;
int calc (int a, int b) {
    int c;
    counter++;
    if (b==3) return (a*a*a);
    else {
        c = calc (a, b/3);
        return (c*c*c);
    }
}
int main () {
    calc (4, 81);
    printf ("%d", counter);
}
```

The output of this program is ______. **[2018]**

25. Consider the following program written in pseudo-code. Assume that *x* and *y* are integers.

```
Count (x,y) {
if (y ! = 1) {
    if (x ! = 1) {
        print ("*");
        Count (x/2, y);
    }
    else {
        y = y-1;
        Count (1024, y);
        }
    }
}
```

The number of times that the print statement is executed by the call count (1024, 1024) is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** C	**3.** A	**4.** D	**5.** D	**6.** D	**7.** D	**8.** C	**9.** B	**10.** A
11. C	**12.** C	**13.** C	**14.** D	**15.** C					

Practice Problems 2

1. D	**2.** C	**3.** B	**4.** B	**5.** C	**6.** C	**7.** A	**8.** C	**9.** C	**10.** A
11. B	**12.** C	**13.** B	**14.** A	**15.** B					

Previous Years' Questions

1. C	**2.** -	**3.** –D	**4.** B	**5.** C	**6.** D	**7.** B	**8.** D	**9.** C	**10.** B
11. C	**12.** 9	**13.** 1.72 to 1.74		**14.** B	**15.** 51	**16.** B	**17.** 230	**18.** 0	**19.** A
20. C	**21.** A	**22.** C	**23.** 23	**24.** 4	**25.** 10230				

Chapter 3

Arrays, Pointers and Structures

LEARNING OBJECTIVES

- *Arrays*
- *Array initialization*
- *Passing array elements to function*
- *Two dimensional arrays*
- *Syntax for 3D array declaration*
- *Pointers*
- *Pointer to pointer*
- *Pointer to void (generic pointer)*
- *Array of pointers*
- *Pointer to function*
- *Dynamic memory management*
- *Memory allocation function*
- *Realloc*
- *Structures*
- *Nesting of structures*
- *Array of structures*
- *Structures & functions*
- *Union*
- *Declaration*
- *Bit fields*

Arrays

In *C* we have the following derived data types:

- Arrays
- Pointers
- Structures
- Unions

Imagine a problem that requires to read, process, and print 10 integers. We can declare 10 variables, each with different name. Having 10 different names creates a problem; we need 10 read and 10 write statements each for different variable.

Definition

An array is a collection of elements of same data type. Array is a sequenced collection. So, we can refer to the elements in array as 0th element, 1st element, and so on, until we get the last element. The array elements are individually addressed with their subscripts/indices along with array name. We place subscript value in square brackets ([ ]) followed by array name. This notation is called indexing.

There is a powerful programming construct, loop, that makes array processing easy. It does not matter if there are 1, 10, 100 or 1000 elements.

We can also use a variable name in subscript, as the value of variable changes; it refers different elements at different times.

Syntax of Array Declaration

```
Data_type array_name_[size];
```

Here, data type says the type of elements in collection, array_name is the name given to collection of elements and size says the number of elements in array.

Example: int marks[6];

Here, 'int' specifies the type of variable, marks specifies name of variable. The number 6 tells the dimension/size. The '[ ]' tells the compiler that we are dealing with array.

Accessing array elements: All the array elements are numbered, starting from 0, thus marks [3] is not the third, but the fourth element.

Example: marks[2] – 3rd element
marks[0] – 1st element
We can use the variable as index.
Thus marks[i] – ith element. As the value of i changes, refers different elements in array.

Summary about Arrays

- An array is a collection of similar elements.
- The first element in array is numbered 0, and the last element is one less than the total size of the array.
- An array is also known as subscripted variable.
- Before using an array, its type and dimension must be declared.
- How big an array is, its elements are always stored in contiguous memory locations.
- Individual elements accessed by index indicating relative position in collection.
- Index of an array must be an integer.

Array Initialization

Syntax

```
Data_type array_name[size] = {values};
```

Example:

```
int  n[6]= {2,4,8,12,20,25};  //  Array  ini-
tialized with list of values
int num[10] = {2,4,12,20,35};
// remaining 5 elements are initialized with
0
// values
int  b[10]  =  {0};  //  Entire  array  elements
initialized with 0.
```

Note:

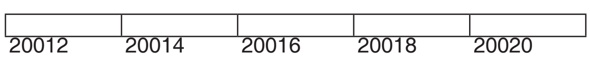
- Till the array elements are not given any specific value, they are supposed to contain garbage values.
- If the number of elements used for initialization is lesser than the size of array, then the remaining elements are initialized with zero.
- Where the array is initialized with all the elements, mentioning the dimension is optional.

Array Elements in Memory

Consider the following declaration – int num[5].

What happens in memory when we make this declaration?

- 10 bytes get received in memory, 2 bytes each for 5 integers.
- Since array is not initialized, all five values in it would be garbage. This happens because the default storage class is auto. If it is declared as static, all the array elements would be initialized with 0.



Note: In *C*, the compiler does not check whether the subscript used for array exceeds the size of array.

Data entered with a subscript exceeding the array size will simply be placed in memory out size the array, and there will be no error/warning message to warn the programmer.

Passing array elements to function

Array elements can be passed to a function by value or by reference.

Example: A program to pass an array by value:

```
void main( )
{
void display(int[  ]);// Declaration
int marks[ ] = {10,15,20,25,30};
display (marks);// function call
}
void display(int n[ ] )// function definition
{
int i;
for(i = 0 ; i < 5 ; i++)
printf("%d ", n[ i ] );
}
```

Output:

10 15 20 25 30

Here, we are passing the entire array by name. The formal parameter to receive is declared as an array, so it receives entire array elements.

To pass the individual elements of an array, we have to use index of element with array name.

Example: display (marks[*i*]); sends only the ith element as parameter.

Example: A program to demonstrate call by reference:

```
void main( )
{
void display (int *);
int marks[ ] = {5, 10 15, 20, 25};
display(&marks[0]);
}
void display(int *p)
{
int i;
for(i = 0; i < 5; i++)
printf("%d ",*(p+i));
}
```

Output:

5 10 15 20 25

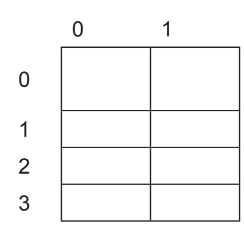
Here, we pass the address of very first element. Hence, the variable in which this address is collected (*p*) is declared as a pointer variable.

Note: Array elements are stored in contiguous memory location, by passing the address of the first element; entire array elements can be accessed.

Two-dimensional Arrays

In *C* a two-dimensional array looks like an array of arrays, i.e., a two-dimensional array is the collection of one-dimensional arrays.

Example: `int x[4][2];`



By convention, first dimension says the number of rows in array and second dimension says the number of columns in each row.

In memory, whether it is one-dimensional or a two-dimensional array, the array elements are stored in one continuous chain.

The arrangement of array elements of a two-dimensional array in memory is shown below:

X[0][0]	x[0][1]	x[1][0]	x[1][1]	x[2][0]	X[2][1]	X[3][0]	X[3][1]
6000	6002	6004	6006	6008	6010	6012	6014

Initialization

We can initialize two-dimensional array as one-dimensional array:

```
int a[4] [2] = {0,1,2,3,4,5,6,7}
```

The nested braces can be used to show the exact nature of array, i.e.,

```
int a[4][2] = {{0,1},{2,3}{4,5},{6,7}}
```

Here, we define each row as a one-dimensional array of two elements enclosed in braces.

Note: If the array is completely initialized with supplied values, then we can omit the size of first dimension of an array (the left most dimension).

- For accessing elements of multi-dimensional arrays, we must use multiple subscripts with array name.
- Generally, we use nested loops to work with multi-dimensional array.

Multidimensional Arrays

C allows array of two or more dimensions and maximum numbers of dimensions a C program can have depends on the compiler, we are using. Generally, an array having one dimension is called 1D array; array having two dimensions is called 2D array and so on.

Syntax:
type array-name[*d*1] [*d*2] [*d*3] [*d*4]…[*dn*];
where dn is the size of last dimension.

Example:
int table[5][5][20];
float arr[5][6][5][6][5];
In our example array “table” is a 3D. (A 3D array is an array of array of array)

Declaration and Initialization of 3D array

A 3D array can be assumed as an array of arrays; it is an array of 2D arrays and as we know 2D array itself is an array of 1D arrays. A diagram can help you to understand this.

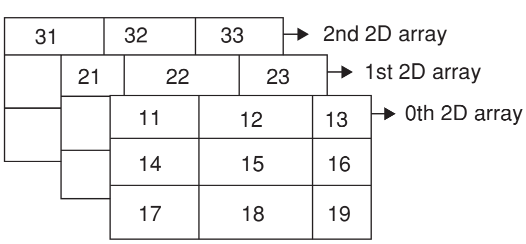


Figure 1 3D array conceptual view

Example:

```
void main( )
{
int i, j, k;
int arr [3] [3] [3] =
{
{11, 12, 13},
{14, 15, 16},
{17, 18, 19}
},
{21, 22, 23},
{24, 25, 26},
{27, 28, 29}
},
{31, 32, 33},
{34, 35, 36},
{37, 38, 39}
}
};
printf(“3D Array Elements \n”);
for (i = 0; i<3; i++)
{
for(j =0; j <3; j++)
{
for (k= 0; k<3; k++)
{
printf (“% d\t”, arr[i][j][k]);
}
printf (“\n”);
}
printf (“\n);
}
}
```

Output: 3D Array Elements

11	12	13
14	15	16
17	18	19
21	22	23
24	25	26
27	28	29
31	32	33
34	35	36
37	38	39

Syntax for 3D Array Declaration

data–type array–name [table] [row] [column];
To store values in any 3D array, first point to table number, row number and lastly to column number.

POINTERS

Pointer is a variable which contains address of another variable. *C*'s clever use of pointers makes it the excellent language.

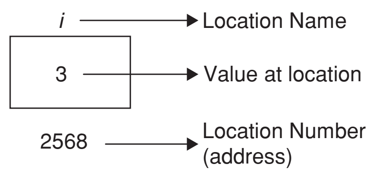
Consider the declaration:

```
int i = 3;
```

The declaration tells the *C* compiler to:

- Reserve space in memory to hold in integer value.
- Associate the name *i* with this memory location.
- Store the value 3 at this location.

Memory map is:



Computer may choose different location at different times for same variable. The important point is the address is a number.

The expression '&i' gives the address of variable '*i*'.

p = &i;

Assigns the address of '*i*' to variable '*p*'.

The variable '*p*' is declared as:

int **p*;

* tells the compiler that variable '*p*' is an address variable.

Memory map of i, **p* is –

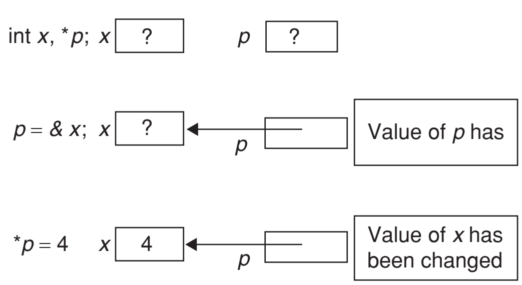
**p*	*i*
2568	3
2720	2568

Now, pointer '*p*' is referring to the variable '*i*'.

The variable '*i*' can be accessed in two ways:

- By using the name of variable.
- By using the pointer variable referring to location '*i*'.

The operator '*' can also be used along with pointer variable in expressions. The operator '*' acts as indirection operator.



Usage of '*p*' refers to value of '*p*', where as '**p*' refers to value at the address stored in '*p*', i.e., value of '*i*'.

Example:
```
int *p;
float *x;
char *ch ;
```

Here, *p*, *x* and ch are pointer variables, i.e., variables capable of holding address. Since addresses are always whole numbers, pointers would always contain whole numbers.

The declaration float **x* does not mean that *x* contains floating value, *x* will contain address of floating point variable. Similarly, 'ch' contains address of char value.

Pointer to Pointer

We know, pointer is a variable that contains address of another variable. Now this variable address might be stored in another pointer. Thus, we now have a pointer that contains address of another pointer, known as pointer to pointer.

Example:

```
void main()
{
int i = 3, *p, **q;
p = &i;
q =&p;
printf("\n Address of i = %u", &i);
printf("\n Address of i = %u", p);
printf("\n Address of i = %u", *q);
printf("\n Address of p= %u", &p);
printf(\n Address of p= %u, q);
printf("\n Address of q = %u", &q)
printf('\n value of i= %d",i);
printf('\n value of i= %d",*(&i));
printf('\n value of i= %d",*p);
printf('\n value of i= %d",**q);
}
```

If the memory map is

***q*	**p*	*i*
2010	2000	3
2050	2010	2000

Then the output is:

Address of *i* = 2000
Address of *i* = 2000
Address of *i* = 2000
Address of *p* = 2010
Address of *p* = 2010
Address of *q* = 2050
Value of *i* = 3
Value of *i* = 3
Value of *i* = 3
Value of *i* = 3

Note: We can extend pointer to a pointer to pointer. In principal, there is no limit on how far we can go on extending this definition.

Pointers for Inter-function Communication

We know that functions can be called by value and called by reference.

- If the actual parameter should not change in called function, pass the parameter-by value.

- If the value of actual parameter should get changed in called function, then use pass-by reference.
- If the function has to return more than one value, return these values indirectly by using call-by-reference.

Example: The following program demonstrates how to return multiple values.

```
void main( )
{
void areaperi(int, int *, int *);
int r;
float a,p;
printf("\n Enter radius of a circle");
scanf("%d", &r);
areaperi(r, &a, &p);
printf("Area = %f", a);
printf("\n Perimeter = %f", p);
}
void areaperi(int x, int *p, int *q)
{
*p = 3.14*x*x;
*q = 2 * 3.14*x;
}
```

Output:
Enter radius of circle 5
Area = 78:500000
Perimeter = 31.400000

Compatibility: Pointers have a type associated with them. They are not just pointer types, but rather are pointers to a specific type. The size of all pointers is same, which is equal to size of int. Every pointer holds the address of one memory location in computer, but size of variable that the pointer references can be different.

Pointer to Void (Generic Pointer)

A pointer to void is a generic type; this can point to any type. Its limitation is that the pointed data cannot be referenced directly. Since void pointer has no object type, so its length is undetermined; it cannot be dereference unless it is cast.

Example: The following example demonstrates generic pointer.

```
void main ( )
{
int a = 10;
float x = 5.7;
void *p;
p = &a;
printf("\n value of a = %d", *((int*)p));
p= &x;
printf ("\n value of x = % f", *((float *)p));
}
```

Output:
value of *a* = 10
value of *x* = 5.700000

Operations can be Performed on Pointers

1. Addition of a number to a pointer.

Example:
```
int i = 4, *j, *k;
  j =&i;
  j = j +1;
  k = j +5;
```

2. Subtraction of a number from a pointer.

Example:
```
int i = 4, * j, * k;
  j = &i; j = j -1;
  k = j - 3;
```

3. Subtraction of one pointer from another. One pointer variable can be subtracted from another (provided both variables point to same array elements). The resulting value indicates the number of bytes (elements) separating (the corresponding array elements).

Example:
```
void main ( )
{
int a[ ] = {5,10,15,20,25} ,*i, *j;
i = &a[0];
j = &a[4];
printf("%d, %d", j-i,*j-*i);
}
```

Output: 4, 20
The expression *j-i* prints 4 but not 8. because j and i pointing to integers that are 4 integers apart.

4. Comparison of two pointer variables. Pointer variables can be compared provided both pointing to the same data type.

Notes: Do not attempt the following operations on pointers:
1. Addition of two pointers.
2. Multiplication of a pointer with a number or another pointer.
3. Division of a pointer with a number or another pointer.

Important points about pointer arithmetic

- A pointer when incremented always points to an immediately next location.
- A pointer when decremented always points to an element precedes the current element.

Notice the difference with:

```
(*p)++
```

Here, the expression would have been evaluated as the value pointed by *p* increased by one. The value of *p* would not be modified if we write

$$\boxed{*p{+}{+} = *q{+}{+};}$$

Because ++ has a higher precedence than *, both *p* and *q* are increased, but because both increase operators (++) are used as postfix and not prefix, the value assigned to **p* is

*q before both *p* and *q* are increased. And then both are increased, it would be equivalent to

```
*p = *q
++p;
++q;
```

Implementation of arrays in C

Array name is the pointer to the first element in array. The following discussion explains how pointers are used for implementing arrays in *C*.

```
int n[ ] = {10,20,30,40,50};
```

n					
	10	20	30	40	50
	5512	5514	5516	5518	5520

- We know that mentioning the array name gets the base address.

  ```
  int *p = n;
  ```

  Now 'p' points to 0^{th} element of array '*n*'.
- 0th element can be accessed as *array_ name.

  ```
  int  x = *n;
  stores n[0] into 'x'.
  ```
- we can say that *array _ name and *(array _ name+0) are same. This indicates the following are same.

  ```
  num[i]
  *(num + i)
  *(i+num)
  ```

  (num is an array; i is an index)

Array of Pointers

The way there can be an array of ints or array of floats, similarly there can be an array of pointers. An array of pointers is the collection of addresses.

These arrays of pointers may point to isolated elements or an array element.

Example 1: Array of pointers pointing to isolated elements:

```
int i = 5, j=10, k =15;
int *ap[3];
ap[0] = &i; ap[1] = &j; ap[2] = &k;
```

Example 2: Array of pointers pointing to elements of an array:

```
int a[ ] = {0,20,45,50,70};
int *p[5], i ;
for(i = 0; i <5 ; i++ )
p[i] = &a[i] ;
```

Example 3: Array of pointers pointing to elements of different arrays;

```
int a[ ] = {5,10,20,25};
int b[ ] = {0,100,200,300,400};
int c[ ] = {50,150,250,350,450};
int *p[3];
p[0] = a; p[1] = b; p[2]=c;
```

Example 4: Array of pointers pointing to 0th element of each row of a two-dimensional array

```
int a[3][2] = {{1,2} {3,4}, {5,6}};
int *p[3];
p[0] = a[0]; p[1] =a[1];p[2] = a[2];
```

Pointer to Function

Function is a set of instructions stored in memory, so the function also contains the base address. This address can hold by using a pointer called pointer to function.

Syntax:

```
return_type  (*function_pointer)(parameter –
list );
```

Example: `int (*fp)(float, char, char);`

Example:

```
// pointer to functions
# include <iostream>
Using name space std;
int addition(int a, int b)
{
return (a + b);
}
int subtraction(int a, int b)
{
return (a – b) ;
}
int operation (int x, int y, int (*funtocall)
(int, int))
{
int g;
g = (*functocall)(x, y);
return (g);
}
int main( )
{
int m, n;
int (*minus)(int, int) = substraction;
m = operation(7, 5, addition);
n = operation(20, m, minus);
cout < < n;
return 0;
}
```

In the example, minus is a pointer to a function that has two parameters of type int. It is immediately assigned to point to the function subtraction, all in a single line.

Example: Program to demonstrate function pointer

```
int add(int, int);
int sub(int, int);
void main( )
{
Int (*fp) (int, int);
fp = add;
```

```
printf("\n 4+5=%d", fp(4,5));
fp = sub;
printf ("\n 4 - 5 = %d", fp(4,5));
}
int add(int x, int y)
{
return x + y;
}
int sub(int x,int y)
{
return x - y;
}
```

Output: 4 + 5 = 9
4 – 5 = –1

Pointer to structure The main usage of pointer to structure is we can pass structure as parameter to function as call by reference.

The other usage is to create linked lists and other dynamic data structures which depend on dynamic allocation.

Consider the declaration

struct employee

```
{
char name[20];
Int age;
float salary;
};
```

struct employee * *p*;

Variable of structures can be accessed using '.' Operator (or) → operator that is

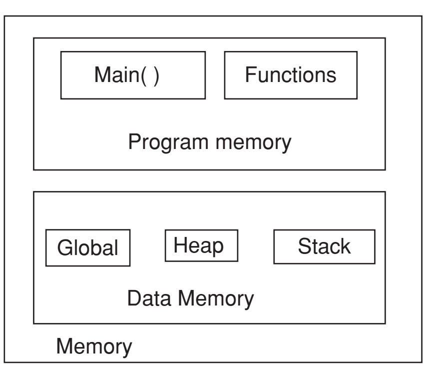
(**p*).age = 20 ; (or) *p* → age = 20;

(**p*).salary = 40, 231.0; (or) *p* → salary = 40,231.0;

Dynamic Memory Management

We can allocate the memory to objects in two ways—static and dynamic allocation. Static memory allocation requires declaration and definition of memory fully specified in the source program. The number of bytes required cannot be changed during run time. Dynamic memory allocation uses predefined functions to allocate and de-allocate memory for data dynamically during the execution of program.

We can refer to dynamically allocated memory only through pointers. Conceptual view of memory:



Memory Allocation Function

- Static memory allocation uses stack memory for variables.
- Dynamic memory management allocates memory from heap.

The following are the four memory management functions available in alloc.h and stdlib.h.

1. **Malloc (Block memory allocation):** Malloc function allocates block of memory that contained the number of bytes specified in parenthesis. It returns 'void' pointer to the first byte of allocated memory. The allocated memory is not initialized. If the memory allocation is not successful then it return NULL pointer.
   Declaration
   void *malloc (size_t size);
   The type size_t is defined as unsigned int in several header files including `stdio.h.`

   Syntax: pointer = (type*) malloc(size);
2. **Calloc (contiguous memory allocation):** Calloc is primarily used to allocate memory for arrays. It initializes the allocated memory with null characters.

   Declaration: void *calloc (size_t ele_count, size_t ele_size);

   Syntax: ptr = (type*)calloc(ele-count,ele-size);
3. **Realloc (reallocation of memory):** The realloc function is highly inefficient. When given a pointer to a previously allocated block of memory, realloc changes the size of block by deleting or extending the memory at the end of block. If the memory cannot be extended, then realloc allocates completely new block, copies the contents from existing memory location to new location, and deletes the old location.

   Declaration: void *realloc (void *ptr, size_t new_size);

   Syntax: ptr = (type*)realloc(ptr, new_ size);
4. **Free (Releasing memory):** When the memory allocated by malloc, calloc or realloc is no longer needed, they can be freed using the function free( ).

   Declaration: void free(void *ptr);

   Syntax: free(ptr);

Free function de-allocates complete memory referenced by the pointer. Part of the memory block cannot be de-allocated.

Structures

Arrays are used to store large set of data and manipulate them but the disadvantage is that all the elements stored in an array are to be of the same data type. When we require using a collection of different data items of different data types, we can use a structure.

- Structure is a method of packing data of different types.
- A structure is a convenient method of handling a group of related data items of different data types.

Syntax for declaration

```
struct sturct_name
{
Data_type_1 var1;
Data_type_2 var2;
:
Data_type_n varn;
};
```

Example:

```
struct lib - books
{
char title [20];
char author[15];
int pages;
float price;
};
```

The keyword struct declares a structure to hold the details of four fields namely title, author, pages and price, these are members of the structures.

We can declare structure variables using the tag name anywhere in the program.

Example: struct lib – books book1, book2, book3;

- Declares book1, book2, book3 as variables of type struct lib _ books, each declaration has four elements of the structure lib_ books.

  Memory map of book1:

Book1		
	Title	20 bytes
	Author	15 bytes
	Pages	2 bytes
	Price	4 bytes

- Memory will not be allocated to the structure until it is instantiated. i.e., till the declaration of a variable to structure.
- To access the members of a structure variable, *C* provides the member of (.) operator.

Example: To access author of book 1 – book1. author

Syntax: `structure_var.member_name;`

- The structures can also be initialized as any other variable of C.

Example: struct lib-books book4={"Let us *C*", "yashwanth", 450, 200.95};

Note: The values must provide in the same order as they appear in structure declaration.

- One structure variable can be assigned to another structure variable.
- Structure variables cannot be compared.

Example:

```
# include <stdio.h>
void main( )
{
Struct s1{
int id_no;
char name[20];
char address[20];
char combination[3];
   int age;
   } newstudent;
printf (" Enter student Information");
printf ("Enter student id _ no");
scanf ("%d", &newstudent.id_no):
printf (" Enter the name of the student");
scanf ("%s", & newstudent.name);
printf (" Enter the address  of the student");
scanf ("%s", &newstudent.address);
printf("Enter   the   combination   of   the
student")';
scanf("%s", &newstudent.combination");
printf (" Enter the age of student);
scanf ("%d ", &newstudent.age");
printf (" student information");
printf  ("  student id_no  =  %d",  newstudent.
id - no);
printf("student   name   =   %s",   newstudent.
name);
printf("student address = %s", newstudent.
address);
printf ("students combination = %s", newstu-
dent. combination);
printf("Age  of  student  =  %d",  newstudent.
age);
}
```

Nesting of Structures

The structures can be nested in two ways:

- Placing the structure variable as a member in another structure declaration.
- Declaration of the entire structure in another structure.

Example:

```
struct date
{
int day;
int month;
int year;
};
struct student
{
int id_no;
char name[20];
char address [20];
int age;
structure date doa;
} oldstudent, newstudent;
```

The structure 'student' contains another structure date as one of its members.

To access the day of date of admission (doa) of old student – oldstudent.doa.day.

Example:

```
struct outer
{
```

```
int o1;
float o2;
struct inner
{
int i1;
float i2;
};
} out1, out2;
```

The innermost members in a nested structure can be accessed by chaining all the concerned structure variables, from outermost to innermost; accessing i1 for out1-out1. inner.i1;

Array of Structures

It is possible to define an array of structures. For example, if we are maintaining information of all the students in the college and if 100 students are studying in the college, we need to use an array than single variables.

Example:

```
structure information
{
int id _ no;
char name[20];
char address[20];
char combination[3];
int age;
}
student[100];
```

Example:

```
# include <stdio.h>
{
struct info
{
int  id _ no;
char name[20];
char address[20];
char combination[3];
int age;
}
struct info std[100];
int, i ,n;
printf (" Enter the number of students");
scanf ("%d", &n);
scanf("Enter id_no, name, address, combina-
tion and age");
for (i = 0; i<n; i ++)
scanf(" %d  %s  %s  %s  %d",  &std[i].id_no,
std[i].name, std[i].address,
std[i]. combination,&std [i].age);
printf("student information");
for (i = 0 ; i < n; i ++)
printf("%d  %s  %s  %  s  %  d",  std[i].id_no,
std[i].name, std[i].address, std[i]. combi-
nation, std[i]. age);
```

Structures and Functions

- An entire structure can be passed as a parameter like any other variable.
- A function can also return a structure variable.

Example:

```
# include <stdio.h>
struct employee
{
int emp_id;
char name[25];
char department[10];
float salary;
};
void main( )
{
static struct employee emp1 = {
12, "shyam", "computer", 7500.00};
/* sending entire employee structure */
display(emp1);
}
/* function to pass entire structure vari-
able */
display(empf)
struct employee empf
{
printf (" %d %s % s %f", empf.empid, empf.
name, empf.department, empf.salary);
}
```

Union

Union, like structure contains members whose individual data types may differ from one another. The members that compose union all share the same storage area within the computer's memory whereas each member within a structure is assigned its own unique storage area. Thus, unions are used to conserve memory.

Declaration

```
union item
{
int m;
float p;
char c;
}Code;
```

This declares a variable code of type union item.

The union contains three members each with a different data type. However, we can use only one of them at a time. The compiler allocates a piece of storage that is large enough to access a union member; we can use the same syntax that we use to access structure members, i.e.,

```
Code.m
Code.p
Code.c
```

are all valid member variables. During accessing, we should make sure that we are accessing the member whose value is currently stored.

Example:

```
union marks
{
float perc;
char grade;
}
main( )
{
union marks student1;
student1.perc = 98.5;
printf("marks are %f address is %16ℓu", stu-
dent1.perc, &student1. perc);
student1. grade = 'c';
printf("grade is %c address is %16ℓu", stu-
dent1. grade, &student1. grade);
}
```

Example:

```
# include <stdio.h>
void main ( )
{
Union u_example
{
float decval;
int p_num;
double my_value;
}U1;
U1.my_value = 125.5;
U1.pnum = 10;
U1.decval = 1000.5f;
printf("decval = %f pnum = %d my_value = % lf
", U1. decval, U1.pnum, U1.my_value);
printf(" U1 size = %d decval size =%d,
pnum size = %d my-value size = % d",
sizeof (U1), sizeof (U1.decval), sizeof
(U1.pnum), sizeof (U1.my_value));
       }
```

Bit Fields

When a program variable '*x*' is declared as int, then '*x*' takes the values from (-2^{15}) to ($2^{15} - 1$), if *x* in the program takes only two values, 1 and 0, which requires only one bit, then the remaining 15 bits are waste.

In order to not to have this wastage, we can use bit fields with the several variables with the small enough maximal values, which can pack into a single memory location

Example:

```
struct student
{
Int gender : 1 ; // gender takes only 0,1
values
Int marriage : 2 ; // marriage takes 4(0, 1,
2, 3) values
Int marks : 7 ; // marks takes values from
0 - 127
}
```

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Output of the following C program is

```
intF(int x, int *py, int **pz)
{
int y, z;
** pz+= 1;
z = *pz;
*py+= 2;
y = *py;
x+ = 3;
return x+y+z;
}
void main( )
{
int c, *b, **a ;
c = 4;
b = &c;
a = &b;
printf( "%d", F(c, b, a));
}
```

(A) 30 (B) 22
(C) 20 (D) Error

2.
```
main( )
{
    char  *ptr;
    ptr = "Hello World";
     printf("%c\n",*&*ptr);
}
```

Output of the above program is
(A) Garbage value
(B) Error
(C) H
(D) Hello world

3.
```
#include <stdio.h>
main( )
{
register a =10;
char b[ ] = "Hi";
printf("%s %d ", b, a);
}
```

Output is
(A) Hi 10 (B) Error
(C) Hi (D) Hi garbage value

4.
```
main ( )
{
    int fun( ) ;
    (*fun)( ) ;
}
int fun( )
{ printf("Hello") ;
}
```
(A) Hello (B) Error
(C) No output (D) H

5. Let B be a two-dimensional array declared as
B : array[1...10] [1...15] of integer;

Assuming that each integer takes one memory location the array is stored in row major order and the first element of the array is stored at location 100, what is the address of the element $B[i]$ $[j]$?

(A) $15i + 10j + 84$ (B) $15i + j - 16$
(C) $15i + j$ (D) $15i + j + 84$

6. Consider the following C program which is supposed to compute the transpose of a given 4×4 matrix M. Note that, there is a Y in the program which indicates some missing statements. Choose the correct option to replace Y in the program.

```
# include <stdio.h>
int M[4][4] = { 8, 10, 9, 16, 12, 13, 11,
15, 14, 7, 6, 3, 4, 2, 1, 5 };
main( )
{
   int i, j, temp;
    for (i = 0; i<4; ++i)
     {
            Y
     }
     for (i=0; i<4; ++i)
     for (j=0; j<4; ++j)
      printf("%d", M[i] [j]);
}
```
(A)
```
for (j=0; j<4; ++j)
    {
       M[j] [i] = temp;
        temp = M[j][i];
         M[j][i] = M[i][j];
        }
```
(B)
```
for (j=0; j<4; ++j)
    {
       temp = M[j][i];
       M[i][j] = M[j][i];
        M[j][i] =  temp;
        }
```
(C)
```
for (j=i; j<4; ++j)
    {
       temp = M[i][j];
       M[i][j] = M[j][i];
        M[j][i] =  temp;
        }
```
(D)
```
for (j=i; j<4; ++j)
        {
       M[i][j] = temp;
        temp = M[j][i];
       M[j][i] = M[i][j] ;
        }
```

7. Consider the C program shown below:
```
# include <stdio.h>
# define print(a) printf("%d", a)
int a;
void z(int n)
{
  n += a;
print (n);
}
void x(int *p)
{
   int a = *p+2;
   z(a) ;
*p = a;
   print(a);
}
main(void)
  {
     a = 6;
     x(&a);
     print(a);
  }
```
The output of this program is
(A) 14 8 6 (B) 16 6 6
(C) 8 6 6 (D) 22 11 12

8. Consider the program below:
```
# include <stdio.h>
int fun(int n, int *p)
{
    int x,y;
    if (n<=1)
     {
         *p = 1;
          return 1;
    }
x = fun(n-1, p);
y = x +p;
*p = x;
return y;
}
int main( )
    {
        int a =15;
    printf( "%d\n", fun(5, &a));
    return 0;
    }
```
The output value is
(A) 14 (B) 15
(C) 8 (D) 95

9. Consider the following C program segment

```
char p[20] ;
int i;
char *s = "string" ;
int l = strlen(s);
for (i=0; i<l; i++)
p[i] = s[l - i] ;
printf("%s", p) ;
```

The output of the program is

(A) string
(B) gnirt
(C) gnirts
(D) No output is printed

10.
```
# include <stdio.h>
main( )
{
    struct AA
      {
            int A = 5;
             char name[ ] = "ANU";
      };
    struct  AA  *p  =  malloc(sizeof(struct
    AA));
                printf("%d",p->A);
                printf("%s",p->name);
}
```

Output of the program is

(A) 5 ANU
(B) Runtime error
(C) Compiler error
(D) Linker error

11. The declaration

```
union u_tag {
int ival;
float fval;
char sval;
} u;
```

denotes *u* is a variable of type *u*_tag and

(A) *u* can have a value of int, float and char
(B) *u* can represent either integer value, float value or character value at a time
(C) *u* can have a value of float but not integer
(D) None of the above

12. If the following program is run from command line as myprog 1 2 3, what would be the output?

```
main (int argc, char *argv[ ])
{
int i;
i = argv [1] + argv [2] - argv [3];
printf ("%d", i);
}
```

(A) 123 (B) 6
(C) 0 (D) Error

13. The following C program is run from the command line as
myprog one two;
what will be the output?

```
main (int argc, char *argv [ ])
{
printf ("%c",**++argv);
}
```

(A) m (B) o
(C) myprog (D) one

14. The following program

```
change(int *);
main( ) {
int a = 4;
change(a);
printf ("%d", a);
}
change(a)
int a;
{
printf("%d", a);
}
```

Outputs

(A) 44 (B) 55
(C) 34 (D) 22

15. What is the output of the following program:

```
main( )
{
    const int x = 10;
    int *ptrx;
    ptrx = &x;
    *ptrx = 20;
    printf ("%d", x);
}
```

(A) 5 (B) 10
(C) Error (D) 20

Practice Problems 2

Directions for questions 1 to 11: Select the correct alternative from the given choices.

1. The following program segment

```
int *i;
*i = 10;
```

(A) Results in run time error
(B) Is a dangling reference
(C) Results in compilation error
(D) Assigns 10 to *i*

2. A $m \times n$ matrix is stored in column major form. The expression which accesses the (ij)th entry of the same matrix is
(A) $n \times (j-1) + i$
(B) $m \times (j-1) + i$
(C) $n \times (m-1) + ij$
(D) $m \times (n-1) + j$

3. int * $S[a]$ is 1D array of integers, which of the following refers to the third element in the array?
(A) $*(S+2)$ (B) $*(S+3)$
(C) $S+2$ (D) $S+3$

4. If an array is declared as char a[10][12]; what is referred to by $a[5]$?
(A) Pointer to 3rd Row
(B) Pointer to 4th Row
(C) Pointer to 5th Row
(D) Pointer to 6th Row

5. The following code is run from the command line as myprog 1 2 3. What would be the output?

```
main(int argc, char *argv[ ])
{
    int i, j = 0;
    for (i = 1; i < argc; i++)
    j = j + atoi (argv [i]);
    printf ("%d", j);
}
```

(A) 123 (B) 6
(C) Error (D) "123"

6. What will be the following C program output?

```
main (int argc, char *argv[ ], char *env
[ ]) {
int i;
for(i = 1; i < argc; i++)
printf ("%s", env[i]);
}
```

(A) List of all arguments
(B) List of all path parameters
(C) Error
(D) List of environment variables

7. The declaration

```
enum colors {
    red,
    blue,
    yellow = 1,
    green
};
```

assigns the value 1 to
(A) Red and Yellow
(B) Blue
(C) Red and blue
(D) Blue and yellow

8. What would be the output of the following program?

```
sum = 0;
for (i = -10; i < 0; i++)
    sum = sum + abs(i);
printf ("%d", sum);
```

(A) 100 (B) −505
(C) 55 (D) −55

9. An integer occupies 2 bytes of memory, float occupies 4 bytes and character occupies 1 byte. A structure is defined as:

```
struct tab {
    char a;
    int b;
    float c;
} table [10];
```

Then the total memory requirement (in bytes) is
(A) 14 (B) 70
(C) 40 (D) 100

10. What are the values of u1 and u2?

```
int u1, u2;
int x = 2;
int *ptr;
u1 = 2*(x + 10);
ptr = &x;
u2 = 2*(*ptr + 10);
```

(A) $u1 = 8, u2 = 16$
(B) $u1 = 23, u2 = 24$
(C) $u1 = 24, u2 = 24$
(D) None of the above

11. What is the output?

```
func(a, b)
int a, b;
{
return (a = (a = = b));
}
main ()
{
int process(), func();
printf("The value of process is %d", pro-
cess (func,3,6));
}
process (pf, val1, val2)
int (*pf) ();
```

```
int val1, val2;
{
return ((*pf) (val1, val2));
}
```

(A) The value of process is 0
(B) The value of process is 3
(C) The value of process is 6
(D) Logical error

Previous Years' Questions

1. Consider the following program in C language:

```
# include < stdio. h>
main ()
{
int i;
int *pi = &i;
scanf ("%d", pi);
printf("%d\n", i + 5);
}
```

Which one of the following statement is TRUE? **[2014]**

(A) Compilation fails
(B) Execution results in a run-time error
(C) On execution, the value printed is 5 more than the address of variable *i*.
(D) On execution, the value printed is 5 more than the integer value entered.

2. Consider the following C function in which size is the number of elements in the array *E*:

```
int MyX (int *E, unsigned int size)
{
int Y = 0;
int Z;
int i, j, k;
for (i = 0; i < size; i++)
Y = Y + E[i];
for (i = 0; i < size; i++)
for (j = 1; j < size; j++)
{
Z = 0;
for (k = i; k < = j; k++)
Z = Z + E[k];
if (Z > Y)
Y = Z;
}
return Y;
}
```

The value returned by the function My X is the **[2014]**

(A) maximum possible sum of elements in any sub - array of array *E*.
(B) maximum element in any sub-array of array *E*.
(C) sum of the maximum elements in all possible sub-arrays of array *E*.
(D) the sum of all the elements in the array *E*.

3. The output of the following C program is _______ **[2015]**

```
void f1 (int a, int b)  {
      int c;
      c=a; a=b; b=c;
}
void f2(int *a, int *b)  {
      int c;
      c=*a; *a=*b; *b=c;
}
int main ( ) {
      int a=4, b=5, c=6;
      f1 (a, b);
      f2 (&b, &c);
      printf("%d", c-a-b);
}
```

4. What is the output of the following C code? Assume that the address of *x* is 2000 (in decimal) and an integer requires four bytes of memory. **[2015]**

```
int main ( ) {
  unsigned int x[4] [3] =
  { {1, 2, 3}, {4, 5, 6}, {7, 8, 9},
  {10, 11, 12}};
  printf ("%u, %u, %u", x + 3, *(x +
  3), *(x + 2) + 3);
}
```

(A) 2036, 2036, 2036 (B) 2012, 4, 2204
(C) 2036, 10, 10 (D) 2012, 4, 6

5. Consider the following function written in the C programming language. **[2015]**

```
void foo(char *a {
      if ( *a && *a != ' '){
            foo(a + 1);
            putchar(*a);
      }
}
```

The output of the above function on input "ABCD EFGH" is

(A) ABCD EFGH (B) ABCD
(C) HGFE DCBA (D) DCBA

6. Consider the following C program segment. **[2015]**

```
#include <stdio.h>
int main()
{
   char s1[7] = "1234", *p;
```

```
    p = s1 + 2;
    *p = '0';
    printf("%s", s1);
}
```

What will be printed by the program?

(A) 12 (B) 120400
(C) 1204 (D) 1034

7. Consider the following C program **[2015]**

```
    #include<stdio.h>
    int main ( )
    {
    static int a[ ] = {10, 20, 30, 40, 50};
    static int *p[ ] = {a, a+3, a+4, a+1, a+2};
    int **ptr = p;
    ptr++;
    printf("%d%d", ptr-p, **ptr);
    }
```

8. Consider the following C program. **[2016]**

```
void f (int, short);
void main( )
{
int i = 100;
short s = 12;
short *p = &s;
_____; // call to f( )
}
```

Which one of the following expressions, when placed in the blank above, will **NOT** result in a type checking error?

(A) $f(s,*s)$ (B) $i = f(i,s)$
(C) $f(i,*s)$ (D) $f(i,*p)$

9. Consider the following *C* program. **[2016]**

```
# include<stdio.h>
void mystery (int *ptra, int *ptrb) {
int *temp;
temp = ptrb;
ptrb = ptra;
ptra = temp;

}
int main ( ) {
int a = 2016, b = 0, c = 4, d = 42;
mystery (&a, &b);
if (a < c)
    mystery(&c, &a);
mystery (&a, &d);
printf("%d\n", a)
}
```

The output of the program is _____.

10. The following function computes the maximum value contained in an integer array p [ ] of size n ($n >= 1$). **[2016]**

```
int max (int *p, int n) {
    int a = 0, b = n – 1;
    while (_____) {
        if (p [a] < = p [b]) {a = a+1;}
        else                 { b = b – 1;}
    }
    return p[a];
}
```

The missing loop condition is

(A) $a\ != n$
(B) $b\ != 0$
(C) $b > (a + 1)$
(D) $b\ != a$

11. The value printed by the following program is ___. **[2016]**

```
void f (int* p, int m) {
m = m +5;
*p = *p + m;
return;
}
void main () {
int i = 5, j = 10;
f(&i, j);
print f ("%d", i +j);
}
```

12. Consider the following program: **[2016]**

```
int f (int *p, int n)
{   if (n < = 1) return 0;
else return max (f (p +1, n – 1), p [0] – p [1] );
}
int main ()
{
int a[ ] = {3,5,2,6,4};
printf ("%d", f(a,5));
}
```

Note: *max (x,y) returns the maximum of x and y.*

The value printed by this program is ______

13. Consider the following C code:

```
# include <stdio.h>
int *assignval (int *x, int val) {
     *x = val;
     return x;
}
void main ( )  {
    int *x = malloc (sizeof (int));
    if (NULL == x) return;
     x = assignval (x, 0);
     if (x)  {
          x  =  (int  *)  malloc
          (sizeof (int));
          if (NULL == x) return;
          x = assignval (x, 10);
     }
    printf("%d\n", *x);
    free (x);
}
```

The code suffers from which one of the following problems: **[2017]**

(A) compiler error as the return of malloc is not typecast appropriately

(B) compiler error because the comparison should be made as x == NULL and not as shown

(C) compiles successfully but execution may result in dangling pointer

(D) compiles successfully but execution may result in memory leak

14. Consider the following C program.

```
# include <<stdio.h>
# include <<string.h>
void printlength (char *s, char *t)
{
   unsigned int c = 0;
   int  len  =  ((strlen(s)  -  strlen
   (t))  > c)  ? strlen  (s)  : strlen
   (t);
   printf ("%d\n", len);
}
void main ( ) {
   char *x = "abc";
   char *y = "defgh";
   printlength (x, y);
}
```

Recall that strlen is defined in string.h as returning a value of type size_t, which is an unsigned int. the output of the program is _________. **[2017]**

15. Given the following binary number in 32-bit (single precision) IEEE-754 format:

00111110011011010000000000000000

The decimal value closest to this floating-point number is **[2017]**

(A) 1.45×10^{1} (B) 1.45×10^{-1}

(C) 2.27×10^{-1} (D) 2.27×10^{1}

16. Match the following:

(P) static char var;	(i) Sequence of memory locations to store addresses
(Q) m = malloc (10); m = NULL;	(ii) A variable located in data section of memory
(R) char *ptr [10];	(iii) Request to allocate a CPU register to store data
(S) register int var1;	(iv) A lost memory which cannot be freed

[2017]

(A) P → (ii), Q → (iv), R → (i), S → (iii)

(B) P → (ii), Q → (i), R → (iv), S → (iii)

(C) P → (ii), Q → (iv), R → (iii), S → (i)

(D) P → (iii), Q → (iv), R → (i), S → (ii)

17. Consider the following function implemented in C:

```
void printxy (int x, int y) {
     int *ptr;
     x = 0;
     ptr = &x;
     y = *ptr;
     *ptr = 1;
     printf ("%d, %d" x, y);
}
```

The output of invoking printxy (1, 1) is **[2017]**

(A) 0, 0 (B) 0, 1

(C) 1, 0 (D) 1, 1

18. Consider the following snippet of a C program. Assume that swap (&x, &y) exchanges the contents of x and y.

```
int main ()  {
       int array[] = {3, 5, 1, 4, 6, 2};
       int done = 0;
       int i;
while (done == 0)  {
    done = 1;
    for (i=0; i <=4; i++)  {
        if (array[i] < array[i+1]) {
            swap(&array[i], &array[i + 1]) ;
            done = 0;
        }
    }
    for (i=5; i >=1; i--)  {
          if (array[i] > array[i-1]) {
               swap(&array[i], &array[i-1]);
               done = 0;
          }
     }
  }
```

```
    printf{"%d", array[3]);
}
```

The output of the program is ________. **[2017]**

19. Consider the following C Program.

```
#include<stdio.h>
#include<string,h>
int main ()  {
            char* c = "GATECSIT2017";
            char* p = c;
            printf{"%d",
            (int) strlen(c+2[p]-6[p]-1)) ;
            return 0;
}
```

The output of the program is __________. **[2017]**

20. Consider the following C program.

```
#include<stdio.h>
struct Ournode {
    char x, y, z;
} ;
Int main () {
    struct Ournode p = {'1', '0', 'a'+2};
    struct Ournode *q = &p;
    printf ("%c, %c", *( (char*)q+1),
                       * ( (char*)q+2) );
    return 0;
}
```

The output of this program is: **[2018]**

(A) `0, c`
(B) `0, a+2`
(C) `'0', 'a+2'`
(D) `'0', 'c'`

21. Consider the following C program:

```
#include<stdio.h>
void fun1 (char *s1, char * s2) {
char *tmp;
tmp = s1;
s1 = s2
s2 = tmp;
}
void fun2 (char **s1, char **s2) {
char *tmp;
tmp = *s1;
*s1 = *s2;
*s2 = tmp;
}
int main () {
char *str1 = "Hi", *str2 = "Bye";
fun1 (str1, str2);
printf ("%s %s ", str1, str2);
fun2 (&str1, &str2);
printf ("%s %s", str1, str2);
return 0;
}
```

The output of the program above is: **[2018]**

(A) Hi Bye Bye Hi
(B) Hi Bye Hi Bye
(C) Bye Hi Hi Bye
(D) Bye Hi Bye Hi

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** C	**3.** A	**4.** A	**5.** D	**6.** C	**7.** A	**8.** C	**9.** D	**10.** C
11. B	**12.** D	**13.** B	**14.** A	**15.** D					

Practice Problems 2

1. B	**2.** B	**3.** A	**4.** D	**5.** B	**6.** D	**7.** D	**8.** C	**9.** B	**10.** C
11. A									

Previous Years' Questions

1. D	**2.** A	**3.** –5	**4.** A	**5.** D	**6.** C	**7.** 140	**8.** D	**9.** 2016	**10.** D
11. 30	**12.** 3	**13.** D	**14.** 3	**15.** C	**16.** A	**17.** C	**18.** 3	**19.** 2	**20.** A
21. A									

Chapter 4

Linked Lists, Stacks and Queues

LEARNING OBJECTIVES

- *Data structure*
- *Linked list*
- *Single-linked list*
- *Double-linked list*
- *Circular linked list*
- *Double circular-linked list*
- *Stack*
- *Queue*
- *Double-ended queue*
- *Circular queue*
- *Priority queue*
- *Array implementation*
- *Linked list implementation*
- *Linked list implementation of priority queue*

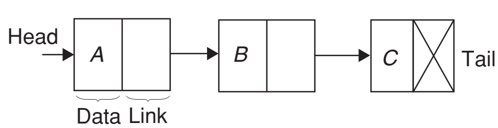
Data Structure

Data structure represents the logical arrangement of data in computer memory for easily accessing and maintenance.

Linked List

A linked list is a data structure that consists of a sequence of nodes, each of which contains data field and a reference (i.e., link) to next node in sequence.

- Generally node of linked list is represented as self-referential structure.
- The linked list elements are accessed with special pointer(s) called head and tail.



- The principal benefit of a linked list over a conventional array is that the list elements can easily be added or removed without reallocation or reorganization of the entire structure because the data items need not be stored contiguously in memory or on disk.
- Linked lists allow insertion and removal of nodes at any point in the list.
- Finding a node that contains a given data, or locating the place where a new node should be inserted may require scanning most or all of the list elements.
- The list element does not have to occupy contiguous memory.
- Adding, insertion or deletion of list elements can be accomplished with the minimal disruption of neighbouring nodes.

Single-linked List

List in which each node contains only one link field.

Node structure

```
struct
{
int ele;
struct node * next;
};
```

typedef struct node Node;

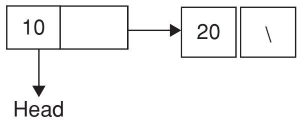
Creating a linked list with two nodes of type list node

Creating a linked list with 2 nodes
struct node

```
{
Int ele;
struct node * next ;
};
typedef struct node Node ;
Node * ptr1, * ptr2;
ptr1 = getnode ();
ptr2 = getnode ();
if((ptr1) && (ptr2))
{
Printf("No memory");
exit(1);
}
Ptr1 → ele = 10;
```

```
Ptr1 → next = ptr2;
Ptr2 → ele = 20;
Ptr2 → next = NULL;
Head = ptr1;
```

the linked list appears as below



Operations on SLL (single-linked list)

- Insert at Head
- Insert at Tail
- Insert in Middle
- Delete Head
- Delete Tail
- Delete Middle
- Search
- Display

Declare two special pointers called head and tail as follows:
Node *Head, *Tail;
Head = Tail = NULL;

Head or tail is NULL represents list is empty.
Steps for Insertion:

1. Allocate memory
2. Read data
3. Adjust references

Insert head element

```
1. void ins _ Head (int x)
2. {
3. Node *temp;
4. temp = (Node *) malloc(sizeof (Node));
5. temp → ele = x;
6. temp → next = Head;
7. Head = temp;
8. if (Tail = = NULL)
9. Tail = Head;
10 }
```

- Step 4 allocates memory
- Step 5 read data
- Steps from 6 to 9 adjust reference
- 'if' condition represents first insertion

Insert tail element

```
1. void ins_tail (int x)
2. {
3. Node *temp;
4. temp = (Node *) malloc (sizeof (Node));
5. temp → ele = x;
6. temp → next = NULL;
7. Tail = temp;
8. if (Head = = NULL)
9. Head = Tail;
10. }
```

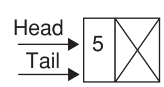
- Step 4 allocates memory
- Step 5 read data
- Steps from 6 to 9 adjust reference
- 'if' condition represents first insertion

Insert in middle/random position of list

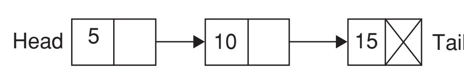
```
1. void ins _ mid (int n, int pos)
2. {
   int i = 1;
3. Node  *  temp,  N,  P;  //N,P  represent
   previous //& next nodes
4. if (Head == NULL)
5. {
6. ins _ head(n);
7. return;
8. }
9. temp = (Node *) malloc(sizeof(Node));
10. temp → ele = n;
11. P = head;
12. while (i < pos -1)
13. {
   P = P → next;
   i++;
   }
14. N = P → next;
15. temp → next = N;
16. P → next = temp;
17. }
```

- step 4 checks, whether the insertion is into an empty list.
- If list is empty, invokes ins–head( ) function.
- If list is not empty, then step 9 allocates memory.
- Step 10 reads data.
- Steps from 11 to 14 make the reference to the previous and next nodes of new node to be inserted.
- Steps 15 and 16 create the reference to new node from previous node and from new node to next node.

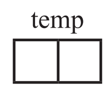
Example 1: Head = Tail = NULL
$n = 5$, P = NULL;
Here the list is empty. So,



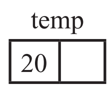
Example 2:



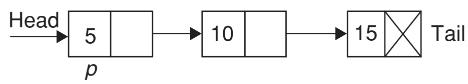
Insert element (n) 20 at position(pos) 3.
In current list, element 5 is the first element, 10 is the second and 15 is the third element.
To insert an element at pos = 3, the new node has to be placed between elements 10 and 15.
Condition in step 4 is false so step 9 executes and allocates memory.



On completion of step 10 –



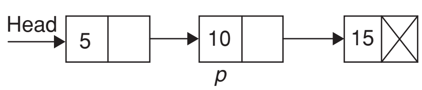
Step 11



Step 12, 13

```
While (i < pos – 1)
{
P = P → next;
i++;
}
i < pos
1 < 2
```

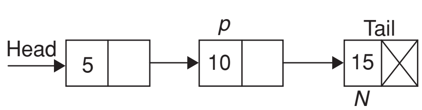
Condition true, so



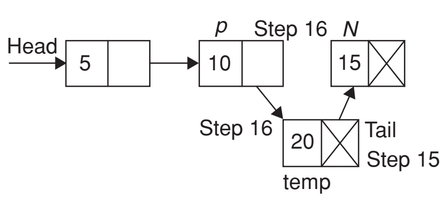
i becomes 2,

2 < 2 // condition false

Step 14 makes a reference to next of previous element.



Steps 15 and 16 execute as follows:



Now the element 20 becomes the 3rd element in the list.

Deletion

- Identify the node
- Adjust the links, such that deallocation of that node does not make the list as unconnected components.
- Return/display element to delete.
- Deallocate memory.

Delete head element

```
1. void del _ head()
2. {
3. int x;
   Node * temp;
4. if (Head = = NULL)
5. {
6. printf("List empty");
7. return;
8. }
9. x = Head → ele;
10. temp = Head;
11. if (Head = = Tail)
12. Head = Tail = NULL;
13. else
14. Head = Head → next;
15. printf ("Deleted element "%d", x);
16. free(temp);
17. }
```

Step 4	–	Checks for list empty
Step 9	–	Reads element to delete
Step 10	–	Head referred by temp pointer
Step 11	–	Checks for last deletion
Step 14	–	Moves the head pointer to next element in the list
Step 15	–	Displays element to delete
Step 16	–	Deallocates memory

Delete tail element

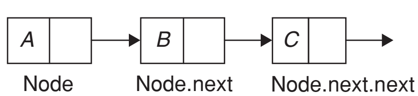
```
1. void del _ tail()
2. {
3. int x;
4. Node * temp;
5. if (Head = = NULL)
6. {
7. printf("\n list empty")
8. return ;
9. }
10. temp = Head;
11. while(temp → next ! = Tail)
12. temp = temp → next;
13. x = Tail → ele;
14. Tail = temp;
15. temp = temp → next;
16. Tail → next = NULL;
17. printf("\n Deleted element : %d", x)
18. free (temp);
19. }
```

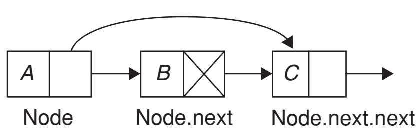
Step 4	– Checks for list empty
Step 10, 11, 12	– Move the temp pointer to last but one node of the list
Step 13	– Reads tail element to delete
Step 14	– Moves tail pointer to last but one node
Step 15	– Moves the temp pointer to last node of the list
Step 16	– Removes the reference from tail node to temp node, i.e., tail node becomes the last element
Step 17	– Displays elements to delete
Step 18	– Deallocate memory

Delete middle element

```
1. void del _ mid (int pos)
2. {
3. int i = 1, x;
4. Node * temp P, N;
5. if(Head = = NULL)
6. {
7. printf ("\n list empty")
8. return;
9. }
10. P = head;
11. while (i < pos -1)
12. {
   P = P → next;
   i++ ;
   }
13. temp = P → next;
14. N = temp → next;
15. P → next = N;
16. x = temp → ele;
17. printf("\n Element to Delete %d", x);
18. free(temp);
19. }
```

Step 5 – Checks for empty list
Step 10, 11, 12 – Move previous pointer *P* to previous node of node to delete.
Step 13 – Temp points to node to delete
Step 14 – N points to temp next
Step 15 – Creates link from *P* to *N*
Steps 16, 17, 18 – Read and display elements to delete and deallocate memory.





Linked list using dynamic variables

Node in the linked list contains data part that is ele and link part which points to the next node, and some other external pointer will be pointing to this as these take some storage, a programmer when creating a list, should check with the available storage. For this we make use of get node ()

Function which is defined as follows:

```
struct node
{
int ele
struct node * next ;
};
typedef struct node Node;
Node getnode ()
{
Node ptr;
ptr = (Node *) malloc (size of (struct node)):
return (ptr);
}
```

If ptr returns NULL, then it is underflow (there is no available memory) otherwise, it returns start address of memory location.

Search an element

```
1. void search (int x)
2. {
3. Node * temp = head;
4. int  c = 1;
5. while (temp! = NULL)
6. {
7. if (temp → ele = = x)
8. {
9. printf("\n Element found at % d", c);
10. break;
11. }
12. c++;
13. }
14. if (temp = = NULL)
15. printf("\n search unsuccessful");
16. }
```

Step 7 – Checks temp data with search element. Repeats this step until the element is found or reaches the last node
Step 9 – Displays the position of search element in the list, if found
Step 14, 15 – Represents search element not exists in list

Display

```
1. void display ( )
2. {
3. Node *temp = Head;
4. printf("\n list elements: ");
5. while (temp ! = NULL)
6. {
7. printf("%d", temp → ele);
8. temp = temp → next;
9. }
10. }
```

Step 7 – Displays temp data
Step 8 – Moves temp pointer to next node

Algorithm to reverse direction of all links of singly liked list

Consider a linked list '*L*' with head as pointer pointing to the first node contains data element 'ele' and a pointer called 'next' which points to the next node.

Reverse is the routine which will reverse the list, there are three node pointers *P*, *Q*, *R* with *P* pointing to the first node, *Q* pointing to NULL.

```
1. START
2. if (P = NULL)
   1. print ("List is null");
   2. Exit
3. While (P)
4. R = Q;
5. Q = P;
6. P = P → next;
7. Q → next = R
8. End While
9. Head = Q;
10. STOP
```

Double-linked List (DLL)

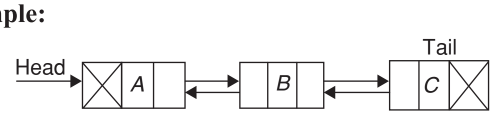
Double-linked list is a linked list in which, each node contains data part and two link fields.

Node structure:

```
struct Dnode
{
struct Dnode *prev;
int ele;
struct Dnode *next;
};
```

- prev – points to previous node in list
- next – points to next node in list
- The operations which can be performed in SLL can also be preformed on DLL.
- The major difference is that we have to adjust double reference as compared to SLL.
- We can traverse or display the list elements in forward as well as in reverse direction.

Example:



Circular-linked List (CLL)

Circular-linked list is completely same as SLL, except, in CLL the last (Tail) node points to first (Head) node of list.

So, the Insertion and Deletion operation at Head and Tail are little different from SLL.

Double Circular-linked List (DCL)

Double circular-linked list can be traversed in both directions again and again. DCL is very similar to DLL, except the last node's next pointer points to first node of list and first node's previous pointer points to last node of list.

So, the insertion and deletion operations at head and tail in DCL are little different in adjusting the reference as compared to DLL.

Storing ordered table as linked list: The table is stored as a linked list, it is retrieved and stored with two pointers, one pointer will point to node holding a record having the smallest key and other pointer performs the search.

Stack

A stack is a last in first out (LIFO) abstract data type and data structure. A stack can have any abstract data type as an element, but is characterized by only two fundamental operations.

√ PUSH
√ POP

- The PUSH operation adds an item to the top of the stack, hiding any items already on the stack or initializing the stack if it is empty.
- The POP operation removes an item from the top of the stack, and returns the poped value to the caller.
- Elements are removed from the stack in the reverse order to the order of their insertion. Therefore, the lower elements are those that have been on the stack for longest period.

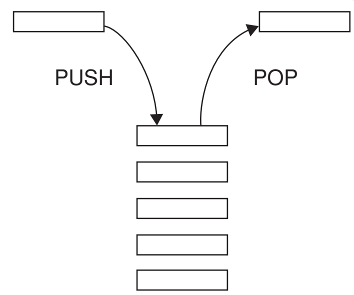


Figure 1 Simple representation of a stack

Implementation

A stack can be easily implemented either through an array or a linked list. The user is only allowed to POP or PUSH items onto the array (or) linked list.

1. **Array Implementation:** Array implementation aims to create an array where the first element inserted is placed st[0] which will be deleted last.

   The program must keep track of position top (last) element of stack.

   Operations

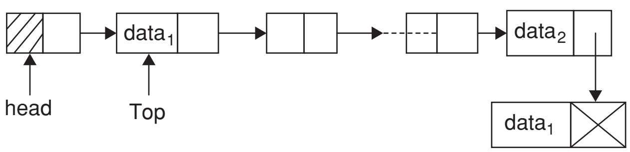
   Initially Top = –1;//represents stack empty

```
(i) Push (S, N, TOP, x)
    {
    if (TOP = = N - 1)
    printf("overflow");
    else
    TOP = TOP + 1;
    S[TOP] = x;
    }
(ii) POP (S, N, TOP, x)
    {
    if (TOP = = -1)
    printf("underflow");
    else
    x = S[TOP]
    TOP = TOP - 1
    return x;
    }
```

2. **Dynamic Implementation:** The Array implementation is also called static implementation, because the stack size is fixed.
   The stack implementation using linked list is called dynamic implementation, because the stack size can grow and shrink as the elements added or removed from the stack.
   - The PUSH operation on stack is same as insert head in SLL.
   - The POP operation is same as delete head in SLL.

Algorithm to add and delete to a link stack and link queue

Link stack:



The linked stack with head and top pointers is shown above

The algorithm to push the elements into stack is given below, the method push (item)

Steps:

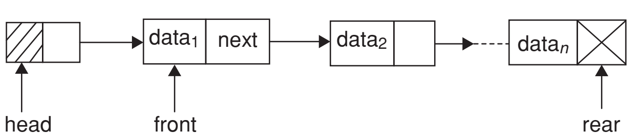
```
1. ptr = getnode (Node)
2. ptr.data = item
3. ptr.next = Top
4. Top = new
5. Head.next = Top
6. Stop.
```

for deletion of elements from stack, its algorithm is pop(), it is given below

Steps:

```
1. if (Top = NULL)
   1. print "stack is empty"
   2. exit
2. Else
   1. ptr = Top.next
   2. item = Top.data
   3. Head.next = ptr
   4. Top = ptr
3. End if
4. Stop.
```

Linked queue representation



The linked queue with head, front and rear point is shown above.

The algorithm to enqueue the elements into queue is given below, the method enqueue (item)

Steps:

```
1. ptr = getNode (Node)
2. ptr.data = item
3. ptr.next = NULL
4. if (front = NULL)
      front = ptr
      else
      rear.next = ptr;
5. end if
6. rear = ptr
7. Stop
```

For deletion of elements from queue that is ptr dequeue () is given below

Steps:

```
1. if (front = NULL)
   1. print "underflow".
   2. exit
2. ptr = front;
3. front = ptr.next
4. Head.next = front
5. item = ptr.data
6. free(ptr)
7. end.
```

Uses of Stack

- Function calls: When a function is called all local storage for the function is allocated on system 'stack', and return address also pushed on to system stack.
- Recursion stacks can be used to implement recursion if the programming language does not provide recursion facility.
- Reversing a list
- Parsing: Stacks are used by compilers to check the syntax of program.
- For evaluating expressions.

Expression Notations

Infix expression: Here binary operator comes between the operands.

Postfix expression: Here the binary operator comes after both the operands.

Example: *ab+*

Prefix expression: Here the binary operator comes before both the operands.

Example: *+ab*

Infix to postfix conversion

- If operand, output to postfix expression
- If operator, push it onto stack
- In case of parenthesis, when an opening parenthesis is read, it is pushed onto stack and when a closing parenthesis is read, all operators up to the first opening parenthesis must be popped from the stack into the post fix notation.

Example: $(A + (B - C))*D$

i/p	Postfix notation	Stack
(		(
A	*A*	(
+	*A*	(+
(	*A*	(+(
B	*AB*	(+(
–	*AB*	(+(–
C	*ABC*	(+(–
)	*ABC*–	(+
)	*ABC*–+	–
*	*ABC*–+	*
D	*ABC*–+*D*	*
	ABC–+*D**	

Evaluation of postfix expression

We use operand stack for evaluation. Scan the post fix expression,

- When an operand encounters while scanning, push on to stack.
- While scanning post fix expression, if operator found then
  - Pop top two operands from stack
  - Perform the operation on those two operands
  - Push, result on to stack top
- Finally, the stack contains only one value, which represents result of the expression.

Example: 6 2 3 + – 3 8 2 / + 2 3 +

Symbol	OP1	OP2	Value	Operand stack
6				6
2				6, 2
3				6, 2, 3
+	2	3	5	6,5
–	6	5		1
3				1, 3
8				1, 3, 8
2				1, 3, 8, 2
/	8	2	4	1, 3, 4
+	3	4	7	1, 7
*	1	7	7	7
2				7, 2
*	7	2	14	14
3				14, 3
+	14	3		17

Result is 17.

Performing add, delete operations on stack (multiple stack)

Let us consider an array whose size is 'max'
with multiple stack *A*, *B* having top *A* and top *B*, push and pop operations on one stack *A* is given below.

Algorithm for push A(x)

Initially *A*[Max], top *A* = –1, top *B* = MAX;

```
1. if (top A = top B)
   a. print " overflow"
   b. exit
2. top A = top A + 1
3. A[top A] = x
4. stop
```

Algorithm for pop A(x)

```
1. if (top A = - 1)
   a. print "underflow"
   b. exit
2. y = A[top A]
3. top A = top A - 1
4. return y
5. stop
```

Algorithm for push B(x)

```
1. if (top B - 1 = top A)
   a. print "overflow"
   b. exit
2. top B = top B -1
3. A[top B] = x
4. stop
```

Algorithm for pop B(x)

```
1. if (top B = max)
   a. print "underflow"
   b. exit
2. y = A [top B]
3. top B = top B - 1
4. return y
5. stop
```

QUEUE

A queue is an ordered collection of items from which items may be deleted at one end (called that front of queue) and into which items may be inserted at the other end (called rear of queue).

Queue is a linear data structure maintains the data in first in–first out (FIFO) order.

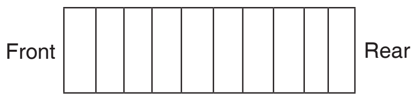
Implementation

Queue can be implemented in the following ways:

1. Array static implementation: queue cannot be extended beyond the array size.
2. Linked list dynamic implementation: Queue size increases as the elements added/inserted to queue. Queue shrinks when an element deleted from queue.

Array Implementation

const int SIZE = 10;
int *q*[SIZE];
int $f = -1$, $r = -1$; //$f = r = -1$ represents queue empty



Insertion

```
1. void insert (int x)
2. {
3. if (r = = SIZE -1)
4. {
5. printf("Q FULL")
6. return;
7. }
8. r++;
9. q[r] = x;
10. if (f = = -1)
11. f = r;
12. }
```

Step 3 – Checks for queue full
Step 8 – Increments rear (*r*)
Step 9 – Inserts '*x*' into queue
Step 10 – Checks whether insertion is first
Step 11 – If first insertion, updates front (*f*)

Deletion

```
1. void deletion()
2. {
3. int x;
4. if (f = = -1)
5. {
6. if ("\n Q Empty");
7. return;
8. }
9. x = q[f];
10. if (f = = r)
11. f = r = -1;
12. else
13. f++;
14. printf("\n deleted element %d", x);
15. }
```

Step 4 – Checks for queue empty
Step 9 – Deletes '*q*' front element
Step 10 – Checks whether queue having only one element
Step 11 – Rear and front initializes to –1, if queue is having only one element
Step 13 – Queue front points to next element
Step 14 – Deleted element is printed

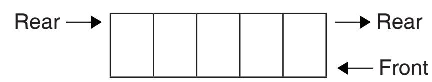
Display

```
1. void display( )
2. {
3. int i = f;
4. if (f = = -1)
5. {
6. printf("Queue Empty");
7. return;
8. }
9. printf ("\n Queue Elemetns");
10. for(; i < = r; i++)
11. printf(" %d", q[i]);
12. }
```

Step 4 – Checks for '*q*' empty
Step 10 and 11 – Display '*q*' elements

Double-ended Queue

A double-ended queue (deque) is an abstract data structure that implements a queue for which elements can only be added to or removed from the front (head) (or) rear (tail) end.



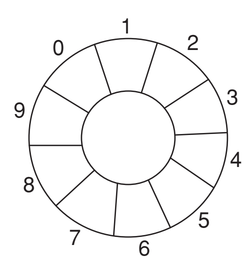
Insertions and deletions are possible at both ends.

Linked List Implementation Double-ended Queue

- Insert – Front is same as insert – Head
- Insert – Rear is same as insert – Tail
- Delete front is same as delete – Head
- Delete – Rear is same as delete – Tail

Circular Queue

As the items from a queue get deleted, the space for that item is reclaimed. Those queue positions continue to be empty. This problem is solved by circular queues. Instead of using a linear approach, a circular queue takes a circular approach; this is why a circular queue does not have a beginning or end.



The advantage of using circular queue over linear queue is efficient usage of memory.

Algorithm to implement addition and deletion from circular queue

Circular Queue Insertion:
To add an element '*X*' to a Queue '*Q*' of size '*N*' with front and rear pointers as '*F*' and '*R*' is done with insert (*X*), Initially $F = R = 0$.
Insert (*X*)

Steps:

```
1. if (((R = N ) & & (F = 1)) or ((R +
   1) = F))
   a. print "overflow"
   b. exit
2. if (R = N)
   then R = 0;
   Else
      R = R + 1;
3. Q[R] = x;
4. if (F = 0)
   F = 1
5. Stop.
```

To delete an element we implement an algorithm delete (). '*y*' contains the deleted element.

delete()

Steps:

```
1. if (F = 0)
   a. print "underflow"
   b. exit
2. y = Q[F]
3. if (F = R)
   F = R = 0
   else
   If (F = N)
   F = 1
   Else
   F = F + 1
4. Return y
5. Stop.
```

Priority Queue

In priority queue, the intrinsic ordering of elements does determine the results of its basic operations.

There are two types of priority queues.

- Ascending priority queue is a collection of items in which items can be inserted arbitrarily and from which only the smallest items can be removed.
- Descending priority queue is similar but allows deletion of the largest item.

Array Implementation

- The insertion operation on priority queue selects the position to the element to insert.
- Makes the position empty/free by moving the existing element (if required).
- Place the element in required position.
- Deletion operation simply deletes front of queue.

Linked-list Implementation

- Insertion operation create a node
- Reads element into node
- Find out the location
- Insert the node into list, by adjusting the reference
- Deletion operation simply deletes head elements, making the head next as head element

Linked-list Implementation of Priority Queue

- Insertion in queue is same as insert-tail of queue
- Deletion from queue is same as delete head

Exercises

Practice Problems I

Directions for questions 1 to 16: Select the correct alternative from the given choices.

1. If the array representation of a circular queue contains only one element then

(A) front = rear (B) front = rear + 1
(C) front = rear − 1 (D) front = rear = NULL

2. The five items *P*, *Q*, *R*, *S* and *T* are pushed in a stack, one after another starting from *P*. The stack is popped four times, and each element is inserted in a queue. The two elements are deleted from the queue and pushed back on the stack. Now one item is popped from the stack. The popped item is ______.

(A) *P* (B) *Q*
(C) *R* (D) *S*

3. What are the contents of the stack (initially the stack is empty) after the following operations?

PUSH (A)
PUSH (B)
PUSH (C)
POP
PUSH(D); POP; POP;
PUSH(E)
PUSH(F)
POP

(A) ABE (B) AE
(C) A (D) ABCE

4. Consider the below code, which deletes a node from the beginning of a list:

```
void deletefront()
{
if(head == NULL)
return;
else
{
..........
.........
.........
}
}
```

Which lines will correctly implement else part of above code?

(A) `if (head → next = = NULL)`
    `head = head → next;`
(B) `if (head = = tail)`
    `head = tail = NULL;`
    `else`
    `head = head → next;`
(C) `if (head = = tail = = NULL)`
    `head = head → next;`
(D) `head = head → next;`

5. When a new element is inserted in the middle of linked list, then the references of _____ to be adjusted/updated.
(A) those nodes that appear after the new node
(B) those nodes that appear before the new node
(C) head and tail nodes
(D) those nodes that appear just before and after the new node

6. The following C function takes double-linked list as an argument. It modifies the list by moving the head (first) element to tail of the list.

```
typedef struct node
{
    struct node *p;
    int data;
    struct node *n;
} Node;
Node * Move - to - last (Node *head)
{
Node * temp, * prev, * next;
if (head = = NULL)||(head → n = = NULL))
return head;
temp = head;
prev = head;
head = head → n;
while (prev → n! = NULL)
{
X;
}
Y;
return head;
}
```

(A)
```
X: prev = prev → n;
Y: prev → n = temp;
temp → p = prev;
temp → n = NULL;
head → P = NULL;
```
(B)
```
X: next = prev → n;
Y: prev → n = temp;
temp → p = prev;
```
(C)
```
X: prev = prev → n;
Y: prev → n = temp;
temp → n = NULL;
head → p = NULL;
```
(D)
```
X: next = prev → n;
prev = prev → n;
Y: prev → n = Next;
next → n = head;
temp → n = NULL;
```

7. Which of the following program segment correctly inserts an element at the front of the linked list. Assume that Node represents linked list node structure, value is the element to be inserted.

(A)
```
temp = (Node *)malloc (sizeof (Node));
temp → data = value;
temp → next = head;
head = temp;
```
(B)
```
temp = (Node *)malloc(sizeof (Node*)
);
temp → data = value;
temp → next = head;
head = temp;
```
(C)
```
temp = (Node *)malloc (sizeof (Node));
head = temp;
temp → next = head;
temp → data = value;
```
(D)
```
temp = (Node *)malloc (sizeof (Node
*);
temp → data = value;
head = temp;
temp → next = head;
```

8. Consider the following program segment:

```
struct element
{
    int x;
    struct element *link;
}
void shuffle(struct element *head)
{
struct *p, *q;
int t;
if (!head || !head → link) return;
p= head ; q = head → link;
while(q)
{
t = p → x;
p→ x = q → x;
q → x = t;
p = q → link;
q = p?  p : 0;
}
}
```

The function called with list containing 10, 15, 20, 25, 30, 35, 40 in given order. What will the order of elements of the list, after executing the function shuffle?

(A)	10	15	20	25	30	35	40
(B)	40	35	30	25	20	15	10
(C)	20	15	10	25	40	35	30
(D)	15	10	25	20	35	30	40

9. Primary ADT's are
(A) Linked list only
(B) Stack only
(C) Queue only
(D) All of these

10. Linked list uses NULL pointers to signal
(A) end of list (B) start of list
(C) Either (A) or (B) (D) Neither (A) nor (B)

11. Which of the following is essential for converting an infix to postfix form efficiently?
(A) Operator stack (B) Operand stack
(C) Both (A) and (B) (D) Parse tree

12. Stacks cannot be used to
(A) Evaluate postfix expression
(B) Implement recursion
(C) Convert infix to postfix
(D) Allocate resource like CPU by the operating system

13. Linked list can be sorted
(A) By swapping data only
(B) By swapping address only
(C) Both (A) and (B)
(D) None of these

14. Linked list are not suitable for implementing
(A) Insertion sort
(B) Binary search
(C) Radix sort
(D) Polynomial manipulation

15. Insertion of node in a double-linked list requires how many changes to previous (prev) and next pointers?
(A) No changes (B) 2 next and 2 prev
(C) 1 next and 1 prev (D) 3 next and 3 prev

16. Minimum number of stacks required to implement a queue is
(A) 1 (B) 2
(C) 3 (D) 4

Practice Problems 2

Directions for questions 1 to 11: Select the correct alternative from the given choices.

1. Stack is useful for implementing _____.
(A) radix sort
(B) breadth first search
(C) quick sort
(D) recursion

2. Which is true about linked list?
(A) A linked list is a dynamic data structure.
(B) A linked list is a static structure.
(C) A stack cannot be implemented by a linear linked list.
(D) None of the above

3. The process of accessing the data stored in a tape is similar to manipulating data on a _____.
(A) stack (B) list
(C) queue (D) heap

4. Which of the following is used to aid in evaluating a prefix expression?
(A) Queue (B) Heap
(C) Stack (D) Hash

5. Select the statement which best completes the sentence — 'Abstract data type is…'
(A) a data type which is abstract in nature
(B) a kind of data type
(C) data structure
(D) a mathematical model together with a set of operations defined on it

6. Which of the following data structures may give an overflow error, even through the current number of elements in it is less than its size?
(A) Simple queue (B) Circular queue
(C) Stack (D) None of these

7. In a circular linked list, insertion of a record involves the modification of _____.
(A) no pointer (B) four pointers
(C) two pointers (D) All of the above

8. Among the following, which one is not the right operation on a stack?
(A) Remove the item that is inserted latest into the stack.
(B) Add an item to the stack.
(C) Remove the first item that is inserted into the stack, without deleting other elements.
(D) None of the above

9. Among the following which one is not the right operation on dequeue?
(A) Inserting an element in the middle of a dequeue.
(B) Inserting an element at the front of a dequeue.
(C) Inserting an element at the rear of a dequeue.
(D) None of the above

10. A linear list in which elements can be added or removed at either end but not in the middle is _____.
(A) queue
(B) dequeue
(C) array
(D) tree

11. The post fix notation of $A/B * * C + D * E - A * C$ is
(A) $ABC * * /DE * + AC * -$
(B) $ABC * * D/E * + AC + -$
(C) $ABC * * /DE * AC + -$
(D) $ABC * * /DE * + AC + -$

Previous Years' Questions

1. An abstract data type (ADT) is **[2005]**
(A) same as an abstract class.
(B) a data type that cannot be instantiated.
(C) a data type for which only the operations defined on it can be used, but none else.
(D) All of the above

2. An implementation of a queue Q, using two stacks S_1 and S_2, is given below:

```
void insert (Q, x)   {
  push (S1, x);
}
void delete (Q)   {
  if (stack-empty (S2)) then
    if (stack-empty (S1)) then    {
      print ("Q is empty");
      return;
    }
    else while(!(stack-empty(S1)))
    {
        x = pop (S1);
        push(S2, x);
    }
  x = pop (S2);
}
```

Let n insert and m (″ n) delete operations be performed in an arbitrary order on an empty queue Q. Let x and y be the number of push and pop operations performed respectively in the process. Which one of the following is true for all m and n? **[2006]**
(A) $n + m$ ″ $x < 2n$ and $2m$ ″ y $n + m$
(B) $n + m$ ″ $x < 2n$ and $2m$ ″ y $2n$
(C) $2m$ ″ $x < 2n$ and $2m$ ″ y $n + m$
(D) $2m$ ″ $x < 2n$ and $2m$ ″ y $2n$

3. The following postfix expression with single digit operands is evaluated using a stack:
8 2 3 ∧ / 2 3 ∗ + 5 1 ∗ −
Note that ∧ is the exponentiation operator. The top two elements of the stack after the first ∗ is evaluated are: **[2007]**
(A) 6 and 1 (B) 5 and 7
(C) 3 and 2 (D) 1 and 5

4. The following C function takes a single-linked list of integers as a parameter and rearranges the elements of the list. The function is called with the list containing the integers 1, 2, 3, 4, 5, 6, 7 in the given order. What will be the contents of the list after the function completes execution?

```
struct node {
int value;
struct node *next;
};
void rearrange (struct node *list) {
struct node *p, *q;
int temp;
if (!list || !list -> next) return;
p = list; q = list -> next;
while (q) {
temp = p -> value; p -> value = q ->
value;
    q -> value = temp; p = q → next;
    q = p?p -> next : 0;
}
}
```

[2008]
(A) 1, 2, 3, 4, 5, 6, 7 (B) 2, 1, 4, 3, 6, 5, 7
(C) 1, 3, 2, 5, 4, 7, 6 (D) 2, 3, 4, 5, 6, 7, 1

5. Suppose a circular queue of capacity $(n - 1)$ elements is implemented with an array of n elements. Assume that the insertion and deletion operations are carried out using REAR and FRONT as array index variables, respectively. Initially, REAR = FRONT = 0. The conditions to detect queue full and queue empty are **[2012]**
(A) Full: (REAR + 1) mod n = = FRONT
Empty: REAR = = FRONT
(B) Full: (REAR + 1) mod n = = FRONT
Empty: (FRONT + 1) mod n = = REAR
(C) Full: REAR = = FRONT
Empty: (REAR + 1) mod n = = FRONT
(D) Full: (FRONT + 1) mod n = = REAR
Empty: REAR = = FRONT

6. Consider the C program below **[2015]**

```
#include <stdio.h>
int *A, stkTop;
int stkFunc (int opcode, int val)
{
  static int size=0, stkTop=0;
  switch (opcode) {
    case -1: size = val; break;
    case 0: if (stkTop < size)
  A[stkTop++] =
       val; break;
    default: if (stkTop) return A[-
    -stkTop];
  }
  return -1;
}
   int main ( )
   {
   int B[20]; A = B; stkTop = -1;
   stkFunc (-1, 10);
   stkFunc (0, 5);
   stkFunc (0, 10);
   printf ("%d\n", stkFunc(1, 0) +
   stkFunc(1, 0));
   }
```

The value printed by the above program is _____

7. The result of evaluating the postfix expression 10 5 + 60 6/* 8 – is **[2015]**
(A) 284 (B) 213
(C) 142 (D) 71

8. Let *Q* denote a queue containing sixteen numbers and *S* be an empty stack.

Head (*Q*) returns the element at the head of the queue **Q without** removing it from **Q**. Similarly Top(*S*) returns the element at the top of **S without** removing it from *S*.

Consider the algorithm given below.

```
while Q is not Empty do
        if S is Empty OR Top(S) ≤ Head (Q)
        then
                x : = Dequeue (Q)
                Push (S, x);
        else
                x : = Pop (S);
                enqueue (Q, x);
        end
end
```

The maximum possible number of iterations of the while loop in the algorithm is ____ . **[2016]**

9. The attributes of three arithmetic operators in some programming language are given below.

Operator	Precedence	Associativity	Arity
+	High	Left	Binary
–	Medium	Right	Binary
*	Low	Left	Binary

The value of the expression

2 – 5 + 1 – 7 * 3 in this language is ______. **[2016]**

10. A circular queue has been implemented using a singly linked list where each node consists of a value and a single pointer pointing to the next node. We maintain exactly two external pointers **FRONT** and **REAR** pointing to the front node and the rear node of the queue, respectively. Which of the following statements is/are CORRECT for such a circular queue, so that insertion and deletion operations can be performed in O (1) time?

I. Next pointer of front node points to the rear node.
II. Next pointer of rear node points to the front node.

[2017]

(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

Answer Keys

Exercises

Practice Problems 1

1. A **2.** C **3.** B **4.** B **5.** D **6.** A **7.** A **8.** D **9.** D **10.** A
11. A **12.** D **13.** C **14.** B **15.** B **16.** B

Practice Problems 2

1. D **2.** A **3.** C **4.** C **5.** D **6.** A **7.** C **8.** C **9.** A **10.** B
11. A

Previous Years' Questions

1. C **2.** A **3.** A **4.** B **5.** **6.** 15 **7.** C **8.** 256 **9.** 9 **10.** B

Chapter 5

Trees

LEARNING OBJECTIVES

- *Tree*
- *2-Tree*
- *Binary tree*
- *Properties of binary trees*
- *Complete binary tree*
- *Full binary tree*
- *Binary tree representation*
- *Linked representation*
- *Binary search tree*
- *Binary tree traversing methods*
- *AVL tree*
- *Binary heap*
- *Max-heap*
- *Min-heap*
- *Expression tree*

Tree

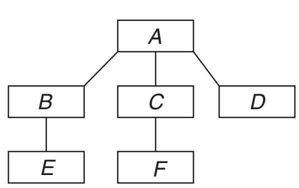
Tree is non-linear data structure designated at a special node called root and elements are arranged in levels without containing cycles.

(or)

The tree is

1. Rooted at one vertex
2. Contains no cycles
3. There is a sequence of edges from any vertex to any other
4. Any number of elements may connect to any node (including root)
5. A unique path traverses from root to any node of tree
6. Tree stores data in hierarchical manner
7. The elements are arranged in layers

Example:



- Root node is A.
- A's children are B, C and D.
- E, F and D are leaves.
- Nodes B, C are called as intermediate nodes.
- A is parent of B, C and D.
- B is parent of E and C is parent of F.
- Number of children of a node is called degree of node.

2-Tree

A tree in which every node contains either 0 or 2 children.

Binary Tree

It is a special type of tree where each node of tree contains either 0 or 1 or 2 children.

(or)

Binary Tree is either empty, or it consists of a root with two binary trees called left-sub tree and right sub-tree of root (left or right or both the sub trees may be empty).

Properties of binary tree

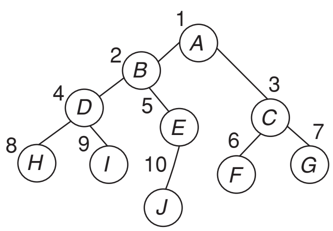
- Binary tree partitioned into three parts.
- First subset contains root of tree.
- Second subset is called left subtree.
- Another subset is called right subtree.
- Each subtree is a binary tree.
- Degree of any node is 0/1/2.
- The maximum number of nodes in a tree with height 'h' is $2^{h+1}-1$.
- The maximum number of nodes at level 'i' is 2^{i-1}.
- For any non-empty binary tree, the number of terminal nodes with n_2, nodes of degree 2 is $N_0 = n_2 + 1$
- The maximum number of nodes in a tree with depth d is $2^d - 1$.

Types of binary tree

Complete binary tree It is a binary tree, in which at every level, except possibly the last, is completely filled and all nodes at the last level are as left as possible.

Example:

Level	Height	Depth
1	3	1
2	2	2
3	1	3
4	0	4



For the given tree:

- Having 4 levels
- Height of the tree is 3
- Depth of the tree is 4
- The numbers at each node represents level order index.
- The level order Index, are assigned to nodes in the following manner
- Root of the tree is '1'
- For a node 'x', the LOI is (2 * LOI (parent)), if 'x' is left child of its parent.
- For a node 'y', the LOI (2 * LOI (Parent) +1), if 'y' is right child of its parent.

Now complete binary tree can be defined as a binary tree, which contains a sequence of numbers to its nodes as LOI's without any break in sequence.

Full binary tree It is a binary tree, for which all leaf nodes are at same level and all intermediate nodes contains exactly 2 children.

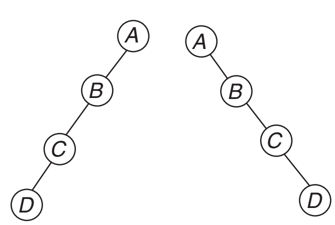
(or)

A tree with depth 'K' contains exactly $2^K - 1$ nodes.

Strictly binary tree A binary tree in which every node contains exactly 0 or 2 children.

Skewed binary tree A binary tree in which elements are added only in one direction.

Example:



Left-skewed Right-skewed

Application

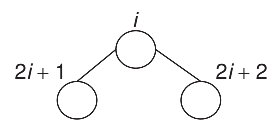
- A binary tree is useful data structure when two way decisions must be made at each point of process.

Binary tree representation

The binary trees can be represented in two ways.

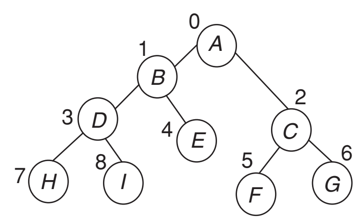
- Array
- Linked list

Array representation The elements of a binary tree are placed in an array using the level order index of each element.



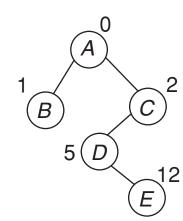
When LOI of Root is 0:

Example 1:



0	1	2	3	4	5	6	7	8
A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*

Example 2:

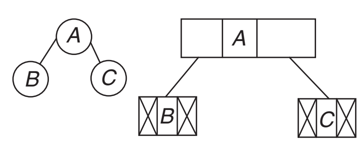


0	1	2	3	4	5	6	7	8	9	10	11	12
A	*B*	*C*			*D*							*E*

Linked representation Each node contains one data field and two link fields. Fist link point to the left child and another point to the right child.

In absence of any child, corresponding link field contains NULL.

Example:



Trade-off's between array and linked, representations

- Array representation is somewhat simpler. It must ensure elements are placed in array at proper position.
- Linked representation requires pointer to its left and right child.
- Array representation saves memory for almost complete binary trees.
- Linked representation allocates the number and nodes equal to the number of elements in tree.
- Array representation does not work efficiently for skewed binary trees.
- Array representation limits the size of binary tree to the array size.
- In linked representation, tree can be extended by adding an element dynamically and can be shrinked by deleting an element dynamically.

Binary search tree

It is a special type of binary tree that satisfies the following properties.

- All the elements of left sub tree of root are smaller than root.
- All the elements of right sub tree of root are greater than root.
- The above two properties satisfy for each subtree.

Example:

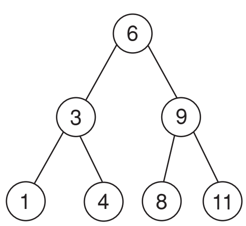


Figure 1 A data structure to encode binary search tree

The binary search tree node contains three fields, data field, left child, right child. Left child is a pointer which points to the predecessor of the node and right child is a pointer which points to the successor of the node.

A data structure to encode binary search tree is

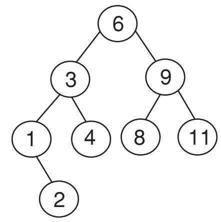
Left child	Data	Right child

The declaration is

```
Struct node
{
Struct node * left child;
Int data;
Struct node * Right child;
};
```

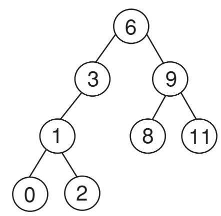
Insertion If a value to be inserted is smaller than the root, value, it must go in the left subtree, if larger it must go in the right subtree. This reasoning applies recursively until we reach a node where the required subtree does not exist and that is where we place the new value.

Example: It must go in 6's left subtree, 3's left subtree, 1's right subtree, 1 has no right subtree, so we make a singleton with 2 and it becomes 1's right subtree.

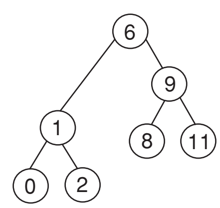


Deletion:

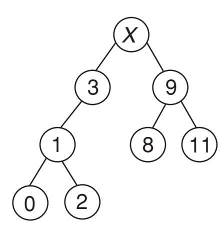
1. If a leaf node has to be deleted, just delete it and the rest of the tree is exactly as it was, so it is still a BST.
2. Suppose the node we are deleting has only one sub tree
   Example, In the following tree, '3' has only one sub-tree



To delete a node with 1 subtree, we just 'link past' the node, i.e., connect the parent of the node directly to the node's only subtree. This always works, whether the one subtree is on the left or on the right. Deleting 3 gives us.



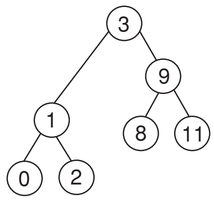
3. Deletion of node which has 2 subtrees
   Example: Delete 6.



Choose value '*X*'

1. Everything in the left subtree must be smaller than *X*.
2. Everything in the right subtree must be bigger than *X*.

We must choose X to be the largest value in the left subtree. In our example, 3 is the largest value in the left subtree. So we replace root node 6 with 3.



Note: We could do the same thing with the right subtree. Just use the smallest value in the right subtree.

Notes:

- The largest element in left subtree is the right most element.
- The smallest element in right subtree is the left most element.

Binary tree traversing methods

The binary tree contains 3 parts:

V – root

L – Left subtree

R – Right subtree

Pre-order: (*V*, *L*, *R*)

- Visit root of the tree first
- Traverse the left - subtree in pre-order
- Traverse the right - subtree in preorder

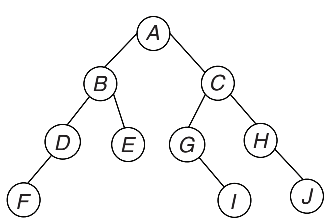
In-order: (*L*, *V*, *R*)

- Traverse the left – subtree in in-order
- Visit Root of the tree
- Traverse right - sub tree in in-order

Post-order: (*L*, *R*, *V*)

- Traverse the left subtree in post-order.
- Traverse the Right - subtree in post-order
- Visit root of the tree

Example 1:



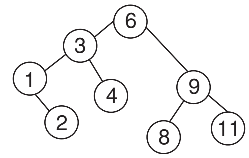
Pre-order: *A B D F E C G I H J*

In-order: *F D B E A G I C H J*

Post-order: *F D E B I G J H C A*

Pre-order, In-order and post-order uniquely identify the tree.

Example 2:



Pre-order:	6 3 1 2 4 9 8 11
In-order:	1 2 3 4 6 8 9 11
Post-order:	2 1 4 3 8 11 9 6

Points to remember

- Pre-order traversal contains root element as first element in traverse list.
- Post-order traversal contains root element as last in traversal list.
- For BST, in-order traversal is a sorted list.
- A unique binary tree can constructed if either pre-order or post-order traversal list provided with In order traversal list.
- If either pre-order or post-order only given then BST cannot be constructed.

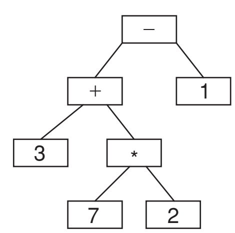
Applications

1. Binary trees can represent arithmetic expressions.
   - An infix expression will have a parent operator and two children operands.

Consider the expression $((3 + (7 * 2)) - 1)$

Each parenthesised expression becomes a tree.

Each operand is a leaf, each operator is an internal node.

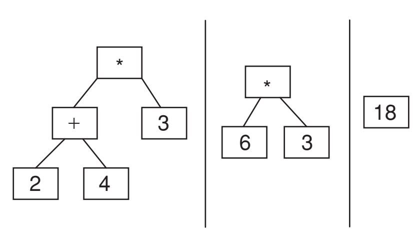


2. To evaluate the expression tree:

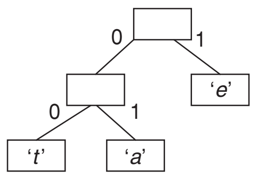
   Take any two leaves

   Apply the parents operator to them

   Replace the operator with the value of the sub expression.



3. Binary trees in a famous file compression algorithm Huffman coding tree
    - Each character is stored in a leaf
    - The code is found by following the path 0 go left, 1 go right.
    - *a* is 01
    - *e* is 1



AVL Tree

An AVL tree is a self-balancing binary search tree, in which the heights of the two child subtrees of any node differ by atmost one.

Insertions and deletions may require the tree to be rebalanced by one or more tree rotations.

- The balance factor of a node is the height of its left subtree minus the height of its right subtree (sometimes opposite) and a node with balance factor—1, 0 or −1 is considered balanced. A node with any other balance factor is considered unbalanced and requires rebalancing the tree.
- The balance factor is either stored directly at each node or computed from the heights of the subtrees.

Insert operations

Step I: Insert a node into the AVL tree as it is inserted in a BST.

Step II: Examine the search path to see if there is a pivot node.

Three cases may arise

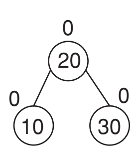
Case I: There is no pivot node. No adjustment required.

Case II: The pivot node exists and the subtree of the pivot node to which the new node is added has smaller height. No adjustment required.

Case III: The pivot node exists and the subtree to which the new node is added has the larger height, Adjustment required.

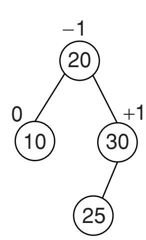
Example: The numbers at each node represents balance factor.

Example 1:



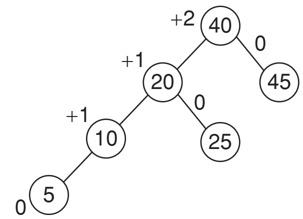
AVL tree

Example 2:



AVL tree

Example 3:



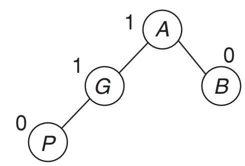
Not an AVL tree

Example 3 is not an AVL tree, because the balance factor of root node is +2.

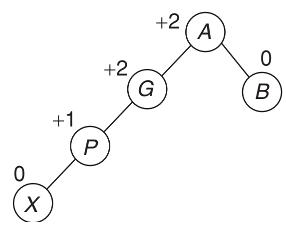
AVL tree becomes height in-balanced tree in following cases:

1. Left-Left case: An insertion in left subtree of left child of pivot node.

Example:

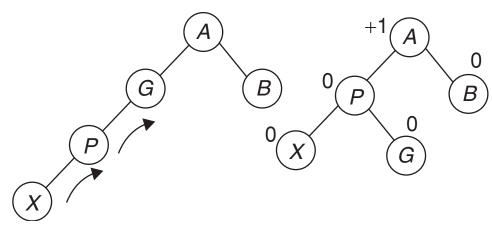


Insert '*X*' as left to node '*P*'. Here '*G*' is pivot node.



Solution:

To make the tree as balanced tree, perform **Left–Left Rotation** as follows:

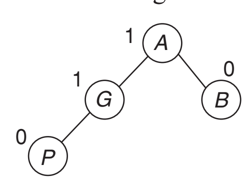


In left–left rotation

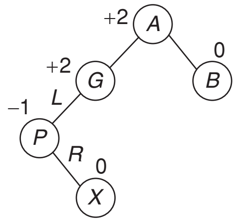
- Intermediate node '*P*' becomes root of subtree.
- Root of subtree '*G*' (pivot) becomes right subtree.
- New node '*X*' remains same as left child of '*P*'.

Left–Right Case

An insertion of left subtree of right child of pivot node.

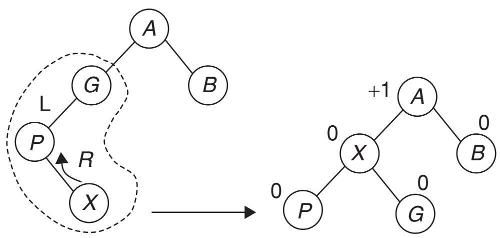


Example 1: Insert '*X*' as right child of '*P*'.



Is not an AVL tree. Height in-balance at node '*G*'.

Solution:
Perform Left–Right Rotation, to balance the height of tree.



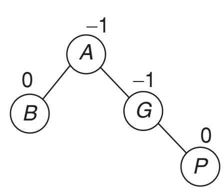
In Left–Right rotation:

- New node '*X*' becomes root of subtree.
- Root of subtree '*G*' (pivot) becomes right child of '*X*'.
- Intermediate node '*P*' becomes left child of new node.

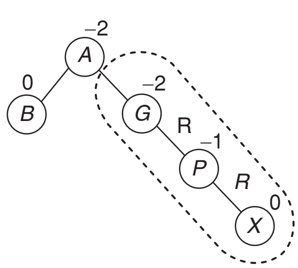
Right–Right case

An insertion of right subtree of right child of pivot node.

Example:

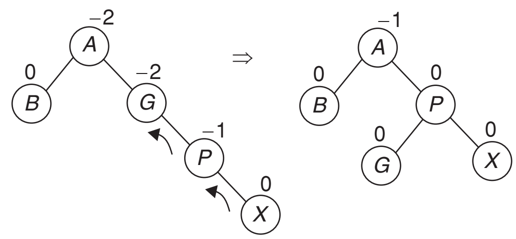


Insert '*X*' as right child of '*P*'



Is not an AVL tree, because of height in-balance at node '*G*'.

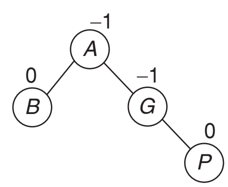
Solution:
To make the tree as balanced tree, perform the right–right rotation as follows:



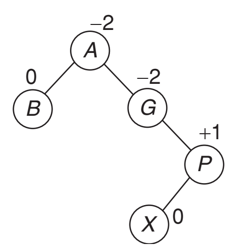
In Right–Right rotation:

- Intermediate node '*P*' becomes root of subtree.
- Root of subtree '*G*' (pivot) becomes left child of '*P*'.
- New node '*X*' remains as right child to '*P*'.

Right–Left case
An insertion of right subtree of left child of pivot node.

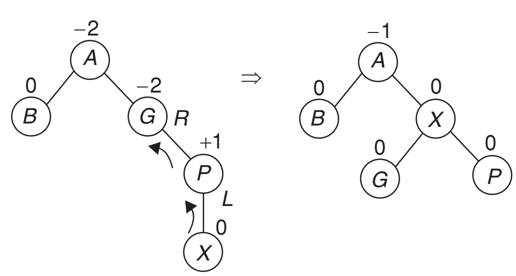


Insert '*X*' as left child of '*P*'



Is not AVL tree, because height in-balance at node '*G*'.

Solution:
To make the above tree as balanced, perform Right–Left rotation as follows:



In Right–Left Rotation:

- New node '*X*' becomes root of subtree.
- Root of subtree '*G*' (pivot) becomes left child of '*X*'.
- Intermediate node '*P*' becomes right of '*X*'.

Note: Left–Right and Right–Left rotation are also called as double rotations.

BINARY HEAP

A binary heap is a heap data structure created using a binary tree. It can be seen as a binary tree with two additional constraints.

The shape property: The tree is a complete binary tree; that is, all levels of the tree, except possibly the last one (deepest) level of the tree is not complete, the nodes of that level are filled, from left to right.

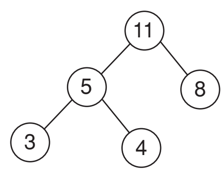
Max-Heap

A heap in which each node is greater than or equal to its children is called max-heap. Max-Heap generally used for heap sort.

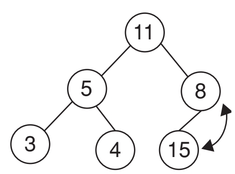
Min-Heap

A heap in which, each node is smaller than or equal to its children is called Min-Heap. Min-heap generally used to implement priority queue.

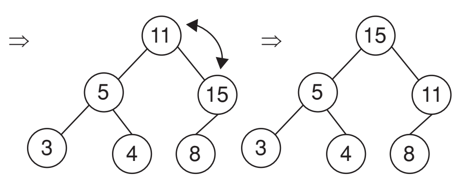
Note: By default heap represent Max-Heap:



Insert 15:

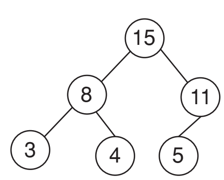


Is not satisfying heap property. So Heapify

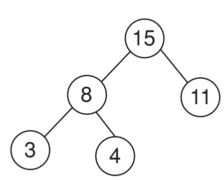


Delete 5: Deletion of a node from heap is always deletes a leaf node.

So interchange the value of last leaf node with node 5.



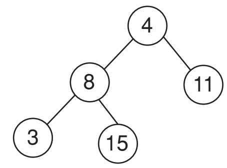
Now delete node '5'



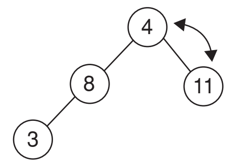
Is satisfying heap property.

Delete 15:

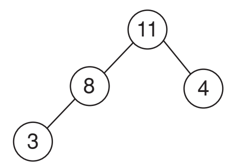
Interchange 4 and 15



Now delete Node '15'



Is not satisfying heap property. So heapify



Note: Insertion or deletion operation on a heap may require heapify process.

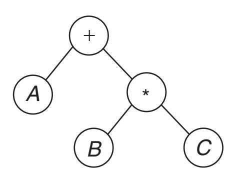
Expression Tree

The expressions can also represented by using a binary tree called expression tree.

Expression tree contains:

- Operators as intermediate nodes.
- Operands as leaf nodes (or) childs to operator nodes.
- The operator at lowest level will be having highest priority.

Example: $A + B * C$



Traversing:

Pre-order: $+ A * B C$

In-order: $A + B * C$

Post-order: $A B C * +$

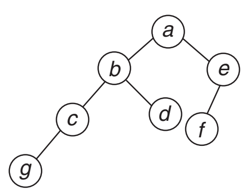
Note: In-order traversal of expression tree generates In-fix expression. Similarly pre-order and post-order generates prefix and postfix, respectively.

Exercises

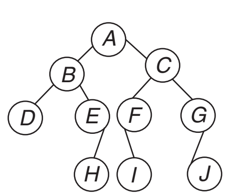
Practice Problems 1

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. A binary tree T has n leaf nodes. The number of nodes of degree two in T is ____.
   (A) n (B) $n - 1$
   (C) $\log n$ (D) $n + 1$
2. How many numbers of binary tree can be created with 3 nodes which when traversed in post-order gives the sequence C, B, A?
   (A) 3 (B) 5
   (C) 8 (D) 15
3. A binary search tree contains the values 3, 6, 10, 22, 25, 30, 60, 75. The tree is traversed in pre-order and the values are printed out. Which of the following sequence is a valid output?
   (A) 25 6 3 10 22 60 30 75
   (B) 25 6 10 3 22 75 30 60
   (C) 25 6 75 60 30 3 10 22
   (D) 75 30 60 22 10 3 6 25
4. Figure shows a balanced tree. How many nodes will become unbalanced when a node is inserted as a child of the node 'g'?



   (A) 7 (B) 2
   (C) 3 (D) 8
5. A full binary tree with n non-leaf nodes contains
   (A) $2n$ nodes (B) $\log_2 n$ node
   (C) $n + 1$ nodes (D) $2n + 1$ nodes
6. Which of the following list of nodes corresponds to a post order traversal of the binary tree shown below?



   (A) *A B C D E F G H I J* (B) *J I H G F E D C B A*
   (C) *D H E B I F J G C A* (D) *D E H F I G J B C A*
7. Which of the following sequence of array elements forms as heap?
   (A) {23, 17, 14, 6, 13, 10, 1, 12, 7, 5}
   (B) {23, 17, 14, 6, 13, 10, 1, 5, 7, 12}
   (C) {23, 17, 14, 7, 13, 10, 1, 5, 6, 12}
   (D) {23, 17, 14, 7, 13, 10, 1, 12, 5, 6}
8. What is the maximum height of any AVL tree with 7 nodes? Assume that the height of a tree with a single node is 0.
   (A) 2 (B) 3
   (C) 4 (D) 5
9. A binary search tree is generated by inserting in order the following integers:
   55, 15, 65, 5, 25, 59, 90, 2, 7, 35, 60, 23.
   The number of nodes in the left subtree and right subtree of the root respectively are
   (A) 8, 3 (B) 7, 4
   (C) 3, 8 (D) 4, 7
10. In a complete binary tree of n nodes, how far are the most distant two nodes? Assume each in the path counts as 1.
    (A) about $\log_2 n$ (B) about $2\log_2 n$
    (C) about $3\log_2 n$ (D) about $4\log_2 (n)$
11. A complete binary tree of level 5 has how many nodes?
    (A) 20 (B) 63
    (C) 30 (D) 73

Common data for questions 12 and 13: A 3-ary max-heap is like a binary max-heap, but instead of 2 children, nodes have 3 children. A 3-ary heap can be represented by an array as follows:

The root is stored in the first location, $a[0]$, nodes in the next level from left to right is stored from $a[1]$ to $a[3]$ and so on. An item x can be inserted into a 3-ary heap containing n items by placing x in the location $a[n]$ and pushing it up the tree to satisfy the heap property.

12. Which one of the following is a valid sequence of elements in an array representing 3-ary max-heap?
    (A) 1, 3, 5, 6, 8, 9 (B) 9, 6, 3, 1, 8 , 5
    (C) 9, 3, 6, 8, 5, 1 (D) 9, 5, 6, 8, 3, 1
13. Suppose the elements 7, 2, 10 and 4 are inserted, in that order, into the valid 3-ary max-heap found in the above question. Which one of the following is the sequence of items in the array representing the resultant heap?
    (A) 10, 7, 9, 8, 3, 1, 5, 2, 6, 4
    (B) 10, 9, 8, 7, 6, 5, 4, 3, 2, 1
    (C) 10, 9, 4, 5, 7, 6, 8, 2, 1, 3
    (D) 10, 8, 6, 9, 7, 2, 3, 4, 1, 5
14. Consider the nested representation of binary trees : (X Y Z) indicated Y and Z are the left and right subtrees respectively, of node X(Y and Z may be null (or) further nested) which of the following represents a valid binary tree?

(A) (1 2 (4 5 6 7)) (B) (1(2 3 4)5 6)7
(C) (1(2 3 4) (5 6 7)) (D) (1(2 3 NULL)(4 5))

15. A scheme for storing binary trees in an array X is as follows:

Indexing of X starts at 1 instead of 0. The root is stored at $X[\ 1\ ]$. For a node stored at $X[i]$, the left child, if any, is stored in $X[\ 2i\ ]$ and the right child, if any, in $X[2i + 1]$. To store any binary tree on 'n' vertices the minimum size of X should be

(A) $2n$ (B) n
(C) $3n$ (D) n^2

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. A binary search tree contains the values 1, 2, 3, 4, 5, 6, 7, 8. The tree is traversed in pre-order and the values are printed out. Which of the following is a valid output?
(A) 53124786 (B) 53126487
(C) 53241678 (D) 53124768

2. A binary search tree is generated by inserting in order the following integers : 50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24. The number of nodes in the left subtree and right subtree of the root respectively are:
(A) (4, 7) (B) (7, 4)
(C) (8, 3) (D) (3, 8)

3. A full binary tree (with root at level 0) of height h has a total number of nodes equal to:
(A) 2^h (B) $2^{h+1} - 1$
(C) $2^h - 1$ (D) 2^{h-1}

4. The number of null pointers of a binary tree of n nodes is :
(A) $n + 1$ (B) $n(n + 1)$
(C) n^2 (D) $2n$

5. Which of the following is false?
(A) A tree with n nodes has $(n - 1)$ edges.
(B) A labeled rooted binary tree can be uniquely constructed, given its post-order, in-order traversal results.
(C) The complete binary tree with n internal nodes has $(n + 1)$ leaves.
(D) The maximum number of nodes in a binary tree of height h is $(2^{h+1} - 1)$.

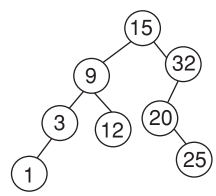
6. The maximum number of nodes in a binary tree at level i is
(A) 2^i (B) $2^i - 1$
(C) $2^i + 1$ (D) $\log_2 i + 1$

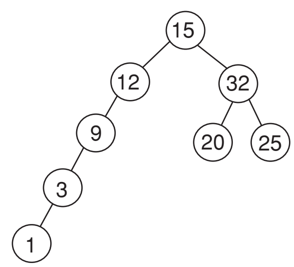
7. The number of leaf nodes in a rooted tree of n nodes, with each node having 0 or 3 children is
(A) $\frac{n}{3}$ (B) $\frac{(n-1)}{3}$
(C) $\frac{(n-1)}{2}$ (D) $\frac{(2n+1)}{3}$

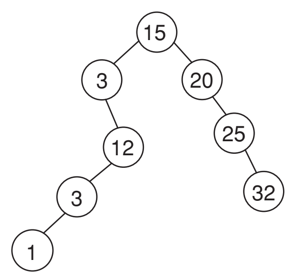
8. A complete n-ary tree is one in which every node has 0 or n children. If x is the number of internal nodes of a complete n-ary tree, the number of leaves in it is given by
(A) $x(n - 1) + 1$
(B) $xn + 1$
(C) $xn - 1$
(D) $x(n + 1) - 1$

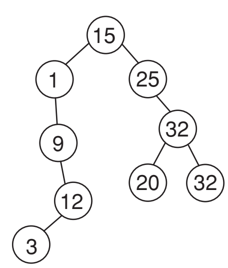
Common data for questions 9 and 10:

9. Insert the keys into a binary search tree in the order specified 15, 32, 20, 9, 3, 25, 12, 1. Which one of the following is the binary search tree after insertion of all elements?

(A)


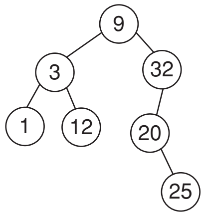
(B)


(C)


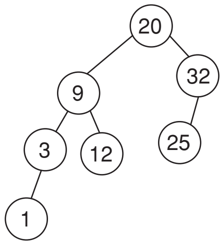
(D)


10. Which of the following is the binary tree after deleting 15?

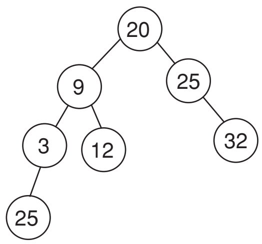
(A)



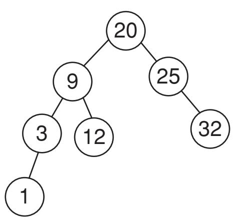
(B)



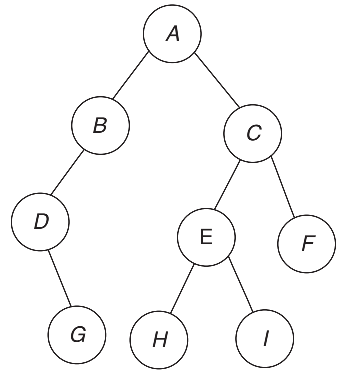
(C)



(D)



For questions 11, 12 and 13 below, use this figure



11. What is the post-order expression?
(A) ABDGCEJHIF (B) GDBHIEFCA
(C) DGBAHEICF (D) ABHIEFCDG

12. What is the pre-order expression?
(A) ABDGCEHIF (B) ABHIEFCDG
(C) DGBAHEIFCF (D) GDBHIEFCA

13. What is the in-order expression?
(A) ABDGCEHIF (B) GDBHIEFCA
(C) DGBAHEICF (D) ABHIEFCDG

14. In a 3-ary tree every internal node has exactly 3 children. The number of leaf nodes in such a tree with 6 internal nodes will be
(A) 13 (B) 12 (C) 11 (D) 10

15. Minimum number of swaps needed to convert the array 89, 19, 14, 40, 17, 12, 10, 2, 5, 7, 11, 6, 9, 70 into a max heap
(A) 2 (B) 3 (C) 1 (D) 0

Previous Years' Questions

1. In a binary tree with n nodes, every node has an odd number of descendants. Every node is considered to be its own descendant. What is the number of nodes in the tree that have exactly one child? **[2010]**
(A) 0 (B) 1
(C) $(n-1)/2$ (D) $n-1$

2. The following C function takes a singly-linked list as input argument. It modifies the list by moving the last element to the front of the list and returns the modified list. Some part of the code is left blank.

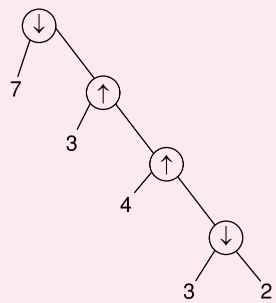
```
typedef struct node {
int value;
struct node *next;
}     Node;
Node *move_to_front(Node *head) {
Node *p, *q;
if ((head = = NULL || (head->next = = NULL)) return head;
q = NULL; p = head;
while (p-> next !=NULL) {
            q = p;
            p = p->next;
}


return head;
}
```

Choose the correct alternative to replace the blank line. **[2010]**
(A) `q = NULL; p->next = head; head = p;`
(B) `q->next = NULL; head = p; p->next = head;`
(C) `head = p; p->next = q; q->next = NULL;`
(D) `q->next = NULL; p->next = head; head = p;`

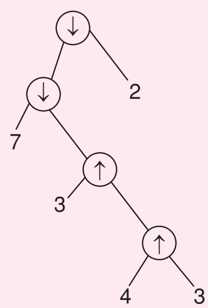
3. Consider two binary operators '↑' and '↓' with the

precedence of operator ↓ being lower than that of the operator ↑. Operator ↑ is right associative while operator ↓ is left associative. Which one of the following represents the parse tree for expression (7 ↓ 3 ↑ 4 ↑ 3 ↓ 2)? **[2011]**

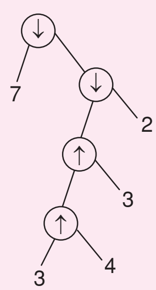
(A)



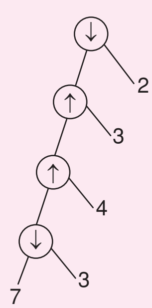
(B)



(C)



(D)



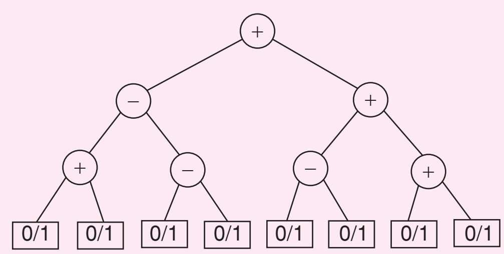
4. The height of a tree is defined as the number of edges on the longest path in the tree. The function shown in the pseudocode below is invoked as "height(root)" to compute the height of a binary tree rooted at the tree pointer "root".

```
int height(treeptr n)
{if (n == NULL) return -1;
if (n → left ==NULL)
if (n → right == NULL) return 0;
else return [B1] ;       //Box 1
else {h1 = height (n → left);
if (n → right== NULL) return (1 + h1);
else {h2 = height (n → right);
return [B2] ;     //Box 2
          }
     }
}
```

The appropriate expressions for the two boxes B_1 and B_2 are **[2012]**

(A) B_1: (1 + height ($n \to$ right))
 B_2: (1 + max (h_1, h_2))
(B) B_1: (height ($n \to$ right))
 B_2: (1+ max(h_1, h_2))
(C) B_1: height ($n \to$ right)
 B_2: max(h_1, h_2)
(D) B_1: (1 + height ($n \to$ right))
 B_2: max(h_1, h_2)

5. Consider the expression tree shown. Each leaf represents a numerical value, which can either be 0 or 1. Over all possible choices of the values at the leaves, the maximum possible value of expression represented by the tree is ______. **[2014]**



6. Consider the pseudocode given below. The function **Dosomething()** takes as argument a pointer to the root of an arbitrary tree represented by the *leftMostChild-rightSibling* representation. Each node of the tree is of type **treenode.** **[2014]**

```
type def struct treeNode* treeptr;
struct treeNode
{
treeptr leftMostChild, rightSibling;
};
int Dosomething (treeptr tree)
```

```
{
int value = 0;
if (tree ! = NULL){
if (tree - > leftMostChild = = NULL)
value = 1;
else
value=Dosomething(tree->leftMostChild);
value = value +Dosomething (tree - > right
Sibling);
}
return (value);
}
```

When the pointer to the root of a tree is passed as the argument to **DoSomething**, the value returned by the function corresponds to the
(A) Number of internal nodes in the tree
(B) Height of the tree
(C) Number of nodes without a right sibling in the tree
(D) Number of leaf nodes in the tree

7. The height of a tree is the length of the longest root-to-leaf path in it. The maximum and minimum number of nodes in a binary tree of height 5 are **[2015]**

(A) 63 and 6, respectively
(B) 64 and 5, respectively
(C) 32 and 6, respectively
(D) 31 and 5, respectively

8. Which of the following is/are correct inorder traversal sequence(s) of binary search tree(s)? **[2015]**
I. 3, 5, 7, 8, 15, 19, 25
II. 5, 8, 9, 12, 10, 15, 25
III. 2, 7, 10, 8, 14, 16, 20
IV. 4, 6, 7, 9, 18, 20, 25
(A) I and IV only (B) II and III only
(C) II and IV only (D) II only

9. Consider a max heap, represented by the array: 40, 30, 20, 10, 15, 16, 17, 8, 4 **[2015]**

Array Index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is
(A) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35
(B) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15
(C) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15
(D) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

10. A binary tree *T* has 20 leaves. The number of nodes in *T* having two children is ______ **[2015]**

11. Consider a binary tree *T* that has 200 leaf nodes. Then, the number of nodes in *T* that have exactly two children are ______. **[2015]**

12. While inserting the elements 71, 65, 84, 69, 67, 83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is **[2015]**
(A) 65 (B) 67
(C) 69 (D) 83

13. Consider the following New–order strategy for traversing a binary tree: **[2016]**
- Visit the root;
- Visit the right subtree using New – order;
- Visit the left subtree using New – order;

The New – order traversal of the expression tree corresponding to the reverse polish expression
3 4 * 5 – 2 ^ 6 7 * 1 + – is given by:
(A) + – 1 6 7 * 2 ^ 5 – 3 4 *
(B) – + 1 * 6 7 ^ 2 – 5 * 3 4
(C) – + 1 * 7 6 ^ 2 – 5 * 4 3
(D) 1 7 6 * + 2 5 4 3 * – ∧ –

14. Let *T* be a binary search tree with 15 nodes. The minimum and maximum possible heights of *T* are: **[2017]**
Note: The height of a tree with a single node is 0.
(A) 4 and 15 respectively
(B) 3 and 14 respectively
(C) 4 and 14 respectively
(D) 3 and 15 respectively

15. The pre-order traversal of a binary search tree is given by 12,8,6,2,7,9,10,16,15,19,17,20. Then the post-order traversal of this tree is: **[2017]**
(A) 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20
(B) 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12
(C) 7, 2, 6, 8, 9,10, 20, 17, 19, 15, 16, 12
(D) 7, 6, 2, 10, 9, 8, 15, 16, 17, 20, 19, 12

16. The postorder traversal of a binary tree is 8, 9, 6, 7, 4, 5, 2, 3, 1. The inorder traversal of the same tree is 8, 6, 9, 4, 7, 2, 5, 1, 3. The height of a tree is the length of the longest path from the root to any leaf. The height of the binary tree above is ______. **[2018]**

17. The number of possible min-heaps containing each value from {1, 2, 3, 4, 5, 6, 7} exactly once is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** B	**3.** A	**4.** C	**5.** D	**6.** C	**7.** C	**8.** B	**9.** B	**10.** B
11. B	**12.** D	**13.** A	**14.** C	**15.** A					

Practice Problems 2

1. D	**2.** B	**3.** B	**4.** A	**5.** C	**6.** B	**7.** D	**8.** A	**9.** A	**10.** B
11. B	**12.** A	**13.** C	**14.** A	**15.** B					

Previous Years' Questions

1. A	**2.** D	**3.** B	**4.** A	**5.** 6	**6.** D	**7.** A	**8.** A	**9.** B	**10.** 19
11. 199	**12.** B	**13.** C	**14.** B	**15.** B	**16.** 4	**17.** 80			

Test

Programming and Data Structure (Part I)

Time: 60 min.

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. What is the behavior of following code?

auto int *I*;

int main( )

```
{
}
```

(A) Compiler error (B) No error
(C) Linker error (D) Runtime error

2. What is the output of following code:

#define scanf "%S is a string"

int main( )

```
{
printf(scanf, scanf);
}
```

(A) is a string is a string
(B) %S is a string is a string
(C) %S is a string %S is a string
(D) Syntax error

3.
```
void rec_fun (int n, int sum)
{
int k = 0, j = 0;
if(n = = 0) return;
k = n%10; j = n/10;
sum+ = k;
rec _fun(j, sum);
printf("%d\t", k);
}
main ( )
{
int a = 2048; sum = 0;
rec _fun(a, sum);
printf("%d", sum);
}
```

What does the above program print?
(A) 8 4 0 2 14
(B) 8 4 0 2 0
(C) 2 0 4 8 18
(D) 2 0 4 8 0

4.
```
int fun (int * p, int n)
{
if (n < = 0) return 0;
else
if(*P % 2 = = 0)
return *p + fun(p + 1, n - 1);
else
return *p - fun(p + 1, n -1);
}
main( )
{
int arr[ ] = {56, 48, 55, 10, 49, 14};
printf("% d", fun(arr, 6));
}
```

Which of the following is the output of above function?
(A) 110 (B) –122
(C) 114 (D) 108

5. Consider the function:

```
int fun (int n)
{
static int i = 1;
if (n > = 5) return n;
n = n + i;
i ++;
return fun (n);
}
```

The value returned by *f*(3) is
(A) 6 (B) 7
(C) 8 (D) 9

6. Which of the following code will change a lower case letter to an upper case?
(A) char $C_2 = (C_1 > = `A' \& C_1 < `Z')?$ $`a' + `C_1' - `A': C_1);$
(B) char $C_2 = (C_1 > = `a' \& C_1 < = `z')?$ $`A' - `a' + `C_1') : C_1;$
(C) char $C_2 = (C_1 >= `a' \&\& C_1 <= `z')?$ $`A' + `C_1' - `a': C_1;$
(D) char $C_2 = (C_1 > = `A' \& \& C_1 < = `Z')?$ $`A' - `C_1' + `a': C_1;$

7. The following is a program to find the average length of several lines of text. What should be the lines of code corresponding to 'SecA' and 'SecB'.

```
main( )
{int n, count = 0, sum = 0;
float avg;
secA
```

```
{
count++;
sum + = n;
}
avg = (float) sum/count;
}
int linecount (void)
{
char line [80];
int count = 0;
while (secB)
{
count ++;
}
return (count);
}
```

(A) Sec A: while ($n > 0$)
Sec B: line [count]! = 0
(B) Sec A: while (linecount ( ) > 0)
Sec B: (line [count] = getch ( ) ! = '\n')
(C) Sec A: while ((n = linecount( ) > 0)
Sec B: (line [count] = getch ( ) ! = '\n')
(D) None of these

8. What is the meaning of following declaration? int(*f_1) ( );

(A) f_1 is a function which returns a pointer to an integer number.
(B) f_1 is a pointer to a function which returns an integer number.
(C) f_1 is a function which takes an integer pointer.
(D) f_1 is a pointer to an integer number.

9. Which of the following represents the statement: "x is a pointer to a group of one dimensional 20 element arrays".

(A) int *x[20]; (B) int *x[10] [20];
(C) int **x[20]; (D) int (*x) [20];

10. What will be the output of the following code?

```
int number[ ] = {18, 20, 22, 24};
main( )
{
int *q;
q = number;
q + = 4;
printf("%d",*q);
}
```

(A) 24 (B) Compiler error
(C) syntax error (D) –24

11.
```
main( )
{
char fname[ ] = "TIME 4 EDUCATION";
time4(fname);
}
time4(char fname[5])
{
fname + = 7;
printf("%s", fname);
fname = 2;
printf("%s", fname);
}
```

What is the output of above code?

(A) 4 EDUCATION
(B) EDUCATION
(C) TIME 4 EDUCATION
(D) EDUCATION ME 4 EDUCATION

12. What will be output of following code?

```
int main( )
{
extern int x;
x = 13;
printf ("%d", x);
return 0;
}
```

(A) 13
(B) Vary from compiler
(C) Linker error
(D) Undefined symbol

13. What is the output of the following code?

```
main( )
{
int x = y = z = 100;
int i;
i = x > y < z;
printf("% d", i);
}
```

(A) 0 (B) 1
(C) error (D) No output

14. In the following code, how many times 'while' loop will be executed?

```
int count = 0;
while (count < 32767)
count++;
```

(A) 32767
(B) 32766
(C) infinite times
(D) varies from complier to compiler

15. #define SUM(x)$x + x * x$

#define DIF(x)$x * x - x$

```
int main( )
{
float y = SUM(5) /DIF(5);
printf("%f", y);
}
```

(A) 1.5 (B) 0
(C) 2 (D) 1

16. Which of the following is a correct description of void (* ptr[10]) ( ) ;

(A) ptr is an array of 10 pointers to functions returning type void.
(B) ptr is an array of 10 functions returning pointers of type void.
(C) ptr is an array of 10 functions returning void *.
(D) ptr is an array of data elements of type void.

17.

```
int main( )
{
int p = 2, g = 2;
printf(" %d%d", p < < g, p > > g);
}
```

Output of above code is

(A) 1 16 (B) 4 0
(C) 16 4 (D) 8 0

18. Which of the following is equivalent expression for $P = a * 16 + b/8$;

(A) $P = (a << 4) + (b >> 2)$
(B) $P = (a >> 4) - (b << 2)$
(C) $P = (a << 4) + (b >> 3)$
(D) $P = (a << 4) + (b << 3)$

19. What is the output of the below code?

```
main( )
{
int I = 0, *j = &I;
f1(j);
*j = *j + 10;
printf(" %d%d", I, j);
}
f1(int *k)
{
*k+ = 15;
}
```

(A) 20 55 (B) 25 25
(C) 45 55 (D) 35 35

20.

```
int rec_f2(int r)
{
if (r = = 1 || r= = 0)
return 1;
if (r % 2 = = 0)
return(rec_f2 (r/2)+ 2);
else return ((rec_f2(r - 1) + 3);
}
main( )
{
printf ("%d", rec_f2(7));
}
```

Which of the following is the output of above program?

(A) 10 (B) 11
(C) 13 (D) 0

21.

```
int guest(int a);
int host(int b);
main( )
{
int p = 50, q = 100, r ;
for (r = 0; r < 2; r ++)
{
q = guest(p) + host(p);
printf(" %d", q);
}
}
int guest (int a)
{
int y;
y = host(a);
return(y);
}
int host (int a)
{
static int y = 0;
y = y + 1;
return (a + y);
}
```

The output of above code will be

(A) 103 107 (B) 107 103
(C) 110 107 (D) 107 110

22. Choose the best matching between groups A and B:

Group A		Group B	
1	Volatile	P	Queue
2	Function pointer	Q	Auto
3	Default	R	Guest and host
4	FIFO	S	Switch–case

(A) 1 – S, 2 – Q, 3 – R, 4 – P
(B) 1 – Q, 2 – P, 3 – R, 4 – S

(C) 1 – Q, 2 – R, 3 – S, 4 – P
(D) 1 – R, 2 – Q, 3 – P, 4 – S

23. Which of the following expression represents the statement: "*P* is a function that accepts a pointer to a character array".

(A) int *p*(char **a*[ ]); (B) int (**p*) (char (**a*)[ ]);
(C) int **p* (char **a*); (D) int *p*(char (**a*) [ ]);

24.
```
void arr_fun(int [ ] [3]);
main( )
{
int x[3][3] = {{10, 20, 30}, {40, 50, 60},
{70, 80, 90}};
arr_fun(x);
printf("%d", x[2][1]);
}
void arr_fun (int y[ ][3])
{
++y;
y[1][1] = 9;
}
```
What is the output of the above code?

(A) 80 (B) 90
(C) 70 (D) None of these

25.
```
#define F – 1
#define T 1
#define N 0
main( )
{
if(N)
printf(" %s", "GOOD");
else
if(F)
printf(" %s", "MORNING");
else
printf(" %s", "GOOD NIGHT");
}
```
Output of above code will be

(A) GOOD
(B) MORNING
(C) GOOD MORNING
(D) GOOD NIGHT

26. An external variable

(i) is defined once and declared in other functions.
(ii) is globally accessible by all functions.
(iii) cannot be static
(iv) is defined after main( )

(A) (i) and (ii) (B) (i), (ii) and (iii)
(C) (ii), (iii) and (iv) (D) (i), (ii), (iii), (iv)

Common Data for Questions 27 and 28:

Consider the following code:
```
int f(int P)
{
if(P < = 0) return 1;
if(P% 10 = = 0)
return f(P – 2); //X
else
return f(P – 3); //Y
}
main( )
{
printf(" %d", f(30));
}
```

27. What will be the output of above code?

(A) 10 (B) 50
(C) 100 (D) 1

28. What will be the output if the lines labeled *X* and *Y* are changed as follows:

X: return $3 + f(P/2)$;
Y: retrun $2 + f(P/3)$;

(A) 4 (B) 6
(C) 8 (D) 10

Common data for Questions 29 and 30:
```
1. float fn1(float n, int a)
2. {
3. float P, S;
   int I;
4. for(S = x, P = 1, I = 1; P = P * x * x;
5. I < a; I ++)
   S = S + P/(l);
6. return (S);
7. }
8. int f(int x)
9. {
10. int f = 1;
11. for (int i = 1; i < = n; i++)
12. f = f * i;
13. return (f);
14. }
```

29. Output of above code for fn1(2.1, 20) is

(A) $x + \frac{x^2}{2!} + \frac{x^4}{4!} + ...a + 2.1$

(B) $\frac{x - x^3}{3!} + \cdot a + 2.1$

(C) $1+\frac{x}{1!}+\frac{x^2}{2!}+$

(D) $1+\frac{x}{1!}+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdot a+2.1$

30. When the statement numbered 4, 5, 6, 7 are replaced by

```
{for (S = 1, P = -x, I = 1; I < a; i++)
P = P * x * x -1;
S = S + P/f(I);
}
```

What will be the approximation of $f(x)$?

(A) $1+x-x^2+x^3\ldots$

(B) $1-\frac{x}{1!}+\frac{x^3}{3!}-$

(C) $1-\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$

(D) $1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots$

Answer Keys

1. A	**2.** B	**3.** A	**4.** C	**5.** A	**6.** C	**7.** C	**8.** B	**9.** D	**10.** B
11. D	**12.** C	**13.** B	**14.** C	**15.** D	**16.** A	**17.** D	**18.** C	**19.** B	**20.** B
21. A	**22.** C	**23.** B	**24.** A	**25.** B	**26.** B	**27.** D	**28.** D	**29.** A	**30.** D

Programming and Data Structures Test 1

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Which one of the following is true about a strict binary tree with 16 leaf nodes?
   (A) has exactly 2^5 nodes
   (B) has atmost $2^5 - 1$ nodes
   (C) has atmost 30 nodes
   (D) has exactly 31 nodes

2. Consider 2 sorted lists of size 50 and 60 respectively. Find the number of comparisons needed in the worst case by merge sort algorithm?
   (A) 109 (B) 50
   (C) 60 (D) 3000

3. The print statement printf("%*d*", ++10); prints
   (A) 11 (B) 10
   (C) An error message (D) Garbage value

4. Consider the following C code segment

```
if(ptr!= NULL)
        *ptr = NULL; else  *ptr = NULL;
```

Which of the following has the same effect as the above code?

```
   (I) if(ptr)
        *ptr = NULL; else *ptr = NULL
  (II) *ptr = NULL;
 (III) if(!ptr)
        *ptr = NULL; else *ptr = NULL;
  (IV) if(ptr= NULL)
        *ptr = NULL; else *ptr = NULL;
```

   (A) I and III (B) I and IV
   (C) I, III and IV (D) I, II, III and IV

5. Which of the following statements are true?
   (I) Casting refers to implicit type conversion
   (II) Coercion refers to implicit type conversion
   (III) Casting refers to explicit type conversion
   (IV) Coercion refers to explicit type conversion
   (A) I and II (B) II and III
   (C) I and IV (D) III and IV

6. For implementing queue using linked list, the enqueue and dequeue operations are performed as
   (A) Enqueue after the last element, dequeue the first element
   (B) Enqueue in front of the first element, dequeue the first element
   (C) Enqueue after the last element, dequeue the last element
   (D) Enqueue in front of the first element, dequeue the last element

7. The tree transversal in which the node is processed before the recursive call to the child nodes complete?
   (A) Postorder (B) Preorder
   (C) Inorder (D) Level order

8. What is the number of '*n*' node binary trees with items 1, 2, …, *n* that have identical postorder and inorder transversals?
   (A) $\frac{n+1}{2}$ (B) n
   (C) 1 (D) $n!$

9. Which of following uses the unweighted single source shortest path problem?
   (A) Breadth First Search (B) Dijkstra's algorithm
   (C) Kruskal's algorithm (D) Depth First Search

10. Which one of the following data structure is best suited to check whether an arithmetic expression has balanced parentheses?
    (A) List (B) Queue
    (C) Stack (D) Tree

11. What is the output returned by following code?

```
# include <stdio.h>
int main( )
{
for(;NULL;)
printf ("TIME");
return 0;
}
```

    (A) T (B) TIME
    (C) Infinite loop (D) Compilation Error

12. Which of the following statements are TRUE?
    (I) Recursive programs are efficient
    (II) Binary search using a linear linked list is efficient
    (III) The worst case Time complexity of Quick sort is $O(n^2)$
    (IV) As the number of entries in the hash table increases, the number of collisions also increase.
    (A) II and III (B) I and III
    (C) III and IV (D) II and IV

13. If the processor supports only the immediate and direct addressing modes, then which one of the following programming language feature cannot be implemented on that processor?
    (A) Arrays
    (B) Pointers
    (C) Records
    (D) Recursive procedures with local variables

14. Which one of the following is true about the result returned by function under value-result and reference parameter passing conventions?
    (A) May differ in the presence of exceptions
    (B) Do not differ
    (C) Differ in the presence of loops
    (D) Differ in all cases

15. If L_1 is deterministic context free and L_2 is regular then which of the following is also deterministic context free?

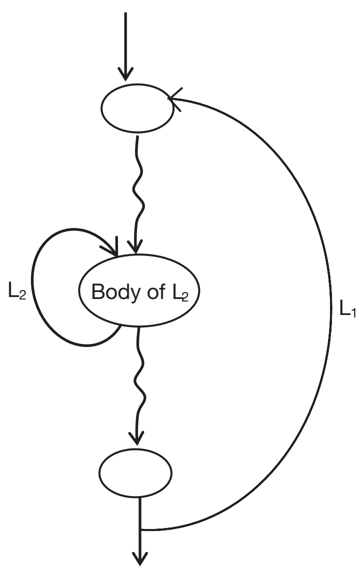
(i) $L_1 \cup L_2$ (ii) $L_1 \cap L_2$
(iii) L_2
(A) (i), (ii) Only (B) (ii), (iii) Only
(C) (ii) Only (D) (i), (ii) and (iii)

16. Consider the following C code segment

```
main ()
{
static int i;
for(putchar('G'); putchar('A');
putchar('E'))
    {
putchar ('T');
if(i == 2)
   break;
else
   i++;
   }
}
```

The output will be
(A) Syntax error (B) GAETETET
(C) GATEATEATE (D) GATEATEAT

17. Consider the below flow graph, which has two loops: L_1 and L_2



Let variables x, y are assigned registers in L_1 but not in L_2, then which of the following actions are TRUE?
(i) Store x, y on entry to L_2
(ii) Load x, y on entry to L_2
(iii) Load x, y on exit from L_2
(iv) Store x, y on exit from L_2
(A) (i), (ii)
(B) (i), (iii)
(C) (ii), (iv)
(D) (iii), (iv)

18. Consider the following recursive function

$$\text{fun}(n) = \begin{cases} 2 & \text{if } n = 0 \\ x & \text{if } n = 1 \\ 2\,\text{fun}\,(n-1) + 4\,\text{fun}\,(n-2) & \text{if } n \geq 2 \end{cases}$$

If the value of fun(4) is 88, then the value of 'x' is
(A) –1 (B) 0
(C) 2 (D) 1

19. What will be the output of following code?

```
# include<stdio.h>
int main ( ) {
int b, a = 6;
b = - a - - ;
printf ("%d %d", b, a);
}
```

(A) 6 5 (B) –5 5
(C) – 6 5 (D) compilation error

20. Which of the following is the prefix expression for the given infix expression?
$A - B * C + (D/E)/F * G$
(A) $+ - A * BC * / DE / FG$
(B) $+ - A * BC / DE / * FG$
(C) $+ - A * BC * || DEFG$
(D) $- A * BC + * || DEFG$

21. The following postfix expression with single digit operands is evaluated using a stack 4 2 2 ∗ / 2 3 ^ + 5 –
The top two elements of the stack after the evaluation of ^ are (^ is the exponentiation operator)
(A) (8, 1) (B) (9, 1)
(C) (5, 9) (D) (5, 1)

22. A circular queue of capacity (m –1) elements is implemented with an array of m elements. Insertion and deletion operation are done at rear and front respectively. Initially rear and front are initialized to zero. The condition to check whether the queue is full and empty are
(A) full :(rear + 1) mod m == front
  empty :(front + 1)mod m == rear
(B) full:(front +1) mod m == rear
  empty: rear == front
(C) full : (rear +1) mod m == front
  empty : rear == front
(D) full : rear == front
  empty : (rear +1) mod m == front

23. The number of comparisons needed to search a single linked list of length 'n' for a given element in worst case scenario is?
(A) $\frac{n}{2}$ (B) $\log_2 n - 1$
(C) $\log_2 n$ (D) n

24. Consider the following C code segment and predict the correct output ?

```
# define cube(x) (x*x*x)
main ()
```

```
{
int i, j = 4;
i = cube (j ++);
printf ("%d %d", i, j);
}
```

(A) 64 7 (B) 64 4
(C) 125 7 (D) 343 7

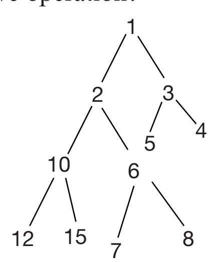
25. What will be the output of the following C code segment?

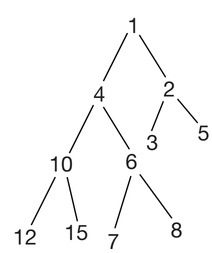
```
# define m(a, b) (((a)>(b))?(a):(b))
main ()
{
int x = 10, y = 5, z = 0;
z = m(++x, y++);
printf("%d %d %d", x, y, z)
}
```

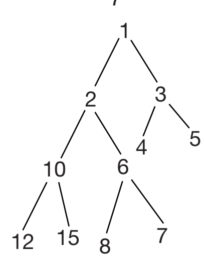
(A) 11 6 11 (B) 12 7 12
(C) 13 6 13 (D) 12 6 12

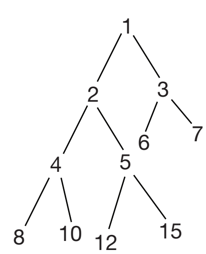
26. Which of the following sequence of array elements form a valid binary max heap?
(A) 35, 21, 10, 14, 15, 9, 6
(B) 50, 18, 14, 15, 17, 16, 12
(C) 40, 20, 14, 15, 21, 4, 11
(D) 9, 6, 4, 2, 1, 5, 3

27. Construct a binary min-heap with following elements inserted in order 5, 10, 2, 6, 8, 4, 3, 12, 15, 1, 7
What will be the resultant binary min-heap after the above operation?

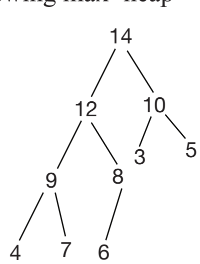
(A) 

(C) 

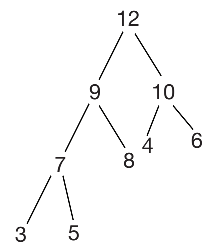
(B) 

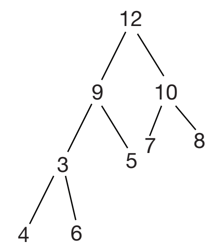
(D) 

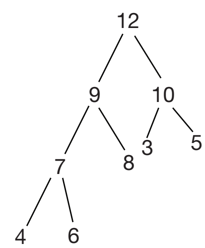
28. Consider the following max–heap

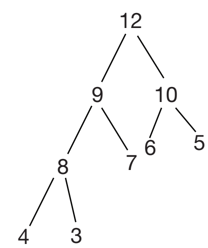


On deleting the element '14' from the max–heap, the resultant heap will be

(A) 

(C) 

(B) 

(D) 

29. Consider a complete binary tree of 15 nodes. How far are the most distant two nodes? (Assume each path count as 1)
(A) about 2 $\log_2 15$ (B) about $\log_2 15$
(C) about 3 $\log_2 15$ (D) about 4 $\log_2 15$

30. Let the last vertex visited of a complete binary tree in preorder, inorder and postorder are denoted by LPRE, LIN, LPOST respectively. Then which of the following is always true?
(A) LPRE = LPOST (B) LIN = LPRE
(C) LIN = LPOST (D) None of the above

31. Consider the following pairs of traversals on a labeled binary tree. Which of these pairs identify a tree uniquely?
(I) Inorder and Postorder
(II) Preorder and Postorder
(III) Preorder and Inorder
(IV) Level order and Postorder
(A) II only (B) I and III
(C) III only (D) IV only

Common Data for Questions 32 and 33:

```
Consider the following C code segment
int x, y, z = 0;
void fun(void);
main()
{
static int x = 4; ---------------S1
fun();
x++;
fun();
printf("\n %d %d" , x, y);
}
void fun(void)
} static int x = 6; ------------S2
int y =2;
x + = ++y;
```

```
printf("\n %d %d", x, y);
}
```

32. What output is generated by the given code segment?

(A) 9 3
9 3
5 0

(B) 9 3
12 3
5 0

(C) 9 3
9 3
9 3

(D) 9 3
12 4
5 garbage value

33. What output is generated by the given code segment if S_1 is replaced by auto int $x = 4$; S_2 is replaced by register int $x = 6$;

(A) 9 3
12 3
5 0

(B) 9 3
12 3
4 0

(C) 9 3
9 3
5 0

(D) 9 3
9 3
5 Garbage value

Linked Data Questions 34 and 35

Construct a binary search tree with elements 55, 40, 65, 60, 35, 20, 25, 70, 50, 10, 15. Inserted in order.

34. What will be the output of post order transversal of the BST?
(A) 15, 10, 25, 20, 35, 40, 50, 60, 70, 65, 55
(B) 15, 10, 25, 20, 35, 50, 40, 60, 70, 65, 55
(C) 15, 10, 25, 20, 35, 40, 60, 50, 70, 65, 55
(D) 15, 10, 25, 20, 35, 40, 70, 65, 60, 50, 55

35. In the above binary search tree, to locate the number 15, which of the following probe sequence is possible?
(A) 55, 40, 35, 25, 10, 15 (B) 55, 40, 50, 20, 10, 15
(C) 55, 40, 35, 20, 10, 15 (D) 55, 60, 35, 25, 10, 15

Answer Keys

1. D	**2.** A	**3.** C	**4.** D	**5.** B	**6.** A	**7.** B	**8.** C	**9.** A	**10.** C
11. B	**12.** C	**13.** B	**14.** A	**15.** D	**16.** D	**17.** B	**18.** D	**19.** C	**20.** C
21. A	**22.** C	**23.** D	**24.** A	**25.** D	**26.** A	**27.** C	**28.** C	**29.** A	**30.** D
31. B	**32.** B	**33.** C	**34.** B	**35.** C					

Hints and Explanations

1. Number of nodes of strict binary tree
$= (2n - 1)$; $n \rightarrow$ number of leaf nodes Choice (D)

2. $m + n - 1$
That is $50 + 60 - 1 = 109$ Choice (A)

8. Unique tree can be created using postorder and inorder transversals
$\therefore$ Only 1 tree is possible with same item that have identical post order and in order Choice (C)

11. Here NULL is micro constant, value of this symbolic constant is '0' as defined in stdio.h.
So for loop should not execute any time because initial condition is false. But it is bug of turbo C compiler. Loop will execute one time and it will print: TIME
Choice (B)

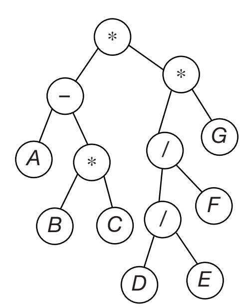
15. L_1 is DCFL
L_2 is regular.
Every regular language is DCFL. $L_1 \cup L_2$ and $L_1 \cap L_2$ are also DCFL's. Choice (D)

17. As the variables belongs to L_1 we need to store x, y on entry to L_2 and load x, y on exit from L_2. Choice (B)

18. $T(4) = 2T(3) + 4T(2)$
$T(3) = 2T(2) + 4T(1)$
$T(2) = 2T(1) + 4T(0) \rightarrow 8 + 2x$
Back substitute the value of $T(2)$
$\therefore \quad x = 1$ Choice (D)

19. $b = -a$ - -
$b = -6$ - -
$b = -6$
Now value of variable 'a' will be 5.
Choice (C)

20.

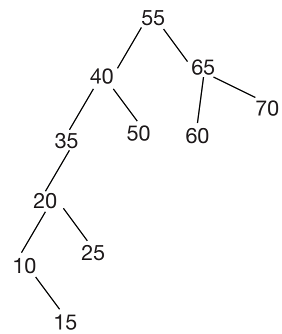


Prefix : $* - A * BC * \| DEFG$ Choice (C)

24. $i = 4 * 4 * 4 = 64$
$j = 4 + 3 = 7$ Choice (A)

25. The line $z = m(++x, y++)$ is replaced by
$z = (++x) > (y++)\ ?\ (++x) : (y++)$ Choice (D)

34.



15, 10, 25, 20, 35, 50, 40, 60, 70, 65, 55 Choice (B)

Programming and Data Structures Test 2

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Consider the following C program

```
main ()
{
char c = 'i'
while (c++ < = 'p')
putchar (xyz);
}
```

What should replace '*xyz*' such that the output will be: ijklmnop

(A) c (B) c – 1
(C) c++ (D) – c

2. Which of the following is/are true regardless of implementations?

I. sizeof(short) equals sizeof(int)
II. sizeof(int) equals sizeof(unsigned)
III. sizeof(double) is not less than sizeof(float)

(A) I only (B) II and III
(C) I and II (D) I, II and III

3. Consider the following type definition:
typedef char *c*[10];
c Array [6];
What will be the sizeof(Array)?
(Assume one character occupies 1 byte)

(A) 16 bytes (B) 60 bytes
(C) 10 bytes (D) 30 bytes

4. For implementing stack using linked list, the push and pop operations are performed as

(A) Push after the last element, pop the last element
(B) Push in front of the first element, pop the last element
(C) Push in front of the first element, pop the first element
(D) Push after the last element, pop the first element

5. Which of the following tree traversals have worst case complexity more than linear time?

(A) Level order (B) Preorder
(C) Inorder (D) None of the above

6. The stack is not used to implement:

(A) Level order (B) infix
(C) Polish (D) Reverse Polish

7. Which of the following data structures has the least height

(A) B–tree of order 4 (B) B–tree of order 3
(C) B–tree of order 5 (D) B–tree of order 6

8. Which one of the following algorithm solves the positive weighted single source shortest path problem?

(A) Breadth first search (B) Depth first search
(C) Kruskal's algorithm (D) Dijkstra's algorithm

9. The minimum spanning tree problem uses:

(A) Breadth first search (B) Dijkstra's algorithm
(C) Kruskal's algorithm (D) Depth first search

10. Heap allocation is required for languages that:

(A) support dynamic data structures
(B) support recursion
(C) use dynamic scope rules
(D) None of the above

11.

```
# include <stdio.h>
int main ( )
{
extern int X;
x = 40;
printf ("%d", X);
}
```

(A) 0 (B) 40
(C) Error (D) Garbage value

12. Forward declaration is absolutely necessary:

(A) if the function call precedes its definition
(B) if the function call precedes its definition and the function returns a non-integer quantity
(C) if a function returns a non-integer quantity
(D) None of the above

13. Faster access to non-local variable is achieved using an array of pointers to activation records is called a/an:

(A) Queue (B) Stack
(C) Heap (D) Activation tree

14. A binary tree *T* has 64 leaf nodes. The number of nodes of degree 2 in *T* is:

(A) 64 (B) $\log_2 64$
(C) 63 (D) 32

15. The number of leaf nodes in a rooted tree of 10 nodes with each node having 0 or 3 children is:

(A) 4 (B) 5
(C) 6 (D) 7

16. Consider the following C code segment

```
main()
{
static int i;
while(printf("%d",printf("hi")))
{
printf ("by");
if (i == 2)
break;
else
i++;
}
}
```

The output will be:

(A) hi2bybyby (B) hi2byhi2byhi2by
(C) hi2byhi2byhi2byhi2by (D) Syntax error

17. Consider the following for loop
for (putchar('*G*'); putchar (0);putchar ('*E*'))
putchar ('*T*');
The above loop will be executed
(A) Infinite times (B) 2 times
(C) 0 time (D) Syntax error

18. Consider below code fragment:

```
Test (list, n)
Nodeptr *list;
int n;
{
    Nodeptr  p, q;
    int m;
    q = NULL;
    p = *list;
    while (p! = NULL)
      if(p ® info = = n)
      {
        p = p → next;
        if(q = = NULL)
      {
        freenode(*list);
        *list = p;
      }
      else
        deleteafter(q, &m);
      }
      else
      {
        q = p;
        p = p → next;
      }
    }
```

deleteafter (*q*, & *m*) next node after *q* will be deleted and deleted data will be stored in *m*. freenode (*list) frees a node. Then what is the function of Test?
(A) Deletes first and last elements of the list.
(B) Deletes all nodes whose 'info' field contains the value '*n*'.
(C) Deletes all nodes whose next 'info' field contains the value '*n*'.
(D) Delete '*n*' nodes from the list.

19. What is the output of the following?
enum coin {penny, nickel, dime, quarter = 25, half – dollar, dollar}; printf ("%d %d", dime, dollar);
(A) 2,5 (B) 2,27
(C) 3,27 (D) 2,25

20. Convert the given infix expression into post fix $A - B * C + (D / E) * F / G$
(A) $ABC * - DE / F * G / +$
(B) $ABC * - DE / FG * / +$
(C) $ABC * -$ DE $/ + F * G /$
(D) $ABC * - DE / + FG / *$

21. What will be the output of the following C code?

```
# define square(x) (x * x)
int main ( )
{
int i, j = 3
i = square(j+2)
printf("%d", i);
}
```

(A) Error (B) 25
(C) 11 (D) None of the above

22. What will be the output of the following *C* code segment?

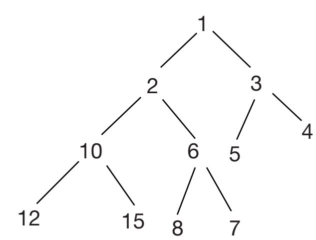
```
# define function(i, j) i##j
main ( )
{
int var1 = 10, var12 = 30;
printf("%d",function(var1, 2));
}
```

(A) Error (B) 30
(C) 0 (D) 12

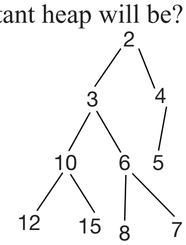
23. Which of the following sequence of array elements form a valid binary min-heap?
(A) 4, 7, 10, 8, 9, 12, 13
(B) 23, 25, 28, 24, 32, 30
(C) 14, 17, 25, 30, 15, 24, 33
(D) 60, 80, 70, 65, 85, 90, 95

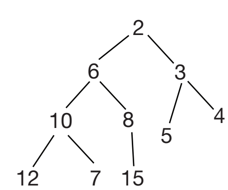
24. Construct a binary max heap with elements 4, 14, 3, 6, 9, 5, 10, 12, 7, 8 inserted inorder
Which of the following is the correct sequence of element of binary max-heap in the array, representation
(A) 14, 12, 10, 9, 8, 3, 5, 7, 4, 6
(B) 14, 12, 10, 9, 8, 3, 5, 4, 7, 6
(C) 14, 12, 10, 8, 9, 5, 3, 4, 7, 6
(D) 14, 12, 10, 8, 9, 4, 3, 5, 7, 6

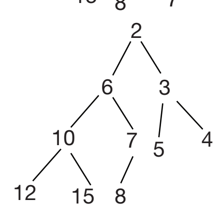
25. Consider the following min-heap

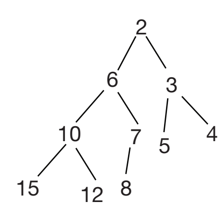


On deleting the element '1' from the min-heap, the resultant heap will be?

(A)


(B)


(C)


(D)


26. The inorder and preorder transversal of binary tree are *l j m i n k o* and *i j l m k n o* respectively. Then the post-order traversal of the binary tree is:
(A) *m l j o n k i* (B) *m l j n o k i*
(C) *l m n o j k i* (D) *l m j n o k i*

27. How many distinct binary search trees can be created out '*n*' distinct keys?
(A) $n!$ (B) n^2
(C) $\frac{2nC_n}{n+1}$ (D) $2n + 1$

28. Find the maximum number of nodes in a binary tree of height 13? (height of a binary tree is the maximum number of edges in any root to leaf path)
(A) $2^{12} - 1$ (B) $2^{14} - 1$
(C) 2^{13} (D) 2^{14}

29. The following sequence of operations is performed on a stack push(30), push(40), pop, push(30), push(40), pop, pop, pop, push(40), push(30), pop, pop
What is the sequence of popped out values?
(A) 40, 40, 30, 40, 30, 30
(B) 30, 40, 40, 30, 40, 30
(C) 40, 40, 30, 30, 30, 40
(D) 40, 30, 40, 30, 30, 40

Common Data Questions 30 and 31:

```
Consider the function
find(int i, int j)
{
return ((i <j) ? 0 : (i - j));
}
```

30. Let *a*, *b* be two non-negative integers. The call find (*a*, find(*a*, *b*)) can be used to find the:
(A) Positive difference of *a*, *b*
(B) Maximum of *a*, *b*
(C) Minimum of *a*, *b*
(D) Sum of *a*, *b*

31. Let *a*, *b* be two non-negative integers, then which of the following calls, finds the positive difference of *a* and *b*?
(A) find(*a*, *b*) + find(*b*, *a*) (B) *a* + find(*a*, *b*)
(C) *b* + find(*a*, *b*) (D) find(*a*, find(*a*, *b*))

Linked Data Questions 32 and 33:
Consider the following C code fragment

```
static char array [4][4] =
{"bat", "tub", "hot", "pig"};
char (*p) [4] = array;
```

32. What will be the output of (∗ (array [2] + 1))
(A) Prints address of '*o*' (B) Prints '*u*'
(C) Prints '*h*' (D) Prints *o*

33. Which of the following prints the character '*g*'?
(A) putchar (∗(ptr + 3)+2)
(B) putchar (∗(∗(ptr + 3)+2))
(C) putchar (∗((array +3) + 2))
(D) Both (B) and (C)

Linked Data Questions 34 and 35:
A binary search tree is generated by inserting the elements in the following order:
20, 25, 30, 15, 18, 12, 35, 8, 23, 5, 13

34. Which one of the following is the valid preorder transversal on the above binary search tree?
(A) 20, 15, 12, 8, 5, 13, 18, 23, 25, 30, 35
(B) 20, 15, 12, 8, 5, 13, 18, 35, 25, 23, 30
(C) 20, 15, 12, 8, 5, 13, 18, 25, 23, 30, 35
(D) 20, 15, 12, 8, 5, 13, 18, 25, 30, 23, 35

35. Which one of the following is the valid post order transversal on the binary search tree?
(A) 5, 8, 13, 12, 18, 15, 23, 35, 25, 30, 20
(B) 5, 8, 13, 12, 18, 15, 23, 25, 30, 20, 35,
(C) 5, 8, 13, 12, 18, 23, 15, 25, 30, 35, 20
(D) 5, 8, 13, 12, 18, 15, 23, 35, 30, 25, 20

Answer Keys

1. B	**2.** B	**3.** B	**4.** C	**5.** D	**6.** A	**7.** D	**8.** D	**9.** C	**10.** A
11. C	**12.** B	**13.** D	**14.** C	**15.** D	**16.** B	**17.** C	**18.** B	**19.** B	**20.** A
21. C	**22.** B	**23.** A	**24.** B	**25.** C	**26.** D	**27.** C	**28.** B	**29.** C	**30.** C
31. A	**32.** D	**33.** D	**34.** C	**35.** D					

Hints and Explanations

3. 10 × 6 = 60 bytes Choice (B)

5. All the other options requires linear time only
Choice (D)

6. Polish → preorder
Reverse polish → post order
Level order → BFS Choice (A)

11. Output of the given program will be "Linker error – undefined symbol *X*"
External variables are declared outside a function.
Choice (C)

12. By default the return type is int.
Choice (B)

14. No. of nodes of degree 2 = $n - 1$
where n – no. of leaf nodes Choice (C)

15. $\lceil 2n + 1)/3 \rceil$

$$\left\lceil \frac{2 \times 10 + 1}{3} \right\rceil = \left\lceil \frac{21}{3} \right\rceil = 7$$ Choice (D)

18. Given code deletes all nodes whose 'info' field contains 'n'. If the 'info' field equal to n delete that node and repeat this for the remaining list. Choice (B)

19.

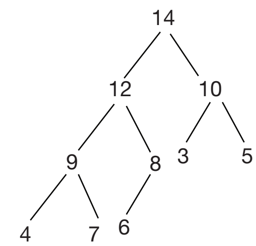
Penny	0
nickel	1
dime	2
quarter	25
half-dollar	26
dollar	27

Choice (B)

21. $(j + \underbrace{2 * j}_{\text{Higher precedence}} + 2)$ Choice (C)

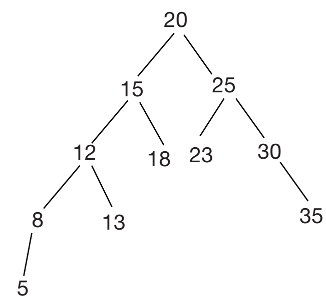
22. $i\#\#j$ performs macro concatenation
$\therefore$ var1, 2 becomes 'var12', the value of var12 is 30
Choice (B)

24.



Choice (B)

34.



Choice (C)

Programming and Data Structures Test 3

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Dynamic checking is performed (at the time of)
   (A) translation (B) execution
   (C) before the execution (D) semantic analysis
2. Which variables are preferable for smaller and faster programs?
   (A) auto variables (B) static variables
   (C) global variables (D) register variables
3. C language implements
   (A)Static scoping (B) dynamic scoping
   (C) both (A) and (B) (D) None of the above
4. Which of the following language doesnot support recursion
   (A) C (B) PROLOG
   (C) PASCAL (D) FORTRAN
5. Consider the following program segment:

```
A (int x)
{
if (x = = 0)
return;
else
{
A(x/10);
printf("%d /t", x);
}
}
```

   What is the output when A (1 2 3 4) is called
   (A) 1 2 3 4 1 2 3 1 1 2 (B) 1 1 2 1 2 3 1 2 3 4
   (C) 1 1 2 3 1 2 1 2 3 4 (D) 1 2 3 4 1 2 3 1 2 1
6. What is the return value when *A* ("TIME") is called from given program segment?

```
int A(char * S)
{
char * P = S;
while (*P! = '/0')
P++;
return P – S; }
```

   (A) 1 (B) 2
   (C) 3 (D) 4
7. The following statement $x + y = z$ in 'C' language is:
   (A) correct (B) invalid
   (C) impossible (D) None of the above
8. The order in which actual arguments are evaluated in a function call is:
   (A) from left to right (B) from right to left
   (C) unpredictable (D) None of the above
9. Which of the following is true for the getche ( ) library function?
   (A) It returns a character when a key is pressed.
   (B) It returns a character when the enter key is pressed.
   (C) It doesn't display a character on the screen.
   (D) It displays a character on the screen.
10. The following declaration float (**A*) (void *,void *) means:
    (A) *A* is function returning a pointer to a float that has two void * arguments.
    (B) *A* is a pointer to a function with float as the arguments.
    (C) *A* is a function with void * as arguments and returns float pointer.
    (D) *A* is a function that has two void * arguments and returns float.

Linked Answer Questions 11 and 12:

Consider the following recursive function:

```
A(m, n)
{
if (m = = 0)
return n + 1;
else if (n = = 0)
A (m – 1, 1);
else
A(m – 1, A(m, n – 1));
}
```

11. What is the value returned when *A* (1, 1) is called:
    (A) 2 (B) 3
    (C) 4 (D) 5
12. The number of function calls when *A* (1, 1) is called:
    (A) 2 (B) 3
    (C) 4 (D) 5
13. Pointers are of unsigned:
    (A) integer data type (B) character data type
    (C) float data type (D) None of the above

Common Data Questions 14 and 15:

Consider the following program segment:

```
int x[3] [5] = {
{1, 2, 3, 4, 5}
{6, 7, 8, 9, 10},
{11, 12, 13, 14, 15}}
int *n = &x;
```

14. The value of $*(*(x + 1) + 3)$ is:
    (A) 7 (B) 8
    (C) 9 (D) None of the above
15. The value of $*(n + 5) + 1$ is:
    (A) 7 (B) 5
    (C) 3 (D) None of the above
16. Consider the declaration char *A* (int (*) (char, float)) int *B*(char, float)

Which of the following function invocation is valid?

(A) A (*B) (B) A (&B)

(C) A (B) (D) None of the above

17. The expression

6 – 2 + 3 – 4 * 2 will evaluate to –1 if

(A) – is left associative and – has precedence over *

(B) – is right associative and – has precedence over *

(C) – is right associative and * has precedence over –

(D) None of the above

18. Which of the following is not the functionality of pre processing?

(A) Macro substitution

(B) Conditional compilation

(C) Inclusion of named files

(D) None of the above

19. Consider the program segment:

```
int x = 10;
A( )
{
int x = 20;
B ( );
}
B( )
{
printf ("%d", x);
}
main ( )
{
A ( )
}
```

What is the out put of above program

(A) 10

(B) 20

(C) prints 10 or 20 (depends on compiler)

(D) compilation error

20. Consider the program segment:

```
int x = 40;
A ( )
{
int y = 60;
B ( );
}
B ( )
{
printf('%d", x);
}
main ( )
{
int x = 50;
A( ) ;
}
```

The output of the above program is:

(A) 40 (B) 50

(C) 60 (D) Compilation error

21. In the above code given in Q. 20, if the above language implements Dynamic scoping then output of the above code is:

(A) 40

(B) 50

(C) 60

(D) None of these

22. Consider the following program:

```
int k;
k = n; // 'n' is some integer
while (k > 0)
k = k/2;
```

The number of comparisons made in the execution of the loop is:

(A) $[\log_2 n]$ (B) $1 + [\log_2 n]$

(C) n (D) $\left[\log\left(\frac{n}{2}\right)\right]+1+1$

23. Consider the following program:

```
A (int x, int y)
{
if (x > = y)
return [1 + A(x - y, y)];
else
return 0;
}
```

What is the functionality of above routine A( )?

(A) It returns the number of function calls

(B) It performs the subtraction between two numbers (x and y).

(C) It performs division of the two given numbers (x and y).

(D) It returns the greater value among the two numbers (x and y).

24. What does the following fragment of C-program print;

char* s = "TIME = HYD";

char* $p = s;$

printf ("%u %u", p, s);

(A) 2000 2006

(B) 2006 2008

(C) 2010 TIME HYD

(D) 2000 2000

25. Consider the following program segment:

```
main ( )
{
static char a[3] [4] = {"time",
   "gate", "2015"}
putchar (**a);
}
```

What will be the output of above program?

(A) prints time

(B) prints gate

(C) prints 2015

(D) None of the above

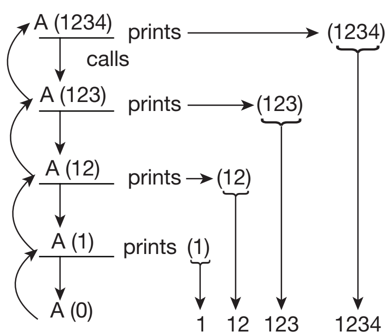
Answer Keys

1. C	**2.** D	**3.** A	**4.** D	**5.** B	**6.** D	**7.** B	**8.** C	**9.** A	**10.** D
11. B	**12.** C	**13.** A	**14.** C	**15.** A	**16.** C	**17.** C	**18.** D	**19.** A	**20.** A
21. B	**22.** B	**23.** C	**24.** D	**25.** D					

Hints and Explanations

1. 'Dynamic checking is performed before the execution. Choice (C)

5.

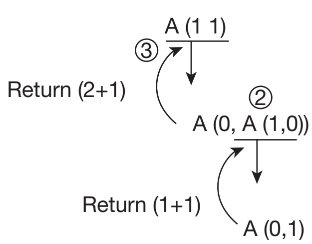


Choice (B)

6. It returns the string length i.e., 4. Choice (D)

7. Left side of assignment operator (=) could not be an expression. Choice (B)

11.



Choice (B)

12. From the above, the function calls are 4. Choice (C)

14. $*(*(x + 1) + 3)$

This is equivalent to $x[1]$ $[3]$, which is 9. Choice (C)

15. $*(n + 5) + 1$

1	2	3	4	5	6	7	8	....
0	1	2	3	4	5	6	7	

The above pointer expression maps to $n[6]$ in memory location which is 7. Choice (A)

16. The function '*A*' means *A* is a function (returning character) whose only argument is a pointer to a function that takes a character and float as arguments and returns as integer. The name of a function can be used as the starting address of the function (i.e., pointer to it). Choice (C)

17. $6 - 2 + 3 - 4 * 2$

$\downarrow$

$= 6 - 2 + 3 - 8$

$\downarrow$

$= 6 - 2 + (-5)$

$= 6 - 2 - 5$

$= 6 - 7 = -1.$ Choice (C)

19. The given program is in 'C' language, it implements static scoping.

When $A(\ )$ is called which calls $B(\ )$ it will check the 'x', values in $B(\ )$. As it is not present, it checks for the 'x' values out of the $B(\ )$ and it will print x as '10'. Choice (A)

20. As 'C' language implements static scoping, when $A(\ )$ is called, which will call $B(\ )$ which has to print 'x', first it will check in $B(\ )$ for the 'x' value then it goes out of the $B(\ )$ (global section) and prints 40. Choice (A)

21. If dynamic scoping is implemented, when $B(\ )$ wants to print 'x' it, first checks for 'x' in $B(\ )$ as it is not present, it checks in the function from where it is called (i.e., in $A(\ )$)

$\therefore$ It prints 50. Choice (B)

22. Here 'k' value is n from the loop, 'k' values will be:

$$n, \frac{n}{2}, \frac{n}{4}, \frac{n}{8}, \ldots, 1$$

let the loop is iterated for 't' times as the sequence is in G.P. then t^{th} term will be 1.

$$\therefore \quad n\left(\frac{1}{2}\right)^{t-1} = 1 = \frac{1}{n}$$

$2^{t-1} = n$

$(t - 1) \log_2 2 = \log_2 n$

$t - 1 = \log_2 n$

$t = 1 + \log_2 n$ Choice (B)

23. The function A( ) returns Quotient.

i.e., it performs division among the two given numbers. Choice (C)

24. Both p and s points to same address, prints same values. Choice (D)

25. **a will map to 1^{st} character in the array it prints't'. Choice (D)

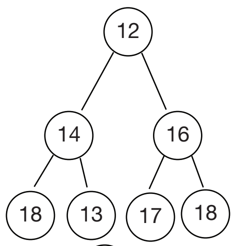
Programming and Data Structures Test 4

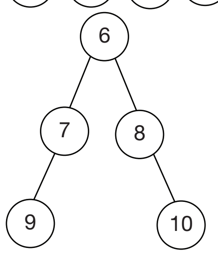
Number of Questions: 25 **Section Marks: 30**

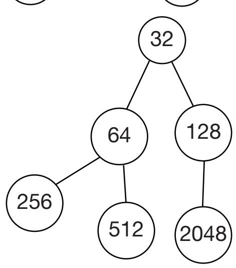
Directions for questions 1 to 25: Select the correct alternative from the given choices.

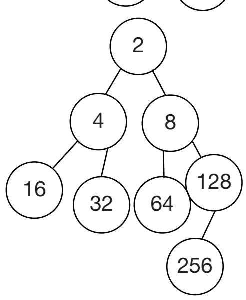
1. Which of the following data structure(s) can be used to represent Stack, Queue, Binary Tree, Hash Table, Heap and Graph?
   (A) Array (B) Singly linked list
   (C) Doubly linked list (D) Both (A) and (C)

2. Which of the following is a min heap?

(A)


(B)


(C)


(D)


3. Which traversal of a binary search tree will give the result in an ascending order of elements?
   (A) Preorder
   (B) In-order
   (C) Post-order
   (D) None of the above (a special code is required to print elements in ascending order)

4. Which of the following cannot be implemented using self-referential structure?
   (A) Graphs (B) AVL trees
   (C) Splay trees (D) B-Trees

5. Which of the following is TRUE about the AVL tree?
   (1) AVL tree is a binary tree.
   (2) The search time of AVL tree will be always less than (or) equal to the search time of binary search tree.
   (3) AVL tree is binary search tree with maximum balancing factor of 2.
   (A) Only (1) (B) Only (1) and (2)
   (C) Only (1) and (3) (D) All the above

6. Which of the following is suitable for the data that contains a relationship between pairs of elements, which is not necessarily hierarchical in nature?
   (A) Heaps (B) AVL Trees
   (C) Graphs (D) B-Trees

7. What is the Number of Null links in a Binary Tree of 'n' nodes?
   (A) n (B) $n + 1$
   (C) $n - 1$ (D) $n/2$

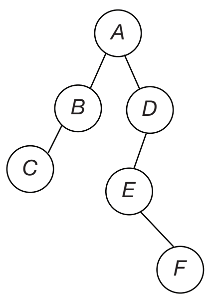
8. What is the number of stacks that should be used to implement a queue?
   (A) 1 (B) 2
   (C) 3 (D) 4

9. Consider the recursive function for fibonacci sequence:
```
fib (n)
{
if (n ==0)
return 0;
else if (n==1)
return 1;
else
return fib (n - 1) + fib (n - 2);
}
```
   What is the Number of additions required, when $fib(n)$ is invoked ?
   (A) $fib(n + 1) - 1$ (B) $fib\ (n + 1) + 1$
   (C) $fib\ (n + 1) - \text{n}$ (D) $fib\ (n + 1) + n$

10. Which of the following tree traversals are necessary to get the unique pattern of a tree structure?
   (A) Only In-order
   (B) In-order and pre-order
   (C) Post-order and pre-order
   (D) All the above

Common data for questions 11 and 12:
```
Consider the following routines:
struct Binary
{
struct Binary * Left ;
int data;
struct Binary * Right;
}
```

```
void F1 (struct Binary *t)
{
if (t)
{
printf("%d", t → data);
F2(t → Left);
F2(t → Right);
}
}
void F2(struct binary * t)
{
if (t)
{
F1(t → Left);
printf("%d", t → data);
F1 (t → Right);
}
}
Let us consider a tree
```



11. If $F1(t)$ is called on the above tree with t as root node (A), what is its output?
(A) *ABCDEF* (B) *ACBFED*
(C) *ABDCFE* (D) *ACBEFD*

12. When $F2(t)$ is called on this tree, with root as the parameter that is ($F2$(root)), what is its output?
(A) *BCADEF* (B) *BCDEFA*
(C) *DEFBCA* (D) *DEFABC*

13. What is the prefix expression of the expression $a + b * c/d \wedge e \wedge f - g$
(A) $-/ * + a\, b\, c$ d $\wedge \wedge e\, f\, g$
(B) $- +$ a/$* b\, c \wedge d \wedge e\, f\, g$
(C) $- + a \wedge \wedge / * b\, c\, d\, e\, f\, g$
(D) $\wedge \wedge + a / * b\, c\, d\, e - f\, g$

14. What does the function DO( ) does on a queue?

```
void DO( )
{
if (!Q is empty)
{
int temp = dequeue ( );
DO ( );
enqueue (temp);
}
}
```

(A) queue remains same
(B) elements will get reversed in a queue
(C) elements are removed and reinserted in the same queue
(D) queue remains empty

15. Fill in the blanks for the routine A( ).

```
A( ) will delete the elements in circular queue.
Initially Front = Rear = -1
A(Q, N, Front, Rear, Y) // N is size of a queue, Q is an array to store circular Queue.
{
if (Front == -1)
{
printf("Queue is underflow");
exit (1);
}
Y = Q(Front);
if(I)
Front = Rear = -1;
else
{
if (Front == N)
(II);
else
Front = Front + 1;
}
```

return *Y;*

(A) I. Front == Rear +1 II. Front = –1
(B) I. Front == Rear II. Front = –1
(C) I. Front == (Rear +1) II. Front = 0
(D) I. Front == Rear II. Front = 0

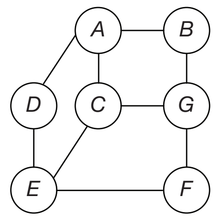
16. Consider the following routine:
struct node

```
{
int data;
struct node → link;
};
void DO (struct node → p, struct node → q)
{
if (q)
{
DO (q, q → link);
q → link = p;
}
else
head = p;
}
```

When DO(head, head → link) is implemented on singly linked list, the resultant list,
(A) is a reversal of given list
(B) remains the same

(C) is a circular linked list
(D) None of these

17. Consider the graph below;



Which of the following orderings are not possible using depth first search graph traversal:
(A) *A C G F E D B* (B) *A D E F C G B*
(C) *A C G B F E D* (D) *C G B A D E F*

18. What is the time complexity for attaining maximum element from Max heap?
(A) $O(n \log n)$ (B) $O(n)$
(C) $O(\log n)$ (D) $O(1)$

19. Minimum number of nodes in AVL tree of height 5 is (with root at height = 1)
(A) 11 (B) 12
(C) 13 (D) 14

20. Consider a Binary search Tree '*T*' in which every node will be having only right child, there are '*n*' nodes \in tree '*T*'. Time complexity for searching an element will be:
(A) $O(\log n)$ (B) $\left(\log\left(\frac{n}{2}\right)\right)$
(C) $O(n)$ (D) $O\left(\frac{n}{2}\right)$

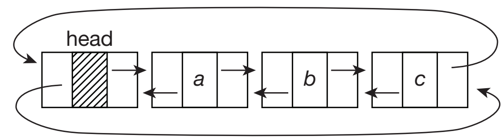
21. Consider post-order and in-order traversal of a tree:
POSTORDER :- DCBGFEA
INORDER :- BDCAFGE
When tree is constructed from above tree traversals then what are the leaf nodes?
(A) *D*, *G* (B) *C*, *F*
(C) *D*, *E* (D) *C*, *G*

Linked Answer for Questions 22 and 23:
Construct an AVL tree with the elements
21, 26, 30, 9, 4, 14, 28, 18

22. What will be the root node for the AVL tree?
(A) 26 (B) 21
(C) 28 (D) 18

23. What will be the post order predecessor of the root node, in the above resultant tree?
(A) 26 (B) 21
(C) 28 (D) 9

Linked Answer for Questions 24 and 25:
Consider the circular doubly linked list given below.



Consider the routine below:

```
struct node
{
sturct node *Lptr ;
int data;
struct node * Rptr;
};
void A ( )
{
struct node → t = head → Rptr;
while (t! = head)
{
swap (t → Lptr, t → Rptr)
t = t → Lptr;
}
swap (head → Lptr, head → Rptr);
}.
```

24. When *A*( ) routine is applied on the given list, the head Right pointer maps to:
(A) Node containing data field as '*a*'
(B) Node containing data field as '*b*'
(C) Node containing data field as '*c*'
(D) The head pointer points to itself

25. To which node *N* (pointer) points when the following operation is performed on the new list formed in the previous question.
(struct node $*$ *N*)
N = Head → Rptr → Rptr → Lptr → Lptr → Rptr;
(A) Head node
(B) Node containing data field as '*a*'
(C) Node containing data field as '*b*'
(D) Node containing data field as '*c*'

Answer Keys

1. A	**2.** C	**3.** B	**4.** A	**5.** B	**6.** C	**7.** B	**8.** B	**9.** A	**10.** B
11. D	**12.** A	**13.** B	**14.** B	**15.** D	**16.** A	**17.** B	**18.** D	**19.** B	**20.** C
21. A	**22.** B	**23.** C	**24.** C	**25.** D					

HINTS AND EXPLANATIONS

1. Array can be used to represent all the above data structures. Choice (A)

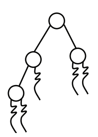
2. Min Heap is a complete Binary tree, with parent node value less than both the child nodes if they exist. Choice (C)

3. Inorder is the traversal of tree which prints data elements in an ascending order. Choice (B)

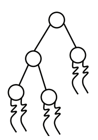
4. Graphs uses adjacency matrix (or) linked representation. Choice (A)

5. AVL tree is binary search tree with balancing factor as 0, +1, –1 and search time as O (log n). Choice (B)

7.



= 5 (for 4 nodes)



= 6 (for 5 nodes)

$\therefore$ For n nodes it will be $(n + 1)$ Null links. Choice (B)

8. 2 stacks required; use one for the Enqueue and the other for Dequeue. Choice (B)

9. Number of additions in a fibonacci sequence will be fib $(n + 1) - 1$. Choice (A)

10. For unique pattern of tree either (pre order + in order) or (post order + in order) are } necessary. Choice (B)

11. Traverse the routines on the tree. Choice (D)

12. Traverse the routines on the tree. Choice (A)

13. $a + b * c / d \wedge e \wedge f - g$

$a + b * c / d \wedge \wedge e f - g$

$a + b * c / \wedge d \wedge e f - g$

$a + * b c / \wedge d \wedge e f - g$

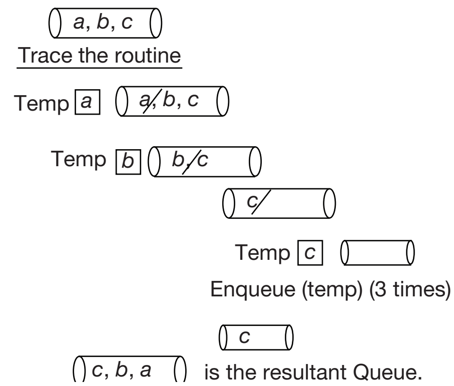
$a + / * b c \wedge d \wedge e f - g$

$+ a / * b c \wedge d \wedge e f - g$

$- + a / * b c \wedge d \wedge e f g.$

Choice (B)

14. Let us consider a queue with 3 elements a, b, c.

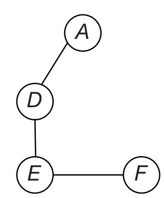


Choice (B)

15. Circular queue deletion routine. Choice (D)

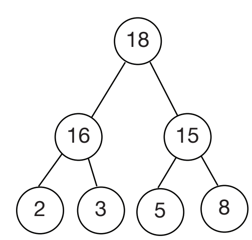
16. It reverses the list. Choice (A)

17. Let us consider option (B)



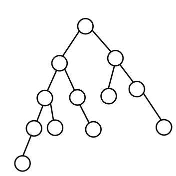
At (F) it can't reach to vertex (C) as it is not adjacent to (F). Choice (B)

18. Let us consider a max heap



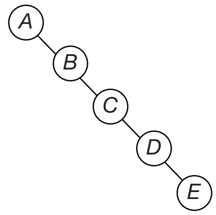
As the maximum element is at the root. To attain root element it takes $O(1)$ time. Choice (D)

19.



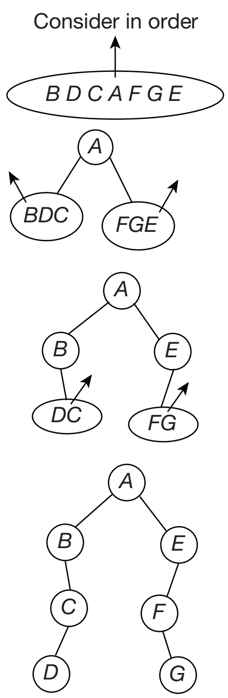
Choice (B)

20. Let us consider 5 nodes



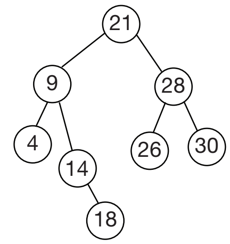
To search an element (E) it has to compare 5 elements
∴ it is $O(n)$. Choice (C)

21. POST: *DCBGFEA*
IN: *BDCAFGE*



Consider in order Choice (A)

22. From the above elements AVL tree will be:



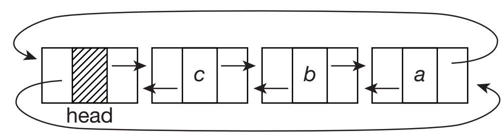
Choice (B)

23. Post order for the above tree is 4 18 14 9 26 30 28 21.
Choice (C)

24. The $A(\ )$ routine will reverse the given list
∴ Head pointer maps to node containing data field as 'c'.

Choice (C)

25.



Traversing the above list with given operation make to point to node containing c. Choice (D)

Chapter 1

Asymptotic Analysis

LEARNING OBJECTIVES

- *Algorithm*
- *Recursive algorithms*
- *Towers of Hanoi*
- *Time complexity*
- *Space complexity*
- *SET representation*
- *TREE representation*
- *Preorder traversal*
- *Post-order traversal*
- *In order traversal*
- *Data structure*
- *Worst-case and average-case analysis*
- *Asymptotic notations*
- *Notations and functions*
- *Floor and ceil*
- *Recurrence*
- *Recursion-tree method*
- *Master method*

Algorithm

An algorithm is a finite set of instructions that, if followed, accomplishes a particular task.

All algorithms must satisfy the following.

- Input: Zero or more quantities are externally supplied.
- Output: Atleast one quantity is produced.
- Definiteness: Each instruction should be clear and unambiguous.
- Finiteness: The algorithm should terminate after finite number of steps.
- Effectiveness: Every instruction must be very basic.

Once an algorithm is devised, it is necessary to show that it computes the correct answer for all possible inputs. This process is called algorithm validation. Analysis of algorithms refers to the task of determining how much computing time and storage an algorithm requires.

Analyzing Algorithms

The process of comparing 2 algorithms rate of growth with respect to time, space, number of registers, network, bandwidth etc is called analysis of algorithms.

This can be done in two ways

1. **Priori Analysis:** This analysis is done before the execution; the main principle behind this is frequency count of fundamental instruction.

   This analysis is independent of CPU, OS and system architecture and it provides uniform estimated values.
2. **Posterior analysis:** This analysis is done after the execution. It is dependent on system architecture, CPU, OS etc. it provides non-uniform exact values.

Recursive Algorithms

A recursive function is a function that is defined in terms of itself. An algorithm is said to be recursive if the same algorithm is invoked in the body.

Towers of Hanoi

There was a diamond tower (labeled *A*) with 64-golden disks. The disks were of decreasing size and were stacked on the tower in decreasing order of size bottom to top. Besides this tower there were 2 other diamond towers (labeled *B* and *C*) we have to move the disks from tower A to tower *B* using tower *C* for intermediate storage. As the disks are very heavy, they can be moved only one at a time. No disk can be on top of a smaller disk .

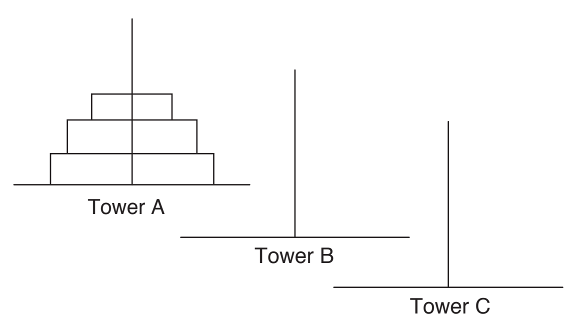
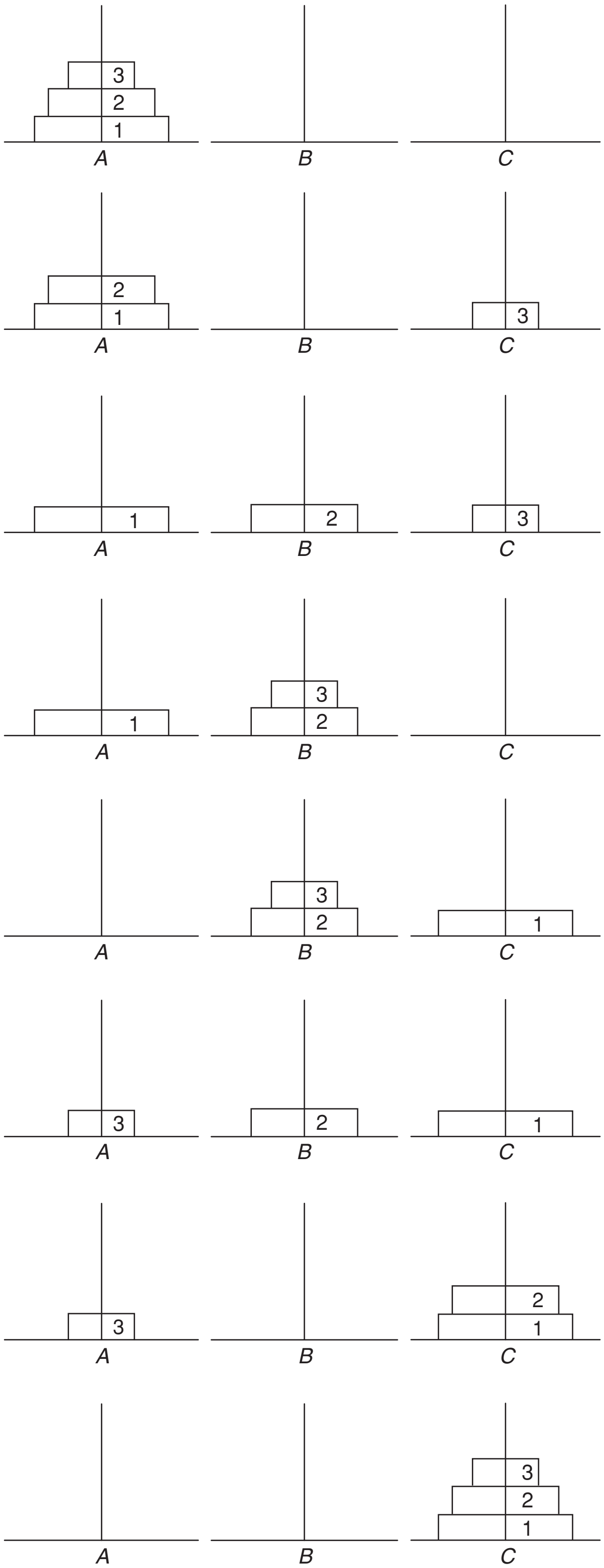


Figure 1 Towers of Hanoi

Assume that the number of disks is '*n*'. To get the largest disk to the bottom of tower *B*, we move the remaining (*n* – 1) disks to tower *C* and then move the largest to tower *B*. Now move the disks from tower *C* to tower *B*.

Example:



To move '3' disks from tower A to tower '*C*' requires 7 disk movements

∴ For '*n*' disks, the number of disk movements required is $2^n - 1 = 2^3 - 1 = 7$

Time complexity

$$T(n) = 1 + 2T(n-1)$$
$$T(n) = 1 + 2(1 + 2\,(T(n-2)))$$
$$T(n) = 1 + 2 + 2^2\,T(n-2)$$
$$T(n) = 1 + 2 + 2^2\,(1 + 2T(n-3))$$
$$T(n) = 1 + 2 + 2^2 + 2^3 + T(n-3)$$
$$T(n) = 1 + 2 + 2^2 + \cdots + 2^{i-1} + 2^i\,\mathrm{T}(n-i)$$

$$T(n) = \sum_{i=0}^{n-1} 2^i$$

The time complexity is exponential, it grows as power of 2.

$$\therefore \quad T(n) \cong O(2^n)$$

Space complexity

The space complexity of an algorithm is the amount of memory it needs to run to completion. The measure of the quantity of input data is called the size of the problem. For example, the size of a matrix multiplication problem might be the largest dimension of the matrices to be multiplied. The size of a graph problem might be the number of edges. The limiting behavior of the complexity as size increases is called the asymptotic time complexity.

- It is the asymptotic complexity of an algorithm which ultimately determines the size of problems that can be solved by the algorithm.
- If an algorithm processes inputs of size '*n*' in time cn^2 for some constant *c*, then we say that the time complexity of that algorithm is $O(n^2)$, more precisely a function $g(n)$ is said to be $O(f(n))$ if there exists a constant *c* such that $g(n) \le c(f(n))$ for all but some finite set of non-negative values for *n*.
- As computers become faster and we can handle larger problems, it is the complexity of an algorithm that determines the increase in problem size that can be achieved with an increase in computer speed.
- Suppose we have 5 algorithms Algorithm 1 – Algorithm 5 with the following time complexities.

Algorithm	Time Complexity
Algorithm – 1	n
Algorithm – 2	$n \log n$
Algorithm – 3	n^2
Algorithm – 4	n^3
Algorithm – 5	$2n$

The time complexity is, the number of time units required to process an input of size '*n*'. Assume that input size '*n*' is 1000 and one unit of time equals to 1 millisecond.

The following figure gives the sizes of problems that can be solved in one second, one minute, and one hour by each of these five algorithms.

Algorithm	Time Complexity	Maximum Problem Size 1 sec	1 min	1 hour
Algorithm – 1	n	1000	6×10^4	3.6×10^6
Algorithm – 2	$n \log n$	140	4893	2.0×10^5
Algorithm – 3	n^2	31	244	1897
Algorithm – 4	n^3	10	39	153
Algorithm – 5	$2n$	9	15	21

From the above table, we can say that different algorithms will give different results depending on the input size. Algorithm – 5 would be best for problems of size $2 \le n \le 9$, Algorithm – 3 would be best for $10 \le n \le 58$, Algorithm – 2 would be best for $59 \le n \le 1025$, and Algorithm – 1 is best for problems of size greater than 1024.

Set Representation

A common use of a list is to represent a set, with this representation the amount of memory required to represent a set is proportional to the number of elements in the set. The amount of time required to perform a set operation depends on the nature of the operation.

- Suppose A and B are 2 sets. An operation such as $A \cap B$ requires time atleast proportional to the sum of the sizes of the 2 sets, since the list representing A and the list representing B must be scanned atleast once.
- The operation $A \cup B$ requires time atleast proportional to the sum of the set sizes, we need to check for the same element appearing in both sets and delete one instance of each such element.
- If A and B are disjoint, we can find $A \cup B$ in time independent of the size of A and B by simply concatenating the two lists representing A and B.

Graph Representation

A graph $G = (V, E)$ consists of a finite, non-empty set of vertices V and a set of edges E. If the edges are ordered pairs (V, W) of vertices, then the graph is said to be directed; V is called the tail and W the head of the edge (V, W). There are several common representations for a graph $G = (V, E)$. One such representation is adjacency matrix, $a|V|X|V|$ matrix M of 0's and 1's, where the ij_{th} element, $m[i, j] = 1$, if and only if there is an edge from vertex i to vertex j.

- The adjacency matrix representation is convenient for graph algorithms which frequently require knowledge of whether certain edges are present.
- The time needed to determine whether an edge is present is fixed and independent of $|V|$ and $|E|$.
- Main drawback of using adjacency matrix is that it requires $|V|^2$ storage even if the graph has only $O(|V|)$ edges.
- Another representation for a graph is by means of lists. The adjacency list for a vertex v is a list of all vertices W adjacent to V. A graph can be represented by $|V|$ adjacency lists, one for each vertex.

Example:

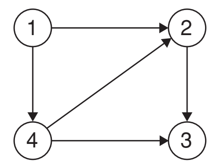


Figure 2 Directed graph

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Figure 3 Adjacency matrix

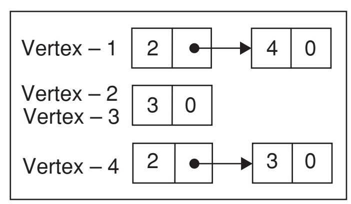


Figure 4 Adjacency lists

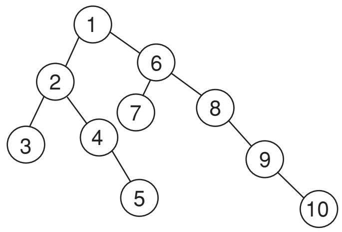
There are edges from vertex – 1 to vertex – 2 and 4, so the adjacency list for 1 has items 2 and 4 linked together in the format given above.

- The adjacency list representation of a graph requires storage proportional to $|V| + |E|$, this representation is used when $|E| << |V|^2$.

Tree Representation

A directed graph with no cycles is called a directed acyclic graph. A directed graph consisting of a collection of trees is called a forest. Suppose the vertex 'v' is root of a sub tree, then the depth of a vertex 'v' in a tree is the length of the path from the root to 'v'.

- The height of a vertex 'v' in a tree is the length of a longest path from 'v' to a leaf.
- The height of a tree is the height of the root
- The level of a vertex 'v' in a tree is the height of the tree minus the depth of 'v'.



	Left child	Right child
1	2	6
2	3	4
3	0	0
4	0	5
5	0	0
6	7	8
7	0	0
8	0	9
9	0	10
10	0	0

Figure 5 A binary tree and its representation

- Vertex 3 is of depth '2', height '0' and the level is 2 (Height of tree – depth of '3' = 4 – 2 = 2).
- A binary tree is represented by 2 arrays: left child and right child.
- A binary tree is said to be complete if for some integer k, every vertex of depth less than k has both a left child and a right child and every vertex of depth k is a leaf. A complete binary tree of height k has exactly $(2^{k+1} - 1)$ vertices.
- A complete binary tree of height k is often represented by a single array. Position 1 in the array contains the root. The left child of the vertex in position 'i' is located at position '$2i$' and the right child at position '$2i + 1$'.

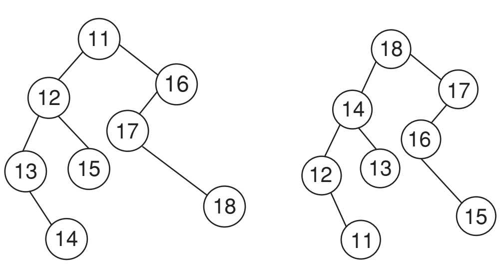
Tree Traversals

Many algorithms which make use of trees often traverse the tree in some order. Three commonly used traversals are pre-order, postorder and inorder.

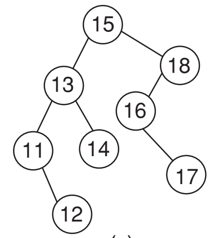
Pre-order Traversal

A pre-order traversal of T is defined recursively as follows:

1. Visit the root.
2. Visit in pre-order the sub trees with roots $v_{1,}\ v_2 \ldots v_k$ in that order.



(a) (b)



(c)

Figure 6 (a) Pre-order, (b) Post-order **(c)** In-order

Post-order traversal

A post-order traversal of T is defined recursively as follows:

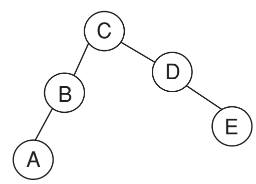
1. Visit in post-order the sub trees with roots v_1, v_2, v_3, … v_k in that order.
2. Visit the root r.

In-order Traversal

An in-order traversal is defined recursively as follows:

1. Visit in in-order the left sub tree of the root 'r'.
2. Visit 'r'.
3. Visit in inorder the right sub tree of r.

Example: Consider the given tree



What are the pre-order, post-order and in-order traversals of the above tree?

Solution: Pre-order – CBADE
Post-order – ABEDC
In-order – ABCDE

Data Structure

A data structure is a way to store and organize data in-order to facilitate access and modifications. No single data structure works well for all purposes, it is important to know the strengths and limitations of several data structures.

Efficiency

Algorithms devised to solve the same problem often differ dramatically in their efficiency. Let us compare efficiencies of Insertion sort and merge sort; insertion sort, takes time equal to C_1n^2 to sort 'n' elements, where C_1 is a constant that does not depend on 'n'. It takes time proportional to n^2, merge sort takes time equal to $C_2n\log n$, C_2 is another constant that also does not depend on 'n'. Insertion sort has a smaller constant factor than merge sort $(C_1 < C_2)$ constant factors are far less significant in the running time.

Merge sort has a factor of 'log n' in its running time, insertion sort has a factor of 'n', which is much larger. Insertion sort is faster than merge sort for small input sizes, once the input size 'n' becomes large enough, merge sort will perform better. No matter how much smaller C_1 is than C_2. There will always be a crossover point beyond which merge sort is faster.

Example: Consider 2 computers, computer A (faster computer), B (slower computer). Computer A runs insertion sort and computer B runs merge sort. Each computer is given 2 million numbers to sort. Suppose that computer A executes one billion instruction per second and computer B executes only 10 million instructions per second, computer A is 100 times faster than computer B ($C_1 = 4$, $C_2 = 50$). How much time is taken by both the computers?

Solution: Insertion sort takes $C_1 * n^2$ time
Merge sort takes $C_2 * n * \log n$ time
$C_1 = 4, C_2 = 50$
Computer A takes

$$\frac{4\times(2\times10^6)^2 \text{ instructions}}{10^9 \text{ instructions/second}} \cong 4000 \text{ seconds}$$

Computer B takes

$$= \frac{50\times2\times10^6\times\log(2\times10^6)\text{ insturctions}}{10^7\text{ instructions/second}}$$
$$= 209 \text{ seconds}$$

By using an algorithm whose running time grows more slowly, even with an average compiler, computer B runs <u>20</u> times faster than computer A. The advantage of merge sort is even more pronounced when we sort ten million numbers. As the problem size increases, so does the relative advantage of merge sort.

Worst-case and average-case analysis

In the analysis of insertion sort, the best case occurs when the array is already sorted and the worst case, in which the input array is reversely sorted. We concentrate on finding the worst-case running time, that is the longest running time for any input of size 'n'.

- The worst-case running time of an algorithm is an upper bound on the running time for any input. It gives us a guarantee that the algorithm will never take any longer.
- The 'average-case' is as bad as the worst-case. Suppose that we randomly choose 'n' numbers and apply insertion sort. To insert an element $A[j]$, we need to determine where to insert in sub-array $A\ [1 \cdots J-1]$. On average half the elements in $A[1 \cdots J-1]$ are less than $A[j]$ and half the elements are greater. So $t_j = j/2$. The average-case running time turns out to be a quadratic function of the input size.

Asymptotic Notations

Asymptotic notations are mostly used in computer science to describe the asymptotic running time of an algorithm. As an example, an algorithm that takes an array of size n as input and runs for time proportional to n^2 is said to take $O(n^2)$ time.

5 Asymptotic Notations:

- O (Big-oh)
- θ (Theta)
- Ω (Omega)
- o (Small-oh)
- ω

How to Use Asymptotic Notation for Algorithm Analysis?

Asymptotic notation is used to determine rough estimates of relative running time of algorithms. A worst-case analysis of any algorithm will always yeild such an estimate, because it gives an upper bound on the running time $T(n)$ of the algorithm, that is $T(n)\ \ g(n)$.

Example:

$a \leftarrow 0$	1 unit	1 time
for $i \leftarrow 1$ to n do{	1 unit	n times
for $j \leftarrow 1$ to i do{	1 unit	$n(n+1)/2$ times
$a \leftarrow a+1$	1 unit	$n(n+1)/2$ times

Where the times for the inner loop have been computed as follows: For each i from 1 to n, the loop is executed i times, so the total number of times is $1+2+3+\cdots+n = \sum_{i=1}^{n} i = n(n+1)/2$

Hence in this case
$T(n) = 1 + n + 2n\ (n+1)/2 = n^2 + 2n + 1$
If we write $g(n) = n^2 + 2n + 1$, then $T(n) \in \theta(g(n))$,
That is $T(n) \in \theta(n^2 + 2n + 1)$, we actually write $T(n) \in \theta(n^2)$, as recommended by the following rule:

- Although the definitions of asymptotic notation allow one to write, for example, $T(n) \in O(3n^2 + 2)$.

We simplify the function in between the parentheses as much as possible (in terms of rate of growth), and write instead $T(n) \in O(n^2)$
For example: $T(n) \in \theta(4n^3 - n^2 + 3)$
$T(n) \in \theta(n^3)$

For instance $O\left(\sum_{i=1}^{n} i\right)$, write $O(n^2)$ after computing the sum.

- In the spirit of the simplicity rule above, when we are to compare, for instance two candidate algorithms A and B having running times ($T_A(n) = n^2 - 3n + 4$ and $T_B(n) = 5n^3 + 3$, rather than writing $T_A(n) \in O(T_B(n))$, we write $T_A(n) \in \theta(n^2)$, and $T_B(n) \in \theta(n^3)$, and then we conclude that A

is better than B, using the fact that n^2(quadratic) is better than n^3(cubic) time, since $n^2 \in O(n^3)$.

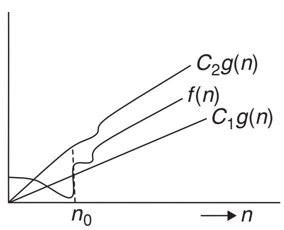
Order of Growth

In the rate of growth or order of growth, we consider only the leading term of a formula. Suppose the worst case running time of an algorithm is $an^2 + bn + c$ for some constants a, b and c. The leading term is an^2. We ignore the leading term's constant coefficient, since constant factors are less significant than the rate of growth in determining computational efficiency for large inputs. Thus we can write, the worst-case running time is $\theta(n^2)$.

We usually consider one algorithm to be more efficient than another if its worst-case running time has a lower order of growth. Due to constant factors and lower order terms, this evaluation may be in error for small inputs. But for large inputs, $\theta(n^2)$ algorithm will run more quickly in the worst-case than $\theta(n^3)$ algorithm.

θ-Notation

A function $f(n)$ belongs to the set $\theta(g(n))$ if there exists a positive constant C_1 and C_2 such that it can be "sand witched" between $C_1g(n)$ and $C_2g(n)$ for sufficiently large n. We write $f(n) \in \theta(g(n))$ to indicate that $f(n)$ is a member of $\theta(g(n))$ or we can write $f(n) = \theta(g(n))$ to express the same notation.



The above figure gives an intuitive picture of functions $f(n)$ and $g(n)$, where we have that $f(n) = \theta(g(n))$, for all the values of 'n' to the right of n_o, the value of $f(n)$ lies at or above $C_1g(n)$ and at or below $C_2g(n)$. $g(n)$ is asymptotically tight bound for $f(n)$. The definition of $\theta(g(n))$ requires that every member $f(n) \in \theta(g(n))$ be asymptotically non-negative, that is $f(n)$ must be non-negative whenever 'n' is sufficiently large.

The θ-notation is used for asymptotically bounding a function from both above and below. We would use θ(theta) notation to represent a set of functions that bounds a particular function from above and below.

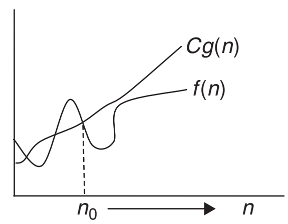
Definition: We say that a function $f(n)$ is theta of $g(n)$ written as $f(n) = \theta(g(n))$ if such exists positive constants C_1, C_2 and n_0 such that $0 \le C_1g(n) \le f(n) \le C_2\, g(n)$, $\forall\, n \ge n_0$.

Example: Let $f(n) = 5.5n^2 - 7n$, verify whether $f(n)$ is $\theta(n^2)$. Lets have constants $c_1 = 9$ and $n_0 = 2$, such that $0 \le f(n) \le C_1\, n^2$, $\forall n \ge n_0$. From example, 4 we have constants $C_2 = 3$, and $n_0 = 2.8$, such that $0 \le C_2\, n^2 \le f(n)$, $\forall n \ge n_0$. To show $f(n)$ is $\theta(n^2)$, we have got hold of two constants C_1 and C_2. We fix the n_0 for θ as maximum $\{2, 2.8\} = 2.8$.

- The lower order terms of an asymptotically positive function can be ignored in determining asymptotically tight bounds because they are insignificant for large n.
- A small fraction of the highest order term is enough to dominate the lower order term. Thus setting C_1 to a value that is slightly smaller than the coefficient of the highest order term and setting C_2 to a value that is slightly larger permits the inequalities in the definition of θ-notation to be satisfied. If we take a quadratic function $f(n) = an^2 + bn + c$, where a, b and c are constants and $a > 0$. Throwing away the lower order terms and ignoring the constant yields $f(n) = \theta(n^2)$.
- We can express any constant function as $\theta(n^0)$, or $\theta(1)$ we shall often use the notation $\theta(1)$ to mean either a constant or a constant function with respect to some variable.

O-Notation

We use O-notation to give an upper bound on a function, within a constant factor.



The above figure shows the intuition behind O-notation. For all values 'n' to the right of n_0, the value of the function $f(n)$ is on or below $g(n)$. We write $f(n) = O(g(n))$ to indicate that a function $f(n)$ is a member of the set $O(g(n))$.

$f(n) = \theta(g(n))$ implies $f(n) = O(g(n))$. Since θ notation is stronger notation than O-notation set theoretically, we have $\theta(g(n)) \subseteq O(g(n))$. Thus any quadratic function $an^2 + bn + c$, where $a > 0$, is in $\theta(n^2)$ also shows that any quadratic function is in $O(n^2)$ when we write $f(n) = O(g(n))$, we are claiming that some constant multiple of $g(n)$ is an asymptotic upper bound on $f(n)$, with no claim about how tight an upper bound it is.

The O-notation is used for asymptotically upper bounding a function. We would use O (big-oh) notation to represent a set of functions that upper bounds a particular function.

Definition We say that a function $f(n)$ is big oh of $g(n)$ written as $f(n) = O(g(n))$ if there exists positive constants C and n_0 such that

$$0 \le f(n) \le Cg(n),\ \forall\ n \ge n_o$$

Solved Examples

Example 1: let $f(n) = n^2$

Then $f(n) = O(n^2)$

$f(n) = O(n^2 \log n)$

$f(n) = O(n^{2.5})$

$f(n) = O(n^3)$

$f(n) = O(n^4) \cdots$ so on.

Example 2: Let $f(n) = 5.5n^2 - 7n$, verity whether $f(n)$ is $O(n^2)$

Solution: Let C be a constant such that

$$5.5n^2 - 7n \leq Cn^2, \text{or } n \geq \frac{7}{c - 5.5}$$

Fix $C = 9$, to get $n \geq 2$
So our $n_0 = 2$ and $C = 9$
This shows that there exists, positive constants $C = 9$ and $n_0 = 2$ such that
$0 \leq f(n) \leq Cn^2$, $\forall\ n \geq n_0$

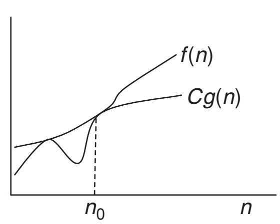
Example 3:

$$h(n) = 3n^3 + 10n + 1000 \log n \in O(n^3)$$

$$h(n) = 3n^3 + 10n + 1000 \log n \in O(n^4)$$

- Using O-notation, we can describe the running time of an algorithm by inspecting the algorithm's overall structure. For example, the doubly nested loop structure of the insertion sort algorithm yields an $O(n^2)$ upper bound on the worst-case running time. The cost of each iteration of the inner loop is bounded from above by $O(1)$ (constant), the inner loop is executed almost once for each of the n^2 pairs.
- $O(n^2)$ bound on worst-case running time of insertion sort also applies to its running time on every input.
- The $\theta(n^2)$ bound on the worst-case running time of insertion sort, however, does not imply a $\theta(n^2)$ bound on the running time of insertion sort on every input, when the input is already sorted, insertion sort runs in $\theta(n)$ time.

Ω *(omega)-notation*



The Ω-notation is used for asymptotically lower bounding a function. We would use Ω(big-omega) notation to represent a set of functions that lower bounds a particular function.

Definition We say that a function $f(n)$ is big-omega of $g(n)$ written as $f(n) = \Omega(g(n))$ if there exists positive constants C and n_0 such that

$$0 \leq Cg(n) \leq f(n)\ ,\ \forall\ n \geq n_0$$

The intuition behind Ω-notation is shown in the above figure. For all values 'n' to the right of n_0, the value of $f(n)$ is on or above $Cg(n)$. For any 2 functions $f(n)$ and $g(n)$ we have $f(n) = \theta(g(n))$ if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$. From the above statement we can say that, $an^2 + bn + c = \theta(n^2)$ for any constants a, b and c, where $a > 0$, immediately implies that

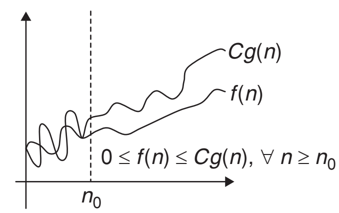
$$\therefore\ an^2 + bn + c = \Omega(n^2)$$

$$\therefore\ an^2 + bn + c = O(n^2)$$

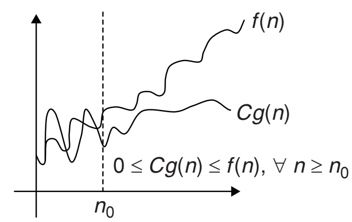
Example 4: Let $f(n) = 5.5n^2 - 7n$.
Verity whether $f(n)$ is $\Omega(n^2)$

Solution: Let C be a constant such that $5.5n^2 - 7n \geq Cn^2$ or $n \geq \frac{7}{5.5 - C}$. Fix $C = 3$, to get $n \geq 2.8$. So, our $n_0 = 2.8$ and $C = 3$

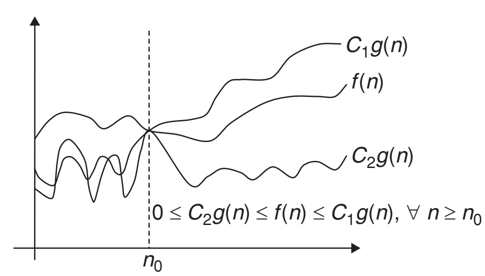
This shows that there exists positive constants $C = 3$ and $n_0 = 2.8$, such that $0 \leq Cn^2 \leq f(n)$, $\forall n \geq n_0$.



(a) $f(n) = O(g(n))$



(b) $f(n) = \Omega(g(n))$



(c) $f(n) = \theta(g(n))$

Figure 7 A diagrammatic representation of the asymptotic notations O, W and q

- Ω-notation describes a lower bound; it is used to bound the best-case running time of an algorithm. The best-case running time of insertion sort is $\Omega(n)$. The running time of insertion sort falls between $\Omega(n)$ and $O(n^2)$, since it falls anywhere between a linear function of 'n' and a quadratic function of 'n'.

- When we say that the running time of an algorithm is $\Omega(g(n))$, we mean that no matter what particular input of size 'n' is chosen for each value of n, the running time on that input is at least a constant times $g(n)$, for sufficiently large 'n'.

O-notation

The asymptotic upper bound provided by O-notation may or may not be asymptotically tight. The bound $2n^3 = O(n^3)$ is asymptotically tight, but the bound $2n = O(n^2)$ is not. We use O-notation to denote an upper bound that is not asymptotically tight.

ω-notation

By analogy, ω-notation is to Ω-notation as o-notation is to O-notation. We use ω-notation to denote a lower bound that is not asymptotically tight.

It is defined as

$f(n) \in \omega(g(n))$ if and only if $g(n) \in o(f(n))$

Comparison of functions

Transitivity

1. $f(n) = \theta(g(n))$ and $g(n) = \theta(h(n))$
$\Rightarrow f(n) = \theta(h(n))$
2. $f(n) = O(g(n))$ and $g(n) = O(h(n))$
$\Rightarrow f(n) = O(h(n))$
3. $f(n) = \Omega(g(n))$ and $g(n) = \Omega(h(n))$
$\Rightarrow f(n) = \Omega(h(n))$
4. $f(n) = o(g(n))$ and $g(n) = o(h(n))$
$\Rightarrow f(n) = o(h(n))$
5. $f(n) = \omega(g(n))$ and $g(n) = \omega(h(n))$
$\Rightarrow f(n) = \omega(h(n))$

Reflexivity

1. $f(n) = \theta(f(n))$
2. $f(n) = O(f(n))$
3. $f(n) = \Omega(f(n))$

Symmetry

$f(n) = \theta(g(n))$ if and only if $g(n) = \theta(f(n))$

Transpose symmetry

1. $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$
2. $f(n) = o(g(n))$ if and only if $g(n) = \omega(f(n))$

Notations and Functions

Floor and Ceil

For any real number 'x', we denote the greatest integer less than or equal to x by $\lfloor x \rfloor$ called as floor of x and the least integer greater than or equal to x by $\lceil x \rceil$ called as ceiling of x.

$x-1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1$ for any integer n,

$$\left\lceil \frac{n}{2} \right\rceil + \left\lceil \frac{n}{2} \right\rceil = n,$$

For any real number $n \ge 0$ and integer $a, b > 0$

$$\left\lceil \frac{\left\lceil \frac{n}{a} \right\rceil}{b} \right\rceil = \left\lceil \frac{n}{ab} \right\rceil$$

$$\left\lfloor \frac{\left\lfloor \frac{n}{a} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{n}{ab} \right\rfloor$$

Polynomials

Given a non-negative integer k, a polynomial in n of degree 'k' is a function $p(n)$ of the form $p(n) = \sum_{i=0}^{k} a_i n^i$

Where the constants $a_0, a_1, \ldots a_k$ are the coefficients of the polynomial and $a_k \ne 0$.

For an asymptotically positive polynomial $p(n)$ of degree k, we have $p(n) = \theta(n^k)$

Exponentials

For all real $a > 0$, m and n, we have the following identities:

$$a^0 = 1$$
$$a^1 = a$$
$$a^{-1} = \frac{1}{a}$$
$$(a^m)^n = a^{mn}$$
$$(a^m)^n = (a^n)^m$$
$$a^m a^n = a^{m+n}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

- For all real x, we have inequality $e^x \ge 1 + x$
- If $x = 0$, we have $1 + x \le e^x \le 1 + x + x^2$

Logarithms

$\lg n = \log_2 n$ (binary logarithm)
$\ln n = \log_e n$ (natural logarithm)
$\lg^k n = (\log n)^k$ (exponentiation)
$\lg \lg n = \lg (\lg n)$ (composition)
For all real $a > 0$, $b > 0$, $c > 0$ and n,
$\log_c (ab) = \log_c a + \log_c b$

$$\log_b a^n = n \log_b a$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log_b (1/a) = -\log_b a$$

$$\log_b a = \frac{1}{\log_a b}$$

$$a^{\log_b^c} = c^{\log_b^a}$$

Factorials

$n!$ is defined for integers $n \geq 0$ as

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ (n-1)! * n & n > 0 \end{cases}$$

A weak upper bound on the factorial function is $n! \leq n^n$ since each of the n terms in the factorial product is almost n.

$$n! = o(n^n)$$
$$n! = \omega(2^n)$$
$$\lg(n!) = \theta(n \log n)$$

Iterated Logarithm

The notation $\lg^* n$ is used to denote the iterated logarithm. Let '$\lg^{(i)} n$' be as defined above, with $f(n) = \lg n$. The logarithm of a non-positive number is undefined, '$\lg^{(i)} n$' is defined only if $\lg^{(i-1)} n > 0$;

The iterated logarithm function is defined as $\lg^* n = \min \{i \geq 0 : \lg^{(i)} n \leq 1\}$. This function is a very slowly growing function.

$$\lg^* 2 = 1$$
$$\lg^* 4 = 2$$
$$\lg^* 16 = 3$$
$$\lg^* 65536 = 4$$
$$\lg^*(2^{65536}) = 5$$

Recurrences

When an algorithm contains a recursive call to itself, its running time can often be described by a recurrence. A recurrence is an equation that describes a function in terms of its value on smaller inputs. For example, the worst-case running time $T(n)$ of the merge-sort can be described as

$$T(n) = \begin{cases} \theta(1) & \text{if } n = 1 \\ 2T(n/2) + \theta(n) & \text{if } n > 1 \end{cases}$$

The time complexity of merge-sort algorithm in the worst-case is $T(n) = \theta(n \log n)$

There are 3 methods to solve recurrence relations:

1. Substitution method
2. Recursion-tree method
3. Master method

Substitution Method

In this method one has to guess the form of the solution. It can be applied only in cases when it is easy to guess the form of the answer. Consider the recurrence relation

$$T(n) = 2T(n/2) + n$$

We guess that the solution is $T(n) = O(n \log n)$ we have to prove that

$$T(n) \leq c\, n \log n \quad (\therefore c > 0)$$

Assume that this bound holds for $\lfloor n/2 \rfloor$

$$T(n/2) \leq c(n/2).\log(n/2) + n$$
$$T(n) \leq 2(c(n/2 \log(n/2)) + n$$
$$\leq cn \log n - cn \log 2 + n$$
$$\leq cn \log n - cn + n$$
$$\leq cn \log \qquad (\therefore c \geq 1)$$

Recursion-tree Method

In a recursion-tree, each node represents the cost of single sub problem somewhere in the set of recursive function invocations. We sum the costs within each level of the tree to obtain a set of per-level costs, and then we sum all the per-level costs to determine the total cost of all levels of the recursion. Recursion trees are useful when the recurrence describes the running time of a divide-and-conquer algorithm.

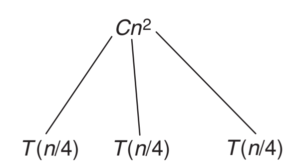
Example:

Consider the given recurrence relation

$$T(n) = 3T(n/4) + \theta(n^2)$$

We create a recursion tree for the recurrence

$$T(n) = 3T(n/4) + Cn^2$$



The Cn^2 term at the root represents the cost at the top level of recursion, and the three sub trees of the root represent the costs incurred by the sub problems of size $n/4$.

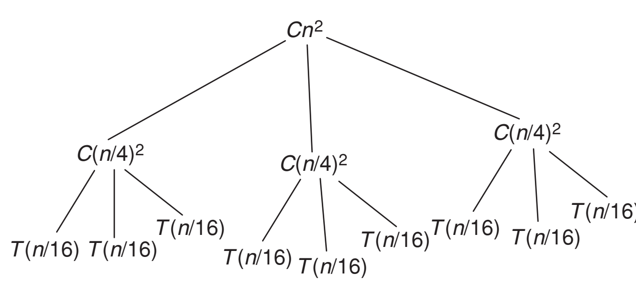


Figure 8 Recursion tree for $T(n) = 3T(n/4) + cn^2$

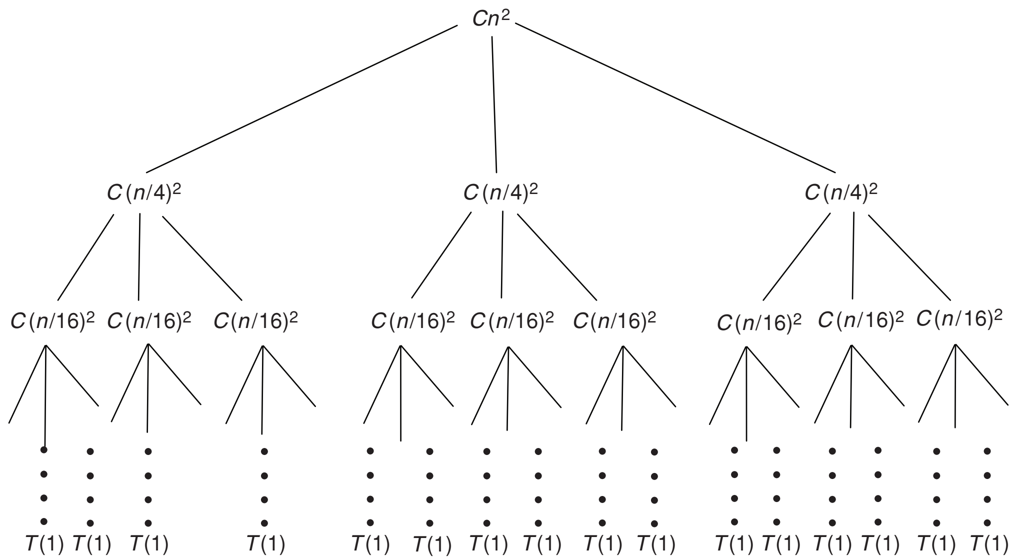


Figure 9 Expanded Recursion tree with height $\log_4^n$ ($\therefore$ levels $\log_4^n + 1$)

The sub-problem size for a node at depth 'i' is $n/4^i$, at this depth, the size of the sub-problem would be $n = 1$, when $n/4^i = 1$ or $i = \log_4^n$, the tree has $\log_4^{n+1}$ levels.

- We have to determine the cost at each level of the tree. Each level has 3 times more nodes than the level above, so the number of nodes at depth 'i' is 3^i.
- Sub problem sizes reduce by a factor of '4' for each level we go down from the root, each node at depth i, for $i = 0, 1, 2 \ldots \log_4^{n-1}$, has a cost of $c(n/4^i)^2$.

Total cost over all nodes at depth i, for $i = 0, 1, \ldots \log_4^{n-1}$

$$= 3^i * c\left(\frac{n}{4^i}\right)^2 = \left(\frac{3}{16}\right)^i cn^2$$

The last level, at depth $\log_4^n$ has 3^i nodes $= 3^{\log_4 n} = n^{\log_4 3}$ each contributing cost $T(1)$, for a total cost of $n^{\log_4 3}\, T(1)$, which is $\theta(n^{\log_4 3})$ cost of the entire tree is equal to sum of costs over all levels.

$$T(n) = cn^2 + \frac{3}{16}cn^2 + \left(\frac{3}{16}\right)^2 cn^2 + \cdots +$$

$$\left(\frac{3}{16}\right)^{\log_4^{n-1}} cn^2 + \cdots + \theta(n^{\log_4 3})$$

$$= \sum_{i=0}^{\log_4^{n-1}} \left(\frac{3}{16}\right)^i cn^2 + \theta(n^{\log_4 3})$$

$$< \sum_{i=0}^{\infty} \left(\frac{3}{16}\right)^i cn^2 + \theta(n^{\log_4 3})$$

$$= \frac{1}{1-\left(\frac{3}{16}\right)} cn^2 + \theta(n^{\log_4 3})$$

$$= \frac{16}{13}{}^2 cn + \theta(n^{\log_4 3}) = O(n^2)$$

Master Method

Let $a \geq 1$ and $b > 1$ be cons-tants, let $f(n)$ be a function and let $T(n)$ be defined on the non-negative integers by the recurrence

$$T(n) = aT(n/b) + f(n)$$

$T(n)$ can be bounded asymptotically as follows

1. If $f(n) = O(n^{\log_b a-\in})$ for some constant $\in > 0$, then $T(n) = \theta(n^{\log_b a})$
2. If $f(n) = \theta(n^{\log_b a})$ then $T(n) = \theta(n^{\log_b a}. \log n)$
3. If $f(n) = \Omega(n^{\log_b a+\in})$ for some constant $\in > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \theta(f(n))$.

Note: In the first case, not only must $f(n)$ be smaller than $n \log_b^a$, it must be polynomially smaller. That is, $f(n)$ must be asymptotically smaller than $n^{\log_b a}$ by a factor of $n^{\in}$, for some constant $\in > 0$.

In the third case, not only must $f(n)$ be larger than $n^{\log_b a}$, it must be polynomially larger and in addition satisfy the regularity condition $af(n/b) \leq Cf(n)$.

Example: Consider the given recurrence relation $T(n) = 9T(n/3) + n$.

To apply master theorem, the recurrence relation must be in the following form:

$$T(n) = aT(n/b) + f(n)$$

$$a = 9, b = 3, f(n) = n$$

$$n^{\log_b a} = n^{\log_3 9} = n^2$$

Since $f(n) = O(n^{\log_3 9-\in})$, where $\in = 1$

We can apply case 1 of the master theorem and the solution is $T(n) = \theta(n^2)$.

Exercises

Practice Problems 1

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. What is the time complexity of the recurrence relation $T(n) = 2T\left(\frac{n}{2}\right) + n^2$?
(A) $\theta(n^2)$ (B) $\theta(n)$
(C) $\theta(n^3)$ (D) $\theta(n \log n)$

2. What is the time complexity of the recurrence relation by using masters theorem $T(n) = 2T\left(\frac{n}{2}\right) + n$?
(A) $\theta(n^2)$ (B) $\theta(n)$
(C) $\theta(n^3)$ (D) $\theta(n \log n)$

3. What is the time complexity of the recurrence relation by using master theorem, $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$
(A) $\theta(n^2)$ (B) $\theta(n)$
(C) $\theta(n^3)$ (D) $(n^{0.51})$

4. What is the time complexity of the recurrence relation using master theorem, $T(n) = 7T\left(\frac{n}{3}\right) + n^2$?
(A) $\theta(n^2)$ (B) $\theta(n)$
(C) $\theta(n^3)$ (D) $(\log n)$

5. Time complexity of $f(x) = 4x^2 - 5x + 3$ is
(A) $O(x)$ (B) $O(x^2)$
(B) $O(x^{3/2})$ (D) $O(x^{0.5})$

6. Time complexity of $f(x) = (x^2 + 5 \log_2 x)/(2x + 1)$ is
(A) $O(x)$ (B) $O(x^2)$
(C) $O(x^{3/2})$ (D) $O(x^{0.5})$

7. For the recurrence relation, $T(n) = 2T(\lfloor \sqrt{n} \rfloor) + \lg n$, which is tightest upper bound?
(A) $T(n) = O(n^2)$ (B) $T(n) = O(n^3)$
(C) $T(n) = O(\log n)$ (D) $T(n) = O(\lg n \lg \lg n)$

8. Consider $T(n) = 9T(n/3) + n$, which of the following is TRUE?
(A) $T(n) = \theta(n^2)$ (B) $T(n) = \theta(n^3)$
(C) $T(n) = \Omega(n^3)$ (D) $T(n) = O(n)$

9. If $f(n)$ is $100 * n$ seconds and $g(n)$ is $0.5 * n$ seconds then
(A) $f(n) = g(n)$ (B) $f(n) = \Omega(g(n))$
(C) $f(n) = w(g(n))$ (D) None of these

10. Solve the recurrence relation using master method: $T(n) = 4T(n/2) + n^2$
(A) $\theta(n \log n)$ (B) $\theta(n^2 \log n)$
(C) $\theta(n^2)$ (D) $\theta(n^3)$

11. Arrange the following functions according to their order of growth (from low to high):
(A) $\sqrt[3]{n}, 0.001n^4 + 3n^3 + 1, 3^n, 2^{2n}$
(B) $3^n, 2^{2n}, \sqrt[3]{n}, 0.001n^4 + 3n^3 + 1$
(C) $2^{2n}, \sqrt[3]{n}, 3^n, 0.001n^4 + 3n^3 + 1$
(D) $\sqrt[3]{n}, 2^{2n}, 3^n, 0.001n^4 + 3n^3 + 1$

12. The following algorithm checks whether all the elements in a given array are distinct:

Input: array $A[0 \ldots n-1]$
Output: true (or) false
For $i \leftarrow 0$ to $n - 2$ do
For $j \leftarrow i + 1$ to $n - 1$ do
if $A[i] = A[j]$ return false
return true

The time complexity in worst case is
(A) $\theta(n^2)$ (B) $\theta(n)$
(C) $\theta(\log n)$ (D) $\theta(n \log n)$

13. The order of growth for the following recurrence relation is $T(n) = 4T(n/2) + n^3$, $T(1) = 1$
(A) $\theta(n)$ (B) $\theta(n^3)$
(C) $\theta(n^2)$ (D) $\theta(\log n)$

14. Time complexity of $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{3}$ is
(A) $\theta(\sqrt{n} \log n)$ (B) $\theta(\sqrt{n} \log \sqrt{n})$
(C) $\theta(\sqrt{n})$ (D) $\theta(n^2)$

15. Consider the following three claims
(I) $(n + k)^m = \theta(n^m)$, where k and m are constants
(II) $2^{n+1} = O(2^n)$
(III) $2^{2n+1} = O(2^n)$

Which one of the following is correct?
(A) I and III (B) I and II
(C) II and III (D) I, II and III

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Arrange the order of growth in ascending order:
(A) $O(1) > O(\log n) > O(n) > O(n^2)$
(B) $O(n) > O(1) > O(\log n) > O(n^2)$
(C) $O(\log n) > O(n) > O(1) > O(n^2)$
(D) $O(n^2) > O(n) > O(\log n) > O(1)$

2. $\sqrt{n} = \Omega(\log n)$ means
(A) To the least $\sqrt{n}$ is log n
(B) $\sqrt{n}$ is log n always
(C) $\sqrt{n}$ is at most log n
(D) None of these

3. Which of the following is correct?
(i) $\theta(g(n)) = O(g(n)) \cap \Omega(g(n))$
(ii) $\theta(g(n)) = O(g(n)) \cup \Omega(g(n))$

(A) (i) is true (ii) is false (B) Both are true
(C) Both are false (D) (ii) is true (i) is false

4. $2n^2 = x\,(n^3)$, x is which notation?
(A) Big-oh (B) Small-oh
(C) Ω – notation (D) θ – notation

5. Master method applies to recurrence of the form $T(n) = a\,T(n/b) + f(n)$ where
(A) $a \geq 1, b > 1$ (B) $a = 1, b > 1$
(C) $a > 1, b = 1$ (D) $a \geq 1, b \geq 1$

6. What is the time complexity of the recurrence relation using master method?

$$T(n) = 4T\left(\frac{n}{2}\right) + n$$

(A) $\theta(n^2)$ (B) $\theta(n)$
(C) $\theta(\log n)$ (D) $\theta(n \log n)$

7. Use the informal definitions of O, θ Ω to determine these assertions which of the following assertions are true.
(A) $n(n+1)/2 \in O(n^3)$ (B) $n(n+1)/2 \in O(n^2)$
(C) $n(n+1)/2 \in \Omega(n)$ (D) All the above

8. Match the following:

(i)	Big-oh	(A)	$\geq$
(ii)	Small-o	(B)	$\leq$
(iii)	Ω	(C)	$=$
(iv)	θ	(D)	$<$
(v)	ω	(E)	$>$

(A) (i) – D, (ii) – A, (iii) – C, (iv) –B , (v) – E
(B) (i) – B, (ii) – D, (iii) – A, (iv) – C, (v) – E
(C) (i) – C, (ii) – A, (iii) – B, (iv) – E, (v) – D
(D) (i) – A, (ii) – B, (iii) – C, (iv) – D, (v) – E

9. Which one of the following statements is true?
(A) Both time and space efficiencies are measured as functions of the algorithm input size.
(B) Only time efficiencies are measured as a function of the algorithm input size.
(C) Only space efficiencies are measured as a function of the algorithm input size.
(D) Neither space nor time efficiencies are measured as a function of the algorithm input size.

10. Which of the following is true?
(A) Investigation of the average case efficiency is considerably more difficult than investigation of the worst case and best case efficiencies.
(B) Investigation of best case is more complex than average case.
(C) Investigation of worst case is more complex than average case.
(D) None of these

11. Time complexity of $T(n) = T(n/3) + T(2n/3) + O(n)$ is
(A) $O(1)$
(B) $O(n \log n)$
(C) $O(\log n)$
(D) $O(n^2)$

12. Solve the recurrence relation to find $T(n)$: $T(n) = 4(n/2) + n$
(A) $\theta(n^2)$ (B) $\theta(\log_2 n)$
(C) $\theta(n^2 \log_2 n)$ (D) $\theta(n^3)$

13. What is the worst case analysis for the given code?

```
int search (int a[ ], int x, int n)
{
int i;
for (i = 0 ; i < n; i ++)
if (a [i] = = x)
return i;
return -1;
}
```

(A) $O(n)$ (B) $O(n \log n)$
(C) $O(\log n)$ (D) $O(n^2)$

14. Find the time complexity of the given code.

```
void f (int n)
{
if (n > 0)
{
f (n/2);
f (n/2);
}
}
```

(A) $\theta(n^2)$
(B) $\theta(n)$
(C) $\theta(n \log n)$
(D) $\theta(2^n)$

15. The running time of the following algorithm procedure
$A(n)$
if $n \leq 2$
return (1)
else
return $(A(\sqrt{n}))$
is described by

(A) $O(\sqrt{n} \log n)$
(B) $O(\log n)$
(C) $O(\log \log n)$
(D) $O(n)$

Previous Years' Questions

1. The median of n elements can be found in $O(n)$ time. Which one of the following is correct about the complexity of quick sort, in which median is selected as pivot? **[2006]**

(A) $\theta(n)$ (B) $\theta(n \log n)$
(C) $\theta(n^2)$ (D) $\theta(n^3)$

2. Given two arrays of numbers $a_1 \dots a_n$ and $b_1 \dots b_n$ where each number is 0 or 1, the fastest algorithm to find the largest span (i, j) such that $a_i + a_i + 1 + \dots + a_j = b_i + b_i + 1 + \dots + b_j$, or report that there is not such span, **[2006]**

(A) Takes $O(3^n)$ and $\Omega(2^n)$ time if hashing is permitted
(B) Takes $O(n^3)$ and $\Omega(n^{2.5})$ time in the key comparison model
(C) Takes $\Theta(n)$ time and space
(D) Takes $O(\sqrt{n})$ time only if the sum of the $2n$ elements is an even number

3. Consider the following segment of C-code:

```
int j, n;
    j = 1;
    while (j <=n)
j = j*2;
```

The number of comparisons made in the execution of the loop for any $n > 0$ is: **[2007]**

(A) $\lceil \log_2 n \rceil + 1$ (B) n
(C) $\lceil \log_2 + n \rceil$ (D) $\lfloor \log_2 n \rfloor + 1$

4. In the following C function, let $n \geq m$.

```
int gcd(n,m)
{
if (n%m = =0) return m;
      n = n%m;
return gcd(m,n);
}
```

How many recursive calls are made by this function? **[2007]**

(A) $\Theta(\log_2 n)$ (B) $\Omega(n)$
(C) $\Theta(\log_2 \log_2 n)$ (D) $\Theta(\sqrt{n})$

5. What is the time *complexity* of the following recursive function:

```
int DoSomething (int n) {
if (n <= 2)
    return 1;
  else
return(DoSomething (floor(sqrt(n)))+ n);}
```
[2007]

(A) $\Theta(n^2)$ (B) $\Theta(n \log_2 n)$
(C) $\Theta(\log_2 n)$ (D) $\Theta(\log_2 \log_2 n)$

6. An array of n numbers is given, where n is an even number. The maximum as well as the minimum of these n numbers needs to be determined. Which of the following is TRUE about the number of comparisons needed? **[2007]**

(A) At least $2n - c$ comparisons, for some constant c, are needed.
(B) At most $1.5n - 2$ comparisons are needed.
(C) At least $n \log_2 n$ comparisons are needed.
(D) None of the above.

7. Consider the following C code segment:

```
int IsPrime(n)
    {
    int i,n;
    for(i=2; i<= sqrt(n); i ++)
      if(n%i = =0)
       {printf("Not Prime\n"); return 0;}
    return 1;
}
```

Let $T(n)$ denote the number of times the *for* loop is executed by the program on input n. Which of the following is TRUE? **[2007]**

(A) $T(n) = O(\sqrt{n})$ and $T(n) = \Omega(\sqrt{n})$
(B) $T(n) = O(\sqrt{n})$ and $T(n) = \Omega(1)$
(C) $T(n) = O(n)$ and $T(n) = \Omega(\sqrt{n})$
(D) None of the above

8. The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity **[2008]**

(A) $\Theta(n)$ (B) $\Theta(m)$
(C) $\Theta(m + n)$ (D) $\Theta(mn)$

9. Consider the following functions:

$f(n) = 2^n$
$g(n) = n!$
$h(n) = n^{\log n}$

Which of the following statements about the asymptotic behavior of $f(n)$, $g(n)$, and $h(n)$ is true? **[2008]**

(A) $f(n) = O(g(n))$; $g(n) = O(h(n))$
(B) $f(n) = \Omega(g(n))$; $g(n) = O(h(n))$
(C) $g(n) = O(f(n))$; $h(n) = O(f(n))$
(D) $h(n) = O(f(n))$; $g(n) = \Omega(f(n))$

10. The minimum number of comparisons required to determine if an integer appears more than $n/2$ times in a sorted array of n integers is **[2008]**

(A) $\Theta(n)$ (B) $\Theta(\log n)$
(C) $\Theta(\log * n)$ (D) $\Theta(1)$

11. We have a binary heap on n elements and wish to insert n more elements (not necessarily one after another) into this heap. The total time required for this is **[2008]**

(A) $\Theta(\log n)$ (B) $\Theta(n)$
(C) $\Theta(n \log n)$ (D) $\Theta(n^2)$

12. The running time of an algorithm is represented by the following recurrence relation: **[2009]**

$$T(n) = \begin{cases} n & n \le 3 \\ T\left(\dfrac{n}{3}\right) + \text{cn} & \text{otherwise} \end{cases}$$

Which one of the following represents the time complexity of the algorithm?

(A) $\theta(n)$ (B) $\theta(n \log n)$
(C) $\theta(n^2)$ (D) $\theta(n^2 \log n)$

13. Two alternative packages A and B are available for processing a database having 10^k records. Package A requires $0.0001\ n^2$ time units and package B requires $10n \log_{10} n$ time units to process n records. What is the smallest value of k for which package B will be preferred over A? **[2010]**

(A) 12 (B) 10
(C) 6 (D) 5

14. An algorithm to find the length of the longest monotonically increasing sequence of numbers in an array $A[0 : n - 1]$ is given below.

Let L denote the length of the longest monotonically increasing sequence starting at index in the array.

Initialize $L_{n-1} = 1$,

For all i such that $0 \le i \le n-2$

$Li \{= 1 + L_{i+1}$, if $A[i] < A\ [i + 1]$,

1 otherwise

Finally the length of the longest monotonically increasing sequence is Max $(L_0, L_1, \ldots L_{n-1})$

Which of the following statements is TRUE? **[2011]**

(A) The algorithm uses dynamic programming paradigm.
(B) The algorithm has a linear complexity and uses branch and bound paradigm.
(C) The algorithm has a non-linear polynomial complexity and uses branch and bound paradigm.
(D) The algorithm uses divide and conquer paradigm.

15. Which of the given options provides the increasing order of asymptotic complexity of functions f_1, f_2, f_3 and f_4? **[2011]**

$f_1(n) = 2^n$

$f_2(n) = n^{3/2}$

$f_3(n) = n \log_2 n$

$f_4(n) = n^{\log_2 n}$

(A) f_3, f_2, f_4, f_1 (B) f_3, f_2, f_1, f_4
(C) f_2, f_3, f_1, f_4 (D) f_2, f_3, f_4, f_1

16. Let $W(n)$ and $A(n)$ denote respectively, the worst-case and average-case running time of an algorithm executed on input of size n. Which of the following is **ALWAYS TRUE**? **[2012]**

(A) $A(n) = \Omega(W(n))$ (B) $A(n) = \Theta(W(n))$
(C) $A(n) = O(W(n))$ (D) $A(n) = o(W(n))$

17. The recurrence relation capturing the optimal execution time of the Towers of Hanoi problem with n discs is **[2012]**

(A) $T(n) = 2T(n - 2) + 2$
(B) $T(n) = 2T(n - 1) + n$
(C) $T(n) = 2T(n/2) + 1$
(D) $T(n) = 2T(n - 1) + 1$

18. A list of n strings, each of length n, is sorted into lexicographic order using the merge sort algorithm. The worst-case running time of this computation is **[2012]**

(A) $O(n \log n)$ (B) $O(n^2 \log n)$
(C) $O(n^2 + \log n)$ (D) $O(n^2)$

19. Consider the following function:

```
int unknown (int n) {
    int i, j, k = 0;
    for (i = n/2; i < = n; i++)
            for (j = 2; j < = n; j = j*2)
                    k = k + n/2;
return (k);
}
```

The return value of the function is **[2013]**

(A) $\Theta(n^2)$ (B) $\Theta(n^2 \log n)$
(C) $\Theta(n^3)$ (D) $\Theta(n^3 \log n)$

20. The number of elements that can be sorted in $\Theta(\log n)$ time using heap sort is **[2013]**

(A) $\Theta(1)$

(B) $\Theta(\sqrt{\log n})$

(C) $\Theta\left(\dfrac{\log n}{\log \log n}\right)$

(D) $\Theta(\log n)$

21. Which one of the following correctly determines the solution of the recurrence relation with $T(1) = 1$

$T(n) = 2T\left(\dfrac{n}{2}\right) + \log n$? **[2014]**

(A) $\theta(n)$ (B) $\theta(n \log n)$
(C) $\theta(n^2)$ (D) $\theta(\log n)$

22. An algorithm performs $(\log N)^{1/2}$ find operations, N insert operations, $(\log N)^{1/2}$ delete operations, and $(\log N)^{1/2}$ decrease-key operations on a set of data items with keys drawn from a linearly ordered set. For a delete operation, a pointer is provided to the record that must be deleted For the decrease – key operation, a pointer is provided to the record that has its key decreased Which one of the following data structures is the most suited for the algorithm to use, if the

goal is to achieve the best total asymptotic complexity considering all the operations? **[2015]**
(A) Unsorted array
(B) Min-heap
(C) Sorted array
(D) Sorted doubly linked list

23. Consider the following C function.

```
int fun1(int n) {
   int i, j, k, p, q=0;
   for (i=1; i<n; ++i) {
         p=0;
         for (j=n; j>1; j=j/2)
               ++p;
         for (k=1; k<p; k=k*2)
               ++q;
         }
         return q;
   }
```

Which one of the following most closely approximates the return value of the function fun1? **[2015]**
(A) n^3 (B) $n(\log n)^2$
(C) $n \log n$ (D) $n \log(\log n)$

24. An unordered list contains n distinct elements. The number of comparisons to find an element in this list that is neither maximum nor minimum is **[2015]**
(A) $\theta(n \log n)$ (B) $\theta(n)$
(C) $\theta(\log n)$ (D) $\theta(1)$

25. Consider a complete binary tree where the left and the right subtrees of the root are max-heaps. The lower bound for the number of operations to convert the tree to a heap is **[2015]**
(A) $\Omega(\log n)$ (B) $\Omega(n)$
(C) $\Omega(n \log n)$ (D) $\Omega(n^2)$

26. Consider the equality $\sum_{i=0}^{n} i^3 =$ and the following choices for X
1. $\theta(n^4)$
2. $\theta(n^5)$
3. $O(n^5)$
4. $\Omega(n^3)$

The equality above remains correct if X is replaced by **[2015]**
(A) Only 1
(B) Only 2
(C) 1 or 3 or 4 but not 2
(D) 2 or 3 or 4 but not 1

27. Consider the following array of elements
<89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100>
The minimum number of interchanges needed to convert it into a max-heap is **[2015]**
(A) 4 (B) 5
(C) 2 (D) 3

28. Let $f(n) = n$ and $g(n) = n^{(1 + \sin n)}$, where n is a positive integer. Which of the following statements is/are correct? **[2015]**
I. $f(n) = O(g(n))$
II. $f(n) = \Omega(g(n))$
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

29. A queue is implemented using an array such that ENQUEUE and DEQUEUE operations are performed efficiently. Which one of the following statements is **CORRECT** (n refers to the number of items in the queue)? **[2016]**
(A) Both operations can be performed in O(1) time.
(B) At most one operation can be performed in O(1) time but the worst case time for the other operation will be Ω (n).
(C) The worst case time complexity for both operations will be Ω (n).
(D) Worst case time complexity for both operations will be Ω (logn).

30. Consider a carry look ahead adder for adding two n - bit integers, built using gates of fan - in at most two. The time to perform addition using this adder is **[2016]**
(A) $\Theta(1)$ (B) $\Theta(\log(n))$
(C) $\Theta\left(\sqrt{n}\right)$ (D) $\Theta(n)$

31. N items are stored in a sorted doubly linked list. For a *delete* operation, a pointer is provided to the record to be deleted. For a *decrease - key* operation, a pointer is provided to the record on which the operation is to be performed.

An algorithm performs the following operations on the list in this order: Θ (N) *delete*, O (logN) *insert*, O (log N) *find*, and Θ (N) *decrease - key*. What is the time complexity of all these operations put together? **[2016]**
(A) $O(\log^2 N)$ (B) O(N)
(C) $O(N^2)$ (D) $\Theta(N^2 \log N)$

32. In an adjacency list representation of an undirected simple graph G = (V,E), each edge *(u,v)* has two adjacency list entries:*[v]* in the adjacency list of u, and *[u]* in the adjacency list of v. These are called twins of each other. A twin pointer is a pointer from an adjacency list entry to its twin. If $|E| = m$ and $|V| = n$, and the memory size is not a constraint, what is the time complexity of the most efficient algorithm to set the twin pointer in each entry in each adjacency list? **[2016]**
(A) $\Theta(n^2)$ (B) $\Theta(n + m)$
(C) $\Theta(m^2)$ (D) $\Theta(n^4)$

33. Consider the following functions from positive integers to real numbers:

$$10,\ \sqrt{n},\ n,\ \log_2 n,\ \frac{100}{n}.$$

The CORRECT arrangement of the above functions in increasing order of asymptotic complexity is: **[2017]**

(A) $\log_2 n, \frac{100}{n}, 10, \sqrt{n}, n$

(B) $\frac{100}{n}, 10, \log_2 n, \sqrt{n}, n$

(C) $10, \frac{100}{n}, \sqrt{n}, \log_2 n, n$

(D) $\frac{100}{n}, \log_2 n, 10, \sqrt{n}, n$

34. Consider the recurrence function

$$T(n) = \begin{cases} 2T\left(\sqrt{n}\right)+1 & n > 2 \\ 2, & 0 < n \leq 2 \end{cases}$$

Then $T(n)$ in terms of Θ notation is **[2017]**

(A) $\Theta(\log \log n)$ (B) $\Theta(\log n)$

(C) $\Theta\left(\sqrt{n}\right)$ (D) $\Theta(n)$

35. Consider the following C function.

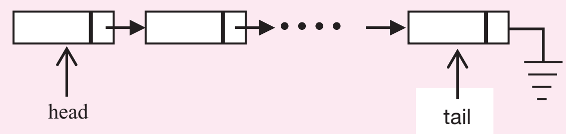
```
int fun (int n)   {
        int i, j;
        for(i = 1; i <= n; i++)   {
               for (j = 1; j < n; j += i) {
                      printf{" %d %d",i, j);
                      }
               }
}
```

Time complexity of fun in terms of Θ notation is **[2017]**

(A) $\Theta\left(n\sqrt{n}\right)$ (B) $\Theta(n^2)$

(C) $\Theta(n \log n)$ (D) $\Theta(n^2 \log n)$

36. A queue is implemented using a non-circular singly linked list. The queue has a head pointer and a tail pointer, as shown in the figure. Let n denote the number of nodes in the queue. Let enqueue be implemented by inserting a new node at the head, and dequeue be implemented by deletion of a node from the tail.



Which one of the following is the time complexity of the most time-efficient implementation of enqueue and dequeue, respectively, for this data structure? **[2018]**

(A) $\theta(1), \theta(1)$ (B) $\theta(1), \theta(n)$

(C) $\theta(n), \theta(1)$ (D) $\theta(n), \theta(n)$

37. Consider the following C code. Assume that unsigned long int type length is 64 bits.

```
unsigned long int fun (unsigned long int n) {
unsigned long int i, j = 0, sum = 0;
for (i = n; i > 1. i = i/2) j++;
for (; j > 1; j = j/2) sum++;
return (sum);
}
```

The value returned when we call fun with the input 2^{40} is: **[2018]**

(A) 4 (B) 5

(C) 6 (D) 40

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** D	**3.** D	**4.** A	**5.** B	**6.** A	**7.** D	**8.** A	**9.** A	**10.** B
11. A	**12.** A	**13.** B	**14.** A	**15.** B					

Practice Problems 2

1. A	**2.** A	**3.** A	**4.** B	**5.** A	**6.** A	**7.** D	**8.** B	**9.** A	**10.** A
11. B	**12.** A	**13.** A	**14.** B	**15.** C					

Previous Years' Questions

1. B	**2.** C	**3.** A	**4.** C	**5.**	**6.** B	**7.** B	**8.** C	**9.** D	**10.** A
11. B	**12.** A	**13.** C	**14.** A	**15.** A	**16.** C	**17.** D	**18.** B	**19.** B	**20.** C
21. A	**22.** A	**23.** D	**24.** D	**25.** A	**26.** C	**27.** D	**28.** D	**29.** A	**30.** B
31. C	**32.** B	**33.** B	**34.** B	**35.** C	**36.** B	**37.** B			

Chapter 2

Sorting Algorithms

LEARNING OBJECTIVES

- *Sorting algorithms*
- *Merge sort*
- *Bubble sort*
- *Insertion sort*
- *Selection sort*
- *Selection sort algorithm*
- *Binary search trees*
- *Heap sort*
- *Sorting–performing delete max operations*
- *Max-heap property*
- *Min-heap property*
- *Priority queues*

Sorting Algorithms

Purpose of sorting

Sorting is a technique which reduces problem complexity and search complexity.

- Insertion sort takes $\theta(n^2)$ time in the worst case. It is a fast inplace sorting algorithm for small input sizes.
- Merge sort has a better asymptotic running time $\theta(n \log n)$, but it does not operate in inplace.
- Heap sort, sorts 'n' numbers inplace in $\theta(n \log n)$ time, it uses a data structure called heap, with which we can also implement a priority queue.
- Quick sort also sorts 'n' numbers in place, but its worst – case running time is $\theta(n^2)$. Its average case is $\theta(n \log n)$. The constant factor in quick sort's running time is small, This algorithm performs better for large input arrays.
- Insertion sort, merge sort, heap sort, and quick sort are all comparison based sorts; they determine the sorted order of an inputarray by comparing elements.
- We can beat the lower bound of Ω ($n \log n$) if we can gather information about the sorted order of the input by means other than comparing elements.
- The counting sort algorithm, assumes that the input numbers are in the set $\{1, 2, \ldots. k\}$. By using array indexing as a tool for determining relative order, counting sort can sort n numbers in $\theta(k + n)$ time. Thus counting sort runs in time that is linear in size of the input array.
- Radix sort can be used to extend the range of counting sort. If there are 'n' integers to sort, each integer has 'd' digits, and each digit is in the set $\{1, 2,\ldots k\}$, then radix sort can sort the numbers in $\theta(d\,(n + k))$ time. Where 'd' is constant. Radix sort runs in linear time.
- Bucket sort, requires knowledge of the probabilistic distribution of numbers in the input array.

Merge Sort

Suppose that our division of the problem yields 'a' sub problems, each of which is $\left(\frac{1}{b}\right)$th size of the original problem. For merge sort, both a and b are 2, but sometimes $a \neq b$. If we take $D(n)$ time to divide the problem into sub problems and $C(n)$ time to combine the solutions of the sub problems into the solution to the original problem. The recurrence relation for merge sort is

$$T(n) = \begin{cases} \theta(1) \text{ if } n \leq \mathrm{c}, \\ aT(n/b) + D(n) + C(n) \text{ otherwise} \end{cases}$$

Running time is broken down as follows:

Divide: This step computes the middle of the sub array, which takes constant time $\theta(1)$.

Conquer: We solve 2 sub problems of size $(n/2)$ each recursively which takes $2T(n/2)$ time.

Combine: Merge sort procedure on an n-element sub array takes time $\theta(n)$.

- Worst case running time $T(n)$ of merge sort

$$T(n) = \begin{cases} 0(1) \text{ if } & n \le 1 \\ aT(n/2) + \theta(n) \text{ if } & n > 1 \end{cases}$$

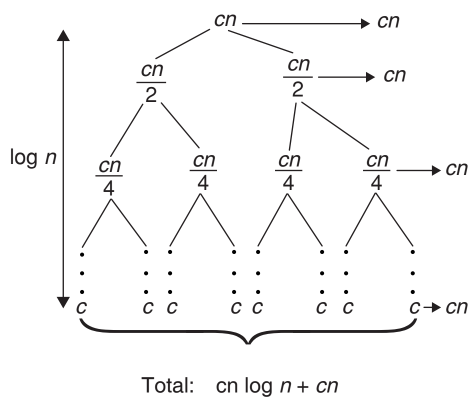


Figure 1 Recurrence tree

The top level has total cost 'cn', the next level has total cost $c(n/2) + c(n/2) = cn$ and the next level has total cost $c(n/4) + c(n/4) + c(n/4) + c(n/4) = cn$ and so on. The ith level has total cost $2^i\ c\ (n/2^i) = cn$. At the bottom level, there are 'n' nodes, each contributing a cost of c, for a total cost of 'cn'. The total number of levels of the 'recursion tree' is $\log n + 1$.

There are $\log n + 1$ levels, each costing cn, for a total cost of $cn\ (\log n + 1) = cn \log n + cn$ ignoring the low–order term and the constant c, gives the desired result of $\theta(n \log n)$.

Bubble Sort

Bubble sort is a simple sorting algorithm that works by repeatedly stepping through the list to be sorted, comparing each pair of adjacent items, and swapping them if they are in the wrong order. The pass through the list is repeated until no swaps are needed, which indicates that the list is sorted. The algorithm gets its name from the way smaller elements 'bubble' to the top of the list.

Example: Take the array of numbers '5 1 4 2 8'and sort the array from lowest number to greatest number using bubble sort algorithm. In each step, elements underlined are being compared.

First pass:

$(\underline{5\ \ 1}\ \ 4\ \ 2\ \ 8) \to (1\ \ 5\ \ 4\ \ 2\ \ 8)$, here algorithm compares the first 2 elements and swaps them

$(1\ \ \underline{5\ \ 4}\ \ 2\ \ 8) \to (1\ \ 4\ \ 5\ \ 2\ \ 8)$, swap $(5 > 4)$

$(1\ \ 4\ \ \underline{5\ \ 2}\ \ 8) \to (1\ \ 4\ \ 2\ \ 5\ \ 8)$, swap $(5 > 2)$

$(1\ \ 4\ \ 2\ \ \underline{5\ \ 8}) \to (1\ \ 4\ \ 2\ \ 5\ \ 8)$, since these elements are already in order, algorithm does not swap them.

Second pass:

$(\underline{1\ \ 4}\ \ 2\ \ 5\ \ 8) \to (1\ \ 4\ \ 2\ \ 5\ \ 8)$

$(1\ \ \underline{4\ \ 2}\ \ 5\ \ 8) \to (1\ \ 2\ \ 4\ \ 5\ \ 8)$, swap since $(4 > 2)$

$(1\ \ 2\ \ \underline{4\ \ 5}\ \ 8) \to (1\ \ 2\ \ 4\ \ 5\ \ 8)$

$(1\ \ 2\ \ 4\ \ \underline{5\ \ 8}) \to (1\ \ 2\ \ 4\ \ 5\ \ 8)$

The array is already sorted, but our algorithm does not know if it is completed. The algorithm needs one whole pass without any swap to know it is sorted.

Third pass:

$(\underline{1\ \ 2}\ \ 4\ \ 5\ \ 8) \to (1\ \ 2\ \ 4\ \ 5\ \ 8)$

$(1\ \ \underline{2\ \ 4}\ \ 5\ \ 8) \to (1\ \ 2\ \ 4\ \ 5\ \ 8)$

$(1\ \ 2\ \ \underline{4\ \ 5}\ \ 8) \to (1\ \ 2\ \ 4\ \ 5\ \ 8)$

$(1\ \ 2\ \ 4\ \ \underline{5\ \ 8}) \to (1\ \ 2\ \ 4\ \ 5\ \ 8)$

Finally the array is sorted, and the algorithm can terminate.

Algorithm

```
void bubblesort (int a [ ], int n)
{
  int i, j, temp;
  for (i=0; i < n-1; i++)
  {
    for (j=0; j < n − 1 − i; j++)
    if (a [j] > a [j + 1])
    {
      temp = a [j + 1];
      a [j + 1] = a [j];
      a [j] = temp;
    }
  }
}
```

Insertion Sort

Insertion sort is a comparison sort in which the sorted array is built one entry at a time. It is much less efficient on large lists than more advanced algorithms such a quick sort, heap sort, (or) merge sort. Insertion sort provides several advantages.

- Efficient for small data sets.
- Adaptive, i.e., efficient for data set that are already substantially sorted. The complexity is $O(n + d)$, where d is the number of inversions.
- More efficient in practice than most other simple quadratic, i.e., $O(n^2)$ algorithms such as selection sort (or) bubble sort, the best case is $O(n)$.
- Stable, i.e., does not change the relative order of elements with equal keys.
- In-place i.e., only requires a constant amount $O(1)$ of additional memory space.
- Online, i.e., can sort a list as it receives it.

Algorithm

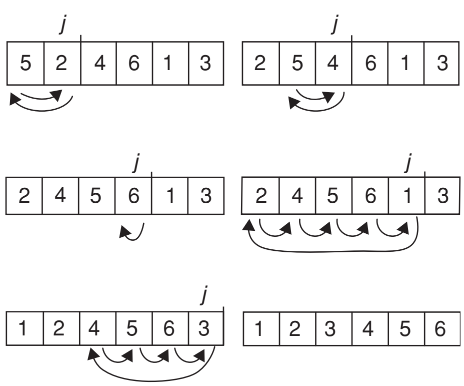
```
Insertion sort (A)
For (j ← 2) to length [A]
Do key ← A [j]
i ←j - 1;
While i > 0 and A [i] > key
{
Do A [i + 1] ← A [i]
i ← i - 1
}
A [i + 1] ← key
```

Every repetition of insertion sort removes an element from the input data, inserting it into the correct position in the already sorted list, until no input element remains. Sorting is typically done in–place. The resulting array after K iterations has the property where the first $k + 1$ entries are sorted. In each iteration the first remaining entry of the input is removed, inserted into the result at the correct position, with each element greater than X copied to the right as it is compared against. X.

Performance

- The best case input is an array that is already sorted. In this case insertion sort has a linear running time (i.e., $\theta(n)$).
- The worst case input is an array sorted in reverse order. In this case every iteration of the inner loop will scan and shift the entire sorted subsection of the array before inserting the next element. For this case insertion sort has a quadratic running time ($O(n^2)$).
- The average case is also quadratic, which makes insertion sort impractical for sorting large arrays, however, insertion sort is one of the fastest algorithms for sorting very small arrays even faster than quick sort.

Example: Following figure shows the operation of insertion sort on the array $A = (5, 2, 4, 6, 1, 3)$. Each part shows what happens for a particular iteration with the value of j indicated. j indexes the 'Current card' being inserted.



Read the figure row by row. Elements to the left of $A[j]$ that are greater than $A[j]$ move one position to the right and $A[j]$ moves into the evacuated position.

Selection Sort

Selection sort is a sorting algorithm, specifically an in-place comparison sort. It has $O(n^2)$ complexity, making it inefficient on large lists.
The algorithm works as follows:

1. Find the minimum value in the list.
2. Swap it with the value in the first position.
3. Repeat the steps above for the remainder of the list (starting at the second position and advancing each time).

Analysis

Selection sort is not difficult to analyze compared to other sorting algorithms, since none of the loops depend on the data in the array selecting the lowest element requires scanning all n elements (this takes $n - 1$ comparisons) and then swapping it into the first position. Finding the next lowest element requires scanning the remaining $n - 1$ elements and so on, for $(n - 1) + (n - 2) + \cdots + 2 + 1 = n(n - 1)/2 \in \theta(n^2)$ comparisons.

Each of these scans requires one swap for $n - 1$ elements (the final element is already in place).

Selection sort Algorithm

First, the minimum value in the list is found. Then, the first element (with an index of 0) is swapped with this value. Lastly, the steps mentioned are repeated for rest of the array (starting at the 2nd position).

Example 1: Here's a step by step example to illustrate the selection sort algorithm using numbers.

Original array: 6 3 5 4 9 2 7
1st pass $\rightarrow$ 2 3 5 4 9 6 7 (2 and 6 were swapped)
2nd pass $\rightarrow$ 2 3 5 4 9 6 7 (no swap)
3rd pass $\rightarrow$ 2 3 4 5 9 6 7 (4 and 5 were swapped)
4th pass $\rightarrow$ 2 3 4 5 6 9 7 (6 and 9 were swapped)
5th pass $\rightarrow$ 2 3 4 5 6 7 9 (7 and 9 were swapped)
6th pass $\rightarrow$ 2 3 4 5 6 7 9 (no swap)

Note: There are 7 keys in the list and thus 6 passes were required. However, only 4 swaps took place.

Example 2: Original array: LU, KU, HU, LO, SU, PU
1st pass $\rightarrow$ HU, KU, LU, LO, SU, PU
2nd pass $\rightarrow$ HU, KU, LU, LO, SU, PU
3rd pass $\rightarrow$ HU, KU, LO, LU, SU, PU
4th pass $\rightarrow$ HU, KU, LO, LU, SU, PU
5th pass $\rightarrow$ HU, KU, LO, LU, PU, SU

Note: There were 6 elements in the list and thus 5 passes were required. However, only 3 swaps took place.

Binary Search Trees

Search trees are data structures that support many dynamic, set operations, including SEARCH, MINIMUM, MAXIMUM, PREDECESSOR, SUCCESSOR, INSERT and DELETE. A search tree can be used as a dictionary and as a priority Queue. Operations on a binary search tree take time proportional to the height of the tree. For a complete binary tree with 'n' nodes, basic operations run in $\theta(\log n)$ worst-case time. If the tree is a linear chain of 'n' nodes, the basic operations take $\theta(n)$ worst-case time.

A binary search tree is organized, in a binary tree such a tree can be represented by a linked data structure in which each node is an object. In addition to key field, each node contains fields left, right and P that point to the nodes corresponding to its left child, its right child, and its parent,

respectively. If the child (or) parent is missing, the appropriate field contains the value NIL. The root node is the only node in the tree whose parent field is NIL.

Binary search tree property

The keys in a binary search tree are always stored in such a way as to satisfy the binary search tree property.

Let 'a' be a node in a binary search tree. If 'b' is a node in the left sub tree of 'a', key $[b] \leq$ key $[a]$

If 'b' is a node in the right sub tree of 'a' then key $[a] \leq$ key $[b]$.

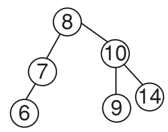


Figure 2 Binary search tree.

The binary search tree property allows us to print out all keys in a binary search tree in sorted order by a simple recursive algorithm called an inorder tree.

Algorithm

INORDER-TREE-WALK (root $[T]$)

INORDER-TREE-WALK (a)

1. If $a \neq$ NIL
2. Then INORDER-TREE-WALK (left $[a]$)
3. Print key $[a]$
4. INORDER-TREE-WALK (right $[a]$)

It takes $\theta(n)$ time to walk an n-node binary search tree, since after the initial call, the procedure is called recursively twice for each node in the tree.

Let $T(n)$ denote the time taken by IN-ORDER-TREE-WALK, when it is called on the root of an n-node subtree.

INORDER-TREE-WALK takes a small, constant amount of time on an empty sub-tree (for the test $x \neq$ NIL).

So $T(1) = C$ for some positive constant C.

For $n > 0$, suppose that INORDER-TREE-WALK is called on a node 'a' whose left subtree has k nodes and whose right subtree has $n - k - 1$ nodes.

The time to perform in order traversal is $T(n) = T(k) + T(n - k - 1) + d$.

For some positive constant 'd' that reflects the time to execute in-order (a), exclusive of the time spent in recursive calls $T(n) = (c + d)\, n + c$.

For $n = 0$, we have $(c + d)\, 0 + c = T(0)$,

For $n > 0$,

$$\begin{aligned} T(n) &= T(k) + T(n - k - 1) + d \\ &= ((c + d)(k + c) + ((c + d)\,(n - k - 1) + c) + d \\ &= (c + d)\, n + c - (c + d) + c + d = (c + d)n + c \end{aligned}$$

Heap Sort

Heap sort begins by building a heap out of the data set, and then removing the largest item and placing it at the end of the partially sorted array. After removing the largest item, it reconstructs heap, removes the largest remaining item, and places, it in the next open position from the end of the partially sorted array. This is repeated until there are no items left in the heap and the sorted array is full. Elementary implementations require two arrays one to hold the heap and the other to hold the sorted elements.

- Heap sort inserts the input list elements into a binary heap data structure. The largest value (in a max-heap) or the smallest value (in a min-heap) is extracted until none remain, the value having been extracted in sorted order.

Example: Given an array of 6 elements: 15, 19, 10, 7, 17, 16, sort them in ascending order using heap sort.

Steps:

1. Consider the values of the elements as priorities and build the heap tree.
2. Start delete Max operations, storing each deleted element at the end of the heap array.

If we want the elements to be sorted in ascending order, we need to build the heap tree in descending order-the greatest element will have the highest priority.

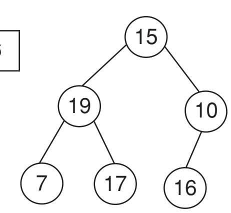
1. Note that we use only array, treating its parts differently,
2. When building the heap-tree, part of the array will be considered as the heap, and the rest part-the original array.
3. When sorting, part of the array will be the heap and the rest part-the sorted array.

Here is the array: 15, 19, 10, 7, 17, 6.

Building the Heap Tree

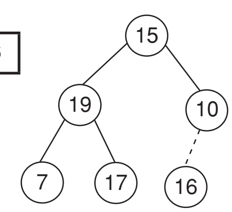
The array represented as a tree, which is complete but not ordered.

15	19	10	7	17	16



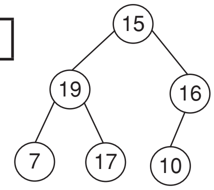
Start with the right most node at height 1 – the node at position 3 = size/2. It has one greater child and has to be percolated down.

15	19	10	7	17	16



After processing array [3] the situation is:

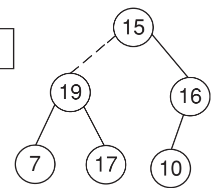
15	19	16	7	17	10



Next comes array [2]. Its children are smaller, so no percolation is needed.

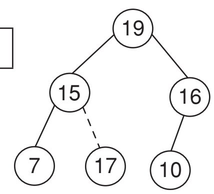
The last node to be processed is array[1]. Its left child is the greater of the children. The item at array [1] has to be percolated down to the left, swapped with array [2].

15	19	16	7	17	10



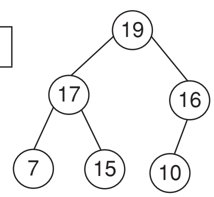
As a result:

19	15	16	7	17	10



The children of array [2] are greater and item 15 has to be moved down further, swapped with array [5].

19	17	16	7	15	10



Now the tree is ordered, and the binary heap is built.

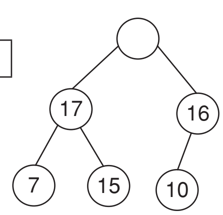
Sorting-performing Delete Max Operations

Delete the top element

Store 19 in a temporary place, a hole is created at the top.

	17	16	7	15	10

19



Swap 19 with the last element of the heap. As 10 will be adjusted in the heap, its cell will no longer be a part of the heap. Instead it becomes a cell from the sorted array

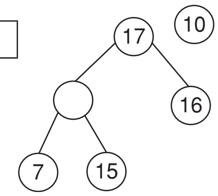
	17	16	7	15	19

10

Percolate down the hole

17		16	7	15	19

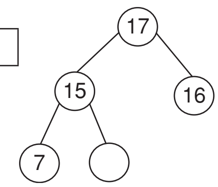
10



Percolate once more (10 is less than 15, so it cannot be inserted in the previous hole)

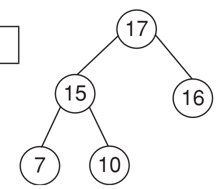
17	15	16	7		19

10



Now 10 can be inserted in the hole

17	15	16	7	10	19



Repeat the step *B* till the array is sorted.

Heap sort analysis

Heap sort uses a data structure called (binary) heap binary, heap is viewed as a complete binary tree. An Array A that represents a heap is an object with 2 attributes: length [*A*], which is the number of elements in the array and heap size [*A*], the number of elements in the heap stored within array *A*.

No element past *A* [heap size [*A*]], where heap size [*A*] ≤ length [*A*], is an element of the heap.

There are 2 kinds of binary heaps:

1. Max-heaps
2. Min-heaps

In both kinds the values in the nodes satisfy a heap-property.

Max-heap property *A*[PARENT (i)] ≥*A*[*i*]
The value of a node is almost the value of its parent. Thus the largest element in a max-heap is stored at the root, and the sub tree rooted at a node contains values no larger than that contained at the node itself.

Min-heap property For every mode '*i*' other than the root [PARENT (i)] ≤ *A*[*i*]. The smallest element in a min-heap is at the root.

Max-heaps are used in heap sort algorithm.
Min-heaps are commonly used in priority queues.

Basic operations on heaps run in time almost proportional to the height of the tree and thus take $O(\log n)$ time

- MAX-HEAPIFY procedure, runs in $O(\log n)$ time.
- BUILD-MAX-HEAP procedure, runs in linear time.
- HEAP SORT procedure, runs in $O(n \log n)$ time, sorts an array in place.
- MAX-HEAP-INSERT
  HEAP- EXTRACT-MAX
  HEAP-INCREASE-KEY
  HEAP-MAXIMUM

  All these procedures, run in $O(\log n)$ time, allow the heap data structure to be used as a priority queue.
- Each call to MAX-HEAPIFY costs $O(\log n)$ time, and there are $O(n)$ such calls. Thus, the running time is $O(n \log n)$
- The HEAPSORT procedure takes time $O(n \log n)$, since the call to BUILD-MAX-HEAP takes time $O(n)$ and each of the $(n-1)$ calls to MAX-HEAPIFY takes time $O(\log n)$.

Priority Queues

The most popular application of a heap is its use as an efficient priority queue.

A priority queue is a data structure for maintaining a set S of elements, each with an associated value called a key. A max-priority queue supports the following operations:

INSERT: INSERT (s, x) inserts the element x into the set S. This operation can be written as $S \leftarrow S\ U\ \{x\}$.

MAXIMUM: MAXIMUM (S) returns the element of S with the largest key

EXTRACT-MAX: EXTRACT-MAX(S) removes and returns the element of S with the largest key.

INCREASE-KEY: INCREASE-KEY(s, x, k) increases the value of element x's key to the new value k, which is assumed to be atleast as large as x's current key value.

One application of max–priority queue is to schedule jobs on a shared computer.

Exercises

Practice Problems 1

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Solve the recurrence relation $T(n) = 2T(n/2) + k.n$ where k is constant then $T(n)$ is

(A) $O(\log n)$ (B) $O(n \log n)$
(C) $O(n)$ (D) $O(n^2)$

2. What is the time complexity of the given code?

```
Void f(int n)
{
if (n > 0)
f (n/2);
}
```

(A) $\theta(\log n)$ (B) $\theta(n \log n)$
(C) $\theta(n^2)$ (D) $\theta(n)$

3. The running time of an algorithm is represented by the following recurrence relation;

$$T(n) = \begin{cases} n & n \le 3 \\ T\left[\dfrac{n}{3}\right] + cn & \text{otherwise} \end{cases}$$

What is the time complexity of the algorithm?

(A) $\theta(n)$ (B) $\theta(n \log n)$
(C) $\theta(n^2)$ (D) $\theta(n^2 \log n)$

Common data for questions 4 and 5:

4. The following pseudo code does which sorting?

$x$sort $[A, n]$

for $j \leftarrow 2$ to n

do key $\leftarrow A\ [\ i\ ]$

$i \leftarrow j - 1$

While $i > 0$ and $A\ [\ i\ ] >$ key

do $A\ [i + i] \leftarrow A\ [\ i\ ]$

$i \leftarrow i - 1$

$A\ [i + 1] \leftarrow$ key

(A) Selection sort (B) Insertion sort
(C) Quick sort (D) Merge sort

5. What is the order of elements after 2 iterations of the above-mentioned sort on given elements?

8	2	4	9	3	6

(A)

2	4	9	8	3	6

(B)

2	4	8	9	3	6

(C)

2	4	6	3	8	9

(D)

2	4	6	3	8	9

Common data for questions 6 and 7:

6. The following pseudo code does which sort?

1. If $n = 1$ done
2. Recursively sort
   A [1…[n/2]]and
   A [[n/2] + 1 … n]
3. Combine 2 ordered lists

(A) Insertion sort (B) Selection sort
(C) Merge sort (D) Quick sort

7. What is the complexity of the above pseudo code?
(A) $\theta(\log n)$ (B) $\theta(n^2)$
(C) $\theta(n \log n)$ (D) $\theta(2^n)$

8. Apply Quick sort on a given sequence 6 10 13 5 8 3 2 11. What is the sequence after first phase, pivot is first element?
(A) 5 3 2 6 10 8 13 11
(B) 5 2 3 6 8 13 10 11
(C) 6 5 13 10 8 3 2 11
(D) 6 5 3 2 8 13 10 11

9. Selection sort is applied on a given sequence:
89, 45, 68, 90, 29, 34, 17. What is the sequence after 2 iterations?
(A) 17, 29, 68, 90, 45, 34, 89
(B) 17, 45, 68, 90, 29, 34, 89
(C) 17, 68, 45, 90, 34, 29, 89
(D) 17, 29, 68, 90, 34, 45, 89

10. Suppose there are log n sorted lists of $\left\lfloor \frac{n}{\log n} \right\rfloor$ elements each. The time complexity of producing sorted lists of all these elements is: (hint: use a heap data structure)
(A) $\theta(n \log \log n)$ (B) $\theta(n \log n)$
(C) $\Omega(n \log n)$ (D) $\Omega(n^{3/2})$

11. If Divide and conquer methodology is applied on powering a Number X^n. Which one the following is correct?
(A) $X^n = X^{n/2} \cdot X^{n/2}$
(B) $X^n = X^{\frac{n-1}{2}} \cdot X^{\frac{n-1}{2}} \cdot X$
(C) $X^n = X^{\frac{n+1}{2}} \cdot X^{\frac{n}{2}}$
(D) Both (A) and (B)

12. The usual $\theta(n^2)$ implementation of insertion sort to sort an array uses linear search to identify the position, where an element is to be inserted into the already sorted part of the array. If binary search is used instead of linear search to identify the position, the worst case running time would be.
(A) $\theta(n \log n)$
(B) $\theta(n^2)$
(C) $\theta(n(\log n)^2)$
(D) $\theta(n)$

13. Consider the process of inserting an element into a max heap where the max heap is represented by an array, suppose we perform a binary search on the path from the new leaf to the root to find the position for the newly inserted element, the number of comparisons performed is:
(A) $\theta(\log n)$ (B) $\theta(\log \log n)$
(C) $\theta(n)$ (D) $\theta(n \log n)$

14. Consider the following algorithm for searching a given number 'X' in an unsorted array $A[1 \cdots n]$ having 'n' distinct values:
(1) Choose an 'i' uniformly at random from $1 \cdots n$
(2) If $A[i] = x$
Then stop
else
goto(1);
Assuming that X is present in A, what is the expected number of comparisons made by the algorithm before it terminates.
(A) n (B) $n - 1$
(C) $2n$ (D) $n/2$

15. The recurrence equation for the number of additions $A(n)$ made by the divide and conquer algorithm on input size $n = 2^K$ is
(A) $A(n) = 2A(n/2) + 1$ (B) $A(n) = 2A(n/2) + n^2$
(C) $A(n) = 2A(n/4) + n^2$ (D) $A(n) = 2A(n/8) + n^2$

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1.

Input Array	Linear Search $W(n)$	Binary search $W(n)$
128 elements	128	8
1024 elements	1024	x

Find x value?
(A) 10 (B) 11
(C) 12 (D) 13

2. Choose the correct one
(i) $\log n$ (ii) n
(iii) $n \log n$ (iv) n^2
(a) A result of cutting a problem size by a constant factor on each iteration of the algorithm.
(b) Algorithm that scans a list of size 'n'.
(c) Many divide and conquer algorithms fall in this category.
(d) Typically characterizes efficiency of algorithm with two embedded loops.
(A) i – b, ii – c, iii – a, iv – d
(B) i – a, ii – b, iii – c, iv – d
(C) i – c, ii – d, iii – a, iv – b
(D) i – d, ii – a, iii – b, iv – c

3. Insertion sort analysis in worst case
(A) $\theta(n)$
(B) $\theta(n^2)$
(C) $\theta(n \log n)$
(D) $\theta(2^n)$

4. From the recurrence relation. Of merge sort $T(n) = 2T(n/2) + \theta(n)$.
Which option is correct?
I. $n/2$ II. 2T III. $\theta(n)$
(a) Extra work (divide and conquer)
(b) Sub-problem size
(c) Number of sub-problems
(A) III – b, II – a, I – c (B) I – b, II – c, III – a
(C) I – a, II – c, III – b (D) I – c, II – a, III – b

5. What is the number of swaps required to sort 'n' elements using selection sort, in the worst case?
(A) $\theta(n)$ (B) $\theta(n^2)$
(C) $\theta(n \log n)$ (D) $\theta(n^2 \log n)$

6. In a binary max heap containing 'n' numbers, the smallest element can be found in time
(A) $O(n)$ (B) $O(\log n)$
(C) $O(\log \log n)$ (D) $O(1)$

7. What is the worst case complexity of sorting 'n' numbers using quick sort?
(A) $\theta(n)$ (B) $\theta(n \log n)$
(C) $\theta(n^2)$ (D) $\theta(n\,!)$

8. The best case analysis of quick sort is, if partition splits the array of size n into
(A) $n/2 : n/m$ (B) $n/2 : n/2$
(C) $n/3 : n/2$ (D) $n/4 : n/2$

9. What is the time complexity of powering a number, by using divide and conquer methodology?
(A) $\theta(n^2)$ (B) $\theta(n)$
(C) $\theta(\log n)$ (D) $\theta(n \log n)$

10. Which one of the following in-place sorting algorithm needs the minimum number of swaps?
(A) Quick sort (B) Insertion sort
(C) Selection sort (D) Heap sort

11. As the size of the array grows what is the time complexity of finding an element using binary search (array of elements are ordered)?
(A) $\theta(n \log n)$ (B) $\theta(\log n)$
(C) $\theta(n^2)$ (D) $\theta(n)$

12. The time complexity of heap sort algorithm is
(A) $n \log n$ (B) $\log n$
(C) n^2 (D) None of these.

13. As part of maintenance work, you are entrusted with the work of rearranging the library books in a shelf in a proper order, at the end of each day. The ideal choices will be_____.
(A) Heap sort (B) Quick sort
(C) Selection sort (D) Insertion sort

14. The value for which you are searching is called
(A) Binary value
(B) Search argument
(C) Key
(D) Serial value

15. To sort many large objects and structures it would be most efficient to _____.
(A) Place them in an array and sort the array
(B) Place the pointers on them in an array and sort the array
(C) Place them in a linked list and sort the linked list
(D) None of the above

Previous Years' Questions

1. What is the number of swaps required to sort n elements using selection sort, in the worst case? **[2009]**
(A) $\theta(n)$
(B) $\theta(n \log n)$
(C) $\theta(n^2)$
(D) $\theta(n^2 \log n)$

2. Which one of the following is the tightest upper bound that represents the number of swaps required to sort n numbers using selection sort? **[2013]**
(A) $O(\log n)$ (B) $O(n)$
(C) $O(n \log n)$ (D) $O(n^2)$

3. Let P be a quick sort program to sort numbers in ascending order using the first element as the pivot. Let t_1 and t_2 be the number of comparisons made by P for the inputs [1 2 3 4 5] and [4 1 5 3 2] respectively. Which one of the following holds? **[2014]**
(A) $t_1 = 5$ (B) $t_1 < t_2$
(C) $t_1 > t_2$ (D) $t_1 = t_2$

4. The minimum number of comparisons required to find the minimum and the maximum of 100 numbers is ———. **[2014]**

5. Suppose P, Q, R, S, T are sorted sequences having lengths 20, 24, 30, 35, 50 respectively. They are to be merged into a single sequence by merging together two sequences at a time. The number of comparisons that will be needed in the worst case by the optimal algorithm for doing this is ———. **[2014]**

6. You have an array of n elements. Suppose you implement quick sort by always choosing the central element of the array as the pivot. Then the tightest upper bound for the worst case performance is **[2014]**
(A) $O(n^2)$ (B) $O(n \log n)$
(C) $\theta(n \log n)$ (D) $O(n^3)$

7. What are the worst-case complexities of insertion and deletion of a key in a binary search tree? **[2015]**

(A) $\theta(\log n)$ for both insertion and deletion
(B) $\theta(n)$ for both insertion and deletion
(C) $\theta(n)$ for insertion and $\theta(\log n)$ for deletion
(D) $\theta(\log n)$ for insertion and $\theta(n)$ for deletion

8. The worst case running times of *Insertion sort, Merge sort* and *Quick sort*, respectively, are: **[2016]**
(A) $\Theta(n \log n)$, $\Theta(n \log n)$,and $\Theta(n^2)$
(B) $\Theta(n^2)$, $\Theta(n^2)$,and $\Theta(n \log n)$
(C) $\Theta(n^2)$, $\Theta(n \log n)$,and $\Theta(n \log n)$
(D) $\Theta(n^2)$, $\Theta(n \log n)$,and $\Theta(n^2)$

9. An operator delete(i) for a binary heap data structure is to be designed to delete the item in the i-th node. Assume that the heap is implemented in an array and i refers to the i-th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element? **[2016]**
(A) $O(1)$ (B) $O(d)$ but not $O(1)$
(C) $O(2^d)$ but not $O(d)$ (D) $O(d2^d)$ but not $O(2^d)$

10. Assume that the algorithms considered here sort the input sequences in ascending order. If the input is already in ascending order, which of the following are **TRUE**? **[2016]**
I. Quicksort runs in $\Theta(n^2)$ time
II. Bubblesort runs in $\Theta(n^2)$ time
III. Mergesort runs in $\Theta(n)$ time
IV. Insertion sort runs in Θ (n) time

(A) I and II only (B) I and III only
(C) II and IV only (D) I and IV only

11. A complete binary min - heap is made by including each integer in [1,1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is depth 0. The maximum depth at which integer 9 can appear is _____ . **[2016]**

Answer Keys

Exercises

Practice Problems I

1. B	**2.** A	**3.** A	**4.** B	**5.** B	**6.** C	**7.** C	**8.** B	**9.** A	**10.** B
11. D	**12.** A	**13.** A	**14.** B	**15.** A					

Practice Problems 2

1. B	**2.** B	**3.** B	**4.** B	**5.** A	**6.** A	**7.** C	**8.** B	**9.** C	**10.** C
11. B	**12.** A	**13.** D	**14.** C	**15.** B					

Previous Years' Questions

1. A	**2.** B	**3.** C	**4.** 148	**5.** 358	**6.** A	**7.** B	**8.** D	**9.** B	**10.** D
11. 8									

Chapter 3

Divide-and-conquer

LEARNING OBJECTIVES

- *Divide-and-conquer*
- *Divide-and-conquer examples*
- *Divide-and-conquer technique*
- *Merge sort*
- *Quick sort*
- *Performance of quick sort*
- *Recurrence relation*
- *Searching*
- *Linear search*
- *Binary search*

Divide-and-Conquer

Divide-and-conquer is a top down technique for designing algorithms that consists of dividing the problem into smaller sub problems hoping that the solutions of the sub problems are easier to find and then composing the partial solutions into the solution of the original problem.

Divide-and-conquer paradigm consists of following major phases:

- Breaking the problem into several sub-problems that are similar to the original problem but smaller in size.
- Solve the sub-problem recursively (successively and independently)
- Finally, combine these solutions to sub-problems to create a solution to the original problem.

Divide-and-Conquer Examples

- Sorting: Merge sort and quick sort
- Binary tree traversals
- Binary Search
- Multiplication of large integers
- Matrix multiplication: Strassen's algorithm
- Closest-pair and Convex-hull algorithm

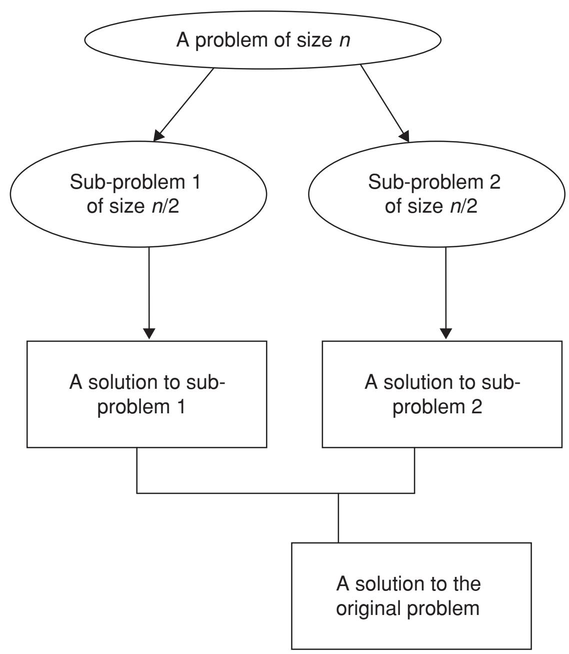


Figure 1 Divide-and-conquer technique.

Merge Sort

Merge sort is a sorting algorithm for rearranging lists (or any other data structure that can only be accessed sequentially, e.g., file streams) into a specified order.

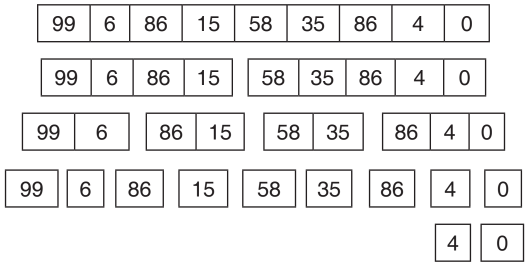
Merge sort works as follows:

1. Divide the unsorted list into two sub lists of about half the size.
2. Sort each of the two sub lists.
3. Merge the two sorted sub lists back into one sorted list
4. The key of merge sort is merging two sorted lists into one, such that if we have 2 lists
   $X(x_1 \le x_2 \le x_3 \cdots \le x_m)$ and
   $Y(y_1 \le y_2 \le y_3 \cdots \le y_n)$ the resulting list is $z\,(z_1 \le z_2 \le \cdots \le z_{m+n})$

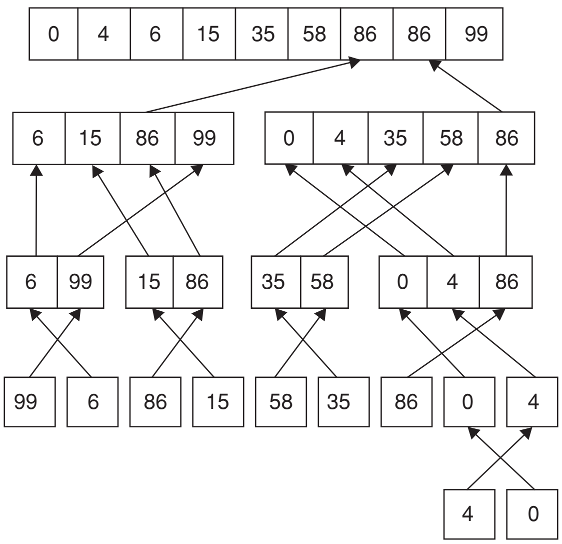
Example 1: $L_1 = \{3, 8, 9\}$, $L_2 = \{1, 5, 7\}$

Merge $(L_1, L_2) = \{1, 3, 5, 7, 8, 9\}$

Example 2:



Merge:



Implementing Merge Sort

Merging is done with a temporary array of the same size as the input array.

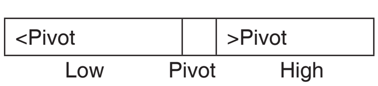
Pro: Faster than in-place since the temp array holds the resulting array until both left and right sides are merged into the temp array then the temp array is appended over the input array.

Con: The memory required is doubled. The double memory merge sort runs $O(N \log N)$ for all cases, because of its Divide-and-conquer approach.

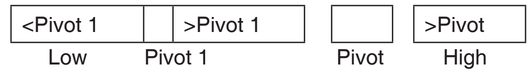
$$T(N) = 2T(N/2) + N$$
$$= O(N \log N)$$

Quick Sort

Quick sort is an example of Divide-and-conquer strategy. In Quick sort we divide the array of items to be sorted into two partitions and then call the quick sort procedure recursively to sort the two partitions, i.e., we divide the problem into two smaller ones and conquer by solving the smaller ones. The conquer part of the quick sort routine looks like this



Make bold



Divide: Partition the array A $[p - r]$ into 2 sub arrays A $[p - q - 1]$ and A $[q + 1 - r]$ such that each element of A $[p - q - 1]$ is less than or equal to A$[q]$, which is, in turn, less than or equal to each element of A $[q + 1 - r]$

Conquer: Sort the 2 sub arrays A $[p - q - 1]$ and A $[q + 1 - r]$ by recursive calls to quick sort.

Combine: Since the sub arrays are sorted inplace, no work is needed to combine them.

Sort left partition in the same way. For this strategy to be effective, the partition phase must ensure that the pivot, is greater than all the items in one part (the lower part) and less than all those in the other (upper) part. To do this, we choose a pivot element and arrange that all the items in the lower part are less than the pivot and all those in the upper part are greater than it. In the general case, the choice of pivot element is first element.

(Here $\lfloor$number of elements/2$\rfloor$ is pivot)

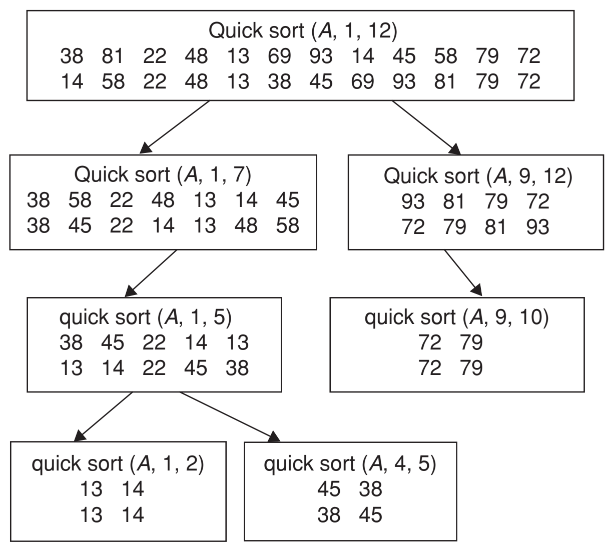


Figure 2 Tree of recursive calls to quick sort.

- Quick sort is a sorting algorithm with worst case running time $O(n^2)$ on an input array of n numbers. Inspite of this slow worst case running time, quick sort is often the best practical choice for sorting because it is efficient on the average: its expected running time is $O(n \log n)$ and the constants hidden in the O-notation are quite small
- Quick sort algorithm is fastest when the median of the array is chosen as the pivot element. This is because the resulting partitions are of very similar size. Each partition splits itself in two and thus the base case is reached very quickly.

Example: Underlined element is pivot.

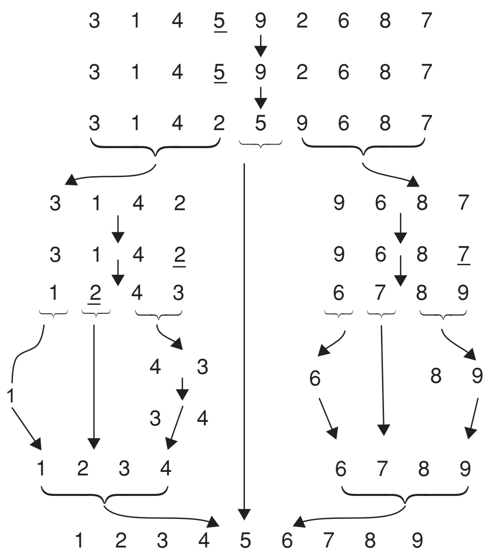


Figure 3 The ideal quick sort on a random array

Performance of Quick Sort

- Running time of quick sort depends on whether the partitioning is balanced or unbalanced, it depends on which elements are used for partitioning. If the partitioning is balanced, the algorithm runs asymptotically as fast as merge sort. If the partitioning is unbalanced, it runs as slowly as insertion sort.
- The worst case of quick sort occurs when the partitioning routine produces one sub-problem with $n - 1$ elements and one with '1' element. If this unbalanced partitioning arises in each recursive call, the partitioning costs $\theta(n)$ time.

Recurrence Relation

$$T(n) = T(n-1) + T(1) + \theta(n)$$
$$(\therefore\ T(0) = \theta(1))$$
$$= T(n-1) + \theta(n)$$

If we sum the costs incurred at each level of the recursion we get an arithmetic series, which evaluates to $\theta(n^2)$.

- Best case partitioning–PARTITION produces 2 sub problems, each of size no more than $n/2$, since one is of size $\lfloor n/2 \rfloor$ and one of size $\lceil n/2 \rceil - 1$

  The recurrence for the running time is then

$$T(n) \le 2T(n/2) + \theta(n)$$

The above Recurrence relation has the solution $T(n) = O(n \log n)$ by case 2 of the master theorem.

- The average–case time of quick sort is much closer to the best than to the worst case

For example, that the partitioning algorithm always produces a 8-to-2 proportional split, which at first seems unbalanced. The Recurrence relation will be

$$T(n) \le T(8n/10) + T(2n/10) + cn$$

The recursion tree for this recurrence has cost 'cn' at every level, until a boundary condition is reached at depth $\log_{10} n = \theta(\log n)$. The recursion terminates at depth $\log_{10/8} n = \theta(\log n)$. The total cost of quick sort is $O(n \log n)$

Searching

Two searching techniques are:

- Linear search
- Binary search

Linear Search

Linear search (or) sequential search is a method for finding a particular value in list that consists of checking every one of its elements, one at a time and in sequence, until the desired one is found. Linear search is a special case of brute force search. Its worst case cost is proportional to the number of elements in the list.

Implementation

```
boolean linear search (int [ ] arr, int target)
{
int i = 0;
while (i < arr. length) {
if (arr [i] = = target){
return true;
}
+ + i;
}
return false;
}
```

Example:

Consider the array

10	7	1	3	–4	2	20

Search for 3

10	7	1	3	–4	2	20
3?						

Move to next element

10	7	1	3	–4	2	20
	3?					

Move to next element

10	7	1	3	–4	2	20
		3?				

Move to next element

10	7	1	3	–4	2	20
			3?			

Element found; stop the search.

Binary Search

A binary search algorithm is a technique for finding a particular value in a linear array, by ruling out half of the data at each

step; a binary search finds the median, makes comparison, to determine whether the desired value comes before or after it, and then searches the remaining half in the same manner. A binary search is an example of Divide-and-conquer algorithm.

Implementation

function binary search (a, value, left, right)

```
{
if right < left
        return not found
      mid: = floor ((right -left)/2) + left
      if a [mid] = value
      return mid
      if value < a[mid]
```

return binary search (a, value, left, mid –1) else return binary search (a, value, mid + 1, right)

```
}
```

Example: Value being searched 123

2 6 7 34 76 123 234 567 677 986

First (1) mid(5) Last(10)

2 6 7 34 76 123 234 567 677 986

First (6) mid(8) Last(10)

2 6 7 34 76 123 234 567 677 986

First (6) Last(7)
Mid (6)

2 6 7 34 76 123 234 567 677 986

First, mid, last (6)

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. How many comparisons are required to search an item 89 in a given list, using Binary search?

4	8	19	25	34	39	45	48	66	75	89	95
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

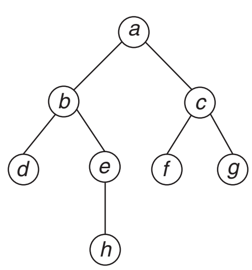
(A) 3 (B) 4
(C) 5 (D) 6

2. Construct a Binary search tree with the given list of elements:

300, 210, 400, 150, 220, 370, 450, 100, 175, 215, 250

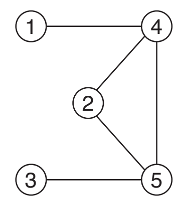
Which of the following is a parent node of element 250?
(A) 220
(B) 150
(C) 370
(D) 215

3. What is the breadth first search order of the given tree?



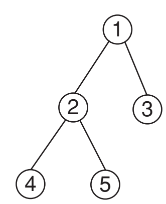
(A) acbhdefg (B) abcdefgh
(C) adbcefgh (D) aebcdfgh

4. What is the depth first search order of the given graph?



(A) 14325
(B) 12435
(C) 14253
(D) 12354

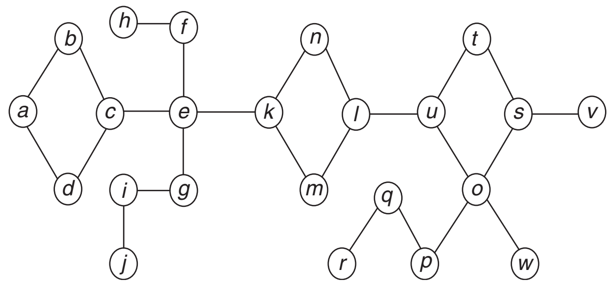
5. When pre-order traversal is applied on a given tree, what is the order of elements?



(A) 1 – 2 – 4 – 5 – 3
(B) 1 – 4 – 2 – 5 – 3
(C) 1 – 2 – 4 – 3 – 5
(D) 1 – 2 – 3 – 4 – 5

6. What is the order of post-order traversal and in-order traversals of graph given in the above question?
(A) 4 – 2 – 5 – 1 – 3 and 4 – 5 – 2 – 3 – 1
(B) 4 – 5 – 2 – 3 – 1 and 4 – 2 – 5 – 1 – 3
(C) 4 – 5 – 2 – 1 – 3 and 4 – 2 – 5 – 1 – 3
(D) 4 – 5 – 2 – 3 – 1 and 4 – 2 – 5 – 3 – 1

7. Find the number of bridges in the given graph

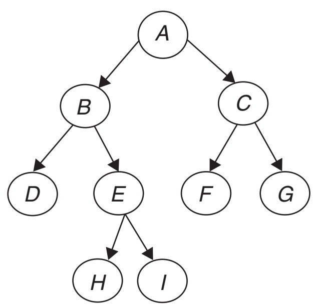


(A) 12 (B) 13
(C) 11 (D) 10

8. Match the following:

I.	In-order	1.	*ABCDEFGHI*
II.	Pre-order	2.	*DBHEIAFCG*
III.	Post-order	3.	*ABDEHICFG*
IV.	Level-order	4.	*DHIEBFGCA*

For the tree



(A) I – 2, II – 3, III – 4, IV – 1
(B) I – 3, II – 1, III – 4, IV – 2
(C) I – 1, II – 2, III – 3, IV – 4
(D) I – 4, II – 3, III – 2, IV – 1

9. A complete *n*-array tree in which each node has '*n*' children (or) no children.

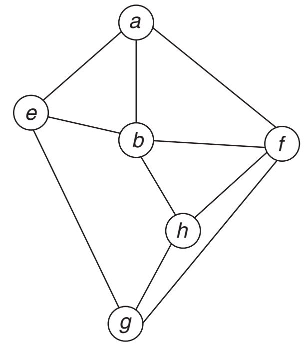
Let '*I*' be the number of internal nodes and '*L*' be the number of leaves in a complete *n*-ary tree.

If $L = 51$ and $I = 10$ what is the value of '*n*'?
(A) 4 (B) 5
(C) 6 (D) Both (A) and (B)

10. A complete *n*-ary tree is one in which every node has 0 (or) *n* children. If '*X*' is the number of internal nodes of a complete *n*-ary tree, the number of leaves in it is given by
(A) $X(n - 1) + 1$ (B) $Xn - 1$
(C) $Xn + 1$ (D) $X(n + 1) + 1$

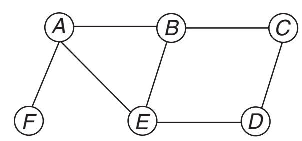
11. The numbers 7, 5, 1, 8, 3, 6, 0, 9, 4, 2 are inserted in the given order into an initially empty binary search tree. The binary search tree uses the usual ordering on natural numbers. What is the in-order traversal sequence of the resultant tree?
(A) 7 5 1 0 3 2 4 6 8 9
(B) 0 2 4 3 1 6 5 9 8 7
(C) 0 1 2 3 4 5 6 7 8 9
(D) 9 8 6 4 2 3 0 1 5 7

12. Consider the following graph:



Among the following sequences
I. *a b e g h f* II. *a b f e h g*
III. *a b f h g e* IV. *a f g h b e*
Which are depth first traversals of the above graph?
(A) I, II and IV only (B) I and IV only
(C) I, III only (D) I, III and IV only

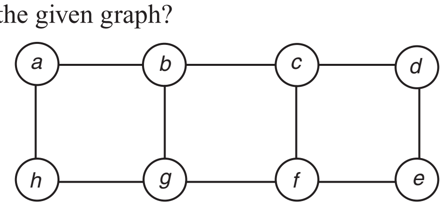
13. The breadth first search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes is



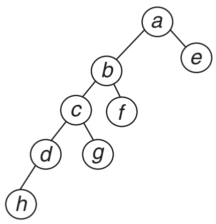
(A) *A B C D E F* (B) *B E A D C F*
(C) *E A B D F C* (D) Both (A) and (B)

14. An undirected graph *G* has '*n*' nodes. Its adjacency matrix is given by an $n \times n$ square matrix.
(i) Diagonal elements are 0's
(ii) Non-diagonal elements are 1's
Which of the following is true?
(A) Graph *G* has no minimum spanning tree
(B) Graph *G* has a unique minimum spanning tree of cost $(n - 1)$
(C) Graph *G* has multiple distinct minimum spanning trees, each of cost $(n - 1)$
(D) Graph *G* has multiple spanning trees of different cost.

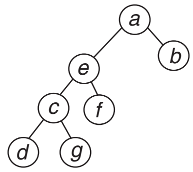
15. Which of the following is the breadth first search tree for the given graph?



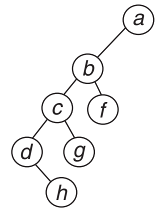
(A)



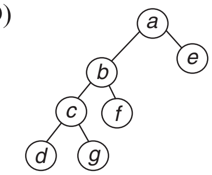
(B)



(C)



(D)



Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Which of the following algorithm design technique is used in finding all pairs of shortest distances in a graph?
(A) Divide-and-conquer
(B) Greedy method
(C) Back tracking
(D) Dynamic programming

2. Let LASTPOST, LASTIN and LASTPRE denote the last vertex visited in a post-order, in-order and pre-order traversals respectively of a complete binary tree. Which of the following is always true?
(A) LASTIN = LASTPOST
(B) LASTIN = LASTPRE
(C) LASTPRE = LASTPOST
(D) LASTIN = LASTPOST = LASTPRE

3. Match the following:

X : Depth first search

Y : Breadth first search

Z : Sorting

a : Heap

b : Queue

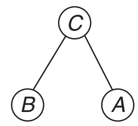
c : Stack

(A) X – a, Y – b, Z – c
(B) X – c, Y – a, Z – b
(C) X – c, Y – b, Z – a
(D) X – a, Y – c, Z – b

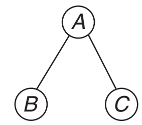
4. Let G be an undirected graph, consider a depth first traversal of G, and let T be the resulting DFS Tree. Let 'U' be a vertex in 'G' and let 'V' be the first new (unvisited) vertex visited after visiting 'U' in the traversal. Which of the following is true?
(A) $\{U, V\}$ must be an edge in G and 'U' is a descendant of V in T.
(B) $\{U, V\}$ must be an edge in 'G' and V is a descendant of 'U' in T.
(C) If $\{U, V\}$ is not an edge in 'G' then 'U' is a leaf in T.
(D) if $\{U, V\}$ is not an edge in G then U and V must have the same parent in T.

5. Identify the binary tree with 3 nodes labeled A, B and C on which preorder traversal gives the sequence C, B, A.

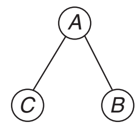
(A)



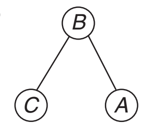
(B)



(C)



(D)

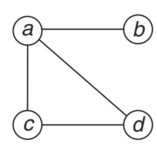


6. Consider an undirected unweighted graph G. Let a breadth first traversal of G be done starting from a node r. Let $d(r, u)$ and $d(r, v)$ be the lengths of the shortest paths from r to u and v respectively in 'G'. If u is visited before v during the breadth first travel, which of the following is correct?
(A) $d(r, u) < d(r, v)$
(B) $d(r, u) > d(r, v)$
(C) $d(r, u) \leq d(r, v)$
(D) None of these

7. In a complete 5-ary tree, every internal node has exactly 5 children. The number of leaves in such a tree with '3' internal nodes are:
(A) 15
(B) 20
(C) 13
(D) Can't predicted

8. Which of the following algorithm is single pass that is they do not traverse back up the tree for search, create, insert etc.
(A) Depth first search
(B) Pre-order traversal
(C) B-tree traversal
(D) Post-order traversal

9. Which of the following is the adjacency matrix of the given graph?



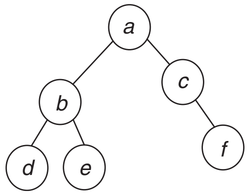
(A) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

10. Which one of the following is the post-order traversal of the given tree?

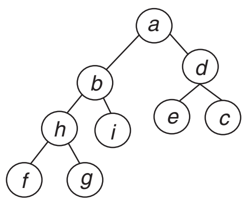


(A) *d e a f c b a* (B) *d e b f c a*
(C) *e b d f c a* (D) *a b c d e f*

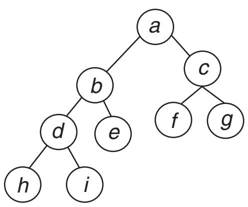
Common data for questions 11 and 12:

11. The pre-order traversal of a tree is *a b d h i e c f g*. Which of the following is the correct tree?

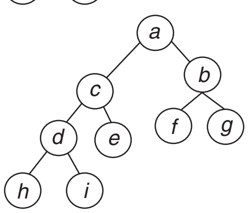
(A)



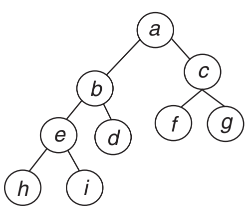
(B)



(C)



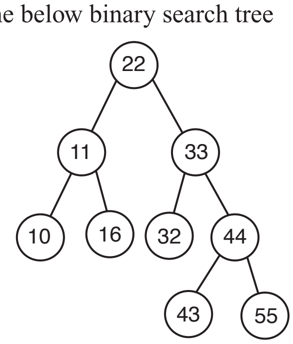
(D)



12. Which of the following is in-order traversal of the above tree?

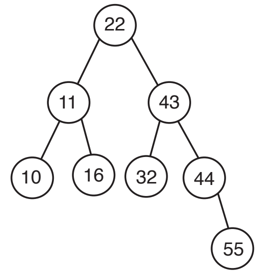
(A) *a b h d e i f g c* (B) *a b d h e i f g c*
(C) *h d i b e a f c g* (D) *i d h b e a f c g*

13. Consider the below binary search tree

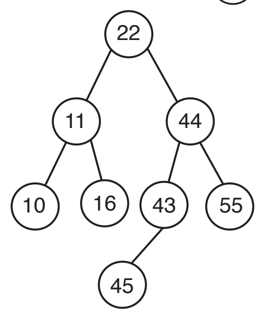


Which of the following is the resultant binary search tree after deletion of 33?

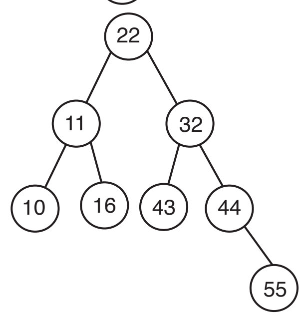
(A)



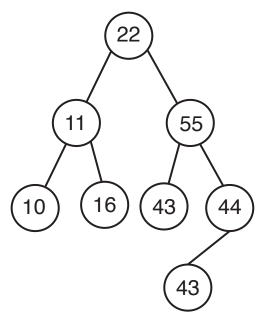
(B)



(C)



(D)



14. Match the following:

I.	Articulation Point	1.	An edge whose removal disconnects graph
II.	Bridge	2.	A vertex whose removal disconnects graph
III.	Bi connected component	3.	Maximal set of edges such that any two edges in the set lie on a common simple cycle

(A) I – 1, II – 2, III – 3 (B) I – 2, II – 1, III – 3
(C) I – 2, II – 3, III – 1 (D) I – 1, II – 2, III – 3

15. If x is the root of an n-node subtree, then the inorder-tree-walk takes

(A) $\theta(n)$ (B) $\theta(n^2)$
(C) $\theta(n^3)$ (D) $\theta(n \log n)$

Previous Years' Questions

1. Which one of the following is the tightest upper bound that represents the time complexity of inserting an object into a binary search tree of n nodes? **[2013]**

   (A) $O(1)$ (B) $O(\log n)$
   (C) $O(n)$ (D) $O(n \log n)$

2. Consider a rooted n node binary tree represented using pointers. The best upper bound on the time required to determine the number of sub trees having exactly 4 nodes is $O(n^a \log^b n)$. The value of $a + 10b$ is ________ **[2014]**

3. Which one of the following is the recurrence equation for the worst case time complexity of the Quicksort algorithm for sorting $n(\geq 2)$ numbers? In the recurrence equations given in the options below, c is a constant. **[2015]**

   (A) $T(n) = 2T(n/2) + cn$
   (B) $T(n) = T(n - 1) + T(1) + cn$
   (C) $T(n) = 2T(n - 1) + cn$
   (D) $T(n) = T(n/2) + cn$

4. Suppose you are provided with the following function declaration in the C programming language.

   int partition (int *a*[ ], int *n*);

   The function treats the first element of *a* [ ] as a pivot, and rearranges the array so that all elements less than or equal to the pivot is in the left part of the array, and all elements greater than the pivot is in the right part in addition, it moves the pivot so that the pivot is the last element of the left part. The return value is the number of elements in the left part.

   The following partially given function in the C programming language is used to find the *k*th smallest element in an array *a* [ ] of size *n* using the partition function. We assume *k* ″ *n*.

   int *k*th_smallest (int *a* [ ], int *n*, int *k*) **[2015]**

```
{
    int left_end = partition(a, n);
    if (left_end+1 == k) {
      return a [left_end];
    )
    if (left_end+1 > k) {
      return kth_smallest (___________);
    } else {
      return kth_smallest (___________);
    }
}
```

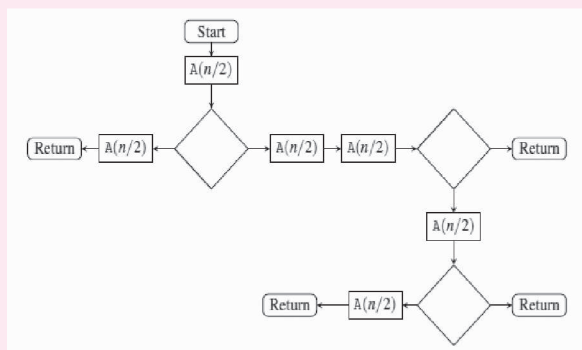
   The missing argument lists are respectively
   (A) (*a*, left_end, *k*) and (*a*+left_end+1, *n*-left_end-1, *k*-left_end-1)
   (B) (*a*, left_end, *k*) and (*a*, *n*-left_end-1, *k*-left_end-1)
   (C) (*a*+left_end+1, *n*-left_end-1, *k*-left_end-1) and (*a*, left_end, *k*)
   (D) (*a*, *n*-left_end-1, *k*-left_end-1) and (*a*, left_end, *k*)

5. Assume that a mergesort algorithm in the worst case takes 30 seconds for an input of size 64. Which of the following most closely approximates the maximum input size of a problem that can be solved in 6 minutes? **[2015]**

   (A) 256 (B) 512
   (C) 1024 (D) 2048

6. The given diagram shows the flowchart for a recursive function A(n). Assume that all statements, except for the recursive calls, have O (1) time complexity. If the worst case time complexity of this function is O (n^{α}), then the least possible value (accurate up to two decimal positions) of α is ____. **[2016]**

   Flowchart for Recursive Function A(n)



7. Let A be an array of 31 numbers consisting of a sequence of 0's followed by a sequence of 1's. The problem is to find the smallest index i such that $A[i]$ is 1 by probing the minimum number of locations in A. The *worst case* number of probes performed by an *optimal* algorithm is _______. **[2017]**

8. Match the algorithms with their time complexities:

Algorithm	Time complexity
(P) Towers of Hanoi with n disks	(i) $\Theta(n^2)$
(Q) Binary search given n sorted numbers	(ii) $\Theta(n \log n)$
(R) Heap sort given n numbers at the worst case	(iii) $\Theta(2^n)$
(S) Addition of two $n \times n$ matrices	(iv) $\Theta(\log n)$

[2017]

   (A) P → (iii), Q → (iv), R → (i), S → (ii)
   (B) P → (iv), Q → (iii), R → (i), S → (ii)
   (C) P → (iii), Q → (iv), R → (ii), S → (i)
   (D) P → (iv), Q → (iii), R → (ii), S → (i)

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** A	**3.** B	**4.** C	**5.** A	**6.** B	**7.** B	**8.** A	**9.** C	**10.** A
11. C	**12.** D	**13.** A	**14.** C	**15.** A					

Practice Problems 2

1. B	**2.** B	**3.** C	**4.** B	**5.** A	**6.** D	**7.** C	**8.** C	**9.** A	**10.** B
11. B	**12.** C	**13.** A	**14.** B	**15.** A					

Previous Years' Questions

1. C	**2.** 1	**3.** B	**4.** A	**5.** B	**6.** 2.2 to 2.4	**7.** 5	**8.** C

Chapter 4

Greedy Approach

LEARNING OBJECTIVES

- Greedy approach
- Knapsack problem
- Fractional knapsack problem
- Spanning trees
- Prim's algorithm
- Kruskal's algorithm
- Tree and graph traversals
- Back tracking
- Graph traversal
- Breadth first traversal
- Depth first search
- Huffman codes
- Task-scheduling problem
- Sorting and order statistics
- Simultaneous minimum and maximum
- Graph algorithms

Greedy Approach

In a greedy method, we attempt to construct an optimal solution in stages.

- At each stage we make a decision that appears to be the best (under some criterion) at the time.
- A decision made at one stage is not changed in a later stage, so each decision should assure feasibility.
- Some problems that use greedy approach are:

1. Knapsack problem
2. Minimum spanning tree
3. Prims algorithm
4. Kruskals algorithm

Knapsack Problem

The knapsack problem is a problem in combinatorial optimization: given a set of items, each with a weight and a value, determine the number of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible. We have n kinds of items, 1 through n. Each kind of item i has a value V_i and a weight W_i, usually assume that all values and weights are non-negative. The maximum weight that we can carry in the bag is W.

Solved Examples

Example 1: (Making change)

Problem:	Accept n dollars, to return a collection of coins with a total value of n dollars.
Configuration:	A collection of coins with a total value of n.
Objective function:	Minimize number of coins returned.
Greedy solution:	Always return the largest coin you can.

- Coins are valued \$.30, \$.020, \$0.05, \$0.01 use a greedy choice property and make \$.40 by using 3 coins.

Solution: \$0.30 + \$0.05 + \$0.05 = \$0.40

Fractional Knapsack Problem

Given: A set S of n items, with each item i having

- b_i – *a* positive benefit
- w_i – *a* positive weight

Goal: Choose items with maximum total benefit but with weight atmost W.

If we are allowed to take fractional amounts, then this is the fractional knapsack problem.

- In this case, let x_i denote the amount we take of item i.
- Objective: Maximize $\sum_{i \in S} b_i(x_i/w_i)$
- Constraint: $\sum_{i \in S} x^i \leq W$

Example 2:

Items

Weight	4 ml	8 ml	2 ml	6 ml	1 ml
Benefit	\$12	\$32	\$40	\$30	\$50
Value (\$ per ml)	3	4	20	5	50

"Knapsack"

10 ml

Solution: 1 ml of 5, 2 ml of 3, 6 ml of 4, 1 ml of 2

- Greedy choice: Keep taking item with highest value (benefit to weight ratio).
- Correctness: suppose there is a better solution, there is an item i with higher value than a chosen item *j*. (i.e., $v_j < v_i$). If we replace some j with i, we get a better solution.

Thus there is no better solution than the greedy one.
$N = 3$, $m = 20$
$(P_1, P_2, P_3) = (25, 24, 15)$
$(W_1, W_2, W_3) = (18, 15, 10)$

Example 3:

	$X_1\ X_2\ X_3$	$\Sigma\ W_iX_i$	$\Sigma\ P_i\ X_i$
1.	1/2 1/3 1/4	9 + 5 + 2.5 = 16.5	12.5 + 8 + 3.75 = 24.25
2.	1 2/15 0	18 + 2 + 0 = 20	25 + 3.2 + 0 = 28.2
3.	0 2/3 1	0 + 10 + 10 = 20	0 + 16 + 15 = 31
4.	0 1 1/2	0 + 15 + 5 = 20	0 + 24 + 7.5 = 31.5

(1), (2), (3), (4) are feasible ones but (4) is the optimum solution.

Spanning Trees

A spanning tree of a graph is just a sub graph that contains all the vertices and is a tree. A graph may have many spanning trees.

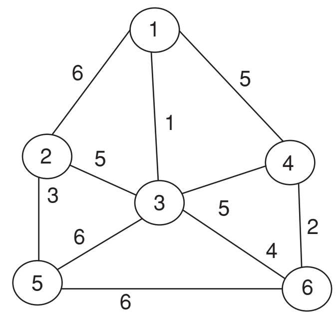
- A sub graph that spans (reaches out to) all vertices of a graph is called a spanning sub graph.
- A sub graph that is a tree and that spans all vertices of the original graph is called a spanning tree.
- Among all the spanning trees of a weighted and connected graph, the one (possibly more) with the least total weight is called a Minimum Spanning Tree (MST).

Prim's Algorithm

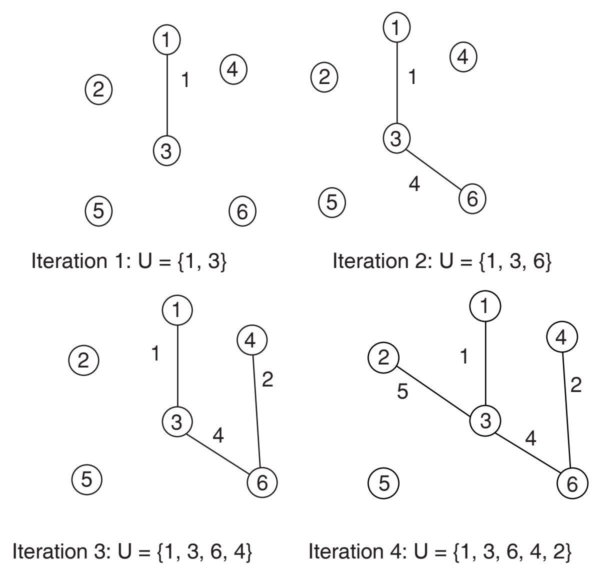
Prim's algorithm is a greedy algorithm that finds a minimum spanning tree for a connected weighted undirected graph. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized. The algorithm continuously increases the size, of a tree, one edge at a time starting with a tree consisting of a single vertex, until it spans all vertices.

- Using a simple binary heap data structure and an adjacency list representation, prim's algorithm can be shown to run in time $O(E \log V)$ where E is the number of edges and V is the number of vertices.

Example:



Start:



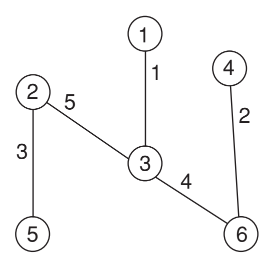
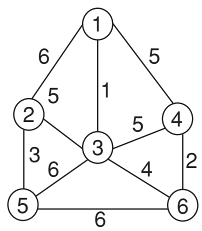


Figure 1 An example graph for illustrating prim's algorithm.

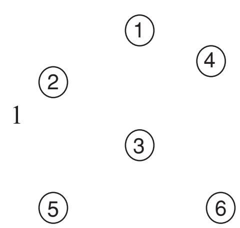
Kruskal's Algorithm

Like prim's algorithm, Kruskal's algorithm also constructs the minimum spanning tree of a graph by adding edges to the spanning tree one-by-one. At all points, during its execution the set of edges selected by prim's algorithm forms exactly one tree. On the other hand the set of edges selected by Kruskal's algorithm forms a forest of trees. Kruskals algorithm is conceptually simple. The edges are selected and added to the spanning tree in increasing order of their weights. An edge is added to the tree only if it does not create a cycle.

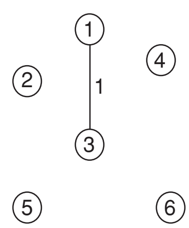
Example:



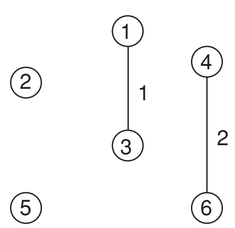
Start:



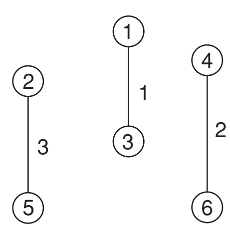
Initial configuration



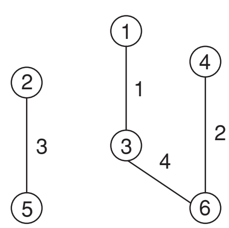
Setp 1: choose (1, 3)



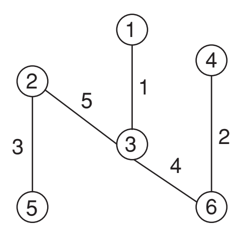
Setep 2: choose (4, 6)



Setep 3: choose (2, 5)



Setep 4: choose (3, 6)



Setep 6: choose (4, 3)

Tree and Graph Traversals

Back Tracking

Backtracking is a general algorithm technique that considers searching every possible combination in order to solve an optimization problem.

Backtracking is also known as depth first search (or) branch and bound. Backtracking is an important tool for solving constraint satisfaction problems, such as crosswords, verbal arithmetic, sudoku and many other puzzles. It is often the more convenient technique for parsing, for the knapsack problem and other combinational optimization problems.

- The advantage of backtracking algorithm is that they are complete, that is they are guaranteed to find every solution to every possible puzzle.

Graph Traversal

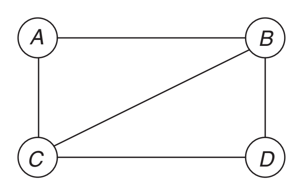
To traverse a graph is to process every node in the graph exactly once, because there are many paths leading from one node to another, the hardest part about traversing a graph is making sure that you do not process some node twice. There are general solutions to this difficulty.

1. When you first encounter a node, mark it as REACHED. When you visit a node, check if it is marked REACHED, if it is, just ignore it. This is the method our algorithms will use.
2. When you process a node, delete it from the graph. Deleting the node causes the deletion of all the arcs that lead to the node, so it will be impossible to reach it more than once.

General traversal strategy

1. Mark all nodes in the graph as NOT REACHED,
2. Pick a starting node, mark it as REACHED, and place it on the READY list.
3. Pick a node on the READY list. Process it remove it from READY. Find all its neighbors, those that are NOT REACHED should marked as REACHED and added to READY.
4. Repeat 3 until READY is empty.

Example:



Step I: $A = B = C = D =$ NOT REACHED
Step II: READY = {A} . A = REACHED
Step III: Process A. READY = {B, C}.
B = C = REACHED
Step IV: Process C. READY = {B, D}.
D = REACHED
Step V: Process B. READY = {D}
Step VI: Process D. READY = { }

The two most common traversal patterns are

- Breadth first traversal
- Depth first traversal

Breadth First Traversal

In breadth first traversal, READY is a QUEUE, not an arbitrary list. Nodes are processed in the order they are reached (FIFO), this has the effect of processing nodes according to their distance from the initial node. First, the initial node is processed. Then all its neighbors are processed. Then all of the neighbors etc.

- Since a graph has no root, we must specify the vertex at which to start the traversal.
- Breadth first tree traversal first visits all the nodes at depth zero (i.e., the root) then all the nodes at depth 1, and so on.

Procedure

First, the starting vertex is enqueued. Then, the following steps are repeated until the queue is empty.

1. Remove the vertex at the head of the queue and call it vertex.
2. Visit vertex
3. Follow each edge emanating from vertex to find the adjacent vertex and call it ‘t_o’. If ‘t_o’ has not already been put into the queue, enqueued it.

Notice, that a vertex can be put into the queue at most once. Therefore, the algorithm must some how keep track of the vertices that have been enqueued.

Procedure for BFS for undirected graph G(V, E)

To perform BFS over a graph, the data structures required are queue (Q) and the visited set (Visited), ‘V’ is the starting vertex.

Procedure for BFS(V)

Steps

1. Visit the vertex ‘V’
2. Enqueue the vertex V
3. while (Q is not Empty)
   (i) V = dequeue ();
   (ii) for all vertices ϑ adjacent to V
      (a) if not visited (ϑ)
         - Enqueue (ϑ)
         - Visit the vertex ‘ϑ’
         - end if.
         - end for
         - end while
         - Stop

Example:

A Unexplored vertex

A Visited vertex

Unexplored edge
Discovery edge
Cross edge

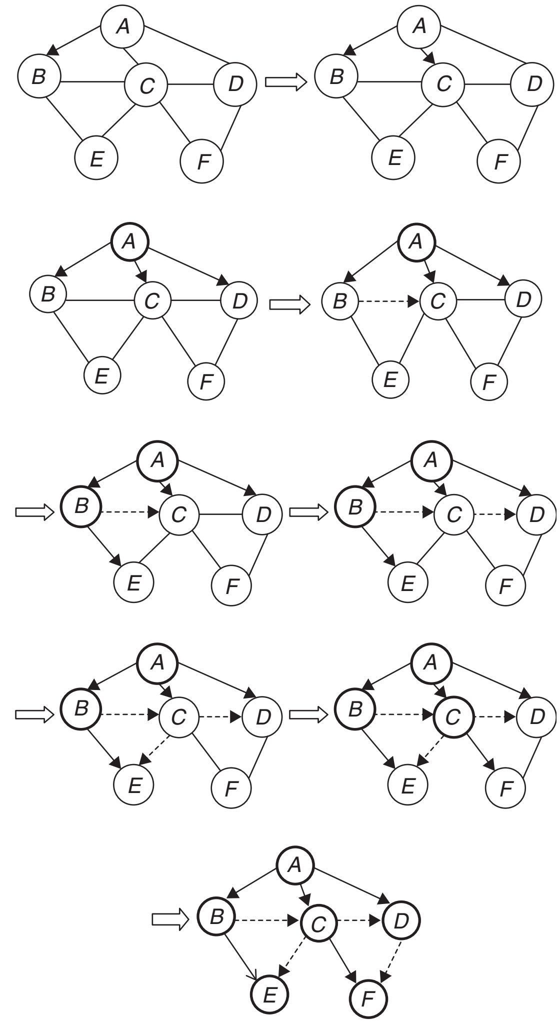


Figure 2 Breadth–first search

Depth First Search

A depth first traversal of a tree always starts at the root of the tree. Since a graph has no root, when we do a depth first traversal, we must specify the vertex at which to begin. A depth first traversal of a tree visits a node and then recursively visits the sub trees of that node similarly, depth first traversal of a graph visits a vertex and then recursively visits all the vertices adjacent to that node. A graph may contain cycles, but the traversal must visit every vertex at most once.

The solution to the problem is to keep track of the nodes that have been visited.

Procedure for DFS for undirected graph G(V, E)

To perform DFS over a graph, the data structures required are stack (S) and the list (visited), ‘V’ is the start vertex.

Procedure for DFS(V)

Steps

1. push the start vertex 'V' into the stack S
2. while (S is not empty)
   (i) pop a vertex V
   (ii) if 'V' is not visited
      (a) visit the vertex
      (b) Store 'V' in visited
      (c) push all the adjacent vertices of 'V' in to visited
   (iii) End if
3. End while
4. Stop.

Example:

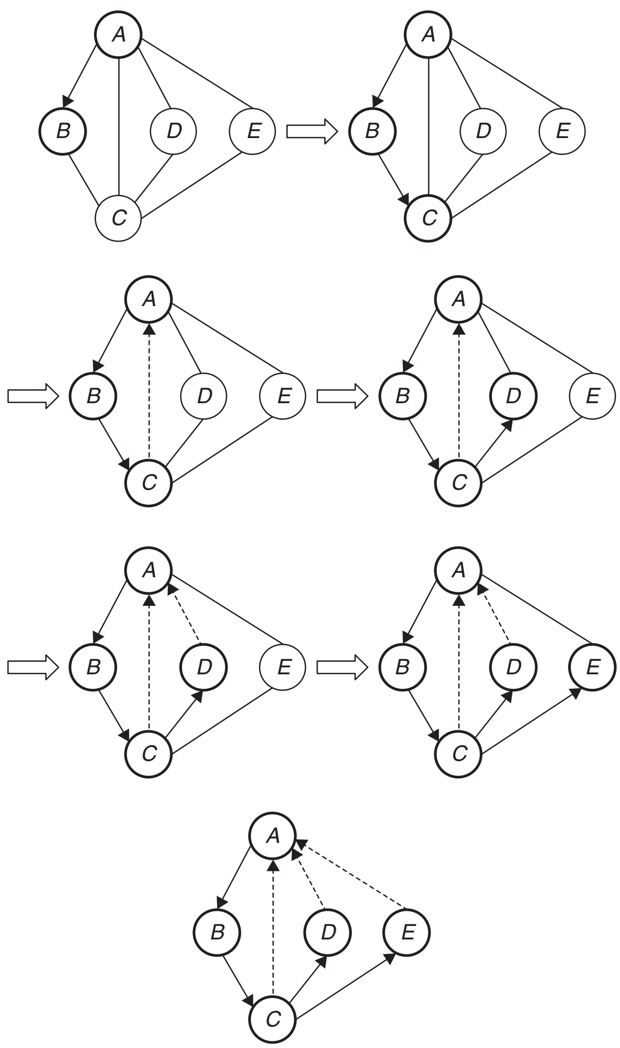
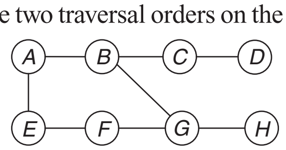


Figure 3 Depth first search

- Let us compare two traversal orders on the following graph:



Initial steps:

READY = [A]. process A. READY = [B, E]. process B.

It is at this point that two traversal strategies differ. Breadth first adds B's neighbors to the back of READY, depth first adds them to the front.

Breadth first

- READY = [E, C, G]
- Process E. READY = [C, G, F]
- Process C. READY = [G, F, D]
- Process G .READY = [F, D, H]
- Process F. READY = [D, H]
- Process D. READY = [H]
- Process H. READY = [ ]

Depth First

- READY = [C, G, E]
- Process C. READY = [D, G, E]
- Process D. READY = [G, E]
- Process G. READY = [H, F, E]
- Process H. READY = [F, E]
- Process F. READY = [E]
- Process E. READY = [ ]

Connected Components

A graph is said to be connected if every pair of vertices in the graph are connected. A connected component is a maximal connected sub graph of 'G'. Each vertex belongs to exactly one connected component as does each edge.

- A graph that is not connected is naturally and obviously decomposed into several connected components (Figure 4). Depth first search does this handily. Each restart of the algorithm marks a new connected component.
- The directed graph in (Figure 5) is "Connected" Part of it can be "Pulled apart" (so to speak, without "breaking" any edges).
- Meaningful way to define connectivity in directed graph is:

'Two nodes U and V of a directed graph $G = (V, E)$ connected if there is a path from U to V', and one from V to U. This relation between nodes is reflective, symmetric and transitive. As such, it partitions V into disjoint sets, called the strongly connected components of the graph. In the directed graph of figure 2 there are four strongly connected components.

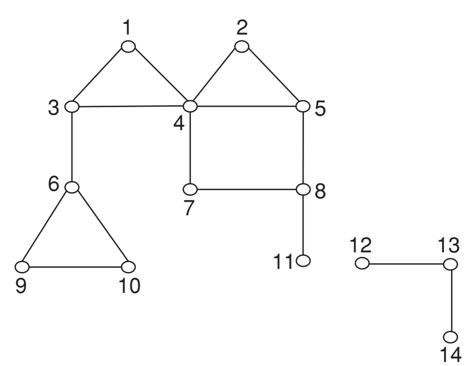


Figure 4 Undirected graph.

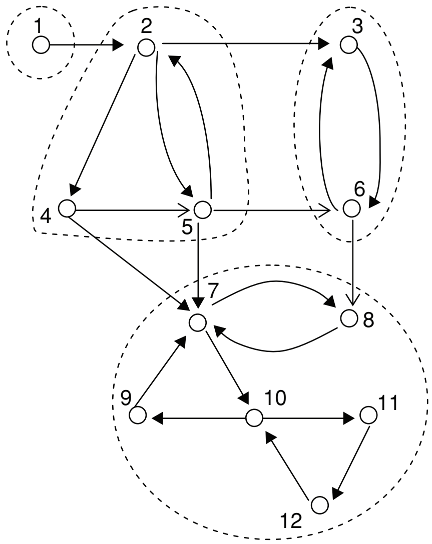
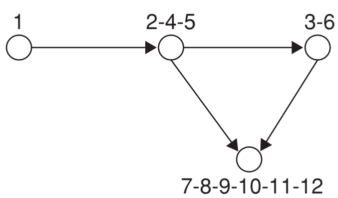


Figure 5 A directed graph and its strongly connected components

If we shrink each of these strongly connected components down to a single node and draw an edge between two of them if there is an edge from some node in the first to some node in the second, the resulting directed graph has to be a directed acyclic graph (DAG) – it has no cycles (figure 6). The reason is simple.

A cycle containing several strongly connected components would merge them all to a single strongly connected component.



Every directed graph is a DAG of its strongly connected components.

Huffman Codes

For compressing data, a very effective and widely used technique is Huffman coding. We consider the data to be a sequence of characters. Huffmans's greedy algorithm uses a table of the frequencies of occurrence of the characters to build up an optimal way of representing each character as a binary string.

Example: Suppose we have a 1,00,000 – character data file, that we wish to store compactly. The characters in the file occur with the frequencies given below:

Character	a	b	c	d	e	f
Frequency	47	12	11	14	10	6

Solution: Two methods are used for compression of data are:

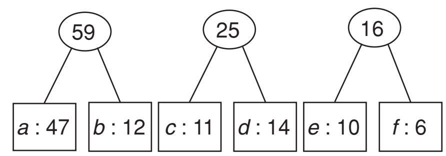
Fixed Length Coding

- Arrange all the characters in sequence (no particular order is followed)
- $a = 47, b = 12, c = 11, d = 14, e = 10, f = 6$

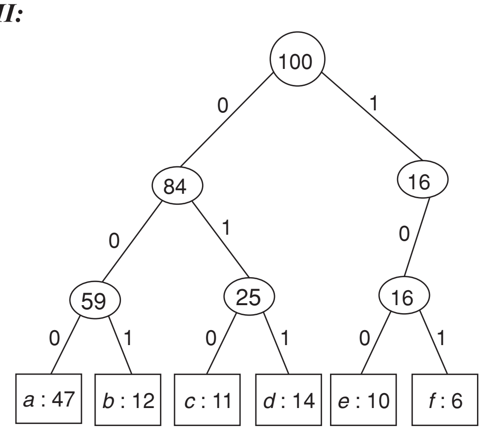
Step I:

a : 47	b : 12	c : 11	d : 14	e : 10	f : 6

Step II:



Step III:



We interpret the binary code word for a character as the path from the root to that character where '0' means 'go to the left child', and 1 means 'go to the right child'.

The above tree is not binary search tree, since the leaves need not appear in sorted order.

Constructing Huffman Code

This algorithm builds the tree T corresponding to the optimal code in a bottom-up manner. It begins with set of $|C|$ leaves and performs a sequence of $|C|$-1 'merging' operations to create the final tree.

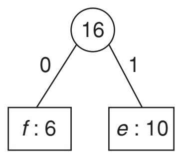
- A min-priority queue Q, keyed on f, is used to identify the 2 least – frequent objects to merge together. The result of the merger of 2 objects is a new object whose frequency is the sum of the frequencies of the 2 objects that were merged.
- In the given example, there are 6 alphabets the initial queue size is $n = 6$, and 5 merge steps are required to build the tree. The final tree represents the optimal prefix code. The code word for a letter is the sequence of edge labels on the path from the root to the letter.

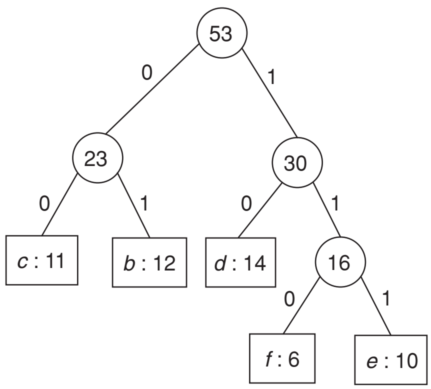
$a = 47, b = 12, c = 11, d = 14, e = 10, f = 6$

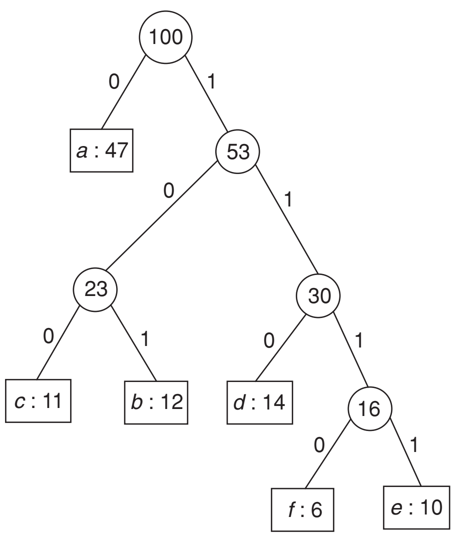
Step I: Arrange the characters in non-decreasing order according to their frequencies

f : 6	e : 10	c : 11	b : 12	d : 14	a : 47

Let x and y be 2 characters in C having the lowest frequencies. Then there exists an optimal prefix code for C in which the code words for x and y have the same length and differ only in the last bit







Analysis: The analysis of the running time of Huffman's algorithm assumes that Q is implemented as a binary min-heap for a set C of 'n' characters, the initialization of Q can be performed in $O(n)$ time using the BUILD – MIN HEAP procedure.

Each heap operation requires $O(\log n)$ time, and this will be performed exactly $(n - 1)$ times, it contributes to $O(n \log n)$ running time. Thus the total running time of HUFFMAN on a set of 'n' characters is $O(n \log n)$.

Task-scheduling problem

This is the problem of optimally scheduling unit – time tasks on a single processor, where each task has a deadline, along with a penalty that must be paid if the deadline is missed. The problem looks complicated, but it can be solved in simple manner using a greedy algorithm.

- A unit – time task is a job, such as a program to be run on a computer, that requires exactly one unit of time to complete.
- Given a finite set S of unit – time tasks, a schedule for S is a permutation of S specifying the order in which these tasks are to be performed.
- The first task in the schedule begins at time '0' and finishes at time 1, the second task begins at time 1 and finishes at time 2, and so on
- The problem of scheduling unit – time tasks with deadlines and penalties for a single processor has the following inputs:

1. A set $S = \{a_1, a_2, \ldots an\}$ of n unit – time tasks:
2. A set of n integer deadlines $d_1, d_2, \ldots d_n$ such that each d_i satisfies $1 \le d_i \le n$ and task a_i is supposed to finish by time d_i.
3. A set of n non-negative weights or penalties w_1, $w_2, \ldots w_n$, such that we incur a penalty of w_i if task a_i is not finished by time d_i and we incur no penalty if a task finishes by its deadline.

Example: Consider the following 7 tasks, T_1, T_2, T_3, T_4, T_5 T_6, T_7. Every task is associated with profit and deadline.

Tasks	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Deadline	4	2	4	3	1	4	5
Profit	75	65	55	45	40	35	30

45	65	55	75	30			
T_4	T_2	T_3	T_1	T_7			
0	1	2	3	4	5	6	7

T_1 has highest profit, so it will be executed first and the deadline of T_1, is '4' so T_1 has to be executed within 4 slots of time, same procedure is applied to other tasks also.

The tasks which are not executed by CPU are T_5 and T_6.

Profit: sum up the profits made by executing the tasks. Profit = 45 + 65 + 55 + 75 + 30 = 270

Analysis: We can use a greedy algorithm to find a maximum weight independent set of tasks. We can then create an optimal schedule having the tasks in A as its early tasks.

This method is an efficient algorithm for scheduling unit – time tasks with deadlines and penalties for a single processor. The running time is $O(n^2)$ using GREEDY METHOD, since each of the $O(n)$ independent checks made by that algorithm takes time $O(n)$.

Sorting and Order Statistics

Minimum and Maximum

This algorithm determines, how many comparisons are necessary to find minimum or maximum of a set of 'n' elements. Usually we can obtain maximum or minimum, by performing

$(n - 1)$ comparisons; examine each element of the set in turn and keep track of the smallest element seen so far.

Consider the following procedure.

Assume that the set of elements reside in an array A where length $[A] = n$

MINIMUM (A)
Min $\leftarrow A[1]$
For $i \leftarrow 2$ to length $[A]$
Do if min $> A[i]$
Then min $\leftarrow A[i]$
Return min.

Simultaneous minimum and maximum

In some applications, we must find both the minimum and the maximum of a set of 'n' elements.

We can find the minimum and maximum independently using $(n - 1)$ comparisons for each, for a total of $(2n - 2)$ comparisons.

- In fact, atmost $3\lfloor n/2 \rfloor$ comparisons are sufficient to find both the minimum and the maximum.
- The strategy is to maintain the minimum and maximum elements seen so far.
- Rather than processing each element of the input by comparing it against the current minimum and maximum at a cost of 2 comparisons per element, we process elements in pairs.
- Compare pairs of elements from the input with each other, and then we compare smaller to the current minimum and the larger to the current maximum, at a cost of 3 comparisons for every 2 elements.
- Setting up initial values for the current minimum and maximum depends on whether 'n' is odd or even. If 'n' is odd, we set both the minimum and maximum to the value of the first element, and then we process the rest of the elements in pairs.
- If 'n' is even, we perform 1 comparison on the first 2 elements to determine the initial values of the minimum and maximum and then process the rest of the elements in pairs.

Analysis: If 'n' is odd the total number of comparisons would be $3\lfloor n/2 \rfloor$.

If 'n' is even, we need 1 initial comparison followed by $\frac{3(n-2)}{2}$ comparisons, for a total of $\frac{3n}{2} - 2$.

$\therefore$ The total number of comparisons is atmost $3\lfloor n/2 \rfloor$

Graph Algorithms

Single Source Shortest Path

In a shortest-path problem, we are given a weighted directed graph $G = (V, E)$ with weight function $W : E \rightarrow R$ mapping edges to real-valued weights. The weight of path $P = < V_O, V_1 \ldots V_K >$ is the sum of the weights of its constituent edges. Shortest-path weight from U to V is defined by

$$\delta(U,V) = \begin{cases} \{\min\{W(P): u \rightarrow v\} & \text{if there is a path from} \\ \infty & \text{'}U\text{' to '}V\text{' otherwise} \end{cases}$$

Edge weights can be interpreted as metrics other than distances. They are often used to represent time, cost, penalties, loss, or any other quantity.

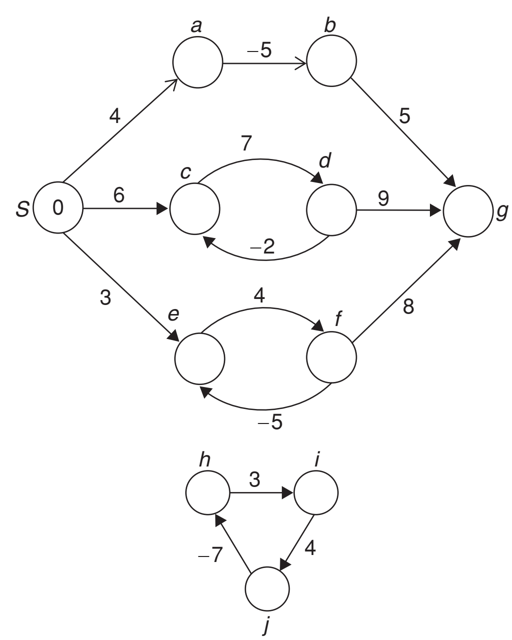
- The breadth first search algorithm is a shortest-path algorithm that works on un weighted graphs, that is, graphs in which each edge can be considered to have unit weight.

Negative-weight edges

Some of the instances of the single-source-shortest-paths problem, there may be edges whose weights are negative.

- If the graph $G = (V, E)$ contains no negative weight cycles reachable from source S, then for all $v \in V$, the shortest – path weight $d(S, V)$ remains well defined, even if it has a negative value.
- If there is a negative-weight cycle reachable from S, shortest-path weights are not well defined.
- No path from 'S' to a vertex on the cycle can be a shortest path - a lesser weight path can always be found that follows the proposed 'shortest' path and then traverses the negative-weight cycle.
- If there is a negative-weight cycle on some path form 'S' to 'V', we define $\delta(S,V) = -\infty$.

Example: Consider the following graph, calculate the shortest distance to all vertices from sources 'S'.



Solution:

- Shortest path from S to a is $\delta(S, a) = W(S, a) = 4$ (because there is only one path from 'S' to 'a')
- Similarly, there is only one path from 's' to 'b' $\delta(S, a) = W(S, a) + W(a, b) = 4 + (-5) = -1$
- Shortest-path from 's' to 'c'

There are infinitely many paths from 'S' to 'c'

1. $<S, c>$
2. $<S, c, d, c>$
3. $<S, c, d, c, d, c>$ and so on

$\delta <S, c> = 6$

$\delta (S, d, d, c) = 6 + 7 - 2 = 11$

$\delta (S, c, d, c, d, c) = 6 + 7 - 2 + 7 - 2 = 16$

$\delta (S, c, d, c, d, c, d, c)$

$= 6 + 7 - 2 + 7 - 2 + 7 - 2 = 21$

The cycle $<c, d, c>$ has weight $= 7 + (-2) = 5 > 0$

The shortest path from 'S' to 'c' is $<s, c>$ with weight $\delta (S, c) = 6$ similarly, the shortest-path from 'S' to 'd' is $<s, c, d>$, with weight $\delta (S, d) = w (S, c) + W(c, d) = 13$

May there are infinitely paths from 'S' to 'e'

1. $<s, e>$
2. $<s, e f, e>$
3. $<s, e, f, e, f, e>$ and so on

Since the cycle $<e, f, e>$ has weight $4 + (-5) = -1 < 0$. However, there is no shortest path from 'S' to 'e' by traversing the negative-weight cycle $<e, f, e>$ arbitrarily many times, we can find paths from 's' to 'e' with arbitrarily large negative weights,

So $\delta(S, e) = -\infty$

Similarly, $\delta(S, f) = -\infty$

- The shortest path from 'S' to 'g':
  'g' can be reachable from 'f'; we can also find paths with arbitrarily large negative weights from 's' to 'g' and $\delta(s, g) = -\infty$
- Vertices 'h', 'i' and 'j' also form a negative - weight cycle. They are not reachable from 'S' so, $\delta(S, h) = \delta(S, i) = \delta(S, j) = \infty$

Dijkstra's Algorithm

Dijkstra's algotithm solves the single-source shortest-path problem on a weighted, directed graph $G = (V, E)$, for the case in which all edge weights are non-negative.

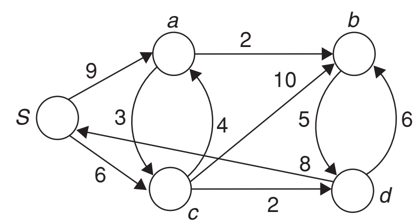
- The running time of Dijkstra's algorithm is lower than that of the Bellman–Ford algorithm.
- Dijkstra's algorithm maintains a set 's' of vertices whose final shortest-path weights from the source 'S' have already been determined.
- The algorithm repeatedly selects the vertex $u \in (V - S)$ with the minimum shortest-path estimate, adds 'u' to 'S'

DIJKSTRA (G, W, S)
INITIALIZE – SINGLE – SOURCE (G, S)
$S \leftarrow \varnothing$
$S \leftarrow V[G]$
While $Q \neq 0$
do $u \leftarrow$ EXTRACT – MIN(Q)
$S \leftarrow S \, U\{u\}$
For each vertex $v \in \text{Adj}[u]$
do RELAX (u, v, w)

The algorithm maintains the invariant that $Q = V - S$ at the start of each iteration of the while loop. Initially the min – priority queue Q contains all the vertices in V. ($\because S = \varnothing$). Each time through the while loop, a vertex 'u' is extracted from $Q = V - S$ and added to set S.

- Each vertex is extracted from Q and added to S exactly once, so the contents of while loop will be executed exactly $|v|$ times.
- Dijkstra's algorithm always chooses the 'closest' or 'lightest' vertex in $(V - S)$ to add to set S, we say that it uses a greedy strategy.

Example: Consider the following graph, what is the shortest path?



Solution:

S	V – S
S	a b c d
S c	a b d
S c d	a b
S c d a	b
S c d a b	∅

Distance from S to all vertices of $(V - S)$

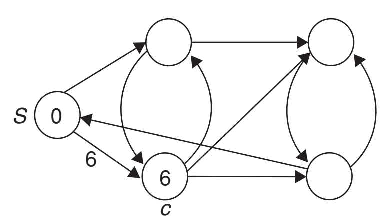
$d[a] = 9$

$d[b] = \infty$

$d[c] = 6$

$d[d] = \infty$

$9, \infty, 6, \infty$ values are given to MIN-PRIORITY Queue 'Q', '6' is returned.

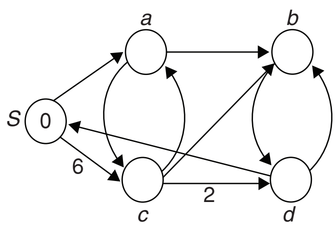


Distance from [Sc] to all vertices of $(V - S)$

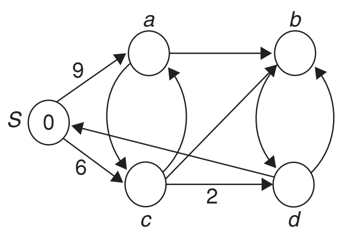
$d[b] = (S - c - b) = 6 + 10 = 16$

$d[a] = \min\{(S - a) = 9, (s - c - a) = 10\} = 9$

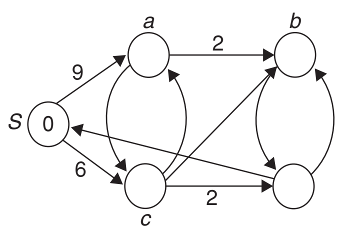
$d[d] = \min\{\infty, (S - c - d) = 6 + 2 = 8\} = 8$



Distance from [s c d] to [ab]
$d[a] = \min\{9, (S - c - d - S - a) = 25\} = 9$
$d[b] = \min\{16, (S - c - d - b) = 14\} = 14$



$d[a] = \min\{14, (s - a - b) = 9 + 2 = 11\} = 11$



Analysis: It maintains the min-priority queue 'Q' by calling three priority-queue operations: INSERT, EXTRACT-MIN, and DECREASE-KEY. We maintain the min-priority queue by taking the vertices being numbered 1 to $|v|$. We store $d[v]$ in the vth entry of an array. Each INSERT and DECREASE-KEY operation takes $O(1)$ time, and each EXTRACT- MIN operation takes $O(v)$ time ($\therefore$ we have to search through the entire array) for a total time of $O(v^2 + E) = O(v^2)$.

- If we implement the min - priority queue with a binary min-heap. Each EXTRACT-MIN operation takes time $O(\log V)$, there are $|V|$ such operations.
- The time to build the binary min-heap is $O(v)$. Each DECREASE-KEY operation takes time $O(\log V)$, and there are still atmost $|E|$ such operations. The total running time is $O((V + E) \log V)$, which is $O(E \log V)$ if all vertices are reachable form the source.
- We can achieve a running time of $O(V \log V + E)$ by implementing the min-priority queue with a Fibonacci heap.

Bellman–Ford Algorithm

Bellman–Ford algorithm solves the single-source shortest-path problems in the case in which edge weights may be negative.

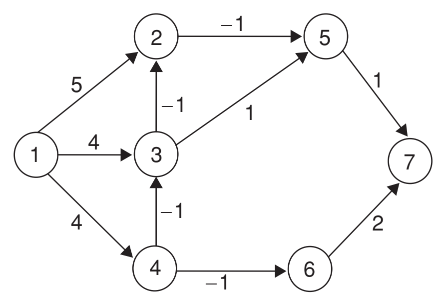
- When negative edge lengths are permitted, we require that the graph have no cycles of negative length. This is necessary to ensure that shortest-paths consist of a finite number of edges.
- When there are no cycles of negative length, there is a shortest-path between any two vertices of an n-vertex graph that has atmost $(n - 1)$ edges on it.
- A path that has more than $(n - 1)$ edges must repeat atleast one vertex and hence must contain a cycle
- Let $\text{dist}^x[u]$ be the length of a shortest-path from the source vertex 'v' to vertex 'u' under the constraint that the shortest-path contains atmost 'x' edges. Then $\text{dist}'[u] = \text{cost}[v, u]$ $1 \le u \le n$ when there are no cycles of negative length we can limit our search for shortest-paths to paths with at most $(n - 1)$ edges. Hence, $\text{dist}^{n-1}[u]$ is the length of an unrestricted shortest-path from 'v' to 'u'.

The Recurrence Relation for dist is:

$$\text{Dist}^k[u] = \min\{\text{dist}^{k-1}[u], \min\{\text{dist}^{k-1}[i] + \text{cost}[i, u]\}\}$$

This recurrence can be used to compute dist^k from dist^{k-1}, for $k = 2, 3, \ldots, n - 1$.

Example: Consider the given directed graph



Find the shortest path from vertex '1' to all other vertices using Bellman–Ford algorithm?

Solution: Source vertex is '1' the distance from '1' to '1' in all '6' iterations will be zero. Since the graph has '7' vertices, the shortest-path can have atmost '6' edges. The following figure illustrates the implementation of Bellman–Ford algorithm:

	1	2	3	4	5	6	7
1	0	5	4	4	∞	∞	∞
2	0	3	3	4	4	3	∞
3	0	2	3	4	2	3	5
4	0	2	3	4	1	3	3
5	0	2	3	4	1	3	2
6	0	2	3	4	1	3	2
7	0	2	3	4	1	3	2

Analysis

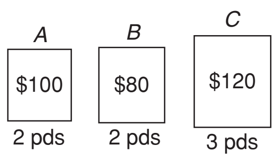
- Each iteration takes $O(n^2)$ time if adjacency matrices are used and $O(e)$ time if adjacency lists are used. Here 'e' is the number of edges in the graph.
- The time complexity is $O(n^3)$ when adjacency matrices are used and $O(N * E)$ when adjacency lists are used.

Exercises

Practice Problems 1

Directions for questions 1 to 14: Select the correct alternative from the given choices.

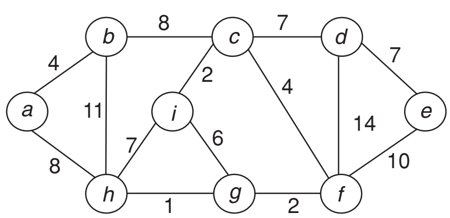
1. A thief enters a store and sees the following:



His knapsack can hold 4 pounds, what should he steal to maximize profit? (Use 0–1 Knapsack).
(A) *A* and *B* (B) *A* and *C*
(C) *B* and *C* (D) *A*, *B* and *C*

2. By using fractional Knapsack, calculate the maximum profit, for the data given in the above question?
(A) 180 (B) 170
(C) 160 (D) 150

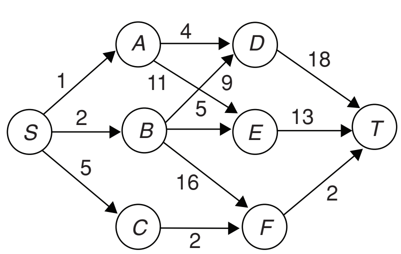
3. Consider the below figure:



What is the weight of the minimum spanning tree using Kruskals algorithm?
(A) 34 (B) 35
(C) 36 (D) 38

4. Construct a minimum spanning tree for the figure given in the above question, using prim's algorithm. What are the first three nodes, added to the solution set respectively (consider '*a*' as starting node).
(A) *b*, *c*, *i* (B) *h*, *b*, *c*
(C) *c*, *i*, *b* (D) *h*, *c*, *b*

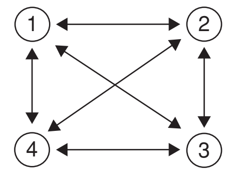
5. Consider the below graph, calculate the shortest distance from '*S*' to '*T*'?



(A) 23 (B) 9
(C) 20 (D) 22

6. Solve the travelling salesman problem, with the given distances in the form of matrix of graph, which of the following gives optimal solution?

C	1	2	3	4
1	0	15	20	25
2	10	0	14	15
3	11	18	0	17
4	13	13	14	0



(A) 1 – 2 – 4 – 3 – 1 (B) 2 – 3 – 4 – 1 – 2
(C) 1 – 4 – 2 – 3 – 1 (D) 2 – 4 – 3 – 1 – 2

7. Calculate the maximum profit using greedy strategy, knapsack capacity is 50. The data is given below:
$n = 3$
$(w_1, w_2, w_3) = (10, 20, 30)$
$(p_1, p_2, p_3) = (60, 100, 120)$ (dollars)? (0/1 knapsack)
(A) 180 (B) 220
(C) 240 (D) 260

Common data for questions 8 and 9: Given that

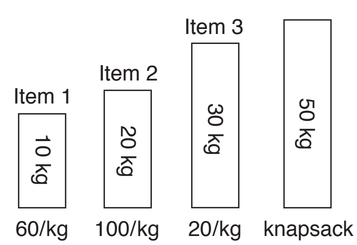
	a	b	c	d	e	f
Frequency	45	13	12	16	9	5
Fixed length code word	000	001	010	011	100	101

8. Using Huffman code, find the path length of internal nodes.
(A) 8 (B) 100
(C) 100 × 8 (D) 100/8

9. Using above answer, external path length will be
(A) 18 (B) 108
(C) 8 (D) None of these

Common data for questions 10 and 11:

10.



Using 0–1 knapsack select a subset of the three items shown, whose weight must not exceed 50 kg. What is the value?
(A) 2220 (B) 2100
(C) 2600 (D) 2180

11. Which of the following gives maximum profit, using fractional knapsack?
(A) $x_1 = 1, x_2 = 1, x_3 = 0$ (B) $x_1 = 1, x_2 = 1, x_3 = 2/3$
(C) $x_1 = 1, x_2 = 0, x_3 = 1$ (D) $x_1 = 1, x_2 = 1, x_3 = 1/3$

12. Using dynamic programming find the longest common subsequence (LCS) in the given 2 sub sequences:

x [1, … , m]
y [1, … , n]
x : A B C B D A B
Y : B D C A B A

Find longest sequence sets common to both.
(A) (BDAB, BCAB, BCBA)
(B) (BADB, BCAB, BCBA)
(C) (BDAB, BACB, BCBA)
(D) (BDAB, BCAB, BBCA)

13. Let C_1, C_2, C_3, C_4 represent coins.

C_1 = 25 paisa
C_2 = 10 paisa
C_3 = 5 paisa
C_4 = 1 paisa

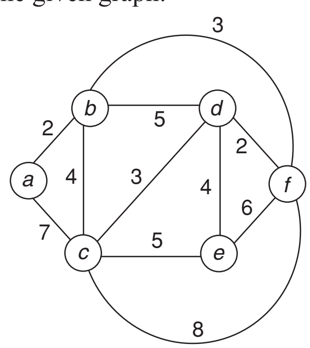
To represents 48 paisa, what is the minimum number of coins used, using greedy approach?
(A) 6 (B) 7
(C) 8 (D) 9

14. Worst-case analysis of hashing occurs when
(A) All the keys are distributed
(B) Every key hash to the same slot
(C) Key values with even number, hashes to slots with even number
(D) Key values with odd number hashes to slots with odd number.

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

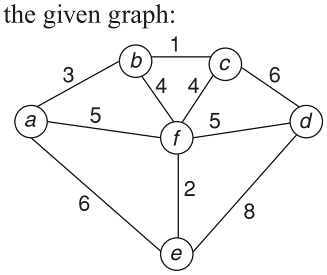
1. Consider the given graph:



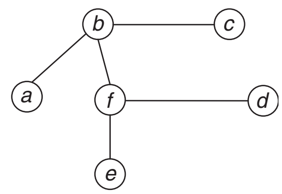
Which one of the following cannot be the sequence of edges added, in that order, to a minimum spanning tree using Kruskal's algorithm?
(A) $(a-b)$, $(d-f)$, $(b-f)$, $(d-c)$, $(d-e)$
(B) $(a-b)$, $(d-f)$, $(d-c)$, $(b-f)$, $(d-e)$
(C) $(d-f)$, $(a-b)$, $(d-c)$, $(b-f)$, $(d-e)$
(D) $(d-f)$, $(a-b)$, $(b-f)$, $(d-e)$, $(d-c)$

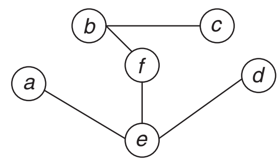
2. The worst case height analysis of B-tree is
(A) $O(n)$
(B) $O(n^2)$
(C) $O(\log n)$
(D) $O(n \log n)$

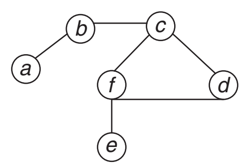
3. Consider the given graph:

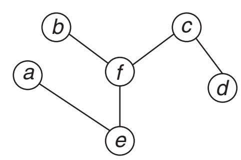


Which of the following is the minimum spanning tree. (If we apply Kruskal algorithm).

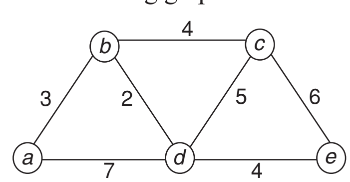
(A)


(B)


(C)


(D)


4. Consider the following graph:



Find the shortest path using Dijkstra's algorithm.
(A) a – b – d – e (B) a – b – c – d
(C) a – c – d – e (D) a – b – c – e

5. Which statement is true about Kruskal's algorithm?
(A) It is a greedy algorithm for the minimum spanning tree problem.
(B) It constructs spanning tree by selecting edges in increasing order of their weights.
(C) It does not accept creation of cycles in spanning tree.
(D) All the above

6. Dijkstra's algorithm bears similarity to which of the following for computing minimum spanning trees?
(A) Breadth first search (B) Prim's algorithm
(C) Both (A) and (B) (D) None of these

7. Which of the following algorithm always yields a correct solution for a graph with non-negative weights to compute shortest paths?
(A) Prim's algorithm (B) Kruskal's algorithm
(C) Dijkstra's algorithm (D) Huffman tree

8. Let the load factor of the hash table is number of keys is n, cells of the hash table is m then

(A) $\propto = n/m$ (B) $\propto = m/n$
(C) $\propto \dfrac{m+1}{n}$ (D) $\propto \dfrac{n+1}{m}$

9. To implement Dijkstra's shortest path algorithm on unweighted graphs so that it runs in linear time, the data structure to be used is:
(A) Queue
(B) Stack
(C) Heap
(D) B-tree

10. The development of a dynamic-programming algorithm can be broken into a sequence of four steps, which are given below randomly.
I. Construct an optimal solution from computed information.
II. Compute the value of an optimal solution in a bottom-up fashion.
III. Characterize the structure of an optimal solution.
IV. Recursively defines the value of an optimal solution.

The correct sequence of the above steps is
(A) I, II, III, IV (B) IV, III, I, II
(C) IV, II, I, III (D) III, IV, II I

11. Let V stands for vertex, E stands for edges.

For both directed and undirected graphs, the adjacency list representation has the desirable property that the amount of memory required is
(A) $\theta(V)$ (B) $\theta(E)$
(C) $\theta(V+E)$ (D) $\theta(V-E)$

12. Which of the following is false?
(A) Adjacency-matrix representation of a graph permits faster edge look up.
(B) The adjacency matrix of a graph requires $\theta(v^2)$ memory, independent of the number of edges in the graph.
(C) Adjacency-matrix representation can be used for weighted graphs.
(D) All the above

13. Dynamic programming is a technique for solving problems with
(A) Overlapped sub problems
(B) Huge size sub problems
(C) Small size sub problems
(D) None of these

14. The way a card game player arranges his cards, as he picks them up one by one is an example of _____.
(A) Bubble sort (B) Selection sort
(C) Insertion sort (D) None of the above

15. You want to check whether a given set of items is sorted. Which method will be the most efficient if it is already in sorted order?
(A) Heap sort (B) Bubble sort
(C) Merge sort (D) Insertion sort

Previous Years' Questions

Data for question 1: We are given 9 tasks $T_1, T_2 \ldots T_9$. The execution of each task requires one unit of time. We can execute one task at a time. Each task T_i has a profit P_i and a deadline D_i. Profit P_i is earned if the task is completed before the end of the D_ith unit of time.

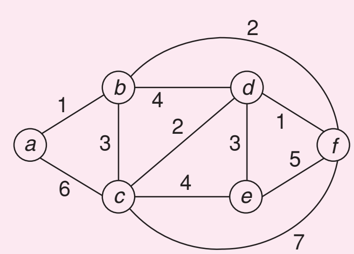
Task	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9
Profit	15	20	30	18	18	10	23	16	25
Deadline	7	2	5	3	4	5	2	7	3

1. What is the maximum profit earned? **[2005]**
(A) 147 (B) 165
(C) 167 (D) 175

2. Consider a weighted complete graph G on the vertex set $\{v_1, v_2, \ldots, v_n\}$ such that the weight of the edge (v_i, v_j) is $2|i-j|$. The weight of the minimum spanning tree is: **[2006]**
(A) $n-1$ (B) $2n-2$
(C) $\binom{n}{2}$ (D) n^2

3. To implement Dijkstra's shortest path algorithm on unweighted graphs so that it runs in linear time, the data structure to be used is: **[2006]**
(A) Queue
(B) Stack
(C) Heap
(D) B-Tree

4. Consider the following graph: **[2006]**



Which one of the following cannot be the sequence of edges added, in that order, to a minimum spanning tree using Kruskal's algorithm?

(A) (a – b), (d – f), (b – f), (d – c), (d – e)
(B) (a – b), (d – f), (d – c), (b – f), (d – e)
(C) (d – f), (a – b), (d – c), (b – f), (d – e)
(D) (d – f), (a – b), (b – f), (d – e), (d – c)

Common data for questions 5 and 6: A 3-ary max-heap is like a binary max-heap, but instead of 2 children, nodes have 3 children. A 3-ary heap can be represented by an array as follows: The root is stored in the first location, $a[0]$, nodes in the next level, from left to right, is stored from $a[1]$ to $a[3]$. The nodes from the second level of the tree from left to right are stored from $a[4]$ location onward. An item x can be inserted into a 3-ary heap containing n items by placing x in the location $a[n]$ and pushing it up the tree to satisfy the heap property.

5. Which one of the following is a valid sequence of elements in an array representing 3-ary max-heap? **[2006]**

(A) 1, 3, 5, 6, 8, 9 (B) 9, 6, 3, 1, 8, 5
(C) 9, 3, 6, 8, 5, 1 (D) 9, 5, 6, 8, 3, 1

6. Suppose the elements 7, 2, 10 and 4 are inserted, in that order, into the valid 3-ary max-heap found in the above question, Q-76. Which one of the following is the sequence of items in the array representing the resultant heap? **[2006]**

(A) 10, 7, 9, 8, 3, 1, 5, 2, 6, 4
(B) 10, 9, 8, 7, 6, 5, 4, 3, 2, 1
(C) 10, 9, 4, 5, 7, 6, 8, 2, 1, 3
(D) 10, 8, 6, 9, 7, 2, 3, 4, 1, 5

7. In an unweighted, undirected connected graph, the shortest path from a node S to every other node is computed most efficiently, in terms of *time complexity*, by **[2007]**

(A) Dijkstra's algorithm starting from S.
(B) Warshall's algorithm
(C) Performing a DFS starting from S.
(D) Performing a BFS starting from S.

8. A complete n-ary tree is a tree in which each node has n children or no children. Let I be the number of internal nodes and L be the number of leaves in a complete n-ary tree. If $L = 41$, and $I = 10$, what is the value of n? **[2007]**

(A) 3 (B) 4
(C) 5 (D) 6

9. Consider the following C program segment where CellNode represents a node in a binary tree: **[2007]**

```
struct CellNode {
  struct CellNode *leftChild;
     int element;
   struct CellNode *rightChild;
};
```

```
int GetValue (struct CellNode *ptr) {
   int value = 0;
   if (ptr != NULL) {
       if ((ptr->leftChild == NULL) &&
            (ptr->rightChild == NULL))
          value = 1;
   else
  value = value + GetValue(ptr->leftChild)
           + GetValue(ptr->rightChild);
     }
  return(value);
```

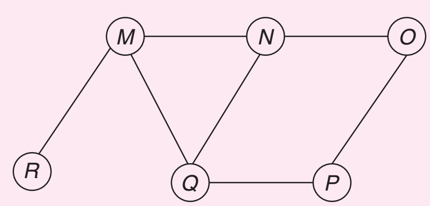
The value returned by GetValue when a pointer to the root of a binary tree is passed as its argument is:

(A) The number of nodes in the tree
(B) The number of internal nodes in the tree
(C) The number of leaf nodes in the tree
(D) The height of the tree

10. Let w be the minimum weight among all edge weights in an undirected connected graph. Let e be a specific edge of weight w. Which of the following is FALSE? **[2007]**

(A) There is a minimum spanning tree containing e.
(B) If e is not in a minimum spanning tree T, then in the cycle formed by adding e to T, all edges have the same weight.
(C) Every minimum spanning tree has an edge of weight w.
(D) e is present in every minimum spanning tree.

11. The Breadth first search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is **[2008]**



(A) MNOPQR (B) NQMPOR
(C) QMNPRO (D) QMNPOR

12. G is a graph on n vertices and $2n - 2$ edges. The edges of G can be partitioned into two edge-disjoint spanning trees. Which of the following is NOT true for G? **[2008]**

(A) For every subset of k vertices, the induced subgraph has atmost $2k - 2$ edges
(B) The minimum cut in G has atleast two edges
(C) There are two edge-disjoint paths between every pair of vertices
(D) There are two vertex-disjoint paths between every pair of vertices

13.

Dijkstra's single source shortest path algorithm when run from vertex a in the above graph, computes the correct shortest path distance to **[2008]**

(A) Only vertex *a*
(B) Only vertices *a*, *e*, *f*, *g*, *h*
(C) Only vertices *a*, *b*, *c*, *d*
(D) all the vertices

14. You are given the post-order traversal, *P*, of a binary search tree on the n elements 1, 2,…, *n*. You have to determine the unique binary search tree that has *P* as its post-order traversal. What is the time complexity of the most efficient algorithm for doing this? **[2008]**

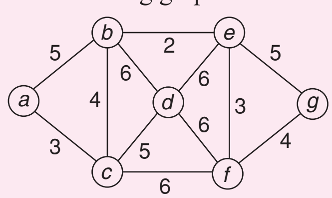
(A) $\Theta(\log n)$
(B) $\Theta(n)$
(C) $\Theta(n \log n)$
(D) None of the above, as the tree cannot be uniquely determined

15. Which of the following statement(*s*) is/are correct regarding Bellman–Ford shortest path algorithm? **[2009]**

P. Always finds a negative weighted cycle, if one exists.
Q. Finds whether any negative weighted cycle is reachable from the source.

(A) P only (B) Q only
(C) Both P and Q (D) Neither P nor Q

16. Consider the following graph: **[2009]**



Which one of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's algorithm? **[2009]**

(A) (b, e) (e, f) (a, c) (b, c) (f, g) (c, d)
(B) (b, e) (e, f) (a, c) (f, g) (b, c) (c, d)
(C) (b, e) (a, c) (e, f) (b, c) (f, g) (c, d)
(D) (b, e) (e, f) (b, c) (a, c) (f, g) (c, d)

Common data for questions 17 and 18: Consider a binary max-heap implemented using an array.

17. Which one of the following array represents a binary max-heap? **[2009]**

(A) {25, 12, 16, 13, 10, 8, 14}
(B) {25, 14, 13, 16, 10, 8, 12}
(C) {25, 14, 16, 13, 10, 8, 12}
(D) {25, 14, 12, 13, 10, 8, 16}

18. What is the content of the array after two delete operations on the correct answer to the previous question? **[2009]**

(A) {14, 13, 12, 10, 8} (B) {14, 12, 13, 8, 10}
(C) {14, 13, 8, 12, 10} (D) {14, 13, 12, 8, 10}

Common data for questions 19 and 20: Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the weight of the edge $\{i, j\}$.

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

19. What is the minimum possible weight of a spanning tree *T* in this graph such that vertex 0 is a leaf node in the tree *T*? **[2010]**

(A) 7 (B) 8
(C) 9 (D) 10

20. What is the minimum possible weight of a path *P* from vertex 1 to vertex 2 in this graph such that *P* contains at most 3 edges? **[2010]**

(A) 7 (B) 8
(C) 9 (D) 10

Common data for questions 21 and 22: A hash table of length 10 uses open addressing with hash function $h(k) = k \bmod 10$, and linear probing. After inserting 6 values into an empty hash table, the table is as shown below:

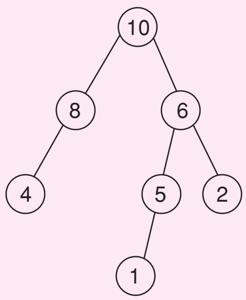
0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

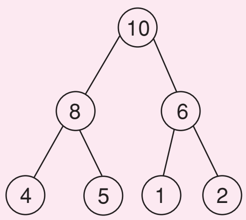
21. Which one of the following choices gives a possible order in which the key values could have been inserted in the table? **[2010]**

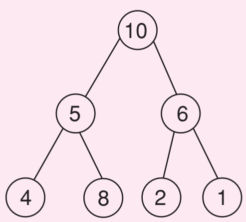
(A) 46, 42, 34, 52, 23, 33
(B) 34, 42, 23, 52, 33, 46
(C) 46, 34, 42, 23, 52, 33
(D) 42, 46, 33, 23, 34, 52

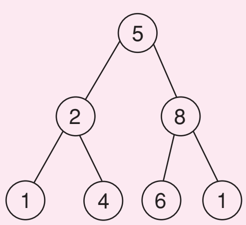
22. How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above? **[2010]**
(A) 10 (B) 20
(C) 30 (D) 40

23. A max-heap is a heap where the value of each parent is greater than or equal to the value of its children. Which of the following is a max-heap? **[2011]**

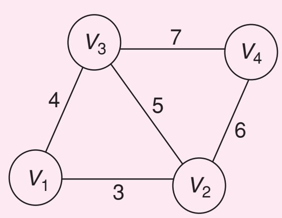
(A)


(B)


(C)


(D)


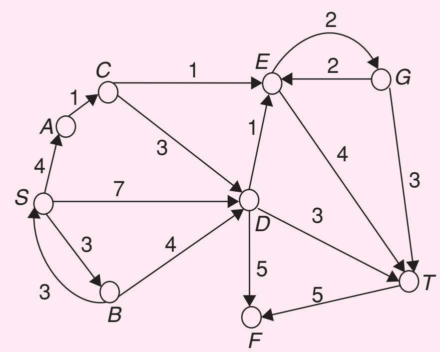
Common data for questions 24 and 25: An undirected graph $G(V, E)$ contains $n(n > 2)$ nodes named $V_1, V_2, \ldots, V_n$. Two nodes V_i, V_j are connected if and only if $0 < |i - j| \leq 2$. Each edge (V_i, V_j) is assigned a weight $i + j$. A sample graph with $n = 4$ is shown below.



24. What will be the cost of the minimum spanning tree (MST) of such a graph with n nodes? **[2011]**
(A) $\frac{1}{12}(11n^2 - 5n)$ (B) $n^2 - n + 1$
(C) $6n - 11$ (D) $2n + 1$

25. The length of the path from V_5 to V_6 in the MST of previous question with $n = 10$ is **[2011]**
(A) 11 (B) 25
(C) 31 (D) 41

26. Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered. **[2012]**



(A) SDT (B) SBDT
(C) SACDT (D) SACET

27. Let G be a weighted graph with edge weights greater than one and G^1 be the graph constructed by squaring the weights of edges in G. Let T and T^1 be the minimum spanning trees of G and G^1, respectively, with total weights t and t^1. Which of the following statements is **TRUE**? **[2012]**
(A) $T^1 = T$ with total weight $t^1 = t^2$
(B) $T^1 = T$ with total weight $t^1 < t^2$
(C) $T^1 \neq T$ but total weight $t^1 = t^2$
(D) None of the above

28. What is the time complexity of Bellman–Ford single-source shortest path algorithm on a complete graph of n vertices? **[2013]**
(A) $\Theta(n^2)$ (B) $\Theta(n^2 \log n)$
(C) $\Theta(n^3)$ (D) $\Theta(n^3 \log n)$

29. Consider the following operation along with Enqueue and Dequeue operations on queues, where k is a global parameter.

```
MultiDequeue(Q) {
m = k
```

```
while (Q is not empty) and (m > 0) {
          Dequeue (Q)
          m = m - 1
    }
}
```

What is the worst case time complexity of a sequence of n queue operations on an initially empty queue? **[2013]**

(A) $\Theta(n)$ (B) $\Theta(n + k)$
(C) $\Theta(nk)$ (D) $\Theta(n^2)$

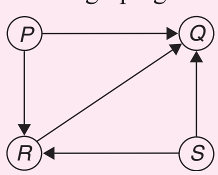
30. The preorder traversal sequence of a binary search tree is 30, 20, 10, 15, 25, 23, 39, 35, 42. Which one of the following is the post order traversal sequence of the same tree? **[2013]**

(A) 10, 20, 15, 23, 25, 35, 42, 39, 30
(B) 15, 10, 25, 23, 20, 42, 35, 39, 30
(C) 15, 20, 10, 23, 25, 42, 35, 39, 30
(D) 15, 10, 23, 25, 20, 35, 42, 39, 30

31. Let G be a graph with n vertices and m edges. What is the tightest upper bound on the running time of depth first search on G, when G is represented as an adjacency matrix? **[2014]**

(A) $\theta(n)$ (B) $\theta(n + m)$
(C) $\theta(n^2)$ (D) $\theta(m^2)$

32. Consider the directed graph given below. **[2014]**



Which one of the following is TRUE?

(A) The graph does not have any topological ordering.
(B) Both PQRS and SRQP are topological orderings.
(C) Both PSRQ and SPRQ are topological orderings.
(D) PSRQ is the only topological ordering.

33. There are 5 bags labeled 1 to 5. All the coins in a given bag have the same weight. Some bags have coins of weight 10 gm. Others have coins of weight 11 gm. I pick 1, 2, 4, 8, 16 coins respectively from bags 1 to 5. Their total weight comes out to 323 gm. Then the product of the labels of the bags having 11 gm coins is ___ **[2014]**

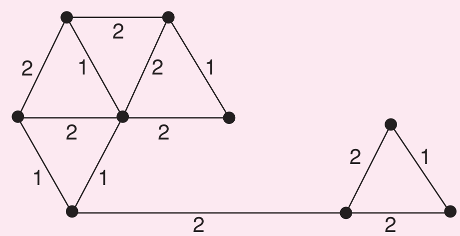
34. A priority queue is implemented as a max-heap. Initially it has 5 elements. The level-order traversal of the heap is : 10, 8, 5, 3, 2. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is **[2014]**

(A) 10, 8, 7, 3, 2, 1, 5 (B) 10, 8, 7, 2, 3, 1, 5
(C) 10, 8, 7, 1, 2, 3, 5 (D) 10, 8, 7, 5, 3, 2, 1

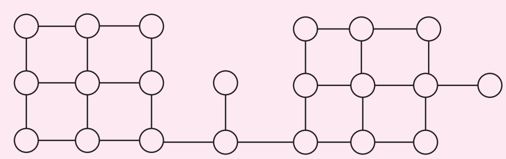
35. Consider the tree arcs of a BFS traversal from a source node W in an unweighted, connected, undirected graph. The tree T formed by the tree acrs is a data structure for computing **[2014]**

(A) The shortest path between every pair of vertices
(B) The shortest path from W to every vertex in the graph
(C) The shortest paths from W to only those nodes that are leaves of T.
(D) The longest path in the graph

36. The number of distinct minimum spanning trees for the weighted graph below is _______. **[2014]**

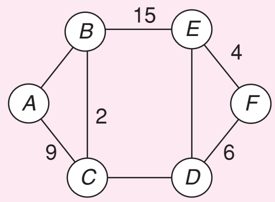


37. Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (Including the initial call) is _______. **[2014]**



38. Suppose we have a balanced binary search tree T holding n numbers. We are given two numbers L and H and wish to sum up all the numbers in T that lie between L and H. suppose there are m such numbers in T. If the tightest upper bound on the time to compute the sum is $O(n^a \log^b n + m^c \log^d n)$, the value of $a + 10b + 100c + 1000d$ is ———. **[2014]**

39. The graph shown below has 8 edges with distinct integer edge weights. The minimum spanning tree (MST) is of weight 36 and contains the edges: $\{(A, C), (B, C), (B, E), (E, F), (D, F)\}$. The edge weights of only those edges which are in the MST are given in the figure shown below. The minimum possible sum of weights of all 8 edges of this graph is _______ **[2015]**



40. Consider two decision problems Q_1, Q_2 such that Q_1 reduces in polynomial time to 3-SAT and 3-SAT reduces in polynomial time to Q_2. Then which one of the following is consistent with the above statement? **[2015]**

(A) Q_1 is in NP, Q_2 is NP hard.
(B) Q_2 is in NP, Q_1 is NP hard.
(C) Both Q_1 and Q_2 are in NP.
(D) Both Q_1 and Q_2 are NP hard.

41. Given below are some algorithms, and some algorithm design paradigms.

1. Dijkstra's Shortest Path	i. Divide and Conquer
2. Floyd-Warshall algorithm to compute all pairs shortest path	ii. Dynamic Programming
3. Binary search on a sorted array	iii. Greedy design
4. Backtracking search on a graph	iv. Depth-first search
	v. Breadth-first search

Match the above algorithms on the left to the corresponding design paradigm they follow.

(A) 1–i, 2–iii, 3–i, 4–v
(B) 1–iii, 2–iii, 3–i, 4–v
(C) 1–iii, 2–ii, 3–i, 4–iv
(D) 1–iii, 2–ii, 3–i, 4–v

42. A Young tableau is a 2D array of integers increasing from left to right and from top to bottom. Any unfilled entries are marked with ∞, and hence there cannot be any entry to the right of, or below a ∞. The following Young tableau consists of unique entries.

1	2	5	14
3	4	6	23
10	12	18	25
31	∞	∞	∞

When an element is removed from a Young tableau, other elements should be moved into its place so that the resulting table is still a Young tableau (unfilled entries maybe filled in with a ∞). The minimum number of entries (other than 1) to be shifted, to remove 1 from the given Young tableau is _______ **[2015]**

43. Which one of the following hash functions on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 for i ranging from 0 to 2020? **[2015]**

(A) $h(i) = i^2 \bmod 10$
(B) $h(i) = i^3 \bmod 10$
(C) $h(i) = (11 * i^2) \bmod 10$
(D) $h(i) = (12 * i) \bmod 10$

44. Let G be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are TRUE? **[2016]**

P : Minimum spanning tree of G does not change.

Q : Shortest path between any pair of vertices does not change.

(A) P only
(B) Q only
(C) Neither P nor Q
(D) Both P and Q

45. Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being 1,2,3,4,5, and 6. The maximum possible weight that a minimum weight spanning tree of G can have is _____ . **[2016]**

46. G = (V,E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/ are **TRUE**? **[2016]**

I. If e is the lightest edge of some cycle in G, then every MST of G includes e

II. If e is the heaviest edge of some cycle in G, then every MST of G excludes e

(A) I only
(B) II only
(C) both I and II
(D) neither I nor II

47. Breadth First Search (BFS) is started on a binary tree beginning from the root vertex. There is a vertex t at a distance four from the root. If t is the n-th vertex in this **BFS** traversal, then the maximum possible value of n is _____. **[2016]**

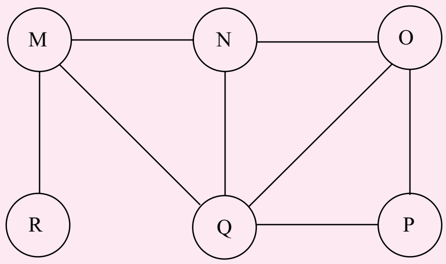
48. Let $G = (V,E)$ be *any* connected undirected edge-weighted graph. The weights of the edges in E are positive and distinct. Consider the following statements:

(I) Minimum spanning Tree of G is always unique.
(II) Shortest path between any two vertices of G is always unique.

Which of the above statements is/are necessarily true? **[2017]**

(A) (I) only
(B) (II) only
(C) both (I) and (II)
(D) neither (I) nor (II)

49. The Breadth First Search (BFS) algorithm has been implemented using the queue data structure. Which one of the following is a possible order of visiting the nodes in the graph below? **[2017]**



(A) MNOPQR
(B) NQMPOR
(C) QMNROP
(D) POQNMR

50. A message is made up entirely of characters from the set $X = \{P, Q, R, S, T\}$. The table of probabilities for each of the characters is shown below:

Character	Probability
P	0.22
Q	0.34
R	0.17
S	0.19
T	0.08
Total	1.00

If a message of 100 characters over X is encoded using Huffman coding, then the expected length of the encoded message in bits is ________. **[2017]**

51. Let G be a simple undirected graph. Let T_D be a depth first search tree of G. Let T_B be a breadth first search tree of G.

Consider the following statements.

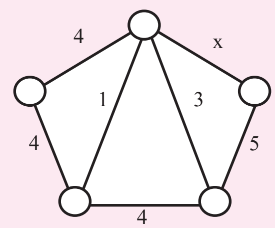
(I) No edge of G is a cross edge with respect to T_D. (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D.)

(II) For every edge (u, v) of G, if u is at depth i and v is at depth j in T_B, then $|i - j| = 1$.

Which of the statements above must necessarily be true? **[2018]**

(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

52. Consider the following undirected graph G:



Choose a value for x that will maximize the number of minimum weight spanning trees (MWSTs) of G. The number of MWSTs of G for this value of x is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. A **2.** A **3.** B **4.** A **5.** B **6.** A **7.** B **8.** A **9.** A **10.** C
11. B **12.** A **13.** A **14.** B

Practice Problems 2

1. D **2.** C **3.** A **4.** A **5.** D **6.** C **7.** C **8.** A **9.** A **10.** D
11. C **12.** C **13.** A **14.** C **15.** D

Previous Years' Questions

1. A **2.** B **3.** C **4.** D **5.** D **6.** A **7.** D **8.** C **9.** C **10.** B
11. C **12.** D **13.** D **14.** B **15.** B **16.** D **17.** C **18.** D **19.** D **20.** B
21. C **22.** C **23.** B **24.** B **25.** C **26.** D **27.** **28.** C **29.** A **30.** D
31. C **32.** C **33.** 12 to 12 **34.** A **35.** B **36.** 6 to 6 **37.** 19 **38.** 110 **39.** 69
40. A **41.** C **42.** 5 **43.** B **44.** A **45.** 7 **46.** B **47.** 31 **48.** A **49.** D
50. 225 **51.** A **52.** 4

Chapter 5

Dynamic Programming

LEARNING OBJECTIVES

- *Dynamic programming*
- *Multi-stage graph*
- *All pairs shortest path problem*
- *Hashing methods*
- *Mid-square method*
- *Folding method*
- *Resolving collisions*
- *Matrix chain multiplication*
- *Longest common subsequence*
- *Optimal substructure of LCS*
- *NP-hard and NP-complete*
- *P-problem*
- *NP-problem*
- *P, NP, and Co-NP*
- *Cooks theorem*
- *Non-deterministic search*

Dynamic Programming

Dynamic programming is a method for solving complex problems by breaking them down into simpler sub problems. It is applicable to problems exhibiting the properties of overlapping sub problems which are only slightly smaller, when applicable; the method takes far less time than naive method.

- The key idea behind dynamic programming is to solve a given problem, we need to solve different parts of the problem (sub problems) then combine the solutions of the sub problems to reach an overall solution. Often, many of these sub problems are the same.
- The dynamic programming approach seeks to solve each sub problem only once, thus reducing the number of computations. This is especially useful when the number of repeating sub problems is exponentially large.
- There are two key attributes that a problem must have in order for dynamic programming to be applicable 'optimal sub structure' and 'overlapping sub-problems'. However, when the overlapping problems are much smaller than the original problem, the strategy is called 'divide-and-conquer' rather than 'dynamic programming'. This is why merge sort-quick sort are not classified as dynamic programming problems.

Dynamic programming is applied for:
- Multi stage graph
- All pairs shortest path

Principle of Optimality

It states that whatever the initial state is, remaining decisions must be optimal with regard to the state following from the first decision.

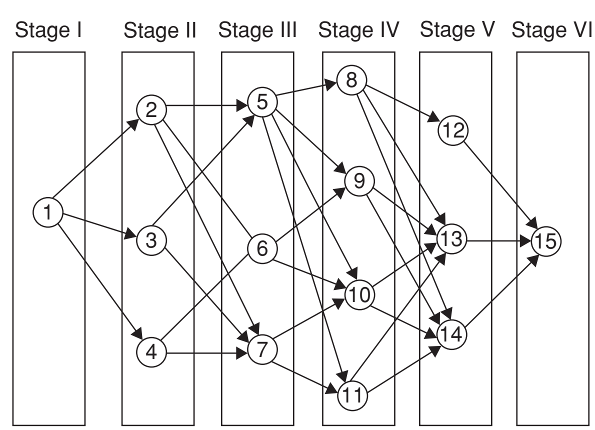
To solve a problem using dynamic programming strategy, it must observe the principle of optimality.

Multi-stage Graph

A multi-stage graph is a graph

- $G = (V, E)$ with V partitioned into $K >= 2$ disjoint subsets such that if (a, b) is in E, then a is in V_i, and b is in V_{i+1} for some sub sets in the partition;
- $|V_1| = |V_K| = 1$ the vertex S in V_1 is called the source; the vertex t is called the sink.
- G is usually assumed to be a weighted graph.
- The cost of a path from node V to node W is sum of the costs of edges in the path.
- The 'multi-stage graph problem' is to find the minimum cost path from S to t.

Example:



Costs of edges

$1 - 2 \rightarrow 10$

$1 - 3 \rightarrow 20$

1 – 4	$\rightarrow$	30
2 – 5	$\rightarrow$	10
2 – 6	$\rightarrow$	20
2 – 7	$\rightarrow$	30
3 – 5	$\rightarrow$	40
3 – 7	$\rightarrow$	50
4 – 6	$\rightarrow$	40
4 – 7	$\rightarrow$	30
5 – 8	$\rightarrow$	10
5 – 9	$\rightarrow$	20
5 – 10	$\rightarrow$	30
5 – 11	$\rightarrow$	40
6 – 9	$\rightarrow$	20
6 – 10	$\rightarrow$	30
7 – 10	$\rightarrow$	30
7 – 11	$\rightarrow$	20
8 – 12	$\rightarrow$	10
8 – 13	$\rightarrow$	20
8 – 14	$\rightarrow$	30
9 – 13	$\rightarrow$	20
9 – 14	$\rightarrow$	10
10 – 13	$\rightarrow$	10
10 – 14	$\rightarrow$	20
11 – 13	$\rightarrow$	10
11 – 14	$\rightarrow$	30
12 – 15	$\rightarrow$	20
13 – 15	$\rightarrow$	10
14 – 15	$\rightarrow$	30

Solution Using Backward Cost

Format: COST (Stage, node) = minimum cost of travelling to the node in stage from the source node (node 1)

Step I:

Cost (I, 1) = 0

Step II:

Cost (II, 2) = cost (I, 1) + cost (1, 2) = 0 + 10 = 10
Cost (II, 3) = cost (I, 1) + cost (1, 3) = 0 + 20 = 20
Cost (II, 4) = cost (I, 1) + cost (1, 4) = 0 + 30 = 30

Step III:

Cost (III, 5) = min {cost (II, 2) + cost (2, 5),
cost (II, 3) + cost (3, 5),
cost (II, 4) + cost (4, 5)
= min $\{10 + 10, 20 + 40, 30 + \infty\}$
= 20 $\rightarrow$ Via path 1 – 2 – 5

Cost (III, 6) = min {cost (II, 2) + cost (2, 6),
cost (II, 3) + cost (3, 6),
cost (II, 4) + cost (4, 6)}
= min $\{10 + 20, 20 + \infty, 30 + 40\}$
= 30 $\rightarrow$ via the path 1 – 2 – 6

Cost (III, 7) = min {cost(II, 2) + cost (2, 7),
Cost (II, 3) + cost (3, 7),
Cost (II, 4) + cost (4, 7)}
= min {10 + 30, 20 + 50, 30 + 30}
= 40 $\rightarrow$ Via the path 1 – 2 – 7

Step IV:

Cost (IV, 8) = min {cost (III, 5) + cost (5, 8),
Cost (III, 6) + cost (6, 8),
Cost (III, 7) + cost (7, 8)}
= min $\{20 + 10, 30 + \infty, 40 + \infty\}$
= 30 $\rightarrow$ Via path 1 – 2 – 5 – 8

Cost (IV, 9) = min {cost (III, 5) + cost (5, 9),
Cost (III, 6) + cost (6, 9),
Cost (III, 7) + cost (7, 9)}
= min $\{20 + 20, 30 + 20, 40 + \infty\}$
= 40 $\rightarrow$ Via the path 1 – 2 – 5 – 9

Cost (IV, 10) = min {cost (III, 5) + cost (5, 10),
Cost (III, 6) + cost (6, 10),
Cost (III, 7) + cost (7, 10}
= min {20 + 30, 30 + 30, 40 + 30}
= 50 $\rightarrow$ Via the path 1 – 2 – 5 – 10

Cost (IV, 11) = min {cost (III, 5) + cost (5, 11)
Cost (III, 6) + cost (6, 11),
Cost (III, 7) + cost (7, 11)}
= min $\{20 + 40, 30 + \infty, 40 + 20\}$
= 60 $\rightarrow$ Via the path 1 – 2 – 5 – 11
or Via the path 1 – 2 – 7 – 11

Step V:

Cost (V, 12) = min {cost (IV, 8) + cost (8, 12)
Cost (IV, 9) + cost (9, 12),
Cost (IV, 10) + cost (10, 12),
Cost (IV, 11) + cost (11, 12)}
= min $\{30 + 10, 40 + \infty, 50 + \infty, 60 + \infty\}$
= 40 $\rightarrow$ Via the path 1 − 2 − 5 – 8 – 12

Cost (V, 13) = min {cost (IV, 8) + cost (8, 13)
Cost (IV, 9) + cost (9, 13),
Cost (IV, 10) + cost (10, 13),
Cost (IV, 11) + cost (11, 13)}
= min {30 + 20, 40 + 20, 50 + 10, 60 + 10}
= 50 $\rightarrow$ Via the path 1 – 2 – 5 – 8 – 13

Cost (V, 14) = min {cost (IV, 8) + cost (8, 14)
Cost (IV, 9) + cost (9, 14),
Cost (IV, 10) + cost (10, 14),
Cost (IV, 11) + cost (11, 14)}
= min {30 + 30, 40 + 10, 50 + 20, 60 + 30}
50 $\rightarrow$ Via the path 1 – 2 – 5 – 9 – 14

Step VI:

Cost (VI, 15) = min {cost (V, 12) + cost (12, 15),
Cost (V, 13) + cost (13, 15),
Cost (V, 14) + cost (14, 15)}
= min {40 + 20, 50 + 10, 50 + 30}
= 60 → Via the path 1 – 2 – 5 – 8 – 13 – 15
(or) 1 – 2 – 5 – 8 – 12 – 15

All Pairs Shortest Path Problem (Floyd–Warshall Algorithm)

A weighted graph is a collection of points (vertices) connected by lines (edges), where each edge has a weight (some real number) associated with it.

Example: A graph in the real world is a road map. Each location is a vertex and each road connecting locations is an edge. We can think of the distance travelled on a road from one location to another as the weight of that edge.

- The Floyd–Warshall algorithm determines the shortest path between all pairs of vertices in a graph.
- The vertices in a graph be numbered from 1 to n. Consider the subset {1, 2, … K} of these n vertices.
- Finding the shortest path from vertex i to vertex j that uses vertex in the set {1, 2, … K} only. There are two situations.
  1. K is an intermediate vertex on the shortest path.
  2. K is not an intermediate vertex on the shortest path.

In the first situation, we can break down our shortest path into two paths: i to K and then K to j. Note that all the vertices from i to K are from the set {1, 2, … K – 1} and that all the intermediate vertices from K to j are from the set {1, 2, … K –1}. Also in the second situation, we simply have that all intermediate vertices are from the set {1, 2, ... K – 1}. Now define the function D for a weighted graph with the vertices {1, 2, … n} as follows.

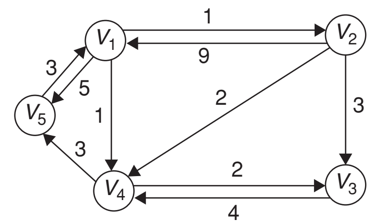
$D(i, j, K)$ = the shortest distance from vertex i to vertex j using the intermediate vertices. In the set {1, 2, … K}

Using the above idea, we can recursively define the function D.

$$D(i, j, K) = W(i, j) \text{ if } K = 0$$
$$\min (D(i, j, K-1), D(i, K, K-1) + D(K, j, K-1)) \text{ if } K > 0$$

- The first line says that if we do not allow intermediate vertices, then the shortest path between two vertices is the weight of the edge that connects them. If no such weightexists, we usually define this shortest path to be of length infinity.
- The second line pertains to allowing intermediate vertices. It says that the minimum path from i to j through vertices {1, 2, … K} is either the minimum path from i to j through vertices {1, 2, … K – 1} OR the sum of the minimum path from vertex i to K through {1, 2, … K – 1} plus the minimum path from vertex K to j through {1, 2, … K – 1}. Since this is the case, we compute both and choose the smaller of these.

Example:



The weight matrix will be

	1	2	3	4	5
1	0	1	∞	1	5
2	9	0	3	2	∞
3	∞	∞	0	4	∞
4	∞	∞	2	0	3
5	3	∞	∞	∞	0

Let $D^{(K)}[i, j]$ = weight of a shortest path from v_i to v_j using only vertices from $\{v_1, v_2, \ldots v_k\}$ as intermediate vertices in the path.

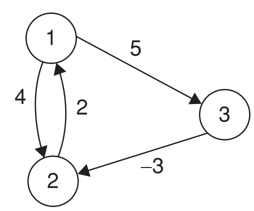
- $D^{(0)} = W$
- $D^{(n)} = D$ which is the goal matrix.

How to compute $D^{(K)}$ from $D^{(K-1)}$?

Case I: A shortest path from v_i to v_j restricted to using only vertices from $\{v_1, v_2, \ldots v_K\}$ as intermediate vertices does not use V_K.
Then $D^{(K)}[i, j] = D^{(K-1)}[i, j]$

Case II: A shortest path from v_i to v_j restricted to using only vertices from $\{v_1, v_2 \ldots v_K\}$ as intermediate vertices does use V_K. Then $D^{(K)}[i, j] = D^{(K-1)}[i, K] + D^{(K-1)}[K, j]$
Since $D^{(K)}[i, j] = D^{(K-1)}[i, j]$
or $D^{(K)}[i, j] = D^{(K-1)}[i, K] + D^{(K-1)}[K, j]$
We conclude: $D^{(K)}[i, j] = \min\{D^{(K-1)}[i, j], D^{(K-1)}[i, K] + D^{(K-1)}[K, j]\}$

Example: 1



$W = D^{\circ} =$

	1	2	3
1	0	4	5
2	2	0	∞
3	∞	–3	0

$P =$

	1	2	3
1	0	0	0
2	0	0	0
3	0	0	0

$K = 1$, vertex 1 can be intermediate node

$D^1\,[2, 3] = \min\,(D^{\circ}[2, 3], D^{\circ}[2, 1] + D^{\circ}[1, 3])$
$= \min\,(\infty, 7)$
$= 7$

$D^1\,[3, 2] = \min\,(D^{\circ}[3, 2], D^{\circ}[3, 1] + D^{\circ}[1, 2])$
$= \min\,(-3, \infty)$
$= -3$

$D^1 =$

	1	2	3
1	0	4	5
2	2	0	7
3	∞	–3	0

$P =$

	1	2	3
1	0	0	0
2	0	0	1
3	0	0	0

$K = 2$, vertices 1, 2 can be intermediate nodes,

$D^2\,[1, 3] = \min\,(D'[1, 3], D'[1, 2] + D'[2, 3])$
$= \min\,(5, 4 + 7) = 5$

$D^2\,[3, 1] = \min\,(D'[3, 1], D'[3, 2] + D'[2, 1])$
$= \min\,(\infty, -3 + 2)$
$= -1$

$D^2 =$

	1	2	3
1	0	4	5
2	2	0	7
3	–1	–3	0

$P =$

	1	2	3
1	0	0	0
2	0	0	1
3	2	0	0

$K = 3$ vertices 1, 2, 3 can be intermediate

$D^3[1, 2] = \min\,(D^2[1, 2], D^2[1, 3] + D^2[3, 2])$
$= \min\,(4, 5 + (-3))$
$= 2$

$D^3[2, 1] = \min\,(D^2[2, 1], D^2[\,2, 3] + D^2[3, 1])$
$= \min\,(2, 7 + (-1))$
$= 2$

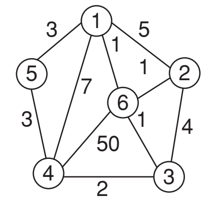
$D^3 =$

	1	2	3
1	0	2	5
2	2	0	7
3	–1	–3	0

$P =$

	1	2	3
1	0	3	0
2	0	0	1
3	2	0	0

Example 2:



The final distance matrix and P

$D^6 =$

	1	2	3	4	5	6
1	0	2(6)	2(6)	4(6)	3	1
2	2(6)	0	2(6)	4(6)	5(6)	1
3	2(6)	2(6)	0	2	5(4)	1
4	4(6)	4(6)	2	0	3	3(3)
5	3	5(4)	5(4)	3	0	4(1)
6	1	1	1	3(3)	4(1)	0

The values in parenthesis are the non-zero P values.

Table 1 *Divide and conquer vs dynamic programming.*

1. This design strategy divides the problem into sub problems, conquer the each sub problem recursively, finally combine all the sub problem solutions, for the original problem.	1. This design strategy chooses an optimal solution for the problem, by recursively defining the value of optimal solution, these values are computed in bottom up fashion or top down fashion.
2. each sub problem is solved recursively, and consumes more time at each sub problem	2. Each sub problem is solved only once and is stored in table
3. Sub problems are independent of each other e.g., Binary search	3. The sub problems are dependent e.g., Traveling sales person problem

Dynamic Programming vs Greedy Method

The main difference between greedy method (*GM*) and dynamic programming (*DP*) methodology is, *DP* considers all possible solutions for the given problem and picks the optimal one. Where as greedy, considers only one set of solutions to the problem.

The other difference between *GM* and *DP* is that, *GM* considers the choice, which is best at that step, which is done at each level of the sub problem. That is, it won't reconsider its choice. The choices reflect only present, won't consider the future choices, where as *DP* tries out all the best alternatives and finds the optimal solution. It implements principle of optimality. At each stage of the problem, it decides based on the previous decision made in the previous stage.

Hashing Methods

Uniform Hash Function

If the keys, K, are integers randomly distributed in [0, r] then hash function $H(K)$ is given as

$$H(K) = \left\lfloor \frac{mk}{r} \right\rfloor$$

$H(K)$ is a uniform hash function

Uniform hashing function should ensure

$$\sum_{K|h(K)=0} P(K) = \sum_{K|h(K)=1} P(K) = \cdots \sum_{K|h(K)=0} P(K) = \frac{1}{m}$$

$P(K)$ = probability that a key K, occurs that is the number of keys that map to each slot is equal.

Division method

Hashing an integer x is to divide x by M and then to use the remainder modulo M. This is called the division method of hashing. In this case the hash function is

$$\boxed{h(x) = x \bmod M}$$

Generally this approach is quite good for just about any value of M. However, in certain situations some extra care is needed in the selection of a suitable value for M. For example, it is often convenient to make M an even number. But this means that $h(x)$ is even if x is even, and $h(x)$ is odd of x is odd. If all possible keys are equiprobable, then this is not a problem. However, if say even keys are more likely than odd keys, the function $h(x) = x \bmod M$ will not spread the hashed values of those keys evenly.

- Let M be a power of two, i.e., $M = 2^k$ for some integer $k > 1$. In this case, the hash function $h(x) = x \bmod 2^k$ simply extracts the bottom k-bits of the binary representation of x. While this hash function is quite easy to compute, it is not a desirable function because it does not depend on all the bits in the binary representation of x.
- For these reasons M is often chosen to be a prime number. Suppose there is bias in the way the keys are created that makes it more likely for a key to be a multiple of some small constant, say two or three. Then making M a prime increases the likelihood that those keys are spread out evenly. Also if M is a prime number, the division of x by that prime number depends on all the bits of x, not just the bottom k-bits, for some small constant k.

Example: Hash table size = 10

Key value = 112

Hash function = $h(k) = k \bmod M$

$= 112 \bmod 10 = 2$

Disadvantage: A potential disadvantage of the division method is due to the property that consecutive keys map to consecutive hash values.

$h(i) = i$

$h(i + 1) = i + 1 \pmod M$

$h(i + 2) = i + 2 \pmod M$

.

.

.

While this ensures that consecutive keys do not collide, it does not mean that consecutive array locations will be occupied. We will see that in certain implementations this can lead to degradation in performance.

Multiplication method

A variation on the middle-square method that alleviates its deficiencies is called, multiplication hashing method. Instead of multiplying the key x by itself, we multiply the key by a carefully chosen constant 'a' and then extract the middle k bits from the result. In this case, the hashing function is

$$\boxed{h(x) = \left\lfloor \frac{M}{W}(ax \bmod W) \right\rfloor}$$

if we want to avoid the problems that the middle-square method encounters with keys having a large number of leading (or) trailing zero's then we should choose an 'a' that has neither leading nor trailing zero's.

Furthermore, if we, choose an 'a' that is relatively prime to W, then there exists another number 'a' such that $aa' = 1 \pmod W$. Such a number has the nice property that if we take a key x, and multiply it by 'a' to get ax, we can recover the original key by multiplying the product again by a', since $a \times a' = aa'x = 1x$.

The multiplication method for creating a hash function operates in two steps:

Step 1: Multiply the key K by a constant A in the range $0 < A < 1$ and extract the fractional part of KA.

Step 2: Multiply this value by M and take the floor of the result.

In short the hash function is

$$h(k) = \lfloor M \cdot (KA \bmod 1) \rfloor$$

Where (KA mod 1) denotes the fractional part of KA, that is $KA \quad \lfloor KA \rfloor$

Example:

Let $m = 10000$, $K = 123456$ and $A = \frac{\sqrt{5}-1}{2}$

$= 0.618033$

Then $h(k) = \lfloor 10000 \cdot (123456 \cdot 0.61803 \bmod 1) \rfloor$

$= \lfloor 10000 \cdot (76300.00412 \bmod 1) \rfloor$

$= \lfloor 10000 \cdot 0.00412 \rfloor = 41$

Practical issues

- Easy to implement
  –On most machines multiplication is faster than division.
  –We can substitute one multiplication by shift operation.
  –We don't need to do floating-point operations.
- If successive keys have a large interval, $A = 0.6125423371$ can be recommended.

Mid-square method

A good hash function to use with integer key values is the mid-square method. The mid-square method squares the key value, and then takes out the middle 'r' bits of the result, giving a value in the range 0 to $2^r - 1$. This works well because most (or) all bits of the key value contribute to the result.

Example:
Consider records whose keys are 4-digit numbers in base 10. The goal is to hash these key values to a table of size 100(i.e., *a* range of 0 to 99).

This range is equivalent to two digits in base 10.

That is $r = 2$. If the input is the number 4567, squaring yields an 8-digit number, 20857489. The middle two digits of this result are 57. All digits of the original key value (equivalently, all bits when the number is viewed in binary) contribute to the middle two digits of the squared value. Thus, the result is not dominated by the distribution of the bottom or the top digit of the original key value. Of course, if the key values all tend to be small numbers, then their squares will only affect the low order digits of the hash value.

Example: To map the key 3121 into a hash table of size 1000, we square it $(3121)^2 = 97\underline{406}41$ and extract 406 as the hash value.

Folding method

The folding method breaks up a key into precise segments that are added to form a hash value, and still another technique is to apply a multiplicative hash function to each segment individually before folding.

Algorithm $H(x) = (a + b + c) \bmod m$. Where a, b, and c represent the preconditioned key broken down into three parts, m is the table size, and mod stands for modulo. In other words: The sum of three parts of the pre conditioned key is divided by the table size. The remainder is the hash key.

Example:
Fold the key 123456789 into a hash table of ten spaces (0 through 9)

We are given $x = 123456789$ and the table size (i.e., $m = 10$)

Since we can break x into three parts any way, we will break it up evenly.

Thus $a = 123$, $b = 456$ and $c = 789$

$H(x) = (a + b + c) \bmod M$

$H(123456789) = (123 + 456 + 789) \bmod 10$

$= 1368 \bmod 10 = 8$

123456789 are inserted into the table at address 8.

The folding method is distribution independent.

Resolving collisions In collision resolution strategy algorithms and data structures are used to handle two hash keys that hash to the same hash keys. There are a number of collision resolution techniques, but the most popular are open addressing and chaining.

- Chaining: An array of linked list, Separate chaining
- Open Addressing: Array based implementation:
  – Linear probing (Linear Search)
  – Quadratic probing (non-linear search)
  – Double hashing (use two hash functions)

Separate chaining Every linked list has each element that collides to the similar slot. Insertion need to locate the accurate slot and appending to any end of the list in that slot wherever, deletion needs searching the list and removal.

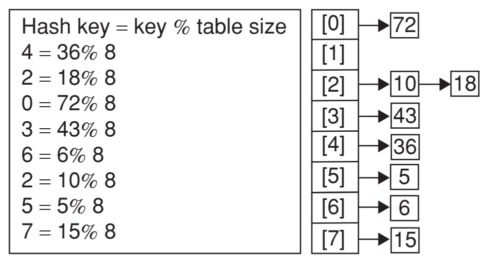


Figure 1 Separate chaining

Open addressing Open addressing hash tables are used to stock up the records straight inside the array. This approach is also known as closed hashing. This procedure is based on probing. Well known probe sequence include:

- Linear probing: In which the interval between probes is fixed often at 1.
- Quadratic probing: In which the interval between probes increases proportional to the hash value (the interval thus increasing linearly and the indices are described by a quadratic function).
- Double hashing: In which the interval between probes is computed by another hash function.

(i) **Linear probing:** Linear probing method is used for resolving hash collisions of values of hash functions by sequentially searching the hash table for a free location. The item will be stored in the next available slot in the table in linear probing. Also an assumption is made that the table is not already full.

This is implemented via a linear search for an empty slot, from the point of collision.

If the physical end of table is reached during the linear search, the search will again get start around to the beginning of the table and continue from there. The table is considered as full, if an empty slot is not found before reaching the point of collision.

[0]	72
[1]	
[2]	18
[3]	43
[4]	36
[5]	
[6]	6
[7]	7

Add the keys 10, 5 and 15 to the previous example.
Hash key = key % table size
2 = 10% 8
5 = 5% 8
7 = 15% 8

[0]	72
[1]	5
[2]	18
[3]	43
[4]	36
[5]	10
[6]	6
[7]	7

Figure 2 Linear probing

Limitation: A problem with linear probe method is primary clustering. In primary clustering blocks of data may possibly be able to form collision. Several attempts may be required by any key that hashes into the cluster to resolve the collision.

(ii) Quadratic probing: To resolve the primary clustering problem, quadratic probing can be used. With quadratic probing, rather than always moving one spot, move i^2 spots from the point of collision where i is the number of attempts needed to resolve the collision.

[0]	49
[1]	
[2]	58
[3]	69
[4]	
[5]	
[6]	
[7]	
[8]	18
[9]	89

89% 10 = 9
18% 10 = 8
49% 10 = 9 → 1 attempt needed → 1^2 = 1 spot
58% 10 = 8 → 2 attempts needed → 2^2 = 4 spot
69% 10 = 9 → 2 attempts needed → 2^2 = 4 spot

Limitation: Maximum half of the table can be used as substitute locations to resolve collisions. Once the table gets more than half full, its really hard to locate an unfilled spot. This new difficulty is recognized as secondary clustering because elements that hash to the same hash key will always probe the identical substitute cells.

(iii) Double hashing: Double hashing uses the idea of applying a second hash function to the key when a collision occurs, the result of the second hash function will be the number of positions from the point of collision to insert. There are some requirements for the second function:

1. It must never evaluate to zero
2. Must make sure that all cells can be probed.

A popular second hash function is:
Hash(key) = R-(Key mod R) where R is a prime number smaller than the size of the table.

Table size = 10 elements
Hash1(key) = key %10
Hash 2(key) = 7 – (key %7)
Insert keys: 89, 18, 49, 58 and 69
Hash key (89) = 89% 10 = 9
Hash key (18) = 18% 10 = 8
Hash key (49) = 49% 10 = 9 (collision)
= (7 – (49% 7)
= (7 – (0))
= 7 positions from [9]

[0]	
[1]	
[2]	
[3]	
[4]	
[5]	
[6]	49
[7]	
[8]	18
[9]	89

Figure 3 Double hashing

Insert keys = 58, 69
Hash key (58) = 58% 10 = 8 a collision!
= (7 – (58% 7) = (7 – 2) = 5 positions from [8]
Hash key (69) = 69% 10 = 9 a collision!
= (7 – (69 % 7)) = (7 – 6) = 1 position from [9]

[0]	69
[1]	
[2]	
[3]	58
[4]	
[5]	
[6]	49
[7]	
[8]	18
[9]	89

Figure 4 Double hashing

Matrix-chain Multiplication

We are given a sequence of n matrices $m_1, m_2 \dots m_n$ to be multiplied. If the chain matrices is $< m_1, m_2, m_3, m_4>$, the product m_1, m_2, m_3, m_4 can be fully parenthesized in 5 distinct ways:

1. $(m_1 (m_2 (m_3 m_4)))$
2. $(m_1 ((m_2 m_3) m_4))$
3. $((m_1 m_2) (m_3 m_4))$
4. $((m_1 (m_2 m_3)) m_4)$
5. $(((m_1 m_2) m_3) m_4)$

The way we parenthesize a chain of matrices can have a dramatic impact on the cost of evaluating the product. We can multiply 2 matrices A and B only if they are compatible i.e., the number of columns of A must equal the number of rows of B. If A is a $(p \times q)$ matrix and B is a $(q \times r)$ matrix, the resulting matrix C is a $(p \times r)$ matrix. The time to compute C is the number of scalar multiplications, which is (pqr).

Example: Consider the problem of a chain $<m_1, m_2, m_3>$ of three matrices. Suppose that the dimensions of the matrices are (6 × 8), (8 × 14), (14 × 20) respectively. Which parenthesization will give least number of multiplications?

Soluation:

(i) $((m_1\, m_2)\, m_3)$

$[m_1]_{6\times 8} \times [m_2]_{8\times 14} = [m_1\, m_2]_{6\times 14}$

Number of multiplications performed

$= 6 \times 8 \times 14 = 672$

$[m_1\, m_2]_{6\times 14} \times [m_3]_{14\times 20} = ((m_1\, m_2)\, m_3)_{6\times 20}$

Number of multiplications performed

$= 6 \times 14 \times 20 = 1680$

Total number of multiplications

$= 672 + 1680 = 2352$

(ii) $(m_1\, (m_2\, m_3))$

$[m_2]_{8\times 14} \times [m_3]_{14\times 20} = [m_2\, m_3]_{8\times 20}$

Number of multiplications performed

$= 8 \times 14 \times 20 = 2240$

$[m_1]_{6\times 8} \times [m_2\, m_3]_{8\times 20} = (m_1\, (m_2\, m_3))_{6\times 20}$

Number of multiplications performed

$= 6 \times 8 \times 20 = 960$

Total number of multiplications $= 960 + 2240 = 3200$

$\therefore$ $((m_1\, m_2)\, m_3)$ gives least number of multiplications.

We need to define the cost of an optimal solution recursively in terms of the optimal solutions to sub problems. For Matrix-chain multiplication problem, we pick as our sub problem the problems of determining the minimum cost of a parenthesization of $A_i\, A_{i+1} \ldots A_j$ for $1 \le i \le j \le n$ let $m[i, j]$ be the minimum number of scalar multiplications needed to compute the matrix $A_{i} \ldots_{j}$; for the full problem, the cost of a cheapest way to compute $A_{1} \ldots_{N}$ would be $m[1, n]$. We can define $m[i, j]$ recursively as follows:

$$\boxed{m\,[i, j] = m\,[i, k] + m\,[k + 1, j] + P_{i-1}\, P_k\, P_j}$$

If $i = j$, the problem is trivial. The chain consists of just one matrix $A_{i} \ldots_{i} = A_i$, so that no scalar multiplications are necessary to compute the product.

Minimum cost of parenthesizing the product $A_i\, A_{i+1} \ldots A_j$ becomes

$$m[i, j] = \begin{cases} 0 \quad if\ i = j \\ \min\{m[i,k] + m[k+1, j] \\ +\ p_{i-1} p_k p_i\} \quad if\ i < j, i \le k < j \end{cases}$$

The $m[i, j]$ values give the costs of optimal solutions to sub problems.

At this point, to write a recursive algorithm based on recurrence to compute the minimum cost $m[1, n]$ for multiplying $A_1\, A_2 \ldots A_n$. However, this algorithm takes exponential time, which is not better than the brute force method of checking each way of parenthesizing the product. The important observation we can make at this point is that we have relatively few sub problems, one problem for each choice of i and j satisfying $1 \le i \le j \le n$ (or) $\binom{n}{2} + n = \theta(n^2)$ in all. The property of overlapping sub problems is the second hallmark of the applicability of dynamic programming.

The first hall mark being optimal substructure.

Algorithm

```
n ← length [p] – 1
for i ← 1 to n
do m[i, i] ← 0
for i ← 2 to n
do for i ← 1 to n – i + 1
do j ← i + i – 1
m[i, j] ← ∞
for k ← i to j – 1
do q ← m [i, k] + m [k + 1, j] + P_(i – 1) P_k P_j
if q < m [i, j]
then m [i, j] ← q
S[i, j] ← k
return m and S
```

It first computes $m[i, j] \leftarrow 0$ for $i = 1, 2 \ldots n$ (the minimum costs for chains of length 1). To compute $m[i, i + 1]$ for $i = 1, 2, \ldots n - 1$(the minimum costs for chains of length $\lambda = 2$ and so on). At each step, the $m[i, j]$ cost computed depends only on table entries $m[i, k]$ and $m[k + 1, j]$ already computed. An entry $m[i, j]$ is computed using the products $P_{i-1}\, P_k\, P_j$ for $k = i, i + 1, \ldots j - 1$. A simple inspection of the nested loop structure of the above algorithm yields a running time of $O(n^3)$ for the algorithm.

Longest Common Subsequence

A sub sequence of a given sequence is just the given sequence with 0 or more elements left out. Formally, given a sequence $x = <x_1, x_2 \cdots x_m>$, another sequence $z = <z_1, z_2 \cdots z_k>$ is a subsequence of x if there exists a strictly increasing sequence $<i_1, i_2 \ldots i_k>$ of indices of x such that for all $j = 1, 2 \cdots k$, we have $x_{ij} = z_j$

Example: $z = < B, C, D, B >$ is a subsequence of $x = <A, B, C, B, D, A, B>$ with corresponding index sequence <2, 3, 5, 7>

Example: Given 2 sequences x and y, we say that a sequence z is a common sub sequence of x and y if z is a sub sequence of both x and y.

If $x = <A, B, C, B, D, A, B>$

$y = <B, D, C, A, B, A>$

The sequence $<B, C, A>$ is a common subsequence of both x and y.

The sequence $<B, C, A>$ is not a longest common subsequence (LCS) of x and y since it has length '3' and the sequence $<B, C, B, A>$, which is also common to both x and y, has length 4. The sequence $<B, C, B, A>$ is an LCS of x and y, as is the sequence $<B, D, A, B>$, since there is no common subsequence of length 5 or greater.

- In the longest-common-sub sequence problem, we are given 2 sequences $x = <x_1, x_2, x_3 \ldots x_m>$ and $y = <y_1, y_2 \ldots y_n>$ and wish to find a maximum length common subsequence of x and y.
- LCS problem can be solved efficiently using dynamic programming.
- A brute force approach to solve the LCS problem is to enumerate all subsequences of x and check each subsequence to see if it is also a subsequence of y, keeping track of the longest subsequence found. Each subsequence of x corresponds to a subset of the indices $\{1, 2 \ldots m\}$ of x. There are 2^m subsequences of x, so this approach requires exponential time, making it impractical for long sequences.
- The classes of sub problems correspond to pairs of 'pre fixes' of 2 input sequences:

Given a sequence $x = <x_1, x_2 \cdots x_m>$, we define the ith prefix of x, for $i = 0, 1, \ldots m$, as

$$x_i = <x_1\, x_2 \ldots x_i>$$

Example: If $x = <A, B, C, B, D, A, D>$, then $x_4 = <A, B, C, B>$ and x_0 is the empty sequence. LCS problem has an optimal sub-structure property.

Optimal Substructure of LCS

Let $x = <x_1, x_2 \ldots x_m>$ and $y = <y_1, y_2 \ldots y_n>$ be sequences and let $z = <z_1, z_2 \ldots z_k>$ be any LCS of x and y then

1. If $x_m = y_n$, then $z_k = x_m = y_n$ and z_{k-1} is an LCS of x_{m-1} and y_{n-1}.
2. If $x_m \neq y_n$, then $z_k \neq x_m$ implies that z is an LCS of x_{m-1} and y.
3. If $x_m \neq y_n$, then $z_k \neq y_n$ implies that z is an LCS of x and y_{n-1}.

NP-HARD AND *NP*-COMPLETE

A mathematical problem for which, even in theory, no shortcut or smart algorithm is possible that would lead to a simple or rapid solution. Instead the only way to find an optimal solution is a computationally intensive, exhaustive analysis in which all possible outcomes are tested. Examples of NP-hard problems include the travelling salesman problem.

P-problem

A problem is assigned to the *P*(polynomial time) class if there exists at least one algorithm to solve that problem, such that number of steps of the algorithm is bounded by a polynomial in n, where n is the length of the input.

NP-problem

A problem is assigned to the *NP* (non-deterministic polynomial time) class if it is solvable in polynomial time by a non-deterministic turing machine.

A *P*-problem (whose solution time is bounded by a polynomial) is always also *NP*. If a problem is known to be *NP*, and a solution to the problem is somehow known, then demonstrating the correctness of the solution can always be reduced to a single *P* (polynomial time) verification. If *P* and *NP* are not equivalent then the solution of *NP*-problems requires (in the worst case) an exhaustive search.

A problem is said to be *NP*-hard, if an algorithm for solving it can be translated into one for solving any other *NP*-problem. It is much easier to show that a problem is *NP* than to show that it is *NP*-hard. A problem which is both *NP* and *NP*-hard is called an *NP*-complete problem.

P versus NP-problems

The *P* versus *NP* problem is the determination of whether all *NP*-problems are actually *P*-problems, if *P* and *NP* are not equivalent then the solution of *NP*-problem requires an exhaustive search, while if they are, then asymptotically faster algorithms may exist.

NP-complete problem

A problem which is both *NP* (verifiable in non-deterministic polynomial time) and *NP*-hard (any *NP*-problem can be translated into this problem). Examples of *NP*-hard problems include the Hamiltonian cycle and travelling sales man problems.

Example:

Circuit satisfiability is a good example of problem that we don't know how to solve in polynomial time. In this problem, the input is a Boolean circuit. A collection of and, or and not gates connected by wires. The input to the circuit is a set of m Boolean (true/false) values $x_1 \ldots x_m$. The output is a single Boolean value. The circuit satisfiability problem asks, given a circuit, whether there is an input that makes the circuit output TRUE, or conversely, whether the circuit always outputs FLASE. Nobody knows how to solve this problem faster than just trying all 2^m possible inputs to the circuit but this requires exponential time.

P, NP, and Co-NP

- *P* is a set of yes/no problems that can be solved in polynomial time. Intuitively *P* is the set of problems that can be solved quickly.
- *NP* is the set of yes/no problems with the following property: If the answer is yes, then there is a proof of this fact that can be checked in polynomial time. Intuitively *NP* is the set of problems where we can verify a YES answer quickly if we have the solution in front of us.

Example: The circuit satisfiability problem is in *NP*.

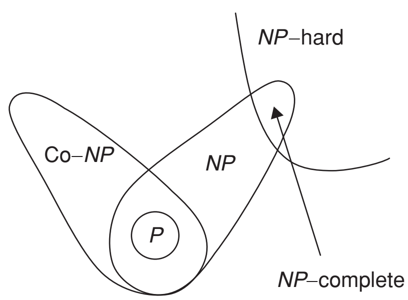
If the answer is yes, then any set of m input values that produces TRUE output is a proof of this fact, we can check the proof by evaluating the circuit in polynomial time.

- Co-*NP* is the exact opposite of *NP*. If the answer to a problem in co-*NP* is no, then there is a proof of this fact that can be checked in polynomial time.

- π is *NP*-hard $\Rightarrow$ if π can be solved in polynomial time, then $P = NP$.

This is like saying that if we could solve one particular *NP*-hard problem quickly, then we could solve any problem whose solution is easy to understand, using the solution to that one special problem as a subroutine. *NP*-hard problems are atleast as hard as any problem in *NP*.

- Saying that a problem is *NP*-hard is like saying 'If I own a dog, then it can speak fluent English'. You probably don't know whether or not I own a dog, but you're probably pretty sure that I don't own a talking dog. Nobody has a mathematical proof that dogs can't speak English. The fact that no one has ever heard a dog speak English is evidence as per the hundreds of examinations of dogs that lacked the proper mouth shape and brain power, but mere evidence is not a proof nevertheless, no sane person would believe me if I said I owned a dog that spoke fluent English. So the statement 'If I own a dog then it can speak fluent English' has a natural corollary: No one in their right mind should believe that I own a dog ! Likewise if a problem is *NP*-hard no one in their right mind should believe it can be solved in polynomial time.



Cooks Theorem

Cook's theorem states that CNFSAT is *NP*-Complete

It means, if the problem is in *NP*, then the deterministic Turing machine can reduce the problem in polynomial time.

The inference that can be taken from these theorems is, if deterministic polynomial time algorithm exists for solving satisfiability, then to all problems present in *NP* can be solved in polynomial time.

Non-deterministic Search

Non-deterministic algorithms are faster, compared to deterministic ones. The computations are fast as it always chooses right step

The following functions are used to specify these algorithms

1. Choice (A), which chooses a random element from set A
2. Failure (A), specifies failure
3. Success ( ), Specifies success

The non-deterministic search is done as follows.

Let us consider an array $S[1 \ldots n]$, $n \geq 1$ we need to get the indice of 'i' such that $S[i] = t$ (or) $i = 0$. The algorithm is given below.

Steps:

1. i = Choice $(1, n)$;
2. if $S[i] = t$, then
   (i) Print (i);
   (ii) Success ( );
3. Print (0)
   failure
4. Stop.

If the search is successful it returns the indice of array '*S*', otherwise it returns '0', the time complexity is $\Omega(n)$.

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Hash the keys 12, 44, 13, 88, 23, 94, 11, 39, 20 using the hash function with chaining $(2k + 5) \bmod 11$, which of the following slots are empty?
(A) 0, 1, 2, 3, 4 (B) 0, 2, 3, 4, 8, 10
(C) 0, 1, 2, 4, 8, 10 (D) 0, 1, 2, 4, 8

2. Using linear probing on the list given in the above question with the same hash function, which slots are not occupied?
(A) 3, 4 (B) 4, 5
(C) 3, 6 (D) 4, 6

3. In hashing, key value 123456 is hashed to which address using multiplication method ($m = 10^4$)?
(A) 40 (B) 41
(C) 42 (D) 44

4. Insert element 14 into the given hash table with double hashing? $h_1(k) = k \bmod 13$, $h_2(k) = 1 + (k \bmod 11)$. The element will occupy, which slot?

0	
1	79
2	
3	
4	69
5	98
6	
7	72
8	
9	
10	
11	50
12	

(A) 7th (B) 8th
(C) 2nd (D) 9th

5. Consider the below given keys:

257145368, 25842354, 12487654, 248645452. Find the hash values of keys using shift folding method?

(A) 770, 221, 153, 345 (B) 221, 770, 153, 345
(C) 760, 770, 153, 345 (D) 815, 770, 153, 345

6. Consider the following two problems on unidirected graphs.

β : Given $G(V, E)$, does G have an independent set of size $|V|-4$?

α : Given $G(V, E)$, does G have an independent set of size 5?

Which of the following is true?

(A) β is in P and α is in NP-Complete
(B) β is in NP-Complete and α is in P
(C) Both α and β are NP-Complete
(D) Both α and β are in P

7. Let S be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial-time reducible to S and S is polynomial-time reducible to R. Which one of the following statements is true?

(A) R is NP-Complete (B) R is NP-Hard
(C) Q is NP-Complete (D) Q is NP-Hard

8. Let FHAM_3 be the problem of finding a Hamiltonian cycle in a graph $G = (V, E)$ with $|V|$ divisible by 3 and DHAM_3 be the problem of determining if a Hamiltonian cycle exists in such graphs. Which of the following is true?

(A) Both FHAM_3 and DHAM_3 are NP-hard
(B) FHAM_3 is NP-hard but DHAM_3 is not
(C) DHAM_3 is NP-hard but FHAM_3 is not
(D) Neither FHAM_3 nor DHAM_3 is NP-hard

9. Consider a hash table of size 7, with starting index '0' and a hash function $(3x + 4) \bmod 7$. Initially hash table is empty. The sequence 1, 3, 8, 10 is inserted into the table using closed hashing then what is the position of element 10?

(A) 1st (B) 2nd
(C) 6th (D) 0th

10. Place the given keys in the hash table of size 13, index from '0' by using open hashing, hash function is $h(k)$ mod 13.

Keys: A, FOOL, HIS, AND

(hint : Add the positions of a word's letters in the alphabet, take $A \to 1$, $B \to 2$, $C \to 3$. $D \to 4$ … $Z \to 26$).

Which of the following shows the correct hash addresses of keys?

(A) A – 1, FOOL – 10, HIS – 9, AND – 6
(B) A – 1, FOOL – 9, HIS – 10, AND – 6
(C) A – 0, FOOL – 6, HIS – 10, AND – 9
(D) A – 0, FOOL – 9, HIS – 9, AND – 6

11. Consider the following input (322, 334, 471, 679, 989, 171, 173, 199) and the hash function is x mod 10 which statement is true?

I. 679, 989, 199 hash to the same value
II. 471, 171, hash to the same value
III. Each element hashes to a different value
IV. All the elements hash to the same value

(A) I Only (B) II Only
(C) I and II (D) III

12. For the input 30, 20, 56, 75, 31, 19 and hash function $h(k) = k$ mod 11, what is the largest number of key comparisons in a successful search in the open hash table.

(A) 4 (B) 3
(C) 5 (D) 2

13. The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an empty hash table of length 10 using open addressing with hash function, $h(k) = k$ mod 10 and linear probing.

Which is the resultant hash table?

(A)

	0
	1
2	2
23	3
13	4
15	5
	6
	7
	8
	9

(B)

	0
3	1
12	2
13	3
	4
15	5
	6
	7
	8
	9

(C)

	0
	1
12	2
13	3
2	4
3	5
23	6
5	7
18	8
15	9

(D)

	0
	1
2	2
3	3
12	4
13	5
23	6
5	7
18	8
15	9

14. Which one of the following is correct?

(A) Finding shortest path in a graph is solvable in poly-nomial time.
(B) Finding longest path from a graph is solvable in poly-nomial time.
(C) Finding longest path from a graph is solvable in polynomial time, if edge weights are very small values.
(D) Both (A) and (B) are correct

15. In the following pair of problems

$$\underset{\text{I}}{\underline{\text{2 CNF Satisfiability}}} \text{ Vs } \underset{\text{II}}{\underline{\text{3 CNF Satisfiability}}}.$$

(A) I is solvable in polynomial time, II is NP complete problem.
(B) II is solvable in polynomial time, I is NP complete problem.
(C) Both are solvable in polynomial time
(D) None can be solved in polynomial time.

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. For *NP*-complete problems
(A) Several polynomial time algorithms are available
(B) No polynomial time algorithm is discovered yet
(C) Polynomial time algorithms exist but not discovered
(D) Polynomial time algorithms will not exist, hence cannot be discovered

2. In the division method for creating hash functions, we map a key k into one of m slots by taking the remainder of k divided by m. That is, the hash function is
(A) $h(k) = m \bmod k$ (B) $h(k) = m \bmod m/k$
(C) $h(k) = k \bmod m$ (D) $h(k) = mk \bmod k$

3. In the division method for creating hash function, which of the following hash table size is most appropriate?
(A) 2 (B) 7
(C) 4 (D) 8

4. Which of the following techniques are commonly used to compute the probe sequence required for open addressing?
(A) Linear probing (B) Quadratic probing
(C) Double hashing (D) All the above

5. Which of the following problems is not *NP*-hard?
(A) Hamiltonian circuit problem
(B) The 0/1 knapsack problem
(C) The graph coloring problem
(D) None of these

6. For problems x and y, y is *NP*-complete and x reduces to y in polynomial time. Which of the following is true?
(A) If x can be solved in polynomial time, then so can y
(B) x is *NP*-hard
(C) x is *NP*-complete
(D) x is in *NP*, but not necessarily *NP*-complete

7. If P_1 is *NP*-complete and there is a polynomial time reduction of P_1 to $P_{2,}$ then P_2 is
(A) NP-complete
(B) Not necessarily NP-complete
(C) Cannot be *NP*-complete
(D) None of these

8. A problem is in *NP*, and as hard as any problem in *NP*. The given problem is
(A) *NP* hard
(B) *NP* complete
(C) *NP*
(D) *NP*-hard $\cap$ *NP*-complete

9. Which of the following is TRUE?
(A) All *NP*-complete problems are *NP*-hard.
(B) If an *NP*-hard problem can be solved in polynomial time, then all *NP*-complete problems can be solved in polynomial time.
(C) *NP*-hard problems are not known to be *NP*-complete.
(D) All the above

10. If a polynomial time algorithm makes polynomial number of calls to polynomial time subroutines, then the resulting algorithm runs in
(A) Polynomial time (B) No-polynomial time
(C) Exponential time (D) None of these

11. If a polynomial time algorithm makes atmost constant number of calls to polynomial time subroutines, then the resulting algorithm runs in
(A) Polynomial time (B) No-polynomial time
(C) Exponential time (D) None of these

12. When a record to be inserted maps to an already occupied slot is called
(A) Hazard
(B) Collision
(C) Hashing
(D) Chaining

13. Worst-case analysis of hashing occurs when
(A) All the keys are distributed
(B) Every key hash to the same slot
(C) Key values with even number, hashes to slots with even number
(D) Key values with odd number hashes to slots with odd number

14. Main difference between open hashing and closed hashing is
(A) Closed hashing uses linked lists and open hashing does not.
(B) Open hashing uses linked list and closed hashing does not
(C) Open hashing uses tree data structure and closed uses linked list
(D) None of the above

15. The worst case scenario in hashing occurs when
(A) All keys are hashed to the same cell of the hash table
(B) The size of hash table is bigger than the number of keys
(C) The size of hash table is smaller than the number of keys
(D) None of the above

Previous Years' Questions

1. Consider a hash table of size seven, with starting index zero, and a hash function $(3x + 4)$ mod7. Assuming the hash table is initially empty, which of the following is the contents of the table when the sequence 1, 3, 8, 10 is inserted into the table using closed hashing? Note that − denotes an empty location in the table. **[2007]**

   (A) 8, −, −, −, −, −, 10 (B) 1, 8, 10, −, −, −, 3
   (C) 1, −, −, −, −, −, 3 (D) 1, 10, 8, −, −, −, 3

Common data for questions 2 and 3: Suppose the letters a, b, c, d, e, f have probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32}$, respectively.

2. Which of the following is the Huffman code for the letter a, b, c, d, e, f? **[2007]**
   (A) 0, 10, 110, 1110, 11110, 11111
   (B) 11, 10, 011, 010, 001, 000
   (C) 11, 10, 01, 001, 0001, 0000
   (D) 110, 100, 010, 000, 001, 111

3. What is the average length of the correct answer to above question? **[2007]**

   (A) 3 (B) 2.1875
   (C) 2.25 (D) 1.9375

4. The subset-sum problem is defined as follows: Given a set S of n positive integers and a positive integer W, determine whether there is a subset of S whose elements sum to W.

   An algorithm Q solves this problem in $O(nW)$ time. Which of the following statements is false? **[2008]**

   (A) Q solves the subset-sum problem in polynomial time when the input is encoded in unary
   (B) Q solves the subset-sum problem in polynomial time when the input is encoded in binary
   (C) The subset sum problem belongs to the class NP
   (D) The subset sum problem is NP-hard

5. Let π_A be a problem that belongs to the class NP. Then which one of the following is TRUE? **[2009]**

   (A) There is no polynomial time algorithm for π_A.
   (B) If π_A can be solved deterministically in polynomial time, then $P = NP$.
   (C) If π_A is NP-hard, then it is NP-complete.
   (D) π_A may be undecidable.

6. The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function $h(k) = k$ mod 10 and linear probing. What is the resultant hash table? **[2009]**

(A)

0	
1	
2	12
3	23
4	
5	15
6	
7	
8	18
9	

(B)

0	
1	
2	12
3	13
4	
5	5
6	
7	
8	18
9	

(C)

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

(D)

0	
1	
2	12,2
3	13,3,23
4	
5	5,15
6	
7	
8	18
9	

Common data for questions 7 and 8: A sub-sequence of a given sequence is just the given sequence with some elements (possibly none or all) left out. We are given two sequences $X[m]$ and $Y[n]$ of lengths m and n, respectively, with indexes of X and Y starting from 0.

7. We wish to find the length of the longest common sub-sequence (LCS) of $X[m]$ and $Y[n]$ as $l(m, n)$, where an incomplete recursive definition for the function $l(i, j)$ to compute the length of the LCS of $X[m]$ and $Y[n]$ is given below:
   $I(i, j) = 0$, if either $i = 0$ or $j = 0$
   = expr1, if $i, j > 0$ and $X[i - 1] = Y[j - 1]$
   = expr2, if $i, j > 0$ and $X[i - 1] \neq Y[j - 1]$

   Which one of the following options is correct? **[2009]**
   (A) expr1 $\equiv I(i - 1, j) + 1$
   (B) expr1 $\equiv I(i, j - 1)$
   (C) expr2 $\equiv \max(I(i - 1, j), I(i, j - 1))$
   (D) expr2 $\equiv \max(I(i - 1, j - 1), I(i, j))$

8. The values of $l(i, j)$ could be obtained by dynamic programming based on the correct recursive definition of $l(i, j)$ of the form given above, using an array $L[M, N]$, where $M = m + 1$ and $N = n + 1$, such that $L[i, j] = l(i, j)$.
   Which one of the following statements would be TRUE regarding the dynamic programming solution for the recursive definition of $l(i, j)$? **[2009]**
   (A) All elements of L should be initialized to 0 for the values of $l(i, j)$ to be properly computed.

(B) The values of $l(i, j)$ may be computed in a row major order or column major order of $L(M, N)$.
(C) The values of $l(i, j)$ cannot be computed in either row major order or column major order of $L(M, N)$.
(D) $L[p, q]$ needs to be computed before $L[r, s]$ if either $p < r$ or $q < s$.

9. The weight of a sequence $a_0, a_1, \cdots, a_{n-1}$ of real numbers is defined as $a_0 + a_1/2 + \cdots + a_{n-1}/2^{n-1}$. A subsequence of a sequence is obtained by deleting some elements from the sequence, keeping the order of the remaining elements the same. Let X denote the maximum possible weight of a subsequence of $a_0, a_1, \ldots, a_{n-1}$. Then X is equal to **[2010]**

(A) $\max(Y, a_0 + Y)$ (B) $\max(Y, a_0 + Y/2)$
(C) $\max(Y, a_0 + 2Y)$ (D) $a_0 + Y/2$

10. Four matrices M_1, M_2, M_3 and M_4 of dimensions $p \times q$, $q \times r$, $r \times s$ and $s \times t$ respectively, can be multiplied in several ways with different number of total scalar multiplications. For example when multiplied as $((M_1 \times M_2) \times (M_3 \times M_4))$, the total number of scalar multiplications is $pqr + rst + prt$. When multiplied $(((M_1 \times M_2) \times M_3) \times M_4)$ the total number of scalar multiplications is $pqr + prs + pst$.

If $p = 10$, $q = 100$, $r = 20$, $s = 5$ and $t = 80$, then the minimum number of scalar multiplications needed is **[2011]**

(A) 248000
(B) 44000
(C) 19000
(D) 25000

11. Assuming $P \neq NP$, which of the following is **TRUE**? **[2012]**

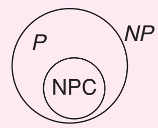
(A) NP-complete = NP
(B) NP-complete $\cap P = \varnothing$
(C) NP-hard = NP
(D) $P = NP$-complete

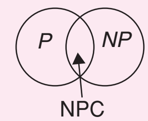
12. Which of the following statements are TRUE?

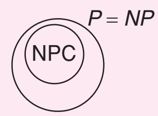
(i) The problem of determining whether there exists a cycle in an undirected graph is in P.
(ii) The problem of determining whether there exists a cycle in an undirected graph is in NP.
(iii) If a problem A is NP-Complete, there exists a non-deterministic polynomial time algorithm to solve A. **[2013]**

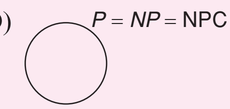
(A) 1, 2 and 3 (B) 1 and 2 only
(C) 2 and 3 only (D) 1 and 3 only

13. Suppose a polynomial time algorithm is discovered that correctly computes the largest clique in a given graph. In this scenario, which one of the following represents the correct Venn diagram of the complexity classes P, NP and NP-complete (NPC)? **[2014]**

(A)


(B)


(C)


(D)


14. Consider a hash, table with 9 slots. The hash function is $h(K) = K \bmod 9$. The collisions are resolved by chaining. The following 9 keys are inserted in the order: 5, 28, 19, 15, 20, 33, 12, 17, 10. The maximum, minimum, and average chain lengths in the hash table, respectively, are **[2014]**

(A) 3, 0 and 1 (B) 3, 3 and 3
(C) 4, 0 and 1 (D) 3, 0 and 2

15. Consider two strings A = 'qpqrr' and B = 'pqprqrp'. Let x be the length of the longest common subsequence (not necessarily contiguous between A and B and let y be the number of such longest common subsequences between A and B. then $x + 10y$ = ______ **[2014]**

16. Suppose you want to move from 0 to 100 on the number line. In each step, you either move right by a unit distance or you take a *shortcut*. A shortcut is simply a pre-specified pair of integers i, j with $i < j$. Given a shortcut i, j if you are at position i on the number line, you may directly move to j. Suppose $T(k)$ denotes the smallest number of steps needed to move from k to 100. Suppose further that there is at most 1 shortcut involving any number, and in particular from 9 there is a shortcut to 15. Let y and z be such that $T(9) = 1 + \min(T(y), T(z))$. Then the value of the product yz is ______ **[2014]**

17. Consider the decision problem 2CNFSAT defined as follows: **[2014]**

$\{\phi \mid \phi$ is a satisfiable propositional formula in CNF with at most two literals per clause$\}$

For example, $\phi = (x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (x_2 \vee x_4)$ is a Boolean formula and it is in 2CNFSAT.

The decision problem 2CNFSAT is

(A) NP-complete
(B) Solvable in polynomial time by reduction to directed graph reach ability.
(C) Solvable in constant time since any input instance is satisfiable.
(D) NP-hard, but not NP-complete

18. Consider a hash table with 100 slots. Collisions are resolved using chaining. Assuming simple uniform

hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions? **[2014]**

(A) $(97 \times 97 \times 97)/100^3$
(B) $(99 \times 98 \times 97)/100^3$
(C) $(97 \times 96 \times 95)/100^3$
(D) $(97 \times 96 \times 95)/(3! \times 100^3)$

19. Match the following **[2014]**

(P) prim's algorithm for minimum spanning tree	(i) Backtracking
(Q) Floyd-Warshall algorithm for all pairs shortest paths	(ii) Greedy method
(R) Mergesort	(iii) Dynamic programming
(S) Hamiltonian circuit	(iv) Divide and conquer

(A) P–iii, Q–ii, R–iv, S–i
(B) P–i, Q–ii, R–iv, S–iii
(C) P–ii, Q–iii, R–iv, S–i
(D) P–ii, Q–i, R–iii, S–iv

20. Given a hash table T with 25 slots that stores 2000 elements, the load factor ∝ for T is ________ **[2015]**

21. Language L_1 is polynomial time reducible to language L_2. Language L_3 is polynomial time reducible to L_2, which in turn is polynomial time reducible to language L_4. Which of the following is/are true? **[2015]**

(1) if $L_4 \in P$, then $L_2 \in P$
(2) if $L_1 \in P$ or $L_3 \in P$, then $L_2 \in P$
(3) $L_1 \in P$, if and only if $L_3 \in P$
(4) if $L_4 \in P$, then $L_1 \in P$ and $L_3 \in P$

22. The Floyd - Warshall algorithm for all -pair shortest paths computation is based on **[2016]**

(A) Greedy paradigm
(B) Divide-and-Conquer paradigm
(C) Dynamic Programming paradigm
(D) Neither Greedy nor Divide-and-Conquer nor Dynamic Programming paradigm.

23. Let A_1, A_2, A_3, and A_4 be four matrices of dimensions 10×5, 5×20, 20×10, and 10×5, respectively. The minimum number of scalar multiplications required to find the product $A_1 A_2 A_3 A_4$ using the basic matrix multiplication method is _____ . **[2016]**

24. Consider the following table:

Algorithms	Design Paradigms
(P) Kruskal	(i) Divide and Conquer
(Q) Quicksort	(ii) Greedy
(R) Floyd-Warshall	(iii) Dynamic Programming

Match the algorithms to the design paradigms they are based on. **[2017]**

(A) (P) ↔ (ii), (Q) ↔ (iii),(R) ↔ (i)
(B) (P) ↔ (iii), (Q) ↔ (i), (R) ↔ (ii)
(C) (P) ↔ (ii), (Q) ↔ (i), (R) ↔ (iii)
(D) (P) ↔ (i), (Q) ↔ (ii), (R) ↔ (iii)

25. Assume that multiplying a matrix G_1 of dimension $p \times q$ with another matrix G_2 of dimension $q \times \underline{r}$ requires pqr scalar multiplications. Computing the product of n matrices $G_1G_2G_3, ..., G_n$ can be done by parenthesizing in different ways. Define $G_i\ G_{i+1}$ as an explicitly computed pair for a given paranthesization if they are directly multiplied. For example, in the matrix multiplication chain $G_1G_2G_3G_4G_5G_6$ using parenthesization $(G_1(G_2G_3))(G_4(G_5G_6))$, G_2G_3 and G_5G_6 are the only explicitly computed pairs.

Consider a matrix multiplication chain $F_1F_2F_3F_4F_5$, where matrices F_1, F_2, F_3, F_4, and F_5 are of dimensions 2×25, 25×3, 3×16, 16×1 and 1×1000, respectively. In the parenthesization of $F_1F_2F_3F_4F_5$ that minimizes the total number of scalar multiplications, the explicitly computed pairs is/are: **[2018]**

(A) F_1F_2 and F_3F_4 only
(B) F_2F_3 only
(C) F_3F_4 only
(D) F_1F_2 and F_4F_5 only

26. Consider the weights and values of items listed below. Note that there is only one unit of each item.

Item no.	Weight (in Kgs)	Value (in Rupees)
1	10	60
2	7	28
3	4	20
4	2	24

The task is to pick a subset of these items such that their total weight is no more than 11 kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt}. A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted by V_{greedy}.

The value of $V_{opt} - V_{greedy}$ is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** A	**3.** B	**4.** D	**5.** A	**6.** C	**7.** C	**8.** A	**9.** B	**10.** B
11. C	**12.** B	**13.** C	**14.** A	**15.** A					

Practice Problems 2

1. B	**2.** C	**3.** B	**4.** D	**5.** B	**6.** C	**7.** A	**8.** B	**9.** D	**10.** C
11. A	**12.** B	**13.** B	**14.** B	**15.** A					

Previous Years' Questions

1. B	**2.** A	**3.** D	**4.** B	**5.** C	**6.** C	**7.** C	**8.** B	**9.** C	**10.** B
11. B	**12.** A	**13.** D	**14.** A	**15.** 34	**16.** 150	**17.** B	**18.** A	**19.** C	**20.** 80
21. C	**22.** C	**23.** 1500	**24.** C	**25.** C	**26.** 16				

Test

Algorithms (Part 2)

Time: 45 min.

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. The worst case running time of an algorithm means
(A) The algorithm will never take any longer.
(B) The algorithm will take less time than running time
(C) The algorithm will run in a finite time
(D) None of the above

2. Analyzing an algorithm involves
(A) Evaluating the complexity
(B) Validating the Algorithm
(C) Both A and B
(D) None of the above

3. $f(n) = \theta(g(n))$ is
(A) $g(n)$ is asymptotic lower bound for $f(n)$
(B) $g(n)$ is asymptotic tight bound for $f(n)$
(C) $g(n)$ is asymptotic upper bound for $f(n)$
(D) None of the above

4. Which case yields the necessary information about an algorithm's behaviour on a random input?
(A) Best-case (B) Worst-case
(C) Average-case (D) Both A and C

5. Algorithms that require an exponential number of operations are practical for solving.
(A) Only problems of very small size
(B) Problems of large size
(C) Problems of any size
(D) None of these

6. Problems that can be solved in polynomial time are called
(A) Tractable (B) Decidable
(C) Solvable (D) Computable

7. Problems that cannot be solved at all by any algorithm are known as
(A) Tractable (B) Undecidable
(C) Untractable (D) Unsolvable

8. Which of the following problems is decidable but intractable?
(A) Hamiltonian circuit (B) Traveling sales man
(C) Knapsack problem (D) All the above

9. Which method is used to solve recurrences?
(A) Substitution method
(B) Recursion-tree method
(C) Master method
(D) All the above

10. Consider the following
(i) Input
(ii) Output
(iii) Finiteness
(iv) Definiteness means clear and unambiguous
(v) Effectiveness
Which of the following is not a property of an algorithm?
(A) (iv) only (B) (iv) and (v) only
(C) (iii) and (iv) only (D) None of the above

11. Finiteness of an algorithm means
(A) The steps of the algorithm should be finite
(B) The algorithm should terminate after finite time
(C) Algorithm must terminate after a finite number of steps
(D) Algorithm should consume very less space

12. Asymptotic analysis on efficiency of algorithm means
(A) The efficiency of the algorithm on a particular machine
(B) How the running time of an algorithm increases as the size increases without bound
(C) How efficiently the algorithm is applied to solve a problem without thinking of input size.
(D) None of the above

13. What is the input size of a problem?
(A) Number of variables used to solve the problem
(B) Number of constants used to solve the problem
(C) it is problem specific that is in case of graph it is number of edges and vertices and so on.
(D) None of these

14. (i) An algorithm must take input
(ii) An algorithm must give out put
Which is true in the following options?
(A) (i) Only (B) (ii) Only
(C) (i) and (ii) Only (D) None of the above

15. As n θ°
Which of the following is efficient?
(A) $\theta(n^3)$ (B) $\theta(n^2)$
(C) $\theta(2^n)$ (D) $\theta(n^4)$

16. Suppose
$T_1(n) = O(f(n))$
$T_2(n) = O(f(n))$
which of the following is true,.
(A) $T_1(n) + T_2(n) = O(f(n))$ (B) $\dfrac{T_1(n)}{T_2(n)} = 0(1)$
(C) $T_1(n) = 0(T_2(n))$ (D) None of these

17. The following program computes n!
Find the complexity?
Input: A non-negative integer
Output: Value of n!
If $n = 0$ return 1
Else return $F(n - 1)\ \theta n$
(A) $\theta(n)$
(B) $\theta(n \log n)$
(C) $\theta(n^2)$
(D) $\theta(n^3)$

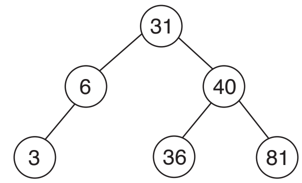
18. Which of the following functions are often referred as 'exponential growth function'?
(A) 2^n, $\log n$
(B) 2^n, $n!$
(C) $n!$, $n \log n$
(D) $n!$, $\log n$

19. Consider the following code
sort (a, n)
```
{
    for i = 1 to n do
    {
            j = i;
    for k = i + 1 to n do
            if (a[k] < a [j]) then j = k;
            t = a[i];
            a[i] = a[j];
            a[j] = t;
            }
    }
```
The above code implements which sorting?
(A) Merge sort
(B) selection sort
(C) Insertion sort
(D) Radix sort

20. Assume that the number of disks in a 'Towers of Hanoi problem' is 'n', with '3' towers, Initially all disks are placed on tower 1, to get the largest disk are placed on tower 1, to get the largest disk to the bottom of 2nd tower, How many moves are required? ($\theta n = 3$)
(A) n
(B) $(n - 1)$
(C) $(n + 1)$
(D) $2n$

21. Each new term in Fibonacci sequence is obtained by taking the sum of the two previous terms. The first term of the sequence is $f_0 = 0$, and the second term $f_1 = 1$. Which of the following gives Fibonacci sequence?
(A) $f_n = f_{n+1} + f_{n-2}$, $n \geq 2$
(B) $f_n = f_{n-1} + f_{n-2}$, $n \geq 2$
(C) $f_n = f_{n-1} + f_{n+1}$, $n \geq 2$
(D) All the above

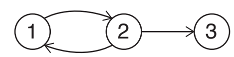
22. Consider the binary search tree



Delete node '31', what would be the parent node in the new binary search tree?
(A) 36
(B) 40
(C) 81
(D) 6

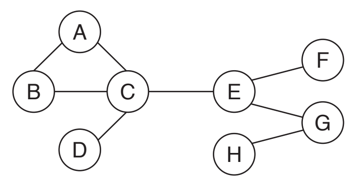
23. Consider the given array [4, 6, 7, 8, 21, 9, 3, 10, 13, 16, 31] after performing '1' delete max operation, on the max heap. What would be the sequence of elements in the array?
(A) 9, 21, 13, 16, 3, 7, 10, 8, 4, 6
(B) 21, 9, 13, 16, 7, 3, 10, 8, 4, 6
(C) 21, 9, 13, 16, 3, 7, 10, 8, 4, 6
(D) 21, 9, 13, 16, 7, 3, 10, 4, 8, 6

24. Consider the given Di-graph



How many strongly connected components does the above graph contain?
(A) 1
(B) 2
(C) 3
(D) many

25. Consider the given graph



Which of the following shows the adjacency matrix of the above graph?

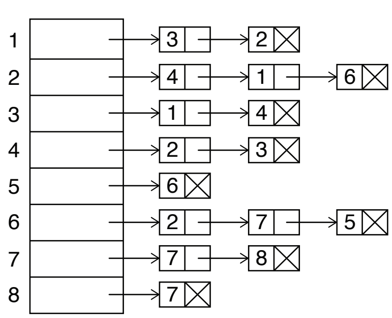
(A)
$$
\begin{array}{c|cccccccc|}
 & A & B & C & D & E & F & G & H \\
\hline
A & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
B & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
C & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
D & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
E & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
F & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
G & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
H & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}
$$

(B)
$$\begin{array}{cc} & \begin{matrix} A & B & C & D & E & F & G & H \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(C)
$$\begin{array}{cc} & \begin{matrix} A & B & C & D & E & F & G & H \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

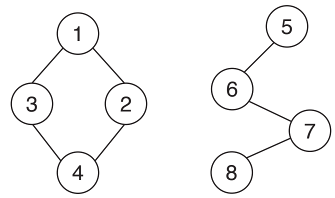
(D)
$$\begin{array}{cc} & \begin{matrix} A & B & C & D & E & F & G & H \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

26. Consider the given adjacency list

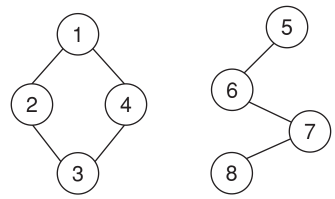


The above list is representation of which of the following graph?

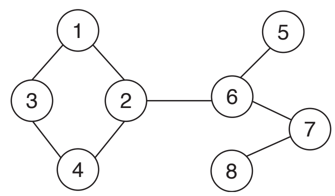
(A)



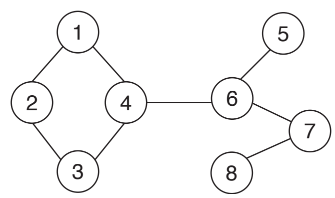
(B)



(C)



(D)



27. Which of the following is FALSE?

(A) In dynamic programming an optimal sequence of decisions is obtained by making explicit appeal to the principle of optimality

(B) In greedy method only one decision sequence is generated.

(C) In dynamic programming, many decision sequences may be generated.

(D) In greedy method many decision sequences are generated.

28. Consider an array $a[n]$ of 'n' numbers that has '$n/2$' distinct elements and '$n/2$' copies of another element, to identify that repeated element, how many steps are required in the worst case?

(A) $n/2$ (B) $n/2 + 1$

(C) $n/2 + 2$ (D) n

29. Match the following, for a very large value of 'n'

I. $36n^3 + 2n^2$

II. $5n^2 - 6n$

III. $n^{1.001} + n \log n$

P. $\theta(n^2)$

Q. $|(n^3)$

R. $\theta(n^{1.001})$

(A) I – P, II – Q, III – R (B) I – Q, II – P, III – R

(C) I – R, II – Q, III – P (D) I – R, II – P, III – R

30. Consider the following code

```
    T(a, n)
{
  for i = 1 to n - 1 do
     for j = i + 1 to n do
           {
                  t = a[i, j];
                  a[i, j] = a[j, i];
                  a[j, i] = t;
           }
  }
```

The above code performs

(A) Matrix multiplication
(B) Matrix addition
(C) Matrix transpose
(D) Matrix chain multiplication

Answers Keys

1. A	**2.** C	**3.** B	**4.** C	**5.** A	**6.** A	**7.** B	**8.** D	**9.** D	**10.** D
11. C	**12.** D	**13.** C	**14.** B	**15.** B	**16.** A	**17.** A	**18.** B	**19.** A	**20.** C
21. B	**22.** A	**23.** B	**24.** B	**25.** A	**26.** C	**27.** D	**28.** C	**29.** B	**30.** C

ALGORITHMS TEST I

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Consider the given properties of Asymptotic Notations:
I. $f(n) = \theta(g(n))$ and $g(n) = \theta(h(n))$
$\Rightarrow$ $f(n) = \theta(h(n))$
II. $f(n) = \theta(g(n))$ if and only if $g(n) = \theta(f(n))$
III. $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$
IV. $f(n) = O(g(n))$ if and only if $g(n) = O(f(n))$
Which of the following are valid?
(A) I, II only (B) I, II, III only
(C) II, III only (D) I, II, III and IV

2. Consider the given Recurrence Relation
$T(n) = 2^n\, T(n/3) + n$
Which of the following is TRUE?
(A) Master theorem cannot be applied because 'a' is not constant.
(B) It comes under case 1 of Master theorem.
(C) It comes under case 2 of Master theorem.
(D) It comes under case 3 of Master theorem.

3. Let 'n' be the number of elements in the queue, then What is the Time complexity of following operations respectively, Enqueue( ), Dequeue ( ), IsEmptyQueue ( ), Delete Queue( )?
(A) $O(n)$, $O(1)$, $O(1)$, $O(n)$
(B) $O(1)$, $O(1)$, $O(n)$, $O(1)$
(C) $O(1)$, $O(1)$, O(1), $O(1)$
(D) $O(n)$, $O(n)$, $O(n)$, $O(1)$

4. Which of the following are Applications of Binary Trees?
I. Huffman coding trees are used in data compression Algorithms.
II. Priority Queues support search and deletion of minimum or maximum on 'n' number of items in (log n) time.
III. Expression trees are used in compilers.
IV. Binary search Tree supports search, insertion and deletion on 'n' number of items in (log n) time (average case)
(A) I, II only (B) I, III, IV only
(C) I, II, III only (D) I, II, III and IV

5. A Traversal is defined as follows:
1. Visit the root
2. While traversing Level 'i', keep all the elements at level '$i + 1$' in queue.
3. Go to the next Level and visit all the nodes at that level.
4. Repeat this until all the levels are completed.
The above defined traversal is
(A) Depth First Traversal
(B) Level order Traversal
(C) Binary Tree Traversal
(D) Binary search Tree Traversal

6. How many different binary trees are possible with '8' nodes?
(A) 256 (B) 128
(C) 248 (D) 64

7. A binary search tree is generated by inserting in order the following integers 66, 72, 46, 48, 9, 8, 40, 36, 18, 7, 5, 91, 88, 49, 6. The number of nodes in the Left sub tree and Right sub tree of the root respectively.
(A) (8, 6) (B) (9, 5)
(C) (10, 4) (D) (11, 3)

8. For a Full Binary tree of height 'h', the sum of the heights of all nodes is ________ ('n' is number of nodes)?
(A) $n - (h - 1)$ (B) $n - (h + 1)$
(C) $n + (h - 1)$ (D) $n + (h + 1)$

9. A graph 'G' has 29 edges and its complement $\bar{G}$ has 7 edges, what is the number of vertices present in graph G?
(A) 7 (B) 8
(C) 9 (D) 10

10. Consider the given statements:
I. Uses priority queue to store unvisited vertices by distance from source.
II. It uses greedy method, means pick the next closest vertex to the source.
III. Does not work with negative weights.
The above statements describe
(A) Bellman Ford Algorithm
(B) Dijkstra's Algorithm
(C) Breadth First search Algorithm
(D) Kruskals Algorithm

11. A complete bipartite graph $k_{m,n}$ is a bipartite graph that has each vertex from one set is adjacent to each vertex to another set, what is the minimum 'vertex cover' for $k_{m,n}$ graphs?
(A) MAX(m, n) (B) MIN(m, n)
(C) m (D) n

12. What is the number of Regions present in the bipartite graph $K_{5,2}$?
(A) 4 (B) 5
(C) 6 (D) 7

13. Consider the following:

	Best Case	Average Case	Worst Case
I.	$O(n)$	$O(n^2)$	$O(n^2)$
II.	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$

Which of the following is TRUE?
(A) Merge sort-II, Selection sort-II
(B) Merge sort-II, Heap sort-II
(C) Heap sort-I, Insertion sort-I
(D) Bubble sort-I, Insertion sort-II

14. Which of the following Algorithm uses Divide-and-Conquer strategy?
(A) Merge sort
(B) Quick sort
(C) Binary search and strassens multiplication
(D) All the above

15. For merging two sorted Lists of sizes 'K' and 'L' into a sorted list of size $K + L$, what is the number of comparisons required?
(A) $O(K)$ (B) $O(L)$
(C) $O(K*L)$ (D) $O(K + L)$

16. Consider the following:
$f(n) = n \log n$
$g(n) = \log (n!)$
$h(n) = 2^{\log_2^n}$
Which of the following is TRUE according to Rate of Growth?
(A) $g(n) \in \Omega\, h(n)$ and $g(n) \in O(f(n))$
(B) $g(n) \in O(f(n))$ and $f(n) \in \Omega(g(n))$
(C) $g(n) \in \Omega(f(n))$ and $f(n) \in \Omega(g(n))$
(D) $h(n) \in O(g(n))$ and $g(n) \in O(f(n))$

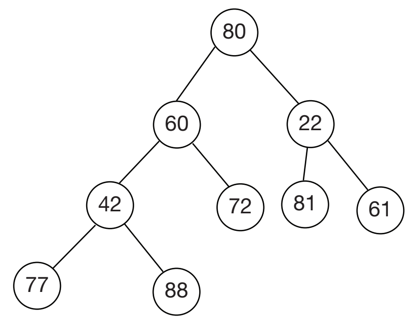
17. Consider the given code:
```
y = y + z;
for (i = 1; i < = n; i++)
k = k + 2;
for (i = 1; i < = n; i++)
{
for(j = 1; j < = n; j++)
x = x + 1;
}
```
What is the time complexity of the given code?
(A) $O(n^3)$ (B) $O(n^2)$
(C) $O(n \log n)$ (D) $O(n^2 \log n)$

18. Consider the given Recurrence Relation
$$T(n) = 3T\left(\frac{n}{9}\right) + n^{0.52}.$$
What is the Time complexity?
(A) $O(n)$ (B) $O(n^2)$
(C) $O(n^{0.52})$ (D) $O(\sqrt{n})$

19. Consider an empty stack of integers. The numbers 6, 7, 1, 4, 3, 2, 8, 9 are pushed on to the empty stack in the above given order from Right to Left. Let Z denote a PUSH operation and 'W' denote a *POP* operation. Which of the following is the sequence of integers Popped out after performing ZZZZWWZZWWZW?
(A) 3, 2, 4, 1, 7 (B) 3, 2, 1, 4, 9
(C) 3, 2, 1, 4, 8 (D) 3, 2, 1, 4, 7

20. The following sequence of operations is performed on a stack, PUSH(70), PUSH(50), POP, PUSH(30), PUSH(50), PUSH(70), POP, POP, PUSH(70), POP, POP, what is the sequence of values popped out?
(A) 50, 70, 50, 70, 70
(B) 50, 50, 70, 30, 70
(C) 50, 70, 50, 70, 30
(D) 50, 70, 50, 30, 70

21. Assume an algorithm for printing the level order data in Reverse order, for the Binary tree shown below:



(A) 80, 60, 22, 42, 72, 81, 61, 77, 88
(B) 80, 22, 60, 61, 81, 72, 42, 88, 77
(C) 77, 88, 42, 72, 81, 61, 60, 22, 80
(D) 88, 77, 61, 81, 72, 42, 22, 60, 80

22. Consider the given code:
```
int fun(struct BinaryTreeNode *root1,
struct Binary Tree Node *root2)
{
if(root1 = = NULL & & root2 = = NULL)
return 1;
if(root1 = = NULL || root2 = = NULL)
return 0;
return (root1 → data = = root2 →
   data &&
fun(root1 → left, root2 → left) &&
fun(root1 → right, root2 → right));
}
```
The above code describes, which of the following task?
(A) Finding the number of nodes with only one child
(B) Finding the number of nodes in a Tree.
(C) Finding, whether two binary trees are structurally identical or not
(D) Finding the number of Leaf nodes in a tree.

23. We are given a set of '6' distinct elements and an unlabelled binary tree with '6' nodes. In how many ways can we populate the tree with the given set, so that it becomes a binary search tree?
(A) 58 (B) 720
(C) 360 (D) 132

24. A binary tree is a tree data structure in which each node has atmost 2 children, that is, the degree of each node can be atmost '2'. If a binary tree has 'n' leaf nodes, then the number of nodes of degree 2 is?

(A) $n - 1$ (B) $2n - 1$
(C) $n + 1$ (D) $\frac{n}{2}$

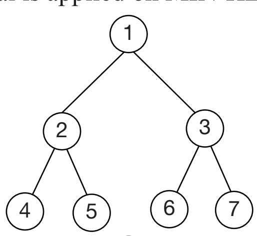
25. For a 5-ary tree (each node can contain maximum of 5 children), what is the maximum possible height with 50 nodes, Assume that height of a single node is '0'?
(A) 25 (B) 30
(C) 49 (D) 50

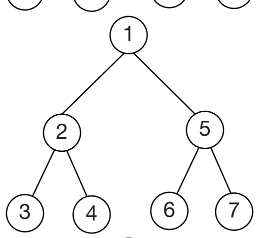
26. For a 7-ary tree (each node can contain maximum of 7 children), what is the maximum possible height with 60 nodes, If we have a restriction that atleast one node should have 7 children?
(A) 14 (B) 21
(C) 49 (D) 53

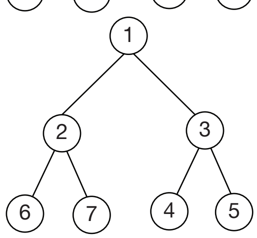
27. For a K-ary tree (each node can contain maximum of K children), what is the maximum possible nodes at height 'h'?
(A) Kh^{+1} (B) K^h
(C) Kh^{-1} (D) $2K^h$

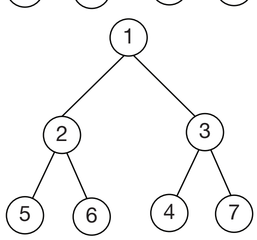
28. Consider any completer Binary tree, what are the minimum and maximum number of elements interms of height 'h'?
(A) 2^{h+1} and 2^{h+2} (B) 2^h and $2^{h+1} - 1$
(C) $2^{h+1} - 1$ and $2^{h+1} + 1$ (D) 2^h and 2^{h+1}

29. Which of the following MIN-HEAP displays the elements in sequence (ascending order) if preorder traversal is applied on MIN-HEAP?

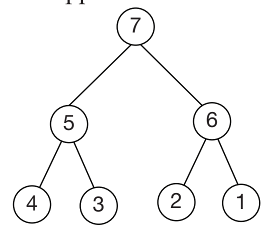
(A)


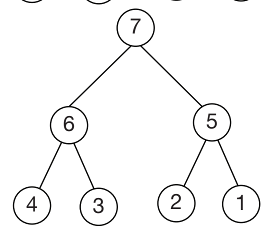
(B)


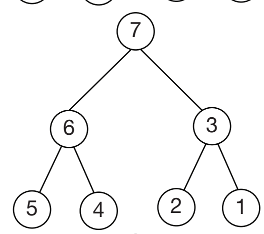
(C)


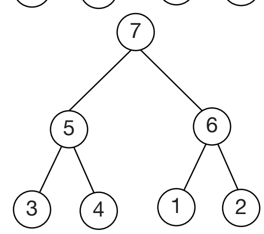
(D)


30. Which of the following MAX-HEAP displays the elements in sequence (descending order) if preorder traversal is applied on MAX-HEAP?

(A)


(B)


(C)


(D)


31. What is the minimum and maximum number of nodes that exist in MIN-HEAP/MAX-HEAP interms of height 'h'?
(A) 2^{h-1} and 2^{h+1} (B) 2^{h-1} and 2^h
(C) 2^h and $2^{h+1} - 1$ (D) $2^{h+1} - 1$ and 2^{h+1}

32. Which of the following cannot be the partition of array elements by using Quick sort Algorithm?

(A) $T(n) = T\left(\frac{6n}{10}\right) + T\left(\frac{4n}{10}\right) + \theta(n)$

(B) $T(n) = T\left(\frac{4n}{5}\right) + T\left(\frac{n}{5}\right) + \theta(n)$

(C) $T(n) = T(n-1) + T(1) + \theta(n)$

(D) $T(n) = T\left(\frac{4n}{5}\right) + T\left(\frac{4n}{5}\right) + \theta(n)$

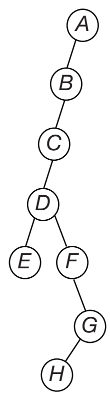
33. A stable sorting is defined as, Assume that A is an array to be sorted, X and Y are having the same key and X appears earlier in the array than Y. That means X is at $A[i]$ and Y is at $A[j]$, where $i < j$, an algorithm is said to be stable if in the output X precedes Y. Which of the following is not stable sorting?
(A) Bubble sort (B) Insertion sort
(C) Merge sort (D) Quick sort

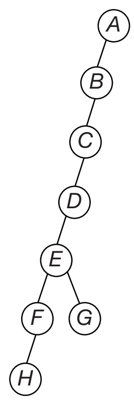
Common Data for Questions 34 and 35:
Consider the given Inorder, preorder, postorder traversals of a tree, but it is not known which is what order

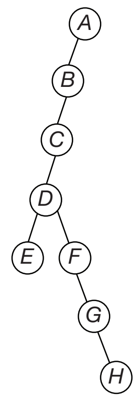
I. *ABCDEGFH*
II. *EDGHFCBA*
III. *EHFGDCBA*

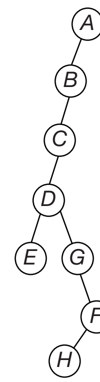
34. Which of the following is TRUE?
(A) I is pre order and II is post order
(B) I is pre order and III is In order
(C) I is pre order and II is In order
(D) II is post order and III is In order

35. Which of the following is the correct tree for the traversals given in the above question?

(A)


(B)


(C)


(D)


Answer Keys

1. B	**2.** A	**3.** C	**4.** D	**5.** B	**6.** C	**7.** D	**8.** B	**9.** C	**10.** B
11. B	**12.** B	**13.** B	**14.** D	**15.** D	**16.** D	**17.** B	**18.** C	**19.** D	**20.** C
21. C	**22.** C	**23.** D	**24.** A	**25.** C	**26.** D	**27.** B	**28.** B	**29.** B	**30.** C
31. C	**32.** D	**33.** D	**34.** C	**35.** D					

Hints and Explanations

1. **Transitivity:**
$f(n) = \theta(g(n))$ and $g(n) = \theta(h(n))$
$\Rightarrow$ $f(n) = \theta(h(n))$ valid for Ω and O notations also.
Symmetry:
$f(n) = \theta(g(n))$ if and only if $g(n) = \theta(f(n))$
Transpose symmetry:
$f(n) = O(g(n))$ if and only if $g(n) = \Omega\ (f(n))$.
Choice (B)

2. $T(n) = aT(n/b) + f(n)$
'a' has to be constant. Choice (A)

3. Time complexity of EnQueue( ) = $O(1)$
Time complexity of DeQueue( ) = $O(1)$
Time complexity of IsEmptyQueue( ) = $O(1)$
Time complexity of DeleteQueue( ) = $O(1)$.
Choice (C)

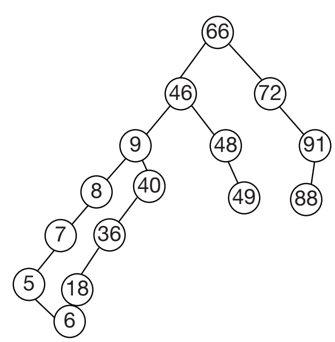
4. All the given statements I, II, III and IV are Applications of Binary Trees. Choice (D)

5. **Depth First Traversal:**
1. Pre order Traversal
2. In order Traversal
3. Post order Traversal

Breadth First Traversal:
1. Level order Traversal
The traversal defined in the given Questions is Level order traversal. Choice (B)

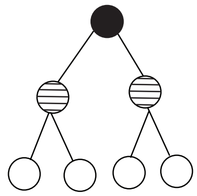
6. With 'n' nodes, it will have the maximum combinations of different trees.
$\Rightarrow$ $2^8 - 8 = 256 - 8 = 248$. Choice (C)

7. Root is '66', the numbers less than 66 will appear in Left sub tree and greater numbers appear in Right sub tree.
$\therefore$ (11, 3)

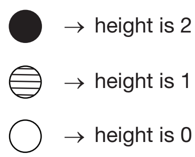


Choice (D)

8. Let us consider the following Full binary tree
$n = 7$ nodes



$n - (h + 1)$
$\Rightarrow \quad 7 - (2 + 1) = 7 - 3 = 4$
$h = 2$ (height)

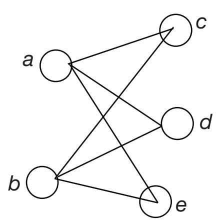


$2 + 1 + 1 + 0 + 0 + 0 + 0 = 4.$ Choice (B)

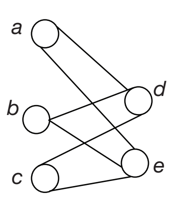
9. $\bar{G}(e) = \dfrac{V(V-1)}{2} - G(e)\; V^2 - V - 72 = 0$
$\Rightarrow \quad V(V - 9) + 8(V - 9) = 0$
$\Rightarrow \quad V = 9.$ Choice (C)

10. The given statements describe Dijkstra's Algorithm.
Choice (B)

11. Let us take
$k_{2,3}$



Minimum vertex cover = $\{a, b\}$
$K_{3,2}$



Minimum vertex cover = $\{d, e\}$
$\therefore \quad$ MIN(m, n). Choice (B)

12. Eulers formula: $|V| + |R| - |E| = 2$
$k_{5,2}$ is planar graph, As we know,
The number of vertices in $k_{m,n}$ graph is $m + n$
$\therefore \quad k_{5,2}$ has 7 vertices
The number of edges in $k_{m,n}$ graph is $m*n$
$\therefore \quad k_{5,2}$ has 10 edges
$|V| + |R| - |E| = 2$
$7 + R - 10 = 2$
$R - 3 = 2$
$R = 5$ Choice (B)

13. Merge sort - II
Heap sort - II
Insertion sort - I, Bubble sort - I, selection sort - I.
Choice (B)

14. All the algorithms follow Divide-and-conquer approach. Choice (D)

15. In the worst case, if there are 4 elements in each list, consider the following lists (if $(i < j)$
print 'i'
else
print 'j')

2 4 6 8	1 3 5 7
i	j

1 comparison

2 4 6 8	1 3 5 7
i	j

1 comparison

2 4 6 8	1 3 5 8
i	j

1 comparison

2 4 6 8	1 3 5 8
i	j

$\therefore \quad$ Almost $(K + L)$ comparisons are required.
Choice (D)

16. $n \log n \geq \log (n!) \geq 2^{\log_2^n}$
option (D) is true.
$2^{\log_2^n} \leq c * \log (n!)$
$\log (n!) \leq c * n \log n.$ Choice (D)

17. $y = y + 2$; // constant time
for ($i = 1$; $i <= n$; $i++$) //executed '$n + 1$' times
$k = k + 2$; // executes n times
for ($i = 1$; $i \leq n$; $i++$)//outer loop executed '$n + 1$' times
for ($j = 1$; $j <= n$; $j++$)//inner loop executed '$n(n + 1)$' times
$x = x + 1$; // executes n times
Total time $= C_0 + C_1 n + C_2 n^2 = O(n^2)$. Choice (B)

18. $T(n) = 3T\left(\dfrac{n}{9}\right) + n^{0.52}$

$T(n) = aT\left(\dfrac{n}{b}\right) + f(n)$

$a = 3, b = 9, f(n) = n^{0.52}$
compare
$n^{\log_b^a}$ Vs $f(n)$
$n^{\log_9^3}$ Vs $n^{0.52}$
$n^{\log_9^{\sqrt{9}}}$ Vs $n^{0.52}$ $f(n)$ is greater than $n^{\log_b^a}$
case 3 of master theorem, so the time complexity is
$T(n) = \theta(f(n))$
$\Rightarrow \quad \theta(n^{0.52})$. Choice (C)

19. Given Integers
6, 7, 1, 4, 3, 2, 8, 9
From Right to Left:
9, 8, 2, 3, 4, 1, 7, 6
Z Z Z Z(4 push operations)

3
2
8
9

2 pop operations
⇒ 3, 2
Again 2 push operations

1
4
8
9

2 pop operations
⇒ 1, 4
One push operation

7
8
9

One pop operation
⇒ 7
∴ popped integers are 3, 2, 1, 4, 7. Choice (D)

20.

50
70

⇒ pop '50'

70
50
30
70

⇒ pop 70, 50

70
30
70

⇒ pop 70, 30
∴ popped sequence = 50, 70, 50, 70, 30.
Choice (C)

21. Level order data in reverse means,
Bottom level data elements from Left to Right should be taken first.
Level order Traversal is, 77, 88, 42, 72, 81, 61, 60, 22, 80. Choice (C)

22. Given two binary trees,
Returns true if they are structurally identical.
∴ If both trees are NULL then return true.
∴ If both trees are not NULL, then compare data and recursively check left and right subtree structures.
Choice (C)

23. With '6' distinct elements and 6 unlabelled binary tree nodes, we can have,

$$n = 6,\ \frac{1}{n+1}\times 2n_{C_n}$$

$$\frac{1}{6+1}\times\frac{(2\times 6)!}{(12-6)!\times 6!}\ \frac{1}{7}\times\frac{12\times 11\times 10\times 9\times 8\times 7}{6\times 5\times 4\times 3\times 2}$$

$= 11 \times 2 \times 3 \times 2 = 132.$ Choice (D)

24. A binary tree with 'n' leaves have $(n-1)$ internal nodes. So $(n-1)$ nodes will have degree '2'. Choice (A)

25. In 5-ary tree each node can contain 0 to 5 children and to get maximum height, we have to keep only one child for each parent, with 50 nodes the maximum possible height we can get is 49. Choice (C)

26. If we have a restriction that atleast one node should have 7 children, then we keep one node with 7 children and remaining all nodes with 1 child. In this case, with 'n' nodes the maximum possible height is $(n-7)$.
∴ $60 - 7 = 53$. Choice (D)

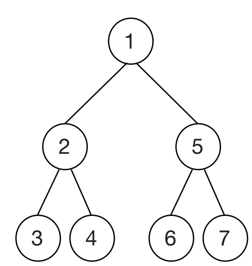
27. If we want to get minimum height, then we need to fill all nodes with maximum children.
Lets take a 4-ary tree.

Height (h)	Maximum Nodes at height, $h = 4^h$
0	1
1	4
2	4 × 4
3	4 × 4 × 4

Choice (B)

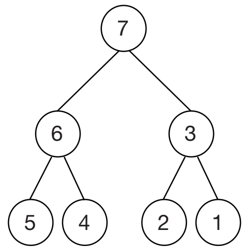
28. In a complete binary tree, all levels contain full nodes except possibly the lowest level.
Maximum = $2^{h+1} - 1$ elements
Minimum = 2^h. Choice (B)

29. Pre order traversal on the following Min-Heap produces elements in ascending order



∴ 1 2 3 4 5 6 7. Choice (B)

30. Pre order Traversal on the following MAX-HEAP produces elements in sequence (descending order)



∴ Pre order = 7 6 5 4 3 2 1. Choice (C)

31. A Heap is a complete binary tree. All the levels, except the lowest, are completely full. So the heap has atleast 2^h elements and atmost $2^{h+1} - 1$ elements. Choice (C)

32. The given array is partitioned into two non-empty sub arrays
In Option (D) 80 percent of elements are in First sub array that means in the second part of array there should be 20 percent elements that is $T\left(\frac{n}{5}\right)$ instead of $T\left(\frac{4n}{5}\right)$. Choice (D)

33. **Quick sort:**
The partitioning step can swap the location of records many times, and thus 2 elements with equal value could swap position in the final output.
Choice (D)

34. Pre order = $A\ B\ C\ D\ E\ G\ F\ H$
In order = $E\ D\ G\ H\ F\ C\ B\ A$
Post order = $E\ H\ F\ G\ D\ C\ B\ A$.
Choice (C)

35. The tree given in option (D) is correct.
Choice (D)

Algorithms Test 2

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. What is the worst case time complexity of search operation on unordered and ordered List, using Linear search Algorithm respectively?

(A) $O(n)$ and $O(1)$
(B) $O(n)$ and $O(\log n)$
(C) $O(n)$ and $O(n)$
(D) $O(\log n)$ and $O(\log n)$

2. Which of the following is the Recurrence relation for binary search and What is the time complexity of that Recurrence Relation respectively?

(A) $T(n) = T(n/2) + \theta(n)$ and $\theta(\log n)$
(B) $T(n) = T(n/2) + \theta(1)$ and $\theta(\log n)$
(C) $T(n) = T(n/2) + \theta(1)$ and $\theta(n \log n)$
(D) $T(n) = T(n/2) + \theta(n)$ and $\theta(n \log n)$

3. Given 'n' numbers randomly, what is the time complexity of calculating median?

(A) $O(\log n)$ (B) $O(n \log n)$
(C) $O(n^2)$ (D) $O(n^2 \log n)$

4. Given an array of 'n' elements, what is the time complexity of Finding a number which appears more than $(n/2)$ times in the given array (if it exists)?

(A) $O(n/2)$ (B) $O(n \log n)$
(C) $O(\log n)$ (D) $O(n^2)$

5. What is the number of comparisons performed to find the smallest and largest keys in an array A of even size and odd size of n and $(n + 1)$ elements respectively?

(A) $\frac{3n}{2} - 2$ and $\frac{3n}{2}$ (B) $\frac{3n}{2}$ and $\frac{3n}{2} - 2$
(C) $\frac{3n}{2} - 2$ and $\frac{3n}{2} - \frac{3}{2}$ (D) $\frac{3n}{2} - 2$ and $\frac{3n}{2} + 1$

6. What is the time complexity of an algorithm, for finding Fourth Largest element in the given input list of n elements?

(A) $2n - 3$ (B) $2n - 4$
(C) $2n - 5$ (D) $2(2n - 5)$

7. The Bellman-Ford algorithm solves the single-source shortest path problem in the case in which edge weights may be negative, what is the time complexity of running Bellman-Ford Algorithm?

(A) $O(V^2)$ (B) $O(V * E)$
(C) $O(V + E)$ (D) $O(E \log V)$

8. Construct a Hash table with size '8', hash function is $h(k) = (2k + 1) \bmod 8$, with elements 3, 8, 16, 9, 1, 4. What is the Location of key '4'?

(A) Location 1 (B) Location 2
(C) Location 4 (D) Location 5

9. Consider the given two strings str1 and str2.
Let str1 = < A B R A C A D A B R O A >
str2 = < Y A B B A D A B B A D O O B A >
Then the length of longest common subsequence would be ___________.

10. Which of the following is the regular expression to the string of the form $a^m b^{2n} c^{3p}$, where $m, n, p \geq 2$.

(A) $a^* b^* c^*$
(B) $aa^* (bb)(bb)^* ccc(ccc)^*$
(C) $aaa^* (bbbb)(bb)^* cccccc(ccc)^*$
(D) $aa(aa)^* (bbbb)(bbbb)^* cccccc (cccccc)^*$

11. Which of the following statements are TRUE.

(i) $3n + 1 \in O(3^n)$
(ii) $100n \log n \in O(n \log n)$
(iii) $2n \neq O(n^k)$; K is a constant.
(iv) $0 < i < j$; $n^j \in O(n^i)$

(A) (i, ii, iii) (B) (i, ii, iv)
(C) (ii, iii) (D) (i, ii)

12. Suppose that we want to encode strings over the 8-character alphabet $C = \{a, b, c, d, e, f, g, h\}$ by using Fixed Length Encoding, and the frequencies are given below:
$a - 11, b - 13, c - 12, d - 10, e - 9, f - 7, g - 5, h - 3$
The number of bits required to store string '*abfeg*' is _______.

13. In Depth First search Algorithm, _____ number of times all the vertices are accessed, in a graph $G(V, E)$:

(A) one (B) two
(C) three (D) four

14. What is the number of substrings of any length excluding empty string, of a given string of length 'n', that can be formed?

(A) n^2 (B) $n \log n$
(C) $\frac{n(n+1)}{2}$ (D) $\frac{n(n-1)}{2}$

15. What is the time complexity of running Bellman-Ford Algorithm on K-Regular graph ($K \geq 3$)?

(A) $O(n^2 \log n)$
(B) $O(n^3)$
(C) $O(2^n)$
(D) $O(n \log n)$

16. Consider the given array

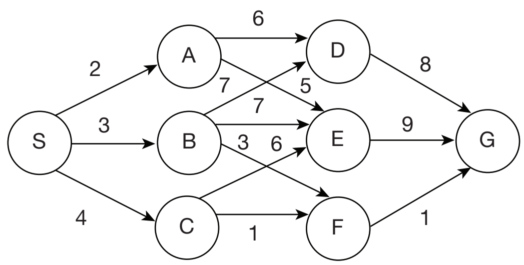
4	2	6	4	2	6

For finding the first element in the array which is repeated, which of the following is TRUE?

(A) Sort the given array, in this sorted array, the first element is the repeated element.
(B) Use brute force method, every element is checked with all the other elements, return the first element which is repeated.

(C) Sort the given array in descending order, in this sorted array the first element is the repeated element.
(D) Both (A) and (B)

17. Consider the given multi stage graph



What is the shortest path from node 'S' to node 'G' using Greedy Approach?

(A) 16 (B) 6
(C) 7 (D) 19

18. Consider the same graph given in the above question, What is the shortest path from node 'S' to node 'G' using Dynamic programming?

(A) 6 (B) 7
(C) 16 (D) 19

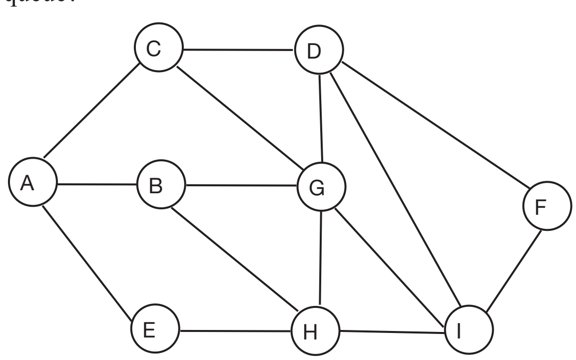
19. Consider the given Huffman code:

110 100 11110 110

Huffman code is constructed for the following set of letters whose frequencies are based on the first 8 Fibonacci numbers?
$a-1, b-1, c-2, d-3, e-5, f-8, g-13, h-21$
Which of the following sequence of letters correctly matches the given Huffman code?

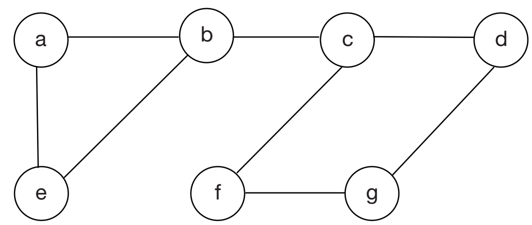
(A) $f\,g\,h\,a\,d$ (B) $f\,g\,h\,c\,e$
(C) $f\,g\,h\,d\,f$ (D) $f\,g\,h\,d\,e$

20. Implement Breadth-First search on the graph given below, starting at vertex A. Assume that the adjacency lists are in sorted order. Example when exploring vertex E, the algorithm considers the edge $E-B$, before $E-C$, $E-F$, $E-G$, or $E-H$. Which of the following is the order of vertices that are enqueued on the FIFO queue?



(A) A, B, C, E, G, H, I, D, F
(B) A, B, C, E, H, G, D, I, F
(C) A, B, C, D, E, G, H, I, F
(D) A, B, C, E, G, H, D, I, F

21. Consider the given graph



Implement Depth First search Algorithm on the given graph, which of the following cannot be the sequence of poped elements?

(A) a, b, c, d, g, f, e (B) c, d, g, f, b, a, e
(C) c, d, g, f, b, e, a (D) f, g, d, c, a, e, b

22. Consider the graph given in the above Question, Implement Breadth First search on the given graph, Which of the following cannot be the sequence of nodes Dequeued?

(A) d, c, g, f, b, a, e (B) g, f, d, c, b, a, e
(C) f, c, g, b, d, a, e (D) e, a, c, d, g, f, b

23. The keys 22, 31, 46, 42, 58, 61, 64, 71, 83, 97 are inserted into an initially empty hash table of length 10 using open addressing with hash function $h(k) = k$ mod 10 and Linear probing. What is the Resultant hash table?

(A)

0	1	2	3	4	5	6	7	8	9
83	31	22	42	61	64	46	71	58	97

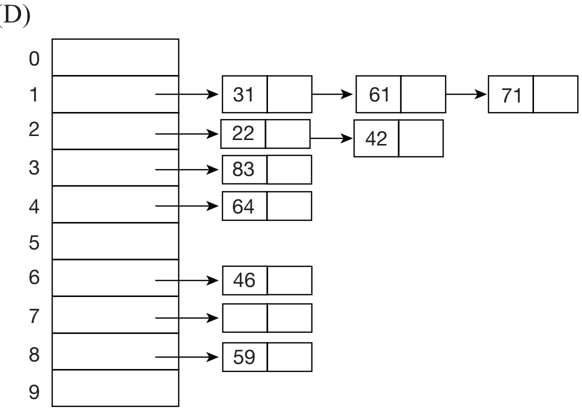
(B)

0	1	2	3	4	5	6	7	8	9
97	31	22	42	61	64	46	71	58	83

(C)

0	1	2	3	4	5	6	7	8	9
	31 61 71	22 42	83	64		46	97	58	

(D)



24. What is the number of bits required to store a file of 200 characters, The frequencies of *a*, *b*, *c*, *d* are as follows *a* – 25, *b* – 50, *c* – 100, *d* – 25 using Fixed Length Encoding?

(A) 200 bits (B) 800 bits
(C) 400 bits (D) 600 bits

25. Consider the data given in the above question what is the number of bits required to store the file using variable Length Encoding scheme?

(A) 200 bits (B) 250 bits
(C) 300 bits (D) 350 bits

26. Consider the tasks T_1, T_2, T_3, T_4, T_5. Following table shows, Deadlines and profits of the given tasks

Task	Deadline	Profit
T_1	2	20
T_2	2	50
T_3	1	30
T_4	1	40
T_5	4	60

Which tasks are not executed?

(A) T_1 and T_2 (B) T_1 and T_5
(C) T_1 and T_3 (D) T_2 and T_5

27. For the data given in the above question, what is the profit made from the tasks executed?

(A) 150 (B) 90
(C) 120 (D) 130

Common Data for questions 28 and 29:

```
Consider the given code:
int fun (int k[ ], int n)
{
int counter = 0, max = 0;
for (int i = 0; i<n; i ++)
{
counter = 0;
for (int j = 0; j <n; j++)
{
if (k[i] = = k[j])
counter ++;
}
if (counter >max)
max = counter;
}
return max;
}
```

28. What is the task performed by the given code?

(A) Counting the number of swaps.
(B) Finding the element which appears maximum number of times in the array.
(C) Finding the element which appears minimum number of times in the array.
(D) Counting the number of comparisons.

29. What is the time complexity in executing the given code?

(A) $O(\log n)$ (B) $O(n \log n)$
(C) $O(n^2)$ (D) $O\left(n\sqrt{n}\right)$

Common Data for Questions 30 and 31:

Consider the following code:

```
void fun1(int k[ ], int n, int X)
{
for (int i = 0; i < n; i++)
{
for (j = i; j < n; j++)
{
if (k[i] + k[j] = = X)
{
printf("Items found")
return;
}
}
}
printf("Items Not found");
}
```

30. The above code performs

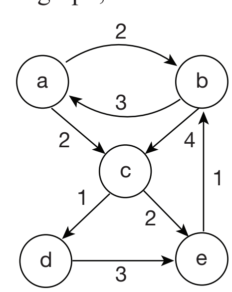
(A) The sum of 2 array elements is equal to given '*X*' value.
(B) Sum of adjacent elements whose sum is equal to given '*X*' value.
(C) The sum of adjacent elements if they are equal.
(D) The sum of adjacent elements which are equal and equal to given '*X*' value.

31. What is the time complexity of given code?

(A) $O(n \log n)$
(B) $O(n^2)$
(C) $O(n^3)$
(D) $O(\log n)$

Common Data for Questions 32 and 33:

Consider the given graph,



32. What is the eccentricity of node '*d*'?

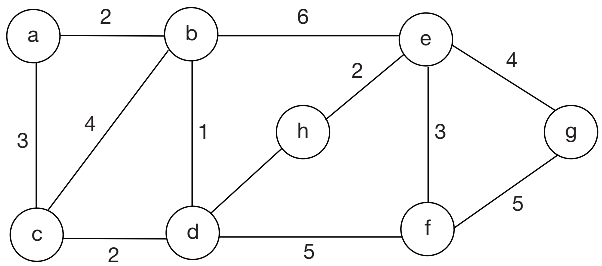
(A) 3 (B) 5
(C) 1 (D) 6

33. What is the center of given graph?

(A) Node '*a*' (B) Node '*c*'
(C) Node '*b*'
(D) Either Node '*d*' or Node '*e*'

Statement for linked Answer questions 34 and 35:

34. Consider the given graph



Which of the following can be the sequence of edges added to minimum spanning tree using "Prims Algorithm"?

(A) $(a-b), (b-d), (d-c), (d-f), (f-g), (f-e), (e-h)$
(B) $(a-b), (b-d), (d-c), (d-f), (f-e), (e-g), (e-h)$
(C) $(e-h), (e-f), (e-g), (f-d), (d-b), (b-a), (d-c)$
(D) $(e-h), (e-f), (e-g), (d-b), (f-d), (b-a), (d-c)$

35. What is the total weight of spanning tree (correct edge sequence) identified in the above question?

(A) 17 (B) 19
(C) 20 (D) 21

Answer Keys

1. C	**2.** B	**3.** B	**4.** D	**5.** C	**6.** D	**7.** B	**8.** D	**9.** 8	**10.** C
11. A	**12.** 15	**13.** B	**14.** C	**15.** B	**16.** B	**17.** A	**18.** A	**19.** C	**20.** D
21. D	**22.** D	**23.** B	**24.** C	**25.** D	**26.** C	**27.** A	**28.** B	**29.** C	**30.** A
31. B	**32.** D	**33.** C	**34.** C	**35.** B					

Hints and Explanations

1. In worst case Linear search takes $O(n)$ time, whether it is ordered list (or) unordered List. Choice (C)

2. Recurrence Relation for Binary search is $T(n) = T(n/2) + \theta(1)$. Time complexity of Binary search is $\theta(\log n)$. Choice (B)

3. Randomly given 'n' numbers are to be sorted first, which takes $O(n \log n)$ time and return the $(n/2)$th element, that is the median of given 'n' random elements. Choice (B)

4. The basic solution is to have two loops and keep track of maximum count for all different elements. If maximum count becomes greater than $n/2$ then break the loops and return the element having maximum count. If maximum count doesn't become more than $n/2$ then majority element doesn't exist.
2 loops(nested) $\Rightarrow O(n^2)$. Choice (D)

5. Number of comparisons

If 'n' is even $\frac{3n}{2} - 2$

If 'n' is odd $\frac{3n}{2} - \frac{3}{2}$. Choice (C)

6. To find First Largest element $(n - 1)$ comparisons are required, for second largest $(n - 2)$, for Third largest $(n - 3)$ and For fourth largest $(n - 4)$.

$\therefore$ Total $\Rightarrow (n - 1) + (n - 2) + (n - 3) + (n - 4)$
$\therefore$ $4n - 10 \Rightarrow 2(2n - 5)$.

Choice (D)

7. The algorithms makes $|V| - 1$ passes over the edges of the graph (E)

$(V - 1) * E$
$O(V * E)$.

Choice (B)

8. **Linear Probing:**

$h(k) = (2k + 1) \bmod 8$
$h(3) = 7 \bmod 8 = 7$
$h(8) = 17 \bmod 8 = 1$
$h(16) = 33 \bmod 8 = 1$
(collision occurred at location 1)
Search for next empty slot from the point of collision.
$h(9) = 19 \bmod 8 = 3$
$h(1) = 3 \bmod 8 = 3$ (collision)
$h(4) = 9 \bmod 8 = 1$ (collision). Choice (D)

9. str1 = A B R A C A D A B R O A

str2 = Y A B B A D A B B A D O O B A

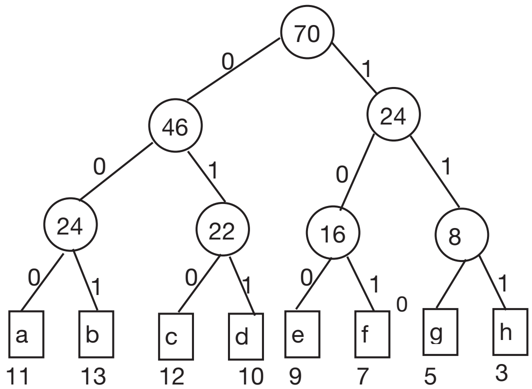
The longest common subsequence would be A B A D A B O A.
The length is '8'.

10. The string form is $a^m\, b^{2n}\, c^{3p}$, $m, n, p \geq 2$.
i.e., The strings are *aabbbbcccccc*, *aaabbbbbbcccccc-ccc*, ...
The regular expression will be
$(aa)a^*\ (bbbb)\ (bb)^*\ (cccccc)(ccc)^*$
(minimum two a's four b's, six c's will be there in the string). Choice (C)

11. $0 < i < j$
$n^j \leq c * n^i$ which is false.

Choice (A)

12. **Fixed length coding:**



If there are 8 characters, 7 merging operations would be performed. In fixed length coding each character takes same number of encoded bits, each character takes 3-bits.

'*a b f e g*', 5 characters ⇒ 5 × 3 = 15 bits.

13. In Depth First search Algorithm, First all elements (V) will be pushed onto stack and followed by (V) pop operations.

∴ Total $V * V = 2V$.

Choice (B)

14. Since the length of the given string is n, there are 'n' sub strings possible (excluding empty string) with first character. Similarly, (n – 1) substrings possible with second character and so on.

Choice (C)

15. K-Regular graph (K_{n+1}-graph) that is complete graph. Hence $O(n^3)$. Choice (B)

16.

4	2	6	4	2	6

Sorted array:

2	2	4	4	6	6

In the sorted array First repeating element is '2', but in the given array 4 is the first repeating element.

Sorted array in descending order:

6	6	4	4	2	2

In the sorted array (descending order), The first repeating element is '6'. Choice (B)

17. Greedy Approach, pick the least possible weight at each phase.

$$S \xrightarrow{2} A \xrightarrow{5} E \xrightarrow{9} G$$

⇒ 2 + 5 + 9 = 16. Choice (A)

18. Shortest distance from node 'u' to node 'v' is shown by $\delta(u, v)$ $\delta(A, G) = \min\{6 + \delta(D, G), 5 + \delta(E, G)\}$

$\delta(C, G) = \min\{1 + \delta(F, G), 6 + \delta(F, G)\}$

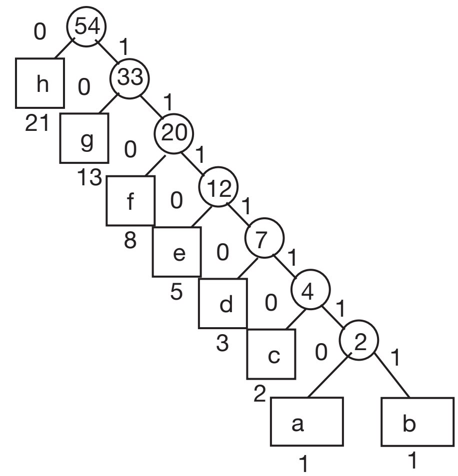
$\delta(E, G) = 9$

⇒ Back substitute the value

$\delta(E, G) = \min\{16, 4\} = 4$

$\delta(S, G) = \min\{16, 7, 6\} = 6.$ Choice (A)

19. $a – 1, b – 1, c – 2, d – 3, e – 5, f – 8, g – 13, h – 21$



Huffman codes

a – 1111110

b – 1111111

c – 111110

d – 11110

e – 1110

f – 110

g – 10

h – 0

Given code <u>110</u> <u>10</u> <u>0</u> <u>11110</u> <u>110</u>

f g h d f. Choice (C)

20. Queue

A

dequeue A, and enqueue its neighbours in lexicographic order

B	C	E

Dequeue B

C	E	G	H

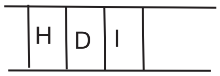
Dequeue C

E	G	H	D

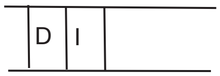
Dequeue E

G	H	D

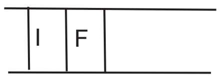
Dequeue *G*



Dequeue *H*



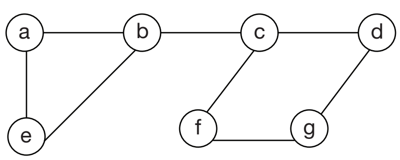
Dequeue *D*



Dequeue *I*

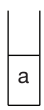
Dequeue *F* Choice (D)

21.

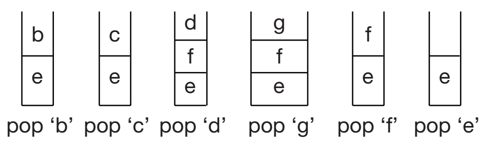


Choice (A)

a, *b*, *c*, *d*, *g*, *f*, *e*



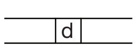
pop '*a*' and push its neighbours



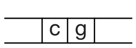
popped elements ⇒ *a*, *b*, *c*, *d*, *g*, *f*, *e*
In the same way check all the options.
Choice (D) is incorrect.

Choice (D)

22. Option (A): *d*, *c*, *g*, *f*, *b*, *a*, *e*

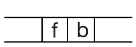


dequeue 'd' and Enqueue its neighbours

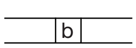


dequeue 'c' and Enqueue its neighbours

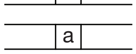
dequeue 'g' and Enqueue its neighbours



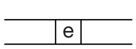
dequeue 'f'



dequeue 'b'



dequeue 'a'



dequeue 'e'

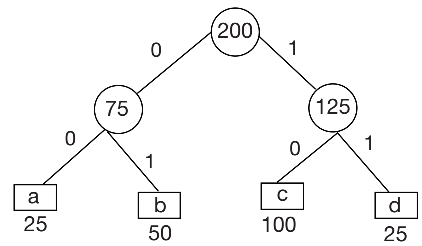
In the same way, check all the option, options (D) is incorrect. Choice (D)

23. $h(k) = k \bmod 10$

0	1	2	3	4	5	6	7	8	9
97	31	22	42	61	64	46	71	58	83

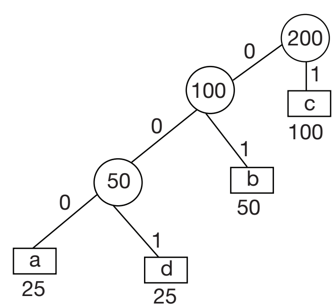
Linear Probing: when collision occurs, search for next empty slot from the place of collision, treat array as circular array. Choice (B)

24. $a - 25$, $b - 50$, $c - 100$, $d - 25$



After Encoding
$a - 00$(2 bits)
$b - 01$(2 bits)
$c - 10$(2 bits)
$d - 11$(2 bits)
Number of bits required
$= 25 \times 2 + 50 \times 2 + 100 \times 2 + 25 \times 2$
$= 50 + 100 + 200 + 50$
$= 400$ bits. Choice (C)

25. Variable Length Encoding is also called "Huffman coding"
Arrange the characters in increasing order
$a - 25$, $d - 25$, $b - 50$, $c - 100$



After Encoding:
$a - 000$ (3 bits)
$b - 01$ (2 bits)
$c - 1$ (1 bit)
$d - 001$ (3 bits)
Number of bits required
$= 25 \times 3 + 50 \times 2 + 25 \times 3 + 100 \times 1$
$= 75 + 100 + 75 + 100$
$= 350$ bits. Choice (D)

26. Arrange the tasks in decreasing order according to their profits.

T_5	T_2	T_4	T_3	T_1
4	2	1	1	2
60	50	40	30	20

T_4	T_2		T_5					

T_3 and T_1 cannot be executed because their deadlines are 1 and 2 respectively. Choice (C)

27. The executed Tasks are T_2, T_4 and T_5 and the profit made is 40 + 50 + 60 = 150. Choice (A)

28. The given code performs, For each input element, check whether there is any element with same value and for each such occurrence, increment the counter. Every time, check the current counter with the max and update it, if this value is greater than counter. Choice (B)

29. There are 2 nested loops, each loop will be executed 'n' times

$\therefore \quad n \times n = O(n^2)$. Choice (C)

30. The given code performs sum of 2 array elements which is equal to given 'X' value. Choice (A)

31. Outer loop executes '$n + 1$' times
Inner loop executes '$n - 1$' times
$\therefore O(n^2)$. Choice (B)

32. Eccentricity of a node is the maximum of minimum path from other nodes to the given node.

$$\max \begin{Bmatrix} \text{Min path } (a-d) = 3 \\ \text{Min path } (b-d) = 5 \\ \text{Min path } (c-d) = 1 \\ \text{Min path } (e-d) = 6 \end{Bmatrix}$$

Eccentricity = 6 Choice (D)

33. Center of a graph is a node with minimum eccentricity.
Eccentricity of Node 'a'

$$\max \begin{Bmatrix} \text{dist } (b-a) = 3 \\ \text{dist } (c-a) = 6 \\ \text{dis } (d-a) = 7 \\ \text{dist } (e-a) = 4 \end{Bmatrix} = 7$$

Eccentricity of Node 'b'

$$\max \begin{Bmatrix} \text{dist } (a-b) = 2 \\ \text{dist } (c-b) = 3 \\ \text{dist } (d-b) = 4 \\ \text{dist } (e-b) = 1 \end{Bmatrix} = 7$$

Eccentricity of Node 'c'

$$\max \begin{Bmatrix} \text{dist } (a-c) = 2 \\ \text{dist } (b-c) = 4 \\ \text{dist } (d-c) = 8 \\ \text{dist } (e-c) = 5 \end{Bmatrix} = 8$$

Eccentricity of Node 'd' is given in the above solution that is '6'
Eccentricity of Node 'e'

$$\max \begin{Bmatrix} \text{dist } (a-e) = 4 \\ \text{dist } (b-e) = 6 \\ \text{dist } (c-e) = 2 \\ \text{dist } (d-e) = 3 \end{Bmatrix} = 6$$

Center of a graph is min of all eccentricities min (7, 4, 8, 6, 6) = Node. Choice (C)

34. If we start Prims Algorithm with edge $(a - b)$ the sequence will be
$(a-b)$, $(b-d)$, $(d-c)$, $(d-f)$, $(f-e)$, $(e-h)$, $(e-g)$
If it starts with edge $(e-h)$ the sequence will be
$(e-h)(e-f)(e-g)$, $(f-d)$, $(d-b)$, $(b-a)$, $(d-c)$.
Choice (C)

35. For the correct sequence
$(e-h)$ $(e-f)$ $(e-g)$, $(f-d)$ $(d-b)$ $(b-a)$ $(d-c)$
2 + 3 + 4 + 5 + 1 + 2 + 2 = 19. Choice (B)

ALGORITHMS TEST 3

Number of Questions: 25 **Section Marks: 30**

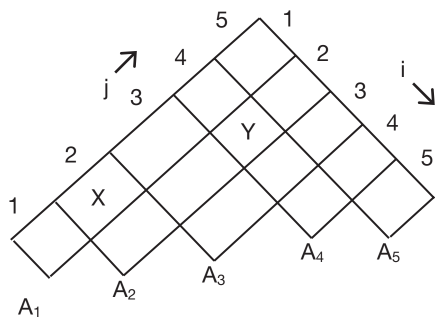
Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Given the following input 564, 816, 726, 903, 1321, 1345, 1498 and the Hash function is $h(k) = k \bmod 3$. Which of the following statements are CORRECT?
I. 564, 816, 726, 903 will hash to same Location.
II. 1321, 1345, 1498 will hash to same Location.
III. 816, 1345, 903 will hash to same Location.
(A) I and III (B) I and II
(C) II and III (D) I, II and III

2. Let M be an integer greater than 1. Which of the following represents the order of growth of the expression $\sum_{i=1}^{n} M^i$ as a function of 'n'?
(A) $\theta(n^M)$ (B) $\theta(M^{2n+1})$
(C) $\theta(M^{n \log n})$ (D) $\theta(M^n)$

Common Data For Questions 3 and 4:

To compute the Matrix product $M_1 M_2$, where M_1 has 'p' rows and 'q' columns and M_2 has 'q' rows and 'r' columns, takes time proportional to 'pqr', and result is a matrix of 'p' rows and 'r' columns. Consider the given 5 matrices, A_1, A_2, A_3, A_4 and A_5 and their dimensions are $P_0 \times P_1$, $P_1 \times P_2$, $P_2 \times P_3$, $P_3 \times P_4$, $P_4 \times P_5$, respectively. The values are $P_0 = 10$, $P_1 = 5$, $P_2 = 15$, $P_3 = 20$, $P_4 = 40$, $P_5 = 25$.



3. MATRIX-CHAIN-ORDER computes the rows from bottom to top and from left to right, The value of X (where $X = M[1, 2]$) is ________
(A) 600 (B) 750
(C) 800 (D) 900

4. The value of Y (where $Y = M[2, 4]$) is ________.
(A) 1500 (B) 5500
(C) 15000 (D) None of the above

5. Which of the following cannot be the time complexity of Quick sort algorithm, under any of the Average, Best, Worst cases?
(A) $O(n \log n)$ (B) $O(n^2)$
(C) $O(n^3)$ (D) $O(\log n)$

6. Consider the following:
I. If $f(n) = O(g(n))$ and $g(n) = O(h(n))$ then $f(n) = \Omega(h(n))$
II. The asymptotically tight upper bound for $T(n) = T(n-2) + 1$ is $O(\log n)$
Which of the following is correct?
(A) I–TRUE, II–TRUE (B) I–TRUE, II–False
(C) I–False, II–TRUE (D) I–False, II–False

7. Let
$$f(n) = 6n + 8n^2 + 200n^3,$$
$$g(n) = n^2 \log n$$
Which of the following is valid?
(A) $f(n) = O(g(n))$ (B) $f(n) = \theta(g(n))$
(C) $f(n) = \Omega(g(n))$ (D) $g(n) = \Omega(f(n))$

8. If $f(n) = n \log n$ then which of the following is FALSE?
(A) $f(n) = O(n^2 \log n)$
(B) $f(n) = \theta(n \log n)$
(C) $f(n) = \Omega(n^2)$
(D) $f(n) = \Omega(\log n * \log n)$

9. What is the average time complexity of the sequential search algorithm where the searched item is sequentially compared to each element in the list of 'n' elements?
(A) $\frac{n}{2}$ (B) $\log n$
(C) $\frac{n+1}{2}$ (D) $\frac{n-1}{2}$

10. Consider the given data:
— Let n be the number of elements
— Number of levels used in sorting: $O(\log n)$
— At each level $O(n)$ amount of work
The given data is related to:
(A) Heap sort (B) Merge sort
(C) Selection sort (D) Bubble sort

11. Consider a graph, with all distinct Edge weights, which of the following is TRUE?
(A) The shortest path is unique between every pair of vertices
(B) The maximal matching is unique
(C) The minimum spanning tree is unique
(D) All the above

12. The Hash function $h(k) = k \bmod 7$ with Linear probing is used to insert the keys 37, 38, 72, 68, 98, 11, 74 into hash table indexed (0 – 6), The location of Key 74 is ________.
(A) 1 (B) 2
(C) 3 (D) 4

13. Given an undirected graph $G = (V, E)$ and a positive integer 'K', does 'G' have 'K' vertices that form

a complete sub graph and if it does, then what is the minimum value of 'K'?

(A) 2 (B) 3
(C) 4 (D) None of the above

14. In the Build-Heap algorithm, the Heapify routine is invoked 'n' times. This indicates that Build-Heap Complexity is ______.

(A) $O(\log n)$ (B) $O(n)$
(C) $O(n \log n)$ (D) $O(n^2)$

15. Give asymptotically tight Big-O bounds for the following recurrence relations. (By using Iterative substitution method)

$$T(n) = T(n-2) + 1 \text{ if } n > 2$$
$$T(n) = 1 \quad \text{if } n \leq 2$$

(A) $O(n \log n)$ (B) $O(n)$
(C) $O(n^2)$ (D) $O(\log n)$

16. Consider the given code:

```
public static int f2(int n){
int x = 0;
for (int i= 0; i < n; i++)
for (int j = 0; j < i * i; j++)
x++;
return x;
}
```

What is the order of growth of the above code?

(A) $O(\log n)$ (B) $O(n^2)$
(C) $O(n^3)$ (D) $O(2^n)$

17. Consider the following:

I. $(3n)! = O(n!^3)$ II. $\log(n!) = \theta(n \log n)$

Which of the following is correct?

(A) I–False, II–False (B) I–False, II–TRUE
(C) I–TRUE, II–False (D) I–TRUE, II–TRUE

18. The Floyd-Warshall all pairs shortest path algorithm for finding the shortest distances between nodes in a graph is an example of:

(A) An iterative based divide - and - conquer
(B) A Dynamic programming formulation
(C) A greedy algorithm
(D) A recursive based divide - and - conquer

19. Which of the following is not having $O(n^2)$ complexity?

(A) $n + 2000n$ (B) $n^{1.999}$
(C) $10^6 n + 2^6 n$ (D) $\frac{n^3}{\sqrt{n}}$

20. Consider the given two strings str1 and str2.
Let str1 = < A B R A C A D A B R O A >
str2 = < Y A B B A D A B B A D O O B A >
Then the length of longest common subsequence would be ___________.

(A) 5 (B) 6
(C) 7 (D) 8

21. What is the computational complexity of the following piece of code:

```
for (i = n; i > 0; i/= 2){
for (int j = 1; j < n; j * = 2){
for (int k = 0; k < n; k + = 2){
      sum + = (i + j * k);
      }
   }
}
```

(A) $O(n^3)$
(B) $O(n^2 \log n)$
(C) $O(\log n * \log n * \log n)$
(D) $O(n * \log n * \log n)$

22. Consider the modified binary search algorithm so that it splits the input not into 2 sets of sizes equal sizes, but into three sets of sizes approximately one-third. What is the recurrence for this ternary search algorithm?

(A) $T(n) = T\left(\frac{n}{2}\right) + T(n-2) + C$

(B) $T(n) = T\left(\frac{n}{3}\right) + C$

(C) $T(n) = T\left(\frac{3n}{4}\right) + T\left(\frac{n}{4}\right) + C$

(D) $T(n) = T\left(\frac{n}{3}\right) + \log n$

23. Sort the following growth rate classes in increasing order of time complexity:
Exponential, quadratic, logarithmic, cubic, and factorial.

(A) Logarithmic, quadratic, cubic, exponential, factorial
(B) Logarithmic, quadratic, cubic, factorial, exponential
(C) Quadratic, cubic, logarithmic, exponential, factorial
(D) Quadratic, cubic, logarithmic, factorial, exponential

24. Determine if 'x' is present in an array of n-elements:

```
for i = 0 to n
if (a[i] = x) return TRUE
else
return false
```

What is the worst case, best case time complexities and space complexity respectively?

(A) $O(n)$, $O(1)$, $O(n)$
(B) $O(n)$, $O(1)$, $O(1)$
(C) $O(n)$, $O(\log n)$, $O(1)$
(D) $O(\log n)$, $O(\log n)$, $O(n)$

25. What is the space complexity of following sorting algorithms respectively?

I. Quick sort II. Merge sort
III. Selection sort IV. Insertion sort

(A) $O(n)$, $O(n)$, $O(1)$, $O(1)$
(B) $O(1)$, $O(n)$, $O(1)$, $O(n)$
(C) $O(1)$, $O(n)$, $O(1)$, $O(1)$
(D) $O(1)$, $O(n)$, $O(n)$, $O(1)$

Answer Keys

1. B	2. D	3. B	4. B	5. D	6. D	7. C	8. C	9. C	10. B
11. C	12. A	13. A	14. D	15. B	16. C	17. A	18. B	19. D	20. D
21. D	22. B	23. A	24. B	25. C					

Hints and Explanations

1. $h(k) = k \bmod 3$

564 mod 3 = 0
816 mod 3 = 0
726 mod 3 = 0
903 mod 3 = 0
1321 mod 3 = 1
1345 mod 3 = 1
1498 mod 3 = 1

564, 816, 726, 903 will hash to same location,
1321, 1345, 1498 will hash to same location.
Statement III is not TRUE. Choice (B)

2. Let M value be '3'

$$\sum_{i=1}^{n} 3^i = 3^1 + 3^2 + 3^3 \ldots + 3^n$$

The sequence is in Geometric Progression.

First term $= 3 = a$

$$r = \frac{3^2}{3} = 3$$

As $r > 1 \Rightarrow \text{sum} = a\frac{(r^n - 1)}{r - 1}$

$$3 \cdot \frac{(3^n - 1)}{3 - 1} = \frac{3(3^n - 1)}{2}$$

$$= \frac{3.3^n - 3 * 1}{2} = \frac{3^{n+1} - 3^1}{2}$$

$= 3^{n+1} \Rightarrow 3^n$ (Ignore constants in Asymptotic Notations)

$\therefore$ $\theta(M^n)$ is the order of growth of given expression. Choice (D)

3. Computing the matrix product $A_{i....k}\, A_{k+1....j}$ takes $P_{i...1}\, P_k\, P_j$ scalar multiplications.

$M[i, j]$

$$= \begin{cases} 0 & \text{if } i = j \\ \min\limits_{i \le k < j} \{M[i,k] + M[k+1, j] + P_{i-1} P_k P_j, & \text{if } i < j\} \end{cases}$$

$X = M[1, 2]$
$i = 1, j = 2$
$\therefore$ $k = 1$ $(\because i \le k < j)$
$P_0 = 10, P_1 = 5, P_2 = 15, P_3 = 20,$
$P_4 = 40, P_5 = 25$
$M[1, 1] = 0$
$M[2, 2] = 0$
$M[3, 3] = 0$
$M[4, 4] = 0$
$M[5, 5] = 0$

$$M[1, 2] = \min_{i \le k < j} [M[i, k] + M[k+1, j] + P_{i-1} P_k P_j$$

$$K = 1 \Rightarrow M[1, 2] = M[1, 1] + M[2, 2] + P_0\, P_1\, P_2$$

$0 + 0 + 10 \times 5 \times 15$
$= 750$ Choice (B)

4. $M[2, 4] \to$ The possible values for 'K' are 2, 3.

(1) $\underset{k=2}{M[2,4]} = M[2, 2] + M[3, 4] + P_1\, P_2\, P_4$

(2) $\underset{k=3}{M[2,4]} = M[2, 3] + M[4, 4] + P_1\, P_3\, P_4$

We need to get the values of $M[3, 4]$ and $M[2, 3]$

$\underset{k=3}{M[3,4]} = M[3, 3] + M[4, 4] + P_2\, P_3\, P_4$

$0 + 0 + 15 \times 20 \times 40 = 12000$

$\underset{k=2}{M[2,3]} = M[2, 2] + M[3, 3] + P_1\, P_2\, P_3$
$= 0 + 0 + 5 \times 15 \times 20$
$= 1500$

Choose min of (1) and (2)

$\underset{k=2}{M[2,4]} = M[2, 2] + M[3, 4] + P_1\, P_2\, P_4$

$0 + 12000 + 5 \times 15 \times 40$
$= 12000 + 3000 = 15000$

$\underset{k=3}{M[2,4]} = M[2, 3] + M[4, 4] + P_1\, P_3\, P_4$
$= 1500 + 0 + 5 \times 20 \times 40$
$= 1500 + 4000 = 5500$

$\therefore$ $M[2, 4] = \min\{15000, 5500\} = 5500$ Choice (B)

5. Average and Best case time complexities are $O(n \log n)$ and $O(n \log n)$

Worst case time complexity is $O(n^2)$

Since $(n \log n)$ and n^2 are less than n^3, $O(n^3)$ is also valid. Choice (D)

6. I. $f(n) = O(g(n))$ and $g(n) = O(h(n))$ then $f(n) = \Omega(h(n))$

Lets assume

$f(n) = n$
$g(n) = n^2$
$h(n) = n^3$
$f(n) \le g(n)$
$\Rightarrow$ $n \le n^2$
$g(n) \le h(n) \Rightarrow n^2 \le n^3$
Then $f(n) \ge h(n)$
$\Rightarrow$ $n \ge n^3$ (false)

II. $T(n) = T(n - 2) + 1$

Gives $O(n)$ time complexity (False) Choice (D)

7. $f(n) = 6n + 8n^2 + 200n^3 \Rightarrow n^3$

$g(n) = n^2 \log n$

Option (A):

$n^3 \leq c * n^2 \log n$ (false)

Option (B):

$n^3 = c * n^2 \log n$ (false)

Option (C):

$n^3 \geq c * n^2 \log n$ (TRUE) Choice (C)

8. Option (A):

$n \log n \leq c * n^2 \log n$ (TRUE)

Option (B):

$n \log n = c * n \log n$ (TRUE)

Option (C):

$n \log n \geq c * n^2$ (False) Choice (C)

9. In an average case, the probability that 'X' is in the K^{th} array slot is $1/n$, hence the average number of comparisons is

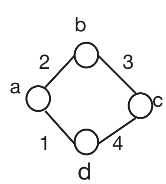
$$\sum_{i=1}^{n}\left(K \times \frac{1}{n}\right) = \frac{1}{n} \times \sum_{k=1}^{n} K$$

$$= \frac{1}{n} * \frac{n(n+1)}{2} = \frac{n+1}{2}$$ Choice (C)

10. In the worst, average, best cases, merge sort procedure will have $O(\log n)$ levels.

In each level $O(n)$ work is required. Choice (B)

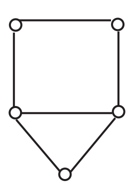
11. Option (A):



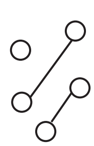
∴ There are 2 shortest paths From 'a' to 'c'

1. $a - b - c$
2. $a - d - c$

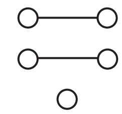
Option (B):



Maximal matching 1



Maximal matching 2



Option (C)

For any graph, there will be unique spanning tree.

Choice (C)

12. Hash function $h(k) = k \bmod 7$

Given elements are: 37, 38, 72, 68, 98, 11, 74

37 mod 7 = 2

38 mod 7 = 3

72 mod 7 = 2

68 mod 7 = 5

98 mod 7 = 0

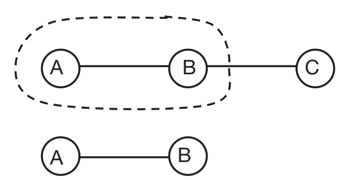
11 mod 7 = 4

74 mod 7 = 4

98	0
74	1
37	2
38	3
72	4
68	5
11	6

Choice (A)

13.



It is a complete sub graph, with $K = 2$. Choice (A)

14. Heapify routine takes $O(\log n)$ time, if it is invoked 'n' times then Build Heap takes $n * \log n \Rightarrow O(n \log n)$.

Choice (D)

15. Given Recurrence Relation:

$T(n) = T(n - 2) + 1$

Assume $n = 16$

$T(16) = T(14) + 1$

$T(14) = T(12) + 1$

$T(12) = T(10) + 1$

$T(10) = T(8) + 1$

$T(8) = T(6) + 1$

$T(6) = T(4) + 1$

$T(4) = T(2) + 1$

$T(2) = 1$

$T(4) = 2$

$T(6) = 3$

$T(8) = 4$

$T(10) = 5$

$T(12) = 6$

$T(14) = 7$

$$T(16) = 8$$

$$\Rightarrow \frac{n}{2} \Rightarrow O\left(\frac{n}{2}\right) \Rightarrow O(n)$$ Choice (B)

16. Each iteration of the inner loop is quadratic in the outer loop variable. The simplest way to do this is to realize we are just summing Si^2, which will just be $O(n^3)$, if we use the integration trick. Choice (C)

17. Let us assume some value for n

$n = 5$

I. $(3 \times 5)! = 15! =$ very large number

$(5!)^3 = 120^3 = 1728000$

$(3n)! < O(n!^3)$

I–false

II. $\log(n!) = \theta(n \log n)$

lets take $n = 8$

$\log(n!) = \log(8!) = \log(40320) = 15.299$

$n \log n = 24$

$\log(n!) < (n \log n)$

II–false Choice (A)

18. The Floyd-warshall all pairs shortest path algorithm in a graph is an example of a dynamic programming formulation.

Choice (B)

19. Option (A)

$n + 2000n \leq c * n^2$

$\Rightarrow \quad n \leq c * n^2$ (TRUE)

Option (B)

$n^{1.999} \leq c * n^2$ (TRUE)

Option (C)

$10^6 n + 2^6 n \leq c * n^2$

$n \leq c * n^2$ (TRUE)

Option (D)

$\frac{n^3}{\sqrt{n}} \leq c * n^2$ (False)

$n^{2.5} \leq c * n^2$(False)

$\therefore$ for a very large value of 'n'

Choice (D)

20. str1 = A B R A C A D A B R O A

str2 = Y A B B A D A B B A D O O B A

The longest common subsequence would be A B A D A B O A.

The length is '8'. Choice (D)

21. In the outer for loop, the variable 'i' keeps halving, so it goes a round ($\log n$) times. For each 'i' next loop goes round also ($\log n$) times, because of doubling the variable j. The inner most loop by 'k' goes round $\frac{n}{2}$ times. Loops are nested, so the bounds may be multiplied to give that the algorithm is $O(n * \log n * \log n)$

Choice (D)

22. By analogy, we divide the elements instead of 2 subsets, we divide them into three subsets.

$$\therefore \quad T(n) = T\left(\frac{n}{3}\right) + c$$ Choice (B)

23. logarithmic $\rightarrow \log n$

cubic $\rightarrow n^3$

quadratic $\rightarrow n^2$

Exponential $\rightarrow 2^n$

Factorial $\rightarrow n!$

$\log n < n^2 < n^3 < 2^n < n!$

Logarithmic, quadratic, cubic, exponential, Factorial.

Choice (A)

24. The procedure given in problem performs linear search,

Worst case $\Rightarrow O(n)$

Best case $\Rightarrow O(1)$

We do not require extra space to search for an element in an array, so space complexity is $O(1)$. Choice (B)

25. Quick sort, selection sort, insertion sort are 'In-place' algorithms means they do not take extra space.

$\therefore O(1), O(1), O(1)$

Merge sort is Not In-Place algorithm.

It needs extra space

$\therefore O(n)$. Choice (C)

Algorithms Test 4

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Suppose 'G' is an undirected graph, Then which of the following is TRUE?

I. If G is a tree, there is a unique path between any 2 vertices in G.

II. If $G = (V, E)$ is connected and $E = V - 1$, then G is a tree.

III. Deleting an edge from a cycle cannot disconnect a graph.

(A) Only II (B) Only III
(C) Only I, II (D) I, II and III

2. Consider an array with 'n' values, to return the maximum sum of 3 values in an array. What is the time complexity of performing this task?

(A) $O(n)$ (B) $O(n^2 \log n)$
(C) $O(n^2)$ (D) $O(\log n)$

3. What is the length of longest increasing subsequence of '6, 4, 5, 2, 7, 11, 8, 12, 13, 9, 10'?

(A) 2 (B) 3
(C) 4 (D) 5

4. Consider 2 sequences, S = gacggattag, and X = gatcggaatag. What is the length of Longest common subsequence?

(A) 7 (B) 8
(C) 9 (D) 11

5. How does the key in a node compared to the keys of its children in

I. Binary search tree

II. A Max HEAP

P. node. key > node.left.key, node.right.key

Q. node.left.key < node.key < node.right.key

Which of the following is correct?

(A) I–P, II–Q (B) I–P, II–P
(C) I–Q, II–P (D) I–Q, II–Q

6. A hash table has 11 slots, uses the hash function $h(k) = k$ mod 11 and collisions are resolved by separate chaining, what is the minimum chain length in the hash table, after inserting these elements:

3, 43, 8, 11, 14, 25

(A) 0 (B) 1
(C) 2 (D) 3

7. Which priority queue implementations, with N elements allow for new entries to be inserted in $O(1)$ time?

(A) Sorted array (B) MAX HEAP
(C) MIN HEAP (D) Unsorted array

8. In order to search for an element in a dynamic set, which of the following techniques is the asymptotically most time efficient in the worst case for the search operation?

(A) Store the element in an unsorted array and apply linear search.
(B) Store the element in a hash table and use hashing.
(C) Store the element in a sorted array and apply binary search.
(D) All the above

9. Which of the following problems have solutions (algorithms) that run in $\theta(n)$ time in the worst case?

(A) Finding the median of 'n' integers
(B) Finding the sum of 'n' integers
(C) Finding the largest of 'n' integers
(D) All the above

10. Consider the following: (Here $h(\ )$ is a hash function).

I. $h(k_1) = h(k_2)$ even for $k_1 \neq k_2$

II. $h(k_1) \neq h(k_2)$ for $k_1 < k_2$ always

III. $h(k_1) = h(k_2)$ for $k_1 > k_2$ always

Which of the following is TRUE?

(A) I only (B) I and II
(C) II and III (D) I, II and III

11. Let 'OPT' be an optimal solution and 'x' the solution we found.

If OPT equals x then we are done otherwise, we can find another optimal solution that 'agrees more with x' which of the following algorithm has in first attempt 'OPT = x'?

(A) Fractional knapsack
(B) Travelling sales person problem
(C) Kruskals algorithm
(D) None of the above

12. Match the following algorithm with its time complexity.

I	Dijkstras algorithm	P.	$O(E + V) \log V$
II	Prims algorithm	Q.	$O(E + V)$
III	Breadth first search		
IV	Depth first search		

(A) I–P, II–P, III–P, IV–Q
(B) I–P, II–P, III–Q, IV–Q
(C) I–Q, II–Q, III–P, IV–P
(D) I–Q, II–P, III–P, IV–Q

13. The distance matrix of a graph with vertices A, B, C, and D is given below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & \infty & \infty \\ \infty & 0 & 2 & 4 \\ 3 & \infty & 0 & 1 \\ 1 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

The shortest path from B to D consists of edge(s) ________.

(A) BC and CD
(B) BD

(C) *AB* and *CD*
(D) There is no shortest path

14. Match the following:

	Sorting algorithm		Programming paradigm
I	Insertion sort	P	Dynamic programming
II	Selection sort	Q	Divide and conquer
III	Merge sort	R	Greedy algorithm
IV	Quick sort	S	Back tracking

(A) I–P, II–R, III–Q, IV–Q
(B) I–P, II–R, III–Q, IV–S
(C) I–R, II–P, III–Q, IV–Q
(D) I–R, II–P, III–Q, IV–S

15. Given a set of objects with (Weight, Profit) pair, and a knapsack of limited weight capacity (M), find a subset of objects for the knapsack to maximize total profit 'p'.
Objects (Weight, Profit)
$$= \{(3, 2), (4, 3), (10, 21), (6, 4)\}$$
$$M = 9 \text{ (using 0/1 knapsack)?}$$
(A) 5 (B) 6
(C) 21 (D) 7

16. There are multiple ways to order the multiplication of 4 matrices A, B, C, D: $(A(BC)D)$, $A(B(CD))$, $(AB)(CD)$, $((AB)C)D$, $A((BC)D)$.
Efficiency depends on number of scalar multiplications, in the case of $(A(BC))D$ it is
$$1 \times 4 \times 3 + 5 \times 1 \times 3 + 5 \times 3 \times 6 = 117$$
In the case of $(A(B(CD)))$, it is
$$4 \times 3 \times 6 + 1 \times 4 \times 6 + 5 \times 1 \times 6 = 126$$
What are the dimensions of A, B, C, D respectively?
(A) 5×1, 1×4, 4×6, 6×3
(B) 4×3, 3×5, 5×6, 6×1
(C) 5×1, 1×4, 4×3, 3×6
(D) 1×4, 4×3, 3×5, 5×6

17. Suppose that the symbols a, b, c, d, e occur with frequencies $\frac{1}{36}, \frac{1}{36}, \frac{1}{12}, \frac{1}{9}, \frac{5}{36}$ then what is the Huffman Encoding of the alphabets a, b, c respectively?
(A) 1101, 111, 1101 (B) 1100, 1101, 111
(C) 1100, 10, 0 (D) 1101, 1100, 111

18. We are given 'n' positive integers $a_1, \ldots, a_n$. The goal is to select a subset of the numbers with maximal sum such that no three consecutive numbers are selected.
Sequence is '7 5 6 3 8 12 9 13 14 10 11'
What is the maximal sum?
(A) 66 (B) 68
(C) 69 (D) 72

19. Consider the coin change problem with coin values 1, 5, 6, Does the greedy algorithm always find an Optional solution?
(A) No, for even numbers
(B) Yes, for odd numbers
(C) Yes, for even numbers
(D) No

20. If a data structure supports an operation 'foo' such that a sequence of 'n' foo operations take $\theta(n \log n)$ time to perform in the worst case, then the amortized time of a 'foo' operation is θ(____)
(A) $\theta(n)$ (B) $\theta(\log n)$
(C) $\theta(\log n)$ (D) $\theta(n)$

21. Consider the following statements:
I. Computing the median of 'n' elements takes $\Omega(n \log n)$ time for any algorithm working in the comparison based model.
II. Let 'T' be a minimum spanning tree of G, then for any pair of vertices 'a' and 'b' the shortest path from 'a' to 'b' in G is the path from 'a' to 'b' in T.
Which of the following is CORRECT?
(A) I–TRUE, II–False (B) I–TRUE, II–TRUE
(C) I–False, II–TRUE (D) I–False, II–False

22. Below are the contents of an array of some sorting algorithm sorting it.
Identify the algorithm from given steps:
12 39 2 94 23 77 52 9
12 $\underline{39}$ 2 94 23 77 52 9
$\underline{2}$ 12 39 94 23 77 52 9
2 12 39 $\underline{94}$ 23 77 52 9
2 12 $\underline{23}$ 39 94 77 52 9
2 12 23 39 $\underline{77}$ 94 52 9
2 12 23 39 $\underline{52}$ 77 94 9
2 $\underline{9}$ 12 23 39 52 77 94
(A) Quick sort (B) Selection sort
(C) Insertion sort (D) Bubble sort

23. Below are the contents of an array of some sorting algorithm sorting it.
Identify the algorithm from given steps:
12 39 02 94 23 77 52 09
1$\underline{2}$ 0$\underline{2}$ 5$\underline{2}$ 2$\underline{3}$ 9$\underline{4}$ 7$\underline{7}$ 3$\underline{9}$ 0$\underline{9}$
$\underline{0}$2 $\underline{0}$9 $\underline{1}$2 $\underline{2}$3 $\underline{3}$9 $\underline{5}$2 $\underline{7}$7 $\underline{9}$4
(A) Selection sort (B) Heap sort
(C) Bubble sort (D) Radix sort

24. Suppose that we have possible keys $[0, 1, \ldots, n^2 - 1]$ for a hash table of size 'n', what is the greatest number of distinct keys the table can hold with each of the following separate chaining, Linear probing, Quadratic probing, collision resolution strategies respectively?
(A) n, n, n^2 (B) n^2, n, n^2
(C) n^2, n, n (D) n^2, n^2, n^2

25. Given these 2 states of an array, the second one results from the first pass of which sorting algorithm?

14	5	21	4	16	31	17	8	11

5	14	4	16	21	17	8	11	31

(A) Insertion sort (B) Selection sort
(C) Merge sort (D) Bubble sort

Answer Keys

1. D	**2.** C	**3.** B	**4.** C	**5.** C	**6.** A	**7.** D	**8.** B	**9.** D	**10.** A
11. C	**12.** B	**13.** A	**14.** A	**15.** B	**16.** C	**17.** B	**18.** C	**19.** D	**20.** C
21. D	**22.** C	**23.** D	**24.** C	**25.** D					

Hints and Explanations

1. All the statements are TRUE. Choice (D)

2. First sort the given array with 'n' values. In the worst case sorting takes $O(n^2)$ time.
Last '3' values in the array can be added in constant time. Choice (C)

3. 6 4 5 $\boxed{2\ 7\ 11}$ $\boxed{8\ 12\ 13}$ 9 10
Longest increasing subsequence length is 3
(i) 2 7 11 (ii) 8 12 13. Choice (B)

4.
S = <u>gac</u> <u>gga</u> t <u>tag</u>
X = <u>ga</u> t c <u>gga</u> a <u>tag</u>
LCS = {g a c g g a t a g}
Length = 9. Choice (C)

5. **Binary search tree:**
node · left. key < node. key < node × right. key
max heap:
node. key > node. left . key, node. right. key.
Choice (C)

6. Hash function is $h(k) = k \bmod 11$
3 mod 11 = 3
43 mod 11 = 10
8 mod 11 = 8
11 mod 11 = 0
14 mod 11 = 3
25 mod 11 = 3
The minimum chain length is '0', from the hash values it is clear that, the chains with slot numbers 1, 2, 4, 5, 6, 7, 9 are empty.
We do not need to construct a hash table to check the chain lengths. Choice (A)

7. To insert an element in a sorted array we need to check for the correct position of the element. In the worst case it will take 'n' comparisons.
In max-heap and min-heap, we need (log n) time to insert an element.
In unsorted array simply keep the element at the end of the array. Choice (D)

8. Searching an element in a hash table using hashing takes constant time usually, it is the efficient search method, but its implementation depends on hash function, table size etc. Choice (B)

9.
- Median of 'n' integers can be finded by traversing the entire list. It takes θ(n) time.
- Sum of 'n' integers take $(n - 1)$ computation which requires $\theta(n)$ time.
- For finding largest of 'n' number it requires $(n - 1)$ comparisions which requires $\theta(n)$ time.

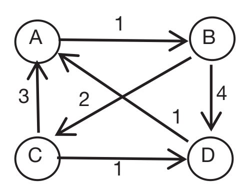
Choice (D)

10. Suppose $h(k) = k \bmod 5$
$k_1 = 10, k_2 = 15$
I. I–TRUE
II. $k_1 = 6, k_2 = 11$
6 mod 5 = 1, 11 mod 5 = 1
II–False
III. $k_1 = 16, k_2 = 14$
16 mod 5 = 1, 14 mod 5 = 4
III–false. Choice (A)

11. Kruskals algorithm is used to construct a minimum spanning tree, in the first attempt itself we will get optimal solution. Choice (C)

12. Dijkstras Algorithm and prims algorithm implementation is similar.
Time complexity is $O(E + V) \log V$
Depth first search and Breadth first search has same time complexity $O(E + V)$. Choice (B)

13. From the given matrix, the graph is drawn



Shortest path from B to $D \Rightarrow = (\overline{BC} - \overline{CD})$
$= 2 + 1 = 3$. Choice (A)

14.
- Insertion sort is a quadratic time sorting algorithm. It is an example of dynamic programming.
- Selection sort is a quadratic time sorting algorithm, It is an example of greedy algorithm.
- Merge sort and Quick sort are examples of Divide and conquer approach. Choice (A)

15. $M = 9$
(i) Weight $= 3 + 4 \le 9$
Profit $= 2 + 3 = 5$
(ii) Weight $= 3 + 6 \le 9$
Profit $= 2 + 4 = 6$. Choice (B)

16. $(A\ (B\ (CD))) = 4 \times 3 \times 6 + 1 \times 4 \times 6 + 5 \times 1 \times 6$
First $(CD) = 4 \times 3 \times 6$
$[C]_{4\times3} \times [D]_{3\times6} = [\text{Res } 1]_{4\times6}$
$(B \text{ Res } 1) = 1 \times 4 \times 6$
$[B]_{1\times4}\ [\text{Res } 1]_{4\times6} = [\text{Res } 2]_{1\times6}$
$(A \text{ Res } 2) = 5 \times 1 \times 6$
$[A]_{5\times1}\ [\text{Res } 2]_{1\times6} = [\text{Res } 3]_{5\times6}$
$\therefore \quad A = 5 \times 1,$

$B = 1 \times 4,$
$C = 4 \times 3,$
$D = 3 \times 6.$ Choice (C)

17. $a = \frac{1}{36} = 0.02$

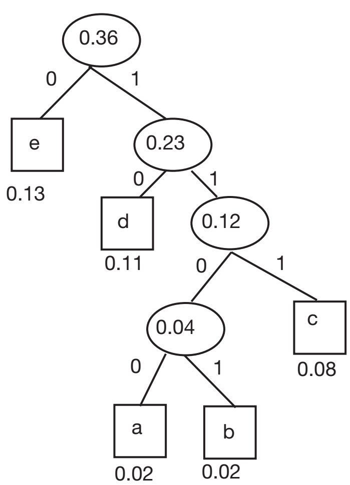
$b = \frac{1}{36} = 0.02$

$c = \frac{1}{12} = 0.08$

$d = \frac{1}{9} = 0.11$

$e = \frac{5}{36} = 0.13$

Huffman tree:



Huffman codes
$a = 1100$, $b = 1101$, $c = 111$, $d = 10$, $e = 0$. Choice (B)

18. $\boxed{7\ 5}$ 6 $\boxed{3\ 8}$ 12 $\boxed{9\ 13}$ 14 $\boxed{10\ 11}$
$12 + 11 + 22 + 21 = 66$
7 $\boxed{5\ 6}$ 3 $\boxed{8\ 12}$ 9 $\boxed{13\ 14}$ 10 $\boxed{11}$
$11 + 20 + 27 + 11 = 69.$ Choice (C)

19. Coin values 1, 5, 6.
Value = 20

(i) $\frac{\text{Greedy Algorithm}}{6+6+6+1+1}$

$\Rightarrow$ 5 coins

(ii) $\frac{\text{Brute-Force}}{5+5+5+5}$

$\Rightarrow$ 4 coins
Value = 21

(iii) $\frac{\text{Greedy Algorithm}}{6+6+6+1+1+1}$

$\Rightarrow$ 6 coins

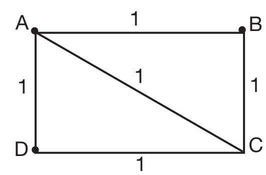
(iv) $\frac{\text{Brute Force}}{5+5+5+5+1}$

$\Rightarrow$ 5 coins. Choice (D)

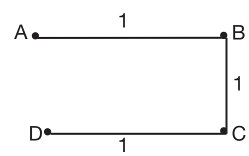
20. Amortized time is $\theta(\log n)$
Worst case time is $\theta(n \log n)$. Choice (C)

21.
- Computing a median of 'n' elements takes $\theta(n)$ time as it has to traverse 'n' elements.
- The shortest path between 2 vertices need not be same in the Minimum spanning tree and in graph.

Consider following graph G



The MST for the above graph 'G' is



The shortest path from A to C is 1 in graph 'G' where as, it is 2 in MST. Choice (D)

22. $\boxed{12\ 39}$ 2 94 23 77 52 9

$\boxed{2\ 12\ 39}$ 94 23 77 52 9

$\boxed{2\ 12\ 39\ 94}$ 23 77 52 9

$\boxed{2\ 12\ 23\ 39\ 94}$ 77 52 9

$\boxed{2\ 12\ 23\ 39\ 77}$ 94 52 9

$\boxed{2\ 12\ 23\ 39\ 77}$ 52 9

2 12 23 39 52 77 94 9

$\therefore$ 2 9 12 23 39 52 77 94

$\therefore$ Insertion sort. Choice (C)

23. Given elements 12 39 02 94 23 77 52 09
First sort elements based on least significant digit

12 02 52 23 94 77 39 09.

Sort the above elements based on second least significant digit.

02 09 12 23 39 52 77 94

$\therefore$ The above sorting algorithm is 'Radix–Sort'.
Choice (D)

24. The available elements are $[0, 1, 2, ..., n^2 - 1]$
There are n^2 elements.
Separate chaining has unlimited space, it can save all the n^2 elements.

Linear probing:
It will have 'n' slots, so maximum we can store 'n' values.
Quadratic probing:
Like linear probing, it will also have 'n' slots.
We can store only 'n' values.

Choice (C)

25. Bubble sort, compares adjacent elements, if not in order swaps them.

14	5	21	4	16	31	17	8	11

5 14 21 4 16 31 17 8 11

5 14 4 21 16 31 17 8 11

5 14 4 16 21 31 17 8 11

5 14 4 16 21 17 31 8 11

5 14 4 16 21 17 8 31 11

5 14 4 16 21 17 8 11 31.

Choice (D)

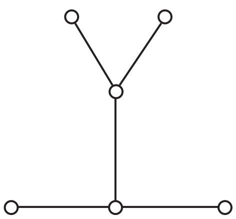
Algorithms Test 5

Number of Questions: 25 **Section Marks: 30**

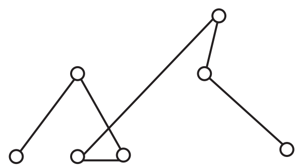
Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following is NOT a tree?

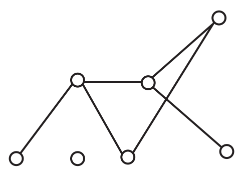
(A)



(B)



(C)



(D) None of the above

2. What is the number of vertices in a tree with 57 edges?

(A) 58 (B) $2^6 - 4$
(C) 56 (D) 57

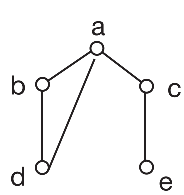
3. Consider 2 graphs G_1 and G_2:

I. G_1 is connected and every edge is a bridge.
II. In G_2 for any pair of vertices in the graph there is one and only one path joining them.

Which of the following is TRUE?

(A) G_1 is a tree but G_2 is not a tree
(B) G_1 is not a tree but G_2 is a tree
(C) Both G_1 and G_2 are trees
(D) Both G_1 and G_2 are not trees

4. How many spanning trees does the following graph contain, (Assume that all edges have same weight)?



(A) 2 (B) 3
(C) 4 (D) 5

5. Suppose we have a graph where each edge value appears atmost twice, then what will be the number (atmost) of minimum spanning Trees of this graph?

(A) 2 (B) 3
(C) 4 (D) 6

6. If we run Dijkstra's algorithm starting from 'S' to find shortest path 'T', consider the following statements:

I. Dijkstra's algorithm returns shortest path with minimum total weight.
II. Dijkstra's algorithm returns shortest path, with minimum number of edges.

Which of the following is TRUE?

(A) I only (B) II only
(C) Both I and II (D) None of the above

7. The reason for using Bellman-Ford algorithm for finding the shortest path between 2 vertices in a graph, when Dijkstras algorithm does the same thing:

I. Bellman-Ford Algorithm is faster than Dijkstra's Algorithm.
II. Bellman-Ford Algorithm works on any directed graph, with negative weights also.

Which of the following is TRUE?

(A) I only (B) II only
(C) Both I and II (D) None of these

8. Consider the following statements:

I. By applying Breadth First search, on a tree with all edges having equal weight; finds minimum distance from root to any node.
II. Use of greedy algorithm to solve the knapsack with fractions is optimal.

Which of the following is TRUE?

(A) I only (B) II only
(C) Both I and II (D) None of the above

9. Consider the set of keys {1, 4, 5, 10, 16, 17, 21}, Draw a binary search tree with height '2,' Then what values will appear at internal nodes? [height of root node is '0']

(A) 10, 5, 16 (B) 10, 4, 17
(C) 4, 17, 21 (D) 5, 10, 17

10. Consider the set of keys {2, 5, 6, 11, 17, 18, 22}, Draw a binary search tree with height '6', then, what will be the child node values (height of root node is 0).

(A) 2 (B) 22
(C) Either 2 or 22 (D) None of the above

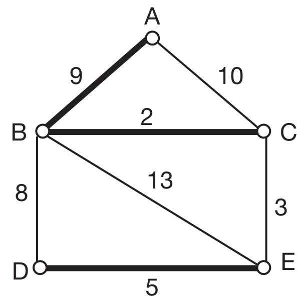
11. The following is a directed graph

$G = (V, E)$:
$V = \{a, b, c, d, e, f, g\}$
$E = \{(a, b), (c, a), (b, c), (c, d), (d, e), (e, f), (e, g), (f, g), (d, g)\}$

Identify the correct strongly connected components, which of the following is TRUE?

(A) There are 3 strongly connected components.
(B) There are 4 strongly connected components.
(C) There are 5 strongly connected components.
(D) None of the above

12. Consider the given graph, with some edges in bold:



The bold edges given in the above graph cannot form a spanning Tree because

(A) The bold edges are not connected
(B) The bold edges are not least weight edges
(C) The kruskals algorithm is not implemented on the above graph.
(D) Both (A) and (B)

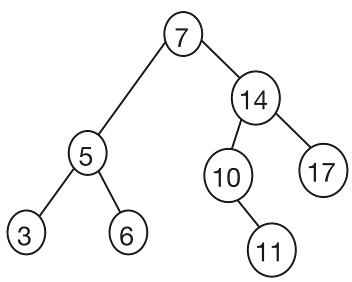
13. Consider the given statements:

I. A digraph is a graph with exactly 2 vertices.
II. A spanning tree of a graph must contain atleast $\frac{n}{2}$ edges always.
III. The sorted edges algorithm for solving the travelling salesman problem always gives optimal result.

Which of the following is TRUE?

(A) I and II (B) II and III
(C) I and III (D) Only II

14. Consider the given AVL-tree,



If the element with value '12' is inserted into above AVL tree, How many rotations are required to balance the tree?

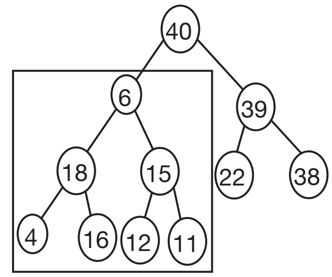
(A) 0 (B) 1
(C) 2 (D) 3

15. In which order, the element must be inserted into an AVL tree, so that, no single Rotation is required, for the given

Elements = {1, 2, 3, 4, 5, 6, 7}

(A) {4, 2, 6, 1, 3, 5, 7}
(B) {4, 2, 1, 3, 5, 6, 7}
(C) {7, 6, 5, 4, 2, 1, 3}
(D) {4, 1, 2, 3, 5, 6, 7}

16. Consider the given tree:



The MAX HEAP property is applied on the above tree, (box area), at node with value '6', which of the following is BEST suitable?

(A) First compare 6 and 18 then swap them if required
(B) First compare 6 and 15 then swap them if required.
(C) First compare 18 and 15, then compare the greater element with '6' and swap them if required.
(D) Any one of the above

17. Consider the following statements about Depth First traversal:

I. Suppose we run DFS (Depth First search) on an undirected graph and find exactly 15 back edges. Then the graph is guaranteed to have at least one cycle.
II. DFS on a directed graph with 'n' vertices and at least 'n' edges is guaranteed to find at least one back edge.

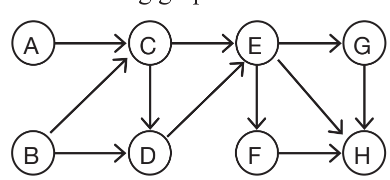
Which of the following is TRUE?

(A) I only (B) II only
(C) Both I and II (D) None of the above

18. Suppose 'G' is a connected, undirected graph, whose edges have positive weights. Let M be a minimum spanning Tree of this graph. We modify the graph by adding '6' to the weight of each edge, which of the following is TRUE?

(A) The order of edges added to minimum spanning tree using kruskal's algorithm, will change.
(B) The modification adds $6(|V| - 1)$ to the total weight of all spanning trees.
(C) The order of edges added to minimum spanning tree using prim's algorithm, will change.
(D) None of the above

19. Consider the following graph:



Which of the following orderings not a valid topological sort of the graph?

(A) BACEDFGH (B) ABCDEFGH
(C) ABCDEGFH (D) BACDEFGH

20. What is the time complexity to print all the keys of a binary search tree in sorted order?

(A) $O(\log n)$ (B) $\theta(n)$
(C) $\theta(n^2)$ (D) $O(n + \log n)$

21. Let T_1, T_2, T_3, T_4, T_4, T_5, T_6 are tasks given along with their deadlines and profits:

Tasks	T_1	T_2	T_3	T_4	T_5	T_6
Deadlines	2	1	3	2	3	1
Profit	6	4	3	7	2	8

Which of the following tasks are not completed?

(A) T_6, T_4, T_3 (B) T_6, T_1, T_2
(C) T_1, T_2, T_5 (D) T_4, T_5, T_6

22. Consider the below program fragment:

```
for i ← 1 to length (A) - 1
{
element ← A [i]
pos ← i
while pos > 0 and A [pos - 1]
{
    A[pos] ← A[pos - 1]
    pos ← pos -1
{
A[pos] ← element.
}
```

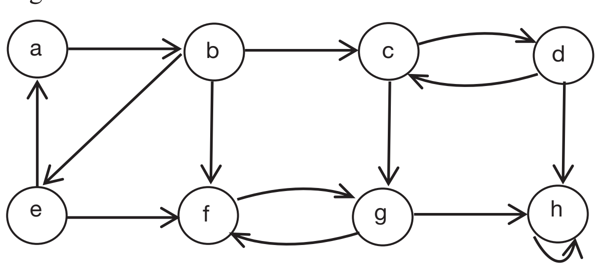
This code implements

(A) Bubble sort (B) Insertion sort
(C) Heap sort (D) Bucket sort

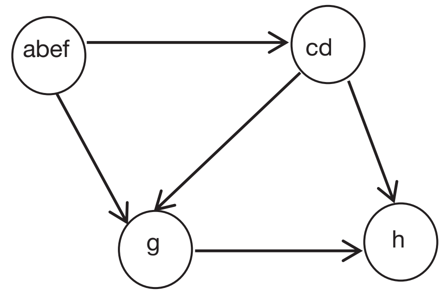
23. What is the best case time complexity of given algorithm in Q. 22?

(A) $\theta(n^2)$ (B) $\theta(\log n)$
(C) $\theta(n)$ (D) $\theta(n \log n)$

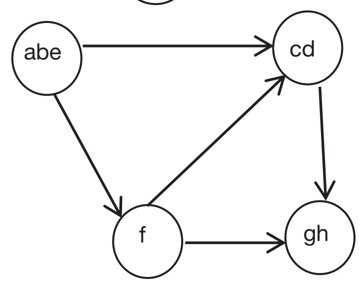
24. Which of the following is an acyclic component graph of given graph using strongly connected components algorithm?



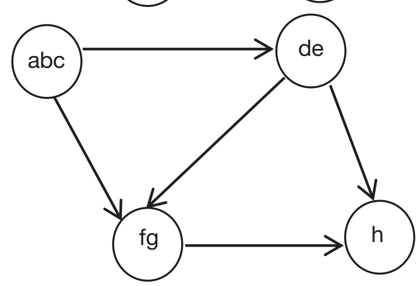
(A)



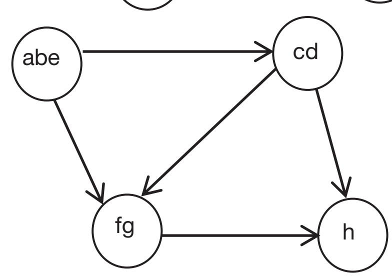
(B)



(A)



(D)



25. Consider an undirected graph

$G = (V, E)$
$V = \{r, s, t, u, v, w, x, y\}$
$E = \{(r,s), (r,v), (s,w), (w,t), (w,x), (t,u), (t,x), (u,y), (u,x)\ (x,y)\}$

Which node has the highest degree?

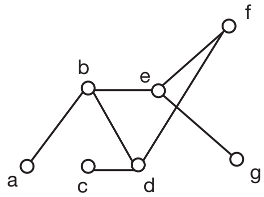
(A) w (B) x
(C) u (D) t

Answer Keys

1. C	**2.** A	**3.** C	**4.** B	**5.** C	**6.** A	**7.** B	**8.** C	**9.** B	**10.** C
11. C	**12.** A	**13.** D	**14.** B	**15.** A	**16.** C	**17.** A	**18.** B	**19.** A	**20.** B
21. C	**22.** B	**23.** C	**24.** D	**25.** B					

Hints and Explanations

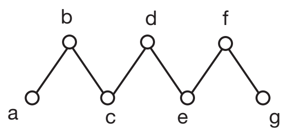
1. A graph without any cycles is called a Tree graph.
Option (C)



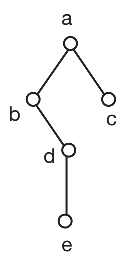
There is a cycle ($b - e - f - d - b$) in the graph.
Choice (C)

2. A tree with 'n' vertices contains (n – 1) edges.
$n - 1 = 57$
Number of vertices, $n = 58$ Choice (A)

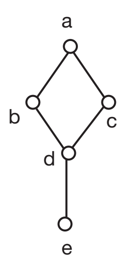
3. **G_1: Example:**



∴ No cycles, It is a tree
G_2: Example:

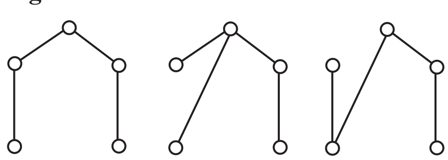


∴ No cycles,
If we have cycle



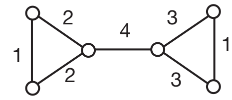
There will be different paths from ($a - e$)
(i) $a - b - d - e$
(ii) $a - c - d - e$
∴ G_2 is also a tree. Choice (C)

4. **Spanning trees:**

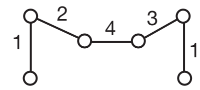


Choice (B)

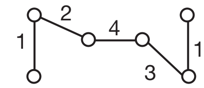
5. Assume the following graph:



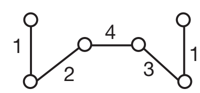
Spanning Tree 1:



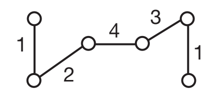
Spanning Tree 2:



Spanning Tree 3:



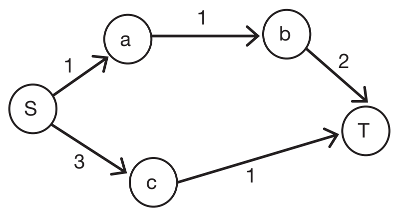
Spanning Tree 4:



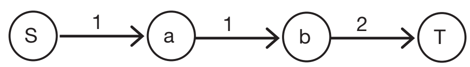
Atmost 4 minimum spanning trees are possible.
Choice (C)

6. I. Dijkstra's algorithm always returns shortest path with minimum total weight.

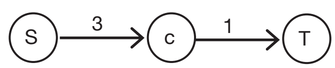
II.



Dijkstra's Algorithm gives, shortest path (length = 3)



But the shortest path

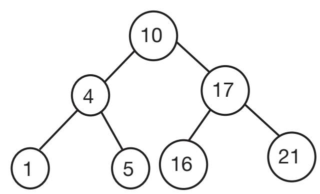


has length = 2
I–TRUE, II–False Choice (A)

7. Dijkstra's is faster than Bellman-Ford, but It cannot work on graphs with negative weight edges.
Choice (B)

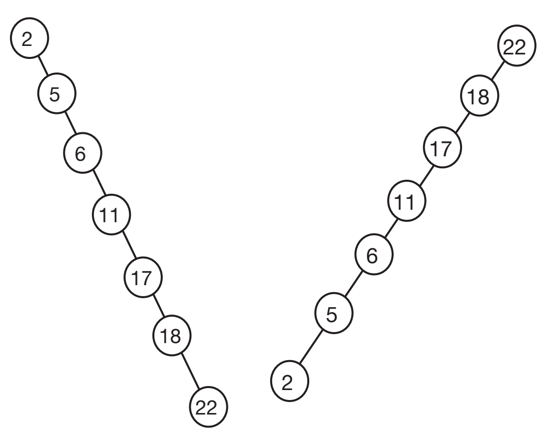
8. Applying BFS on a tree with equal weights results the same tree and it gives minimum distance from root to other nodes. Knapsack fractional problem is optimally solved using greedy design strategy. Choice (C)

9. 1, 4, 5, 10, 16, 17, 21



With height = 2, Internal nodes are 10, 4, 17.
Choice (B)

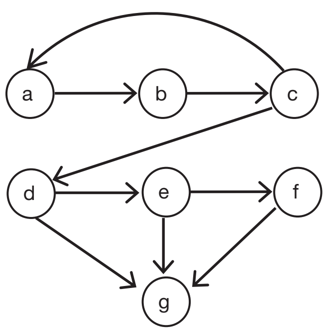
10.



Tree 1: Child 22 Tree 2: Child 2

Choice (C)

11.



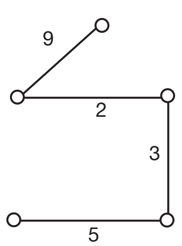
Strongly connected components are:

$\{abc\}$ $\{d\}$ $\{e\}$ $\{f\}$ $\{g\}$

There are five strongly connected components.

Choice (C)

12. Kruskals algorithm is been implemented on the given graph. The bold edges are least weight edges, because the complete spanning tree will contain following edges:



Choice (A)

13. I. Digraph:

There is no limit on number of vertices, In digraph edges will have directions.

I–False

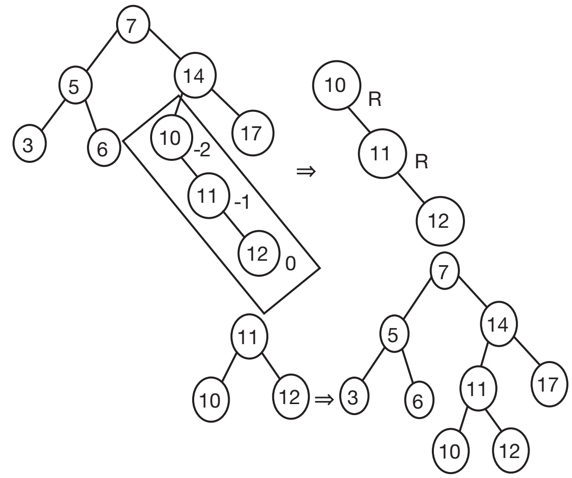
II. Example: **Spanning tree**

II–TRUE

III. There is no optimal solution for Travelling sales person problem.

$\therefore$ Only II is TRUE Choice (D)

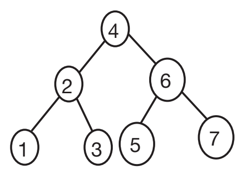
14.



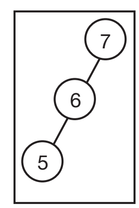
RR–Imbalance: perform Left Rotation.

$\therefore$ 1 Rotation. Choice (B)

15. Option (A):

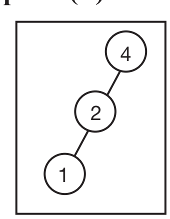


Option (C):



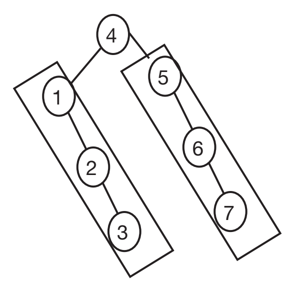
requires LL rotation
for 1st 3 elements

Option (B): **Option (D):**



requires LL rotation

for 1st 3 elements



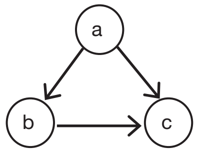
If we have Imbalance, we need to perform Rotation.

Option (A) has no Imbalance. Choice (A)

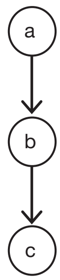
16. First compare the children of a node. If it is a max heap, then compare the child with greater value to parent node, if required swap them. Choice (C)

17. I. If the graph has a back edge, then it has a cycle.

II.



DFS on the above graph starting at '*a*' does not find any back edge.



∴ I–TRUE, II–False Choice (A)

18. The order of edges added to minimum spanning tree does not change, because we are adding same weight to all the edges.
To the total weight we have to add $6(V-1)$, because in a spanning tree, there will be $(V-1)$ edges, For every edge additional weight '6' is added. Choice (B)

19. **Option (A):**
BACEDFGH
There is an edge from $D \to E$
Where as in the order E appears first and D appears next. Which is invalid.
Check the other options in the same manner.
Choice (A)

20. We can print all the keys of a binary search tree in sorted order using inorder traversal which will take $\theta(n)$ time. Choice (B)

21. Arrange the given tasks according to their profits

Tasks	*T*6	*T*4	*T*1	*T*2	*T*3	*T*5
Profit	8	7	6	4	3	2
Deadline	1	2	2	1	3	3

Order of execution of tasks to maximize the profit

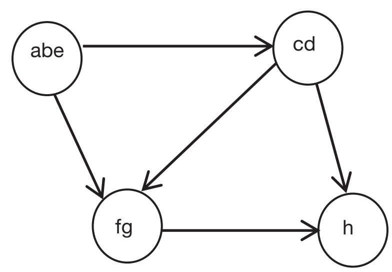
	8	7	3		
	T6	T4	T3		
0	1	2	3	4	5

The uncompleted tasks are *T*1, *T*2, *T*5
Choice (C)

22. The given code implements insertion sort. In insertion sort, every iteration removes an element from the input data and insert it into the correct position in the already sorted list until no input elements remain. Choice (B)

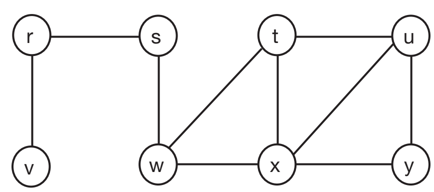
23. The best case input is when the array is already in sorted order. Then time complexity is $\theta(n)$ because in each iteration, the first remaining element of input is only compared with the right-most element of sorted list of the array. Choice (C)

24. Strongly connected components of a directed graph $G = (V, E)$ is a maximal set of vertices $C \subseteq V$ such that for every pair of vertices u and v in C, vertices u and v are reachable from each other. In the given graph the connected components are $\{a, b, e\}$, $\{f, g\}$, $\{c, d\}$ and $\{h\}$.
Hence, the required acyclic component graph will be



Choice (D)

25.



Highest degree node = x(degree = 4) Choice (B)

Chapter 1

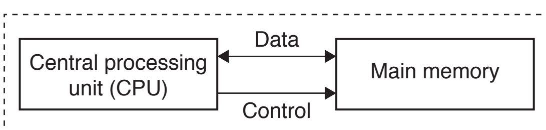
Machine Instructions, Addressing Modes

LEARNING OBJECTIVES

- *Computer*
- *Computer system*
- *Computer component*
- *Machine instruction*
- *Instruction types*
- *Types of operands*
- *Types of operations*
- *Procedure call instruction*
- *Addressing modes*
- *Computer performance*

Computer

A computer is a data-processing machine which is operated automatically under the control of a list of instructions (called a program) stored in its main memory.



Computer System

- A computer system consists usually of a computer and its peripherals.
- Computer peripherals include input devices, output devices and secondary memories.

Computer Architecture: Computer Architecture refers to those attributes of a system visible to a programmer, i.e., the attributes that have direct impact on the logical execution of a program.

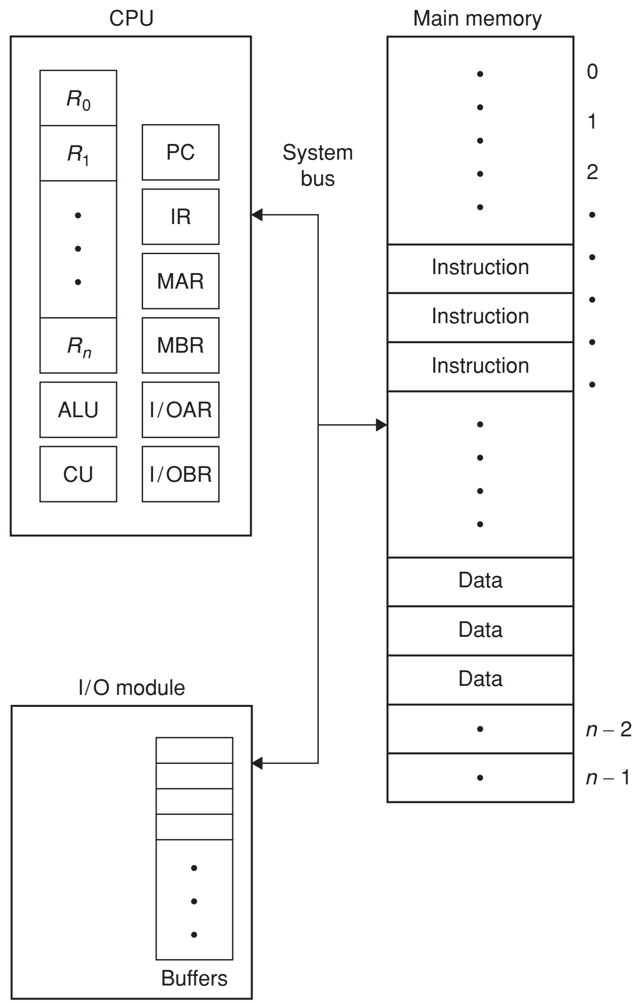
Example: Whether a computer will have a multiply instruction or not.

Computer Organization: Computer organization refers to operational units and their interconnections that realize the architectural specifications.

Example: Whether the multiply instruction will be implemented by a special multiply unit or by a mechanism that makes repeated use of the add unit of the system.

Computer Components

- $R_1, R_2 \ldots R_n$: General Purpose Registers.
- **PC:** Program counter. Holds address of next instruction to be executed. PC = PC + I (I = instruction length)
- **IR:** Instruction Register. It holds the instruction which is fetched from memory.
- **MAR:** Memory Address Register: MAR specifies the address in memory for the next read or write.
- **MBR:** Memory Buffer Register. It contains the data to be written into memory or receives the data read from memory.
- **Input–outputAR:** Input–output Address Register. It specifies a particular input–output device.
- **Input–outputBR:** Input–output Buffer Register. Used for the exchange of data between an input–output module and the CPU.
- **ALU:** Arithmetic and Logic Unit. Used to perform arithmetic and logical operations.



- **CU:** Control Unit. It causes operations to happen within the processor. Also generates timing signals.
- **Memory:** It consists of set of locations, defined by sequential numbered address.
- **Input–output module:** Transfer data from external device to CPU and memory and vice versa.
- **System bus:** A bus that connects major computer components is called a system bus.

Machine Instructions

- The operation of the CPU is determined by the instructions it executes. These instructions are called machine instructions or computer instructions.
- The collection of different instructions that the CPU can execute is referred to as the CPU's instruction set.

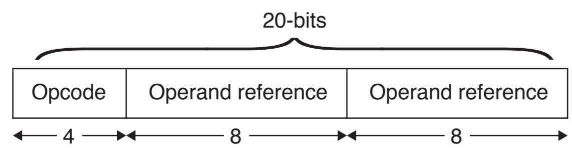
Elements of Machine Instructions

Each instruction must contain the information required by the processor for execution. The elements of a machine instruction are

1. Operation code: Specifies the operation
2. Source operand reference: Inputs for operation.
3. Result operand reference
4. Next instruction reference

Instruction representation

- Each instruction is represented by a sequence of bits.
- Example: 20-bit instruction format:



Instruction Types

Number of addresses

Most of the instructions have one, two or three operand addresses, with the address of next instruction being implicit.

(i) 3-address instructions: Computers with 3-address instruction formats can use each address field to specify either a processor register or a memory operand.

Example: 3-address instruction format for the evaluation of $X = (P + Q) \times (R + S)$ is

ADD R_1, P, Q
ADD R_2, R, S
MUL X, R_1, R_2.

Here R_1, R_2 are processor registers.

Advantage: Shorter programs when evaluating arithmetic expressions.

Disadvantage: The binary coded instructions required too many bits to specify three addresses.

(ii) 2-address instructions: These are most common in commercial computers. Each address field can specify either a processor register or a memory word.

Example: For evaluating $X = (P + Q) \times (R + S)$,

The 2-address instructions are

MOV R_1, P
ADD R_1, Q
MOV R_2, R
ADD R_2, S
MUL R_1, R_2
MOV X, R_1.

(The first symbol of instruction is both source and destination)

(iii) One-address instructions: Use an implied accumulator (AC) register for all data manipulations.

Example: 1-address instructions to evaluate $R = (P + Q) \times (R + S)$.

LOAD P
ADD Q

STORE T
LOAD R
ADD S
MUL T
STORE X.

Here 'T' is a temporary memory location required to store the intermediate result.

(iv) Zero-address instructions: A stack organized computer does not use an address field for the instructions ADD and MUL. The push and pop instructions require an address field to specify the operand that communicates with the stack.

Example: Zero-address instructions for the evaluation of $X = (P + Q) \times (R + S)$

PUSH P
PUSH Q
ADD
PUSH R
PUSH S
ADD
MUL
POP X.

(v) RISC instructions: The instruction set of a reduced instruction set computer (RISC) processor is restricted to the use of load and store instruction when communicating between memory and CPU. All other instructions are executed with in the register of the CPU without referring to memory.

Example: RISC instruction to evaluate,
$X = (P + Q) \times (R + S)$

LOAD R_1, P
LOAD R_2, Q
LOAD R_3, R
LOAD R_4, S
ADD R_1, R_1, R_2
ADD R_3, R_3, R_4
MUL R_1, R_1, R_3
STORE X, R_1

Types of Operands

Machine instructions operate on data. The most important general categories of data are

- Addresses
- Numbers
- Characters
- Logical data.

Types of Operations

The number of different opcodes varies widely from machine to machine. A useful and typical categorization is the following

1. Data transfer
2. Arithmetic
3. Logic
4. Conversion
5. Input–output
6. System control
7. Transfer of control

(i) Data transfer operations: This type of instructions transfers data from one location to another.

Example: move, store, load, exchange, clear, set, push, pop.

(ii) Arithmetic operations: Perform some function in ALU.

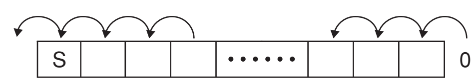
Example: add, subtract, multiply, divide, absolute, negate, increment, decrement

(iii) Logical operations: Perform some logical operation in ALU and set condition codes and flags.

Example: AND, OR, NOT, EX-OR, Test, Compare, set control variables, shift, Rotate.
Let $R_1 = 10100101$, $R_2 = 00001111$ then
(R_1) AND $(R_2) = 00000101$.
AND is also called mask operation.
(R_1) OR $(R_2) = 10101111$.
NOT $(R_1) = 01011010$.
(R_1) EX-OR $(R_2) = 10101010$.

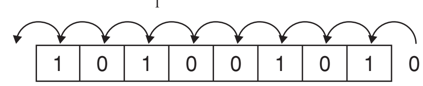
(iv) Shifting and rotating operations: The operations are:

(a) Logical left shift:



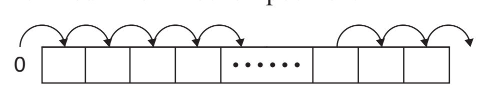
Here the bits of a word are shifted left. The left most bits is lost and 0 is shifted in right most bit position (i.e., bit empty).

Example: $R_1 = 1010\ 0101$
Logical left shift R_1:

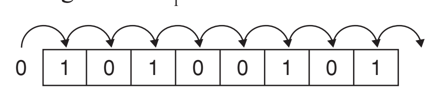


After left shift $R_1 = 0100\ 1010$.

(b) Logical right shift: Here the bits of a word are shifted right. The right most bit lost and '0' is shifted in left most bit position.

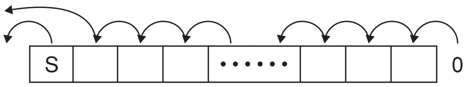


Example: $R_1 = 1010\ 0101$
Logical right shift $R_1 = 0101\ 0010$

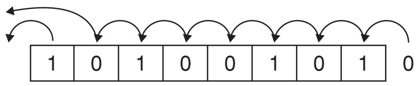


Logical shift operations are useful primarily for isolating fields within a word and also used to displace unwanted information.

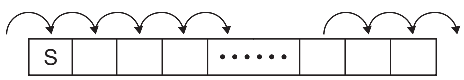
(c) Arithmetic left shift: Arithmetic shift operation treats the data as a signed integer and does not shift the sign bit. In Arithmetic left shift, a logical left shift is performed on all bits but the sign bit, which is retained.



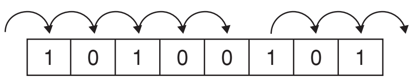
Example: $R_1 = 1010\ 0101$
Arithmetic Left shift $R_1 = 1100\ 1010$.



(d) Arithmetic right shift: Here, the sign bit is replicated into the bit position to its right.



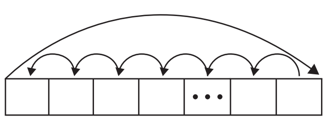
Example: $R_1 = 1010\ 0101$
Arithmetic Right shift $R_1 = 1101\ 0010$



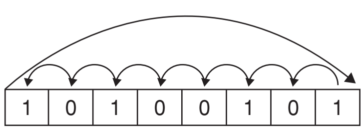
Notes:

1. With numbers in 2's complement form, a right arithmetic shift corresponds to a division by 2, with truncation for odd numbers.
2. Both arithmetic left shift and logical left shift correspond to a multiplication by 2 when there is no overflow.

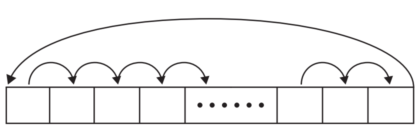
(e) Left rotate (Cyclic left shift): Rotate operations preserve all the bits being operated on. Here the bits from LSB will move one bit position to the left and MSB will placed in LSB position.



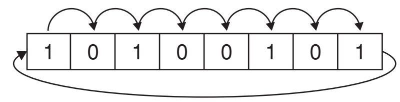
Example: $R_1 = 1010\ 0101$
Left Rotate $R_1 = 0100\ 1011$



(f) Right rotate (Cyclic right shift): Here the bits from MSB will be shifted to one bit position right and LSB is placed in MSB.



Example: R_1: 1010 0101
Right Rotate $R_1 = 1101\ 0010$.



(v) Transfer of control: This type of operations updates the program counter. Used for subroutine call/return, manage parameter passing and linkage.

Example: Jump, jump unconditional, return, execute, skip, skip conditional, Halt, Wait, NOP, etc.

(vi) Input–output operations: These are used to issue a command to input–output module.

Example: input, output, start input–output, test input–output etc.

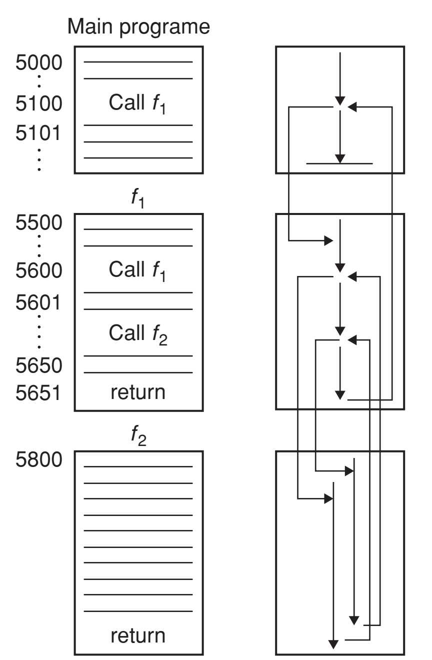
(vii) Conversion operations: These are similar to arithmetic and logical operations. May also involve special logic to perform conversion.

Procedure Call Instruction

A procedure is a self-contained computer program that is incorporated into a large program. It allows us to use the same piece of code many times. The procedure mechanism involves two basic instructions:

1. A call instruction that branches from the present location to the procedure.
2. A return instruction that returns from the procedure to the place from where it was called.

Example:



Call and returns   Execution sequence

We can call a procedure from a variety of points, so the processor must somehow save the return address to return

appropriately. We can store the return address in the following places.

1. Register
2. Start of called procedure
3. Top of stack.

If the register approach is used, call X causes the following actions:

$$RN \leftarrow PC + L$$
$$PC \leftarrow X$$

Where RN is a used to store return address, PC is the program counter and L is instruction length.

To store the return address at the start of the procedure for call X, the following tasks required.

$$X \leftarrow PC + L$$
$$PC \leftarrow X + 1$$

We can pass the parameters using registers or store in memory after call instruction or use stack.

Example 1: Consider the following program fragment in the assembly language of a certain hypothetical processor. The processor has three 8-bit general purpose registers R_1, R_2, R_3.

Instruction	Meaning
X: CMP R_1,0	Compare R_1 and 0, set flags appropriately in status register.
JZ Z	Jump if zero to target Z.
MOV R_2, R_1	Copy contents of R_1 to R_2.
SHR R_1	Shift Right R_1
SHL R_1	Shift left R_1
CMP R_2, R_1	Compare R_2 and R_1 and set flag in status register
JZ Y	Jump if zero to Y.
INC R_3	Increment R_3
Y: SHR R_1	Shift Right R_1 by 1-bit
JMP X	Jump to X
Z: ...	

Let R_1, R_2, R_3 contain the values 3, 0, 0 respectively. What are the final values of R_1, R_2, R_3 when control reaches Z?

(A) 0, 0, 0 (B) 0, 1, 2
(C) 0, 1, 1 (D) 0, 2, 1

Solution: (B)

$R_1 = 0000\ 0011$
$R_2 = 0000\ 0000$
$R_3 = 0000\ 0000$

CMP R_1, 0, As $R_1 \neq 0 \Rightarrow$ Zero flag = 0.
Jump to Z if $Zf = 1$; but $Zf = 0$
MOV R_2, R_1; $R_2 \leftarrow R_1$ i.e., $R_2 = 0000\ 0011$
SHIFT right R_1; $R_1 = 0000\ 0011$
$\Rightarrow$ shr(R_1) = 00000001
SHIFT left R_1; $R_1 = 0000\ 0001 \Rightarrow$ shl (R_1)
$= 0000\ 0010$
Compare R_2, R_1; $R_1 \neq R_2 \Rightarrow ZF = 0$
As Zero flag is not set, increment R_3: $R_3 = 0000\ 0001$
Shift Right R_1; i.e., 0000 0001.
Jump to X.
Compare R_1, 0; As $R_1 \neq 0 \Rightarrow ZF = 0$.
Move R_2, R_1; $R_2 \leftarrow 0000\ 0001$.
Shift right R_1; $R_1 = 0000\ 0000$.
Shift left R_1; $R_1 = 0000\ 0000$.
Compare R_2, R_1; $R_1 \neq R_2 \Rightarrow ZF = 0$.
Increment R_3; $R_3 = 0000\ 0010$.
Shift Right R_1; 0000 0000.
Jump to X.
Compare R_1, 0; As $R_1 = 0 \Rightarrow ZF = 1$
As $ZF = 1$, jump to Z.
$\therefore R_1 = 0$; $R_2 = 1$; $R_3 = 2$.

Addressing Modes

The different ways in which the location of an operand is specified in an instruction are referred to as addressing modes.

Computers use addressing mode techniques for the purpose of accommodating the following provisions:

1. Facilitates pointers to memory.
2. Facilitates counters for loop control
3. Facilitates indexing of data
4. Facilitates program relocation.
5. Reduce the number of bits in the addressing field of the instruction.

The most common addressing techniques are

i. Implied mode
ii. Immediate mode
iii. Direct mode
iv. Indirect mode
v. Register mode
vi. Register Indirect mode
vii. Auto-increment or Auto-decrement mode
viii. Displacement mode
  - PC relative mode
  - Indexed mode
  - Base register mode

(i) Implied mode: Operands are specified implicitly in the definition of the instruction.

Example: CPL (complement accumulator)

Here operand in accumulator is implied in the definition of instruction.

- All register-reference instructions that use an accumulator are implied mode instructions.
- Zero-address instructions in a stack-oriented computer are implied-mode instructions.

(ii) Immediate mode: The operand is specified in the instruction itself. Instruction format in immediate mode is

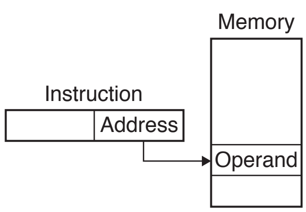
Opcode	Operand

Example: Move A, 50.

- These are useful for initializing registers to constant value or to set initial values of variables.
- No memory reference is required other than the instruction fetch.
- The size of number is restricted to the size of the address (operand) field.

(iii) Direct mode:

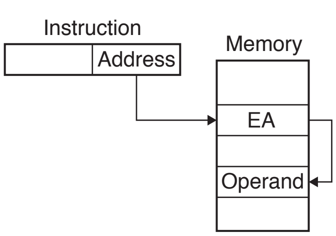
- Here the address of the operand is equal to the address part of the instruction.



- Required only one memory reference and no special calculation required.
- Limitation is limited address space.

(iv) Indirect mode:

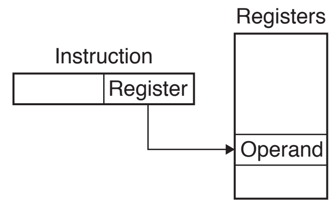
- The address field of the instruction gives the address of the operand which is stored in memory.
- The advantage of this approach is that for a word length of N, an address space of 2^N is available.
- The disadvantage is that the instruction execution requires two memory references to fetch the operand.



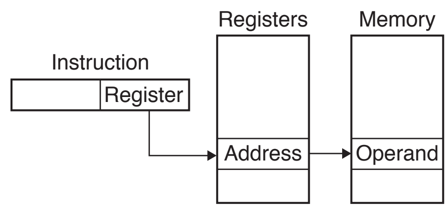
(Here EA is effective address of operand)

(v) Register mode:

- Here the operands are in registers that reside within the CPU.
- Only small address field required in instructions.
- No time consuming memory references are required.
- Address space is very limited.

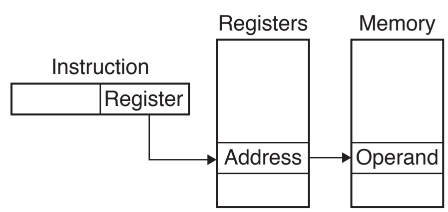


(vi) Register indirect mode: In this mode the instruction specifies a register in the CPU whose contents give the address of the operand in memory. Address field of the instruction uses fewer bits to select a register than would have required to specify a memory address directly.



Effective address: The effective address is defined to be the memory address obtained from the computation based on the addressing mode, consists the actual address of the operand.

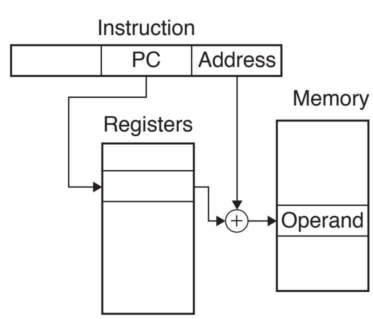
(vii) Auto increment and auto decrement mode: This is similar to register indirect mode except that the register is incremented or decremented after (or before) its value is used to access memory.



- After fetch operand, increment or decrement address.
- Used to access table of data.

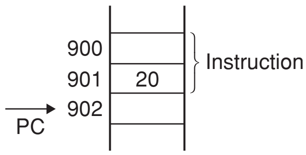
(viii) PC-relative mode: Here the content of the program counter is added to the address part of the instruction to get the effective address.

The address part of the instruction is usually a signed number which can be either positive or negative. The effective address will be a displacement relative to address of the inst ruction.



Effective address = PC + address part

Example: Let PC = 900 and address part of the 2-word instruction = 20.

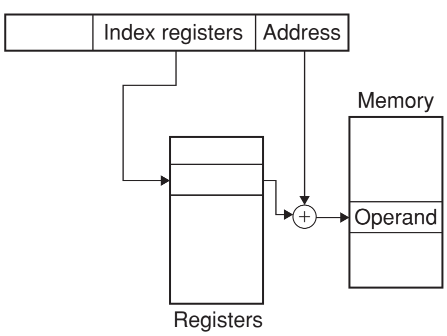


The instruction at location 900 is read memory during fetch phase and the PC will be incremented by instruction length i.e., 2. Then PC = 902.

∴ The effective address using PC-relative = 902 + 20 = 922.

- This addressing mode is used with branch-type instructions.
- Requires shorter address field.

(ix) Indexed mode: Here the content of the index register is added to the address part of the instruction to obtain the effective address. The address field of the instruction defines the beginning address of a data array in memory. The distance between the beginning address and the address of the operand is the index value stored in the index register, is a positive displacement from the address.



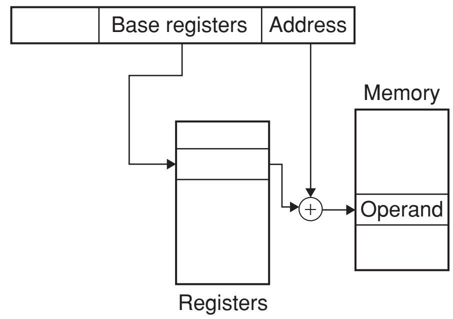
Effective Address = Index Register + Address part of instruction.

This approach is opposite to the interpretation of base-register addressing. This is used to provide an efficient mechanism for performing iterative operations. To store an array using indexed mode, the address part consists of the address of first element of array and the index register specifies the index value.

Example: Let address part of instruction = 300

Index register = 5 and each element of array requires 2 bytes then address of 5th element = $300 + 5 \times 2 = 310$.

(x) Base Register mode: Here the content of a base register is added to the address part of the instruction to obtain the effective address. Base register has a base address and the address field of the instruction gives a displacement relative to the base address.



This addressing mode is used in computers to facilitate the relocation of programs in memory, i.e., when programs and data are moved from one segment of memory to another, as required in multiprogramming system, the address values of instructions must reflect this change of position. With a base register, the displacement values of instructions do not have to change. Only the value of the base register requires updating to reflect the beginning of a new memory segment.

Example 2: If base register is 200 and address part of the instruction is 31, then effective address = 200 + 31 = 231.

If the program's base address is changed from 200 to 400, then new effective address will be 400 + 31 = 431.

Example 3: Match the following:

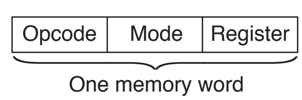
	LIST I		LIST II
P.	P[i] = Q[i];	**1.**	Indexed mode
Q.	while(i++);	**2.**	Immediate mode
R.	int i = 10;	**3.**	Auto increment mode

(A) P – 1, Q – 2, R – 3 (B) P – 2, Q – 3, R – 1
(C) P – 1, Q – 2, R – 2 (D) P – 1, Q – 2, R – 2

Solution: (C)

Array indexing uses indexed mode. For increment operations use Auto increment mode. To initialize variables use immediate mode.

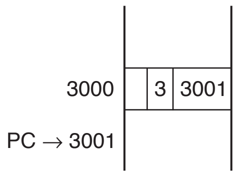
Example 4: The instruction format of a CPU is



Mode and Register together specifies the operand. Register specifies a CPU register and mode specifies an addressing mode. Let mode = 3, specifies that the register contains the address of the operand, after fetching the operand, the contents of register are incremented by 1. An instruction at memory location 3000 specifies mode = 3 and register refers to program counter (PC). Then what is the address of the operand?

(A) 3000 (B) 3001
(C) 3002 (D) Data insufficient

Solution: (B)



$\therefore$ Address of operand = 3001

Example 5: Consider the following machine instruction:

$$\text{MUL } P[R_0], @Q$$

The first operand (destination) '$P[R_0]$' uses indexed addressing mode with R_0 as the index register. The second operand (source) '$@Q$' uses indirect addressing mode. P and Q are memory addresses residing at the second and third words respectively. The first word of instruction specifies the opcode, the index register designation, source and destination addressing modes. During the execution of MUL, the result is stored in destination. How many memory cycles needed during the execution cycle of the instruction?

(A) 3 (C) 5
(B) 4 (D) 6

Solution: (C)
The first operand $P[R_0]$ uses indexed mode. So it requires two memory references: on reference to the address part and next one to obtain the operand. The second operand $@Q$ uses indirect addressing mode, so it requires two memory references, one to obtain the address, and the second, to obtain operand. Finally one more memory reference required to store result. Total five references.

Computer Performance

Response time: The time between the start and completion of a task. This is also referred as execution time.

Throughput: The total amount of work done in a given time.

Performance can be defined as Performance

$$\text{Pertermance} = \frac{1}{\text{Execution time}}$$

CPU execution time or CPU time: This is the time the CPU spends computing for the task and does not include time spent waiting for input–output or running other programs.

CPU time can be divided into

1. User CPU time
2. System CPU time

User CPU time: CPU time spent in the program.

System CPU time: CPU time spent in the operating system performing tasks on behalf of the program.

Clock cycle: Computers are constructed using a clock that determines when events take place in the hardware. These discrete time intervals are called clock cycles.

Clock period: The length of each clock cycle.

Clock rate: Inverse of the clock period.

CPU execution time for a program = CPU clock cycles for a program * clock cycle time

$$= \frac{\text{CPU clock cycles for a program}}{\text{Clock rate}}$$

CPU clock cycles = Instructions for a program * Average clock cycles per instructions.

CPI (clock cycles per instructions): CPI is the average number of clock cycles each instruction takes to execute.

CPU Performance Equation:
CPU time = Instruction count * CPI * Clock cycle time

$$= \frac{\text{Instruction count} * \text{CPI}}{\text{Clock rate}}$$

Exercises

Practice Problems I

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. An instruction is stored at location 301 with its address field at location 300. The address field has the value 400. A processor register R_1 contains the number 200. Evaluate the effective address and Match the following:

(A) Direct	(1) 702
(B) Immediate	(2) 600
(C) Relative	(3) 301
(D) Register Indirect	(4) 400
(E) Index with R_1 as index Register	(5) 200

(A) A – 4, B – 3, C – 1, D – 5, E – 2
(B) A – 3, B – 4, C – 1, D – 5, E – 2
(C) A – 4, B – 3, C – 1, D – 2, E – 5
(D) A – 3, B – 3, C – 1, D – 2, E – 5

2. The two word instruction is stored in memory at an address designated by symbol W. The address field of the instruction (stored at $W + 1$) is designated by the symbol Y. The operand used during the execution of the instruction is stored at address symbolized by Z. An index register contains the value X. State how Z is calculated from the other addresses if the addressing mode of the instruction is

(A) Direct	(1) $Z = \text{Mem}(Y)$
(B) Indirect	(2) $Z = Y + W + 2$

(C) Relative (3) $Z = Y + X$
(D) Indexed (4) $Z = Y$

(A) A – 4, B – 1, C – 2, D – 3
(B) A – 3, B – 1, C – 2, D – 4
(C) A – 4, B – 2, C – 1, D – 3
(D) A – 3, B – 2, C – 1, D – 4

3. A computer has 32-bit instruction and 12-bit addresses. If there are 250 two-address instructions, how many one-address instructions can be formulated?
(A) 6 (B) 256
(C) 12,288 (D) 24,576

4. The memory unit of a computer has 256k words of 32-bits each. The computer has an instruction format with four fields:

1. An operation field
2. A mode field to specify one of 8 addressing modes
3. A Register address field to specify one of 120 processor registers.
4. A memory address.

Then what is the number of bits in each field respectively if the instruction is in one memory word?
(A) 4, 3, 7, 18 (B) 3, 4, 7, 18
(C) 2, 1, 6, 23 (D) 3, 7, 4, 18

Common data for questions 5 to 7: A relative mode branch type of instruction is stored in memory at an address equivalent to decimal 750. The branch is made to an address equivalent to decimal 400.

5. What should be the value of the relative address field of the instruction in decimal?
(A) 351 (B) −351
(C) 350 (D) −350

6. The relative address value in binary using 12-bits, will be
(A) 000101011111 (B) 100101011111
(C) 111010100000 (D) 111010100001

7. What will be the binary value in PC after the fetch phase (in binary)?
(A) 001011101111 (B) 001011101110
(C) 111011101111 (D) 110100010001

Common data for questions 8 to 10: Consider a 16-bit processor in which the following appears in main memory, starting at location 200:

200	Load to AC	Mode
201	500	
202	Next to instruction	

The first part of the first word indicates that this instruction loads a value into an accumulator. The mode field specifies an addressing mode or a source register, R_1, which has a value 400. There is a base register that contain the value 100. The value 500 in location 201, may be the part of address calculation. Assume that location 399 contains the value 999, location 400 contains the value 1000 and so on.

8. What will be the effective address and operand to be loaded by using Register indirect mode?
(A) 200, 400 (B) 400, 1000
(C) 400, 500 (D) 200, 1000

9. What will be the effective address using indirect addressing mode?
(A) 200 (B) 201
(C) 500 (D) Present in 500 location

10. What will be the effective address using immediate addressing mode?
(A) 202 (B) 201
(C) 500 (D) 400

11. A CPU of a computer has 48-bit instructions. A program starts at address $(600)_{10}$. Which one of the following is a legal program counter value in decimal?
(A) 610 (B) 650
(C) 672 (D) 693

12. Consider a new instruction named branch- on-bit-reset (bbr). The instruction '*BBR* R_1, I, label'

Jumps to label, and if bit in position I of register operand, R_1 is zero. The registers of the computer are 16-bits wide and are numbered 0 to 15, position 0 being LSB. Consider the following implementation of this instruction on a processor that does not have *BBR* implemented.

Temp ← R_1 and mask

Branch to label if temp is zero.

The variable 'temp' is a temporary register. For correct implementation, the variable 'mask' must be generated by
(A) mask $\leftarrow 0 \times 1 << I$
(B) mask $\leftarrow 0 \times FFFFFFFF >> I$
(C) mask $\leftarrow I$
(D) mask $\leftarrow 0 \times F$.

13. Consider a hypothetical processor with an instruction of type

LW R_1, 40(R_2)(R_3).

Which during execution reads a 16-bit word from memory and stores it in a 16-bit register R_1. The effective address of the memory location is obtained by the addition of constant 40, contents of R_2 and R_3 registers. Which of the following best reflects the addressing mode implemented by this instruction for the operand in memory?
(A) Immediate addressing
(B) Register addressing
(C) Register indirect scaled addressing
(D) Base with index and displacement addressing

14. Which of the following is true of base-register addressing mode?
(i) It is useful in creating self-relocating code.
(ii) If it is included in an instruction set architecture, then an additional ALU is required for effective address calculation.
(iii) The amount of displacement depends on the content of base register.
(A) (i) only (B) (ii) only
(C) (i) and (ii) only (D) (ii) and (iii) only

15. Which of the following addressing modes are suitable for program relocation at run time?
(i) Direct addressing
(ii) Based register addressing
(iii) PC-relative addressing
(iv) Index register addressing
(A) (i) and (ii) (B) (ii) and (iii)
(C) (iii) and (iv) (D) (ii), (iii) and (iv)

16. In which of the following addressing mode, the address of the operand is inside the instruction?
(A) Implied mode
(B) Absolute addressing mode
(C) Immediate addressing mode
(D) Register addressing mode

17. A certain processor supports only the immediate and the direct addressing modes. Which of the following programming language features can be on this processor?
(i) Pointers
(ii) Arrays
(iii) Initialization
(A) (i) and (ii) (B) (i) and (iii)
(C) (ii) and (iii) (D) (iii) only

18. In which of the following situation, relative addressing mode is useful?
(A) Coroutine writing
(B) Position-independent code writing
(C) Sharable code writing
(D) Interrupt handlers

19. In indexed addressing mode with scaling, the effective address is calculated as
(A) Index + scaling + signed displacement
(B) (Index * scaling) + signed displacement
(C) Index + (scaling * displacement)
(D) (Index + scaling) * displacement

20. Which of the following addressing modes require more number of memory accesses?
(A) DIRECT
(B) IMMEDIATE
(C) INDIRECT
(D) IMPLIED

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. The addressing mode that facilitates access to an operand and whose location is defined relative to the beginning of the data structure in which it appears is
(A) Direct (B) Indirect
(C) Immediate (D) Index

2. Stack addressing is same as
(A) Direct addressing
(B) Indirect addressing
(C) Zero addressing
(D) Relative addressing

3. The Register which contain the Instruction to be executed is called
(A) Instruction register
(B) Memory address register
(C) Index register
(D) Memory data register

4. The Register which keeps track of the execution of a program and which contains the memory address of the next instruction to be executed is called
(A) Instruction register
(B) Program counter
(C) Index register
(D) Memory address register

5. A stack pointer is
(A) A 16-bit register in the microprocessor that indicate the beginning of the Stack Memory
(B) A register that decodes and execute 16-bit arithmetic operation.
(C) The first memory location where a subroutine address is stored
(D) A register in which flag bits are stored.

6. Function of Control Unit in the CPU is
(A) To transfer data to primary storage
(B) To store program instruction
(C) To perform logic operations
(D) To generate timing signals

7. When a subroutine is called the address of the instruction following the CALL instruction stored in the
(A) Stack (B) Accumulator
(C) Program counter (D) Stack pointer

8. In Immediate addressing mode the operand is placed
(A) In the CPU Register
(B) After the OP Code in the instruction
(C) In the memory
(D) In the stack memory

9. When the RET instruction at the end of subroutine is executed
(A) The information where the stack is initialized is transferred to the stack pointer.
(B) The memory address of the RET instruction is transferred to the program counter.
(C) Two data bytes stored in the top two locations of the stack are transferred to the program counter.
(D) Two data bytes stored in the top two location of the stack are transferred to the stack pointer.

10. Match the following:

List I	List II
P. Indirect Addressing	**1.** Loops
Q. Auto decrement Addressing	**2.** Constants
R. Immediate Addressing	**3.** Pointers

(A) P – 1, Q – 3, R – 2
(B) P – 3, Q – 1, R – 2
(C) P – 2, Q – 1, R – 3
(D) P – 3, Q – 2, R – 1

11. An instruction used to set the carry flag in a computer can be
(A) Data control (B) Process control
(C) Logical (D) Data transfer

12. The addressing mode in which the address of the location of the operand is given explicitly as part of the instruction is
(A) Direct addressing mode
(B) Indirect addressing mode
(C) Immediate addressing mode
(D) Register addressing mode

13. The unit that is used to supervise each instructions in the CPU is
(A) Control register (B) Control logic unit
(C) ALU (D) Address register

14. The address of the location to or from which data are to be transferred is called
(A) Memory data register
(B) Memory address register
(C) Program counter
(D) Index register

15. Which register is used as a working area in CPU?
(A) Program counter (B) Accumulator
(C) Stack pointer (D) Instruction register

16. Which of the following statement is false about the PC relative addressing mode?
(A) It allows indexing of array element with same instruction.
(B) It enables reduced instruction size.
(C) It enables faster address calculations than indirect addressing.
(D) It enables easy relocation of data.

17. Which of the following is not an application of logic operations?
(A) Insert new bit values into a register
(B) Change bit value
(C) Delete a group of bits
(D) Shift bit values in a register

18. In which of the following addressing mode, less number of memory references are required?
(A) Immediate (B) Register
(C) Implied (D) All of the above

19. Which of the following is not involved in a memory write operation?
(A) MDR (B) MAR
(C) PC (D) Data bus

20. In ____ addressing mode the instruction contains 8-bit signed offset, address register A_n and index register R_K.
(A) Basic index (B) Full index
(C) Basic relative (D) Full relative

Previous Years' Questions

Common Data for Questions 1 to 3: Consider the following program segment. Here R_1, R_2 and R_3 are the general purpose registers.

Instruction		Operation	Instruction Size (No. of Words)
	MOV R_1, (3000)	$R_1 \leftarrow$ M[3000]	2
LOOP:	MOV R_2, (R_3)	$R_2 \leftarrow$ M[R_3]	1
	ADD R_2, R_1	$R_2 \leftarrow R_1 + R_2$	1
	MOV (R_3), R_2	M[R_3] $\leftarrow R_2$	1
	INC R_3	$R_3 \leftarrow R_3 + 1$	1
	DEC R_1	$R_1 \leftarrow R_1 - 1$	1
	BNZ LOOP	Branch on not zero	2
	HALT	Stop	1

Assume that the content of memory location 3000 is 10 and the content of the register R_3 is 2000. The content of each of the memory locations from 2000 to 2010 is 100. The program is loaded from the memory location 1000. All the numbers are in decimal.

1. Assume that the memory is word addressable. The number of memory references for accessing the data

in executing the program completely is: **[2007]**

(A) 10 (B) 11
(C) 20 (D) 21

2. Assume that the memory is word addressable. After the execution of this program, the content of memory location 2010 is: **[2007]**

(A) 100 (B) 101
(C) 102 (D) 110

3. Assume that the memory is byte addressable and the word size is 32 bits. If an interrupt occurs during the execution of the instruction 'INC R_3', what return address will be pushed on to the stack? **[2007]**

(A) 1005 (B) 1020
(C) 1024 (D) 1040

4. Which of the following is/are true of the auto-increment addressing mode?

(i) It is useful in creating self-relocating code
(ii) If it is included in an Instruction Set Architecture, then an additional ALU is required for effective address calculation
(iii) The amount of increment depends on the size of the data item accessed **[2008]**

(A) (i) only (B) (ii) only
(C) (iii) only (D) (ii) and (iii) only

5. Consider a hypothetical processor with an instruction of type LW R_1, 20(R_2), which during execution reads a 32-bit word from memory and stores it in a 32-bit register R_1. The effective address of the memory location is obtained by the addition of a constant 20 and the contents of register R_2. Which of the following best reflects the addressing mode implemented by this instruction for the operand in memory? **[2011]**

(A) Immediate addressing
(B) Register addressing
(C) Register indirect scaled addressing
(D) Base indexed addressing

6. Consider two processors P_1 and P_2 executing the same instruction set. Assume that under identical conditions, for the same input, a program running on P_2 takes 25% less time but incurs 20% more CPI (Clock cycles per instructions) as compared to the program running on P_1. If the clock frequency of P_1 is 1GHz, then the clock frequency of P_2 (in GHz) is ______. **[2014]**

7. A machine has a 32-bit architecture with 1-word long instructions. It has 64 registers, each of which is 32 bits long. It needs to support 45 instructions, which have an immediate operand in addition to two register operands. Assuming that the immediate operand is an unsigned integer, the maximum value of the immediate operand is ________. **[2014]**

8. Consider a new instruction named branch-on-bit-set (mnemonic bbs). The instruction 'bbs reg, pos, label' jumps to label if bit in position pos of register operand reg is one. A register is 32 bits wide and the bits are numbered 0 to 31, bit in position 0 being the least significant. Consider the following emulation of this instruction on a processor that does not have bbs implemented.

temp ← reg & mask

Branch to label if temp is non-zero.

The variable temp is a temporary register. For correct emulation, the variable mask must be generated by **[2006]**

(A) mask ← 0 × 1 << pos
(B) mask ← 0 × ffffffff >> pos
(C) mask ← pos
(D) mask ← 0 × f

9. Consider a processor with byte-addressable memory. Assume that all registers, including Program Counter (PC) and Program Status Word (PSW), are of size 2 bytes. A stack in the main memory is implemented from memory location $(0100)_{16}$ and it grows upward. The stack pointer (SP) points to the top element of the stack. The current value of SP is $(016E)_{16}$. The CALL instruction is of two words, the first word is the op-code and the second word is the starting address of the subroutine (one word = 2 bytes). The CALL instruction is implemented as follows:

- Store the current value of PC in the stack
- Store the value of PSW register in the stack
- Load the starting address of the subroutine in PC

The content of PC just before the fetch of a CALL instruction is $(5FA0)_{16}$. After execution of the CALL instruction, the value of the stack pointer is **[2015]**

(A) $(016A)_{16}$ (B) $(016C)_{16}$
(C) $(0170)_{16}$ (D) $(0172)_{16}$

10. A processor has 40 distinct instructions and 24 general purpose registers. A 32 - bit instruction word has an opcode, two register operands and an immediate operand. The number of bits available for the immediate operand field is ____. **[2016]**

11. Suppose the functions F and G can be computed in 5 and 3 nanoseconds by functional units U_F and U_G, respectively. Given two instances of U_F and two instances of U_G, it is required to implement the computation $F(G(X_i))$ for $1 \le i \le 10$. Ignoring all other delays, the minimum time required to complete this computation is _____ nanoseconds. **[2016]**

12. Consider a processor with 64 registers and an instruction set of size twelve. Each instruction has five distinct fields, namely, opcode, two source register identifiers, one destination register identifier, and a twelve - bit immediate value. Each instruction must be stored in

memory in a byte - aligned fashion. If a program has 100 instructions, the amount of memory (in bytes) consumed by the program text is ____. **[2016]**

13. Consider the C struct defined below:

```
struct data  {
    int marks [100];
    char grade;
    int cnumber;
};
struct data student;
```

The `base` address of `student` is available in register R1. The field `student.grade` can be accessed efficiently using **[2017]**

(A) Post-increment addressing mode, $(R1)+$
(B) Pre-decrement addressing mode, $-(R1)$
(C) Register direct addressing mode, $R1$
(D) Index addressing mode, $X(R1)$, where X is an offset represented in 2's complement 16-bit representation.

14. Consider a RISC machine where each instruction is exactly 4 bytes long. Conditional and unconditional branch instructions use PC-relative addressing mode Offset specified in bytes to the target location of the branch instruction. Further the Offset is always with respect to the address of the next instruction in the program sequence. Consider the following instruction sequence.

Instr.No.	Instruction
i :	add R2, R3, R4
i+1 :	sub R5, R6, R7
i + 2 :	cmp R1, R9, R10
i + 3 :	beq R1, Offset

If the target of the branch instruction is i, then the decimal value of the Offset is ______. **[2017]**

15. A processor has 16 integer registers ($R0, R1, ..., R15$) and 64 floating point registers ($F0, F1, ..., F63$). It uses a 2-byte instruction format. There are four categories of instructions: Type-1, Type-2, Type-3, and Type-4. Type-1 category consists of four instructions, each with 3 integer register operands ($3Rs$). Type-2 category consists of eight instructions, each with 2 floating point register operands ($2Fs$). Type-3 category consists of fourteen instructions, each with one integer register operand and one floating point register operand ($1R + 1F$). Type-4 category consists of N instructions, each with a floating point register operand ($1F$).

The maximum value of N is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** A	**3.** D	**4.** A	**5.** B	**6.** D	**7.** A	**8.** B	**9.** D	**10.** B
11. C	**12.** A	**13.** D	**14.** A	**15.** B	**16.** B	**17.** B	**18.** B	**19.** B	**20.** C

Practice Problems 2

1. D	**2.** C	**3.** A	**4.** B	**5.** A	**6.** D	**7.** A	**8.** B	**9.** C	**10.** B
11. B	**12.** A	**13.** B	**14.** B	**15.** B	**16.** A	**17.** D	**18.** B	**19.** D	**20.** B

Previous Years' Questions

1. D	**2.** A	**3.** C	**4.** C	**5.** D	**6.** 1.6	**7.** 16383	**8.** A	**9.** D	**10.** 16
11. 28	**12.** 500	**13.** D	**14.** −16	**15.** 32					

Chapter 2

ALU and Data Path, CPU Control Design

LEARNING OBJECTIVES

- *Arithmetic and logic unit*
- *Fixed-point arithmetic operation*
- *Floating point arithmetic operation*
- *BCD*
- *Data path*
- *CPU control design*
- *Instruction cycle*
- *Control unit*
- *Control of processor*
- *Function of control unit*
- *Design of control unit*
- *Types of micro-instructions*
- *Micro-instruction sequencing*
- *RISC and CISC*
- *RISC characteristic*
- *CISC characteristic*

ALU (Arithmetic and Logic Unit)

ALU performs arithmetic and logical operations on data (*see* Figure 1).

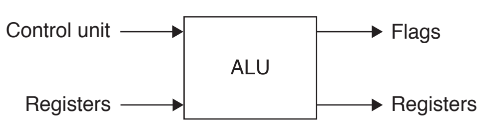


Figure 1 ALU inputs and outputs

- Data are presented to ALU in registers and the results of an operation are stored in registers.
- Registers are temporary storage locations within the processor that are connected by signal paths to ALU.
- The control unit provides signals that control the operation of ALU and the movement of data into and out of the ALU.
- Here we will discuss
  1. Fixed-point arithmetic operations
  2. Floating-point arithmetic operations
  3. BCD data arithmetic operations

Fixed-point Arithmetic Operations

Fixed-point representation

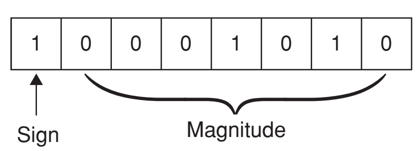
The numbers may be positive, zero or negative. So we have two types of numbers:

Unsigned numbers Only zero and positive integers can be represented. All bits represent magnitude and no need of sign.

Signed numbers In signed representation, the most significant bit represents the sign. If the number is positive, the MSB is 0 and remaining bits represent magnitude. If the number is negative, we have three techniques to represents that number:

1. **Signed magnitude representation:** In signed magnitude representation, the MSB represents sign and remaining bits represents magnitude. If the number is negative then the MSB is 1.

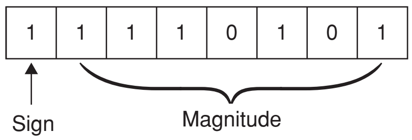
   Example: Signed magnitude representation of –10 =



2. **Signed 1's complement representation:** In signed 1's complement representation, the MSB bit is 1. The remaining bits of its signed magnitude bits are inverted i.e., convert 0's to 1's and 1's to 0's to obtain 1's complement.

Example:

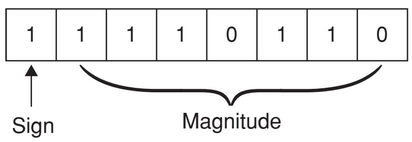
Signed 1's complement (–10) =



3. **Signed 2's complement representation:** To get signed 2's complement representation, add 1 to the signed 1's complement of that number.

Example:

Signed 2's complement (–10) =



Fixed-point arithmetic operations

We will discuss the following operations using signed magnitude data and signed 2's complement data.

1. Addition
2. Subtraction
3. Multiplication
4. Division

Addition and subtraction using signed magnitude data Consider two numbers whose magnitude is represented as A and B. When the signed numbers are added or subtracted, there are eight different conditions to consider, depending on the sign of the numbers and operation performed.

Operation	Add Magnitudes	Subtract Magnitudes ($A > B$)
$(+A) + (+B)$	$+(A + B)$	
$(+A) + (-B)$		$+(A - B)$
$(-A) + (+B)$		$-(A - B)$
$(-A) + (-B)$	$-(A + B)$	
$(+A) - (+B)$		$+(A - B)$
$(+A) - (-B)$	$+(A + B)$	
$(-A) - (+B)$	$-(A + B)$	
$(-A) - (-B)$		$-(A - B)$

Algorithm for addition (subtraction): When the signs of A and B identical (different), add the two magnitudes and attach the sign of A to the result. When the signs of A and B are different (identical), compare the magnitudes and subtract the smaller number from the larger. Choose the sign of result based on magnitudes of A and B.

Example: All eight cases for the numbers $A = 5$, $B = 2$.

$(+A) + (+B) = (+5) + (+2)$
$= 0101 + 0010 = 0111 = +7$
$(+A) + (-B) = (+5) + (-2)$
$= 0101 + 1010$

Take 2's complement of –2 and add it to 5

101
110

1] 011
↑
Discard

$\therefore$ result = + 3 ($A > B$)

$(-A) + (+B) = (-5) + (+2)$
$= 1101 + 0010$

add 2's complement of –5 to 2

011
<u>010</u>
<u>101</u>

As MSB is 1 take 2's complement to get original number i.e., 011.

Result = 1011 = –3 ($\because A > B$)

$(-A) + (-B) = (-5) + (-2)$
$= 1101 + 1010$

101
<u>010</u>

Result = 1111 = –7

Similarly we can perform the subtractions using signed magnitude data.

Hardware implementation:

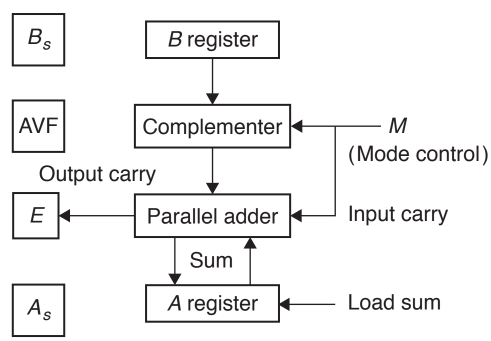


Figure 2 Hardware implementation for addition and subtraction.

Figure 2 shows the hardware implementation for addition and subtraction operations. It consists of registers A and B and sign flip-flops A_s and B_s. Subtraction is done by adding A to the 2's complement of B. The output carry is transferred to flip-flop E and add overflow flip-flop AVF holds the overflow bit when A and B are added. The addition is done through the parallel adder. The output of adder is sent to 'A' register. The complementer provides an output of B or complement of B depending on the state of mode control M. When $M = 0$, the output equal to $A + B$, when $M = 1$, the output equal to $A + \overline{B} + 1$, i.e., $A - B$.

Addition and Subtraction with signed 2's complement data

Addition: In 2's complement representation, addition proceeds as if the two numbers were unsigned integers. If the result of the operation is positive, we get a positive number

in 2's complement form, which is same as in unsigned integer form. If the result of the operation is negative, we get a negative number in 2's complement form.

Example:

+5 = 0101
+2 = 0010
0111 = +7

+5: 0101
−2: 1110
10011 = +3
−5: 101
+2: 0010
1101

As the result is negative, take 2's complement of result to get original number, i.e., 0011 and the answer is −3.

−5: 1011
−2: 1110
11001

As the result is negative take 2's complement to get original number, i.e., 0110 + 1 = 0111.

∴ Answer is −7.

Note: If two numbers are added and they are both positive or both negative, then overflow occurs if the result has the opposite sign.

Subtraction: To subtract subtrahend from minuend, take the 2's complement of subtrahend and add it to the minuend.

Example:

+5: 0101
+2: 0010

To subtract these two numbers add 2's complement of 2 to 5.

+5: 0101
−2: 1110
10011 = +3
+5: 0101
− 2: 1110

2's complement of −2 = 0010.

+5: 0101
+2: 0010
0111 = +7

Similarly for the other cases we can perform the subtraction.

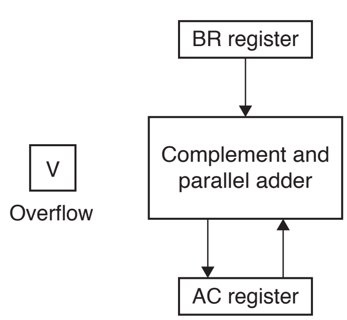


Figure 3 Hardware implementation for signed 2's complement addition and subtraction:

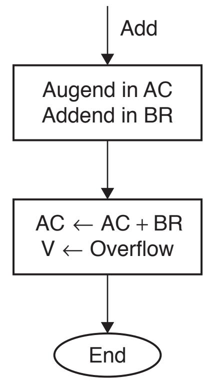


Figure 4 Flowchart for addition in 2's complement form

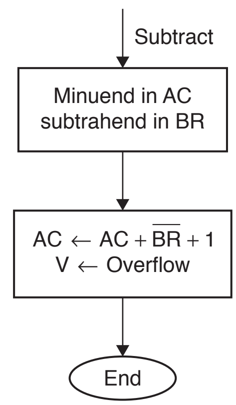


Figure 5 Flowchart for subtraction of 2's complement data

Multiplication of signed magnitude data

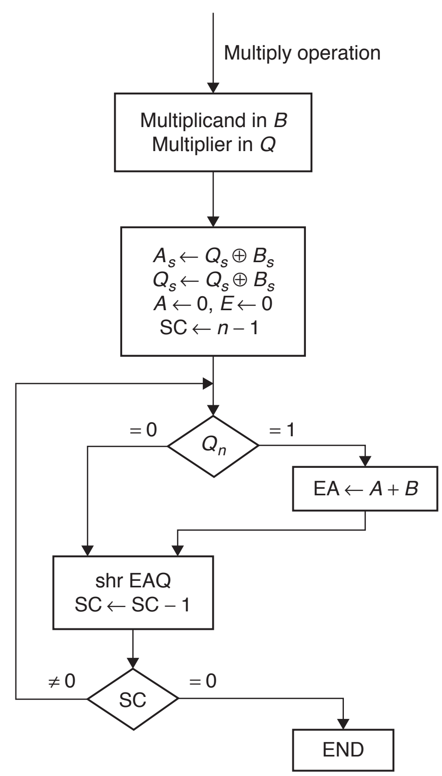


Figure 6 Flowchart for multiplication of signed magnitude data

Multiplication of two fixed point binary numbers in signed magnitude representation is a process of successive shift and add operations (*see* Figure 6).

Example 1: Multiply the two numbers −7 and +8, using 5-bit registers.
−7 = 10111
+8 = 01000

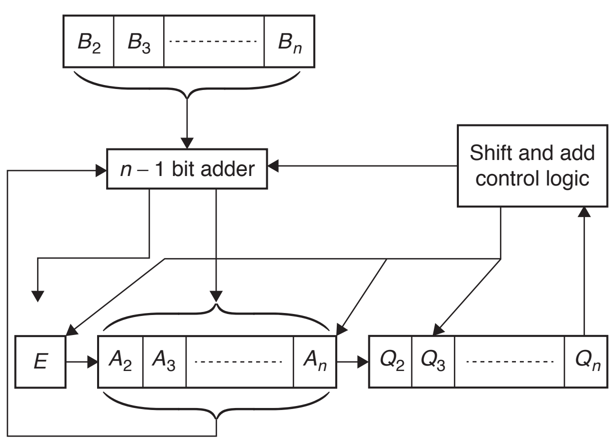
By excluding sign-bits, the multiplicand, B = 0111 and multiplier Q = 1000. Initially A = 0000, SC is sequence counter contains number of bits in multiplier magnitude.

Here SC = 4

Multiplicand **B = 0111**	***E***	***A***	***Q***	**SC**
Multiplier in Q	0	0000	1000	4
Last bit of Q, $Q_n = 0 \Rightarrow$ Shr EAQ	0	0000	0100	3
$Q_n = 0 \Rightarrow$ Shr EAQ	0	0000	0010	2
$Q_n = 0 \Rightarrow$ Shr EAQ	0	0000	0001	1
$Q_n = 1 \Rightarrow$ Add B to A	0	0000 <u>0111</u> 0111	0001	
Shr EAQ	0	0011	1000	0

$B \times Q = 00111000 = 56$
Sign $= Q_s \oplus B_s = 1 \oplus 0 = 1$
∴ Result = –56

Hardware for signed magnitude data multiplication: B_1, A_1, Q_1 represent the respective signs of the registers B, A, Q. Final result will be in AQ, which consist of $2n$-bits. (Here each register has n-bits).



Multiplication of Singed 2's complement data The straight forward multiplication will not work if either the multiplicand or the multiplier is negative. There are number of ways to perform multiplication of signed 2's complement data. One such a technique is *Booth's multiplication algorithm*. The following flowchart depicts about Booth's algorithm (see figure 7).

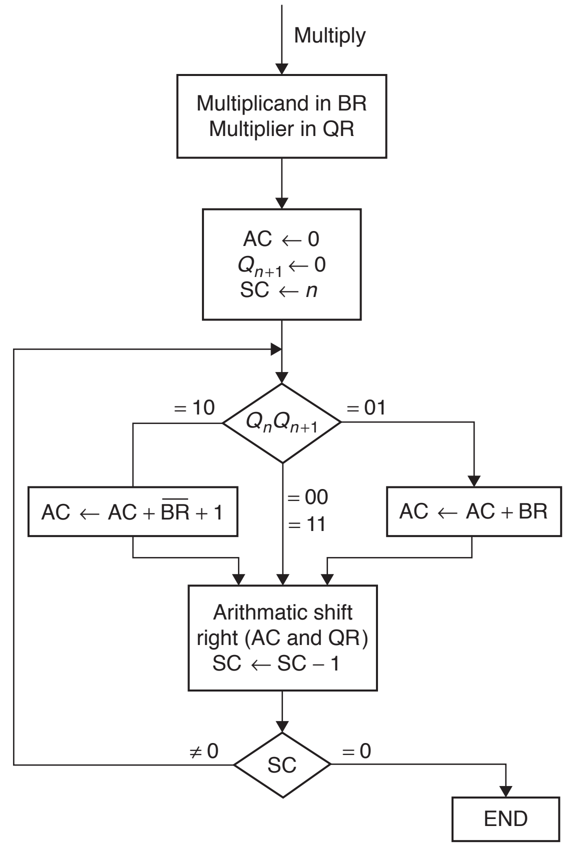


Figure 7 Booth's multiplication algorithm

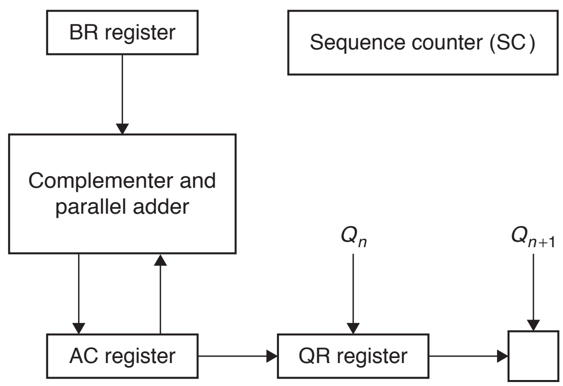
An additional 1-bit register placed logically to the right of the LSB(Q_n) of Q register designated Q_{n+1}.

Example 2: Multiply the two numbers −7 and +8 using booth's algorithm, using 5 bits.
BR = −7 = 11001
QR = +8 = 01000
Initially AC = 00000, Q_{n+1} = 0, SC = 5

$Q_n\,Q_{n+1}$	**BR** = **11001** $\overline{\textbf{BR}}+1 = 00111$	**AC**	**QR**	Q_{n+1}	**SC**
	Initial	00000	01000	0	5
00	ashr(AC and QR)	00000	00100	0	4
00	ashr(AC and QR)	00000	00010	0	3
00	ashr(AC and QR)	00000	00001	0	2
10	subtract BR ashr(AC and QR)	00000 <u>00111</u> 00111 00011	 10000	 1	 1
01	add BR ashr(AC and QR)	00011 <u>11001</u> 11100 11110	 10000 01000	 1 0	 1 0

∴ Result = 1111001000 = –56

Hardware implementation for Booth's algorithm:



Note: These two multiplication algorithms are sequential but we can also do the operation by means of a combinational circuit that forms the product bits all at once. The circuit consist of AND gates and adders.

Division algorithms

Division of signed magnitude data: Division of signed magnitude data is a process of successive compare, shift and subtract operations.

Example:
Dividend = 0111000000
Divisor = 10001

```
10001) 0111000000 (11010
       –10001
        010110
        –10001
         0010100
          –10001
           000110
```

Hardware implementation:

- The hardware implementation of division is same as multiplication, instead of shifting the divisor to the right, the dividend or partial remainder is shifted to the left, thus leaving the two numbers in the required relative position.
- The divisor is stored in the B register and the double-length dividend is stored in registers A and Q. The dividend is shifted to the left and the divisor is subtracted by adding its 2's complement value. The information about the relative magnitude is available in E.
- If $E = 1$, it signifies that $A \geq B$. A quotient bit 1 is inserted into Q_n and the partial remainder is shifted to the left to repeat the process.
- If $E = 0$, it signifies that $A < B$ so the quotient is Q_n remains a 0. The value of B is added to restore the partial remainder in A to its previous value. The partial remainder is shifted to the left and the process is repeated again until all quotient bits are formed.
- Finally, the quotient is in Q and remainder is in A. This method is called *restoring method*.

Divide overflow:

- A divide overflow condition occurs if the high-order half bits of the dividend constitute a number greater than or equal to the divisor.
- A division by zero must be avoided.

Other algorithms for division: Two other methods are available for dividing numbers:

Comparison method: To divide the two numbers A and B in comparison method, they are compared prior to the subtraction operation. If $A \geq B$, B is subtracted from A. If $A < B$ nothing is done. The partial remainder is shifted left and the numbers are compared again.

Non-restoring method: To divide two numbers A and B is non-restoring method, B is not added if the difference is negative but instead, the negative difference is shifted left and then B is added.

Floating-point Arithmetic Operations

Floating-point representation

Fixed-point representation allows representation of numbers with fractional component as well. But this approach has limitations. It is not possible to represent very large numbers and very small numbers in fixed point representation.

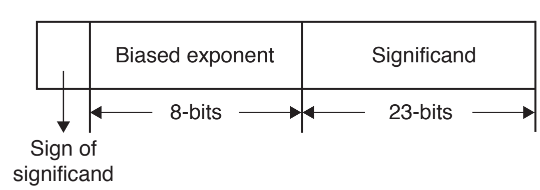
In floating-point representation, the numbers can be represented in the form,

$\pm S \times B^{\pm E}$

The three fields are
Sign: plus or minus
Significand: S
Exponent: E
are stored in a binary word.

The base B is implicit and need not be stored because it is same for all the numbers. It is assumed that the radix point is to the right of the left most or most significant bit of the significand, i.e., there is one bit to the left of the radix point.

32-bit floating-point format: The left most bit stores the sign of the number. The exponent value is stored in next 8-bits. This is represented in biased representation.



Biased representation: In biased representation, a fixed value, called the bias, is subtracted from the exponent field to get the true exponent value.

Bias = $(2^{k-1} - 1)$, where k = number of bits in binary exponent. In IEEE 32-bit floating point representation, bias $= 2^7 - 1 = 127$.

And the range of true exponents is −127 to +128.

The advantage of biased representation is that non-negative floating point numbers can be treated as integers for comparison purposes.

The last portion of word is the siginificand.

Normalized numbers:

- To simplify operation on floating point numbers, it is typically required that they must be normalized.
- A normalized number is one in which the most significant digit of significand is non-zero.
- For base-2 representation, a normalized number is therefore one in which the MSB of the significand is one.
- Normalized non-zero number is one in the form $\pm 1.bbb\cdots b \times 2^{\pm E}$, where b is either binary digit 0 or 1.
- As the MSB is always one, it is unnecessary to store this bit. Thus the 23-bit field is used to store a 24-bit significand with a value in the half open internal (1, 2).
- A number may be normalized by shifting the radix point to the right of the leftmost 1 bit and adjusting the exponent accordingly.

Example: In 32-bit floating representation of $-1.6328125 \times 2^{-20}$,

Sign = 1 (as the number is negative)

$(.6328125)_{10} = (.1010001\ldots)_2$

Exponent = −20

Biased exponent = 127 − 20 = 107

= 1101011

$\therefore -1.6328125 \times 2^{-20}$

= 1 01101011 10100010000000000000000

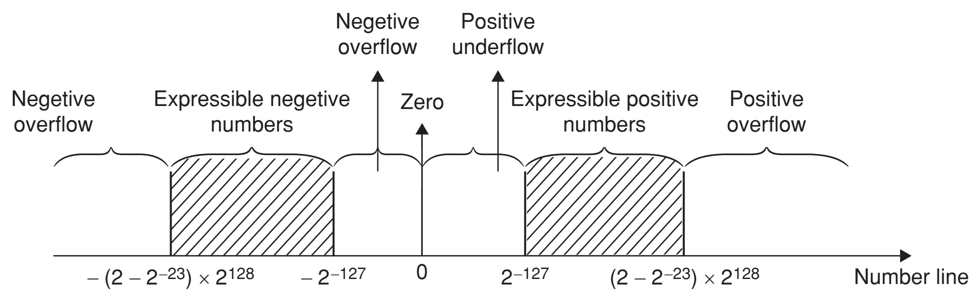
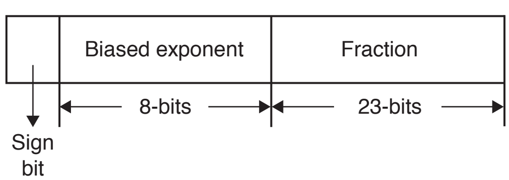
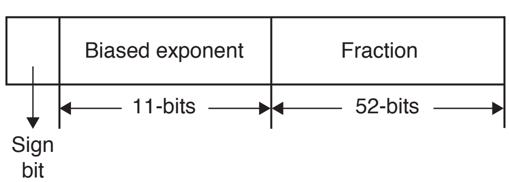


Figure 8 Range of expressible numbers in a 32-bit floating point format

IEEE standard for binary floating-point representation



Single precision format



Double precision Format

Floating-point arithmetic

(i) Addition and subtraction: The algorithm consists the following phases:

1. Check for zeros
2. Align the significands/mantissas
3. Add or subtract the significands
4. Normalize the result

Example:

$(123 \times 10^0) + (234 \times 10^{-2})$

$= 123 \times 10^0 + 2.34 \times 10^0 = 125.34 \times 10^0$

(ii) Multiplication: The steps to multiply two floating point numbers are

1. Check for zeros
2. Add the exponents
3. Multiply the significands
4. Normalize the result

(iii) Division: The steps to divide two floating point numbers are

1. Check for zeros
2. Initialize registers and evaluate the sign
3. Align the dividend
4. Subtract the exponents
5. Divide the significands

Binary-Coded Decimal (BCD) Arithmetic Operations

Computers capable of performing decimal arithmetic must store the data in binary-coded form.

Example: BCD of 239 = 0010 0011 1001

BCD addition

In BCD each digit do not exceed 9, so the sum of two BCD digits cannot be greater than 9 + 9 + 1 = 19, the 1 in the sum being an input carry. When the binary sum of two BCD digits is greater than 1001, we obtain a non-valid BCD representation. The addition of binary 6 (0110) to the binary sum converts it to the correct BCD representation and also produces an output carry as required.

Example:

239 = 0010 0011 1001
426 = 0100 0010 0110
665   0110 0101 1111 >1001
            0110
              0110 0110 0101
         = 665

BCD subtraction

- Perform the subtraction by taking the 9's or 10's complement of the subtrahend and adding it to the minuend.
- The 9's complement of a decimal digit represented in BCD can be obtained by complementing the bits in the coded representation of the digit, provided a correction is included.

There are two possible correction methods:

1. Binary 1010 is added to each complemented digit and the carry discarded after each addition.

Example:

9's complement of 7 = 2
7 in BCD = 0111.
Complement of 7 = 1000
Add 1010 = 1010
1] 0010 = 2
↑
Discard

2. Binary 0110 is added before the digit is complemented.

Example:

BCD of 7 = 0111
Add 0110 = 0110
1101
Complement = 0010 = 2

Example 4: Which of the following multiplier bit pattern of Booth's multiplication algorithm gives worst case performance?

(A) 01010101….0101
(B) 000000….0000
(C) 11111111….1111
(D) 011110111110….01110

Solution: (A)

Booth's multiplication algorithm works well with consecutive 0's or 1's. But it gives worst case performance when the multiplier consists of alternative 0's and 1's. (As 01, 10 pattern leads to addition and subtractions).

Example 5: Consider the following 32-bit floating point representation scheme as shown in the format below:

Sign	Fraction	Exponent
← 1 →	← 24 →	← 7 →

A value is specified by three fields:
Sign field: 1 bit (0 for positive and 1 for negative values)
Fraction: 24-bits (with binary point being at the left end of fraction bits)
Exponent: 7-bits (in excess-64 signed integer representation)
The base of exponentiation is 16. The sign bit is in MSB. Then the normalized floating point representation of − 6.5 is

(A) E8000042 (B) E1000012
(C) D8000841 (D) D0000042

Solution: (D)

Here sign = 1 as the number is negative.
$(-6.5)_{10} = (-0110.1)_2$
$= (-1.101 \times 2^2)$
Fraction = 101000000000000000000000
Exponent = Excess − 64 exponent
= 64 + 2 = 66 = 1000010
$\therefore (-6.5)_{10}$
= 1 1010000000000000000000001000010
= D0000042.

Data Path

Data path consists of the components of the processor that performs arithmetic operations.

Components of data path: ALU is just one data path building block. Other components are

1. Computational Components, which consist of combinational circuits (output follow inputs)

Example: ALU.

2. State components, which consists of sequential circuits (output changes on clock edge)

Example: Registers.

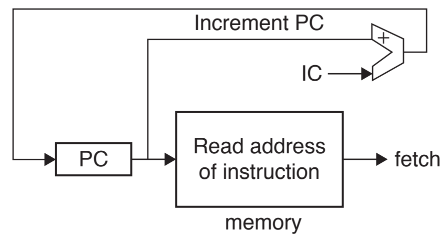
Example: The sequence of steps for the addition of two registers content are

1. $R1_{out}$, X_{in}
2. $R2_{out}$, Choose X, ADDITION, Y_{in}
3. Y_{out}, $R3_{in}$

(Each step executed in a single clock cycle).

- Data path and control unit forms the processing unit of a computer. The Data path includes ALU, multiplexers, all registers (like PC, IR) etc.

Example Data Path Design:

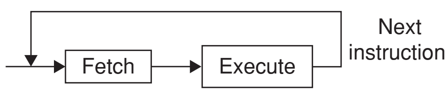


CPU Control Design

Instruction Cycle

A program residing in the memory unit of the computer consists of a sequence of instructions. The program is executed in the computer by going through a cycle for each instruction. Each instruction cycle in turn is subdivided into a sequence of sub cycles. For example, the phases of instruction cycle may be

1. Fetch
2. Decode
3. Read effective address
4. Execute, etc.



Instruction cycle

The cycle will be repeated, till all the instructions are executed.

Each phase is made up of more fundamental operations, called micro-operations.

Example micro-operations: Transfer between registers, simple ALU operation, etc.

Control Unit

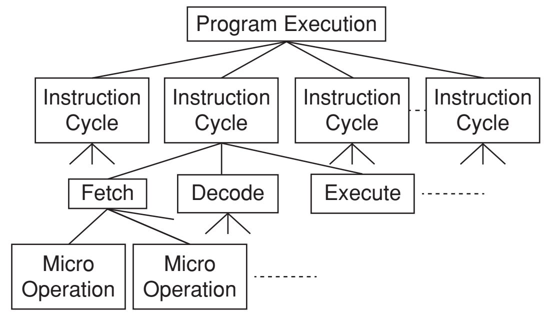
The control unit of a processor performs two tasks:

1. It causes the processor to execute micro-operations in the proper sequence, determined by the program being executed.
2. It generates the control signals that cause each micro-operation to be executed.

We now discuss the micro-operations of various phases of instruction cycle.

Micro-operation

- These are the functional or atomic operations of a processor.



Fetch Cycle: 'Fetch' stage of an instruction occurs at the beginning of each instruction, which causes an instruction to be fetched from memory. The micro-operations involved in fetch phase are

t_1: MAR ← PC (move contents of PC to MAR)
t_2: MBR← memory; PC ← (PC) + I (move contents of MAR location to MBR and increment PC by I)
t_3: IR← (MBR) · (move contents of MBR to IR)

Here *I* is instruction length.
Each micro-operation can be performed within the time of a single time unit.

Execute Cycle: For a machine with *N* different opcodes, there will be *N* different sequence of micro-operations. For the execution of following instruction.
Add R_1, X, the micro-operations will be

t_1: MAR← (IR(Address))
t_2: MBR← memory
t_3: $R_1 \leftarrow (R_1) + (\text{MBR})$

Control of the Processor

(i) Functional requirements of control unit: Let us consider the following concepts to the characterization of a CU.

1. Define the basic elements of the processor.
2. Describe the micro-operations that the processor performs.
3. Determine the functions that the control unit must perform to cause the micro-operations to be performed.

(ii) Basic elements of processor:
- ALU
- Registers
- Internal data path: Used to move data between registers and between register and ALU.
- External data path: Used to link registers to memory and *input–output* modules, often by means of a system bus.
- Control unit: Causes operations to happen within the processor.

(iii) Micro-operations of processor:
- Transfer data from one register to another.
- Transfer data from a register to an external interface.

- Transfer data from an external interface to a register.
- Perform an arithmetic or logic operation, using registers for input and output.

(iv) Control unit tasks:

- **Sequencing:** The control unit causes the processor to step through a series of micro-operations in the proper sequence, based on the program being executed.
- **Execution:** The control unit causes each micro-operation to be executed.

(v) Control signals: For the control unit to perform its function, it must have inputs that allow it to determine the state of the system and outputs that allows it to control the behaviour of the system. These are external specifications of the control unit.

Internally, the control unit must have the logic required to perform its sequencing and execution functions.

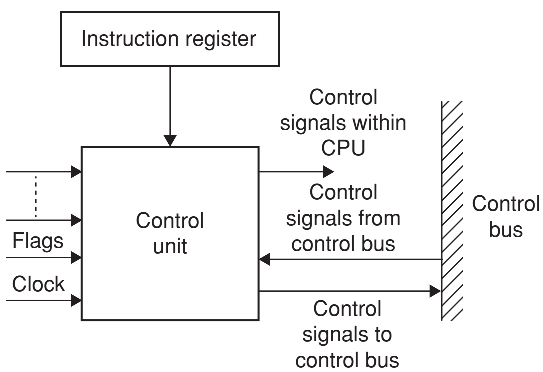


Figure 9 Block diagram of control unit

(a) Clock: This is how the control unit 'keeps time.' The control unit causes one micro-operation to be performed for each clock pulse. This is referred as processor cycle time or clock cycle time.

(b) Instruction registers: The opcode of current instruction is used to determine which micro-operations to perform during the execute cycle.

(c) Flags: Used to determine the status of the processor and outcome of previous ALU operations.

(d) Control signals from control bus: The control bus portion of system bus provides signals to the control unit.

(e) Control signals within the processor:

1. Those that cause data to be moved from register to another.
2. Those that activate specific ALU functions.

(f) Control signals to control bus:

1. Control signals to memory.
2. Control signals to input–output modules.

Totally, there are three types of control signals:

1. Those that activate ALU function.
2. Those that activate a data path.
3. Those that are signals on the external system bus.

Functions of Control Unit

- The control unit directs the entire computer system to carry out stored program instructions.
- The control unit must communicate with both the Arithmetic Logic Unit and Main memory.
- The control unit instructs the arithmetic logic unit by which, logical or arithmetic operation is to be performed.
- The control unit coordinate the activities of the other two units as well as all peripheral and auxiliary storage devices linked to the computer.

Design of Control Unit

Control unit generates control signals using one of the two organizations

(1) Hardwired control unit
(2) Micro-programmed control unit.

Hardwired control unit

- It is implemented as logic circuits (gates, flip-flops, decoders, etc.) in the hardware.
- It is very complicated if we have a large control unit.
- In this organization, if the design has to be modified or changed. It requires changes in wiring among the various components. Thus the modification of all the combinational circuits may be very difficult.

Architecture of hardwired control unit An example hardwired control unit is shown in Figure 9.

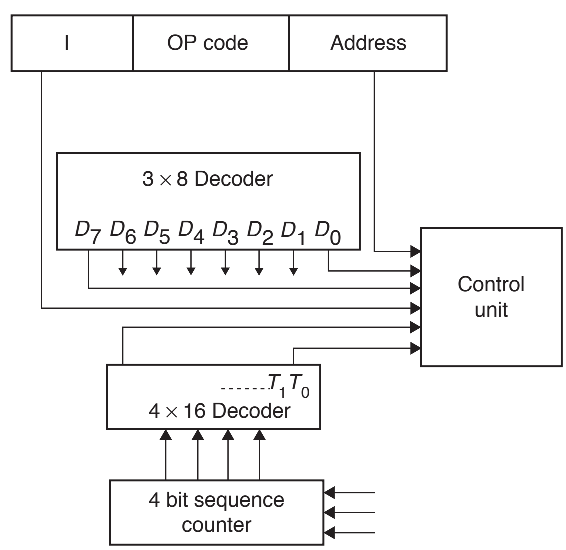


Figure 10 Hardwired control unit

The above control unit consists of:

- Instruction Register
- Number of control logic gates

- Two decoders
- 4-bit sequence counter.
- An instruction read from memory is placed in the instruction register (IR)
- The instruction register is divided into three parts: the *I* bit, operation code and Address part.
- First 12-bits (0-11) to specify an address, next 3-bits specify the operation code (op code) field of the instruction and last left most bit specify the addressing mode *I*.
  $I = 0$ for direct address
  $I = 1$ for indirect address
- First 12-bits are applied to the control logic gates.
- The Opcode bits (12-14) are decoded with 3×8 decoder.
- The eight outputs (D_0 through D_7) from a decoder go to the control logic gates to perform specific operation.
- Last bit 15 is transferred to a *I* flip flop designated by symbol *I*.
- The 4-bit sequence counter SC can count in binary from 0 through15.
- The counter output is decoded into 16 timing pulses T_0 through T_{15}.
- The sequence counter can be incremented by INR input or clear by CLR input synchronically.

Advantages:

- Hardwired control unit is fast because control signals are generated by combinational circuits.
- The delay in generation of control signals depends upon the number of gates.

Disadvantages:

- More is the control signal required by CPU, more complex will be the design of control unit.
- Modifications in control signal are very difficult. That means it requires rearranging of wires in the hardware circuit.
- It is difficult to correct mistake in original design or adding new features.

Micro-programming control unit

- A micro-programmed Control unit is implemented using programming approach. A sequence of micro-operations are carried out by executing a program consisting of microinstructions.
- Micro-program, consisting of micro instructions is stored in the control memory of the control unit.
- Execution of micro-instruction is responsible for generation of a set of control signals.

A micro-instruction consists of:

- One or more micro-instructions to be executed.
- Address of next micro-instruction to be executed.

(a) Micro-operations: The operations performed on the Data stored inside the registers are called Micro-operations.

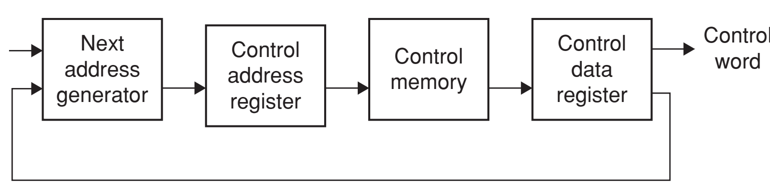
(b) Micro-programs: Micro-programming is the concept for generating control signals using programs. These programs are called Micro-programs.

(c) Micro-instructions: The instructions that make Micro-programs are called micro-instructions.

(d) Micro-code: Micro-program is a group of micro-instructions. Micro-program can also be termed as micro-code.

(e) Control memory: Micro-programs are stored in the read-only memory (ROM). That memory is called control memory.

(f) Architecture of Micro-Programmed Control Unit:



- The address of micro-instruction that is to be executed is stored in the control address register (CAR).
- Micro-instruction corresponding to the address stored in CAR is fetched from control memory and is stored in the control data register (CDR).
- This micro-instruction contains control word to execute one or more micro-operations.
- After the execution of all micro-operations of micro-instructions, the address of next micro-instructions is located.

Advantages:

- The design of micro-program control unit is less complex because micro-programs are implemented using software routines.
- The micro-programmed control unit is more flexible because design modifications, correction and enhancement is easily possible.
- The new or modified instruction set of CPU can be easily implemented by simply rewriting or modifying the contents of control memory.
- The fault can be easily diagnosed in the micro-program control unit using diagnostic tools by maintaining the contents of flags, registers and counters.

Disadvantages:

- The micro-program control unit is slower than hardwired control unit. That means to execute an instruction in micro-program control unit requires more time.

- The micro-program control unit is expensive than hardwired control unit in case of limited hardware resources.
- The design duration of micro-program control unit is more than hardwired control unit for smaller CPU.

Types of Micro-instructions

Micro-instructions can be classified as

Horizontal micro-instruction

- Individual bits in horizontal micro-instructions correspond to individual control lines.
- These are long and allow maximum parallelism since each bit controls a single control line.
- No decoding needed.

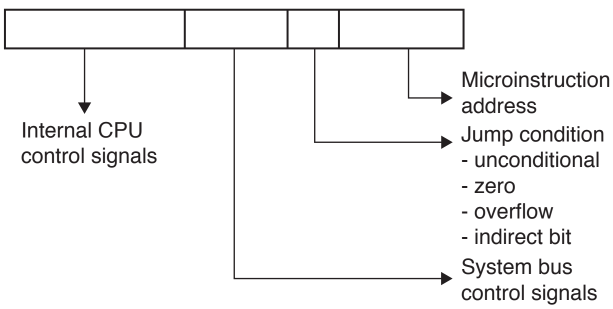


Figure 11 Horizontal micro-instruction format

Vertical micro-instruction

- Here, control lines are coded into specific fields within a micro-instruction.
- Decoders are needed to map a field of k-bits to 2^k possible combinations of control lines.

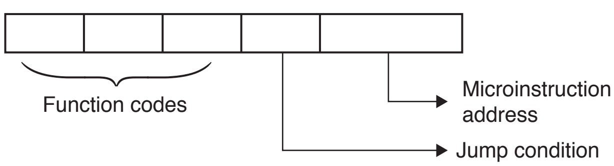


Figure 12 Vertical micro-instruction format

Example: A 3-bit field in a micro-instruction could be used to specify any one of eight possible lines.

- Hence these instructions are much shorter than horizontal ones.
- Control fields encoded in the same field cannot be activated simultaneously. Therefore vertical micro-instructions allow only limited parallelism.
- Decoding is necessary.

Micro-instruction Sequencing

Two concerns are involved in the design of a micro-instruction sequencing technique:

1. The size of micro-instruction: Minimizing size of control memory reduces the cost of that component.
2. The address-generation time:

A desire to execute micro-instructions as fast as possible. In executing a micro program, the address of next micro-instruction to be executed is in one of these categories.

1. Determined by IR
2. Next sequential address
3. Branch

Micro-instructions Execution

The Micro-instruction cycle has two parts:

1. Fetch
2. Execution

The effect of execution of a micro-instruction is to generate control signals. Some of the signals control points internal to the processor. The remaining signals go to the external control bus or other external interface.

Micro-instructions can be classified in a variety of ways.

1. Vertical/horizontal
2. Packed/unpacked
3. Hard/soft micro-programming
4. Direct/indirect encoding.

RISC and CISC

One of the important aspects of computer architecture is the design of the instruction set for the processor. The instruction set chosen for a particular computer determines the way that machine language programs are constructed. There are two categories of computers based on instructions:

1. Complex instruction set computer (CISC)
2. Reduced instruction set computer (RISC)

CISC: A computer with a large number of instructions is classified as a complex instruction set computer.

RISC: A computer which has fewer instructions with simple constructs, so they can be executed much faster with in the CPU without having to use memory as often. This type of computer is classified as RISC.

CISC characteristics

- CISC provides a single machine instruction for each statement, That is written in a high level language so that compilation process is simplified and the over all computer performance improved.
- It has variable length instruction formats.
- It provides direct manipulation of operands residing in memory.
- Some instructions that perform specialized tasks and are used infrequently.
- A large variety of addressing modes

Drawback of CISC architecture As more instructions and addressing modes are incorporated into a computer, the more hardware logic is needed to implement and support them and hence this causes the computations to slow down.

RISC characteristics

- Reduce execution time by simplifying the instruction set of the computer.
- Fewer numbers of instructions
- Relatively fewer addressing modes
- Memory access is limited to load and store instructions.
- All operations are done with in the register of the CPU.
- Fixed - length, easily decoded instruction format.
- Single-cycle instruction execution.
- Hardwired rather than micro-programmed control.
- Relatively large number of registers.
- Uses overlapped register windows to speed - up procedure call and return.
- Efficient instruction pipeline.
- Efficient translation of high - level language programs into machine language programs by the compiler.

Example 6: An instruction set of a processor has 200 signals which can be divided into 5 groups of mutually exclusive signals as follows.

Group 1: 30 Signals
Group 2: 90 Signals
Group 3: 20 Signals
Group 4: 10 Signals
Group 5: 50 Signals
How many bits of the control words can be saved by using vertical micro-programming over horizontal microprogramming?

(A) 27 (B) 173
(C) 200 (D) 227

Solution: Horizontal micro-programming requires 200 signals. But vertical micro-programming uses encoding. So

Group 1 requires 5-bits ($\because 2^5 = 32$)
Group 2 requires 7-bits ($\because 2^7 = 128$)
Group 3 requires 5-bits ($\because 2^5 = 32$)
Group 4 requires 4-bits ($\because 2^4 = 16$)
Group 5 requires 6-bits ($\because 2^6 = 64$)

$\therefore$ Total bits required using vertical micro programming = 27

$\therefore$ Number of bits saved = 200 – 27 = 173

Exercise

Practice Problems I

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Using two's complement arithmetic the resultant of 111100001111 – 110011110011 is
(A) 0010 0001 1111
(B) 0011 0000 1100
(C) 0010 0001 1101
(D) 0010 0001 1100

2. IEEE 32-bit floating point format of 384 is
(A) 0 10000111 00000000000000000000000
(B) 0 10000111 10000000000000000000000
(C) 0 00001000 00000000000000000000000
(D) 0 00001000 10000000000000000000000

3. Consider the following IEEE 32-bit floating point number:
0 01111110 10100000000000000000000.
What is the decimal value equivalent to given number?
(A) 0.25 (B) 3.25
(C) 0.8125 (D) 0.9375

4. What would be the bias value for a base-8 exponent in a 7-bit field?
(A) 8 (B) 16
(C) 63 (D) 64

5. The normalized value of the resultant of $8.844 \times 10^{-3} - 2.233 \times 10^{-1}$ is
(A) -2.144×10^{-1} (B) -0.2144
(C) -2×10^{-1} (D) -0.2

6. Which of the following is the correct sequence of micro-operations to add a number to the AC when the operand is a direct address operand and store the final result to AC?
(A) MAR←(IR(address))
MBR ← memory
$R_1 \leftarrow$ (AC) + (MBR)
(B) MAR ← IR(address)
MBR ← MAR
$R_1 \leftarrow$ (MBR)
$R_2 \leftarrow$ (AC) + (R_1)
AC ← R_2
(C) MAR ← (IR(address))
MBR ← Memory(MAR)
$R_1 \leftarrow$ (MBR)
$R_2 \leftarrow$ (AC) + (R_1)
AC ← (R_2)
(D) MAR ← (IR (address))
MBR ← Memory(MAR)
AC ← (AC) + (MAR)

Statement for linked answer questions 7 to 9: Assume that the control memory is 24 bits wide. The control portion of the micro-instruction format is divided into two fields. A micro-operation field of 13-bits specifies the micro-operation to be performed. An address selection field specifies a condition, based on the flags, that will cause a micro-instruction branch. There are eight flags.

7. How many bits are there in address selection field?
(A) 1 (B) 2
(C) 3 (D) 4

8. How many bits are there in address field?
(A) 8 (B) 9
(C) 13 (D) 24

9. What is the size of control memory in bits?
(A) 256 (B) 768
(C) 3328 (D) 6144

10. A simple processor has 3 major phases to its instructions cycle:
1. Fetch
2. Decode
3. Execute

Two 1-bit flags are used to specify the current phase in hardwired implementation. Will these flags required in micro-programming also?
(A) Yes
(B) No
(C) Cannot predict
(D) Depends on clock cycle time

11. In a 3-bus data path, the micro instructions format will be Opcode src1, src2, desti; The number of operations supported are 8 and the src1, src2 and desti require 20, 16 and 20 bits respectively.
The total number of horizontal microinstructions specified will be
(A) 2^{64} (B) 2^{8}
(C) 2^{56} (D) 2^{61}

12. What is the smallest positive normalized number represented using IEEE single precision floating point representation?
(A) 2^{-128} (B) $1 - 2^{-127}$
(C) 2^{-127} (D) 2^{-126}

13. A micro program control unit is required to generate a total of 30 control signals. Assume that during any micro instruction, almost two control signals are active. Minimum number of bits required in the control word to generate the required control signals will be
(A) 2 (B) 2.5
(C) 10 (D) 12

14. What is the fraction field of the single-precision floating point representation of 6.25?
(A) 1110 1000 0000 0000 0000 000
(B) 1001 0000 0000 0000 0000 000
(C) 1100 0000 0000 0000 0000 000
(D) 0110 0100 0000 0000 0000 000

15. Let the total number of control signals generated are n, then what is the number of bits allocated in control field of vertical micro programming?
(A) $n/2$ (B) n
(C) 2^n (D) $\log_2^{\ n}$

16. In a micro programmed control unit, a control field of one address control instruction has to support two groups of control signals. In group1 it is required to generate either one or none of the 32 control signals. In group 2 at most 5 from the remaining, what will be the number of bits needed for the control field?
(A) 8 (B) 10
(C) 35 (D) 37

17. Assume that the exponent e is constrained to lie in the range $0 \leq e \leq x$, with a bias of q, that the base is b and that the significant is P-digits in length.
What is the largest positive value that can be written is normalized floating point?
(A) $b^{x-q}(1 - b^{-p})$ (B) b^{-q-1}
(C) b^{-q-p} (D) $b^{x-q}(b^{-p} - 1)$

18. By using Booth's Multiplication algorithm. Below two numbers are multiplied:
Multiplicand: 0111 0111 1011 1101
Multiplier: 0101 1010 1110 1110
How many additions/subtractions are required for the multiplication of the above two numbers?
(A) 8 (B) 10
(C) 13 (D) 7

19. Let us assume, we are multiplying two positive integers 1101 and 1011. The multiplicand M is 1101 and Multiplier Q is 1011. What is partial product after second cycle?
(A) 0110 1101 (B) 1001 1110
(C) 0100 1111 (D) 1000 1111

20. The decimal representation of the 2's complement number 1101011 is
(A) 21 (B) –21
(C) 219 (D) 91

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. A microprogrammed control unit
(A) is faster than a hard-wired control unit
(B) facilitates easy implementation of new instructions.
(C) is useful when very small programs are to be run.
(D) usually refers to the control unit of a micro-processor.

2. Micro-program is
(A) the name of a source program in micro computers.
(B) a primitive form of macros used in assembly language programming.
(C) a program of a very small size.
(D) the set of instruction indicating the basic elemental commands which directly control the operation of a system.

3. Programming that actually controls the path of signal or data within the computer is called
(A) System programming
(B) Micro-programming
(C) High-level language programming
(D) Assembly language programming

4. The instruction cycle time in a generic microprocessor is
(A) Longer than the machine cycle time
(B) Shorter than the machine cycle time
(C) Same as the machine cycle time
(D) Double the machine cycle time

5. Microprocessor unit or central processor unit consist of
(A) Control circuitry (B) ALU
(C) Memory (D) All of these

6. The exponent of a floating point number is represented in excess-N code so that
(A) the dynamic range is large
(B) overflow is avoided
(C) the precision is high
(D) the smallest number is represented efficiently

7. Using Booth's algorithm for Multiplication, the Multiplier −14 is coded as
(A) 11110 (B) 01110
(C) 10010 (D) 00010

8. Data Path consists of
(A) Registers (B) ALU
(C) Bus (D) All of these

9. A floating point number that has a '0' in MSB of mantissa is said to have ____
(A) Overflow (B) Underflow
(C) Normalization (D) Positive exponent

10. Let the Binary sum after BCD addition is stored in K, Z_8, Z_4, Z_2, and Z_1 Then the condition for a correction and output carry can be expressed as $C =$
(A) $K + Z_8 Z_4 + Z_8 Z_2$ (B) $K + Z_8 Z_4 + Z_4 Z_2$
(C) $K + Z_8 Z_2 + Z_8 Z_1$ (D) $K + Z_4 Z_2 + Z_2 Z_1$

11. Which of the following is an advantage of biased exponents?
(A) Convenient way to represent exponents
(B) Useful for conversion
(C) Convenient for comparison purposes
(D) All of these

12. Booth multiplication skips over runs of zeros and ones which reduces the number of add and subtract steps needed to multiply two n-bit numbers to n to a variable number whose average value n_{avg} is less than n what will be n_{avg}?
(A) $n/3$ (B) $n/4$
(C) $n/2$ (D) n

13. The sequence of events that happen during a fetch operation is:
(A) PC → memory → IR
(B) PC → MAR → memory → IR
(C) PC → MAR → memory → MDR → IR
(D) PC → memory → MDR → IR

14. Micro-programming is a technique for
(A) Programming input or output routines
(B) Programming the microprocessors
(C) Programming the control steps of a computer
(D) Writing small programs

15. In a micro program ____ specifies the address of Micro-instructions to be executed.
(A) AR (B) PC
(C) SP (D) CAR

16. Which one of the following statements is correct?
(A) Micro-programmed control unit is costlier and slow.
(B) Micro-programmed control unit are cheap and slow.
(C) Micro-programmed control unit is costlier and fast.
(D) Micro-programmed control unit are fast and cheaper.

17. Horizontal micro-instructions have
(A) High degree parallelism, more encoding of control information.
(B) High degree parallelism, little encoding of control information.
(C) Low degree parallelism, more encoding of control information.
(D) Low degree parallelism, little encoding of control information.

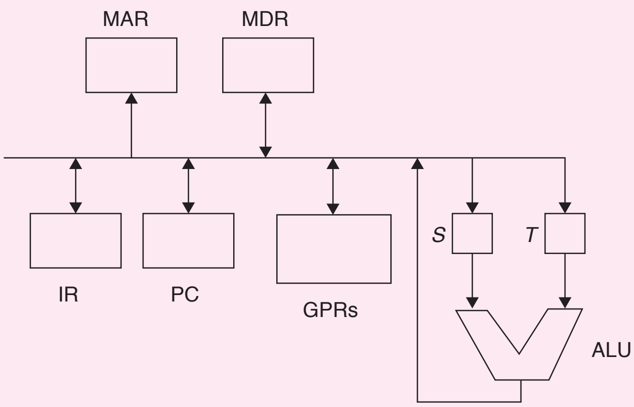
18. A vertical micro-instruction have ____.
(A) Short formats and considerable encoding of control information
(B) Long formats and considerable encoding of control information
(C) Short formats and little encoding of control information
(D) Long formats and little encoding of control information

19. Guard bits are used to
(A) avoid unnecessary loss of MSB
(B) avoid unnecessary loss of LSB
(C) the loss of MSB
(D) the loss of LSB

20. Which of the following is not the essential element of a number represented in floating-point notation?
(A) Exponent (B) Significand
(C) Sign (D) Normalization

Previous Years' Questions

Common data for questions 1 and 2: Consider the following data path of a CPU.



The ALU, the bus and all the registers in the data path are of identical size. All operations including incrementation of the PC and the GPRs are to be carried out in the ALU. Two clock cycles are needed for memory read operation—the first one for loading address in the MAR and the next one for loading data from the memory bus into the MDR.

1. The instruction 'add R_0, R_1' has the register transfer interpretation $R_0 \leftarrow R_0 + R_1$. The minimum number of clock cycles needed for execution cycle of this instruction is
   (A) 2 (B) 3
   (C) 4 (D) 5

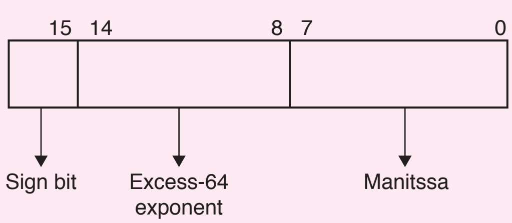
2. The instruction 'call *Rn*, sub' is a two word instruction. Assuming that PC is incremented during the fetch cycle of the first word of the instruction, its register transfer interpretation is

   $Rn \leftarrow$ PC + 1;

   PC $\leftarrow$ M[PC]

   The minimum number of CPU clock cycles, needed during the execution cycle of this instruction is
   (A) 2 (B) 3
   (C) 4 (D) 5

Data for question 3: Consider the following floating-point format.



Mantissa is a pure fraction in sign-magnitude form.

3. The normalized representation for the above format is specified as follows. The mantissa has an implicit 1 preceding the binary (radix) point. Assume that only 0's are padded in while shifting a field.

   The normalized representation of the above number (0.239×2^{13}) is: **[2005]**
   (A) 0*A* 20 (B) 11 34
   (C) 49 *D*0 (D) 4*A E*8

4. In the IEEE floating point representation the hexadecimal value 0*x*00000000 corresponds to **[2008]**
   (A) The normalized value 2^{-127}
   (B) The normalized value 2^{-126}
   (C) The normalized value + 0
   (D) The special value + 0

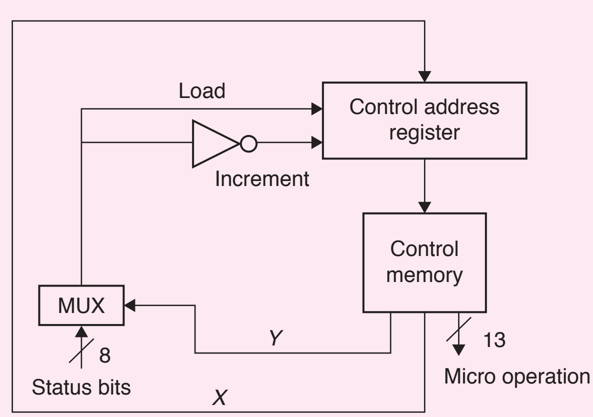
5. *P* is a 16-bit signed integer. The 2's complement representation of *P* is $(F87B)_{16}$. The 2's complement representation of 8**P* is **[2010]**
   (A) $(C3D8)_{16}$ (B) $(187B)_{16}$
   (C) $(F878)_{16}$ (D) $(987B)_{16}$

6. The decimal value 0.5 in IEEE single precision floating point representation has **[2012]**
   (A) fraction bits of 000 … 000 and exponent value of 0
   (B) fraction bits of 000…000 and exponent value of –1
   (C) fraction bits of 100…000 and exponent value of 0
   (D) no exact representation

7. The smallest integer that can be represented by an 8-bit number in 2's complement form is **[2013]**
   (A) –256 (B) –128
   (C) –127 (D) 0

8. Let *A* = 1111 1010 and *B* = 0000 1010 be two 8-bit 2's complement numbers. Their product in 2's complement is **[2004]**
   (A) 1100 0100 (B) 1001 1100
   (C) 1010 0101 (D) 1101 0101

9. The microinstructions stored in the control memory of a processor have a width of 26 bits. Each microinstruction is divided into three fields, a micro-operation field of 13 bits, a next address field (*X*), and a MUX select field (*Y*), there are 8 status bits in the inputs of the MUX **[2004]**



How many bits are there in the X and Y fields, and what is the size of the control memory in number of words?

(A) 10, 3, 1024 (B) 8, 5, 256
(C) 5, 8, 2048 (D) 10, 3, 512

10. Consider the following sequence of micro-operations.

MBR ← PC
MAR ← X
PC ← Y
Memory ← MBR

Which one of the following is a possible operation performed by this sequence? **[2013]**

(A) Instruction fetch
(B) Operand fetch
(C) Conditional branch
(D) Initiation of interrupt service

11. For computers based on three-address instruction formats, each address field can be used to specify which of the following: **[2015]**

(S_1) A memory operand
(S_2) A processor register
(S_3) An implied accumulator register

(A) Either S_1 or S_2
(B) Either S_2 or S_3
(C) Only S_2 and S_3
(D) All of S_1, S_2 and S_3

12. Let X be the number of distinct 16 - bit integers in 2's complement representation. Let Y be the number of distinct 16 - bit integers in sign magnitude representation. They $x - y$ is ______ . **[2016]**

13. The n-bit fixed-point representation of an unsigned real number X uses f bits for the fraction part. Let $i = n - f$. The range of decimal values for X in this representation is **[2017]**

(A) 2^{-f} to 2^i (B) 2^{-f} to $(2^i - 2^{-f})$
(C) 0 to 2^i (D) 0 to $(2^i - 2^{-f})$

14 Consider the C code fragment given below.

```
typedef struct node  {
  int data;
  node* next;
}   node;
void join (node* m, node* n)  {
    node* p = n;
    while (p - >next != NULL)  {
      p = p - >next;
    }
    p - >next = m;
}
```

Assuming that m and n point to valid NULL-terminated linked lists, invocation of join will **[2017]**

(A) append list m to the end of list n for all inputs.
(B) either cause a null pointer dereference or append list m to the end of list n.
(C) cause a null pointer dereference for all inputs.
(D) append list n to the end of list m for all inputs.

15. The representation of the value of a 16-bit unsigned integer X in hexadecimal number system is BCA9. The representation of the value of X in octal number system is **[2017]**

(A) 571244 (B) 736251
(C) 571247 (D) 136251

16. Consider the following processor design characteristics.

I. Register-to-register arithmetic operations only
II. Fixed-length instruction format
III. Hardwired control unit

Which of the characteristics above are used in the design of a RISC processor? **[2018]**

(A) I and II only (B) II and III only
(C) I and III only (D) I, II and III

17. Consider the unsigned 8-bit fixed point binary number representation below:

$$b_7\, b_6 b_5 b_4\, b_3 \cdot b_2\, b_1\, b_0$$

where the position of the binary point is between b_3 and b_2. Assume b_7 is the most significant bit. Some of the decimal numbers listed below cannot be represented exactly in the above representation:

(i) 31.500 (ii) 0.875
(iii) 12.100 (iv) 3.001

Which one of the following statements is true? **[2018]**

(A) None of (i), (ii), (iii), (iv) can be exactly represented
(B) Only (ii) cannot be exactly represented
(C) Only (iii) and (iv) cannot be exactly represented
(D) Only (i) and (ii) cannot be exactly represented

Answer Keys

Exercises

Practice Problems I

1. D	**2.** B	**3.** C	**4.** C	**5.** A	**6.** C	**7.** C	**8.** A	**9.** D	**10.** B
11. A	**12.** D	**13.** C	**14.** B	**15.** D	**16.** B	**17.** A	**18.** B	**19.** B	**20.** B

Practice Problems I

1. B	**2.** D	**3.** B	**4.** C	**5.** D	**6.** D	**7.** C	**8.** D	**9.** B	**10.** A
11. C	**12.** C	**13.** C	**14.** C	**15.** D	**16.** A	**17.** B	**18.** A	**19.** B	**20.** D

Previous Years' Questions

1. B	**2.** B	**3.** D	**4.** D	**5.** A	**6.** B	**7.** B	**8.** A	**9.** A	**10.** D
11. A	**12.** 1	**13.** D	**14.** B	**15.** D	**16.** D	**17.** C			

Chapter 3

Memory Interface, I/O Interface

LEARNING OBJECTIVES

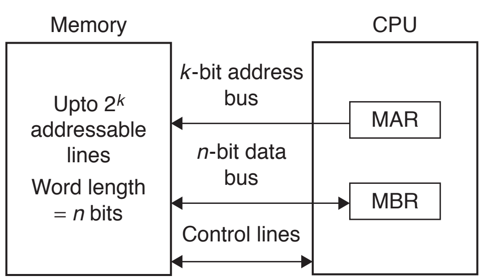
- *Memory interface*
- *RAM*
- *ROM*
- *Memory interfacing*
- *Input–output interfacing*
- *Handshaking*
- *Data transfer mode*
- *Design techniques for interrupts*
- *Direct memory access*
- *Input–output processor*

Memory Interface

Basic Concepts

Computer memory is used to store programs and data. The maximum size of a memory that can be used in any computer is determined by the addressing scheme.

Example: If the memory address has 16-bits, then the size of memory will be 2^{16} Bytes.



If MAR is k-bits long and MDR is n-bits long, then the memory may contain up to 2^k addressable locations and the n-bits of data are transferred between the processor and memory. This transfer takes place over processor bus. The processor bus has

1. Address line
2. Data line
3. Control line

Control line is used for coordinating data transfer.

Processor reads the data from the memory by loading the address of the required memory location into MAR and setting the $R/\overline{W}$ line to 1.

The memory responds by placing the data from the addressed location onto the data lines and confirms the actions. Upon confirmation, the processor loads the data onto the data lines, into MDR register. The processor writes the data into the memory location by loading the address of this location into MAR and loading the data into MDR sets the $R/\overline{W}$ line to 0.

- **Memory Access Time:** It is the time that elapses between the initiation of an operation and the completion of that operation.
- **Memory Cycle Time:** It is the minimum time delay that required between the initiations of two successive memory operations.

RAM (Random Access Memory)

In RAM, if any location that can be accessed for a read/write operation in fixed amount of time, it is independent of the location's address:

- Memory cells are usually organized in the form of array, in which each cell is capable of storing one bit of information.
- Each row of cells constitutes a memory word and all cells of a row are connected to a common line called as word line.
- The cells in each column are connected to sense/write circuit by two bit lines.

The data input and data output of each sense/write circuit are connected to a single bidirectional data line that can be connected to a data bus.

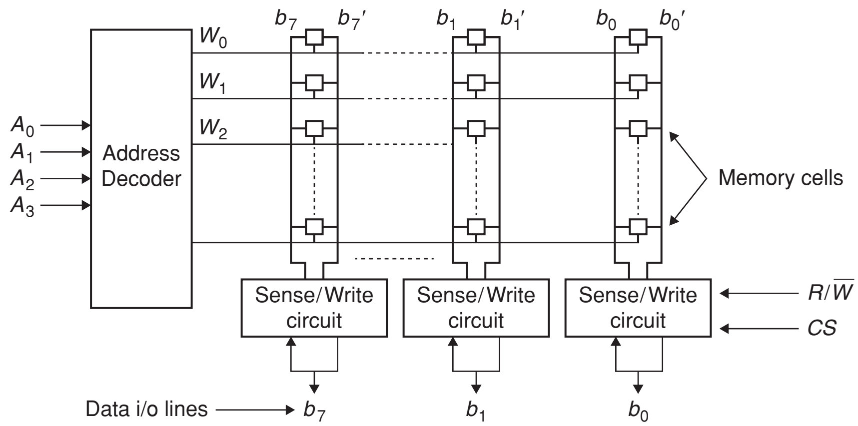


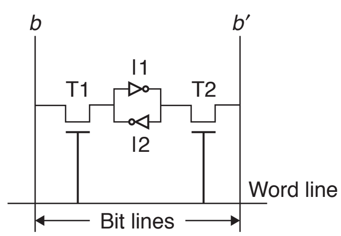
Figure 1 Organization of bit cells in a memory chip

- $R/\overline{W}$: Specifies the required operation.
- CS: Chip select input selects a given chip in the multi-chip memory system.

Static memories

Memories that consist of circuits capable of retaining their state as long as power is applied are known as static memories.

SRAM (static RAM) SRAM consists of two inverters, two transistors. In order to read the state of the SRAM cell, the word line is activated to close switches T1 and T2.



Advantages of SRAM:

1. It has low power consumption, because the current flows in the cell only when the cell is being activated or accessed.
2. SRAM can be accessed quickly.

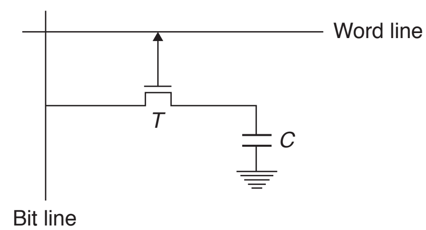
Disadvantages of SRAM: SRAMs are said to be volatile memories, because their contents are lost when the power is interrupted.

DRAM (Dynamic RAM) Less expensive RAMs can be implemented if simplex cells are used, such cells cannot retain their state indefinitely. Hence they are called dynamic RAMs.

The information stored in a dynamic memory cell in the form of a charge on a capacitor and this charge can be maintained only for tens of milliseconds.

The contents must be periodically refreshed by restoring the capacitor charge to its full value.

Example: Single-transistor dynamic memory cell:



If charge on capacitor > threshold value, then bit line will have ‘1’. If charge on capacitor < threshold value, then bit line will have ‘0’.

DRAM	SRAM
1. Volatile	1. Volatile
2. Simple to build and slower than SRAM	2. Faster than DRAM
3. Need refresh circuitry	3. More expensive to build
4. Favoured for large memory units	4. Favoured for cache memory

Latency It is the amount of time it takes to transfer a word of data to or from the memory.

- For the transfer of a single word, the latency provides the complete indication of memory performance.
- For a block transfer, the latency denotes the time it takes to transfer the first word of data.

Bandwidth It is defined as the number of bits or bytes that can be transferred in one second.

Note: All dynamic memories have to be refreshed.

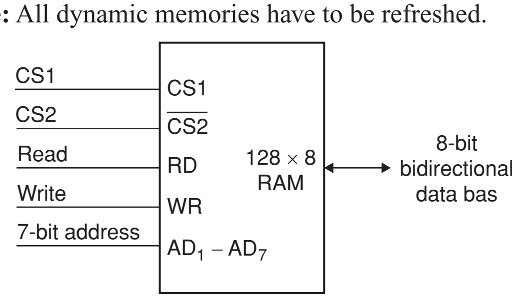


Figure 2 RAM chip block diagram

Read-only Memory (ROM)

Both SRAM and DRAM chips are volatile, which means that they lose the stored information if power is turned off. If the normal operation involves only reading of stored data, use ROM memory.

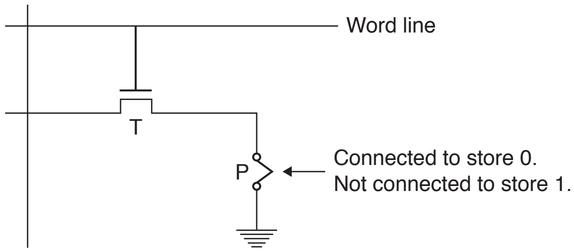


Figure 3 ROM cell

Types of ROM

Different types of non-volatile ROM are:

1. **PROM (Programmable ROM):**
   - Allows the data to be loaded by the user.
   - Less expensive, faster, flexible.
2. **EPROM (Erasable PROM):**
   - Allows the stored data to be erased and new data to be loaded.
   - Flexible, retain information for a long time.
   - Contents erased by UV light.
3. **EEPROM (Electrically Erasable PROM):**
   - Programmed and erased electrically.
   - Allows the erasing of all cell contents selectively.
   - Requires different voltage for erasing, writing and reading of stored data.
4. **Flash memory:** Allows to read the contents of a single cell but it is only possible to write the entire contents of a block.

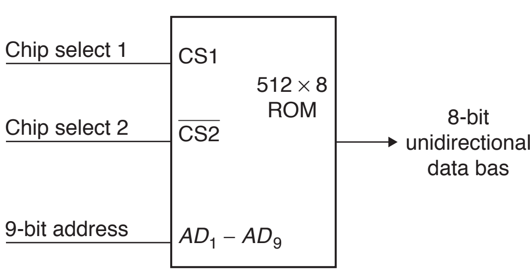


Figure 4 Block diagram of ROM chip

Memory Interfacing

The interfacing circuit enables the access of processor to memory. The function of memory interfacing is that the processor should be able to read from and write into a given register of a memory chip. To perform this, the microprocessor should be

1. able to select the chip.
2. identify the register.
3. enable the appropriate buffer.

Input–output Interfacing

Basic Concepts of I/O Module

I/O module contains logic for performing a communication function between the peripherals and the bus. The peripherals are not connected to the system bus directly. The reasons for this are

1. Peripherals are electromechanical and electromagnetic devices and their manner of operation is different from the operation of the CPU and memory, which are electronicdevices. So a conversion of signal values may be required.
2. The data transfer rate of peripherals is usually slower than the transfer rate of the CPU and hence a synchronization mechanism may be needed.
3. Data codes and formats in peripherals differ from the word format in the CPU and memory.
4. The operating modes of peripherals are different from each other and each must be controlled so as not to disturb the operation of other peripherals connected to the CPU.

To resolve these differences, computer systems include special hardware components between the CPU and peripherals to supervise and synchronize all input and output transfers. These components are called 'interface' units.

By using this interfacing,

1. interface to the processor and memory via the system bus or central switch.
2. interface to one or more peripheral devices by tailored data links.

Input–output devices

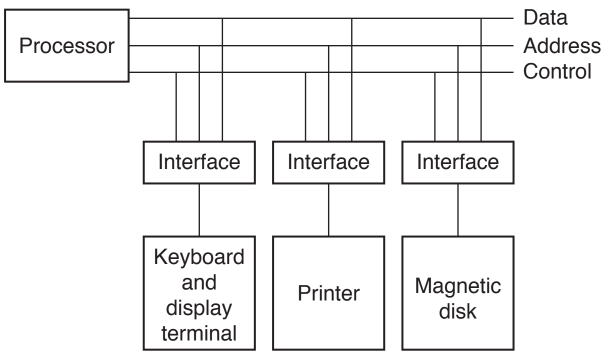
- Input and Output devices provide a means for people to make use of a computer.
- Some I/O devices function as an interface between a computer system and other physical system.

Input–output interface

Input/output Interface provides a method for transferring information between internal storage (such as memory and CPU Register) and external I/O devices. It resolves the difference between the computer and peripheral devices.

Input–output bus and interface modules

Each peripheral has an interface module associated with it. The interface module decodes the device address (device code), decodes signals for the peripheral controller, synchronizes the data flow and supervises the transfer rate between peripheral and CPU or memory.



Function of buses

1. **Memory bus:** It is used for information transfer between CPU and main memory.
2. **I/O bus:** It is used for information transfers between CPU and I/O devices through their I/O interface.

Isolated versus memory mapped I/O

1. **Isolated I/O:**
   - Separate I/O read/write control lines in addition to memory read/write control lines.
   - Separate (isolated) memory and I/O address space
   - Distinct input and output instructions.
2. **Memory-mapped I/O:**
   - A single set of Read/write control lines (i.e., no distinction between memory and I/O transfer).
   - Memory and I/O address share the common address space (reduces memory address range available).
   - No specific input or output instruction.
   - The same memory reference instructions can be used for I/O transfer.
   - Considerable flexibility in handling I/O operations.

Asynchronous serial transfer

In serial data transmission, each bit in the message is sent in sequence one at a time. Serial transmission can be synchronous or asynchronous.

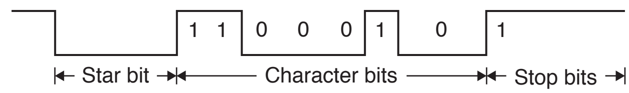
In synchronous transmission, the two units share a common clock frequency and bits are transmitted continuously at the rate dictated by the clock pulses.

In asynchronous transmission, binary information is sent only when it is available and the line remains idle when there is no information to be transmitted.

In serial asynchronous transmission technique, each character consists of three parts:

1. start bits
2. character bits
3. stop bits

Example:



A transmitted character can be detected by the receiver from the knowledge of the transmission rules:

1. When a character is not being sent, the line is kept in the 1-state.
2. The initiation of a character transmission is detected from the start bit, which is always 0.
3. The character bits always follow the start bit.
4. After the last bit of the character is transmitted, a stop bit is detected when the line returns to the 1-state for at least one bit time.
   - The baud rate is defined as the rate at which serial information is transmitted and is equivalent to the data transfer in bits per second.

Strobe control Employs a single control line to time each transfer. Strobe may be activated by either the source or the destination units.

(a) Source initiated transfer:

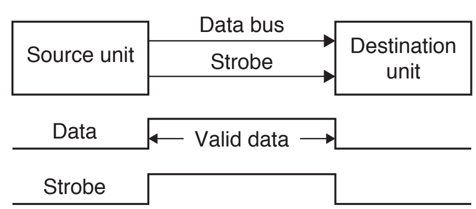
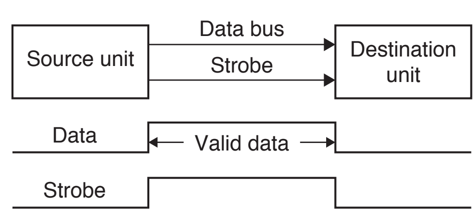


Figure 5 Source initiated strobe for data transfer

- The data bus carries the binary information from source unit to the destination unit.
- The strobe is a single line that informs the destination unit when a valid data word is available in the bus.

(b) Destination initiated strobe for data transfer:



- The destination unit activates the strobe pulse, informing the source to provide the data. The source unit responds by placing the requested binary information on the data bus.
- The data must be valid and remain in the bus long enough for the destination unit to accept it.
- The falling edge of the strobe pulse can be used again to trigger a destination register. The destination unit then disables the strobe.

Handshaking

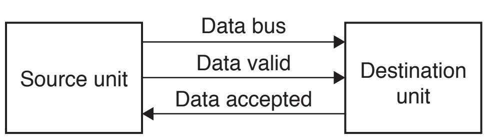
Disadvantage of strobe method: Source unit which initiated the transfer has no way of knowing whether the destination unit has actually received the data item that was placed in the bus.

The handshake method solves this problem by introducing a second control signal that provides a reply to the unit that initiates the transfer.

Principle of two-wire handshaking: One control line is in the same direction as the data flow in the bus from the source to the destination. It is used by the source unit to inform the destination unit whether there are valid data in the bus.

The other control line is in the other direction from the destination to the source. It is used by the destination unit to inform the source whether it can accept data.

The sequence of control during the transfer depends on the unit that initiates the transfer.



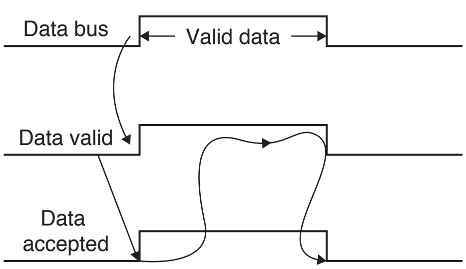


Figure 6 Source initiated transfer using hand shaking

Similarly a destination unit may also initiate the transfer.

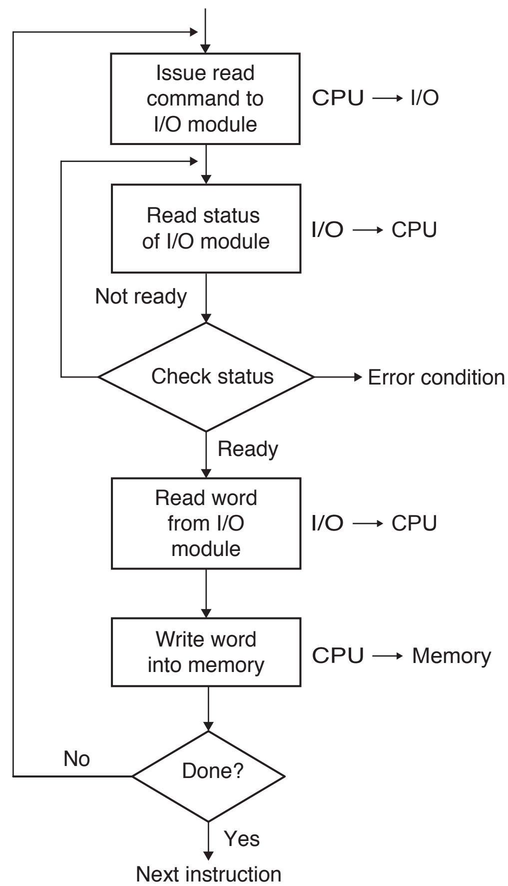
Advantage: Handshaking scheme provides a high degree of flexibility and reliability because the successful completion of a data transfer relies on active participation by both units.

Modes of transfer

There are three different data transfer modes between the central computer (CPU or Memory) and peripherals:

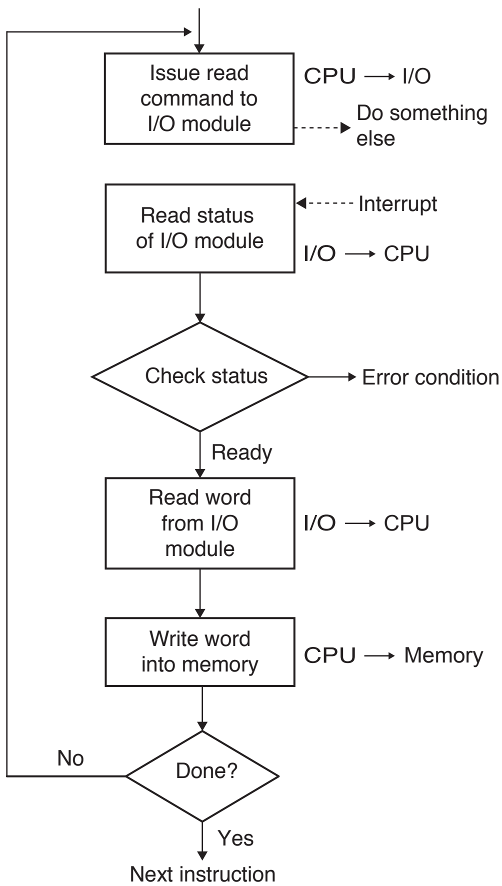
1. Program-controlled I/O
2. Interrupt-initiated I/O
3. Direct memory access

Program-controlled input–output With programmed I/O, the I/O module will perform the requested action and then set the appropriate bits in the I/O status register. The I/O module takes no further action to alert the CPU. In particular it does not interrupt the CPU. Thus, it is the responsibility of the CPU to periodically check the status of the I/O module until it finds that the operation is complete.



Interrupt initiated input–output The problem with programmed I/O is that the CPU has to wait a long time for the I/O module of concern to be ready for either reception or transmission of data. The CPU, while waiting must repeatedly interrogate the status of the I/O module. As a result, the level of the performance of the entire system is severely degraded.

An alternation is for the CPU to issue an I/O command to a module and then go on to do some other useful work. The I/O module will then interrupt the CPU to request service when it is ready to exchange data with the CPU. The CPU then executes the data transfer, as before and then resumes its former processing.



Interrupt Processing: In all computers, there is a mechanism by which the normal processing of the processor is interrupted by other modules like I/O, memory. The interrupts may be of the following class:

1. Program: Generated by some condition that occurs as a result of an instruction execution.

   Examples: Arithmetic overflow, division by zero, etc.
2. Timer: Generated by timer within the processor.
3. I/O: Generated by an I/O controller, to signal normal completion of an operation or to signal a variety of error conditions.
4. Hardware failure: Generated by a failure such as power failure or memory parity error.

Interrupts are provided primarily as a way to improve processing efficiency.

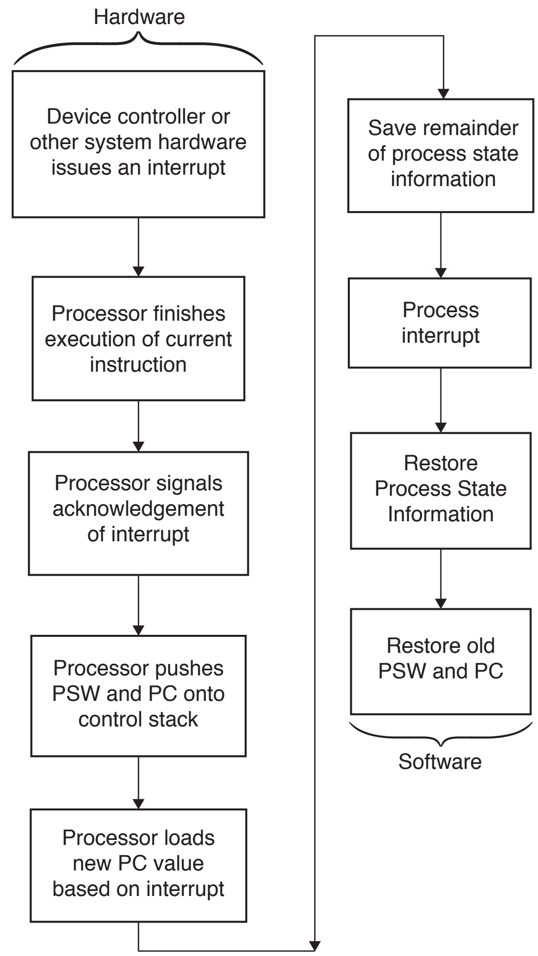


Figure 7 Interrupt Processing Flowchart

Consider the following figures, which show the contents of memory and registers before and after interrupt instruction processing.

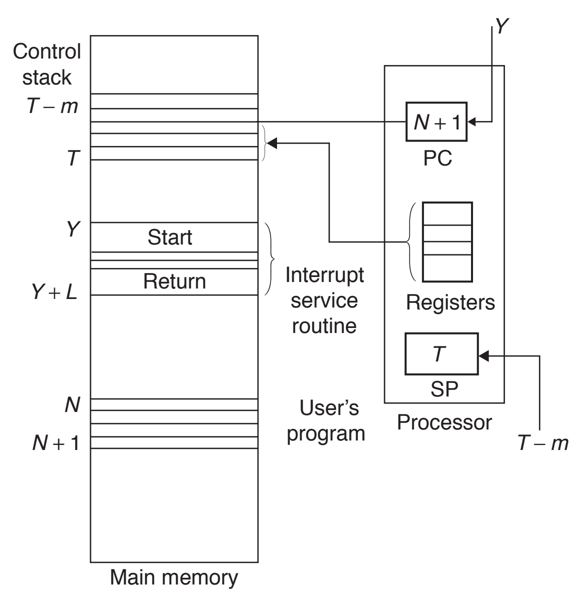


Figure 8 Interrupt occurs after instruction at location *N*

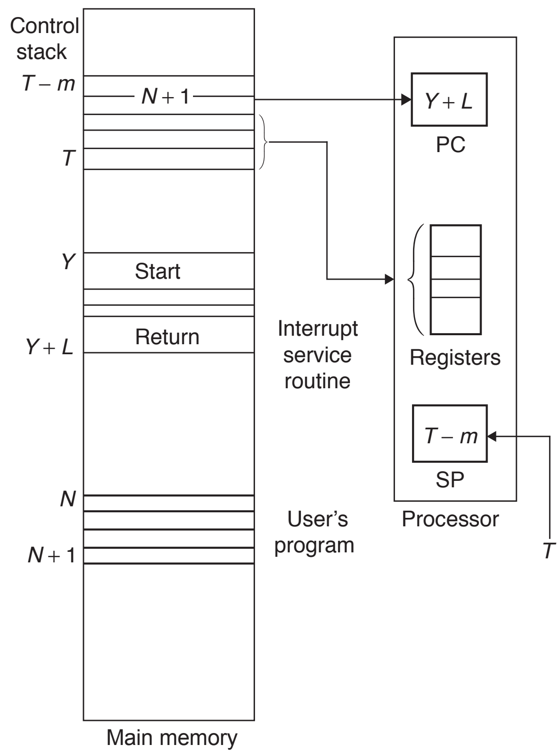


Figure 9 Return from interrupt

Interrupt priority: Priority determines which interrupt is to be served first when two or more requests are made simultaneously. Priority also determines which interrupts are permitted to interrupt the computer while another is being serviced. Higher priority interrupts can make requests while servicing a lower priority interrupt.

Design techniques for interrupts: Two design issues arise in implementing interrupt I/O:

1. Since there will almost invariably be multiple I/O module, how does the CPU determine which device issued the interrupt.
2. If multiple interrupts have occurred, how does the CPU decide which one to process.

Four general categories of techniques are there which are common in use:

(a) Multiple interrupt lines: In this technique, multiple interrupt lines are provided between the CPU and the I/O modules. However, it is impractical to dedicate more than a few bus lines or CPU pins to interrupt lines. Consequently, even if multiple lines are used, it is likely that each line will have multiple I/O modules attached to it. Thus, one of the other three techniques must be used one each line.

(b) Software poll: When the CPU detects an interrupt, it braches to an interrupt-service routine whose job is to poll each I/O module to determine which module generated the interrupt. The poll could be in the form of a separate command line. The CPU receives the command and places the address of a particular I/O module on the address lines. The I/O module responds positively if it set the interrupt. Alternatively, each I/O module could contain an addressable status register. The CPU then read the status register of each I/O module to identify the interrupting module. Once the correct module is identified, the CPU branches to a device service routine specified to that device. It is time consuming.

(c) Daisy chain: Daisy chain in effect provides a hardware poll. For interrupts all I/O modules share a common interrupt request line. The interrupt acknowledge line is daisy chained through the modules. When the CPU is interrupted, it sends out an interrupt acknowledgement. This signal propagates through a series of I/O modules until it gets to a requesting module. The requesting module typically responds by placing a word on the data lines. This word is referred to as a vector and is either the address of the I/O module or some other unique identifier. In either case, the CPU uses the vector as a pointer to the appropriate device-service routine. This avoids the need to execute a general interrupt-service routine first. This technique is referred to as a vectored Interrupt.

(d) Bus arbitration: Bus arbitration is also another technique which makes use of vectored Interrupts. With bus arbitration, an I/O module must first gain control of the bus before it can raise the interrupt request line. Thus only one module can raise the line at a time. When the CPU detects the interrupt, it responds on the interrupt acknowledge line. The requesting module then places its vector on the data lines.

Direct Memory Access (DMA)

Drawbacks of programmed and interrupt-driven I/O:

1. The I/O transfer rate is limited by the speed with which the processor can test and service a device.
2. The processor is tied up in managing I/O transfer; a number of instructions must be executed for each I/O transfer.

Both methods have an adverse impact on both processor activity and I/O transfer rate.

When large volume of data is to be moved, a more efficient technique is required: Direct Memory Access (DMA).

DMA function: DMA involves an additional module on the system bus.

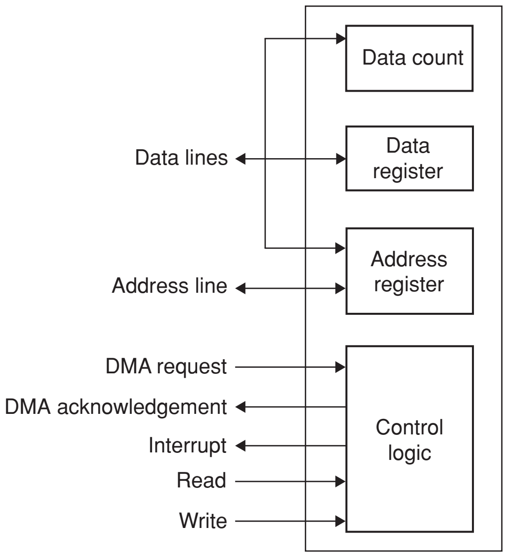
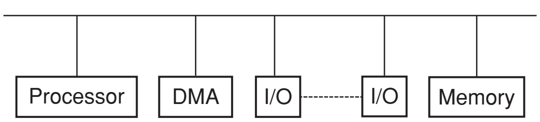


Figure 10 DMA Block Diagram

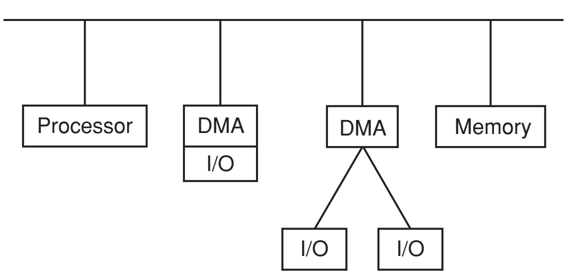
The DMA module is capable of mimicking the processor and indeed, of taking over control of the system from the processor. It needs to do this to transfer data to and from memory over the system bus. For this purpose, the DMA module must use the bus only when the processor does not need it or it must force the processor to suspend operation temporarily. The latter technique is more common and is referred as cycle stealing.

DMA configurations

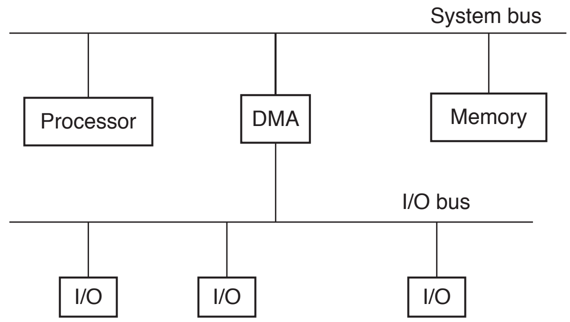
1. **Single bus, detached DMA**
   - Inexpensive, inefficient
   - Each transfer of a word consumes two bus cycles.



2. **Single bus, integrated DMA I/O** There is a path between the DMA module and one or more I/O modules that does not include system bus.



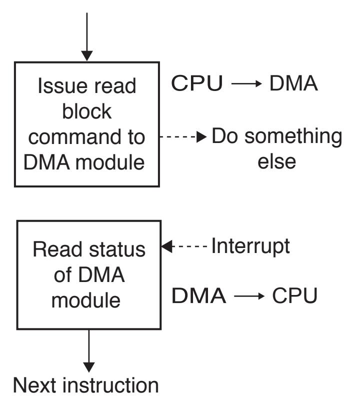
3. **I/O bus**
   - Reduces the number of I/O interfaces in the DMA module to one.
   - Easily expandable configuration.



With DMA, when the CPU wishes to read or write a block of data, it issues a command to the DMA module, by sending the following information to the DMA module.

1. Whether a read or write is requested.
2. The address of the I/O device involved.
3. The starting location in memory to read from or write to.
4. The number of words to be read or written.

The CPU then continues with other work. It has delegates this I/O operation to the DMA module, and that module will take care of it. The DMA module transfers the entire block of data, one word at a time, directly to or from memory, without going through the CPU. When the transfer is complete, the DMA module sends an interrupt signal to the CPU. Thus, the CPU is involved only at the beginning and end of the transfer.



DMA transfer can either happen as:

1. **Burst transfer**
   - A block sequence consisting of a number of memory words is transferred in continuous burst.
   - DMA controller is master of memory Buses.
   - This mode of transfer is needed for fast devices such as Magnetic Disks, where transmission cannot be stopped or slowed down.
2. **Cycle stealing**
   - CPU is usually much faster than I/O (DMA), thus CPU uses the most of the memory cycles.
   - DMA controller steals the memory cycles from CPU.
   - For those stolen cycles, CPU remains idle.

- For those slow CPU, DMA Controller may steal most of the memory.
- Cycle stealing, which may cause CPU remain idle long time.

Input–Output Processor (IOP)

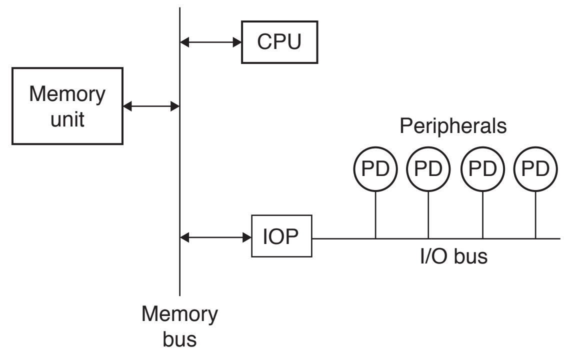
An IOP is a processor, having a direct memory access capability, used to communicate with I/O devices.

In this configuration, the computer system can be divided into a memory unit and a number of processors comprised of the CPU and one or more IOPs.

Each IOP takes care of input and output tasks, relieving the CPU from the house keeping chores involved in I/O transfers.

- IOP is similar to a CPU except that it is designed to handle the details of I/O processing.
- Unlike DMA, the IOP can fetch and execute its own instructions.

The following figure shows a computer with two processors:



Exercises

Practice Problems I

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Consider a DRAM that must be given a refresh cycle 64 times per ms. Each refresh operation requires 150 ns, a memory cycle requires 250 ns. What is the approximate percentage of the memory's total operating time must be given to refreshes?
(A) 1% (B) 2%
(C) 9% (D) 60%

2. A DMA controller transfers 16-bit words to memory using cycle stealing. The words are assembled from a device that transmits characters at a rate of 2400 characters per second. The CPU is fetching and executing instructions at an average rate of 1 million instructions per second. By how much time will the CPU be slowed down because of the DMA transfer?
(A) 0.6% (B) 0.1%
(C) 0.12% (D) 0.24%

3. A system is based on a 16-bit microprocessor and has two I/O devices. The I/O controllers for this system use separate control and status registers. Both devices handle data on a one-byte-at-a time basis. The first device has two status lines and three control lines. The second device has three status lines and four control lines. How many 16-bit I/O control module registers do we need for status reading and control of each device?
(A) 1, 2 (B) 2, 1
(C) 2, 2 (D) 1, 1

4. In a programmed I/O technique, the processor is stuck in a wait loop doing status checking of an I/O device. To increase efficiency, the I/O software could be written so that the processor periodically checks the status of the device. If the device is not ready, the processor can jump to other tasks. After some timed interval, the processor comes back to check status again. Let us assume that above scheme is used for outputting data one character at a time to a printer that operates at 10 characters per second (CPS). Which of the following statement is true if its status is scanned every 200 ms?
(A) The printing speed is increased by 5 CPS
(B) The printing rate is slowed to 5 CPS
(C) The printing rate is at 10 CPS only
(D) The printing rate is at 20 CPS

5. Consider a system employing interrupt-driven I/O for a particular device that transfers data at an average of 8 KB/s on a continuous basis. The interrupt processing takes about 100 μs and the I/O device interrupts processor for every byte. Let assume that the device has two 16-byte buffers and interrupts the processor when one of the buffer is full. While executing the ISR, the processor takes about 8 μs for the transfer of each byte. Then what is the fraction of processor time is consumed by this I/O device?
(A) 8% (B) 11%
(C) 50% (D) 65%

6. A 32-bit computer has two selector channels one multiplexor channel. Each selector channel supports two magnetic disks and three magnetic tape units. The multiplexor channel has two line printers, two card readers and 10 VDT terminals connected to it. Assume the following transfer rates:

Disk drive: 1000 KB/sec
Magnetic tape drive: 300 KB/sec
Line printer: 6.2 KB/sec
Card reader: 2.4 KB/sec
VDT: 1 KB/sec

What is the maximum aggregate I/O transfer rate of this system?
(A) 1625.6 KB/s (B) 1327.2 KB/s
(C) 2027.2 KN/s (D) 2327.2 KB/s

7. Consider a disk drive with 16 surfaces, 512 tracks per surface and 512 sectors per track, 1 kilo bytes per sector and a Rotation speed of 3000 RPM. The disk is operated in cycle stealing mode where by whenever one 4 byte word is ready it is sent to memory; similarly, for writing, the disk interface read a 4 byte word from the memory in each DMA cycle. The memory cycle time is 40 nsec. Find the maximum percentage of time that the CPU gets blocked during DMA operation?
(A) 2.62% (B) 26.21%
(C) 0.26% (D) 0.52%

8. How many RAM chips of size (256 K × 1-bit) are needed to build a 1 M Byte memory?
(A) 16 (B) 8
(C) 32 (D) 24

9. Four memory chips of 16 × 4 size have their address bases connected together. The whole system will have a size of
(A) 16 × 8 (B) 64 × 64
(C) 16 × 16 (D) 256 × 1

10. In which of following I/O techniques, there will be no interrupt?
(A) Programmed I/O (B) Interrupt-driven I/O
(C) DMA (D) Both (B) and (C)

11. The capacity of a memory unit is defined by the number of words multiplied by the number of bits/word. How many separate address and data lines are needed for a memory 16K × 16?
(A) 10 address, 4 data lines
(B) 14, 4
(C) 14, 16
(D) 14, 14

12. The main problem of strobe asynchronous data transfer is
(A) it employs a single control line
(B) it is controlled by clock pulses in the CPU.
(C) the falling edge again used to trigger
(D) no way of knowing whether the destination has received the data item.

13. Which of the following DMA transfer modes and interrupt handling mechanisms will enable the highest I/O bandwidth?
(A) Block transfer and polling interrupt
(B) Cycle stealing and polling interrupt
(C) Block transfer and vectored interrupt
(D) Transparent DMA and vectored interrupt

14. Which of the following enables peripherals to pass a signal down the bus to the next device on the bus during polling of the device?
(A) Interrupt vectoring (B) Cycle stealing
(C) DMA (D) Daisy chain

15. What will be the response of the CPU, on receiving an interrupt from an input/output device?
(A) It hands over the control of address bus and data bus to the interrupting device.
(B) It branches off to the interrupt service routine after completion of the current instruction.
(C) It halts for a predetermined time.
(D) It branches off to the interrupt service routine immediately.

16. What is the bandwidth of memory system that has a latency of 50ns, a pre charge time of 10ns and transfers 2 bytes of data per access?
(A) 60 B/sec (B) 1.67 B/sec
(C) 1.67×10^7 B/sec (D) 3.33×10^7 B/sec

17. A hard disk is connected to a 50MHz processor through a DMA controller. Assume that the initial set-up of a DMA transfer takes 2000 clock cycles for the processor and also assume that the handling of the interrupt at DMA completion requires 1000 clock cycles for the processor. The hard disk has a transfer rate of 4000 K bytes/sec and average block size transferred is 8 K bytes. What fraction of the processor time is consumed by the disk, if the disk is actively transferring 100% of the time?
(A) 1% (B) 1.5%
(C) 2% (D) 3%

18. A device with transfer rate of 20KB/sec is connected to a CPU. Data is transferred byte wise. Let the interrupt overhead is 6 micro seconds. The byte transfer time between the device interface register and CPU or memory is negligible. What is minimum performance gain of operating the device under interrupt mode over operating it under program-controlled mode?
(A) 6 (B) 8
(C) 10 (D) 12

19. A DMA module is transferring characters to main memory from an external device at 76800 bits per second. The processor can fetch instructions at a rate of 2 million instructions per second. How much will the processor be slowed down due to DMA activity? (Express this as a percent of the time from when there is a conflict between DMA and the CPU)
(A) 0.24% (B) 0.48%
(C) 0.96% (D) 0.50%

20. Let us suppose that we want to read 2048 bytes in programmed I/O mode of CPU. The bus width is 32-bits. Each time an interrupt occurs from Hard disk drive and it taken 4 μsec to service it. How much CPU time is required to read 2048 bytes?
(A) 512 msec (B) 768 msec
(C) 1024 msec (D) 2048 msec

Practice Problems 2

Directions for questions 1 to 21: Select the correct alternative from the given choices.

1. Memory which is ultraviolet erasable is
(A) RAM (B) EPROM
(C) PROM (D) EEPROM

2. Memory which is electrically erasable is
(A) EPROM (B) EEPROM
(C) ROM (D) PROM

3. The minimum time delay that is required between the initiation of two successive memory operations is called
(A) Memory access time (B) Transmission time
(C) Seek Time (D) Memory cycle

4. The memory that is programmed at the time of manufacture is
(A) RAM (B) PROM
(C) ROM (D) EEPROM

5. The disadvantage of dynamic RAM over static RAM is
(A) High power consumption
(B) Higher bit density
(C) Need to refresh the capacitor charge every once in two milliseconds.
(D) Variable speed

6. If an error is detected, a part of the memory can be erased in
(A) PROM (B) EPROM
(C) EAROM (D) EROM

7. What are sequences of events in source initiated hand shaking transfer?
(A) Source enable data valid, destination enable data accepted, source disable data valid, destination disable data accepted
(B) Source disable data valid, destination enable data accepted, source disable data valid, destination enable data accepted
(C) Source disable data valid, destination Disable data valid, source enable data valid, destination enable data accepted.
(D) Source disable data valid, destination enable data valid, source enable data valid, destination disable data accepted.

8. Processor needs software interrupt to
(A) return from subroutine
(B) implement co-routines
(C) test the interrupt system of the processor
(D) obtain system services which need execution of privileged instructions

9. A microcomputer has primary memory of 512 KB. what is the exact number of bytes contained in this memory?
(A) 512×1000 (B) 512×100
(C) 512×1024 (D) 512×1028

10. The number of address lines required in a microprocessor which has to access 1 K bytes of memory is
(A) 6 (B) 4
(C) 10 (D) 8

11. Software interrupt is
(A) used to stimulate an external device
(B) generated by an external device
(C) Both (A) and (B)
(D) None of these

12. The bus that is used to transfer data from main memory to peripheral devices and vice-versa is
(A) Control bus (B) input bus
(C) output bus (D) DMA bus

13. The bus which is connected between the CPU and the main memory that permits transfer of information between the CPU and main memory is called
(A) memory bus (B) address bus
(C) control bus (D) DMA bus

14. An interrupt in which the external device supplies the interrupt requests as well as its address is called
(A) maskable interrupt
(B) vectored interrupt
(C) designated interrupt
(D) non-maskable interrupt

15. A temporarily ignored interrupt is called
(A) designated interrupt
(B) maskable interrupt
(C) non-maskable interrupt
(D) low priority interrupt

16. Which of the following device is used to connect a peripheral to a bus?
(A) control register
(B) interface
(C) communication protocol
(D) None of these

17. Which of the following is true for the daisy scheme of connecting input/output devices?
(A) It gives non-uniform priority to various devices.
(B) It gives uniform priority to all devices.
(C) It is only useful for connecting slow devices to a processor device.
(D) It requires a separate interrupt pin on the processor for each device.

18. In direct memory access data are directly transferred
(A) from CPU to input/output device and memory
(B) from an input/output device to memory only.
(C) from memory to an input/output device only.
(D) from an input/output device to the memory or vice versa

19. Which one of the following is true for a CPU having a single interrupt request line and a single interrupt grant line?
(A) Vectored interrupt multiple interrupting devices are always possible.
(B) Vectored interrupts are not possible but multiple interrupting devices are possible
(C) Vectored interrupts and multiple interrupting devices are sometimes possible
(D) Vectored interrupt is possible but multiple interrupting devices are not possible

20. In which of the following I/O, there is a single address space for memory locations and I/O devices
(A) Isolated I/O
(B) Memory mapped I/O
(C) DMA
(D) Both (A) and (B)

21. ____ signal used to interrupt processor and to execute service routine that takes an error recovery action.
(A) Strobe (B) Handshaking
(C) Polling (D) Time out

Previous Years' Questions

1. A device with data transfer rate 10 KB/sec is connected to a CPU. Data is transferred byte-wise. Let the interrupt overhead be 4 μsec. The byte transfer time between the device interface register and CPU or memory is negligible. What is the minimum performance gain of operating the device under interrupt mode over operating it under program-controlled mode? **[2005]**
(A) 15 (B) 25
(C) 35 (D) 45

2. A computer handles several interrupt sources of which the following are relevant for this question.
- Interrupt from CPU temperature sensor (raises interrupt if CPU temperature is too high)
- Interrupt from Mouse (raises interrupt if the mouse is moved or a button is pressed)
- Interrupt from Keyboard (raises interrupt when a key is pressed or released)
- Interrupt from Hard Disk (raises interrupt when a disk read is completed)

Which one of these will be handled at the HIGHEST priority? **[2011]**
(A) Interrupt from Hard Disk
(B) Interrupt from Mouse
(C) Interrupt from Keyboard
(D) Interrupt from CPU temperature sensor

The following information pertains to 3 and 4: Consider the following program segment for a hypothetical CPU having three user registers R_1, R_2 and R_3.

Instruction	Operation	Instruction Size (in words)
MOV R_1, 5000;	$R_1 \leftarrow$ Memory [5000]	2
MOV R_2 (R1);	$R_2 \leftarrow$ Memory [(R_1)]	1
ADD R_2, R3;	$R_2 \leftarrow R_2 + R_3$	1
MOV 6000, R_2;	Memory [6000] $\leftarrow R_2$	2
HALT;	Machine halts	1

3. Consider that the memory is byte addressable with size 32 bits, and the program has been loaded starting from memory location 1000 (decimal). If an interrupt occurs while the CPU has been halted after executing the HALT instruction, the return address (in decimal) saved in the stack will be **[2004]**
(A) 1007 (B) 1020
(C) 1024 (D) 1028

4. Let the clock cycles required for various operations be as follows:
Register to/from memory transfer: 3 clock cycles
ADD with both operands in register: 1 clock cycle
Instruction fetch and decode: 2 clock cycles per word
The total number of clock cycles required to execute the program is **[2004]**
(A) 29 (B) 24
(C) 23 (D) 20

5. A CPU generally handles an interrupt by executing an interrupt service routine **[2009]**
(A) As soon as an interrupt is raised.
(B) By checking the interrupt register at the end of fetch cycle.
(C) By checking the interrupt register after finishing the execution of the current instruction.
(D) By checking the interrupt register at fixed time intervals.

6. A hard disk with a transfer rate of 10 Mbytes/second is constantly transferring data to memory using DMA. The processor runs at 6000 MHz, and takes 300 and 900 clock cycles to initiate and complete DMA transfer respectively. If the size of the transfer is 20 Kbytes, what is the percentage of processor time consumed for the transfer operation?
(A) 5.0% (B) 1.0%
(C) 0.5% (D) 0.1%

7. On a non-pipelined sequential processor, a program segment, which is a part of the interrupt service routine, is given to transfer 500 bytes from an I/O device to memory.

Initialize the address register.

Initialize the count to 500

LOOP: Load a byte from device

Store in memory at address given by address register.

Increment the address register

Decrement the count

If count! = 0 go to LOOP

Assume that each statement in this program is equivalent to a machine instruction which takes one clock cycle to execute if it is a non-load/store instruction. The load-store instructions take two clock cycles to execute.

The designer of the system also has an alternate approach of using the DMA controller to implement the same transfer. The DMA controller requires 20 clock cycles for initialization and other overheads. Each DMA transfer cycle takes two clock cycles to transfer one byte of data from the device to the memory.

What is the approximate speedup when the DMA controller based design is used in place of the interrupt driven program based input-output? **[2011]**

(A) 3.4 (B) 4.4
(C) 5.1 (D) 6.7

8. Which of the following statements about synchronous and asynchronous I/O is NOT true? **[2008]**
(A) An ISR is invoked on completion of I/O in synchronous I/O but not in asynchronous I/O
(B) In both synchronous and asynchronous I/O, an ISR (Interrupt Service Routine) is invoked after completion of the I/O
(C) A process making a synchronous I/O call waits until I/O is complete, but a process making an asynchronous I/O call does not wait for completion of the I/O
(D) In the case of synchronous I/O, the process waiting for the completion of I/O is woken up by the ISR that is invoked after the completion of I/O

9. A main memory unit with a capacity of 4 megabytes is built using 1M × 1-bit DRAM chips. Each DRAM chip has 1K rows of cells with 1K cells in each row. The time taken for a single refresh operation is 100 nanoseconds. The time required to perform one refresh operation on all the cells in the memory unit is **[2010]**
(A) 100 nanoseconds
(B) 100 * 2^{10} nanoseconds
(C) 100 * 2^{20} nanoseconds
(D) 3200 * 2^{20} nanoseconds

10. A processor can support a maximum memory of 4GB, where the memory is word - addressable (a word consists of two bytes). The size of the address bus of the processor is atleast ___ bits. **[2016]**

11. The size of the data count register of a DMA controller is 16 bits. The processor needs to transfer a file of 29,154 kilobytes from disk to main memory. The memory is byte addressable. The minimum number of times the DMA controller needs to get the control of the system bus from the processor to transfer the file from the disk to main memory is _____. **[2016]**

12. The following are some events that occur after a device controller issues an interrupt while process *L* is under execution.
(P) The processor pushes the process status of *L* onto the control stack.
(Q) The processor finishes the execution of the current instruction.
(R) The processor executes the interrupt service routine.
(S) The processor pops the process status of *L* from the control stack.
(T) The processor loads the new PC value based on the interrupt.

Which one of the following is the correct order in which the events above occur? **[2018]**
(A) QPTRS (B) PTRSQ
(C) TRPQS (D) QTPRS

13. A 32-bit wide main memory unit with a capacity of 1 GB is built using 256 M × 4-bit DRAM chips. The number of rows of memory cells in the DRAM chip is 2^{14}. The time taken to perform one refresh operation is 50 nanoseconds. The refresh period is 2 milliseconds. The percentage (rounded to the closest integer) of the time available for performing the memory read write operations in the main memory unit is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** C	**3.** D	**4.** B	**5.** B	**6.** C	**7.** B	**8.** C	**9.** C	**10.** A
11. C	**12.** D	**13.** A	**14.** D	**15.** B	**16.** D	**17.** D	**18.** B	**19.** B	**20.** D

Practice Problems 2

1. B	**2.** B	**3.** A	**4.** C	**5.** A	**6.** C	**7.** B	**8.** D	**9.** C	**10.** C
11. A	**12.** D	**13.** A	**14.** B	**15.** B	**16.** B	**17.** A	**18.** D	**19.** B	**20.** B
21. D									

Previous Years' Questions

1. B	**2.** D	**3.** D	**4.** B	**5.** C	**6.** D	**7.** A	**8.** A	**9.** D	**10.** 31
11. 456	**12.** A	**13.** 59 to 60							

Chapter 4

Instruction Pipelining

LEARNING OBJECTIVES

- *Flynn's classification*
- *Pipelining*
- *Six-stages of pipelining*
- *Pipeline performance*
- *Pipeline hazards*
- *Structural hazards*
- *Data hazards*
- *Control hazards*
- *Conditional branch*
- *Dealing with branches*

Flynn's Classification

In parallel processing, the system is able to perform concurrent data processing to achieve faster execution time. A classification introduced by M.J. Flynn considers the organization of a computer system by the number of instructions and data items that are manipulated simultaneously. According to this classification, there are four major groups of computers:

1. **Single instruction stream, single data stream (SISD):**
   - Single computer containing a control unit, a processor unit and a memory unit
   - Instructions executed sequentially; parallel processing may be achieved by multiple functional units or by pipeline processing.
2. **Single instruction stream, Multiple data stream (SIMD):**
   - Include many processing units under the supervision of a common control unit.
   - All processors receive the same instruction from the control unit but operate on different items of data.
3. **Multiple instruction stream, Single data stream (MISD):**
   - No practical system has been constructed.
4. **Multiple instruction stream, Multiple data stream (MIMD):**
   - Capable of processing several programs at the same time.

One type of parallel processing that does not fit Flynn's classification is pipelining.

Pipelining

Pipelining is a technique of decomposing a sequential process into sub operations, with each subprocess being executed in special dedicated segment that operates with all other segments.

In pipelining, new inputs are accepted at one end before previously accepted inputs appear as outputs at the other end.

Two-stage Pipeline

As a simple approach, consider subdividing instruction processing into two stages:

1. Fetch instruction
2. Execute instruction
   - There are times during the execution of an instruction when main memory is not being accessed. This time can be used to fetch the next instruction in parallel with the execution of the current one. This is called instruction prefetch or fetch overlap.
   - This process will speed up instruction execution. If the fetch and execute stages were of equal duration, the instruction cycle time would be halved.
   - But there are some problems in this technique:
     (i) The execution time is generally longer than the fetch time.
     (ii) A conditional branch instruction makes the address of the next instruction to be fetched unknown.
   - These two factors reduce the potential effectiveness of the two-stage pipeline, but some speed up occurs. To gain further speed up, the pipeline must have more stage(s).

Six-stage Pipeline

Let the six stages be

F: Fetch Instruction
D: Decode Instruction
C: Calculate Operand Address
O: Operand Fetch

E: Execute instruction
W: Write operand

Then the time line diagram for seven instructions is shown below:

Clock cycle	1	2	3	4	5	6	7	8	9	10	11	12
Instruction i	F	D	C	O	E	W						
$i+1$		F	D	C	O	E	W					
$i+2$			F	D	C	O	E	W				
$i+3$				F	D	C	O	E	W			
$i+4$					F	D	C	O	E	W		
$i+5$						F	D	C	O	E	W	
$i+6$							F	D	C	O	E	W

Execution Time for the seven instructions with pipelining $=\left(\frac{t_{ex}}{6}\right)\times 12 = 2 * t_{ex}$. Where t_{ex} in the execution time required for each instruction.

- A deeper pipeline means that there are more stages in the pipeline. This generally means that the processor's frequency can be increased as the cycle time is lowered. This happen because there are fewer components in each stage of the pipeline, so the propagation delay is decreased for the overall stage.
- An instruction pipeline is said to be fully pipelined if it can accept a new instruction in every clock cycle.
- A pipeline that is not fully pipelined has wait cycle that delays the progress of the pipeline.

Advantages of pipelining:

- The cycle time of the processor is reduced, thus increasing instruction issue rate in most cases.
- Some combinational circuits such as adders or multipliers can be made faster by adding more circuitry. If pipelining is used instead it can save circuitry versus a more complex combinational circuit.

Limitations of pipelining:

1. If the stages are not of equal duration, there will be some waiting involved at various stages.
2. Conditional branch instruction may invalidate several instruction fetches.
3. The contents of one stage may depend on the contents of other stages of previous instructions, which is still in pipeline.

Pipeline Performance

The cycle time τ of an instruction pipeline is the time needed to advance a set of instructions one stage through the pipeline.

$$\text{Cycle time} = \max[\tau_i] + d = \tau_m + d,\ 1 \le i \le K$$

where τ_i = Time delay of the circuitry in the i^{th} stage of the pipeline.

τ_m = maximum stage delay.
K = number of stages in instruction pipeline
d = time delay of a latch, needed to advance signals and data from one stage to the next.

Suppose that n instructions are processed without any branches. Let $T_{k,n}$ be the total time required for a pipeline with K stages to execute n instructions. Then

$T_{k,n} = [K+(n-1)]\tau$

Example 1: Let $n = 7$, $K = 6$, $\tau = 1$. Then $T_{k,n} = [6 + (7 - 1)] \times 1 = 12$ cycles.

Now consider a processor with equivalent functions but no pipeline and assume that the instruction cycle time is $k\tau$. The speed up factor for the instruction pipeline compared to execution without the pipeline is defined as

$$S_k = \frac{T_{1,n}}{T_{k,n}} = \frac{nk\tau}{[(k+(n-1)]\tau} = \frac{nk}{k+(n-1)}$$

Note: Larger the number of stages, greater the potential for speed up. But practically, the potential gains of additional pipeline stages are countered by increase in cost, delays between stages, and the fact that braches will be encountered requiring the flushing of the pipeline.

Arithmetic pipeline:

- Pipeline arithmetic units are usually found in very high-speed computers.
- These are used to implement floating point operations, multiplication of fixed point numbers and similar computations encountered in scientific problems.

Pipeline Hazards

- Pipeline hazards are situations that prevent the next instruction in the instruction stream from executing during its designated clock cycle. The instruction is said to be stalled. When an instruction is stalled, all instructions later in the pipeline than the stalled instructions are also stalled. Instructions earlier than the stalled one can continue. No new instructions are fetched during the stall.

Note: Keeping a pipeline at its maximal rate is prevented by pipeline hazard.

Different types of Hazards:

1. Structural Hazards
2. Data Hazards
3. Control Hazards

Structural Hazards

Structural hazards occur when a certain resource is requested by more than one instruction at the same time.

Example 2: Instruction MVI B, X fetches in the O stage operand X from memory. The memory does not accept another access during that cycle.

Clock cycle	1	2	3	4	5	6	7	8	9	10	11
MVI *B*, *X*	F	D	C	O	E	W					
Instruction *i* + 1		F	D	C	O	E	W				
i + 2			F	D	C	O	E	W			
i + 3				stall	F	D	C	O	E	W	
i + 4						F	D	C	O	E	W

Penalty: 1 cycle

Certain resources are duplicated in order to avoid structural hazards (ALU, floating-point unit) can be pipelined themselves in order to support several instructions at a time. A classical way to avoid hazards at memory access is by providing separate data and instruction caches.

Note: Structural hazards are due to resource conflict.

Data Hazards

In a pipeline execution of two instructions I_1 and I_2 a certain stage of the pipeline I_2 needs the result produced by I_1, but this result has not yet been generated, then we have a data hazard.

Example 3: I_1: ADD R_3, R_2 $\quad R_3 \leftarrow R_3 + R_2$

I_2: MUL R_1, R_3 $\quad R_1 \leftarrow R_1 * R_3$

Clock cycle	1	2	3	4	5	6	7	8	9	10	11	12
ADD R_3, R_2	F	D	C	O	E	W						
MUL R_1, R_3		F	D	C	stall	stall	O	E	W			
Instruction *i* + 2			F	D			C	O	E	W		

Penalty: 2 cycles Before executing the O stage (operand fetch stage), the MUL instruction is stalled until the ADD instruction has written the result into R_3.

Data dependencies

Data dependency exists between two instructions if the data data used by an instruction depends on the data created by other instructions.

Two type of dependencies exist between instructions:

1. True data dependency
2. Name dependencies
   (i) Anti-dependency
   (ii) Output dependency

True data dependency

- This is also called as Read-After-Write Hazard (RAW).
- This type of dependency occurs when the value produced by an instruction is required by a subsequent instruction.
- This is also known as a flow dependency because dependency is due to flow of data in a program.

Example: ADD R_3, R_2, R_1; $R_3 \leftarrow R_2 + R_1$

SUB R_4, R_3, 1; $R_4 \leftarrow R_3 - 1$

- Here R_3 is read before it is written by 'ADD' instruction.
- In RAW hazard $(i + 1)^{st}$ instruction tries to read a source before it is written by 'i^{th}' instruction. So $(i + 1)^{st}$ instruction incorrectly gets the old value.
- This kind of hazard can be reduced by using forwarding (or Bypassing).

Name dependencies

1. **Anti-dependency:**
   - This is also called as Write-After-Read hazard
   - This kind of dependency occurs when an instruction writes to a location which has been read by a previous instruction.
   - Here $(i + 1)^{st}$ instruction tries to write an operand before it is read by i^{th} instruction. So i^{th} instruction incorrectly gets the new value.

Example: I_1: ADD R_3, R_2, R_1 ;$R_3 \leftarrow R_2 + R_1$

I_2: SUB R_2, R_5, 1; $R_2 \leftarrow R_5 - 1$

I_2 must not produce its result in R_2 before I_1 read R_2, otherwise I_1 would use the value produced by I_2 rather than the previous value of R_2.

2. **Output dependency:**
   - This is also called as Write-After-Write (WAW) hazard.
   - This dependency occurs when a location is written by two instructions.
   - i.e., $(i + 1)$th instruction tries to write an operand before it is written by i^{th} instruction.

   The writes end up being performed in the wrong order.

Example: I_1: ADD R_3, R_2, R_1; $R_3 \leftarrow R_2 + R_1$

I_2: SUB R_2, R_3, 1; $R_2 \leftarrow R_3 - 1$

I_3: R_3, R_2, R_5; $R_3 \leftarrow R_2 + R_5$

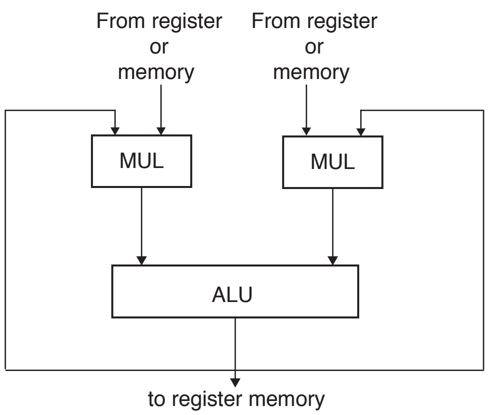
There is a possibility of WAW hazard between I_1 and I_3.

Handling data dependency There are ways to handle date dependency.

1. Hardware interlocks
2. Operand forwarding
3. Delayed load

1. **Hardware interlocks:**
   - To avoid data dependency, insert hardware interlock.
   - An interlock is a circuit that detects instructions whose source operands are destinations of instructions farther up in the pipeline.
2. **Operand forwarding:**
   - Uses special hardware to detect a conflict and then avoid it by routing the data through special paths between pipeline segments.

- Some of the penalty produced by data hazards can be avoided using a technique called forwarding (Bypassing).
- The ALU result is always fed back to the ALU input. If the hardware detects that the value needed for the current operation is the one produced by the previous operation (but which has not yet been written back). It selects the forwarded result as the ALU input, instead of the value read from register or memory.



Clock cycle	1	2	3	4	5	6	7	8
ADD R_3, R_2	F	D	C	O	E	W		
MUL R_1, R_3		F	D	C	stall	O	E	W

Penalty: 1 cycle After the E stage of the MUL instruction the result is available by forwarding. Therefore the penalty is reduced to one cycle.

Delayed Load: Here the compiler of a computer will detect the data conflicts and reorder the instructions as necessary to delay the loading of the conflicting data by inserting no-operation instruction.

Control Hazards

Control hazards are produced by Branch Instructions.

Unconditional branch

- Jump loop
- Loop

Clock cycle	1	2	3	4	5	6	7	8	9	10	11
Loop	F	D	C	O	E	W					
Loop		stall	stall	stall	F	D	C	O	E	W	
Loop + 1						F	D	C	O	E	W

Penalty: 3 cycles

- The instruction following the branch is fetched before the D stage is finished in 2nd clock. It is not known that a branch is executed. Later the fetched instruction is discarded.
- After the O stage of the branch instruction the address of the target is known and it can be fetched.

Conditional branch

Example: ADD B; $A \leftarrow A + B$
JZ Loop

Loop: If condition satisfies and branch is taken:

Clock cycle	1	2	3	4	5	6	7	8	9	10	11	12
ADD *B*	F	D	C	O	E	W						
JZ Loop		F	D	C	O	E	W					
Loop			stall	stall	stall	F	D	C	O	E	W	

Penalty:3 cycles

At this moment both the condition (set by ADD) and the target address are known.

If condition not satisfied and branch not taken:

Clock cycle	1	2	3	4	5	6	7	8	9	10	11	12
ADD *B*	F	D	C	O	E	W						
JZ Loop		F	D	C	O	E	W					
Instruction $i + 1$			F	stall	stall	D	C	O	E	W		

Penalty:2 cycles

At this moment the condition is known and instruction i + 1 can go on.

- With conditional branch, we have a penalty even if the branch has not been taken. This is because we have to wait until the branch condition is available.

Dealing with branches

One of the major problems in designing an instruction pipeline is the occurrence of branch instructions. A variety of approaches have been taken for dealing with braches

1. Multiple streams
2. Prefetch branch target
3. Branch target buffer
4. Loop buffer
5. Branch prediction
6. Delayed branch

Multiple streams A branch instruction may cause to choose one of two instructions to fetch next, then allow the pipeline to fetch both instructions, making use of streams.

There are two problems with this approach:

1. With multiple pipelines there are contention delays for access to the registers and to memory.
2. Additional branch instructions may enter the pipeline before the original branch decision is resolved.

Prefetch branch target When a conditional branch is recognized, the target of the branches is prefetched, in addition to the instruction following the branch. This target is then saved until the branch instruction is executed.

Branch targets buffer (BTB) BTB is an associative memory included in the fetch segment of the pipeline. Each entry in the BTB consists of the address of a previously executed branch instruction and target instructions for that branch.

It also stores the next few instructions after the branch target instruction.

When the pipeline decodes a branch instruction, it searches the associative memory BTB for the address of the instruction. If it is in BTB, the instruction is available directly and prefetch continues from the new path. If the instruction is not in BTB, the pipeline shifts to a new instruction stream and stores the target instruction in the BTB.

Advantage: Branch instructions that occurred previously are readily available in the pipeline without interruption.

Loop Buffer A loop buffer is a small, very high speed memory maintained by the instruction fetch stage of the pipeline and containing the n most recently fetched instructions in sequence. If a branch is to be taken, the hardware first checks whether the branch target is within the buffer. If so, the next instruction is fetched from the buffer. The Advantages of loop buffer are

1. Loop buffer will contain some instructions sequentially ahead of the current instruction fetch address. Thus instructions fetched in sequence will be available without the usual memory access time.
2. If the branch occurs to a target just a few locations ahead of the address of the branch instruction, the target will already be in the buffer.
3. This strategy is well suited in dealing with loops.

Branch prediction Various techniques can be used to predict whether a branch will be taken or not. The common techniques are

Static:
1. Predict never taken Static
2. Predict always taken
3. Predict by opcode

Dynamic:
4. Taken/not taken switch
5. Brach history table

- The first two approaches are static, i.e., no dependency on execution history. Here always assume that the branch will not be taken and continue to fetch instructions in sequence, or always assume that the branch will be taken and always fetch from the branch target.
- The third approach is also static. Takes the decision based on the opcode of the branch instruction in a program.
- Dynamic branch strategies attempt to improve the accuracy of prediction by recording the history of conditional branch instructions in a program.

(a) Taken/not taken switch:

- Use two bits to record the result of the last two instances of the execution of the associated instruction or record a state in some other fashion.

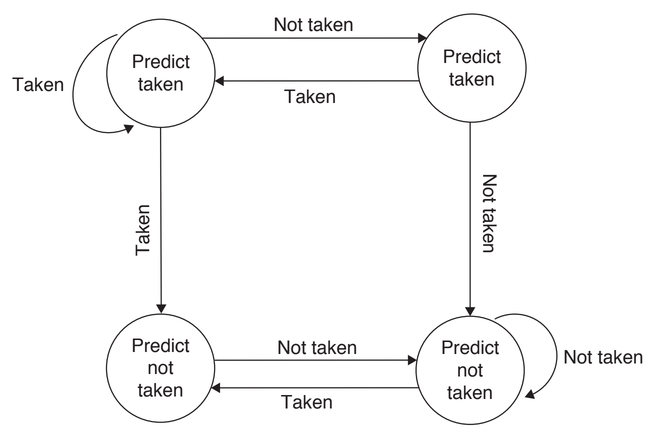


Figure 1 Branch prediction state diagram

- As long as each succeeding conditional branch instruction that is encountered is taken, the decision process predicts that the next branch will be taken.
- If a single prediction is wrong, the algorithm continues to predict that the next branch is taken.
- Only if two successive branches are not taken does the algorithm shift to not taken branch.

Drawback: If the decision is made to take the branch, the target instruction cannot be fetched until the target address, which is an operand in the conditional branch instruction is decoded.

(b) Branch history table: It is a small cache memory associated with the instruction fetch stage of the pipeline.

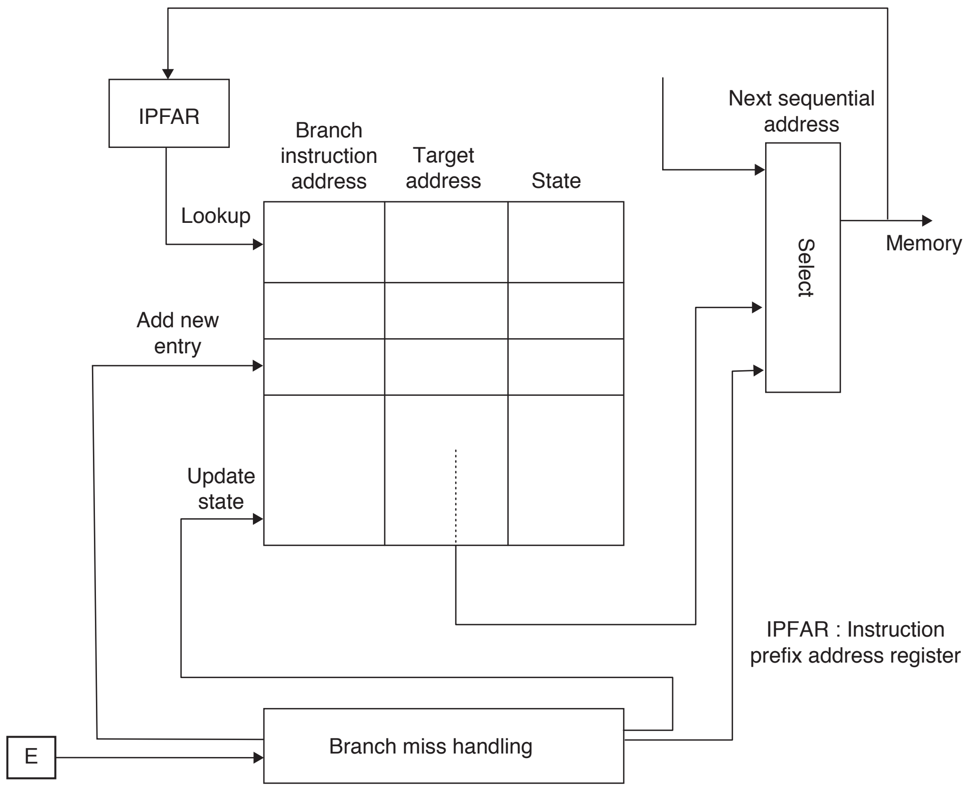


Figure 2 Branch history table

- Each entry in the table consist of three elements:
  1. The address of branch instruction.
  2. Some number of history bits that record the state of use of that instruction.
  3. Information about target instruction.

Delayed branch A complier detects the branch instructions and rearranges the machine language code sequence by inserting useful instructions that keep a pipeline operating without interruptions.

Exercises

Practice Problems I

Directions for questions 1 to 21: Select the correct alternative from the given choices.

Common data for questions 1 and 2: An unpipelined processor with eight number cycle time and pipeline batches with 1 ns latency is given.

1. Find the cycle times of pipelined versions of the processor with 2, 4, 8 and 16 stages if the Data path logic is evenly divided among the pipeline stages.
(A) 5, 3, 2, 1.5 (B) 4, 2, 1, 0.5
(C) 8, 4, 2, 1 (D) 10, 6, 4, 3

2. What is the latency of each of the pipelined versions of the processor?
(A) 4, 2, 1, 0.5 (B) 10, 6, 4, 3
(C) 5, 3, 2, 1.5 (D) 10, 12, 16, 24

3. A 4-stage pipeline has the stage delays as 110, 120, 130, and 140 nanoseconds respectively. Registers that are used between the stages have a delay of 2 nanoseconds each. Assuming constant clocking rate. Find the total time taken to process 1000 instructions on this pipeline.
(A) 7.1 ms (B) 14.24 ms
(C) 28 ms (D) 2000 ms

4. Consider a pipelined processor with the following four stages:
IF: instruction fetch
ID: Instruction decode
EX: Execute
WB: Write back
The IF, ID and WB stages takes one clock cycle each to complete the operation. The number of clock cycles for EX stage depends on the instruction; for I_1 and I_3 one clock cycle is needed and for I_2 three clock cycles are needed. Find the number of clock cycles taken to complete the following sequence of instructions?

I_1:	ADD R_0,	R_1,	R_2	$R_0 \leftarrow R_1 + R_2$
I_2:	MUL R_2,	R_3,	R_4	$R_2 \leftarrow R_3 \times R_4$
I_3:	SUB R_4,	R_5,	R_6	$R_4 \leftarrow R_5 - R_6$

(A) 7 (B) 8
(C) 6 (D) 9

5. A CPU has five stage pipelines and runs at 1 GHz frequency. Instruction fetch happens in the first stage of the pipeline. A conditional branch instruction computes the target address and evaluates the condition in the third stage of the pipeline. The processor stops fetching new instructions following a conditional branch until the branch outcome is known. A program executes 10^9 instructions. Out of which 10% are conditional branches. If each instruction takes one cycle to complete on average then find the total execution time of the program?

(A) 1 sec (B) 1.2 sec
(C) 1.4 sec (D) 1.8 sec

6. Consider a four stage pipeline processor, number of cycles needed by the four instructions I_1, I_2, I_3 and I_4 in stages S_1, S_2, S_3 and S_4 are shown below:

	S_1	S_2	S_3	S_4
I_1	2	1	1	1
I_2	1	3	2	2
I_3	2	1	1	3
I_4	1	2	2	2

What is the number of cycles needed to execute the instructions in the order:
$I_1 : I_2 : I_3 : I_4$

(A) 8 (B) 12
(C) 14 (D) 15

7. A non-pipelined system takes 50 ns to process a task; the same task can be processed in a six-segment pipeline with a clock cycle of 10 ns. Speedup ratio of 100 tasks for pipeline is

(A) 1.62 (B) 3.21
(C) 4.76 (D) 8.21

8. Consider a pipelined processor with the following four stages:
IF: Instruction fetch
ID: Instruction decode
EX: Execute
WB: Write back

The IF, ID and WB stages takes one clock cycle each to complete the operation. The number of clock cycles for EX stage depends on the instruction; for I_1 and I_3 one clock cycle is needed and for I_2 three clock cycles are needed. The number of clock cycles taken to complete the following sequence of instructions is

I_1:	ADD R_0, R_1, R_2,	$R_0 \leftarrow R_1 + R_2$
I_2:	MUL R_2, R_0, R_4,	$R_2 \leftarrow R_0 \times R_4$
I_3:	SUB R_4, R_5, R_2,	$R_4 \leftarrow R_5 - R_2$

(A) 7 (B) 8
(C) 9 (D) 10

9. Following are the sequence of stages in a pipeline CPU:
(1) IF: Instruction fetch from instruction memory
(2) RD: Instruction decode and register read
(3) EX: Execute ALU operation for data and address computation
(4) MA: Data memory access, for write access, the register read at RD stage is used.
(5) WB: Register write back

Consider the following sequence of instructions:
LOAD R_1, M[loc]
ADD R_1, R_1, R_1
ADD R_2, R_1, R_2
Let each stage take one clock cycle.

What is the number of clock cycles taken to complete the above sequence of instructions starting from the fetch of first instruction?

(A) 18 (B) 15
(C) 13 (D) 10

10. Which of the following can cause a hazard for a pipelined CPU with a single ALU?
(i) The $(j + 1)^{st}$ instruction uses the result of the j^{th} instruction as an operand.
(ii) The j^{th} and $(j + 1)^{st}$ instructions require the ALU at the same time.
(iii) The execution of a conditional jump instruction.
(iv) The execution of non-conditional jump instruction.

(A) (i) and (ii) (B) (ii) and (iii)
(C) (i), (ii) and (iii) (D) (i), (ii), (iii) and (iv)

11. Given an unpipelined processor with a 10 ns cycle time and pipeline latches with 0.5 ns latency, how many stages of pipelining are required to achieve cycle time of 2 ns?

(A) 5.5 (B) 6.67
(C) 7 (D) 6

12. In a 4-stage pipeline,
IF – instruction fetches
ID – instruction decode and fetch operands
EX – Execute
WB – write back
ADD, SUB take one clock cycle, MUL take three clock cycles. Then for
ADD R_2, R_1, R_0 $R_2 \leftarrow R_1 + R_0$
MUL R_4, R_3, R_2 $R_4 \leftarrow R_3 * R_2$
SUB R_6, R_5, R_4 $R_6 \leftarrow R_5 - R_4$
Number of clock cycles required using operand forwarding technique are

(A) 8 (B) 12
(C) 10 (D) 14

13. Consider an instruction sequence of length 'n' that is streaming through a K-stage instructions pipeline. Let P be the probability of encountering a conditional or

unconditional branch instruction and let q be the probability that execution of a branch instruction I_B causes a jump to a non-consecutive address. Assume that each such jump requires the pipeline to be cleared, destroying all ongoing instruction processing, when I_B emerges from the last stage. Also assume that T is the cycle time. Then which of the following expression correctly specifies the time required for this pipeline?
(A) $pqnk\tau + (1 - pq)\,[K + (n - 1)]\tau$
(B) $(1 - pq)\,[k + (n - 1)]\tau + pqn\tau$
(C) $pqnk\tau + (1 - pq)\,n[k + (n - 1)]\tau$
(D) $pqn + (1 - pq)n\,[k + (n - 1)]\tau$

14. If T_m is maximum stage delay of an m-stage pipeline with time delay of the latch is d then cycle time is
(A) T_m/d (B) $T_m + d$
(C) $2T_m + d$ (D) $T_m \times d$

15. Pipelining is a general technique for increasing processor ____ without requiring large amounts of extra hardware.
(A) turnaround time (B) waiting time
(C) latency (D) throughput

16. A 4-stage instruction pipeline executes a 100 instruction program. The probability of occurrence of a conditional or unconditional branch is 0.4 and the probability of execution of a branch instruction I_B causing a jump to a non-consecutive address is 0.1. Then the speed up factor for the instruction pipeline compared to execution without pipeline is
(A) 2.14 (B) 6.23
(C) 3.21 (D) 3.48

17. A non-pipelined processor has a clock rate of 2.5GHz and an average cycles per instruction of 4. An upgrade to the processor introduces a five stage pipeline. However, due to internal pipeline delays, such as latch delay, the clock rate of the new processor has to be reduced to 2GHz. What is the MIPS rate for each of these processors respectively.
(A) 625, 400 MIPS (B) 625, 2000 MIPS
(C) 3125, 2000 MIPS (D) 3125, 400 MIPS

18. Consider the following sequence of instructions:
I_1: MUL R_1, R_2 $R_1 \leftarrow R_1 * R_2$
I_2: SUB R_3, 1 $R_3 \leftarrow R_3 - 1$
I_3: ADD R_3, R_4 $R_3 \leftarrow R_3 + R_4$
I_4: BEZ Target Branch if zero
I_5: MOVE R_3,10 $R_3 \leftarrow 10$
⋮
Target:
Which of the following instruction will be placed in delayed slot to reduce penalty in a 6-stage pipeline? (Assume that the branch outcome will be known during 5th stage)
(A) I_1 (B) I_2
(C) I_3 (D) I_5

19. Consider the following sequence of instructions:
ADD R_1, R_2 $R_1 \leftarrow R_1 + R_2$
BEZ Target Branch if Zero
MUL R_3, R_4 $R_3 \leftarrow R_3 * R_4$
MOVE R_1,10 $R_1 \leftarrow 10$
⋮
Target:
Assume that this program executed on a 6-stage pipelined processor and each stage required 1 clock cycle.
Let us suppose that "branch not taken" Prediction is used but the prediction is not fulfilled, then the penalty will be (branch outcome is known at 5th stage)
(A) 1 clock cycle (B) 2 clock cycles
(C) 3 clock cycles (D) 4 clock cycles

20. Suppose 40% of the instructions are loads and half the time they are followed by instruction that depends on value loaded. If this hazard causes single cycle delay, how must faster is ideal pipelined machine (CPI = 1) than real one? (Ignore other stalls)
(A) 1 time (B) 1.2 times
(C) 1.5 times (D) 1.15 times

21. Assume that a pipelined processor has three categories of instructions: Branch, load/store, other. If it is a branch instruction it will take 3 clock cycles, if it is a load/store instruction it will take 4 clock cycles and all other instructions require 6 clock cycles. A program consisting of 10% Branch instructions, 10% of load/store instructions is executed on this processor. Then the number of clock cycles required for the execution of the program is
(A) 2.45 (B) 3.61
(C) 4.66 (D) 5.5

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. The time required for the five functional units, which operated in each of the five cycles are 10 ns, 7 ns, 10 ns, 10 ns and 8 ns. Assume that pipelining add 1 ns of overhead. The speed up of pipeline compared to unpipeline is
(A) 4.5 times (B) 1.1 times
(C) 4.1 times (D) 2.4 times

2. Which of the following is a technique of decomposing a sequential process into sub operations with each subprocess being executed in a special dedicated segment that operates concurrently with each other?
(A) Straight line sequencing
(B) Random sequencing
(C) Pipelining
(D) Serial execution

3. Which of the following statements is incorrect?
(A) Latency is the number of time units between two initiations in a pipelined architecture.
(B) If initiations are of different but fixed reservation tables, the architecture is known as static pipelined configuration.
(C) A collision in a pipelined architecture is an attempt by two different initiations to use the same stage at the same time.
(D) None of the above

4. Which of the following technique is used in a pipelined processor, when there is a conditional branch?
(A) Loop butter (B) Branch prediction
(C) Delayed Branch (D) All of the above

5. Which of the following cases, leads to a pipelined computer architecture?
(A) The evaluation of each basic function is relatively independent of the previous one.
(B) The sub-functions are closely related to each other.
(C) The evaluation of each sub function requires approximately the same sequence.
(D) All of the above

6. The performance of a pipelined processors is degraded if
(A) the pipeline stages have different delays.
(B) consecutive instructions are dependent on each other.
(C) the pipeline stages share hardware resources.
(D) All of the above

7. The following is a limit on how much the performance of a processor can be improved using pipelining:
(A) the number of pipeline stages
(B) data dependencies
(C) branch delays
(D) All of the above

8. A pipeline processor consists of a sequence of 'm' data processing circuits called ____, which collectively perform a single operation on a stream of data operands passing through them.
(A) stages (B) pipelines
(C) latches (D) None of the above

9. A five-stage pipelined CPU has the following sequence of stages:
IF – Instruction fetch from memory
RD – Decode instruction
EX – Execute
MA – Data memory access
WB – Register write back

Consider the following instruction sequence:
I_1: Load R_0 $\quad R_0 \Leftarrow M$
I_2: ADD R_1, $\quad R_1\, R_1 \Leftarrow R_1 + R_1$
I_3: SUB R_2, $\quad R_3\, R_2 \Leftarrow R_2 - R_3$

Each stage takes one clock cycle.
Number of clock cycles to execute the program is
(A) 8 (B) 10
(C) 7 (D) 15

Common data for questions 10 and 11: Given an unpipelined processor with a 10 number cycle time and pipeline latches with 0.5 ns latency.

10. Which are the cycle times of pipelined versions of the processors with 2, 4, 8 and 16 stages if the data path logic is evenly divided among the pipeline stages?
(A) 5.0, 3.0, 1.5, 1.0
(B) 5.5, 3.0, 1.75, 1.125
(C) 4.0, 5.0, 6.0, 7.0
(D) None of the above

11. What is the latency of each of the pipelined versions of the processor with 2, 4, 8 and 16 stages?
(A) 10, 11, 12, 14 ns
(B) 10, 10, 11, 11 ns
(C) 11, 12, 14, 18 ns
(D) None of the above

12. Assume an unpipelined processor has a 1 ns clock cycle and it uses 5 cycles for ALU operations and branches. And 6 clock cycles for memory operations. A program has 40%, 30%, and 20% of ALU operations, branch instructions and memory operations respectively. If we are using pipelining it adds 0.2 ns overhead. Then what is the speedup of pipelining compared to unpipelined processor?
(A) 1.2 (B) 3.91
(C) 4.7 (D) 2.5

13. Consider a five-stage pipeline processor in which each instructions on an average has 2 clock cycle stalls. Then the speed up of this pipelined processor compared to an unpipelined processor is
(A) 2.5 (B) 1.67
(C) 0.4 (D) 5

14. Pipelining strategy is called to implement
(A) instruction execution
(B) instruction prefetch
(C) instruction decoding
(D) instruction manipulation

15. If an Instruction 'j' tries to read a source operand before instruction 'i' writes it. Then it is a ____ type of hazard.
(A) WAR (B) RAW
(C) WAW (D) None of these

16. What is the average instruction processing time of a five-stage instruction pipeline for 32 instructions if conditional branch instructions occur as follows: I_2, I_5, I_7, I_{25}, I_{27}.
(A) 1.97 (B) 1.67
(C) 1.75 (D) 1.25

17. Consider the execution of 1000 instructions on a five-stage pipeline machine. Then the speed-up due to the use of pipelining given that the probability of an instruction being a branch is 0.2.
(A) 1.77 (B) 2.6
(C) 2.77 (D) 3.2

18. If an instructions following a branch (taken or not taken) have a dependency on the branch and cannot be executed until the branch is executed, then the dependency is
(A) True data dependency
(B) Procedural dependency
(C) Resource conflict
(D) Output dependency

19. 'A two-stage instruction pipeline unlikely to cut the instruction cycle time in half, compared with the use of no pipeline.' The statement is
(A) Always true (B) Always False
(C) Can't predict (D) Some times true

20. Write after read dependency is also known as
(A) True dependency (B) Anti-dependency
(C) Output dependency (D) Inverse dependency

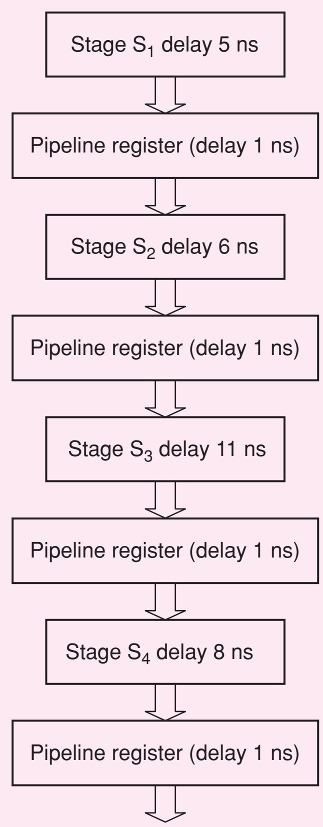
Previous Years' Questions

1. Consider a 6-stage instruction pipeline, where all stages are perfectly balanced. Assume that there is no cycle time overhead of pipelining. When an application is executing on this 6-stage pipeline, the speedup achieved with respect to non-pipelined execution if 25% of the instructions incur 2 pipeline stall cycles is _____. **[2014]**

2. Consider the following processors (ns stands for nano-seconds). Assume that the pipeline registers have zero latency.
P1: Four-stage pipeline with stage latencies 1 ns, 2 ns, 2 ns, 1 ns.
P2: Four-stage pipeline with stage latencies 1 ns, 1.5 ns, 1.5 ns, 1.5 ns.
P3: Five-stage pipeline with stage latencies 0.5 ns, 1 ns, 1 ns, 0.6 ns, 1 ns.
P4: Five-stage pipeline with stage latencies 0.5 ns, 0.5 ns, 1 ns, 1 ns, 1.1 ns.
Which processor has the highest peak clock frequency? **[2014]**
(A) P1 (B) P2
(C) P3 (D) P4

3. An instruction pipeline has five stages, namely, instruction fetch (IF), instruction decode and register fetch (ID/RF), instruction execution (EX), memory access (MEM), and register write back (WB) with stage latencies 1 ns, 2.2 ns, 2 ns, 1 ns, and 0.75 ns, respectively (ns stands for nano seconds). To gain in terms of frequency, the designers have decided to split the ID/RF stage into three stages (ID, RF1, RF2) each of latency 2.2/3 ns. Also, the EX stage is split into two stages (EX1, EX2) each of latency 1 ns. The new design has a total of eight pipeline stages. A program has 20% branch instructions which execute in the EX stage and produce the next instruction pointer at the end of the EX stage in the old design and at the end of the EX2 stage in the new design. The IF stage stalls after fetching a branch instruction until the next instruction pointer is computed . All instructions other than the branch instruction have an average CPI of one in both the designs. The execution times of this program on the old and the new design are *P* and *Q* nanoseconds, respectively. The value of *P*/*Q* is _____. **[2014]**

4. Consider an instruction pipeline with four stages (S_1, S_2, S_3 and S_4) each with combinational circuit only. The pipeline registers are required between each stage and at the end of the last stage. Delays for the stages and for the pipeline registers are as given in the figure.



What is the approximate speed up of the pipeline in steady state under ideal conditions when compared to the corresponding non-pipeline implementation? **[2011]**

(A) 4.0 (B) 2.5
(C) 1.1 (D) 3.0

5. Register renaming is done in pipelined processors **[2012]**
(A) as an alternative to register allocation at compile time
(B) for efficient access to function parameters and local variables
(C) to handle certain kinds of hazards
(D) as part of address translation

6. Consider an instruction pipeline with five stages without any branch prediction: Fetch Instruction (FI), Decode Instruction (DI), Fetch Operand (FO), Execute Instruction (EI) and Write Operand (WO). The stage delays for FI, DI, FO, EI and WO are 5 ns, 7 ns, 10 ns, 8 ns and 6 ns, respectively. There are intermediate storage buffers after each stage and the delay of each buffer is 1 ns. A program consisting of 12 instructions $I_1, I_2, I_3, \ldots, I_{12}$ is executed in this pipelined processor. Instruction I_4 is the only branch instruction and its branch target is I_9. If the branch is taken during the execution of this program, the time (in ns) needed to complete the program is **[2013]**
(A) 132 (B) 165
(C) 176 (D) 328

Common data for Questions 7 and 8: Delayed branching can help in the handling of control hazards

7. For all delayed conditional branch instructions, irrespective of whether the condition evaluates to true or false **[2008]**
(A) The instruction following the conditional branch instruction in memory is executed
(B) The first instruction in the fall through path is executed
(C) The first instruction in the taken path is executed
(D) The branch takes longer to execute than any other instruction

8. The following code is to run on a pipelined processor with one branch delay slot:
I_1: ADD $R_2 \leftarrow R_7 + R_8$
I_2: SUB $R_4 \leftarrow R_5 - R_6$
I_3: ADD $R_1 \leftarrow R_2 + R_3$
I_4: STORE Memory $[R_4] \leftarrow R_1$
BRANCH to Label if $R_1 = 0$
Which of the instructions I_1, I_2, I_3 or I_4 can legitimately occupy the delay slot without any other program modification? **[2008]**
(A) I_1 (B) I_2
(C) I_3 (D) I_4

9. A 5 stage pipelined CPU has the following sequence of stages:
IF - Instruction fetch from instruction memory
RD - Instruction decode and register read
EX - Execute: ALU operation for data and address computation
MA - Data memory access - for write access, the register read at RD state is used
WB - Register write back
Consider the following sequence of instructions.
I_1: $L\ R_0$, loc$_1$; $R_0 <= M[\text{loc}_1]$
I_2: $A\ R_0, R_0$; $R_0 <= R_0 + R_0$
I_3: $S\ R_2, R_0$; $R_2 <= R_2 - R_0$
Let each stage take one clock cycle.
What is the number of clock cycles taken to complete the above sequence of instructions from the fetch of I_1?
(A) 8 (B) 10
(C) 12 (D) 15

10. Consider a non-pipelined processor with a clock rate of 2.5 gigahertz and average cycles per instruction of four. The same processor is upgraded to a pipelined processor with five stages; but due to the internal pipeline delay, the clock speed is reduced to 2 gigahertz. Assume that there are no stalls in the pipeline. The speed up achieved in this pipelined processor is ______. **[2015]**

11. Consider the sequence of machine instructions given below:

MUL R_5, R_0, R_1
DIV R_6, R_2, R_3
ADD R_7, R_5, R_6
SUB R_8, R_7, R_4

In the above sequence, R_0 to R_8 are general purpose registers. In the instructions shown, the first register stores the result of the operation performed on the second and the third registers. This sequence of instructions is to be executed in a pipelined instruction processor with the following 4 stages: (1) Instruction Fetch and Decode (IF), (2) Operand Fetch (OF), (3) Perform Operation (PO) and (4) Write back the result (WB). The IF, OF and WB stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD or SUB instruction, 3 clock cycles for MUL instruction and 5 clock cycles for DIV instruction. The pipelined processor uses operand forwarding from the PO stage to the OF stage. The number of clock cycles taken for the execution of the above sequence of instructions is _______. **[2015]**

12. Consider the following reservation table for a pipeline having the stages S_1, S_2 and S_3.

Time →

	1	2	3	4	5
S_1	X				X
S_2		X		X	
S_3			X		

The minimum average latency (MAL) is ______ **[2015]**

13. Consider the following code sequence having five instructions l_1 to l_5. Each of these instructions has the following format. **[2015]**

OP Ri, Rj, Rk

Where operation *OP* is performed on contents of registers *Rj* and *Rk* and the result is stored in register *Ri*.

l_1: ADD R_1, R_2, R_3
l_2: MUL R_7, R_1, R_3
l_3: SUB R_4, R_1, R_5
l_4: ADD R_3, R_2, R_4
l_5: MUL R_7, R_8, R_9

Consider the following three statements.

S_1: There is an anti-dependence instructions between instructions l_2 and l_5

S_2: There is an anti-dependence between Instructions l_2 and 1_4

S_3: |Within an instruction pipeline an anti-dependence always creates one or more stalls

Which one of the above statements is/are correct?

(A) Only S_1 is true
(B) Only S_2 is true
(C) Only S_1 and S_3 are true
(D) Only S_2 and S_3 are true

14. The stage delays in a 4 - stage pipeline are 800, 500, 400 and 300 picoseconds. The first stage (with delay 800 picoseconds) is replaced with a functionally equivalent design involving two stages with respective delays 600 and 350 picoseconds. The throughput increase of the pipeline is ___ percent. **[2016]**

15. Consider a 3 GHz (gigahertz) processor with a three - stage pipeline and stage latencies τ_1,τ_2 and τ_3 such that $\tau_1 = 3\tau_2/4 = 2\tau_3$. If the longest pipelines stage is split into two pipeline stages of equal latency, the new frequency is ____ GHz, ignoring delays in the pipeline registers. **[2016]**

16. Instruction execution in a processor is divided into 5 stages, *Instruction Fetch* (IF), *Instruction Decode* (ID), *Operand Fetch* (OF), *Execute* (EX), and *Write Back* (WB). These stages take **5, 4, 20, 10**, and **3 nanoseconds (ns)** respective. A pipelined implement action of the processor requires buffering between each pair of consecutive stages with a delay of **2 ns**. Two pipelined implementations of the processor are contemplated:

(i) A navie pipeline implementation (NP) with 5 stages and
(ii) An efficient pipeline (EP) where the OF stage is divided into stages OF1 and OF2 with execution times of **12 ns** and **8 ns** respectively.

The speedup (correct to two decimal places) achieved by EP over NP in executing 20 independent instructions with no hazards is ______. **[2017]**

17. The instruction pipeline of a RISC processor has the following stages: Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Write back (WB). The IF, ID, OF and WB stages take 1 clock cycle each for every instruction. Consider a sequence of 100 instructions. In the PO stage, 40 instructions take 3 clock cycles each, 35 instructions take 2 clock cycles each, and the remaining 25 instructions take 1 clock cycle each. Assume that there are no data hazards and no control hazards.

The number of clock cycles required for completion of execution of the sequence of instructions is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** D	**3.** B	**4.** B	**5.** B	**6.** D	**7.** C	**8.** D	**9.** C	**10.** D
11. A	**12.** A	**13.** A	**14.** B	**15.** D	**16.** D	**17.** B	**18.** A	**19.** B	**20.** B
21. D									

Practice Problems 2

1. C	**2.** C	**3.** B	**4.** D	**5.** D	**6.** D	**7.** D	**8.** A	**9.** C	**10.** B
11. C	**12.** B	**13.** B	**14.** B	**15.** B	**16.** C	**17.** C	**18.** B	**19.** A	**20.** B

Previous Years' Questions

1. 4	**2.** C	**3.** 1.54	**4.** B	**5.** C	**6.** B	**7.** A	**8.** D	**9.** A	**10.** 3.2
11. 13	**12.** 3	**13** B	**14.** 33.0 : 34.0		**15.** 4	**16.** 1.51	**17.** 219		

Chapter 5

Cache and Main Memory, Secondary Storage

LEARNING OBJECTIVES

- *Characteristics of memory system*
- *Memory hierarchy*
- *Locality of reference*
- *Cache memory*
- *Basic operation of cache*
- *Elements of cache design*
- *Replacement algorithm*
- *Secondary storage*
- *Disk*
- *Diskette*
- *Magnetic tape*
- *Optimal memory*

Characteristics of Memory System

1. **Location:** The term refers to whether memory is internal or external to the computer. The location of memory may be
   - Processor
   - Internal (main)
   - External (secondary)
2. **Capacity:** The capacity of internal memory is expressed in terms of bytes. The capacity specified using
   - Word size
   - Number of words
3. **Unit of transfer**
   - For internal memory, the unit of transfer is equal to the number of data lines into and out of the memory module. The unit of transfer need not equal a word or an addressable unit.
   - For external memory, data are often transferred in much larger units than a word, and these are referred to as blocks.
4. **Access method:** The various methods of accessing units of data are

   (i) Sequential access: Memory is organized into units of data, called records.

   Example: Magnetic tapes

   (ii) Direct access: Individual blocks or records have a unique address based on physical location.

   Example: Magnetic disks

   (iii) Random access: Each addressable location in memory has a unique, physically wired-in addressing mechanism. The time to access a given location is independent of the sequence of prior accesses and is constant.

   Example: Main memory

   (iv) Associative: This is a random access type of memory that enables one to make a comparison of desired bit locations within a word for a specified match.
5. **Performance:** Three performance parameters are:

   (i) Access time (latency):
   - For random access memory, this is the time it takes to perform a read or write operation.
   - For non-random-access memory, access time is the time it takes to position the read-write mechanism at the desired location.

   (ii) Memory cycle time: For a random access memory it consists of the access time plus any additional time required before a second access can commence.

   (iii) Transfer rate: This is rate at which data can be transferred into or out of memory unit.

For Random access memory,

$$\text{Transfer rate} = \frac{1}{\text{Cycle Time}}$$

For non-random access memory, $T_N = T_A + \frac{N}{R}$

Where, T_N = Average time to read or write N-bits.
T_A = Average access time
N = Number of bits
R = Transfer rate in bits per second

6. **Physical type:** The physical type of a memory will be
   i. Semiconductor
   ii. Magnetic
   iii. Optical
   iv. Magneto-optimal
7. **Physical characteristics:** The memory may be
   - Volatile/non-volatile
   - Erasable/non-erasable
8. **Organization:** There is a trade–off among the three key characteristic of memory.
   i. Cost
   ii. Capacity
   iii. Access time

Memory Hierarchy

Consider the following memory hierarchy, which shows the various memory components.

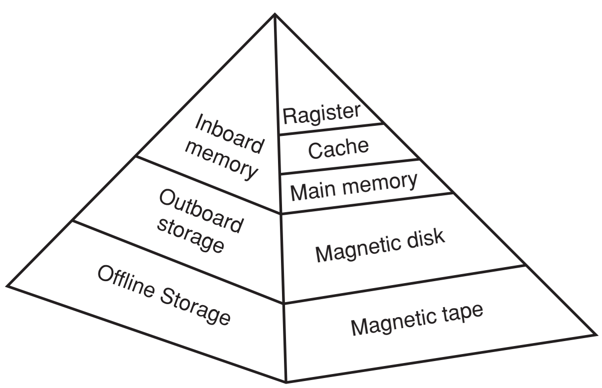


Figure 1 Memory Hierarchy

As one goes down the hierarchy, the following occur:

1. Decreasing cost per bit
2. Increasing capacity
3. Increasing access time
4. Decreasing frequency of access of the memory by the processor.

Locality of Reference

During the course of execution of a program, memory references by the processor, for both instructions and data, tend to cluster. This is referred as principal of locality.

(i) **Registers:** The fastest, smallest and most expensive type of memory consists of the registers internal to the processor.

(ii) **Main memory:** The principal internal memory system of the computer is main memory. Each location in main memory has a unique address.

(iii) **Cache:** Main memory is usually extended with a higher speed, smaller cache. The cache is not visible to the programmer or, indeed, to the processor. It is a device for staging the movement of data between main memory and processor registers to improve performance.

These three forms of memory are volatile and employ semi conductor technology.

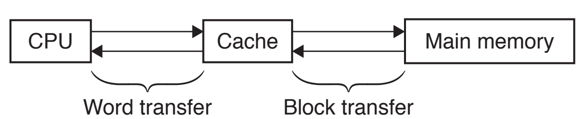
(iv) **Magnetic tapes and disks:** Data are stored more permanently on external mass storage devices, of which the most common are hard disk and removable media.
- External, non-volatile memory is also referred to as secondary or auxiliary memory.
- Used to store program and data files, which are visible to the programmer in the form of files and records.

Cache Memory

The locality of reference property states that over a short interval of time, the address generated by a typical program refer to a few localized areas of memory repeatedly, while the remainder of memory is accessed relatively infrequently (Because of frequent loops and subroutine calls).

If the active portions of the program and data are placed in a fast small memory, the average memory access time can be reduced, thus reducing the total execution time of the program. Such a fast small memory is referred to as a cache memory.

Cache memory is intended to give memory speed approaching that of the fastest memories available and at the same time provide a large memory size at the price of less expensive types of semiconductor memories. The following figure shows the structure of cache/main memory system.



The fundamental idea of cache organization is that by keeping the most frequently accessed instructions and data in the fast memory, the average memory access time will approach the access time of cache.

Basic Operation of Cache

- When the CPU need to access memory, the cache is examined. If the word is found in cache, it is read otherwise main memory is accessed to read the word.

- The performance of cache memory is measured in terms of hit ratio.
- When the CPU refers to memory and find the word in cache, it is called hit.
- If the word is not found in cache and is in Main Memory, it is called miss.

$$\text{Hit ratio} = \frac{\text{hits}}{\text{hits} + \text{misses}}$$

Average access time = $hc + (1 - h)(c + m)$m

Where, $c \rightarrow$ Cache access time

$m \rightarrow$ Main memory access time

$h \rightarrow$ hit ratio

- Let main memory consists of up to 2^n addressable words, with each word having a unique n-bit address.
- For mapping purposes, this memory is considered to consist of a number of fixed length blocks of K words each.

$$\therefore \text{ Number of blocks } (M) = \frac{2^n}{K}$$

- The cache consists of C lines.
- Each line contains K words, plus a tag of a few bits.
- The number of words in a line is referred to as the line size.
- The number of lines is considerably less than the number of main memory blocks i.e., $C << M$.
- Each line includes a tag that identifies which particular block is currently being stored.

  The tag is usually a portion of the main memory address.

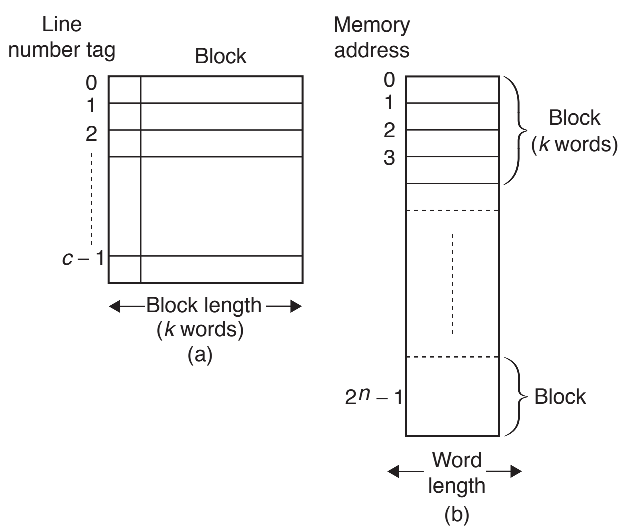


Figure 2 (a) Cache, (b) Main memory

Elements of Cache Design

1. Cache size
2. Mapping function
   - Direct
   - Associative
   - Set-associative
3. Replacement algorithm
4. Write policy
   - Write through
   - Write back
   - Write once
5. Line size
6. Number of caches
   - Single or two level
   - Unified or split

Cache size

The size of the cache to be small enough so that the overall average cost per bit is close to that of main memory alone and large enough so that the overall average access time is close to that of the cache alone.

Mapping function

Because there are fewer cache lines than main memory blocks, an algorithm is needed for mapping main memory blocks into cache lines. Three techniques can be used for mapping.

(i) Direct
(ii) Associative
(iii) Set-associative

Direct mapping Maps each block of main memory into only one possible cache line. Figure 2 illustrates the general mechanism. The mapping is expressed as

$i = j$ modulo m, where

i = cache line number

j = main memory block number

m = number of lines in the cache

For purpose of cache access, each main memory address can be viewed as consisting of three fields.

- The least significand w bits identify a unique word or byte within a block of main memory.
- The remaining s-bits specify one of the 2^s blocks of main memory. The cache logic interpret these s-bits as a tag of s-r bits. (most significand portion)
- A line field of r-bits, to identify one of 2^r lines.

To summarize,

Address length = $(s + w)$ bits

Number of Addressable units = 2^{s+w} words or bytes

Block size = line size = 2^w words or bytes

Number of blocks in main memory = $\frac{2^{s+w}}{2^w} = 2^s$ Number of lines in cache = $M = 2^r$.

Size of Tag = $(s - r)$ bits

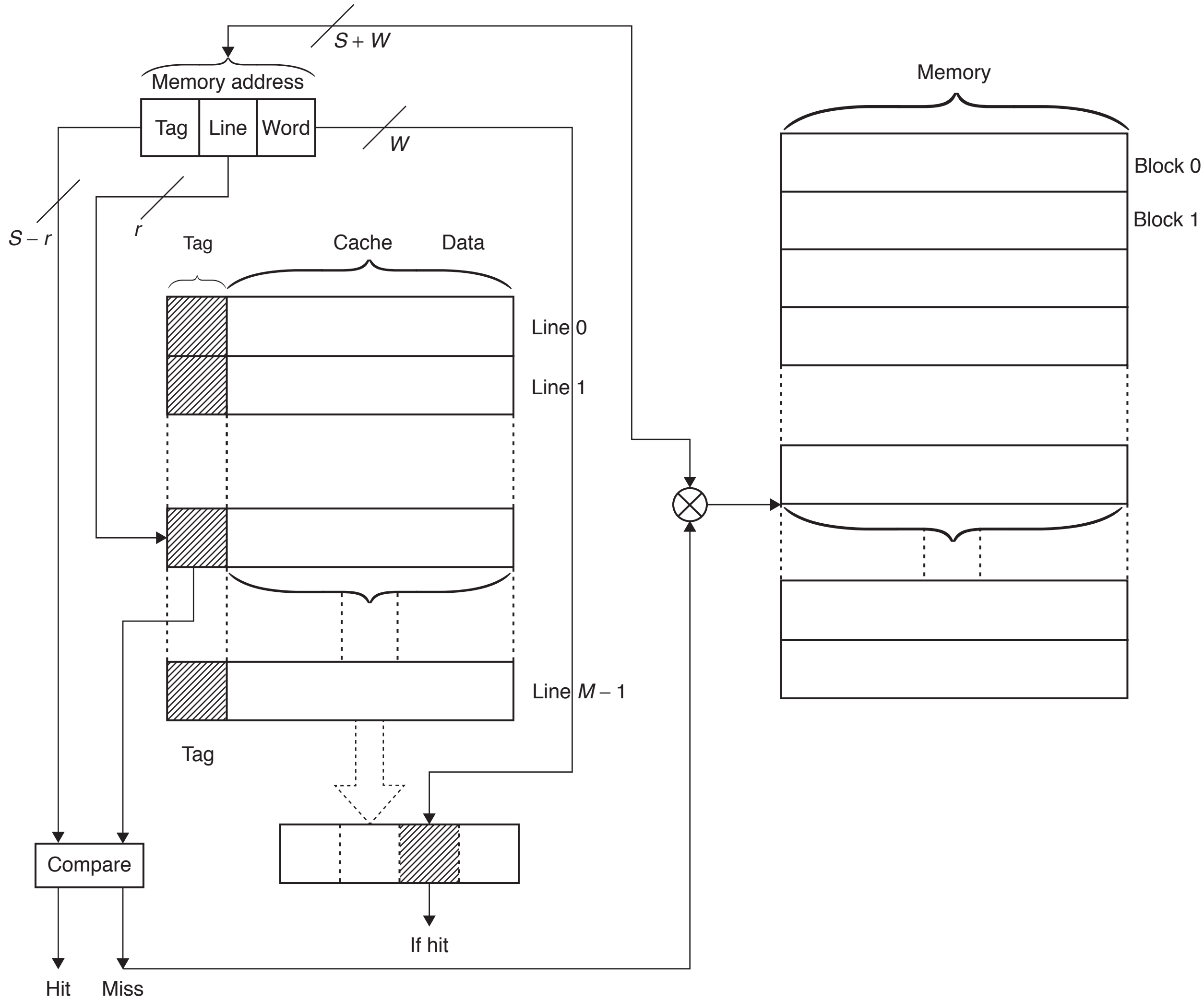


Figure 3 Direct Mapping

The effect of this mapping is that blocks of main memory are assigned to lines of the cache as follows:

Cache Line	Main Memory Blocks Assigned
0	0, m, $2m$, … $2^s - m$
1	1, $m+1$, $2m+1$, … $2^s - m + 1$
.	
.	
.	
.	
$m-1$	$m-1$, $2m-1$, $3m-1$,… $2^s - 1$

Example 1: Let cache capacity = 64 KB
Line size = 4 B
Main memory capacity = 16 MB = 2^{24} B
Using direct mapping, Address length = $s + w$ = 24-bits
Block size = 2^2 B

Number of blocks in main memory $= \dfrac{2^{24}}{2^2} = 2^{22}$

Number of lines in cache $= m = 2^r = \dfrac{2^{16}}{2^2} = 2^{14}$

$\therefore$ Size of tag $= s - r = 22 - 14 = 8$
$\therefore$ Main memory address =

Tag	Line	Word
8	14	2

The mapping becomes

Cache Line	Starting Memory Address of Blocks (Hexa)
0	00000, 010000, … FF0000
1	000004, 010004, … FF0004
.	.
.	.
.	.
.	.
$2^{14} - 1$	00FFFC, 01FFFC,… FFFFFC

Note: No two blocks that map into the same line number have the same tag number.

Advantages:
- Simple and cheap
- The tag field is short; only those bits have to be stored which are not used to address the cache.
- Access is very fast.

Disadvantages: A given block fits into a fixed cache location, i.e., a given cache line will be replaced whenever there is a reference to another memory block which fits to the same line, regardless what the status of the other cache line is.

This can produce a low hit ratio, even if only a very small part of the cache is effectively used.

Associative mapping This technique overcomes the disadvantage of direct mapping by permitting each main memory block to be loaded into any line of the cache. Here the cache control logic interprets a memory address as two fields.

1. Tag
2. Word

Figure shows associative mapping technique:

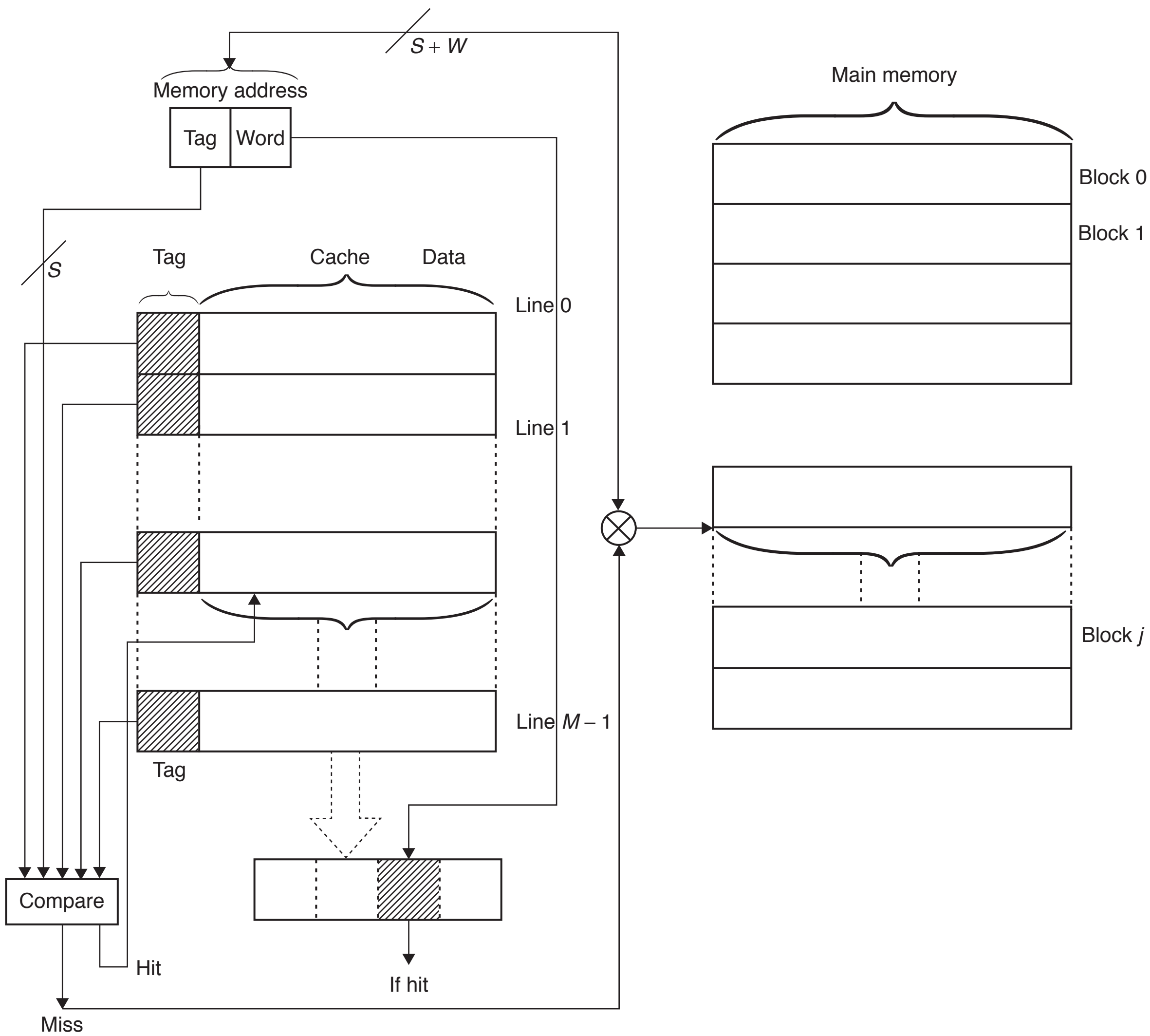


Figure 4 Associative mapping

To determine whether a block is in the cache, the cache control logic must simultaneously examine every line tag for a match. No field in the address corresponds to line number, so that the number of lines in the cache is not determined by the address format.

To summarize,

Address length = $(s + w)$ bits
Number of addressable units = 2^{s+w} words or bytes
Block size = line size = 2^w words or bytes
Number of blocks in main memory = $\frac{2^{s+w}}{2^w} = 2^S$
Number of lines in cache = undetermined
Size of tag = s-bits

Example 2: Cache size = 64 KB
Line size = 4 B
Main memory capacity = 16 MB

Number of blocks in main memory $= \dfrac{2^{24}}{2^2} = 2^{22}$.

$\therefore$ Size of tag = 24 – 2 = 22-bits

For example, the tag of the hexadecimal main memory address 16339C is 058CE7

Main memory address =

Tag	Word
22	2

Advantages: Associative mapping provides the highest flexibility concerning the line to be replaced when a new block is read into a cache.

Disadvantages:

- Complex
- The tag field is long
- Fast access can be achieved only using high performance associative memories for the cache, which is difficult and expensive.

Set-associative mapping: It exhibits strengths of both the direct and associative approaches and reduces their disadvantages.

Here the cache is divided into V sets, each of which consists of K lines

i.e., $m = V \times K$

$i = j$ modulo V

Where i = cache set number

j = main memory block number

m = number of lines in cache

As there are K lines in each set, this is referred as K-way set associative mapping. The cache control logic interprets a memory address simply as three fields.

1. Tag
2. Set
3. Word

The d set bits specify one of $V = 2^d$ sets. The S-bits of the tag and the set fields specify one of the 2^S blocks of main memory.

Figure 3 shows Set-associative mapping.

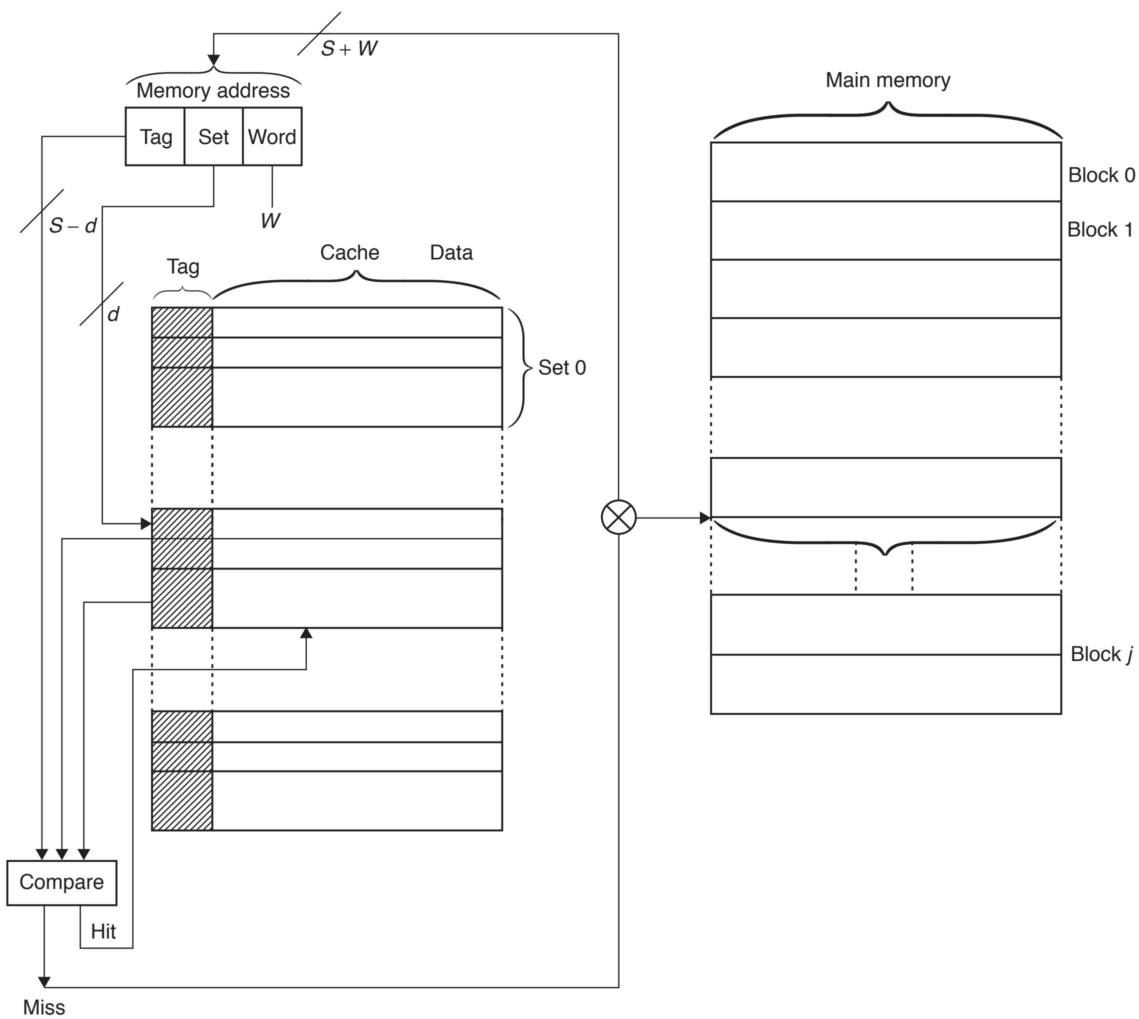


Figure 5 *K*-way set associative cache

Here the tag in a memory address is much smaller and is only compared to the K tags within a single set. To summarize,

Address length = $(s + w)$ bits
Number of Addressable units = 2^{s+w} words or bytes.
Block size = line size = 2^w words or bytes.

Number of blocks in main memory = $\frac{2^{s+w}}{2^w} = 2^s$

Number of lines in set = K
Number of sets $V = 2^d$
Number of lines in cache = $KV = K \times 2^d$
$\therefore$ Size of tag = $(s - d)$ bits

Example 3: Cache capacity = 64 KB
Block size = 4 B
Main memory capacity = 16 MB

Number of blocks in main memory $= \frac{2^{24}}{2^2} = 2^{22}$

For 2-way set associative mapping,
Number of lines in a set $K = 2$
Number of sets = $V = 2^d$

Number of lines in cache = $K \times 2^d = \frac{2^{16}}{2^2}$

$= 2^{14}$
$\Rightarrow \quad 2 \times 2^d = 2^{14}$
$\Rightarrow \quad 2^d = 2^{13}$
$\Rightarrow \quad d = 13$
$\therefore \quad$ Size of Tag = 22 − 13 = 9
Hence main memory address =

Tag	Set	Word
9	13	2

In practice, 2 and 4-way set associative mapping are used with very good results. Larger sets do not produce further significant performance improvement.

If a set consist of a single line, i.e., it is direct mapping; If there is one single set consisting of all lines i.e., it is associative mapping.

Replacement algorithms

Once the cache has been filled, when a new block is brought into the cache, one of the existing blocks must be replaced. For direct mapping, there is only possible line for any particular block, and no choice is possible.

For associative and set associative techniques, a replacement algorithm is needed. Four of the most common replacement algorithms are

(i) LRU (Least recently used): Replaces the block in the set that has been in the cache longest with no reference to it.
(ii) FIFO (First-in-first-out): Replace the block in the set that has been in the cache longest.
(iii) LFU (Least frequently used): Replace the block in the set that has experienced the fewest references.
(iv) Random

Write policy

When a block that is resident in the cache is to be replaced, there are two cases to consider.

(i) If the old block in the cache has not been altered, then it may be over-written with a new block without first writing out the old block.
(ii) If at least one write operation has been performed on a word in that line of the cache, then main memory must be updated by writing the line of cache out of the block of memory before bringing in the new block.

The write policies are

(a) Write through: All write operations are made to main memory as well as to the cache, ensuring that main memory is always valid.
Drawback: Creates substantial memory traffic

(b) Write back: This technique minimizes memory writes. It updates are made only in the cache. When a block is replaced it is written back to main memory if and only if it is updated.
Drawback: There some portions of main memory are invalid and hence accesses by I/O modules can be allowed only through the cache.

Line size

Larger blocks reduce the number of blocks that fit into a cache. Because each block fetch overwrites older cache contents, a small number of blocks results in data being over written shortly after they are fetched.

As a block becomes larger, each additional word is farther from the requested word, therefore less likely to be needed in the near future.

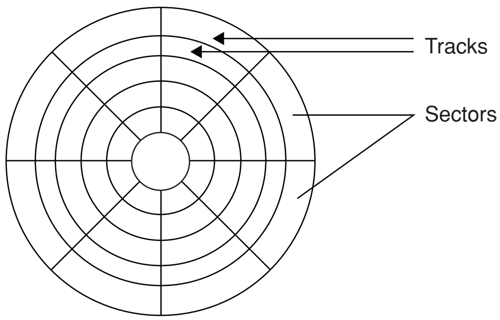
Number of caches

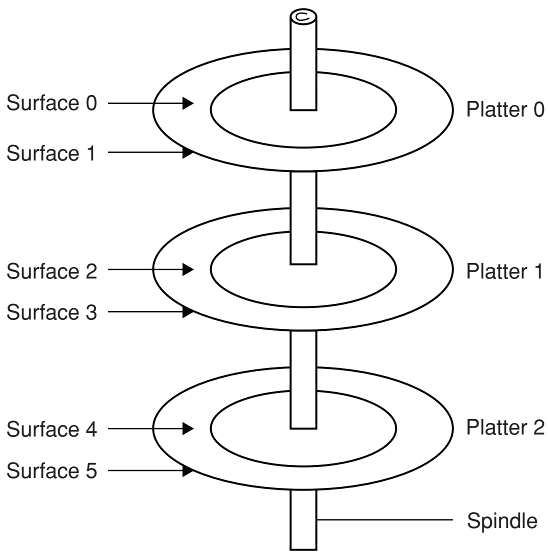
Multilevel caches We may have on-chip cache as well as external cache. This is a two level cache organization, with the internal cache designated as level 1, and external cache designated as level 2.

Secondary Storage

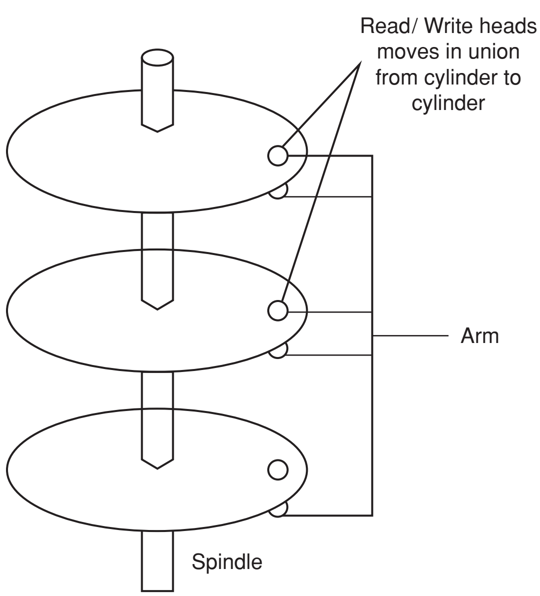
Disk

Disk consists of platters, each with two surfaces. Each surface consists of concentric rings called tracks. Each track consists of sectors separated by gaps.





Disk operation: The disk surface spins at a fixed rotational rate. There is a read/write head attached to the end of the arm and flies over the disk surface on a thin cushion of air. By moving radially the arm can position the read/write head over any track.



Disk access time: Average time to access some target sector:

$T_{ae} = T_{\text{avg seek}} + T_{\text{avg rotation}} + T_{\text{avg transfer}}$

Where $T_{\text{avg seek}}$ is typical 9 ms.

$$T_{\text{avg rotation}} = \frac{1}{2} \times \frac{1}{\text{RPMS}} \times 60 \text{ Sec/1min}$$

$$T_{\text{avg rotation}} = \frac{1}{\text{RPM}} \times \frac{1}{(\text{avg sector/track})} \times 60 \text{ Sec/1min}$$

Notes:

1. Seek time is the Time to position heads over cylinder containing target sectors ($T_{\text{avg seek}}$).
2. Rotational Latency is the time waiting for first bit of target sector to pass under read/write head. ($T_{\text{avg rotation}}$).
3. Transfer Time is the time to read the bits in the target sector ($T_{\text{avg transfer}}$).
   - Data are recorded on the surface of a hard disk made of metal coated with magnetic material.
   - The disks and the drive are usually built together and encased in an air tight container to protect the disk from pollutants such as smoke particle and dust. Several disks are usually started on a common drive shaft with each disk having its own read/write head.

Diskette

Data are recorded on the surface of a floppy disk made of polyester coated with magnetic material.

A special diskette drive must be used to access data stored in the floppy disk. It works much like a record turntable of Gramophone.

Main features

- Direct access
- Cheap
- Portable, convenient to use

Main Standards

- $5^1/_4$ inch capacity ≅ 360 KB/ disk
- $3^1/_2$ inch capacity ≅ 1.44 MB/disk (about 700 pages of A_4 text)

Magnetic Tape

Magnetic tape is made up from a layer of plastic which is coated with iron oxide. The oxide can be magnetized in different directions to represent data. The operation uses a similar principle as in the case of a tape recorder.

Main features

- Sequential access (access time about 1.55)
- High value of storage (50 MB/tape)
- Inexpensive

It is often used for Batch up or archive purpose.

Optimal Memory

CD-ROM (Compact disk ROM): The disk surface is imprinted with microscopic holes which record digital information. When a low-powered power beam shines on the surface, the intensity of the reflected light changes as it encounters a hole. The change is detected by a photo sensor and converted into a digital signal.

- Huge capacity: 775 MB/disk (≈550 diskette)
- Inexpensive replication, cheap production.
- Removable, read only.
- Long access time (could be half a second)

WORM (Write Once Read Memory) CD: A lower beam of modest intensity equipped in the disk drive is used to imprint the hole pattern.

- Good for archival storage by providing a permanent record of large volumes of data.

Erasable Optical Disk: Combination of Laser technology and magnetic surface technique.

- Can be repeatedly written and overwritten.
- High reliability and longer life than magnetic disks.

Exercise

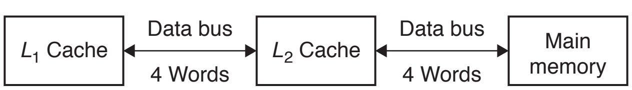
Practice Problems 1

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Find the number of bits in the cache index and tag for a direct mapped cache of size 32 KB with block size of 32 bytes. The CPU generates 48-bit addresses.
(A) 33,15 (B) 15,10
(C) 10,33 (D) 15,33

2. Given the cache access time is 200 ns and the memory access time is 400 ns. If the effective access time is 20% greater than the cache access time, what is the hit ratio?
(A) 80% (B) 20%
(C) 40% (D) 100%

3. A computer system has an L_1 cache, an L_2 cache and a main memory unit connected as shown below. The block size in L_1 cache is 2 words. The block size in L_2 cache is 8 words. The memory access times are 2 nanoseconds, 20 nanoseconds and 200 nanoseconds for L_1 cache, L_2 cache and main memory unit, respectively.



When there is a miss in L_1 cache and a hit in L_2 cache, a block is transferred from L_2 cache to L_1 cache. What is the time taken for this transfer?
(A) 22 ns (B) 44 ns
(C) 66 ns (D) 88 ns

4. In direct memory management, CPU references address of 15-bits. Main memory size is 512 * 8 and cache memory size is 128 * 8. Tag and line are respectively
(A) 9, 7 (B) 7, 9
(C) 15, 7 (D) 7, 15

5. Consider a cache with 64 blocks and a block size of 16 bytes. The byte address of 1200 maps to ____ block number.
(A) 10 (B) 11
(C) 64 (D) 16

6. In a cache memory, cache line is 64 bytes. The main memory has latency of 32 ns and bandwidth of 1 GB/sec. Then the time required to fetch the entire cache line from main memory is
(A) 32 ns (B) 64 ns
(C) 96 ns (D) 128 ns

7. A set associative cache consists of 64 lines or slots divided into four-line sets. Main memory contains 4K blocks of 128 word each. Then the number of bits present in tag, set and word fields are respectively.
(A) 7, 6, 7 (B) 6, 7, 7
(C) 4, 8, 7 (D) 8, 4, 7

8. A 2-way set-associative cache has lines of 32 bytes and a total size of 16 KB. The 32 MB main memory is byte addressable. Then which of the following two memory addresses mapped to same set?
(A) 10D6A32, 035C3A2
(B) 2A36D01, 2A3C530
(C) 10D63A2, 035C3A0
(D) 2A36D08, 0A3C538

9. Let the cache memory capacity is 64 KB and main memory capacity is 16 MB. Let block size is 4 bytes. Then the tag, line, word fields in hexadecimal notation for the main memory address cccccc using direct mapped cache will be
(A) cc, ccc, c (B) cc, 3333, 0
(C) cc, ccc, 0 (D) cc, 333, 30

10. Consider a 32-bit microprocessor that has an on-chip 16 KB four-way set associative cache. Assume that the cache has a line size of four 32-bit words. Then the word in the memory location ABCDE8F8 will be mapped to
(A) 143rd set (B) 815th set
(C) 255th set (D) 0th set

11. Given the following specifications for an external cache memory:
Four-way set associative, Line size of two 16-bit words; Able to accommodate a total of 4K 32-bit words from

main memory. Used with a 16-bit processor that issues 24-bit address. Then the number of bits used to represent set field is
(A) 2-bits (B) 10-bits
(C) 12-bits (D) 14-bits

12. Consider a machine with a byte addressable main memory of 2^{16} bytes and block size of 8 bytes. Assume that a direct mapped cache consisting of 32 lines is used with this machine. Then in what line would bytes with the address 1100 0011 0011 0100 is stored?
(A) slot 3 (B) slot 4
(C) slot 6 (D) slot 12

13. A computer system contains a main memory of 32 K 16-bit words. It also has a 4 K-word cache divided into four line sets with 64 words per line. The processor fetches words from locations 0, 1, 2…, 4351 in that order. It then repeats this fetch sequence 10 more times. The cache is 10 times faster than main memory. Then the improvement resulting from the use of the cache is (assume an LRU policy is used for block replacement)
(A) 0.63 (B) 0.45
(C) 1.21 (D) 2.18

14. Consider an L_1 cache with an access time of 1ns and a hit ratio of $H = 0.95$. Suppose that we can change the cache design such that we increase H to 0.98, but increase access time to 1.5ns. Which of the following condition is met for this change to result in improved performance?
(A) Next level memory access time must be less than 16.67
(B) Next level memory access time must be greater than 16.67
(C) Next level memory access time must be less than 50
(D) Next level memory access time must be greater than 50

15. Consider a single-level cache with an access time of 2.5 ns and a line size of 64 bytes and a hit ratio of $H = 0.95$. Main memory uses a block transfer capability that has a first-word (4 bytes) access time of 50 ns and an access time of 5ns for each word thereafter. What is the access time when there is a cache miss?
(A) 130 ns (B) 149.4 ns
(C) 2.375 ns (D) 8.875 ns

16. The tag, block and word fields of main memory address using direct mapping technique for 2048 main memory blocks, 128 blocks of cache memory and block size of 16:
(A) 4, 7, 4 (B) 7, 4, 4
(C) 11, 7, 4 (D) Data insufficient

17. Let H_1 is level 1 cache hit ratio, H_2 is level 2 cache hit ratio, C_1 is the time required to access Level 1 cache, C_2 is the time required to access Level 2 cache and M is the time required to access Main memory. Then the average access time required by the processor is
(A) $H_1C_1 + (1 - H_1)\, H_2(C_2) + (1 - H_1)\,(1 - H_2)\,(M)$
(B) $H_1C_1 + (1 - H_1)\, H_2(C_1 + C_2) + (1 - H_1)\,(1 - H_2)\,(C_1 + C_2 + M)$
(C) $H_1C_1 + H_1H_2(C_1 + C_2) + H_1H_2(C_1 + C_2 + M)$
(D) $H_1C_1 + (1 - H_1)\, H_2(C_1.C_2) + (1 - H_1)\,(1 - H_2)\,(C_1.C_2.M)$

18. If $p = 2^m$ be the number of lines in cache and $b = 2^n$ be the size of each block, then total words that can be stored in cache memory is given by
(A) 2^{m+n} (B) 2^{m-n}
(C) $m + n$ (D) $p + b$

19. Cache memory enhances
(A) memory capacity
(B) memory access time
(C) secondary storage capacity
(D) secondary storage access time

20. Which of the following property allows the processor to execute a number of clustered locations?
(A) Spatial (B) Temporal
(C) Inclusion (D) Coherence

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. If average access time of CPU is 20 ns, access time of main memory is 110 ns and the cache access time is 10 ns. What is the hit ratio?
(A) 100% (B) 90%
(C) 80% (D) 70%

2. A hard disk spins at 180 revolutions per minute. What is the average rotational latency?
(A) 0.16 sec (B) 0.32 sec
(C) 0.2 sec (D) 0.4 sec

3. A disk pack have 16 surfaces, with 128 tracks per surface and 256 sectors per track. 512 bytes of data are stored in a bit serial manner in a sector. The number of bits required to specify a particular sector in the disk is
(A) 4 (B) 7
(C) 11 (D) 19

4. A disk has 19456 cylinders, 16 heads and 63 sectors per track. The disk spins at 5400 rpm. Seek time between adjacent tracks is 2 ms. Assuming the read/write head is already positioned at track 0, how long does it take to read the entire disk?
(A) 48 min (B) 58 min
(C) 64 min (D) 72 min

5. A certain moving arm disk storage with one head has following specifications:
Number of tracks/recording surface = 200
Disk Rotation Speed = 2400 rpm
Track storage capacity = 62500-bits
The average latency time (assuming that head can move from one track to another only by traversing the entire track) is
(A) 0.125 sec (B) 1.25 sec
(C) 0.0125 sec (D) 12.5 sec

6. In Memory management system, cache memory access time is 100 ns and main memory access time is 200 ns. Number of CPU references is 100 and number of hits is 10. Average access time is
(A) 150 ns (B) 100 ns
(C) 190 ns (D) 280 ns

7. The seek time of disk is 40 m sec. It rotates at the rate of 40 rps. The capacity of each track is 400 words. The access time is
(A) 50 m sec (B) 53 m sec
(C) 60 m sec (D) 63 m sec

8. An Associated cache and one million word main memory are divided into 256 word blocks. How many blocks are there?
(A) 2^8 (B) 2^{12}
(C) 2^{20} (D) 2^{28}

9. The average access time of a disk is
(A) Seek time + Rotational latency time
(B) Seek time
(C) Rotational latency + transfer time + seek time
(D) Rotation latency + transfer time.

10. What will be the size of the memory whose last memory location is FFFF?
(A) 64 k (B) 32 k
(C) 10 k (D) 24 k

11. Data from a cassette tape is obtained by ____ accessing method.
(A) Parallel (B) Serial
(C) Sequential (D) Random

12. For a memory system, the desirable characteristics is/are
(A) Speed and reliability
(B) Durability and compactness
(C) Low power consumption
(D) All of these

13. The memory that has the shortest access time is
(A) Magnetic bubble (B) Magnetic core memory
(C) Cache memory (D) RAM

14. Cache memory
(A) has greater capacity than RAM.
(B) enhances secondary storage access time.
(C) is faster to access than registers.
(D) is faster to access than main memory

15. Consider a disk pack with 16 surfaces 128 tracks per surface and 256 sectors per track. 512 bytes of data are stored in bit and serial manner, Then the capacity of the disk is
(A) 256 MB (B) 256 KB
(C) 512 MB (D) 64 MB

16. Principle of locality justifies the use of
(A) Cache (B) DMA
(C) Disk (D) RAM

17. The main memory of a computer has $2ab$ blocks while cache has $2a$ blocks. If the cache uses the set associative mapping scheme with two blocks per set, then block k of main memory maps to the set:
(A) (k mod b) of the cache (B) (k mod a) of cache
(C) (k mod $2a$) of cache (D) (k mod $2ab$) of cache

18. Which of the following factors do not affect the hit ration of cache?
(A) Block replacement algorithms.
(B) Block frame size
(C) Cycle counts
(D) Main memory size

19. In which of the following mapping function, there is no need of replacement algorithm?
(A) Direct Mapping
(B) Set-associative mapping
(C) Full associative mapping
(D) Both (A) and (B)

20. In a direct mapping, the index field equals to
(A) Sum of tag and word fields
(B) Sum of block and word fields
(C) Sum of tag an block fields
(D) Same as block field

Previous Years' Questions

1. Consider a small two-way set-associative cache memory, consisting of four blocks. For choosing the block to be replaced, use the least recently used (LRU) scheme. The number of cache misses for the following sequence of block addresses is 8, 12, 0, 12, 8 **[2004]**

(A) 2 (B) 3
(C) 4 (D) 5

2. Consider a direct mapped cache of size 32 KB with block size 32 bytes. The CPU generates 32 bit addresses. The number of bits needed for cache indexing and the number of tag bits are respectively. **[2005]**

(A) 10, 17 (B) 10, 22
(C) 15, 17 (D) 5, 17

Common data for questions 3 and 4: Consider two cache organizations: The first one is 32 KB 2-way set associative with 32-byte block size. The second one is of the same size but direct mapped. The size of an address is 32 bits in both cases. A 2-to-1 multiplexer has a latency of 0.6 ns while a k-bit comparator has a latency of k/10 ns. The hit latency of the set associative organization is h_1 while that of the direct mapped one is h_2.

3. The value of h_1 is: **[2006]**

(A) 2.4 ns (B) 2.3 ns
(C) 1.8 ns (D) 1.7 ns

4. The value of h_2 is: **[2006]**

Data for question 5: Consider a machine with a byte addressable main memory of 2^{16} bytes. Assume that a direct mapped data cache consisting of 32 lines of 64 bytes each is used in the system. A 50 × 50 two-dimensional array of bytes is stored in the main memory starting from memory location 1100 H. Assume that the data cache is initially empty. The complete array is accessed twice. Assume that the contents of the data cache do not change in between the two accesses.

5. Which of the following lines of the data cache will be replaced by new blocks in accessing the array for the second time? **[2007]**

(A) line 4 to line 11 (B) line 4 to line 12
(C) line 0 to line 7 (D) line 0 to line 8

6. For inclusion to hold between two cache levels L_1 and L_2 in a multi-level cache hierarchy, which of the following are necessary? **[2008]**

(i) L_1 must be a write-through cache
(ii) L_2 must be a write-through cache
(iii) The associativity of L_2 must be greater than that of L_1
(iv) The L_2 cache must be atleast as large as the L_1 cache

(A) (iv) only (B) (i) and (iv) only
(C) (i), (ii) and (iv) only (D) (i), (ii), (iii) and (iv)

Common data for questions 7, 8 and 9: Consider a machine with a 2-way set associative data cache of size 64K-bytes and block size 16-bytes. The cache is managed using 32-bit virtual addresses and the page size is 4Kbytes. A program to be run on this machine begins as follows:

```
double ARR [1024] [1024] ;
        int i, j;
/* Initialize array ARR to 0.0 */
 for (i = 0; i < 1024; i++)
 for (j = 0; j < 1024; j++)
 ARR [i] [j] = 0.0;
```

The size of double is 8 Bytes. Array ARR is located in memory starting at the beginning of virtual page 0XFF000 and stored in row major order. The cache is initially empty and no pre-fetching is done. The only data memory references made by the program are those to array ARR.

7. The total size of the tags in the cache directory is **[2008]**

(A) 32K-bits (B) 34K-bits
(C) 64K-bits (D) 68K-bits

8. Which of the following array elements has the same cache index as ARR [0] [0]? **[2008]**

(A) ARR [0] [4] (B) ARR [4] [0]
(C) ARR [0] [5] (D) ARR [5] [0]

9. The cache hit ratio for this initialization loop is **[2008]**

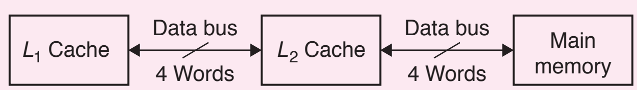
(A) 0% (B) 25%
(C) 50% (D) 75%

10. Consider a 4-way set associative cache (initially empty) with total 16 cache blocks. The main memory consists of 256 blocks and the request for memory blocks is in the following order: **[2009]**
0, 255, 1, 4, 3, 8, 133, 159, 216, 129, 63, 8, 48, 32, 73, 92, 155.

Which one of the following memory block will NOT be in cache if LRU replacement policy is used?

(A) 3 (B) 8
(C) 129 (D) 216

Common data questions 11 and 12: A computer system has an L_1 cache, an L_2 cache, and a main memory unit connected as shown below. The block size in L_1 cache is 4 words. The block size in L_2 cache is 16 words. The memory access times are 2 nanoseconds. 20 nanoseconds and 200 nanoseconds, for L_1 cache, L_2 cache and main memory unit respectively.



11. When there is a miss in L_1 cache and a hit in L_2 cache, a block is transferred from L_2 cache to L_1 cache. What is the time taken for this transfer? **[2010]**

(A) 2 ns (B) 20 ns
(C) 22 ns (D) 88 ns

12. When there is a miss in both L_1 cache and L_2 cache, first a block is transferred from main memory to L_2 cache, and then a block is transferred from L_2 cache to L_1 cache. What is the total time taken for these transfers? **[2010]**

(A) 222 ns (B) 888 ns
(C) 902 ns (D) 968 ns

13. An 8 KB direct-mapped write-back cache is organized as multiple blocks, each of size 32 bytes. The processor generates 32-bit addresses. The cache controller maintains the tag information for each cache block comprising of the following.

1 Valid bit
1 Modified bit
As many bits as the minimum needed to identify the memory block mapped in the cache.

What is the total size of memory needed at the cache controller to store meta-data (tags) for the cache? **[2011]**

(A) 4864 bits (B) 6144 bits
(C) 6656 bits (D) 5376 bits

Common data for questions 14 and 15: A computer has a 256 KB, 4-way set associative, write back data cache with block size of 32 bytes. The processor sends 32 bit addresses to the cache controller. Each cache tag directory entry contains, in addition to address tag, 2 valid bits, 1 modified bit and 1 replacement bit.

14. The number of bits in the tag field of an address is **[2012]**

(A) 11 (B) 14
(C) 16 (D) 27

15. The size of the cache tag directory is **[2012]**

(A) 160 K-bits (B) 136 K-bits
(C) 40 K-bits (D) 32 K-bits
(A) 2.4 ns (B) 2.3 ns
(C) 1.8 ns (D) 1.7 ns

16. In a k-way set associative cache, the cache is divided into v sets, each of which consists of k lines. The lines of a set are placed in sequence one after another. The lines in set s are sequenced before the lines in set $(s + 1)$. The main memory blocks are numbered 0 onwards. The main memory block numbered j must be mapped to any one of the cache lines from **[2013]**

(A) $(j \bmod v) * k$ to $(j \bmod v) * k + (k - 1)$
(B) $(j \bmod v)$ to $(j \bmod v) + (k - 1)$
(C) $(j \bmod k)$ to $(j \bmod k) + (v - 1)$
(D) $(j \bmod k) * v$ to $(j \bmod k) * v + (v - 1)$

17. An access sequence of cache block addresses is of length N and contains n unique block addresses. The number of unique block addresses between two consecutive accesses to the same block address is bounded above by K. what is the miss ratio if the access sequence is passed through a cache of associativity $A \geq K$ exercising least-recently used replacement policy? **[2014]**

(A) n/N (B) $1/N$
(C) $1/A$ (D) K/n

18. A 4-way set -associative cache memory unit with a capacity of 16 KB is built using a block size of 8 words. The word length is 32-bits. The size of the physical address space is 4 GB. The number of bits for the TAG field is _______. **[2014]**

19. In designing a computer's cache system, the cache block (or cache line) size is an important parameter. Which one of the following statements is correct in this context? **[2014]**

(A) A smaller block size implies better spatial locality.
(B) A smaller block size implies a smaller cache tag and hence lower cache tag overhead.
(C) A smaller block size implies a larger cache tag and hence lower cache hit time.
(D) A smaller block size incurs a lower cache miss penalty.

20. Consider a main memory system that consists of 8 memory modules attached to the system bus, which is one word wide. When a write request is made, the bus is occupied for 100 nanoseconds (ns) by the data, address, and control signals. During the same 100 ns, and for 500 ns thereafter, the addressed memory module executes one cycle accepting and storing the data. The (internal) operation of different memory modules may overlap in time, but only one request can be on the bus at any time. The maximum number of stores (of one word each) that can be initiated in 1 millisecond is _______. **[2014]**

21. If the associativity of processor cache is doubled while keeping the capacity and block size unchanged, which one of the following is guaranteed to be NOT affected? **[2014]**

(A) Width of tag comparator
(B) Width of set index decoder
(C) Width of way selection multiplexer
(D) Width of processor to main memory data bus

22. The memory access time is 1 nanosecond for a read operation with a hit in cache, 5 nanoseconds for a read operation with a miss in cache, 2 nanoseconds for a write operation with a hit in cache and 10 nanoseconds for a write operation with a miss in cache. Execution of a sequence of instructions involves 100 instruction fetch operations. 60 memory operand read operations

and 40 memory operand write operations. The cache - hit ratio is 0.9. The average memory access time (in nanoseconds) in executing the sequence of instructions is _______. **[2014]**

23. Assume that for a certain processor, a read request takes 50 nanoseconds on a cache miss and 5 nanoseconds on a cache hit. Suppose while running a program, it was observed that 80% of the processor's read requests result in a cache hit. The average read access time in nanoseconds is _______ **[2015]**

24. A computer system implements a 40-bit virtual address, page size of 8 kilobytes, and a 128-entry translation look-aside buffer (TLB) organized into 32 sets each having four ways. Assume that the TLB tag does not store any process id. The minimum length of the TLB tag in bits is ______ **[2015]**

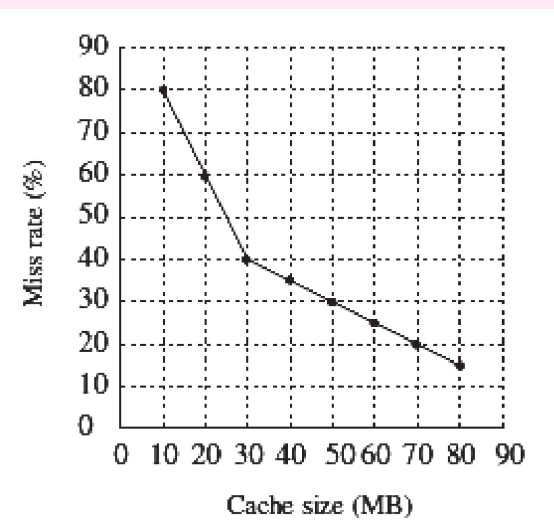
25. Consider a machine with a byte addressable main memory of 2^{20} bytes, block size of 16 bytes and a direct mapped cache having 2^{12} cache lines. Let the addresses of two consecutive bytes in main memory be $(E201F)_{16}$ and $(E2020)_{16}$. What are the tag and cache line address (in hex) for main memory address $(E201F)_{16}$? **[2015]**

(A) E, 201 (B) F, 201
(C) E, E20 (D) 2, 01F

26. The width of the physical address on a machine is 40 bits. The width of the tag field in a 512KB 8-way set associative cache is ____ bits. **[2016]**

27. A file system uses an in - memory cache to cache disk blocks. The miss rate of the cache is shown in the figure. The latency to read a block from the cache is 1 ms and to read a block from the disk is 10ms. Assume that the cost of checking whether a block exists in the cache is negligible. Available cache sizes are in multiples of 10MB.

The smallest cache size required to ensure an average read latency of less than 6 ms is ____ MB. **[2016]**



28. Consider a two-level cache hierarchy with L1 and L2 caches. An application incurs **1.4** memory accesses per instruction on average. For this application, the miss rate of L1 cache is **0.1**; the L2 cache experiences on average 7 misses per 1000 instructions. The miss rate of L2 expressed correct to two decimal places is ______. **[2017]**

29. Consider a 2-way set associative cache with 256 blocks and uses LRU replacement. Initially the cache is empty. Conflict misses are those misses which occur due to contention of multiple blocks for the same cache set. Compulsory misses occur due to first time access to the block. The following sequence of accesses to memory blocks

(0, 128, 256, 128, 0, 128, 256, 128, 1, 129, 257, 129, 1, 129, 257, 129)

is repeated 10 times. The number of *conflict misses* experienced by the cache is _____. **[2017]**

30. A cache memory unit with capacity of N words and block size of B words is to be designed. If it is designed as a direct mapped cache, the length of the TAG field is 10 bits. If the cache unit is now designed as a 16-way set-associative cache, the length of the TAG filed is __________bits. **[2017]**

31. In a two-level cache system, the access times of L_1 and L_2 caches are 1 and 8 clock cycles, respectively. The miss penalty from the L_2 cache to main memory is 18 clock cycles. The miss rate of L_1 cache is twice that of L_2. The average memory access time (AMAT) of this cache system is 2 cycles. The miss rates of L_1 and L_2 respectively are: **[2017]**

(A) 0.111 and 0.056 (B) 0.056 and 0.111
(C) 0.0892 and 0.1784 (D) 0.1784 and 0.0892

32. The read access times and the hit ratios for different caches in a memory hierarchy are as given below.

Cache	Read access time (in nanoseconds)	Hit ratio
I-cache	2	0.8
D-cache	2	0.9
L2-cache	8	0.9

The read access time of main memory is 90 nanoseconds. Assume that the caches use the referred- word-first read policy and the write back policy. Assume that all the caches are direct mapped caches. Assume that the dirty bit is always 0 for all the blocks in the caches. In execution of a program. 60% of memory reads are for instruction fetch and 40% are for memory operand fetch. The average read access time in nanoseconds (up to 2 decimal places) is __________. **[2017]**

33. Consider a machine with a byte addressable main memory of 2^{32} bytes divided into blocks of size 32

bytes. Assume that a direct mapped cache having 512 cache lines is used with this machine. The size of the tag field in bits is________. **[2017]**

34. The size of the physical address space of a processor is 2^P bytes. The word length is 2^W bytes. The capacity of cache memory is 2^N bytes. The size of each cache block is 2^M words. For a K-way set-associative cache memory, the length (in number of bits) of the tag field is: **[2018]**

(A) $P - N - \log_2 K$
(B) $P - N + \log_2 K$
(C) $P - N - M - W - \log_2 K$
(D) $P - N - M - W + \log_2 K$

Answer Keys

Exercises

Practice Problems 1

1. D	**2.** A	**3.** D	**4.** A	**5.** B	**6.** C	**7.** D	**8.** C	**9.** B	**10.** A
11. B	**12.** C	**13.** D	**14.** B	**15.** A	**16.** A	**17.** B	**18.** A	**19.** B	**20.** A

Practice Problems 2

1. B	**2.** A	**3.** D	**4.** B	**5.** C	**6.** D	**7.** B	**8.** B	**9.** C	**10.** A
11. C	**12.** D	**13.** C	**14.** D	**15.** A	**16.** A	**17.** B	**18.** D	**19.** A	**20.** B

Previous Years' Questions

1. C	**2.** A	**3.** A	**4.** D	**5.** C	**6.** B	**7.** D	**8.** B	**9.** C	**10.** D
11. D	**12.** D	**13.** D	**14.** C	**15.** A	**16.** A	**17.** A	**18.** 20	**19.** D	
20. 10000		**21.** D	**22.** 1.68	**23.** 14	**24.** 22	**25.** A	**26.** 24	**27.** 30	**28.** 0.05
29. 76	**30.** 14	**31.** A	**32.** 4.72	**33.** 18	**34.** B				

Test

Computer Organization and Architecture

Time: 60 min.

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. Which of the following register keeps track of instruction execution sequence?
(A) Accumulator (B) Program counter
(C) Stack pointer (D) Instruction register

2. Consider the following Register Transfer Language:

$$R_1 \leftarrow R_1 + M[R_2 + R_3]$$

Where R_1, R_2 and R_3 are the CPU registers and 'M' is a memory location in primary memory, which addressing mode is suitable for above register transfer language?
(A) Indirect (B) Direct
(C) Indexed (D) Displacement

3. Which of the following is/are advantage(s) of using a multiple-bus architecture over a single-bus architecture?
(i) Multiple-bus architecture reduces propagation delay.
(ii) Multiple-bus architecture reduces bottleneck effects.
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

4. Which of the following statement is false with respect to Booth's Multiplication Algorithm?
(i) Corrections required for the final result.
(ii) Sign bit is protected due to internal arithmetic shift.
(iii) More space required to maintain the sum.
(A) (i), (ii) only (B) (ii), (iii) only
(C) (i), (iii) only (D) (i), (ii), (iii)

5. After selective complement of $A = 1100$ with $B = 0101$, the resultant A will be
(A) 0000 (B) 1100
(C) 0101 (D) 1001

6. Which type of shift operation always keeps the sign bit unchanged?
(A) Logical shift (B) Arithmetic shift
(C) Circular shift (D) Any right shift

7. Consider the register transfer language instructions:
$AC \leftarrow M[R_1]$;
$R_1 \leftarrow R_1 + 1$;
Which addressing mode is specified by the instructions?
(A) Register addressing mode
(B) Register indirect mode
(C) Auto-increment mode
(D) Relative mode

8. Which of the following statement is true?
(A) Floating point representation is better than fixed point representation.
(B) Fixed point representation is better than floating point representation.
(C) Datapath is same as ALU.
(D) Both (A) and (C)

9. Which of the following statements correctly specifies about overflow?
(i) When adding two unsigned numbers the carry-out, from the MSB position serves as the overflow indicator.
(ii) Overflow can occur only by adding two signed numbers that have the same sign.
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

10. A certain processor supports only the immediate and direct addressing modes. Which of the following programming language features cannot be implemented on this processor?
(A) Pointers (B) Arrays
(C) Records (D) All of these

11. The special purpose storage location(s) used by both ALU and CU are
(A) Decoders (B) Demultiplexers
(C) Registers (D) Buffers

12. Which of the following is a component of the datapath of Von Neumann machine?
(i) Registers
(ii) ALU input bus
(iii) ALU I/O registers
(A) (i), (ii) only (B) (ii), (iii) only
(C) (i), (ii), (iii) (D) None of these

13. Which of the following is/are false with respect to single-bus datapath?
(i) It is simplest and least expensive.
(ii) No limit on the amount of data transfer in a single clock cycle.
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

14. If we have shifted the significant to the right by a single position, then
(A) Add one to the exponent
(B) Subtract one from the exponent
(C) Don't change the exponent
(D) Data insufficient

15. In which addressing mode, the effective address of the operand is generated by adding a constant value to the content of a register?
(A) Absolute mode (B) Indirect mode
(C) Immediate mode (D) Index mode

16. What is the number of instructions required to add '*n*' numbers and store the result in memory using only one-address instructions?
(A) n (B) $n - 1$
(C) $n + 1$ (D) independent of n

17. Which unit of a computer system executes program, communicates with and often controls the operation of other subsystems?
(A) CPU (B) ALU
(C) I/O module (D) DMA

18. The multiplicand register and multiplier register of a hardware circuit implementing booth's algorithm have 1001 and 1100 respectively. The resultant will be
(A) 10011100 (B) 00011100
(C) 01101100 (D) 00010010

19. A floating point number has sign bit 0, Excess-64 exponent is 1010100 and fractional part is 0000000000011011. After converting this number to normalized form, the exponent (in decimal) will be
(A) 20 (B) 9
(C) 31 (D) 0

20. In IEEE floating point single precision representation, the number of bits in the fractional part is
(A) 24
(B) 23
(C) 32
(D) Depends on the architecture

21. After multiplying the binary numbers 010111 and 110110 using booth's multiplication algorithm, the resultant will be
(A) –1242 (B) 1242
(C) 230 (D) –230

22. The IEEE standard 754 single precision floating point representation of $(0.000000110110100101)_2$ is.
(A) 0 10000111 11011010010100000000000
(B) 0 01111001 11011010010100000000000
(C) 0 10000110 10110100110100000000000
(D) 0 01111000 10110100101000000000000

23. How many clock cycles are required to perform two-operand operations using one bus datapath?
(A) 1 (B) 2
(C) 3 (D) Can't be determined

24. Which of the following is a rounding mode in IEEE754 standard?
(i) round to 0
(ii) round towards $+\infty$
(iii) round towards $-\infty$
(iv) round to nearest representable number
(A) (i), (iv) only (B) (ii), (iii) only
(C) (i), (ii) only (D) (i), (ii), (iii), (iv)

25. What is the normalized form of $0.00000110 \times 16^{101}$?
(A) 1.10×16^{107} (B) 1.10×16^{95}
(C) 0.110×16^{94} (D) 0.110×16^{106}

26. What is the biased representation of –7, using 4-bits for the bias?
(A) 0111 (B) 1111
(C) 0000 (D) 1001

27. What is the total resultant after adding $A = -7$ and $B = -6$ using signed two's complement representation?
(A) 0100 (B) 11101
(C) 1101 (D) Overflow occurs

28. What is the total number of additions and subtractions required using Booths multiplication algorithm for the multiplier 00011110?
(A) 1 (B) 2
(C) 30 (D) Can't be determined

Common data questions 29 and 30: Consider a 12-bit floating point format in which base $b = 2$, a 5-bit exponent e with a bias = 16 and 6-bit normalized mantissa m. Given two floating point numbers:

A = 0 10001 011011
B = 1 01111 101010

29. After adding A and B, the resultant will be
(A) 1 10001 000000
(B) 1 10001 000001
(C) 0 10001 000000
(D) 0 10001 000001

30. After subtracting B from A, the resultant will be
(A) 1 10001 110101 (B) 0 10001 110101
(C) 1 10001 110110 (D) 0 10001 110110

Answers Keys

1. B	**2.** C	**3.** C	**4.** C	**5.** D	**6.** B	**7.** C	**8.** A	**9.** C	**10.** D
11. C	**12.** C	**13.** B	**14.** A	**15.** D	**16.** C	**17.** A	**18.** B	**19.** B	**20.** B
21. D	**22.** D	**23.** B	**24.** D	**25.** B	**26.** C	**27.** D	**28.** B	**29.** C	**30.** D

COMPUTER ORGANIZATION AND ARCHITECTURE TEST I

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. On which of the following factors does the CPU execution time of a program depends?
(i) CPU clock cycles required for a program.
(ii) Clock cycle time.
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (iii)

2. Which of the following component(*s*) is not related to the performance of a computer?
(i) CPU execution time for a program
(ii) Instruction count
(iii) Clock cycles per instruction
(iv) Clock cycle time
(A) (ii) only (B) (iii) and (iv)
(C) (i), (iv) (D) None of these

3. Which of the following metrics will be affected by the Instruction set Architecture?
(A) Instruction count (B) Clock rate
(C) CPI (D) All of the above

4. Which of the following combination illustrates the basic design principles of the Hardware design of a computer?
(i) Simplicity favours regularity
(ii) Smaller is faster
(iii) Make the common case fast
(iv) Good design demands good compromises
(A) (i), (iii) (B) (ii), (iv)
(C) (i), (ii), (iv) (D) (i), (ii), (iii), (iv)

5. To convert a 16 - bit number to a 32 - bit equivalent, take the MSB of the number and replicate it to fill the new bits of the 32 - bit number. The old bits are copied into the right portion of the new word. This is known as
(A) Sign expansion
(B) Sign extension
(C) Size casting
(D) Conversion from 16 bit number to 32 bit number is not possible

6. A number in scientific notation that has no leading zeros is called a ——— number.
(A) Floating point (B) Normalized
(C) Overflow (D) De-normalized

7. Consider a 5 - stage pipeline :
1. Instruction fetch 2. Register Read
3. ALU operation 4. Data Access
5. Register write
The time taken for memory access is 200 ns, CPU operation is 200 ns, register read or write is 100 ns. Consider a program with the following kind of instructions:
I_1: Load word
I_2: Store word
I_3: ALU operation
I_4: BEQ (Branch if equal)
If each instruction takes exactly one clock cycle, under ideal conditions, what is the time between I_1 and I_4 with out pipelining?
(A) 800 ns (B) 2100 ns
(C) 2400 ns (D) 600 ns

8. For the previous problem, what is the time between I_1 and I_4 using pipelining?
(A) 2100 ns (B) 2400 ns
(C) 800 ns (D) 600 ns

9. The speed up of pipelined processor compared to non pipelined processor is
——— the number of pipeline stages.
(A) always equal to (B) always less than
(C) less than (D) greater than

10. Pipelining improves the performance by
(A) increasing instruction throughput.
(B) decreasing the execution time of an individual instruction.
(C) Both (A) and (B)
(D) replacing complex instructions with simple instructions.

11. Assume that an enhancement is made to a computer that improves some mode of execution by a factor of 10. This new fast mode is used 50% of the time, measured as a percentage of the execution time when the fast mode is in use. What is the over all speed up we can achieve?
(A) 5.5 (B) 5.2
(C) 4.8 (D) 5.0

12. A bus protocol requires 20 ns for devices to make requests, 10 ns for arbitration and 30 ns to complete each operation. How many operations can be completed per second?
(A) $10 * 10^6$ (B) $17 * 10^6$
(C) $20 * 10^6$ (D) $33 * 10^6$

13. A process has five interrupt lines, numbered 0 – 4 and a policy that low numbered interrupts have priority over higher numbered ones. The processor starts with no interrupts pending and the following sequence of interrupts occurs: 4, 3, 0, 1, 2, 3
Assume that handling any interrupt takes enough time that two more interrupts arrive while the first interrupt is being handled, until all of the interrupts have arrived and that interrupts cannot interrupt each other.
In what order the interrupts are handled ?
(A) 0, 1,2, 3, 3, 4 (B) 4, 3, 0, 1, 2, 3
(C) 4, 0, 1, 3, 2, 3 (D) 4, 0, 1, 2, 3, 3

14. Which of the following mapping technique will provide simpler hardware, simple instruction set and provides all addressing modes?
 (i) Memory mapped I/O
 (ii) Isolated I/O
 (A) (i) only (B) (ii) only
 (C) Both (i) and (ii) (D) Neither (i) nor (ii)

15. Which of the following factors limits the maximum transfer rate of DMA?
 (A) Speed of I/O devices
 (B) Erroneous transfer of block
 (C) Speed of bus
 (D) All the above

16. An address field in an instruction contains decimal value 10. Where is the corresponding operand located for direct addressing?
 (A) In the location '10'.
 (B) '10' is the operand itself.
 (C) '*PC*+ 10' location.
 (D) Data insufficient

17. Suppose we had a block transfer from an I/O device to memory. The block consists of 1024 words and one word can be transferred to/from memory at a time. The number of interrupts needed to transfer a block using DMA is _____.
 (A) 0 (B) 1
 (C) 1024 (D) Unpredictable

18. For the above problem, what is the number of interrupts needed to transfer a block using interrupt driven I/O?
 (A) 0 (B) 1
 (C) 1024 (D) Unpredictable

19. For the Q.no :17, what is the number of interrupts needed to transfer a block using 'programmed I/O'?
 (A) 0 (B) 1
 (C) 1024 (D) Unpredictable

20. The number of times does the processor need to refer to memory when it fetches and executes an indirect address mode instruction (if the instruction is not a branch instruction) is:
 (A) 0 (B) 1
 (C) 2 (D) 3

21. Which of the following is a valid difference between a program branch instruction and a subroutine call?
 (i) A branch instruction changes the contents of PC.
 (ii) A branch instruction do not save the contents of PC before changing, Where as a subroutine will save PC.
 (A) (i) only (B) (ii) only
 (C) Both (i) and (ii) (D) Neither (i) nor (ii)

22. If there are no push and pop operations in an instruction set, then which of the following is TRUE?
 (A) CPU can't able to use the stack.
 (B) CPU can use the stack to perform its internal functions.
 (C) The programmer can use the stack explicitly.
 (D) None of the above

23. Match the following:

Addressing mode		Basic advantage	
1.	Immediate	a.	No memory reference
2.	Direct	b.	Simple
3.	Indirect	c.	Large address space
4.	Register	d.	Flexibility

 (A) 1 – b, 2 – a, 3 – c, 4 – a
 (B) 1 – a, 2 – a, 3 – d, 4 – b
 (C) 1 – a, 2 – b, 3 – c, 4 – a
 (D) 1 – a, 2 – b, 3 – d, 4 – c

24. What is the 8 - bit biased notation representation for –45?
 (A) 01010010 (B) 10101101
 (C) 11010011 (D) 11010010

25. Which of the following are the disadvantages of device polling?
 (i) Polling consumes execution resources even when there are no I/O requests to handle.
 (ii) There is a need to schedule the polling frequency.
 (A) (i) only (B) (ii) only
 (C) Both (i) and (ii) (D) Neither (i) nor (ii)

Answer Keys

1. C	**2.** D	**3.** D	**4.** D	**5.** B	**6.** B	**7.** C	**8.** D	**9.** C	**10.** A
11. A	**12.** B	**13.** D	**14.** A	**15.** D	**16.** A	**17.** B	**18.** C	**19.** A	**20.** D
21. C	**22.** B	**23.** C	**24.** A	**25.** C					

Hints and Explanations

1. CPU execution time for a program = CPU clock cycles for a program * clock cycle time. Choice (C)

2. All the four are the basic measurements at different levels in the computer. Choice (D)

3. The instruction set architecture affects all the three aspects of CPU performance; it affects the instructions needed for a function, the cost in cycles of each instruction and the overall clock rate of the processor.
Choice (D)

4. The design principles of Hardware of a computer are
1. Simplicity
2. Smaller number of registers
3. Commonly occurring things needs to be constant.
4. Some compromises required for good design

Choice (D)

5. 8-bits

1	1	1	1	1	0	0	0

↓ 16 bits

1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0

Choice (B)

7. Without using pipelining the time taken for a single instruction is the sum of the times required by all the stages of the pipeline.
i.e., $IF + RR + ALU + DA + RW$
$= 200 + 100 + 200 + 200 + 100 = 800$ns
Each instruction requires one clock cycle.
Time between I_4 and I_1 is 800 ns * 3(instructions)
= 2400 ns. Choice (C)

8. Using pipelining I_2 will start IF after completion of I_1's IF.
So the maximum stage delay 200 ns is considered as time between I_1 and I_2.
(i.e., any two instructions)
Time between I_1 and I_4 is 3×200 ns = 600 ns.

Choice (D)

9. The speed up of pipelined processor is equal to number of pipeline stages under ideal conditions. But in general, it will be less than the number of stages.

Choice (C)

10. Pipelining improves performance by increasing instruction throughout rather than decreasing the execution time of an individual instruction. Choice (A)

11. Speed of after enhancement
$= 0.5 * 10 + (1 – 0.5)$
$= 5 + 0.5 = 5.5$. Choice (A)

12. Time taken for an operation= 20 ns + 10 ns + 30 ns = 60 ns. Number of operations completed per second
$= \dfrac{1}{60\times10^{-9}}$
$= 17 * 10^6$ operations/sec Choice (B)

13. Lower numbered interrupts have high priority.
Let initially 4 is handled, mean while, 3 and 0 will occur. Give priority to 0.
While 0 is handled, 1 and 2 will occur with 3 also in pending Queue. Next handle 1, 2, 3, 3.
∴ Order is 4, 0, 1, 2, 3, 3. Choice (D)

14. Memory mapped I/O provides
- Simple hardware
- Simple instruction set
- All address modes available. Choice (A)

15. DMA does not use CPU but its transfer rate is limited because of
1. low speed of I/O devices
2. low speed of bus
3. less internal buffering
4. erroneous Disk. Choice (D)

16. In direct addressing mode, the address of operand is directly present in the instruction itself. So location 10 contains the data. Choice (A)

17. DMA will generate an interrupt after completion of block transfer.
∴ One interrupt will be generated. Choice (B)

18. In interrupt driven technique, interrupt will be generated after transfer of every word.
∴ 1024 interrupts. Choice (C)

19. In programmed I/O, no interrupt will be generated.
The CPU will check the status bits.
∴ Zero interrupts. Choice (A)

20 '3' memory references required.
one for fetching instruction and '2' for fetching operand. Choice (D)

21. A branch instruction do not save PC before changing PC, where as a subroutine saves for restoring them later. Choice (C)

22. CPU can use stack even if there are no PUSH and POP instructions. PUSH and POP are used by the programmer. Choice (B)

23. Immediate, Register – No memory reference
Direct – simple.
Indirect – large address space. Choice (C)

24. The bias using 8-bits is 127.
Biased value of –45 is
–45 + 127 = 82 = 01010010. Choice (A)

25. Polling wastes the resources and the frequency of polling also need to be scheduled. Choice (C)

Computer Organization and Architecture Test 2

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following memory access methods, will accurately predict the amount of time it will take to receive the data after requesting it?
(A) Sequential access (B) Direct access
(C) Random access (D) Both (B) and (C)

2. In which of the following access method, any location can be accessed in a fixed amount of time after specifying its address?
(A) Sequential access (B) Random access
(C) Direct access (D) None of the above

3. Which of the following statement is TRUE with respect to write-through and write-back techniques?
(A) Write-back scheme is faster than write-through.
(B) Write-through improves performance better than write-back.
(C) Write-through is more complex to implement than write-back
(D) All the above

4. Increasing the degree of associativity of a cache memory will
(A) increases the miss rate.
(B) decreases the miss rate.
(C) increases the number of sets.
(D) decreases the number of elements per set.

5. Which of the following address information is required to identify whether a word in the cache corresponds to the requested word or not?
(A) Tag (B) Index
(C) Line number (D) offset

6. Which of the following factors will be better effected because of larger cache memory?
(i) Hit rate (ii) Cost
(iii) Speed (iv) Power consumption
(v) Reliability
(A) (ii), (iv), (v) (B) (ii), (iii)
(C) (i) only (D) (i), (iii), (iv)

7. If the depth of a pipeline increases, then which of the following is true?
(i) Clock frequency increases
(ii) CPI increases
(iii) Branch mis-prediction penalty reduced.
(A) (i), (ii) only (B) (ii), (iii) only
(C) (i), (iii) only (D) (i), (ii), (iii)

8. Which of the following are the necessary conditions for an n-stage pipeline is n times faster than a non pipelined processor?
(A) Equal cycle time (of the stages).
(B) All instructions take equal number of cycles.
(C) No stalls between instructions.
(D) All of the above

9. Which of the following technique is generally used to avoid WAR and WAW hazards?
(A) By executing an instruction only when the operands are available.
(B) By Register renaming.
(C) Delayed pranching.
(D) All the above

10. Assume a memory access to main memory on a cache miss takes 10 ns and a memory access to the cache on a cache hit takes 1 ns. If 90% of the processors memory requests result in a cache hit, what is the average memory access time?
(A) 2 ns (B) 1 ns
(C) 1.9 ns (D) 1.4 ns

11. Consider the following set of instructions:
I_1: $R_1 \leftarrow R_2 + R_3$
I_2: $R_3 \leftarrow R_1 + R_2$
I_3: $R_1 \leftarrow R_1 * R_3$
The time taken by the 4-stages of the pipeline is given below:

	Fetch	Decode	Execute	Write
I_1	1	2	2	1
I_2	1	2	1	2
I_3	1	1	2	1

What is the number of cycles needed to execute above instructions (use operand forwarding)?
(A) 9 (B) 10
(C) 11 (D) 12

12. Consider the execution of n instructions using k stages of a pipeline, each stage requires 't' time units. Which of the following expression provides the speed up measure of the pipeline?
(A) $\frac{nk}{n+k-1}$ (B) $\frac{nk}{n+k}t$
(C) $\frac{n}{(n+k-1)}$ (D) $\frac{k}{n+k}$

13. Consider 2 MB of RAM and 4 KB of cache with block size of 16 B. What are the sizes of fields used in associative mapping technique?
(A) 16, 5 (B) 8, 9, 4
(C) 9, 8, 4 (D) 17, 4

14. Consider a cache, which requires 2 clock cycles. If there is a cache miss, it will stall the processor for an additional 5 clock cycles. Then what will be the hit rate to achieve an average memory access of 3 clock cycles?

(A) 80% (B) 85%
(C) 90% (D) 95%

15. A digital computer has a memory unit of 64 K × 16 and a cache memory of 1 K words. The cache uses direct mapping with a block size of 4 words. The number of bits present in each line of cache are________.
(A) 20 bits (B) 22 bits
(C) 26 bits (D) 33 bits

16. Consider a cache of 4 K blocks, a 4 word block size and a 32 bit address main memory. What is the total number of tag bits per set for 4 way set associative cache?
(A) 18 (B) 36
(C) 72 (D) 64

17. A 5-stage pipeline contains IF, ID, FO (fetch operands), EX, WB stages. Each stage takes 1 clock cycle. Consider the execution of the following two instructions on this pipeline:
I_1: ADD a, b; $a \leftarrow a + b$
I_2: MUL c, a; $c \leftarrow c * a$
What is the number of clock cycles required for the execution using operand forwarding?
(A) 6 (B) 7
(C) 8 (D) 9

18. Consier a main memory address of p-bits and an associated direct-mapped cache of having 2^n blocks. Block size is 2^m bytes. Then the Tag field is of ______ bits.
(A) $p - (n + m + 2)$ (B) $p + n + m$
(C) $p - n + m$ (D) $p - n - m$

19. A 2-way set-associative cache is made up of 32-bit words, has 4 words per line and 4096 sets. Then the cache capacity in bytes is
(A) 32 K (B) 64 K
(C) 128 K (D) 256 K

20. A main memory of a computer has 4096 blocks, each consisting of 128 words. Each word is of 32-bits in size. Then the number of bits present in main memory address is _____ bits.
(A) 18 (B) 19
(C) 20 (D) 21

21. Consider a computer, whose address has M bits (using byte addressing), the cache data size is C bytes, the block size is $B = 2^b$ bytes and the cache is k-way set-associative. Then the number of bits in the set field of the address is given by
(A) $\log_2^c - \log_2^M$ (B) $\log_2^c - \log_2^M - b$
(C) $\log_2^M - b$ (D) $\log_2^B - \log_2^M$

22. Consider a 5-stage pipeline with the stage delays as shown below:

Stage	Delay
Fetch	30 ns
Decode	40 ns
Execute	35 ns
Memory	50 ns
Write	10 ns

Latch delay between the pipeline stages is 2 ns.
What are the cycle time and latency values (in ns) of an instruction respectively?
(A) 50, 50 (B) 52, 260
(C) 50,1 (D) 52, 1

23. Consider the execution of the following program on a 5 stage pipeline with the stages Fetch (F), Decode (D), Execute (E), Memory (M) and write (W).
ADD R_1, R_2, 10; $R_1 \leftarrow R_2 + 10$
LOAD R_2, 4 (R_1); $R_3 \leftarrow$ M[R_1 + 4]
ADD R_4, R_2, R_3,; $R_4 \leftarrow R_2 + R_3$
STORE R_4, 8(R_1); $M[8 + R_1] \leftarrow R_4$
LOAD R_5, 0(R_6); $R_5 \leftarrow M(R_6)$
OR R_5, R_1, R_4,; $R_5 \leftarrow R_1$ or R_4
If each stage requires one clock cycle, the number of stalls required for the program execution with forwarding are
(A) 0 (B) 1
(C) 2 (D) 3

24. The outcome of a branch is given as:
T, T, NT, T
Where T: Taken
NT: Not Taken
What is the accuracy of 'always taken' predictor for the above sequence of branch outcomes?
(A) 100% (B) 75%
(C) 50% (D) 25%

25. The outcome of a branch is given as:
T, T, NT, T.
Where T : Taken
NT: Not Taken
What is the accuracy of '2-bit predictor' for the above sequence of branch outcome?
(If the 2-bit predictor starts with T)
(A) 100% (B) 75%
(C) 50% (D) 25

Answer Keys

1. C	**2.** B	**3.** A	**4.** B	**5.** A	**6.** C	**7.** A	**8.** D	**9.** B	**10.** C
11. B	**12.** A	**13.** D	**14.** A	**15.** B	**16.** C	**17.** A	**18.** D	**19.** C	**20.** D
21. B	**22.** B	**23.** B	**24.** B	**25.** B					

Hints and Explanations

1. Random access uses a decoder to instantly access the location regardless of current state of the memory where as sequential and Direct methods depends on current data read/write position. Choice (C)

2. Choice (B)

3. Write-back scheme is faster than write-through write-back improves performance but is complex to implement. Choice (A)

4. Increasing the degree of associativity of a cache will decrease the miss rate, decreases the number of sets and increases the number of elements per set. Choice (B)

5. Choice (A)

6. With large cache memories, the hit rate is improved. Cost increases, speed reduced, power consumption is high and reliability is reduced. Choice (C)

7. If there is an increase in pipeline depth then clock frequency, CPI, mis-prediction penalty increases. Choice (A)

8. Choice (A), (B), (C) are all the necessary conditions for an n stage pipeline to be efficient.
Equal cycle time means no structural hazards. Equal number of cycles means no control hazards.
No stalls means no data hazards. Choice (D)

9. Register renaming is used to avoid WAR and WAW hazards. Choice (B)

10. Miss time = 10 ns
Hit time = 1 ns
90% are hits.
Average memory access time
= 0.9 * 1 + 0.1 * 10
= 0.9 + 1 = 1.9 ns. Choice (C)

11.

	1	2	3	4	5	6	7	8	9	10
I_1	F	D	D	E	E	W				
I_2		F			D	D	E	W	W	
I_3			F				D	E	E	W

∴ 10 clock cycles required. Choice (B)

12. Choice (A)

13. RAM size = 2 MB = 2^{21} B

21 bits	
Tag	Offset

Block size = 16 B = 2^4 B
⇒ offset = 4
⇒ Tag = 21 – 4 = 17. Choice (D)

14. Cache hit requires 2 clock cycles.
Cache miss requires 7 clock cycles.
Average memory access requires 3 clock cycles.
Let hit rate is x, then $3 = x * 2 + (1 - x)7$
⇒ $3 = 2x + 7 - 7x$
⇒ $5x = 4$
⇒ $x = \frac{4}{5} \times 100 = 80\%$ Choice (A)

15. Main memory capacity is 64 K × 16.
It has 16-bit address

16 bits		
Tag	Line	word

Block size = 4 words = 2^2
⇒ word size = 2
Number of lines in cache $= \frac{2^{10}}{2^2} = 2^8$
∴ line field width = 8
⇒ Tag = 16 – 10 = 6
Each line cache holds
Tag + Data = 6 + 16-bits = 22-bits. Choice (B)

16. Main memory address has 32-bits.

Tag	Set	Offset

Block size = 4 words = 4 × 4 Bytes = 2^4 B
⇒ offset = 4
Number of blocks in cache = 4 K = 2^{12}
Number of sets $= \frac{2^{12}}{4} = 2^{10}$
⇒ set field width = 10
∴ Tag = 32 – (10 + 4) = 18
Number of tag bits per set is 18 * 4 = 72 bits. Choice (C)

17.

	1	2	3	4	5	6	7	8	9
I_1	IF	ID	FO	EX	WB				
I_2		IF	ID	FO	EX	WB			

∴ number of clock cycles required = 6. Choice (A)

18.

p-bits		
Tag	Line	Offset
	n	m

$n\ m$
⇒ Tag = $p - (n + m)$ bits. Choice (D)

19. Number of sets = 4096
Number of words = 4 words
Word length = 32-bits = 4 B

$\Rightarrow$ Number of words (in bytes) = 4 * 4 B = 16B

The cache is 2-way set associative.

Cache capacity = 2 * 4096 * 16

$= 2^1 * 2^{12} * 2^4$

$= 2^{17}$ B = 128 KB. Choice (C)

20. Number of blocks = 4096

Number of words per block = 128

Word size = 32 bits = 4 B

Number of words per block

(in bytes) = 128 * 4 B = 512 B

Main memory capacity = 4096 * 512 B

$= 2^{12} * 2^9\text{B} = 2^{21}$ B

$\therefore$ 21-bits required for main memory address.

Choice (D)

21. Cache size = C bytes

Block size, $B = 2^b$ bytes

The cache is m-way set-associative.

Number of sets in cache $= \dfrac{C}{M \times B}$

Bits required for set field is $\log_2^{\left(\frac{C}{M}\times 3\right)}$

$= \log_2^C - \log_2^{(M\times B)} = \log_2^C - \log_2^M - \log_2^B$

$= \log_2^C - \log_2^M - \log_2^{2^b}$

$= \log C - \log M - b$. Choice (B)

22. Cycle time = maximum stage delay + latch delay

= 50 + 2 = 52 ns

Latency = number of stages * cycle time

= 5 * 52 = 260 ns. Choice (B)

23.

	1	2	3	4	5	6	7	8	9	10	11
I_1	F	D	E	M	W						
I_2		F	D	E	M	W					
I_3			F		D	E	M	W			
I_4				F		D	E	M	W		
I_5					F		D	E	M	W	
I_6						F		D	E	M	W

One stall caused by 2^{nd} and 3^{rd} instructions.

Choice (B)

24. Given predictor always predicts about the branch 'Taken'.

Prediction	T	T	T	T
Outcome	T	T	NT	T
Result	True	True	False	True

1 out of 4 predictions is wrong.

i.e., Accuracy of predictor $= \dfrac{3}{4}$

* 100 = 75%. Choice (B)

25. Using 2-bit predictor, the predictor will change its decision with two successive wrong predictions.

Initial prediction: T

Prediction	T	T	T	T
Outcome	T	T	NT	T
Result	True	True	False	True

Accuracy $= \dfrac{3}{4}$

* 100 = 75%. Choice (B)

Computer Organization and Architecture Test 3

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. A computer 'A' uses the following 32-bit floating point representation of real numbers.

S	Mantissa	Exponent
31	30 … 8	7 … 0

Computer 'B' uses the following floating point representation scheme.

S	Mantissa	Exponent
31	30 … 7	6 … 0

Which of the following statement is TRUE with regard to computer B's method of representing floating-point numbers over computer A's method?

(A) Both range and precision are decreased.
(B) The range is decreased but the precision is increased.
(C) The range is increased but the precision is decreased.
(D) Both range and precision remain same.

2. The minimum number of control bits required to execute a microprogram which has 50 control signals is ________.

(A) 5 (B) 6
(C) 50 (D) 100

3. Consider the following instructions:
BEQ R_0, R_1, L_1
The opcode is BEQ, which means Branch if Equal. The instruction checks the equality of R_0, R_1 and if both are equal jump to L_1. Which of the following correctly specifies given instruction but with much greater branching distance?

(A) compare R_0, R_1 if zero jump to L_1
(B) LOAD R_0 Branch if equal R_1, L_1
(C) Branch if not equal R_0, R_1, L_2 Jump L_1 L_2:
(D) Branch if not Equal R_0, R_1, L_1 Jump L_2 L_1:

4. Consider a loop branch that branches 100 times in a row, then it is not taken once. Assume that the prediction bit for this branch remains in the prediction buffer. The prediction accuracy for this branch by using 1-bit branch history table prediction scheme (in percentage) is ________.

(A) 90 (B) 95
(C) 98 (D) 99

5. The biased exponent value for double precision floating point numbers is _________.

(A) 1023 (B) 1024
(C) 256 (D) 255

6. Consider the following instruction:
ADD R_1, (R_2, R_3)
Here R_1, R_2, R_3 are registers
The Sum of data present in the R_2 and R_3 is stored in a 32-bit register R_1.
Which of the following mode best reflects the mode of second operand?

(A) Indexed mode
(B) Base Register mode
(C) Base with index mode
(D) Base with index and offset mode

7. Consider an instruction format with fields containing zeros:

0000	0000 0000	0000 0000	0000 0000 0000
Opcode	Destination Register	Source Register	Immediate value

What is the maximum possible number of operations, registers with the given instruction format?

(A) 4, 8 (B) 4, 16
(C) 16, 256 (D) 16, 65536

8. A 64 M-bit DRAM organized as 4 M addresses of 16-bit words each. A memory system is built using 128 M addresses of 256-bit words each. How many DRAM chips that are required for this memory system?

(A) 16 (B) 32
(C) 128 (D) 512

9. Consider a hypothetical 64-bit micro-processor having 64-bit instructions composed of two fields: Opcode: 2-bytes. Immediate operand or an operand address: remaining bytes.
What is the maximum directly addressable memory capacity (in bytes)?

(A) 2^{16} (B) 2^{48}
(C) 2^{64} (D) 2^{24}

10. Consider a machine with a byte addressable main memory of 4 GB and a block size of 16 bytes. Assume that a direct mapped cache consisting of 32 lines is used with this machine. How many bytes of main memory can be stored in the cache?

(A) 4096 (B) 2^{20}
(C) 512 (D) 736

11. Consider a dynamic RAM that must be given a refresh cycle 64 times per ms. Each refresh operation requires 150 ns; a memory cycle requires 250 ns. What percentage of the memory's total operating time is given to refreshes?

(A) 1% (B) 3%
(C) 5% (D) 10%

12. A DMA module is transferring characters to memory using cycle stealing, from a device transmitting at

9600 bps. The processor is fetching instructions at the rate of 1 million instructions per second (1 MIPS). By how much the processor is slowed down due to the DMA activity?

(A) 0.0012% (B) 0.01%
(C) 0.001% (D) 0.12%

13. Match list-A with list-B and select the correct answer using the code given below the list:

	List–A		List–B
a.	Cache	1.	Printer
b.	DMA I/O	2.	Disk
c.	Interrupt I/O	3.	High speed RAM

(A) a–1, b–2, c–3 (B) a–2, b–3, c–1
(C) a–3, b–2, c–1 (D) a–3, b–1, c–2

14. The speed gained by a 'p' segment pipeline executing 'q' tasks is:

(A) $\dfrac{(q+p-1)}{pq}$ (B) $\dfrac{p+q}{pq-1}$

(C) $\dfrac{pq}{p+q-1}$ (D) $\dfrac{p+q}{pq+1}$

15. The IEEE 32-bit floating point format of –6 is:

(A) 1 10000001 10000000000000000000000
(B) 1 00000010 00000000000000000000000
(C) 1 00000001 10000000000000000000000
(D) 1 10000001 00000000000000000000000

16. Consider below code segment:

LOAD R_1, M[1000]
LOAD R_2, M[1002]
ADD R_3, R_1, R_2
STORE R_3, M[1008]
LOAD R_4, M[1004]
ADD R_5, R_1, R_4
STORE R_5, M[1010]

Assume that this program is executed on a pipelined processor with 5 stages: FI (Fetch Instruction), RD(Read registers while Decoding), EX (Execute the operation or calculate the address), MEM (Access an operand in data memory), WR (write result into a register).

The first operand of each instruction will be the destination. What is the minimum number of stalls that will be there in the reordered code of given code if the processor uses forwarding also?

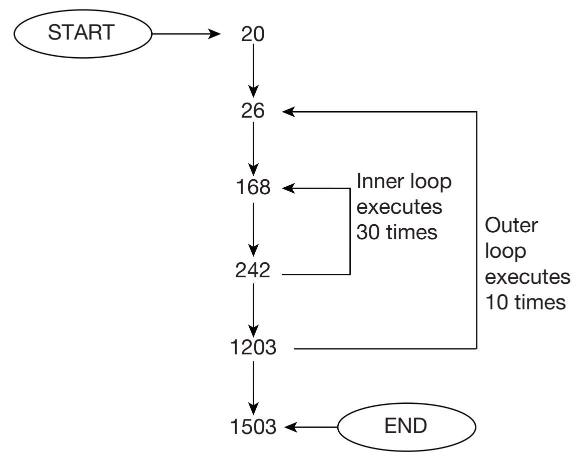
(A) 0 (B) 1
(C) 2 (D) 3

17. Consider a 2 × 8 two-dimensional array of elements, M. Assume that each element in the array occupies one word and the array elements are stored in column-major order in the main memory from location 2000 to location 2015. The cache consists of 8 blocks and each block will have just two words. Assume also that whenever needed, LRU replacement policy can be used. What is the percentage of cache utilization for Direct mapped, Associative and 2-way set-Associative respectively, if the processor accesses the following elements?

$M_{0,0}$, $M_{0,1}$, $M_{0,2}$, $M_{0,3}$, $M_{0,4}$, $M_{0,5}$, $M_{0,6}$, $M_{0,7}$, $M_{1,0}$, $M_{1,1}$, $M_{1,2}$, $M_{1,3}$, $M_{1,4}$, $M_{1,5}$, $M_{1,6}$, $M_{1,7}$.

(A) 50%, 100%, 50% (B) 50%, 50%, 50%
(C) 100%, 100%, 100% (D) 100%, 100%, 50%

18. Consider the given program structure, which is in Main memory:



The memory addresses are in decimal. The program consists of two loops along with start and end statements. All the instructions in the program are executed in sequential manner. The program is to be run on a computer that has an instruction cache of size 1 K words organized in a direct-mapped manner. The main memory size is 64 K words with block size of 128 words. Let the cycle time of main memory is 10 ns and cycle time of the cache is 1 ns.

Then the total time needed for reading instructions from main memory to the cache during the execution of the program (in nano seconds) is ________.

(A) 1280 (B) 4864
(C) 48640 (D) 61440

19. A pipelined processor has two branch delay slots, An optimizing compiler can fill one of these slots 75% of the time and can fill the second slot only 20% of the time. What is the ratio of improvement in performance achieved by this optimization to without optimization, assuming that 10% of the instructions executed are branch instructions?

(A) 1.102 (B) 1.086
(C) 0.9 (D) 1.105

20. Consider a system with a 4 KB, 4-way set associative cache memory with 128 lines, a 1024 word array where each element is a 32-bit word. When a program accesses this array with a scale of D, it means starting with the first element, the program accesses every

D^{th} element. For example, for $D = 1$, the program accesses every element, for $D = 2$, the program accesses every second element and so on. Assuming a cache that is initially empty and a program makes one pass over this array with a scale of D, what is the miss rate generated for $D = 8$?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$
(C) $\frac{1}{8}$ (D) $\frac{1}{16}$

21. A computer has 32-bit instructions and 12-bit addresses. If there are 240 two-address instructions, how many one-address operations can be formulated?
(A) 4096 (B) 65536
(C) 8192 (D) 131072

22. Consider a pipelined processor with a 5-stage pipeline. Assume that all instructions take 5 cycles. The dynamic instruction count by type, as a percentage of the total, is as follows:
10% store instructions
20% load instructions
30% branch instructions
40% ALU instructions
What is the ideal speed-up due to pipelining for this processor?
(A) 2.5 (B) 5
(C) 10 (D) 50

23. For the data given in Q. No. 22, let stalls due to data hazards occur only under two reasons.
A stall of two cycles occur when a load instruction is followed by an ALU instruction that uses the result of load. This scenario exists for 40% of the load instructions.
A stall of three cycles occur when a branch instruction is preceded by an ALU operation whose result is used as a branch condition. This scenario exists for 50% of the branch instructions. What is the decrease in the ideal speed up of pipelining only due to data hazards?
(A) 50% (B) 75.6%
(C) 18.9% (D) 37.8%

24. Given
$x = (0100\ 0110\ 1101\ 1000\ 0000\ 0000\ 0000\ 0000)_2$ and
$y = (1011\ 1110\ 1110\ 0000\ 0000\ 0000\ 0000\ 0000)_2$,
representing single precision IEEE 754 floating point numbers. Then the respective values of $x + y$ and $x * y$ in decimal (approximately) are:
(A) 27647.5625 and –12096
(B) 27647.5625 and –24192
(C) 13823.75 and 24192
(D) 13823.75 and –24192

25. A 5-stage pipeline has the following stages:
IF: Instruction fetch
ID: Instruction decode and register file read
EX: Execution or address calculation
MEM: Data memory access
WB: Write back
The following code executed on this pipeline:

Instruction	Operation
ADD R_1, R_2, R_3	$R_1 \leftarrow R_2 + R_3$
SUB R_4, R_1, R_5	$R_4 \leftarrow R_1 - R_5$
LOAD R_6, 200(R_1)	$R_6 \leftarrow M[200 + R_1]$
ADD R_7, R_1, R_6	$R_7 \leftarrow R_1 + R_6$

Use forwarding to resolve data hazards. Then the number of stalls that will occur because of data hazards in given code is ______.
(A) 0 (B) 1
(C) 2 (D) 3

26. Consider a floating point representation: $c.r^e$ where c represents coefficient register of size 10 in which MSB bit is used to represent sign, r represents radix and e represents contents of exponent register of size 5, in which MSB is used to represent sign. Then the contents of coefficient and exponent registers for the number +1001.110 will be:
(A) 0100111000, 00100 (B) 0001001110, 00100
(C) 1001110000, 10100 (D) 1001110000, 10101

27. Consider a 32-bit microprocessor, with a 16-bit external data bus, driven by an 8 MHz input clock. Assume that this microprocessor has a bus cycle whose minimum duration equals four input clock cycles. What is the minimum data transfer rate across the bus that this microprocessor can sustain, in bytes?
(A) 2 MB/sec (B) 4 MB/sec
(C) 6 MB/sec (D) 8 MB/sec

28. Consider a bus structure in which a single internal bus connects the ALU and all processor registers.
Which of the following represents the correct sequence of micro-operations to add a number to the accumulator when the number is an indirect address operand?
(A) t_1: MAR ← (IR(address))
t_2: MBR ← memory
t_3: Y ← (MBR)
t_4: $Z \leftarrow (AC) + (Y)$
t_5: $AC \leftarrow (Z)$
(B) t_1: MAR ← (IR(address))
t_2: MBR ← memory
t_3: $Z \leftarrow (AC)$ + (MBR)
t_4: $AC \leftarrow (Z)$
(C) t_1: MBR ← (IR(address))
t_2: $Z \leftarrow (AC)$ + (MBR)
t_3: $AC \leftarrow (Z)$
(D) t_1: MAR ← (IR(address))
t_2: MBR ← memory
t_3: MAR ← (MBR)
t_4: MBR ← memory

t_5: $Y \leftarrow (MBR)$
t_6: $AC \leftarrow (AC) + (Y)$
t_7: $AC \leftarrow (Z)$

29. For the data given in Q. No. 28, what is the number of clock cycles required for above operation if each micro-operation takes one clock cycle?
(A) 5 (B) 4
(C) 3 (D) 7

30. The access time of a cache memory is 100 ns and that of main memory is 1 μs. 80% of the memory requests are for read and others are for write. Hit ratio for read only accesses is 0.9. A write-through procedure is used. The average access time of the system for both read and write requests is:
(A) 160 ns (B) 200 ns
(C) 360 ns (D) 720 ns

31. Consider a new instruction named branch-on-bit-set. The instruction
"bbs reg, pos, label"
jumps to label if bit in position 'pos' of register operand 'reg' is one. A register is 32 bits wide and the bits are numbered 0 to 31, bit in position 0 being the least significant. Consider the following emulation of this instruction on a processor that does not have branch-on-bit-set implemented.
result ← reg & mark
Branch to label if result is non-zero.
The variable 'result' is temporary register. For correct emulation the variable mask must be generated by:
(A) mask ← 0*xf* (B) mask ← *pos*
(C) mask ← 0*xffff* >> *pos* (D) mask ← 0*x*1<< *pos*

32. Consider a computer with the following characteristics: Total main memory: 1 MB; word size is 1 byte; Block size of 16 bytes; and cache size of 64 kbytes; for the main memory address *F*0010, what will be the cache line bits for a direct mapped cache?
(A) 1111 (B) 0000
(C) 0000 0000 0001 (D) 000000001000

33. Consider a system in which a data transfer over a bus takes 500 ns. Transfer of bus control in either direction, from processor to I/O device or vice versa, takes 250 ns. One of the I/O devices has a data transfer rate of 50 kb/s and employs DMA. Data is transferred one byte at a time. Suppose we employ DMA in a cycle-stealing mode. When transferring a block of 128 bytes, it would tie up the bus for (in micro seconds) ________.
(A) 250 (B) 0.2
(C) 128 (D) 0.4

34. A PC relative mode branch instruction is 2 bytes long. The address of the instruction, in decimal, is 356029. Determine the branch target address if the signed displacement in the instruction is –31.
(A) 356029 (B) 356030
(C) 356031 (D) 356000

35. Consider a system employing interrupt driven I/O for a particular device that transfers data on an average of 10 KB/sec on a continuous basis. Assume that interrupt processing takes 100 μs (i.e., the time to jump to the ISR, execute it and return to the main program). Determine what fraction of processor time is consumed by this I/O device if it interrupts for every byte.
(A) 1 (B) 10
(C) 100 (D) 0.8

Answer Keys

1. B	**2.** B	**3.** C	**4.** C	**5.** A	**6.** C	**7.** C	**8.** D	**9.** B	**10.** C
11. A	**12.** D	**13.** C	**14.** C	**15.** A	**16.** A	**17.** C	**18.** D	**19.** B	**20.** B
21. B	**22.** B	**23.** D	**24.** A	**25.** B	**26.** A	**27.** B	**28.** D	**29.** D	**30.** C
31. D	**32.** C	**33.** D	**34.** D	**35.** A					

Hints and Explanations

1. The range of a floating point number depends on exponent size. As exponent size in '*B*' decreases, the range also decreases.
The precision specifies number of digits after decimal point, which increases in '*B*' computer. Choice (B)

2. Using Vertical Microprogramming, there will be 6-control bits to provide 50 control signals.
($\because 2^6 = 64 > 50$) Choice (B)

3. Given instruction BEQ R_0, R_1, L_1.
This instruction can be replaced by following instructions, to achieve greater branching distance:
BNE R_0, R_1, L_2
Jump L_1, L_2:
We get much greater branching distance by using 1-address instruction: Jump L_1.
We can't use zero-address as we need to specify the opcode 'jump'
By using the instruction 'Jump L_1' the address space is more so we can specify much greater branching distance. Choice (C)

4. Branch history table is a small memory, which contains a bit that says whether the branch was recently taken or not.

In the given problem, the loop executes 100 times but in the 101^{th} time the loop is not taken.
But it will be incorrectly predicted by 1-bit branch history table prediction as the loop is taken for 100 times. In the 1^{st} iteration also the predictor specifies incorrect branch prediction as the bit is set to 'not taken' in the exit stage of last execution of the program.

$\therefore$ 2 wrong predictions, out of 101 predictions.

$\therefore$ Prediction Accuracy percentage

$= \frac{99}{101} \times 100 = 98\%$ Choice (C)

5. In Double precision floating point format,
Number of bits for exponent = 11

$\therefore$ Biased exponent $= 2^{k-1} - 1$,
Where k is the number of bits used for exponent.
Here $k = 11$, hence Biased exponent
$= 2^{10} - 1 = 1023$. Choice (A)

6. If we assume R_2 as Base register, R_3 as Index register, then the mode will be Base with index mode.
Choice (C)

7. Number of bits in opcode = 4

$\Rightarrow$ Number of operations possible $= 2^4 = 16$
Number of bits in src/dest. Registers = 8

$\therefore$ Number of registers $= 2^8 = 256$ Choice (C)

8. Each DRAM has $4M$ addresses of 16-bit words.

$\Rightarrow$ DRAM capacity $= 4M \times 16$
Memory system capacity $= 128M \times 256$

$\therefore$ Number of chips required $= \frac{128M \times 256}{4M \times 16} = 512$
Choice (D)

9. Instruction size = 64-bits
Opcode size = $2B$ = 16-bits
Operand/Address field = 64 – 16 = 48-bits.
The 48-bits can be used to specify a particular address.
$\therefore$ Maximum directly addressable memory $= 2^{48}$
Choice (B)

10. Number of lines in cache = 32
There are 32 lines in cache. Each line will have 16 bytes.
$\therefore$ Total bytes of memory in cache
$= 16 \times 32 = 512$ bytes Choice (C)

11. The DRAM has given a refresh cycle 64 times per ms.
Time required for one refresh operation = 150 ns
In 1 ms, the time required to refresh is 64×150 ns = 9600 ns

$\therefore$ The fraction of time devoted to memory refresh is
$\frac{9600 \times 10^{-9}}{10^{-3}} = 0.0096$

$\therefore$ Approximate percentage of the memory's total operating time given to refreshes is 1%.
Choice (A)

12. The DMA is transmitting at a rate of 9600 bits per second i.e., it is transmitting $\frac{9600}{8} = 1200$ characters per second. The processor is processing at a rate of 1 million instructions per second i.e., It will take $\frac{1}{10^6} = 1\mu$ second to process a single instruction.
A single character will be processed by DMA in
$\frac{1}{1200} \approx 833\ \mu s$

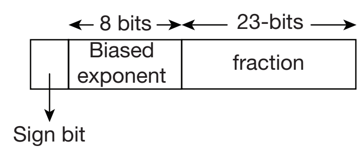
$\therefore$ Slow down of processor due to DMA
$= \frac{1}{833} \times 100 = 0.12\%$ Choice (D)

13. Cache is high speed RAM, DMA I/O is used with disk, Interrupt I/O is used with printer. Choice (C)

14. Without pipelining, execution time $= pq$
with pipelining, execution time $= p + q - 1$

$\therefore$ speed up $= \frac{pq}{p+q-1}$ Choice (C)

15. IEEE 32-bit floating point representation will be in the form of:



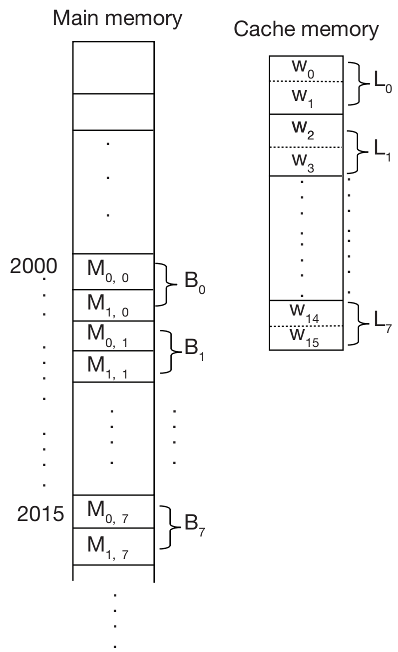
6 = 110
For –6 sign bit is 1.
$6 = 110 = 1.10 \times 2^{010}$
Exponent = 010
Biased exponent = 127 + 2 = 129 = 10000001
Fraction = 10000000000000000000000
Hence IEEE 32-bit floating point representation of –6 is:
110000001
10000000000000000000000 Choice (A)

16. In given code stalls occur before the ADD instructions. 2 stalls are there in given code. To minimize stalls, we reorder the code. In the reordered code place LOAD R_4, $M[1004]$ before ADD R_3, R_1, R_2.
The resultant code will be:

LOAD R_1, $M[1000]$
LOAD R_2, $M[1002]$
LOAD R_4, $M[1004]$
ADD R_3, R_1, R_2
ADD R_5, R_1, R_4
STORE R_3, $M[1008]$
STORE R_5, $M[1010]$

No stalls in the reordered code. Choice (A)

17. The Array $M_{2\times 8}$ is stored in column-major order in the main memory, i.e.,



In direct mapping, B_0 is placed in L_0, B_1 in L_1, B_2 in L_2, ..., B_7 in L_7 to access the elements $M_{0,0} - M_{0,7}$
The remaining elements are already in cache. All those accesses will be hits.
As all the 8-blocks used, cache utilization is 100%.
In Associative mapping, each block of main memory will be placed at anywhere in the cache lines. Also 8-accesses will be misses and the remaining will be hits. No need of replacement and all lines of cache will be used.

∴ Cache utilization = 100%
In 2-way set-Associative mapping, two blocks will be treated as a single set. There will be 4-sets.

Cache memory

L_0L_1	Set 0
L_2L_3	Set 1
L_4L_5	Set 2
L_6L_7	Set 3

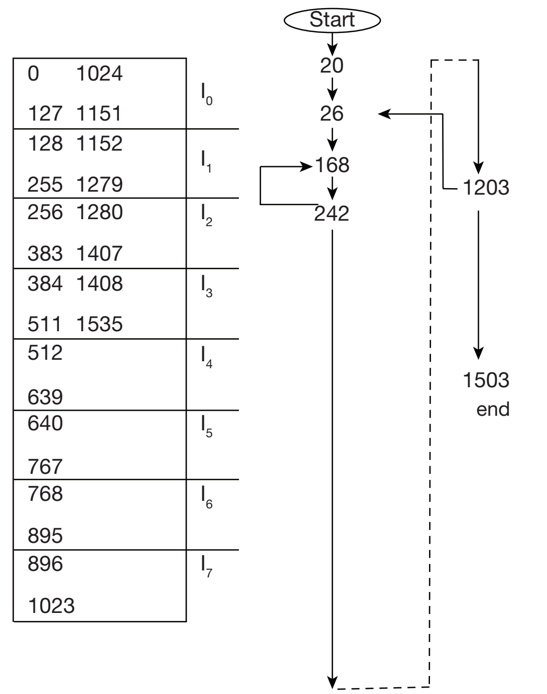
All the four sets are used. Set 0 consists B_0, B_4, set 1 contains B_1, B_5. Like this all sets are used. Hence cache utilization is 100%. Choice (C)

18. Given main memory size = 64 K = 2^{16} words
Block size = 128 = 2^7 words
Cache size = 1 K = 2^{10} words
Word field size = 7

$$\text{Number of lines} = \frac{2^{10}}{2^7} = 2^3$$

∴ Line field size = 3
⇒ Tag = 16 – (7 + 3) = 6

The cache memory is shown below with main memory block addresses.



Hence the sequence of reads from the main memory blocks into cache line is:

$$\text{Line: } \underbrace{0, 1, 2, 3, 4, 5, 6, 7, 0, 1,}_{\text{Pass 1 outer loop}} \underbrace{0, 1, 0, 1,}_{\text{Pass 2}} \ldots \underbrace{0, 1, 0, 1, 2, 3}_{\text{Pass 10}}$$

i.e., in pass1 of outer loop the lines 0, 1, 2, 3, 4, 5, 6, 7, 0, 1 will be accessed. In pass 2 0, 1 are accessed for (0 – 127), (128 – 255) and again 0, 1 are accessed for (1024 – 1151), (1152 – 1279).
In last pass 0, 1, 0, 1, 2, 3 lines will be accessed.

∴ Total time for reading the blocks of main memory into the cache
= (10 + 9 × 4 + 2) × 128 × 10
= 61440 n sec. Choice (D)

19. Let the number of instruction be 100.
Without optimization, time required to execute 100 Instructions = 100 + 10 × 2 = 100 + 20 = 120
With optimization time required
= (120 – 0.75 ×10 – 0.20 × 10) = 110.5
∴ Improvement using optimization

$$= \frac{120}{110.5} = 1.086.$$ Choice (B)

20. Number of lines in cache = 128
As the cache is a 4-way set associative, each set contains 4-blocks.

$$\therefore \quad \text{Number of sets} = \frac{128}{4} = 32$$

So there will be 32 misses.

But the program accesses every 8th word.

$\therefore$ Number of misses $= \frac{32}{8} = 4$

Hence miss rate $= \frac{1}{4}$ Choice (B)

21. Instruction size = 32-bits
Two address instruction format will be

opcode	address1	address2
8	12	12

There will be 2^8 possible combinations of operations.
Two address instructions = 240
Operations for single address instructions
= 256 – 240 = 16
Single address instruction format will be

opcode	address
	12 bits

Total one address operations = $16 \times 2^{12} = 65536$.
Choice (B)

22. The ideal speed up of a pipelined processor is equal to the number of stages in the pipeline.
$\therefore$ As there are 5-stages the speed will be 5.
Choice (B)

23. Ideal speed up = 5
Speed up due to Data hazards

$$= \frac{\text{Pipeline Depth}}{1 + (\text{load frequency} \times \text{load penalty}) + (\text{branch frequency} \times \text{branch penalty})}$$

$$= \frac{5}{1 + (0.2 \times 0.4 \times 2) + (0.3 \times 0.5 \times 3)}$$

$$= \frac{5}{1 + 0.16 + 0.45} = 3.11$$

$\therefore$ Speed up $= \frac{5 - 3.11}{5} \times 100 = 37.8\%$ Choice (D)

24. Given x = 0100 0110 1101 1000 0000 0000 0000 0000
y = 1011 1110 1110 0000 0000 0000 0000 0000
IEEE 754 floating point representation numbers has the following format:

Sign	Biased exponent	Mantissa
1	8	23

For x, sign bit = 0 $\Rightarrow$ sign = +
Biased exponent = 100 01101 = 141
Exponent = 141 – 127 (Bias) = 14
Mantissa = 1.101 1000 0000 0000 0000 0000
$\therefore$ x = +1.101 1000 0000 0000 0000 0000 $\times 2^{14}$
= +27648
For y, sign bit =1 $\Rightarrow$ sign = –
Biased exponent = 011 1110 1 = 125
Exponent = 125 – 127 = –2
Mantissa =1.110 0000 0000 0000 0000 0000
$\therefore$ y = –1.110 0000 0000 0000 0000 0000 $\times 2^{-2}$
= –0.4375
$x + y$ = 27648 – 0.4375 = 27647.5625
$x * y$ = 27648 * 0.4375 = –12096 Choice (A)

25. Given code:
I_1: ADD R_1, R_2, R_3
I_2: SUB R_4, R_1, R_5
I_3: LOAD R_6, 200(R_1)
I_4: ADD R_7, R_1, R_6
I_2, I_3, I_4 are dependent on I_1. I_4 dependent on I_3.
The execution chart is shown below:

	1	2	3	4	5	6	7	8	9	10
I_1	IF	ID	EX	MEM	WB					
I_2		IF	ID	EX	MEM	WB				
I_3			IF	ID	EX	MEM	WB			
I_4				IF	Stall	ID	EX	MEM	WB	

$\therefore$ R_1 can be forwarded to I_2, I_3 and I_4, but R_6 (in I_3) cannot be forwarded to I_4 immediately, so one stall occurs. Choice (B)

26. Given number +1001.110

Then c =

0	1	0	0	1	1	1	0	0	0

$\underbrace{0}_{\text{sign}}\ \underbrace{1\,0\,0\,1\,1\,1\,0\,0\,0}_{\text{fraction}}$

e =

0	0	1	0	0

$\underbrace{0}_{\text{sign}}\ \underbrace{0\,1\,0\,0}_{\text{exponent}}$

The decimal point will be at 4th position from left.
Choice (A)

27. Input clock frequency = 8 MHz

clock cycle $= \frac{1}{8 \text{ MHz}} = 125$ ns

Given that bus cycle = 4 × clock cycle
= 4 × 125 = 500 ns
Data bus width = 16-bits = 2 B
$\therefore$ 2 B can be transferred for every 500 ns.

$\therefore$ Transfer rate $= \frac{2 \text{ B}}{500 \text{ ns}} = 4$ MB/sec Choice (B)

28. As the number is an indirect address operand, the effective address will present in the given address location. So we refer two times to get the actual operand from memory. Hence the correct sequence of micro-operation is Choice (D)

29. As there are 7 micro-operations and each micro-operation takes one clock cycle, the total number of clock cycles for addition with an indirect address operand = 7.
Choice (D)

30. Access time of cache = 100 ns
Access time of main memory = 1 μs = 10^3 ns

Read requests = 80%
Write requests = 20%
Hit ratio for read only access = 0.9
Write-through means main memory is updated while updating cache memory.
Average access time of the system
$= 0.8(0.9 \times 100 + 0.1(100 + 1000)) + 0.2 \times 1000$
$= 160 + 200 = 360$ ns Choice (C)

31. The instruction "bbs reg, pos, label" jumps to label it bit in position 'pos' of register operand 'reg' is one.
To implement this instruction as
result ← reg & mask
We are performing 'and' operation between 'reg' and 'mask'.
The mask must be '1' to check a position value of 'reg'. And it is left shifted POS number of times to align that 1 to the required position of reg.
For example,
Reg = A091 = 1010 0000 1001 0001
POS = 2
Using bbs, we won't jump to label as 2^{nd} bit is zero using mask ← ox1 << POS
← ox1 << 2
← ox 0100
Now we perform AND between reg and mask. As the result is zero we won't jump to label. Choice (D)

32. Main memory = 1 MB = 2^{20} B
Word size = 1 B
Block size = 16 B = 2^4 B
Cache size = 64 KB = 2^{16} B
For Direct mapped cache:

$$\underbrace{\boxed{\text{tag}}\boxed{\text{line}}\boxed{\text{word}}}_{20}$$

word size = 4

Number of blocks in cache $= \dfrac{2^{16}}{2^4} = 2^{12}$

∴ Line = 12
tag + line = 20 – 4 = 16
Tag = 16 – 12 = 4
Given address F0010:

$$F0010 = \underset{\text{tag}}{\underline{1111}}\ \underset{\text{line}}{\underline{000000000001}}\ \underset{\text{word}}{\underline{000}}$$

∴ Line address = 0000 0000 0001 Choice (C)

33. The time required to transfer one byte is:
= 250 + 500 + 250
= 1000 ns = 1 μs.
To transfer 128 bytes, we require 128 μs.
Choice (D)

34. Instructions size = 2 B
Current instructions address = 356029
PC always points to the address of the next instruction.
i.e., 356029 + 2 = 356031
In PC-relative addressing mode, the content of PC is added to the displacement, which is given as –31.
So the branch target address
= 356031 – 31 = 356000 Choice (D)

35. Data transferred = 10 kB/sec
= 10000 Bytes/sec
Given that interrupts are generated for every 1 Byte.
There will be 10000 interrupts and rate of interrupts

$= \dfrac{1}{10000} = 100$ μs

∴ Fraction of processor time consumed by this I/O

$= \dfrac{100}{100} = 1$ Choice (A)

Computer Organization and Architecture Test 4

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. The primary memory of a computer consists of M locations, and the computer has a 4-way set associative cache consisting of 'C' locations. On an average, how many different locations in primary memory map to a particular location in the cache?

(A) 4 (B) $\frac{M}{C}$

(C) $\frac{4M}{C}$ (D) M

2. Which of the following statement is FALSE?

(A) The Program Counter and Stack Pointer are updated when a procedure call is executed on a processor.

(B) When a Procedure call is executed on a processor, data cache is flushed to avoid aliasing problems.

(C) A direct mapped cache can have a higher miss-rate than an associative mapped cache, each are of same size (i.e., same number of blocks).

(D) Programs with high temporal locality have a low cache miss rate penalty because, exactly the same addresses are re-referenced.

3. Let multiplicand be 23, multiplier be 29 and each number is represented using 6-bits. Then the difference between the total number of add/subtract operations required using unsigned binary multiplication and booths algorithm will be ________.

(A) 0 (B) 1

(C) 2 (D) 3

4. Consider a cache with a line size of 16 bytes and a main memory that requires 20 ns to transfer a 4-byte word. For any line that is written at least once before being swapped out of the cache, what is the average number of times that the line must be written before being swapped out for a write-back cache to be more efficient than a write through cache?

(A) more than 2 times (B) more than 3 times

(C) more than 4 times (D) more than 8 times

5. A certain pipelined RISC machine has 8 general purpose registers R_0, R_1, ..., R_7 and supports the following operations:

ADD $RS1$, $RS2$, Rd (Adds $RS1$ and $RS2$ and put the sum in Rd.)

MUL $RS1$, $RS2$, Rd (Multiply $RS1$ by $RS2$ and put the product in Rd.)

An operation normally takes one cycle; however, an operation takes two cycles if it produces a result required by the immediately following operation in an operation sequence. Consider the expression $xy + xz + xyz$, where variables x, y, z are located in registers R_0, R_1, R_2. The contents of these registers must not be modified, then the minimum number of clock cycles required for an operation sequence that compute the value of $xy + xz + xyz$ is ______.

(A) 5 (B) 6

(C) 7 (D) 8

6. Consider two processors, 5-stage pipeline P_5 and 7-stage pipeline P_7 to implement the same instruction set. Processor P_5 has a clock cycle of 20 ns and P_7 has a clock cycle of 17.5 ns. Which of the following is/are TRUE?

I. P_7 pipeline has better maximum throughput than P_5's pipeline.

II. The latency of a single instruction is shorter on P_7's pipeline than on P_5's pipeline.

III. Programs executing on P_7 will always run faster than the programs executing on P_5.

(A) I only (B) II only

(C) I and III only (D) II and III only

7. What is the size of the cache memory required for a direct-mapped cache with 16 KB of data and 16 B blocks, assuming a 32-bit address of main memory?

(A) 145 K bits

(B) 146 K bits

(C) 147 K bits

(D) 148 K bits

8. In a computer the exponents of a floating point number are represented as 'excess-64' integers. Two such exponents are input to a conventional 7-bit parallel adder. Which of the following should be accomplished in order to obtain a sum that is also in excess-64 notation?

(A) An end-around carry should be added to LSB.

(B) The adder outputs should be computed bitwise.

(C) The most significant adder output bit should be complemented.

(D) The adder outputs should be left unchanged.

9. The format of a double operand instruction of a CPU is shown below:

opcode	src data	dest data
← 6-bits →	← 4-bits →	← 4-bits →

20 double-operand instructions and 40 single-operand instructions must be implemented and the opcode field must identify 3-groups of n-operand instructions. The total number of zero-operand instructions that can be implemented is ______.

(A) 704 (B) 11264

(C) 16384 (D) 10496

10. A CPU has 32-bit memory address and a 256KB cache memory. The cache is organized as a 4-way set-associative cache with a cache block size of 4 words (32-bits per word). The smallest addressable unit is a byte. Then the percentage of cache memory used for tag bits is _________.

(A) 11.11 (B) 12
(C) 80.8 (D) 88.89

11. Assume that we have three significant decimal digits. What is the sum of 3.67 and 3.45×10^2 using guard and round digits and the sum of the two numbers without using guard and round digits respectively?

(A) 348, 349 (B) 349, 348
(C) 350, 347 (D) 348, 348

12. If an encoded micro-instruction format is used, then how a 10-bit micro-operation field can be divided into subfields to specify 41 different actions efficiently?

(A) 5, 5 (B) 2, 4, 4
(C) 5, 3, 2 (D) 5, 4, 1

13. The two-stages of a pipelined processor are Bus Interface Unit (BIU) and Execution Unit (EU). The BIU fetches instructions from a 4-byte instruction queue. It also calculates address, fetches operands and writes results on to memory as requested by EU. If no such requests are outstanding and the bus is free, the BIU fills vacancies (if any) in the instruction queue. When the EU completes execution of an instruction, it passes results to BIU and proceeds to the next instruction. Let the tasks performed by the BIU and EU take about equal time. The factor by which pipelining improves the performance of the processor (Ignore the effect of branch Instructions) is _________.

(A) 2 (B) 4
(C) 5 (D) 8

14. Consider a processor that includes a base with indexed addressing mode. Suppose an instruction is encountered that employs this addressing mode and specifies a displacement of 2970, in decimal. Currently the base and index register contain the decimal numbers 58022 and 8 respectively. Then the address of operand would be _________.

(A) 58022 (B) 60992
(C) 61000 (D) 2978

15. Consider a RAID level 5 organization comprising of five disks, with the parity for sets of four blocks on four disks stored on the fifth disk. The number of blocks that are accessed to write one block of data is _________.

(A) 3 (B) 4
(C) 5 (D) 6

16. Consider a static two-issue pipeline for MIPS. The two issue packets are, one ALU/branch instruction, one load/store instruction. Which of the following is the correct re-ordering of the below loop to avoid as many pipeline stalls as possible? Assume branches are predicted, so that control hazards are handled by the hardware.

loop: Load R_0, $M[100]$;	$R_0 \leftarrow M[100]$
ADD R_0, R_0, 5;	$R_0 \leftarrow R_0 + 5$
Store R_0, $M[100]$;	$M[100] \leftarrow R_0$
ADD R_1, R_1, –4;	$R_1 \leftarrow R_1 - 4$
BNE R_1, 0, loop;	branch if $R_1 \neq 0$

(A) loop: Load R_0, $M[100]$
ADD R_1, R_1, –4
ADD R_0, R_0, 5
BNE R_1, 0, loop; Store R_0, $M[100]$
(Both in single cycle).

(B) loop: Load R_0, $M[100]$
ADD R_1, R_1, –4
BNE R_1, 0, loop; ADD R_0, R_0, 5
(Both in single cycle)
Store R_0, $M[100]$

(C) Loop: Load R_0, $M[100]$
ADD R_0, R_0, 5
ADD R_1, R_1, –4
Store R_0, $M[100]$
BNE R_1, 0, loop

(D) loop: ADD R_1, R_1, –4
BNE R_1, 0, loop
Load R_0, $M[100]$
ADD R_0, R_0, 5; Store R_0, $M[100]$;
(Both in single clock cycle)

17. Consider a single level cache with an access time of 2.5 ns, a line size of 64 Bytes and a hit ratio of 0.95. Main memory uses a block transfer capability that has a first word (4 bytes) access time of 60 ns and an access time of 5 ns for each word thereafter.
What is the access time when there is a cache miss, Assume that the cache waits until the line has been fetched from main memory and then re-executes for a hit.

(A) 9.25 ns (B) 9.375 ns
(C) 9.625 ns (D) 9.5 ns

18. For the data given in Q.No:17, Suppose that increasing the line size to 128 bytes increases hit ratio to 0.97, then the average memory access time

(A) increases
(B) decreases
(C) remains constant
(D) will be unpredictable

19. Consider an instruction pipeline with five stages without any branch prediction: Fetch Instruction (FI), Decode instruction (DI), Fetch operand (FO), Execute instruction (EI) and Write operand (WO). The stage delays for FI, DI, FO, EI and WO are 3 ns, 4 ns, 6 ns, 2 ns and 4 ns respectively. There are intermediate storage buffers after each stage and delay of each buffer is 1 ns. A program consisting of 10 instructions $I_1, I_2, ..., I_{10}$ is executed in this pipelined processor. Instruction I_3,

is the only branch instruction and its branch target is I_7. If the branch is taken during the execution of this program, the time needed to complete the program (in nano seconds) is ________.

(A) 49 (B) 56
(C) 98 (D) 105

20. Consider the following program segment for a hypothetical CPU having three user registers R_1, R_2 and R_3.

Instruction	Operation	Instruction size (in words)
MOV R_1, 6000	$R_1 \leftarrow$ M[6000]	2
MOV R_2,(R_1)	$R_2 \leftarrow$ M[(R_1)]	1
SUB R_2, R_3	$R_2 \leftarrow R_2$ - R_3	1
MOV 6000, R_2	M[6000] $\leftarrow R_2$	2
HALT	Machine halts	1

Let the clock cycles required for various operations be as follows:
Instruction fetch and decode: 2 clock cycles per word.
Register to/from memory transfer: 3 clock cycles.
SUB with both operands in registers: 1 clock cycle
Then the total number of clock cycles required to execute the program will be ________.

(A) 2 (B) 23
(C) 24 (D) 19

21. Consider a 32KB, two-way set associative cache with 64-byte block size. The CPU generates 32-bit addresses, A 2-to-1 multiplexer has a latency of 0.5ns while a K-bit comparator has a latency of K/10 ns, then the hit latency of the cache is _______.

(A) 1.8 (B) 2.3
(C) 2.4 (D) 3

22. According to IEEE standard, a 32-bit, single precision, floating point number N is defined to be:
$N = (-1)^S \times 1.\, M \times 2^{E\text{-}127}$
Where S: The sign bit
M: Fractional mantissa
E: The Biased Exponent
A floating point number is stored as SEM, where S, E and M are stored in 1-bit, 8-bits and 23-bits respectively.
What is the value of the floating point number $C0E80000$ (hexa decimal notation)?

(A) –7.25 (B) –29
(C) –0.453 (D) –7

23. For the data given in Q.No. 22, using given notation, what is the hexa decimal equivalent of 565?

(A) 048D4000 (B) 440D4000
(C) 480D4000 (D) 444D4000

24. Consider an 8 KB direct-mapped write-back data cache with 16 byte blocks. The elements of two arrays A and B are 8 bytes long. Array A has 5 rows and 100 columns and array B has 5 rows and 101 columns. (Initially cache is empty). To execute below code, how many cache misses will occur? (Arrays are stored in row-major order).
for (i = 0; i < 5; i++)
for (j = 0; j < 100; j++)
A[i][j] = B[j] [0] * B[j+1][0];

(A) 251 (B) 351
(C) 451 (D) 551

25. Consider a processor with a 5-stage MIPS pipeline (IF, ID, EX, MEM and WB) with no forwarding and each stage takes 1 cycle. The register file can be written and then read in same cycle. In the below program, the destination is the first (left most) register.

Instruction	Operation
LD R_0, (R_2)	$R_0 \leftarrow M(R_2)$
LD R_2, 4(R_3)	$R_2 \leftarrow M[R_3 + 4]$
ADD R_3, R_2, R_0	$R_3 \leftarrow R_2 + R_0$
SUB R_3, R_0, R_0	$R_3 \leftarrow R_0 - R_0$
OR R_4, R_3, 0	$R_4 \leftarrow R_3$ OR 0
SW R_3, 5(R_2)	$R_3 \leftarrow M[R_2 + 5]$

How many stall cycles do the program incur?

(A) 1 (B) 2
(C) 3 (D) 4

26. Consider a deeper pipeline in which, it takes atleast three pipeline stages before the branch-target address is known and an additional cycle before the branch condition is evaluated, assuming no stalls on the registers in the conditional comparison. The 3-stage delay leads to the branch penalties for three simple branch prediction schemes, which are listed below:

Prediction Scheme	Frequency
Unconditional Branch	5%
Conditional branch, Taken	5%
Conditional branch, Untaken	10%

Branch scheme	Penalty (unconditional)	Penalty (taken)	Penalty (Untaken)
Flush Pipeline	2	3	3
Predicted Taken	2	2	3
Predicted Untaken	2	3	0

Find the effective addition to CPI arising from each branch scheme for this pipelining (By taking above frequencies into count)?

(A) 0.5, 0.5, 0.3 (B) 0.3, 0.4, 0.6
(C) 0.55, 0.5, 0.25 (D) 1.3, 1.4, 1.6

27. Assume that the number of clock cycles for a polling operation is 200. Consider a processor which executes at 60 MHz, what is the overhead of polling in percentage

(i) For a mouse that is polled 20 times per second.

(ii) For a hard disk transferring data in 1 word chunks at 1 MB/sec.

(A) 0.006%, 100% (B) 0.0067%, 87.4%
(C) 0.0067%, 43.7% (D) 0.0033%, 43.7%

28. A CPU, which addresses the data through its 8 registers in one of the 15 different modes, is to be designed to support 15 arithmetic instructions, 10 logic instructions, 24 data-moving instructions, 6 branch instructions, 5 control type instructions. Of these instructions respectively 20%, 60%, 50%, 50% and 60% are either single-operand or no-operand instructions and the rest are of double-operand type. What is the minimum size of the CPU's instruction word?

(A) 15-bits (B) 20-bits
(C) 19-bits (D) 21-bits

29. Consider the following sequence of micro-operations:

MAR ← (IR (address))
MBR ← (PC)
PC ← (IR(address))
Memory ← (MBR)
PC ← (PC) + I
I is instruction Length

Which one of the following is a possible operation performed by this sequence?

(A) Instruction fetch
(B) Initiation of interrupt service
(C) Branch-and-save address
(D) Increment and skip if zero.

30. Suppose that a 1 GHz processor needs to read 2000 bytes of data from a particular I/O device. The I/O device supplies 1 byte of data every 0.02 ms. The code to process the data and store it in a buffer takes 1000 cycles.

If the processor detects that a byte of data is ready through polling and a polling iteration takes 80 cycles, how many cycles does the entire operation take?

(A) 42000 K cycles (B) 40000 K cycles
(C) 40080 K cycles (D) 39920 K cycles

31. For the data given in Q. No. 30, If instead, the processor is interrupted when a byte is ready and the processor spends the time between interrupts on another task; then how many cycles are saved by using interrupt-driven over polling technique. The overhead of handling an interrupt is 200 cycles.

(A) 39680 K cycles (B) 39600 K cycles
(C) 38960 K cycles (D) 37680 K cycles

32. A CPU has 64 KB direct mapped cache with 128-byte block size. Suppose X is a two dimensional array of size 512 × 512 with elements that occupy 8-bytes each. Consider the following C-code segment:

```
for (i = 0; i < 512 ; i ++)
{
for (j = 0; j < 512 ; j ++)
{
a+ = X [i] [j];
}
}
```

Initially array X is not in the cache and i, j, a are in registers. The number of cache misses experienced by given code fragment is ________.

(A) 8196 (B) 16384
(C) 32768 (D) 65536

33. Consider the execution of the following sequence of instructions on a five-stage pipeline consisting of:

IF: Instruction Fetch
ID: Instruction Decode
OF: Operand Fetch
IE: Instruction Execution
IS: Store results

Instruction	Operation
LOAD R_1, 1;	$R_1 \leftarrow 1$
LOAD R_2, 5;	$R_2 \leftarrow 5$
SUB R_2, R_2, 1;	$R_2 \leftarrow R_2 - 1$
ADD R_3, R_1, R_2;	$R_3 \leftarrow R_1 + R_2$
ADD R_6, R_4, R_5	$R_6 \leftarrow R_4 + R_5$
SL M[1000], R_3;	$M[1000] \leftarrow R_3$
ADD R_7, R_4, R_6;	$R_7 \leftarrow R_4 + R_6$

The speedup achieved by executing this program on a pipelined processor compared to a sequential processor (Let each stage takes one clock cycle) is _______.

(A) 1.365 (B) 2
(C) 2.1875 (D) 3.236

34. In a program, an array is declared as int A[1024]. Each array element is 2 bytes in size. In main memory the array is stored at the address 0X00000000. This program is run on a computer that has a direct mapped data cache of size 4 KB, with line size of 8 bytes. Which of the following element of the array conflict with the element A[0] in the data cache?

(A) A[256] (B) A[512]
(C) A[63] (D) No conflict

35. For the data given in Q. No. 34, If the program accesses the elements of this array one by one in reverse order that is starting with the last element and ending with the first element, how many data cache misses would occur?

(Assume that the data cache is initially empty and that no other data or instruction accesses are to be considered)

(A) 64 (B) 256
(C) 512 (D) 1024

Answer Keys

1. C	**2.** B	**3.** A	**4.** C	**5.** B	**6.** A	**7.** B	**8.** C	**9.** D	**10.** A
11. B	**12.** C	**13.** A	**14.** C	**15.** B	**16.** A	**17.** B	**18.** B	**19.** C	**20.** C
21. B	**22.** A	**23.** B	**24.** B	**25.** B	**26.** C	**27.** B	**28.** B	**29.** C	**30.** C
31. D	**32.** B	**33.** C	**34.** D	**35.** B					

Hints and Explanations

1. If primary memory has 'M' locations and cache has 'C' locations then each location on the cache is responsible for M/C locations in direct mapping. For 4-way set associative, the cache can hold $4 \times \frac{M}{C}$ locations.
Choice (C)

2. Choice (B)

3. Multiplicand (M) = 23 = 010111
Multiplier (Q) = 29 = 011101
Using unsigned binary multiplication, perform addition if
Q_0 = 1 i.e., LSB of Q register is 1 so number of additions equal to number of '1's in Q. Here number of 1's in Q = 4.
In booth algorithm, perform addition or subtraction if Q_0, Q_{-1} = 01 or 10 respectively. i.e., whenever a bit change occurred, perform either addition or subtraction.
Multiplier (Q) = 0 1 1 1 0 1 0 ← initially Q_{-1} = 0
∴ 4 ADD/SUB operations required.
Hence difference is zero. Choice (A)

4. In write-back case, each dirty line is written back once, i.e., at a swap-out time, taking $\frac{16}{4} \times 20 = 80$ ns.
In write-through case, each update of the line requires that one word be written out to main memory, taking 20 ns.
∴ If the average line that gets written atleast once gets written more then $\frac{80}{20} = 4$ times before swap out, then write back is more efficient. Choice (C)

5. $R_0 \leftarrow x$, $R_1 \leftarrow y$, $R_2 \leftarrow z$
To compute the value $xy + xz + xyz$ on this RISC processor, the operations are
I_1: MUL R_0, R_1, R_3 ... $R_3 \leftarrow xy$
I_2: MUL R_0, R_2, R_4 ... $R_4 \leftarrow xz$
I_3: MUL R_3, R_2, R_5 ... $R_5 \leftarrow xyz$
I_4: ADD R_3, R_4, R_6 ... $R_6 \leftarrow xy + xz$
I_5: ADD R_5, R_6, R_6 ... $R_6 \leftarrow xy + xz + xyz$
Each of I_1, I_2, I_3, I_4 require one clock cycle. I_5 requires two clock cycles as R_6 is used in I_5.
∴ Minimum clock cycles required = 6 Choice (B)

6. Increasing number of stages will increase the throughput but increases latency of single instruction. (III) Condition not valid always. Choice (A)

7. Data in cache = 16 KB = 2^{14}B
Block size = 16 B
Number of blocks $= \frac{2^{14}}{2^4} = 2^{10}$
CPU generates 32-bit addresses.

Tag	Block	Word

Word field requires 4-bits as block size is 16 B. Block field requires 10-bits as there are 2^{10} blocks.
Tag = 32 – (10+4) = 18-bits
∴ Cache size = $2^{10} \times (4 \times 32 + 18)$ = 146 K bits
(as each block has 4 × 32-bit data and 18-bits tag).
Choice (B)

8. Let the two exponents to be added are 13 and 15.
13 in excess - 64 = 13 + 64 = 77 = 1001101
15 in excess - 64 = 15 + 64 = 79 = 1001111
Output from 7-bit parallel adder = 0011100
But 13 + 15 + 64 = 92 = 1011100
Hence complement the MSB of the output.
Choice (C)

9. Opcode field size = 6-bits
∴ Total number of operations possible = 2^6 = 64
Out of these 64, only 20 operations are used by double-operand instructions.
Remaining = 64 – 20 = 44.
In single-operand instructions, src data will be included in opcode field.
∴ Possible single-operand operations = 44×2^4
Out of these only 40 are used for single-operand instructions.
∴ Remaining = 41×2^4
(∵ Three 2^4's are used for single operand instruction)
In zero-operand instructions there will be no operands, so it will be included in opcode field.
∴ Zero-operand instructions
$= 41 \times 2^4 \times 2^4 = 10496$ Choice (D)

10. Address generated by CPU has 32-bits.
Cache memory = 256 KB = 2^{18} Bytes
Block size = 4 words
= 4 × 32 bits (∵ 1 word = 32-bits)
$= \frac{4 \times 32}{8}$ bytes = 16 bytes = 2^4 B

$$\text{Number of lines} = \frac{2^{18}}{2^4} = 2^{14}$$

$$\text{Number of sets} = \frac{2^{14}}{2^2} = 2^{12}$$

Tag	Set	Word
16	12	4

32 bits

Percentage of the cache memory is used for tag bits

$$= \frac{16 \times 2^{14}}{(16+128) \times 2^{14}} \times 100 \text{ } (\because \text{ each block has 128-bits})$$

$$= \frac{16}{16+128} \times 100 = 11.11\%$$ Choice (A)

11. Given numbers 3.67 and 3.45×10^2. To add these two numbers, first align the exponents.
i.e., shift the smaller number to right.
$3.67 = 0.0367 \times 10^2$
The guard digit holds 6 and round digit holds 7.
The sum will be
0.0367
3.4500
3.4867
$\therefore$ Sum $= 3.4867 \times 10^2$
Here we need to round two digits; after rounding sum will be $3.49 \times 10^2 = 349$
Without round and guard digits,
3.45
0.03
3.48
$\therefore$ Sum $= 3.48 \times 10^2 = 348$ Choice (B)

12. 10-bit micro-operation field can be divided into 3 subfields 5, 3, 2 to yield 31, 7, 3 actions respectively. Choice (C)

13. Each stage takes equal amount of time. Let 'x' be the time taken by BIU and EU stages.
Time taken to execute 'n' instructions on this pipeline $= [2 + n - 1]\, x = (n+1)x$
Without pipelining time taken will be $= 2.n.x$

$$\text{Speed up ratio} = \frac{2nx}{(n+1)x}$$

For large number of instructions, $n >> x$.

$$\therefore \text{ Speed up ratio} = \frac{2n}{n} = 2.$$

Hence pipeline improves the performance by a factor of 2. Choice (A)

14. The addressing mode is base with indexing mode.
Displacement = 2970
Base register value = 58022
Index register value = 8
Address of operand = 2970 + 58022 + 8 = 61000
Choice (C)

15. To write one block of data the tasks performed are:
1. Read the parity
2. Read target block old data
3. Compute new parity
4. Write new block
5. Write new parity

$\therefore$ 4 times the blocks are accessed. Choice (B)

16. The first three instructions have data dependencies and so do the last two. The best reorder for these instructions is:

Loop:	ALU/branch instruction	LOAD/STORE instruction	Clock cycle
		LOAD R_0, M[100]	1
	ADD R_1, R_1, –4		2
	ADD R_0, R_0, 5		3
	BNE R_1, 0, Loop	STORE R_0, M[100]	4

Hence only one pair of instructions has both issue slots used. Choice (A)

17. Cache access time = 2.5 ns
Line size = 64 Bytes
Hit ratio = 0.95
Access time of first byte in main memory = 60 ns
Next word access time = 5 ns

$$\text{Number of words in each line} = \frac{64}{4} = 16 \text{ words}$$

For 1st word 60 ns and for remaining 15 words, 15×5 ns required.
Cache access time when there is a miss
$= 0.95 \times 2.5 + 0.05\,(2.5 + 60 + 15 \times 5 + 2.5)$
$= 9.375$ ns Choice (B)

18. New line size = 128 bytes
$H = 0.97$

$$\text{Now, words in each line} = \frac{128}{4} = 32$$

First word requires 60 ns and remaining 31 words require 31×5 ns.
$\therefore$ Cache access time when there is a miss
$= 0.97 \times 2.5 + 0.03 \times (2.5 + 60 + 31 \times 5 + 2.5)$
$= 9.025$
$\therefore$ Average memory access time reduced.
Choice (B)

19. Stage delays for FI, DI, FO, EI and WO are 3, 4, 6, 2, 4 ns respectively.
Maximum time is taken by FO. i.e., 6 ns and additional 1 ns is required for delay. So total time for an instruction to pass from one stage to another is 7 ns. The instruction execution sequence is $I_1, I_2, I_3, I_7, I_8, I_9, I_{10}$.
When I_3, is in its execution stage, we detect the branch and when I_3 is in 'WO' stage we fetch I_7.
Execution time for $I_1, I_2, I_3 = [5 + (3 - 1)]\, 7 = 49$ ns.

Execution time for $I_7, I_8, I_9, I_{10} = [5 + (4 – 1)]7 = 56$ ns
But we fetch I_7 in I_3's WO stage, so we saved 7 ns
Hence total time = 49 + 56 – 7 = 98 ns.

Choice (C)

20. Each instruction requires fetch and decode stage.

	Fetch & Decode	Operation
MOV R_1, 6000	2 × 2	3
MOV R_2, (R_1)	2	3
SUB R_2, R_3	2	1
MOV 6000, R_2	2 × 2	3
HALT	2	–

∴ Total clock cycles required = 7 + 5 + 3 + 7 + 2 = 24

Choice (C)

21. Cache capacity = 32 KB = 2^{15}B
Block size = 64B = 2^6B
∴ word size = 6-bits.

The cache is 2-way set associative, so $2 \times 2^v = \frac{2^{15}}{2^6}$

⇒ $2^V = 2^8$
∴ set size = 8-bits
CPU generates 32-bit addresses.

← 32 →

Tag	Set	Word
← 18 →	← 8 →	← 6 →

First a 2-to-1 multiplexer is required whose latency is 0.5 ns. After that the tag bits are compared using a comparator.

For 18-bit comparator $\frac{18}{10}$ ns required.

∴ Hit latency = 0.5 + 1.8 = 2.3 ns

Choice (B).

22. Given number $C0E80000$
In binary, it will be

← 32 →

1 | 100 00001 | 110 1000 0000 0000 0000 0000

sign, exponent, mantissa

$S = 1$
$E = 100\ 00001 = 129$
Mantissa = 110 1000 0000 0000 0000 0000
∴ $N = (–1)^1 \times 1.1101 \times 2^{129–127}$
$= –1.1101 \times 2^2$
$= –111.01 = –7.25$

Choice (A)

23. 565 = 1000110101
$N = (–1)^0 \times 1.000110101 \times 2^{136–127}$
IEEE standard binary equivalent of
N = <u>0 100</u> <u>0 100</u> <u>0 000</u> <u>1101</u> <u>0100</u> <u>0000</u> <u>0000</u> <u>0000</u>
4 4 0 D 4 0 0 0
= 440D4000

Choice (B)

24. Elements of "A" are written in the order in which they are stored in memory. So A will benefit from spatial locality. The even values of j will miss and odd values will hit. i.e., Each block consists two values. (For example a[0] [0] , a[0] [1] or a[0] [2], a[0] [3] etc.)
Since A has 5 rows and 100 columns, its accesses will lead to $5 \times \frac{100}{2} = 250$ misses.
The array 'B' does not benefit from spatial locality since the accesses are not in the order it is stored. But it will benefit from temporal locality.
The misses due to 'B' will be for $B[j + 1]$ [0] accesses when i = 0 and also the first access to $B[j][0]$ when j = 0. Since j goes from j = 0 to 99 when i = 0, accesses to B lead to 100 + 1 or 101 misses.
∴ Total cache misses = 250 + 101 = 351.

Choice (B)

25. There are only two stall cycles. One is after 2nd load as R_2 is immediately required in ADD instruction. Another stall is after 'SUB' as $R3$ is immediately required by 'OR' instruction. Choice (B)

26. We find the CPI's of each prediction by multiplying the relative frequencies with respective penalties.

Branch scheme	**Frequency**	**Unconditional**	**Conditional, Taken**	**Conditional, Untaken**
Stall Pipeline	5%	$\frac{5}{100} \times 2 = 0.1$	$\frac{5}{100} \times 3 = 0.15$	$\frac{10}{100} \times 3 = 0.3$
Predicted Taken	5%	$\frac{5}{100} \times 2 = 0.1$	$\frac{5}{100} \times 2 = 0.1$	$\frac{10}{100} \times 3 = 0.3$
Predicted Untaken	10%	$\frac{5}{100} \times 2 = 0.1$	$\frac{5}{100} \times 3 = 0.15$	$\frac{10}{100} \times 0 = 0$

Total branches frequency = 5 + 5 + 10 = 20%

Using stall pipeline, additional CPI = 0.1 + 0.15 + 0.3 = 0.55
Using predicted taken, additional CPI = 0.1 + 0.1 + 0.3 = 0.5
Using predicted Untaken, additional CPI = 0.1 + 0.15 + 0 = 0.25 Choice (C)

27. Clock cycles per second used for polling a mouse = 20 × 200 = 4000
Fraction of processor cycles used for polling

$$= \frac{4000}{60 \times 10^6} \times 100 = 0.0067\%$$

Rate of polling for hard disk $\dfrac{\frac{1\text{ MB}}{\text{sec}}}{4\text{ bytes per access}}$

= 262144 polling access per second
Clock cycles = 262144 × 200 = 52428800
∴ Fraction of CPU cycles

$$= \frac{52428800}{60 \times 10^6} \times 100 \times 100 = 87.4\%$$ Choice (B)

28. Double-operand instruction format will be

Opcode	Src data	Dest Data

Data will consist both mode and register field.
There are 8-registers, so 3-bits required. For 15-different modes 4-bits required.
∴ Each data field requires 7-bits.
Now we need to find opcode size.

	Instruction	Single/ no-operand	Double-operand
Arithmetic	15	$3\left(\frac{20}{100} \times 15\right)$	12
Logic	10	6	4
Data-moving	24	12	12
Branch	6	3	3
Control	5	3	2

∴ Total double operand instructions = 12 + 4 + 12 + 3 + 2 = 33
6-bits required for double operand. (∵ $2^6 > 33$).
∴ Minimum size of CPU's instruction word = 6 + 7 + 7 = 20-bits. Choice (B)

29. Given micro-operations specifies Branch-and-save address. The address part in instruction is extracted. Next instruction address is saved to memory location and execution continues at location (PC) + I. Choice (C)

30. Polling is same as programmed-I/O
Number of cycles required for polling
= 1 × 10^9 × 0.02 × 10^{-3} = 20000
Before processing the data it will be
20000 – 1000 = 19000 cycles
Each polling iteration takes 80 cycles.
∴ Number of polls required for one byte $= \frac{19000}{80}$
= 237.5 polls ≃ 238 polls
Cycles spend on polling = 238 × 80 = 19040 cycles
∴ Each byte thus take 19040 + 1000 = 20040 cycles
To read 2000 bytes it requires
2000 × 20040 = 40080000
= 40080 K cycles Choice (C)

31. Each byte takes 200 + 1000 = 1200 cycles
i.e., 1200 cycles to process a byte.
For 2000 bytes it will required
1200 × 2000 = 2400000 cycles
Number of cycles saved using interrupt method
= 40080000 – 2400000 = 37680 K cycles Choice (D)

32. Cache size = 64 KB = 2^{16} B
Block size = 128-Bytes = 2^7 B
Array size = 512 × 512
Each element requires 8 bytes.
Memory required to store X is $512 \times 512 \times 8 = 2^{21}$ Bytes.
Number of Blocks required to store 2^{21} Bytes

$$= \frac{2^{21}}{2^7} = 2^{14} = 16384$$

∴ Number of cache misses = 16384 Choice (B)

33. Time required to execute given program on a sequential processor = 7 × 5 = 35

On a pipelined processor the execution sequence is shown below:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
I1	IF	ID	OF	IE	IS											
I2		IF	ID	OF	IE	IS										
I3			IF	ID			OF	IE	IS							
I4				IF	ID					OF	IE	IS				
I5					IF	ID					OF	IE	IS			
I6						IF	ID						OF	IE	IS	
I7							IF	ID						OF	IE	IS

∴ Total number of clock cycles required = 16

∴ Speed-up

$$= \frac{\text{Time required on sequential processor}}{\text{Time required on pipelined processor}} = \frac{35}{16}$$

= 2.1875 Choice (C)

34. Array is $A[1024]$
Array requires 1024 × 2 bytes.
Cache size = 4 KB = 2^{12} B
Line size = 8 bytes

Number of lines $= \frac{2^{12}}{2^3} = 2^9$ lines

Array requires 2048 bytes

i.e., $\frac{2048}{8}$ = 256 blocks. As number of lines is greater than the blocks required for array storage, there will be no conflict with element $A[0]$.

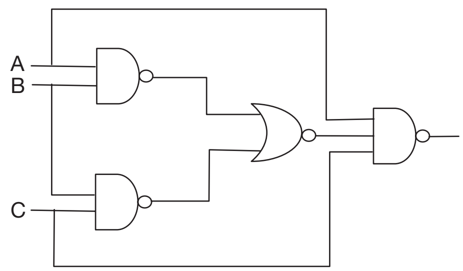
Choice (D)

35. As the array requires 2048 bytes i.e., $\frac{2048}{8}$ = 256 blocks, 256 misses will be there. Choice (B)

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. The output of the following circuit is:



(A) $\overline{A}\overline{B}+\overline{B}\overline{C}$ (B) $\overline{A}\overline{B}+\overline{B}\overline{C}+\overline{A}\overline{C}$
(C) $\overline{A}+\overline{B}+\overline{C}$ (D) $\overline{AB+BC}$

2. Match the following:
List I (Numbers in Decimal)
List II (equivalents in signed 2's complement representation)

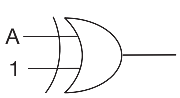
	List-I		List-II
P.	– 43	1.	01100000
Q.	– 78	2.	00110110
R.	+ 54	3.	111010101
S.	+ 96	4.	10110010

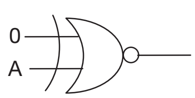
(A) P–3, Q–4, R–1, S–2
(B) P–4, Q–3, R–1, S–2
(C) P–3, Q–4, R–2, S–1
(D) P–4, Q–3, R–1, S–2

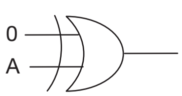
3. Which one of the following function will satisfy the property, "Dual of function = complement of the function"?
(A) $f(A, B, C) = \Sigma m\ (0, 1, 2, 3)$
(B) $f(A, B, C) = \Sigma m\ (4, 5, 6, 7)$
(C) $f(A, B, C) = \Sigma m\ (0, 2, 4, 6)$
(D) $f(A, B, C) = \Sigma m\ (0, 1, 6, 7)$

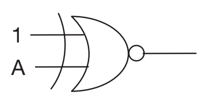
4. The number of min terms for the function
$F(a, b, c, d, e) = b + cd$ is:
(A) 24 (B) 20
(C) 32 (D) 16

5. Which of the following will work like an inverter?

(P) 

(Q) 

(R) 

(S) 

(A) P, Q (B) Q, R
(C) R, S (D) P, S

6. Perform the following operation in 2's complement signed representation, and specify the result in 2's complement signed notation.

$$(-13)_{10} + (-28)_{10} = ?$$

(A) 10101001 (B) 11010111
(C) 11010110 (D) 00101001

7. Convert the following number to base 9.

$$(1101222.201121)_3$$

(A) 1358.647 (B) 4172.647
(C) 4178.153 (D) 1358.153

8. Which of the following is usually regarded as a bottleneck to von-Neumann computer Architecture?
(A) ALU
(B) Instruction set
(C) Processor/memory interface
(D) Control unit

9. Which of the following is the largest storage unit in a usual memory hierarchy?
(A) Cache memory (B) Main memory
(C) Register (D) Hard disk

10. Which type of cache miss does not occur in fully-Associative cache memory?
(A) Capacity miss (B) Conflict miss
(C) Compulsory miss (D) Cold start miss

11. Which of the following statement is FALSE?
(A) Pipelining does not improve the execution time of a single task.
(B) Pipelining improves the throughput of the total work load.
(C) Pipeline speed is limited by the slowest pipeline stage.
(D) In pipelining, only one task is processed at a time.

12. What is the execution time per stage of a pipeline that has 4 equal stages and a mean overhead of 12 cycles?
(A) 3 cycles (B) 4 cycles
(C) 6 cycles (D) 12 cycles

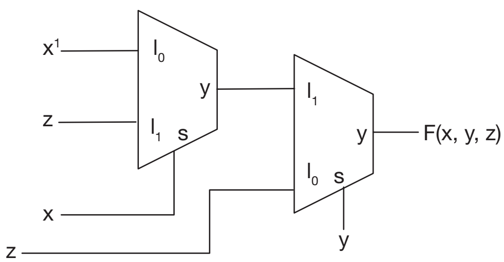
13. How many bits are needed to represent a direct address on a 64-bit machine?
(A) 6-bits (B) 64-bits
(C) 32-bits (D) 2^{64}-bits

14. A program has 5% divide instructions. Any non-divide instruction takes one cycle. All divide instructions take 25 cycles. What percent of CPU time is spent just doing divides?
(A) 100% (B) 95%
(C) 56.82% (D) 28.41%

15. Which of the following is not an advantage of memory mapped technique?
(i) Simple hardware
(ii) Simple instruction size
(iii) All address modes available
(iv) More memory address space
(A) (i), (iv) only (B) (iv) only
(C) (i), (ii), (iii) only (D) (iii), (iv) only

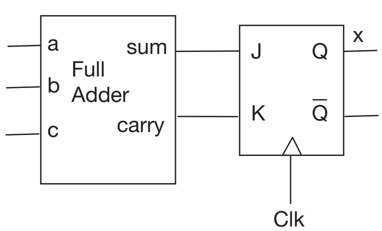
16. How many number of 2-input NAND gates are required to implement $f(A, B, C, D) = \Sigma m$ (1, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15)?
(A) 4 (B) 3
(C) 2 (D) 1

17. The output of the following Multiplexer circuit is:



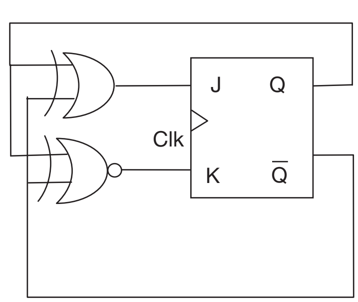
(A) $x^1 + yz$ (B) $x^1 y + z$
(C) $(x^1 + y)z$ (D) $x^1 y^1 + yz + xy^1 z$

18. Initially $Q_n = 0$, $\bar{Q}_n = 1$, , after clock pulse $Q_{n+1} = 1$, $\bar{Q}_{n+1} = 0$, which of the following correctly specifies about the inputs a, b, c?



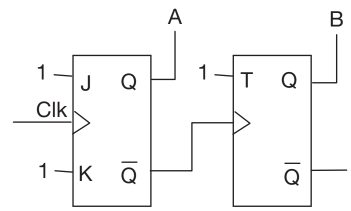
(A) two or more inputs should be 1.
(B) only one input has to be 1, or all inputs should be 1.
(C) c should be zero, a, b, can be either 11 or 00
(D) All inputs should be zero.

19. The states of Q, $\bar{Q}$ after a clock pulse are:



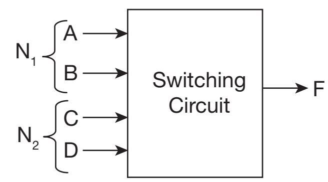
(A) 0, 1 (B) 1, 0
(C) 1, 1
(D) cannot be determined without initial states

20. The initial state of counter is $AB = 01$. What is the output (AB) after first clock pulse?



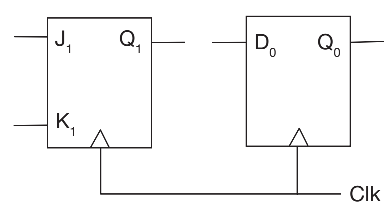
(A) 10 (B) 11
(C) 01 (D) 00

21. A switching circuit has four inputs as shown below. A and B represent the first and second bits of a binary number N_1, C and D represent the first and second bits of a binary number N_2.
The output is 1 only, if the product $N_1 \times N_2$ is less than or equal to 2.
The minimum POS form of $F(A, B, C, D)$ is



(A) $\bar{A}\bar{B} + \bar{C}\bar{D} + \bar{A}\bar{C} + \bar{A}\bar{D} + \bar{B}\bar{C}$
(B) $(A + C)(A + B + D)(B + C + D)$
(C) $(\bar{A} + \bar{D})(\bar{A} + \bar{B} + \bar{D})(\bar{B} + C + \bar{D})$
(D) $(\bar{A} + \bar{C})(\bar{A} + \bar{B} + \bar{D})(\bar{B} + \bar{C} + \bar{D})$

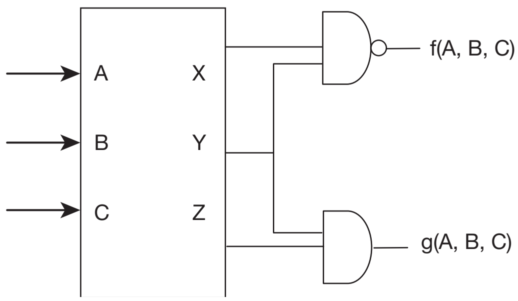
22. The synchronous counter which follows ($Q_1\ Q_0$) $00 \to 10 \to 11 \to 01 \to 00$ by using JK-flip flop and D-flip flop, has inputs as



(A) $J_1 = \bar{Q}_1, K_1 = Q_0, D_0 = Q_0$
(B) $J_1 = Q_1, K_1 = \bar{Q}_1, D_0 = Q_0$
(C) $J_1 = \bar{Q}_0, K_1 = Q_0, D_0 = Q_1$
(D) $J_1 = Q_0, K_1 = \bar{Q}_0, D_0 = Q_1$

23. The minimum SOP form of:
(A) $\bar{P} + QR$ (B) $\bar{P} + Q$
(C) $\bar{P}\bar{Q} + R$ (D) P

24. A combinational circuit has 3 inputs A, B, C, and 3 outputs X, Y, Z. And the functions $f(A, B, C)$ and $g(A, B, C)$ are generated from the combinational logic circuit as shown here with NAND, AND gates. Find the least possible minterm expression for $Y(A, B, C)$.
$f(A, B, C) = \Sigma m(1, 3, 4, 5, 7)$
$g(A, B, C) = \Sigma m(4, 6)$
$Y(A, B, C) = \Sigma m(?)$



(A) $\Sigma m(0, 2, 3, 6, 7)$ (B) $\Sigma m(1, 3, 4, 5, 7)$
(C) $\Sigma m(0, 2, 4, 6)$ (D) $\Sigma m(0, 2, 4)$

25. From the above data, for which input A, B, C. The outputs all X, Y, Z will become 111 (i.e., $XYZ = 1$)
(A) 010 (B) 101
(C) 100 (D) 110

26. What is the speed up of the pipeline, which is executing 10 tasks. Consider the mean overhead of the pipeline as 5 and an execution time per stage as 1 cycle.
(A) 4.5 (B) 4
(C) 3 (D) 5

27. Consider a 32-bit computer that has an on-chip 16 Kbyte four-way set-associative cache. Assume that the cache has a line size of four 32-bit words. Then what will be Tag, Set and Word fields respectively:
(A) 18, 8, 6 (B) 18, 10, 4
(C) 20, 8, 4 (D) 20, 10, 2

28. Consider a memory of 64 blocks (labelled 0 through 63) and a cache of 8 lines (labelled 0 through 7). Using direct mapping, which of the following blocks of memory contend for line 2 of the cache?
(i) Block 10 (ii) Block 15
(iii) Block 20 (iv) Block 55
(v) Block 42 (vi) Block 63
(A) (i), (iii), (v) only (B) (i), (v) only
(C) (i), (ii), (iv) only (D) All the six

29. Convert the decimal number –30.375 to IEEE 754 Floating-point format. Which of the following correctly specifies the hexa-decimal equivalent of converted number?
(A) C1F30000 (B) 82830000
(C) 02830000 (D) 41F30000

30. Consider a main memory system that consists of Eight memory modules attached to the system bus, which is one word wide. When a write request is made the bus in occupied for 100 ns by the data, address and control signals. During the same 100 ns and for 500 ns thereafter, the addressed memory module executes one cycle accepting and storing the data. The internal operation of different memory modules may overlap in time, but only one request can be on the bus at any time. What is the maximum number of stores that can be initiated in one second?
(A) 10^9 (B) 10^7
(C) 10^5 (D) 10^2

31. Suppose a bus protocol requires 15 ns for devices to make requests, 15 ns for arbitration and 20 ns to complete each operation. How many operations can be completed per second?
(A) 10^7 (B) 2×10^7
(C) 5×10^7 (D) 50

32. A computer truncates the significant to four decimal places and gives the results in normalized form. What is the resultant of $(0.2233 \times 10^2) + (0.6688 \times 10^1)$?
(A) 0.2901×10^2 (B) 2.9018×10^1
(C) 0.2902×10^2 (D) 29.018

33. For each of the following cases, specify whether SRAM or DRAM would be more appropriate building block for the memory system?
(i) A memory system where performance is the most important goal.
(ii) A memory system where cost is the most important goal.
(A) SRAM, SRAM
(B) SRAM, DRAM
(C) DRAM, SRAM
(D) DRAM, DRAM

Linked answer questions 34 and 35:
Consider an unpipelined processor. Assume that it has 1 ns clock cycle and it uses 3 cycles for ALU operations, 5 cycles for branch instructions and 4 cycles for memory operations. Assume that the relative frequencies of these operations are 80%, 10% and 10% respectively. Suppose that due to clock skew and set up, pipelining the processor adds 0.1 ns of overhead to the clock.

34. What is the average instruction execution time on a pipelined processor?
(A) 4.3 ns (B) 4.4 ns
(C) 1.1 ns (D) 1 ns

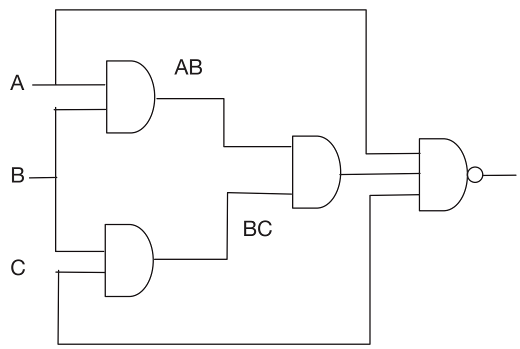
35. By ignoring latency impact, what is the speedup gain in the instruction execution rate using pipeline?
(A) 4.3 (B) 3.9
(C) 1.1 (D) 1

Answer Keys

1. C	**2.** C	**3.** D	**4.** B	**5.** A	**6.** B	**7.** A	**8.** C	**9.** D	**10.** B
11. D	**12.** B	**13.** B	**14.** C	**15.** B	**16.** B	**17.** B	**18.** B	**19.** B	**20.** B
21. D	**22.** C	**23.** A	**24.** C	**25.** D	**26.** B	**27.** C	**28.** B	**29.** A	**30.** B
31. B	**32.** A	**33.** B	**34.** C	**35.** B					

Hints and Explanations

1. In the circuit NAND – NOR structure can be redrawn as AND – AND Structure.



$\overline{A \cdot ABC \cdot C} = \overline{ABC} = \bar{A} + \bar{B} + \bar{C}$ Choice (C)

2. Positive numbers will be represented in their original binary magnitude but sign bit will be zero (0) to make it as positive number.
Negative Numbers are represented as 2's complement of their positive number representation in 2's complement signed number.
+ 43 = 00101011
– 43 = 11010101 (By taking 2's complement)
+ 78 = 01001110
– 78 = 10110010 (By taking 2's complement)
+ 54 = 00110110
+ 96 = 01100000 Choice (C)

3. Dual of function can be obtained by making min terms to max terms, i.e., 0 to 1, 1 to 0.
Dual of
000 $\leftrightarrow$ 111
001 $\leftrightarrow$ 110
010 $\leftrightarrow$ 101
011 $\leftrightarrow$ 100
$f = \Sigma m\ (0, 1, 6, 7)$
Dual is $f^D = \pi\ m\ (7, 6, 1, 0)$
$= \pi\ m\ (0, 1, 6, 7)$
$= \Sigma\ m\ (2, 3, 4, 5)$
Which is equal to $\bar{f}$ (complement of f). Choice (D)

4. $F = b + cd$
The term 'b' = - b - - - will have 16 min terms, the term cd = - - cd - will have 8 min terms out of those 4 min terms, have 'b'.
So remaining are 8 – 4 = 4
So total min terms = 16 + 4 = 20 Choice (B)

5. P is $1 \oplus A = \bar{A} \cdot 1 + A \cdot \bar{1} = \bar{A}$
Q is $A \odot 0 = \bar{A} \cdot 1 + A \cdot 0 = \bar{A}$
R is $0 \oplus A = \bar{A} \cdot 0 + A \cdot \bar{0} = A$
S is $1 \odot A = 1 \cdot A + 0 \cdot \bar{A} = A$
So P, Q work like inverter. Choice (A)

6. $(-13)_{10} + (-28)_{10} = (-41)_{10}$
The operation in 2's complement signed representation also given the same answer.
$(-41)_{10}$ is signed 2's complement representation is 11010111 ($\because$ +41 = 00101001) Choice (B)

7. Base 9 and base 3 are related ($3^2 = 9^1$). 2 digits of base 3 is equal to base 9.

Base-3	Base-9
01	1
02	2
10	3
11	4
12	5
20	6
21	7
22	8
100	10

Given number in base 3 is
$\underline{01}\ \underline{10}\ \underline{12}\ \underline{22} \cdot \underline{20}\ \underline{11}\ \underline{21}$
1 3 5 8 · 6 4 7
$= (1358 \cdot 647)_9$ Choice (A)

8. In Von-Neumann computer architecture, most of the time is used for transferring data or instructions from memory to CPU. This is the bottleneck of Von-Neumann computer. Choice (C)

9. Choice (D)

10. In full-Associative cache, we can place any block of main memory at any line of cache. So there will be no conflict miss. Choice (B)

11. In pipelining, several tasks are processed simultaneously. Choice (D)

12. If there are k stages and each stage take T cycles then $(k - 1)\ T$ = Pipeline mean overhead
$\Rightarrow$ $(4 - 1) \times T = 12$
$\Rightarrow$ $T = 4$ cycles. Choice (B)

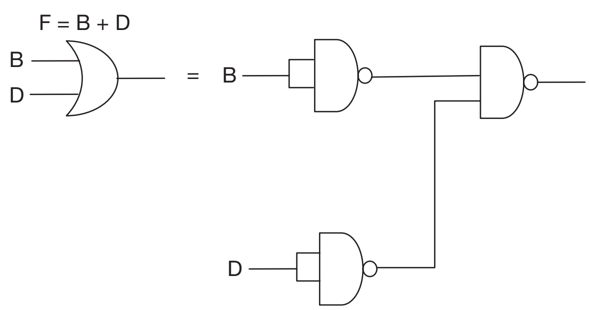
13. A 64-bit machine has 2^{64} locations. To address these locations, we require 64-bits. Choice (B)

14. Total time required to execute the given program
$= (0.95) * 1 + (0.05) * 25 = 0.95 + 1.25 = 2.2$
Time taken for Divide Operations $= 0.05 * 25 = 1.25$
$\therefore$ Percentage of CPU time spent on divides
$= \frac{1.25}{2.2} * 100 = 56.82\%.$ Choice (C)

15. Using memory mapped technique, the memory address space will be reduced. Choice (B)

16. $f(A, B, C, D) = \Sigma m\,(1, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15)$

AB \ CD	00	01	11	10
00		1	1	
01	1	1	1	1
11	1	1	1	1
10		1	1	



$F = B + D$. So 3, 2–input NAND gates are required to implement $F(A, B, C, D)$

Choice (B)

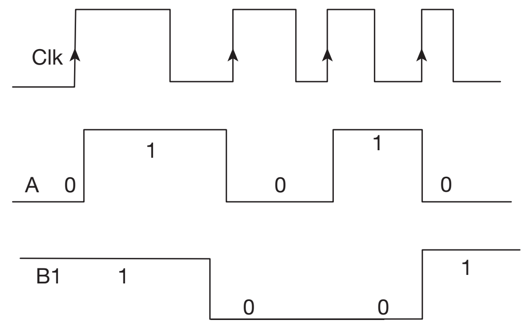
17. The first multiplexer output is
$y = I_0\bar{s} + I_1 s$
$= x^1 \cdot x^1 + zx = x^1 + zx = x^1 + z$
$F = (x^1 + z)y + z \cdot y^1 = x^1 y + yz + y^1 z$
$= x^1\, y + (y + y^1)z = x^1\, y + z$ Choice (B)

18. $Q_n = 0, Q_{n+1} = 1$
i.e., the state gets toggled i.e., when $J = 1, K = 1$
or the next state is set, $Q_{n+1} = 1$ i.e., when $J = 1, K = 0$
so $J = 1, k = x$ (either zero or 1)
so sum should be one, carry either zero or one.
So a, b, c should have only one 1 (sum = 1, carr$y$ = 0)
All inputs a, b, c can be 1 so that sum = 1, carr$y$ = 1.
Choice (B)

19. $J = Q_n \oplus \bar{Q}_n$(EXOR of $Q, \bar{Q}$) $= 1$
$K = Q_n \,\Theta\, \bar{Q}_n$(EXNOR of $Q, \bar{Q}$) $= 0$
When $J = 1, K = 0$, next clock pulse will give
$Q = 1,\ \bar{Q} = 0$ Choice (B)

20. Given circuit is an Asynchronous counter, as clock is connected to rising edge.

$\bar{Q}$ connected to rising edge means this is an UP counter A is LSB, B is MSB.
$\bar{Q}_A$ connected to rising edge.
i.e., for every falling edge of Q_A, B will change so $AB = 11$



Clk	BA
0	10
1	11
2	00
3	01
4	10

$\bar{Q}_A$ connected to rising edge.
i.e., for every falling edge of Q_A, B will change so $AB = 11$ Choice (B)

21.

N_1	AB \ CD (N_2)	00 (0)	01 (1)	11 (3)	10 (2)
0	00	1	1	1	1
1	01	1	1		1
3	11	1			
2	10	1	1		

The max terms will be (for POS form)

AB \ CD	00	01	11	10
00				
01			0	
11		0	0	0
10			0	0

So $F = (\bar{A} + \bar{C})(\bar{B} + \bar{C} + \bar{D})(\bar{A} + \bar{B} + \bar{D})$ Choice (D)

22.

Present state		Next state		Inputs		
Q_1	Q_0	Q_1	Q_0	J_1	K_1	D_0
0	0	1	0	1	X	0
1	0	1	1	X	0	1
1	1	0	1	X	1	1
0	1	0	0	0	X	0

From the above table,
Writing the inputs interms of present state
$D_0 = Q_1, J_1 = \bar{Q}_0,\ K_1 = Q_0$ Choice (C)

23. Given function is in standard POS Form
$F(P, Q, R) = \pi M\ (5, 6, 4) = \Sigma m\ (0, 1, 2, 3, 7)$

P \ QR	00	01	11	10
0	1	1	1	1
1			1	

$F\ (P, Q, R) = \bar{P} + QR$ Choice (A)

24. When we consider two min terms product,
$m_i, m_j = 0$ if $i \neq j$
e.x., $A\bar{B}C \cdot A\bar{B}\bar{C} = 0$ $m_i, m_j \neq 0$ if $i = j$
e.x., $A\bar{B}\bar{C}\ A\bar{B}\bar{C} = A\bar{B}\bar{C}$
If two functions ANDed with min terms, then resultant is common min terms of the two functions.
Example:
If $f_1\ (a, b, c) = m_1 + m_2 + m_3$
$f_2\ (a, b, c) = m_3 + m_4$
$f_1 \cdot f_2 = (m_1 + m_2 + m_3)\ (m_3 + m_4)$
$= m_1 \cdot m_3 + m_1 \cdot m_4 + m_2 \cdot m_3 + m_2 m_4 + m_3 \cdot m_3 + m_3 \cdot m_4$
$= m_3$ (all other terms will be zero)
In the given problem $f(A, B, C) = \overline{X \cdot Y}$
$= \Sigma m(1, 3, 4, 5, 7)$
So $X, Y = \Sigma m\ (0, 2, 6)$ (the remaining min terms)
$g(A, B, C) = Y \cdot Z = \Sigma m\ (4, 6)$
X and Y are having (0, 2, 6) min terms in common, i.e., Y will have all these 3 min terms.
Similarly Y and Z have (4, 6) min terms in common,
So Y will have these 2 min terms also.
So $Y = \Sigma m\ (0, 2, 4, 6)$ Choice (C)

25. From the above solution
$f(A, B, C) = \overline{XY} = \Sigma m\ (1, 3, 4, 5, 7)$
$X \cdot Y = \Sigma m\ (0, 2, 6)$
$g\ (A, B, C) = Y\,Z = \Sigma m(4, 6)$
The min terms common to $X\,Y$ and $Y\,Z$ are
$XY \cdot YZ = XYZ = \Sigma m\ (6)$
i.e., The output $X = 1, Y = 1, Z = 1$ only for the min term 6, i.e., $ABC = 110$. Choice (D)

26. Pipeline mean overhead = 5
Let there are k stages.
Given each stage take 1 cycle.
$(k - 1)T$ = Pipeline overhead
$\Rightarrow\ (k - 1) \times 1 = 5$
$\Rightarrow\ k = 6$

$$\text{Speed up} = \frac{\text{nonpipeline execution time}}{\text{pipeline execution time}}$$

$$= \frac{10 \times 6}{(10 + 6 - 1)} = \frac{60}{15} = 4$$ Choice (B)

27.

Tag	Set	Word
20	8	4

32-bits

32-bits
Cache capacity = 16 KB = 2^{14} bytes
Line size = 4 × 32-bit words
= 4 × 4 Bytes = 16 Bytes = 2^4B

$$\text{Number of lines in cache} = \frac{2^{14}}{2^4} = 2^{10}$$

4 × Number of sets = 2^{10}
$\Rightarrow$ Number of sets = 2^8
$\therefore$ Set size = 8
Tag = 32 – (8 + 4) = 20. Choice (C)

28. Main memory has 64-blocks (0 – 63).
Cache has 8 lines (0 – 7).
To place a particular block of main memory in a line of cache, use the following formula:
Cache line number = Block number % number of cache lines.
In cache line number 2, at a time any one of the blocks 2, 10, 18, 26, 34, ..., 58 can be placed. Choice (B)

29. $-30.375 = -11110.011 = -1.1110011 \times 2^4$
Sign = 1
Mantissa = 11100110000000000000000
Exponent = 4
Biased exponent = 127 + 4 = 131 = 10000011
$\therefore$ –30.375 in IEEE standard is

$$\begin{array}{cccccccc} \underline{1100} & \underline{0001} & \underline{1\,111} & \underline{0011} & \underline{0000} & \underline{0\,000} & \underline{0\,000} & \underline{0\,000} \\ C & 1 & F & 3 & 0 & 0 & 0 & 0 \end{array}$$

$= (C1F30000)_{16}$. Choice (A)

30. First write operation requires 600 ns. (100 ns for bus, 500 ns for store)
During the store operation of first write another request may be given to bus.
So we can start a write operation after every 100 ns.
$\therefore$ Maximum number of stores that can be initiated in

$$\text{one second} = \frac{1}{100\text{ ns}} = \frac{10^9}{10^2} = 10^7$$ Choice (B)

31. Total time taken for one operation = 15 + 15 + 20 = 50 ns

One operation requires 50×10^{-9} sec
In one second, number of operations completed

$= \dfrac{1}{50 \times 10^{-9}} = 2 \times 10^{7}$. Choice (B)

32. 0.2233×10^{2}
$+0.6688 \times 10^{1}$
?

First equate the exponents
$0.6688 \times 10^{1} = 0.06688 \times 10^{2}$
0.22330×10^{2}
0.06688×10^{2}
0.29018×10^{2}

The resultant must have four decimal places (by truncation)
$\therefore$ Sum is 0.2901×10^{2}. Choice (A)

33. For high performance use SRAM and for low cost use DRAM. Choice (B)

34. Average instruction time on a pipelined processor
= 1 ns + 0.1 ns = 1.1 ns. Choice (C)

35. Average instruction time on a non-pipelined processor
$= 1 + ((3 \times 0.8) + (5 \times 0.1) + (4 \times 0.1)) = 4.3$ ns

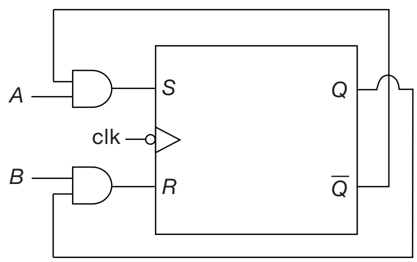
Speed up $= \dfrac{4.3}{1.1} = 3.9$ Choice (B)

Digital Logic and Computer Organization and Architecture Test 6

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices

1. The following flip flop is

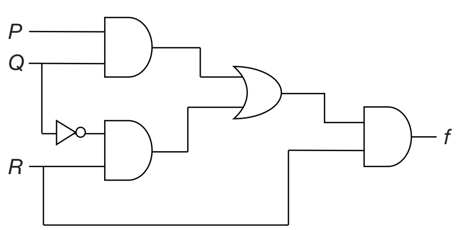


(A) D flip flop when $A = 0$
(B) D flip flop when $A = B$
(C) T flip flop when $A = 0$
(D) T flip flop when $A = B$

2. If $a \le b$, which of the following is true
(P) $a^1 + b = 1$ (Q) $a^1 \le b^1$
(R) $ab^1 = 0$
(A) P, Q (B) P, R
(C) Q, R (D) P, Q, R

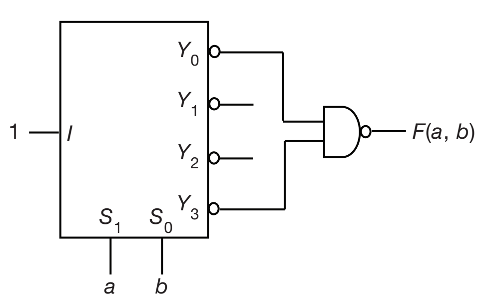
3. If $a = (b + c)\,(b^1 + c^1)$, then the value of b is:
(A) $(a^1 + c)\,(a + c^1)$ (B) $a^1c + ac^1$
(C) $a^1c^1 + a^1c$ (D) $(a^1 + c^1)(a + c^1)$

4. Calculate switching function realized by this network in minimized SOP form



(A) $PQ + \bar{Q}R$ (B) $PQR + \bar{Q}R$
(C) $PR + \bar{Q}R$ (D) $P\bar{Q} + R$

5.



In the above circuit the function $F(a, b)$ in POS form
(A) $(a + b)(a^1 + b^1)$ (B) $(a^1 + b)(a + b^1)$
(C) $(a + b^1)(a^1 + b^1)$ (D) $(a + b)(a^1 + b)$

6. A combinational circuit takes input in the range of 000 to 101, (remaining combinations are unused) The output of the circuit is 1 when ever input is a power of 2 (in decimal). Find the minimized expression for output, if x, y, z are inputs of the combinational circuit.
(A) $x^1\, y \oplus z^1$ (B) $(x + y) \odot z$
(C) $x^1\, y^1 \oplus z$ (D) $(x + y) \oplus z$

7. A boolean function defined as $f(a, b, c, d, e) = b + d\,(c + ae)$
How many number of min terms will be present for the function $f(a, b, c, d, e)$?
(A) 20 (B) 24
(C) 28 (D) 16

8. The characteristic equation of the flip-flop with functional table given below is, $(Q_{n+1}) =$

X Y	Q_{n+1}
0 0	$\overline{Q_n}$
0 1	1
1 0	0
1 1	Q_n

(A) $\bar{X}Q_n + Y\bar{Q}_n$ (B) $\bar{X}Q_n + YQ_n$
(C) $\bar{X}Q_n + \bar{Y}Q_n$ (D) $\bar{X}\bar{Q}_n + YQ_n$

9. Consider reading a block from a disk using DMA technique. Then which of the following factors will limit the rate of transfer?
(i) Limiting speed of I/O devices
(ii) Speed of bus
(iii) Too small internal buffering space
(A) (i), (ii) only (B) (ii), (iii) only
(C) (i), (iii) only (D) (i), (ii), (iii)

10. If a normal processor memory Read (RD) and write (WR) control outputs are connected to I/O interface adapters, then which type of I/O technique is used?
(A) Memory mapped
(B) Isolated I/O
(C) Both Memory mapped and Isolated I/O
(D) Interrupt-driven I/O

11. What is the signed decimal equivalent of the 2's complement number: 00001111
(A) –15 (B) –240
(C) +15 (D) +241

12. Right arithmetic shift of ones complement of –6 is (in 8-bits)
(A) 00000011 (B) 01000011
(C) 01111100 (D) 11111100

13. LRU is an effective cache replacement policy primarily because

(A) Programs exhibit locality of reference.
(B) Programs usually have small working sets.
(C) Programs read data much more frequently than write data.
(D) Program can generate addresses that collide in the cache.

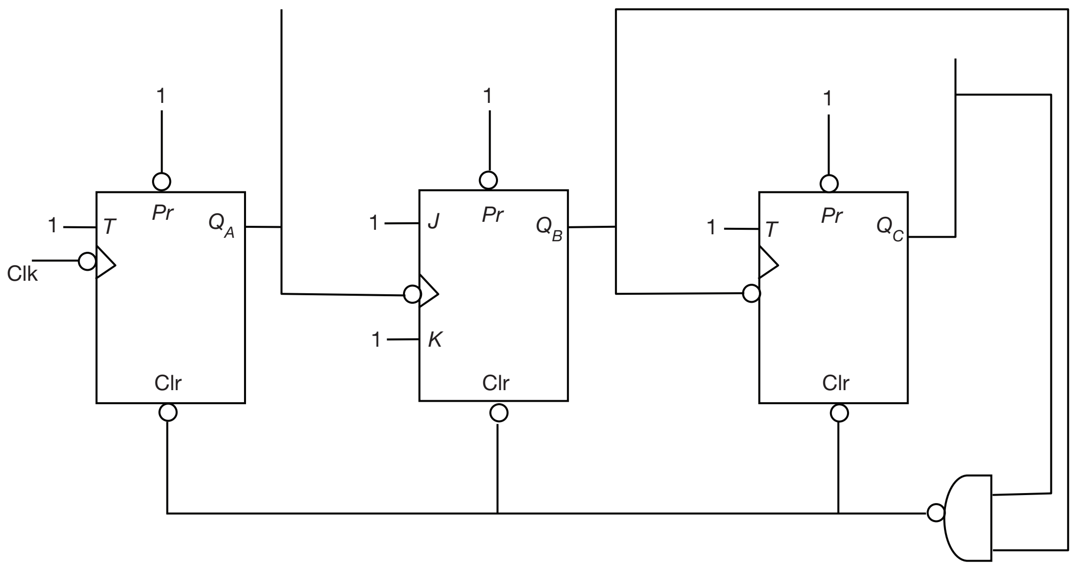
14. "The purpose of cache memory is to increase the size of main memory". This statement is

(A) Always true (B) Always False
(C) Sometimes true (D) Sometimes false

15. A memory has 2^{36} addressable locations. What is the smallest width in bits that the address can be, while still being able to address all 2^{36} locations?

(A) 36-bits (B) 72-bits
(C) 18-bits (D) 9-bits

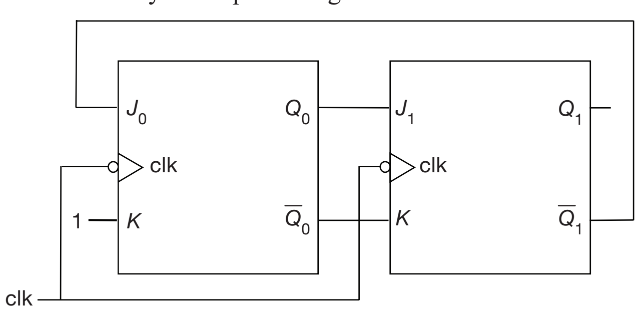
16.



A counter formed with T, JK flip flops, as shown above, preset (Pr), clear (clr) are active low, asynchronous inputs, T, J, K are synchronous inputs. The modulus of the counter is:

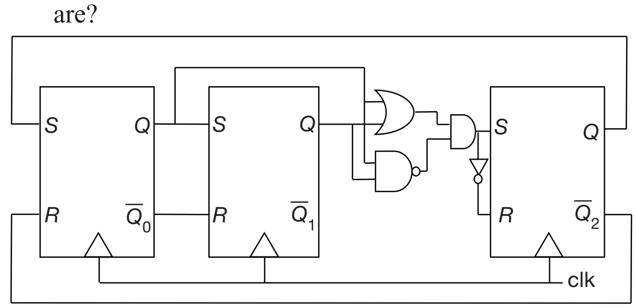
(A) 4
(B) 5
(C) 6
(D) 7

17. The flip flops used in this circuit are master-slave JK flip flops. If the counter is initially at reset state after how many clock pulses it gets reset?



(A) 2 (B) 3
(C) 4 (D) 1

18. The counter shown above has initial state $Q_2\ Q_1\ Q_0$ = 111(7), then the next states of ($Q_2\ Q_1\ Q_0$) (in decimal) are?



(A) 7, 6, 2, 1, 4, 3, 5 (B) 7, 3, 5, 2, 6, 1, 4
(C) 7, 3, 2, 4, 1, 6, 5 (D) 7, 5, 1, 4, 2, 3, 6

19. The max term expression of a four variable even function is?

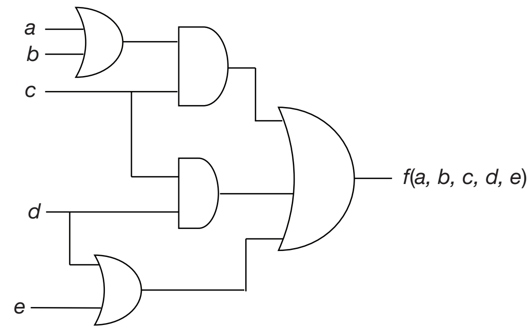
(A) πM (0, 2, 4, 6, 8, 10, 12, 14)
(B) πM(1, 3, 5, 7, 9, 11, 13, 15)
(C) πM (0, 3, 5, 6, 9, 10, 12, 15)
(D) πM (1, 2, 4, 7, 8, 11, 13, 14)

20. If $(3.5)_{\text{base } 6} + (2.3)_{\text{base } 6} = (X)_{\text{base } 6}$ then the value of X is:

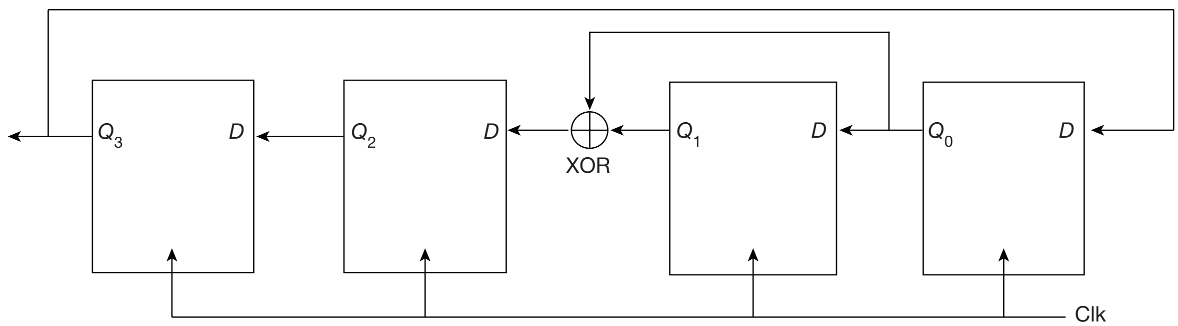
(A) 5.8 (B) 10.2
(C) 6.2 (D) 5.6

21. If the following circuit is converted to all-NAND network, then How many number of NAND gates are required? (inverted inputs are available)

(A) 3 (B) 4
(C) 5 (D) 6

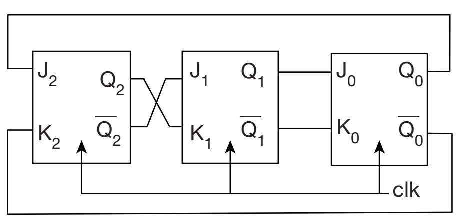


22. A 4-bit shift register is connected as shown with initial states $Q_3\ Q_2\ Q_1\ Q_0$ = 0100. What is state $Q_3\ Q_2\ Q_1\ Q_0$ after 5 clock pulses?



(A) 0111 (B) 1100
(C) 1111 (D) 1001

23.



For the counter shown above the initial state is $Q_2Q_1Q_0$ is 101, what is state after 5 clock pulses?

(A) 111 (B) 011
(C) 100 (D) 000

24. A cache access requires one clock cycle and handling cache miss stalls the processor for an additional 4 clock cycles. Which of the following cache hit rates comes closest to achieve an average memory access of 2 cycles?

(A) 75% (B) 80%
(C) 85% (D) 90%

25. Consider a byte-addressable system with 16-bit addresses associated a cache, which has a two-way set-associative write-back cache with perfect LRU replacement. The Tag store requires a total of 4608 bits of storage. What is the block size of the cache?

(A) 4 B (B) 5 B
(C) 32 B (D) Data insufficient

26. Consider a system employing interrupt-driven I/O for a particular device that transfers data at an average of 10 KB/sec on a continuous basis. Assume that interrupt processing takes about 200 μs. What fraction of processor time is consumed by this I/O device if it interrupts for every byte?

(A) 0.2 (B) 2
(C) 20 (D) 200

27. Consider a program which takes 200 seconds to execute. Of his time, 30% is used for multiplication, 40% for memory access instructions and 30% for other tasks. To enhance the performance, make multiply instructions run four times faster than before. What will be the speed up be if you improve the multiply instruction time?

(A) 0.29 (B) 0.71
(C) 1.29 (D) 1.71

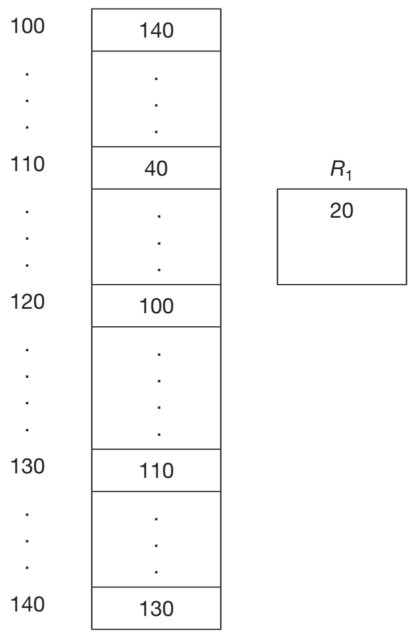
28. Consider the following code sequence (First operand is destination):

LOOP: LOAD R_0, 0(R_{10})
MUL R_0, R_0, R_2
LOAD R_4, 0(R_{11})
ADD R_0, R_0, R_4
STORE R_0, 0(R_{11})
SUB R_{10}, R_{10}, #8
SUB R_{11}, R_{11}, #8
BNEZ R_{10}, LOOP

How many 'WAW' hazards are there in given code?

(A) 8 (B) 5
(C) 3 (D) 0

29. Suppose we have an instruction 'LOAD 100'.
The memory and register R_1 contain the values as given below:

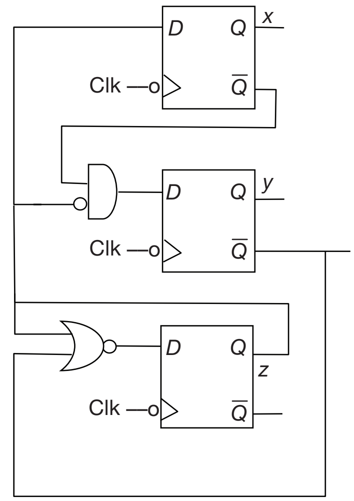


Assuming R_1 is implied in the indexed addressing mode, what will be the actual value loaded into accumulator in immediate, direct, indirect and indexed mode respectively:
(A) 140, 140, 130, 100 (B) 100, 140, 130, 100
(C) 140, 100, 130, 20 (D) –, 100, 140, 120

30. How many bits would you need, to address a $2G \times 32$ memory, if the memory is byte-addressable and word addressable respectively?
(A) 36, 31 (B) 31, 31
(C) 31, 36 (D) 36, 36

31. Suppose a computer using direct mapped cache has 2^{20} words of main memory and a cache of 32 blocks, where each cache block contains 16 words. To which cache block will the memory reference $(0DB63)_{16}$ map?
(A) 3 (B) 22
(C) 32 (D) 10

Common Data for Questions 32 and 33:
Consider the following circuit involving three D-type flip flops used in a certain type of counter configuration.



32. If at some instance prior to the occurrence of the clock edge x, y and z have a value 0, 1, and 1, what shall be the value of xyz after the clock edge?
(A) 000 (B) 100
(C) 010 (D) 101

33. If all the flip flops were reset to 0 at power on, what is the total number of distinct outputs (states) represented by xyz generated by the counter?
(A) 4 (B) 5
(C) 6 (D) 7

Common Data for Questions 34 and 35:
A processor requires 2000 cycles to perform a context switch and start an interrupt handler or 1000 cycles to poll an I/O device. An I/O device attached to that processor makes 100 requests per second, each of which takes 10,000 cycles to resolve once the handler has been started. By default, the processor polls every 0.5 ms if it is not using interrupts.

34. How many cycles per second does the CPU spend handling I/O from the device if interrupts are used?
(A) 1400000 (B) 1200000
(C) 14000 (D) 12000

35. How many cycles per second are spent on I/O if polling is used?
(A) 1000000 (B) 2000000
(C) 3000000 (D) 5000000

Answer Keys

1. D	**2.** B	**3.** B	**4.** C	**5.** B	**6.** D	**7.** A	**8.** D	**9.** D	**10.** A
11. C	**12.** D	**13.** A	**14.** B	**15.** A	**16.** B	**17.** B	**18.** C	**19.** D	**20.** B
21. A	**22.** D	**23.** A	**24.** A	**25.** C	**26.** B	**27.** C	**28.** C	**29.** B	**30.** A
31. B	**32.** B	**33.** A	**34.** A	**35.** C					

Hints and Explanations

1. The characteristic equation of *SR* flip flop is:

$Q_{n+1} = S + \bar{R}$
Q_n

Given (from circuit) $S = A \cdot \bar{Q}_n, R = B \cdot Q_n$

$Q_{n+1} = A\bar{Q}_n + \bar{B} \cdot \bar{Q}_n$
Q_n

$= A\bar{Q}_n + (\bar{B} + \bar{Q}_n)Q_n$

$Q_{n+1} = A\bar{Q}_n + \bar{B}$
Q_n

The equation is like JK flip flop equation.

(JK equation $Q_{n+1} = J\bar{Q}_n + \bar{K}Q_n$)

So it works like *T* flip flop if $A = B$. Choice (D)

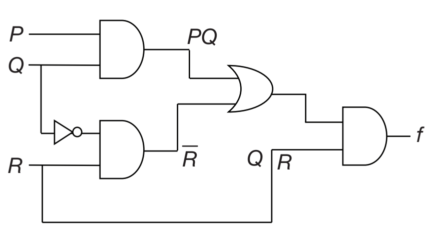
2. If $a \leq b$ then

a	b	$a^1 + b$	ab^1
0	0	1	0
0	1	1	0
1	1	1	0

($ab = 10$ is not valid condition for $a \leq b$)
We can observe that $a^1 \geq b^1$, so only (*P*) $a^1 + b = 1$, (*R*) $ab^1 = 0$ are true. Choice (B)

3. From given $a = (b + c)(b^1 + c^1)$
$= bc^1 + b^1c$
$= b \oplus c$
EXOR with *c* both sides
$a \oplus c = b \oplus c \oplus c = b \oplus 0 = b$
So, $b = a \oplus c = a^1c + ac^1$ or $(a + c)(a^1 + c^1)$.
Choice (B)

4.



$f = (PQ + \bar{Q}R)R = PQR + \bar{Q}R$

$= (PQ + \bar{Q})R = (P + \bar{Q})R$

$= PR + \bar{Q}R$ Choice (C)

5. Given circuit is Demultiplexer

Output equation are $Y_0 = I\,\bar{S}_1\,\bar{S}_0$

$Y_1 = I\,\bar{S}_1\,S_0$

$Y_2 = I\,S_1\,\bar{S}_0$

$Y_3 = I\,S_1\,S_0$

$F(a, b) = \overline{\overline{Y_0} \cdot \overline{Y}_3} = Y_0 + Y_3 = 1 \cdot a^1b^1 + 1 \cdot ab$

$= a^1\,b^1 + ab = (a^1 + ab)(b^1 + ab)$

By applying $x + yz = (x + y)(x + z) = (a^1 + b)(a + b^1)$
Choice (B)

6. If the output of the circuit is *F* then $F(x, y, z) = \Sigma(1, 2, 4) + d(6, 7)$ in the given range 000 to 101, only for inputs 001, 010, 100 output is 1 (min terms) 110, 111 are unused combinations (don't cares)

x \ yz	00	01	11	10
0		1		1
1	1		X	x

$F(x, y, z) = xz^1 + yz^1 + x^1y^1z$
$= (x + y)z^1 + x^1y^1z$
$= (x^1y^1)^1z^1 + (x^1y^1)z = (x + y)z^1 + (x + y)^1z$
$= x^1\,y^1 \odot z = (x + y) \oplus z$ Choice (D)

7. $f = b + cd + ade$
a b c d e
b = - 1 - - - → 16 min terms (2^4)
cd = - - 1 1 - → 8 min terms (2^3)
in these 8 terms, 4 terms will have $b = 1$,
so $8 - 4 = 4$ terms
ade = 1 - - 1 1 → 4 min terms (2^2)
these 4 terms are already present in first 16 terms so no need to count them. Total = 16 + 4 = 20 Choice (A)

8. Given functional table is same as JK flipflop with $J = \bar{X}, K = \bar{Y}$, so $Q_{n+1} = \bar{X}\bar{Q}_n + Y$
Q_n Choice (D)

9. Choice (D)

10. Choice (A)

11. Given number: 00001111. The sign is '0', so the number is positive.
∴ Decimal equivalent = +15. Choice (C)

12.

–6	=	1	0	0	0	0	1	1	0

(SM)

In one's complement,
– 6 = 11111001
Arithmetic Right shift of –6 = 11111100
(sign bit must be same). Choice (D)

13. LRU replaces locations that have the lowest probability of being accessed in the cache. Choice (A)

14. Cache memory doesn't extend the size of main memory but useful in faster program execution. Cache memory contains only some replica of main memory. Choice (B)

15. Choice (A)

16. Given circuit is an Asynchronous counter (ripple counter). All flip flops are in toggle mode.
Q_A is LSB (clk input is connected to Q_A)
Negative edge Clk is connected from *Q*, so it works as UP counter.

$Q_C\, Q_B\, Q_A$ is the sequence, UP counter.
The NAND gate connected from $Q_B\, Q_C$
i.e., whenever output $Q_C\, Q_B\, Q_A$ becomes 110 or 111, the NAND gate output is zero.
i.e., the same zero applied to the active low asynchronous inputs pr, clr of Q_A, QB, Q_C.
so initial state is 001

Clk	Q_C	Q_B	Q_A
0	0	0	1
1	0	1	0
2	0	1	1
3	1	0	0
4	1	0	1
5	0	0	1

So total 5 different states.
∴ Given circuit is mod-5 counter. Choice (B)

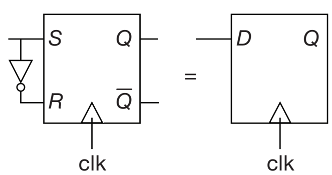
17. Master slave flip flop also works like normal flip flop only, to eliminate race around condition in JK latch, we go for JK master slave flip flop.

From the circuit $J_0 = \bar{Q}_1, K_0 = 1, J_1 = Q_0, K_1 = \bar{Q}_0$

clk	Q_1 Q_0	$J_1 = Q_0$	$K_1 = \bar{Q}_0$	$J_0 = \bar{Q}_1$	$K_0 = 1$
0	0 0	0	1	1	1
1	0 1	1	0	1	1
2	1 0	0	1	0	1
3	0 0				

∴ 3 clk pulses required to reset again. Choice (B)

18.



We can observe that SR connected in D flip flop mode, S, R are connected with normal (Q), complemented ($\bar{Q}$) output forms.

We can draw the circuit as follows
From given circuit $D_0 = Q_2$, $D_1 = Q_0$

$$D_2 = (Q_1 + Q_0)(\overline{Q_1 \cdot Q_0}) = (Q_1 + Q_0)(\bar{Q}_1 + \bar{Q}_0) = Q_1 \oplus Q_0$$

clk	Q_2 Q_1 Q_0	$D_2 = Q_1 \oplus Q_0$	$D_1 = Q_0$	$D_0 = Q_2$
0	1 1 1	0	1	1
1	0 1 1	0	1	0
2	0 1 0	1	0	0
3	1 0 0	0	0	1
4	0 0 1	1	1	0
5	1 1 0	1	0	1
6	1 0 1	1	1	1
7	1 1 1			

∴ The state sequences of given circuit are 7, 3, 2, 4, 1, 6, 5, 7 . . . Choice (C)

19. Even function is a Boolean function, which will be equal to 1, if the input variables have an even number of 0's.

(odd function is a Boolean function for which output is 1, for input combinations with odd number of 1'*s*)

So for 4 variables, even number of zeroes occur for input combinations 0000, 0011, 0101, 0110, 1001, 1010, 1100, 1111

For these combinations, even function output is 1.

So remaining terms are max terms.

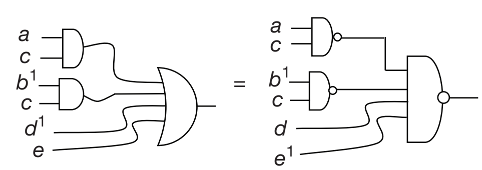
So $f_{\text{even}} = \prod M(1, 2, 4, 7, 8, 11, 13, 14)$

Choice (D)

20. 3.5
$+\underline{2.3}$
?
$5 + 3 = 8$
$(8)_{10} = (12)_6$, so result is 2 and carry is 1.
1
3.5
$\underline{+2.3}$
?.2
$1 + 3 + 2 = 6$
$(6)_{10} = (10)_6$, so result is 0 and carry is 1
3.5
$+\underline{2.3}$
10.2 Choice (B)

21. Given function has to be in SOP form, so it can be implemented by AND-OR gates, same as NAND-NAND gates

$f(a, b, c, d, e) = (a + b^1)c + cd^1 + (d^1 + e)$
$= ac + b^1c + cd^1 + d^1 + e$



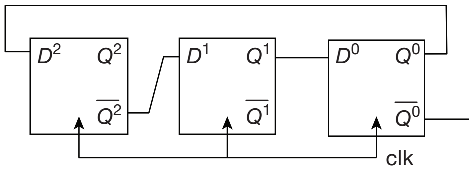
$= ac + b^1\, c + d^1 + e$ Choice (A)

22.

	Present state				Q_2	$Q_1 \oplus Q_0$	Q_0	Q_3
Clk	Q_3	Q_2	Q_1	Q_0	D_3	D_2	D_1	D_0
0	0	1	0	0	1	0	0	0
1	1	0	0	0	0	0	0	1
2	0	0	0	1	0	1	1	0
3	0	1	1	0	1	1	0	0
4	1	1	0	0	1	0	0	1
5	1	0	0	1	0	1	1	1
6	0	1	1	1				

Choice (D)

23. Given circuit can be drawn as



As JK flip flop is used in Data mode $(J = \bar{K})$

Clk	Q_2 Q_1 Q_0
0	1 0 1
1	1 0 0
2	0 0 0
3	0 1 0
4	0 1 1
5	1 1 1

Choice (A)

24. Average access time = 2
Cache access time = 1
Time to handle miss = 5
Let cache hit rate = x
$2 = 1 \times x + 5(1 - x)$
$\Rightarrow \quad x + 5 - 5x = 2$
$\Rightarrow \quad -4x = -3$
$\Rightarrow \quad x = \frac{3}{4} = 0.75$
$\therefore$ Hit ratio = 75%. Choice (A)

25. In 2-way set-associative cache,
t(Tag) + s(Set) + w(Word) = 16
For LRU, use 1-bit per set.
For Valid bit, Dirty bit use 1 bit per block for each.
Let tag has t-bits per block.
Number of sets = 2^s
Number of blocks = 2×2^s
Tag store size = $2^s + 2 \times 2^s \times (2 + t)$
Given tag store size = 4608 bits
$2^s + 4 \times 2^s + 2^{s+1} \times t = 4608$
$\Rightarrow \quad 5 \times 2^s + t \times 2^{s+1} = 2^{12} + 2^9$
$\Rightarrow \quad 2^s(2t + 5) = 2^9 \times 9$
$\Rightarrow \quad s = 9, t = 2 \Rightarrow w = 5$
$\therefore$ Block size = 2^5 = 32 bytes. Choice (C)

26. The device generates 10000 interrupts per second i.e., one for every 100 μs.
Each interrupt requires 200 μs, then fraction of processor time consumed $= \frac{200}{100} = 2.$ Choice (B)

27. Speed up $= \dfrac{1}{(1 - \text{fraction}) + \dfrac{\text{Fraction}_{\text{enh}}}{\text{Speedup}_{\text{enh}}}}$

$= \dfrac{1}{(1 - 0.3) + \dfrac{0.3}{4}}$ Choice (C)

28. There are three WAW hazards.
(1) LOAD R_0, $0(R_{10})$
MUL R_0, R_0, R_2
(2) MUL R_0, R_0, R_2
ADD R_0, R_0, R_4
(3) LOAD R_0, $0(R_{10})$
ADD R_0, R_0, R_4. Choice (C)

29. Immediate: Data is present in instruction i.e., $AC = 100$
Direct: Instruction contains effective address of operand i.e., $AC = M[100] = 140$
Indirect: Instruction contains address of address of operand.
i.e., $AC = M[M[100]] = M[140] = 130$
Indexed: Address part of instruction is added with IR.
i.e., $AC = M[IR + 100] = M[20 + 100] = M[120] = 100.$
Choice (B)

30. Memory capacity : $2G \times 32 = 2^{31} \times 2^5$
Byte Addressable requires 36-bits for an address.
Word addressable requires 31-bits. Choice (A)

31. Main memory = 2^{20}
Number of blocks in cache = 32
Block size = 16 words

Tag	Line	Word
11	5	4

Given address $(0DB63)_{16}$

	0000 1101101	10110	0011
=	Tag	line	word

Line = 10110 = $(22)_{10}$. Choice (B)

32. From the given circuit, inputs are
As all flip flops are D flip flops, output (as of input) will be same as D-state.
Given initially $xyz = 011$.
So $D_x = 1, D_y = 0, D_z = 0$
So next state $xyz = 100$. Choice (B)

33.

Clk	x	y	z	$D_x = z$	$D_y = \overline{xz}$	$D_z = \bar{z}y$
0	0	0	0	0	1	0
1	0	1	0	0	1	1
2	0	1	1	1	0	0
3	1	0	0	0	0	0
4	0	0	0			

After 4 clk pulses again come back to original state (000) so number of states are 4. Choice (A)

34. I/O device generates 100 requests.
Each request requires an interrupt.
The time taken for one request is
= 2000 + 10000 + 2000 = 14000 (to start handler, execute handler, switch to original program).
Total there are 100 requests so cycles required
= 14000 × 100 = 1400000. Choice (A)

35. The processor polls for every 0.5 ms. In one second there will be $\frac{1}{0.5\times10^{-3}}$ polls i.e., 2000 times.

Each poll requires 1000 cycles.

So it requires 2000000 cycles.

There are 100 requests, each requires 10000 cycles, total 1000000 cycles to complete.

$\therefore$ Total time spent on I/O each second
= 2000000 + 1000000 = 3000000. Choice (C)

Chapter 1

ER Model and Relational Model

LEARNING OBJECTIVES

- *Data model*
- *Schemas*
- *Three-schema architecture*
- *ER model*
- *Types of attributes*
- *Mapping cardinality*
- *Complex attributes*
- *Entity types, entity sets and value sets*
- *Weak entity set*
- *Relational database*
- *NULL in tuples*
- *Inherent constraint*
- *Referential and entity integrity constraint*

Introduction

A database is a collection of related data. By data, we mean facts that can be recorded and that have implicit meaning.

Example: Consider the names, telephone numbers and addresses of the people. We can record this data in an indexed address book and store it as Excel file on a hard drive using a personal computer. This is a collection of related data with an implicit meaning and hence is a database.

A database management system (DBMS) is a collection of programs that enables users to create and maintain a database. The DBMS is a general-purpose software system that facilitates the processes of defining, constructing, manipulating and sharing databases among various users and applications.

1. Defining the database involves specifying the data types, structures, and constraints for the data to be stored in the database.
2. Constructing the database is the process of storing the data itself on some storage medium that is controlled by the DBMS
3. Manipulating a database includes such functions as querying the database to retrieve specific data, updating the database to reflect changes.
4. Sharing a database allows multiple users and programs to access the database concurrently.
5. Fundamental characteristics of the database approach is that it provides some level of data abstraction by hiding details of data storage that are not needed by users.

Data Model

A data model is a collection of concepts that can be used to describe the structure of a database. It provides the necessary means to achieve abstraction.

Schemas

In any data model, it is important to distinguish between the description of the database and the database itself. The description of a database is called the *database schema*, which is specified during database design and is not expected to change frequently.

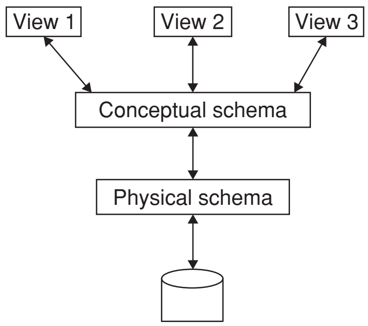
The actual data in a database may change frequently, for example the student database changes every time we add a student or enter a new grade for a student. The data in the database at a particular moment in time is called a *database state* or *snapshot*. It is also called the *current set of occurrences* or *instances in the database*.

The distinction between database schema and database state is very important. When we define a new database, we specify its database schema only to the DBMS. At this point, the corresponding database state is the empty state with no data. We get the initial state of the database when the database is first loaded with the initial data. The DBMS stores the description of the schema constructs and constraints, also called the *metadata* in the DBMS catalog so that DBMS software can refer to the schema whenever it needs. The schema is sometimes called the *intension*, and the database state an *extension* of the schema.

Three-schema Architecture

The goal of the three-schema architecture is to separate the user applications and the physical database.

Levels of Abstraction



1. The external or view level includes a number of external schemas or user views. Each external schema describes the part of the database that a particular user group is interested in and hides the rest of the database.
2. The conceptual level has a conceptual schema, which describes the structure of the whole database for a community of users. The conceptual schema hides the details of physical storage structures and concentrates on describing entities, data types, relationships, user operations, and constraints.
3. The internal level has an internal schema, which describes the physical storage structures of the database. It describes the complete details of data storage and access paths for the database.

ER Model

Entity relationship model is a popular high-level conceptual data model. This model and its variations are used for the conceptual design of database applications, and many database design tools employ its concepts. ER model describes data as entities, relationships and attributes. The basic object that the ER model represents is an entity.

Entity

It is an object that exists and is distinguishable from other objects
(or)
Entity is a "thing" in the real world with an independent existence.
(or)
An entity is something that has a distinct, separate existence, although it need not be a material existence. In particular, abstractions and legal functions are usually regarded as entities. In general, there is no presumption that an entity is animate.

1. An object with a physical existence
   Example: A particular person, car, house, employee.
2. An object with a conceptual existence
   Example: A company, a job, a university course. Each entity has attributes, the particular properties that describe it.
   Example: An employee entity can be described by employee's name, age, address, salary and job.

Entity Set

Set of entities of same type that shares the same properties.

Example: All persons, all companies etc.

Example: Entity sets of customer and loan

Table 1 *Customer Entity Set*

Customer-id	Cust-name	Cust-street	City
C-143	John	MG Road	Sec.bad
C-174	Mary	SP Road	Hyd.bad
C-183	Tony	KD Road	Sec.bad
C-192	Satya	SG Road	Eluru

Table 2 *Loan Entity Set*

Loan-no	Amount
L-30	$3000
L-31	$4000
L-32	$3500
L-33	$4500
L-34	$5000

An entity is represented by a set of attributes and by a descriptive properties possessed by all members of an entity set.

Types of Attributes

1. Simple versus composite
2. Single valued versus multivalued
3. Stored versus derived.

*Composite attribute***:** Composite attributes can be divided into smaller subparts, which represent more basic attribute that has their own meaning (Figure 1).

Example: A common example is Address, it can be broken down into a number of subparts such as street address, city, postal code; street address is further broken down into Number, street Name and Apartment number.

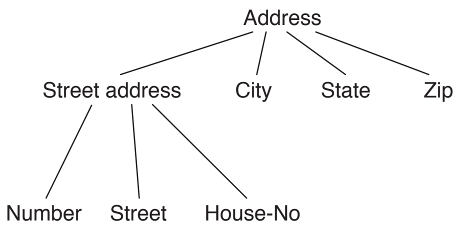


Figure 1 A hierarchy of composite attributes.

Street address is a composite attribute. Attributes that are not divisible are called simple (or) atomic attributes.

Single-valued versus multivalued attributes: Most attributes have a single value for a particular entity, such attributes are called *single-valued attribute*.

Example: Age is a single-valued attribute

Multivalued attributes: An attribute can have a set of values for the same entity.

Example: College degrees attribute for a person
Example: Name is also a multivalued attribute (Figure 2).

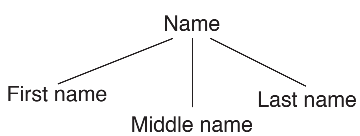


Figure 2 Multivalued attribute.

Stored versus derived attributes: Two (or) more attribute values are related.

Example: Age can be derived from a person's date of birth. The age attribute is called derived attribute and is said to be derivable from the DOB attribute, which is called a stored attribute.

Domain: The set of permitted values for each attribute.

Example: A person's age must be in the domain {0-130}

Relationship Sets

A relationship is an association among several entities. Relationship sets that involve two entity sets are binary. Generally, most relationships in databases are binary. Relationship sets may involve more than two entity sets.

Example: Employee of a bank may have responsibilities at multiple braches, with different jobs at different branches, then there is a ternary relation between employee, job and branch.

Mapping Cardinality

For a binary relationship set, mapping cardinality must be:

1. One-to-one
2. One-to-many
3. Many-to-one
4. Many-to-many

One-to-one: An entity in *A* is associated with at most one entity in *B* and an entity in *B* is associated with at most one entity in *A* (Figure 3).

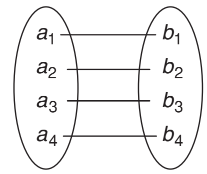


Figure 3 One-to-one relationship set.

One-to-many: An entity in *A* is associated with any number of entities in *B*. But an entity in *B* is associated with at most one entity in *A* (Figure 4).

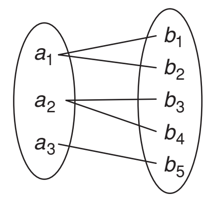


Figure 4 One-to-many relationship set.

Many-to-one: An entity in *A* is associated with at most one entity in *B*. But an entity in *B* can be associated with any number of entities in *A* (Figure 5).

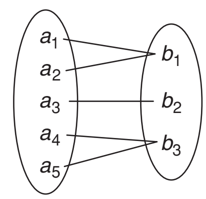


Figure 5 Many-to-one relationship set.

Many-to-many: An entity in *A* is associated with any number of entities in *B*. But an entity in *B* can be associated with any number of entities in *A* (Figure 6).

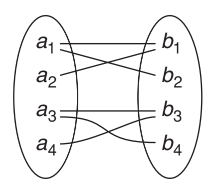
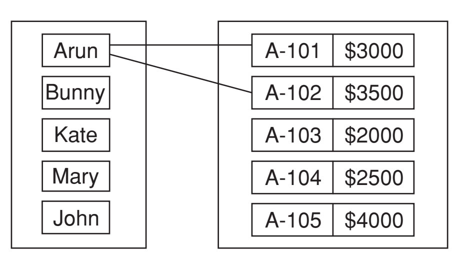


Figure 6 Many-to-many relationship set.

Example: One customer can have multiple accounts
Customer(c-Name) (Acc. no, Amount)

Table 3 *Example of One-to-many Relationship Set*



In Table 3, many-to-one relationship is not possible.

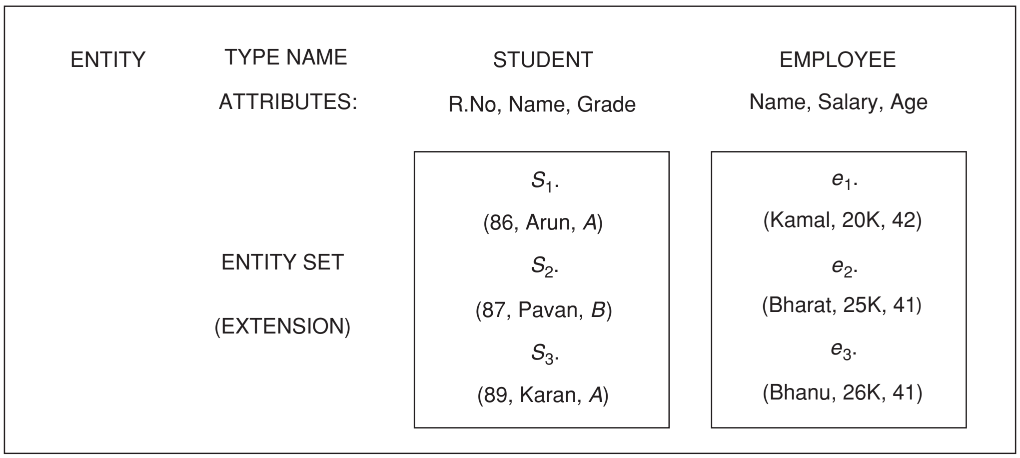
Complex Attributes

Composite and multivalued attributes can be nested in an arbitrary way. We can represent arbitrary nesting by grouping components of a composite attribute between parentheses () and separating the components with commas, and by displaying multivalued attributes between braces { }. Such attributes are called complex attributes.

Example: A person can have more than one residence and each residence can have multiple phones, an attribute AddressPhone for a person can be specified as shown below. {AddressPhone ({Phone (Areacode, phoneNumber)}, Address (StreetAddress (StreetNumber, streetName, ApartmentNumber), city, state, zip))}

Entity types, entity sets and value sets

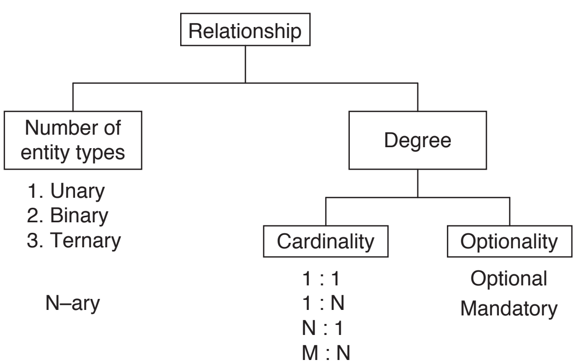
An entity type defines a collection of entities that have the same attributes. Each entity type in the database is described by its name and attribute. The following figure shows two entity types, named STUDENT and EMPLOYEE and a list of attributes for each



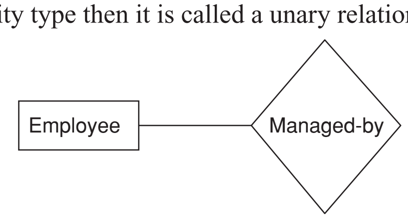
The collection of all entities of a particular entity type in the database at any point in time is called an *entity set*.

Types of relations

1. Unary relation.
2. Binary relation.
3. Ternary relation.
4. Quadnary relation.
5. N-ary relation



Unary relation If a relationship type is between entities in a single entity type then it is called a unary relationship type.

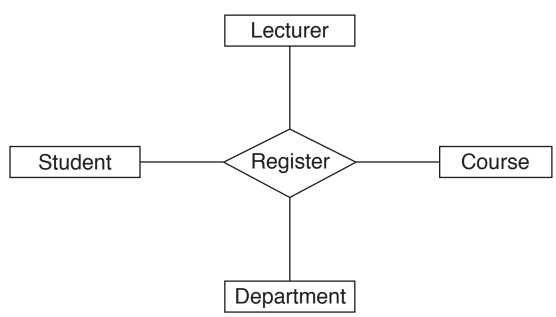


In employee entity, we will have all the employees including 'manager', this relation indicates, employees are managed by manager.

Binary relation If a relationship type is between entities in one type and entities in another type then it is called a *binary relation*, because two entity types are involved in the relation.

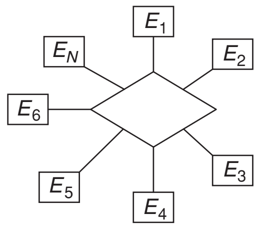
The above relation indicates that customers purchased product (or) products are purchased by customers.

Quadnary relation If a relationship type is among entities of four different types, then it is called *quadnary relation*.



In the above ER-diagram, any two entities can have a relation.

N-ary relation 'N' number of entities will participate in a relation, and each entity can have a relation with all the other entities.

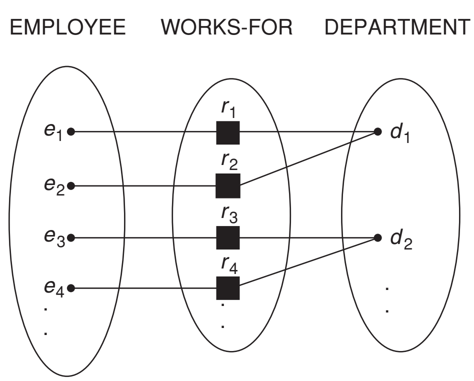


Cardinality Ratio and Participation Constraints

The cardinality ratio for a binary relationship specifies the maximum number of relationship instances that an entity can participate in

1. The participation constraint specifies whether the existence of an entity depends on its being related to another entity via the relationship type. This constraint specifies the minimum number of relationship instances that each entity can participate in, and is some times called the *minimum cardinality constraint*.
2. There are two types of participation constraints:
   a. Total participation
   b. Partial participation

Example: If a company policy states that every employee must work for a department, then an employee entity can exist only if it participates (or) works for at least one department.



Every entity in the EMPLOYEE set must be related to a DEPARTMENT entity via WORK-FOR. Total participation is also called *existence dependency*. If we do not expect every employee to manage a department, so the participation of EMPLOYEE in the relationship type is partial, means not necessarily all employees' entities are related to some department entity.

Cardinality ratio and participation constraints are taken together as the structural constraints of a relationship type. In ER diagrams, total participation is displayed as a double line connecting the participating entity type to the relationship, whereas partial participation is represented by a single line.

ER Diagrams (Figure 7 and 8)

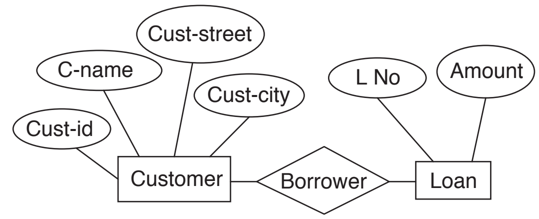
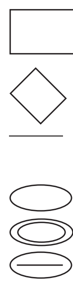


Figure 7 ER diagram.

Notations:



→ Represents entity
→ Represents relationship sets
→ Represents links between attributes to entity sets and entity sets to relationship sets
→ Represents attributes
→ Represents multivalued attributes
→ Represents primary key attribute

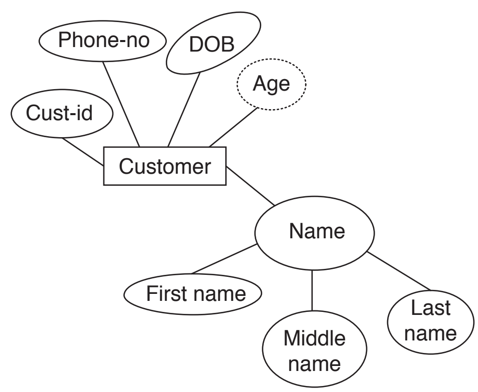
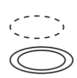
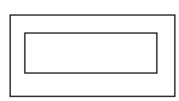


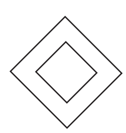
Figure 8 ER diagram.



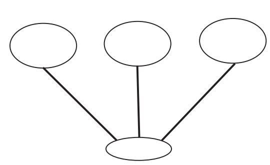
→ Derived attribute
→ Multivalued attribute



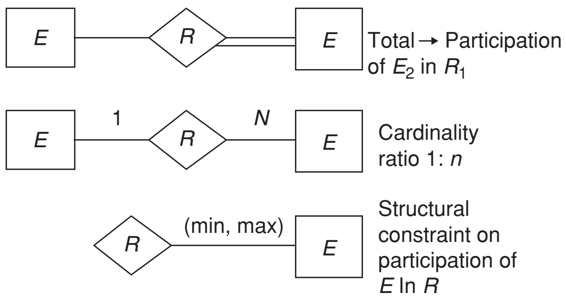
→ Weak entity



→ Weak entity set relation



→ Composite attribute



Cardinality Constraints

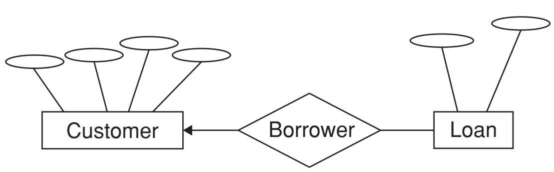
One-to-one

Each entity of one entity set is related to at most one entity of the other set. Only one matching record exists between two tables.

Example: Assume each owner is allowed to have only one dog and each dog must belong to one owner. The own relationship between dog and owner is one-to-one. One-to-one relationships can often combine the data into one table.

Examples:

1. One birdfeeder is located in one place in the yard.
2. One state has one governor.
3. One yard has one address.
4. One patient has one phone number.
2. One student has one ID.



One customer is associated to one loan via borrower

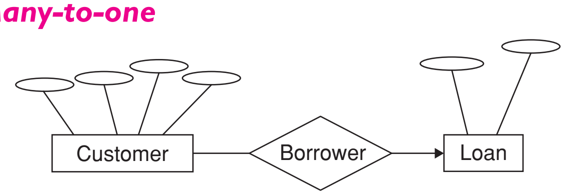
One-to-many

Examples:

1. One birdfeeder is visited by many birds.
2. One student can have many degrees.
3. One Book can be written by many authors.
4. One yard contains many bird feeders.
5. One patient has many prescriptions.

In the one-to-many relationship, a loan is associated with one customer via borrower.

Many-to-one

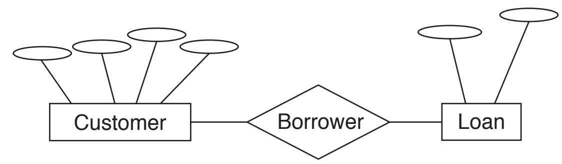


A customer is associated with at most one loan via borrower.

Many-to-many

Examples:

1. Many students are taught by many teachers.
2. Many patients are treated by many doctors.
3. Many medications are taken by many patients.
4. Many customers buy many products.
5. Many books are written by many authors.



Customer is associated with several loans and loan is associated with several customers.

ER Diagram with a Ternary Relationship

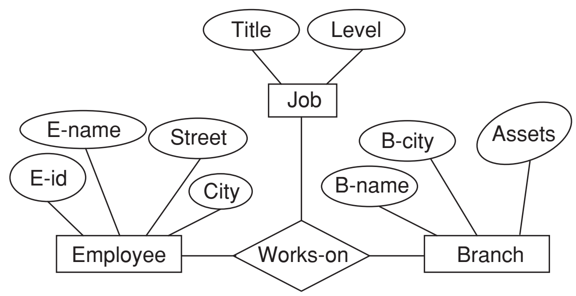
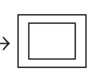


Figure 9 ER diagram with a ternary relationship.

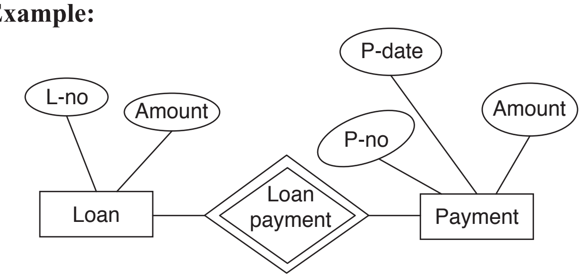
Weak Entity Set

An entity set that does not have a primary key is called *weak entity set*.

Weak entity set is represented by → 

Underline the primary key of a weak entity with a dashed line.

Example:



Relational Database

Relational Model

The relational model represents the database as a collection of relations. When a relation is thought of as a table of values, each row in the table represents a collection of related data values. Each row in the table represents a fact that typically corresponds to a real-world entity. The table name and

column names are used to help in interpreting the meaning of the values in each row.

In formal relational model terminology, a row is called a *tuple*, a column header is called an *attribute*, and the table is called a *relation*.

Domain

A domain is a set of atomic values. A common method of specifying a domain is to specify a data type from which the data values forming the domain are drawn. It is also useful to specify a name for the domain, to help in interpreting its values

Example:

1. *Set of telephone numbers*: The set of valid numbers in a particular country.
2. *Employee id numbers*: The set of valid employee numbers in a company.
3. *Names*: The set of character strings that represent names of persons.
4. *Grade-point average*: Possible values of computed grade point averages, each must be real (floating point) number between 0 and 4
5. *Research department names*: The set of research department names in a specialization, such as computer science, chemistry and applied mathematics.
6. *Research department codes*: The set of Research department codes, such as CS, CHE, AM.

The preceding is called *logical definitions of domains*. The data type for research department names is set of character strings that represent valid department names. A domain is thus given a name, data type, and format. Additional information for interpreting the values of a domain can also be given, for example a numeric domain such as person weights should have the units of measurements, such as kilograms or pounds.

1. The relational model is often described as having the following three aspects:
   - *Structural aspect*: The data in the database is perceived by the user as tables.
   - *Integrity aspect*: Those tables has to satisfy certain integrity constraints.
   - *Manipulative aspect*: The operators available to the user for manipulating those tables, for purposes of data retrieval, these operators derive tables form tables, the most important operators are 'SELECT', 'PROJECT' and JOIN.

Relation Schema

A relation schema 'R' denoted by $R(A_1, A_2, \ldots A_n)$ is made up of a relation name R and a list of attributes $A_1, A_2, \ldots A_n$. Each Attribute A_i is the name of role played by some domain D in the relation schema R. D is called the domain of A_i and is denoted by dom(A_i).

A relation schema is used to describe a relation and R is called the name of this relation. The degree of a relation is the number of attributes 'n' of its relation schema.

Example:

A relation schema of degree '7', which describes an employee is given below:

EMPLOYEE (Name, EId, HomePhone, Address, Office phone, Age, Salary)

Using the data type of each attribute, the definition is written as:

EMPLOYEE (Name: String, EId: INT, Homephone: INT, Address: String, OfficePhone: String, Age: Real, Salary: INT)

For this relation schema, EMPLOYEE is the name of the relation, which has '7' attributes.

2. A relation 'r' of the relation schema $R(A_1, A_2 \ldots A_n)$, also denoted by $r(R)$, is a set of n-tuples. $r = \{t_1, t_2 \ldots t_m\}$. Each n-tuple 't' is an ordered list of n values $t = <V_1, V_2 \ldots N_n>$, where each value V_i, $1 \le i \ldots n$, is an element of dom(A_i) or is a special null value.
3. The ith value in tuple t, which corresponds to the attribute A_i, is referred to as $t[A_i]$.
4. The following figure shows EMPLOYEE relation. Each type in a relation represents a particular employee entity. We display the relation as a table where each tuple is shown as a row and each attribute corresponds to a column header, indicating a role or interpretation of the values in that column. Null values represent attributes whose values are unknown or do not exist for some individual EMPLOYEE tuple.

Employee

Name	EId	Home Phone	Address	Office Phone	Age	Salary
Mahesh	30-01	870-223366	Warangal	NULL	35	40 k
Ramesh	30-02	040-226633	Hyderabad	NULL	36	40 k
Suresh	30-03	040-663322	Kolkata	040-331123	35	42 k
Dinesh	30-04	040-772299	Bangalore	040-321643	36	40 k

Fig: The attributes and tuples of a relation EMPLOYEE. The earlier definition of a relation can be restated as follows:
A relation $r(R)$ is a mathematical relation of degree 'n' on the domains dom(A_1), . . . , dom(A_n), which is a subset of the Cartesian product of the domains that define R:

$$r(R)\ C\ (\text{dom}(A_1) \times \text{dom}(A_2) \ldots \times \text{dom}(A_n))$$

Characteristics of Relations

There are certain characteristics that make a relation different from a file or a table.

Ordering of tuples in a relation: A relation is defined as a set of tuples. Tuples in a relation do not have any particular order. In a file, records are stored on disk in order. This ordering indicates first, second, i^{th}, and last records in the file. Similarly, when we display a relation as a table, the rows are displayed in a certain order.

Tuple ordering is not part of a relation definition, because a relation attempts to represent facts at a logical or abstract level. Many logical orders can be specified on a relation. Tuples in the EMPLOYEE relation could be logically ordered by values of Name or by EID or by Age or by some other attribute. When a relation is implemented as a file or displayed as a table, a particular ordering may be specified on the records of the file or the rows of the table.

NULL in Tuples

Each value in a tuple is an atomic value; that is, it is not divisible into components. Hence, composite and multivalued attributes are not allowed. This model is sometimes called the *flat relational model*. Multivalued attributes must be represented by separate relations, and composite attributes are represented only by their simple component attributes in the basic relational model.

NULLS are used to represent the values of attributes that may be unknown or may not apply to tuple. For example, some student tuples have null in their office phones because they do not have an office. In this case, the meaning behind NULL is not applicable. If a student has a NULL for home phone, it means either he/she does not have a home phone or he/she has one but we do not know it, in this case the meaning of NULL is 'Unknown'.

Relational Model Constraints

In a relational database, there will be many relations, and the tuples in those relations are related in various ways. There are many restrictions or constraints on the actual values in a database state.

Constraints on database can generally be divided into three main categories as follows:

1. Constraints that are inherent in the data model, we call them *inherent model-based constraints*.
2. Constraints that can be directly expressed in the schemes of the data model, by specifying them in the DDL (Data Definition Language). We call these *schema-based constraints*.
3. Constraints that cannot be directly expressed in the schemas of the data model, and they must be expressed and enforced by the application programs are *application-based constraints*.

Inherent Constraint

The constraint that a relation cannot have duplicate tuples is an *inherent constraint*. Another important category of constraints is data dependencies, which include 'functional dependencies' and 'multivalued dependencies'. They are used mainly for testing the 'goodness' of the design of a relational database and are utilized in a process called normalization.

Schema-based Constraints

1. Domain constraints
2. Key constraints
3. Constraints on nulls (Not null constraint)
4. Entity integrity constraints
5. Referential integrity constraints
6. Unique constraint
7. Check constraint

Domain constraints

Domain constraints specify that within each tuple, the value of each attribute 'X' must be an atomic value from the domain dom(X).

The data types associated with domains include standard numeric data types for integers

1. Short integer
2. Integer
3. Long integer
4. Real numbers
    - Float
    - Double-precision float
5. Characters
6. Booleans
7. Fixed-length strings
8. Variable-length strings
9. Date, time, time stamp
10. Money data types

Key constraints

A relation is a set of tuples. All elements of a set are distinct; hence, all tuples in a relation must also be distinct. This means no two tuples can have the same combination of values for all their attributes.

1. There are other subsets of attributes of a relation schema R with the property that no two tuples in any relation state 'r' of R should have the same combination of values of these attributes.

   Suppose that we denote one subset of attributes by 'SK', then for two distinct tuples t_1 and t_2 in a relation state 'r' of R, we have the following constraint:

$$t_1[\text{SK}] = t_2[\text{SK}]$$

2. Any such set of attributes SK is called a *super key* of the relation schema *R*.

   SK specifies a uniqueness constraint that no two distinct tuples in any state *r* or *R* can have the same value for SK.
3. Every relation has at least one default super key, the set of all its attributes. *A* key, '*K*' of a relation schema *R* is a super key of *R* with the additional property that removing any attributes '*X*' from *K* leaves a set of attributes *K*' that is not a super key of *R* any more.

A key satisfies the following two constraints:

1. Two distinct tuples in any state of the relation cannot have identical values for all the attributes in the key.
2. A super key from which we cannot remove any attributes and still have the uniqueness constraints mentioned in above condition is known as a *minimal super key*.

   The first condition applies to both keys and super keys. The second condition is required only for keys.

Example: Consider the employee relation in Page no. 9. The attribute set {EId} is a key of employee because no two employee tuples can have the same value for EId.

Any set of attributes that include EId will form a super key.

1. {EId, Homephone, Name}
2. {EId, Age, Salary}
3. {Name, EId, Address}

However, the super key {EId, Name, Age} is not a key of EMPLOYEE, because removing Name or age or both from the set leaves us with a super key. Any super key formed from a single attributes is also a key. A key with multiple attributes must require all its attributes to have the uniqueness property.

A relation schema may have more than one key. In that case, each of the keys is called a *candidate key*.

Example: Employee relation has three candidate keys.
{Name, EId, Homephone}

One of the candidate keys is chosen as primary key of the relation.

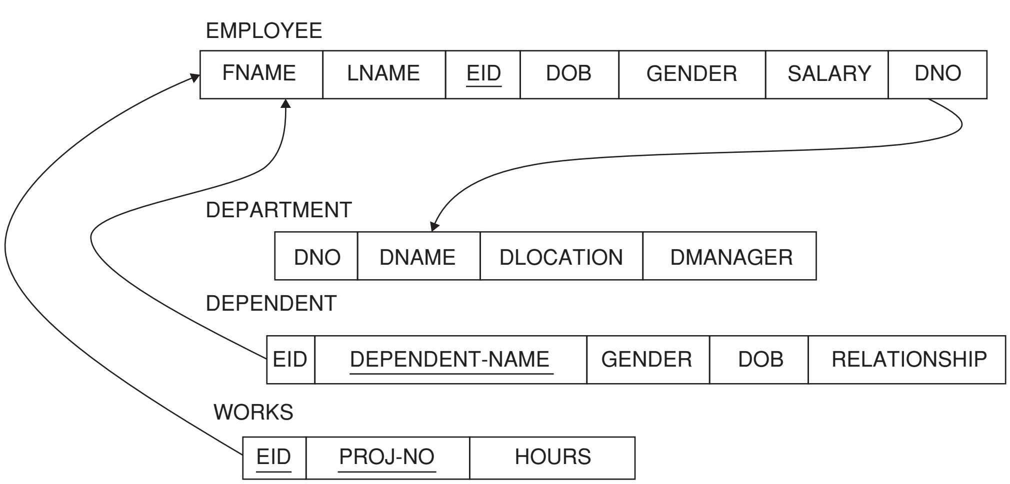
Another constraint on attributes specifies whether null values are permitted in tuples or not. If we want some tuples to have a valid (or) non-null value, we need to use NOT NULL constraint on that attribute.

Referential and entity integrity constraint

The entity Integrity constraint states that no primary key value can be null. If we have NULL values in the primary key column, we cannot identify some tuples in a relation.

1. Key constraints and entity Integrity constraints are specified on individual relations
2. Referential integrity constraint is specified between relations and used to maintain the consistency among tuples in the two relations.
3. Referential Integrity constraints states that a tuple in one relation that refers to another relation must refer to an existing tuple in that relation.
4. To understand the concept of Referential Integrity, first we have to understand the concept of FOREIGN KEY.
5. Suppose we have two relations R_1 and R_2. A set of attributes FK in relation schema R_1 is a foreign key of R_1 that references relation R_2 if it satisfies the following two rules:
   - The attributes in FK have the same domains as the primary key attributes PK of R_2, FK will have to refer to PK.
   - A value of FK in a tuple t_1 of the current state $r_1(R_1)$ either occurs as a value of PK for some tuple t_2 in the current state $r_2(R_2)$ or is null. We have t_1[FK] = t_2[PK], and we say that the tuple t_1 references to the tuple t_2. In this definition, R_1 is called the *referencing relation* and R_2 is the *referenced relation*.
6. A *foreign key* can refer to its own relation. We can diagrammatically display referential integrity constraints by drawing a directed Arc from each foreign key to the relation it references. The arrow head may point to the primary key of the referenced relation.

Example:



7. *Referential integrity rule*: The database must not contain any unmatched foreign key values.

If '*B*' References '*A*', then A must exist.

NOT NULL constraint

1. NOT NULL constraint restricts a column from having a NULL value. NOT NULL constraint can be applied to any column in a table.
2. We cannot give NULL values under that column
3. NOT NULL Constraint enforces a column to contain a proper value.
4. This constraint cannot be defined at table level.

Example: CREATE TABLE
student(RNo:INT Name:varchar(70) NOT NULL age:INT)
Suppose a row is inserted into the following table,

Insert into student values <11, NULL, 20>

In the schema, we enforced NOT NULL constraint on Name column, means Name cannot have NULL value, when the above insert command is executed, the system gives, NOT NULL constraint violation.

UNIQUE constraint

The column on which UNIQUE constraint is enforced should not have any duplicate values.

1. UNIQUE constraint can be enforced on any column except the primary key column.
2. By default primary key column will not accept any duplicate values that are handled by key constraint.
3. UNIQUE constraint can be applied at column level or table level.

Example: CREATE TABLE
student (RNo:INT Name:varchar(60) Grade:CHAR(1))
Assume that the table contains following tuples

Student

R no.	Name	Grade
11	Sita	B
12	Anu	A
13	Bala	A

Suppose the following tuple is inserted into the student table.

1. Insert into student values <14, 'Anu', '*B*'>
2. UNIQUE constraint is enforced on Name column, in the student table we have 'Anu', and again the new tuple contains name 'Anu', this Insert command violates the UNIQUE constraint.

CHECK constraint

This constraint is used to restrict a value of a column between a range.

1. When a value is inserted into particular column, before storing that, a check will be performed to see whether the values lie within the specified range.
2. If the value entered is out of range, it will not accept and violation happens.
3. It is like checking a condition before saving data into a column.

Example: Create table student (RNo:INT CHECK (Rno>0)
Name:varchar(60)
Dept:varchar(4))
Suppose the following tuple is inserted into student table.

1. Insert into student values <-4, 'Bhanu', 'CS'>
2. CHECK constraint is enforced on RNo column, the RNo should be greater than '0', but '-4' is given.
3. CHECK constraint is violated.

Creating table from a table: A view is called a *derived table* (or) *virtual table*, because the data stored in views is taken from already existing tables.

A view can also be defined as a logical subset of data from one or more tables.

Syntax:

```
CREATE    view view-name As
SELECT    column-names
FROM      table-name
WHERE     condition
```

Example: Consider the following table "sales".

Sales

Order-Id	Order Name	Previous Balance	Customer
21	Order 3	3000	Ana
22	Order 4	1000	Adam
23	Order 5	3000	Brat
24	Order 6	2000	John
25	Order 7	2000	Ana
26	Order 8	4000	Ana

Query to create a view:

```
CREATE view sales-view As
SELECT    *
FROM      Sales
WHERE     customer = 'Ana'
```

1. The data fetched from select statement will be stored in an object called 'sales-view'.
2. To display the contents stored in view, execute the following statement.

```
SELECT    *
FROM      Sales-view
```

Removal of specific rows:
Consider the following SQL query:

```
Delete    *
FROM   Sales
```

The above query will delete all the tuples from sales.

To remove specific rows, we have to specify the condition in WHERE clause.

Consider the table "sales" given in the above example.

Remove the rows from sales table whose previous balance is 3000.

SQL query:

```
Delete    *
FROM      Sales
WHERE     Previous-balance = 3000
```

Output:

Order Id	Order Name	Previous Balance	Customer
22	Order 4	1000	Adam
24	Order 6	2000	John
25	Order 7	2000	Ana
26	Order 8	4000	Ana

Referential actions: Referential Integrity can be violated if the value of any foreign key refers to a tuple that does not exist in the referenced relation.

When certain violations occur, we need to perform some alternate action. Those actions are as follows:

1. ON DELETE CASCADE
2. ON UPDATE CASCADE
3. ON DELETE SET NULL
4. ON DELETE SET DEFAULT

Example: Consider the given database:

SUPPLIERS:

Supplier Number	Supplier Name	Status	City
SN1	Suma	30	Hyderabad
SN2	Hari	20	Chennai
SN3	Anu	10	Hyderabad
SN4	Mahesh	20	Bombay
SN5	Kamal	30	Delhi

PARTS:

Part number	Part name	Colour	Weight	City
PN1	X	Red	13.0	Chennai
PN2	Y	Green	13.5	Bombay
PN3	X	Yellow	13.2	Hyderabad
PN4	Y	Green	14.1	Calcutta
PN5	Z	Red	14.3	Hyderabad
PN6	Z	Blue	14.2	Bombay

PROJECT:

Project Number	Project Name	City
PJ1	Display	Chennai
PJ2	OCR	Bombay
PJ3	RAID	Chennai
PJ4	SORTER	Hyderabad
PJ5	EDS	Chennai
PJ6	Tape	Bombay
PJ7	Console	Hyderabad

SHIPMENTS:

Supplier Number	Part Number	Project Number	Quantity
SN1	PN1	PJ1	300
SN1	PN1	PJ4	400
SN2	PN3	PJ1	350
SN2	PN3	PJ2	450
SN2	PN3	PJ3	640
SN2	PN3	PJ4	320
SN2	PN3	PJ5	330
SN2	PN3	PJ6	520
SN2	PN3	PJ7	480
SN2	PN5	PJ2	460
SN3	PN3	PJ1	440
SN3	PN4	PJ2	410
SN4	PN6	PJ3	310
SN4	PN6	PJ7	320
SN5	PN2	PJ2	340
SN5	PN2	PJ4	350
SN5	PN5	PJ5	360
SN5	PN5	PJ7	370
SN5	PN6	PJ2	380
SN5	PN1	PJ4	420
SN5	PN3	PJ4	440
SN5	PN4	PJ4	450
SN5	PN5	PJ4	400
SN5	PN6	PJ4	410

Consider the following statement:

```
DELETE FROM SUPPLIER
WHERE SUPPLIER – NUMBER = 'SN1'
```

It deletes the supplier tuple for supplier 'SN1'. The database has some other tables which have 'SN1' tuple (Shipments table). The application does not delete those suppliers, then it will find a violation, and an exception will be raised.

An alternate approach is possible, one that might be preferable in some cases, and that is for the system to perform an appropriate 'compensating action' that will guarantee that the overall result does still satisfy the constraint. In the example, the compensating action would be for the system to delete the shipments for supplier SN1 "automatically".

We can achieve this effect by extending the foreign key as indicated below:

```
CREATE TABLE SHIPMENT{.....}.......
FOREIGN KEY {SUPPLIER – NUMBER}
REFERENCES
SUPPLIER ON DELETE CASCADE
```

The specification ON DELETE CASCADE defines a delete rule for this particular foreign key, and the specification CASCADE is the referential action for that delete rule. The meaning of these specifications is that a DELETE operation on the suppliers relvar will 'Cascade" to delete matching tuples (if any) in the shipments relvar as well.

Same procedure is applied for all the referential actions.

Triggers

Triggers are precompiled procedures that are stored along with the database and invoked automatically whenever some specified event occurs.

Suppose we have a view called HYDERABAD - SUPPLIER defined as follows:

```
CREATE VIEW HYDERABAD-SUPPLIER
AS SELECT SUPPLIER - NUMBER, SUPPLIER-
NAME, STATUS
FROM SUPPLIER
WHERE CITY = 'HYDERABAD',
```

Normally, if the user tries to insert a row into this view, SQL will actually insert a row into the underlying base table SUPPLIERS with CITY value whatever the default is for the CITY column. Assuming that default is not Hyderabad, the net effect is that the new row will not appear in the view; therefore, let us create a triggered procedure as follows:

```
CREATE TRIGGER HYDERABAD -
SUPPLIER - INSERT
INSTEAD OF INSERT ON HYDERABAD
- SUPPLIER
REFERENCING NEW ROW AS R
FOR EACH ROW
INSERT INTO SUPPLIERS (SUPPLIER -
NUMBER, SUPPLIER - NAME, STATUS, CITY)
VALUES (R. SUPPLIER - NUMBER, R.
SUPPLIER - NAME, R. STATUS, 'HYDERABAD');
```

Inserting a row into the view will now cause a row to be inserted into the underlying base table with CITY value equal to Hyderabad inserted of the default value.

In general, CREATE TRIGGER specifies, among other things, an event, a condition, and an action.

The event is an operation on the database ("INSERT ON HYDERABAD - SUPPLIER" in the example)

1. The "condition" is a Boolean expression that has to evaluate to TRUE in order for the action to be executed.
2. The 'action' is the triggered procedure ("INSERT INTO SUPPLIERS . . .)
3. The event and condition together are sometimes called the *triggering event*. The combination of all three (event, condition, and action) is usually called a trigger.
4. Possible events include INSERT, DELETE, UPDATE, reaching end-of-transaction (COMMIT) reaching a specified time of day, exceeding a specified elapsed time, violating a specified constraint, etc.
5. A database that has associated triggers is sometimes called an active database.

Base Table Constraints

SQL-base table constraints are specified on either CERATE TABLE or ALTER TABLE. Each such constraint is a candidate key constraint, a foreign key constraint, or a CHECK constraint.

Candidate keys: An SQL candidate key definition takes one of the following two forms:

```
PRIMARY KEY (< column name comma list>)
UNIQUE (< column name comma list>)
The following example illustrates base table constraints of all three kinds:
CREATE TABLE SHIPMENTS
(SUPPLIER-NUMBER.      SUPPLIER-NUMBER
NOT NULL, PART - NUMBER PART-NUMBER
NOT NULL, QUANTITY NOT NULL
PRIMARY KEY (SUPPLIER - NUMBER,
PART  NUMBER)
FOREIGN KEY (SUPPLIER-NUMBER)
REFERENCES SUPPLIERS
ON DELETE CASCADE
ON UPDATE CASCADE,
FOREIGN KEY (PART-NUMBER)
REFERENCES PARTS
ON DELETE CASCADE
ON UPDATE CASCADE
CHECK(QUANTITY ≤ QUANTITY (0) AND
QUANTITY ≤ QUANTITY (1000));
```

A check constraint of the form CHECK (< column name > IS NOT NULL) can be replaced by a simple NOT NULL specification in the definition of the column.

Exercises

Practice Problems I

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Consider the following two tables T_1 and T_2. Show the output for the following operations:

Table T_1

P	Q	R
11	a	6
16	b	9
26	a	7

Table T_2

A	B	C
11	b	7
26	c	4
11	b	6

What is the number of tuples present in the result of given algebraic expressions?

(i) $T_1 \bowtie_{T1.P = T2.A} T_2$

(A) 2 (B) 3
(C) 4 (D) 5

(ii) $T_1 \bowtie_{T1.Q = T2.B} T2$

(A) 2 (B) 3
(C) 4 (D) .5

(iii) $T_1 \bowtie_{(T1.p = T2.A \text{ AND } T1.R = T2.C)} T_2$

(A) 1 (B) 2
(C) 3 (D) .4

2. Suppose $R_1(A, B)$ and $R_2(C, D)$ are two relation schemas. Let R_1 and R_2 be the corresponding relation instances. B is a foreign key that refers to C in R_2. If data in R_1 and R_2 satisfy referential integrity constraints, which of the following is true?

(A) $\pi_B(R_1) - \pi_c(R_2) = \phi$

(B) $\pi_C(R_2) - \pi_B(R_1) = \phi$

(C) $\pi_B(R_1) - \pi_C(R_2) \neq \phi$

(D) Both A and B

3. Consider the following relations:

A, B and C

A

Id	Name	Age
12	Arun	60
15	Shreya	24
99	Rohit	11

B

Id	Name	Age
15	Shreya	24
25	Hari	40
98	Rohit	20
99	Rohit	11

C

Id	Phone	Area
10	2200	02
99	2100	01

How many tuples does the result of the following relational algebra expression contain? Assume that the scheme of $(A \cup B)$ is the same as that of A.

$(A \cup B) \bowtie_{A.Id > 40 \vee c.Id < 15} C$

(A) 6 (B) 7
(C) 8 (D) 9

4. Consider the relations A, B and C given in Question 3. How many tuples does the result of the following SQL query contain?

```
SELECT   A.Id
FROM           A
WHERE    A.Age>
ALL            (SELECT B.Age
FROM           B
WHERE    B.Name = 'Arun')?
```

(A) 0 (B) 1
(C) 2 (D) 3

5. Consider a database table T containing two columns X and Y each of type integer. After the creation of the table, one record ($X = 1$, $Y = 1$) is inserted in the table. Let MX and MY denote the respective maximum value of X and Y among all records in the table at any point in time. Using MX and MY, new records are inserted in the table 128 times with X and Y values being MX + 1, 2 * MY + 1, respectively. It may be noted that each time after the insertion, values of MX and MY change. What will be the output of the following SQL query after the steps mentioned above are carried out?

SELECT Y FROM T WHERE $X = 7$;?

(A) 15 (B) 31
(C) 63 (D) 127

6. Database table by name loan records is given below:

Borrower	Bank Manager	Loan Amount
Ramesh	Sunderajan	10000
Suresh	Ramgopal	5000
Mahesh	Sunderajan	7000

What is the output of the following SQL Query SELECT count (*)

FROM((SELECT Borrower, Bank-manager

FROM Loan-Records)AS S

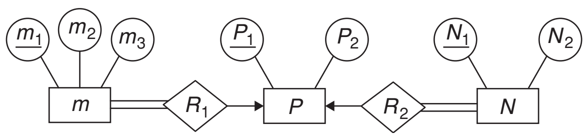
NATURAL JOIN

(SELECT Bank-manager, Loan-Amount

FROM Loan-Records) AS T);

(A) 3 (B) 4
(C) 5 (D) 6

7. Consider the following ER diagram:



What is the minimum number of tables needed to represent M, N, P, R_1, R_2?

(A) 2 (B) 3
(C) 4 (D) 5

8. Let E_1 and E_2 be two entities in an ER diagram with simple single-valued attributes. R_1 and R_2 are two relationships between E_1 and E_2, where R_1 is one-to-many and R_2 is many-to-many. R_1 and R_2 do not have any attributes of their own. What is the minimum number of tables required to represent this situation in the relational model?

(A) 2 (B) 3
(C) 4 (D) 5

9. The following table has two attributes A and C where A is the primary key and C is the foreign key referencing A with ON DELETE CASCADE.

A	C
2	4
3	4
4	3
5	2
7	2
9	5
6	4

What is the set of all tuples that must be additionally deleted to preserve referential integrity when the tuple (2, 4) is deleted?

(A) (5, 2),(7, 2) (B) (5, 2), (7, 2),(9, 5)
(C) (5, 2),(9, 5) (D) (2, 4), (7, 2)

10. Consider the following SQL query

SELECT DISTINCT $a_1, a_2, a_3 \ldots a_n$

FROM $R_1, R_2 \ldots R_m$

WHERE P

For any arbitrary predicate P, this query is equivalent to which relational algebra expression?

(A) $\Pi_{a_1, a_2, \ldots a_n}(\sigma_p R_1 \times R_2 \times \ldots \times R_m)$

(B) $\sigma_{a_1, a_2, \ldots a_n}(\sigma_p R_1 \times R_2 \times \ldots \times R_m)$

(C) $\sigma_{a_1, a_2, \ldots a_n}(\Pi_p R_1 \times R_2 \times \ldots \times R_m)$

(D) $\Pi_{R_1, R_2, \ldots Rm}(\sigma_p a_1 \times a_2 \times \ldots \times (a_n)$

11. Consider the following relation schema pertaining to a student's database:

Student (Rollno, name, address)

Enroll (Rollno, courseno, coursename)

Where the primary keys are shown underlined. The number of tuples in the student and Enroll tables are 120 and 6, respectively. What are the maximum and minimum number of tuples that can be present in

(student * Enroll)

Where '*' denotes natural join?

(A) 6, 6 (B) 6, 120
(C) 120, 6 (D) 120, 120

12. A relational schema for a train reservation database is given below

Table 4 *Passenger*

Pid	P Name	Age
0	'Sachin'	65
1	'Rahul'	66
	'Sourav'	67
3	'Anil'	69

Table 5 *Reservation*

Pid	Class	Tid
0	AC	8200
1	AC	8201
2	SC	8201
5	AC	8203
1	SC	8204
3	AC	8202

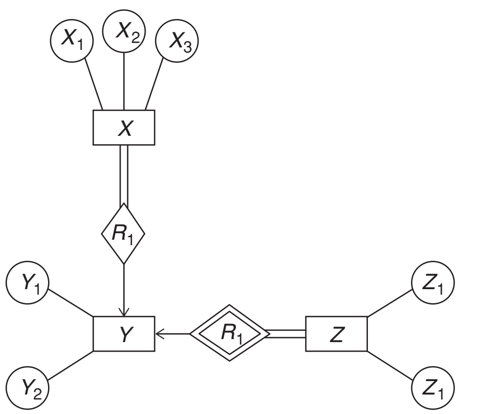
What pid's are returned by the following SQL query for the above instance of the tables?

```
SELECT   pid
FROM     Reservation
WHERE    class = 'AC' AND
         EXISTS (SELECT *
                 FROM passenger
         WHERE age > 65 AND
         Passenger.pid = Reservation.pid)
```

(A) 0, 1 (B) 1, 3
(C) 1, 5 (D) 0, 3

13. Given {customer} is a candidate key, [customer name, customer street} is another candidate key then
(A) {customer id, customer name} is also a candidate key.
(B) {customer id, customer street} is also a candidate key.
(C) {customer id, customer name, customer street} is also a candidate key.
(D) None

Common data for questions 14 and 15: Consider the following diagram,



14. The minimum number of tables needed to represent X, Y, Z, R_1, R_2 is
(A) 2 (B) 3
(C) 4 (D) 5

15. Which of the following is a correct attribute set for one of the tables for the correct answer to the above questions?
(A) $\{X_1, X_2, X_3, Y_1\}$ (B) $\{X_1, Y_1, Z_1, Z_2\}$
(C) $\{X_1, Y_1, Z_1\}$ (D) $\{M_1, Y_1\}$

16. UPDATE account SET
DA = basic * .2,
GROSS = basic * 1.3,
Where basic > 2000;
(A) The above query displays DA and gross for all those employees whose basic is ≥ 2000
(B) The above query displays DA and gross for all employees whose basic is less than 2000
(C) The above query displays DA as well as gross for all those employees whose basic is >2000
(D) Above all

17. Which of the following query transformations is correct?
R_1 and R_2 are relations C_1, C_2 are selection conditions and A_1 and A_2 are attributes of R_1
(A) $\sigma_{C1}(\sigma_{C1}(R_1)) \to \sigma_{C2}(\sigma_{C2}(R_1))$
(B) $\sigma_{C1}(\sigma_{A1}(R_1)) \to \sigma_{A1}(\sigma_{C1}(R_1))$
(C) $\pi_{A2}(\pi_{A}(R_1)) \to \pi_{A}(\pi_{A2}(R_1))$
(D) All the above

18. Consider the following query select distinct $a_1, a_2, \ldots a_n$ from $r_1, r_2 \ldots r_m$ where P for an arbitrary predicate P, this query is equivalent to which of the following relational algebra expressions:
(A) $\pi_{a_1 \ldots a_n} \sigma_p (r_1 \times r_2 \times \ldots \times r_m)$
(B) $\pi_{a_1 \ldots a_n} \sigma_p (r_1 \times r_2 \times r_3 \times \ldots . \times r_m)$
(C) $\sigma_{a_1 \ldots a_n} \pi_p (r_1 \times r_2 \times \ldots \times r_m)$
(D) $\sigma_{a_1 \ldots a_n} \pi_p (r_1 \times r_2 \times \ldots \times r_m)$

19. The relational algebra expression equivalent to the following tuple calculus expression
$\{a\}\ a \in r \wedge (a[A] = 10 \wedge a[B] = 20)$ is
(A) $\sigma_{(A = 10\ r\ B = 20)}$r
(B) $\sigma_{(A = 10)}(r) \cup \sigma_{(B = 20)}(r)$
(C) $\sigma_{(A = 10)}(r) \cap \sigma_{(B = 20)}(r)$
(D) $\sigma_{(A = 10)}(r) - \sigma_{(B = 20)}(r)$

20. Which of the following is/are wrong?
(A) An SQL query automatically eliminates duplicates.
(B) An SQL query will not work if there are no indexes on the relations.
(C) SQL permits attribute names to be repeated in the same relation
(D) All the above

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. If *ABCDE* are the attributes of a table and *ABCD* is a super key and *ABC* is also super key then
(A) *A B C* must be candidate key
(B) *A B C* cannot be super key
(C) *A B C* cannot be candidate key
(D) *A B C* may be candidate key

2. The example of derived attribute is
(A) Name if age is given as other attribute
(B) Age if date_of_birth is given as other attribute
(C) Both (A) and (B)
(D) None

3. The weak entity set is represented by
(A) box
(B) ellipse
(C) diamond
(D) double outlined box

4. In entity relationship diagram double lines indicate
(A) Cardinality
(B) Relationship
(C) Partial participation
(D) Total participation

5. An edge between an entity set and a binary relationship set can have an associated minimum and maximum cardinality, shown in the form 1… h where 1 is the minimum and h is the maximum cardinality A minimum value 1 indicates:
(A) total participation (B) partial participation
(C) double participation (D) no participation

6. Let R be a relation schema. If we say that a subset k of R is a super key for R, we are restricting R, we are restricting consideration to relations $r(R)$in which no two district tuples have the same value on all attributes in K. That is if t_1 and t_2 are in r and $t_1 \uparrow t_2$
(A) $t_1[k] = 2t_2[K]$ (B) $t_2[K] = 2t_1[k]$
(C) $t_1[k] = t_2[k]$ (D) $t_1[k] \uparrow t_2[k]$

7. Which one is correct?
(A) Primary key ⊂ Super key ⊂ Candidate key
(B) Candidate key ⊂ Super key ⊂ Primary key
(C) Primary key ⊂ Candidate key ⊂ Super key
(D) Super key ⊂ Primary key ⊂ Candidate key

8. If we have relations $r1(R1)$ and $r2(R2)$, then $r1$ ($r2$ is a relation whose schema is the
(A) concatenation (B) union
(C) intersection (D) None

9. Match the following:

I	Empid	1	Multivalued
II	Name	2	Derived
III	Age	3	Composite
IV	Contact No.	4	Simple

(A) I – 4, II – 3, III – 2, Iv – 1
(B) I – 3, II – 2, III – 4, Iv – 1
(C) I – 2, II – 1, III – 4, Iv – 3
(D) I – 1, II – 3, III – 2, IV – 4

10. Match the following:

I	Double-lined ellipse	1	Multivalued attribute
II	Double line	2	Total participation
III	Double-lined box	3	Weak entity set
IV	Dashed ellipse	4	Derived attribute

(A) I – 1, II – 2, III – 3, IV – 4
(B) I – 2, II – 3, III – 4, IV – 1
(C) I – 3, II – 4, III – 2, IV – 2
(D) I – 4, II – 3, III – 2, IV –1

11. The natural join is a
(A) binary operation that allows us to combine certain selections and a Cartesian product into one operation
(B) unary operations that allows only Cartesian product
(C) query which involves a Cartesian product and a projection
(D) None

12. The number of entities participating in the relationship is known as
(A) maximum cardinality (B) composite identifiers
(C) degree (D) None

13 A minimum cardinality of 0 specifies
(A) non-participation
(B) partial participation
(C) total participation
(D) zero participation

14. What is not true about weak entity?
(A) They do not have key attributes.
(B) They are the examples of existence dependency.
(C) Every existence dependency results in a weak entity
(D) Weak entity will have always discriminator attributes

15. Which one is the fundamental operation in the relational algebra?
(A) Natural join (B) Division
(C) Set intersection (D) Cartesian product

16. For the given tables

A

X	Y
a_1	b_1
a_2	b_1
a_1	b_2
a_2	b_2

B

Y
b_1
b_2

$A \div B$ will return
(A) a_1, a_2 (B) a_1
(C) a_2 (D) None

17. The number of tuples selected in the above answer is
(A) 2 (B) 1
(C) 0 (D) 4

Common data for questions 18 and 19: Consider the following schema of a relational database.

Emp (empno, name, add)

Project (Pno, Prame)

Work on (empno, Pno)

Part (partno, Pname, qty, size)

Use (empno, pno, partno, no)

18. ((name(emp) ((name(emp ⋈ workon) displays
(i) The names of the employees who are not working in any project
(ii) The names of the employees who were working in every project.
(A) Only (i) (B) Only (ii)
(C) Both (A) and (B) (D) None

19. List the partno and names of the parts used in both the projects DBMS & MIS:

(A) $\sigma_{partno,\ pname,}{}^{(part} \bowtie {}^{(}\pi_{partno} (\sigma_{pname} = \text{“DBMS”}^{(project)} \bowtie$ ${}^{use).}\cap \pi_{partno} (\sigma_{pname} = \text{“MIS”}^{(project)} \bowtie {}^{use)})$

(B) (partno, pname(part $\bowtie$ ((partno ((pname = “DBMS”(project $\bowtie$ use) ((partno(σ_{pname} = “MIS”$^{(project)} \bowtie {}^{use)}$))

(C) $\pi_{partno,\,pname}(\text{part} \bowtie (\sigma_{partno} (\sigma_{pname} = \text{“DBMS”}^{(project} \bowtie$ ${}^{use)}$ ((partno ((pname = “MIS”(project) $\bowtie$ use)))

(D) None

20. The following query shows. SELECT job-status, sum (basic – salary) AVG (basic – salary) from employees group by job – status.

(i) It shows job status, sum, AVG of all data

(ii) It shows job status, Sum, AVG, with group by clause in use.

(A) only (i)
(B) only (ii)
(C) both (A) and (B)
(D) None

Previous Years' Questions

1. Let E_1 and E_2 be two entities in an E/R diagram, with simple single-valued attributes. R_1 and R_2 are two relationships between E_1 and E_2, where R_1 is one-to-many and R_2 is many-to-many. R_1 and R_2 do not have any attributes of their own. What is the minimum number of tables required to represent this situation in the relational model? **[2005]**

(A) 2
(B) 3
(C) 4
(D) 5

2. The following table has two attributes A and C where A is the primary key and C is the foreign key referencing A with on-delete cascade.

A	C
2	4
3	4
4	3
5	2
7	2
9	5
6	4

The set of all tuples that must be additionally deleted to preserve referential integrity when the tuple (2, 4) is deleted is: **[2005]**

(A) (3, 4) and (6, 4)
(B) (5, 2) and (7, 2)
(C) (5, 2), (7, 2) and (9, 5)
(D) (3, 4), (4, 3) and (6, 4)

3. Which of the following tuple relational calculus expression(s) is/are equivalent to $\forall t \in r\ (P(t))$?

I. $\neg \exists t \in r\ (P(t))$

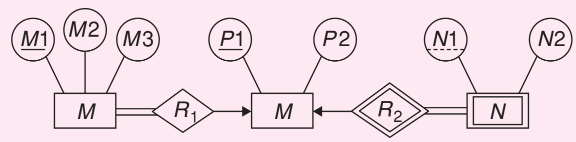
II. $\exists t \notin r\ (P(t))$

III. $\neg \exists t \in r\ (\neg P(t))$

IV. $\exists t \in r\ (\neg P(t))$ **[2008]**

(A) I only
(B) II only
(C) III only
(D) III and IV only

Common data for questions 4 and 5: Consider the following ER diagram:



4. The minimum number of tables needed to represent M, N, P, R_1, R_2 is? **[2008]**

(A) 2
(B) 3
(C) 4
(D) 5

5. Which of the following is a correct attribute set for one of the tables for the correct answer to the above question? **[2008]**

(A) $\{M1, M2, M3, P1\}$
(B) $\{M1, P1, N1, N2\}$
(C) $\{M1, P1, N1\}$
(D) $\{M1, P1\}$

6. Consider a relational table with a single record for each registered student with the following attributes.

1. *Registration_Num*: Unique registration number of each registered student
2. *UID*: Unique identity number, unique at the national level for each citizen
3. *BankAccount_Num*: Unique account number at the bank. A student can have multiple accounts or joint accounts. This attribute stores the primary account number
4. *Name*: Name of the student
5. *Hostel_Room*: Room number of the hostel

Which of the following options is incorrect? **[2011]**

(A) *BankAccount_Num* is a candidate key
(B) *Registration_Num* can be a primary key
(C) *UID* is a candidate key if all students are from the same country
(D) If S is a super key such that $S \cap UID$ is NULL then $S \cup UID$ is also super key.

7. Given the basic ER and relational models, which of the following is incorrect? **[2012]**

(A) An attribute of an entity can have more than one value
(B) An attribute of an entity can be composite
(C) In a row of a relational table, an attribute can have more than one value
(D) In a row of a relational table, an attribute can have exactly one value or a NULL value

8. An ER model of a database consists of entity types A and B. These are connected by a relationship R which does not have its own attribute, Under which one of the following conditions, can the relational table for R be merged with that of A? **[2017]**
(A) Relationship R is one-to-many and the participation of A in R is total.
(B) Relationship R is one-to-many and the participation of A in R is partial.
(C) Relationship R is many-to-one and the participation of A in R is total.
(D) Relationship R is many-to-one and the participation of A in R is partial.

9. In an Entity-Relationship (ER) model, suppose R is a many-to-one relationship from entity set $E1$ to entity set $E2$. Assume that $E1$ and $E2$ participate totally in R and that the cardinality of $E1$ is greater than the cardinality of $E2$.
Which one of the following is true about R? **[2018]**
(A) Every entity in $E1$ is associated with exactly one entity in $E2$.
(B) Some entity in $E1$ is associated with more than one entity in $E2$.
(C) Every entity in $E2$ is associated with exactly one entity in $E1$.
(D) Every entity in $E2$ is associated with at most one entity in $E1$.

Answer Keys

Exercises

Practice Problems 1

1. (i) B (ii) A (iii) A			**2.** A	**3.** B	**4.** D	**5.** D	**6.** D	**7.** B	**8.** B
9. B	**10.** A	**11.** A	**12.** B	**13.** D	**14.** A	**15.** A	**16.** C	**17.** B	**18.** B
19. C	**20.** D								

Practice Problems 2

1. D	**2.** B	**3.** C	**4.** A	**5.** A	**6.** D	**7.** C	**8.** A	**9.** A	**10.** A
11. A	**12.** C	**13.** C	**14.** A	**15.** D	**16.** A	**17.** A	**18.** A	**19.** C	**20.** B

Previous Years' Questions

1. B	**2.** C	**3.** D	**4.** B	**5.** A	**6.** A	**7.** C	**8.** C	**9.** A

Chapter 2

Structured Query Language

LEARNING OBJECTIVES

- *Relational algebra*
- *Select operator*
- *Project operator*
- *Set operators*
- *Union compatible relations*
- *Union operation*
- *Aggregate operators*
- *Correlated nested queries*
- *Relational calculus*
- *Tuple relational calculus*
- *Tuple relational calculus*
- *DML*
- *Super key*
- *SQL commands*

Relational Algebra

1. A set of operators (unary or binary) that take relation instances as arguments and return new relations.
2. Gives a procedural method of specifying a retrieval query
3. Forms the core component of a relational query engine
4. *SQL* queries are internally translated into *RA* expressions
5. Provides a framework for query optimization

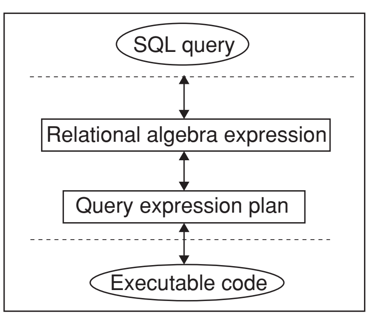
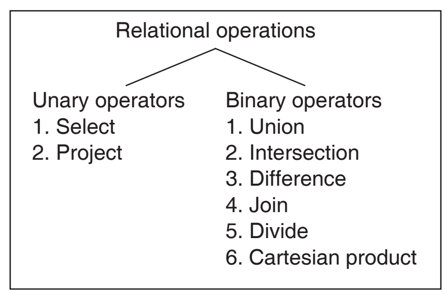


Figure 1 *Role of relational algebra in DBMS:*

Relational Operations

A collection of simple 'low-level' operations used to manipulate relations.

1. It provides a procedural way to query a database.
2. Input is one (or) more relations.
3. Output is one relation.



Select Operator (σ)

Select operator is an unary operator. It can be used to select those tuples of a relation that satisfy a given condition.

Notation: $\sigma_{\theta}(r)$

σ: Select operator(read as sigma)

θ: Selection condition

r: Relation name

Result is a relation with the same scheme as r consisting of the tuples in r that satisfy condition θ

Syntax: $\sigma_{\text{condition}}$ (relation)

Example:

Table 2.1 *Person*

Id	Name	Address	Hobby
112	John	12, SP Road	Stamp collection
113	John	12, SP Road	Coin collection
114	Mary	16, SP Road	Painting
115	Brat	18, GP Road	Stamp collection

$\sigma_{\text{Hobby = 'stamp. Collection'}}$(person)

The above given statement displays all tuples (or) records with hobby 'stamp collection'.

Output:

Id	Name	Address	Hobby
112	John	12, SP Road	Stamp collection
115	Brat	18, GP Road	Stamp collection

Selection condition can use following operators:
$<, \leq, >, \geq, =, \neq$

1. <attribute> operator <attribute>
2. <attribute> operator <constant>

Example: Salary ≥ 1000

3. <Condition> AND/OR <condition>

Example: (Experience > 3) AND (Age < 58)

4. NOT <condition>

Selection operation examples:

1. $\sigma_{\text{Id > 112 OR Hobby = 'paint'}}$ (person)
   It displays the tuples whose ID > 112 or Hobby is paint
2. $\sigma_{\text{Id > 112 AND Id < 115}}$ (person)
   It displays tuples whose ID is greater than 112 and less than 115
3. $\sigma_{\text{NOT (hobby = 'paint')}}$ (person)
   It displays tuples whose hobby is not paint
4. $\sigma_{\text{Hobby} \neq \text{'paint'}}$ (person)
   It displays tuples whose hobby is not paint, displays all tuples other than hobby paint.
   Selection operator: Produces table containing subset of rows of argument table which satisfies condition.

Project Operator (π)

The project operator is unary operator. It can be used to keep only the required attributes of a relation instance and throw away others.

Notation: $\pi_{A2,\ A2,\ \ldots\ Ak(r)}$ Where $A_1, A_2, \ldots A_K$ is a list L of desired attributes in the scheme of r.

Result = $\{(V_1, V_2, \ldots V_K)/V_i \in \text{DOM}(A_i), 1 \leq i \leq k$ and there is some tuple t in r, such that $t.A_1 = v_1, t.A_2 = v_2, \ldots t.A_K = V_K\}$

$\pi_{\text{Attribute List}}$ (Relation)

Take table 2.1 as reference.

1. π_{Name} (person)

Output:

Name
John
Mary
Bart

In the output table, John name has appeared once, project operation eliminated duplicates.

2. $\pi_{\text{Name, address}}$ (person)

Output:

Name	Address
John	12, SP Road
Mary	16, SP Road
Bart	18,GP Road

Expressions:

$\pi_{\text{id, name}}(\sigma_{\text{hobby = 'stamp collection' OR Hobby= 'coin collection'}}(\text{person}))$

Output:

Id	Name
112	John
115	Bart

The above given relational algebra expression gives Ids, names of a person whose hobby is either stamp collection (or) coin collection.

Set Operators

Union (∪), Intersection (∩), set difference (–) are called *set operators*. Result of combining two relations with a set operator is a relation ⇒ all its elements must be tuples having same structure. Hence scope of set operations is limited to union compatible relations.

Union Compatible Relations

Two relations are union compatible if

1. Both have same number of columns
2. Names of attributes are same
3. Corresponding fields have same type
4. Attributes with the same name in both relations have same domain.
5. Union compatible relations can be combined using Union, Intersection, and set difference.

Example:

Consider the given tables.
Person (SSN, Name, Address, Hobby)
Professor (Id, Name, office, phone)
person and professor tables are not union compatible.

Union

The result of union will be a set consisting of all tuples appearing in either or both of the given relations. Relations cannot contain a mixture of different kinds of tuples, they must be 'tuple – homogeneous'. The union in the relational algebra is not the completely general mathematical union; rather, it is a special kind of union, in which we require the two input relations to be of the same type.

The general definition of relational union operator:

Given are two relations '*a*' and '*b*' of the same type. The union of those two relations, a union *b*, is a relation of the same type, with body consisting of all tuples '*t*' such that '*t*' appears in a or *b* or both.

* Union operation eliminates duplicates.

Here is a different but equivalent definition:

Given are two relations '*a*' and '*b*' of the same type. The union of those two relations, *a* union *b*, is a relation of the same type, with body consisting of all tuples t such that t is equal to (i.e., is a duplicate of) some tuple in *a* or *b* or both.

Union Operation (U)

When union operation is applied on two tables it gives all the tuples in both without Repetition.

Example:

Table 2 *Result of union operation*

R

Roll. no.	Name	Semester	Percentage
22	Arun	7	45%
31	Bindu	6	55%
58	Sita	5	35%

S

Roll no.	Name	Semester	Percentage
28	Suresh	4	65%
31	Bindu	6	55%
44	Pinky	4	75%
58	Sita	5	35%

$R \cup S$

Roll no.	Name	Semester	Percentage
22	Arun	7	45%
31	Bindu	6	55%
58	Sita	5	35%
44	Pinky	4	75%
28	Sita	5	35%

Intersection

Like union, Intersection operator requires its operands to be of the same type. Given are two relations *a* and *b* of the same type, then, the intersection of those two relations, '*a*' INTERSECT '*b*', is a relation of the same type, with body consisting of all tuples t such that t appears in both '*a*' and '*b*'.

Intersection operation returns tuples which are common to both tables

Table 3 *Result of intersection operation*

$R \cap S =$

Roll no.	Name	Semester	Percentage
31	Bindu	6	55%
58	Sita	5	35%

Difference

Like union and intersection, the relational difference operator also requires its operands to be of the same type. Given are two relations '*a*' and '*b*' of the same type, Then, the difference between those two relations, '*a*' MINUS '*b*' (in that order), is a relation of the same type, with body consisting of all types t such that t appears in a and not *b*.

1. MINUS has a directionality to it, just as subtraction does in ordinary arithmetic (e.g., '6 – 3' and '3 – 6' are not the same thing)
2. Redundant duplicate rows are always eliminated from the result of UNION, INTERSECTION, EXCEPT operations.
3. SQL also provides the qualified variants UNION ALL, INTERSECT ALL and EXCEPT ALL, where duplicates are retained

Set difference operation returns the tuples in the first table which are not matching with the tuples of other table.

Table 4 *Result of R – S*

R – S =

Roll no.	Name	Semester	Percentage
22	Arun	7	45%

Table 5 *Result of S – R*

S – R =

Roll no.	Name	Semester	Percentage
28	Suresh	4	65%
44	Pinky	4	75%

* R – S ≠ S –R (both are different)

Example:

A

Supplier number	Supplier name	Status	City
SN1	MAHESH	40	HYDERABAD
SN3	SURESH	40	HYDERABAD

B

Supplier number	Supplier number	Status	City
SN3	SURESH	40	HYDERABAD
SN4	RAMESH	30	CHENNAI

UNION ($A \cup B$)

Supplier number	Supplier name	Status	City
SN1	MAHESH	40	HYDERABAD
SN3	SURESH	40	HYDERABAD
SN4	RAMESH	30	CHENNAI

INTERSECTION ($A \cap B$)

Supplier number	Supplier name	Status	City
SN3	SURESH	40	HYDERABAD

DIFFERENCE ($A - B$)

Supplier name	Supplier name	Status	City
SN1	MAHESH	40	HYDERABAD

DIFFERENCE ($B - A$)

Supplier name	Supplier name	Status	City
SN4	RAMESH	30	CHENNAI

Cartesian Product

The Cartesian product of two sets is the set of all ordered pairs such that in each pair, the first element comes from the first set and the second element comes from second set.

The result consists of all the attributes from both of the two input headings. We define the Cartesian product of two relations '*a*' and '*b*', as

'*a*' times '*b*', where *a* and *b* have no common attribute names (If we need to construct the Cartesian product of two relations that do have any such common attribute names, therefore, we must use the RENAME operator first to rename attributes appropriately).

The Cartesian product operation is also known as CROSS PRODUCT. This is also a binary set operation, but the relations on which it is applied need not to be union compatible. This operation is used to combine tuples from two relations in a combinational fashion.

Example:

R

A	B
X_1	X_2
X_3	X_4

S

C	D
Y_1	Y_2
Y_3	Y_4

$R \times S =$

A	B	C	D
X_1	X_2	Y_1	Y_2
X_1	X_2	Y_3	Y_4
X_3	X_4	Y_1	Y_2
X_3	X_4	Y_3	Y_4

Example: Transcript (StuId, coursecode, semester, grade) Teaching (ProfId, coursecode, semester)

$$\pi_{\text{stuId,coursecode}} (\text{Transcript}) \times \pi_{\text{profId,coursecode}} (\text{Teaching})$$

The above expression returns

Table 6 *Result of cross product*

Stu Id	Course code	Prof Id	Course code
...	...	...	...

Aggregate Operators

SQL Supports the usual aggregate operators COUNT, SUM, AVG, MAX, MIN, EVERY and ANY, but there are a few SQL-specific points.

1. The argument can optionally be preceded by the keyword DISTINCT, for example SUM (DISTINCT column -name) to indicate that duplicates are to be eliminated before the aggregation is done. For MAX, MIN, EVERY and ANY, however, DISTINCT has no effect and should not be specified.
2. The operator COUNT (*), for this DISTINCT is not allowed, and is provided to count all rows in a table without any duplicate elimination.
3. Any NULLS in the argument column are eliminated before the aggregation is done, regardless of whether DISTINCT is specified, except in the case of COUNT (*), where nulls behave as if they were values.
4. After NULLS if any have been eliminated, if what is left is an empty set, COUNT returns zero. The other operators return NULL.

AVG, MIN, MAX, SUM, COUNT

These functions operate on the multiset of values of column of a relation and returns a value.

1. Find the average account balance at the Perryridge branch.

Solution: SELECT AVG (balance) FROM account WHERE branch.name = 'perryridge'

2. Find the number of tuples in customer relation.

Solution: SELECT count (*) FROM customer

3. Find the number of depositors for each branch.

Solution: SELECT branch.name, COUNT (distinct customer-name) FROM depositor, account WHERE depositor. account-no = account account-no GROUPBY branch-name.

Nested Queries

Some queries require that existing values in the database be fetched and then used in a comparison condition.

Such queries can be conveniently formulated by using nested queries, which are complete SELECT – FROM –WHERE blocks within the WHERE clause of another query. The other query is called the *outer query*.

In-Comparison Operator

The comparison operator IN, which compares a value '*v*' with a set of values '*V*' and evaluates to TRUE if '*v*' is one of the elements in *V*.

Example: Consider the given database scheme and the statement:

EMPLOYEE

FNAME	INITIAL	LNAME	ENO	DOB	ADDRESS	SALARY	DNO

DEPARTMENT

D NAME	DNO	MANAGER-NO

DEPARTMENT–LOCATIONS

DNO	D-LOCATION

PROJECT

PNAME	PNO	P-LOCATION	DNO

WORKS–ON

ENO	PNO	HOURS

Example: Select distinct PNO from project where PNO IN (select PNO from project, department, employee where P. DNO = D. DNO AND MANAGER. NO = ENO AND LNAME = 'RAMYA')

The first query selects. The project numbers that have a 'Ramya' involved as manager, while the second selects the project numbers of projects that have a 'Ramya' involved as worker.

If a nested returns a single value, in such cases, it is permissible to use = instead of IN for the comparison operator.

In general, the nested query will return a table, Which is a set of multiset of tuples.

* SQL allows the use of tuples of values in comparisons by placing them within parentheses.

Example: SELECT DISTINCT ENO FROM WORKS - ON WHERE (PNO, HOURS) IN (SELECT PNO, HOURS FROM WORKS - ON WHERE ENO = 929).

This query will select the employee numbers of all employees who work on the same (PROJECT, HOURS) combination on some project a particular employee whose ENO = '929' works on. In this example, the IN operator compares the subtuple of values in parentheses (PNO, HOURS) for each tuple in works on with the set of union-compatible tuples produced by the nested query.

Correlated nested queries: Nested queries can be evaluated by executing the sub query (or) Inner query once and substituting the resulting value (or) values into the WHERE clause of the outer query.

1. In co-related nested queries, the inner query depends on the outer query for its value.
2. Sub-query is executed repeatedly, once for each row that is selected by the outer query.
3. A correlated subquery is a sub query that contains a reference to a table that also appears in the outer query.

Example: Consider the following correlated nested query:

SELECT *
FROM table1
WHERE col1 ≥ ALL
(SELECT col1 FROM table2 WHERE table2. col2 = table1. col2)

1. The subquery contains reference to a column of table1, even though the sub-queries FROM clause does not mention a table table1
2. SQL has to check outside the sub-query and find Table 1 in the outer query
3. Suppose that Table 1 contains a row where col1 = 3 and col2 = 4 and Table 2 contains a row where col1 = 5 and col2 = 4
4. The expression

   WHERE col1 ≥ All (SELECT col1 FROM table2

   3 ≥ 5 (false)

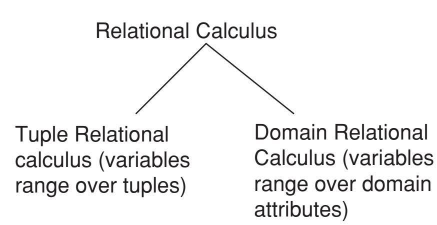
   (WHERE condition TRUE) Table1. col2 = table2. col2 4 = 4

   So the expression as a whole is FALSE.
5. It is evaluated from outside to inside

Relational Calculus

Relational calculus can define the information to be retrieved

1. In this, there is no specific series of operations.
2. Relational algebra defines the sequence of operations.
3. Relational calculus is closer to how users would formulate queries, in terms of information requirements, rather than in terms of operations.



4. Relational calculus is based on predicate logic, gives the usual quantifiers to construct complex queries.

Tuple Relational Calculus

Example: Employee

E Id	F Name	L Name	S alary
201	John	James	3000
202	Brat	Frank	2000
203	Mary	Jennifer	3000
204	Adam	Borg	2000
205	Smith	Joyce	1000

Query 1: Display the employees whose salary is above 2000.

$\{E \mid \exists E \in \text{Employee}(E.\text{salary} > 2000)\}$

Output:

E Id	F Name	L Name	Salary
201	John	James	3000
203	Mary	Jennifer	3000

Query 2: Display the employee Ids whose salary is above 1000 and below 3000.

$\{P \mid \exists E \in \text{Employee } ((E.\text{salary} > 1000 \wedge E.\text{salary} < 3000) \wedge P.\text{EId} = E.\text{EId})\}$

P is a table, in which EIds are stored, from tuples which satisfies the given condition.

TUPLE RELATIONAL CALCULUS

A non-procedural language, where each query is of the form $\{t \mid p(t)\}$. It is a set of all tuples *t* such that predicate *p* is true for *t*, *t* is a tuple variable, *t* [*A*] denotes the value of tuple '*t*' on attribute A.

$$\{T \mid p\ (T)\}$$

T is a tuple and *P* (*T*) denotes a formula in which tuple variable *T* appears.

1. $\forall \times (P(X))$

   $\forall$ is called the universal or 'for all' quantifier because every tuple in 'the universe of' tuples must make *F* true to make the quantified formula true.
   Only true if $p(X)$ is true for every *X* in the universe.

   Example: $\forall \times (\text{x.color} = \text{'Red'})$
   means everything that exists is red.

   Example: $\forall \times ((x \in \text{Boats}) \Rightarrow (X.\text{ color} = \text{'Red'}))$
   '$\Rightarrow$' is a logical implication. $a \Rightarrow b$ means that if a is true, *b* must be true
   (or)
   $\exists \times \in \text{Boats } (X.\text{color} = \text{'Red'})$
   For every '*x*' in the boats relation, the color must be red.

2. $\exists \times (P(X))$

   $\exists$ is called the *existential* or '*there exists*' quantifier because any tuple that exists in 'the universe of' tuples may take *F* true, to make the quantified formula true.

   Example: $\exists \times ((X \in \text{Boats}) \wedge \text{X.color}) = \text{'Red'})$
   There exists a tuple *X* in the boats relation whose color is red.
   (or)
   $\exists \times \in \text{Boats (X.color} = \text{'Red')}$

Examples:

1. Find all sailors with rating above 8.

Sailors:

Sid	Sname	Rating	Age
28	Yuppy	9	35
35	Rubber	8	55
44	grove	5	35
58	rusty	10	35

Solution: $\{s \mid s \in \text{sailors} \wedge s.\text{ rating} > 8\}$

Output

$R =$

Sid	Sname	Rating	Age
28	yuppy	9	35
58	rusty	10	35

2. Find names and ages of sailors with rating > 8.

Solution: $\{R \mid \exists S \in \text{sailors (s.rating} > 8 \wedge \text{R.sname} = \text{s.name} \wedge \text{R.age} = \text{s.age})$

Output:

sname	age
yuppy	35
rusty	35

Join Operation in Tuple Relational Calculus

Examples:

3. Find sailors rated > 7 who have reserved boat = 103.

Solution: $\{S \mid S \in \text{sailors} \wedge \text{s.rating} > 7 \wedge \exists R\ (R \in \text{reserves} \wedge \text{R.sid} = \text{s.sid} \wedge \text{R.bid} = 103)\}$

4. Find sailors rated > 7 who have reserved a red boat.

Solution: $\{S \mid S \in \text{sailors} \wedge \text{s.rating} > 7 \wedge \exists R\ (R \in \text{reserves} \wedge \text{R.sid} = \text{s.sid} \wedge \exists B(\text{ Boats} \wedge \text{B.bid} = \text{R.bid} \wedge \text{B.color} = \text{'Red'}))\}$

Division Operation in Tuple Relational Calculus

Examples

1. Find sailors who have reserved all boats.

Solution: $\{S \mid S \in \text{sailors} \wedge \forall B \in \text{boats } (\exists R \in \text{reserves (s.sid} = \text{R.sid} \wedge \text{B.bid} = \text{R.bid}))\}$

Domain Relational Calculus

1. Tuple relational and domain relational are semantically similar.
2. In *TRC*, tuples share an equal status as variables, and field referencing can be used to select tuple parts.

3. In *DRC* formed variables are explicit.
4. *DRC* query has the following form.
   $\{<x_1, x_2, \ldots x_n> / P(<x_1, x_2, \ldots, x_n>\}$
   Result included all tuples $<x_1, x_2, \ldots x_n>$
   That make the formula $p(<x_1, x_2, \ldots x_n>)$ true.
5. Formula given in *DRC* is recursively defined. First start with simple atomic formula and expand the formulas by using the logical connectives.
6. A variable that is not bound is free.
7. The variable $X_1, X_2, \ldots, X_n$ that appear in the left side of '/' must be the only free variable in the formula $p(\ldots)$.

Example: Consider the employee table given in the above Example.
The use of quantifiers $\exists x$ and $\forall x$ in a formula is said to bind x

Query 1: Display the Employees whose salary is above 2000?

$$\{<I, F, L, S>/<I, F, L, S> \in \text{Employee} \wedge S > 2000\}$$

Query 2: Display the EIds of Employees, whose salary is above 1000 and below 3000?

$$\{<I>/\exists F, L, S(<I, F, L, S> \in \text{Employee} \wedge (S > 1000 \wedge S < 3000))\}$$

SQL (Structured Query Language)

When a user wants to get some information from a database file, he/she can issue a query. A query is a user-request to retrieve data (or) information with a certain condition. SQL is a query language that allows user to specify the conditions (instead of algorithms)

Concept of SQL

The user specifies a certain condition. The program will go through all the records in the database file and select those records that satisfy the condition. The result of the query will be stored in the form of a table.

Features of SQL

1. SQL is a language of database. It includes database creation, deletion, fetching rows and modifying rows.
2. SQL is a structured query language for storing, manipulating and retrieving data stored in relational database.
3. It allows users to describe the data.
4. It allows users to create and drop database and tables.
5. It allows users to create view, functions in a database.
6. Allows users to set permissions on tables and views.
7. The standard SQL commands to interact with relational database are CREATE, SELECT, UPDATE, INSERT, DROP and DELETE.
8. The commands can be classified as follows:
   - *Data query language*: SELECT – It retrieves particular rows which satisfies the given condition.
   - *Data definition language*: CREATE, ALTER, DROP.
   - *Data manipulation language*: INSERT, UPDATE, DELETE

Features

1. Strong data protection
2. Robust transactional support
3. High performance
4. High availability
5. Security and flexibility to run anything
6. Easy to manage
7. User friendly

General Structure

SELECT ... FROM ... WHERE
SQL is divided into two languages

1. DML (data manipulation language)
   SELECT: Extracts data from a database table.
   UPDATE: Updates data in a database table.
   DELETE: Deletes data from a database table.
   INSERT INTO: Inserts new data into database table.
2. DDL (data definition language)
   CREATE TABLE - creates a new database table.
   ALTER TABLE - Alters a database table.
   DROP TABLE - deletes a database table.
   CREATE INDEX - Creates an index (search key).
   DROP INDEX - Deletes an index.
   RENAME – Changes the name of the table.

Types of keys:

1. Candidate key
2. Primary key
3. Super key
4. Foreign key
5. Composite primary key

In relational database, 'keys' play a major role. Keys are used to establish and identify relation between relations (or) tables.

Keys are used to ensure that each record within a table can be uniquely identified by combining one or more fields (or) column headers within a table.

Candidate key: A candidate key is a column or set of columns in a table that contains unique values, with these we can uniquely identify any database record without referring to any other columns data.

Each table may have one or more candidate keys, among the available candidate keys, one key is preserved for primary key.

A candidate key is a subset of a super key.

Example: Student

StudentId	First name	Last name	Course Id
CS00345	Jim	Black	C2
CS00254	Carry	Norris	C1
CS00349	Peter	Murray	C1
CS00196	John	Mc Cloud	C3
CS00489	Brat	Holland	C4
CS00553	Mary	Smith	C5

In the above table, we have studentId that uniquely identifies the students in a student table. This would be a candidate key.

In the same table, we have student's first name and last name, which are also candidate keys.

1. If we combine first name and last name then also it becomes a candidate key.
2. Candidate key must (have)
   - Unique values
   - No null values
   - Minimum number of fields to ensure uniqueness.
   - Uniquely identify each record in the table.
3. The candidate keys which are not selected for primary key are known as secondary keys or alternative keys.

Primary key: A primary key is a candidate key that is most suitable (or) appropriate to become main key of the table.

1. It is a special relational database table column ((or) combination of columns)
2. Primary key main features are
   - It must contain a unique value for each row of data.
   - It cannot contain null values.

Example: We can choose primary key as studentId which is mentioned in the table given in above example.

Composite primary key: A key that consists of two or more attributes that uniquely identify an entity is called *composite key* or *composite primary key.*

Example: Customer

Cust-Id	Order-Id	Sale-details
C1	O – 2	Sold
C1	O – 3	Sold
C2	O – 2	Sold
C2	O – 3	Sold

Composite primary key is {cust-Id, order-Id}

Super key: A super key is a combination of attributes that can be uniquely used to identify a database record. A table can have any number of super keys.

1. Candidate key is a special subset of super keys.

Example: Customer

Customer name	Customer Id	SSN	Address	DOB

Assume that we can guarantee uniqueness only for SSN field, then the following are some of the super keys possible.

1. {Name, SSN, DOB}
2. {ID, Name, SSN}

In a set of attributes, there must be at least one key (could be primary key or candidate key)

Foreign key: A foreign key is a column or group of columns in a relational database table that provides connectivity between data in two tables.

1. The majority of tables in a relational database system adhere to the concept of foreign key.
2. In complex databases, data must be added across multiple tables, thus the link or connectivity has to be maintained among the tables.
3. The concept of Referential Integrity constraint is derived from Foreign key.

Example: Emp

EId	EName	Dept – No

Dept

Dept-No	DName

In the above specified tables, Dept-No is common to both the tables, In Dept table it is called as primary key and in Emp table it is called as *foreign key*.

These two tables are connected with the help of 'Dept-No' field

1. For any column acting as a foreign key, a corresponding value should exist in the link (or) connecting table.
2. While inserting data and removing data from the foreign key column, a small incorrect insertion or deletion destroys the relationship between the two tables.

SQL Commands

SELECT statement

The most commonly used SQL command is SELECT statement. The SQL SELECT statement is used to query or retrieve data from a table in the database. A query may retrieve information from specified columns or from all of the columns in the table. To create a simple SQL SELECT statement, you must specify the column(s) names and the table name.

Syntax: SELECT column-name (s) from table name

Example: Persons

Lastname	Firstname	Address	City
Hansen	Ola	SpRoad,-20	Hyd
Svendson	Tove	GPRoad,-18	Secbad
Petterson	Kari	RpRoad,-19	Delhi

1. SELECT lastname FROM persons

   Output:

Lastname
Hansen
Svendson
Petterson

2. SELECT lastname, firstname FROM persons

Output:

Lastname	Firstname
Hansen	Ola
Svendson	Tove
Petterson	Kari

DISTINCT statement

Returns distinct values. It eliminates duplicate values.

Syntax: Select DISTINCT column_name (s) from table-name

Example: Orders

Company	Order.No
IBM	3412
DELL	5614
WIPRO	4412
DELL	4413

1. SELECT company FROM orders

Output:

Company
IBM
DELL
WIPRO
DELL

2. SELECT DISTINCT company FROM orders

Company
IBM
DELL
WIPRO

WHERE statement

The WHERE clause is used when you want to retrieve specific information from a table excluding other irrelevant data. By using WHERE clause, we can restrict the data that is retrieved. The condition provided in the WHERE clause filters the rows retrieved from the table and gives only those rows which were expected. WHERE clause can be used along with SELECT, DELETE, UPDATE statements.

The WHERE clause is used to specify a selection condition. All conditions are specified in this clause.

Syntax: SELECT column FROM table WHERE column operator value.

Operates used in where clause:
=
< > (not equal) (or) ! =
>
<
> =
< =
BETWEEN - Between an inclusive range.
LIKE - Search for a pattern

Example: Persons

Lastname	Firstname	Address	City	Year
Hansen	Ola	SPRoad, 16	Hyd	1956
Svendson	Tiva	GPRoad, 18	Sec	1977
Smith	Ole	RPRoad, 19	Hyd	1986
Petterson	Kari	SPRoad, 17	Sec	1985

1. SELECT * FROM persons
   Output: It displays the entire table

   2. SELECT * FROM persons WHERE city = 'Hyd'

   Output:

Lastname	Firstname	Address	City	Year
Hansen	Ola	SPRoad, 16	Hyd	1956
Smith	Ole	RPRoad, 19	Hyd	1986

LIKE condition

The LIKE operator is used to list all rows in a table whose column values match a specified pattern. It is useful when you want to search rows to match a specific pattern, or when you do not know the entire value. For this purpose, we use a wildcard character '%'.

The LIKE condition is used to specify a search for a pattern in a column.

A '%' sign can be used to define wildcards (missing letters in the pattern) both before and after the pattern.

Syntax: SELECT column FROM table WHERE column LIKE pattern

1. SELECT * FROM persons WHERE Firstname LIKE 'O%'

Solution: SQL statement will return persons with first names that start with a letter '*O*'

Output:

Lastname	Firstname	Address	City	Year
Hansen	Ola	SPRoad, 16	Hyd	1956
Smith	Ole	RPRoad, 19	Hyd	1986

2. SELECT * FROM persons WHERE Firstname LIKE '%a'

Solution: SQL statement will return persons whose first name ends with letter 'a'.

Output:

Last name	First name	Address	City	Year
Hansen	Ola	SPRoad, 16	Hyd	1956
Svendson	Tiva	GPRoad, 18	Sec. bad	1977

3. SELECT * FROM persons WHERE firstname LIKE '%la%'

Solution: SQL statement returns persons whose firstname contains 'la'. The word sequence 'la' may come at any place in the word.

Output:

Last name	First Name	Address	City	Year
Hansen	Ola	SPRoad, 16	Hyd	1956

String operations

1. '%idge%' matches 'Rockridge', 'Ridgeway', 'Perryridge'.
2. '______' matches a string of three characters.
3. '______%' matches a string of at least rhree characters.

INSERT INTO statement

This statement is used to insert new rows into a table. While inserting a row, if you are adding values for all the columns of the table you need not specify the column(s) name in the SQL query. But you need to make sure the order of the values is in the same order as the columns in the table. When adding a row, only the characters or data values should be enclosed with single quotes and ensure the data type of the value and the column matches. One can specify the columns for which you want to insert data

Syntax: INSERT INTO table-name (column1, column2 ...) VALUES (value 1, value2 ...)

1. INSERT INTO persons VALUES ('Hetland', 'Camilla', 'HPRoad 20', 'Hyd')

Output:

Last name	First Name	Address	City
Hansen	Ola	S.P Road 16	Hyd
Svesdon	Tiva	GP Road 18	Secbad
Smith	Ole	RP Road 19	Hyd
Petterson	Kari	SP Road 17	Secbad
Hetlan	Camilla	HPRoad, 20	Hyd

2. Insert data into specified columns
   INSERT INTO persons (Lastname, Address) VALUES ('Rasmussen', 'street 67')

Output:

Last name	First Name	Address	City
Hansen	Ola	SP Road 16	Hyd
Svesdon	.Tiva	GP Road 18	Secbad
Smith	Ole	RP Road 19	Hyd
Petterson	Kari	SP Road 17	Secbad
Hetlan	Camilla	HP Road 20	Hyd
Rasmussen		Street 67	

UPDATE

The update statement is used to modify the data in a table.

Syntax: UPDATE table_name
SET Column_name = new_value
WHERE column_name = some_value.

1. Add a first name (Nine) to the person whose last name is 'Rasmussen'?

Solution: UPDATE person SET Firstname = 'Nine' WHERE Lastname = 'Rasmussen'

2. Change the address and add the name of the city as Hyd of a person with last name Rasmussen?

Solution: UPDATE person
SET Address = 'street 12',
city = 'Hyd'
WHERE Lastname = 'Rasmussen'

DELETE statement

The DELETE statement is used to delete rows from a table. The WHERE clause in the SQL delete command is optional, and it identifies the rows in the column that gets deleted. If you do not include the WHERE clause, all the rows in the table will be deleted.

Syntax: DELETE FROM table_name
WHERE column_name = some_value

1. Delete all rows?

Solution: DELETE * FROM table_name

Cartesian product

The Cartesian product of two sets is the set of all ordered pairs of elements such that the first element in each pair belongs to the first set and the second element in each pair belongs to the second set. It is denoted by cross(X).

For example, given two sets:

$S1 = \{1, 2, 3\}$ and $S2 = \{4, 5, 6\}$

The Cartesian product $S1 \times S2$ is the set

$\{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$

Example:

Female	
Name	Job
Komal	Clerk
Ankita	Sales

Male	
Name	Job
Rohit	Clerk
Raju	Sales

Assume that the tables refer to male and female staff, respectively. Now, in order to obtain all possible inter-staff marriages, the cartesian product can be taken.

Male-Female

Female name	Female job	Male name	Male job
Komal	Clerk	Rohit	Clerk
Komal	Clerk	Raju	Sales
Ankita	Sales	Rohit	Clerk
Ankita	Sales	Raju	Sales

Examples:

1. Find the Cartesian product of borrower and loan?

Solution: SELECT * FROM borrower, loan

2. Find the name, loan-no, and loan amount of all customers having a loan at the Perryridge branch?

Solution: SELECT customer_name, borrower. loan_ number, amount FROM borrower, loan WHERE borrower. loan-no = Loan.loan_no AND branch_name = 'perryridge'

3. Find all loan numbers for loans made at the perryridge branch with loan amount greater than 1200?

Solution: SELECT loan-no FROM loan WHERE branch. name = 'perryridge' AND amount > 1200

Comparison operator

Relation algebra includes six comparison operators (=, < >, <, >, < =, > =). These are proposition forming operators on terms. For example, $x <> 0$ asserts that x is not equal to 0. It also includes three logical operators (AND, OR, NOT). These are proposition forming operators on propositions.

Example: $x > 0$ and $x < 8$
Comparison results can be combined using the logical connections AND, OR NOT

1. Find the loan-no of those loans with amounts between 90,000 and 1,00,000?

Solution: SELECT loan-no FORM loan WHERE amount BETWEEN 90,000 AND 1,00,000. SQL allows renewing relations and attributes using 'AS' clause

2. Find the name, loan-no and loan amount of all customers, rename the column name loan-no as loan.id?

Solution: SELECT customer.name, borrower.loan no AS loan.id, amount FROM borrower, loan, WHERE borrower. loan-no = loan.loan-no

Ordering of Tuples

It lists the tuples in alphabetical order.

Example: List in alphabetic order, the names of all customers having a loan in Perryridge branch?

Solution: SELECT customer-name FROM borrower WHERE branch.name = 'perryridge' ORDERBY customer-name

We may specify 'desc' for descending order (or) 'asc' for ascending order. - 'asc' is default.

Example: ORDERBY customer-name desc.

Join (⋈)
SQL Join is used to get data from two (or) more tables, which appear as single table after joining.

1. Join is used for combining columns from two or more tables by using values common to both tables.
2. Self Join: A table can also join to itself is known as self join. Types of JOIN
   1. INNER JOIN
   2. OUTER JOIN
      (i) LEFT OUTER JOIN
      (ii) RIGHT OUTER JOIN
      (iii) FULL OUTER JOIN

1. INNER JOIN (or) EQUI JOIN

It is a simple JOIN in which result is based on matching tuple, depending on the equality condition specified in the query.

Syntax: SELECT Column-names FROM table name1 INNER JOIN table name 2 WHERE table name 1. Column name = table name 2.column – name.

Example: Class

SID	Name
11	Ana
12	Bala
13	Sudha
14	adam

Info

SID	City
11	Bangalore
12	Delhi
13	Hyderabad

SELECT *
FROM Class INNER JOIN Info
WHERE Class.SID = Info.SID

Result:

SID	Name	SID	City
11	Ana	11	Banglore
12	Bala	12	Delhi
13	Sudha	13	Hyderabad

NATURAL JOIN:
NATURAL JOIN is a type of INNER JOIN which is based on column having same name and same data type present in two tables on which join is performed.

Syntax: SELECT *
FROM table-name1 NATURAL JOIN table-name 2

Example: Consider the tables class and Info, and the following Query

SELECT *
FROM class NATURAL JOIN Info

Result:

SID	Name	City
11	Ana	Bangalore
12	Bala	Delhi
13	Sudha	Hyderabad

Both tables being joined have SID column (same name and same data type), the tuples for which value of SID matches in both the tables, appear in the result.

Dangling tuple: When NATURAL JOIN is performed on two tables, there would be some missing tuples in the result of NATURAL JOIN

Those missing tuples are called *Dangling tuples*. In the above example, the number of dangling tuples is 1 that is

14	Adam

OUTER JOIN: Outer Join is based on both matched and unmatched data.

LEFT OUTER JOIN: Left outer Join returns the tuples available in the left side table with the matched data of 2 tables and null for the right tables column.

Example: Consider the table's class and Info

SELECT *
FROM class LEFT OUTER JOIN Info
ON(class.SID = Info. SID)

Result:

SID	Name	City
11	Ana	Banglore
12	Bala	Delhi
13	Sudha	Hyderabad
14	adam	NULL

RIGHT OUTER JOIN: RIGHT OUTER JOIN returns the tuples available in the Right side table with the matched data of 2 tables and NULL for the left table's column.

Example: Class 1

SID	Name
16	Arun
17	Kamal

Info 1

SID	City
16	Chennai
17	Noida

Query:

SELECT *
FROM Class1 RIGHT OUTER JOIN Info1
ON(class1.SID = Info1.SID)

Result:

SID	Name	City
16	Arun	Chennai
18	NULL	Noida

FULL OUTER JOIN: The full outer Join returns the tuples with the matched data of two tables, remaining rows of both left table and Right table are also included.

Example: Consider the tables class 1 and Info1

Query:

SELECT *
FROM class1 FULL OUER JOIN Info1
ON(class1.SID = Info1.SID)

Result:

SID	Name	City
16	Arun	Chennai
17	Kamal	NULL
18	NULL	Noida

ALTER command: ALTER command is used for altering the table structure

1. It is used to add a new column to existing table.
2. To rename existing column.
3. ALTER is used to drop a column.
4. It is used to change data type of any column or modify its size.

Add new column: By using alter command, we can add a new column to the table.

Syntax: ALTER table table-name ADD(column-name data type).

Example: Consider a student table.

SID	S Name	Grade

Add a new column called address

```
ALTER table student ADD (address char);
```

Example: Add multiple columns, parent-name, course-Name, date-of-birth to student table.

```
ALTER table student ADD (parent-name
varchar(60), course-Name varchar(20),
date-of-birth date);
```

Example: Change the data type of column address to varchar?

```
ALTER table student modify(address varchar(30))
```

Example: Rename a column address to Location

```
ALTER table student rename address to Location
```

TRUNCATE command: Truncate command removes all tuples from a table, this command will not destroy the tables structure.

Syntax: Truncate table table-name

DROP Command: DROP query removes a table completely from database. This command will destroy the table structure.

Syntax: Drop table table-name

Rename: This command is used to rename a table.

Syntax: Rename table old-table-name to new-table-name.

Example: Rename table Employee to New-Employee.

DROP a column: Alter command can be combined with DROP command to remove columns from a table.

Syntax: alter table table-name DROP(column-name)

Example: Alter table student DROP (grade)

Exercises

Practice Problems I

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Consider the given table called *Persons*

P-Id	Lastname	Firstname	Address	City
1	Hansen	ola	Timoteivn -10	Sandnes
2	Svendson	Tove	Brazil-50	Sandnes
3	Petterson	Kari	Storgt-20	Stavanger
4	Joseph	ole	Brazil-20	Sandnes

Write a query to select the persons with first name 'Tove' and last name 'Svendson'?

(A) SELECT *
FROM Persons
WHERE first-name='tove'
AND last-name='svendson'

(B) SELECT *
FROM Persons
WHERE first-name='tove'
OR last-name='svendson'

(C) SELECT first-name
FROM Persons
WHERE first-name='tove'
AND last-name='svendson'

(D) SELECT last-name
FROM Persons
WHERE first-name='tove'
AND last-name='svendson'

2. Write a query to select only the persons with last name 'Svendson' and the first name equal to 'Tove' or 'ola'?

(A) SELECT *
FROM Persons
WHERE last-name='svendson'
AND first-name='tove'

(B) SELECT *
FROM Persons
WHERE last-name='svendson'
AND (first-name='tove' OR first-name='ola')

(C) SELECT *
FROM Persons
WHERE last-name='svendson'
AND (first-name='tove' AND first-name='ola')

(D) SELECT *
FROM Persons
WHERE last-name='svendson'
OR (first-name='tove' AND first-name='ola')

3. Write an SQL statement to add a new row, but only in specified columns, for the persons table add data into columns 'P-Id', 'Last name' and the 'First name' with values (5, Teja, Jakob)?

(A) INSERT INTO Persons VALUES(5,'teja','jakob')
(B) INSERT INTO Persons VALUES(5,teja,jakob)
(C) INSERT INTO Persons (P-Id, last-name, first-name) VALUES(5,'teja','jakob')
(D) INSERT INTO Persons(P-Id, last-name, first-name) VALUES(5,teja,jakob)

4. Write an SQL statement:

(i) To select the persons living in a city that starts with 'S' from the 'Persons' table?

(A) SELECT *
FROM Persons
WHERE city LIKE 's__'.

(B) SELECT *
FROM Persons
WHERE city LIKE 's%'.

(C) SELECT *
FROM Persons
WHERE city LIKE '%s'.

(D) SELECT *
FROM Persons
WHERE city LIKE '_s%'.

(ii) To select the persons living in a city that contains the pattern 'tav' from 'Persons' table?

(A) SELECT *
FROM Persons
WHERE city LIKE '_tav_'.

(B) SELECT *
FROM Persons
WHERE city LIKE '_tav%'.

(C) SELECT *
FROM Persons
WHERE city LIKE '%tav_'.

(D) SELECT *
FROM Persons
WHERE city LIKE '%tav%'.

(iii) To select the persons whose last name starts with '*b*' or '*s*' or '*p*' ?

(A) SELECT *
FROM Persons
WHERE last-name LIKE '*b-s-p*'

(B) SELECT *
FROM Persons
WHERE last-name LIKE '*b%s%p*'

(C) SELECT *
FROM Persons
WHERE last-name LIKE '*b%s%p%*'

(D) SELECT *
FROM Persons
WHERE last-name LIKE '[*bsp*]%'

5. Consider the given table called 'Persons'

P-Id	Last-name	First-name	Address	City
1	Hansen	ola	Timoteivn-10	Sandnes
2	Svendson	Tove	Brazil-50	Sandnes
3	Petterson	Kari	Storgt-20	Stavanger

and the 'Orders' table

O-Id	Order No	P-Id
11	77895	3
12	44678	3
13	22456	1
14	24562	1
15	34764	5

perform NATURAL JOIN operation on both the tables and what is are the O_Id's displayed in the result?

(A) 11, 12, 13 (B) 11, 13, 14
(C) 11, 12, 13, 14 (D) 12, 13, 14

6. Write an SQL to perform FULL JOIN operation on both 'Person' and 'Orders' tables and What is the number of tuples in the Result?

(A) 4 (B) 5
(C) 6 (D) 7

7. Consider the given table 'Result'.

Student Name	Marks
A	55
B	90
C	40
D	80
E	85
F	95
G	82

(i) Find out the students who have scored more than 80 marks, and display them in descending order according to their marks?

(A) SELECT student-name,marks
FROM Result
WHERE marks > 80
ORDERBY marks DESC

(B) SELECT *
FROM Result
WHERE marks > 80
ORDERBY marks DESC

(C) SELECT student-name,marks
FROM Result
WHERE marks > 80
ORDERBY marks

(D) (A) and (B)

(ii) From the above table, find out the top-most three students.

(A) SELECT student-name
FROM Result
ORDERBY marks DESC > 3

(B) SELECT student-name
FROM Result
ORDERBY marks DESC = 3

(C) SELECT student-name
FROM Result
ORDERBY marks DESC limit 3

(D) None of these

8. From the table 'Results', Identify the suitable SQL expression?

(i) Find out the student Who stood 2nd?

(A) SELECT student-name
FROM Result
ORDERBY marks DESC limit 2

(B) SELECT student-name
FROM Result
ORDERBY marks DESC limit 1,1

(C) SELECT student-name
FROM Result
ORDERBY marks DESC limit 1,2

(D) SELECT student-name
FROM Result
ORDERBY marks DESC limit 2,1

(ii) Find out how many students scored > = 80.

(A) SELECT COUNT(*)
FROM Result
WHERE marks > = 80

(B) SELECT COUNT
FROM Result
WHERE marks > = 80

(C) SELECT SUM(*)
FROM Result
WHERE marks > = 80

(D) SELECT SUM
FROM Result
WHERE marks > = 80

9. Consider the given tables:

Customer

Customer name	Customer street	Customer city
Sonam	Mirpurroad	Dhaka
Sonam	Aga KhaRoad	Bogra
Anusha	XYZRoad	Kanchi
Nandy	MirpurRoad	Dhaka

Account

Account number	Customer name	Balance
A-101	Anusha	1000
A-102	Anusha	1500
A-103	Sonam	2000
A-104	Nandy	2500

From the customer table, find out the names of all the customers who live in either Dhaka or Bogra?

(A) SELECT customer-name
FROM customer
WHERE customer-city='dhaka' OR customer-city='bogra'

(B) SELECT customer-name
FROM customer
WHERE customer-city=dhaka OR customer-city='bogra'

(C) SELECT customer-name
FROM customer
WHERE customer-city='dhaka' AND customer-city='bogra'

(D) SELECT customer-name
FROM customer
WHERE customer-city='dhaka' EXIST customer-city='bogra'

10. Consider the given tables

Loan

Loan Number	Branch Name	Amount
L-101	Dhaka	1000
L-103	Khulna	2000

Borrower:

Customer name	Loan number
Sonam	L-101
Nandy	L-103
Anusha	L-103

(i) What are the number of tuples present in the result of cross product of the above two tables?

(A) 4 (B) 5
(C) 6 (D) 7

(ii) Find the loan-numbers from loan table where branch-name is Dhaka?

(A) SELECT loan-number
FROM loan
WHERE branch-name='dhaka'

(B) SELECT loan-number
FROM branch-name='dhaka'

(C) SELECT loan-number
FROM Loan × Borrower

(D) Both (A) and (C)

11. (i) Find all customers who have only accounts but no loans.

(A) SELECT customer-name
FROM depositor LEFT OUTER JOIN Borrower ON
Depositor.customer-name=Borrower.customer-name
WHERE loan-number IS NULL

(B) SELECT customer-name
FROM depositor LEFT OUTER JOIN Borrower ON
Depositor.customer-name = Borrower.customer-name
WHERE loan-number=NULL

(C) SELECT customer-name
FROM depositor RIGHT OUTER JOIN Borrower ON
Depositor.customer-name=Borrower.customer-name
WHERE loan-number IS NULL

(D) SELECT customer-name
FROM depositor RIGHT OUTER JOIN Borrower ON
Depositor.customer-name=Borrower.customer-name
WHERE loan-number=NULL

(ii) Find the names of all customers who have either an account or loan but not both.

Borrower

Customer name	Loan no.
Sonam	L-101
Sonam	L-102
Anusha	L-103

Depositor

Customer name	Account no.
Anusha	A-102
Sonam	A-103
Nandy	A-104

(A) SELECT customer name
FROM depositor FULL OUTER JOIN
Borrower ON
Depositor.customer-name=Borrower.customer-name
WHERE loan-number IS NULL OR Account-number=NULL

(B) SELECT customer-name
FROM depositor FULL OUTER JOIN
Borrower ON
Depositor.customer-name = Borrower.customer-name
WHERE loan-number IS NULL OR Account-number IS NULL

(C) SELECT customer-name
FROM depositor FULL OUTER JOIN
Borrower ON
Depositor.customer-name = Borrower.customer-name
WHERE loan-number = NULL OR Account-number = NULL

(D) SELECT customer-name
FROM depositor FULL OUTER JOIN Borrower ON
Depositor.customer-name = Borrower.customer-name
WHERE loan-number=NULL OR Account-number IS NULL

12. Consider the following 'employee' table

Employee name	Branch name	Branch city	Salary
A	DU	Dhaka	1000
B	DU	Dhaka	2000
C	BUET	Dhaka	3000
D	KUET	Khulna	4000
E	KU	Khulna	5000
F	RU	Rajshahi	6000

(i) Find the distinct number of branches appearing in the employee relation.

(A) SELECT COUNT(branch-name)
FROM Employee

(B) SELECT COUNT(DISTINCT branch-name)
FROM Employee

(C) SELECT DISTINCT COUNT(branch-name)
FROM Employee

(D) SELECT COUNT(*)
FROM Employee

(ii) Find the total salary of all employees at each branch of the bank.

(A) SELECT branch-name, SUM(salary)
FROM Employee
GROUP BY Branch-city

(B) SELECT branch-name, SUM(salary)
FROM Employee
GROUP BY Branch-name

(C) SELECT SUM(salary)
FROM Employee
GROUP BY Branch-name

(D) SELECT branch-name, SUM(salary)
FROM Employee

(iii) Find branch city, branch name Wise total salary, average salary and also number of employees.

(A) SELECT branch-city, branch-name,
SUM (salary), AVG(salary),
COUNT (Employee-name)
FROM Employee
GROUP BY branch-city, branch-name

(B) SELECT branch-city, branch-name,
SUM (salary), AVG (salary),
COUNT (Employee-name)
FROM Employee
GROUP BY branch-city

(C) SELECT branch-city, branch-name,
SUM (salary), AVG (salary), COUNT (Employee-name)
FROM Employee
GROUP BY branch-name

(D) SELECT branch-name, SUM (salary),
AVG (salary), COUNT (Employee-name)
FROM Employee
GROUP BY branch-city, branch-name

Common data for questions 13 to 15: Consider the SHIPMENTS relation and write the SQL statements for the below

SUPPLIERS

Supplier number	Supplier name	Status	City
SN1	Suma	30	Hyderabad
SN2	Hari	20	Chennai
SN3	Anu	10	Hyderabad
SN4	Mahesh	20	Bombay
SN5	Kamal	30	Delhi

PARTS

Part number	Part name	Color	Weight	City
PN1	X	Red	13.0	Chennai
PN2	Y	Green	13.5	Bombay
PN3	X	Yellow	13.2	Hyderabad
PN4	Y	Green	14.1	Calcutta
PN5	Z	Red	14.3	Hyderabad
PN6	Z	Blue	14.2	Bombay

PROJECT

Project number	Project name	City
PJ1	Display	Chennai
PJ2	OCR	Bombay
PJ3	RAID	Chennai
PJ4	SORTER	Hyderabad
PJ5	EDS	Chennai
PJ6	Tape	Bombay
PJ7	Console	Hyderabad

SHIPMENTS

Supplier number	Part number	Project number	Quantity
SN1	PN1	PJ1	300
SN1	PN1	PJ4	400
SN2	PN3	PJ1	350
SN2	PN3	PJ2	450
SN2	PN3	PJ3	640
SN2	PN3	PJ4	320
SN2	PN3	PJ5	330
SN2	PN3	PJ6	520
SN2	PN3	PJ7	480
SN2	PN5	PJ2	460
SN3	PN3	PJ1	440
SN3	PN4	PJ2	410
SN4	PN6	PJ3	310
SN4	PN6	PJ7	320
SN5	PN2	PJ2	340
SN5	PN2	PJ4	350
SN5	PN5	PJ5	360
SN5	PN5	PJ7	370
SN5	PN6	PJ2	380
SN5	PN1	PJ4	420
SN5	PN3	PJ4	440
SN5	PN4	PJ4	450
SN5	PN5	PJ4	400
SN5	PN6	PJ4	410

13. (i) For each part supplied, get the part number and the total shipment quantity?

(A) SELECT shipments.part-number, SUM (shipments.quantity)
FROM Shipments
GROUP BY shipments.part-number

(B) SELECT SUM(shipments.quantity)
FROM Shipments
GROUP BY shipments.part-number

(C) SELECT shipments.part-number, SUM (shipments.quantity)
FROM Shipments
GROUP BY shipments.quantity

(D) SELECT shipments.part-number, SUM (shipments. part-number)
FROM Shipments
GROUP BY shipments.part-number

(ii) Get part numbers for parts supplied by more than two suppliers?

(A) SELECT shipments.part-number
FROM Shipments
GROUP BY shipments.part-number
HAVING COUNT(shipments.supplier-number) > 2

(B) SELECT shipments.part-number
FROM Shipments
GROUP BY shipments.part-number
HAVING COUNT(shipments.supplier-number)>=2

(C) SELECT shipments.part-number
FROM Shipments
GROUP BY shipments.part-number>2

(D) SELECT shipments.part-number, COUNT (shipments.supplier-number)>2
FROM Shipments
GROUP BY shipments.part-number

(iii) Get supplier names for suppliers who supply part PN3?

(A) SELECT DISTINCT suppliers.supplier-name
FROM Supplier
WHERE suppliers.supplier-number IN (SELECT Shipments.supplier-number
FROM Shipments
WHERE Shipments.part-number='PN3')

(B) SELECT DISTINCT suppliers.supplier-name
FROM Supplier
WHERE suppliers.supplier-number NOT IN(SELECT Shipments.supplier-number
FROM Shipments
WHERE Shipments.part-number='PN3')

(C) SELECT DISTINCT suppliers.supplier-name
FROM Supplier
WHERE suppliers.supplier-number EXCEPT (SELECT Shipments.supplier-number
FROM Shipments
WHERE Shipments.part-number='PN3')

(D) SELECT DISTINCT suppliers, supplier-name
FROM Supplier
WHERE suppliers.supplier-number
UNION
SELECT Shipments.supplier-number
FROM Shipments
WHERE Shipments.part-number='PN3'

14. (i) Get supplier names for suppliers who supply at least one blue part.

(A) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE suppliers.supplier-number
IN (SELECT Shipments.supplier-number
FROM Shipments
WHERE Shipments.part-number
IN (SELECT Parts.part-number
FROM Parts
WHERE Parts.color='Blue'))

(B) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE suppliers.supplier-number
IN (SELECT Shipments.supplier-number
FROM Shipments

WHERE Shipments.part-number NOT
IN(SELECT Parts.part-number
FROM Parts
WHERE Parts.color='Blue'))

(C) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE suppliers.supplier-number NOT
IN(SELECT Shipments.supplier-number
FROM Shipments
WHERE Shipments.part-number
IN (SELECT Parts.part-number
FROM Parts
WHERE Parts.color='Blue'))

(D) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE suppliers.supplier-number
IN (SELECT Shipments.supplier-name
FROM Shipments
WHERE Shipments.part-number
IN (SELECT Parts.part-number
FROM Parts
WHERE Parts.color='Blue'))

(ii) Get supplier numbers for suppliers with status less than the current maximum status in the suppliers table:

(A) SELECT Suppliers.supplier-number
FROM suppliers
WHERE Suppliers.status < (SELECT MAX (Suppliers.status)
FROM Suppliers)

(B) SELECT Suppliers.supplier-number
FROM suppliers
WHERE Suppliers.status<=(SELECT MAX (Suppliers.status)
FROM Suppliers)

(C) SELECT Suppliers.supplier-number,
MAX (Suppliers.status)
FROM suppliers
WHERE Suppliers.status

(D) SELECT Suppliers.supplier-number
FROM suppliers
WHERE Suppliers.status=MAX(Suppliers.status)

(iii) Get supplier names for suppliers who supply part PN2?

(A) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE EXIST(SELECT *
FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
AND
Shipments.part-number='PN2')

(B) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE NOT EXIST(SELECT *
FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
AND
Shipments.part-number='PN2')

(C) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE EXIST(SELECT *
FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
OR
Shipments.part-number='PN2')

(D) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE EXIST(SELECT *
FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
UNION
Shipments.part-number='PN2')

15. (i) Get supplier names for suppliers who do not supply part PN2.

(A) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE NOT EXIST(SELECT *
FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
AND
Shipments.part-number='PN2')

(B) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE EXIST(SELECT *
FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
AND
Shipments.part-number='PN2')

(C) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE EXCEPT(SELECT *
FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
AND
Shipments.part-number='PN2')

(D) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE NOT EXIST(SELECT *
FROM Shipments

WHERE Shipments.supplier-number = suppliers.supplier-number
OR
Shipments.part-number='PN2')

(ii) Get supplier names for suppliers who supply all parts.

(A) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE NOT EXIST(SELECT *
FROM Part
WHERE NOT EXIST(SELECT * FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
AND
Shipments.part-number=Parts.part-number))

(B) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE EXIST(SELECT *
FROM Part
WHERE NOT EXIST(SELECT * FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
AND
Shipments.part-number=Parts.part-number))

(C) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE NOT EXIST(SELECT *
FROM Part
WHERE EXIST(SELECT * FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
AND
Shipments.part-number=Parts.part-number))

(D) SELECT DISTINCT suppliers.supplier-name
FROM Suppliers
WHERE EXIST(SELECT *
FROM Part
WHERE EXIST(SELECT * FROM Shipments
WHERE Shipments.supplier-number = suppliers.supplier-number
AND
Shipments.part-number=Parts.part-number))

(iii) Get part numbers for parts that either weigh more than-16 pounds or are supplied by supplier *SN*3, or both?

(A) SELECT parts.part-number
FROM parts
WHERE Parts.weight>18
UNION
SELECT Shipments.part-number
FROM shipments
WHERE Shipments.supplier-number='SN2'

(B) SELECT parts.part-number
FROM parts
WHERE Parts.weight>18
UNION
SELECT Shipments.supplier-name
FROM shipments
WHERE Shipments.supplier-number='*SN*2'

(C) SELECT parts.part-number
FROM parts
WHERE Parts.weight>18
UNION
SELECT Shipments.part-number,Shipments.supplier-name
FROM shipments
WHERE Shipments.supplier-number='*SN*2'

(D) SELECT parts.part-Number, parts.color
FROM parts
WHERE Parts.weight>18
UNION
SELECT Shipments.part-number
FROM shipments
WHERE Shipments.supplier-number='*SN*2'

Common data for questions 16 and 17: Consider the following relation: Teach

Name	Address	course
Zohar	40B,east city	MD
Nisha	16/2, hyd	BDS
Zohar	40B, East city	MS
Ravi	New York	MBA

16. The teacher with name Zohar teaching the course MS?
(A) σ_{Name} = 'Zohar' teach = MS.
(B) π_{Name} = 'Zohar' teach = MS.
(C) σ_{name} = 'Zohar' and course = 'MS' (teach).
(D) π_{Name} = 'Zohar' and course = 'MS' (teach).

17. Select the names of courses taught by Zohar?
(A) $\pi_{\text{course}}(\sigma_{\text{Name} = \text{'Zohar'}}(\text{Teach}))$
(B) $\sigma_{\text{course}}(\pi_{\text{Name} = \text{'Zohar'}}(\text{Teach}))$
(C) $\pi_{\text{course}}(\sigma_{\text{Name} = \text{'MD'}}(\text{Teach}))$
(D) None

18. Consider the join of a relation *A* with a relation *B*. If *A* has *m* tuples and *B* has *n* tuples. Then the maximum and minimum sizes of the join respectively are.
(A) mn and $m + n$
(B) $m + n$ and $(m - n)$
(C) mn and m
(D) mn and 0

19. Match the following:

I	Set intersection	1	$R \mid\times\mid S$
II	Natural join	2	$r - (r - s)$
III	Division	3	$\leftarrow$
IV	Assignment	4	$\pi_{R-S}(r) - \pi_{R-S}$ $(\pi_{R-S}(r) \times s)$ $-\pi_{R-S}, s(r)$

(A) I – 2, II – 1, III – 4, IV – 3
(B) I – 3, II – 4, III – 2, IV – 1
(C) I – 1, II – 2, III – 3, IV – 4
(D) I – 2, II – 3, III – 4, IV – 1

20. Which one is correct for division operations for relation r and s

(A) $r \div s$
(B) $\pi_{R-S}(r) - \pi_{R-S}((\pi_{R-S}(r) \times s) - \pi_{R-S}), s(r)$
(C) Temp 1 $\leftarrow \pi_{R-S}(r)$
Temp 2 $\leftarrow \pi_{R-S}(\text{temp1} \times s) - \pi_{R-S}, s(r)$
result = temp 1 – temp 2
(D) All the above

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. The correct order of SQL expression is
(A) Select, group by, where, having
(B) Select, where, group by, having
(C) Select, group by, having, where
(D) Select, having, where, group by

2. Which one is not a query language?
(A) SQL (B) QBE
(C) Data log (D) MySQL

3. Like '*a b* \ % *c d*' escape '\' matches all the strings
(A) Ending with *a b c d*
(B) Beginning with *a b c d*
(C) Beginning with *a b c d*
(D) Beginning with *a b* % *c d*

4. '_ _ _%' matches any string of
(A) At least three characters
(B) At most three characters
(C) Exactly three characters
(D) exactly three characters ending with %

5. Which of the following are set operations?
(i) Union
(ii) Intersection
(iii) Set Difference
(iv) Cartesian Product
(A) (i), (ii), (iii)
(B) (i), (iii), (iv)
(C) (i), (iii), (ii), (iv)
(D) (i), (ii), (iv)

6. What is the purpose of project operation?
(A) It selects certain columns
(B) It selects certain rows
(C) It selects certain strings
(D) It selects certain integers

Common data for questions 7 and 8: Person

Id	Name	Age	Hobby
11	Anu	21	Stamp Collection
22	Kamal	32	Painting
33	Ravi	24	Dancing
44	Ram	22	Singing

7. Select the persons whose hobby is either painting (or) singing.
(A) $\sigma_{\text{Hobby = 'painting' OR Hobby = 'singing'}}$ (person)
(B) $\sigma_{\text{Hobby = 'painting',' singing'}}$ (person)
(C) $\sigma_{\text{Hobby = 'painting' OR 'singing'}}$ (person)
(D) All are correct

8. Select the persons whose age is above 21 and below 32:
(A) $\sigma_{\text{age > 21 AND age < 32}}$ (person)
(B) $\sigma_{21 < \text{age} < 32}$ (person)
(C) $\sigma_{\text{age > 21 OR age < 32}}$ (person)
(D) $\sigma_{\text{age < 21 AND age > 32}}$ (person)

Common data for questions 9 and 10: Consider the following relation: Teach

Name	course	Rating	Age
Zohar	MD	7	35
Nisha	BDS	8	27
Zohar	MS	7	34
Ravi	MBA	9	33

9. Select the teachers whose rating is above 7 and whose age is less than 32?
(A) $s_{\text{Rating > 7 AND Age < 32}}$ (Teach)
(B) $s_{\text{Rating} \geq \text{7 AND Age < 32}}$ (Teach)
(C) $s_{\text{Rating > 7 AND < 32}}$ (Teach)
(D) Both (A) and (B)

10. Select the courses with rating above 7?
(A) $\pi_{\text{course}}(\sigma_{\text{rating > 7}}(\text{Teach}))$
(B) $\sigma_{\text{course}}(\pi_{\text{rating > 7}}(\text{Teach}))$
(C) $\pi_{\text{name, course}}(\sigma_{\text{rating > 7}}(\text{Teach}))$
(D) None

Common data for questions 11 and 12: Consider the following schema of a relational database employee (<u>empno</u>, ename, eadd) project (<u>pno</u>, pname) Work–on (empno, pno) Part(<u>partno</u>, partname, qty-on-hand, size) Use (empno, pno, partno, number)

11. Display the names of the employees who are working on a project named 'VB'.
(A) $\sigma_{\text{name}}(\text{employee} \bowtie (\sigma_{\text{pname = 'VB'}}\ \text{project}) \bowtie \text{worked on})$
(B) $\sigma_{\text{name}}(\text{employee} \bowtie (\pi_{\text{pname = 'VB'}}(\text{project}) \bowtie \text{work on})$
(C) $\pi_{\text{name}}(\text{employee} \bowtie (\sigma_{\text{pname = 'VB'}}(\text{project}) \bowtie \text{work on})$
(D) $\pi_{\text{name}}(\text{employee} \bowtie (\pi_{\text{pname}} = \text{'VB'}(\text{project}) \bowtie \text{work on})$

12. Display the names of the people who are not working for any project.

(A) π_{name} (employee ⋈(π_{name} (employee + work on)

(B) π_{name} (employee – π_{name} (employee ∩ work on)

(C) π_{name} (employee – π_{name} (employee ⋈ work on)

(D) σ_{name} (employee – σ_{name} (employee ⋈ work on)

13. Consider the following tables:

A	B	C	D
b	c	e	f
a	b	i	j
b	c	g	h
b	c	a	d
d	i	g	h
d	j	j	k
d	i	e	f

C	D
e	f
g	h

R ÷ S

A	B
b	c
d	i

Which of the following statements is true?

(A) $R \div S = p_{A,B}(R) - p_{A,B}(p_{A,B}(R) \times S + R)$

(B) $R \div S = p_{A,B}(R) - p_{A,B}(p_{A,B}R \times S - R)$

(C) $R \div S = p_{A,B}(R) - p_{A,B}((p_{A,B}(R) \times S) - R)$

(D) $R \div S = p_{A,B}(R) - p_{A,B}(p_{A,B}(R) \times R - S)$

Common data for questions 14 and 15: Consider the following schema of a relational data base
student (sno, name, address)
project (pno, Pname) work-on (sno, pno)
Part (part no, part name, qtyon hand size)
Use (sno, pno, part no, number)

14. List the names of the students who are participating in every project and have used every part.

(A) σ_{name}(student ⋈(((Workon) ÷ σ_{pro}(project)) ∩ ($\sigma_{sno,\ part\ no}$(use) ÷ $\sigma_{part\ no}$ (part)))

(B) π_{name}(student ⋈(((Workon) ÷ π_{pro}(project)) ∩ ($\pi_{sno,\ partno}$(use) ÷ $\sigma_{part\ no}$ (part)))

(C) π_{name}(student ⋈(((Workon) ÷ π_{partno} (project)) ∩ ($\pi_{sno,\ partno}$(use) ÷ $\sigma_{part\ no}$ (part)))

(D) π_{name}(student ∞ (((Workon) ÷ π_{pro}(project)) ∪ ($\pi_{ssno,\ partno}$ (use) ÷ $\pi_{part\ no}$ (part)))

15. The following query gives π_{name} (employee ⋈(work on ÷ π_{pro} ($\sigma_{Pname\ =\ 'MS'\ AND\ 'MD'}$(project)))

(A) Names of the students who are working in either projects 'MS' or 'MD'

(B) Names of the students who are working in both the projects 'MS' or 'MD'

(C) Names of the students who are not working in any of the projects 'MS' or 'MD'

(D) None of the above

16. 'All rows corresponding to students whose sno's are between 10 and 20

(i) Select * form student where SNo are between 5 AND 10

(ii) Select * from student where SNO IN(5, 10)

(A) Only (i) (B) Only (ii)

(C) Both (A) and (B) (D) None

17. UPDATE account SET
DA = basic * .2,
GROSS = basic * 1.3, Where basic > 2000;

(A) The above query displays *DA* and gross for all those employees whose basic is ≥ 2000

(B) The above query displays *DA* and Gross for all employees whose basic is less than 2000

(C) The above query displays updated values of *DA* as well as gross for all those employees whose basic is > 2000

(D) All the above

18. Given two union compatible relations $R_1(A, B)$ and $R_2(C, D)$, what is the result of the operation $R_1\ A = CAB = DR_2$?

(A) $R_1 \cup R_2$ (B) $R_1 \times R_2$

(C) $R_1 - R_2$ (D) $R_1 \cap R_2$

19. Which of the following queries finds the clients of banker Agassi and the city they live in?

(A) $\pi_{client \cdot cname \cdot Ccity}(\sigma_{client.cname}$ = customer c name ($\sigma_{Banker \cdot name = Aggassi}$ (client × customer)

(B) $\pi_{Client.c\ city}$ ($\sigma_{Banker\ name\ =\ 'Aggasi'}$(client × customer)

(C) $\pi_{client \cdot c\ name \cdot Cucity}(\sigma_{client.c\ name\ =\ 'Aggasi'}$ ($\sigma_{client \cdot name\ =\ Cutomer}$ (client × customer)

(D) $\pi_{\cdot c\ name \cdot Cucity}(\sigma_{Bankers}$ name = name ($\sigma_{Banker.\ =\ agassi}$ (client × customer)

20. Consider the following schema pertaining to students data
Student (<u>rno,</u> name, add)
Enroll (rno, <u>Cno,</u> Cname) Where the primary keys are shown Underlined. The no. of tuples in the student and Enroll tables are 120 and 8 respectively. What are the maximum and minimum no. of tuples that can be present in (student * Enroll) where '*' denotes natural join.

(A) 8, 8 (B) 120, 8

(C) 960, 8 (D) 960, 120

Previous Years' Questions

1. Consider the relation account (customer, balance) where customer is a primary key and there are no null values. We would like to rank customers according to decreasing balance. The customer with the largest balance gets rank 1, ties are not broke but ranks are skipped; if exactly two customers have the largest balance they each get rank 1 and rank 2 is not assigned.

Query 1: select A.customer, count (B.customer) from account *A*, account *B* where A.balance <= B.balance group by A.customer
Query 2: select A.customer, 1 + count (B.balance) from account *A*, account *B* where A.balance < B.balance group by A.customer Consider these statements about Query1 and Query2.

1. Query1 will produce the same row set as Query2 for some but not all databases.
2. Both Query1 and Query2 are correct implementation of the specification.
3. Query1 is a correct implementation of the specification but Query2 is not.
4. Neither Query1 nor Query2 is a correct implementation of the specification.
5. Assigning rank with a pure relational query takes less time than scanning in decreasing balance order assigning ranks using ODBC.

Which two of the above statements are correct? **[2006]**

(A) 2 and 5 (B) 1 and 3
(C) 1 and 4 (D) 3 and 5

2. Consider the relation enrolled (student, course) in which (student, course) is the primary key, and the relation paid (student, amount) where student is the primary key. Assume no null values and no foreign keys or integrity constraints. Given the following four queries:

Query1: select student from enrolled where student in (select student from paid)

Query2: select student from paid where student in (select student from enrolled)

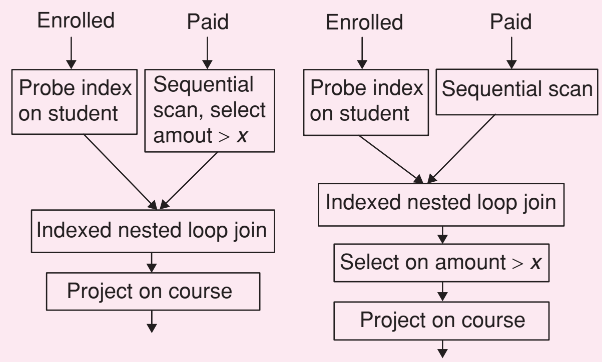
Query3: select E.student from enrolled E, paid P where E.student = P.student

Query4: select student from paid where exists (select * from enrolled where enrolled.student = paid.student)

Which one of the following statement is correct? **[2006]**

(A) All queries return identical row sets for any database
(B) Query2 and Query4 return identical row sets for all databases but there exist databases for which Query1 and Query2 return different row sets
(C) There exist databases for which Query3 returns strictly fewer rows than Query2
(D) There exist databases for which Query4 will encounter an integrity violation at runtime

3. Consider the relation enrolled (student, course), in which (student, course) is the primary key, and the relation paid (student, amount) where student is the primary key. Assume no null values and no foreign keys or integrity constraints. Assume that amounts 6000, 7000, 8000, 9000 and 10000 were each paid by 20% of the students. Consider these query plans (plan 1 on left, plan 2 on right) to 'list all courses taken by students who have paid more than *x*'



A disk seek takes 4 ms, disk data transfer bandwidth is 300 MB/s and checking a tuple to see if amount is greater than x takes 10 μs. Which of the following statements is correct? **[2006]**

(A) Plan 1 and Plan 2 will not output identical row sets for all databases
(B) A course may be listed more than once in the output of Plan 1 for some databases
(C) For $x = 5000$, Plan 1 executes faster than Plan 2 for all databases
(D) For $x = 9000$, Plan 1 executes slower than Plan 2 for all databases

4. Information about a collection of students is given by the relation **studinfo** (studId, name, sex). The relation **enroll** (studId, courseId) gives which student has enrolled for (or taken) what course(s). Assume that every course is taken by at least one male and at least one female student. What does the following relational algebra expression represent?

$$\Pi_{\text{courseId}}((\Pi_{\text{studid}}(\sigma_{\text{sex = 'female'}}(\text{studInfo})) \times \Pi_{\text{courseId}}(\text{enroll})) - \text{enroll}) \quad \textbf{[2007]}$$

(A) Courses in which all the female students are enrolled
(B) Courses in which a proper subset of female students are enrolled.

(C) Courses in which only male students are enrolled.
(D) None of the above

5. Consider the relation **employee** (<u>name</u>, sex, supervisorName) with *name* as the key. *supervisorName* gives the name of the supervisor of the employee under consideration. What does the following Tuple Relational Calculus query produce?

$$e \cdot \text{name} \mid \text{employee}(e) \wedge$$
$$(\forall x)[\neg \text{employee}(x) \vee x \cdot \text{supervisor Name} \neq e \cdot \text{name} \vee$$
$$x \cdot \text{sex} = \text{"male"}]\}$$ **[2007]**

(A) Names of employees with a male supervisor.
(B) Names of employees with no immediate male subordinates.
(C) Names of employees with no immediate female subordinates.
(D) Names of employees with a female supervisor.

6. Consider the table **employee** (<u>empId</u>, name, department, salary) and the two queries Q_1, Q_2 below. Assuming that department 5 has more than one employee, and we want to find the employees who get higher salary than anyone in the department 5, which one of the statements is **TRUE** for any arbitrary employee table?

Q_1: SELECT e.empId
FROM employee e
WHERE not exists
(Select * From employee s where s.department = '5' and s.salary >=e.salary)

Q_2: SELECT e.empId
FROM employee *e*
WHERE e.salary > Any
(Select distinct salary From employee *s* Where s.department = '5') **[2007]**

(A) Q_1 is the correct query
(B) Q_2 is the correct query
(C) Both Q1 and Q2 produce the same answer.
(D) Neither Q_1 nor Q_2 is the correct query

7. Let *R* and *S* be two relations with the following schema
R (<u>*P*</u>, <u>*Q*</u>, *R*1, *R*2, *R*3)
S (<u>*P*</u>, <u>*Q*</u>, *S*1, *S*2)
Where $\{P, Q\}$ is the key for both schemas. Which of the following queries are equivalent?

I. $\Pi_P (R \bowtie S)$
II. $\Pi_P (R) \bowtie \Pi_P (S)$
III. $\Pi_P (\Pi_{P,Q} (R) \cap \Pi_{P,Q} (S))$
IV. $\Pi_P (\Pi_{P,Q} (R) - (\Pi_{P,Q} (R) - (\Pi_{P,Q} (S)))$ **[2008]**

(A) Only I and II (B) Only I and III
(C) Only I, II and III (D) Only I, III and IV

8. Let *R* and *S* be relational schemes such that $R = \{a,b,c\}$ and $S = \{c\}$. Now consider the following queries on the database:

I. $\pi_{R-S}(r) - \pi_{R-S}(\pi_{R-S}(r) \times s - \pi_{R-S,S}(r))$
II. $\{t \mid t \in \pi_{R-S}(r) \wedge \forall u \in s(\exists v \in r(u = v[s] \wedge t = v[R-S]))\}$
III. $\{t \mid t \in \pi_{R-S}(r) \wedge \forall v \in r(\exists u \in s(u = v[s] \wedge t = v[R-S]))\}$
IV. SELECT R.a, R.b
FROM *R*, *S*
WHERE R.c = S.c

Which of the above queries are equivalent? **[2009]**

(A) I and II (B) I and III
(C) II and IV (D) III and IV

Common data for questions 9 and 10: Consider the following relational schema: Suppliers (sid: integer, sname: string, city: string, street: string) Parts(pid: integer, pname: string, color: string) Catalog (sid: integer, pid: integer, cost: real)

9. Consider the following relational query on the above database:

SELECT S.sname
FROM Suppliers S
WHERE S.sid NOT IN (SELECT C.sid
FROM Catalog C
WHERE C.pid NOT IN (SELECT P.pid FROM Parts P
WHERE P.color <> 'blue'))

Assume that relations corresponding to the above schema are not empty. Which one of the following is the correct interpretation of the above query? **[2009]**

(A) Find the names of all suppliers who have supplied a non-blue part.
(B) Find the names of all suppliers who have not supplied a non-blue part.
(C) Find the names of all suppliers who have supplied only blue parts.
(D) Find the names of all suppliers who have not supplied only blue parts.

10. A relational schema for a train reservation database is given below

Passenger (pid, pname, age)
Reservation (pid, cass, tid)

Table :Passenger
Table :Reservation

Pid	pname	Age
0	'Sachin'	65
1	'Rahul'	66
2	'Sourav'	67

Pid	class	tid
0	'AC'	8200
1	'AC'	8201
2	'SC'	8201

3	'Anil'	69

5	'AC'	8203
1	'SC'	8204
3	'AC'	8202

What pids are returned by the following SQL query for the above instance of the tables?

SELECT pid
FROM Reservation
WHERE class = 'AC' AND
EXISTS (SELECT *
  FROM Passenger
  WHERE age > 65 AND
  Passenger.pid = Reservation.pid) **[2010]**

(A) 1, 0 (B) 1, 2
(C) 1, 3 (D) 1, 5

11. Consider a relational table r with sufficient number of records, having attributes $A_1, A_2, \ldots A_n$ and let $1 \le p \le n$. Two queries $Q1$ and $Q2$ are given below.

$Q1$: $\pi_{A_1 \ldots A_n}(\sigma_{A_p=c}(r))$ where c is a constant.

$Q2$: $\pi_{A_1 \ldots A_n}(\sigma_{c_1 \le A_p \le c_2}(r))$ where c_1 and c_2 are constants.

The database can be configured to do ordered indexing on A_P or hashing on A_p. Which of the following statements is TRUE? **[2011]**

(A) Ordered indexing will always outperform hashing for both queries
(B) Hashing will always outperform ordered indexing for both queries.
(C) Hashing will outperform ordered indexing on $Q1$, but not on $Q2$.
(D) Hashing will outperform ordered indexing on $Q2$, but not on $Q1$.

12. Database table by name Loan_Records is given below.

Borrower	Bank manager	Loan amount
Ramesh	Sunderajan	10000.00
Suresh	Ramgopal	5000.00
Mahesh	Sunderajan	7000.00

What is the output of the following SQL query?
SELECT count (*)
FROM (Select Borrower, Bank_Manager FROM Loan Records) AS S
NATURAL JOIN
(SELECT Bank_Manager, Loan_Amount FROM Loan_Records) AS T; **[2011]**

(A) 3 (B) 9
(C) 5 (D) 6

13. Consider a database table T containing two columns X and Y each of type *integer*. After the creation of the table, one record ($X = 1$, $Y = 1$) is inserted in the table.

Let MX and MY denote the respective maximum values of X and Y among all records in the table at any point in time. Using MX and MY, new records are inserted in the table 128 times with X and Y values being $MX + 1$, $2 * MY + 1$ respectively. It may be noted that each time after the insertion, values of MX and MY change. What will be the output of the following SQL query after the steps mentioned above are carried out?

SELECT Y FROM T WHERE $X = 7$; **[2011]**

(A) 127 (B) 255
(C) 129 (D) 257

14. Which of the following statements are true about an SQL query?

P: An SQL query can contain a HAVING clause even if it does not have a GROUP BY clause
Q: An SQL query can contain a HAVING clause only if it has a GROUP BY clause
R: All attributes used in the GROUP BY clause must appear in the SELECT clause
S: Not all attributes used in the GROUP BY clause need to appear in the SELECT clause **[2012]**

(A) P and R (B) P and S
(C) Q and R (D) Q and S

15. Suppose $R_1(\underline{A}, B)$ and $R_2(\underline{C}, D)$ are two relation schemas. Let r_1 and r_2 be the corresponding relation instances. B is a foreign key that refers to C in R_2. If data in r_1 and r_2 satisfy referential integrity constraints, which of the following is always true? **[2012]**

(A) $\Pi_B(r_1) - \Pi_C(r_2) = \varnothing$
(B) $\Pi_C(r_2) - \Pi_B(r_1) = \varnothing$
(C) $\Pi_B(r_1) = \Pi_C(r_2)$
(D) $\Pi_B(r_1) - \Pi_C(r_2) \ne \varnothing$

Common data for questions 16 and 17: Consider the following relations A, B and C:

(A)

Id	Name	Age
12	Arun	60
15	Shreya	24
99	Rohit	11

(B)

Id	Name	Age
15	Shreya	24
25	Hari	40
98	Rohit	20
99	Rohit	11

(C)

Id	Phone	Area
10	2200	02
99	2100	01

16. How many tuples does the result of the following SQL query contain?
SELECT A.Id
FROM A
WHERE A. Age > ALL (SELECT B. Age
FROM B
WHERE B. Name = 'Arun') **[2012]**
(A) 4 (B) 3
(C) 0 (D) 1

17. How many tuples does the result of the following relational algebra expression contain? Assume that the schema of $A \cup B$ is the same as that of A.
$(A \cup B) \bowtie_{A.Id > 40 \vee C.Id < 15} C$ **[2012]**
(A) 7 (B) 4
(C) 5 (D) 9

18. Consider the following relational schema. Students (rollno: integer, sname: string) Courses (courseno: integer, cname: string) Registration(rollno:integer,courseno: integer, percent: real)

Which of the following queries are equivalent to this query in English?

'Find the distinct names of all students who score more than 90% in the course numbered 107'

(I) SELECT DISTINCT S.sname FROM Students as S, Registration as R WHERE R.rollno=S.rollno AND R.courseno=107 AND R.percent>90
(II) $\pi_{sname}(\sigma_{courseno=107 \wedge percent>90}$ Registration $\bowtie$ Students)
(III) $\{T \mid \exists S \in$ Students, $\exists R \in$ Registration (S.rollno=R.rollno $\wedge$ R.courseno=107 $\wedge$ R.percent>90$\wedge$T.sname=S.sname)}
(IV) $\{<S_N> \mid \exists S_R \exists R_P\ (<S_R, S_N> \in$ Students $\wedge <S_R, 107, R_P> \in$ Registration $\wedge R_P > 90)\}$ **[2013]**
(A) I, II, III and IV (B) I, II and III only
(C) I, II and IV only (D) II, III and IV only

19. Given the following statements:
S_1: A foreign key declaration can always be replaced by an equivalent check assertion in SQL.
S_2: Given the table R (a, b, c) where a and b together form the primary key, the following is a valid table definition.
CREATE TABLE S (
a INTEGER
d INTEGER,
e INTEGER,
PRIMARY KEY (d),
FOREIGN KEY (a) references R)
Which one of the following statements is CORRECT? **[2014]**
(A) S_1 is TRUE and S_2 is FALSE
(B) Both $S1$ and S_2 are TRUE
(C) S_1 is FALSE and S_2 is TRUE
(D) Both S_1 and S_2 are FALSE

20. Given the following schema:
Employees (emp–id, first-name, last– name, hire–date, dept–id, salary)
Departments (dept–id, dept–name, manager–id, location–id)
you want to display the last names and hire dates of all latest hires in their respective departments in the location ID 1700. You issue the following query:
SQL > SELECT last–name, hire–date
FROM employees
WHERE (dept–id, hire–date) IN
(SELECT dept–id, MAX (hire–date)
FROM employees JOIN departments USING (dept–id)
WHERE location–id = 1700
GROUP BY dept–id);
What is the outcome? **[2014]**
(A) It executes but does not give the correct result.
(B) It executes and gives the correct result.
(C) It generates an error because of pair wise comparison.
(D) It generates an error because the GROUP BY clause cannot be used with table joins in a subquery.

21. Given an instance of the STUDENTS relation as shown below:

Student ID	Student Name	Student Email	Student Age	CPI
2345	Shankar	shaker @ math	X	9.4
1287	Swati	swati @ ee	19	9.5
7853	Shankar	shankar @ cse	19	9.4
9876	Swati	swati @ mech	18	9.3
8765	Ganesh	ganesh@ civil	19	8.7

For (StudentName, StudentAge) to be a key for this instance, the value X should NOT be equal to _____. **[2014]**

22. Consider a join (relation algebra) between relations ($r(R)$) and ($s(S)$) using the nested loop method. There are three buffers each of size equal to disk block size, out of which one buffer is reserved for intermediate results. Assuming size $r(R) <$ *size* $s(S)$, the join will have fewer number of disk block accesses if **[2014]**
(A) Relation $r(R)$ is in the outer loop
(B) Relation $s(S)$ is in the outer loop
(C) Join selection factor between $r(R)$ and $s(S)$ is more than 0.5
(D) Join selection factor between $r(R)$ and $s(S)$ is less than 0.5

23. SQL allows duplicate tuples in relations, and correspondingly defines the multiplicity of tuples in the result of joins. Which one of the following queries always gives the same answer as the nested query shown below:

Select * from R where a in (select S. a from S) [2014]

(A) Select R.* from R, S where $R.\ a = S.\ a$

(B) Select distinct R * from R, S where $R\ .\ a = S\ .\ a$

(C) Select R.* from R, (select distinct a from S) as $S1$ where $R.a = S1.a$

(D) Select R.* from R, S where $R.a = S.a$ and is unique R

24. What is the optimized version of the relation algebra expression $\pi_{A_1}(\pi_{A_2}(\sigma_{F_1}(\sigma_{F_2(r)})))$, where A_1, A_2 are sets of attributes in r with $A_1 \subset A_2$ and F_1, F_2 are Boolean expressions based on the attributes in r? **[2014]**

(A) $\pi_{A_1}(\sigma_{(F_1 \wedge F_2)(r)})$

(B) $\pi_{A_1}(\sigma_{(F_1 \vee F_2)(r)})$

(C) $\pi_{A_2}(\sigma_{(F_1 \wedge F_2)(r)})$

(D) $\pi_{A_2}(\sigma_{(F_1 \vee F_2)(r)})$

25. Consider the relational schema given below, where **eId** of the relation **dependent** is a foreign key referring to **empId** of the relation **employee**. Assume that every employee has at least one associated dependent in the **dependent** relation.

Consider the following relational algebra query:

employee (<u>empId</u>, empName, empAge)

dependent (<u>depId, eId</u>, depName, depAge)

$\pi_{\text{empId}}(\text{employee})- \pi_{\text{empId}}(\text{employee} \bowtie_{(\text{empId} = \text{eID}) \wedge (\text{empAge} \leq \text{depAge})} \text{dependent})$

The above query evaluates to the set of empIds of employees whose age is greater than that of **[2014]**

(A) some dependent.

(B) all dependents.

(C) some of his/her dependents.

(D) all of his/her dependents.

26. Consider the following relational schema:

employee (<u>empId</u>, empName, empDept)

customer(<u>custId</u>, custName, salesRepid, rating)

salesRepId is a foreign key referring to empId of the employee relation. Assume that each employee makes a sale to at least one customer. What does the following query return?

```
SELECT empName
FROM employee E
WHERE NOT EXISTS
(SELECT custId
FROM customer C
WHERE C.salesRepId = E.empId
AND C.Rating < > 'GOOD');
```
[2014]

(A) Names of all the employees with at least one of their customers having a 'GOOD' rating.

(B) Names of all the employees with at most one of their customers having a 'GOOD' rating.

(C) Names of all the employees with none of their customers having a 'GOOD' rating.

(D) Names of all the employees with all their customers having a 'GOOD' rating.

27. SELECT operation in SQL is equivalent to **[2015]**

(A) The selection operation in relational algebra

(B) The selection operation in relational algebra, except that SELECT in SQL retains duplicates.

(C) The projection operation in relational algebra.

(D) The projection operation in relational algebra, except that SELECT in SQL retains duplicates.

28. Consider the following relations:

Student

Roll No	Student Name
1	Raj
2	Rohit
3	Raj

Performance

Roll No	Course	Marks
1	Math	80
1	English	70
2	Math	75
3	English	80
2	Physics	65
3	Math	80

Consider the following SQL query.

```
SELECT S.Student_Name, sum (P.Marks)
FROM Student S, Performance P
WHERE S.Roll_No = P.Roll_No
GROUP BY S.Student_Name
```

The number of rows that will be returned by the SQL query is ______ **[2015]**

29. Consider two relations $R_1(A, B)$ with the tuples (1, 5), (3, 7) and $R_2(A, C) = (1, 7), (4, 9)$. Assume that $R(A, B, C)$ is the full natural outer join of R_1 and R_2. Consider the following tuples of the form (A, B, C): a = (1, 5, null), b = (1, null, 7), c = (3, null, 9), d = (4, 7, null), e = (1, 5, 7), f = (3, 7, null), g = (4, null, 9). Which one of the following statements is correct? **[2015]**

(A) R contains a, b, e, f, g but not c, d.

(B) R contains all of a, b, c, d, e, f, g.

(C) R contains e, f, g but not a, b.

(D) R contains e but not f, g.

30. Consider the following relation

Cinema (theater, address, capacity)

Which of the following options will be needed at the end of the SQL query

SELECT P_1.address

FROM Cinema P_1

such that it always finds the addresses of theaters with maximum capacity? **[2015]**

(A) WHERE P_1.capacity > = All (select P_2. Capacity from Cinema P_2)

(B) WHERE P_1.capacity >= Any (select P_2. Capacity from Cinema P_2)

(C) WHERE P_1.capacity > All (select max(P_2. capacity) from Cinema P_2)

(D) WHERE P_1.capacity > Any (select max(P_2. capacity) from Cinema P_2)

31. Which of the following is NOT a superkey in a relational schema with attributes *V, W, X, Y, Z* and primary key *VY*? **[2016]**

(A) *V XYZ* (B) *V WXZ*
(C) *V WXY* (D) *V WXYZ*

32. Consider a database that has the relation schema EMP (EmpId, EmpName and DeptName). An instance of the schema EMP and a SQL query on it are given below.

EMP		
EmpId	**EmpName**	**DeptName**
1	XYA	AA
2	XYB	AA
3	XYC	AA
4	XYD	AA
5	XYE	AB
6	XYF	AB
7	XYG	AB
8	XYH	AC
9	XYI	AC
10	XYJ	AC
11	XYK	AD
12	XYL	AD
13	XYM	AE

```
SELECTIVE AVG(EC.Num)
FROM EC
WHERE (DeptName, Num) IN
        (SELECT DeptName, COUNT (EmpId) AS
                                EC(DeptName, Num)
         FROM EMP
         GROUP BY DeptName)
```

The output of executing the SQL query is _______. **[2017]**

33. Consider a database that has the relation schemas EMP(EmpId, EmpName, DeptId), and DEPT(DeptName, DeptId), Note that the DeptId can be permitted to be NULL in the relation EMP. Consider the following queries on the database expressed in tuple relational calculus.

(I) $\{t \mid \exists u \in \text{EMP}(t[\text{EmpName}] = u[\text{EmpName}] \wedge \forall v \in \text{DEPT}(t[\text{DeptId}] \neq v[\text{DeptId}]))\}$

(II) $\{t \mid \exists u \in \text{EMP}(t[\text{EmpName}] = u[\text{EmpName}] \wedge \exists v \in \text{DEPT}(t[\text{DeptId}] \neq v[\text{DeptId}]))\}$

(III) $\{t \mid \exists u \in \text{EMP}(t[\text{EmpName}] = u[\text{EmpName}] \wedge \exists v \in \text{DEPT}(t[\text{DeptId}] = v[\text{DeptId}]))\}$

Which of the above queries are safe? **[2017]**

(A) (I) and (II) only
(B) (I) and (III) only
(C) (II) and (III) only
(D) (I), (II) and (III)

34. Consider a database that has the relation schema CR (studentName, CourseName). An instance of the schema CR is as given below.

CR	
StudentName	**CourseName**
SA	CA
SA	CB
SA	CC
SB	CB
SB	CC
SC	CA
SC	CB
SC	CC
SD	CA
SD	CB
SD	CC
SD	CD
SE	CD
SE	CA
SE	CB
SF	CA
SF	CB
SF	CC

The following query is made on the database.

$T1 \leftarrow \pi_{CouraseName}(\sigma_{StudentName='SA'}(\text{CR}))$

$T2 \leftarrow CR \div T1$

The number of rows in $T2$ is _______. **[2017]**

35. Consider the following database table named *top_scorer*.

top_scorer

player	country	goals
Klose	Germany	16
Ronaldo	Brazil	15
G Miiller	Germany	14
Fontaine	France	13
Pelé	Brazil	12
Klinsmann	Germany	11
Kocsis	Hungary	11
Batistuta	Argentina	10
Cubillas	Peru	10
Lato	Poland	10
Lineker	England	10
T Muller	Germany	10
Rahn	Germany	10

Consider the following SQL query:

```
SELECT ta.player FROM top_scorer AS ta
WHERE ta.goals >ALL (SELECT tb.goals
    FROM top_scorer AS tb
    WHERE tb.country = 'Spain')
AND ta.goals >ANY (SELECT tc.goals
    FROM top_scorer AS tc
    WHERE tc. country = 'Germany')
```

The number of tuples returned by the above SQL query is _________. **[2017]**

36. Consider the following two tables and four queries in SQL.

Book (isbn, bname), Stock (isbn, copies)

Query 1: SELECT B.isbn, S.copies
FROM Book B INNER JOIN Stock S
ON B.isbn = S.isbn;

Query 2: SELECT B.isbn, S.copies
FROM Book B LEFT OUTER
JOIN Stock S
ON B.isbn = S.isbn;

Query 3: SELECT B.isbn, S.copies
FROM Book B RIGHT OUTER
JOIN Stock S
ON B.isbn = S.isbn;

Query 4: SELECT B.isbn, S.copies
FROM Book B FULL OUTER
JOIN Stock S
ON B.isbn = S.isbn;

Which one of the queries above is certain to have an output that is a superset of the outputs of the other three queries? **[2018]**

(A) Query 1 (B) Query 2
(C) Query 3 (D) Query 4

37. Consider the relations $r(A, B)$ and $s(B, C)$, where $s \cdot B$ is a primary key and $r \cdot B$ is a foreign key referencing $s \cdot B$. Consider the query

$$Q: r \bowtie (\sigma_{B<5}(S))$$

Let LOJ denote the natural left outer-join operation. Assume that r and s contain no null values.

Which one of the following queries is NOT equivalent to Q? **[2018]**

(A) $\sigma_{B<5}(r \bowtie s)$ (B) $\sigma_{B<5}(r \text{ LOJ } s)$
(C) $r \text{ LOJ } (\sigma_{B<5}(s))$ (D) $\sigma_{B<5}(r) \text{ LOJ } s$

Answer Keys

Exercises

Practice Problems 1

1. A **2.** B **3.** C **4.** (i) B (ii) D (iii) D **5.** C **6.** C **7.** (i) A (ii) C
8. (i) B (ii) A **9.** A **10.** A **11.** (i) A (ii) B **12.** (i) B (ii) B (iii) A
13. (i) A (ii) A (iii) A **14.** (i) A (ii) A (iii) A **15.** (i) A (ii) A (iii) A **16.** C
17. A **18.** D **19.** A **20.** D

Practice Problems 2

1. B **2.** D **3.** D **4.** A **5.** C **6.** A **7.** A **8.** A **9.** A **10.** A
11. C **12.** C **13.** C **14.** C **15.** C **16.** B **17.** C **18.** D **19.** B **20.** A

Previous Years' Questions

1. C **2.** A **3.** C **4.** B **5.** C **6.** B **7.** D **8.** A **9.** A **10.** C
11. C **12.** C **13.** A **14.** C **15.** A **16.** B **17.** A **18.** A **19.** D **20.** B
21. 19 **22.** A **23.** C **24.** A **25.** D **26.** D **27.** D **28.** 2 **29.** C **30.** A
31. B **32.** 2.6 **33.** D **34.** 4 **35.** 7 **36.** D **37.** C

Chapter 3

Normalization

LEARNING OBJECTIVES

- Normalization
- Anomalies
- First normal form
- Functional dependency
- Inference rules
- Second normal form
- Third normal form
- Higher normal forms (Boyce-Codd normal form)
- Fifth normal form
- Courses

Normalization

Database design theory includes design standards called *normal forms*. The process of making data and tables match these standards is called *normalizing data* or *data normalization*. By normalizing data, we eliminate redundant information and organize table to make it easier to manage the data and make future changes to the table and database structure. This process removes the insertion, deletion, and modification anomalies. In normalizing your data, we usually divide large tables into smaller, easier to maintain tables. We can then use the technique of adding foreign keys to enable connections between the tables.

Data normalization is part of the database design process and is neither specific nor unique to any particular RDBMS. These are in order, such as first, second, third, Boyce-Codd, fourth, and fifth normal forms. Each normal form represents an increasingly stringent set of rules; that is, each normal form assumes that the requirements of the preceding forms have been met. Many relational database designers feel that, if their tables are in third normal form, most common design problems have been addressed. However, the higher-level normal forms can be of use and are included here.

Database normalization is the process of removing redundant data from tables to improve storage efficiency, data integrity and scalability.

1. In the relational model, methods exists for quantifying how efficient a database is, these classifications are called q'.
2. Normalization generally involves splitting existing tables into multiple ones, which must be rejoined (or) linked each time a query is issued.
3. Edgar F. Codd originally established three normal forms: 1NF, 2NF, 3NF. There are others also, but 3NF is widely considered to be sufficient for most applications, most tables when reaching 3NF are also in BCNF (Boyce–Codd normal form).

Table 1

Title	Author 1	Author 2	I SBN	Subject	Pages	Publisher
Database system concepts	Abraham Silber schatz	Henry F. Korth	0072958863	My SQL, computers	1160	McGraw-Hill
OS concepts	Abraham Silberschatz	Henry F. Korth	0471694665	Computers	990	McGraw-Hill

Problems:

1. This table is not very efficient with storage.
2. This design doesn't protect data integrity.
3. This table doesn't scale well.

Anomalies

An anomaly is a variation that differs in some way from what is said to be normal, with respect to maintaining a database.

1. The basic operations performed on Databases are Record insertion, Record updation, Record deletion.

2. It is desirable for these operations to be straight forward and efficient.
3. When relations are not fully normalized they exhibit anomalies.
4. The design goal of database is too easily to understand and to maintain.
5. Anomalies are problems that occur in un-normalized databases where all the data is stored in one table.

Types of anomalies

There are three types of anomalies that can arise in the database because of redundancy as follows:

1. Insertion anomaly
2. Deletion anomaly
3. Updation anomaly

Insertion anomaly An insertion anomaly occurs when particular attributes cannot be inserted into the database without the presence of other attributes.

Example: Consider the following table: Sales

Sales-Rep-Id	Name	Hire-Date	Client
1	Ana	1/1/2015	Madison
2	Sudha	2/4/2014	Peterson
3	Joey	3/2/2014	John
* New			

Insertion anomaly occurs in the above table which stores records for a company's sales representatives and the clients for whom they are responsible.

1. It is not possible to add records for newly hired Sales representatives until they have been assigned to one or more clients.
2. If we insert a record for newly hired, client column will be NULL, which is a required field for the table.
3. It is not possible to record newly hired in the table during training.

Deletion anomaly Deletion anomaly occurs when some particular attributes are lost because of the deletion of other attributes.

Example: Consider the following table 'course'.

S No	C No	S Name	Course
S41	C9201	John	Sales
S42	C9401	Brat	Finance
S40	C9201	Amit	Sales
S43	C9608	Arun	Accounts

Execute the following *SQL* query:

```
Delete     *
From       course
Where      S No = S43
```

If we delete a tuple where SNo = S43, he is the only (or) last student in the accounts department, we will lose data about student 'S43, Arun' as well as data about Accounts course that is 'C9608, Accounts'.

Updation anomaly An updation anomaly occurs when one or more instances of duplicated data are updated but not all.

Example: Consider the 'course' table given in the above example.

If we want to update course – No (Cno) of sales C9201 to C8686, in the course table.

1. It might happen that, the tuple with S No = S41 updated its CNo to C8686, but not the tuple with SNo = S43.
2. Inconsistency occurs in the table, because for the same course sales we have 2 different course Numbers.

Determining keys For a table 'R', its schema R consists of all attributes of R, we say X is a key to R if $X \to R$ means

X determines R
R is dependent upon X
If you know x then you know R

Example: Consider a relation schema R(ABCDE) and the functional dependencies:

$AC \to D$

$B \to E$

$DA \to B$

The closure of AC determines all the attributes present in Relation R, so the key for R is 'AC'.

$AC^+ =$	$\{AC\}$	(self determination)
	$\{ACD\}$	$(AC \to D)$
	$\{ACDB\}$	$(DA \to B)$
	$\{ACDBE\}$	$(B \to E)$

$\therefore$ key = AC

Any attribute which does not appear on the right-hand-side of a given functional dependency appears in any one of the candidate keys.

1. From the above example, neither A (or) C appears in the right hand side of any functional dependency.

First Normal Form (1NF)

In Table 1, we have two violations of 1NF such as:

1. More than one author field and
2. Subject field contains more than one piece of information with more than one value in a single field; thus, it would be very difficult to search for all books on a given subject.

Table 2 *1NF table*

Title	Author	ISBN	Subject	Pages	Publisher
Database system concept	Abrahem Silbers Chatt	0072958863	My SQL	1160	McGraw-Hill
Database system concept	Henry K. Forth	0072958863	Computers	1160	McGraw-Hill
OS concepts	Henry K. Forth	0471694665	Computers	990	McGraw-Hill
OD concepts	Abraham Silber Schatz	0471694665	Computers	990	McGraw-Hill

In Table 2, we have two rows for a single book. Additionally, we would be violating the second NF. A better solution to the problem would be to separate the data into separate tables—an author table and a subject table to store our information, removing that information from the book table.

Table 3 *Subject table*

Subject–ID	Subject
1	My SQL
2	computers

Table 4 *Author table*

Author–ID	Last Name	First name
1	Silberschatz	Abraham
2	Korth	Henry

Table 5 *Book table*

ISBN	Title	Pages	Publisher
00729 58863	Database System Concepts	1160	McGraw-Hill
04716 94665	OS concepts	990	McGraw-Hill

Each table has a primary key, used for joining tables together when querying the data.

A table is in first normal form (1NF) if there are no repeating groups. A repeating group is a set of logically related fields or values that occur multiple times in one record. The sample tables below do not comply with first normal form. Look for fields that contain too much data and repeating group of fields.

EMPLOYEES_PROJECTS_TIME

A table with fields containing too much data.

Employee ID	Name	Project	Time
EN1-26	Sean O'Brien	30-452-T3, 30-457-T3, 32-244-T3	0.25, 0.40, 0.30
EN1-33	Amy Guya	30-452-T3, 30-382-TC, 32-244-T3	0.05, 0.35, 0.60
EN1-35	Steven Baranco	30-452-T3, 31-238-TC	0.15, 0.80
EN1-36	Elizabeth Roslyn	35-152-TC	0.90
EN1-38	Carol Schaaf	36-272-TC	0.75
EN1-40	Alexandra Wing	31-238-TC, 31-241-TC	0.20, 0.70

The example above is also related to another design issue, namely, that each field should hold the smallest meaningful value and that there should not be multiple values in a single field.

Why is this table design a problem?

There would be no way to sort by last names or to know which allocation of time belonged to which project.

EMPLOYEES_PROJECTS_TIME

Table 5 *A table with repeating groups of fields.*

Emp ID	Last Name	First Name	Project1	Time1	Project2	Time2	Project3	Time3
EN1-26	O'Brien	Sean	30-452-T3	0.25	30-457-T3	0.40	32-244-T3	0.30
EN1-33	Guya	Amy	30-452-T3	0.05	30-382-TC	0.35	32-244-T3	0.60
EN1-35	Baranco	Steven	30-452-T3	0.15	31-238-TC	0.80		
EN1-36	Roslyn	Elizabeth	35-152-TC	0.90				
EN1-38	Schaaf	Carol	36-272-TC	0.75				
EN1-40	Wing	Alexandra	31-238-TC	0.20	31-241-TC	0.70		

If an employee was assigned to a fourth project, you would have to add two new fields to the table. Also, it would be very difficult to total the amount of time devoted to a particular project.

The design problems addressed are very common, particularly among new designers who are accustomed to tracking data in a spreadsheet. Often, when building a spreadsheet, we arrange the data horizontally, laying it out across the spreadsheet. When designing tables, we have to think more vertically. Similar data belongs in the same column or field with a single value in each row.

Now we will take the table you saw above and redesign it so it will comply with first normal form.

Look at the repeating groups of data. Identify tables and fields that will hold this data without the repeating groups. Think vertically and remember that similar data belongs in the same field.

Enter the sample data from the table to make sure you don't have repeating groups. If necessary, include foreign key field(s) to connect the tables.

EMPLOYEES

EmployeeID	Last Name	First Name
EN1-26	O'Brien	Sean
EN1-33	Guya	Amy
EN1-35	Baranco	Steven
EN1-36	Roslyn	Elizabeth
EN1-38	Schaaf	Carol
EN1-40	Wing	Alexandra

PROJECTS_EMPLOYEES_TIME

Project Num	EmployeeID	Time
30-328-TC	EN1-33	0.35
30-452-T3	EN1-26	0.25
30-452-T3	EN1-33	0.05
30-452-T3	EN1-35	0.15
31-238-TC	EN1-35	0.80
30-457-T3	EN1-26	0.40
31-238-TC	EN1-40	0.20
31-241-TC	EN1-40	0.70
32-244-T3	EN1-33	0.60
35-152-TC	EN1-36	0.90
36-272-TC	EN1-38	0.75

Mark the primary key field(s) and foreign keys in each table. Shown below with * indicating the Primary key.

EMPLOYEES

EmployeeID	Last Name	First Name
EN1-26	O'Brien	Sean
EN1-33	Guya	Amy
EN1-35	Baranco	Steven
EN1-36	Roslyn	Elizabeth
EN1-38	Schaaf	Carol
EN1-40	Wing	Alexandra

PROJECTS_EMPLOYEES_TIME

Project Num	EmployeeID	Time
30-328-TC	EN1-33	0.35
30-452-T3	EN1-26	0.25
30-452-T3	EN1-33	0.05
30-452-T3	EN1-35	0.15
31-238-TC	EN1-35	0.80
30-457-T3	EN1-26	0.40
31-238-TC	EN1-40	0.20
31-241-TC	EN1-40	0.70
32-244-T3	EN1-33	0.60
35-152-TC	EN1-36	0.90
36-272-TC	EN1-38	0.75

If an employee was assigned to an additional project, it would involve merely adding a new record. Also, it would be much easier to search for a particular project number as they are all held in a single column.

Functional Dependency

A functional dependency is a relationship between fields so that the value in Field *A* determines the value in Field *B*, and there can be only one value in Field *B*. In that case, Field *B* is functionally dependent on Field *A*. Consider the following sample table:

Airport	City
National	Washington, DC
JFK	New York
LaGuardia	New York
Logan	Boston
Dulles	Washington, DC

Each airport name is unique and each airport can be in only one city. Therefore, City is functionally dependent on Airport. The value in the Airport field determines what the value will be in the City field (making Airport the determinant field) and there can be only one value in the City field. This does not need to work in the reverse. As shown in the table, a city can have more than one airport, so Airport is not functionally

dependent on City; the value in City does not necessarily determine what the value in Airport will be.

You will sometimes see a functional dependency written in this format:

Determinant field(s) → Functionally dependent field

as in:

Airport → City

Functional dependency describes the relationship between attributes in a relation.

Example: If A and B are attributes of relation R, and B is functionally dependent on A ($A \rightarrow B$) if each value of A is associated with one value of B.

$$A \xrightarrow[\text{Dependent on A}]{\text{B is functionally}} B$$

Determinant refers to the attributed (or) group attributes on the left-hand side of the arrow of a functional dependency.

Inference Rules

The following inference rules IR 1 through IR 6 form a complete set for inferring functional and multi-valued dependencies from a given set of dependencies

Assume that all attributes are included in a 'universal' relation schema $R = \{A_1, A_2, \ldots A_N)$ and that X, Y, Z and W are subsets of R.

IR 1 (reflexive rule): if $X \supseteq Y$, then $X \rightarrow Y$
IR 2 (Augmentation rule): $\{X \rightarrow Y\} = XZ \rightarrow YZ$
IR 3 (transitive rule): $\{X \rightarrow Y, Y \rightarrow Z\} = X \rightarrow Z$
IR 4 (complementation rule): $\{X \rightarrow \rightarrow Y\} = \{X \rightarrow \rightarrow (R - (X \cup Y))\}$
IR5 (augmentation rule for MVD's): if $X \rightarrow \rightarrow y$ and $W \rightarrow Z$ then $WX \supseteq YZ$
IR6 (transitive rule for MVD's): $\{X \rightarrow \rightarrow Y, Y \rightarrow \rightarrow Z\} = X \rightarrow \rightarrow (Z - Y)$

Second Normal Form

A table is said to be in second normal form if it is in first normal form and each non-key field is functionally dependent on the entire primary key.

Look for values that occur multiple times in a non-key field. This tells us that we have too many fields in a single table.

Example: In the example below, see all the repeating values in the name and Project Title fields. This is an inefficient way to store and maintain data. In a well-designed database, the only data that is duplicated is in key fields used to connect tables. The presumption is that the data in key fields will rarely change, while the data in non-key fields may change frequently.

A table with a multifield primary key and repeating data in non-key fields

EmployeeID	Last Name	First Name	Project Number	Project Title
EN1-26	O'Brien	Sean	30-452-T3	STAR manual
EN1-26	O'Brien	Sean	30-457-T3	ISO procedures
EN1-26	O'Brien	Sean	31-124-T3	Employee handbook
EN1-33	Guya	Amy	30-452-T3	STAR manual
EN1-33	Guya	Amy	30-482-TC	Web Site
EN1-33	Guya	Amy	31-241-TC	New catalogue
EN1-35	Baranco	Steven	30-452-T3	STAR manual
EN1-35	Baranco	Steven	31-238-TC	STAR prototype
EN1-36	Roslyn	Elizabeth	35-152-TC	STAR pricing
EN1-38	Schaaf	Carol	36-272-TC	Order system
EN1-40	Wing	Alexandra	31-238-TC	STAR prototype
EN1-40	Wing	Alexandra	31-241-TC	New catalogue

If a ProjectTitle changed, we would have to edit it in several records. And what would happen in this table if the EmployeeID was part of the primary key and we wanted to add a new ProjectNum and ProjectTitle even though no employees had yet been assigned?

The primary key cannot contain a null value so you couldn't add the new project. Additionally, if a project ended and you wanted to delete it, you would have to delete the individual values because, if we deleted the records containing the titles and an employee was assigned to only that project, you would also delete that employee's record, something that we may not want to do.

In the above example, the asterisks indicate the fields that make up the primary key of this table as it now stands. A multifield primary key is necessary because neither the EmployeeID nor the ProjectNum fields contain unique values.

The reason there are repeated values in LastName, FirstName, and ProjectTitle is that these fields are dependent

on only part of the primary key. The value in EmployeeID determines what the value in LastName will be, but the value in ProjectNum has nothing to do with it. Similarly, the value in ProjectNum determines the value in ProjectTitle, but EmployeeID does not. These non-key fields relate to only part of the primary key. They are not functionally dependent on the entire primary key.

The solution to this lies in breaking the table into smaller tables that do meet second normal form. You will find that more tables are the solution to most problems encountered during data normalisation.

EMPLOYEES

EmployeeID	Last Name	First Name
EN1-26	O'Brien	Sean
EN1-33	Guya	Amy
EN1-35	Baranco	Steven
EN1-36	Roslyn	Elizabeth
EN1-38	Schaaf	Carol
EN1-40	Wing	Alexandra

EMPLOYEES_PROJECTS

EmployeeID	Project Num
EN1-26	30-452-T3
EN1-26	30-457-T3
EN1-26	31-124-T3
EN1-33	30-328-TC
EN1-33	30-452-T3
EN1-33	32-244-T3
EN1-35	30-452-T3
EN1-35	31-238-TC
EN1-36	35-152-TC
EN1-38	36-272-TC
EN1-40	31-238-TC
EN1-40	31-241-TC

Now we'll take the table above and design new tables that will eliminate the repeated data in the non-key fields.

1. To decide what fields belong together in a table, think about which field determines the values in other fields. Create a table for those fields and enter the sample data.
2. Think about what the primary key for each table would be and about the relationship between the tables. If necessary, add foreign keys or a junction table.
3. Mark the primary key for each table and make sure that you don't have repeating data in non-key fields.

PROJECTS

Projec tNum	Project Title
30-452-T3	STAR manual
30-457-T3	ISO procedures
30-482-TC	Web site
31-124-T3	Employee handbook
31-238-TC	STAR prototype
31-238-TC	New catalog
35-152-TC	STAR pricing
36-272-TC	Order system

Examine the tables to make sure there are no repeating values in non-key fields and that the value in each non-key field is determined by the value(s) in the key field(s). This removes the modification anomaly of having the repeated values.

Third Normal Form

A table is said to be in third normal form if it is in second normal form (2NF) and there are no transitive dependencies.

A transitive dependency is a type of functional dependency in which the value in a non-key field is determined by the value in another non-key field and that field is not a candidate key. Again, look for repeated values in a non-key field as in the following example.

A table with a single field primary key and repeating values in non-key fields.

Project Num	Project Title	Project Mgr	Phone
30-452-T3	STAR manual	Garrison	2756
30-457-T3	ISO procedures	Jacanda	2954
30-482-TC	Web site	Friedman	2846
31-124-T3	Employee handbook	Jones	3102
31-238-TC	STAR prototype	Garrison	2756
31-241-TC	New catalog	Jones	3102
35-152-TC	STAR pricing	Vance	3022
36-272-TC	Order system	Jacanda	2954

The phone number is repeated each time a manager's name is repeated. It is dependent on the manager, which is dependent on the project number (a transitive dependency).

The Project Manager field is not a candidate key, because the same person manages more than one project. Again, the solution is to remove the field with repeating data to a separate table.

Take the above table and create new tables to fix the problem.

1. Think about which fields belong together and create new tables to hold them.
2. Enter the sample data and check for unnecessarily (not part of primary key) repeated values.
3. Identify the primary key for each table and, if necessary, add foreign keys.

PROJECTS

Project Num	Project Title	Project Mgr
30-452-T3	STAR manual	Garrison
30-457-T3	ISO procedures	Jacanda
30-482-TC	Web site	Friedman
31-124-T3	Employee handbook	Jones
31-238-TC	STAR prototype	Garrison
31-241-TC	New catalog	Jones
35-152-TC	STAR pricing	Vance
36-272-TC	Order system	Jacanda

MANAGERS

Project Manager	Phone
Friedman	2846
Garrison	2756
Jacanda	2954
Jones	3102
Vance	3022

Reexamine your tables to make sure there are no unnecessarily repeating values in non-key fields and that the value in each non-key field is determined by the value(s) in the key field(s). In most cases, 3NF should be sufficient to ensure that your database is properly normalised.

Higher Normal Forms (Boyce–Codd Normal Form)

A table is in third normal form (3NF), and all determinants are candidate keys.

Boyce–Codd normal form (BCNF) can be thought of as a 'new' third normal form. It was introduced to cover situations that the 'old' third normal form did not address. The mean of a determinant (determines the value in another field) and candidate keys (qualify for designation as primary key). This normal form applies to situations where you have overlapping candidate keys.

If a table has no non-key fields, it is automatically in BCNF (Figure 1). Look for potential problems in updating existing data (modification anomaly) and in entering new data (insertion anomaly).

Imagine that we were designing a table for a college to hold information about courses, students, and teaching assistants. We have the following business rules:

1. Each course can have many students.
2. Each student can take many courses.
3. Each course can have multiple teaching assistants (TAs).
4. Each TA is associated with only one course.
5. For each course, each student has one TA.

Some sample data:

COURSES_STUDENTS_TA's

CourseNum	Student	TA
ENG101	Jones	Clark
ENG101	Grayson	Chen
ENG101	Samara	Chen
MAT350	Grayson	Powers
MAT350	Jones	O'Shea
MAT350	Berg	Powers

To uniquely identify each record, we could choose CourseNum + Student as a primary key. This would satisfy third normal form also because the combination of CourseNum and Student determines the value in TA. Another candidate key would be Student + TA. In this case, you have overlapping candidate keys (Student is in both). The second choice, however, would not comply with third normal form, because the CourseNum is not determined by the combination of Student and TA; it only depends on the value in TA. This is the situation that Boyce-Codd normal form addresses; the combination of Student + TA could not be considered to be a candidate key.

If we wanted to assign a TA to a course before any students enrolled, we couldn't because Student is part of the primary key. Also, if the name of a TA changed, would have to update it in multiple records. If assume have just these fields, this data would be better stored in three tables: one with CourseNum and Student, another with Student and TA, and third with CourseNum and TA.

COURSES

Course Num	Student
ENG101	Jones
ENG101	Grayson
ENG101	Samara
MAT350	Grayson
MAT350	Jones
MAT350	Berg

STUDENTS

Student	TA
Jones	Clark
Grayson	Chen
Samara	Chen
Grayson	Powers
Jones	O'Shea
Berg	Powers

TA's

*CourseNum	*TA
ENG101	Clark
ENG101	Chen
MAT350	O'Shea
MAT350	Powers

Figure 1 Tables that comply with BCNF.

Fourth Normal Form

A table is in Boyce-Codd normal form (BCNF) and there are no multi-valued dependencies.

A *multi-valued dependency* occurs when, for each value in field *A*, there is a set of values for field *B* and a set of values for field *C* but fields *B* and *C* are not related.

Look for repeated or null values in non-key fields. A multi-valued dependency occurs when the table contains fields that are not logically related. An often used example is the following table:

MOVIES

Movie	Star	Producer
Once Upon a Time	Julie Garland	Alfred Brown
Once Upon a Time	Mickey Rooney	Alfred Brown
Once Upon a Time	Julie Garland	Muriel Humphreys
Once Upon a Time	Mickey Rooney	Muriel Humphreys
Moonlight	Humphrey Bogart	Alfred Brown
Moonlight	Julie Garland	Alfred Brown

A movie can have more than one star and more than one producer. A star can be in more than one movie. A producer can produce more than one movie. The primary key would have to include all three fields, and so this table would be in BCNF. But you have unnecessarily repeated values, with the data maintenance problems that causes and you would have trouble with deletion anomalies.

The Star and the Producer really aren't logically related. The Movie determines the Star and the Movie determines the Producer. The answer is to have a separate table for each of those logical relationships: one holding Movie and Star and the other with Movie and Producer, as shown below:

STARS

*Movie	*Star
Once Upon a Time	Julie Garland
Once Upon a Time	Mickey Rooney
Moonlight	Humphrey Bogart
Moonlight	Julie Garland

PRODUCERS

*Movie	*Producer
Once Upon a Time	Alfred Brown
Once Upon a Time	Muriel Humphreys
Moonlight	Alfred Brown

Above, showing tables that comply with 4NF

Below is another example of a common design error, and it's easily spotted by all the missing or blank values.

PROJECTS_EQUIPMENT

Dept Code	Project Num	Project Mgr ID	Equipment	Property ID
IS	36-272-TC	EN1-15	CD-ROM	657
IS			VGA desktop monitor	305
AC	35-152-TC	EN1-15		
AC			Dot-matrix printer	358
AC			Calculator with tape	239
TW	30-452-T3	EN1-10	486 PC	275
TW	30-457-T3	EN1-15		
TW	31-124-T3	EN1-15	Laser printer	109
TW	31-238-TC	EN1-15	Handheld scanner	479
RI			Fax machine	775
MK			Laser printer	858
MK			Answering machine	187
TW	31-241-TC	EN1-15	Standard 19200 bps modem	386
SL			486 Laptop PC	772
SL			Electronic notebook	458

A table with many null values (Note: It also does not comply with 3NF and BCNF).

It is the same problem here because not all of the data is logically related. As usual, the answer is more tables: one to hold the information on the equipment assigned to departments (with PropertyID as the primary key) and another with projects and departments. We would now the business rules to know whether a project might involve more than one department or manager and be able to figure out the primary key. Assuming a project can have only one manager and be associated with only one department, the tables would be as follows:

EQUIPMENT

*Property ID	Equipment	DeptCode
657	CD-ROM	IS
305	VGA desktop monitor	IS
358	Dot-matrix printer	AC
239	Calculator with tape	AC
275	486 PC	TW
109	Laser printer	TW
479	Handheld scanner	TW
775	Fax machine	RI
858	Laser printer	MK
187	Answering machine	MK
386	Standard 19200 bps modem	TW
772	486 Laptop PC	SL
458	Electronic notebook	SL

PROJECTS_EQUIPMENT

Project Num	Project Mgr ID	Dept Code
36-272-TC	EN1-15	IS
35-152-TC	EN1-15	AC
30-452-T3	EN1-10	TW
30-457-T3	EN1-15	TW
31-124-T3	EN1-15	TW
31-238-TC	EN1-15	TW
31-241-TC	EN1-15	TW

Figure 2 Tables that eliminate the null values and comply with 4NF.

Fifth Normal Form

A table is in fourth normal form (4 NF) and there are no cyclic dependencies.

A *cyclic dependency* can occur only when you have a multifield primary key consisting of three or more fields. For example, let's say your primary key consists of fields *A*, *B*, and *C*. A cyclic dependency would arise if the values in those fields were related in pairs of *A* and *B*, *B* and *C*, and *A* and *C*.

Fifth normal form is also called *projection-join normal form*. A *projection* is a new table holding a subset of fields from an original table. When properly formed projections are joined, they must result in the same set of data that was contained in the original table.

Look for the number of records that will have to be added or maintained

Following is some sample data about buyers, the products they buy, and the companies they buy from.

BUYING

Buyer	Product	Company
Chris	Jeans	Levi
Chris	Jeans	Wrangler
Chris	Shirts	Levi
Lori	Jeans	Levi

Figure 3 A table with cyclic dependencies.

The primary key consists of all three fields. One data maintenance problem that occurs is that you need to add a record for every buyer who buys a product for every company that makes that product or they can't buy from them. That may not appear to be a big deal in this sample of two buyers, two products, and two companies ($2 \times 2 \times 2 = 8$ total records). But what if we went to 20 buyers, 50 products, and 100 companies ($20 \times 50 \times 100 = 100{,}000$ potential records)? It quickly gets out of hand and becomes impossible to maintain.

We might solve this by dividing this into the following two tables:

BUYERS

Buyer	Product
Chris	jeans
Chris	shirts
Lori	jeans

PRODUCTS

Product	Company
jeans	Wrangler
jeans	Levi
shirts	Levi

However, if you joined the two tables above on the Product field, it would produce a record not part of the original data set (it would say that Lori buys jeans from Wrangler). This is where the projection-join concept comes in.

The correct solution would be three tables:

BUYERS

*Buyer	*Product
Chris	jeans
Chris	shirts
Lori	jeans

PRODUCTS

*Product	*Company
jeans	Wrangler
jeans	Levi
shirts	Levi

COMPANIES

*Buyer	*Company
Chris	Levi
Chris	Wrangler
Lori	Levi

Figure 4 Tables that comply with 5NF.

When the first two tables are joined by Product and the result joined to the third table by Buyer and Company, the result is the original set of data.

Exercises

Practice Problems I

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Consider the given functional dependencies

$A \to B$

$BC \to DE$

$AEF \to G$

Which of the following is true?

(A) Functional dependency $ACF \to DG$ implied by the set
(B) Functional dependency $ACF \to DG$ cannot be implied by the set
(C) Functional dependency $AB \to G$ implied by the set
(D) Both (B) and (C)

2. Consider the given relation

DNAME	DNO	MGRNO	LOCATION
RESEARCH	5	333	{BANGLORE,DELHI, HYDERABAD}
ADMINISTRATION	4	987	{CHENNAI}
EXECUTIVES	1	885	{HYDERABAD}

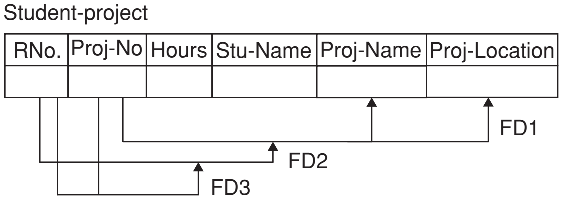
Department

The given relation is

(A) is not in 1NF (B) in 1NF
(C) in 2NF (D) in 3NF

3. Consider the given Relational scheme

Student-project

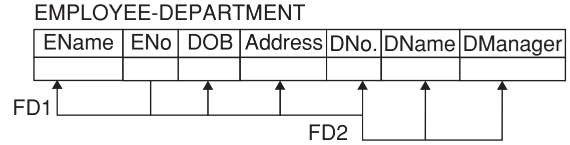


Which functional dependencies are violating 2NF property?

(A) FD1 (B) FD2
(C) FD3 (D) Both A and B

4. Consider the given relation

EMPLOYEE-DEPARTMENT



Which functional dependencies are violating 3NF?

(A) FD1 (B) FD2
(C) Both (D) None of these

5. Consider the given relation $R(A, B, C, D)$and functional dependencies:

$FD = (AB \to C$

$C \to B$

$C \to D)$

Determine the key, prime attributes and non-prime attributes.

(A) $\{A\}, \{AB\}, \{CDE\}$
(B) $\{AB, AC\}, \{ABC\}, \{D\}$
(C) $\{AB, BC\}, \{ABC\}, \{D\}$
(D) $\{AB, AC\}, \{AB\}, \{D\}$

6. Consider the given relation and functional dependencies

$R(ABCDE)$

$FD = (ABD \to C$

$BC \to D$

$CD \to E)$

Determine the key, prime attributes, non-prime attributes and the normal form of the relation?

(A) $\{AB, AD\}, \{ABCD\}, \{E\}$
(B) $\{ABC, ABD\}, \{ABCD\}, \{E\}$
(C) $\{AB, AD\}, \{ABC\}, \{DE\}$
(D) $\{ABC, ABD\}, \{AB\}, \{CDE\}$

7. Consider the given relation and functional dependencies
$R(ABC)$
$FD = (AB \to C$
$C \to A)$
The relation is in which normal form?
(A) 1NF (B) 2NF
(C) 3NF (D) BCNF

8. Consider the given relation and its functional dependencies:
$R(ABCDE)$
$FD = (AB \to C$
$C \to E$
$B \to D$
$E \to A)$
The relation is further decomposed into two relations:
$R_1(BCD), R_2(ACE)$
(A) Decomposition is lossy and dependency preserving
(B) Decomposition is lossless and dependency preserving
(C) Decomposition is lossy and not dependency preserving
(D) Decomposition is lossless and not dependency preserving

9. Consider the following relational instance:

X	Y	Z
1	4	2
1	5	3
1	6	3
3	2	2

Which of the following functional dependencies are satisfied by the instance?
(A) $xy \to z$ and $z \to y$
(B) $yz \to x$ and $y \to z$
(C) $yz \to x$ and $x \to z$
(D) $xz \to y$ and $y \to x$

10. Consider the following functional dependencies:
DOB $\to$ Age
Age $\to$ Eligibility
Name $\to$ RNo
RNo $\to$ Name
CourseNo $\to$ CourseName
CourseNo $\to$ Instructor
(RNo, CNo) $\to$ Grade
The relation (RNo, Name, DOB, Age) is in which normal form?
(A) 1NF (B) 2NF
(C) 3NF (D) BCNF

11. Consider the given functional dependencies:
$AB \to CD$
$AF \to D$
$DE \to F$
$C \to G$
$F \to E$
$G \to A$
Which of the following is false?
(A) $\{CF\}^+ = \{ACDEFG\}$
(B) $\{BG\}^+ = \{ABCDG\}$
(C) $\{AF\}^+ = \{ACDEFG\}$
(D) $\{AB\}^+ = \{ABCDG\}$

12. What should be the key to make the given relation to be in BCNF? The dependencies for the following, 'Grades' relation are GRADES (student-Id, course#, semester#, Grade) student-Id, course#, semester# $\to$ Grade
(A) student-Id
(B) course#
(C) semester#
(D) student-Id, course#, semester #

13. What normal from is the following relation in?
STORE_ITEM (SKU, promotionID, vendor, style, price)
SKU, promotionID $\to$ vendor, style, price
SKU $\to$ vendor, style
(A) 1NF (B) 2NF
(C) 3NF (D) 4NF

14. What normal form is the following relation in?
Only H, I can act as the key
STUFF (H, I, J, K, L, M, N, O)
$H, I \to$ J,K,L
$J \to M$
$K \to N$
$L \to O$?
(A) 1NF (B) 2NF
(C) 3NF (D) BCNF

15. What normal form the following relation is in?
STUFF2(D, O, N, T, C, R, Y)
$D, O \to N, T, C, R, Y$
$C, R \to D$
$D \to N$?
(A) 1NF (B) 2NF
(C) 3NF (D) BCNF

16. The given table is in the BCNF form, convert it to the 4th normal form.

Employee	Skill	Language
Jones	Electrical	French
Jones	Electrical	German
Jones	Mechanical	French
Jones	Mechanical	German
Smith	Plumbing	Spanish

(A)

Employee	Skill

(B)

Employee	Language

(C)

Skill	Language

(D) Both A and B

17. For a database relation $x(a, b, c, d)$, where all the domains of a, b, c, d, include only atomic values, only the following FDs and those that can be inferred from them hold.
$a \to b, c \to d$
the relation is
(A) In 1st NF but not in 2nd NF
(B) In 2nd NF but not in 3rd NF
(C) In 2nd NF
(D) In 3rd NF

18. Which of the following FDs are satisfied by the instance from the below relation:

A	B	C
2	8	4
2	10	6
2	12	6
6	4	4

(A) AB → C and C → B
(B) BC → A and B → C
(C) BC → A and A → C
(D) AC → B and B → A

19. Consider the following database:
Course # → Title
Course # time → location
Emp – ID → T –Name salary
is in
(A) 3NF (B) 2NF
(C) 1NF (D) BCNF

20. Consider the following schema
$A = (w, x, y, z)$ and the dependencies are
$W \to X, X \to Y, Y \to Z$, and $Z \to W$
Let $A = (A_1$ and $A_2)$ be a decomposition such that $A_1 \cap A_2 = \phi$
The decomposition is
(A) In 1NF and in 2NF
(B) In 2NF and not in 3NF
(C) In 2NF and in 3NF
(D) Not in 2NF and in 3NF

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Integrity constraints ensures that changes made to the database by authorized users do not result in
(A) Loss of FDs
(B) Loss of keys
(C) Loss of tables
(D) Loss of data consistency

2. Relation $R = (\overline{A}, B, C, D)$ with AB as primary key. Choose one FD such that R should be in 1NF but not in 2NF
(A) $AB \to C$
(B) $AB \to D$
(C) $A \to D$
(D) $AB \to CD$

3. A normalized relation (1NF) can be retrieved from unnormalized relation by removing
(A) repeating groups (B) duplicate tuples
(C) transitive dependency (D) primary key

4. A relation will be in 2NF, if we
(A) remove repeating groups
(B) remove partial dependency
(C) remove transitive dependency
(D) have overlapping candidate key

5. Relation $R = (A, B, C, D)$ with AB as primary key, choose the FD so that R should be in 2NF but not in 3NF.
(A) $D \to C$ (B) $AB \to C$
(C) $AB \to D$ (D) $A \to B$

6. If a relation is in 2NF, then it can be in 3NF by removing
(A) repeating groups
(B) partial dependencies
(C) transitive dependencies
(D) overlapping dependencies

7. BCNF can be achieved from 3NF by removing
(A) repeating groups
(B) partial dependencies
(C) transitive dependencies
(D) overlapping dependencies

8. Which one of the following is not possible?
(A) Relation is in BCNF but not in 4NF
(B) Relation is in 3NF but not in BCNF
(C) Relation is in 2NF but not in 3NF
(D) Relation is in 3NF but not in 2NF

Common data for questions 9 and 10: Let R be a relation schema R (A, B, C, D);

$F = \{AB \rightarrow CD;\ C \rightarrow A\}$ F is the set of functional dependencies

9. How many prime attributes are there?
(A) 1 (B) 2
(C) 3 (D) 4

10. The highest normal form of the above relation is
(A) 1NF (B) 2NF
(C) 3NF (D) 4NF

Linked answer questions

11. For a given relation schema $R = \{A, B, C, D, E\}$
$A \rightarrow BC$
$CD \rightarrow E$
$B \rightarrow D$
$E \rightarrow A$
Which of the following is not a candidate key?
(A) A (B) B
(C) E (D) BC

12. For the above answer, what is the closure?
(A) BD (B) ABC
(C) $ABCDE$ (D) BC

13. Consider the following functional dependencies:
$A \rightarrow B$
$C \rightarrow D$
$B \rightarrow E$
$F \rightarrow A$
The relation (A, B, C, D) is
(A) in second normal form, but not in third normal form
(B) in third normal form, but not in $B\ C\ N\ F$
(C) in $B\ C\ N\ F$
(D) None of the above

Common data for questions 14 and 15:
$R = (A, B, C, D, E, F)$
FDs = $A \rightarrow B$
$C \rightarrow$ DF
$AC \rightarrow E$
$D \rightarrow F$

14. Determine the key from the given FDs:
(A) AB (B) AC
(C) ACB (D) ACD

15. Decompose the FDs into 2NF
(A) $R_1(AB)\ R_2(CDF)\ R_3(ACE)$
(B) $R_1(AB)\ R_2(CDEF)$
(C) $R_1(ABC)\ R_2(CDF)$
(D) $R_1(AB)\ R_2(CD)\ R_3(EF)$

16. For a database relation $x(a, b, c, d)$, where all the domains of a, b, c, d, include only atomic values, only the following FDs and those that can be inferred from them hold.
$a \rightarrow b,\ c \rightarrow d$
The relation is decomposed into $R_1(ab)$, $R_2(cd)$. Which of the following is true, The decomposition
(A) is dependency preserving
(B) is not dependency preserving
(C) is loss less
(D) Both A and C

17. Which of the following FDs are satisfied by the instance from the below relation?

A	B	C
4	12	8
4	14	10
4	16	10
10	8	8

(A) $AB \rightarrow C$ and $C \rightarrow B$
(B) $BC \rightarrow A$ and $B \rightarrow C$
(C) $BC \rightarrow A$ and $A \rightarrow C$
(D) $AC \rightarrow B$ and $B \rightarrow A$

18. Indicate which of the following statements are false: 'A relational database, which is in 3NF still have undesirable data redundancy because there may exist.
(A) Below all
(B) Non trivial FDs involving prime attributes on the right side.
(C) Non-trivial FDs involving prime attributes on the left side
(D) Non-trivial FDs involving only prime attributes

19. Consider the following database:
SOFTWARE (software-vendor, product, Release-date, systemReq, warranty)
FD: (software-vendor, product, Releasedate) $\rightarrow$ system Req, price, Warranty.
Which of the following are non–prime attributes?
(A) SystemReq
(B) Price
(C) Warranty
(D) All the above

20. Consider a relation schema R (A, B, C, D, E, X, Y) with the following FDs
$F = \{0 \rightarrow A, XD \rightarrow C, DA \rightarrow B, A \rightarrow X, XE \rightarrow B, E \rightarrow A, B \rightarrow D, DA \rightarrow B, EB \rightarrow C, AB \rightarrow C, Y \rightarrow B, C \rightarrow B\}$
is in
(A) 2NF
(B) 3NF
(C) 4NF
(D) BCNF

Previous Years' Questions

1. Which one of the following statements is false? **[2007]**
(A) Any relation with two attributes is in BCNF
(B) A relation in which every key has only one attribute is in 2NF
(C) A prime attribute can be transitively dependent on a key in a 3NF relation.
(D) A prime attribute can be transitively dependent on a key in a BCNF relation.

2. Consider the following relational schemas for a library database:

Book (Title, Author, Catalog_ no, Publisher, Year, Price)

Collection (Title, Author, Catalog_ no)

with the following functional dependencies:

I. Title Author $\rightarrow$ Catalog_no

II. Catalog_no $\rightarrow$ Title Author Publisher Year

III. Publisher Title Year $\rightarrow$ Price

Assume {Author, Title} is the key for both schemas. Which of the following statements is true? **[2008]**
(A) Both Book and Collection are in BCNF
(B) Both Book and Collection are in 3NF only
(C) Book is in 2NF and Collection is in 3NF
(D) Both Book and Collection are in 2NF only

3. The following functional dependencies hold for relations $R(A, B, C)$ and $S(B, D, E)$
$B \rightarrow A,$
$A \rightarrow C$

The relation R contains 200 tuples and the relation S contains 100 tuples. What is the maximum number of tuples possible in the natural join $R \bowtie S$? **[2010]**
(A) 100 (B) 200
(C) 300 (D) 2000

4. Which of the following is true? **[2012]**
(A) Every relation in 3NF is also in BCNF
(B) A relation R is in 3NF if every non-prime attribute of R is fully functionally dependent on every key of R
(C) Every relation in BCNF is also in 3NF
(D) No relation can be in both BCNF and 3NF

Common data questions 5 and 6: Relation R has eight attributes ABCDEFGH, Fields of R contain only atomic values.

$F = \{CH \rightarrow G, A \rightarrow BC, B \rightarrow CFH, E \rightarrow A, F \rightarrow EG\}$ is a set of functional dependencies (FDs) so that F^+ is exactly the set of FDs that hold for R.

5. How many candidate keys does the relation R have? **[2013]**
(A) 3 (B) 4
(C) 5 (D) 6

6. The relation R is **[2013]**
(A) in 1NF, but not in 2NF
(B) in 2NF, but not in 3NF
(C) in 3NF, but not in BCNF
(D) in BCNF

7. Assume that in the suppliers relation above, each supplier and each street within a city has a unique name, and (sname, city) forms a candidate key. No other functional dependencies are implied other than those implied by primary and candidate keys. Which one of the following is true about the above schema? **[2009]**
(A) The schema is in BCNF
(B) The schema is in 3NF but not in BCNF
(C) The schema is in 2NF but not in 3NF
(D) The schema is not in 2NF

8. Consider the relation scheme $R = (E, F, G, H, I, J, K, L, M, N)$ and the set of functional dependencies $\{\{E, F\} \rightarrow \{G\}, \{F\} \rightarrow \{I, J\}, \{E, H\} \rightarrow \{K, L\}, \{K\} \rightarrow \{M\}, \{L\} \rightarrow \{N\}\}$ on R. What is the key for R? **[2014]**
(A) $\{E, F\}$ (B) $\{E, F, H\}$
(C) $\{E, F, H, K, L\}$ (D) $\{E\}$

9. Given the following two statements:

S_1: Every table with two single-valued attributes is in 1NF, 2NF, 3NF and BCNF

S_2: $AB \rightarrow C, D \rightarrow E, E \rightarrow C$ is a minimal cover for the set of functional dependencies $AB \rightarrow C, D \rightarrow E, AB \rightarrow E, E \rightarrow C$

Which one of the following is correct? **[2014]**
(A) S_1 is true and S_2 is false
(B) Both S_1 and S_2 are true
(C) S_1 is false and S_2 is true
(D) Both S_1 and S_2 are false

10. The maximum number of super-keys for the relation schema R (E, F, G, H) with E as the key is ______. **[2014]**

11. A *prime attribute* of a relation scheme R is an attribute that appears **[2014]**
(A) in all candidate keys of R
(B) in some candidate key of R
(C) in a foreign key of R
(D) only in the primary key of R

12. Consider an entity-Relationship (ER) model in which entity sets E_1 and E_2 are connected by an m:n relationship R_{12}. E_1 and E_3 are connected by a 1:n (1 on the side of E_1 and n on the side of E_3) relationship R_{13}.

E_1 has two single-valued attributes a_{11} and a_{12} of which a_{11} is the key attribute. E_2 has two single-valued

attributes a_{21} and a_{22} of which a_{21} is the key attribute. E_3 has two single-valued attributes a_{31} and a_{32} of which a_{31} is the key attribute. The relationships do not have any attributes.

If a relational model is derived from the above ER model, then the minimum number of relations that would be generated if all the relations are in 3 NF is _______. **[2015]**

13. Consider the relation $X(P, Q, R, S, T, U)$ with the following set of functional dependencies

```
F = {
    {P, R} → {S, T}
    {P, S, U} → {Q, R}
    }
```

Which of the following is the trivial functional dependency in $F+$, where $F+$ is closure of F? **[2015]**

(A) $\{P, R\} \rightarrow \{S, T\}$
(B) $\{P, R\} \rightarrow \{R, T\}$
(C) $\{P, S\} \rightarrow \{S\}$
(D) $\{P, S, U\} \rightarrow \{Q\}$

14. A database of research articles in a journal uses the following schema. **[2016]**

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE)

The primary key is (VOLUME, NUMBER, STARTPAGE,ENDPAGE) and the following functional dependencies exist in the schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) → TITLE

(VOLUME, NUMBER) → YEAR

(VOLUME, NUMBER,
STARTPAGE,ENDPAGE) → PRICE

The database is redesigned to use the following schemas.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, PRICE)

(VOLUME, NUMBER, YEAR)

Which is the weakest normal form that the new database satisfies, but the old one does not?

(A) 1NF (B) 2NF
(C) 3NF (D) BCNF

15. Consider the following database table water_schemes: **[2016]**

water_schemes		
scheme_no	**District name**	**Capacity**
1	Ajmer	20
1	Bikaner	10
2	Bikaner	10
3	Bikaner	20
1	Churu	20
2	Churu	20
1	Dungargarh	10

The number of tuples returned by the following SQL query is ______ .

with total (name, capacity) **as**
select district _ name, **sum** (capacity)
from water _ schemes
group by district _ name
with total _avg (capacity) **as**
select avg (capacity)
from total
select name
from total, total _ avg
where total . capacity ≥ total_avg. capacity

16. The following functional dependencies hold true for the relational schema $R\ \{V, W, X, Y, Z\}$:

$$V \rightarrow W$$
$$VW \rightarrow X$$
$$Y \rightarrow VX$$
$$Y \rightarrow Z$$

Which of the following is irreducible equivalent for this set of set of functional dependencies? **[2017]**

(A) $V \rightarrow W$, $V \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow Z$
(B) $V \rightarrow W$, $W \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow Z$
(C) $V \rightarrow W$, $V \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$
(D) $V \rightarrow W$, $W \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$

17. Consider the following tables T1 and T2.

T1

P	Q
2	2
3	8
7	3
5	8
6	9
8	5
9	8

T2

R	S
2	2
8	3
3	2
9	7
5	7
7	2

In table T1, **P** is the primary key and **Q** is the foreign key referencing **R** in table T2 with on-delete cascade and on-update cascade. In table T2, **R** is the primary key and **S** is the foreign key referencing **P** in table Tl

with on-delete set NULL and on-update cascade. In order to delete record ⟨3, 8⟩ from table T1, the number of additional records that need to be deleted from table T1 is __________. **[2017]**

18. Consider the following four relational schemas. For each schema, all non-trivial functional dependencies are listed. The underlined attributes are the respective primary keys.

Schema I:

Registration (<u>rollno,</u> courses)

Field 'courses' is a set-valued attribute containing the set of courses a student has registered for.

Non-trivial functional dependency:

Rollno → courses

Schema II:

Registration (<u>rollno, courseid</u>, email)

Non-trivial functional dependencies:

Rollno, courseid → email

email → rollno

Schema III:

Registration (<u>rollno, courseid</u>, marks, grade)

Non-trivial functional dependencies:

Rollno, courseid → marks, grade

Marks → grade

Schema IV:

Registration (<u>rollno, courseid,</u> credit)

Non-trivial functional dependencies:

Rollno, courseid → credit

Courseid → credit

Which one of the relational schemas above is in 3NF but not in BCNF? **[2018]**

(A) Schema I
(B) Schema II
(C) Schema III
(D) Schema IV

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** A	**3.** D	**4.** B	**5.** B	**6.** B	**7.** C	**8.** D	**9.** B	**10.** A
11. C	**12.** D	**13.** A	**14.** B	**15.** A	**16.** D	**17.** A	**18.** B	**19.** C	**20.** C

Practice Problems 2

1. D	**2.** C	**3.** A	**4.** B	**5.** A	**6.** C	**7.** D	**8.** D	**9.** C	**10.** C
11. B	**12.** A	**13.** D	**14.** B	**15.** A	**16.** A	**17.** C	**18.** C	**19.** D	**20.** B

Previous Years' Questions

1. D	**2.** C	**3.** A	**4.** C	**5.** B	**6.** A	**7.** B	**8.** B	**9.** A	**10.** 8
11. B	**12.** 4	**13.** C	**14.** B	**15.** 2	**16.** A	**17.** 0	**18.** B		

Chapter 4

Transaction and Concurrency

LEARNING OBJECTIVES

- *Transactions and concurrency control*
- *Transaction*
- *Transaction properties*
- *Uncommitted data*
- *Transaction processing systems*
- *Concurrency control with locking methods*
- *Two-phase locking to ensure serializability*
- *Concurrency control with time stamping methods*
- *Concurrency control with optimistic methods*
- *Recoverability*
- *Equivalence of schedules*
- *Testing for conflict serializability*

Introduction

A transaction is a logical unit of work. It begins, with the execution of a BEGIN TRANSACTION operation, and ends with the execution of a COMMIT or ROLLBACK operation. The logical unit of work that is, a transaction does not necessarily involve just a single database operation. Rather, it involves a sequence of several such operations as follows:

1. Database updates are kept in buffers in main memory and not physically written to disk until the transaction commits. That way, if the transaction terminates unsuccessfully, there will be no need to undo any disk updates.
2. Database updates are physically written to disk as part of the process of honouring the transaction's COMMIT request. That way if the system subsequently crashes, we can be sure that there will be no need to redo any disk updates.

Transactions and Concurrency Control

Database transactions reflect real-world transactions that are triggered by events, such as buying a product, registering for a course, or making a deposit in your checking account. Transactions are likely to contain many parts, for example, a sales transaction consists of at least two parts.

UPDATE inventory by subtracting number of units sold from the PRODUCT table's available quantity on hand and UPDATE the ACCOUNTS RECEIVABLE table in order to bill the CUSTOMER. All parts of a transaction must be completed to prevent data integrity problems. Therefore, executing and managing transactions are important database system activities.

Concurrency control is the management of concurrent transactions execution. When many users are able to access the database, the number of concurrent transactions tends to grow rapidly; as a result, concurrency control is especially important in multiuser database environments.

Transaction

A transaction is a logical unit of work that must be either entirely completed or aborted, no intermediate states are acceptable, that is, multicomponent transactions like the previously mentioned sale, must not be partially completed. If you read from and/or write to (update) the database, you create a transaction. Another example is using SELECT, to generate a list of table contents. Many real-world database transactions are formed by two or more database requests. A database request is the equivalent of a single SQL statement in an application program or transaction. Each database request generates several input/output operations. A transaction that changes the contents of a database must alter the database from one consistent state to another. A consistent database state is one in which all data integrity constraints are satisfied.

Example:

1. Checking an account balance:

   ```
   SELECT ACC_NUM, ACC_BALANCE
   FROM CHECKACC
   WHERE ACC_NUM = '0908110638';
   ```

   Even though we did not make any changes to the CHECKACC table, the SQL code represents a transaction, because we accessed the database.

2. Registering a credit sale of 100 units of product *X* to customer *Y* in the amount of \$500.00 first, product X's quantity on hand (QOH) needs to be reduced by 100.

   UPDATE PRODUCT
   SET PROD_QOH = PROD_QOH_100
   WHERE PROD_CODE = '*x*';
   Then, \$500 needs to be added to customer Y's accounts receivable
   UPDATE ACCT_RECEIVABLE
   SET ACCT_RECEIVABLE = ACCT_BALANCE + 500
   WHERE ACCT_NUM = '*Y*';

In Example 2, both the SQL, transactions must be completed in order to represent the real-world sales transaction. If both transactions are not completely executed, the transaction yields an inconsistent database.

If a transaction yields an inconsistent database, the DBMS must be able to recover the database to a previous consistent state.

Transaction Properties

All transactions must display atomicity, consistency, isolation and durability. These are known as ACID properties of transactions.

Atomicity

It requires that all operations of a transaction be completed; if not, the transaction is aborted. Therefore, a transaction is treated as a single, logical unit of work.

Consistency

It describes the result of the concurrent execution of several transactions. The concurrent transactions are treated as though they were executed in serial order. This property is important in multiuser and distributed database, where several transactions are likely to be executed concurrently.

Isolation

It means that the data used during the execution of a transaction cannot be used by a second transaction until the first one is completed. Therefore, if a transaction T_1 is being executed and is using the data item X_1, that data item cannot be accessed by any other transaction ($T_2 \dots T_n$) until T_1 ends. This property is particularly useful in multiuser database environment, because several different users can access and update the database at the same time.

Durability

It indicates the permanence of the database's consistent state. When a transaction is completed, the database reaches a consistent state, and that state cannot be lost, even in the event of the system's failure.

Transaction Management with SQL

The ISO standard defines a transaction model based on two SQL statements: COMMIT and ROLLBACK. The standard specifies that an SQL transaction automatically begins with a transaction-initiating SQL statement executed by a user or program (e.g., SELECT, INSERT, UPDATE). Changes made by a transaction are not visible to other concurrently executing transactions until the transaction completes. When a transaction sequence is initiated, it must continue through all succeeding SQL, statements until one of the following four events occur:

1. A COMMIT statement ends the transaction successfully, making the database changes permanent.
   A new transaction starts after COMMIT with the next transaction initiating statement.
2. For programmatic SQL, successful program termination ends the final transaction successfully, even if a commit statement has not been executed (equivalent to COMMIT)
3. For programmatic SQL, abnormal program termination aborts the transaction (equivalent to ROLLBACK)

Example:
UPDATE PRODUCT
SET PROD_QOH = PROD_QOH_100
WHERE PROD_CODE = '345TYX';

UPDATE ACCREC
SET AR_BALANCE = AR_BALANCE + 3500
WHERE AR_NUM = '60120010';
COMMIT;

Concurrency Control

The coordination of simultaneous execution of transactions in a multiprocessing database system is known as *concurrency control*. The objective of concurrency control is to ensure the serializability of transaction in a multiuser database environment. Concurrency is important, because the simultaneous execution of transactions over a shared database can create several data integrity and consistency problems. Three main problems are lost updates, uncommitted data and inconsistent retrievals.

Lost Updates

Consider the following two concurrent transactions where PROD_QOH represents a particular PRODUCT's quantity on hand. (PROD_QOH is an attribute in the Product table)

Assume the current PROD_QOH value for the product concerned is 35.

Table 1

Transaction	Computation
T_1: purchase 100 units	PROD_QOH = PROD_QOH + 100
T_2: sell 30 units	PROD_QOH = PROD_QOH – 30

Table 1 shows the serial execution of these transactions under normal circumstances, yielding the correct answer: PROD_QOH = 105. But suppose that a transaction is able to read a product's PROD_QOH value from the table before a previous transaction (using the same product) has been committed. The sequence depicted in Table 2 shows how the cost update problem can arise. Note that the first transaction (T_1) has not yet been committed when the second transaction (T_2) is executed. Therefore T_2 still operates on the value 135 to disk which is promptly over written by T_2. As a result, the addition of 100 units is "lost" during the process.

Uncommitted Data

Data are not committed when two transactions, T_1 and T_2 are executed concurrently and the first transaction (T_1) is rolled back after the second transaction (T_2) has already accessed the uncommitted data, thus violating the isolation property of transaction. Consider the same transactions from T_1 and T_2, from above. However, this time T_1 is rolled back to eliminate the addition of the 100 units. Because T_2 subtracts 30 from the original 35 units, the correct answer should be 5.

Table 2

	Computation
T_1: purchase 100 units	PROD_QOH = PROD_QOH + 100 (Rolled Back)
T_2: sell 30 units	PROD_QOH = PROD_QOH_30

Table 2 shows how, under normal circumstances, the serial execution of these transactions yield the correct answer. The uncommitted data problem can arise when the ROLLBACK is completed after T_2 has begun its execution.

Inconsistent Retrievals

Inconsistent retrievals occur when a transaction calculates some summary (aggregate) functions over a set of data, while other transactions are updating the data. The problem is that the transaction might read some data before they are changed and other data after they are changed, thereby yielding inconsistent results.

Example:

1. T_1 calculates the total PROD_QOH of the products stored in the PRODUCT table
2. At the same time, T_2 updates the PROD_QOH for two of the PRODUCT table's products (T_2 represents the correction of a typing error: the user added 30 units to product 345TYX's PROD_QOH but meant to add the 30 units to 125TYZ's PROD_QOH to correct the problem, the user subtracts 30 from product 345TYX's PROD_QOH and adds 30 to product 125TYZ's PROD_QOH).

The computed answer 485 is obviously wrong, because we know the correct answer to be 455.

Transaction Processing Systems

Transaction processing systems are systems with large databases and hundreds of concurrent users that are executing database transactions. For example, banking, credit card processing, stock markets, supermarket checkout, etc.

They require high availability and fast response time for hundreds of concurrent users.

1. A transaction includes one or more database access operations. These can include insertion, deletion, modification, or retrieval operations.
2. *Basic operations*: The basic database access operations that a transaction can include are as follows:
   - read_item (X): Reads a database item named X into a program variable.
   - Write_item (X): Writes the value of program variable X into the database item named X

Executing a read_item (X): Command includes the following steps:

1. Find the address of the disk block that contains item X
2. Copy that disk block into a buffer in main memory (if that disk block is not in main memory buffer).
3. Copy item X from the buffer to the program variable named x

Executing a write_item (X) command includes the following steps:

1. Find the address of the disk block that contains item X
2. Copy that disk block into a buffer in main memory (if that disk block is not in main memory buffer)
3. Copy item X from the program variable named X into its correct location in the buffer
4. Store the updated block from the buffer back to disk.

Step 4 actually updates the database on disk.

The decision about when to store back a modified disk block that is in a main memory buffer is handled by the recovery manager of the DBMS in cooperation with the underlying operating system.

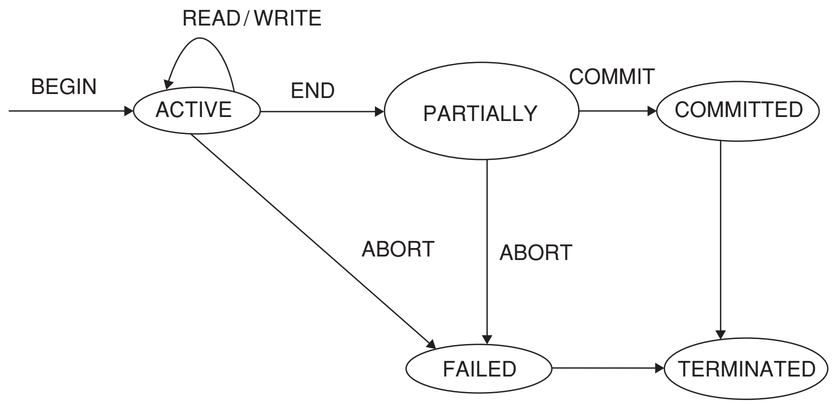


Figure 1 Transactions execution state transition diagram.

For the purpose of recovery, the system needs to keep track of when the transaction starts, terminates, and commits or aborts. Hence, the recovery manager keeps track of the following operations:

1. BEGIN_TRANSACTION: It shows the beginning of Execution of a transaction.
2. READ/WRITE: These specify read or write operations on the database items.
3. END_TRANSACTION: This specifies that READ and WRITE operations have ended and marks the end of transaction execution.
4. COMMIT_TRANSACTION: This shows a successful end of the transaction so that any changes executed by the transaction can be safely committed to the database and will not be undone.
5. ROLL BACK OR ABORT: This shows that the transaction has ended unsuccessfully, so that any changes or effects that the transaction may have applied to the database must be undone.
6. ACTIVE STATE: A transaction goes into an active state immediately after it starts execution where it can issue READ and WRITE operations.
7. PARTIALLY COMMITTED: When the transaction ends, it moves to the partially committed state
8. COMMIT: A transaction reaches its commit point when all its operations that access the database have been executed successfully, and the effect of all the transaction operations on the database have been recorded in the log. Beyond the commit point, the transaction is said to be committed, and its effect is assumed to be permanently recorded in the database.
9. FAILED_STATE: A transaction can go to the failed state if the transaction is aborted during its active state. The transaction may then have to be rolled back to undo the effect of its WRITE operations on the database.
10. TERMINATIED: The terminated state corresponds to the transactions leaving the system.

Concurrency Control with Locking Methods

A lock guarantees exclusive use of a data item to a transaction. In general, if transaction T_1 holds a lock on a data item (e.g., an employee's salary) then transaction T_2 does not have access to that data item. A transaction acquires a lock prior to data access; the lock is released (unlocked) when the transaction is completed, so that another transaction can lock the data item for its exclusive use. All lock information is managed by a lock manager, which is responsible for assigning and policing the locks used by the transactions.

Lock Granularity

Lock granularity indicates the level of lock use. Locking can take place at the following levels: database level, table level, page level, row level and field (or attribute) level.

Database Level

In a database-level lock, the entire database is locked, thus preventing the use of any tables in the database by transaction T_2 while transaction T_1 is being executed. Transaction T_1 and T_2 cannot access the database concurrently, even if they use different tables. This level of locking is suitable for batch processes, but it is not unsuitable for online multiuser DBMSs.

Table Level

In a table-level lock, the entire table is locked, preventing access to any row by transaction T_2 while transaction T_1 is using Table 2 transactions can access the same database, as long as they access different tables. Transactions T_1 and T_2 cannot access the same table even if they try to use different rows, T_2 must wait until T_1 unlocks the table.

Page level

In a page level lock, the DBMS will lock an entire disk page (a disk page or page is the equivalent of a disk block, which can

be described as a (referenced) section of a disk). Transactions T_1 and T_2 access the same table while locking different disk pages. If T_2 requires the use of a row located on a page that is locked by T_1, T_2 must wait until the page is unlocked by T_1.

Row level

The row-level lock is much less restrictive than the locks discussed earlier. The DBMS allows concurrent transactions to access different rows of the same table, even if the rows are located on the same page. A lock exists for each row in each table of the database.

Field level

The field-level lock allows concurrent transactions to access same row as long as they require the use of different fields (attributes) within the row. Although, field-level locking clearly yields the most flexible multi user data access, it requires a high level of computer over head.

Lock Types

1. Binary locks
2. Shared/Exclusive locks

Binary Locks

A binary lock has only two states: locked (1) or unlocked (0). If an object, that is, a database, table, page, or row is locked by a transaction, no other transaction can use that object. If an object is unlocked, any transaction can lock the object for its use. As a rule, a transaction must unlock the object after its termination. Every database operation requires that the affected object be locked. Therefore, every transaction requires a lock and unlock operation for each data item that is accessed. Such operations are automatically scheduled by the DBMS, the user need not concerned about locking or unlocking data items. Binary locks are now considered too restrictive to yield optimal concurrency conditions. For example if two transaction want to read the same database object, the DBMS will not allow this to happen, even though neither transaction updates the database (and therefore, no concurrency problems can occur) concurrency conflicts occur only when two transactions execute concurrently and one of them updates the database.

Shared/Exclusive locks

The tables "shared' and "exclusive" indicate the nature of the lock. The following table comparatively explains both locks.

Exclusive Locks	Shared Locks
An *exclusive lock* exists when access is specifically reserved for the transaction that locked the object.	A *shared lock* exists when concurrent transactions are granted READ access on the basis of a common lock.
The exclusive lock must be used when the potential for conflict exists.	A shared lock produces no conflict as long as the concurrent transactions are read only.
(An exclusive lock is issued when a transaction wants to write (update) a data item and no locks are currently held on that data item by any other transaction.	A shared lock is issued when a transaction wants to read data from the database and no exclusive lock is held on that data item.

Using the shared/exclusive locking concept, a lock can have three states: unlocked, shared (READ) and exclusive (WRITE). 2 READ transactions can be safely executed and shared locks allow several READ transactions to concurrently read the same data item. For example, if transaction T_1 has a shared lock on data item X, and transaction T_2 wants to read data item X, T_2 may also obtain a shared lock on data item X.

If transaction T_2 updates data item X, then an exclusive lock is required by T_2 over data item X. The exclusive lock is granted if and only if no other locks are held on the data item. Therefore, if a shared or exclusive lock is already held on data item X by transaction T_1, an exclusive lock cannot be granted to transaction T_2.

Potential problems with locks

Although locks prevent serious data inconsistencies, their use may lead to two major problems:

1. The resulting transaction schedule may not be serializable.
2. The schedule may create deadlocks. Database *deadlocks* are the equivalent of a traffic gridlock in a big city and are caused when *two transactions wait for each other to unlock data.*

Both problems can be solved. Serializability is guaranteed through a locking protocol known as two-phase locking and deadlocks can be eliminated by using deadlock detection, and prevention techniques. We shall examine these techniques next.

Two-phase Locking to Ensure Serializability

The *two-phase locking protocol* defines how transactions acquire and relinquish locks. It guarantees serializability, but it does NOT prevent deadlocks. The two phases are as follows:

1. A *growing phase*, in which a transaction acquires all the required locks without unlocking any data. Once all locks have been acquired, the transaction is in its locked point.
2. A *shrinking phase*, in which a transaction releases all locks and cannot obtain any new lock.

The two-phase locking protocol is governed by the following rules"

1. Two transactions cannot have conflicting locks.
2. No unlock operation can precede a lock operation in the same transaction.
3. No data are affected until all locks are obtained, that is, until the transaction is in its locked point.

DEADLOCKS

Deadlocks exist when two transactions, T_1 and T_2, exist in the following mode:

1. T_1 would like to access data item X and then data item Y. (So far, T_1 has locked data item X and T_1 is in progress, it will eventually require to lock data item Y.)
2. T_2 needs to access data items X and Y, to begin. (So far, T_2 has locked data item Y.)

If T_1 has not unlocked data item X, T_2 cannot begin; if T_2 has not unlocked data item Y, T_1 cannot continue. Consequently, T_1 and T_2 wait indefinitely, each waiting for the other to unlock the required data item. Such in a real-world DBMS, many transactions can be executed simultaneously, thereby increasing the probability of generating deadlocks. Note that deadlocks are possible only if one of the transactions wants to obtain an exclusive lock on a data item; no deadlock condition can exist among shared locks.

Three basic techniques exist to control deadlocks:

1. *Deadlock prevention*: A transaction requesting a new lock is aborted if there is a possibility that a deadlock can occur. If the transaction is aborted, all the changes made by this transaction are ROLLED BACK, and all locks obtained by the transaction are released. The transaction is then rescheduled for execution. Deadlock prevention works because it avoids the conditions that lead to deadlocking.
2. *Deadlock detection*: The DBMS periodically tests the database for deadlocks. If a deadlock is found, one of the transactions (the "victim") is aborted (ROLLED BACK and restarted), and the other transaction continues.
3. *Deadlock avoidance*: The transaction must obtain all the locks it needs before it can be executed.

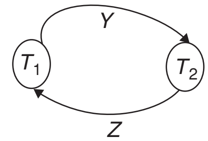
The best deadlock control method depends on the database environment. For example, if the probability of deadlocks is low, deadlock detection is recommended. However, if the probability of deadlocks is high, deadlock prevention is recommended. If response time is not high on the system priority list, deadlock avoidance might be employed.

Deadlock occurs when each transaction T in a set of two or more tractions is waiting for some item that is locked by some other transaction T^1 in the set. Each transaction in the set is on a waiting queue, waiting for one of the other transactions in the set to release the lock on an item

Example:

T_1	T_2
Write lock (z)	
	Read lock (z)
	Read lock (y)
Write lock (y)	

Transaction T_1 is waiting for Y which is locked by Transaction T_2 and transaction T_2 is waiting for z which is locked by transaction T_1. The below graph is called *wait for graph*.



Deadlock Prevention

There are number of deadlock prevention schemes that make a decision about what to do with a transaction involved in a possible deadlock situation:

1. Should it be blocked and made to wait
2. Should it be aborted
3. Should the transaction pre-empt and abort another transaction.

CONCURRENCY CONTROL WITH TIME STAMPING METHODS

The *time stamping* approach to scheduling concurrent transactions assigns a global unique time stamp to each transaction. The time stamp value produces an explicit order in which transactions are submitted to the DBMS. Time stamps must have two properties: uniqueness and monotonicity. *Uniqueness* ensures that no equal time stamp values can exist, and *monotonicity* ensures that time increases.

All database operations (READ and WRITE) within the same transaction must have the same time stamp. The DBMS executes conflicting operations in time stamp order, thereby ensuring serializability of the transactions. If two transactions conflict, one often is stopped, rescheduled, and assigned a new time stamp value.

The concept of transaction time stamp $TS(T)$, which is a unique identifier assigned to each transaction. The time stamps are based on the order in which transactions start. If transaction T_1 starts before transaction T_2, then $TS(T_1) < TS(T_2)$

1. Older transaction will have the smaller time stamp value
2. Two schemes that prevent deadlock are
   - Wait-die
   - Wound-wait

Suppose that transaction T_k tries to lock an item x but is not able to because x is locked by some other transaction T_L with a conflicting lock.

The rules followed by these schemes are as follows:

1. Wait-die: If T_s $(T_k) < T_s$ (T_L), then (T_k older than T_L) T_k is allowed to wait; otherwise (T_k younger than T_L) abort T_k and restart it later with the same time stamp
2. Wound-wait: If T_s $(T_k) < T_s$ (T_L) then (T_k older than T_L) abort T_L and restart it later with the same time stamp; otherwise (T_k younger than T_L) T_k is allowed to wait

Both schemes end up aborting the younger of the two transactions that may be involved in a deadlock

Concurrency Control with Optimistic Methods

Optimistic methods are based on the assumption that the majority of the database operations do not conflict. A transaction is executed without restrictions until it is committed. Each transaction moves through *two* or *three* phases:

Read phase: The transaction reads the database, executes the needed computations, and makes the updates to a private copy of the database values.

Validation phase: The transaction is validated to assure that the changes made will not affect the integrity and consistency of the database.

If the validation test is positive, transaction goes to the Write Phase.

If the validation test is negative, transaction is restarted, and changes are discarded.

Write phase: The changes are permanently applied to the database.

Serializability

Serializabilty is accepted as 'criterion for correctness' for the interleaved execution of a set of transactions, such an execution is considered to be correct if and only if it is serializable.

1. A set of transactions is serializable if and only if it is equivalent to some serial execution of the same transactions
2. A serial execution is one in which the transactions are run one at a time in some sequence

Schedule: Given a set of transactions, any execution of those transactions interleaved or otherwise is called a *schedule*.

1. Executing the transactions one at a time, with no interleaving constitutes a serial schedule. A schedule that is not serial is an interleaved schedule (or) non-serial schedule.
2. Two schedules are said to be equivalent if and only if they are guaranteed to produce the same result as each other. Thus, a schedule is serializable, and correct, if and only if it is equivalent to some serial schedule.

Two-phase Locking Theorem

If all transactions obey the two phase locking protocol, then all possible interleaved schedule are serializable.

1. Before operating on any object (it could be a database tuple), a transaction must acquire a lock on the object
2. After releasing a lock, a transaction must never go on to acquire any more locks.

A transaction that obeys this protocol thus has two phases: a lock acquisition or "growing phase and a lock releasing or "shrinking" phase

Let '*I*' be an interleaved schedule involving some set of transactions $T, T_2, T_3, \ldots T_n$.

If '*I*' is serializable, then there exists at least one serial schedule '*S*' involving $T_1, T_2, \ldots T_n$ such that '*I*' is equivalent to '*S*' is said to be a serialization of '*I*'

Let T_i and T_j be any two distinct transactions in the set $T_1, T_2, T_3, \ldots T_n$. Let T_i precede T_j in the serialization '*S*'. In the interleaved schedule *I*, then the effect must be as if T_i really did execute before T_j. In other words, if *A* and *B* are any two transactions involved is some serializable schedule, then either A logically precedes *B* or *B* logically precedes *A* in that schedule, that is, either *B* can see A's output or *A* can see B's. If the effect is not as if either A ran before *B* or *B* ran before A, then the schedule is not serializable and not correct.

1. A schedule '*S*' of '*n*' transactions $T_1, T_2 \ldots T_n$ is an ordering of the operations of the transactions subject to the constraint that, for each transaction T_i that participates in '*S*', the same order in which they occur in T_i.
2. For the purpose of recovery and concurrency control, we are mainly interested in the 'read item' and 'write item' operations of the transactions, as well as the COMMIT and ABORT operations. A shorthand notation for describing a schedule uses the symbols, '*R*', '*W*', '*C*' and 'A' for the operations read item, write item, commit, and abort respectively, and appends as subscript the transition-id (transaction number) to each operation in the schedule

Example: The schedule of the given set of transactions can be written as follows:

T_1	T_2
Read item (x);	
$X = X - N$;	
	Read item (x);
	$X = X + M$;
Write item (x)	
Read item (y)	
	Write item (x)
	Write item (y)
$Y = Y + N$	
Write item (y)	Commit

Schedule:

$$S: R_1(X); R_2(X); W_1(X); R_1(Y); W_2(X); W_2(Y); W_1(Y); C_2$$

Conflicts: Two operations in a schedule are said to have conflict if they satisfy all three conditions, if

1. they belong to different transactions
2. they access the same data item
3. at least one of the operations is a write item

For example, In the schedule 'S' given above, the operations $r_1(x)$ and $w_2(x)$ conflict, as do

The operations $r_2(x)$ and $w_1(x)$ and the operations $w_1(x)$ and $w_2(x)$. However the operations $r_1(x)$ and $r_2(x)$ do not conflict, since they are both read operations;

The operations $w_1(x)$ and $w_2(y)$ do not conflict because they operate on distinct data items x and y. The operations $r_1(x)$ and $w_1(x)$ do not conflict, because they belong to the same transaction.

Complete schedule: A schedule S of 'n' transactions T_1, T_2, $T_3 \ldots T_n$ is said to be a complete schedule if the following conditions hold.

1. The operations is 'S' are exactly those operations in $T_1, T_2, \ldots T_n$ including a commit or abort operation as the last operation for each transaction in the schedule.
2. For any pair of operations from the same transaction T_i, their order of appearance in 'S' is the same as their order of appearance in T_i
3. For any two conflicting operations, one of the two must occur before the other in the schedule.

The preceding condition (3) allows for two non-conflicting operations to occur in the schedule without defining which occurs first, thus leading to the definition of a schedule as a partial order of the operations in the 'n' transactions.

It is difficult to encounter complete schedules in a transaction processing system, because new transactions are continually being submitted to the system. Hence, it is useful to define the concept of the 'committed projection $C(S)$ of schedule S; which include only the operations in S that belong to committed transactions, that is, transaction T_i whose commit operation is C_i.

Recoverability

Recoverability ensures that once a transaction T is committed, it should never be necessary to roll back T. The schedules that theoretically meet this criterion are called *recoverable schedules* and those that do not are called *non-recoverable*, and hence should not be permitted

A schedule 'S' is recoverable if no transaction T in 'S' commits until all transactions T^1 that have written an item that T reads have committed

A transaction T reads from transaction T^1 in a schedule S if some item x is first written by T^1 and later read by T. In addition, T^1 should not have been aborted before T reads item x, and there should be no transactions that write x after T^1 writes it and before T reads it (unless those transactions, if any, have aborted before T reads x)

Example:
Consider the given schedule, check whether it is recoverable or not:
S: $R_1(X)$; $R_2(X)$; $W_1(X)$; $R_1(Y)$; $W_2(X)$; C_2, $W_1(Y)$; C_1?

Solution:
The given schedule can also be represented as follows:

T_1	T_2
$R_1(x)$	
	$R_2(x)$
$W_1(x)$	
$R_1(y)$	
	$W_2(x)$
	C_2
$W_1(y)$	
C_1	

There are two *WR* conflicts, if the schedule consists of *RW* conflict, then we may say that the schedule is not recoverable (if the transaction which is performing read operation commits first)

Cascadeless Schedule

In a recoverable schedule, no committed transaction ever needs to be rolled back. It is possible for a phenomenon known as cascading rollback to occur, when an uncommitted transaction has to be rolled back because it read an item from a transaction that failed.

This is illustrated in the following schedule:

Example:
S: $R_1(X)$; $W_1(X)$; $R_2(X)$; $R_1(Y)$; $W_2(X)$; $W_1(Y)$; A_1, A_2
The above schedule is represented as follows:

T_1	T_2
$R_1(x)$	
$W_1(x)$	
	$R_2(x)$
$R_1(y)$	
	$W_2(x)$
$W_1(y)$	
A_1	
	A_2

Transaction T_2 has to be rolled back because it reads item x from T_1, and T_1 is then aborted, because cascading rollback

can be quite time consuming since numerous transactions can be rolled back. It is important to characterize the schedules where this phenomenon is guaranteed not to occur.

A scheduled is said to be cascadeless if every transaction in the schedule reads only items that were written by committed transaction. In this case, all items read will not be discarded, so no cascading rollback will occur.

Strict Schedule

A schedule is called *strict schedule*, in which transactions can neither read nor write an item x until the last transaction that wrote x has committed or aborted

1. All strict schedules are cascadeless
2. All cascadeless schedules are recoverable

Equivalence of Schedules

There are several ways to define equivalence of schedules as follows:

1. Result equivalent
2. Conflict equivalent
3. View equivalent

Result Equivalent

Two schedules are called *result equivalent* if they produce the same final state of the database. However, two different schedules may accidentally produce the same final state.

Example: Check whether the two schedules are result equivalent or not:

S_1	S_2
Read item (x);	Read item (x)
$X = X + 20$;	$X = X * 1.1$;
Write item (x);	Write item (x);

Solution:
Schedules S_1 and S_2 will produce the same final database state if they execute on a database with an initial value of $x = 200$; but for other initial values of x, the schedules are not result equivalent

For two schedules to be equivalent, the operations applied to each data item affected by the schedules should be applied to that item in both schedules in the same order. The other two definitions of equivalence of schedules generally used are conflict equivalence and view equivalence

Conflict Equivalence

Two schedules are said to be conflict equivalent if the order of any two conflicting operations is the same in both schedules. If two conflicting operations are applied in different orders in two schedules, the effect can be different on the database or on other transactions in the schedule, and hence the schedules are not conflict equivalent.

Example:

S_1	
$r_1(x)$	
	$W_2(x)$

S_3	
$W_1(x)$	
	$W_2(x)$

S_2	
	$W_2(x)$
$r_1(x)$	

S_4	
	$W_2(x)$
$W_1(x)$	

The value read by $r_1(x)$ can be different in the two schedules. Similarly, if two write operations occur in the order $w_1(x)$, $w_2(x)$ in s_3, and in the reverse order $w_2(x)$, $w_1(x)$ in s_4, the next $r(x)$ operation in the two schedules will read potentially different values.

Testing for Conflict Serializability

The following algorithm can be used to test a schedule for conflict serializability. The algorithm takes read item and write item operations in a schedule to construct a precedence graph or serialization graph, which is a directed graph $G(N, E)$ here N is a set of Nodes $N = \{T_1, T_2, \ldots. T_n\}$ and E is a set of directed edges $E = \{e_1, e_2, \ldots.. E_m\}$ There is one node in the graph for each transaction. T_i in the schedule. Each edge e_i in the graph is of the form $(T_j \rightarrow T_k)$, 1 ″ j ″ n, 1 ″ k ″ n,

Where T_j is the starting node of e_i and T_k is the ending node of e_i.

Edge is created if one of the operations in T_j appears in the schedule before some conflicting operation in T_k

Algorithm

1. For each transaction T_i participating in schedule S, create a node labelled T_i in the precedence graph
2. For each case in S where T_j executes a read item (x) after T_i executes a write item (x), create an edge ($T_i \rightarrow T_j$) in the graph
3. For each case in S where T_j executes a write item (x) after T_i executes a read item (x), create an edge ($T_i \rightarrow T_j$) in the graph
4. For each case in S where T_j executes a write item (x) after T_i executes a write item (x), create an edge ($T_i \rightarrow T_j$) in the precedence graph
5. The schedule S is serializable, if the precedence graph contains no cycles

A cycle in a directed graph is a sequence of edges $C = ((T_j \rightarrow T_k), (T_k \rightarrow T_p), \ldots (T_i \rightarrow T_j))$

With the property that the starting node of each edge, except the first edge is the same as the ending node of the previous edge, and the starting node of the first edge is the same as the ending node of the last edge.

Example: Check whether the given schedule is conflict serializable or not by drawing precedence graph:

T_1	T_2	T_3
$R_1(x)$		
	$W_2(x)$	
		$R_3(x)$
$W_1(x)$		
		$W_3(x)$
$R_1(x)$		

Solution:

First identify the conflicts:

$(T_1 \rightarrow T_2)$ WR conflict

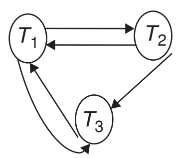
$(T_2 \rightarrow T_1)$ WW conflict

$(T_2 \rightarrow T_3)$ RW conflict

$(T_3 \rightarrow T_1)$ WR conflict

$(T_1 \rightarrow T_3)$ WW conflict

Take transactions as nodes in the precedence graph:



The precedence graph has cycle, which says that the schedule is not serializable.

View Equivalence and View Serializability

View equivalence is less restrictive compared to conflict equivalence. Two schedules S and S' are said to be view equivalent if the following three conditions hold:

1. The same set of transactions participate in S and S', and S and S' include the same operations of those transactions
2. For any operation $r_i(x)$ of T_i in S, if the value of x read by the operation has been written by an operation $w_j(x)$ of T_j, the same condition must hold for the value of x read by operation $r_i(x)$ of T_j in S'
3. If the operation $w_k(y)$ of T_k is the last operation to write Y in S, then $W_k(y)$ of T_k must also be the last operation to write item Y in S'

The idea behind view equivalence is that as long as each read operation of a transaction reads the result of the same write operation in both schedules, the write operations of each transaction must produce the same result. Hence the read operation is said to see the same view in both schedules:

1. A schedule S is said to be view serializable if it is view equivalent to a serial schedule.
2. All conflict serializable schedules are view serializable, but vice versa is not true.

Exercises

Practice Problem 1

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Consider the given schedules S_1 and S_2

S_1: $r_1(x)$, $r_1(y)$, $r_2(x)$, $r_2(y)$, $w_2(y)$, $w_1(x)$

S_2: $r_1(x)$, $r_2(x)$, $r_2(y)$, $w_2(y)$, $r_1(y)$, $w_1(x)$

Which schedule is conflict serializable?

(A) S_1 (B) S_2
(C) S_1 and S_2 (D) None of these

2. Consider the given schedule with three transactions T_1, T_2 and T_3:

T_1	T_2	T_3
$r_1(x)$		
	$r_2(y)$	
		$r_3(y)$
	$w_2(y)$	
$w_1(x)$		
		$w_3(x)$
	$r_2(x)$	
	$w_2(x)$	

Which of the following is correct serialization?

(A) $T_2 \rightarrow T_1 \rightarrow T_3$ (B) $T_1 \rightarrow T_3 \rightarrow T_2$
(C) $T_3 \rightarrow T_1 \rightarrow T_2$ (D) None of these

3. Consider the three data items D_1, D_2 and D_3 and the following execution of schedules of transactions T_1, T_2 and T_3:

T_1	T_2	T_3
	$R(D_2)$	
	$R(D_2)$	
	$W(D_2)$	
		$R(D_2)$
		$R(D_3)$
$R(D_1)$		
$W(D_1)$		
		$W(D_2)$
		$W(D_3)$
	$R(D_1)$	
$R(D_2)$		
$W(D_2)$		
	$W(D_1)$	

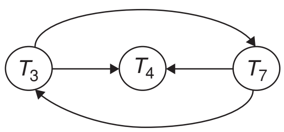
Which of the following is true?

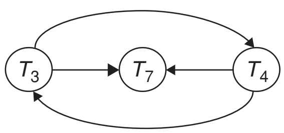
(A) The schedule is conflict serializable
(B) The schedule is not conflict serializable
(C) The schedule has deadlock
(D) Both (A) and (C)

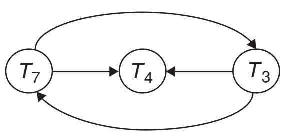
4. Consider the given schedule

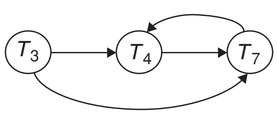
T_3	T_4	T_7
$R(Q)$		
	$W(Q)$	
$W(Q)$		
		$R(Q)$
		$W(Q)$

Which of the following is the correct precedence graph for the above schedule?

(A) 

(B) 

(C) 

(D) 

5. Consider two Transactions T_1 and T_2 and four schedules: S_1, S_2, S_3 and S_4 of T_1 and T_2:

T_1: $r_1(x), w_1(x), w_1(y)$

T_2: $r_2(x), r_2(y), w_2(y)$

S_1: $r_1(x), r_2(x), r_2(y), w_1(x), w_1(y), w_2(y)$

S_2: $r_1(x), r_2(x), r_2(y), w_1(x), w_2(y), w_1(y)$

S_3: $r_1(x), w_1(x), r_2(x), w_1(y), r_2(y), w_2(y)$

S_4: $r_2(x), r_2(y), r_1(x), w_1(x), w_1(y), w_2(y)$

Which schedules are conflict serializable in the given schedules?

(A) S_1 and S_2
(B) S_1 and S_3
(C) S_2 and S_3
(D) S_1 and S_4

6. Consider the following transactions with data items P and Q initialized to '0':

T_1: read(P)

Read(Q)

if $p = 0$ then $Q = Q + 1$

Write(Q)

T_2: read(Q)

Read(P)

if $Q = 0$ then $p = p + 1$

Write (P)

Any non-serial interleaving of T_1 and T_2 for concurrent execution leads to

(A) a serializable schedule
(B) a schedule that is not conflict serializable
(C) a conflict serializable schedule
(D) a schedule for which a precedence graph cannot be drawn

7. Consider the concurrent execution of two transactions T_1 and T_2, if the initial values of x, y, M and N are 200, 100, 10, 20 respectively. What are the final values of x and y?

T_1	T_2
read-item(x)	
$x = x - N$	
	read-item(x)
	$x = x + M$
Write-item(x)	
read-item(y)	
	Write-item(x)
$y = y + N$	
Write-item(y)	

(A) 220, 110 (B) 210, 120
(C) 220, 120 (D) 210, 110

8. For the above data, if the transactions are executed in serial manner, what would be the values of X and Y at the end of the serial execution of T_1 and T_2?

T_1	T_2
Read-item(x)	
$X = X - N$	
Write-item(x)	
Read-item(y)	
$Y = Y + N$	
Write-item(y)	
	Read-item(x)
	$X = X + M$
	Write-item(x)

(A) 190, 120
(B) 180, 120
(C) 190, 110
(D) 180, 110

9. Consider the given two transactions T_1 and T_2:

T_1: $r_1(x), w_1(x), r_1(y)$

T_2: $r_2(x), r_2(y), w_2(x), w_2(y)$

Which of the following schedules are complete schedules?

(A) $r_1(x), r_2(x), w_1(x), r_1(y), r_2(y), w_2(x), w_2(y)$
(B) $r_2(x), r_1(x), r_2(y), w_1(x), w_2(x), r_1(y), w_2(y)$
(C) $r_1(x), r_1(y), r_2(x), r_2(y), w_1(x), w_2(x), w_2(y)$
(D) All the above

10. Consider the given schedule with data-locks on data-items, check whether it has dead-lock or not. The locks are shared-lock(S) and Exclusive-lock(X). Shared-lock is also called Read-lock, Exclusive-lock is also called Write-lock. Read and Write operations are denoted by *R* and *W*, respectively.

T_1	T_2	T_3	T_4
S(A)			
R(A)			
	X(B)		
	W(B)		
S(B)			
		S(C)	
		R(C)	
	X(C)		
			X(B)
		X(A)	

Which of the following is incorrect?

(A) $T_1 \rightarrow T_2$ (B) $T_3 \rightarrow T_1$
(C) $T_2 \rightarrow T_3$ (D) $T_4 \rightarrow T_3$

11. Consider the three transactions T_1, T_2 and T_3 and the schedule S_1 as given below. Draw the serializability (precedence) graph for S_1, and state whether the schedule is serializable or not. If a schedule is serializable, which one of the following is equivalent serial schedule?

T_1: $r_1(x), r_1(z), w_1(x)$
T_2: $r_2(z), r_2(y), w_2(z), w_2(y)$
T_3: $r_3(x), r_3(y), w_3(y)$

S_1: $r_1(x), r_2(z), r_1(z), r_3(x), r_3(y), w_1(x), w_3(y), r_2(y), w_2(z), w_2(y)$?

(A) $r_3(x), r_3(y), w_3(y), r_1(x), r_1(z), w_1(x), r_2(z), r_2(y), w_2(z), w_2(y)$
(B) $r_1(x), r_1(z), w_1(x), r_2(z), r_2(y), w_2(z), w_2(y), r_3(x), r_3(y), w_3(y)$
(C) $r_2(z), r_2(y), w_2(z), w_2(y), r_3(x), r_3(y), w_3(y), r_1(x), r_1(z), w_1(x)$
(D) $r_2(z), r_2(y), w_2(z), w_2(y), r_1(x), r_1(z), w_1(x), r_3(x), r_3(y), w_3(y)$

12. Consider the data given in the above question. Draw the precedence graph for S_2 and state whether each schedule is serializable or not. If a schedule is serializable, which of the following is equivalent serial schedule?

S_2: $r_1(x), r_2(z), r_3(x), r_1(z), r_2(y), r_3(y), w_1(x), w_2(z), w_3(y), w_2(y)$

(A) $r_3(x), r_3(y), w_3(y), r_1(x), r_1(z), w_1(x), r_2(z), r_2(y), w_2(z), w_2(y)$
(B) $r_1(x), r_1(z), w_1(x), r_2(z), r_2(y), w_2(z), w_2(y), r_3(x), r_3(y), w_3(y)$
(C) $r_2(z), r_2(y), w_2(z), w_2(y), r_3(x), r_3(y), w_3(y), r_1(x), r_1(z), w_1(x)$
(D) $r_2(z), r_2(y), w_2(z), w_2(y), r_1(x), r_1(z), w_1(x), r_3(x), r_3(y), w_3(y)$

13. Consider schedule S_3, which is a combination of transactions T_1, T_2 and T_3 from Q. No.11.

S_3: $r_1(x), r_2(z), r_1(z), r_3(x), r_3(y), w_1(x), c_1, w_3(y), c_3, r_2(y), w_2(z), w_2(y), c_2$?

Which of the following is true?

(A) Recoverable and conflict serializable
(B) Recoverable but not conflict serializable
(C) Conflict serializable but not Recoverable
(D) Not recoverable and not conflict serializable

14. Consider the given schedule:

S_4: $r_1(x), r_2(z), r_1(z), r_3(x), r_3(y), w_1(x), w_3(y), r_2(y), w_2(z), w_2(y), c_1, c_2, c_3$:

Which of the following is true?

(A) Recoverable and conflict serializable
(B) Recoverable but not conflict serializable
(C) Conflict serializable but not Recoverable
(D) Not recoverable and not conflict serializable

15. Which of the following is correct for the below compatibility matrix?

Mode of Locks Currently held by other transactions		**Shared-Lock**	**Exclusive-Lock**
	S		
	X		

S – shared - Lock, X – Exclusive - Lock

(A)

	S	X
S	No	No
X	Yes	No

(B)

	S	X
S	Yes	No
X	No	No

(C)

	S	X
S	Yes	Yes
X	No	No

(D)

	S	X
S	No	Yes
X	No	No

16. Consider the following schedule with locking:

T_1	T_2
Lock – $X(A)$	
$R(A)$	
$W(A)$	
	Lock – $X(B)$
	$R(B)$
	$W(B)$
	Lock – $X(A)$
Lock – $X(B)$	

Which of the following is true?
(A) schedule is in Dead–Lock state
(B) schedule is conflict serializable
(C) schedule is not conflict serializable
(D) Both A and B

17. Consider the given set of transactions:

T_1	T_2
	SELECT AVG (balance)
	FROM Account
INSERT INTO Account	
VALUES (487, 2000);	
COMMIT	
	SELECT AVG (balance)
	FROM Account
	COMMIT

The above problem is a case of
(A) READ UNCOMMITTED
(B) RAD COMMITTED
(C) REPEATABLE READ
(D) DIRTY READ

18. Consider the given set of transactions

T_1	T_2
UPDATE ACCOUNT	
SET balance = balance – 1000	
WHERE number = 586;	
	SELECT AVG (balance)
	FROM Account
ROLL BACK	
	COMMIT

The above problem is a case of
(A) READ UNCOMMITTED
(B) READ COMMITTED
(C) DIRTY READ
(D) BOTH A and C

19. Consider the following set of transactions

T_1	T_2
	SELECT AVG (balance)
	FROM Account
UPDATE Account	
SET balance = balance – 4000	
WHERE number = 586;	
COMMIT	
	SELECT AVG (balance)
	FROM Account
	COMMIT

The above problem is a case of
(A) READ UNCOMMITTED
(B) READ COMMITTED
(C) REPEATABLE READ
(D) DIRTY READ

20. Consider the following schedule with locks on data items:

T_1	T_2	T_3
$X(A)$		
	$X(A)$	
		$X(A)$
$S(B)$		
	$S(B)$	

Which of the following is incorrect?
(A) $T_2 \rightarrow T_1$
(B) $T_3 \rightarrow T_2$
(C) $T_3 \rightarrow T_1$
(D) $T_1 \rightarrow T_3$

Practice Problem 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Which of the following is false with respect to B^+ -trees of order p?
(A) Each internal node has at most p tree pointers.
(B) Each leaf node has at most $\left\lceil \left(\frac{p}{2}\right) \right\rceil$ values.
(C) Each internal node, except the root, has at least $\left\lceil \left(\frac{p}{2}\right) \right\rceil$ tree pointers.
(D) All leaf nodes are at same level.

2. Consider below transactions:

T_1	T_2
Read – item(X);	
$X := X - N$;	
	Read – item (X);
	$X := X + M$;
Write – item(X);	
Read – item (Y);	
	Write – item (X);
$Y := Y + N$;	
Write – item (Y);	

Which of the following problem will occur during the concurrent execution of the above transactions?

(A) Lost update problem because of incorrect X.
(B) Lost update problem because of incorrect Y.
(C) Dirty read problem because of incorrect X.
(D) Dirty read problem because of incorrect Y.

3. Consider the scheduled:

S: $r_1(X)$; $r_2(X)$; $w_1(X)$; $r_1(Y)$; $w_2(X)$; C_2; $w_1(Y)$; C_1;

This schedule is

(A) Recoverable (B) Non-recoverable
(C) Strict schedule (D) Both (A) and (C)

4. Consider below schedule:

T_1	T_2
Read – item(X);	
$X := X - N$;	
	Read – item (X);
	$X := X + M$;
Write – item(X);	
Read – item (Y);	
	Write – item (X);
$Y := Y + N$;	
Write – item (Y);	

This schedule is

(A) Serializable
(B) Not serializable
(C) Under dead lock
(D) Both (B) and (C)

5. Let, current number of file records = r

maximum number of records = bfr

current number of file buckets = N

Then what will be the file load factor?

(A) $\frac{r}{(bfr * N)}$ (B) $r + (bfr * N)$

(C) $r * (bfr * N)$ (D) $r * (bfr + N)$

6. Match the following:

	LIST I		LIST II
1.	Primary index	A.	Ordered key field
2.	Clustering index	B.	Non-ordered field
3.	Secondary index	C.	Ordered non-key field

(A) 1 – A, 2 – B, 3 – C
(B) 1 – A, 2 – C, 3 – B
(C) 1 – C, 2 – B, 3 – A
(D) 1 – C, 2 – A, 3 – B

7. Consider a file with 30,000 fixed length records of size 100 bytes stored on a disk with block size 1024 bytes. Suppose that a secondary index on a non-ordering key field is constructed with key field size 9 bytes and block pointer 6 bytes. What will be the number of blocks needed for the index?

(A) 68 (B) 442
(C) 1500 (D) 3000

8. Match the following:

	Index type		Number of Index entries
1.	Primary Index	A.	Blocks in data file
2.	Clustering index	B.	Record in data file
3.	Secondary key index	C.	Distinct index filed values

(A) 1 – A, 2 – B, 3 – C (B) 1 – A, 2 – C, 3 – B
(C) 1 – C, 2 – B, 3 – A (D) 1 – C, 2 – A, 3 – B

9. Which of the following is true with respect to B – Tree of order p?

(A) Each node has at most p tree pointers.
(B) Each node, except the root and leaf nodes, has at least $\left\lceil \frac{p}{2} \right\rceil$ tree pointers.
(C) All leaf nodes are at the same level.
(D) All of these.

10. What is the amount of unused space in allocation of unspanned fixed records of size R on a block of size B bytes?

(A) $B - R$ (B) $B - \left\lfloor \frac{B}{R} \right\rfloor$

(C) $B - \left(\left\lfloor \frac{B}{R} \right\rfloor * R\right)4$ (D) $\left(\left\lfloor \frac{B}{R} \right\rfloor * R\right) - B$

11. What is the average time required to access a record in a file consisting of b blocks using unordered heap linear search?

(A) b (B) $b/2$
(C) $\log_2^b$ (D) b^2

12. Consider a file of fixed length records of size R bytes. If the block size is B bytes, then the blocking factor will be

(A) $B \times R$ records (B) $\left\lfloor \frac{B}{R} \right\rfloor$ records

(C) $\left\lceil \frac{B}{R} \right\rceil$ records (D) $B + R$ records

13. Consider the following relation instance:

P	Q	R
1	4	2
1	5	3
1	6	3
3	2	2

Which of the following FDs are satisfied by the instance?

(A) $PQ \rightarrow R$ and $R \rightarrow Q$ (B) $QR \rightarrow P$ and $Q \rightarrow R$

(C) $QR \rightarrow P$ and $P \rightarrow R$ (D) $PR \rightarrow Q$ and $Q \rightarrow P$

14. Consider an ordered file with 30,000 records stored on a disk with block size of 1024 bytes. The records are of fixed size and are of unspanned, with record length 100 bytes. What is the number of accesses required to access a data file using binary search?

(A) 10 (B) 12

(C) 1500 (D) 3000

15. What is the blocking factor for an index if the ordering key field size is 9 bytes and block pointer is 6 bytes long, and the disk block size is 1024 bytes?

(A) 114 (B) 171

(C) 341 (D) 68

16. For a set of n transactions, there exist ______ different valid serial schedules

(A) n (B) n^2

(C) $n/2$ (D) $n!$

17. The number of possible schedules for a set of n transactions is

(A) lesser than $n!$ (B) much larger than $n!$

(C) $n!$ (D) None

18. Which one of the following is conflict operation?

(A) Reads and writes from the same transaction

(B) Reads and writes from different transaction

(C) Reads and writes from different transactions on different data items.

(D) Reads and writes from different transaction on same data.

19. The following schedule S: $r_3(x), r_2(x), w_3(x), r_1(x), w_1(x)$ is conflict equivalent to serial schedule

(A) $T_1 \rightarrow T_3 \rightarrow T_1$ (B) $T_2 \rightarrow T_1 \rightarrow T_3$

(C) $T_1 \rightarrow T_2 \rightarrow T_3$ (D) None

20. The following schedule S: $R_1(x), R_2(x), W_1(x), W_2(x)$ is

(A) Conflict serializable (B) View serializable

(C) Both (D) None

Previous Years' Questions

1. Consider the following four schedules due to three transactions (indicated by the subscript) using *read* and *write* on a data item x, denoted by $r(x)$ and $w(x)$, respectively. Which one of the them is conflict serializable? **[2014]**

(A) $r_1(x); r_2(x); w_1(x); r_3(x); w_2(x)$

(B) $r_2(x); r_1(x); w_2(x); r_3(x); w_1(x)$

(C) $r_3(x); r_2(x); r_1(x); w_2(x); w_1(x)$

(D) $r_2(x); w_2(x); r_3(x); r_1(x); w_1(x)$

2. Consider the following schedule S of transactions T_1, T_2, T_3, T_4:

T_1	T_2	T_3	T_4
	Reads (X)		
		Writes (X) Commit	
Writes (X) Commit			
	Writes (Y) Reads (Z) Commit		
			Reads (X) Reads (Y) Commit

Which one of the following statements is correct? **[2014]**

(A) S is conflict-serializable but not recoverable

(B) S is not conflict-serializable but is recoverable

(C) S is both conflict-serializable and recoverable

(D) S is neither conflict-serializable nor it is recoverable

3. Consider the following transaction involving two bank accounts x and y.

read (x) ; $x := x - 50$; write (x) ; read(y); $y := y + 50$; write(y)

The constraint that the sum of the accounts x and y should remain constant is that of **[2015]**

(A) Atomicity (B) Consistency

(C) Isolation (D) Durability

4. Consider a simple checkpointing protocol and the following set of operations in the log.

(start, T_4); (write, T_4, y, 2, 3); (start, T_1); (commit, T_4); (write, T_1, z, 5, 7);

(checkpoint);

(start, T_2); (write, T_2, x, 1, 9); (commit, T_2); (start, T_3), (write, T_3, z, 7, 2);

If a crash happens now and the system tries to recover using both undo and redo operations. What are the contents of the undo list and the redo list? **[2015]**

(A) Undo: T_3, T_1; Redo: T_2
(B) Undo: T_3, T_1; Redo: T_2, T_4
(C) Undo: none; Redo: T_2, T_4, T_3, T_1
(D) Undo: T_3, T_1, T_4; Redo: T_2

5. Consider the following partial schedule S involving two transactions T_1 and T_2. Only the read and the write operations have been shown. The read operation on data item P is denoted by read(P) and the write operation on data item P is denoted by write(P)

Time Instance	Transaction – id	
	T_1	T_2
1	read(A)	
2	write(A)	
3		read(C)
4		write(C)
5		read(B)
6		write(B)
7		read(A)
8		commit
9	read(B)	

Schedule S

Suppose that the transaction T_1 fails immediately after time instance 9. Which one of the following statements is correct? **[2015]**

(A) T_2 must be aborted and then both T_1 and T_2 must be re-started to ensure transaction atomicity.
(B) Schedule S is non-recoverable and cannot ensure transaction atomicity.
(C) Only T_2 must be aborted and then re-started to ensure transaction atomicity.
(D) Schedule S is recoverable and can ensure atomicity and nothing else needs to be done.

6. Which one of the following is **NOT** a part of the ACID properties of database transactions? **[2016]**

(A) Atomicity
(B) Consistency
(C) Isolation
(D) Deadlock - freedom

7. Consider the following two phase locking protocol. Suppose a transaction T accesses (for read or write operations), a certain set of objects $\{O_1, \ldots, O_k\}$. This is done in the following manner: **[2016]**

Step 1. T acquires exclusive locks to $O_1, \ldots O_k$ in increasing order of their addresses.

Step 2. The required operations are performed.

Step 3. All locks are released.

This protocol will

(A) guarantee serializability and deadlock-freedom.
(B) guarantee neither serializability nor deadlock-freedom.
(C) guarantee serializability but not deadlock-freedom.
(D) guarantee deadlock-freedom but not serializabilty.

8. Suppose a database schedule S involves transactions $T_1, \ldots T_n$. Construct the precedence graph of S with vertices representing the transactions and edges representing the conflicts. If S is serializable, which one of the following orderings of the vertices of the precedence graph is guaranteed to yield a serial schedule? **[2016]**

(A) Topological order
(B) Depth - first order
(C) Breadth - first order
(D) Ascending order of transaction indices

9. Consider the following database schedule with two transactions T_1 and T_2.

$S = r_2(X); r_1(X); r_2(Y); w_1(X); r_1(Y); w_2(X); a_1; a_2$

Where $r_i(Z)$ denotes a *read* operation by transaction T_i on a variable Z, $w_i(Z)$ denotes a *write* operation by T_i on a variable Z and *ai* denotes an *abort* by transaction T_i.

Which one of the following statements about the above schedule is **TRUE**? **[2016]**

(A) S is non - recoverable
(B) S is recoverable, but has a cascading abort
(C) S does not have a cascading abort
(D) S is strict.

10. In a database system, unique timestamps are assigned to each transaction using Lamport's logical clock. Let $TS(T_1)$ and $TS(T_2)$ be the timestamps of transactions T_1 and T_2 respectively. Besides, T_1 holds a lock on the resource R, and T_2 has requested a conflicting lock on the same resource R. The following algorithm is used to prevent deadlocks in the database system assuming that a killed transaction is restarted with the same timestamp.

if $TS(T_2) < TS(T_1)$ then
    T_1 is killed
else T_2 waits.

Assume any transaction that is not killed terminates eventually. Which of the following is TRUE about the database system that uses the above algorithm to prevent deadlocks? **[2017]**

(A) The database system is both deadlock-free and starvation-free.
(B) The database system is deadlock-free, but not starvation-free.

(C) The database system is starvation-free, but not deadlock-free.
(D) The database system is neither deadlock-free nor starvation-free.

11. Two transactions T_1 and T_2 are given as

$$T_1 : r_1(X)w_1(X)r_1(Y)w_1(Y)$$

$$T_2 : r_2(Y)w_2(Y)r_2(Z)w_2(Z)$$

where $r_i(V)$ denotes a *read* operation by transaction T_i on a variable V and $w_i(V)$ denotes a *write* operation by transaction T_i on a variable V. The total number of conflict serializable schedules that can be formed by T_1 and T_2 is _______. **[2017]**

Answer Keys

Practice Problem I

1. B	**2.** B	**3.** B	**4.** B	**5.** C	**6.** B	**7.** B	**8.** A	**9.** A	10. D
11. A	12. A	13. A	14. C	15. B	16. D	17. C	18. D	19. B	20. D

Practice Problem I

1. B	**2.** A	**3.** A	**4.** D	**5.** A	**6.** B	**7.** B	**8.** B	**9.** D	**10.** C
11. B	**12.** B	**13.** B	**14.** B	**15.** D	**16.** D	**17.** C	**18.** D	**19.** A	**20.** D

Previous Years' Questions

1. D	**2.** C	**3.** B	**4.** A	**5.** B	**6.** D	**7.** A	**8.** A	**9.** C	**10.** A
11. 54									

Chapter 5

File Management

LEARNING OBJECTIVES

- *Files*
- *Memory hierarchies*
- *Description of disk devices*
- *File records*
- *Sorted files*
- *Hashing techniques*
- *Extendible hashing*
- *Index update*
- *Clustering index*
- *B-Trees*
- *B+Trees*
- *Over flow in internal node*

FILES

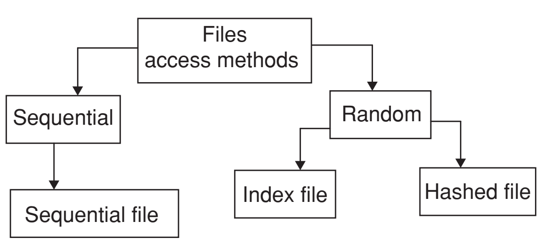
Databases are stored on magnetic disks as files of records. Computer storage media form a storage hierarchy that includes two main categories.

Primary storage This category includes storage media that can be operated on, directly by CPU, such as the computer main memory and cache memory. Primary storage provides fast access but is of limited storage capacity.

Secondary storage This category includes magnetic disks, optical disks, and tapes. These devices usually have a larger capacity, less cost, and slower access to data. Data in secondary storage cannot be processed directly by the CPU, it must be copied into primary storage.

File Structure

Taxonomy of file structure



Sequential file A sequential file is one in which records can only be accessed sequentially, one after another from beginning to end. Records are stored contiguously on the storage device.

Index files These files are used to access a record in the file. The entire index file is loaded into main memory data and indexes are stored in the same file. The term 'index file' is used as a synonym for the term 'database file'. The index file contains parameters that specify the name and location of file used to store DB.

Indexing Indexing mechanism is used to speed up access to desired data. An index file consists of records (called *index entries*) of the form.

Search-key	Pointer

Index files are typically much smaller than the original file.

Ordered indices In ordered index, index entries are stored, sorted on the search-key value.
Example: Author catalogue in library.

MEMORY HIERARCHIES

At the primary storage level, the memory hierarchy includes cache memory which is a static RAM.

The next level of primary storage is DRAM (dynamic RAM) which provides the main work area for the CPU for keeping programs and data and is called the *main memory*.

At the secondary storage level, the hierarchy includes magnetic disks, as well as mass storage in the form of CD-ROM (compact disk read-only memory) and tapes. Programs reside in DRAM and large permanent databases reside on secondary storage.

Another form of memory, *flash memory*, is non-volatile. Flash memories are high-density, high-performance memories using EEPROM (electrically erasable programmable read-only

memory) technology. The advantage of flash memory is the fast access speed, the disadvantage is that an entire block must be erased and written over at a time. Finally, magnetic tapes are used for archiving and backup storage of data.

Description of Disk Devices

Magnetic disks are used for storing large amounts of data. The capacity of a disk is the number of bytes it can store. A disk is single sided if it stores information on only one of its surfaces and double sided if both surfaces are used. To increase storage capacity, disks are assembled into a disk pack, which may include many disks and hence many surfaces. Information is stored on a disk surface in concentric circles with small width, each having a distinct diameter. Each circle is called a *track*. For disk packs, the tracks with the same diameter on the various surfaces are called a *cylinder* because of the shape they would form if connected in space.

A track usually contains a large amount of information; it is divided into smaller blocks (or) sectors. The division of track into equal-sized disk blocks (or pages) is set by the operating system during disk formatting.

Blocks are separated by fixed-size inter-block gaps, which include specially coded control information written during disk initialization. This information is used to determine which block on the track follows each inter block gap. Transfer of data between main memory and disk takes place in units of disk blocks. The hardware address of a block is the combination of a cylinder number, track number and block number is supplied to the disk I/O hardware.

The actual hardware mechanism that reads or writes a block is the disk read/write head, which is part of a system called a *disk drive*. A disk is mounted in the disk drive, which includes a motor that rotates the disk. To transfer a disk block, given its address, the disk controller must first mechanically position the read/write head on the correct track. The time required to do this is called the *seek time*. There is another delay called *rotational delay* or *latency*; the beginning of the desired block rotates into position under the read/write head. It depends on the RPM of the disk. Finally, some additional time is needed to transfer the data, which is called *block-transfer time*. Hence, the total time needed to locate and transfer an arbitrary block, given its address is the sum of the seek time, rotational delay and block transfer time. The seek time and rotational delay are usually much larger than the block transfer time.

File Records

Data is usually stored in the form of *records*. Each record consists of a collection of related data values or items where each value is of one or more bytes and corresponds to a particular field of the record. Records describe entities and their attributes.

Record type A collection of field names and their corresponding data types constitutes a record type (or) record format.

A file is a sequence of records. If every record in the file has exactly the same size (in bytes), the file is said to be made up of fixed-length records. If different records in the file have different sizes, the file is said to be made up of variable-length records.

Spanned Versus Unspanned Records

The records of a file must be allocated to disk blocks because a block is the unit of data transfer between disk and memory. When the block size is larger than the record size, each block will contain numerous records, although some files may have unusually large records that cannot fit in one block.

Suppose that the block size is B bytes. For a file of fixed-length records of size R bytes, with $B \geq R$, we can fit

$$\boxed{bfr = \lfloor B/R \rfloor \text{ records per block}}$$

The value *bfr* is called the *blocking factor* for the file. Some times R may not divide B exactly, so we have some unused space in each block equal to $B - (bfr * R)$ bytes. To utilize this unused space, we can store part of a record on one block and the rest on another. A pointer at the end of the first block points to the block containing the remainder of the record in case it is not the next consecutive block on disk. This organization is called *spanned*, because records can span more than one block. Whenever a record is larger than a block, we must use a spanned organization. If records are not allowed to cross block boundaries, the organization is called *unspanned*. This is used with fixed-length records having $B > R$, because it makes each record start at a known location in the block. For variable-length records, either a spanned or an unspanned organization can be used.

For variable-length records using spanned organization, each block may store a different number of records. In this case, the blocking factor *bfr* represents the average number of records per block for the file. We can use *bfr* to calculate the number of blocks '*b*' needed for a file of '*r*' records.

$b = \lceil (r/bfr) \rceil$ blocks

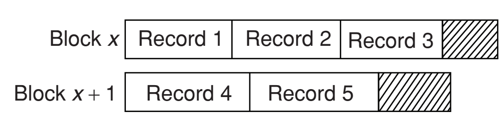


Figure 1 Unspanned records

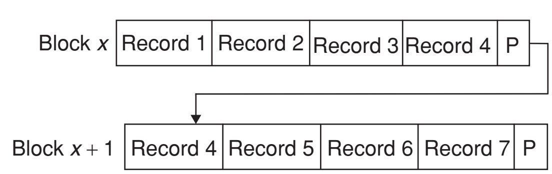


Figure 2 Spanned

There are several standard techniques for allocating the blocks of a file on disk. In contiguous allocation, the file blocks are allocated to consecutive disk blocks. In linked

allocation, each file block contains a pointer to the next file block. A combination of the two allocates clusters of consecutive disk blocks, and the clusters are linked. Clusters are sometimes called file *segments* (or) *extents*. Another possibility is to use indexed allocation, where one or more index blocks contain pointers to the actual file blocks.

Sorted Files (Ordered Records)

We can physically order the records of a file on disk based on the values of the one of their fields called the *ordering field*. This leads to an ordered or sequential file. If the ordering field is also a key field of the file, a field guaranteed to have a unique value in each record, then the field is called the *ordering key for the file*.

Advantages

1. Reading the records in order of the ordering key values becomes extremely efficient, because no sorting is required.
2. Finding the next record from the current one in order of the ordering key usually requires no additional block access, because the next record is in the same block as the current one.
3. Using a search condition based on the value of an ordering key field results in faster access when the binary search technique is used. This constitutes an improvement over linear searches, although it is not often used for disk files.

A binary search for disk files can be done on the blocks rather than on the records. Suppose that a file has 'b' blocks numbered 1, 2 ,…, b, the records are ordered by ascending value of their ordering key field and we are searching for a record whose ordering key field value is K. Assuming that disk addresses of the file blocks are available in the file header, the binary search usually accesses $\log_2^{(b)}$ blocks, whether the record is found (or) not, an improvement over linear searches, where, on the average, (b/2) blocks are accessed when the record is found and 'b' blocks are accessed when the record is not found.

Type of Organization	Access Method	Average Time to Access a Specific Record
Heap (unordered)	Sequential scan (linear search)	$b/2$
Ordered	Ordered scan	$b/2$
Ordered	Binary search	$\log_2 b$

Ordered files are rarely used in database applications unless an additional access path, called a *primary index*, is used; this results in an indexed sequential file. This further improves the random access time on the ordering key field.

Hashing Techniques

The other type of primary file organization is based on hashing, which provides very fast access to records on certain search conditions. This organization is usually called a *hash file*.

The search condition must be an equality condition on a single field, called the *hash field* of the file. If the hash is also a key field of the file, in which case it is called the *hash key*.

The idea behind hashing is to provide a function 'h', called a hash function or randomizing function, which is applied to the hash field value of a record and yields the address of the disk block in which the record is stored. We need only a single-block access to retrieve that record.

Example:

	NAME	RNO	CLASS	GRADE
0				
1				
2				
3				
.			.	
.			.	
.			.	
$m-2$				
$m-1$				

Internal Hashing

For internal files, hashing is implemented as a hash table through the use of an array of records. Suppose that the array index range is from 0 to M – 1, then we have M slots whose addresses corresponds to the array indexes. We choose a hash function that transforms the hash field value into an integer between 0 and M – 1. One common hash function is the $h(K) = K \bmod M$ function, which returns the remainder of an integer hash field value K after division by M; this value is then used for the record address.

Non-integer hash field values can be transformed into integers before the mod function is applied. For character strings, the numeric (ASCII) codes associated with characters can be used in the transformation.

A collision occurs when the hash field value of a record that is being inserted hashes to an address that already contains a different record. In this situation, we must insert the new record in some other position, since its hash address is occupied. The process of finding another position is called *collision resolution*. There are different methods for collision resolution as follows:

Open addressing Proceeding from the occupied position specified by the hash address, the program checks the subsequent positions in order until an unused (empty) position is found.

Chaining For this method, various overflow locations are kept, usually by extending the array with a number of overflow positions. In addition, a pointer field is added to each record location. A collision is resolved by placing the new record in an unused overflow location and setting the pointer of the occupied hash address location to the address of that overflow location. A linked list of overflow records for each hash address is thus maintained.

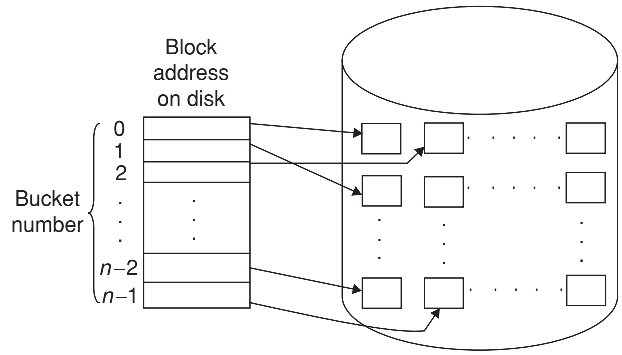
Multiple hashing The program applies a second hash function if the first results in a collision. If another collision results, the program uses open addressing or applies a third hash function and then uses open addressing if necessary.

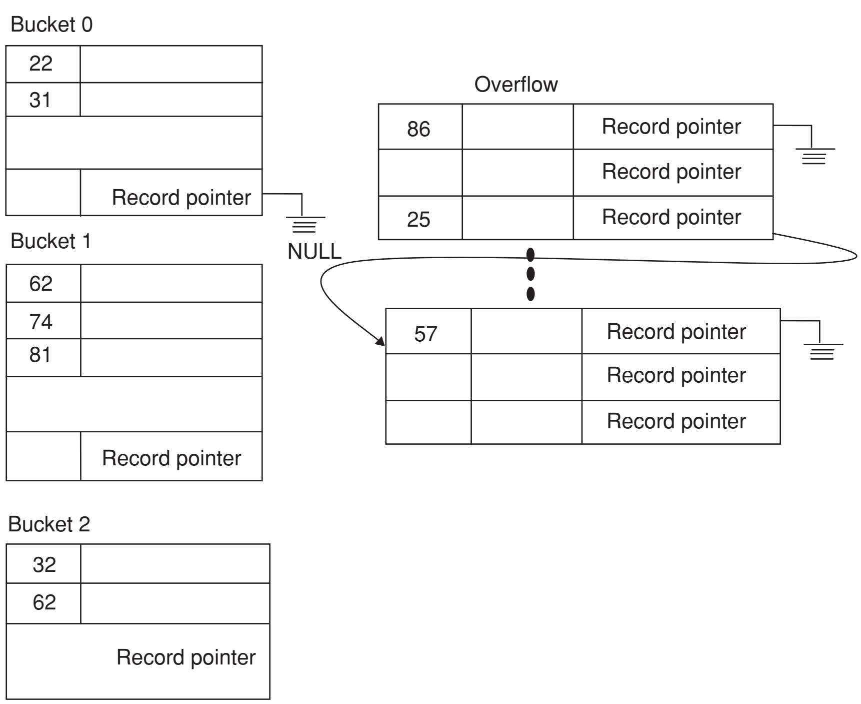
If we expect to have '*r*' records to store in the table, we should choose *M* locations for the address space such that (r/M) is between 0.7 and 0.9. It may also be useful to choose a prime number for *M*, since it has been demonstrated that this distributes the hash addresses better over the address space when the 'mod' hashing function is used. Other hash functions may require *M* to be a power of 2.

External Hashing

Hashing for disk files is called *external hashing*. To suit the characteristics of disk storage, the target address space is made of buckets, each of which holds multiple records. A bucket is either one disk block or a cluster of contiguous blocks. The hashing function maps a key into a relative bucket number, rather than assigning an absolute block address to the bucket. A table maintained in the file header converts the bucket number into the corresponding disk block address.

The collision problem is less severe with buckets, because as many records as will fit in a bucket can hash to the same bucket without causing problems. If the capacity of bucket exceeds, we can use a variation of chaining in which a pointer is maintained in each bucket to a linked list of overflow records for the bucket. The pointers in the linked list should be record pointers, which include both a block address and a relative record position within the block.





The hash function is $h(k) = k \bmod 10$ and the hashing scheme described above is called *static hashing* because a fixed number of buckets *M* is allocated. It can be a drawback for dynamic files. Suppose that we allocate *M* buckets for the address space and let '*m*' be the maximum number of records that can fit in one bucket, then at most ($m * M$) records will fit in the allocated space. If the number of records turns out to be substantially fewer than ($m * M$), we are left with a lot of unused space.

If the number of records increases to substantially more than ($m * M$), numerous collisions will result and retrieval will be slowed down because of the long lists of overflow

records. In either case, we may have to change the number of blocks M allocated and then use a new hashing function (based on the new value of M) to redistribute the records. These organizations can be quite time consuming for large files. Newer dynamic file organizations based on hashing allows the number of buckets to vary dynamically with only localized reorganization.

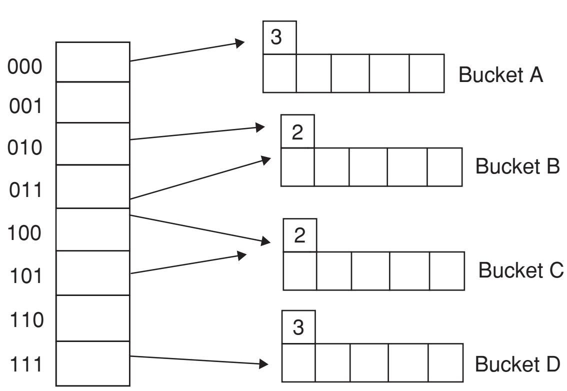
Hashing Techniques with Dynamic File Expansion

The disadvantage of static hashing is that the hash address space is fixed. Hence, it is difficult to expand or shrink the file dynamically. The first scheme is extendible hashing, It stores an access structure in addition to the file hence it is similar to indexing. The main difference is that the access structure is based on the values that result after application of the hash function to the search field. In indexing, the access structure based on the values of the search field itself. The second technique, called *linear hashing*, does not require additional access structure.

These hashing schemes take advantage of the fact that the result of applying a hashing function is a non-negative integer and hence can be represented as a binary number. The access structure is built on the binary representation of the hashing function result, which is a string of bits. We call this the *hash value of a record*. Records are distributed among buckets based on the values of the leading bits in their hash values.

Extendible Hashing

In extendible hashing, a type of directory, an array of 2^d bucket addresses is maintained, where d is called the *global depth of the directory*. The integer value corresponding to the first (high-order) d bits of a hash value is used as an index to the array to determine a directory entry, and the address in that determines the bucket in which the corresponding records are stored. Several directory locations with the same first d' bits for their hash values may contain the same bucket address if all the records that hash to these locations fit in a single bucket. A local depth d' is stored with each bucket specifies the number of bits on which the bucket contents are based.



The value of d can be increased and decreased by one at a time, thus doubling or halving the number of entries in the directory array. Doubling is needed if a bucket, whose local depth d' is equal to the global depth d, overflows. Halving occurs if $d > d'$ for all the buckets after some locations occur. Most record retrievals require two block accesses: one to the directory and the other to the bucket.

The main advantage of extendible hashing is the performance of the file does not degrade as the file grows, as opposed to static external hashing where collisions increases and the corresponding chaining causes additional accesses. No space is allowed in extendible hashing for future growth, but additional buckets can be allocated dynamically as needed. The space overhead for the directory table is negligible.

Another advantage is that splitting causes minor reorganization in most cases, since only the records in one bucket are redistributed to the two new buckets. The only time a reorganization is more expensive is when the directory has to be doubled (or) halved.

A disadvantage is that the directory must be searched before accessing the buckets themselves, resulting in two block accesses instead of one in static hashing.

Indexing

Indexes are auxiliary access structures, which are used to speed up the retrieval of records in response to certain search conditions. The index structure typically provides secondary access paths, which provide alternative ways of accessing the records without affecting the physical placement of records on disk. They enable efficient access to records based on the indexing fields that are used to construct the index.

Any field of the file can be used to create an index and multiple indexes on different fields can be constructed on the same file. To find a record or records in the file based on a certain selection criterion on an indexing field, one has to initially access the index, which points to one or more blocks in the file where the required records are located. The most prevalent types of indexes are based on ordered files (single-level indexes) and tree data structures (multilevel indexes, B^+ trees).

Dense Index files: Index record appears for every search-key value in the file.

Brighton	
Downtown	
Mianus	
Perryridge	

A-217	Brighton	750
A-101	Downtown	600
A-110	Downtown	300
A-215	Mianus	400
A102	Perryridge	800

Figure 3 Dense index file.

Sparse index files These files contain index records for only some search-key values. Applicable when records are sequentially ordered on search key.

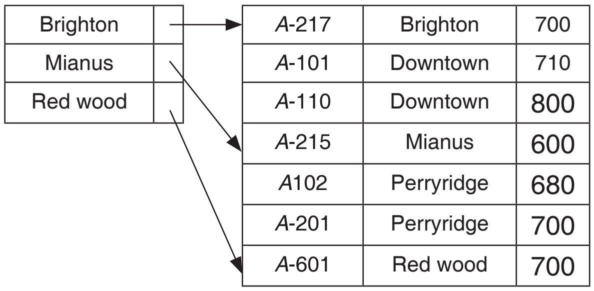


Figure 4 Sparse index file.

Compared to dense index, sparse index takes less space and less maintenance over head for insertions and deletions. It is slower than dense index for locating records.

Index Update

Record deletion If delete key was the only record in the file with its particular search-key value, the search key is deleted from the index also.

In dense index, delete the search key.

In spare index, if deleted key value exists in the index, the value is replaced by next search-key value in the file. If the next search-key value already has an index entry, the entry is deleted instead of being replaced.

Record insertion In dense index, if the search-key value doesn't appear in the index insert it.

If index stores an entry for each block of the file, no change needs to be made to the index unless a new block is created. If a new block is created, the first search-key value appearing in the new block is inserted into the index.

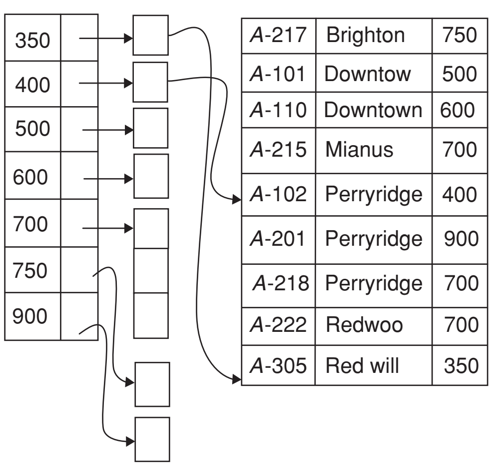


Figure 5 Secondary index.

Secondary index example:

1. Index record points to a bucket that contains pointers to all the actual records with that particular search – key value
2. secondary index have to be dense

Single-level Ordered Indexes

A file with a given record structure consisting of several fields (or attributes), an index access structure is usually defined on a single field of a file is called an *indexing field* or *indexing attribute*. The index typically stores each value of the index field along with a list of pointers to all disk blocks that contain records with that field value. The values in the index are ordered so that we can do a binary search on the index.

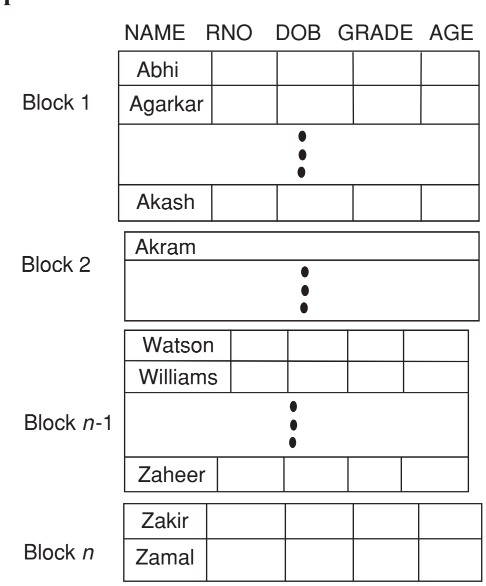
The index file is much smaller than the data file, so searching the index using a binary search is reasonably efficient. Multilevel indexing does away the need for a binary search at the expense of creating indexes to the index itself.

Types of Ordered Indexes

1. Primary index
2. Clustering index
3. Secondary index

Primary index A primary index is an ordered file whose records are of fixed length with two fields. The first field is of the same data type as the ordering key field called the *primary key of the data file*, and the second field is a pointer to a disk block (block address). There is one index entry (index record) in the index file for each block in the data file. Each index entry has the value of the primary key field for the record in a block and a pointer to that block as its two field values. The two field values of index entry i is $<k(i), p(i)>$.

Example:



To create a primary index on the ordered file shown in the above figure, we use the NAME field as primary key, because that the ordering key field on the file (assuming that each value of NAME is unique). Each entry in the index has a NAME value and a pointer. Some sample index entries are as follows:

$< k(1) = (\text{Abhi}), p(1) = \text{address of block } 1 >$
$< k(2) = (\text{Akram}), p(2) = \text{address of block } 2 >$
$< k(3) = (\text{Brat}), p(3) = \text{address of block } 3 >$

The below figure illustrates this primary index. The total number of entries in the index is the same as the number of disk blocks in the ordered data file. The first record in each block of the data file is called the *anchor record of the block (or) block anchor*.

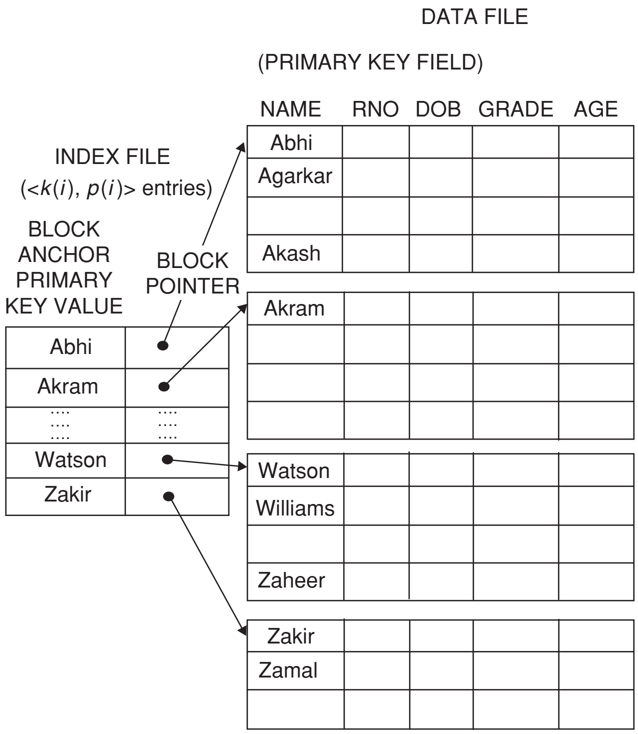


Figure 6 Primary index on the ordering key field of the file.

Indexes can also be characterized as dense or sparse. A dense index has an index entry for every search-key value (every record) in the data file. A sparse (non-dense) index has index entries only for some of the search values. A primary index is non-dense (sparse) index, since it includes an entry for each disk block of the data file and the keys of its anchor record rather than for every search value.

The index file for primary index needs fewer blocks than does the data file, for two reasons as follows:

1. There are fewer index entries than there are records in the data file.
2. Each index entry is typically smaller in size than a data record because it has only two fields. So more index entries than data records can fit in one block.

A binary search on the index file requires fewer block accesses than a binary search on the data file. The binary search for an ordered data file required $\log_2{}^b$ block accesses. But if the primary index file contains b_i blocks, then to locate a record with a search-key value requires a binary search of that index and access to the block containing that record, a total of $\log_2{}^b{}_i + 1$ accesses.

A record whose primary key value is k lies in the block whose address is $p(i)$, where $k(i) \le k \le k(i + 1)$. The ith block in the data file contains all such records because of the physical ordering of the file records on the primary key field. To retrieve a record, given the value k of its primary key field, we do a binary search on the index file to find the appropriate index entry i, and then retrieve the data file block whose address is $p(i)$.

The following example illustrates the saving in block accesses that is attainable when a primary index is used to search for a record.

Example: Suppose that we have an ordered file with r = 24,000 records stored on a disk with block size B = 512 bytes. File records are of fixed size and are unspanned, with record length R = 120 bytes.

The blocking factor for the file would be $bfr = \lfloor B/R \rfloor$

$$= \left\lfloor \frac{512}{120} \right\rfloor = \lfloor 4.26 \rfloor = 4 \text{ records per block}$$

The number of blocks needed for the file is

$$b = \left\lceil \left(\frac{r}{bfr} \right) \right\rceil = \left\lceil \frac{24,000}{42} \right\rceil = 6000 \text{ blocks}$$

A binary search on the data file would need

$\lceil \log_2{}^b = \log_2{}^{6000} \rceil = 13$ block accesses

Example: For the above data, suppose that the ordering key field of the file is V = 7 bytes long, a block pointer, P = 5 bytes long, and we have constructed a primary index for the file.

The size of each index entry is $R_i = (7 + 5) = 12$ bytes, so the blocking factor for the index is $bfr_i = \lfloor (B/R_i) \rfloor$

$\lfloor 512/12 \rfloor = \lfloor 42.66 \rfloor = 42$ entries per block.

The total number of index entries r_i is equal to number of blocks in the data file, which is 6000. The number of index blocks is hence

$b_i = \lceil (r_i/bfr_i) \rceil = \lceil 6000/42 \rceil = 142$ blocks

To perform a binary search on the index file would need $\lceil \log_2 b_i \rceil = \lceil \log_2 142 \rceil = 8$ block accesses. To search for a record using the index, we need one additional block access to the data file for a total of '9' block accesses.

Disadvantage: A major problem with a primary index is insertion and deletion of records. If we attempt to insert a record in its correct position in the data file, we have to not only move records to make space for the new record but also change some index entries.

Clustering Index If records of a file are physically ordered on a non-key field, which does not have a distinct value for each record, that field is called the *clustering field*. We can create a different type of index called *clustering index* to

speed up the retrieval of records that have the same value for the clustering field.

A clustering index is also an ordered file with two fields, the first field is of the same type as the clustering field of the data file, and the second field is a block pointer.

There is one entry in the clustering index for each distinct value of the clustering field, containing the value and a pointer to the first block in the data file that has a record with that value for its clustering field.

The record insertion and deletion still cause problems, because the data records are physically ordered. To alleviate the problem of insertion, reserve a whole block (or a cluster of contiguous blocks) for each value of the clustering field, all records with that value are placed in the block (or block cluster). A clustering index is an example of a non-dense index, because it has an entry for every distinct value of the indexing field which is a non-key.

Secondary Index A secondary index provides a secondary means of accessing a file for which some primary access already exists. The secondary index may be on a field which is a candidate key and has a unique value in every record, or a non-key with duplicate values. The index is an ordered file with two fields. The second field is either a block pointer or a record pointer. There can be many secondary indexes for the same file.

First consider a secondary index access structure on a key field that has a distinct value for every record such a field is some times called a *secondary key*.

The records of the data file are not physically ordered by values of the secondary key field, we cannot use block anchors. That is why an index entry is created for each record in the data file, rather than for each block, as in the case of a primary index.

B^- Trees

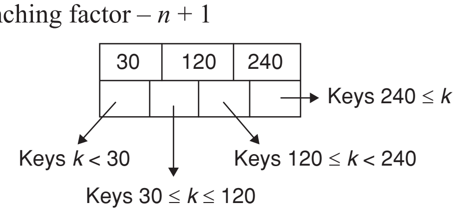
1. A commonly used index structure
2. Non-sequential, 'balanced'
3. Adapts well to insertions and deletions
4. Consists of blocks holding at most n keys and $n + 1$ pointers.
5. We consider a variation actually called a B^+ *tree*

B^+ Trees

B^+ trees are a variant of B^- trees. In B^+ trees data stored only in leaves, leaves form a sorted linked list.

Parameter – n

Branching factor – $n + 1$

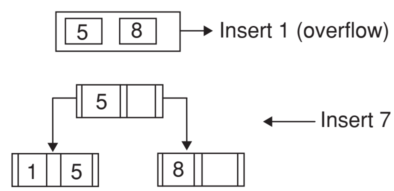


Each node (except root) has at least $n/2$ keys. B^-tree stands for balanced tree. All the paths through a B^-tree from root to different leaf nodes are of the same length (balanced path length). All leaf nodes are at the same depth level.

This ensures that number of disk accesses required for all the searches are same. The lesser the depth (level) of an index tree, the faster the search.

Insertion into B^+ tree

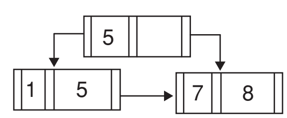
Given nodes 8 5 1 7 3 12 Initially start with root node (has no children)



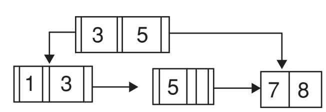
Overflow in Leaf Node

Split the leaf node First, j = ceiling $((p_{leaf} + 1)/2)$ entries are kept in the original node and the remaining moved to the new leaf.

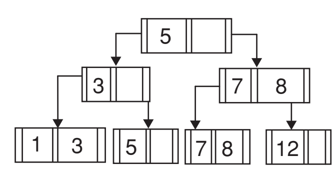
1. Create a new internal node, and jth index value is replicated in the parent internal node.
2. A pointer is added to the newly formed leaf node.



Insert 3 → overflow



Insert 12 (overflow, split propagates, new level)



Overflow in Internal Node

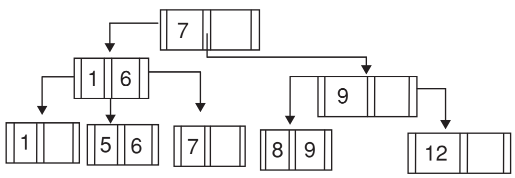
Split the internal node, the entries up to P_j where j = floor $((p + 1)/2)$ are kept in the original node and remaining moved to the new internal node

1. Create a new internal node and the jth index value is moved to the parent internal node (without replication)
2. Pointers are added to the newly formed nodes.

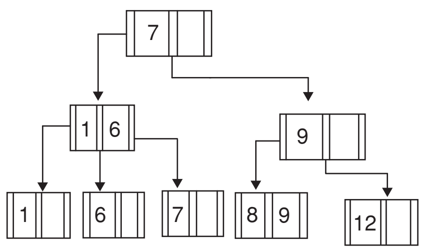
3. B^+ tree ensures some space always left in nodes for new entries. Also makes sure all nodes are at least half full.

Deletion in B^+ Trees

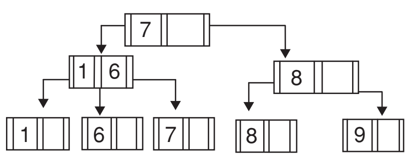
Delete 5,12,9 from the below B^+ tree:



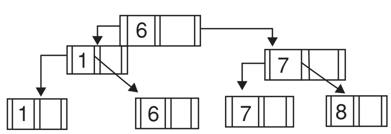
Delete 5:



Delete 12:
Under flow has occurred, so redistribute.



Delete 9: Underflow (merge with left) redistribute.



Advantages

1. B^- *Trees and* B^+ *trees*: B^- tree is a data structure used for external memory.
2. B^- trees are better than binary search trees if data is stored in external memory.
3. Each node in a tree should correspond to a block of data.
4. Each node can store many data items and has many successors.
5. The B^- tree has fewer levels but search for an item takes more comparisons at each level.
6. If a B^- tree has order 'd', then each node (except root) has at least d/2 children, then the depth of the tree is at most $\log_{d/2}$ (size) + 1.
7. In the worst case, we need $(d - 1)$ comparisons in each node (using linear search)
8. Fewer disk accesses are required compared to binary Tree.
9. The usual data structure for an index is the B^+ tree.
10. Every modern DBMS contains some variant of B^- trees in addition with other index structures depending on the application.
11. B^- trees and B^+ trees are one and the same. They differ from B^- trees in having all data in the leaf blocks.
12. Compared to binary trees, B^- trees will have higher branching factor.
13. Binary trees can degenerate to a linear list, B^- trees are balanced, so this is not possible.
14. In B^+ tree, the values in inner nodes are repeated in the leaf nodes.
15. The height of the tree might decrease, because the data pointer is needed only in the leaf nodes, we can also get a sorted sequence.
16. In B^- trees, all leaves have the same distance from root hence B^- trees are balanced. This ensures that the chain of links followed to access a leaf node is never too long.
17. The time complexity of search operation in B^- tree (tree height) is $O(\log n)$, where 'n' is the number of entries.
18. Advantage of B^+ tree automatically reorganizes itself with small and local changes while doing insertions and deletions, reorganization of entire file is not required to maintain performance.
19. Disadvantage of B^+ tree, extra Insertion and deletion overhead, space overhead.
20. B^+ trees can be used as dynamic multilevel Indexes.

Exercises

Practice Problems I

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Consider the following specifications of a disk. Block size of a disk is 512 bytes, inter-block gap size is 128 bytes Number of blocks per track is 20 and number of tracks per surface is 400.

(i) What is the capacity of disk including Inter block gap?

(A) 124000 (B) 1260000
(C) 5120000 (D) 512000

(ii) What is the capacity of disk excluding Inter block gap?

(A) 25400 (B) 25600
(C) 25800 (D) 25900

2. Consider the following specifications of a disk. Block size of disk is 512 bytes, 2000 tracks per surface, 50 sectors per track and 5 double sided platters.

(i) What is the capacity of track in bytes?
(A) 4096000 (B) 4086000
(C) 4076000 (D) 4066000

(ii) What is the capacity of surface in bytes?
(A) 25600000 (B) 512000
(C) 5120000 (D) 51200000

(iii) What is the capacity of disk in bytes?
(A) 512*10^4 (B) 512*10^5
(C) 512*10^6 (D) 512*10^7

(iv) How many cylinders does it have?
(A) 512 (B) 1000
(C) 2000 (D) 2048

(v) Identify the In valid disk block size from below:
(A) 2048 (B) 51200
(C) 4098 (D) 4096

3. What is the order of internal node of B^+ tree suppose that a child pointer takes 6 bytes, the search field value takes 14 bytes and the block size is 512 bytes?
(A) 23 (B) 24
(C) 25 (D) 26

4. The order of a leaf node in a B^+ tree is the maximum number of (value, data, record pointer) pairs it can hold. Given that block size is 1 k bytes (1024 bytes), data record pointer is 7 bytes long, the value field is '9' bytes long and block pointer is 6 bytes.
(A) 63 (B) 64
(C) 65 (D) 66

5. The following key values are inserted into a B^+ tree in which order of the internal nodes is 3, and that of the leaf nodes is 2, in the sequence given below. The order of internal nodes is the maximum number of tree pointers in each node, and the order of leaf nodes is the maximum number of data items that can be stored in it. The B^+ tree is initially empty. 10, 3, 6, 8, 4, 2, 1. What is the maximum number of times leaf nodes would get split up as a result of these insertions?
(A) 3 (B) 4
(C) 5 (D) 6

6. For the same key values given in the above question, suppose the key values are inserted into a B^- tree in which order of the internal nodes is 3 and that of leaf nodes is 2. The order of internal nodes is the maximum number of tree pointers in each node and the order of leaf nodes is the maximum number of data items that can be stored in it. The B^- tree is initially empty. What is the maximum number of times leaf nodes would get split up as a result of these insertions?
(A) 1 (B) 2
(C) 3 (D) 4

7. Suppose that we have an ordered file with 45,000 records stored on a disk with block size 2048 bytes. File records are of fixed size and are unspanned with record length 120 bytes.

(i) What is the blocking factor?
(A) 16 (B) 17
(C) 18 (D) 19

(ii) What is the number of blocks needed for the file?
(A) 2642 (B) 2644
(C) 2646 (D) 2648

(iii) How many block accesses are required to search for a particular data file using binary search?
(A) 10 (B) 11
(C) 12 (D) 13

8. Suppose that the ordering key field of the file is 12 bytes long, a block pointer is 8 bytes long, and we have constructed a primary index for the file. Consider the file specifications given in the above questions.

(i) What is the size of each index entry?
(A) 16 (B) 18
(C) 20 (D) 22

(ii) What is the blocking factor for the index?
(A) 101 (B) 102
(C) 103 (D) 104

(iii) What is the total number of index entries?
(A) 2642 (B) 2644
(C) 2646 (D) 2648

(iv) What is the number of index blocks?
(A) 22 (B) 24
(C) 26 (D) 28

(v) How many block accesses are required, if binary search is used?
(A) 3 (B) 4
(C) 5 (D) 6

9. For the file specifications given in Q. No. 7, if we construct secondary index on a non-ordering key field of the file that is 12 bytes long, a block-pointer of size 8 bytes, each index entry is 20 bytes long and the blocking factor is 102 entries per block.

(i) What is the total number of index blocks?
(A) 422 (B) 424
(C) 442 (D) 444

(ii) How many block accesses are required to access the secondary index using binary search?
(A) 6 (B) 7
(C) 8 (D) 9

10. For the file specifications given in Q. No. 8, if we construct a multilevel index, number of 1st-level blocks are 442, blocking factor is 102, each index entry is 20 bytes long.

(i) What is the number of 2nd-level blocks?
(A) 4 (B) 5
(C) 6 (D) 7

(ii) What is the number of 3rd-level blocks?
(A) 0 (B) 1
(C) 2 (D) 3

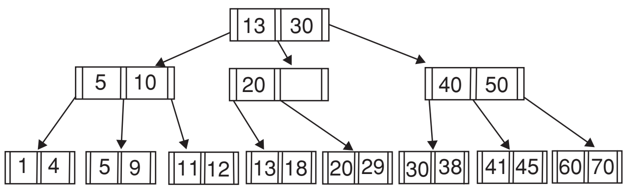
11. Construct a B^+ tree for (1,4,7,10,17,21,31) with $n = 4$, which nodes will appear two times in a final B^+ tree?

(A) 17,7,20 (B) 17,7,20,25
(C) 17,20,25 (D) 7,17.25

12. Suppose the hash function is $h(x) = x\bmod 8$ and each bucket can hold at most two records. The extendable hash structure after inserting 1, 4, 5 ,7, 8, 2, 20, what is the local depth of '4'?

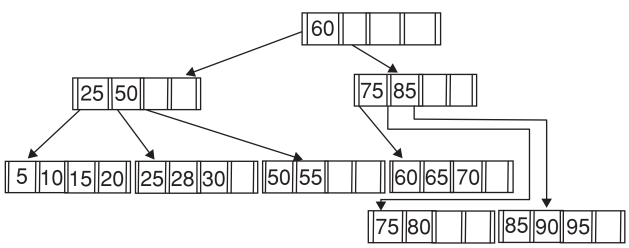
(A) 0 (B) 1
(C) 2 (D) 3

13. Consider the given B^+ tree, insert 19 into the tree, what would be the new element in level 2?



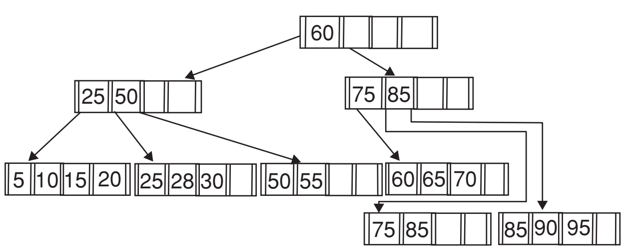
(A) 13 (B) 18
(C) 20 (D) 29

14. Consider the given B^+ tree, delete 70 and 25 from the tree, what are the elements present in level 2? (∴ root is at level 1)



(A) 25, 50, 75 (B) 25, 50,75, 85
(C) 28, 50, 75, 85 (D) 28, 50, 65, 75

15. Delete 60 from the above given tree (Q. No. 14). After deletion, what is the total number of nodes present in the tree?



(A) 5 (B) 6
(C) 7 (D) 8

16. What will be the number of index records/block?

(A) 68 (B) 65
(C) 69 (D) None

17. What will be the number of index blocks?

(A) 442 (B) 440
(C) 400 (D) None

18. Consider the following:

Block size = 1025 bytes

Record length in data file = 100 bytes

Total number of records = 30000

Search key = 9 bytes

Pointer = 6 bytes

What is the number of index blocks?

(A) 44 (B) 45
(C) 46 (D) None

19. Which of the following is maximum search time $t_{\max}$ in B^- trees?

(A) $t_{\max} = a\log_2\left(\frac{N}{2}\right)\left[\frac{a+d}{\log_2 m} + \frac{bm}{\log_2 m} + c\right]$

(B) $t_{\max} = a\log_2\left(\frac{N}{2}\right)\left[\frac{a+d}{\log_2 m} + \frac{bm}{\log_2 m} + c\right]$

(C) $t_{\max} = a\log_2 N\left[\frac{a+d}{\log_2 m} + \frac{bm}{\log_2 m} + c\right]$

(D) $t_{\max} = a\log_2(N)\left[\frac{a}{\log_2 m} + \frac{bm}{\log_2 m} + c\right]$

20. Consider a B^+ tree. A child pointer takes 3 bytes, the search field value takes 7 bytes, and the block size is 256 bytes. What is the order of the internal node?

(A) 63 (B) 64
(C) 65 (D) 66

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Which of the following is true?
   (A) Every conflict serializable is view serializable
   (B) Every view serializable is conflict serializable
   (C) Both A and B
   (D) A schedule can be either only conflict serializable or only view-serializable.

2. Which one is the 2-phase locking rule?
   (A) Two transactions cannot have conflicting locks
   (B) No unlock operation can precede a lock operation in the same transaction.
   (C) No data is/are affected until all locks are obtained and until the transaction is in its locked point.
   (D) All of the above

3. If Transaction T_i has obtained an exclusive mode lock on item Q, then
   (A) T_i can read Q
   (B) T_i can write Q
   (C) T_i can read and write
   (D) Neither read nor write

4. Phantom phenomenon is
   (A) A transaction retrieves a collection of objects but sees same result.
   (B) A transaction retrieves a collection of objects but sees different results.
   (C) Transaction T_1 waits for T_2 and T_2 waits for T_1
   (D) This problem arises when the transaction has not locked all the objects.

5. We can avoid the starvation of transactions by granting locks by following manner:

   When a transaction T_i requests a lock on a data item Q in a particular mode M, the concurrency control manager grants the lock provided that
   (A) There is no other transaction holding a lock on Q in a mode that conflicts with M.
   (B) There is no other transaction that is waiting for a lock on Q,
   (C) (A) and (B)
   (D) None

6. Which one is correct?
   (A) Upgrading can take place only in shrinking phase
   (B) Upgrading can take place only in growing phase.
   (C) Downgrading can take place only in growing phase
   (D) (A) and (C) both

7. A simple but widely used scheme automatically generates the appropriate lock and unlock instructions for a transaction, on the basis of read and write requests from the transaction:
   (A) When a transaction T_i issues a read (Q) operation, the system issues a lock $s(Q)$ instruction followed by the read instruction.
   (B) When T_i issues a write Q operation, the system checks to see whether T_i already holds a shared lock on Q. If it does, then the system issues an upgrade Q instruction followed by the write Q instruction, otherwise the system issues a lock -$X(Q)$ instruction, followed by the write Q instruction.
   (C) All locks obtained by a transaction are unlocked after that transaction commits or aborts.
   (D) All of the above

8. Which one is correct?
   (A) A lock manager can be implemented as a process that receives messages from transactions and sends messages in reply.
   (B) It uses linked list of records.
   (C) It uses hash table called lock table.
   (D) All of the above

Common data questions 9 and 10: Transaction T_1 has 5 instructions. Transaction T_2 has 3 instructions.

9. The number of non-serial transactions will be
   (A) 15 (B) 8
   (C) 2 (D) 56

10. The number of serial transaction schedules will be
    (A) 15 (B) 8
    (C) 2 (D) 56

11. In a heap file system, which of the following function finds 'average number of blocks to be read'?
    (A) $\frac{i}{n} = \frac{1}{2}(1+n) = \frac{n}{2}$
    (B) $\sum_{i=1}^{n} \frac{i}{n} = \frac{1}{2}(1+n) = \frac{n}{2}$
    (C) $\sum_{i=0}^{n-1} \frac{i}{n} = \frac{1}{2}(1+n) = \frac{n}{2}$
    (D) All of the above

12. What is the disadvantage in one directory per user?
    (A) Different applications can be divided into separate groups.
    (B) Different applications cannot be divided into separate groups
    (C) All files are in a single group
    (D) All of the above

13. What are the possible violations if an application program uses isolation-level 'Read uncommitted'?
    (A) Dirty read problem
    (B) Non-repeatable read problem
    (C) Phantom phenomenon
    (D) All of the above

14. The two-phase locking protocol
(A) ensures serializability
(B) issues locks in two phases
(C) unlocks in two phases
(D) All of the above

15. The point in the schedule where the transaction has obtained its final lock (the end of its growing phase) is called the
(A) block point (B) critical section
(C) growing point (D) lock point

16. Which of the following is not a problem of file management system?
(A) Data redundancy
(B) Lack of data independence
(C) Program dependence
(D) All of the above

17. Which of the following is/are true about master list of an index file?
(i) Is sorted in ascending order
(ii) A number is assigned to each record.
(A) Only (i) (B) Only (ii)
(C) Both (i) and (ii) (D) None of the above

18. To have a file, holding a list is necessary to
(i) Identify the records in the list
(ii) Identify the name, and type of the fields of each record.
(iii) Decide which fields will be used as sort of index keys.
(A) Only (i) and (ii)
(B) Only (i) and (iii)
(C) Only (ii) and (iii)
(D) All of the above

19. Two files may be joined into a third file, if the following is true:
(A) if they have row in common
(B) if they have a field in common
(C) Both (A) and (B)
(D) None

20. The minimum number of record movements required to merge four files w(with 10 records), x(with 20 records), y(with 15 records) and z(with 5 records) is:
(A) 50 (B) 40
(C) 30 (D) 35

Previous Years' Questions

1. A clustering index is defined on the fields which are of type **[2008]**
(A) non-key and ordering
(B) non-key and non-ordering
(C) key and ordering
(D) key and non-ordering

2. A B^- tree of order 4 is built from scratch by 10 successive insertions. What is the maximum number of node splitting operations that may take place? **[2008]**
(A) 3 (B) 4
(C) 5 (D) 6

3. Consider a file of 16384 records. Each record is 32 bytes long and its key field is of size 6 bytes. The file is ordered on a non-key field, and the file organization is unspanned. The file is stored in a file system with block size 1024 bytes, and the size of a block pointer is 10 bytes. If the secondary index is built on the key field of the file, and a multilevel index scheme is used to store the secondary index, the number of first-level and second-level blocks in the multilevel index are respectively **[2008]**
(A) 8 and 0 (B) 128 and 6
(C) 256 and 4 (D) 512 and 5

4. The following key values are inserted into a B^+ tree in which order of the internal node s is 3, and that of the leaf nodes is 2, in the sequence given below. The order of internal nodes is the maximum number of tree pointers in each node, and the order of leaf nodes is the maximum number of data items that can be stored in it. The B^+ tree is initially empty.

10, 3, 6, 8, 4, 2, 1

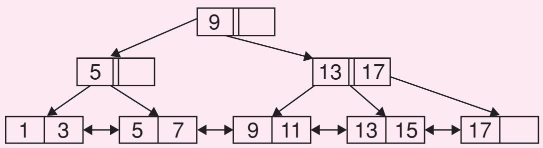
The maximum number of times leaf nodes would get split up as a result of these insertions is **[2009]**
(A) 2 (B) 3
(C) 4 (D) 5

5. Consider a B^+ tree in which the maximum number of keys in a node is 5. What is the minimum number of keys in any non-root node? **[2010]**
(A) 1 (B) 2
(C) 3 (D) 4

6. An index is clustered, if **[2013]**
(A) it is on a set of fields that form a candidate key.
(B) it is on a set of fields that include the primary key.
(C) the data records of the file are organized in the same order as the data entries of the index.
(D) the data records of the file are organized not in the same order as the data entries of the index.

7. A file is organized so that the ordering of data records is the same as or close to the ordering of data entries in some index. Then that index is called **[2015]**
(A) Dense (B) Sparse
(C) Clustered (D) Unclustered

8. With reference to the B+ tree index of order 1 shown below, the minimum number of nodes (including the Root node) that must be fetched in order to satisfy the following query: "Get all records with a search key greater than or equal to 7 and less than 15" is ________ **[2015]**



9. Consider a B^+ tree in which the search key is 12 bytes long, block size is 1024 bytes, record pointer is 10 bytes long and block pointer is 8 bytes long. The maximum number of keys that can be accommodated in each non-leaf node of the tree is ________. **[2015]**

10. B+ Trees are considered **BALANCED** because **[2016]**

(A) The lengths of the paths from the root to all leaf nodes are all equal.
(B) The lengths of the paths from the root to all leaf nodes differ from each other by at most 1.
(C) The number of children of any two non - leaf sibling nodes differ by at most 1.
(D) The number of records in any two leaf nodes differ by at most 1.

11. In a B+ tree, if the search-key value is 8 bytes long, the block size is 512 bytes and the block pointer size is 2 bytes, then the maximum order of the B+ tree is __________. **[2017]**

Answer Keys

Exercises

Practice Problems 1

1. (i) C (ii) A **2.** (i) B (ii) D (iii) C (iv) C (v) C **3.** C **4.** A **5.** C **6.** B
7. (i) B (ii) D (iii) C **8.** (i) C (ii) B (iii) D (iv) C (v) C **9.** (i) C (ii) D
10. (i) B (ii) B **11.** B **12.** D **13.** B **14.** C **15.** B **16.** A **17.** A **18.** B
19. A **20.** C

Practice Problems 2

1. A **2.** D **3.** C **4.** B **5.** C **6.** B **7.** D **8.** D **9.** D **10.** C
11. B **12.** B **13.** D **14.** D **15.** D **16.** D **17.** B **18.** D **19.** B **20.** B

Previous Years' Questions

1. A **2.** C **3.** C **4.** C **5.** B **6.** C **7.** C **8.** 5 **9.** 50 **10.** A
10. 52

Test

Databases

Time: 60 min.

1. Which of the following are used in DBMS files?
(i) Data dictionary (ii) DML
(iii) Query language (iv) Transaction log
(A) (i) and (ii) (B) (ii) and (iii)
(C) (iii) and (iv) (D) (i) and (iv)

2. Which among the following is not a problem of file management system?
(A) Data redundancy
(B) Lack of data independence
(C) Program independence
(D) None of these

3. A transparent DBMS
(A) cannot hide sensitive information from users
(B) keeps its logical structure hidden from users
(C) keeps its physical structure hidden from users
(D) All of the above

4. If the field size is too small, for the longest piece of data to be entered,
(A) database program will be freezed
(B) field will automatically expand
(C) part of the data will be cut off
(D) All of the above

5. Which of the following functional dependencies are satisfied by the instance from the below relation?

A	B	C
1	7	3
1	9	5
1	11	5
5	3	3

(A) $AB \to C$ and $C \to B$
(B) $BC \to A$ and $B \to C$
(C) $BC \to A$ and $A \to C$
(D) $AC \to B$ and $B \to A$

6. Let E_1 and E_2 be two entities in an E/R diagram with single-valued attributes, R_1 and R_2 are two relationships between E_1 and E_2, R_1 is one to many R_2 is many-to-one. R_1 and R_2 do not have any attributes of their own. What is the minimum number of tables required to represent this situation in the relation model?
(A) 2 (B) 3
(C) 4 (D) 5

7. Which of the following is true about DBMS?
(i) Low-level DMLs are record-at-a time
(ii) High-level DMLs are set oriented or set-at-a time
(iii) Query in high-level DML specify which data to retrieve rather than how.
(iv) When used as standalone, DML is called 'host language'
(A) (i) only (B) (i) and (iii)
(C) (i), (ii) and (iii) (D) (iii) and (iv)

8. In which of the following, the structure of data files is stored?
(A) Metadata (B) Database catalog
(C) Database schema (D) Data model

9. A schedule is a collection of
(A) Data models (B) Transactions
(C) Schemas (D) Tables

10. Select from the following which matches the term 'Impedance mismatch problem':
(A) In compatibility of storage and data structure
(B) Mismatch in user authentication
(C) File structure mismatching
(D) None of these

11. Which of the following is not a/an integrity constraint?
(A) Entity integrity
(B) Candidate key constraint
(C) Business rules
(D) None of the above

12. Select from the following which is concerned with 'Query Optimizer':
(A) Extracts DML commands from an application program in a high-level language
(B) Parsing and analyzing interactive query
(C) Rearrangement and reordering of operations and elimination of redundancies
(D) Performance monitoring

13. Which of the following does not belong to database model?
(A) Relational Model (B) Distributed Model
(C) Hierarchical Model (D) Network Model

14. What is the correct sequence of database design process?
(i) Create conceptual schema
(ii) Data model mapping
(iii) Requirement collection and analysis
(iv) Physical design
(A) iii $\to$ i $\to$ ii $\to$ iv
(B) iii $\to$ ii $\to$ i $\to$ iv
(C) i $\to$ ii $\to$ iii $\to$ iv
(D) i $\to$ iii $\to$ ii $\to$ iv

15. Consider the following schema definitions
Employee {Name, SSN, Address, DNo}
Department {DName, DNumber, Manager, SSN}
Which among the following expressions represent the query $\Pi_{\text{name, address}}(\sigma_{\text{Dname = 'Res'} \wedge \text{DNumber = DNo}}$ (Department $\bowtie$ Employee)?

(A) Retrieve the name and address of employees who work for the project no 'Dno'
(B) Retrieve the name and address of all employees who control the 'Res' department.
(C) Retrieve the name and address of all employees who work for the 'Res' department.
(D) None of these

16. Select from the following which closely resembles the concept 'Degree of a relationship':
(A) Number of entities participating in a relation
(B) Number of entity types participating in a relation
(C) Number of strong entity types in a relation
(D) Number of weak entity types in a relation

17. Consider the following statements in a database:
(i) No primary key value can be NULL
(ii) A tuple in one relation which refers to another relation must refer to an existing tuple in that relation
(iii) The value of x determines the value of y in all states of a relation, where x and y are two attributes of the relation Which of the following combinations matches the given statements in order?
(A) Referential integrity, functional dependency, entity integrity.
(B) Functional dependency, entity integrity, referential integrity
(C) Entity integrity, functional dependency, referential integrity.
(D) Entity integrity, referential integrity, functional dependency

18. Consider the following relation schemas:

Works (emp_name, comp_name,salary)

Livesin (emp_name, street, city)

Location (comp_name, city)

Manager (manager_name)

What is returned by the following relational algebra expression

$\pi_{\text{emp\_name}}(\sigma_{\text{comp-name=Time} \wedge \text{Works.emp\_name=livesin.emp\_name}}$

(Works $\bowtie$ Livesin)
(A) Names of all employees who work for TIME
(B) Names of all employees of TIME who lives in the same city
(C) Names of people who live in the same city
(D) None of these

19. Consider the following SQL query:
Select distinct $a_1, a_2, \ldots, a_n$ from $r_1, r_2 \ldots r_m$ where P
This query is equivalent to one of the following relational algebra expression:

(A) $\pi_{a_1, a_2 \ldots a_n} \sigma_P (r_1 \times r_2 \times \cdots \times r_m)$

(B) $\pi_{a_1, a_2 \ldots a_n} \sigma_P (r_1 r_2 \times \cdots \times r_m)$

(C) $\pi_{a_1, a_2 \ldots a_n} \sigma_P (r_1 \cup r_2 \cup \times \ldots \times \cup r_m)$

(D) $\pi_{a_1, a_2 \ldots a_n} \sigma_P (r_1 r_2 \times \ldots \times r_m)$

20. Let $R_1(A, B, C)$ and $R_2(D, E)$ be two relation schemas with primary keys A and D and C be a foreign key in R_1 referring to R_2. Suppose there is no violation of the above referential integrity constraint in the instances r_1 and r_2, which of the following relational algebra expression would necessarily produce an empty relation?
(A) $\pi_D(r_2) - \pi_C(r_1)$
(B) $\pi_C(r_1) - \pi_E(r_2)$
(C) $\pi_D(r_1 \bowtie_{C=D} r_2) - \pi_B(r_1)$
(D) $\pi_C(r_1 \bowtie_{C=E} r_2)$

21. Let r be an instance for the schema $R = (A, B, C, D)$. Let $r_1 = \pi_{A,B,C}(r) r_2 = \pi_{A,D}(r)$ and $S = r_1 \bowtie r_2$. Also given that the decomposition of r into r_1 and r_2 is lossy, which of the following is true?

(A) $S \subset r$	(B) $r \cup s = r$
(C) $r \subset s$	(D) $r \bowtie s = s$

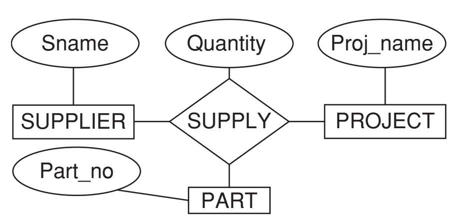
22. Which of the following is/are logical database structures?

(A) Network	(B) Tree
(C) Chain	(D) All of the above

23. A relational database management system manages data in more than one file at a time by using which of the following combinations?
(A) Tables and tuples
(B) Relations and tuples
(C) Tables and Relations
(D) Attributes and tuples

24. Let Emp = (Name, ID, ADDRESS, PHONE, SPOUSE, LIVINGAT) be a relation scheme with following FDs, which one of the following is a key
ADDRESS $\rightarrow$ Phone
SPOUSE $\rightarrow$ NAME
SPOUSE, ADDRESS $\rightarrow$ PHONE
NAME $\rightarrow$ ID
(A) ADDRESS, PHONE
(B) SPOUSE, ADDRESS
(C) NAME, SPOUSE
(D) NAME, ADDRESS

25. Consider the following E-R diagram



Select the most appropriate statement from the following for the above ER diagram:

(A) Represents a ternary relationship
(B) Represents a binary relationship
(C) Represents a ternary relationship with instances of the form (s, j, p)
(D) Represents 1 – to – many relationships

26. If two relations R_1 and R_2 are such that they are of the same degree and domain of the corresponding fields are also the same, then which one of the following is true about R_1 and R_2?
(A) $R_1 \subset R_2$
(B) $R_1 \cup R_2 = R_2 \cup R_1$
(C) R_1 and R_2 are union compatible
(D) None of these

Common data questions for 27 and 28: Let Employee and Guests be two relations with attributes (id, mobl_no, name, address) and (id, mob–no, comps_working, shifts) Relations respectively {id, mob_no} is the key for both.

27. Which of the following queries are equivalent?
(i) π_{id} (Employee $\bowtie$ Guests)
(ii) π_{id} (Employee) $\bowtie$ π_{id}(Guests)
(iii) π_{id}{(Employee–Guest) $\cap$ Guest–Employee)}
(iv) π_{id} { $\pi_{\text{id, mob}}$ (Employee) $\cap$ $\pi_{\text{id, mob}}$ (Guest)}
(A) (ii) and (iii)
(B) (ii), (iii) and (iv)
(C) (i), (ii) and (iv)
(D) (ii) and (iv) only

28. What does the following relational algebra expression represent?
π_{id} ($\pi_{\text{id, mob\_no}}$(Employee–Guests))
(A) Id of all employees working with the company
(B) Id of all permanent employees
(C) Id of part time employees
(D) None of these

Common data for questions 29 and 30:

29. Let R_1 and R_2 be two relations with attributes a_1 and a_2. P_1 and P_2 be two predicates.
Select the expression from the following which is wrong:
(A) $\sigma_{P_1}(\sigma_{P_1}(R_1)) \to \sigma_{P_2}(\sigma_{P_2}(R_1))$
(B) $\sigma_{P_1}(\pi_{a_1}(R_1)) \to \pi(\sigma_{a_1}(\sigma_{P_1}(R_1)))$
(C) $\sigma_{P_1}(R_1 \cup R_2) \to \sigma_{P_1}(R_1) \cup \sigma_{P_2}(R_2)$
(D) $\pi_{a_2}(\sigma_{a_1}(R_1)) \to \sigma_{P_1}(\pi_{a_2}(R_1))$

30. Select from the following corresponding TRC for the wrong expression in the above question:
(A) $\{t/ \exists u, R_1(t[P_1]) = R_2(u[P_1])\}$
(B) $\{t/ \forall u, R_1(t[P_1]) = R_1(u[P_1])\}$
(C) $\{t/ \exists u, R_1(t[P_1]) \neq R_2(u([P_1])\}$
(D) $\{t/\square(t \in R_1)\}$

Answer Keys

1. D	**2.** D	**3.** C	**4.** C	**5.** B	**6.** B	**7.** C	**8.** B	**9.** B	**10.** A
11. B	**12.** C	**13.** B	**14.** A	**15.** C	**16.** A	**17.** D	**18.** C	**19.** A	**20.** A
21. C	**22.** D	**23.** C	**24.** B	**25.** C	**26.** C	**27.** C	**28.** B	**29.** A	**30.** B

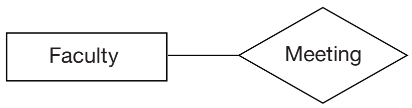
Database Test I

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

Questions 1 to 15 carry one mark each:

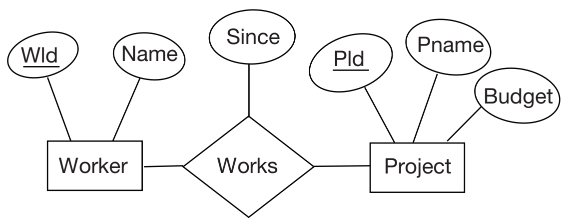
1. Consider the given ER-Diagram



The given ER-Diagram represents

(A) Unary Relationship (B) Binary Relationship
(C) Ternary Relationship (D) None of the above

2. Consider the given ER-Diagram:



Which of the following is the descriptive Attribute?

(A) WId (B) Since
(C) PId (D) {WId, PId}

3. Which of the following statements is FALSE about "Weak Entity"?

(A) A weak entity can be identified uniquely only by considering some of its attributes in conjunction with the primary key of another Entity.
(B) The owner entity set and the Weak entity set must participate in a one-to-many relationship set.
(C) One owner entity is associated with one or more weak entities, but each weak entity has a single owner.
(D) The weak entity set may/may not have total participation in the identifying relationship set.

4. Consider the given Relation schema:
Student (RNo: integer, sname: string, login: string, age: integer, grade: char(1), parent-name: string, percentage: Real).
What is the "Arity" of given Relation schema?

(A) 2, 3, 1, 1 (B) 7
(C) 3, 4 (D) 6

5. Which of the following specifies "Cardinality" of a Relation?

(A) The number of fields in a Relation
(B) The number of columns in a Relation
(C) The number of Tuples in a Relation
(D) Both (A) and (C)

6. Consider the following table:
Sailor

Sname	Rating	Age
Yashu	9	35
Lalit	10	45
Yashu	9	40
Bose	8	41

How many tuples are returned by following expression,
$\pi_{\text{Sname, Rating}}$ (Sailor)

(A) 4 (B) 3
(C) 2 (D) 1

7. Which of the following statement is FALSE, for 2 Relations R and S?

(A) RXS returns a Relation with all the fields of R in the same order as they appear in R followed by all the fields of S in the same order as they appear in S.
(B) The fields in RXS have the same domains as the corresponding fields in R and S.
(C) $R \cap S = R - (R - S)$
(D) $R \cap S = S - (S - R)$

8. Which of the following correctly describes "Prime Attribute"?

(A) It should be a part of primary key
(B) It should be a part of any candidate key
(C) It should be a part of every candidate key
(D) None of the above.

9. Let X, Y and Z denote sets of attributes over a relation schema R. Match the following.

I. If $X \to Y$, then $XZ \to YZ$ for any Z
II. If $X \supseteq Y$, then $X \to Y$
III. If $X \to YZ$, then $X \to Y$ and $X \to Z$
P. Reflexivity
Q. Augmentation
R. Decomposition

(A) I–Q, II–P, III–R (B) I–P, II–R, III–Q
(C) I–P, II–Q, III–R (D) I–R, II–Q, III–P

10. Which of the following are additional features of SQL?

(A) SQL has language constructs for specifying views, also known as virtual tables, using the CREATE VIEW Statement
(B) SQL and Relational databases can interact with new technologies such as XML and OLAP.
(C) SQL has Language constructs for creating triggers
(D) All the above

11. Consider the given Functional Dependencies for Employee – Project Relation:
ENo $\to$ EName
PNo $\to$ PName, Location

{ENo, PNo} $\rightarrow$ Hours
How many attributes appear in the closure of ENo, (ENo^+)?
(A) 1 (B) 2
(C) 3 (D) 5

12. Magnetic tapes are sequential access devices, to access the n^{th} block on tape _____?
(A) We must scan over the preceding $(n - 1)$ blocks
(B) We can scan directly n^{th} block
(C) We must scan atleast $\frac{n}{2}$ blocks before scanning n^{th} block
(D) We must scan atmost $\frac{n}{2}$ blocks before scanning n^{th} block

13. What is the unused space in each block, for fixed length record of size 'R' bytes with $B \geq R$ (B = Block Size), we can fit $\lfloor B/R \rfloor$ ($bfr = \lfloor B/R \rfloor$) records in one block?
(A) $B - (bfr * R)$ (B) $(B - bfr * R) + 1$
(C) $B + (bfr * R)$ (D) $\frac{B}{bfr} + R$

14. To utilize the unused space in each block, we can store part of a record on one block and the rest on another, A pointer at the end of the first block points to the block containing the remaining part of record, what this organization is called?
(A) Unspanned (B) Spanned
(C) Distributed (D) Collabarative

15. For variable length records using spanned organization, each block may store different number of records. The blocking factor "bfr" represents the average number of records per block for the file, what is the number of blocks 'b' needed for a file of 'r' records?
(A) $b = \left\lceil \frac{bfr}{r} \right\rceil$ (B) $b = \left\lceil \frac{r}{bfr} \right\rceil$
(C) $b = \lceil r \times bfr \rceil$ (D) $b = \left\lceil \frac{2 \times r}{bfr} \right\rceil$

16. Which of the following SQL Query is valid to increment the age of the student whose student Id (Sid) is 63078?
(A) Update student S
SET S.age = S.age + 1;
WHERE S.Sid = 63078
(B) Select S.age + 1
From student S
Where S.Sid = 63078
(C) Update S.age = S.age + 1
WHERE S.Sid = 63078
(D) Both (A) and (B)

17. Consider the given Relation worker
Worker

WId	Name	Service	Age

To Rename table as Employee or WId as EId, service as Experience, which of the following is not valid?
(A) ρ_{Employee} worker
(B) $\rho_{\text{(EID, Experience)}}$ worker
(C) $\rho_{\text{(EID, Name, Experience, Age)}}$ worker
(D) $\rho_{\text{Employee (EID, Name, Experience, Age)}}$ worker

18. Consider the given schema for sailors
sailors(Sid : integer, Sname : string, Rating : integer, age : real).
Which of the following SQL Queries is invalid?
(A) SELECT S.Rating, (S.age)
FROM Sailors S
WHERE S.Rating = 10
(B) SELECT S.Rating, MAX (S.age)
FROM Sailors S
GROUPBY S.Rating
(C) SELECT S.Sname, MAX (S.age)
FROM Sailors S
WHERE S.Rating > 7
(D) SELECT AVG (S.Age)
FROM Sailors S
WHERE S.Rating > 8

19. Consider the schema given in the above Question,
SELECT Sname
FROM Sailors
WHERE Rating IN(6, 8, 9, 10)
What is retrieved by the above query?
(A) The names of sailors whose Rating is between 6 and 10.
(B) The names of sailors whose Rating is any one of 6, 8, 9, 10.
(C) Both (A) and (B)
(D) None of the above

20. Let 'R' be a Relation schema and Let x and y be non empty sets of attributes in R. An instance 'r' of R satisfies the FD $-x \rightarrow y$. For which of the following tuples t_1 and t_2 are in r?
(A) $t_1 \cdot x = t_2 \cdot x$ and $t_1 \cdot y = t_2 \cdot y$
(B) $t_1 \cdot x = t_2 \cdot y$ and $t_1 \cdot y = t_2 \cdot y$
(C) $t_1 \cdot x = t_2 \cdot x$ and $t_1 \cdot y = t_2 \cdot x$
(D) $t_1 \cdot x = t_2 \cdot y$ and $t_1 \cdot x = t_2 \cdot y$

21. Consider the following Relational instance?

A	B	C	D
a_1	b_1	c_1	d_1
a_1	b_1	c_1	d_2
a_3	b_2	c_3	d_1
a_2	b_3	c_2	d_4

Which of the following are satisfied by the given instance?
(A) $AB \rightarrow C$ (B) $AB \rightarrow CD$
(C) $AC \rightarrow D$ (D) $D \rightarrow AC$

22. Consider the given Relation R

R

W	X	Z
W_1	X_1	Z_1
W_2	X_2	Z_2
W_3	X_2	Z_1

$\pi_{wx}(R)\ x\ \pi_{xz}(R)$, what is the number of tuples returned by the given expression?

(A) 2 (B) 3
(C) 4 (D) 5

23. For the Relation 'R' given in the above question, if we perform $\pi_{wx}(R) ⟕ \pi_{xz}(R)$ (⟕ : left outer join), what is the number of tuples that appear in the Result?

(A) 0 (B) 3
(C) 5 (D) 6

24. Consider the given SQL Query:
Select DISTINCT ENo
From works
Where (PNo, Hours) IN (Select PNo, Hours
From works where ENo = '788');
What is returned by the above query?

(A) The employee numbers of all employees who work on the same (PNo, Hours) combination on some project on which Employee ENo 788 is working.
(B) The employee numbers of all employees who work on the same (PNo, Hours) combination whose ENo is 788.
(C) Both (A) and (B)
(D) None of the above

25. Retrieve the Employee numbers (ENo) of all employees who work on project numbers 11, 22, 23, 24?

(A) SELECT ENo
FROM Works
Where PNo IN(11, 22, 23, 24)
(B) Select DISTINCT ENo
From Works
Where PNo IN(11, 22, 23, 24)
(C) Select ENo
FROM Works
Where PNo = 11 AND PNo = 22 AND PNo = 23 AND PNo = 24.
(D) Select *
FROM works
Where PNo = 11 OR PNo = 22 OR PNo = 23 OR PNo = 24.

26. Which of the following correctly specifies the "JOIN" Operation on 2 tables Employee and Department, the common field in both tables is DNo?

(A) Select *
From (Employee JOIN Department ON Employee.DNo = Department.DNo)
(B) Select *
From (Employee JOIN Department)
(C) Select *
From Employee, Department
(D) Select *
From (Employee JOIN Department IN Employee.DNo = Department.DNo)

27. Consider the given Functional Dependencies for Employee – project Relation
ENo → EName
PNo → PName, Location
{ENo, PNo} → Hours
How many attributes are included in the closure of $\{ENo, PNo\}^+$?

(A) 2 (B) 4
(C) 5 (D) 6

Common data for Questions 28 and 29:
Consider the 2 tables T_1 and T_2

Table T_1

P	Q	R
20	X	50
25	Y	80
35	X	60

Table T_2

A	B	C
20	Y	60
35	Z	30
20	Y	50

28. What is the number of tuples returned from
$T_1 \bowtie (T_1 \cdot P = T_2 \cdot A \text{ AND } T_1 \cdot R = T_2 \cdot C) T_2$?

(A) 0 (B) 1
(C) 2 (D) 3

29. What is the number of tuples returned from
$T_1 \bowtie (T_1 \cdot P = T_2 \cdot A \text{ OR } T_1 \cdot R = T_2 \cdot C)\ T_2$?

(A) 1 (B) 2
(C) 3 (D) 4

Common data for Questions 30 and 31:
Consider the given Relation worker:

Name	Age	Project-Number
Anu	26	10
Bala	24	20
Sudhir	28	10
Shreya	24	10
Bharat	19	20
Srinath	21	30
Raj	22	30
Mishra	21	20
Phani	19	40

30. What is the Result of the following SQL Query
SELECT Project-Number, MIN (Age)

FROM Worker
GROUPBY Project-Number
HAVING COUNT (*) ≥ 3

(A)

Project-Number	MIN (Age)
10	24
20	19

(B)

Project-Number	MIN (Age)
10	24
20	19
30	21
40	19

(C)

Project-Number	MIN (Age)
10	24
20	19
30	21

(D)

Project-Number	MIN (Age)
20	19
40	19

31. In the Result of following SQL query
SELECT Age, count (*)
FROM Worker
GROUPBY Age
What is the number of tuples?
(A) 4 (B) 5
(C) 6 (D) 9

Common data for Questions 32 and 33:
Consider the given 2 tables:

Sailor-1:

Sname	Rating	Age
Yashu	9	35
Lalit	10	45
Bose	8	41
Ana	7	40

Sailor-2

Sname	Rating	Age
Raj	10	40
Kamal	10	41
Ana	7	40

32. Which of the following Tuple is not part of sailor-1 and sailor-2?

(A)

Yashu	9	35	Kamal	10	41

(B)

Lalit	10	45	Raj	10	40

(C)

Ana	7	40	Kamal	7	40

(D)

Bose	8	41	Lalit	10	45

33. What is the number of tuples appear in sailor-1 SET DIFFERENCE sailor-2?
(A) 0 (B) 1
(C) 2 (D) 3

Common Data for Questions 34 and 35:
Consider the given 2 tables Employee and Department and primary keys are shown with underline.

Employee

EId	Name	DNo	Age
0326	Kumar	5	38
0429	Nilesh	4	36
0589	Phani	4	34
0679	Raja	3	36
0588	Deepak	2	37

Department

DNo	DName
1	Accounts
2	Sales
3	Marketing
4	Executives
5	Research

34. Insert into Employee values < 0589, 'Anurag', 3, 37 >, The above operation violates which constraint?
(A) NOT NULL Constraint
(B) KEY Constraint
(C) Entity Integrity Constraint
(D) Referential Integrity Constraint

35. Delete Department tuple with DNo = 5, The above operation violates which constraint?
(A) NOT NULL Constraint
(B) KEY Constraint
(C) Referential Integrity Constraint
(D) Entity Integrity Constraint

Answer Keys

1. A	**2.** B	**3.** D	**4.** B	**5.** C	**6.** B	**7.** D	**8.** B	**9.** A	**10.** D
11. B	**12.** A	**13.** A	**14.** B	**15.** B	**16.** D	**17.** B	**18.** C	**19.** B	**20.** A
21. A	**22.** D	**23.** C	**24.** A	**25.** B	**26.** A	**27.** D	**28.** B	**29.** D	**30.** A
31. C	**32.** D	**33.** D	**34.** B	**35.** C					

Hints and Explanations

1. If only one entity is participating in a relationship that relation is called Unary Relation. Choice (A)
2. Descriptive attributes are used to record information about the relationship, rather than about any one of the participating entities. Suppose we wish to record that a particular worker is working for a project since 1993; This information is captured by adding an attribute "since" to works. Choice (B)
3. The weak entity set must have total participation in the identifying relationship set. Choice (D)
4. The degree is also called "Arity" of a relation, is the number of fields (or) columns, There are '7' fields. Choice (B)
5. The "cardinality" of a relation instance is the number of tuples in it. The degree of the relation is the number of fields (or) columns. Choice (C)
6. π-project, eliminates duplicates

Sname	Rating
Yashu	9
Lalit	10
Bose	8

$\therefore$ Hence '3' tuples. Choice (B)

7. $R \cup S = S - (S - R)$ is not TRUE. Choice (D)
8. Prime Attributes should be part of any candidate key. Choice (B)
9. Choice (A)
10. All the statements are additional features of SQL. Choice (D)
11. $ENo^{+} = \{ENo, EName\}$. Choice (B)
12. We must scan over the preceding $(n - 1)$ blocks. Choice (A)
13. $B - (bfr * R)$ is the unused space in each block. Choice (A)
14. It is called spanned organization. Choice (B)
15. $b = \left\lceil \left(\frac{r}{bfr} \right) \right\rceil$ Choice (B)
16. We can modify the column values in an existing row using the UPDATE command. We can increment the age of the student with Sid 63078

UPDATE Student S
SET S.age = S.age + 1
WHERE S.Sid = 63078

(or)

Select S.age + 1
From Student S
Where S.Sid = 63078. Choice (D)

17. $\rho_{(EID,\ Experience)}$ worker
It will replace WID with EID and Name with Experience which is NOT Valid. Choice (B)
18. If select clause uses an aggregate operation, then it must only use aggregate unless the query contains a GROUP BY clause. Choice (C)
19. It retrieves the names of sailors whose Rating is any one of the given Ratings 6, 8, 9, 10. Choice (B)
20. $t_1 \cdot x = t_2 \cdot x$ and $t_1 \cdot y = t_2 \cdot y$. Choice (A)
21. The given instance satisfies $AB \to C$. Choice (A)
22. $\pi_{wx}(R)$

W	X
W_1	X_1
W_2	X_2
W_3	X_2

$\pi_{XZ}(R)$

X	Z
X_1	Z_1
X_2	Z_2
X_2	Z_1

$\pi_{wx}(R) (\bowtie) \pi_{xz}(R)$

W	X	Z
W_1	X_1	Z_1
W_2	X_2	Z_2
W_2	X_2	Z_1
W_3	X_2	Z_2
W_3	X_2	Z_1

Choice (D)

23. All tuples in $\pi_{wx}(R)$ has atleast one matching in $\pi_{xz}(R)$, so same number of tuples as $\pi_{wx} \bowtie \pi_{xz}(R)$, returned in the result. Choice (C)
24. The employee numbers of all employees who work on the same (PNO, Hours) combination on some project on which employee ENO 788 is working. Choice (A)
25. Some employees may work on more than one project, so DISTINCT Key word will eliminate duplicates in the output. Choice (B)
26. The concept of Joining Relations was incorporated into SQL to permit users to specify a table resulting from a Join operation in the FROM clause of a query. Choice (A)
27. $\{ENo, PNo\}^{+} = \{ENo, PNo, EName, PName, Location, Hours\}$. Choice (D)

28. $T_1 \bowtie T_2$

P	Q	R	A	B	C
20	X	50	20	Y	60
20	X	50	35	Z	30
20	X	50	20	Y	50
25	Y	80	20	Y	60
25	Y	80	35	Z	30
25	Y	80	20	Y	50
35	X	60	20	Y	60
35	X	60	35	Z	30
35	X	60	20	Y	50

Only 1(3^{rd} tuple). Choice (B)

29. Refer the table given in the above solution.

$T_1 \cdot P = T_2 \cdot A$

1^{st}, 3^{rd}, 8^{th} tuples

$T_1 \cdot R = T_2 \cdot C$

7th tuple

∴ Total '4' tuples. Choice (D)

30. GROUP-BY Project-Number gives

Table-1

Name	Age	Project-Number
Anu	26	10
Sudhir	28	10
Shreya	24	10

Table-2

Name	Age	Project-Number
Bala	24	20
Bharat	19	20
Mishra	21	20

Table-3

Name	Age	Project-Number
Srinath	21	30
Raj	22	30

Table-4

Name	Age	Project-Number
Phani	19	40

Table-1 and Table-2 are satisfying the condition count (*) ≥ 3. Choice (A)

31. GROUP-BY Age returns

Anu	26

Sudhir	28

Bala	24
Shreya	24

Bharat	19
Phani	19

Srinath	21
Mishra	21

Raj	21

Result:

Age	Count (*)
19	2
21	2
22	1
24	2
26	1
28	1

Choice (C)

32. The tuple $\boxed{\text{Bose} \mid 8 \mid 41}$ cannot have cross product with any tuple in sailor-1. Choice (D)

33. It returns the tuples that are present in sailor-1 but not in sailor-2, $\boxed{\text{Ana} \mid 7 \mid 40}$ is present in sailor-1 and sailor-2 also, so this will not appear in the Result of sailor-1 SET DIFFERENCE sailor-2. Choice (D)

34. It violates KEY constraint, because EID is primary key and EID 0589 is already present in the employee table. Choice (B)

35. Employee table references to Department table, Absence of DNo = 5 causes Referential Integrity Constraint violation. Choice (C)

Database Test 2

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. If the database operations in a transaction do not update the database but only retrieve data, what is that transaction called?
(A) Write-only operation (B) Read-only operation
(C) Named data item (D) Granularity

2. Consider the 2 interleaved transactions T_1 and T_2:

T_1	T_2
$r_1(x)$	
$w_1(x)$	
	$r_2(x)$
$r_1(y)$	
$w_1(y)$	
	$r_2(y)$
C_1	
	C_2

Which of the following is schedule for the above?
(A) $r_1(x), r_2(x), w_1(x), r_1(y), w_1(y), r_2(y), C_1, C_2$
(B) $r_1(x), w_1(x), r_2(x), r_1(y), w_1(y), r_2(y), C_1, C_2$
(C) $r_1(x), w_1(x), r_2(x), r_1(y), w_1(y), r_2(y), C_2, C_1$
(D) $r_1(x), w_1(x), r_2(x), r_1(y), r_2(y), w_1(y), C_1, C_2$

3. Two operations in a schedule are said to have conflict if
I. They belong to different transactions.
II. They access the same data item.
III. Atleast one of the operations is write operation.
IV. Atleast one of the operations is Read operation.
Which of the following is TRUE?
(A) I and II only (B) II and III only
(C) I, II and III only (D) II, III and IV only

4. A Non recoverable schedule must contain which of the following conflict?
(A) RR conflict (B) RW conflict
(C) WR conflict (D) WW conflict

5. Consider the given schedule:

T_1	T_2
$r_1(x)$	
$w_1(x)$	
$r_1(y)$	
C_1	
	$r_2(x)$
	$w_2(x)$
	C_2

The given schedule is
(A) Cascadeless schedule
(B) strict schedule
(C) Non-serial schedule
(D) Non-Recoverable schedule

6. In a schedule, transactions can neither read nor write an item 'X' until the last transaction that wrote X has committed or Aborted, what is that schedule?
(A) Cascadeless schedule (B) Strict schedule
(C) Recoverable schedule (D) Non serial schedule

7. Consider the given schedule

T_1	T_2
$R(x)$	
$x = x - N$	
$w(x)$	
	$r(x)$
	$w(x)$
$r(y)$	

What is the number of WR conflicts present in the given schedule?
(A) 0 (B) 1
(C) 2 (D) 3

8. Which of the following protocol is based on locking data items to prevent concurrent transactions from interfering with one another, and enforcing an additional condition that guarantees serializability?
(A) 1 PL (B) 2 PL
(C) 3 PL (D) 4 PL

9. In Random access, for accessing a disk block on a disk, what is the Rotational delay if disk rotates at 10000 rpm (rotations per minute)?
(A) 6 m/sec (B) 9 m/sec
(C) 3 m/sec (D) 12 m/sec

10. Whenever 2 independent 1:N relationships A:B and A:C are mixed in the same relation, then which of the following is TRUE?
(A) A Transitive Dependency may arise
(B) A Partial Dependency may arise
(C) A Multi-valued Dependency may arise
(D) None of the above

11. Which of the following is TRUE about 2NF?
(A) A relation schema R is in 2 NF, if every non prime attribute A in R is not partially dependent on any key of R.
(B) If the primary key contains a single attribute, the test need not be applied at all.
(C) Test for 2NF involves testing for functional dependencies whose left-hand side attributes are part of the primary key.
(D) All the above

12. Consider the following *FD*'s:

$X \rightarrow Y$

$WY \rightarrow Z$

If pseudo transitive rule is applied on given *FD*'s, what is the result?

(A) $X \rightarrow Z$ (B) $XW \rightarrow Z$

(C) $XY \rightarrow Z$ (D) $XWY \rightarrow YZ$

13. What is the average time to access a specific Record in a file using sequential scan (Linear search) on unordered file? (Assume that there are '*b*' blocks)

(A) $b/2$ (B) b

(C) $\log_2 b$ (D) $2b$

14. In which hashing technique, a type of directory, an array of '2^d' bucket addresses is maintained, where '*d*' is called the global depth of the directory?

(A) Internal Hashing (B) Extendible Hashing

(C) Separate Chaining (D) Static Hashing

15. Assume that, a primary index is built on a file with 45 blocks, what is the number of block accesses required to search for a specific Record in a file (Using Binary search)?

(A) 45 (B) 23

(C) 6 (D) 7

16. Consider the given 2 schedules:

S_a: $r_1(x), w_1(x), r_2(x), w_1(x), w_2(x), r_1(y), C_1, C_2$

S_b: $r_1(x), w_1(x), r_2(x), w_2(x), r_1(x), C_1, C_2$

Which of the following is TRUE?

(A) Both are conflict serializable

(B) Only S_a is conflict serializable

(C) Only S_b is conflict serializable

(D) Both are not conflict serializable

17. Consider the given transaction T_1, that follows 2-phase Locking protocol:

T_1
1. Read – Lock(X)
2. Read – item (X)
3. Write – Lock(Y)
4. Unlock(X)
5. Read – item(Y)
6. Y = X + Y
7. Write – item(Y)
8. Unlock(Y)

Which of the following is TRUE?

(A) Record 1 to Record 3 is Growing phase

(B) Record 4 to Record 8 is shrinking phase

(C) Record 1 to Record 4 is Expanding phase

(D) Both (A) and (B)

18. Consider a disk with track size 70 Kbytes and it rotates at 3600 rpm, what is the transfer rate in bytes/m sec?

(A) 3500 (B) 3600

(C) 4200 (D) 4500

19. What is the time required to transfer consecutively '*k*' non contiguous blocks that are on the same cylinder, where '*S*' is seek time, '*rd*' is rotational delay and '*btt*' is block transfer time?

(A) $S + (k * (rd + btt))$ msec

(B) $(S + rd + btt)$ msec

(C) $S + rd + (k * btt)$ msec

(D) $S + (k * rd) + btt$ msec

20. What is the estimate for transferring consecutive blocks (include interblock gap), the disk manufacturer provides a bulk transfer rate (btr) that takes the gap size into account when reading consecutively stored blocks (B), and the gap size is *G* bytes?

(A) btr = $(B/(B + G))$ * tr bytes/msec

(B) btr = $(B/(B + G))$ bytes/msec

(C) btr = $(B + G)$ + tr bytes/msec

(D) btr = $((B + G)/B)$ + tr bytes/msec

21. Match the following:

I. 1 NF

II. 2 NF

III. 3 NF

P. No Non key attribute should be functionally dependent on a part of the primary key.

Q. Relation should have no nonatomic attributes or nested relations.

R. Relation should not have a non key attribute functionally determined by another non key attribute.

(A) I–P, II–Q, III–R

(B) I–Q, II–P, III–R

(C) I–Q, II–R, III–P

(D) I–R, II–Q, III–R

22. Consider the given Relation EMP and given functional dependencies:

$F = (E \rightarrow N$

$P \rightarrow A, L$

$E\,P \rightarrow H)$

Which of the following is Incorrect closure set with respect to *F*?

(A) $E^+ = \{E, N\}$

(B) $P^+ = \{P, A, L\}$

(C) $\{EP\}^+ = \{E, N, P, A, L, H\}$

(D) $\{EH\}^+ = \{E, N, H, P, A\}$

23. Consider the functional dependencies given in the above question, EMP Relation is in which Normal Form?

(A) 1 NF (B) 2 NF

(C) 3 NF (D) BCNF

24. Consider the given 2 tables:

Person

Name	Mobile
Anil	9848463 9440072
Raj	9004432

Student

SName	Project	
	P No.	Hours

Which of the following is TRUE?
(A) Both tables are in 1 NF
(B) Both tables are not in 1 NF
(C) Person is in 1 NF but not student
(D) Person is in 2 NF and student is in 1 NF

25. Consider the schema $R(ABCDE)$ and functional dependencies $A \rightarrow BE$, $C \rightarrow D$, then the decomposition of R into $R_1(ABE)$, $R_2(CD)$ is
(A) Dependency preserving and lossless join.
(B) Dependency preserving and not lossless join.
(C) Lossless join but not dependency preserving.
(D) Not Dependency preserving and Not lossless join.

26. Consider a relation $R(A, B, C, D, E)$ with the following Dependencies:
$A \rightarrow C$
$BCD \rightarrow E$
$DE \rightarrow B$
Identify the candidate keys from the following options?
(A) AD, ADB (B) ADB, ADC
(C) ADB, ADE (D) ADC, ADE

27. Consider the Functional Dependencies given in the above question, which of the following are "Prime Attributes"?
(A) $ABCD$ (B) $ABCE$
(C) $ABDE$ (D) $ACDE$

28. Consider a file with fixed length records, one sample record is given below,

Name	E No	Salary	Department

1 14 18 22 32

The file contains 32000 records, and it is stored on a disk with block size 1024 Bytes, what is the blocking factor?
(A) 16 (B) 32
(C) 64 (D) 128

29. Consider a file with Fixed length Records, one sample record is given below.

Name	E No	Salary	Department	Remarks	Total

1 14 18 22 32 64

The file contains 30000 records, what is the number of blocks required to store the file, on a disk with block size 2048 bytes?
(A) 648 (B) 840
(C) 938 (D) 968

30. Construct a B-tree with 6, 4, 5, 9, 7, 2 values, order of B-tree is $P = 3$, what is the maximum number of node splits?
(A) 1 (B) 2
(C) 3 (D) 4

31. Consider a disk with block size $B = 512$ bytes, number of tracks per surface is 400, this disk pack consists of 16 double sided disks, How many cylinders are there?
(A) 100 (B) 200
(C) 400 (D) 800

Common Data Questions 32 and 33:
Consider the given schedules
S_1: $r_1(P)$, $r_2(R)$, $r_1(R)$, $r_3(P)$, $r_3(Q)$, $w_1(P)$, C_1, $w_3(Q)$, C_3, $r_2(Q)$, $w_2(R)$, $w_2(Q)$, C_2
S_2: $r_1(P)$, $r_2(R)$, $r_1(R)$, $r_3(P)$, $r_3(Q)$, w_1(P), $w_3(Q)$, $r_2(Q)$, $w_2(R)$, $w_2(Q)$, C_1, C_2, C_3.

32. Which of the following is/are Recoverable schedule(s)?
(A) Only S_1 (B) Only S_2
(C) Both S_1 and S_2 (D) Neither S_1 nor S_2

33. Which of the following is/are "cascadeless" schedules(s)?
(A) Only S_1
(B) Only S_2
(C) Both S_1 and S_2
(D) Neither S_1 nor S_2

Linked Answer Questions 34 and 35:
Consider the given table:
Student

Sname	Course	Department
Arun	C_1	D_1
Arun	C_2	D_1
Arun	C_1	D_2
Arun	C_2	D_2

34. Student table has multivalved dependencies, which of the following is correct 4 NF Decomposition?
(A) Student 1 (Sname, course)
Student 2 (Course, Department)
(B) Student 1 (Sname, Department)
Student 2 (course, Department)
(C) Student 1 (sname, course)
Student 2 (sname, Department)
(D) Student 1 (sname, course)
Student 2 (sname, Department)
Student 3 (Course, Department)

35. Which of the following is TRUE, for the above identified correct answer?
(A) student 1 ⋈ student 2, returns student table
(B) student 1 X student 2 returns student table
(C) student 1 ⋈ student 2 returns student table
(D) None of the above

Answer Keys

1. B	**2.** B	**3.** C	**4.** B	**5.** A	**6.** B	**7.** B	**8.** B	**9.** C	**10.** C
11. D	**12.** B	**13.** A	**14.** B	**15.** D	**16.** D	**17.** D	**18.** C	**19.** A	**20.** A
21. B	**22.** D	**23.** A	**24.** B	**25.** B	**26.** C	**27.** C	**28.** B	**29.** C	**30.** B
31. C	**32.** A	**33.** A	**34.** C	**35.** A					

Hints and Explanations

1. Read only operation do not update the database but only retrieves data. Choice (B)

2. Correct schedule is:
$r_1(x)$, $W_1(x)$, $r_2(x)$, $r_1(y)$, w_1 (y), $r_2(y)$, C_1, C_2
Choice (B)

3. Two operations in a schedule are said to have conflict if they satisfy I, II and III.
Choice (C)

4. Recover ability can be decided by the presence of 'RW' conflict in a schedule.
Choice (B)

5. A schedule is said to be cascadeless, if every transaction in the schedule reads only items that were written by committed transactions. Choice (A)

6. More restrictive type of schedule is called a strict schedule, in which transactions can neither read nor write an item 'X' until the last transaction that wrote 'X' has committed or aborted.
Choice (B)

7.

RW

$r_1(x)$, $w_1(x)$, $r_2(x)$, $w_2(x)$, $r_1(y)$

WR

WW

$\therefore$ One WR conflict is present. Choice (B)

8. 2 PL (2 phase Locking protocol) ensures serializability. Choice (B)

9. If the speed of disk rotation is 'P' revolutions per minute (rpm), then the average rotational delay 'rd' is given by
$rd = (1/2) * (1/P)$ min $= (60 * 1000)/(2 * P)$ msec
$rd = 3$ msec Choice (C)

10. If 2 independent 1:N relationships A:B and A:C are mixed in the same Relation, a multi valued Dependency may arise. Choice (C)

11. All the statements are TRUE.
Choice (D)

12. $X \rightarrow Y$
$WY \rightarrow Z$
$XW \rightarrow Z$ Choice (B)

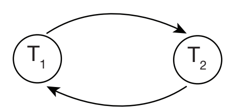
13. The average access time in block accesses to find a specific record in a file with 'b' blocks on unordered file is 'b/2'. Choice (A)

14. Hashing Technique that allows Dynamic File expansion is extendible hashing, Choice (B)

15. First search will be conducted on primary index. By using binary search,
$\Rightarrow \left\lceil \log^{45} \right\rceil \cong 6$ block accesses
6 block accesses in primary index + 1 block access in original file.
$\therefore$ 7 block accesses. Choice (D)

16. S_a: $r_1(x)$, $w_1(x)$, $r_2(x)$, $w_1(x)$, $w_2(x)$, $r_1(y)$, C_1, C_2

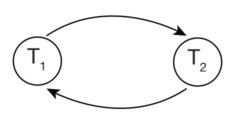
Precedence Graph:



$\therefore$ 'S_a' is not conflict serializable, since cycle is present in Precedence graph.

S_b: $r_1(x)$, $w_1(x)$, $r_2(x)$, $w_2(x)$, $r_1(x)$, C_1, C_2

Precedence Graph:



$\therefore$ S_b is not conflict serializable, since cycle is present in precedence graph. Choice (D)

17. **Expanding or growing phase:** During which new locks on items can be acquired but none can be released.
Shrinking phase: During which existing locks can be released but no new locks can be acquired.
Choice (D)

18. Transfer rate = $(70 \times 1000)/(60 \times 1000/3600)$

$$\frac{70 \times 1000}{\frac{1000}{60}} = \frac{70 \times 60 \times 1000}{1000} = 4200 \text{ bytes/msec}$$

Choice (C)

19. To transfer consecutively 'k' non contiguous blocks that are on the same cylinder, we need approximately, $S + (k * (rd + btt))$ msec. Choice (A)

20. Transferring consecutive blocks (including inter block gap)
$\Rightarrow$ btr = $(B/(B + G))$ * tr bytes/msec Choice (A)

21. I – Q, II – P, III – R is correct match. Choice (B)

22. $\{EH\}^+$ cannot determine P and A. Choice (D)

23. The key for EMP is $\{EP\}$ and the functional Dependencies. $E \rightarrow N$
$P \rightarrow A, L$ are partial dependencies.
$\therefore$ EMP is not in 2NF i.e., it is in 1 NF. Choice (A)

24. Both tables are not in 1 NF, person table has multiple values under mobile column for one name, In student relation, there is another relation project, A relation within a Relation is not allowed in 1 NF. Choice (B)

25. It is Dependency preserving, $A \rightarrow BE$ is in R_1 and $C \rightarrow D$ is in R_2.
Checking for loss less join:

	A	B	C	D	E
R_1(ABE)	*	*			*
R_2(CD)			*	*	

It is Not Loss Less JOIN. Choice (B)

26. $A \rightarrow C$
$BCD \rightarrow E$
$DE \rightarrow B$

L	M	R
AD	BCE	

$\{AD\}^+ = \{ADC\}$
$ADB^+ = \{ADBCE\}$
$ADC^+ = \{ADC\}$
$ADE^+ = \{ADECB\}$ Choice (C)

27. Candidate keys are: ADB, ADE
Prime Attributes are part of any candidate key, $ABDE$
Choice (C)

28. Blocking factor is defined as, the number of records stored on one block.

$$Bfr = \left\lfloor \frac{1024}{32} \right\rfloor = \left\lfloor \frac{2^{10}}{2^5} \right\rfloor = 2^5 = 32 \text{ Records}$$

Choice (B)

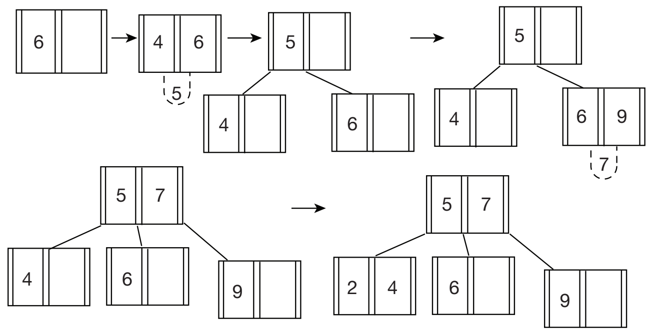
29. Blocking Factor

$$= \left\lfloor \frac{2048}{64} \right\rfloor = \left\lfloor \frac{2^{11}}{2^6} \right\rfloor = 2^5 = 32 \text{ records/block}$$

Number of blocks required:
1 block --------- 32 records
x ----------------- 30000 records

$$x = \left\lceil \frac{30000}{32} \right\rceil = \lceil 937.5 \rceil = 938 \text{ blocks}$$ Choice (C)

30. Order $P = 3$



$\therefore$ 2 Node splits Choice (B)

31. Cylinders = Number of tracks that is 400 Choice (C)

32. S_1: $r_1(P), r_2(R), r_1(R), r_3(P), r_3(Q), W_1(P), C_1,$
$w_3(Q), C_3, r_2(Q), w_2(R), w_2(Q), C_2$

To check recoverability, search for RW conflict. There is one RW conflict, T_3 is performing write operation so it has to commit first.
$\therefore$ S_1 is Recoverable.
S_2: $r_1(P), r_2(R), r_1(R), r_3(P), r_3(Q), w_1(P),$
$w_3(Q), r_2(Q), w_2(R), w_2(Q), C_1, C_2, C_3.$

T_3 has to commit first but T_2 has committed first.
$\therefore$ S_2 is not a recoverable schedule. Choice (A)

33. S_1:

T_1	T_2	T_3
$r_1(P)$		
	$r_2(R)$	
$r_1(R)$		
		$r_3(P)$
		$r_3(Q)$
$w_1(P)$		
C_1		
		$w_3(Q)$
		C_3
	$r_2(Q)$	
	$w_2(R)$	
	$w_2(Q)$	
	C_2	

S_1 is cascadeless, because, data is taken from committed transactions.

S_2:

T_1	T_2	T_3
$r_1(P)$		
	$r_2(R)$	
$r_1(R)$		
		$r_3(P)$
		$r_3(Q)$
$w_1(P)$		
		$w_3(Q)$
	$r_2(Q)$	
	$w_2(R)$	
	$w_2(Q)$	
C_1		
	C_2	
		C_3

T_3 should commit before T_2

$\therefore$ S_2 is not cascadeless schedule.

Choice (A)

34. Multi valued dependencies:
Sname $\rightarrow\rightarrow$ Course, Sname $\rightarrow\rightarrow$ Department
4NF decomposition is as follows
Student 1(Sname, course)
Student 2(Sname, Department)

Choice (C)

35. Student 1 $\bowtie$ student 2 returns student table.

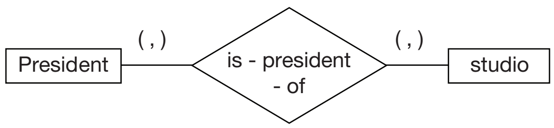
Choice (A)

Database Test 3

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

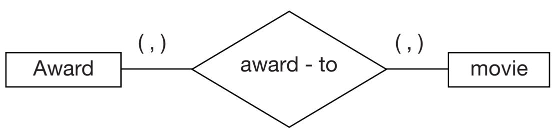
1. A studio can have at most one president and a president can preside at most one studio.



Which of the following is correct cardinality ratio, for the above description?

(A) (0, 1), (1, *n*) (B) (0, 1), (0, 1)
(C) (1, *n*), (0, 1) (D) (1, *n*), (1, *n*)

2. A movie award goes exactly to one movie. A movie may be awarded multiple awards (but may be none at all). Which of the following is correct cardinality ratio, for the above description?



(A) (0, 1), (1, *n*) (B) (1, 1), (1, *n*)
(C) (1, 1), (0, ∗) (D) (1, 1), (1, ∗)

3. Match the following:

I. Conceptual Database design
II. Logical Database design
III. Physical Database design
P. Transforms the conceptual schema into the data model supported by the DBMS.
Q. Design indexes, table distribution, buffer size etc.
R. Produces the initial model.

(A) I–R, II–P, III–Q (B) I–P, II–Q, III–R
(C) I–Q, II–P, III–R (D) I–Q, II–R, III–P

Common Data for Questions 4 and 5:

CREATE TABLE Emp (EId INT
Name CHAR(50)
mgr-Id : INT
SSNo: INT UNIQUE
DNo: INT
Salary: INT
Primary key (EId)
Foreign key DNo References Dept (DNo))

4. If a tuple is inserted into the table Emp, which violation would never occur?

(A) Key constraint
(B) Entity Integrity constraint
(C) Referential Integrity constraint
(D) Unique constraint

5. If a tuple value is updated in Emp table, which violations would never occur?

(A) Key constraint
(B) Referential Integrity constraint
(C) UNIQUE Constraint
(D) None of the above

6. Student (RNo, Name, Marks, Subject)
For each Subject, Retrieve the Subject, maximum marks in that subject.
In the SQL query which clause will not appear?

(A) SELECT
(B) GROUPBY
(C) FROM
(D) WHERE

7. Which of the following is NOT a Relational algebra operator used in basic queries?

(A) JOIN (B) PROJECT
(C) SELECT (D) LOCATE

8. What does the asterisk (∗) in SQL mean?

(A) It is a standard symbol used to start a query.
(B) It is a wild card symbol that requests inclusion of all attributes.
(C) It is an interruption symbol used to abort a query if it takes longer to process than a prescribed time length.
(D) None of the above

9. Consider the given functional dependencies

$R(A, B, C, D)$:
$FD = \{A \rightarrow B$
$C \rightarrow \text{B}$
$D \rightarrow B\}$

The given Relation is in which Normal Form?

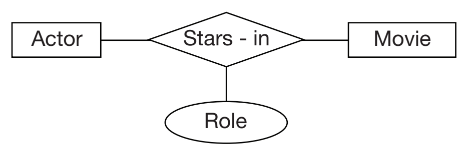
(A) 1 NF (B) 2 NF
(C) 3 NF (D) BCNF

10. For a Relation scheme R(ABC), Assume that all attributes are prime attributes, Then minimum '*R*' is in which Normal Form?

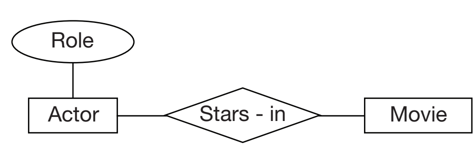
(A) 1 NF (B) 2 NF
(C) 3 NF (D) BCNF

11. Which of the following model, describes the fact that a role is stored for every pair of actor '*X*' and movie '*Y*' such that '*X*' starred in '*Y*'.

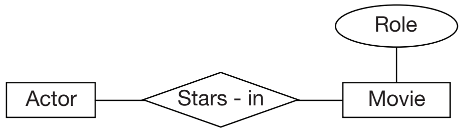
(A)



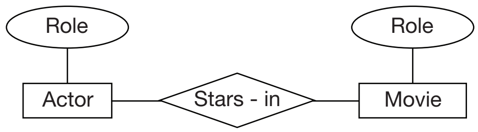
(B)



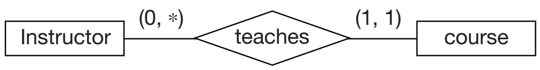
(C)



(D)



12. Consider the given ER-Diagram:



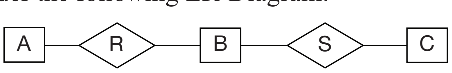
Which of the following is TRUE?
(A) one instructor teaches many courses, but each course is run by exactly one instructor.
(B) One instructor teaches atmost one course.
(C) An instructor may not take even a single course.
(D) One instructor teaches one course, but each course is run by many instructors.

13. Consider the given Functional dependencies.
For a Relation R(ABCDE):
$A \to B$
$B \to C$
$BC \to A$
$A \to D$
$E \to A$
$D \to E$
Which of the following is not a key?
(A) B (B) C
(C) D (D) E

14. Consider a Relation $R(ABCDE)$ with following functional dependencies:
$ABC \to DE$
$D \to AB$
What is the number of candidate keys for the Relation R?
(A) 0 (B) 1
(C) 2 (D) 3

15. Suppose relation $R(A, B)$ currently has tuples {(2, 3), (2, 4), (4, 5)} and Relation $S(B, C)$ currently has {(3, 6), (5, 7), (8, 9)}. What is the number of tuples in the result of the SQL query
SELECT * FROM R NATURAL OUTER JOIN S?
(A) 2 (B) 3
(C) 4 (D) 5

16. Consider the following ER-Diagram:



What is the minimum number of tables required to represent given ER-Diagram?
(A) 1 (B) 2
(C) 3 (D) 4

17. To find the EId of the Employees that are managed by people who are managed by the employee with EId 456.
Consider the following Queries:
I. SELECT E.EId
FROM Emp E, Emp F
WHERE E.MgrId = F.EId AND F.MgrId = 456
II. SELECT EId
FROM Emp
WHERE MgrId IN (SELECT EId
FROM Emp
WHERE MgrId = 456)
Which query will correctly get the desired set of Employee IDs?
(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

18. Consider the following functional dependencies
$AB \to C$
$C \to D$
$AB \to D$
Which of the following is CORRECT minimal set?
(A) $A \to C$ $C \to D$
(B) $AB \to C$ $C \to D$
(C) $B \to C$ $C \to D$
(D) $AB \to C$ $AB \to D$

19. Consider the following functional dependencies:
$R(ABCD)$
$AB \to C$
$C \to A$
$D \to B$
$AB \to D$
What are the keys possible for Relation R?
(A) AC, BD, CD (B) AB, BC, CD
(C) AB, AD, BC, CD (D) AC, BC, BD

20. Sailor (Sid, Sname, Rating, age)
Reserves (Sid, bid)
Boats (bid, bname, color)
To select the names of sailors who have reserved all the boats, which Relational algebra operation is used specifically, to do the under lined part?
(A) OUTER JOIN (B) UNION
(C) DIVISION (D) EXCEPT

21. Student1 (RNo, Name, CNo, CName)
Student2 (RNo, Name, CNo, CName)
I. $\pi_{\text{Name}}\,(\sigma_{\text{CNO} = \text{'}C2\text{'}}$ (student1 – student2))
II. $\pi_{\text{Name}}\,(\sigma_{\text{CNO} = \text{'}C2\text{'}}$ (student2 – student1))
Which of the following is TRUE in most of the cases about Number of tuples appear in the result of I and II?
(A) I ≠ II (B) I < II
(C) I > II (D) I = II

22. Consider the following:
I. $\{P / \exists\, S \in$ student $\exists\, D \in$ Department (S.RNo = D.RNo. ∧ D. DNo. = 5 ∧ P.Sname = S.Sname)

II. $\{< S_N > / \exists\ S_R, S_N\ (< S_R, S_N > \in \text{student} \wedge \exists\ D_R, D_N\ (< D_R, D_N > \in \text{Department} \wedge S_R = D_R\ D_N = 5))\}$

Which of the following is TRUE about given expressions?

(A) I – Tuple Relational Calculus
II – Relational algebra

(B) I – Tuple Relational Calculus
II – Domain Relational Calculus

(C) I – Domain Relational Calculus
II – Tuple Relational Calculus

(D) I – Relational Algebra
II – Domain Relational Calculus

23. Consider 2 Relations *R* and *S*, If we want to Retrieve all the tuples from both *R* and *S* without losing any tuple, which Relational Algebra operator is used?

(A) NATURAL JOIN
(B) LEFT OUTER JOIN
(C) INNER JOIN
(D) FULL OUTER JOIN

24. Assume that table *A* has 5 columns, table *B* has 4 columns, one column is common in both the tables. What is the number of columns appear in the result of NATURAL JOIN, LEFT OUTER JOIN, FULL OUTER JOIN respectively?

(A) 8, 9, 9 (B) 8, 8, 9
(C) 8, 8, 8 (D) 8, 9, 8

25. Consider the following Query in English:

"Retrieve the names of students who scored more than 90% but whose age is not more than 15?"

Which Relational operator is compulsory in SQL?

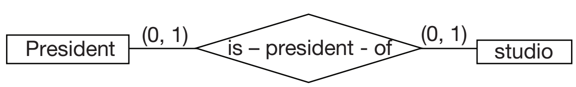
(A) INTERSECTION (B) NATURAL JOIN
(C) OUTER JOIN (D) SET-DIFFERENCE

Answer Keys

1. B	**2.** C	**3.** A	**4.** C	**5.** D	**6.** D	**7.** D	**8.** B	**9.** A	**10.** C
11. A	**12.** A	**13.** B	**14.** B	**15.** C	**16.** C	**17.** C	**18.** B	**19.** C	**20.** C
21. A	**22.** B	**23.** D	**24.** C	**25.** D					

Hints and Explanations

1.



Choice (B)

2. "*" may be used as maximum if there is no limit (0, *) means no restriction at all (general relationship)

– "A movie award goes exactly to one movie" (1, 1)
– A movie may be awarded multiple awards but may be none at all (0, *). Choice (C)

3. The correct match is I – R, II – P, III – Q.
Choice (A)

4. – If we insert some value which already exists, under primary key column, key constraint is violated.
– If we insert NULL value under primary key column Entity Integrity constraint is violated.
– If we insert some value which already exists under the column 'SSNO' which is declared as UNIQUE in the schema, UNIQUE constraint is violated.
Choice (C)

5. All violations are possible with updation operation. Refer above solution. Choice (D)

6. The query will be
SELECT Subject, MAX(Marks)
FROM Student
GROUPBY Subject Choice (D)

7. LOCATE is NOT a Relational algebra operator.
Choice (D)

8. It is a wild card symbol that requests inclusion of all attributes. Choice (B)

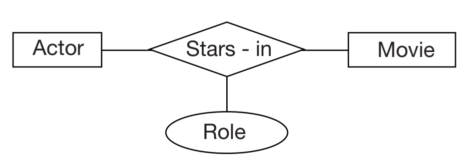
9. $A \to B$
$C \to B$
$D \to B$

L	M	R
ACD		B

$(ACD)^+ = \{ACDB\}$ Prime Attributes = ACD
Non prime Attributes = B
$A \to B$
$\downarrow$ Non prime Attribute
Partial Key
It violates 2NF.
$\therefore$ The Relation is in 1 NF. Choice (A)

10. If All attributes are prime attributes, then a Relation *R* is in minimum 3 NF. Choice (C)

11.



Describes the fact that a Role is stored for every pair of actor 'X' and 'Y' such that X starred in Y. Choice (A)

12. One instructor teaches many courses (0, ∗). Each course is run by exactly one Instructor (1, 1).
Choice (A)

13. $A \to B$
$B \to C$
$BC \to A$
$A \to D$
$E \to A$
$D \to E$

L	M	R
	ABCD E	

$A^+ = \{ABCDE\}$
$B^+ = \{BCADE\}$
$C^+ = \{C\}$
$D^+ = \{DEABC\}$
$E^+ = \{EABCD\}$
∴ 'C' is not the key. Choice (B)

14.

L	M	R
C	ABD	E

$C^+ = \{C\}\ x$
$CA^+ = \{CA\}$
$CB^+ = \{CB\}$
$CD^+ = \{CDABE\}$
Candidate key = CD Choice (B)

15.
R S

A	B
2	3
2	4
4	5

B	C
3	6
5	7
8	9

R NATURAL OUTER JOIN S

A	B	C
2	3	6
2	4	NULL
4	5	7
NULL	8	9

∴ 4-Tuples appear in the Result. Choice (C)

16. Minimum, for every entity one table is required. Since there are 3 entities A, B, C, we need minimum 3 tables.
Choice (C)

17. Both queries will correctly give the desired set of Employee Ids. Choice (C)

18. $AB \to C$
$C \to D$
$AB \to D$
Minimal cover:
$AB \to C$
$C \to D$ Choice (B)

19. $AB \to C$
$C \to A$
$D \to B$
$AB \to D$

L	M	R
	ABCD	

$A^+ = \{A\}$
$B^+ = \{B\}$
$C^+ = \{CA\}$
$D^+ = \{DB\}$
$AB^+ = \{ABCD\}$
$AC^+ = \{AC\}$
$AD^+ = \{ADBC\}$
$BC^+ = \{BCAD\}$
$BD^+ = \{BD\}$
$CD^+ = \{CDAB\}$
Keys are AB, AD, BC, CD. Choice (C)

20. If an SQL query contains "Select – all" term, DIVISION operation is used. Choice (C)

21. (Student1 – Student2) ≠ (Student2 – Student1) option (B) and option (C) depends on the number of tuples in first specified Relation. Choice (A)

22. Both Queries return the names of students Whose department number is 5.
I – Tuple Relational Calculus
II – Domain Relational Calculus Choice (B)

23. FULL OUER JOIN returns all the tuples of Both the tables. Choice (D)

24. Table A has 5 columns
Table B has 4 columns
One column is common to both the tables. In the Result of NATURAL JOIN, LEFT OUTER JOIN, FULL OUTER JOIN, duplicate columns are not allowed, (5 columns + 3 columns) = 8 columns. Choice (C)

25. The query will be as follows
SELECT name
FROM student
WHERE marks > 90
SET DIFFERENCE
SELECT name
FROM student
WHERE age > 15. Choice (D)

Chapter 1

Finite Automata and Regular Languages

LEARNING OBJECTIVES

- Fundamentals
- Languages
- Operations
- Finite state machine
- NFA with ε-moves
- Conversion of NFA to DFA
- Minimization of DFA
- Equivalence between NFA and DFA
- Mealy and Moore machines
- Equivalence of Moore and Mealy machine
- Regular languages
- Constructing FA for given RE
- Pumping lemma for regular sets
- Closure properties of regular sets
- Regular grammar

FUNDAMENTALS

Alphabet: An alphabet is a finite non-empty set of symbols.

Example: Portion of a calculator: {0, 1, 2, 3 … 9, ÷, =, –, +, ×, (,)}

Note: 1. At least one symbol is necessary.

2. 'Σ' denote Alphabet.

String: A string over an alphabet '*A*' is a finite ordered sequence of symbols from '*A*'. The length of string is number of symbols in string, with repetitions counted.

Example: If Σ = {0 – 9, ÷, =, –, +, × (,)} then Strings valid: 12 + 34, 90 × 10, (1 + 2) × (1 ÷ 3)

Strings Invalid: sin (45), log (10) etc. These strings are not valid because sin ( ), log ( ) are not defined over the alphabet set.

Note: Repetitions are allowed.
Length of |12 + 34| = 5(1, 2, +, 3, 4)

- The Empty string denoted by 'ε', is the (unique) string of length zero.

Note: Empty string, $\varepsilon \neq$ empty set, $\varnothing$.

- If S and T are sets of strings, then $ST = \{xy | x \in S \text{ and } y \in T\}$

Given an alphabet A,

$A^o = \{\varepsilon\}$

$A^{n+1} = A.A^n$

…

$$A^* = \bigcup_{n=0}^{\infty} A^n$$

Languages

- A language 'L' over Σ is any finite or infinite set of strings over Σ.
- The elements in L are strings – finite sequences of symbols.
- A language which does not contain any elements is called 'empty language'.

Note: Empty language, { } ≠ {ε}, empty string because { } = $\varnothing \neq \varepsilon$ i.e., Empty language resembles empty set i.e., $\varnothing$.

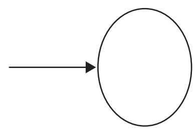
- A language L over an alphabet A is subset of A^* i.e., $L \subset A^*$.

Example 1: Language (L) for strings that consists of only 0's or only 1's and have an odd length over alphabet {0, 1} is

(A) {0, 1, 00, 11, 000, 111 …}
(B) {00, 11, 01, 10 …}
(C) {000, 101, 110, 111 …}
(D) {0, 1, 000, 111, 11111, 00000 …}

Solution: (D)
Only 0's → should have only 0's. It should not be combination of 0's and 1's.
Only 1's → should have only 1's. It should not be combination of 0's and 1's.
Odd length → only odd number of 0's or odd number of 1's i.e., length of string should be odd.

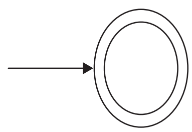
An Empty Languages An empty language is a language which does not accept any strings includinge. The Finite automata for empty language can be represented as



(i.e., One state, non-accepting and no transitions).

A language which only accepts (ε)

E: The language which only accepts 'ε' can be represented as



This machine accepts E – only.

Σ^***:** The set of all strings over an alphabet Σ will be denoted by Σ^*.

Σ^+: This will denote the set $\Sigma^* - \{\varepsilon\}$.

Ex: If $\Sigma = \{0, 1\}$ then

$\Sigma^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, \ldots\}$

$\Sigma^+ = \{0, 1, 00, 01, 10, 11, 000, 001,\}$

Operations

Operations on strings

1. **Concatenation:** Combines two strings by putting one after other.

Example 2: Two strings are defined as x = java, y = script. The concatenation ($x.y$) of two strings results in ________.

(A) scriptjava (B) javascript
(C) jascriptva (D) scrijavapt

Solution: (B)

$x.y$ = java.script = javascript

Note: Concatenation of empty string with any other string gives string itself.

i.e., $x.\varepsilon = \varepsilon.x = x$

2. **Substring:** If 'w' is a string, then 'v' is a substring of 'w' if there exists string x and y such that $w = xvy$.

'x' is called 'prefix' and y is called suffix of w.

Example 3: String, w = 'gymnastics' is defined with prefix, x = '*gym*' and suffix, y = '*cs*'. The substring of the given string is ________

(A) nasti (B) mnas
(C) gymnastics (D) ics

Solution: (A)

Because, $w = xvy$

$\Rightarrow$ gymnastics = gymvcs

$\therefore v$ = nasti

3. **Kleen star operation:** Let 'w' be a string, w^* is set of strings obtained by applying any number of concatenations of w with itself, including empty string.

Example: $a^* = \{\varepsilon, a, aa, aaa, \ldots\}$

4. **Reversal:** If 'w' is a string, then w^R is reversal of string spelled backwards.

Rules:

- $x = (x^R)^R$
- $(xz)^R = z^R \cdot x^R$

Example 4: A string, x is defined as, x = butter. Then $(x^R)^R$ is ________

(A) butter (B) rettub
(C) butret (D) retbut

Solution: (A)

$x \longrightarrow$ butter

$x^R \longrightarrow$ rettub

$(x^R)^R \longrightarrow$ butter.

Operations on languages

1. **Union:** Given some alphabet Σ, for any two languages, L_1, L_2 over Σ, the union $L_1 \cup L_2$ of L_1 and L_2 is the language, $L_1 \cup L_2 = \{w \in \Sigma^* | w \in L_1 \text{ or } w \in L_2\}$
2. **Intersection:** Given some alphabet Σ, for any two languages L_1, L_2 over Σ, the intersection $L_1 \cap L_2$ of L_1 and L_2 is language, $L_1 \cap L_1 = \{w \in \Sigma^* | w \in L_1 \text{ and } w \in L_2\}$
3. **Difference:** Given some alphabet Σ, for any two languages L_1, L_2 over Σ, the difference $L_1 - L_2$ of L_1 and L_2 is language, $L_1 - L_2 = \{w \in \Sigma^* | w \in L_1 \text{ and } w \notin L_2\}$

Note: Difference is also called 'Relative Complement.'

A special case of difference is obtained when $L_1 = \Sigma^*$, in which case. Complement $\overline{L}$ of language, L is defined as, $\overline{L} = \{w \in \Sigma^* | \, w \notin L\}$

4. **Concatenation:** Given an alphabet Σ, for any two languages L_1, L_2 over Σ, the concatenation $L_1 \, L_2$ of L_1 and L_2 is language

$L_1L_2 = \{w \in \Sigma^* | \exists \, u \in L_1, \exists \, v \in L_2, w = uv\}$

Properties:

$L\varnothing = \varnothing = \varnothing L$

$L\,\{\varepsilon\} = L = \{\varepsilon\}$ L

$(L_1 \cup \{\varepsilon\})L_2 = L_1L_2 \cup L_2$

$L_1\,(L_2 \cup \{\varepsilon\}) = L_1L_2 \cup L_1$

$L^n\,L = LL^n = L^{n+1}$

Note: $L_1L_2 \neq L_2L_1$

Example 5: Let $L_1 = \{00, 11\}$, $L_2 = \{01, 10\}$. Then $L_1 o L_2$ = ________

(A) {00, 11, 01, 10}
(B) {0001, 0010, 1101, 1110}
(C) {0001, 0010, 11, 01, 10}
(D) {00, 1101, 1110, 11, 10}

Solution: (B)

$L_1 o L_2 = \{00, 11\} \text{ o } \{01, 10\} = \{00.01, 00.10, 11.01, 11.10\} = \{0001, 0010, 1101, 1110\}$

5. **Kleen * closure (L*):** Given an alphabet Σ, for any language L over Σ, the * closure L^* of L is language,

$L^* = U_{n \geq 0} L^n$

6. **Kleen + closure (L^+):** The kleen +closure, L^+ of L is the language, $L^+ = U_{n\geq 1} L^n$

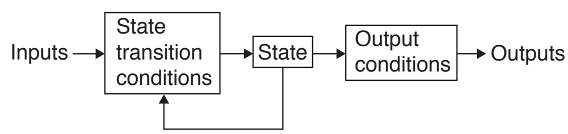
$L^* = L^0 \cup L^1 \cup L^2 \cup \ldots L^n \cup \ldots$
$L^+ = L^1 \cup L^2 \cup L^3 \ldots \cup L^n \cup \ldots$

Properties:

$\varnothing^* = \{\varepsilon\}$
$L^+ = L^*L$
$(L^*)^* = L^*$
$L^*\, L^* = L^*$

Finite State Machine (FSM)

- FSM is simplest computational model of limited memory computers.
- FSM is designed to solve decision problems i.e., to decide whether given input satisfies certain conditions.
- The next state and output of a FSM is a function of input and of current state.

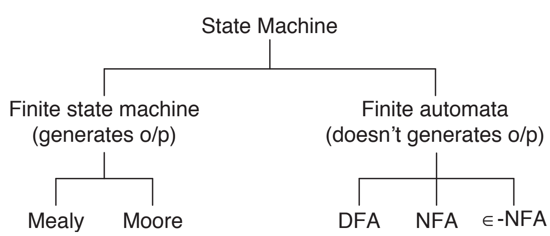


Types of FSM:

1. Melay machine.
2. Moore machine

Finite Automata (FA):

- FA is a state machine that comprehensively captures all possible states and transitions that a machine can take while responding to a stream (sequence) of input symbols.
- FA is recognizer of 'regular languages'.



Types of FA

1. Deterministic Finite Automata (DFA):

- DFA machine can exists in only one state at any given time.
- DFA is defined by 5-tuple: $\{Q, \Sigma, q_0, F, \delta\}$, where

$Q \rightarrow$ Finite number of states (elements)
$\Sigma \rightarrow$ Finite set of symbols (alphabets)
$q_o \rightarrow$ Start/Initial state
$F \rightarrow$ Set of final states.
$\delta \rightarrow$ Transition function, which is a mapping between

$$\delta: Q \times \Sigma \rightarrow Q.$$

How to use DFA:

Input: A word w in Σ^*
Question: Is w acceptable by DFA?

Steps:

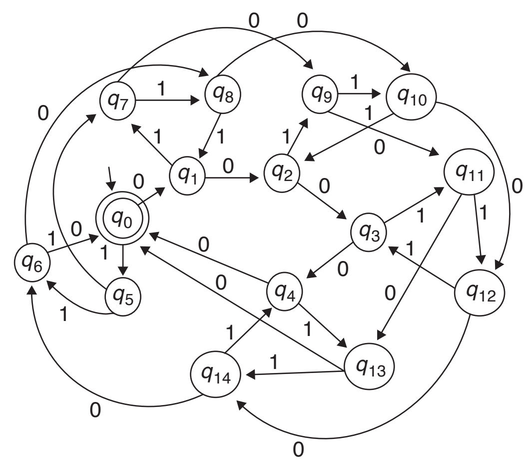
- Start at 'initial state', q_o.
- For every input symbol in sequence w, do.
- Compute the next state from current state, given the current input symbol in w and transition function.
- If after all symbols in 'w' are consumed, the current state is one of the final states (f) then accept 'w';
- Otherwise, reject w.

Transition diagram: State machines are represented by directed graphs called transition (state) diagrams.

- The vertices denoted by single circle represent the state and arcs labeled with input symbol correspond to transition.
- The final states are represented with double circles.

Transition Table: Transition function can be represented by tables.

Example 6: The following finite state machine accepts all those binary strings in which the numbers of 0's and 1's are respectively.



(A) Divisible by 3 and 2
(B) Odd and even
(C) Divisible by 5 and 3
(D) Divisible by 2 and 3

Solution: (C)
Number of 0's is divisible by 5.
Number of 1's is divisible by 3.

Table *Transition Table*

Current State	0	1
$\rightarrow q_0$	q_1	q_5
q_1	q_2	q_7
q_2	q_3	q_9
q_3	q_4	q_{11}
q_4	q_0	q_{13}
q_5	q_7	q_6
q_6	q_8	q_0
q_7	q_9	q_8
q_8	q_{10}	q_1

q_9	q_{11}	q_{10}
q_{10}	q_{12}	q_2
q_{11}	q_{13}	q_{12}
q_{12}	q_{14}	q_3
q_{13}	q_0	q_{14}
q_{14}	q_6	q_4

Note: Minimum number of states for k-divisibility is k-states.
In above example, $q_0 - q_{14} \to 15$ – states.
$\therefore \quad 5 \times 3 = 15$
The given binary strings have number of 0's divisible by 5 and number of 1's divisible by 3.

2. Non-deterministic finite Automata (NFA):

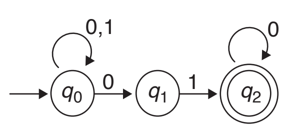
- The machine can exist in multiple states at the same time.
- Each transition function maps to a set of states.
- NFA is defined by 5-tuple: $\{Q, \Sigma, q_0, F, \delta\}$, where

$Q \to$ Finite number of states (elements)
$\Sigma \to$ Finite set of symbols. (Alphabets)
$q_o \to$ Start/Initial state
$F \to$ Set of final states.
$\delta \to$ Transition function which is a mapping between
$\delta = Q \times \Sigma \to 2^Q$

How to use NFA:
Input: a word w in Σ^*
Question: Is w accepted by NFA?
Steps:

- Start at 'start state' q_0.
- For every input symbol in the sequence, *w* does.
- Determine all possible next states from current state, given the current input symbol in *w* and transition function.
- If after all symbols in *w* are consumed, at least one of the current states is a final state then accept *w*.
- Otherwise, reject *w*.

Example 7: What is the language, *L* generated by the below NFA, given strings defined over alphabet, $\Sigma = \{0, 1\}$.



(A) Strings that end with '0'
(B) Strings that start with '0' and end with '0'
(C) Strings that contain '01' as substring
(D) Strings that contain '01' as substring and end with '0'

Solution: (D)

State	0	1
$\to q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\varnothing$	q_2
q_2	$\{q_2\}$	$\varnothing$

String: 0100100

$$q_0 \xrightarrow{0} \{q_0, q_1\}$$
$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_0 \xrightarrow{0} \{q_0, q_1\}$$
$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{0} \{q_0, q_1\}$$
$$\xrightarrow{q_0 1} q_0 \xrightarrow{0} q_0 \xrightarrow{0} \{q_0, q_1\} \text{ (Non-deterministic)}$$
$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{0} \{q_0, q_1\}$$
$$q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_2 \xrightarrow{0} q_2$$

Table 2 *Difference between NFA and DFA*

DFA	NFA
1. All transitions are deterministic i.e., each transition leads to exactly one state.	1. Transitions could be non-deterministic i.e., a transition could lead to a subset of states.
2. For each state, the transition on all possible symbols should be defined.	2. For each state, not all symbols necessarily have to be defined.
3. Accepts input if last state is in 'F'.	3. Accepts input if one of last states is in 'F'.
4. Practical implementation is feasible.	4. Practical implementation has to be deterministic (so needs conversion to DFA).

Relation between DFA and NFA

- A language '*L*' is accepted by a DFA if and only if it is accepted by a NFA.
- Every DFA is special case of a NFA.

Example 8: Let N_f and D_f denote the classes of languages accepted by non-deterministic finite automata and deterministic finite automata respectively. Which one of following is true?

(A) $D_f \subset N_f$ (B) $D_f \supset N_f$
(C) $D_f = N_f$ (D) $D_f \in N_f$

Solution: (C)
According to 'subset construction', every language accepted by NFA is also accepted by some DFA.
$\therefore \quad D_f = N_f$

NFA WITH $\in$-MOVES

- $\in$-transitions in finite automata allows a state to jump to another state without consuming any input symbol.

Conversion and Equivalence:

$\in$-NFA $\to$ NFA $\to$ DFA

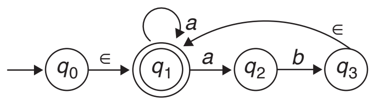
NFA without $\in$-moves:

- Two FA, $N_\in$ and N are said to be equivalent, if $L(N_\in) = L(N)$ i.e., any language described by some $N_\in$, there is an N that accepts the same language.
- For $N_\in = (Q, Z, \delta, q_0, F)$ and $N = (Q, \Sigma', \delta', q_0, F')$, Find
- $\delta'(q, a) = \in$-closure $(\delta(\in\text{-closure}(q), a))$

- $F' = \{F \cup \{q_0\}\}$, if $\in$-closure (q_0) contains a member of $F = F$, otherwise.

Note: When transforming $N_\in$ to N, only transitions are required to be changed and states remains same.

Example 9: Consider following NFA with $\in$-moves.



If given NFA is converted to NFA without $\in$-moves, which of following denotes set of final states?

(A) $\{q_0, q_1\}$ (B) $\{q_1, q_2\}$
(C) $\{q_1, q_2, q_3\}$ (D) $\{q_1\}$

Solution: Let $N = (Q, \Sigma^1, \delta^1, q_0, F^1)$
$F^1 = F \cup \{q_0\}$
ε-closure $(q_0) = \{q_0, q_1\}$
$\therefore F^1 = \{q_1\} \cup \{q_0, q_1\} = \{q_0, q_1\}$
Conversion $N_\in \rightarrow N$:
To compute, δ^1
$\in$-closure $(q_0) = \{q_0, q_1\}$, $\in$-closure $(q_3) = \{q_3, q_1\}$
$\delta^1(q_0, a) = \{q_1, q_2\}$, $\delta^1(q_0, b) = \varnothing$, $\delta^1(q_2, a) = \varnothing$.
$\delta^1(q_1, a) = \{q_1, q_2\}$, $\delta^1(q_1, b) = \varnothing$, $\delta^1(q_2, b) = \{q_1, q_3\}$
$\delta^1(q_3, a) = \{q_1, q_2\}$, $\delta^1(q_3, b) = \varnothing$

Table 3 *Transition Table*

State \ Input	a	b
$\rightarrow q_0$	$\{q_1, q_2\}$	$\varnothing$
q_1	$\{q_1, q_2\}$	$\varnothing$
q_2	$\varnothing$	$\{q_1, q_3\}$
q_3	$\{q_1, q_2\}$	$\varnothing$

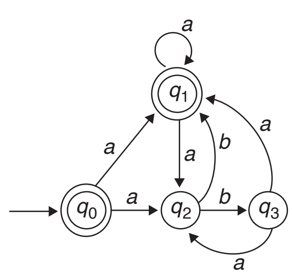


Figure 1 Transition diagram

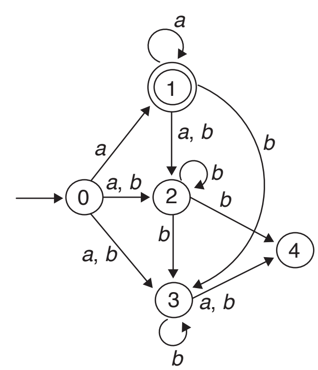
Conversion of NFA to DFA

Let a NFA be defined as, $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$
The equivalent DFA, $D = (Q_D, \Sigma, \delta_D, q_0, F_D)$ where:
Step I: $Q_D = 2^{Q_N}$; i.e., Q_D is set of all subsets of Q_N i.e., it is power set of Q_N.
Step II: F_D is set of subsets S of Q_N such that $S \cap F_N \neq \varnothing$. i.e., F_D is all sets of N's states that include atleast one accepting state of N.
Step III: For each set, $S \leq Q_N$ and for each input symbol a in Σ: $\delta_D(S, a) = \cup_{P \in S} \delta_N(P, a)$

That is, to compute $\delta_D(S, a)$, look at all states P in S, see what states N goes to starting from P on input a, and take the union of all those states.

Note: For any NFA, N with 'n' states, the corresponding DFA, D can have 2^n states.

Example 10: What is the number of final states in DFA constructed from the given NFA?



(A) 1 (B) 2
(C) 3 (D) 4

Solution:

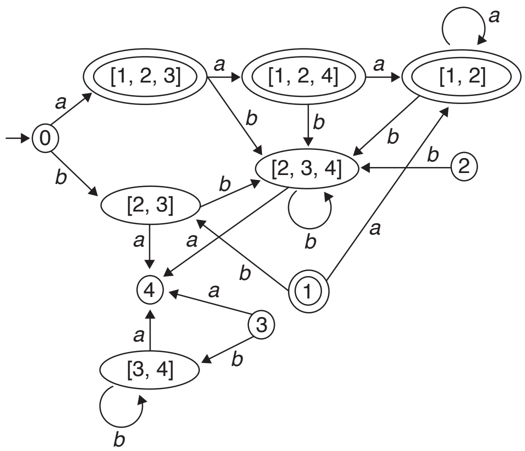
Table 4 *Transition Table of NFA*

State \ Input	a	b
$\rightarrow 0$	{1, 2, 3}	{2, 3}
1	{1, 2}	{2, 3}
2	$\varnothing$	{2, 3, 4}
3	{4}	{3, 4}
4	$\varnothing$	$\varnothing$

Table 5 *Transition Table of DFA*

State \ Input	a	b
$\rightarrow$0	[1, 2, 3]	[2, 3]
1	[1, 2]	[2, 3]
2	$\varnothing$	[2, 3, 4]
3	4	[3, 4]
4	$\varnothing$	$\varnothing$
[1, 2]	[1, 2]	[2, 3, 4]
[2, 3]	[4]	[2, 3, 4]
[3, 4]	[4]	[3, 4]
[1, 2, 3]	[1, 2, 4]	[2, 3, 4]
[1, 2, 4]	[1, 2]	[2, 3, 4]
[2, 3, 4]	[4]	[2, 3, 4]

Hence final states in obtained DFA is '4'.
DFA is: Choice (D)

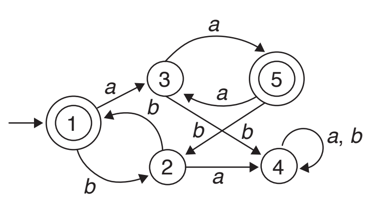


Minimization of DFA

Given a DFA, $M = (Q, \Sigma, \delta, q_0, F)$, we construct a reduced DFA, $M' = (Q', \Sigma', \delta', q'_0, F')$ as follows

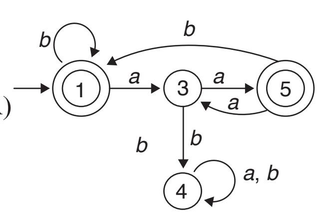
1. Remove all inaccessible states. All states that are unreachable from the initial state are removed.
2. Consider all pairs of states (p, q), If $p \in F$ and $q \in F$ or vice versa mark the pair (p, q) as distinguishable.
3. Repeat until no previously unmarked pairs are marked. For all pairs (p, q) and all $a \in \Sigma$, compute $\delta(p, a) = p_a$ and $\delta(p, q) = q_a$. If the pair (p_a, q_a) is marked as distinguishable mark (p, q) as distinguishable.
4. Find the sets of all indistinguishable states, say $\{q_i, q_j, \cdots q_k\}$, $\{q_\ell, q_m, \cdots q_n\}$, etc. For each set $\{q_i, q_j, \ldots q_k\}$ of such indistinguishable states, create a state labelled $ij \ldots k$ for M.
5. For each transition rule of M of the from $\delta(q_r, q) = q_p$, find the sets to which q_r and q_p belong. If $q_r \in \{q_i, q_j, \ldots q_k\}$ and $q_p \varepsilon \{q_\ell, q_m, \cdots q_n\}$, add a rule to δ: $\delta'(ij \ldots k, a) = \ell m \ldots n$.

Example 11: A DFA with alphabet $\Sigma = \{a, b\}$ is given below:

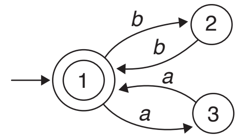


Which of the following is valid minimal DFA which accepts same language as given DFA?

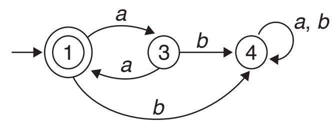
(A)



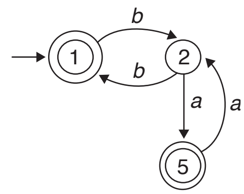
(B)



(C)



(D)



Solution: (B)

Initially, $\{1, 5\}$, $\{2, 3, 4\}$

Depending on next states and inputs, the partitions of states can be as: $\{\{1, 5\}, \{2\}, \{3\}, \text{and } \{4\}\}$

Since, 1 to 5 have same transition, unite $\{1, 5\}$

State 4 is dead state $\to$ It has transition only to itself. Since, $\{2\}$, $\{3\}$ are singletons, they exist.

$\therefore$ States in minimized DFA are $\{1, 2, \text{and } 3\}$

$\{1\} \to \{1, 5\}$

For transitions, since $1 \xrightarrow{a} 3$, $1 \xrightarrow{b} 2$ in given DFA, in minimized DFA, transitions are added from $1 \xrightarrow{a} 3$, $1 \xrightarrow{b} 2$. Also, since $2 \xrightarrow{b} 1$, $3 \xrightarrow{a} 1$ in given DFA, the minimized DFA, transitions are added from $2 \xrightarrow{b} 1$, $3 \xrightarrow{a} 1$.

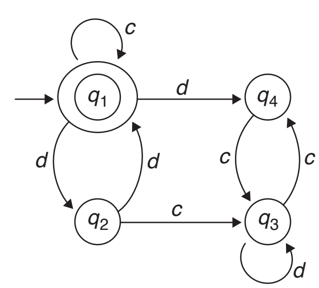
Equivalence Between NFA and DFA

There is a DFA_D for any NFA_N i.e., $L(D) = L(N)$.

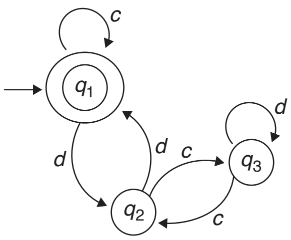
Construction:

- In DFA or NFA, whenever an arrow is followed, there is a set of possible states. This set of states is a subset of Q.
- Track the information about subsets of states that can be reached from initial state after following arrows.
- Consider each subset of states of NFA as a state of DFA and every subset of states containing a final state as a final state of DFA.

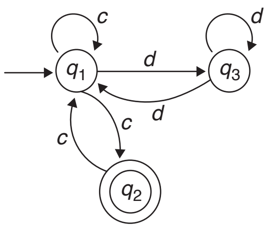
Example 12: Which of following is equivalent DFA for the NFA given below:



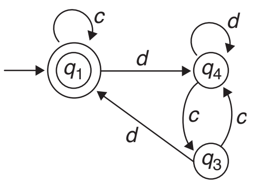
(A)



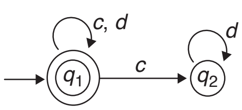
(B)



(C)



(D)



Solution: (A)

Table 6 *Transition Table of NFA*

δ	***c***	***d***
$\rightarrow q_1$	q_1	$\{q_2, q_4\}$
q_2	q_3	q_1
q_3	q_4	q_3
q_4	q_3	$\varnothing$

Table 7 *Transition Table of DFA*

δ	***c***	***d***
$\rightarrow q_1$	q_1	q_2
q_2	q_3	q_1
q_3	q_2	q_1

Table 8 *Common Table*

δ	***c***	***d***
(q_1, q_1)	(q_1, q_1)	(q_2, q_4, q_2)
(q_2, q_2)	(q_3, q_3)	(q_1, q_1)
(q_3, q_3)	(q_4, q_2)	(q_3, q_3)
(q_4)	q_3	$\varnothing$

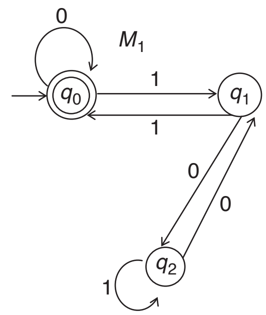
Equivalence of Finite Automatas:

- Two automatas A and B are said to be equivalent if both accept exactly the same set of input strings.
- If two automatas M_1 and M_2 are equivalent then

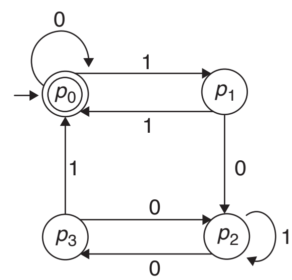
(i) If there is a path from the start state of M_1 to a final state of M_1 labeled $a_1a_2 \ldots a_k$ then there is a path from the start state of M_2 to the final state of M_2 labeled $a_1a_2\ldots a_k$.

(ii) If there is a path from the start state of M_2 to a final state M_2 labeled $b_1b_2 \ldots b_i$ then there is a path from the start state of M_1 to the final state of M_1 labeled $b_1b_2 \ldots b_i$.

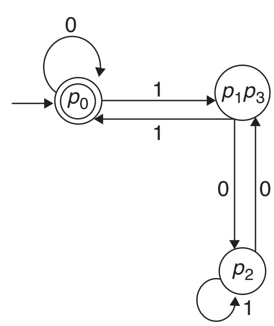
Example:



M_2:



In M_2, states p_1 and p_3 are equivalent (as both are reaching either final or non-final states with same input). After minimizing M_2, we will get



$\therefore$ M_1 and M_2 are equivalent.

Union: The union of two languages L and M is the set of strings that are in both L and M.

Ex: $L = \{0, 1\}$, $M = \{111\}$

$L\ U\ M = \{0, 1{,}111\}$.

Concatenation: The concatenation of Languages L and M is the set of strings that can be formed by taking any string in L and concatenating it with any string in M.

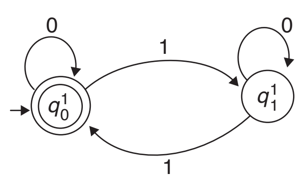
Example: $L = \{0, 1\}$, $M = \{\varepsilon, 010\}$

$LM = \{0, 1, 0010, 1010\}$.

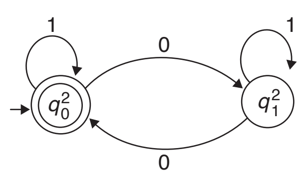
Closure, Star or Kleen star of a language L:
Kleen star is denoted as L^*. It represents the set of strings that can be formed by taking any number of strings from L with repetition and concatenating them. It is a Unary operator.
L^0 is the set; we can make selecting zero strings from L.
$L^0 = \{\varepsilon\}$
L^1 is the language consisting of selecting one string from L.
L^2 is the language consisting of concatenations selecting two strings from L.
…
L^* is the union of L^0, L^1 , …L^∞.
Ex: $L = \{0,10\}$
$L^* = \{0,00,000,10,010, \ldots\}$

Intersection:
Let two DFAs M_1 and M_2 accept the languages L_1 and L_2.
$M_1 = (Q_1, \Sigma, \delta_1, q_0^1, .F_1)$
$M_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2)$
The intersection of M_1 and M_2 can be given as
$M = (Q_1, \Sigma, \delta, q_0\ F)$
Q = Pairs of states, one from M_1 and one from M_2 i.e.,
$Q = \{(q_1, q_2) \mid q_1 \in Q_1 \text{ and } q_2 \in Q_2\}$
$Q = Q_1 \times Q_2$.
$q_0 = (q_0^1, q_0^2)$
$\delta\ (q_i^1\ q_j^2), x) = (\delta_1(q_1^1, x), \delta_2(q_j^2, x))$
$F = \{(q_1, q_2) \mid q_1, \in F_1 \text{ and } q_2 \in F_2\}$

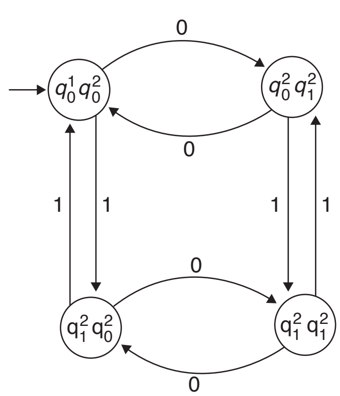
Example:
M_1: Strings with even number of 1's.



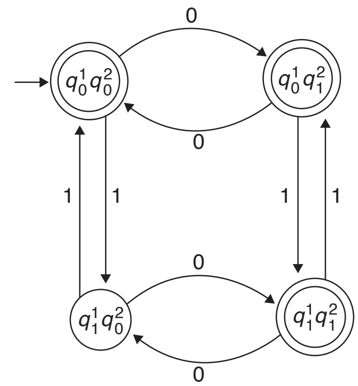
M_2: Strings with odd number of 0's.



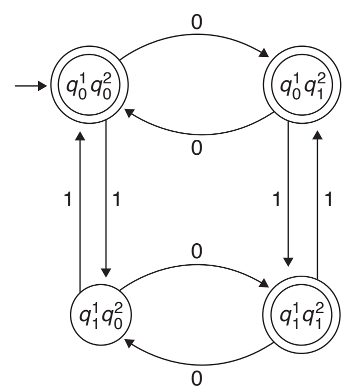
$M_1 \cap M_2$: Strings with even number of 1's and odd number of 0's.



Union of M_1 and M_2:



Difference: The difference of L_1 and L_2 can be given as $L_1 - L_2$ with $M = (Q, \Sigma, \delta, q_0, F)$.
$Q = Q_1 \times Q_2$
$q_0 = (q_0^{\ 1}, q_0^{\ 2})$
$\delta((q_i^{\ 1}, q_j^{\ 2}), x) = (\delta_1\ (q_i^{\ 1}, x), \delta_2(q_j^{\ 2}, x))$
$F = \{(q_1, q_2) \mid q_1, \in F_1 \text{ and } q_2 \notin F_2\}$



Reversing a DFA:

- M is a DFA which recognizes the language L.
- M^R will accept the language L^R.

To construct M^R:

- Reverse all transitions
- Turn the start state to final state
- Turn the final states to start state.
- Merge states and modify the FA,

such that the resultant contain a single start state.

Mealy and Moore Machines

Moore Machine

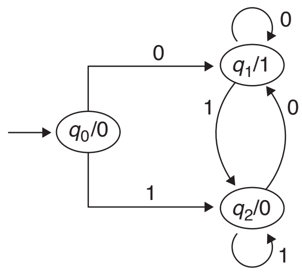
A moore machine is a finite state machine, where outputs are determined by current state alone.

A Moore machine associates an output symbol with each state and each time a state is entered, an output is obtained simultaneously. So, first output always occurs as soon as machine starts.

Moore machine is defined by 6-tuples:
$(Q, \Sigma, \delta, q_0, \Delta, \lambda)$, where
$Q \rightarrow$ Finite set of states
$\Sigma \rightarrow$ Finite set of input symbols
$\Delta \rightarrow$ It is an output alphabet
$\delta \rightarrow$ Transition function, $Q \times \Sigma \rightarrow Q$ (state function)
$\lambda \rightarrow$ Output function, $Q \rightarrow \Delta$ (machine function)
$q_0 \rightarrow$ Initial state of machine

Note: The output symbol at a given time depends only on present state of moore machine.

Example 13: The language generated by the following moore machine is:



(A) 2's complement of binary number.
(B) 1's complement of binary number.
(C) Has a substring 101.
(D) Has a substring 110.

Solution: (B)
Binary number: 1011
1's complement: 0100

$$q_0 \xrightarrow{1/0} q_2, \xrightarrow{0/1} q_1 \xrightarrow{1/0} q_2 \xrightarrow{1/0} q_2$$

$1 \rightarrow 0,\ 0 \rightarrow 1,\ 1 \rightarrow 0,\ 1 \rightarrow 0$

Mealy Machine

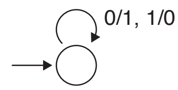
- A mealy machine is a FSM, where outputs are determined by current state and input.
- It associates an output symbol with each transition and the output depends on current input.
- Mealy machine is defined on 6-tuples: $(Q, \Sigma, \delta, q_0, \Delta, \lambda)$, where

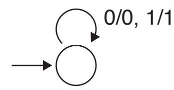
Q – Finite set of states.
Σ – Finite set of input symbols.
δ – $(Q \times \Sigma \rightarrow Q)$ is transition function.
$q_0 \rightarrow q_0 \in Q$ is initial state.
$\Delta \rightarrow$ Finite set of output symbols.
$\lambda \rightarrow$ Output function, $\lambda(Q \rightarrow \Delta)$

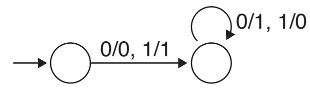
Note: In Moore machine, for input string of length n, the output sequence consists of $(n + 1)$ symbols.

In Mealy machine, for input string of length n, the output sequence also consists of 'n' symbols.

Example 14: Let $(\text{Me})^2$ mean that given a Mealy machine, an input string is processed and then output string is immediately fed into the machine (as input) and reprocessed. Only this second resultant output is considered as the final output of $(\text{Me})^2$. If final output string is same as original input string then $(\text{Me})^2$ has an identity property. Consider following machines.

(i)


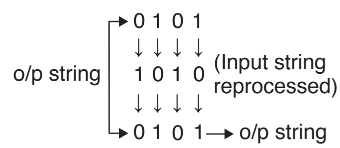
(ii)


(iii)


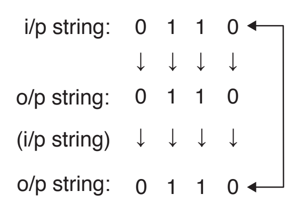
Which of above machines have identity property?
(A) (i) only
(B) (i) and (ii) but not (iii)
(C) (i) and (iii) but not (ii)
(D) All have identity property

Solution: (D)

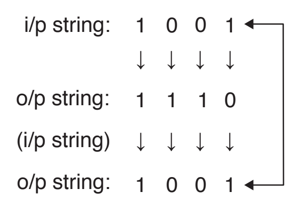
(i) Consider i/p string



(ii)



(iii)



Equivalence of Moore and Mealy machine

(a) Mealy machine equivalent to Moore machine:
If $M_1 = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ is a Moore machine, then there is a Mealy machine M_2 equivalent to M_1.

Proof: Let $M_2 = (Q, \Sigma, \Delta, \delta, \lambda^1, q_0)$ and define $\lambda^1(q, a)$ to be $\lambda(\delta(q, a))$ for all states q and input symbol 'a'.

Then M_1 and M_2 enter the same sequence of states on the same input, and with each transition M_2 emits the o/p that M_1 associates with the state entered.
Let us consider Mealy Machine

Present State	Next State			
	$a = 0$		$a = 1$	
	Input State	Output	Input State	Output
$\rightarrow q_1$	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

To convert the Mealy machine to Moore machine,

- We look into the next state column for any state, say q_i and determine the number of different outputs associated with q_i in next column.
- Split q_i into several different states, the number of such states being equal to the number of different outputs associated with q_i.

Present State	Next State			
	$a = 0$		$a = 1$	
	Input State	Output	Input State	Output
$\rightarrow q_1$	q_3	0	q_{20}	0
q_{20}	q_1	1	q_{40}	0
q_{21}	q_1	1	q_{40}	0
q_3	q_{21}	1	q_1	1
q_{40}	q_{41}	1	q_3	0
q_{41}	q_{41}	1	q_3	0

- The pair of states and outputs in the next state column can be rearranged as:

Present state	Next State $a = 0$	Next State $a = 1$	output
$\rightarrow q_1$	q_3	q_{20}	1
q_{20}	q_1	q_{40}	0
q_{21}	q_1	q_{40}	1
q_3	q_{21}	q_1	0
q_{40}	q_{41}	q_3	0
q_{41}	q_{41}	q_3	1

Moore machine equivalent to Mealy machine

Let $M_1 = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ be a Mealy machine. Then there is a machine M_2 equivalent to M_1

Proof: Let $M_2 = (QX\Delta, \Sigma, \Delta, \delta^1, \lambda^1, [q_0, b_0])$, where b_0 is an arbitrary selected member of Δ.

That is, the states of M_2 are pairs $[q, b]$ consisting of a state of M_1 and output symbol, Define δ^1 $([q, b], a) = [\delta(q, a), \lambda, (q, a)]$ and λ^1 $([q, b]) = b$.

The second component of a state $[q, b]$ of M_2 is the output made by M_1 on some transition into state q.

Only the first components of M_2's states determine the moves made by M_2.

Every induction on 'n' shows that if M_1 enters states $q_0, q_1 \ldots q_n$ on inputs $a_1, a_2 \ldots a_n$ and emits output $b_1, b_2, \ldots b_n$ then M_2 enters states $[q_0, b_0], [q_1, b_1] \ldots [q_n, b_n]$ and emits outputs $b_0, b_1 \ldots b_n$.

Let us consider the Moore machine

Present State	Next State $a = 0$	Next State $a = 1$	Output
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

- To convert Moore into Mealy machine, we must follow the reverse procedure of converting Mealy machine into Moore machine.
- For every input symbol we form, the pair consisting of the next state and the corresponding output and reconstruct the table for Mealy machine.
- For example, the state q_3 and q_1 in the next state column should be associated with outputs 0 and 1, respectively.

The Transition table for Mealy machine is:

Present state	Next State			
	$a = 0$		$a = 1$	
	state	output	state	output
$\rightarrow q_0$	q_3	0	q_1	1
q_1	q_1	1	q_2	0
q_2	q_2	0	q_3	0
q_3	q_3	0	q_0	0

Regular Languages

The set of regular languages over an alphabet Σ is defined recursively as below. Any language belonging to this set is a regular language over Σ.

Definition of set of regular languages

- Basis clause: $\varnothing$, $\{\varepsilon\}$, $\{a\}$ for any symbol $a \in \Sigma$, are regular languages.
- Inductive clause: If L_r and L_s are regular languages, then $L_r \cup L_s$, $L_r \cdot L_s$, L_r^* are regular languages.
- External clause: Nothing is a regular language, unless it is obtained from above two clauses.

Regular language: Any language represented by regular expression(s) is called a regular language.

Ex: The regular expression a^* denotes a language which has $\{\varepsilon, a, aa, aaa, \ldots\}$

Regular expression

- Regular expressions are used to denote regular languages.
- The set of regular expressions over an alphabet Σ is defined recursively as below. Any element of that set is a regular expression.

- Basis clause: $\varnothing$, $\in$, a are regular expression corresponding to languages $\varnothing$, $\{\in\}$, $\{a\}$ respectively where a is an element of Σ.
- Inductive clause: If r and s are regular expression corresponding to languages L_r and L_s then $(r + s)$, (rs) and (r^*) are regular expressions corresponding to the languages $L_r \cup L_s$, $L_r \cdot L_s$ and L_r^* respectively.
- External clause: Nothing is a regular expression, unless it is obtained from above two clauses.

Closure property of regular expressions The iteration or closure of a regular expression R, written as R^* is also a regular expression.

Ex: $\Sigma = \{a\}$ then a^* denotes the closure of Σ.

$a^* = \{\varepsilon, a, aa, aaa, \ldots\}$

Conventions on regular expressions

1. The operation '*' has highest precedence over concatenation, which has precedence over union (+).
   i.e., $RE\ (a + (b(c^*))) = a + bc^*$
2. The concatenation of K r's, where r is a regular expression is written as r^k. The language corresponding to r^k is L_r^k. Where L_r is language corresponding to regular expression r i.e., $rr = r^2$
3. r^+ is a regular expression to represent L_r^+

Note: A regular expression is not unique for a language i.e., regular language corresponds to more than one regular expression.

Example 15: Give regular expression for set of strings which either have 'a' followed by some b's or all b's also containing 'ε'.

(A) $b^* + ab^*$ (B) $a^* + ba^*$
(C) $(\varepsilon) + (\varepsilon + a)\ b^+$ (D) $b^* + ab^* + \varepsilon$

Solution: (C)

The regular expression is, $r = ab^+ + b^+ + \varepsilon = b^+(a + \varepsilon) + \varepsilon$.

Identity rules for regular expressions:

1. $\varnothing + R = R$
2. $\varnothing \cdot R = R\varnothing = \varnothing$
3. $\varepsilon R = R\varepsilon = R$
4. $\varnothing^* = \varepsilon$ and $\varepsilon^* = \varepsilon$
5. $R + R = R$
6. $RR^* = R^*\ R = R^+$
7. $\varepsilon + RR^* = R^*$ and $\varepsilon + R^*\ R = R^*$
8. $(R^*)^* = R^*$
9. $R^*\ R^* = R^*$
10. $\varepsilon + R^* = R^*$
11. $(R + \varepsilon)^* = R^*$
12. $R^*(\varepsilon + R)^* = (\varepsilon + R)^*\ R^* = R^*$
13. $R^*\ R + R = R^*\ R$
14. $(P + Q)R = PQ + QR$ and $R(P + Q) = RP + RQ$
15. $(P + Q)^* = (P^*\ Q^*)^* = (P^* + Q^*)^*$
16. $(PQ)^*\ P = P\ (QP)^*$
17. R is given as, $R = Q + RP$ has unique solution, $R = QP^*$. This is Arden's theorem.
18. $(P + Q)^* = (P^* + Q) = (P + Q^*)$

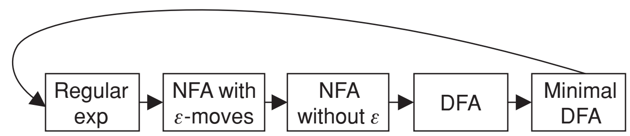
Example 16: If r_1 and r_2 are regular expressions denoting languages L_1 and L_2 respectively then which of following is false?

(A) $(r_1)|\ (r_2)$ is regular expression denoting $L_1 \cup L_2$.
(B) $(r_1)\ (r_2)$ is regular expression denoting $L_1 \cdot L_2$.
(C) $\varnothing$ is not a regular expression.
(D) $\{r_1\}^*$ is regular expression denoting L_1^*.

Solution: (C)

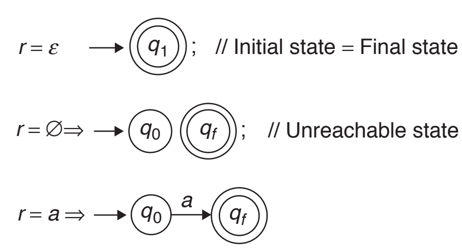
CONSTRUCTING FA FOR GIVEN RE

- Relationship between FA and RE.



Identities:

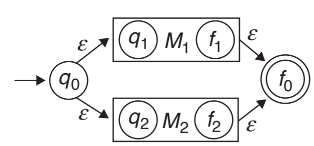
Basis:



Induction:

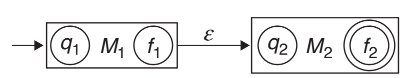
- **Union:** $L(r) = L\ (r_1) + L\ (r_2)$ i.e., $L\ (M) = L\ (M_1) \cup L\ (M_2)$

Let $M_1 = (Q_1, \Sigma_1, \delta_1, q_1, \{f_1\})$, $M_2 = (Q_2, \Sigma_2, \delta_2, q_2, \{f_2\})$ with $L\ (M_1) = L\ (r_1)$ and $L\ (M_2) = L\ (r_2)$, then $M = (Q_1 \cup Q_2 \cup \{q_0, f_0\}, \Sigma_1 \cup \Sigma_2, \delta, q_0, \{f_0\})$



- **Concatenation:**

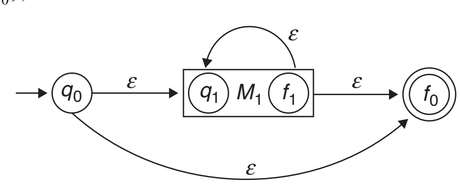
$L(r) = L\ (r_1) \cdot L\ (r_2)$ i.e., $L\ (M) = L\ (M_1) \cdot L\ (M_2)$



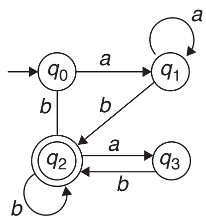
- **Closure:**

$L(r) = L(r)^*$ i.e., $L\ (M) = L\ (M_1)^*$

Let $M_1 = (Q_1, \Sigma_1, \delta_1, q_1, \{f_1\})$ then $L\ (M) = (Q_1 \cup \{q_0, f_0\}, \Sigma_1, \delta, q_0, \{f_0\})$



Example 17: The regular expression generated by the given FA.



(A) $(a + ba^*)\ b^*$ (B) $(aa^*b + bb^*)\ b^*$
(C) $(b + ab^*)\ a^*$ (D) $(ab + ba)^*$

Solution: (B)
q_2 is final state which is obtained with input symbol only 'b'. So, (C) or (D) is not true.
In (A) $\rightarrow$ ba^* is not defined in given *FA*. Instead bb^* is defined.

Pumping Lemma for Regular Sets

Theorem Let 'L' be an arbitrary regular language. Then there exists a positive integer, P with following property:

Given an arbitrary member, w of L having length at least P (i.e., $|w| \geq P$), w can be divided into 3-parts, $w = xyz$ $\exists$

- $|y| \geq 1$ (the middle part is non-empty)
- $|xy| \leq P$ (the first two parts have length atmost P)
- For each, $i \geq 0$, $xy^iz \in L$ (removing or repeating the middle part produces member of L)

Proof Let L be an arbitrary regular language. Then there is a *FA*, say M that decides L.
Let P be the number of states of M.
Let w be an arbitrary member of L, having length 'n' with $n \geq P$.
Let $q_0, q_1, \ldots q_n$ be states that M on input w. That is, for each i, after reading the first i symbols of w, M is at q_i.
q_0 is initial state of M. Also, since $w \in L$, q_n is a final state of M.
Let $x = w_1 \ldots w_c$, $y = w_{c+4} \ldots w_d$, $z = w_{d+1} \ldots w_n$. Then:

- $|y| \geq 1$
- $|xy| \leq P$
- M transitions from q_0 to q_c on x.
- M transitions from q_c to q_c on y.
- M transitions from q_c to q_n on z.

Thus, for every $i \geq 0$, M transitions from q_0 to q_n on xy^iz and so, xy^iz is a member of L.

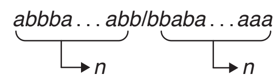
Note:

- Pumping lemma is used to verify that given language is not regular.
- Pumping lemma follows pigeon hole principle.

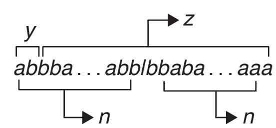
Example 18: The language, L is defined as:
$L = \{w_1w_2 : w_1, w_2 \in \{a, b\}^*, |w_1| = |w_2|\}$. Is the language regular?

(A) Regular
(B) Not regular
(C) Cannot be determined
(D) None of these

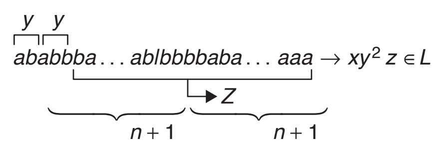
Solution: (A)
Fix pumping length, $K = 2$
For every proper strings in L, $(2n \geq 2)$



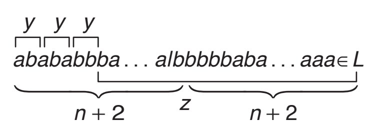
- Split in x, y, z with desired properties.



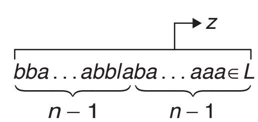
- Let $x = \varepsilon$, y = first two symbols, z = rest.



- xy^3z:



- $xy^oz \rightarrow$



$\therefore$ For every $i \geq 0$, $xy^iz \in L$. Hence given language is regular.

Closure Properties of Regular Sets

1. **Union:** If L and M are regular languages, LUM is regular language closed under union.
2. **Concatenation and Kleen closure:** If L and M are regular languages, L.M is regular language and L^* is also regular.
3. **Intersection:** $L \cap M$ is regular, if L and M are regular languages.
4. **Difference:** $L - M$ contains strings in 'L' but not M, where L and M are regular languages.
5. **Complementation:** The complement of language L is $\Sigma^*\!-\!L$.

Note: Since Σ^* is surely regular, the complement of a regular language is always regular. Where Σ^* is a universal language.

6. **Homomorphism:** If L is a regular language, h is homomorphism on its alphabet then $h\,(L) = \{h\,(w)\ |w$ is in $L\}$ is also a regular language.

Regular grammar

- **Grammar:** Generative description of a language.
- **Automaton:** Analytical description.
- A grammar is a 4-tuple, G = (V, Σ, R, S) where V: alphabet (variable) (non-terminals)

$\Sigma \subseteq V$ is set of terminal symbols.
$R \subseteq (V^+ \times V^*)$ is a finite set of production rules.
$S \in V - \Sigma$ is start symbol.

Notation

- Elements of $V - \Sigma$: $A, B, \ldots$
- Elements of Σ: $a, b \ldots$
- Rules $(\alpha, \beta) \in R$: $\alpha \to \beta$ or $\alpha \xrightarrow[G]{} \beta$
- Start symbol is written as S.
- Empty word: ε

Example 19: The regular expression that describe the language generated by grammar, $G = (\{S, A, B\}, \{a, b\}, S, \{S \to Aab, A \to Aab|B, B \to a\}$

(A) $(ab)^* a$
(B) $aab(ab)^*$
(C) $ab^* aa$
(D) $(a + ba)^*$

Solution: (B)
$S \to Aab \to Aab\ ab \to A\ ab\ abab \to Bababab$
$\to aababab \to aab(ab)^*$

Union of two Regular languages:
If L_1 and L_2 are two languages then
$L_1 \cup L_2 = \{w/w \in L_1 \text{ or } w \in L_2\}$
The union of two regular languages is also a regular language.
Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, f_1)$
$M_2 = (Q_2, \Sigma, \delta_2, q_2, f_2)$
$M = M_1 U M_2$ can be given as
$M = (Q, \Sigma, \delta, q_0, f)$.
Where $Q = \{(r_1, r_2) \mid r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$
i.e., Q is the Cartesian product of sets Q_1 and Q_2.
Σ is the alphabet, is the same in M_1 and M_2.
$\Sigma = \Sigma_1 \cup \Sigma_2$.
δ is the transition function given as:
$\delta(r_1, r_2), a = (\delta_1(r_1, a)\ \delta_2(r_2, a))$.
q_0 is the pair (q_1, q_2).
F is the set of pairs in which either member is an accept state of M_1 or M_2.
$F = \{(r_1, r_2) \mid r_1 \in F_1 \text{ or } r_2 \in F_2\}$

Types of Grammars

- Type 0: Unrestricted, recursively enumerable languages.
- Type 1: Context-sensitive grammar.
- Type 2: Context free grammar.
- Type 3: Regular grammar.

Type 0: Recursively enumerable grammar: (Turing Machine) (TM):

Every production rule is of form: $\alpha \to \beta$, where α and β are in $(V \cup T)^*$, i.e., there can be any strings of terminals and non-terminals (no-restriction).

Type 1: Context-sensitive Grammar: (Linear bounded automaton) (LBA):

Every production rule is of form, $\alpha \to \beta$ are in $(V \cup T)^*$ and $\alpha \neq \varepsilon$ and $|\beta| \geq |\alpha|$ i.e., any strings of terminals and non-terminals and length of string that can appear on RHS of production must be greater than or equal to length of string that can appear on LHS of production.

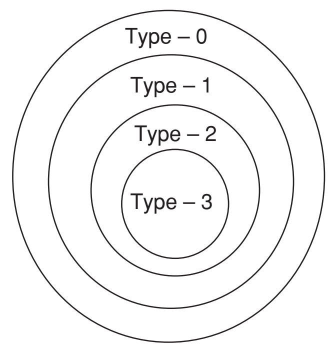
Type 2: Context-free grammar: (Push down automaton) (PDA):

Every production rule is of form, $A \to \alpha$ where α is in $(V \cup T)^*$ i.e., LHS of rule is single non-terminal and RHS can be any string of terminals and non-terminals.

Type 3: Regular grammar: (Finite automaton) (FA):

Every production is of form, $A \to aB$ or $A \to a$ where A and $B \in V$ and $a \in T$. That is, LHS of rule is non-terminal and RHS can be terminal (or) terminal followed by non-terminal.

Relationship between types of grammar:



- Regular sets are properly contained in CFL (Context Free Languages).
- The CFL's not containing empty string ε, are properly contained in CSL. (Context sensitive language).
- The CSL's are properly contained in Recursively enumerable languages.
- RG $\subset$ CFG $\subset$ CSL $\subset$ REG

Left-linear Grammar:
All productions have form: $A \to Bx$ or $A \to x$

Right-linear Grammar:
All productions have the form: $A \to xB$ or $A \to x$.

Note:

- The regular grammars characterize the regular sets i.e., a language is regular if and only if it has a left-linear grammar or if and only if it has a right-linear grammar.
- If L has a regular grammar, then L is a regular set.
- If L is a regular set, then L is generated by some left-linear grammar and by some right-linear grammar.

Arden's theorem: Let P and Q be two regular expressions over Σ. If P does not contain 'ε' then the following equation in R, namely $R = Q + RP$ has a unique solution given by $R = QP^*$.

Arden's Theorem to obtain regular expression from given transition diagram: The following steps are used to find the RE recognized by transition system.

The following assumptions are made regarding the transition system.

(i) The transition graph does not have ε-moves
(ii) It has only one initial state, q_o.
(iii) The states in the transition diagram are $q_o, q_1, q_2, \ldots q_n$.
(iv) Q_i, the regular expression represents the set of strings accepted by a system even though q_i is the final state.
(v) ${}^{a}ij$ denotes the regular expression representing the set of labels of edges from q_i to q_j. When there is no such edge ${}^{a}ij = \phi$.

We will get the following set of equations.

$$Q_1 = Q_1\alpha_{11} + Q_2\alpha_{12} + \ldots Q_n\alpha_{n1} + \varepsilon$$
$$Q_2 = Q_1\alpha_{12} + Q_2\alpha_{22} + \ldots Q_n\alpha_{n2}$$
$$\vdots$$
$$Q_n = Q_1\alpha 1n + Q_2\alpha_{2n} + \ldots Q_n\alpha_{nn}.$$

By Repeatedly applying substitutions and Arden's theorem, we can express Q_i in terms of α_{ij}'s.

For getting the set of strings recognized by the transition system, we have to take the union of all Q_i's corresponding to final states.

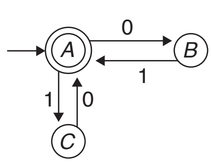
Construction of Regular Grammar from FA

Step I: Associate suitable variables like A, B, C … with states of automata.

Step II: Obtain the productions of the grammar as: If $\delta(A, a) = B$ then add production $A \to aB$ to list of productions of grammar, if B is a final state, then add either $A \to a$ or $B \to \varepsilon$, to list of productions of grammar.

Step III: The variable associated with initial state of automata is start symbol of grammar.

Example 20: Regular grammar generating language accepted by below automata is



(A) $A \to 0B|1C|\varepsilon$
$B \to 1A$
$C \to 0A$
(B) $A \to 1B|0C|\varepsilon$
$B \to 1A$
$C \to 0A$
(C) $A \to B|C|\varepsilon$
$B \to 1$
$C \to 0$
(D) $A \to 0A|1B|\varepsilon$
$B \to 1C$
$C \to 0A$

Solution: (A)
$A \to 0B$, $A \to 1C$, $B \to 1A$, $C \to 0A$
$\therefore$ A is final state, $A \to \varepsilon$

$\therefore A \to 0B\|1C\|\varepsilon$		$A \to 0B\|1C$
$B \to 1A$	(or)	$B \to 1A\|1$
$C \to 0A$		$C \to 0A\|0$

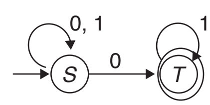
Construction of FA from given regular grammar

Given a regular grammar, G; a regular expression specifying $L(G)$ can be obtained directly as follows:

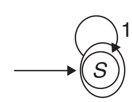
- Replace the '$\to$' symbol in productions of grammar by '=' symbol, to get set of equations.
- Solve the set of equations obtained above to get the value of variable, S, where S is start symbol of grammar, result is regular expression specifying $L(G)$.

Example 21: The Regular grammar and *FA* for given regular expression $\phi^* 1^* U\,(0\phi)^*$ is ____

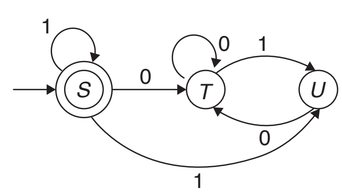
(A) $S \to 0S|1S|0$
$T \to 1T|\varepsilon$



(B) $S \to 1S|\varepsilon$

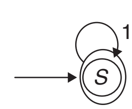


(C) $S \to 0T|1S|\varepsilon$
$T \to 0T|1U|\varepsilon$
$U \to 0T|1S$



(D) Cannot be determined

Solution: (B)
$\varnothing^*\,1^* \cup (0\varnothing)^* = \varnothing^* \cdot 1^* \cup \varnothing^* = \varepsilon \cdot 1^* \cup \varepsilon = 1^*.$

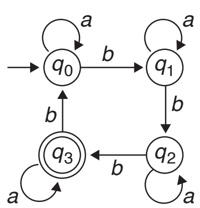


Exercises

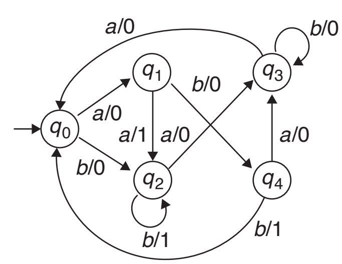
Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Find a regular expression for
   $L = \{uvu: u, v \in \{a, b\}^*, |u| = 2\}$
   (A) $(ab)^*a(ab)^*$
   (B) $(aa)^*ab(aa)^*$
   (C) $aa(a + b)^*bb + bb(a + b)^*aa$
   (D) $aa(a + b)^*aa + ab(a + b)^* ab + ba (a + b)^* ba + bb (a + b)^*bb$
2. Consider the regular expression, $R = 10 + (0 + 11)0^* 1$. The minimum number of states in any DFA accepting this regular expression is:
   (A) 5 (B) 4
   (C) 3 (D) 6
3. The following DFA accepts the set of all strings over $\{a, b\}$ that



   (A) Contains number of b's divisible by 3.
   (B) Contain number of a's and b's divisible by 3
   (C) Contain number of b's congruent to 3 modulo 4.
   (D) Contain any number of a's and b's
4. Consider the grammar, $S \rightarrow SS/a$. To get string of n terminals, the number of productions to be used is
   (A) n^2 (B) n
   (C) 2^{n+1} (D) $2n–1$
5. The language L is defined as, $L = \{a^i\, b^j\, c^{2j} |\ i \geq 0, j \geq 0\}$. Is this language L regular?
   (A) Yes (B) No
   (C) Cant be determined (D) None of these
6. The language, L is defined by set of strings over $\{a, b\}^*$ in which number of a's is a perfect cube. What is the nature of language, L?
   (A) Regular (B) Non-regular
   (C) Cant be determined (D) None of these
7. The language, L is defined over $\Sigma = \{0 – 7\}$. The string include 7, 16, 43, 61, 223, … The language generated is:
   (A) Alternate odd and even numbers
   (B) Octal representation of a number
   (C) Divisible by 7.
   (D) Octal representation of a number divisible by 7.
8. The language L, is defined as set of strings that start and end with equal number of a's and contain any number of b's. The grammar $L(G)$ for language L is defined with productions as:
   (A) $S \rightarrow aBa$
   $B \rightarrow \varepsilon|bB$
   (B) $S \rightarrow aB$
   $B \rightarrow a|bB$
   (C) $S \rightarrow aT|bS$
   $T \rightarrow aT|bT|a|b$
   (D) $S \rightarrow B|aSa$
   $B \rightarrow \varepsilon|bB$
9. If the regular set A is represented by $A = ((01)^*1^*)^*$. And the regular set B is represented by $B = (01 + 1)^*$, which of the following is true?
   (A) $A \subset B$
   (B) $B \subset A$
   (C) $A = B$
   (D) A and B are incomparable
10. The language, L that is generated over $\Sigma = \{0, 1\}$ for regular expression $L(r) = (0 + 10)^*\, 1\, (1 + 10)^*$
   (A) Any string whose number of 1's length is greater than or equal to 3.
   (B) Any string that has no substring 110.
   (C) Any string that has no substring 00 after first 11.
   (D) Any string that has only one occurrence of substring 010.
11. The R.E $L(r) = (a^+b^*)\ U\ \varepsilon$. Is the grammar with productions generated over non-terminals $\{S, A\}$ ambiguous?
   (A) Yes (B) No
   (C) Can't be determined (D) None
12. The number of states in the obtained Moore machine while converting the given mealy to Moore are:



   (A) 5 (B) 6
   (C) 4 (D) 7
13. The language L is defined as $L = \{0^i 1^j / i \neq j\}$ over $\{0, 1, 2\}$, $A = \{0^i\ 1^j / i \geq 0, j \geq 0\}$ and $B = \{0^i\ 1^j / i = j\}$. For language, L to be non-regular. What should be relation between A, B, L?
   (A) $B = (A \cup L)^c$ (B) $B = A \cup L$
   (C) $B = A \cap \overline{L}$ (D) $B = A^c$
14. Which of following grammars are unambiguous?
   (A) $S \rightarrow (S)\ S|[S]\ S|\varepsilon$ (B) $S \rightarrow S(S)S|\varepsilon$
   (C) $S \rightarrow aS|Sa|a$ (D) $S \rightarrow a|Sa|bSS|Ssb|SbS$

15. What will be number of final states obtained in DFA for language $L = \{w/w$ contains at least two 0's and atmost one 1$\}$ over $\Sigma = \{0, 1\}$.

(A) 2 (B) 1
(C) 3 (D) 4

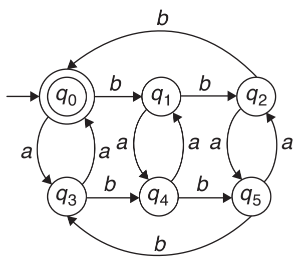
Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Which of the following Regular expression is equal to given regular expression: $(b + aa^* b) + (b + aa^*b)(a + ba^* b)^* (a + ba^* b)$

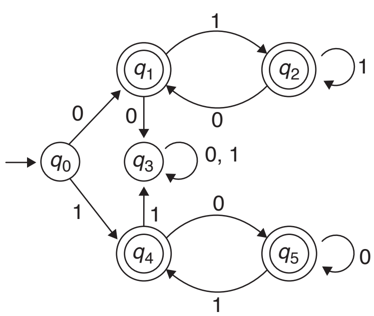
(A) $Ab (b + baa^*)$ (B) $a^*b (a + ba^* b)$
(C) $a^* b (a + ba^* b)^*$ (D) $ab (b + aa^*b)^*$

2. The following DFA accepts set of all strings over $\{a, b\}$ that contain



(A) Number of a's even and number of b's odd.
(B) Consecutive a's and b's
(C) Contain bbb as substring
(D) Number of a's even and number of b's divisible by three.

3. The regular language $L(r)$ for the given FSM is:

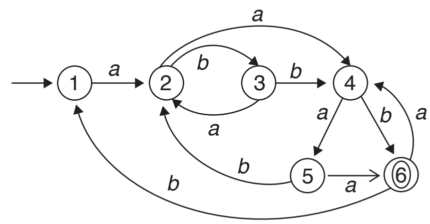


(A) It can start with zero followed by any number of 1's but no two consecutive 0's.
(B) It can start with 1, followed by any number of 0's but no two consecutive 1's.
(C) It is a combination of 0's and 1's but no two consecutive 0's or 1's.
(D) Both (A) and (B).

4. The language, L is defined as a set of non-palindromes over $\{a, b\}$. Is L regular?

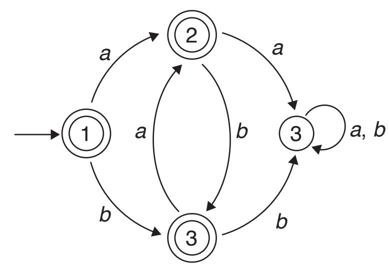
(A) Yes (B) No
(C) Cannot be determined (D) None of above

5. The DFA, for language, L over $\Sigma = \{a, b\}$ is given below. What will be number of states in minimized DFA.



(A) 4 (B) 6
(C) 2 (D) 3

6. The minimal DFA given below is defined for language, $L = \{w \in \{a, b\}^*\}$ over $\Sigma = \{a, b\}$. The 'L' is:



(A) Strings that contain equal number of a's and b's that have adjacent characters same.
(B) Contains adjacent characters same
(C) No two adjacent characters are same
(D) Starts and ends with same character that have adjacent character same.

7. The regular grammar $L(G)$ contains productions, P for language, $L = \{w \in \{a, b\}^* /$ there is at least one a$\}$ are:

(A) $S \to aS|bS|a|aT$
$T \to aT|bT|a|b$
(B) $S \to aS|bS|\varepsilon$
(C) $S \to aBb|bB$
$B \to a|b$
(D) $S \to bB$
$B \to b|\varepsilon$

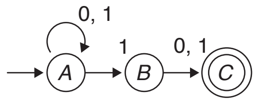
8. The regular expression for a language is defined as $((a^* b)^* (bc^*)^*)$. The total number of final states obtained in both NFA and DFA are respectively:

(A) 4, 2 (B) 1, 3
(C) 1, 5 (D) 2,3

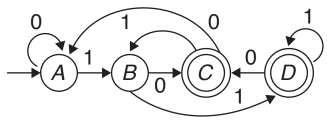
9. The language, L is defined as $\{w/w$ has n occurrences of 0's where n mod 5 is 3$\}$ over $\Sigma = \{0, 1\}$. The number of final states obtained in the DFA for L is:

(A) 4 (B) 5
(C) 1 (D) 2

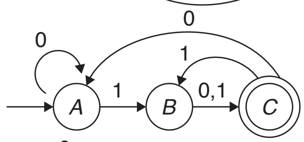
10. Which of the following is an equivalent DFA for the following NFA?



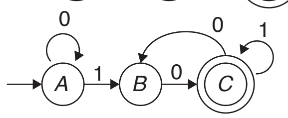
(A)



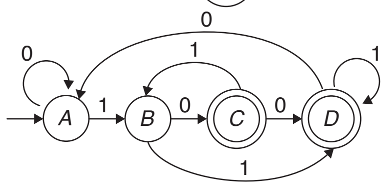
(B)



(C)



(D)



11. A regular grammar over alphabet $\Sigma = \{a, b, c, d\}$ whose language, is set of strings that contain exactly two b's is:

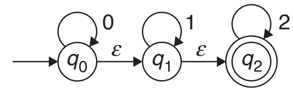
(A) $S \to aS|bS|cS|dA$
$A \to aA|bA|cA|dA|\varepsilon$

(B) $S \to aS|cS|dS|bB$
$B \to aB|cB|dB|bC,$
$C \to aC|cC|dC|\varepsilon$

(C) $S \to aS|bS|cS|dA$
$A \to aA|bB|cC$
$B \to b$
$C \to c$

(D) None of above

12. The following NFA contains ε-moves with 5, transitions. If this NFA with ε-moves is converted to NFA without ε-moves, what will be total number of transitions in obtained NFA?



(A) 5 (B) 4
(C) 6 (D) 3

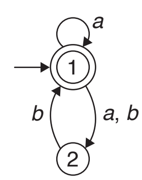
13. The regular expression, $r = (a + b)^*$. One more regular expression which represents same regular expression 'r' is:

(A) $a^* + b^*$ (B) $a^* \cdot b^*$
(C) $a^*(ba^*)^*$ (D) $(a + b)^* (a + b)$

14. The Regular grammar, $L(G)$ is defined for L with productions as $S \to Aab$, $A \to Aab|aB$, $B \to a$. What is Language generated by $L(G)$?

(A) Containing alternative a's and b's
(B) Containing alternative a's and b's, begins with an 'a' and ends with a 'b'.
(C) 'aa' followed by at least one set of alternating ab's.
(D) Consecutive aa's followed by 'b'.

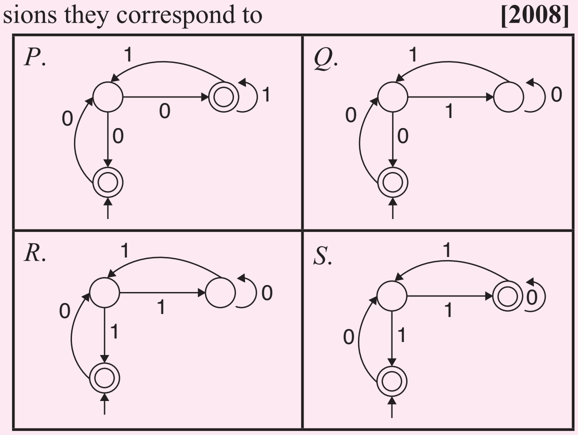
15. The number of final states in DFA after converting the NFA given below is:



(A) 4 (B) 2
(C) 3 (D) 1

Previous Years' Questions

1. Match the following NFAs with the regular expressions they correspond to **[2008]**



1. $\in + 0(01^*1 + 00)^*01^*$
2. $\in + 0(10^*1 + 00)^*0$
3. $\in + 0(10^*1 + 10)^*1$
4. $\in + 0(10^*1 + 10)^*10^*$

(A) P–2, Q–1, R–3, S–4
(B) P–1, Q–3, R–2, S–4
(C) P–1, Q–2, R–3, S–4
(D) P–3, Q–2, R–1, S–4

2. Which of the following are regular sets?

I. $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$
II. $\{a^n b^m \mid n = 2m\}$
III. $\{a^n b^m \mid n \neq m\}$
IV. $\{xcy \mid x, y \in \{a, b\}^*\}$ **[2008]**

(A) I and IV only (B) I and III only
(C) I only (D) IV only

3. Which one of the following languages over the alphabet $\{0, 1\}$ is described by the regular expression: $(0 + 1)^*0(0 + 1)^*0(0 + 1)^*$? **[2009]**

(A) The set of all strings containing the substring 00.
(B) The set of all strings containing atmost two 0's.
(C) The set of all strings containing at least two 0's.

(D) The set of all strings that begin and end with either 0 or 1.

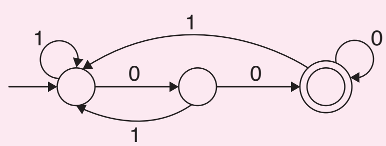
4. Which one of the following is FALSE? **[2009]**
(A) There is a unique minimal DFA for every regular language.
(B) Every NFA can be converted to an equivalent PDA.
(C) Complement of every context-free language is recursive.
(D) Every non-deterministic PDA can be converted to an equivalent deterministic PDA.

5. Match all items in Group 1 with correct options from those given in Group 2. **[2009]**

Group 1		Group 2	
P.	Regular expression	1.	Syntax analysis
Q.	Pushdown automata	2.	Code generation
R.	Dataflow analysis	3.	Lexical analysis
S.	Register allocation	4.	Code optimization

(A) P–4, Q–1, R–2, S–3
(B) P–3, Q–1, R–4, S–2
(C) P–3, Q–4, R–1, S–2
(D) P–2, Q–1, R–4, S–3

6.



The above DFA accepts the set of all strings over {0, 1} that **[2009]**
(A) Begin either with 0 or 1
(B) End with 0
(C) End with 00
(D) Contain the substring 00.

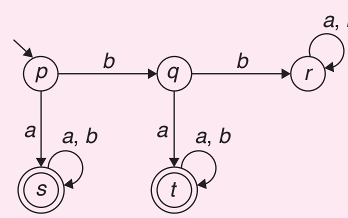
7. Let $L = \{w \in (0 + 1)^* \mid w$ has even number of 1's$\}$, i.e., L is the set of all bit strings with even number of 1's. Which one of the regular expressions below represents L? **[2010]**
(A) $(0^*10^*1)^*$
(B) $0^*(10^*10^*)^*$
(C) $0^*(10^*1^*)^*0^*$
(D) $0^*1(10^*1)^*10^*$

8. Consider the languages $L_1 = \{0^i1^j \mid i \neq j\}$. $L_2 = \{0^i1^j \mid i = j\}$, $L_3 = \{0^i1^j \mid i = 2j + 1\}$. $L_4 = \{0^i1^j \mid i \neq 2j\}$. Which one of the following statements is true? **[2010]**
(A) Only L_2 is context free
(B) Only L_2 and L_3 are context free
(C) Only L_1 and L_2 are context free
(D) All are context free

9. Let w be any string of length n in $\{0, 1\}^*$. Let L be the set of all substrings of w. What is the minimum number of states in a non-deterministic finite automaton that accepts L? **[2010]**
(A) n-1
(B) n
(C) n+1
(D) 2^{n-1}

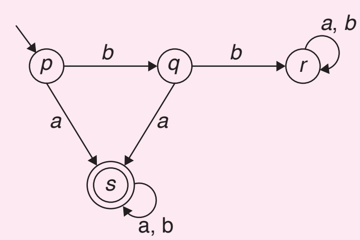
10. Let P be a regular language and Q be a context-free language such that $Q \subseteq P$ (For example let P be the language represented by the regular expression p^*q^* and Q be $\{p^nq^n\}$ $n \in N\}$, Then which of the following is ALWAYS regular? **[2011]**
(A) $P \cap Q$
(B) $P - Q$
(C) $\Sigma^* - P$
(D) $\Sigma^* - Q$

11. A deterministic finite automaton (DFA) D with alphabet $\Sigma = \{a, b\}$ is given below.

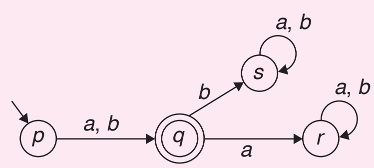


Which of the following finite state machines is a valid minimal DFA which accepts the same language as D? **[2011]**

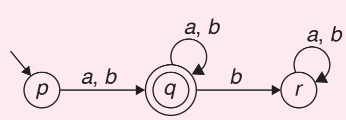
(A)



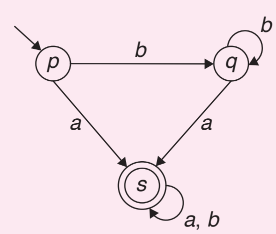
(B)



(C)

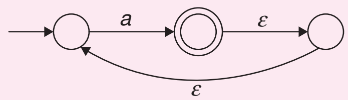


(D)



12. Given the language $L = \{ab, aa, baa\}$, which of the following strings are in L^*? **[2012]**
(1) *abaabaaabaa*
(2) *aaaabaaaa*
(3) *baaaaabaaaab*
(4) *baaaabaa*
(A) 1, 2 and 3
(B) 2, 3 and 4
(C) 1, 2 and 4
(D) 1, 3 and 4

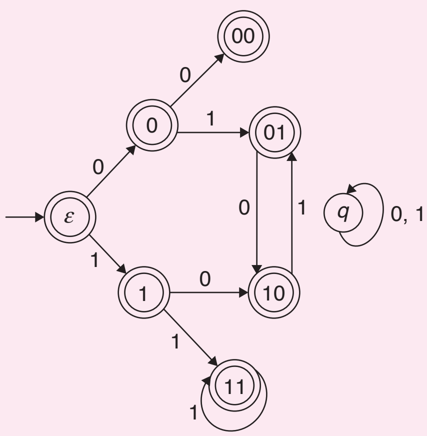
13. What is the complement of the language accepted by the NFA shown below?



Assume $\Sigma = \{a\}$ and ε is the empty string. **[2012]**

(A) $\varnothing$ (B) $\{\varepsilon\}$
(C) a^* (D) $\{a, \varepsilon\}$

14. Consider the set of strings on {0, 1} in which, every substring of 3 symbols has atmost two zeros. For example, 001110 and 011001 are in the language, but 100010 are not. All strings of length less than 3 are also in the language. A partially completed DFA that accepts this language is shown below.



The missing arcs in the DFA are **[2012]**

(A)

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

(B)

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

(C)

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

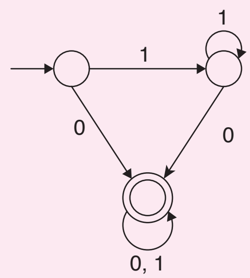
(D)

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

15. Consider the languages $L_1 = \Phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* U L_1^*$? **[2013]**

(A) $\{\in\}$ (B) Φ
(C) a^* (D) $\{\in, a\}$

16. Consider the DFA A given below

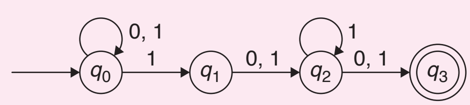


Which of the following are FALSE?

1. Complement of $L(A)$ is context-free.
2. $L(A) = L((11^*0 + 0)(0 + 1)^*0^*1^*)$
3. For the language accepted by A, A is the minimal DFA.
4. A accepts all strings over {0, 1} of length at least 2. **[2013]**

(A) 1 and 3 only (B) 2 and 4 only
(C) 2 and 3 only (D) 3 and 4 only

17. Consider the finite automaton in the following figure. **[2014]**



What is the set of reachable states for the input string 0011?

(A) $\{q_0, q_1, q_2\}$ (B) $\{q_0, q_1\}$
(C) $\{q_0, q_1, q_2, q_3\}$ (D) $\{q_3\}$

18. If $L_1 = \{a^n | n \geq 0\}$ and $L_2 = \{b^n | n \geq 0\}$, consider the statements **[2014]**

(I) $L_1 \cdot L_2$ is a regular language
(II) $L_1 \cdot L_2 = \{a^n b^n | n \geq 0\}$

Which one of the following is CORRECT?

(A) Only (I) (B) Only (II)
(C) Both (I) and (II) (D) Neither (I) nor (II)

19. Let $L_1 = \{w \in \{0, 1\}^* | w$ has at least as many occurrences of (110)'s as (011)'s}. Let $L_2 = \{w \in \{0, 1\}^* | w$ has at least

as many occurrences of (000)'s as (111)'s}. Which one of the following is TRUE? **[2014]**

(A) L_1 is regular but not L_2
(B) L_2 is regular but not L_1
(C) Both L_1 and L_2 are regular
(D) Neither L_1 nor L_2 are regular

20. The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is ______. **[2014]**

$a^*b^*(ba)^*a^*$

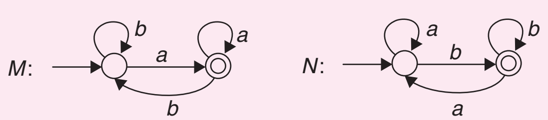
21. Let Σ be finite non-empty alphabet and let 2^{Σ^*} be the power set of Σ^*. Which one of the following is TRUE? **[2014]**

(A) Both 2^{Σ^*} and Σ^* are countable
(B) 2^{Σ^*} is countable and Σ^* is uncountable
(C) 2^{Σ^*} is uncountable and Σ^* is countable
(D) Both 2^{Σ^*} and Σ^* are uncountable

1. $\varepsilon + 0\,(01^*\,1 + 00)^*\,01^*$
2. $\varepsilon + 0\,(10^*\,1 + 00)^*\,0$
3. $\varepsilon + 0\,(10^*\,1 + 10)^*\,1$
4. $\varepsilon + 0\,(10^*\,1 + 10)^*\,10^*$

(A) $P-2, Q-1, R-3, S-4$
(B) $P-1, Q-3, R-2, S-4$
(C) $P-1, Q-2, R-3, S-4$
(D) $P-3, Q-2, R-1, S-4$

22. Consider the DFAs M and N given above. The number of states in a minimal DFA that accepts the language $L(M) \cap L(N)$ is _______ **[2015]**



23. The number of states in the minimal deterministic finite automaton corresponding to the regular expression $(0 + 1)^*(10)$ is ______ **[2015]**

24. Which of the following languages is/are regular? **[2015]**

L_1: $\{wxw^R | w, x \in \{a, b\}^*$ and $|w|, |x| > 0\}$, w^R is the reverse of string w

L_2: $\{a^nb^m | m \neq n$ and $m, n \geq 0\}$

L_3: $\{a^pb^qc^r | p, q, r \geq 0\}$

(A) L_1 and L_3 only
(B) L_2 only
(C) L_2 and L_3 only
(D) L_3 only

25. Consider the alphabet $\Sigma = \{0, 1\}$, the null/empty string λ and the sets of strings X_0, X_1 and X_2 generated by the corresponding non-terminals of a regular grammar. X_0, X_1 and X_2 are related as follows

$X_0 = 1X_1$
$X_1 = 0X_1 + 1X_2$
$X_2 = 0X_1 + \{\lambda\}$

Which one of the following choices precisely represents the strings in X_0? **[2015]**

(A) $10(0^* + (10)^*)1$
(B) $10(0^* + (10)^*)^*1$
(C) $1(0 + 10)^*1$
(D) $10(0 + 10)^*1 + 110(0 + 10)^*1$

26. Let L be the language represented by the regular expression $\Sigma^*\ 0011\ \Sigma^*$ where $\Sigma = \{0, 1\}$. What is the minimum number of states in a DFA that recognizes $\overline{L}$ (complement of L)? **[2015]**

(A) 4
(B) 5
(C) 6
(D) 8

27. Which of the following languages is generated by the given grammar? **[2016]**

$S \rightarrow aS \mid bS \mid \varepsilon]$

(A) $\{a^n\,b^m \mid n, m \geq 0\}$
(B) $\{w \in \{a, b\}^* \mid$ w has equal number of a's and b's$\}$
(C) $\{a^n \mid n \geq 0\}\ U\ \{b^n \mid n \geq 0\}\ U\ \{a^nb^n \mid n \geq 0\}$
(D) $\{a, b\}^*$

28. Which of the following decision problems are undecidable? **[2016]**

I. Given NFAs N_1 and N_2, is $L(N_1) \cap L(N_2) = \Phi$?
II. Given a CFG $G = (N, \Sigma, P, S)$ and a string $x \in \Sigma^*$, does $x \in L(G)$?
III. Given CFGs G_1 and G_2, is $L(G_1) = L(G_2)$?
IV Given a TM M, is $L(M) = \Phi$?

(A) I and IV only
(B) II and III only
(C) III and IV only
(D) II and IV only

29. Which one of the following regular expressions represents the language: *the set of all binary strings having two consecutive 0's and two consecutive 1s?* **[2016]**

(A) (0+1)* 0011 (0+1)* + (0+1)* 1100 (0+1)*
(B) (0+1)* (00(0+1)*11 + 11 (0+1)*00) (0+1)*
(C) (0+1)* 00 (0+1)* + (0+1)* 11 (0+1)*
(D) 00 (0+1)* 11 + 11 (0+1)* 00

30. The number of states in the minimum sized **DFA** that accepts the language defined by the regular expression (0+1)* (0+1) (0+1)* is ______ . **[2016]**

31. Language L_1 is defined by the grammar: $S_1 \rightarrow aS_1b \mid \in$
Language L_2 is defined by the grammar: $S_2 \rightarrow abS_2 \mid \in$

Consider the following statements:
P:L_1 is regular
Q:L_2 is regular
Which one of the following is TRUE? **[2016]**
(A) Both P and Q are true
(B) P is true and Q is false
(C) P is false and Q is true
(D) Both P and Q are false

32. Consider the following two statements:

I. If all states of an NFA are accepting states then the language accepted by the NFA is Σ^*.

II. There exists a regular language A such that for all languages B, A $\cap$ B is regular.

Which one of the following is **CORRECT**? **[2016]**
(A) Only **I** is true
(B) Only **II** is true
(C) Both **I** and **II** are true
(D) Both **I** and **II** are false

33. Consider the language L given by the regular expression $(a + b)^*b\ (a + b)$ over the alphabet $\{a, b\}$. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is ________. **[2017]**

34. The minimum possible number of states of a deterministic finite automaton that accepts the regular language $L = \{w_1aw_2 | w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is ___________. **[2017]**

35. Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the $\in$-NFA whose transition table is given below:

δ	$\in$	a	b
$\rightarrow q_0$	$\{q_2\}$	$(q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	$\varnothing$	$\varnothing$
q_3	$\varnothing$	$\varnothing$	$(q_2\}$

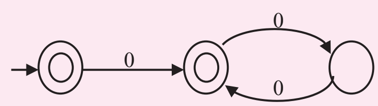
Then $\hat{\delta}\ (q_2, aba)$ is **[2017]**
(A) $\varnothing$ (B) $\{q_0, q_1, q_3\}$
(C) $\{q_0, q_1, q_2\}$ (D) $\{q_0, q_2, q_3\}$

36. Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true? **[2018]**
(A) $k \geq 2^n$ (B) $k \geq n$
(C) $k \leq n^2$ (D) $k \leq 2^n$

37. Given a language L, define L^i as follows:

$$L^0 = \{\varepsilon\}$$
$$L^i = L^{i-1}.\ L \text{ for all } i > 0$$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$. Consider the language L_1 (over alphabet 0) accepted by the following automaton.



The order of L_1 is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. D	**2.** B	**3.** C	**4.** D	**5.** B	**6.** B	**7.** D	**8.** D	**9.** C	**10.** C
11. A	**12.** D	**13.** C	**14.** A	**15.** A					

Practice Problems 2

1. C	**2.** D	**3.** D	**4.** B	**5.** B	**6.** C	**7.** A	**8.** C	**9.** C	**10.** A
11. B	**12.** C	**13.** C	**14.** C	**15.** B					

Previous Years' Questions

1. C	**2.** A	**3.** C	**4.** D	**5.** B	**6.** C	**7.** B	**8.** D	**9.** C	**10.** C
11. A	**12.** C	**13.** B	**14.** D	**15.** A	**16.** D	**17.** A	**18.** A	**19.** A	**20.** C
21. C	**22.** 1	**23.** 3	**24.** A	**25.** C	**26.** B	**27.** D	**28.** C	**29.** B	**30.** 2
31. C	**32.** B	**33.** 4	**34.** 8	**35.** C	**36.** D	**37.** 2			

Chapter 2

Context Free Languages and Push Down Automata

LEARNING OBJECTIVES

- *Context free grammar*
- *Context free language*
- *Ambiguity in context free grammars*
- *Removing ε-productions*
- *Removing unit productions*
- *Normal forms*
- *Chomsky's normal form*
- *Greiback normal form*
- *Closure properties of CFL's*
- *Push down automata*
- *PDAs accepting by final state and empty stack are equivalent*
- *Converting CFG to PDA*
- *Deterministic PDA*

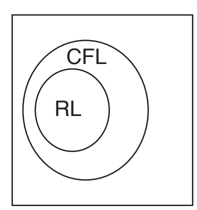
Context Free Grammar

- A context free grammar (CFG) is a finite set of variables (non-terminals) each of which represents a language. The language represented by variables is described recursively in terms of each other. The primitive symbols are called terminals.
- The rules relating variables are called productions. A typical production states that the language associated with a given variable contains strings that are formed by concatenating strings from languages of certain other variables.
- CFG is a collection of three things;
  An alphabet Z of letters called terminals.
  A set of symbols called non-terminals, one of which is a start symbol, S.
  A finite set of productions of the form:
  One terminal $\to$ finite set of terminals and/or non-terminals.
- A CFG is defined as: $G = (V, T, P, S)$
  Where
  $V \to$ Finite set of variables (non-terminals)
  $T \to$ Finite set of terminals (symbols)
  $P \to$ Finite set of productions, each, production is of the form,
  $A \to \alpha, A \in V, \alpha \in (V \cup T)^*$
  $S \to$ Start symbol

Context Free Language (CFL)

The language generated by CFG is a set of all strings of terminals that can be produced from start symbols, using the productions as substitutions. A language generated by a CFG is called context free language (CFL).

Note: Every regular grammar is context free, so a regular language (RL) is also context free.
Family of RL's is proper subset of CFL's.
i.e., RL $\subset$ CFL

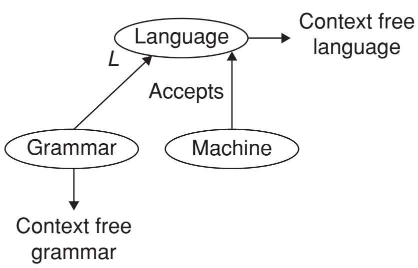


Solved Examples

Example 1: What is the language that is generated by CFG, $G = S \to AB|A \to +/–|B \to CB/C|C \to 0/1/2/ \dots 9$.
(A) Set of all rational numbers
(B) Set of all integers
(C) Set of all natural numbers
(D) Set of all complex numbers

Solution: (B)
$S \to AB|A \to +/–|B \to CB/C|C \to 0/1/2/ \dots 9$
Consider-18 (integer)

$S \rightarrow AB$
$\rightarrow -B$
$\rightarrow -CB$
$\rightarrow -1B$
$\rightarrow -18$



Ambiguity in Context Free Grammars

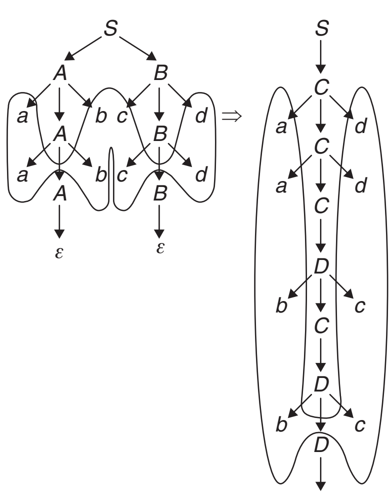
A CFG, G is called ambiguous if there is $w \in L(G)$ such that w has (at least) two different parse trees with respect to G.

Example 2: The language, $L = \{a^n\ b^n\ c^m\ d^m / n \geq 0,\ m \geq 0\} \cup \{a^n b^m c^m d^n / n \geq 0,\ m \geq 0\}$ is designed in CFG, G. The Grammar is
(A) Ambiguous
(B) Unambiguous
(C) Cannot be determined
(D) None of above

Solution: (A)
CFG G for given language L is:

$S \rightarrow AB|C$
$A \rightarrow aAb|\varepsilon$
$B \rightarrow cBd|\varepsilon$
$C \rightarrow aCd|$D
$D \rightarrow bDc|\varepsilon$

It's an inherently ambiguous grammar.
Consider string, aabbccdd



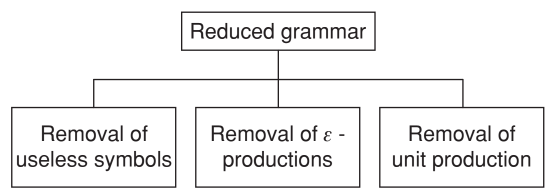
Note:
- A context free language with property that all grammars that generate it are ambiguous is inherently ambiguous.
- Inherently ambiguous grammars cannot convert to unambiguous grammars.

Minimization of Context Free Grammar

- Grammar may consist of some extra symbols (non-terminals). Having extra symbols unnecessarily increases the length of grammar.
- Simplification of grammar means reduction of grammar.

The properties of reduced grammar are:

1. Each variable (non-terminal) and each terminal of G appears in the derivation of some word in L.
2. There should not be any production as $X \rightarrow Y$ where X and Y are non-terminals.
3. If ε is not in language L, then there need not be production $X \rightarrow \varepsilon$.



Removal of Useless Symbols

- Any symbol is useful when it appears on right hand side, in the production rule and generates some terminal string. If no such derivation exists, then it is supposed to be a useless symbol.
- A symbol P is useful, if there exists some derivation

$$S^* \Rightarrow \alpha PB \text{ and } \alpha PB \overset{*}{\Rightarrow} W$$

Then P is said to be useful symbol.

Example 3: A grammar G', is generated by removing useless symbols from G defined below. The obtained G' contains productions:

$S \rightarrow aA|bB$
$A \rightarrow aA|a$
$B \rightarrow bB$
$D \rightarrow ab|Ea$
$E \rightarrow aC|d$

(A) $S \rightarrow aA$
$A \rightarrow aA|a$
(B) $S \rightarrow aS|bA|C$
$A \rightarrow a$
$C \rightarrow aCd$
(C) $S \rightarrow aA|bB$
$A \rightarrow aA|a$
$B \rightarrow bB$
(D) Cannot remove useless symbols

Solution:

$S \rightarrow aA \rightarrow aaA \rightarrow aaaA \rightarrow aaaa$ ✓

$B \rightarrow bB \rightarrow bbB \rightarrow bbbB \rightarrow bbbbB$ ..... (string cannot be generated)

$\therefore$ B is useless

D and E cannot be generated from 'S'. So, eliminate. Hence G' contains

$\therefore$ $S \rightarrow aA$

$A \rightarrow aA|a$

Removing ε-Productions

A production of the form $A \rightarrow \varepsilon$ is called an ε-production. If A is a non-terminal and $A \rightarrow (^*)\ \varepsilon$, then A is called a 'nullable non-terminal'. So eliminate such productions without changing meaning of grammar.

Example 4: The grammar, G is given below. The CFG generated after eliminating ε-production is:

$S \rightarrow ABC$
$A \rightarrow BC|a$
$B \rightarrow bAC|\in$
$C \rightarrow cAB|\in$

(A) $S \rightarrow ABC|AB|BC|CA$
$A \rightarrow BC|B|C$
$B \rightarrow bAC|bA|bC$
$C \rightarrow cAB|cA|cB$

(B) $S \rightarrow ABc$
$A \rightarrow BC$
$B \rightarrow bAC$
$C \rightarrow cAB$

(C) $S \rightarrow ABC|BC|AC|AB|A|B|C$
$A \rightarrow BC|B|C|a$
$B \rightarrow bAC|bA|bC|b$
$C \rightarrow cAB|cA|cB|c$

(D) None of these

Solution (C)

$B \rightarrow \in, C \rightarrow \in$

$\Rightarrow$ $A \rightarrow \in$

$\therefore$ Remove ε-productions and obtained CFG is Choice (C).

Removing Unit Productions

- A production of form $A \rightarrow B$, where A and B are both non-terminals, is called a 'unit production'.
- Presence of unit production in a grammar increases the cost of derivation.

Example 5: The total number of productions obtained by removing unit production from the Grammar,

$A \rightarrow PQ$
$P \rightarrow 0$
$Q \rightarrow R|1$
$R \rightarrow S$
$S \rightarrow W|1R$
$W \rightarrow 2|P1$

(A) 9 (B) 2
(C) 3 (D) 5

Solution: (A)

$A \rightarrow PQ$

$Q \rightarrow R \rightarrow S \rightarrow W \rightarrow 2$

$\Rightarrow$ $Q, R, S \rightarrow$ Unit production

$A \rightarrow PQ$

$P \rightarrow 0$

$Q \rightarrow R|1$
$\quad\downarrow$

$\Rightarrow$ $Q \rightarrow 2|P1|1R|1$ (substitute the production of R, S, W)

$\therefore$ $A \rightarrow PQ$

$P \rightarrow 0$

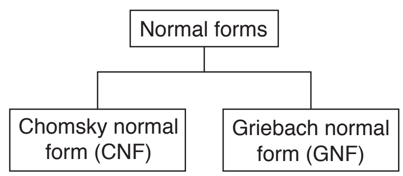
$Q \rightarrow 2|\ P1|\ 1R|\ 1$

$R \rightarrow 2|\ P1|\ 1R$

$\therefore$ 9 – Productions

Normal Forms

- It is necessary to have a grammar in some specific form so, grammar normalization is needed.



That is, There should be fixed number of terminals and non-terminals, in CFG.

Chomsky's Normal Form (CNF)

- A context free grammar (CFG), $G = (V, \Sigma, R, S)$ is said to be in CNF, if and only if every rule in R is of one of the following forms
  1. $A \rightarrow a$, for some $A \in V$ and some $a \in \Sigma$
  2. $A \rightarrow BC$, for some $A \in V$ and $B, C \in V \cup \{S\}$
  3. $S \rightarrow \varepsilon$
- Every rule either replaces a variable by a single character or by a pair of variables except the start symbol and the only rule that can have the empty word as it's right hand side must have start symbol as it's left hand side.

Note: Every parse tree for a grammar in CNF must be a binary tree and the parse tree for any non-empty word cannot have any leaves labeled with ε in it.

Transforming of a grammar to CNF

- In order to construct the grammar G in CNF that is equivalent to a given grammar G, first identify how exactly G can violate the rules for a CNF. Since CNF only restricts the rules in G, see only at R. The 'bad' cases of rules are:
- $A \rightarrow uSv$ where $A \in V$ and $u, v \in (V \cup \Sigma)^*$. The start symbol must not appear on the right-hand side of any rule. This is called 'start symbol rule'.

- **To remove 'start symbol rule',** add a new symbol, so make it the start symbol in new grammar G_1, and add the single rule $S_0 \to S$ to R to get the rules for G_1. Since S_0 does not appear in any rules, the new grammar has no start symbol rules.
  - $A \to \varepsilon$ where $A \in V \cup \{S\}|$. The only symbol that can be replaced by the word is start symbol. This is called 'ε-rules'.
- **To remove 'ε-rules',** identify all variables that can yield the empty string, either directly or indirectly.

  These variables are 'nullable'. Remove all direct rules $A \to \varepsilon$ from the grammar and fix up the grammar by removing all occurrences of nullable variables from the right hand sides of all rules.

  $A \to B$ where $A, B \in V$. The only rules involving variables on the right-hand side must have exactly two of them. This is called 'unit rules'.
- **To remove 'Unit rules',** identify a set of unit pairs. These are pairs of symbols (A, B), where $A \overset{*}{\Rightarrow} B$. Then remove all unit rules by copying right-hand sides. If there is a rule $A \to B$, (A, B) is a unit pair. Then, if there is a rule $B \to W$, derive W from A by $A \to B$, $B \to W$. To remove the unit rule and still generate an equivalent grammar, add the right-hand side W to the rules for A directly, $A \to W$.

  $A \to W$ where $A \in V$, $W \in (V \cup \Sigma)^*$ and W contains at least one character and at least one variable. The only rules where character appear on right-hand side must have exactly one character as right-hand side. This is called 'mixed rules'.
- **To remove 'mixed rules',** Let $A \to W \in R_3$ is a mixed rules. Then write W as $W = V_0\, C_1 V_1 \ldots V_{n-1} C_n V_n$, where $C_i \in \Sigma$ are occurrences of characters, and the $V_i \in V^*$ are strings of only variables. Then add a new symbol, C_i to V_4 for every character C_i and add the rules $C_i \to c_i$ to R_4. Finally define $W^1 := V_0\, C_1\, V_1 \ldots V_{n-1}\, C_n\, V_n \in V^*$ and add rules $A \to W'$ to R_4. If the rule $A \to W$ is part of the derivation for some word, replace that single rule by applying rule $A \to W'$ first and then replacing all C_i by c_i using their respective rules.

  $\boxed{A \to w}$ Where $A \in V$ and $W \in (V \cup \Sigma)^*$ with $|w| > 2$. Rules must have one symbol (character) or two variables (two variables as right hand side). These are called long rules.
- **To remove 'long rules',** Let $A \to B_1 \ldots B_n$ be a long rule, i.e., $n > 2$. B_i is all variables. Break up every single long rule, into several 'short' rules, by introducing new 'helper variables' and splitting right hand side from left to right: add new symbols $A_1, \ldots A_{n-2}$ to set of variables and add following rules to R_5: $A \to B_1 A_1, A_1 \to B_2 A_2, \ldots A_{n-2} \to B_{n-1} B_n$.

Example 6: Consider grammar, $G = S \to ASB$, $A \to aAS|a|\varepsilon$, $B \to SbS|A|bb$. The CNF generated contains ____ non-terminals.

(A) 5 (B) 6
(C) 9 (D) 11

Solution: (C)

Add new start state:

$S_0 \to S$
$S \to ASB$
$A \to aAS|a|\varepsilon$
$B \to SbS|A|bb$

Eliminate ε-rules

$A \to \varepsilon$:
$S_0 \to S$
$S \to ASB|SB$
$A \to aAS|a|aS$
$B \to SbS|A|bb|\varepsilon$

Eliminate $B \to \varepsilon$:
$S_0 \to S$
$S \to ASB|SB|S|AS$
$A \to aAS|a|aS$
$B \to SbS|A|bb$

Remove Unit rules:

$B \to A$:
$S_0 \to S$
$S \to ASB|SB|S|AS$
$A \to aAS|a|aS$
$B \to SbS|bb|aAS|a|aS$

$S \to S$:
$S_0 \to S$
$S_0 \to ASB|SB|AS$
$A \to aAS|a|aS$
$B \to SbS|bb|aAS|a|aS$

$S_0 \to S$:
$S_0 \to ASB|SB|AS$
$S \to ASB|SB|AS$
$A \to aAS|a|aS$
$B \to SbS|bb|aAS|a|aS$

Replace rules which have more than two symbols:

$S_0 \to ASB$: $S_0 \to AU_1$ and $U_1 \to SB$
$\therefore S_0 \to AU_1|SB|\,AS$
$S \to AU_1|SB|\,AS$
$A \to aAS|a|aS$
$B \to SbS|bb|aAS|a|aS$
$U_1 \to SB$

$A \to aAS \Rightarrow A \to aU_2$ and $U_2 \to AS$ and $B \to SbS$
$\Rightarrow B \to SU_3$ and $U_3 \to bS$
$\therefore S_0 \to AU_1|SB|AS$
$S \to AU_1|SB|\,AS$
$A \to aU_2|a|aS$
$B \to SU_3|bb|aU_2|a|aS$
$U_1 \to SB$
$U_2 \to AS$
$U_3 \to bS$

Eliminate rules which have terminals and variables or two terminals.

Let $V_1 \to a$, $V_2 \to b$
$\therefore S_0 \to AU_1|AS|SB$
$S \to AU_1|SB|AS$
$A \to V_1U_2|a|V_1S$

$B \rightarrow SU_3|V_2V_2|V_1U_2|a|V_1S$
$U_1 \rightarrow SB$
$U_2 \rightarrow AS$
$U_3 \rightarrow V_2S$
$V_1 \rightarrow a$
$V_2 \rightarrow b$
$\therefore$ Nine non-terminals.

Greiback Normal Form (GNF)

- A CFG, $G = (V, T, R, S)$ is said to be in GNF, if every production is of form $A \rightarrow a\alpha$ where $a \in T$, $\alpha \in V^*$, i.e., α is a string of zero or more variables.
- Left recursion in R can be eliminated by following schema: If $A \rightarrow A\alpha_1 |A\alpha_2| \ldots |A\alpha_r|\beta_1|\beta_2| \ldots |\beta_s$, then replace the above rules by
  (i) $A \rightarrow \beta_i|\beta_iZ$, $1 \leq i \leq s$
  (ii) $Z \rightarrow \alpha_i|\ \alpha_iZ$, $1 \leq i \leq r$
- If $G = (V, T, R, S)$ is a CFG, then another CFG, $G_1 = (V_1, T, R_1, S)$ can be constructed in GNF $\exists$ $L(G_1) = L(G) - \{\varepsilon\}$.

The step wise algorithm is as follows:

1. Eliminate null production, unit productions and useless symbols from the grammar G and then construct a $G^1 = (V^1, T, R^1, S)$ in CNF generating the language $L(G^1) = L(G) - \{\varepsilon\}$.
2. Rename the variables like $A_1, A_2, \ldots A_n$ starting with $S = A_1$.
3. Modify the rules in R^1, so that if $A_i \rightarrow A_j\gamma \in R^1$ then $j > i$.
4. Starting with A_1 and proceeding to A_n, can be obtained as:
   (a) Assume that productions have been modified so that for $1 \leq i \leq k$, $A_i \rightarrow A_j\gamma \in R^1$ only if $j > i$
   (b) If $A_k \rightarrow A_j\gamma$ is a production with $j < k$, generate a new set of productions substituting for A_j, the body of each A_j production.
   (c) Repeating (b) atmost $k - 1$ times, obtains rules of the form $A_k \rightarrow A_p\gamma$, $p \geq k$.
   (d) Replace rules $A_k \rightarrow A_k\gamma$ by removing left-recursion.
5. Modify the $A_i \rightarrow A_j\gamma$ for $i = n - 1, n - 2, \ldots 1$ in desired form at same time change z production rules.

Example 7: A grammar G is defined with rules $S \rightarrow XA|BB$, $B \rightarrow b|SB$, $X \rightarrow b$, $A \rightarrow a$. The normalized GNF of G contains ____ productions.

(A) 17 (B) 19
(C) 5 (D) 16

Solution: (B)

1. The Grammar, G is already in CNF.
2. Re-label with variables
   S with A_1
   X with A_2
   A with A_3
   B with A_4
   Grammar, G now is:
   $A_1 \rightarrow A_2A_3|A_4A_4$
   $A_4 \rightarrow b|A_1A_4$
   $A_2 \rightarrow b$
   $A_3 \rightarrow a$
3. Identify all productions which do not conform to any of the types listed below:
   $A_i \rightarrow A_j\, x_k \ \exists\, j > i$
   $Z_i \rightarrow A_j\, x_k \ \exists\, j \leq n$
   $A_i \rightarrow a\, x_k \ \exists\, x_k \in V^*$ and $a \in T$
4. $A_4 \rightarrow A_1\, A_4 \ldots$ identified
5. $A_4 \rightarrow A_1\, A_4|b$
   To eliminate A_1, use substitution rule, $A_1 \rightarrow A_2\, A_3|A_4A_4$
   $\therefore$ $A_4 \rightarrow A_2\, A_3\, A_4|A_4\, A_4A_4|b$
   Substitute $A_2 \rightarrow b$
   $\therefore$ $A_4 \rightarrow b\, A_3\, A_4|A_4\, A_4\, A_4|b$
   $A_4 \rightarrow A_4\, A_4\, A_4$ is left recursive. So, remove left recursion i.e., $A_4 \rightarrow b\, A_3\, A_4|b|bA_3A_4Z|bZ$
   $Z \rightarrow A_4A_4|A_4\, A_4Z$
6. Now, $G = A_1 \rightarrow A_2\, A_3|A_4A_4$
   $A_4 \rightarrow b\, A_3\, A_4|b|b\, A_3\, A_4\, Z|b\, Z$
   $Z \rightarrow A_4\, A_4|A_4\, A_4Z$
   $A_2 \rightarrow b$
   $A_3 \rightarrow a$
7. A_1, Z are not in GNF. So,
   For $A_1 \rightarrow A_2A_3|A_4A_4$:
   Substitute for A_2 and A_4 to convert it to GNF
   $A_1 \rightarrow b\, A_3|\ b\, A_3\, A_4\, A_4|b\, A_4|b\, A_3\, A_4\, Z\, A_4|b\, Z\, A_4$
   For $Z \rightarrow A_4\, A_4|A_4\, A_4\, Z$
   substitute for A_4 to convert it to GNF
   $Z \rightarrow b\, A_3\, A_4\, A_4|b\, A_4|b\, A_3\, A_4\, Z\, A_4|b\, Z\, A_4|b\, A_3\, A_4\, A_4\, Z|b\, A_4\, Z|b\, A_3\, A_4\, Z\, A_4\, Z|b\, Z\, A_4\, Z$
   $\therefore$ Final GNF is:
   $A_1 \rightarrow b\, A_3|b\, A_3\, A_4\, A_4|b\, A_4|\ b\, A_3\, A_4\, Z\, A_4|b\, Z\, A_4$
   $A_4 \rightarrow b\, A_3\, A_4|b|b\, A_3\, A_4\, Z|b\, Z$
   $A_2 \rightarrow b$
   $A_3 \rightarrow a$
   $Z \rightarrow b\, A_3\, A_4\, A_4|b\, A_4|b\, A_3\, A_4\, Z\, A_4|b\, Z\, A_4|b\, A_3\, A_4\, A_4\, Z|\ b\, A_4\, Z|b\, A_3\, A_4\, Z\, A_4\, Z|b\, Z\, A_4\, Z$
   $\therefore$ 19 productions.

Pumping Lemma for Context Free Languages

Let 'L' be context free language. There exists some integer, m $\exists$ $\forall$w in L, with $|w| \geq m$, $w = uvxyz$ with $|vxy| \leq m$ and $|vy| \geq 1$ $\exists$ $u\, v^i\, x\, y^i\, z \in L$ $\forall$ $i = 0, 1, 2, 3, \ldots$

Note: Pumping lemma is used to show that a language is Not context free.

Example 8: The language $\{a^n\, b^m\, c^n\, d^{(n+m)}: m, n \geq 0\}$ is

(A) Regular
(B) Context free but not regular
(C) Neither context free nor regular
(D) Cannot be determined

Solution: (C)

$L = \{a^n b^m c^n d^{(n+m)}: m, n \geq 0\}$

Clearly, L is not regular because, number of a's and number of b's must be known to compute number of d's.

'L' is not context free because, Let $w = a^M b^M c^M d^{2M}$. Clearly neither v nor y can cross regions and include more than one letter, since if that happened; letters obtained will be out of order when pumped.

So, consider cases, where v and y fall within a single region. Consider 4-regions corresponding to a, b, c and d.

(1, 1) $\rightarrow$ change number of a's and they won't match c's any more.

(1, 2) $\rightarrow$ If v is not empty, change a's and they won't match with c's. If y is non-empty, number of b's changed won't have right number of d's.

(1, 3), (1, 4) $\rightarrow$ ruled out. $\because$ $|v\,x\,y| \leq M$

(2, 2) $\rightarrow$ Change number of b's and they won't match right number of d's.

(2, 3) $\rightarrow$ If v is non-empty, change number of b's without changing number of d's. If y is not empty, change c's and they'll no longer match a's.

(2, 4) $\rightarrow$ ruled out $\because$ $|v\,x\,y| \leq M$

(3, 3) $\rightarrow$ Change number of c's and they won't match a's.

(3, 4) $\rightarrow$ If v is not empty change c's and they won't match a's. If y is not empty, change d's without changing b's.

(4, 4) $\rightarrow$ change d's without changing a's or b's.

$\therefore$ L is not context free.

Closure Properties of CFL's

1. **CFL's are closed under union:** For CFL's L_1, L_2 with CFG's G_1, G_2 and start variables S_1, S_2. The grammar of Union $L_1 \cup L_2$ has new start symbol S and additional production $S \rightarrow S_1 | S_2$
2. **CFL's are closed under concatenation:** For CFL's L_1, L_2 with CFG's G_1, G_2 and start variables S_1, S_2. The grammar of concatenation L_1L_2 has new start variables S and additional production: $S \rightarrow S_1\, S_2$
3. **CFL's are closed under star operation:** For CFL L, with CFG G and start variable S. The grammar of the start operation L^* has new start variable S_1 and additional production:

$$S_1 \rightarrow S\, S_1 | \varepsilon$$

4. **CFL's are not closed under intersection:** If L_1, L_2 are two context free languages, $L_1 \cap L_2$ not necessarily be context free.
5. **CFL's are not closed under complement:** If L is context free language, $\overline{L}$ not necessarily be context free.
6. **Intersection of CFL's and regular language: (regular closure):** If L_1 is a CFL and R_2 is a regular language then $L_1 \cap L_2$ is a CFL.

Example 9: The language, $L_1 = \{a^n\, b^n : n \geq 0\}$ and $L_2 = \{a^{100}\, b^{100}\}$. The relation $L_1 \cap \overline{L_2}$ is ______

(A) Regular
(B) Context free
(C) Regular but not context fee
(D) Cannot be determined

Solution: (B)

$L_1 = \{a^n\, b^n : n > 0\}$ is context free

$L_2 = \{a^{100}\, b^{100}\}$ is regular

$\overline{L_2} = \{(a+b)^*\} - \{a^{100}b^{100}\}$ is regular

$\{a^n\, b^n\}$ context free

$\overline{L_2} = \{(a+b)^*\} - \{a^{100}b^{100}\}$ is regular

$\{a^n b^n\} \cap \overline{L_2}$ $\rightarrow$ context free

$\{a^n b^n\} \cap \overline{L_2} = \{a^n b^n : n \neq 100,\ n \geq 0\} = L$ is context free:

Table 1 *Comparing Regular and Context free Languages:*

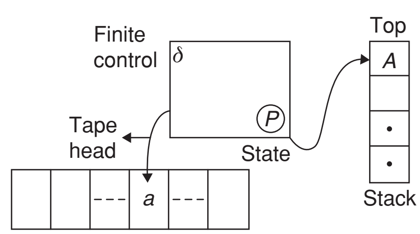
Regular Language	CFL
Regular expression or regular grammar	Context free grammar
Recognize the language	Parses the language
These are DFSA's	These are NDPDA's
Minimize FSA's	Find deterministic grammar.
Closed under: Concatenation Union Kleen star Complement Intersection	Closed under: Concatenation Union Kleen star

Push Down Automata (PDA)

A push down automata is merely a finite automata with a stack added to it.

PDA is used to generate context free language.

The stack allows for unbounded memorization.



Input tape: The tape is divided into finitely many cells. Each cell contains a symbol in an alphabet, Σ.

Stack: The stack head always scans the top symbol of the stack. It performs two basic operations.

- Push: Add a new symbol at the top
- Pop: Read and remove the top symbol

Tape head: The head scans at a cell on the tape and can read a symbol on the cell. In each move, the head can move to the right cell.

Finite control: The finite control has finitely many states which form a set Q. For each move, the state is changed according to the evaluation of transition function.

A PDA is defined as: $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

Where Q: set of States

Σ: Input alphabet

Γ: Stack symbol

δ: Transition function

q_0: Start state

z_0: Initial stack top symbol

F: Final/accepting states

Transition functions δ: $Q \times \Gamma \times \Sigma \Rightarrow Q \times \Gamma$

Q: Old state

Γ: Stack top

Σ: Input symbol

Q: New state, Γ: New stack top

PDA's instantaneous description (IDs): A PDA has a configuration at any given instance: (q, w, y)

$q \rightarrow$ current state

$w \rightarrow$ remainder of input (i.e., unconsumed part)

$y \rightarrow$ current stack contents as a string from top to bottom of the stack.

If $\delta(q, a, x) = \{P, A\}$ is a transition, then following are also true:

- $(q, a, x) \vdash (P, \varepsilon, A)$
- $(q, aw, xB) \vdash (p, w, AB)$

Note: 1. $\rightarrow$: Turnstile notation and represents one move.

2. $\vdash^*$: represents sequence of moves.

Principles about IDs:

1. If for a PDA, $(q, x, A) \vdash^* (p, y, B)$, then for any string $w \in \Sigma^*$ and $\gamma \in \Gamma^*$, it is also true that: $(q, xw, A\gamma) \vdash^* (p, yw, B\gamma)$
2. If for a PDA, $(q, xw, A) \vdash^* (p, yw, B)$, then it is also true that: $(q, x, A) \vdash^* (p, y, B)$

Acceptance by PDA: There are two types of PDAs that one can design:

- Those that accept by final state or
- Those that accept by empty stack

PDAs that accept by final state: For a PDA, P, the language accepted by P, denoted by $L(P)$ by final state, is:

$$\{w | (q_0, w, z_0) \vdash^* (q, \in, A)\} \exists q \in F$$

PDAs that accept by empty stack: For a PDA P, the language accepted by P, denoted by $N(P)$ by empty stack, is:

$$\{w | (q_0, w, z_0) \vdash^* (q, \varepsilon, \varepsilon)\}, \text{ for any } q \in Q.$$

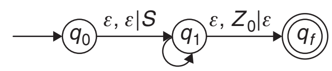
Example 10: Consider the grammar $S \rightarrow aTb | b, T \rightarrow Ta | \varepsilon$. The PDA constructed contains _____ states.

(A) 4 (B) 3 (C) 5 (D) 2

Solution: (B)

$S \rightarrow aTb|b$

$T \rightarrow Ta|\varepsilon$



$\varepsilon, S|aTb$

$\varepsilon, T|Ta$

$\varepsilon, S|b$

$\varepsilon, T|\varepsilon$

$a, a|\varepsilon$

$b, b|\varepsilon$

Let $S \rightarrow q_0, T \rightarrow q_1$

Consider string "aab" $S \rightarrow aTb \rightarrow aTab \rightarrow aab$

$\delta(q_0, aab, z_0) \vdash \delta(q_0, \varepsilon\, aab, z_0)$

$\vdash \delta(q_1, aab, q_0 z_0)$

$\vdash \delta(q_1, aab, aTb\, z_0)$

$\vdash \delta(q_1, ab, Tbz_0)$

$\vdash \delta(q_1, ab, aTbz_0)$

$\vdash \delta(q_1, b, Tbz_0)$

$\vdash \delta(q_1, b, \in bz_0)$

$\vdash \delta(q_1, b, bz_0)$

$\vdash \delta(q_1, \in, z_0)$

$\vdash \delta(q_f, \in) \rightarrow$ acceptance

PDAs accepting by final state and empty stack are equivalent:

$P_F \rightarrow$ PDA accepting by final state,

$P_F = (Q_F, \Sigma, \Gamma, \delta_F, q_0, z_0, F)$

$P_N \rightarrow$ PDA accepting by empty stack

$P_N = (Q_N, \Sigma, \Gamma, \delta_N, q_0, z_0)$

- For every P_N, $\exists P_F \exists L(P_F) = L(P_N)$
- For every P_F, $\exists P_N \exists L(P_N) = L(P_F)$

Converting CFG to PDA

The PDA simulates the left most derivation on a given w, and upon consuming it fully it either arrives at acceptance (by empty stack) or non-acceptance.

The steps to convert CFG to PDA are:

1. Push right hand side of the production on to stack, with left most symbol at the stack top.
2. If stack top is the left most variable, then replace it by all its productions (each possible substitution will represent a distinct path taken by non-deterministic PDA (NPDA).
3. If stack top has a terminal symbol and if it matches with the next symbol in the input string, then pop it. Follow from step-1 again to complete all productions.

Example 11: The CFG, G of a language L is $S \rightarrow AB$, $A \rightarrow aAb|\varepsilon$, $B \rightarrow cB/\varepsilon$. The PDA generated by G contains _____ states.

(A) 5 (B) 4 (C) 3 (D) 1

Solution: (C)

$S \rightarrow AB$
$A \rightarrow aAb|\varepsilon$
$B \rightarrow cB|\varepsilon$
$\Rightarrow \delta(q_0, w, S) = (q_1, AB)$
$\delta(q_1, w, A) = (q_1, aAb)$
$\delta(q_1, \varepsilon, A) = \delta(q_1, \varepsilon)$
$\delta(q_1, w, B) = (q_1, cB)$
$\delta(q_1, \varepsilon, B) = \delta(q_2, \varepsilon) \rightarrow$ accept
$\therefore$ $\{q_0, q_1, q_2\}$ 3-states.

Converting a PDA into a CFG

Given: $G = (V, T, P, S)$ Initial stack symbol (S) same as start variable in grammar
Output: $P_N = (\{q\}, T, V \cup T, \delta, q, S)$, where δ is

- If q_0 is start state in PDA and q_n is final state of PDA then $[q_0, z, q_n]$ becomes a start state of CFG. Here z represents stack symbol.
- The production rule for the ID of the form $\delta(q_i, a, z_0) = (q_{i+1}, z_1, z_2)$ can be obtained as:

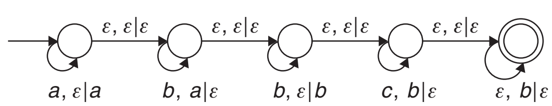
$$\delta(q_i, z_0, q_{i+k}) \rightarrow a(q_{i+1}, z_1, q_m)\ (q_m, z_2, q_{i+k})$$

Where q_{i+k}, q_m represents the intermediate staes, z_0, z_1, z_2 are stack symbols and a is input symbol.

- The production rule for the ID of the form $\delta(q_i, a, z_0) = (q_{i+1}, \varepsilon)$ can be converted as

$$(q_i, z_0, q_{i+1}) \rightarrow a$$

Example 12: The PDA, P for language L is generated as:



The CFG for P is:

(A) $S \rightarrow S_1S_2$
$S_1 \rightarrow aS_2b$
$S_2 \rightarrow c|\varepsilon$
(B) $S \rightarrow S_1bc$
$S_1 \rightarrow a|\varepsilon$
(C) $S \rightarrow S_1S_2$
$S_1 \rightarrow aS_1b|\varepsilon$
$S_2 \rightarrow bS_2|bS_2c|\varepsilon$
(D) $S \rightarrow aS_1c$
$S_1 \rightarrow b|\varepsilon$

Solution: (C)
The language, L generated by given PDA is

$$L = \{a^n\, b^n\, b^m\, c^p : m \geq p \text{ and } n, p \geq 0\}$$

It can be generated by following rules:

$$S \rightarrow S_1S_2$$

$S_1 \rightarrow aS_1b|\varepsilon \rightarrow S_1$ generates $a^n\, b^n$
$S_2 \rightarrow bS_2|bS_2c|\varepsilon \rightarrow S_2$ generates $b^m\, c^p$

Deterministic PDA (Deterministic CFL)

$$\left\{\begin{array}{c}\text{Deterministic}\\ \text{context free}\\ \text{languages}\\ \text{(DPDA)}\end{array}\right\} \subseteq \left\{\begin{array}{c}\text{Context-free}\\ \text{Languages}\\ \text{PDAs}\end{array}\right\}$$

- Every DPDA is also a PDA.
- A context free language 'L' accepted by PDA may or may not be accepted by DPDA.

A PDA, $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ is deterministic if there is no configuration for which M has choice of more than one move. That is, it must satisfy the following conditions:

1. For any $q \in Q$, $a \in \Sigma\varepsilon$ and $s \in \Gamma\, \varepsilon$, the set $\delta(q, a, s)$ has almost one element. (Doesn't allow two or more transitions from same state).
2. For any $q \in Q$, and $s \in \Gamma\varepsilon$, if $\delta(q, \varepsilon, s) \neq \phi$, then $\delta(q, a, s) = \phi$ for every $a \in \Sigma$ and $\delta(q, a, \varepsilon) = \phi$ for all $a \in \Sigma\varepsilon$.
3. For any $q \in$ Q and $a \in \Sigma$, if $(q, a, \varepsilon) \neq \phi$, then $\delta(q, a, s) = \phi$ for all $s \in \Gamma$ and $\delta(q, \varepsilon, t) = \phi$ for all $t \in \Gamma_{\varepsilon.}$
4. For any $q \in Q$, if $\delta(q, \varepsilon, \varepsilon) \neq \phi$, then $\delta(q, a, t) = \phi$ for all $a \in \Sigma\varepsilon$ and $t \in \Gamma_\varepsilon$ (except when $a = \varepsilon$, $t = \varepsilon$).

Rule-2 says that if there is a transition from state q that reads character, s from stack but doesn't read other input, other transitions from q, that don't read stack are not allowed and other transitions from q that read s from the stack and read the input are not allowed either.

Rule-3 says that if there is a transition from state q that reads character a, but doesn't read stack, other transitions from q that don't read the input are not allowed and other transitions from q that read 'a' from input and read the stack are not allowed either.

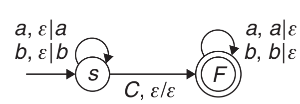
Rule-4 says that if there is a transition from q that doesn't read either input or stack, all other transitions from q are not allowed.

Example 13: A language, L is defined as: $L = \{wcw^R : w \in (a, b)^*\}$. What is Nature of language L?

(A) CFL and DCFL (B) Only CFL
(C) Only DCFL (D) None of these

Solution: (A)

$$L = \{x = wcw^R \text{ for } w \in (a, b)^*\}$$



Clearly, obtained PDA is also DPDA in sense; there is no choice in transitions.

$\therefore$ Hence L is CFL and DCFL.

Exercises

Practice Problems 1

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Consider the grammar, $G = (V, \Sigma, R, S)$ where $V = \{a, b, S, A\}$, $\Sigma = \{a, b\}$, $R = \{S \to AA, A \to AAA, A \to a, A \to bA, A \to Ab\}$ How many strings can be generated by $L(G)$ that can be produced by derivations of four or fewer steps?

(A) 5 (B) 10 (C) 14 (D) 8

2. Consider the following languages L_1, L_2 and L_3:

$L_1 = \{a^n b^m c^{n+m} | n, m \geq 0\}$

$L_2 = \{a^n b^{n+1} c^{n+2} | n \geq 0\}$

$L_3 = \{a^n b^n c^m | n, m \geq 0\}$

Which of following statement is true?

(A) L_1, L_2, L_3 are context free languages
(B) L_1, L_2 are context free but not L_3
(C) L_1, L_3 are context free but not L_2
(D) L_1, L_2, L_3 are not context free languages.

3. The language, $L = \{b_i \# b_{i+1} : b_i$ is i in binary, $i \geq 1\}$ is:

(A) Regular
(B) Context free
(C) Regular and context free
(D) Neither context free nor Regular

4. The CFG, $G : A \to BAB|B|\varepsilon$, $B \to 00|\varepsilon$. The CFG is normalized using CNF. The obtained G', contains ___ rules.

(A) 11 (B) 14 (C) 12 (D) 13

5. The language $L = \{0^{2^i} : i \geq 1\}$ is:

(A) Context free
(B) DCFL
(C) Both CFL and DCFL
(D) Not context free language

6. The context free grammar, G is defined with production rules $S \to EcC'|aAE|AU$, $A \to aA|\varepsilon$, $B \to bB|\varepsilon$, $C' \to cC'|\varepsilon$, $E \to aEc|F$, $F \to bFc|\varepsilon$, $U \to aUc|V$, $V \to bVc|bB$ What is the language generated by L?

(A) $L = \{a^n b^m c^k : k \neq n + m\}$
(B) $L = \{a^n b^m c^k : k = n + m\}$
(C) $L = \{a^n b^m c^k : k > n + m\}$
(D) $L = \{a^n b^m c^k : k < n + m\}$

7. Consider the grammar, $G \equiv S \to abScB|\varepsilon$, $B \to bB|b$. What language does it generate?

(A) $L(G) = \{(ab)^n (cb)^m | n = m\}$
(B) $L(G) = \{a^n b^n (cb)^m | n \neq m\}$
(C) $L(G) = \{(ab)^n (cb^m)^n | n \geq 0, m > 0\}$
(D) $L(G) = \{\{ab)^n (cb^m)^n | n \geq 0, m \geq 0\}$

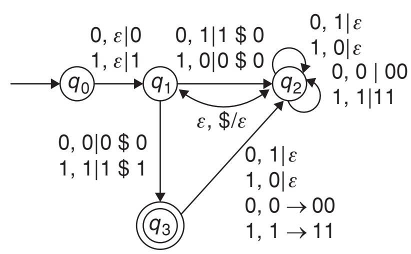
8. The language, $L = \{0^i 1^j 2^k | i \neq j$ or $j \neq k\}$. The CFG, G generated by L contains ___ rules.

(A) 23 (B) 20
(C) 21 (D) 19

9. The DPDA constructed to accept language, L with property $L = L_1 \cup L_2$ where $L_1 = \{10^n 1^n | n > 0\}$, $L_2 = \{110^n 1^{2n} | n > 0\}$ contains ___ states.

(A) 4 (B) 5
(C) 6 (D) 7

10. The PDA is designed as:



What is the language generated by the above PDA?

(A) Binary strings that have same number of 0's and 1's.
(B) Binary strings that start with 00 and end with 11 and have same number of 0's and 1's.
(C) Binary strings that start and end with the same symbol and have same number of 0's and 1's.
(D) Binary strings that start with 11 and end with 00 and have same number of 0's and 1's.

11. The language, $L = (ba^{m_1}ba^{m_2}b \cdots ba^{m_n} : n \geq 2, m_1, \ldots m_n \geq 0$ and $m_i \neq m_j$ for some $i, j)$. What is nature of 'L'?

(A) Regular
(B) Context free but not regular
(C) Regular but not context free
(D) Neither context free nor regular

12. Two languages L_1, L_2 are defined as:

$L_1 = \{a^i b^j c^k : i, j, k \geq 0, i = j\}$

$L_2 = \{a^i b^j c^k : i, j, k \geq 0, j = k\}$ which of following statements are true?

(i) $L_1 \cap L_2$ is context free
(ii) $L_1 \cap L_2 = \{a^n b^n c^n | n \geq 0\}$
(iii) L_1, L_2 are context free
(iv) Only L_1 is context free

(A) All are true (B) (i), (ii) are true
(C) (iii), (iv) are true (D) (ii), (iii) are true

13. The language generated by grammar:

$S \to Te|Ue$, $T \to cTd|cT|\varepsilon$, $U \to cUd|Ud|dd$. is

(A) $L = \{c^n d^m e : m \geq n\}$
(B) $L = \{c^n d^m e : m = n\}$
(C) $L = \{c^m d^n e^m : m \geq n + 2\}$
(D) None of these

14. Remove null productions, useless symbols from the following grammar result in:

$S \to ABC$

$A \to aBC$

$B \to C|\varepsilon$

$C \to cd|DCF$

$D \to dD|\varepsilon$

$E \to eFE$
$F \to eC$

(A) $S \to ABC|AC$
$A \to aBC|aC$
$B \to C$
$C \to cd|DCF|CF$
$D \to dD|d$
$F \to eC$

(B) $S \to aBCc$
$A \to aBC$
$B \to cD|dDEF|dEF$
$C \to cD|dDEF|dEF$
$F \to eB$
$D \to dD|d$
$E \to eFE|e$

(C) $S \to aBCBC|aBC$
$B \to cD|dDEF|dEF$
$F \to eB$
$C \to dD|d$
$D \to e$
$F \to CD|dDEF$

(D) None of these

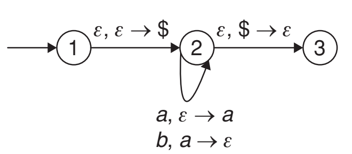
15. Let the language L_1, L_2 are defined as: L_1: $\{a^i\ b^{2i}\ c^j |\ i, j \geq 0\}$, $L_2 = \{a^i\ b^{2i}\ a^i |\ i \geq 0\}$. Which of following is true?
(A) L_1, L_2 are context free
(B) Only L_1 is context free
(C) Only L_2 is context free
(D) Neither L_1 nor L_2 is context free

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Consider the alphabet $\Sigma = \{a, b, c, (,), \cup, ^*, \phi\}$. Then context free grammar that generates all strings in Σ^* that are regular expressions over $\{a, b\}$ is:
(A) $S \to S^*|a|b|SS$
(B) $S \to \phi|a|b|S$
(C) $S \to \phi|a \cup b|S^*$
(D) $S \to \phi|S^*|a|b|(S)|S \cup S|SS$

2. The PDA for language, L is designed below. The CFG generated contains _____ productions.



(A) 5 (B) 4
(C) 3 (D) 6

3. The language, L generated by the following grammar, $S \to SS|\ AAA|\varepsilon$, $A \to aA|\ Aa|\ b$ is
(A) $(a^*\ b^*)^*$ (B) $(a^*\ b^*\ b^*\ a^*)^*$
(C) $a^*\ b^*\ a^*$ (D) $(a^*\ b\ a^*\ b\ a^*\ b\ a^*)^*$

4. The grammar, G is defined with rules $S \to S_1|S_2$, $S_1 \to S_1b|Ab|\varepsilon$, $A \to aAb|\ ab$, $S_2 \to S_2a|\ Ba|\varepsilon$, $B \to bBa|\ ba$. The CNF is applied on G. The obtained grammar, G' contains ____ rules.
(A) 24 (B) 23
(C) 21 (D) 20

5. The language, $L = \{b^{n^2} : n \geq 1\}$ is:
(A) CFL but not DCFL
(B) DCFL but not CFL
(C) Only DCFL
(D) Not CFL

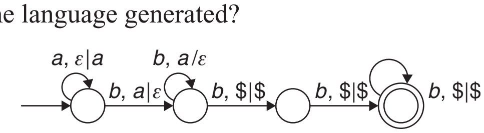
6. Consider the grammar, $G = S \to aSc|B$, $B \to bBc|\varepsilon$ The language, L generated by G is
(A) $L = \{a^n\ b^m\ c^k: k = n + m\}$
(B) $L = \{a^n\ b^m\ c^k: k \neq n + m\}$
(C) $L = \{a^n\ b^m\ c^k: k > n + m\}$
(D) $L = \{a^n\ b^m\ c^k: k < n + m\}$

7. The grammar, G is defined with productions: $S \to 0A|1B$, $A \to 0AA|1S|1$, $B \to 1BB|0S|0$
The grammar, G_2 is defined with productions: $S \to AB|aaB$, $A \to a|Aa$, $B \to b$
Which grammar is/are ambiguous?
(A) Only G_1
(B) Only G_2
(C) Both G_1 and G_2
(D) Both G_1 and G_2 are unambiguous

8. The language, $L_1 = \{0^n\ 1^n |\ n > 0\}$ and $L_2 = \{0^n\ 1^{2n} |\ n > 0\}$. The CFG generated for $L_1 \cup L_2$ is:
(A) $S \to 0\ A\ 1|0\ A\ 1\ 1$
$A \to 0|1|\varepsilon$
(B) $S \to 0\ A\ 1\ 1$
$A \to 0|1|\varepsilon$
(C) $S \to 0\ A\ 1|0\ B\ 1\ 1$
$A \to 0\ A\ 1|\varepsilon$
$B \to 0\ B\ 1\ 1|\varepsilon$
(D) $S \to 0\ A\ 1\ 1|0\ 1\ 1$
$A \to 0|1|\varepsilon$

9. The NPDA constructed to accept language, L with property, $L = L_1 \cup L_2$, where $L_1 = \{1^n\ 0^n |\ n > 0\}$, $L_2 = \{0^n\ 1^{2n} |\ n \geq 0\}$ contains _____ final states.
(A) 3 (B) 1
(C) 2 (D) 4

10. The DPDA for language, L is designed below. What is the language generated?



(A) $L = \{a^n b^m : m = n\}$
(B) $L = \{a^n b^m : m = n + 2\}$
(C) $L = \{a^n b^m : m \geq n + 2\}$
(D) $L = \{a^n b^m : m \leq n + 2\}$

11. The CFG, G is defined with rules: $S \to AB|CD$, $A \to A00|\varepsilon$, $B \to B11|1$, $C \to C00|0$, $D \to D11|\varepsilon$. The language generated by G is
(A) $L = \{0^n\ 1^n | n \geq 0\}$
(B) $L = \{0\ 0^n\ 1\ 1^n | n > 0\}$
(C) $L = \{0^n\ 1^m | n + m \text{ is odd}\}$
(D) $L = \{0^n\ 1^m | n + m \text{ is even}\}$

12. The languages, L_1, L_2, L_3 are defined as: $L_1 = \{a^n\ b^m\ c^{n+m} | n, m \geq 0\}$, $L_2 = \{a^n\ b^n\ c^m | n, m \geq 0\}$, $L_3 = \{a^n\ b^n\ c^{2n} | n \geq 0\}$, Which of the following statements are true?
(i) L_1, L_2 are context free
(ii) L_1, L_3 are context free
(iii) $L_3 = L_1 \cap L_2$
(iv) L_1, L_3 are context free but not L_2
(A) (i), (ii) (B) (i), (iii)
(C) (ii), (iii) (D) (iii), (iv)

13. The language, L_1 and L_2 are defined as $L_1 = \{a^n\ b^n : n \geq 0$ and n is not a multiple of $5\}$ and $L_2 = \{0^n \# 0^{2n} \# 0^{3n} | n \geq 0\}$. Which of following is true?
(A) L_1 and L_2 are context free
(B) Only L_1 is context free
(C) Only L_2 is context free
(D) Neither L_1 nor L_2 is context free

14. The language, L_1 and L_2 are defined as $\overline{L_1} = \{(0^n 1^n)^m | m, n > 0\}$, $L_2 = \{0^n\ 1^n\ 0^n\ 1^n | n \geq 0\}$ which of following is true?
(A) L_1 and L_2 are context free
(B) Only $\overline{L_1}$ is context free
(C) Only L_2 is context free
(D) Neither L_1 nor L_2 is context free

15. The language L_1, L_2 are defined as $L_1 = \{0^i\ 1^i\ 0^j\ 1^j | i, j > 0\}$, $L_2 = \{1^k\ 0^i\ 1^i\ 0^j\ 1^j\ 0^k | i, j, k > 0\}$. Which of following is true?
(A) L_1 and L_2 are context free
(B) Only L_1 is context free
(C) Only L_2 is context free
(D) Neither L_1 nor L_2 is context free

Previous Years' Questions

1. Match the following: **[2008]**

E. Checking that identifiers are declared before their use	P. $L = \{a^n b^m c^n d^m | n \geq 1, m \geq 1\}$
F. Number of formal parameters in the declaration of a function agrees with the number of actual parameters in use of that function	Q. $X \to XbX|XcX|dXf|g$
G. Arithmetic expressions with matched pairs of parentheses	R. $L = \{wcw | w \in (a|b)^*\}$
H. Palindromes	S. $X \to bXb|cXc|\varepsilon$

(A) E − P, F − R, G − Q, H − S
(B) E − R, F − P, G − S, H − Q
(C) E − R, F − P, G − Q, H − S
(D) E − P, F − R, G − S, H − Q

2. Consider the languages L_1, L_2 and L_3 as given below.
$L_1 = \{0^p\ 1^q | p, q \in N\}$,
$L_2 = \{0^p\ 1^q | p, q \in N \text{ and } p = q\}$ and
$L_3 = \{0^p\ 1^q\ 0^r | p, q, r \in N \text{ and } p = q = r\}$. Which of the following statements is NOT TRUE? **[2011]**
(A) Push Down Automata (PDA) can be used to recognize L_1 and L_2.
(B) L_1 is a regular language.
(C) All the three languages are context free
(D) Turing machines can be used to recognize all the languages.

3. Which of the following problems are decidable? **[2012]**
(1) Does a given program ever produce an output?
(2) If L is a context free language, then, is $\overline{L}$ also context free?
(3) If L is a regular language, then, is $\overline{L}$ also regular?
(4) If L is recursive language, then, is $\overline{L}$ also recursive?
(A) 1, 2, 3, 4 (B) 1, 2
(C) 2, 3, 4 (D) 3, 4

4. Consider the following languages.
$L_1 = \{0^p\ 1^q\ 0^r | p, q, r \geq 0\}$
$L_2 = \{0^p\ 1^q\ 0^r | p, q, r \geq 0, p \neq r\}$
Which one of the following statements is **FALSE**? **[2013]**
(A) L_2 is context-free
(B) $L_1 \cap L_2$ is context-free
(C) Complement of L_2 is recursive
(D) Complement of L_1 is context-free but not regular

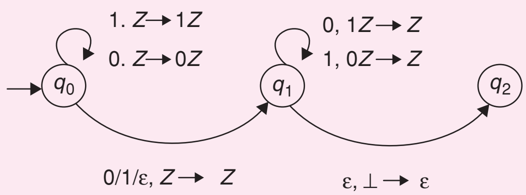
5. Which one of the following is **TRUE**? **[2014]**
(A) The language $L = \{a^n\ b^n | n \geq 0\}$ is regular
(B) The language $L = \{a^n | n \text{ is prime}\}$ is regular
(C) The language $L = \{w | w$ has $3k + 1b$'s for some $k \in N$ with $\Sigma = \{a, b\}\}$ is regular
(D) The language $L = \{ww | w \in \Sigma^*$ with $\Sigma = \{0, 1\}\}$ is regular.

6. Consider the following languages over the alphabet $\Sigma = \{0, 1, c\}$.
$L_1 = \{0^n\ 1^n | n \geq 0\}$
$L_2 = \{wcw^r | w \in \{0, 1\}^*\}$
$L_3 = \{ww^r | w \in \{0, 1\}^*\}$

Here w^r is reverse of the string w. Which of these languages are deterministic context-free languages? **[2014]**

(A) None of the languages
(B) Only L_1
(C) Only L_1 and L_2
(D) All the three languages

7. Consider the NPDA $<Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, \perp\}, \delta, q_0, \perp, F = \{q_2\}>$, where (as per usual convention) Q is the set of states, Σ is the input alphabet, Γ is the stack alphabet, δ is the state transition function, q_0 is the initial state, $\perp$ is the initial stack symbol, and F is the set of accepting states. The state transition is as follows:



Which one of the following sequences must follow the string 1011 00 so that the overall string is accepted by the automation? **[2015]**

(A) 10110 (B) 10010
(C) 01010 (D) 01001

8. Which of the following languages are context-free? **[2015]**

$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$
$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$
$L_3 = \{a^m b^n \mid m = 2n + 1\}$

(A) L_1 and L_2 only (B) L_1 and L_3 only
(C) L_2 and L_3 only (D) L_3 only

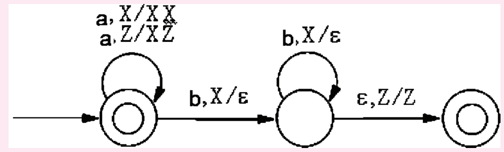
9. Consider the following context-free grammars:

G_1: $S \rightarrow aS|B$, $B \rightarrow b|bB$
G_2: $S \rightarrow aA|bB$, $A \rightarrow aA|B|$ ε, B $|bB$ε

Which one of the following pairs of languages is generated by G_1 and G_2, respectively? **[2016]**

(A) $\{a^m b^n \mid m > 0$ or $n > 0\}$ and $\{a^m b^n \mid m > 0$ and $n > 0\}$
(B) $\{a^m b^n \mid m >$ 0and $n > 0\}$ and $\{a^m b^n \mid m > 0$ or $n \geq 0\}$
(C) $\{a^m b^n \mid m \geq 0$ or $n > 0\}$ and $\{a^m b^n \mid m > 0$ and $n > 0\}$
(D) $\{a^m b^n \mid m \geq 0$ and $n > 0\}$ and $\{a^m b^n \mid m > 0$ or $n > 0\}$

10. Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a, b\}$ and stack alphabet $= \{X,Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.



Which one of the following is TRUE? **[2016]**

(A) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata.
(B) $L = \{a^n \mid n \geq 0) \cup \{a^n b^n \mid n \geq 0\}$ and is not accepted by any deterministic PDA.
(C) L is not accepted by any Turing machine that halts on every input.
(D) $L = \{a^n | n \geq 0\} \cup \{a^n b^n | n \geq 0\}$ and is deterministic context-free.

11. Consider the following languages:

$L_1 = \{a^n b^m c^{n+m} : m, n \geq 1\}$
$L_2 = \{a^n b^n c^{2n} : n \geq 1\}$

Which one of the following is TRUE? **[2016]**

(A) Both L_1 and L_2 are context - free.
(B) L_1 is context - free while L_2 is not context - free
(C) L_2 is context - free while L_1 is not context - free.
(D) Neither L_1 nor L_2 is context - free.

12. Consider the following context-free grammar over the alphabet $\Sigma = \{a, b, c\}$ with S as the start symbol:

$S \rightarrow abScT \mid abcT$
$T \rightarrow bT \mid b$

Which one of the following represents the language generated by the above grammar? **[2017]**

(A) $\{(ab)^n(cb)^n \mid n \geq 1\}$
(B) $\{(ab)^n cb^{m_1} cb^{m_2} \ldots cb^{m_n} \mid n, m_1, m_2, \ldots, m_n \geq 1\}$
(C) $\{(ab)^n(cb^m)^n \mid m, n \geq 1\}$
(D) $\{(ab)^n(cb^n)^m \mid m, n \geq 1\}$

13. If G is a grammar with productions

$$S \rightarrow SaS \mid aSb \mid bSa \mid SS \mid \in$$

Where S is the start variable, then which one of the following strings is not generated by G? **[2017]**

(A) *abab* (B) *aaab*
(C) *abbaa* (D) *babba*

14. Consider the context-free rammers over the alphabet $\{a, b, c\}$ given below. S and T are non-terminals.

$$G_1: S \rightarrow aSb|T, T \rightarrow cT|\in$$
$$G_2: S \rightarrow bSa|T, T \rightarrow cT|\in$$

The language $L(G_1) \cap L(G_2)$ is **[2017]**

(A) Finite
(B) Not finite but regular
(C) Context-Free but not regular
(D) Recursive but not context-free.

15. Consider the following languages over the alphabet $\Sigma = \{a, b, c\}$.

Let $L_1 = \{a^n b^n c^m | m, n \geq 0\}$ and $L_2 = \{a^m b^n c^n | m, n \geq 0\}$. Which of the following are context-free languages? **[2017]**

I. $L_1 \cup L_2$
II. $L_1 \cap L_2$

(A) I only (B) II only
(C) I and II (D) Neither I nor II

16. Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT? **[2017]**

I. $L_1 \cup L_2$ is context-free.
II. $\overline{L_1}$ is context-free.
III. $L_1 - R$ is context-free.
IV. $L_1 \cap L_2$ is context-free.

(A) I, II and IV only (B) I and III only
(C) II and IV only (D) I only

17. Identify the language generated by the following grammar, where S is the start variable. **[2017]**

$$S \to XY$$
$$X \to aX|a$$
$$Y \to aYb|\in$$

(A) $\{a^m b^n | m \geq n, n > 0\}$ (B) $\{a^m b^n | m \geq n, n \geq 0\}$
(C) $\{a^m b^n | m > n, n \geq 0\}$ (D) $\{a^m b^n | m > n, n > 0\}$

18. Consider the following languages.

$L_t = \{a^p \mid p \text{ is a prime number}\}$
$L_2 = \{a^n b^m c^{2m} | n \geq 0, m \geq 0\}$
$L_3 = \{a^n b^n c^{2n} | n \geq 0\}$
$L_4 = \{a^n b^n \mid n \geq 1\}$

Winch of the following are CORRECT? **[2017]**

I. L_1 is context-free but not regular.
II. $L_2$is not context-free.
III. $L_3$is not context-free but recursive.
IV. L_4 is deterministic context-free.

(A) I, II and IV only (B) II and III only
(C) I and IV only (D) III and IV only

19. Consider the following languages:

I. $\{a^m b^n c^p d^q \mid m + p = n + q$, where $m, n, p, q \geq 0\}$
II. $\{a^m b^n c^p d^q \mid m = n$ and $p = q$, where $m, n, p, q \geq 0\}$
III. $\{a^m b^n c^p d^q \mid m = n = p$ and $p \neq q$, where $m, n, p, q \geq 0\}$
IV. $\{a^m b^n c^p d^q \mid mn = p + q$, where $m, n, p, q \geq 0\}$

Which of the languages above are context-free? **[2018]**

(A) I and IV only (B) I and II only
(C) II and III only (D) II and IV only

Answer Keys

Exercises

Practice Problems 1

1. D **2.** C **3.** D **4.** B **5.** D **6.** A **7.** C **8.** B **9.** D **10.** C
11. B **12.** D **13.** D **14.** A **15.** B

Practice Problems 2

1. D **2.** C **3.** D **4.** A **5.** D **6.** A **7.** C **8.** C **9.** C **10.** C
11. C **12.** B **13.** B **14.** B **15.** C

Previous Years' Questions

1. C **2.** C **3.** D **4.** D **5.** C **6.** C **7.** B **8.** B **9.** D **10.** D
11. B **12.** B **13.** D **14.** B **15.** A **16.** B **17.** C **18.** D **19.** B

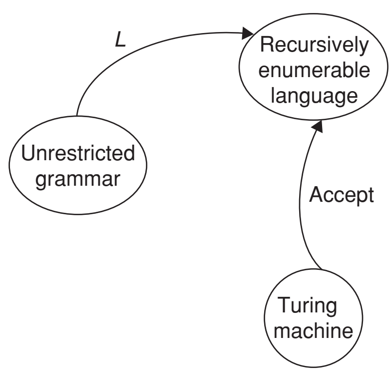
Chapter 3

Recursively Enumerable Sets and Turing Machines, Decidability

LEARNING OBJECTIVES

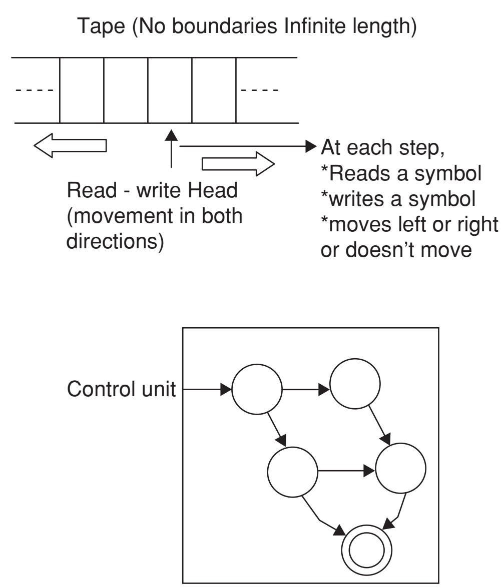
- *Turing machines*
- *Model of turing machine*
- *Types of turing machines*
- *Offline turing machine*
- *Universal turing machine*
- *Recursively enumerable languages*
- *Recursive language*
- *Undecidability*
- *Church's hypothesis*
- *Halting problem*
- *Post's correspondence problem*
- *7 P problems*
- *NP problems*
- *NP – complete problem*
- *NP – hard problem*
- *Closure properties of formal languages*

Turing Machines



A Turing machine is a kind of state machine, which is much more powerful in terms of languages it can recognize. At any time, the machine is in any one of the finite number of states. Instructions for a turing machine include the specification of conditions, under which the machine will make transitions from one state to other.

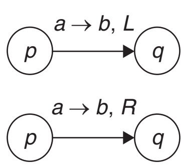
Model of Turing Machine



- A TM (turing machine) consists of Tape, Head, control unit.
- **Tape:** A tape is divided into a sequence of numbered cells, Each cell contains a symbol and cells that have not been written before are assumed to be filled with a blank symbol (B). The set of symbols of tape is denoted by Γ. The tape is assumed to be arbitrarily extensible to the left as well as to the right.
- **Head:** In a single step, a tape head reads the contents of a cell on the tape (reads a symbol), replaces it with some other characters (writes a symbol) and repositions itself to the next cell to the right or to the left of the one it has just read or does not move (moves left or right or does not move).
- **Control unit:** The reading from the tape or writing into the tape is determined by the control unit. It contains a finite set of states, Q. The states are:
  1. Initial state, q_0
  2. Halt state, h: This is state in which TM stops all further operations. There can be one or more halt states in a TM.
  3. Other states.

Note: A TM on entering the halt state stops making moves and whatever string is there on the tape, will be taken as the output, irrespective of whether the position of head is at the end or in the middle of the string on the tape.

Transition Diagram of TM



Reads a symbol
writes a symbol
Move Right (R)
Move Left (L)
No Move (N)

Specification of TM

5-Tuple specification:
TM = (state1, Read symbol, write symbol, $L/R/N$, state 2).
7 - Tuple specification of TM:
A TM, M is represented as a 7-tuple:
$M = (Q, \Sigma, \Gamma, \delta, q_0, B, h)$ where
$Q \rightarrow$ Finite set of states
$\Sigma \rightarrow$ Finite set of non-blank symbols
$\Gamma \rightarrow$ Set of tape characters
$q_o \rightarrow q_o \in Q$, initial state
$B \rightarrow$ Blank character
$h \rightarrow h \subseteq Q$, final state
$\delta \rightarrow$ Transition function, $Q \times \Gamma \rightarrow Q \times \Gamma\ x\ \{L, R, N\}$

String classes in TM

Every TM, over the alphabet Σ, divides set of input string w into three classes:

1. **Accept (TM):** It is the set of all strings $w \in \Sigma^* \ni$ if the tape initially contains w and the TM is then run, then TM ends in a halt state.
2. **LOOP (TM):** It is the set of all strings, $w \in \Sigma^* \ni$ if the tape initially contains w and the TM is then run, then the TM loops forever (infinite loop).
3. **Reject (TM):** It is the set of all strings $w \in \Sigma^* \ni$, any of the following 3-cases arise.

Case I: There may be a state and a symbol under the tape head, for which δ does not have a value.

Case II: If the head is reading the left most cell (i) containing the symbol x, the state of TM is say q, then δ (q, x) suggests a move to the left of the current cell. However as there is no cell to the left, no move is possible.

Case III: If TM enters an infinite loop or if a TM rejects a given string w, because of above two cases, TM crashes (terminates unsuccessfully).

Languages Accepted by a TM

- The language accepted by TM is the set of accepted strings $w \in \Sigma^*$.
- Formally, let $M = (Q, \Sigma, \Gamma, \delta, q_0, B, h)$ be a TM. The language accepted by M denoted by $L(M)$ is defined as, $L(M)$ = $\{w/w \in z^*$ and if $w = a_1 \ldots a_n$ then, $(q_o, \varepsilon, a_1, a_2, \ldots a_n)$ $(h, b, \ldots, b_{i-1}, b_j\, b_n)$ for some $b_1, b_2 \ldots b_n \in N^* \ni\}$

  $L(M) = \{W: q_o\ w \vdash^* x_1 h x_2\}$
- There are three types of turing machine related languages:
  1. **Turing Acceptable language:** A language, L over some alphabet is said to be turing acceptable language if there exists a TM, $M \ni L = L(M)$
  2. **Turing Decidable Language:** A language L over Σ i.e., $L \subseteq \Sigma^*$ is said to be turing decidable, if both languages, L and its complement $\Sigma^* - L$ are turing acceptable.
  3. **Recursively Enumerable Language:** A language L is recursively enumerable, if it is accepted by a TM.

Example 1: Let M be a turing machine has $M = (Q, \Gamma, \Sigma, \delta, S, B, F)$ with $Q = \{q_o, q_1, q_2, q_3, q_4\}$, $\Gamma = \{a, b, X, Y, \#\}$, $\Sigma = \{a, b\}$, $S = q_0$, $B = \#$, δ given by:

	a	b	X	Y	#
q_0	(q_1, X, R)	–	–	(q_3, Y, R)	
q_1	(q_1, a, R)	(q_2, Y, L)	–	(q_1, Y, R)	
q_2	(q_2, a, L)	–	(q_0, X, R)	(q_2, Y, L)	
q_3	–	–	–	(q_3, Y, R)	$(q_4, \#, R)$
q_4	–	–	–	–	–

Which of following is true about M?
(A) M halts on L having 'baa' as substring
(B) M halts on L having 'bab' as substring
(C) M halts on $L = \{a^n b^n / n \geq 1\}$
(D) M halts on L not having 'bbaa' as substring.

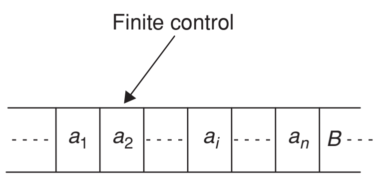
Solution: (C)
M accepts $a^n b^n$.

Example: *aaabbb*

$(q_0, \in, aaabbb)$	$\rightarrow$	$(q_1, XXXYY, b)$
$(q_1\ X, aabbb)$	$\rightarrow$	$(q_2\ XXXY, YY)$
$(q_1, Xa, abbb)$	$\rightarrow$	$^{1}(q_2, XXX, YYY)$
(q_1, Xaa, bbb)	$\rightarrow$	$(q_2, XY, XYYY)$
$(q_1, Xa, aYbb)$	$\rightarrow$	(q_0, XXX, YYY)
$(q_2, X, aaYbb)$	$\rightarrow$	$(q_3, XXXY, YY)$
$(q_2, \in, XaaYbb)$	$\rightarrow$	$(q_3, XXXYY, Y)$
$(q_o, X, aaYbb)$	$\rightarrow$	$(q_3, XXXYYY, \in)$
$(q_1, XX, aYbb)$	$\rightarrow$	$(q_4, XXXYYY\#, \in)$

Types of Turing Machines

Two-way infinite turing machine

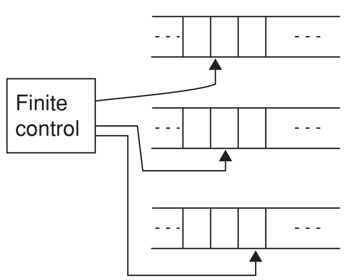


- A TM with a two-way infinite tape is denoted by $M = (Q, \Sigma, \Gamma, \delta, q_o, B, F)$, as in original model.
- The tape is infinite to the left as well as to the right.

If $\delta(q, x) = (p, Y, L)$ then $q\ x\ \alpha \vdash_m pBY$. The tape, is infinte towards left.

If $\delta\ (q, x) = (p, B, R)$ then $q\ x\ \alpha \vdash_m, p\alpha$ the is infinite towards right.

Multiple turing machines



- A multiple TM consists of a finite control with k tape heads and k-tapes, each tape is infinite in both directions, on a single move, depending on the state of the finite control and the symbol scanned by each of tape heads, the machine can,
  - change state
  - print new symbol on each of the cells scanned by its tape head
  - move each of its tape heads, independently, one cell to the left or right or keep it stationary.
- Initially, the input appears on the first tape and other tapes are blank.

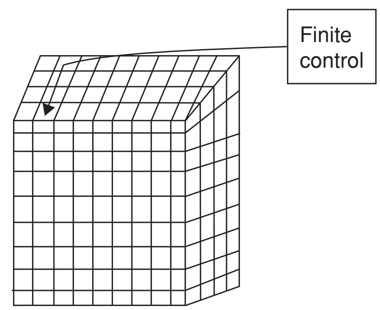
Non-deterministic turing machines

- A non-deterministic turing machine is a device with a finite control and a single one way infinite tape.
- For a given state and a tape symbol scanned by the tape head, the machine has a finite number of choices for next move.

Note: Non-deterministic TM is not permitted to make a move in which the next state is selected from one choice, and the symbol printed and direction of head motion are selected from other choices.

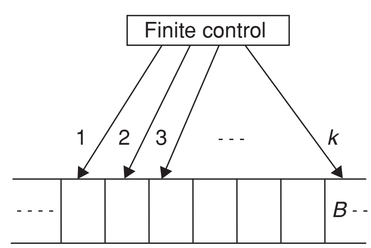
- The non-deterministic TM accepts its input if any sequence of choices of moves leads to an accepting state.

Multi-dimensional TM's



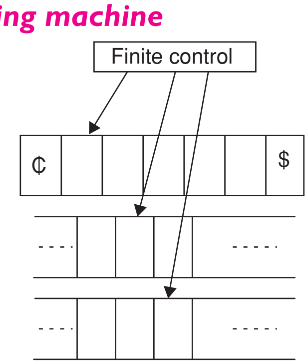
- The tape consists of a k-dimensional array of cells infinite in all 2k directions, for some fixed k.
- Depending on the state and the symbol scanned, the device changes it's state, prints a new symbol and moves its tape head in one of the $2k$ directions, either positively or negatively, along one of the k-axes.

Multihead TM



- A K-head TM has some fixed 'K' number of heads. The heads are numbered from 1 through k, and a move of the TM depends on the state and on the symbol scanned by each head.

Offline turing machine



- An offline TM is a multi tape TM, whose input tape is read only. The input is surrounded by end markers, ¢ on left and \$ on right. The TM is not allowed to move the input tape head off the region between ¢ and \$.

Multi stack machine

- A deterministic two stack machine is a deterministic TM with a read only input and two storage tapes.

Note:

- All these types of TM's does not add any language accepting power and all these are equivalent to the basic model.
- Any language accepted by a 2-PDA can be accepted by some TM and any language accepted by a TM can be accepted by some 2-PDA. Accepting power of a TM = accepting power of a computer.
- Any language accepted by a PDA with n stacks ($n \geq 2$), can also be accepted by some TM.

Example 2: Consider the following statement about L:

1. L is accepted by multi-tape turing machine M_1.
2. L is also accepted by single tape turing machine M_2.

Which of following statement is correct?

(A) Acceptance by M_2 is slower by $O(n^2)$
(B) Acceptance of M_2 is slower by $O(n)$
(C) Acceptance of M_2 is faster by $O(n)$
(D) Acceptance of M_2 is faster by $O(n^2)$

Solution: (A)
While simulating multi-tape TM on a single tape TM the head has to move at least $2k$ cells per move, where k is the number of tracks on single tape TM. Thus for k moves, $\sum_{i=1}^{k} 2i = 2k^2$.
Which means quadratic slow down?
Thus, acceptance of multi-tape is faster by $O(n^2)$.

Universal turing machine

A Universal turing machine is a turing machine that can simulate an arbitrary turing machine on arbitrary input.

- The machine consists of an input output relation to the machine computes.
- The input is given in binary form on the machine tape and the output consists of the contents of the tape when the machine halts.
- The contents of the tape will change based on the Finite State Machine (FSM) inside the TM.
- The problem with TM is that a different machine will be constructed for every new computation to be performed.
- A UTM can simulate any other machine.

Combining turing machines

If TM_1 and TM_2 are turing machines, then we can combine these machines and create a Turing machine which will first behave like TM_1 and TM_2.

To combine two turing machines follow below steps:

1. Change all states in TM_2, so that they do not conflict with the state names in TM_1.
2. Change all halts in TM_1's transition table to the new name of the start state of TM_2.
3. Append TM_2's transition table to the foot of TM_1's transition table.

- If TM_1 and TM_2 are combined in this way, we will write it as $TM_1 \rightarrow TM_2$.

So this new machine starts off in the initial state of TM_1, operates as per TM_1 until TM_1 would halt then it launches TM_2 and operates a TM_2, until TM_2 would halt.

Recursively Enumerable languages

- A language L over the alphabet Σ is called 'recursively enumerable' if there is a TM, M that accept every word in L and either rejects or loops for every word in language L', the complement of L.
  Accept $(M) = L$
  Reject (M) + Loop $(M) = L'$.
- When TM, M is still running on some input of recursively enumerable languages, it is not decided that M will eventually accept, if let it run for long time or M will run forever (in loop).

Recursive language

- A language is said to be recursive, if there exists a TM which will halt and accept when presented with any input string $w \in \Sigma^*$, only if the string is in the language otherwise will halt and reject the string.
- Thus, for turing decidable language L, there is a TM which halts for a large number of inputs w belonging to L.
- A TM that always halts is known as a decider or a total turing machine and is said to decide the recursive language. The recursive language is also called as recursive set of decidable.
- A language accepted by a TM is said to be recursively enumerable language. The subclass of recursively enumerable sets are said to be recursive sets or recursive language.

Note:

- All recursive languages are also recursively enumerable.
- There may be languages which are recursively enumerable but not recursive.
- Set of all possible words over the alphabet of the recursive language is a recursive set.

- Set of all possible words, over the alphabet of the recursive enumerable language, is a recursively enumerable set.

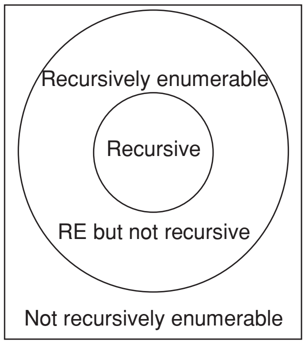


Figure 1 Relationship between the recursive, RE and non-RE languages.

Properties of Recursive and Recursively Enumerable Languages

- If a language L is recursive, then there is a TM T that accepts it and always halts.
- If L and L_I are both recursively enumerable, then L and L_I are recursive.
- Union of two recursive languages is recursive.
- Recursively enumerable languages are closed under union.
- If L, L_1 and L_2 are recursive languages, then so are $L_1 \cup L_2$, $L \cap L_2$, L_1L_2, L^*, $L_1 \cap L_2$ and $L_1 - L_2$.
- If L, L_1 and L_2 are recursively enumerable languages, then so are $L_1 \cup L_2$, L^*, $L_1 \cap L_2$, L_1L_2.
- If Σ is an alphabet, $L \subseteq \Sigma^*$, is a recursively enumerable language and $\Sigma^* - L$ is recursively enumerable, then L is recursive.

Example 3: If $\Sigma = \{0,1\}$, the canonical order is $\{\in,0,1,00,01,10,11,000,\ldots\}$ where w is the i^{th} word and M_j is TM whose code is the integer j, written in binary. The language generated is $L(M_j)$. The diagonalized language, L_d is a.

(A) Recursively enumerable language but not recursive
(B) Recursive language
(C) Non-recursively enumerable language
(D) Both (a) and (c)

Solution: (C)
Non-recursively enumerable language.

Non-recursively enumerable language

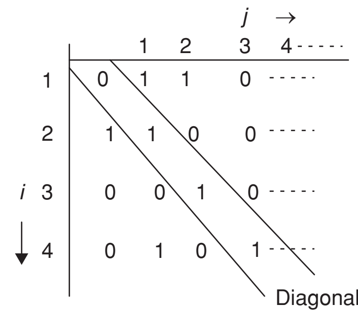
Non-Recursively Enumerable Language: A language which is not accepted by any turing machine is non-recursively enumerable.

Example: Power set of an infinite set.

- These languages cannot be defined by any effective procedure.

For any non-empty Σ, there exist languages that are not Recursively Enumerable.

Infinite table for all i and j is:



To guarantee that no TM accepts L_d:

w_i is in L_d if and only if the (i, i) entry is 0, that is, if M_i does not accept w_i.

Suppose that some TM M_j accepted L_d. Then it contradicts if w_j is in L_d, (j, j) entry is 0, implying that w_j is not in $L\,(M_j)$ and contradicting $L_d = L\,(M_j)$.

If w_i is not in L_d, then the (j, j) entry is 1, implying that w_i is in $L(M_j)$, which again contradicts $L_d = L(M_j)$, as w_j is either in or not in L_d, assumption, $L_d = L(M_j)$ is false.

Thus no TM in the list accepts L_d, Hence L_d is non-recursively enumerable language.

Decidable: A problem with two answers (Yes/No) is decidable if the corresponding language is recursive.

Example:

1. $A_{DFA} = \{(M, w)\ M$ accepts the input string $w\}$.

- A Language L is turing decidable, if there exists a TM M such that on input x, M accepts if $x \in L$ and M rejects otherwise. L is called undecidable if it is not decidable.
- Decidable Languages correspond to algorithmically solvable Decision problems.
- Undecidable language corresponds to algorithmically unsolvable decision problems.

Closure properties of decidable languages

- Decidable Languages are closed under complement, union, intersection, concatenation and star (closure) operations.

Note 1: A language is decidable if both the language and its complement are recognizable.

Note 2: Turing Decidable languages are Recursive languages.

Undecidability

There are problems that can be computed. There are also problems that cannot be computed. These problems which cannot be computed are called 'computationally undecidable problems'.

Church's Hypothesis

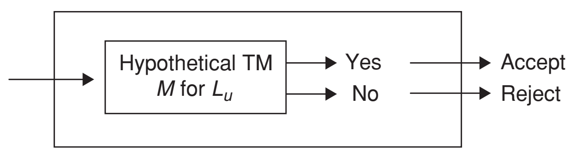
There is an assumption that the intuitive notion of computable functions can be identified with partial recursive functions.

However, this hypothesis cannot be proved. The computability of recursive function is based on following assumptions:

1. Each elementary function is computable.
2. Let 'f' be a computable function and 'g' be another function which can be obtained by applying an elementary operation to f, then g becomes a computable function.
3. Any function becomes computable, if it is obtained by rule (1) and (2).

Undecidability of the universal languages

- The universal language, L_u is a recursively enumerable language but not recursive.



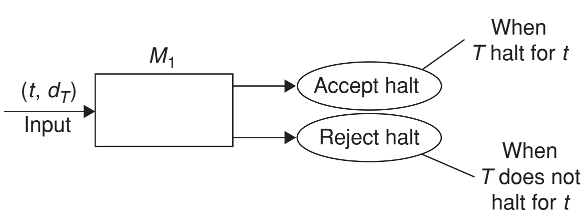
Halting Problem

The given configuration of TM is required to state halting problem. The output of TM can be:

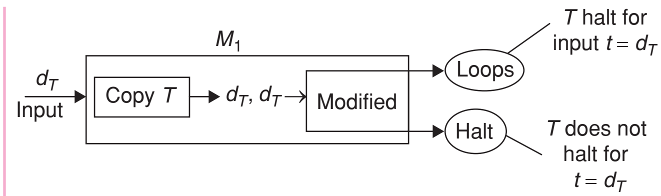
1. **Halt:** The machine starting at this configuration will halt after a finite number of states.
2. **No Halt:** The machine starting at this configuration never reaches a halt state, no matter how long it runs.

- The halting problem is unsolvable because, let, there exists a TM, M, which decides whether or not any computation by a TM, T will ever halt when a description d_T of T and tape t of T is given. That means the input to machine M, will be (machine, tape) pair. Then for every input (t, dT) to M_I if T halt for input t, M_I also halts which is called accept halt.

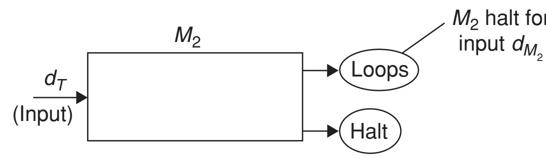
Similarly if T does not halt for input t then the M_1 will halt which is called reject halt.



- Consider another Turing Machine, M_2 which takes an input d_T. It first copies d_T on its tape and then this duplicated tape information is given as input to M_1. But M_1 is a modified machine.



Replace T by M_2 i.e., $M_2 = T$



That's means, a machine M_1, which can tell whether any other TM will halt on particular input does not exist. Hence halting problem is unsolvable.

Post's Correspondence Problem (PCP)

The Undecidability of strings is determined with the help of Post's Correspondence Problem (PCP).

'The PCP consists of two lists of strings that are of equal length over the input Σ. The two lists are $A = w_1, w_2, w_3, \ldots w_n$ and $B = x_1, x_2, \ldots x_n$ then there exists a non-empty set of integers $i_1, i_2, \ldots i_n$ such that $w_1, w_2, \ldots w_n = x_1, x_2, \ldots x_n$'.

To solve PCP, try all the combinations of $i_1, i_2, \ldots i_n$ to find the $w_i = x_i$ then, PCP has a solution.

Example 4: What is the solution for the following system of post correspondence problem. $A = \{100, 0, 1\}$ $B = \{1, 100, 00\}$

(A) 1113322 (B) 1311322
(C) 2233111 (D) No solution

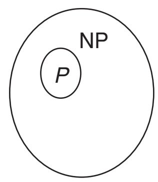
Solution: (B)
The string is:
A1A3A1A1A3A2A2 = 100 + 1 + 100 + 100 + 1 + 0 + 0 = 1001100100100,
B1B3B1B1B3B2B2 = 1 + 00 + 1 + 1 + 00 + 100 + 100 = 1001100100100.

Problems

- P stands for deterministic polynomial time. A deterministic machine at each time executes an instruction. Depending on instruction, it then goes to next state which is unique. Hence, time complexity of deterministic TM is the maximum number of moves made by M in processing an input string of length n, taken over all inputs of length n.
- A language, L is said to be in class P, if $\exists$ a (deterministic) TM, M is of time complexity $P(n)$ for some polynomial P and M accepts L.
- Class P consists of those problems that are solvable in polynomial time by a deterministic TM.

NP Problems

- NP stands for non-deterministic polynomial time.
- A language, L is in class NP, if there is a non-deterministic TM, M is of time complexity $P(n)$ for some polynomial P and M accepts L.
- Class NP consists of problems for which solutions are verified quickly. P consist of problems which can be solved quickly.



- NP languages are closed under union, Intersection, concatenation, Kleen star.
- NP problems are classified into two types:
  1. NP-complete
  2. NP-hard problems.

Example: Vertex (Graph) coloring problem, Travelling salesman problem, the vertex cover problem, the Hamiltonian circuit problem.

NP-Complete Problem

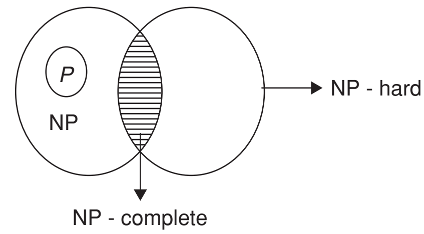
- A class of problems are known as NP-complete problems whose status is unknown. No polynomial time has yet been discovered for NP-complete problems nor has any one been able to prove that no polynomial time exists for any of them. These are hardest of NP-problems. The P and NP-complete problems are disjoint.

Example: (Cook's Theorem) SAT is NP-complete, Bin packing problem, Knapsack Problem.

- A language L is said to be NP-complete if $L \in NP$ and if every $L^I \in$ NP is polynomial-time reducible to L.

A language L_1 is said to be polynomial time reducible to some language L_2 if there exists a DTM by which any w_1 in the alphabet of L_1 can be transformed in polynomial time to a w_2 in the alphabet of L_2 in such a way that $w_1 \in L_1$ if $w_2 \in L_2$. It follows that if some L_1 is NP-complete and polynomial time reducible to L_2, then L_2 is also NP-complete.

NP-Hard Problem



- A problem that is NP-hard has a property that all problems that are in NP can be reduced in polynomial time to it.
- A language, L in NP-hard complete if and only if,

Condition 1: For every language, L^I in NP, there is a polynomial time reduction of L^I to L.

Condition 2: L is not necessarily in NP.

Table 1 *NP-Hard versus NP-complete problems:*

NP-Hard	NP-Complete
(1) A decision problem P_i is NP-hard if every problem in NP is polynomial time reducible to P_i.	(1) A Decision problem P_i is NP-complete if it is NP-hard and is also in class NP itself.
(2) In terms of symbols 'P_i' is NP-hard if for every $P_j \to$ NP	(2) In terms of symbols, 'P_i' is NP-complete, if P_i is NP-hard and $P_j \to$ NP
(3) P_i is 'as hard as' all the problem in NP	(3) P_i is one of the hardest problems in NP
(4) If any problem in NP is proved intractable, then P_i must also be intractable	(4) If any one ever shows that as NP-complete problem is also intractable, then every NP-complete problem is also intractable.

Example 5: Which of following is FALSE?

(A) $\{< x, y > \mid x$ and y are integers, gcd $(x, y) = 1\}$ is a NP class problem.
(B) CLIQUE is a NP class problem.
(C) Eulerian PATH is a P class problem
(D) Dijkstra's algorithm is a problem in P.

Solution: (A)
Choice (A) is a P class problem.
Consider the following table:

D – Decidable, U – Undecidable,? – Open question,T – Trivially Decidable Question	Regular Sets	DCFL's	CFL's	CSL's	Recursive Sets	Recursively Enumerable Sets
(1) Membership problem?	D	D	D	D	D	D
(2) Emptiness problem?	D	D	D	U	U	U
(3) Completeness problem is $L = \Sigma^*$?	D	D	D	U	U	U
(4) Equality problem?	D	?	U	U	U	U
(5) Subset problem is $L_1 \subseteq L_2$?	D	U	U	U	U	U
(6) Is L Regular?	T	D	U	U	U	U
(7) Is the intersection of two languages, a language, of the same type?	T	U	U	T	T	T
(8) Is the complement of a language, also a language of the same type?	T	T	U	?	T	U
(9) Is L is finite or infinite?	D	D	D	U	U	U

Table 2 *Closure properties of formal languages*

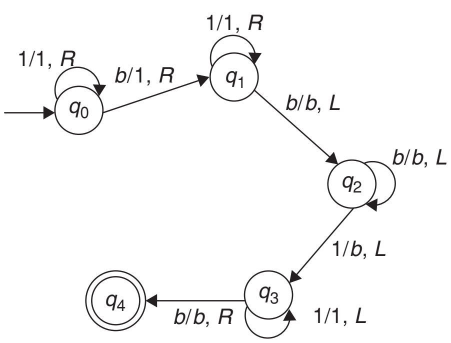
	Regular sets	DCFL'S	CFL'S	CSL'S	Recursive sets	Recursively enumerable sets
(1) Union	*Y*	*N*	*Y*	*Y*	*Y*	*Y*
(2) Concatenation	*Y*	*N*	*Y*	*Y*	*Y*	*Y*
(3) Kleen star	*Y*	*N*	*Y*	*Y*	*Y*	*Y*
(4) Intersection	*Y*	*N*	*N*	*Y*	*Y*	*Y*
(5) Complementation	*Y*	*Y*	*N*	*Y*	*Y*	*N*
(6) Homomorphism	*Y*	*N*	*Y*	*N*	*N*	*Y*
(7) Inverse Homomorphism	*Y*	*Y*	*Y*	*Y*	*Y*	*Y*
(8) Reversal	*Y*	*N*	*Y*	*Y*	*Y*	*Y*
(9) Substitution	*Y*	*N*	*Y*	*Y*	*N*	*Y*
(10) Intersection with regular ets	*Y*	*Y*	*Y*	*Y*	*Y*	*Y*

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. The TM M over $\Sigma = \{1\}$ is given below



What does M generate?
(A) The output is total recursive multiplication function.
(B) The output is addition of two integers.
(C) The output is subtraction of two integers.
(D) The output should be w_1w_2 if input = (w_1w_2) a pair of words.

2. Consider language,

A = {<M>: M is a DFA which doesn't accept any string containing odd number 1's}

Which of following is true about A?
(A) A is Trivially decidable (B) A is undecidable
(C) A is decidable (D) None of these

3. Consider EQ_{CFG} = {<G_1G_2>: G_1, G_2 are CFGs and $L(G_1) = L(G_2)$}. Which of following is true about EQ_{CFG}?
(A) Recognizable (B) Co-Recognizable
(C) Un-recognizable (D) None of the above.

4. A language is given as INFINITE_{DFA} = {<A>: A is a DFA and $L(A)$ is an infinite language}. Which of following is true?
(A) Un-decidable (B) Decidable
(C) Trivially decidable (D) None of above.

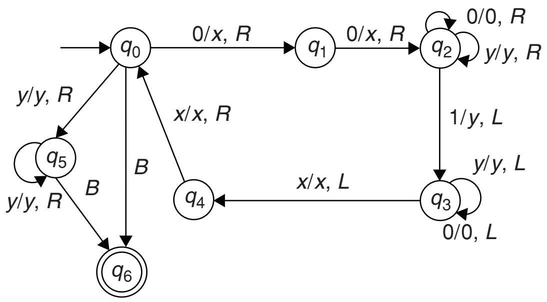
5. A TM designed over an alphabet {0, 1, #}, where 0 indicates blank, which takes a non-null string of 1's and #'s and transfer's the right-most symbol to the left-most end contains-states. (Ex: 000#1#1#1000 … becomes 0001#1#1#000)
(A) 4 (B) 3
(C) 6 (D) 5.

6. Which of following statements are true?
(i) Let K, L be decidable languages. The concatenation of languages, K, L is also decidable language.
(ii) Let L be Turing recognizable language. Then the complement, L^1 is also Turing recognizable language.
(A) (i) and (ii) (B) Only (ii)
(C) Both are false (D) Only (i)

7. Let T_i: denote i th TM. Given, X determines whether $X \in S$, Where the set S is defined inductively as follows: If $u \in S$, then $u^2 + 1$, $3u + 2$ and $u!$ are all members of S. Which of following is true about the given decision problem?
(A) Decidable (B) Un-decidable
(C) Trivially decidable (D) No solution.

8. Fermat's last theorem asserts that there are no integer solution (x, y, z, n) to equation $x^n + y^n = z^n$ satisfying $x, y > 0$ and $n > 2$. Which of the following is true regarding the halting problem?
(A) Decidable
(C) Un-decidable
(C) Trivially decidable
(D) May or may not have solution.

9. The TM, T is designed as



Which of following is true?

(A) T halts on 0^n1^n, $n \geq 0$

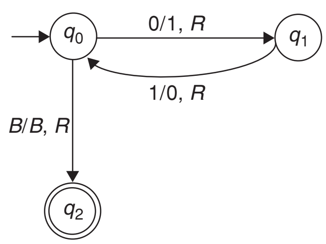
(B) T halts on (01) $(0^n 1^n)$, $n \geq 0$

(C) T halts on $0^{n^2}1^{n^2}$, $n \geq 0$

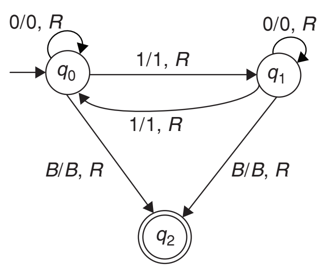
(D) T halts on $0^{2n}1^n$, $n \geq 0$

10. Design TM, which reads an input and starts inverting 0's to 1's till the first 1. The first 1 also inverted. After it has inverted first 1, it read the next symbols and keeps them as they are till the next 1. After encountering 1, it starts repeating the cycle by inverting the symbol till next 1. It halts when it encounters a blank symbol?

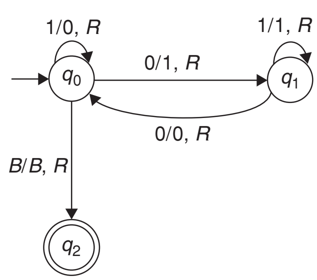
(A)



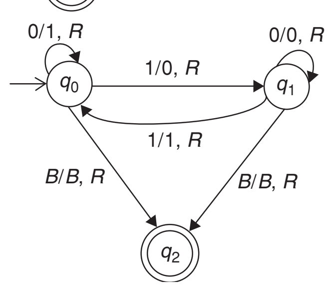
(B)



(C)



(D)



11. Consider three problems, P_1, P_2 and P_3. It is known that P_1 has polynomial time solution, P_2 is NP-complete and P_3 is in NP. Which one of the following is true?

(A) P_3 has polynomial time solution if P_1 is polynomial time reducible to P_3.

(B) P_3 is NP-complete if P_3 is polynomial time reducible to P_2.

(C) P_3 is NP complete if P_2 is reducible to P_3

(D) P_3 has polynomial time complexity and P_3 is reducible to P_2.

12. Let FHAM be the problem of finding a Hamiltonian cycle in a graph G and DHAM be the problem of determining if a Hamiltonian cycle exists in a graph. Which one of the following is true?

(A) Both FHAM and DHAM are NP-hard.

(B) FHAM is NP-hard, but DHAM is not.

(C) DHAM is NP-hard but FHAM is not.

(D) Neither DHAM nor FHAM is NP-hard.

13. The solution for the system of post correspondence problem, $A = \{ba, abb, bab\}$, $B = \{bab, bb, abb\}$ is

(A) 1312212 (B) 15234434

(C) 1311322 (D) No solution.

14. A language, prefix_free REX = $\{R/R$ is a regular expression where $L(R)$ is prefix_free$\}$. Which of following is true about prefix _free REX?

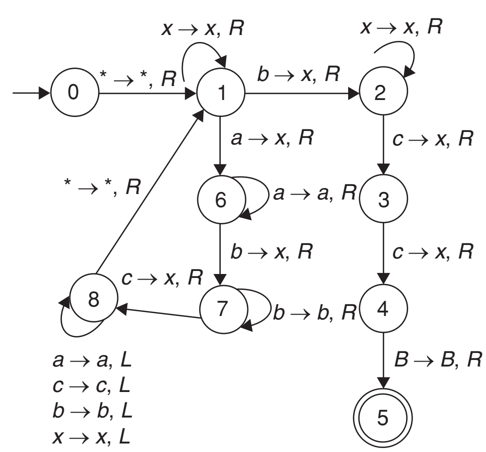
(A) Decidable

(B) Un-decidable

(C) Trivially decidable.

(D) Can't be determined.

15. The TM, M is designed as:



Which of following is true about M?

(A) M is designed for $a^n b^n c^n$, $n \geq 0$

(B) M is designed for $a^{n^2} b^{n^3} c^{n^4}$, $n \geq 0$

(C) M is designed for $a^n b^{n+1} c^{n+2}$, $n \geq 0$

(D) M is designed for $a^n b^n c^n$, $n > 0$

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Consider the language, $A\varepsilon_{\_CFG} = \{<G>: G$ is a CFG that generates$\varepsilon\}$. Which of the following is true?
(A) Undecidable
(B) Decidable
(C) Trivially decidable.
(D) None of the above.

2. The TM is designed with input and output as binary form. (# represents blank). The turing machine TM (M) is

	0	1	#
$q0$	($q1$, 0, R)	(q, 1, R)	ϕ
$q1$	($q1$, 0, R)	(q, 1, R)	($q2$, #, L)
$q2$	($q3$, #, L)	($q3$, #, L)	ϕ
$q3$	($q3$, 0, L)	($q3$, 1, L)	($q4$, #, L)
$q4$	ϕ	ϕ	ϕ

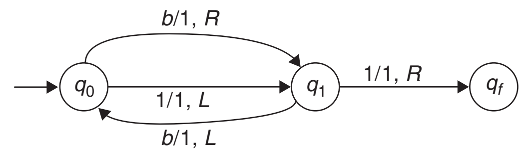
Which of following is true?
(A) M accepts $2n$
(B) M accepts n^2
(C) M replaces left most symbol with #
(D) M replaces right most symbol with #

3. The TM is designed with 3-characters 0, 1, # to compute function $f(n) = 2n$. Input and output are to be in binary form and string represented by 'n' is enclosed between two #'s on left and right of it. b is blank symbol. TM contains _____ states.
(A) 4 (B) 3
(C) 2 (D) 1

4. The language $\{1^n | n$ is a prime number$\}$ is
(A) Undecidable
(B) Decidable
(C) Trivially decidable
(D) None of the above

5. Which of following statement(s) are true?
(i) Let L be Turing decidable language. Then the complement $\bar{L}$ is also Turing decidable language.
(ii) Let K and L be two Turing recognizable languages. The intersection, $K \cap L$ is also Turing recognizable language.
(A) Both (i) and (ii)
(B) Only (i)
(C) Only (ii)
(D) Neither (i) nor (ii) are true.

6. For the following two-way infinite TM, the equivalent one-way TM contains _____ states.



(A) 7 (B) 6
(C) 5 (D) 4

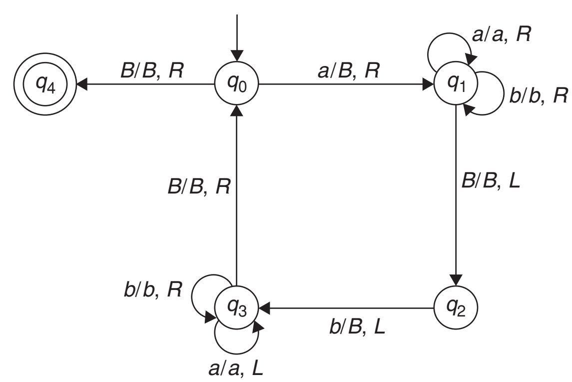
7. L contains at least two strings. Which of following is true?
(A) L has recursively enumerable sets and recursive.
(B) L is recursive.
(C) L has recursively enumerable sets but not recursive.
(D) L does not contain recursively enumerable sets and also is not recursive.

8. Consider the following TM:

Input / State	0	1	B
→ q_0	($q0$, 1, R)	($q0$, 0, R)	($q1$, B, R)
$q1$	–	–	–

What does TM generates?
(A) It display's the negative of given binary number.
(B) It computes one's complement of a binary number.
(C) It computes two's complement of a binary number
(D) It generates double the 0's as 1's.

9. Consider the following TM, M:



Which of following is true?
(A) M halts on $a^{n+1}\, b^n$, $n \geq 0$.
(B) M halts on a^{n^2}, b^{n^3} $n \geq 0$.
(C) M halts on $(ab)\,(a^n)$, $n \geq 0$.
(D) M halts on $a^n\, b^n$, $n \geq 0$.

10. A TM, M is designed generates language $L = \{a^n b^m : n \geq 1$ and $n \neq m\}$. The number of states used are ________
(A) 5 (B) 6
(C) 7 (D) 4

11. Consider three decision problems p_1, p_2 and p_3. It is known that p_1 is decidable, p_2 is undecidable. Which one of following is true?
(A) p_3 is decidable if p_1 is reducible to p_3
(B) p_3 is undecidable if p_3 is reducible to p_2
(C) p_3 is undecidable if p_2 is reducible to p_3
(D) p_3 is decidable if p_3 is reducible to p_2's complement.

12. Which one of following is not decidable?
(A) Given a TM, M, a string S, and an integer K, M accepts S with in K-steps.
(B) Equivalence of two given Turing machines.
(C) Language accepted by a given DFSA is non-empty.
(D) Language accepted by a CFG is non-empty.

13. What is the solution for the correspondence system with two lists $x = \{b, bab^3, ba\}$ and $y = \{b^3, ba, a\}$
(A) 1312213 (B) 2113
(C) 3112 (D) No solution.

14. Given a Turing machine M, a state 'q'and a string 'w'. To determine whether M ever reaches state q when started with input w from its initial state is?
(A) Decidable
(B) Un-decidable
(C) Trivially decidable.
(D) Can not be determined.

15. Given a Turing machine, M to determine whether M ever moves its head to the left when started with input W is:
(A) Decidable
(B) Un-decidable
(C) Trivially decidable.
(D) Can not be determined.

Previous Years' Questions

1. For $s \in (0 + 1)^*$, let $d(s)$ denote the decimal value of s (e.g., $d(101) = 5$). **[2006]**

Let $L = \{s \in (0+1)^* | d(s) \bmod 5 = 2 \text{ and } d(s) \bmod 7 \neq 4\}$

Which one of the following statements is true?
(A) L is recursively enumerable, but not recursive
(B) L is recursive, but not context-free
(C) L is context-free, but not regular
(D) L is regular

2. Which of the following is true for the language $\{a^p \mid p$ is a prime$\}$? **[2008]**
(A) It is not accepted by a Turing Machine
(B) It is regular but not context-free
(C) It is context-free but not regular
(D) It is neither regular nor context-free, but accepted by a Turing machine

3. If L and $\overline{L}$ are recursively enumerable then L is **[2008]**
(A) regular
(B) context-free
(C) context-sensitive
(D) recursive

4. Let $L = L_1 \cap L_2$, where L_1 and L_2 are languages as defined below:

$L_1 = \{a^m b^m c a^n b^n \mid m, n \geq 0\}$

$L_2 = \{a^i b^j c^k \mid i, j, k \geq 0\}$

Then L is **[2009]**
(A) Not recursive
(B) Regular
(C) Context free but not regular
(D) Recursively enumerable but not context free.

5. Let L_1 be a recursive language. Let L_2 and L_3 be languages that are recursively enumerable but not recursive. Which of the following statements is not necessarily true? **[2010]**
(A) $L_2 - L_1$ is recursively enumerable
(B) $L_1 - L_3$ is recursively enumerable
(C) $L_2 \cap L_1$ is recursively enumerable
(D) $L_2 \cup L_1$ is recursively enumerable

6. Which of the following statements is/are FALSE? **[2013]**
1. For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine.
2. Turing recognizable languages are closed under union and complementation.
3. Turing decidable languages are closed under intersection and complementation.
4. Turing recognizable languages are closed under union and intersection.

(A) 1 and 4 only (B) 1 and 3 only
(C) 2 only (D) 3 only

7. Let L be a language and $\overline{L}$ be its complement. Which one of the following is NOT a viable possibility? **[2014]**
(A) Neither L nor $\overline{L}$ is recursively enumerable ($r. e$)
(B) One of L and $\overline{L}$ is r.e. but not recursive, the other is not r. e.
(C) Both L and $\overline{L}$ are r.e. but not recursive
(D) Both L and $\overline{L}$ are recursive

8. Let $A \leq_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B. Which one of the following is FALSE? **[2014]**
(A) If $A \leq_m B$ and B is recursive then A is recursive.
(B) If $A \leq_m B$ and A is undecidable then B is undecidable.
(C) If $A \leq_m B$ and B is recursively enumerable then A is recursively enumerable.

(D) If $A \leq_m B$ and B is not recursively enumerable then A is not recursively enumerable.

9. Let <*M*> be the encoding of a Turing machine as a string over $\Sigma = \{0, 1\}$. Let L = {<*M*>|*M* is a *Turing machine that accepts a string of length* 2014}. Then, L is
(A) Decidable and recursively enumerable
(B) Undesirable but recursively enumerable
(C) Undesirable and not recursively enumerable
(D) Decidable but not recursively enumerable

10. For any two languages L_1 and L_2 such that L_1 is context-free and L_2 is recursively enumerable but not recursive, which of the following is/are necessarily true? **[2015]**
I. $\overline{L_1}$ (complement of L_1) is recursive
II. $\overline{L_2}$ (complement of L_2) is recursive
III. $\overline{L_1}$ is context-free
IV. $\overline{L_1} \cup L_2$ is recursively enumerable
(A) I only (B) III only
(C) III and IV only (D) I and IV only

11. Consider the following statements.
I. The complement of every Turning decidable language is Turing decidable.
II. There exists some language which is in NP but is not Turing decidable.
III. If L is a language in NP, L is Turing decidable.
Which of the above statements is/are true? **[2015]**
(A) Only II (B) Only III
(C) Only I and II (D) Only I and III

12. Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that $\bar{y}$ reduces to W, and Z reduces to $\bar{x}$ (reduction means the standard many-one reduction). Which one of the following statements is **TRUE?** **[2016]**
(A) W can be recursively enumerable and Z is recursive.
(B) W can be recursive and Z is recursively enumerable.
(C) W is not recursively enumerable and Z is recursive.
(D) W is not recursively enumerable and Z is not recursive.

13. Consider the following types of languages: L_1: Regular, L_2: Context - free, L_3: Recursive, L_4: Recursively enumerable. Which of the following is / are TRUE? **[2016]**
I. $\overline{L_3} \cup L_4$ is recursively enumerable
II. $\overline{L_2} \cup L_3$ is recursive
III. $L^*_1 \cap L_2$ is context - free
IV. $L_1 \cup \overline{L_2}$ is context - free
(A) I only (B) I and III only
(C) I and IV only (D) I, II and III only

14. Consider the following languages. **[2016]**
L_1= {<*M*> | *M* takes at least 2016 steps on some input},
L_2= {<*M*> | *M* takes at least 2016 steps on all inputs} and
L_1= {<*M*> | *M* accepts ε}
where for each Turing machine *M*, <*M*> denotes a specific encoding of M. Which one of the following is TRUE?
(A) L_1 is recursive and L_2, L_3 are not recursive
(B) L_2 is recursive and L_1, L_3 are not recursive
(C) L_1, L_2 are recursive L_3 is not recursive
(D) L_1, L_2, L_3 are recursive

15. Let A and B be finite alphabets and let # be a symbol outside both A and B. Let f be a total function from A^* to B^*. We say f is *computable* if there exists a turning machine M which given an input x in A^*, always halts with $f(x)$ on its tape. Let L_f denote the language $\{x \# f(x) | x \in A^*\}$. Which of the following statements is true: **[2017]**
(A) f is computable if and only if L_f is recursive.
(B) f is computable if and only if L_f is recursively enumerable.
(C) If f is computable then L_f is recursive, but not conversely.
(D) If f is computable then L_f is recursively enumerable, but not conversely.

16. Let $L(R)$ be the language represented by regular expression R. Let $L(G)$ be the language generated by a context free grammar G. Let $L(M)$ be the language accepted by a Turing machine M. Which of the following decision problems are undecidable? **[2017]**
I. Given a regular expression R and a string w, is $w \in L(R)$?
II. Given a context-free grammar G, is $L(G) = \emptyset$?
III. Given a context-free grammar G, is $L(G) = \Sigma^*$ for some alphabet Σ ?
IV. Given a Turing machine M and a string w, is $w \in L(M)$?
(A) I and IV only (B) II and III only
(C) II, III and IV only (D) III and IV only

17. The set of all recursively enumerable languages is: **[2018]**
(A) Closed under complementation.
(B) Closed under intersection.
(C) A subset of the set of all recursive languages.
(D) An uncountable set.

18. Consider the following problems. $L(G)$ denotes the language generated by a grammar G. $L(M)$ denotes the language accepted by a machine M.

(I) For an unrestricted grammar G and a string w, whether $w \in L(G)$

(II) Given a Turing machine M, whether $L(M)$ is regular

(III) Given two grammars G_1 and G_2, whether $L(G_1) = L(G_2)$

(IV) Given and NFA N, whether there is a deterministic PDA P such that N and P accept the same language.

Which one of the following statements is correct? **[2018]**

(A) Only I and II are undecidable
(B) Only III is undecidable
(C) Only II and IV are undecidable
(D) Only I, II and III are undecidable

Answer Keys

Exercises

Practice Problems 1

1. D	**2.** C	**3.** B	**4.** B	**5.** D	**6.** D	**7.** A	**8.** D	**9.** D	**10.** D
11. C	**12.** A	**13.** D	**14.** A	**15.** C					

Practice Problems 2

1. B	**2.** D	**3.** B	**4.** B	**5.** A	**6.** B	**7.** C	**8.** B	**9.** D	**10.** B
11. C	**12.** B	**13.** B	**14.** B	**15.** A					

Previous Years' Questions

1. D	**2.** D	**3.** D	**4.** C	**5.** B	**6.** C	**7.** C	**8.** D	**9.** B	**10.** D
11. D	**12.** C	**13.** D	**14.** C	**15.** A	**16.** D	**17.** B	**18.** D		

Test

Theory of Computation

Time: 60 min.

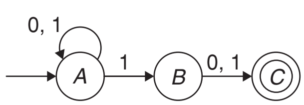
Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. Phrase structure languages, context-sensitive languages, context-free languages and regular languages are commonly referred to as languages of type-0, 1, 2, and 3 respectively. Then, Chomsky's Hierarchy states that
(A) type-0 $\supseteq$ type-1 $\supseteq$ type-2 $\supseteq$ type-3
(B) type-0 $\supset$ type-1 $\supset$ type-2 $\supset$ type-3
(C) type-0 $\subset$ type-1 $\subset$ type-2 $\subset$ type-3
(D) type-0 $\subseteq$ type-1 $\subseteq$ type-2 $\subseteq$ type-3

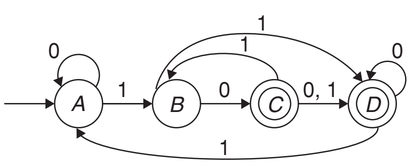
2. Let L be a language recognizable by a finite automaton. The language Reverse $(L) = \{x$ such that x is the reverse of y where $y \in L\}$ is a
(A) Regular language
(B) Context-sensitive language
(C) Context-free language
(D) Phrase-structure language

3. Which of the following statement is true?
(A) It is possible to construct an NFA with more number of states than its equivalent minimum DFA.
(B) There can be a DFA with more than one start state.
(C) Both (A) and (B)
(D) None of these

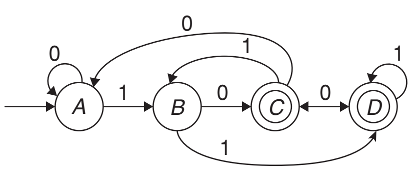
4. Which of the following is an equivalent DFA for the NFA shown below:



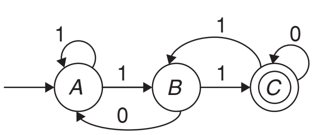
(A)



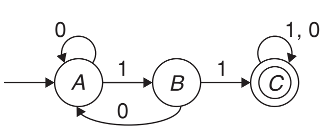
(B)



(C)

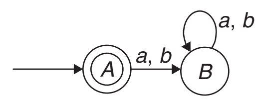


(D)



5. Which one of the following regular expressions over $\{0, 1\}$ denotes the set of strings not containing 100 as a substring?
(A) $0^*(1^*0)^*$ (B) $0^*1^*01^*$
(C) $(0^*(10 + 1)^*)^*$ (D) 0^*1010^*

6. The following transition diagram of a finite automaton accepts



(A) All word over sigma (a, b) such that symbol a and b alternate.
(B) Only empty string.
(C) Only the λ_1 meaning this automaton accepts no string of length greater than zero.
(D) All words over sigma (a, b) except λ.

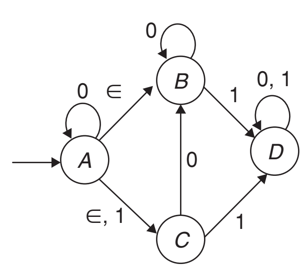
7. Sentence that can be generated from the following production grammar is
$S \rightarrow aS/bA$
$A \rightarrow d/ccA$
(A) *aabccccd* (B) *ababccccd*
(C) *bccddd* (D) *aaccdb*

8. Pumping lemma is generally used for proving
(A) A given grammar is regular
(B) A given grammar is non-regular
(C) Whether two given regular expression are equivalent or not
(D) Both (A) and (C)

9. Finite state machine can recognize
(A) Only context-free grammar
(B) Only regular grammar
(C) Any unambiguous grammar
(D) Any grammar

10. Which of the following is false?
(A) Regular sets are closed under reversal.
(B) Regular sets are closed under substitution.
(C) Regular sets are closed under intersection.
(D) None of these

11. For the DFA shown below $\hat{\delta}(A, 01)$ will be



(A) $\{B, D\}$ (B) $\{C, D\}$
(C) $\{A, B, C\}$ (D) $\{A, B, C, D\}$

12. 'NFA can be simulated by a DFA'. The statement is
(A) True (B) False
(C) Depends on NFA (D) Depends on DFA

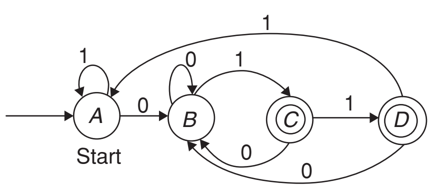
13. Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at least
(A) N^2 (B) 2^N
(C) $2N$ (D) $N!$

14. Let $M = (K, \Sigma, \delta, S, F)$ be a finite state automaton, Where
$K = \{A, B\}$
$\Sigma = \{a, b\}$
$S = A$
$F = \{B\}$,
$\delta(A, a) = A$
$\delta(A, b) = B$
$\delta(B, a) = B$ and
$\delta(B, b) = A$.

A grammar to generate the language accepted by M can be specified as $G = (V, \Sigma, R, S)$, where

$V = K \cup \Sigma$, and $S = A$. Which one of the following set of rules will make $L(G) = L(M)$?
(A) $\{A \to aB, A \to bA, B \to bA, B \to aA, B \to \in\}$
(B) $\{A \to aA, A \to bB, B \to aB, B \to bA, B \to \in\}$
(C) $\{A \to bB, A \to aB, B \to aA, B \to bA, B \to \in\}$
(D) $\{A \to aA, A \to bA, B \to aB, B \to bA, A \to \in\}$

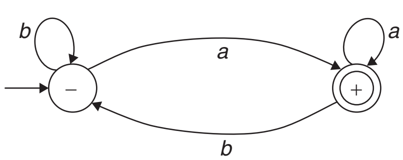
15 A deterministic finite automaton M shown below has a start state A and accepting state D. Which of the following regular expression denotes the set of all words accepted by M?



(A) 0 0 1 (B) 1 0* 1* 10
(C) 1* 0* 0 0 1 (D) (0/1)* 0 1 1

16. Which of the following regular expression is/are true?
(A) $(x^*)^* = x^*$ (B) $(x + y)^* = x^* + y^*$
(C) $x^*y^* = x^* + y^*$ (D) All of these

17. Consider the FA shown in the figure given below, where '–' is the start sate and '+' is the ending state. The language accepted by the FA is



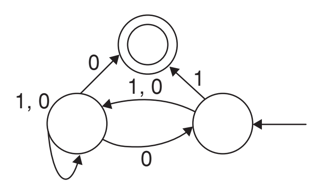
(A) $(a + b)^*b$ (B) $(a + b)^*a$
(C) a^*b (D) a^*b^*

18. Which of the following statement is false?
(A) The family of regular language is closed under the complementary operation.
(B) If L is a regular language, $L_1 = \{UV: U \in L, |V| = 2\}$ is also regular.
(C) If L is a regular language, $L_1 = \{UV: U \in L, V \in L^R\}$ is also regular.
(D) None of these

19. Which of the following is false?
(A) $L = \{0^i\, 1^m 2^m: i \geq 1, m \geq 1\}$ over $\Sigma = \{0, 1, 2\}$ is regular.
(B) $L = \{a^n\, b^l\, a^k, k \geq n + l\}$ is not regular.
(C) $L = \{UWW^2V: U, V, W \in \{a, b\}^+\}$ is regular.
(D) $L = \{a_n b_k : n > k\} \cup \{a_n b_k : n \neq k - 1\}$ is not regular.

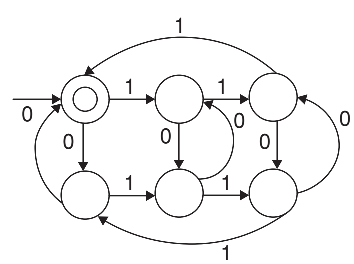
20. Consider a DFA over $\Sigma = \{a, b\}$ accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of states that the DFA will have?
(A) 16 (B) 15
(C) 48 (D) 8

21. For the NFA M given below. Let the language, accepted by M be L. Let L_1 be the language accepted by the NFA M_i, obtained by changing the accepting state of M to a non-accepting state and by changing the non-accepting states of M to accepting states. Which of the following statement is true?



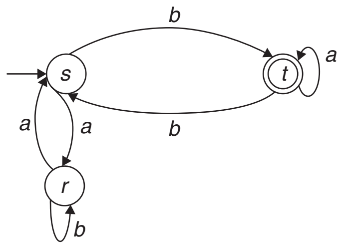
(A) $L_1 = A$ (B) $L_1 \subseteq L$
(C) $L_1 = \{0, 1\}^*$ (D) $L_1 = (0, 1\}^* - L$

22. The following finite state machine accepts all those binary strings in which the number of 1's and 0's are respectively.



(A) Divisible by 3 and 2
(B) Odd and even
(C) Even and odd
(D) Divisible by 2 and 5

23. In the automaton below, s is the start state and t is only final state.



Consider the strings

$U = a\ b\ b\ a\ b\ a$

$V = b\ a\ b$ and

$W = a\ a\ b\ b$

Which of the following statement is true?

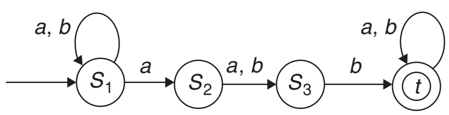
(A) The automaton accepts U and V but not W.

(B) The automaton accepts each of U, V and W.

(C) The automaton rejects U, V and W.

(D) The automaton accepts U but rejects V and W.

24. Which regular expression best describe the language accepted by the non-deterministic automaton below?



(A) $(a + b)^*a(a + b)b$

(B) $(a + b)^*a(a + b)b(a + b)^*$

(C) $(abb)^*$

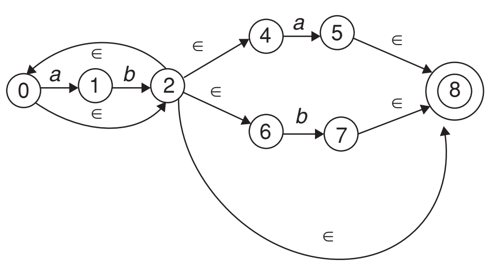
(D) $(a + b)^*$

25. Which of the following strings are accepted by the regular expression:(0/1)* 0(0/1) (0/1)

(A) 000 or 001 (B) 001 or 010

(C) 010 or 011 (D) All the above

26.



The above diagram represents NFA of regular expression.

(A) $(ab)^*\ (a/b/\in)$. (B) $(ab)^*\ (a/b)$.

(C) $(ab)^*\ (a/\in)$. (D) $(ab)^*\ (b/\in)$.

27. If 'a' is a terminal and S, A, B are three (3) non-terminals, then which of the following is regular grammar.

(A) $A \to a\ B/a\ A$ (B) $A \to B\ a/B\ a\ a$

(C) $A \to a\ B$ (D) $S \to \in$
$B \to bA$ $A \to a\ S/b$

28. Consider the grammar

$S \to ABc/Abc$

$BA \to AB$

$Ab \to ab$

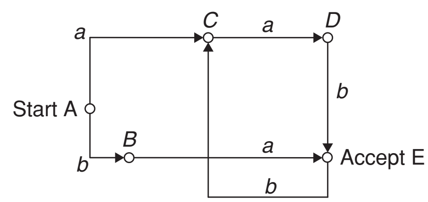
$Aa \to aa$

Which of he following sentences can be derived by this grammar?

(A) aab (B) $abcc$

(C) $abab$ (D) abc

29.



The language recognized by the following finite automation is

(A) $aabb^* + bab^*$

(B) $(aab\ (bab^*))^*$

(C) $(aab + ba)\ (bab)^*$

(D) $(aab^* + bab^*)^*$.

30. From the following regular expressions over an alphabet $\{a, b\}$ given below, which can yield all the possible strings over $\sum (a, b)$?

(i) (a^*b^*)

(ii) $(a + b)^*$

(A) Only (i) (B) Only (ii)

(C) Both (A) and (B) (D) None of these

Answers Keys

1. B	**2.** A	**3.** A	**4.** B	**5.** B	**6.** B	**7.** A	**8.** B	**9.** B	**10.** D
11. B	**12.** A	**13.** B	**14.** B	**15.** D	**16.** A	**17.** B	**18.** D	**19.** A	**20.** B
21. C	**22.** A	**23.** D	**24.** B	**25.** D	**26.** A	**27.** D	**28.** D	**29.** C	**30.** B

Theory of Computation Test I

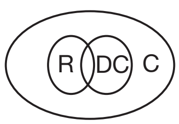
Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

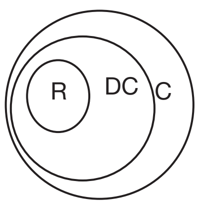
1. Which of the following is FALSE?
I: Any regular language is context free language.
II: There exists a DPDA (Deterministic Push Down Automata) for every CFL.
(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

2. Which of the following figure correctly specifies the relation between regular languages (R), deterministic CFL's (DC) and CFL's (C)?

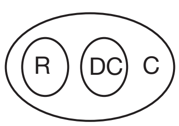
(A)



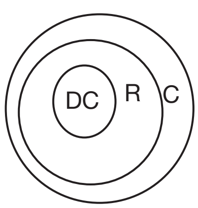
(B)



(C)



(D)



3. Consider the regular expression, $RE = ab^* + ba^*$. Then the reversal of RE, given by RE^R is equal to:
(A) $ab^* + ba^*$ (B) a^*b
(C) $b^*a + a^*b$ (D) $(a^*b)\ (b^*a)$

4. The statement: 'For every regular language L, every subset of L is regular as well' is:
(A) TRUE
(B) False
(C) TRUE, only if L is CFG
(D) False, only if L is CFG.

5. Which of the following strings are not accepted by the language L $(((010 \cup 10)^*1)^*)$?
(A) 1 (B) 0101
(C) 10101110 (D) 01001011

6. Which of the following is FALSE?
(A) $(L_1^*)^* = L_1^*$
(B) $L_1^* = (L_1L_1)^*$
(C) If $L_1 \subset L_2$ then $L_1^* \subset L_2^*$
(D) If L_1 is finite then it is regular.

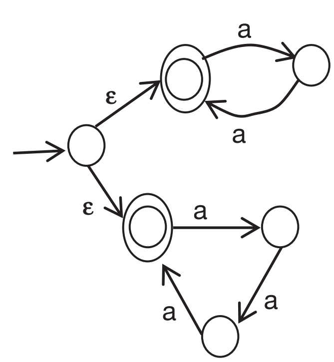
7. Which of the following strings are generated by the regular expression,
$$R = (ab\varepsilon)^*\ (a + b)\ ba?$$
(i) ε (ii) aba
(iii) $ababba$ (iv) $abababa$
(A) (i), (iii) (B) (ii), (iii)
(C) (ii), (iv) (D) (i), (iv)

8. The regular expression $R = (ab\varepsilon)^*\ ((a + b) \cdot \phi)\ ba$ is equivalent to
(A) $(ab\varepsilon)^*\ (a + b)\ ba$ (B) $(ab\varepsilon)^*\ ba$
(C) ba (D) ϕ

9. Which of the following identity is FALSE for regular expressions?
(i) $R + R = R$ (ii) $RR^* = R^*R$
(iii) $\varepsilon R = R\varepsilon = R$
(A) (i), (ii), (iii) (B) (i) only
(C) (ii), (iii) (D) None of these

10. Which of the following language is Regular?
I: $\{ww | w \in \{0, 1\}^*\}$
II: $\{w/w = w^R, w \in \{0, 1\}^*\}$
III: $\{ww^R/w \in \{0, 1^*\}$
IV: Set of all strings with un-equal number of 0's and 1's.
(A) I, III only (B) II, IV only
(C) IV only (D) None of the above

11. Consider the following NFA:

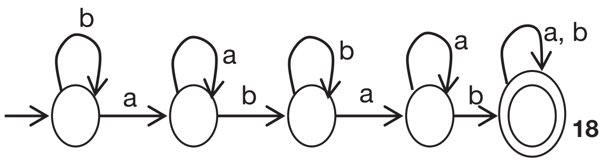


Which of the following gives the language accepted by given NFA?
(A) All strings of the form a^k, $k \geq 0$.
(B) All strings of the form $\{a^m a^n | m \geq 0, n \geq 0\}$
(C) All strings of the form $\{a^m a^n \mid m$ is a multiple of 2 and n is multiple of 3$\}$
(D) All strings of the form $\{a^k \mid k$ is a multiple of 2 or 3$\}$

12. Which of the following is not context-free language?
(i) $\{a^k \mid k$ is a perfect square$\}$

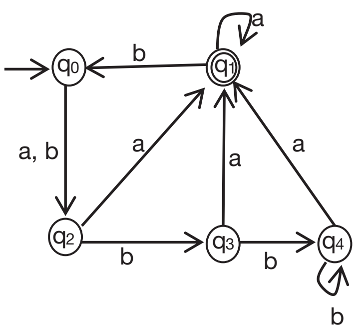
(ii) $\{a^i b^j c^i d^j \mid i, j \geq 0\}$
(iii) $\{a^i b^{2i} a^i \mid i \geq 0\}$
(A) (i), (ii) (B) (ii), (iii)
(C) (i), (iii) (D) (i), (ii), (iii)

13. Consider the following DFA D:



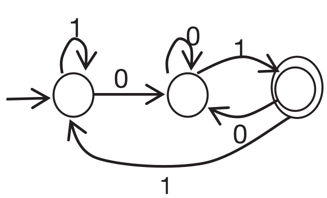
Which of the following is TRUE?
(i) D accepts all strings which contain the sub-word 'ab' two times only.
(ii) D accepts all strings which terminate with 'b'.
(iii) The strings *baabb*, *abbba* are not in the language.
(A) (i), (ii) only (B) (iii) only
(C) (ii), (iii) (D) (i), (iii)

14. Consider the following FA:



Which of the following states are equivalent?
(i) q_0, q_1 (ii) q_2, q_3
(iii) q_3, q_4 (iv) q_0, q_4
(A) (i), (ii) (B) (ii), (iv)
(C) (iii) only (D) (ii), (iii)

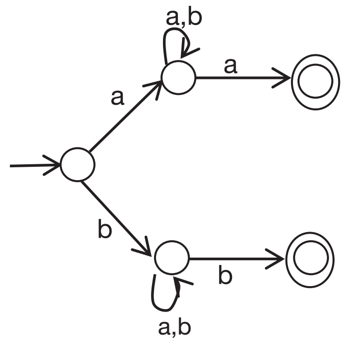
15. Consider the following FA:



The number of states in the minimized FA is
(A) 3 (B) 2
(C) 1 (D) None of the above

16. Which of the following represents a language in automata theory?
(i) Σ^* (ii) ε
(iii) ϕ (iv) $\{\phi\}$
(v) $\{\varepsilon\}$
(A) (i), (iii) only (B) (i), (iii), (v) only
(C) (ii), (iv) only (D) (iv), (v) only

17. Consider the following NFA:



The number of states in its equivalent DFA is
(A) 4 (B) 5
(C) 6 (D) 7

18. Which of the following language is regular?
(i) $\{x = y + z \mid x, y, z$ are binary integers and x is the sum of y and $z\}$
(ii) $\{w \mid w$ is a binary representation of a number greater than 3$\}$
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

19. Consider the following grammar:
S $\rightarrow$ PaP
P $\rightarrow$ ε|PaPbP|PbPaP|Pa|aP
What is the language generated by this grammar over $\{a, b\}$?
(A) Set of all strings with more a's than b's
(B) Set of all strings with more b's than a's
(C) Set of all strings with twice a's than b's
(D) Set of all strings with equal number of a's and b's.

20. Consider a regular language L. A new language DELchar(L) = {W|W is some string from L with exactly one character deleted} is defined. Then DELchar(L) is
(A) a regular language
(B) a CFG but not regular
(C) neither CFG nor regular
(D) not accepted by a PDA

21. Consider the following grammar:
S $\rightarrow$ a Sb | P
P $\rightarrow$ bP | Pa | ε
Which of the following language represents the grammar?
(A) $\{a^n b^n \mid n \in N\}$
(B) $\{a^n b^m b^n \mid m, n \in N\}$
(C) $\{a^n b^m a^p b^n \mid n, m, p \in N\}$
(D) $\{a^n b^m \mid n > m\}$

22. Let M_1 and M_2 are two DFA's with 5-tuple format as given below:

$$M_1 = \{Q_1, \Sigma, \delta_1, S_1, F_1)$$
$$M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$$

where Q_1, Q_2 are set of states; Σ is the alphabet set; δ_1, δ_2 are transition functions; S_1, S_2 are start states;

F_1, F_2 are final states. Then which of the following are necessary for $L(M_1) = L(M_2)$?

(i) $Q_1 = Q_2$ (ii) $F_1 = F_2$
(iii) $S_1 = S_2$ (iv) $\delta_1 = \delta_2$
(A) (i), (ii) (B) (i), (ii), (iii)
(C) (i), (ii), (iii), (iv) (D) (iii), (iv)

23. Which of the following language is both regular and context free?

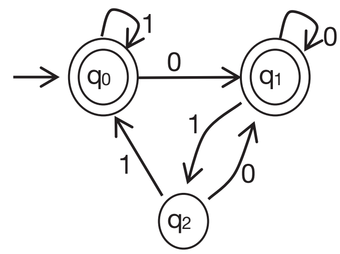
(i) $\{a^n (bc)^n : n \geq 0\}$ (ii) $\{a^n a^n a^n : n \geq 0\}$
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

24. Match the following:

List I		List II	
1	$r^*s + s$	A	r^*
2	ϕ^*	B	ϕ
3	ε^*	C	ε
4	$(\varepsilon + r)^*$	D	$r^* s$

(A) 1–*a*, 2–*b*, 3–*c*, 4–*d*
(B) 1–*d*, 2–*c*, 3–*c*, 4–*a*
(C) 1–*a*, 2–*c*, 3–*c*, 4–*d*
(D) 1–*d*, 2–*c*, 3–*b*, 4–*a*

25. Consider the following FA:



What is the language accepted by above FA?

(A) $\{w|w \in \{0, 1\}^*$ and w do not end with 1$\}$
(B) $\{w|w \in \{0, 1\}^*$ and w contains more zeros than 1's$\}$
(C) $\{w|w \in \{0, 1\}^*$ and w do not end with 01$\}$
(D) $\{w|w \in \{0, 1\}^*$ and w do not have consequent 0's and 1's$\}$

Answer Keys

1. B	**2.** B	**3.** C	**4.** B	**5.** C	**6.** B	**7.** C	**8.** D	**9.** D	**10.** D
11. D	**12.** D	**13.** B	**14.** D	**15.** A	**16.** B	**17.** B	**18.** B	**19.** A	**20.** A
21. C	**22.** C	**23.** B	**24.** B	**25.** C					

Hints and Explanations

1. Every regular language is a CFL but not vice versa. Every CFL has a PDA but that PDA need not be a deterministic PDA. Choice (B)

2. Regular $\subset$ DCFL $\subset$ CFL Choice (B)

3. Given regular expression,

$$RE = ab^* + ba^*$$
$$RE^R = (ab^* + ba^*)^R$$
$$= (ab^*)^R + (ba^*)^R$$
$$= (b^*)^R a^R + (a^*)^R b^R$$
$$= (b^R)^*a + (a^R)^*b$$
$$= b^*a + a^*b$$

Choice (C)

4. Given statement is false.
Ex: $L = \{a, b\}^*$; subset of L is $\{a^n b^n \mid n \in N\}$, which is not regular. Choice (B)

5. Given language,

$$L(((010 \cup 10)^*1)^*)$$

The regular expressions which are accepted by 'L' are of the form $((m)^*1)^*$, where $m = 010 \cup 10$.
Any string in L is either ε or ends with '1'. Choice (C)

6. $L_1^* = (L_1 L_1)^*$ is false.
Let $L_1 = \{a\}$ then L_1^* will have 'a' but $(L_1 L_1)^*$ do not have 'a'.
Choice (B)

7. Given regular expression,

$$R = (abe)^* (a + b) ba$$

'ε' is not accepted.
'*aba*' is accepted.
'*ababba*' is not accepted.
'*abababa*' is accepted. Choice (C)

8. $\phi R = R\phi = \phi$, so $(a + b)\phi = \phi$
$\phi \cdot ba = \phi$, $(ab\varepsilon)^* . \phi = \phi$ Choice (D)

9. All the three are valid identities of regular expressions. Choice (D)

10. None of the four languages is regular. Finite automata cannot check the equality of two substrings of a string. Choice (D)

11. Given NFA accepts number of a's which is a multiple of 2 or 3. Choice (D)

12. $\{a^k \mid k$ is a perfect square$\}$
This is not context-free. The PDA can't check whether a number is perfect square or not.
$\{a^i b^j c^i d^j \mid i, j \geq 0\}$
Not context free. PDA can check the equality of a, b and c, d or a, d and b, c but not a, c and b, d.
$\{a^i b^{2i} a^i \mid i \geq 0\}$ is also not recognized by a PDA.
Choice (D)

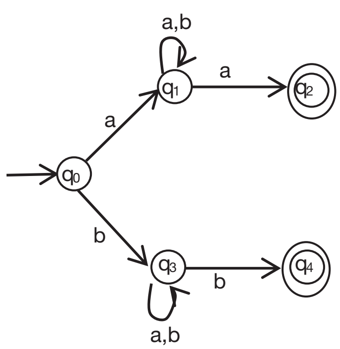
13. Given DFA accepts all strings which contain at least two sub-words '*ab*'. It do not accept *baabb*, *abbba*. Choice (B)

14. In given FA, non-final states are $\{q_0, q_2, q_3, q_4\}$ and final state is $\{q_1\}$.
A state is equivalent to another state if both are either non-final or final states.
Also each transition from those states leads to either final state or non final state only.
$\therefore q_0, q_1$ are not equivalent.
q_0, q_4, are not equivalent.
q_2, q_3 are equivalent as both are non-final states and q_2, q_3 with '*a*' reaches a final state and q_2, q_3 with '*b*' reaches a non-final state.
Similarly, q_3, q_4 are also equivalent. Choice (D)

15. Given FA is the minimal FA. No minimization is possible. ($\because$ With transition '1', both the non-final states are reaching a final and non-final state). Choice (A)

16. Σ^*, ϕ(empty), $\{\varepsilon\}$ are languages. Choice (B)

17. Given NFA,



The DFA equivalent to given NFA is given as,

	a	b
$\to [q_0]$	$[q_1]$	$[q_3]$
$[q_1]$	$[q_1, q_2]$	$[q_1]$
$[q_3]$	$[q_3]$	$[q_3, q_4]$
$[q_1, q_2]$	$[q_1, q_2]$	$[q_1]$
$[q_3, q_4]$	$[q_3]$	$[q_3, q_4]$

$\therefore$ Number of states in the equivalent DFA is 5. Choice (B)

18. (i) is not regular. (This will be shown using pumping Lemma). (ii) is regular and the expression is given as $0^*1\ (0 + 1)\ (0 + 1)\ (0 + 1)^*$ Choice (B)

19. Given grammar is

$$S \to PaP$$
$$P \to e|PaPbP|PbPaP|Pa|aP$$

Consider some derivations:

1. $S \to PaP$
   $\to a$
2. $S \to PaP$
   $\to PaaP$
   $\to aa$
3. $S \to PaP$
   $\to PaPbPaP$
   $\to aba$
4. $S \to PaP$
   $\to baa$

$\therefore$ More *a*'s than *b*'s. Choice (A)

20. DELChar (L) is a regular language. Choice (A)

21. Given grammar,

$$S \to aSb|P$$
$$P \to bP|Pa|e$$

The strings generated by given grammar are
$S \to P$
$\to \varepsilon$
$S \to aSb$
$\to aPb$
$\to ab$
$S \to a\ Sb$
$\to aPb$
$\to aPab$
$\to aab$
$S \to aSb$
$\to aPb$
$\to abPb$
$\to abb$
$S \to aSb$
$\to aaSbb$
$\to aabbb$
In the starting and end of the string we need to have equal number of *a*'s and *b*'s. in between there will be any number of *b*'s and *a*'s.
$\therefore$ The language accepted by given grammar is $\{a^n\, b^m\, a^p\, b^n \mid n, m, p \in N\}$. Choice (C)

22. $L(M_1) = L(M_2)$ if $Q_1 = Q_2$; $F_1 = F_2$; $S_1 = S_2$; $\delta_1 = \delta_2$. Choice (C)

23. (i) is CFL but not regular.
(Checking the equality of *a*'s and (*bc*) is not done using FA)
(ii) is regular and CFL.
$\{a^{3n} \mid n \geq 0\}$ is regular.
Every regular language is CFL. Choice (B)

24.
$$r^*\, s + s = r^*s$$
$$\phi^* = \varepsilon^* = \varepsilon$$
$$(\varepsilon + r)^* = r^*$$
Choice (B)

25. Given FA do not accept the strings which will terminate with '01'. Choice (C)

Theory of Computation Test 2

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following is TRUE for Turing Machines?
   (i) Turing machines accepts Type-0 languages.
   (ii) Turing machines are used for computing functions.
   (iii) Turing machines are used for determining the undecidability of certain languages.
   (A) (i), (ii) only (B) (i), (iii) only
   (C) (ii), (iii) only (D) (i), (ii), (iii)

2. Which of the following operations are not performed on Turing machines?
   (A) Writing a new symbol in the cell being currently scanned.
   (B) Moving to the cell left of the present cell.
   (C) Moving to the cell right of the present cell.
   (D) None of the above

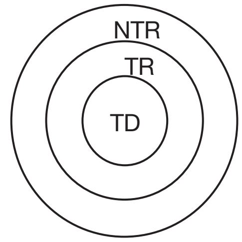
3. If the languages L_1 and L_2 are decidable then which of the following are also decidable?
   (i) $\bar{L}_1 \cup \bar{L}_2$ (ii) $\bar{L}_1 \cap \bar{L}_2$
   (A) (i) only (B) (ii) only
   (C) Both (i) and (ii) (D) Neither (i) nor (ii)

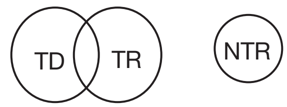
4. If L_1 and L_2 are Turing-recognizable languages then which of the following are not Turing-recognizable?
   (i) $L_1 \cup L_2$ (ii) $L_1 \cap L_2$
   (iii) $L_1 - L_2$ (iv) $\bar{L}_1$
   (v) L_1^* (vi) $L_1 \circ L_2$
   (A) (iv), (v) (B) (iii), (iv)
   (C) (i), (ii), (vi) (D) (ii), (iv), (v)

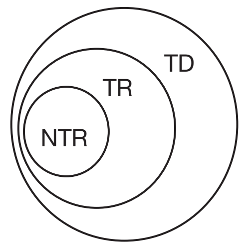
5. Which of the following statement is FALSE?
   (A) The set of decidable languages is closed under symmetric difference.
   (B) If L_1 is undecidable and L_2 is decidable then the symmetric difference of L_1 and L_2 is undecidable.
   (C) The intersection of a recognizable language and an unrecognizable language is always unrecognizable.
   (D) All of the above

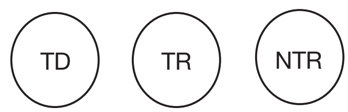
6. If L_1 and L_2 are recursive languages, then which of the following also belongs to recursive languages?
   (i) $L_1 \cup L_2$
   (ii) $L_1 L_2$
   (iii) $\bar{L}_1$
   (A) (i) only (B) (i), (ii)
   (C) (ii), (iii) (D) (i), (ii), (iii)

7. Which of the following correctly specifies the relation between Turing decidable (TD), Turing recognizable (TR) and 'not Turing recognizable' (NTR) languages?

(A)


(B)


(C)


(D)


8. Which of the following DFA problems are Decidable?
   (A) The language of all DFA's with an empty language.
   (B) The language of all pairs of DFA's that have the same language.
   (C) $A_{DFA} = \{<D, \omega>|D$ is a DFA, ω is a word and D accepts $\omega\}$
   (D) All of the above

9. Which of the following language is accepted by a Turing machine?
   (i) $\{a^n \# a^n \# a^n : n \geq 0\}$ and $\Sigma = \{a, \#\}$
   (ii) $\{a^n \# b^{2^n} : n \geq 0\}, \Sigma = \{a, b\}$
   (A) (i) only (B) (ii) only
   (C) Both (i) and (ii) (D) Neither (i) nor (ii)

10. For which of the following languages, no Turing-machines will exist?
   (A) Recursive languages
   (B) Recursively-enumerable languages
   (C) Non-Recursively enumerable.
   (D) None of the above

11. If L_1 is a recursive language and L_2 is a recursively enumerable language then which of the following is also recursively enumerable?
   (i) $L_1 \cup L_2$ (ii) $L_1 \cap L_2$
   (iii) $L_1 L_2$ (iv) $L_1 - L_2$
   (v) $L_2 - L_1$
   (A) (i), (ii), (iii) (B) (i), (iii), (iv)
   (C) (iii), (iv), (v) (D) (i), (ii), (iii), (v)

12. Which of the following is False, if A is mapping reducible to B?
(A) If B is decidable then A is decidable.
(B) If B is recursively enumerable then A is recursively enumerable.
(C) If A is undecidable then B is undecidable.
(D) None of the above

13. Which of the following language is Recursive?
(A) $L_1 = \{<M>|M$ is a TM and there exists an input on which M halts in less than $|<M>|$ steps.$\}$
(B) $L_2 = \{<M> \mid M$ is a TM and $|L(M)| \le 3\}$
(C) $L_3 = \{<M> \mid M$ is a TM and $|L(M)| \ge 3\}$
(D) All of the above

14. The language $\{<M> \mid M$ is a Turing machine and $L(M)$ is finite$\}$ is
(A) recursive
(B) recursively enumerable
(C) not recursively-enumerable
(D) decidable

15. Which of the following problem is Recursive?
(i) $\{<M> \mid M$ is a DFA and $L(M)$ is finite$\}$
(ii) $\{<M> \mid M$ is a DFA and $L(M) = \Sigma^*\}$
(iii) $\{<M, x> \mid M$ is a DFA and M accepts $x\}$
(A) (i), (ii) (B) (iii) only
(C) (ii), (iii) (D) (i), (ii), (iii)

16. Which of the following is not accepted by a PDA but accepted by a TM?
(A) $\{a^n b^n c a^n b^n \mid n > 0\}$
(B) $\{a^n b^{2n} c^{3n} \mid n \ge 0\}$
(C) $\{a^n b^{n+m} c^m \text{d}^m \mid m, n \ge 0\}$
(D) All of the above

17. Which of the following has same power as Turing machine?
(A) NDFA (B) 2-PDA
(C) k-tape TM (D) Both (B) and (C)

18. The language $L = \{a^n b^n c^n \mid n \ge 0\}$ can be accepted by a:
(i) 2-PDA (ii) TM
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

19. If $A \le_m B$ (i.e., A is reducible to B) and if B is regular language then does it implies that A is a regular language?
(A) Yes
(B) No
(C) Data insufficient
(D) Reduction is not possible with regular languages.

20. If a language L is recursively enumerable and $L \le_m \overline{L}$ then, which of the following is TRUE?
(A) L is recursive
(B) L may or may not be recursive
(C) L is undecidable
(D) None of these

21. Which of the following language(s) is/are undecidable?
$L_1 = \{<M> \mid M$ is a TM and $L(M)$ is a CFL$\}$
$L_2 = \{<M> \mid M$ is a TM and $L(M)$ is finite$\}$
(A) L_1 only (B) L_2 only
(C) Both L_1 and L_2 (D) Neither L_1 nor L_2

22. For which of the following language(s) a Turing machine exists?
(i) $\varnothing$ (ii) $\{\varepsilon\}$
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

23. A language L is recursive if
(A) L is recursively enumerable.
(B) $\overline{L}$ is recursively enumerable.
(C) Both L and $\overline{L}$ are recursively enumerable.
(D) L is not recursively enumerable.

24. If a Language L is recursively enumerable but not recursive then $\overline{L}$
(A) is recursive
(B) is recursively enumerable
(C) is not recursively enumerable
(D) is decidable

25. A decision problem is undecidable if
(i) it has no algorithm to solve
(ii) it has no TM that halts on every input
(iii) it specifies a language which is not recursive.
(A) (i), (ii) (B) (ii) only
(C) (i), (iii) (D) (i), (ii), (iii)

Answer Keys

1. D	**2.** D	**3.** C	**4.** B	**5.** C	**6.** D	**7.** A	**8.** D	**9.** C	**10.** C
11. D	**12.** D	**13.** A	**14.** C	**15.** D	**16.** D	**17.** D	**18.** C	**19.** B	**20.** A
21. C	**22.** C	**23.** C	**24.** C	**25.** D					

Hints and Explanations

1. Turing machines accepts type-0 languages. They are used for computing functions and for determining the undecidability of languages. Choice (D)
2. A Turing machine can write into some cell, move left or right. Choice (D)
3. If a language is decidable then its complement is also decidable.
   The union and intersection of two decidable languages is also decidable. Choice (C)
4. Turing-recognizable languages (i.e. Recursively-enumerable) are not closed under subtraction and complement operations. Choice (B)
5. Decidable languages are closed under symmetric difference.
   If L_1 is undecidable and L_2 is decidable then the difference of L_1 and L_2 is undecidable.
   Let L_1 is a recognizable language and $L_1 = \emptyset$, L_2 is unrecognizable. Then $L_1 \cap L_2 = \emptyset$ which is recognizable.
   Choice (C)
6. Recursive languages are closed under union, intersection, complementation, concatenation and Kleene closure. Choice (D)
7. Regular $\subset$ Context-free $\subset$ TD $\subset$ TR $\subset$ NTR.
   Choice (A)
8. Some of the decidable properties of DFA are
   - Universality
   - Kleene-closedness
   - Inclusion
   - Disjointness
   - Membership
   - Emptyness
   - Equivalance

   ∴ All the problems in given choices are Decidable.
   Choice (D)
9. A TM can be designed to accept, $\{a^n \# a^n \# a^n\}$. In the input write x for an 'a'. Traverse till # and again write x for an 'a' and traverse till next # and write x for an 'a'. Traverse left till left most x and repeat this for next 'a'. If no more a's left in any part, accept the language.
   Similarly, a TM can be constructed for $\{a^n \# b^{2^n} : n \geq 0\}$. Replace a with x and traverse till # and replace n b's with x and traverse left till x and repeat this. If no more symbols left then accept the language. Choice (C)
10. Recursive: TM always halts.
    Recursively enumerable: TM either halts or halts in non-final state or loops.
    Non-Recursively-enumerable: No TM exists.
    Choice (C)
11. $L_1 - L_2$ is not recursively enumerable. Choice (D)
12. If A is a mapping reducible to B, then based on some basic Theorems,
    - B is decidable then A is decidable
    - A is undecidable then B is undecidable
    - B is Turing-recognizable then A is Turing recognizable.

    ∴ All are TRUE. Choice (D)
13. L_1 is a recursive language. First find the length of M and store it. Then it runs M on all inputs of length atmost $|<M>|$ steps and accepts if M accepts atleast one of the strings within the specified number of steps.
    Choice (A)
14. Given language is not recursively enumerable and undecidable. Choice (C)
15. All the given problems are decidable and hence recursive. Choice (D)
16. $\{a^n\, b^n\, ca^n\, b^n . | n > 0\}$. PDA can check equality of number of a's and b's.
    But it can't able to compare this number of a's and b's with next a's and b's.
    But this can be accepted by a TM. Similarly for $\{a^n\, b^{2n}\, c^{3n} | n \geq 0\}$ and $\{a^n\, b^{n+m}\, c^m\, d^m | \, m, n \geq 0\}$.
    Choice (D)
17. A TM has same power as k-tape TM ($k \geq 1$) and 2-PDA has same power as TM. Choice (D)
18. A 2-PDA (i.e., 2-stack PDA) has same power as TM. A TM can accept L. Choice (C)
19. Given $A \leq_m B$.
    B is regular then it does not imply that A is regular.
    Ex: $\{a^n\, b^n \mid n \geq 0\} \leq_m \{a^n \mid n \geq 0\}$. Choice (B)
20. Given L is recursively enumerable and $L \leq_m \bar{L}$.
    As $L \leq_m \bar{L}$ then $\bar{L} \leq_m L$.
    ($\because A \leq_m B$ then $\bar{A} \leq_m \bar{B}$).
    L is RE, so $\bar{L}$ is also RE. As both L and $\bar{L}$ are RE then L is recursive. Choice (A)
21. $\{<M, w>$: M is a TM and M accepts input string $w\}$ is a known undecidable problem. We can reduce this to L_1 and L_2 also. So both L_1 and L_2 are undecidable.
    Choice (C)
22. A TM can exist for both languages. (i) rejects everything. (ii) accepts '$\in$' only. Choice (C)
23. If a language L and its complement $\bar{L}$ are recursively enumerable, then L is recursive. Choice (C)
24. Both L and $\bar{L}$ are recursively enumerable if L is recursive. Choice (C)
25. An undecidable problem has no algorithm, no TM and is also not recursive. Choice (D)

Chapter 1

Processes and Threads

LEARNING OBJECTIVES

- *Basics of operating systems*
- *Services of OS*
- *Evolution of operating systems*
- *Processes*
- *Processes and process control blocks*
- *Process states*
- *Process creation*
- *Suspended processes*
- *OS control structures*
- *Process attributes*
- *Modes of execution*
- *Threads*
- *Thread functionality*
- *Thread synchronization*

Basics of Operating System

An operating system (OS) is a program that controls the execution of application programs and acts as an interface between applications and the computer hardware. The three objectives of an OS are as follows:

1. *Convenience*: An OS makes a computer more convenient to use.
2. *Efficiency*: An OS allows the computer system resources to be used in an efficient manner.
3. *Ability to evolve*: An OS should be constructed in such a way as to permit the effective development, testing and introduction of new system functions without interfering with service.

OS as a User–Computer Interface

Consider the below figure (Figure 1) which shows the hardware and software used in providing applications to a user in a layered fashion.

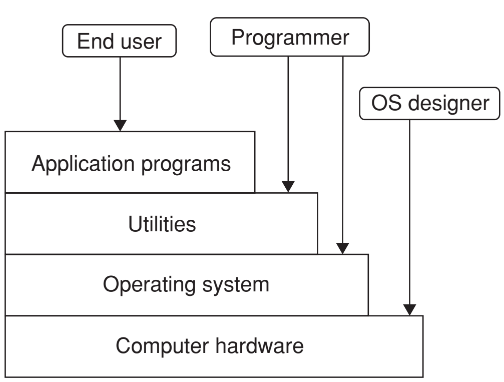


Figure 1 Layers and views of a computer system.

The end user of the application is not concerned with the details of computer hardware. Utilities implement the frequently used functions that assist in program creation, the management of files and control of input/output (I/O) devices. The most important collection of system programs comprises the OS. The OS masks the details of the hardware from the programmer and provides the programmer with a convenient interface for using the system.

Services of an OS

1. *Program development*: An OS provides a variety of facilities and services as editors and debuggers to assist programmers in creating programs.
2. *Program execution*: An OS handles the scheduling duties of program execution for the user.
3. *Access to I/O devices*: An OS provides a uniform interface that hides the details of I/O devices so that the programmers can access the I/O devices using simple reads and writes.
4. *Controlled access to files*: In a system with multiple users, an OS provides protection mechanism to control access to the files.
5. *System access*: For shared or public systems, an OS controls access to the system as a whole and to specific system resources.
6. *Error detection and response*: An OS must provide a response that clears the error condition with the least impact on running applications.
7. *Accounting*: A good OS will collect usage statistics for various resources and monitor performance parameters (viz., response time).

OS as Resource Manager

A computer is a set of resources for the movement, storage and processing of data and for the control of these functions. An OS is responsible for managing these resources.

1. An OS functions in the same way as ordinary computer software, that is, it is a program or suite of programs executed by the processor.
2. An OS frequently relinquishes control and must depend on the processor to allow it to regain control.

The OS directs the processor in the use of the other system resources and in the timing of its execution of other programs.

Figure 2 shows the resources that are managed by an OS.

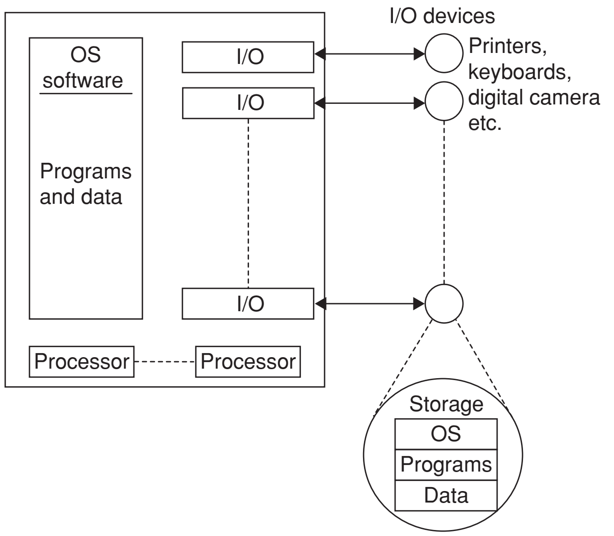


Figure 2 OS as a resource manager.

A portion of the OS lies in the main memory. This includes the kernel or nucleus, which contains the most frequently used functions in the OS and, at a given time, other portions of the OS currently in use. The remainder of main memory contains user programs and data. The allocation of this resource is controlled jointly by the OS and memory management hardware in the processor.

Evolution of OSs

Serial processing

With the earliest computers, the programmer interacted directly with the computer hardware. There was no OS. Programs in machine code were loaded via the input devices. This mode of operation is termed as *serial processing*. Problems with this system are scheduling and setup time.

Simple batch systems

- It requires the grouping up of similar jobs, which consist of programs, data and system commands.
- Users have no control over results of a program.
- Off-line debugging.

Multiprogrammed batch systems

In view of simultaneous execution of multiple programs, it improves system throughput and resource utilization.

Example: Windows XP, 98

- **Multitasking OS:** A running state of a program is called a *process* or a *task*. The concept of managing a multitude of simultaneously active programs, competing with each other for accessing the system resources is called *multitasking*.
- Serial multitasking or context switching is the simplest form of multitasking.

Example: Windows NT, Linux

- **Multiuser OS:** It is defined as multiprogramming OS that supports simultaneous interaction with multiple users.

Example: Linux, Unix, a dedicated transaction processing system (viz., railway reservation system).

- **Multiprocessing OS:** The term *multiprocessing* means multiple CPUs performing more than one job at one time.

The term 'multiprogramming' means situation in which a single CPU divides its time between more than one job.

Time sharing systems

In this kind of OS, the processor time is shared among multiple users. The CPU switches rapidly from one user to another user; each user is given an impression that he/she has his own computer while it is actually one computer shared among many users.

If there are n users actively requesting service at one time, each user will only see on the average $1/n$ of the effective computer capacity, not counting OS overhead.

Bootstrap Bootstrap is an initial program which runs, when a computer is powered up (or) restarted. The task is to initialize system aspects (CPU registers to device controllers to memory contents). It is stored within the computer hardware known as *firm ware* (EEPROM).

Processes

Processes and Process Control Blocks

Process A process is an instance of a program in execution. Two essential elements of a process are as follows:

1. Program code
2. Set of data

At any given point in time, while the program is executing, the process can be uniquely characterized by a number of elements, including the following:

1. Identifier
2. State
3. Priority

4. Program counter
5. Memory pointers
6. Context data
7. I/O status information
8. Accounting information

This information is stored in a data structure, typically called a *process control block*, that is created and managed by the OS.

Process control block (PCB) It contains sufficient information so that it is possible to interrupt a running process and later resume execution as if the interruption has not occurred.

Process States

The behaviour of an individual process can be characterized by listing the sequence of instructions that executed for that process. This listing is referred to as a *trace* of the process. Also the behaviour of a processor is shown by listing the traces of the various processes that are interleaved.

Dispatcher A dispatcher is a small program that switches the processor from one process to another.

Two-state process model In the simplest possible process model (Figure 3), at any time, a process is either being executed by a processor or not, that is, a processor may be in one of two states: *running* or *not running*.

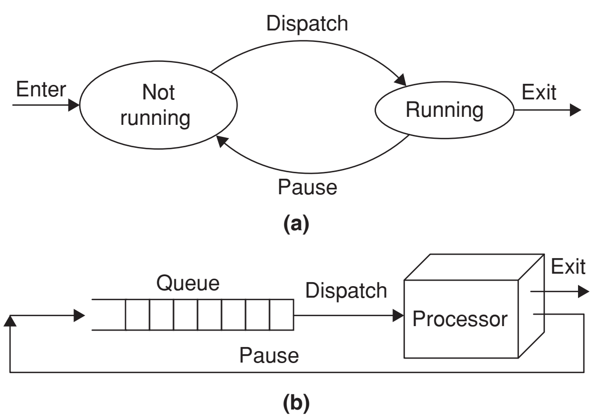


Figure 3 Two-state process model. (a) State transition diagram, (b) Queuing diagram

When the OS creates a new process, it creates the PCB for the process and enters that process into the system in the *not running* state. The process exists, is known to the OS, and is waiting for an opportunity to execute. From time to time, the currently running process will be interrupted and the dispatcher portion of the OS will select some other process to run. The former process moves from the *running* state to the *not running* state and one of the other processes moves to the *running* state.

Processes that are not running must be kept in some sort of queue, waiting their turn to execute. Figure 3(b) shows the structure. There is a single *queue* in which each entry is a pointer to the PCB of a particular process.

Creation and Termination of Processes

Process creation

When a new process is to be added to those currently being managed, the OS builds the data structures that are used to manage the process and allocates address space in main memory to the process. The common events which lead to process creation are as follows:

1. New batch job
2. Interactive logon
3. Created by OS to provide a service
4. Spawned by existing process

When the OS creates a process at the explicit request of another process, the action is referred to as *process spawning*. When one process spawns another, the former is referred to as the *parent process* and the spawned process is referred to as the *child process*.

Process termination

The following are the reasons for process termination:

1. *Normal completion*: Process executes an OS service call to indicate its completion.
2. *Time limit exceeded*: The process has run longer than the specified total time limit.
3. *Memory unavailable*: The process requires more memory than the system can provide.
4. *Bounds violation*: The process tries to access a memory location that it is not allowed to access.
5. *Protection error*: The process attempts to use a resource that is not allowed to access.
6. *Arithmetic error*: The process tries a prohibited computation.
7. *Time overrun*: The process has waited longer than a specified maximum for a certain event to occur.
8. *I/O failure:* Error occurs during input or output.
9. *Invalid instruction*: The process attempts to execute a non-existent instruction.
10. *Privileged instruction*: The process attempts to use an instruction reserved for OS.
11. *Data misuse*: A piece of data is of the wrong type or is not initialized.
12. *Operator or OS intervention.*
13. *Parent termination.*
14. *Parent request.*

Five-state model

The five states in Figure 4 are as follows:

1. *New*: The process is created but not admitted to the pool of executable processes.
2. *Running*: Process in execution, that is, it is using CPU.
3. *Blocked*: Waiting for some event to occur (i.e., I/O) before it can continue execution.

4. *Ready*: Process is ready for execution. Just it is waiting.
5. *Exit*: The process has been aborted by parent process or has finished its execution.

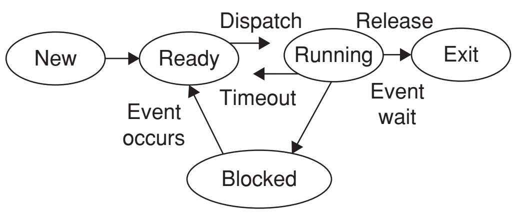


Figure 4 Process states.

Figure 4 indicates the types of events that lead to each state transition for a process. The possible transitions are as follows:

1. NULL → New: A new process is created to execute a program.
2. New → Ready: The OS will move a process from the New state to the Ready state when it is prepared to take on an additional process.
3. Ready → Running: When it is time to select a process to run, the OS chooses one of the processes in the Ready state.
4. Running → Exit: The currently running process is terminated by the OS if the process indicates that it has completed or if it aborts.
5. Running → Ready: The reasons for this transition are
   - Running process has reached the maximum allowable time for uninterrupted execution.
   - As the OS assigns different levels of priority to different processes, there will be pre-emption.
   - A process may voluntarily release control of the processor.
6. Running → Blocked: A process is put in the blocked state if it requests something for which it must wait.
7. Blocked → Ready: This transition occurs when the event for which the process has been waiting occurs.
8. Ready → Exit: A parent may terminate a child process at any time.
9. Blocked → Exit: Parent may terminate any blocked process.

Queuing model for five-state model

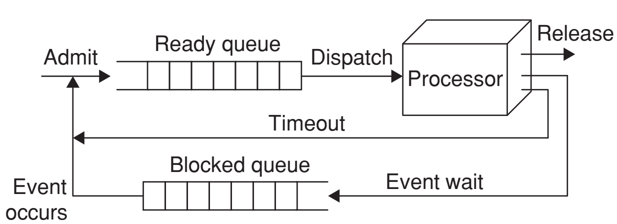


Figure 5 Single blocked queue.

Suspended Processes

Need for swapping In five-state process model using multiple blocked queues, the memory holds multiple processes. Moreover, the processor can move to another process when one process is blocked. But the processor is so much faster than I/O that it will be common for all of the processes in memory to be waiting for I/O. Thus, even with multiprogramming, a processor could be idle most of the time.

Then we can extend the main memory to accommodate more processes, but it is not an efficient solution. Another solution to this problem is swapping. Swapping involves moving part or all of a process from main memory to disk. When none of the processes in main memory is in the ready state, the OS swaps one of the blocked processes out onto disk into a suspend queue. The OS then brings in another process from the suspend queue or it honours a new process request. Then execution continues with the newly arrived process. With the use of swapping, another state is added to the process in the behaviour model.

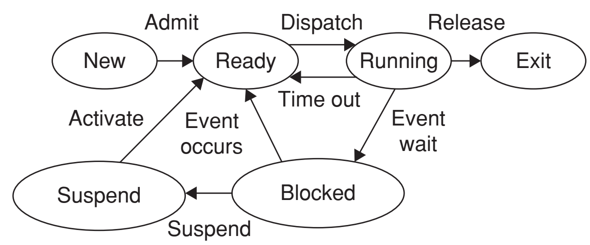


Figure 6 Process state-transition diagram with suspend state.

The four distinguishable states in this process model are as follows:

1. *Ready*: The process is in main memory and is available for execution.
2. *Blocked*: The process is in main memory and awaiting an event.
3. *Blocked/suspend*: The process is in secondary memory and awaiting an event.
4. *Ready/suspend*: The process is in secondary memory but is available for execution as soon as it is loaded into main memory.

Figure 7 shows the process state model with two suspend states:

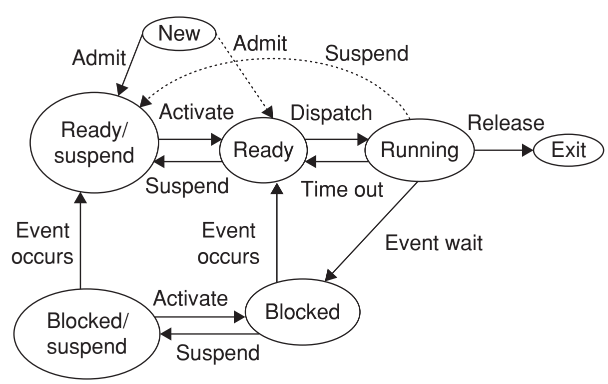


Figure 7 Process state transition diagram with suspend state.

Uses of Suspension

Characteristics of suspended process are as follows:

1. The process is not immediately available for execution.
2. The process may or may not wait on an event.
3. The process was placed in a suspend state by either itself, a parent or the OS.

Reasons for process suspension

- *Swapping*: To Release sufficient main memory.
- *Other OS reason*: OS may suspend a background process.
- *Interactive User Request*: A user may wish to suspend execution of a program.
- *Timing*: A process may be executed periodically and may be suspended.
- *Parent Process request*: A parent process may wish to suspend execution of a descendent.

OS Control Structures

If the OS is to manage processes and resources, it must have information about the current status of each process and resources. The OS constructs and maintains tables of information about each entity that it is managing. Figure 8 shows the general structure of OS control tables:

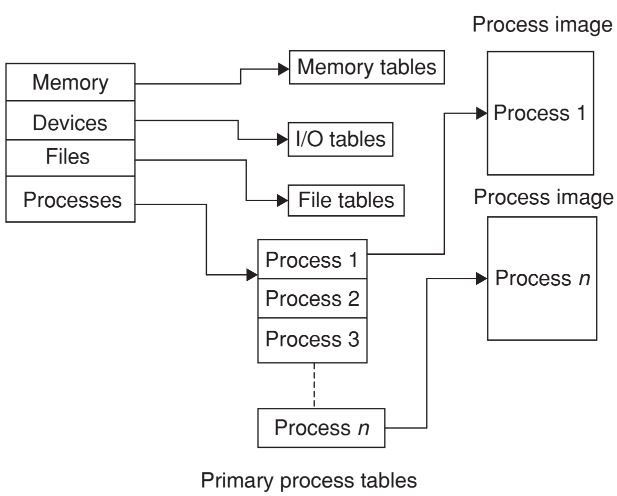


Figure 8 OS control tables.

Different tables maintained by the OS are

1. Memory
2. I/O
3. File
4. Process

Memory tables: These tables are used to keep track of both main and secondary memory.

I/O tables: These are used by the OS to manage the I/O devices and channels of the computer system.

File tables: These tables provide information about the existence of files, their location on secondary memory, their current status and other attributes.

Process tables: An OS must maintain process tables to manage processes.

Process Control Structures

The OS must know about

1. Process location
2. Process attributes

Process Location

The collection of program, data, stack and attributes is referred as process image.

The location of a process image will depend on the memory management scheme being used. The process image is maintained as a contiguous or continuous block of memory. This block is maintained in secondary memory, usually disk, so that the OS can manage the process, at least a small portion of its image must be maintained in main memory. To execute the process, the entire process image must be loaded into main memory or at least virtual memory. Thus the OS needs to know the location of each process on disk and for each such process that is in the main memory, the location of that process is in main memory.

For this, the OS maintains process tables. There is a primary process table with one entry for each process. Each entry contains, at least, a pointer to a process image.

Process Attributes

The typical information required by the OS for each process is as follows:

1. Process identification
2. Process state information
3. Process control information

Process identification Each process is assigned a unique numeric identifier, which may simply be an index into the primary process table. The identifier for a PCB includes the following:

1. Identifier of the process
2. Identifier of the process that created current process
3. User identifier

Process state information It consists of the contents of processor registers. It includes details of

1. User-visible register
2. Control and status registers
3. Stack pointers

Process control information It consists of the additional information needed by the OS to control and coordinate the various active processes. It includes the following:

1. Scheduling and state information
2. Data structuring
3. Interprocess communication

4. Process privileges
5. Memory management
6. Resource ownership and utilization

Process Control

Modes of Execution

Most processors support at least two modes of execution as follows:

1. More-privileged mode
2. Less-privileged mode

Two modes are required to protect the OS and key OS tables from interference by user programs.

1. ***More-privileged mode:*** This is also referred as *system mode*, *control mode* or *kernel mode*. Certain instructions can only be executed in Kernel mode (e.g., reading or altering a control register, viz., PSW, primitive I/O instructions, etc.).

   The Kernel of the OS is a portion of the OS and encompasses the important system functions.

   The functions of an OS kernel are as follows:
   - Process management
   - Memory management
   - I/O Management
   - Support functions
2. ***Less-privileged mode:*** This is also referred as *user mode*, because user programs typically would execute in this mode.

   In this mode, the software has complete control of the processor and all its instructions, registers and memory.

Process creation If the OS decides to create a table, it has to proceed as follows:

1. Assign a unique process identifier to the new process.
2. Allocate space for the process.
3. Initialize the PCB
4. Set the appropriate linkages.
5. Create or expand other data structures.

Process switching In process switching, a running process is interrupted and the OS assigns another process to the running state and turns control over to that process. The design issues are as follows:

1. When to switch processes
2. Mode switching
3. Change of process state

When to switch processes A process switch may occur anytime that the OS has gained control from the currently running process. The mechanisms for interrupting the execution of a process are as follows:

1. Interrupt
2. Trap
3. Supervisor call

Interrupt When an interrupt occurs, the control is first transferred to an interrupt handler, which does some basic housekeeping and then branches to an OS routine that is concerned with the particular type of interrupt that has occurred (e.g., clock interrupt, I/O interrupt, memory fault, etc.).

Trap Trap related to an error or exception condition gets generated within the currently running process. If the error is fatal, the currently running process is moved to exit state and a process switch occurs, otherwise the action of the OS will depend on the nature of the error and design of the OS.

Supervisor call The OS may be activated by a supervisor call from the program being executed. The case of system call may place the user process in blocked state.

Mode switching If the processor identifies that any interrupt is pending, then

1. it sets the PC to the starting address of an interrupt handler program.
2. it switches from user mode to Kernel mode so that the interrupt processing code may include privileged instructions.

During this process, the context of the process, that has been interrupted, is saved into that PCB of the interrupted program. The context of a program includes PC, other processor registers and stack information.

The occurrence of an interrupt does not necessarily mean a process switch.

Change of process state The mode switch is a concept distinct from that of the process switch.

A mode switch may occur without changing the state of the process that is currently in Running state. In that case, the context saving and subsequent restoral involve little overhead. However, if the currently running process is to be moved to another state then the OS must make substantial changes in its environment. Thus, the process switch, which involves a state change, requires more effort than a mode switch.

System call In order to access the OS services, an interface is required which is provided by the system call.

All the system call routines are executed in Kernal mode. Whenever the system call is invoked, the process status word is changed from user mode to Kernal mode ($0 \rightarrow 1$).

System calls are of six types as follows:

1. File system
2. Process
3. Scheduling
4. Interprocess communications
5. Socket
6. Miscellaneous

Execution of the OS

There are three possibilities to consider about OS execution:

1. Separate Kernel (Figure 9)
2. OS functions execute within user processes (Figure 10)
3. OS functions execute as separate processes (Figure 11)

Separate Kernel

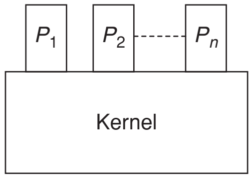


Figure 9 OS as separate Kernel.

Here the Kernel of the OS is executed outside of any process. When currently running process is interrupted or issues a supervisor call, the mode context of this process is saved and control is passed to the Kernel.

Execution within user process

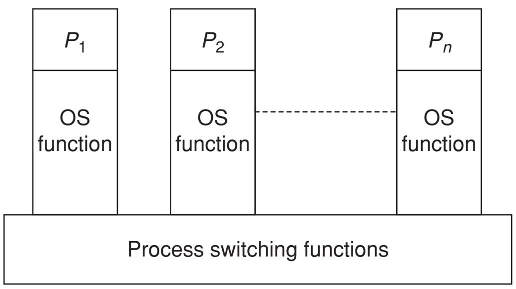


Figure 10 OS functions execute within user processes.

We execute virtually all OS software in the context of user process. To pass control from a user program to the OS, the mode context is saved and a mode switch takes place to an OS routine, that is, a process switch is not performed, just a mode switch within the same process.

Process-based OS

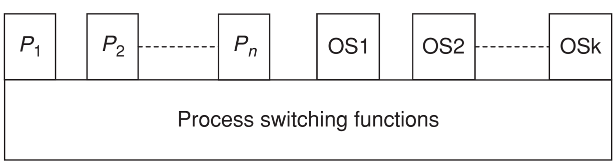


Figure 11 OS functions execute as separate process.

Here the OS is a collection of system processes. This approach encourages the use of modular OS with minimal, clean interface between the modules.

Threads

A *thread* is a basic unit of CPU utilization. It comprises a thread ID, a program counter, a register set and a stack.

Multithreading

It refers to the ability of an OS to support multiple, concurrent paths of execution within a single process.

The threads which belong to same process can share their

1. Code section
2. Data section
3. Other OS resources

If a process has multiple threads of control, it can perform more than one task at a time. Figure 12 shows single threaded and multithreaded process models:

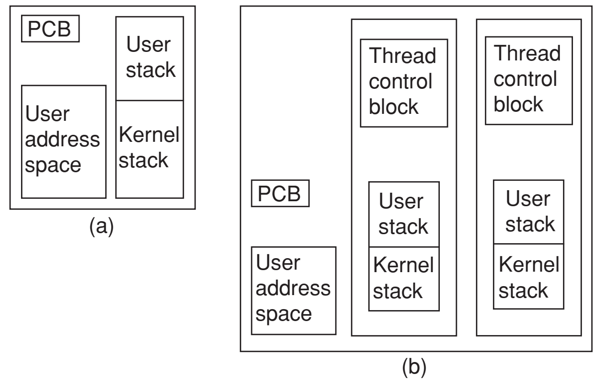


Figure 12 Process models. (a) Single-thread process model (b) Multithreaded process model.

As OS based on its design will be in one of the following manners (Figure 13):

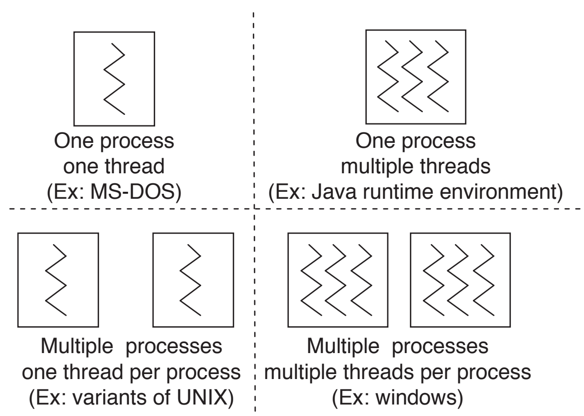


Figure 13 Threads and processes.

The threads of a process consist of the following:

1. Thread execution state.
2. Saved thread context when not running
3. An execution stack
4. Some pre-thread static storage for local variables.
5. Access to the memory and resources of its process, shared with all other threads in that process.

All the threads of a process share the state and resources of that process.

Benefits of multithreaded programming

1. *Responsiveness*: Multithreading an interactive application may allow program to continue running even if

part of it is blocked or is performing a lengthy operation, thereby increasing responsiveness to the user.

2. *Resource sharing*: Threads share the memory and the resources of the process to which they belong by default.
3. *Economy*: It is economical to create a new thread in an existing process than to create a brand-new process. It takes less time to context switch between two threads of same process than to switch between processes. Also the time to terminate a thread is less than process termination.
4. *Scalability*: Multithreading on a multi-CPU machine increases parallelism.

Applications that benefit from thread As the threads take advantages of multiple processors, image processing which can be done in parallel, will execute in threads.

Animation rendering is another thread application, where each frame can be rendered in parallel, as each one is independent of other GUI programming will execute at least two threads when it is processing large number of files.

Applications that cannot benefit from thread The main drawbacks of threads is if kernel is single threaded, system call of one thread will block the whole process, in which CPU will be idle during the blocking period.

The other major drawback is security as it is possible that a thread can overwrite the stack as the other thread, as they were meant to cooperate on a single task. Applications that are developed using PHP does not support multithreading at the server side.

Thread Functionality

The key states for threads are as follows:

1. Running
2. Ready
3. Blocked

There are four basic thread operations associated with a change in thread state:

1. *Spawn*: When a new process is spawned, a thread for that process is also spawned.

   Also, a thread within a process may spawn another thread within the same process.
2. *Block*: When a thread needs to wait for an event, it will block. Then the processor may turn to the execution of another ready thread in the same or different process.
3. *Unblock*: When an event for which a thread is blocked occurs, the thread is moved to Ready queue.
4. *Finish*: When a thread completes, its register context and stacks are deallocated.

Multithreading on a Uni-processor

On a uni-processor, multiprogramming enables the interleaving of multiple threads within multiple processes. For example, consider the execution of three threads A, B, C in two processes on a single processor which are interleaved (Figure 14).

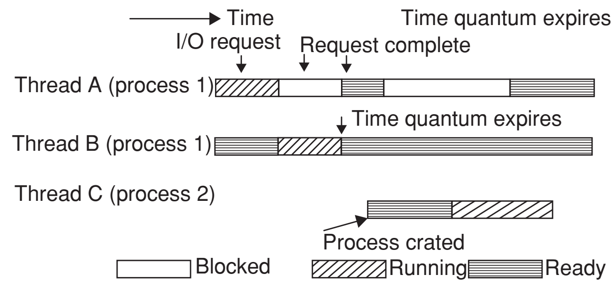


Figure 14 Multithreading on a uni-processor.

Execution passes from one thread to another, either when the currently running thread is blocked or its time slice is exhausted.

Resources used in thread creation and process creations

As process has heavy weight, when it is created, new address space is required, which includes stack, heap and data section, etc. If a process shares the memory, then the IPC is expensive.

The thread is a light-weight process, if it doesn't require any new resources, as it will share the process resources to which it belongs. The major benefit of this is, several threads belong to same activity and can run under same address space.

Thread Synchronization

All of the threads of a process share the same address space and other resources, such as open files. Any alteration of a resource by one thread affects the environment of the other threads in the same process. So, synchronization mechanism is required to coordinate the activities of all the threads within a process.

The techniques used for thread synchronization is the same as process synchronization techniques, which has been discussed later in this book.

Types of Threads

User-Level Threads

All thread management is done by the application. The Kernel is not aware of the existence of threads. Thread creation and scheduling are done in user space. User-level threads (Figure 15) are fast to create and manage. User-level library provides support for creating, managing and scheduling threads. In single-threaded Kernel, blocking system call from user level thread will block the entire process, even if other threads are ready to run.

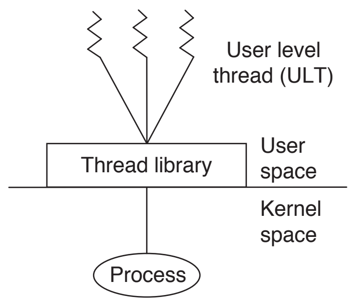


Figure 15 Pure user-level threads.

Advantages of ULTs

1. ULT creation does not require Kernel mode privileges.
2. ULT scheduling can be application specific.
3. ULTs can run on any OS.

Disadvantages of ULTs

1. When a ULT executes a system call, not only is that thread blocked, but also all of the threads within the process are blocked.
2. In a pure ULT strategy, a multithreaded application cannot take advantage of multiprocessing.

Kernel-level Threads

Kernel-level threads (KLTs) are supported directly by the OS. The creation, scheduling, management are done by kernel in kernel space. They are slower to create and manage. In a multiprocessor, Kernel can schedule threads on different processors.

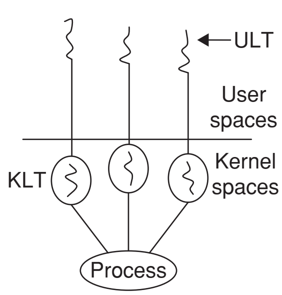


Figure 16 Pure kernel-level threads.

Advantages of KLTs

1. Kernel can simultaneously schedule multiple threads from the same process on multiple processors.
2. If one thread in a process is blocked, the kernel can schedule another thread of the same process.
3. Kernel routines themselves multithreaded.

Disadvantages of KLTs

The transfer of control from one thread to another within the same process requires a mode switch to the Kernel.

Combined Approach

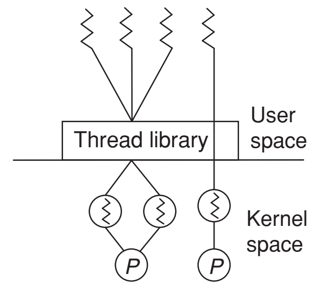


Figure 17 Combined approach.

In combined approach (Figure 17), multiple threads within the same application can run in parallel on multiple processors and a blocking system call need not block the entire process.

Relationship between the threads and processes

1. *One-to-one relationship*: Each thread of execution is a unique process with its own address and resources.
   Example: Traditional UNIX.
2. *Many-to-one relationship*: A process defines an address space and dynamic resources ownership. Multiple threads may be created and executed within that process.
   Example: Windows NT, Solaris, Linux.
3. *One-to-many relationship*: A thread may migrate from one process environment to another. This allows a thread to be easily moved among distinct systems.
   Example: Emerald
4. *Many-to-many relationship*: It combines the attributes of M:1 and 1:M cases.
   Example: TRIX

Threading Issues

fork( ) and exec( ) System Calls

A fork( ) system call is used to create a separate, duplicate process. In UNIX, each process is identified by its process identifier, which is a unique integer. A new process is created by fork( ) system call.

The new process consists of a copy of the address space of the original process. This mechanism allows the parent process to communicate easily with its child process. Both processes continue execution at the instruction after the fork( ), with one difference: the return code for the fork( ) is zero for the new process, whereas the process identifier of the child is returned to the parent. The exec( ) system call is used after a fork( ) system call by one of the two processes to replace the processes memory space with a new program.

If one thread in a program calls fork( ), then UNIX chooses two alternatives as follows:

1. Duplicates all the threads
2. Duplicates only the thread that invoked the fork( ) system call.

If a thread invokes the exec( ) system call, the program specified in the parameter to exec( ) will replace the entire process including all threads.

Cancellation

Thread cancellation is the task of terminating a thread before it has completed. A thread that is to be cancelled is often referred to as the *target thread*. Cancellation of a thread may occur in two different scenarios as follows:

1. *Asynchronous cancellation*: One thread immediately terminates the target thread.
2. *Deferred cancellation*: The target thread periodically checks whether it should terminate, allowing it an opportunity to terminate itself in an orderly fashion.

Microkernels

Microkernel (Figure 18) is a small OS core that provides the foundation for modular extensions.

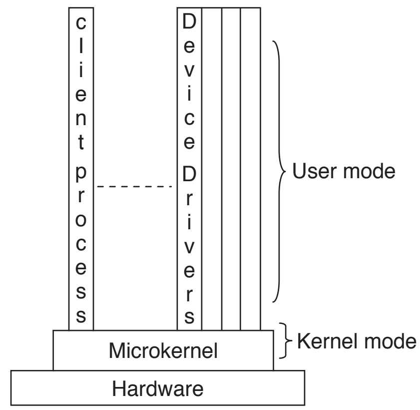


Figure 18 Microkernel architecture.

Advantages of Microkernel Organization

- Uniform interfaces
- Extensibility
- Flexibility
- Portability
- Reliability
- Distributed system support
- Support for object oriented OS

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. In fork( ) system call, the return value to the parent process and the child process are respectively
(A) PID of child process, 1
(B) PID of child process, 0
(C) PID of child, PID of parent process
(D) PID of parent process, PID of child

2. Which of the following is not an advantage of thread?
(A) Inter process communication
(B) Less memory space occupied by thread
(C) Less time to create and terminate than a process
(D) Context switching is faster

3. A process executes the following segment of code

for (i = 1; i < 10; i ++) fork( );

The number of new processes created is
(A) 1024 (B) 1023
(C) 1025 (D) 1028

4. For each of the following transitions, between process states, which transition is not possible?
(A) Running → Ready
(B) Blocked → Suspend
(C) Ready → Ready/Suspend
(D) Blocked → Running

5. An operating system can be mapped to a five-state process model. A new event has been designated as capable to pre-empt the existing processes in order to trigger a new process to complete. Select the correct statement from below:
(A) A new state need to be added to the existing transition model to accommodate the changes.
(B) The existing model still holds good.
(C) Both the states and transitions of the existing model have to be changed.
(D) Only the transitions need to be modified.

6. The advantage of having multiple threads over multiple processes is
(i) Less time for creation
(ii) Less time for termination
(iii) Less time for switching
(iv) Kernel not involved in communication among threads
(A) (i), (ii), (iii) (B) (i), (ii), (iv)
(C) (ii), (iii), (iv) (D) (i), (ii), (iii), (iv)

7. Select the correct sequence of steps taken by the processor when an interrupt occurs
(i) Switch from user mode to kernel mode.
(ii) Set the program counter to the first instruction of the interrupt handling routine.
(iii) Save the current context.

(A) (i), (ii), (iii)
(B) (i), (iii), (ii)
(C) (iii), (ii), (i)
(D) (ii), (i), (iii)

8. What are the necessary steps for a new process creation?
(i) Assign an identifier to the new process.
(ii) Suspend all other processes.
(iii) Allocate space for the process.
(iv) Initialize process control block.
(v) Update process-related data structures.
(vi) Update process state information wherever necessary.
(vii) Set the process to user mode
(viii) Notify all the machines in the network about the new process.
(ix) Set the state of the new process as suspended

(A) All except (ii), (v), (vi), (viii)
(B) All except (ii), (vii), (viii), (ix)
(C) All except (iv), (v), (vii), (ix)
(D) All except (iii), (iv), (vii), (ix)

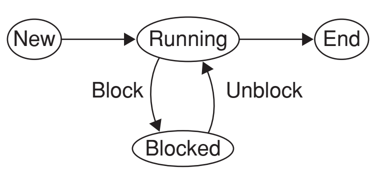
9. A processor, while executing the instruction sequence of user mode process, received n interrupts. If no other activity is reported to processor during the execution of the instruction sequence, what is the number of mode switches and process switches experienced?

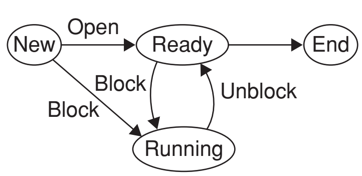
(A) $2n$, $2n$ (B) n, n
(C) $2n$, 0 (D) n, 0

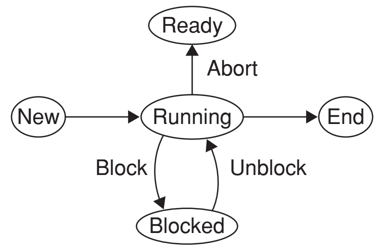
10. Assume a program needs to implement threads, what are the resources that need to be encompassed in critical section?
(i) Global variables
(ii) Local variables
(iii) Static variables
(iv) Function parameters that are passed as reference pointers
(v) Global constants

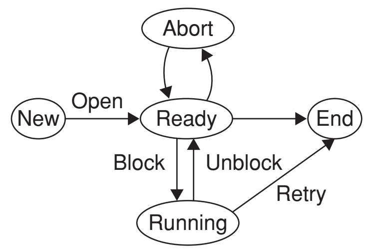
(A) (i), (ii), (iii), (v)
(B) (i), (iv), (v)
(C) (i), (iii), (v)
(D) (i), (iii), (iv)

11. Which of the following is an appropriate four-state model for a process?

(A)


(B)


(C)


(D)


12. Which of the following are considered as disadvantage of user level thread (ULT)?
(i) Calls made from system will block all threads in a process
(ii) When scheduled in a multiprocessing environment only one thread per process can be executed at a time.
(iii) ULTs cannot communicate with each other in a process.
(iv) The cost of creating a thread is high

(A) (i), (ii), (iii) (B) (i), (iii), (iv)
(C) (i), (ii) only (D) (ii), (iii), (iv)

13. Choose from below, advantages of kernel-level threads:
(i) Kernel can simultaneously schedule multiple threads from the same process on multiple processors.
(ii) Kernel routines can be multithreaded.
(iii) If one thread of a process is blocked then kernel can schedule another thread from the same process.

(A) (i) only (B) (i), (ii)
(C) (i), (ii), (iii) (D) (i), (iii)

14. Assume that part of a program takes long time to execute. Select an option from below that can enhance performance:
(i) Implement the part that takes long time as a separate process and use the results as needed from the main program.
(ii) Implement both the parts as two different threads in the same process.

(A) (i) only
(B) (ii) only
(C) Both (i) and (ii)
(D) Neither (i) nor (ii)

15. What is a kernel-level thread?
(i) Threads that are spawned by OS Kernel
(ii) Threads that are launched by user by directly accessing the kernel

(A) (i) only (B) (ii) only
(C) Both (i), (ii) (D) Neither (i) nor (ii)

Practice Problems 2

Directions for questions 1 to 14: Select the correct alternative from the given choices.

1. Many-to-many multithreading model is used in which of the following operating system?
(A) Windows NT/2000 with Thread Fibre
(B) Windows 95
(C) Windows 98
(D) Solaris Green Threads

2. Which of the following does not interrupt a running process?
(A) Device (B) Timer
(C) Scheduler (D) Power failure

3. Which of the following need not be saved on a context switch between processes?
(A) General purpose registers
(B) Translation look aside buffer
(C) Program counter
(D) All of the above

4. Which of the following actions is/are typically not performed by the OS when switching context from process A to process B?
(A) Saving current register values and restoring the register values for process B
(B) Changing address translation tables
(C) Swapping out the memory image of process A to the disk
(D) Both (B) and (C)

5. For each thread in a multithreaded process, there is a separate
(A) Process control block
(B) User address space
(C) User and kernel stack
(D) Kernel space only

6. When a supervisor call is received
(A) Mode switch happens
(B) Process switch happens
(C) Both (A) and (B)
(D) Neither (A) nor (B)

7. What is the purpose of jacketing?
(A) Convert non-blocking system call to blocking system call
(B) Convert blocking system call to non-blocking system call
(C) Convert blocking system call into a new thread
(D) Convert non-blocking system call into a new thread

8. Which of the following statements is/are always true?
(i) Time taken for mode switch is always greater than process switch.
(ii) Time taken for mode switch is always less than process switch.
(iii) Time taken for mode switch is always equal to process switch.
(A) (i) and (iii) (B) (ii) and (iii)
(C) Only (i) (D) Only (ii)

9. Which of the following is the property of time sharing systems?
(i) Multiple user access
(ii) Multiprogramming
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

10. Which of the following is/are not a valid reason for process creation?
(i) Created by OS
(ii) Interactive logon
(iii) Privileged instruction
(A) (i), (ii) (B) (ii), (iii)
(C) (i), (iii) (D) (iii) only

11. Which of the following is/are reason(s) for blocking a running process?
(i) A call from the running program to a procedure that is a part of OS code.
(ii) A running process may initiate an I/O operation.
(iii) A user may block a running process.
(A) (i), (ii) only (B) (ii), (iii)only
(C) (i), (iii) only (D) (iii) only

12. If the OS is pre-empting a running process because a higher priority process on blocked/suspend queue has just become unblocked, then the running process moved to ______ queue.
(A) Suspend (B) Ready/suspend
(C) Blocked (D) Blocked/suspend

13. Which of the following is used to call an OS function?
(A) Interrupt (B) Trap
(C) Supervisor call (D) All of these

14. Which of the following is a general component of a thread?
(i) Thread ID
(ii) Register set
(iii) User stack
(iv) Kernel stack
(A) (i), (iii), (iv) (B) (i), (ii), (iv)
(C) (i), (ii), (iii) (D) (i), (ii), (iii), (iv)

Previous Years' Questions

1. Consider the following statements with respect to User-level threads and Kernel-supported threads. **[2004]**

(1) Context switch is faster with Kernel-supported threads

(2) For user-level threads, a system call can block the entire process

(3) Kernel supported threads can be scheduled independently

(4) User level threads are transparent to the Kernel

Which of the above statements are true?

(A) 2, 3 and 4 only (B) 2 and 3 only

(C) 1 and 3 only (D) 1 and 2 only

2. Which one of the following is true for a CPU having a single interrupt request line and a single interrupt grant line? **[2005]**

(A) Neither vectored interrupt nor multiple interrupting devices are possible

(B) Vectored interrupts are not possible but multiple interrupting devices are possible

(C) Vectored interrupts and multiple interrupting devices are both possible

(D) Vectored interrupt is possible but multiple interrupting devices are not possible

3. Normally user programs are prevented from handling I/O directly by I/O instructions in them. For CPUs having explicit I/O instructions, such I/O protection is ensured by having the I/O instructions privileged. In a CPU with memory mapped I/O, there is no explicit I/O instruction. Which one of the following is true for a CPU with memory mapped I/O? **[2005]**

(A) I/O protection is ensured by operating system routine(s)

(B) I/O protection is ensured by a hardware trap

(C) I/O protection is ensured during system configuration

(D) I/O protection is not possible

4. Consider the following code fragment: if (fork ()==0) **[2005]**

```
{ a = a + 5; printf("%d,%d\n",a,&a);}
else{a=a-5;printf("%d,%d\n",a,&a);}
```

Let u, v be the values printed by the parent process, and x, y be the values printed by the child process. Which one of the following is *true*?

(A) $u = x + 10$ and $v = y$

(B) $u = x + 10$ and $v \neq y$

(C) $u + 10 = x$ and $v = y$

(D) $u + 10 = x$ and $v \neq y$

5. Consider the following statements about user-level threads and Kernel-level threads. Which one of the following statements is *false*? **[2007]**

(A) Context switch time is longer for Kernel-level threads than for user level threads.

(B) User level threads do not need any hardware support.

(C) Related Kernel-level threads can be scheduled on different processors in a multi-processor system.

(D) Blocking one Kernel-level thread blocks all related threads.

6. Which of the following statements about synchronous and asynchronous I/O is NOT *true*? **[2008]**

(A) An ISR is invoked on completion of I/O in synchronous I/O but not in asynchronous I/O

(B) In both synchronous and asynchronous I/O, an ISR (Interrupt Service Routine) is invoked after completion of the I/O

(C) A process making a synchronous I/O call waits until I/O is complete, but a process making an asynchronous I/O call does not wait for completion of the I/O

(D) In the case of synchronous I/O, the process waiting for the completion of I/O is woken up by the ISR that is invoked after the completion of I/O

7. A process executes the following code

for (i=0; i<n; i++) fork( );

The total number of child processes created is **[2008]**

(A) n (B) $2^n - 1$

(C) 2^n (D) $2^{n+1} - 1$

8. A CPU generally handles an interrupt by executing an interrupt service routine **[2009]**

(A) As soon as an interrupt is raised.

(B) By checking the interrupt register at the end of fetch cycle.

(C) By checking the interrupt register after finishing the execution of the current instruction.

(D) By checking the interrupt register at fixed time intervals.

9. A thread is usually defined as 'light weight process' because an operating system (OS) maintains smaller data structures for a thread than for a process. In relation to this, which of the following is true? **[2011]**

(A) On per-thread basis, the OS maintains only CPU register state

(B) The OS does not maintain a separate stack for each thread

(C) On per thread basis, the OS does not maintain virtual memory state.

(D) On per thread basis, the OS maintains only scheduling and accounting information.

10. Let the time taken to switch between user and kernel modes of execution be t_1 while the time taken to switch between two processes be t_2. Which of the following is *true*? **[2011]**

(A) $t_1 > t_2$
(B) $t_1 = t_2$
(C) $t_1 < t_2$
(D) Nothing can be said about the relation between t_1 and t_2

11. A process executes the code

```
fork();
fork();
fork();
```

The total number of **child** processes created is **[2012]**

(A) 3 (B) 4
(C) 7 (D) 8

12. Which one of the following is **FALSE**? **[2014]**

(A) User level threads are not scheduled by the Kernel.
(B) When a user level thread is blocked, all other threads of its process are blocked.
(C) Context switching between user level threads is faster than context switching between Kernel-level threads.
(D) Kernel-level threads cannot share the code segment.

13. Threads of a process share **[2017]**

(A) global variables but not heap.
(B) heap but not global variables.
(C) neither global variables nor heap.
(D) both heap and global variables.

14. Which of the following is/are shared by all the threads in a process? **[2017]**

I. Program counter
II. Stack
III. Address space
IV. Registers

(A) I and II only (B) III only
(C) IV only (D) III and IV only

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** A	**3.** B	**4.** D	**5.** D	**6.** D	**7.** C	**8.** B	**9.** C	**10.** D
11. A	**12.** C	**13.** C	**14.** C	**15.** A					

Practice Problems 2

1. A	**2.** C	**3.** B	**4.** C	**5.** C	**6.** A	**7.** B	**8.** D	**9.** C	**10.** D
11. A	**12.** B	**13.** C	**14.** D						

Previous Years' Questions

1. A	**2.** C	**3.** A	**4.** D	**5.** D	**6.** A	**7.** B	**8.** C	**9.** C	**10.** C
11. C	**12.** D	**13.** D	**14.** B						

Chapter 2

Interprocess Communication, Concurrency and Synchronization

LEARNING OBJECTIVES

- *Principles of concurrency*
- *Process interaction*
- *Mutual exclusion*
- *Semaphores*
- *Binary semaphore*
- *Mutual exclusion using semaphores*
- *Progress using semaphores*
- *Classical problems of synchronization*
- *Dining philosophers problem*
- *Monitors*
- *Message passing*
- *Indirect addressing*
- *Mutual exclusion using message passing*

Basic Concepts

Multiprogramming: It deals with the management of multiple processes within a uniprocessor system.

Multiprocessing: It deals with the management of multiple processes within a multiprocessor.

The fundamental operating system (OS) design is *concurrency*. Concurrency encompasses a host of design issues, including communication among processes, sharing of and competing for resources, synchronization of the activities of multiple processes and allocation of processor time to processes.

Principles of Concurrency

There are two examples for concurrent processing as follows:

1. In a single-processor multiprogramming system, processes are interleaved in time to yield the appearance of simultaneous execution.
2. In a multiprocessor system, it is possible not only to interleave the execution of multiple processes but also to overlap them.

There are two problems with these techniques:

1. Problem with sharing of global resources
2. Problem with allocation of resources optimally.
3. Problem with locating a programming error as results is not deterministic and reproducible.

Example:

```
void process( )
{
in = getchar( );
out = in;
putchar(out);
}
```

The procedure 'process' reads a character and prints it. Let us suppose that we have a uniprocessor system, with single user. Let the user running multiple applications and all applications use the procedure for reading and printing, that is, all the applications share common procedure for efficient and close interaction among them. But this sharing leads to problems. For example,

1. Let the process P_1 invokes 'process' and is interrupted immediately after 'getchar' returns its value and stores it in 'in'. Here the most recently entered character '*C*' is stored in variable 'in'.
2. Now, suppose the process P_2 is activated and it invokes 'process', which runs to conclusion, inputting and then displaying a single character, *D*, on the screen.
3. The process P_1 is resumed. By this time, the value '*C*' has been overwritten in 'in' and therefore lost. Instead 'in' contains '*D*', which is transferred to 'out' and displayed.

Here the problem is with sharing a global variable. To avoid these types of problems, we impose some rules like, only one process

at a time may enter 'process' and that once in 'process' the procedure must run to completion before it is available for another process.

This problem is also applicable to multiprocessor systems.

Race condition: A race condition occurs when multiple processes or threads read and write data items so that the final result depends on the order of execution of instructions in the multiple processes.

Example: Let P_1 and P_2 be two processes that share global variables a and b, with initial values $a = 0$, $b = 1$. At some point in its execution, P_1 executes $a = a + b$ and P_2 executes $b = a + b$.
If P_1 executes before P_2, then $a = 1$, $b = 2$.
If P_2 executes before P_1, then $b = 1$, $a = 1$.

OS Concerns for Concurrency

1. The OS must be able to keep track of various processes using PCBs.
2. The OS must allocate and deallocate various resources for each active process.
3. The OS must protect the data and physical resources of each process.
4. The functioning of a process and the output it produces must be independent of the speed at which its execution is carried out relative to the speed of other concurrent processes.

Process Interaction (IPC)

Process classification There are two types of processes as follows:

1. Independent/isolated
2. Cooperating

Independent Process: It cannot affect or be affected by the execution of another process.

Cooperating Process: It can affect or be affected by the execution of another process.

We can classify the ways in which processes interact on the basis of the degree to which they are aware of each other's existence. There are three types of process interaction as follows:

1. Process unaware of each other
2. Process indirectly aware of each other
3. Processes directly aware of each other

Competition Among Processes for Resources

1. This situation arises when processes unaware of each other.
2. There is no exchange of information between the competing processes.
3. But the execution of one process may affect the behaviour of competing processes.
4. With competing processes, there will be three control problems as follows:

Need for mutual exclusion

Example: Suppose two or more processes require access to a single non-sharable resource, such as a printer. Then that resource is referred as *critical resource* and the portion of the program that uses it is called *critical section* of the program. In the case of printer, only one process will have the control of printer while it prints an entire file.

Possibility of deadlock

Example: Two processes waiting for each other indefinitely for the release of resources.

Possibility of starvation

Example: One process is denied access to a particular resource which is required for the execution of that process.

Control of competition inevitably involves the OS, because it is the OS that allocates resources.

Cooperation Among Processes by Sharing

This situation arises when the processes are indirectly aware of each other. Processes may use and update the shared data without reference to other processes but know that other processes may have access to same data. So the control mechanism must ensure the integrity of the shared data. Problems with this type of sharing are

1. Mutual exclusion
2. Deadlock
3. Starvation
4. Data coherence

Data coherence

Suppose two items of data p and q are maintained in the relationship $p = q$, that is, any program that updates p and q values must maintain the relationship.

$$\begin{aligned} \text{Let } P_1 : p &= p + 1; \\ q &= q + 1; \\ P_2 : p &= p * 2; \\ q &= q * 2; \end{aligned}$$

Let initially the state is consistent, that is, $p = 2$, $q = 2$

Then the concurrent execution of P_1 and P_2 with mutual exclusion on p, q will be $p = p + 1;$

$$\begin{aligned} q &= q * 2; \\ q &= q + 1 \\ p &= p * 2; \end{aligned}$$

The final values of p and q will be $p = 6$, $q = 5$.

So the consistency is not maintained.

Cooperation Among Processes by Communication

This situation arises when processes are aware of each other directly. All the processes communicate with each other to synchronize or coordinate the various activities. Problems with this communication are as follows:

1. Deadlock
2. Starvation

Basic Definitions

Atomic operations: A sequence of one or more statements that appears to be indivisible, that is, no process will interrupt the operation.

Critical section A section of code within a process that requires access to shared resources and that must not be executed while another process is in a corresponding section of code.

Deadlock A situation in which two or more processes are unable to proceed because each is waiting for one of the others to do something.

Mutual exclusion The requirement that when one process is in a critical section that accesses shared resources, no other process may be in a critical section that accesses any of those shared resources.

Race condition A situation in which multiple threads or processes read and write a shared data item and the final result depends on the relative timing of their execution.

Starvation A situation in which a runnable process is overlooked indefinitely by the scheduler; although it is able to proceed, it is never chosen.

Critical Section

1. A section of code or set of operations, in which process may be changing shared variables, updating a common file or a table etc.
2. For the process that execute concurrently, it should ensure that execution of critical section should be made atomic. Atomic means that either an operation in the critical section should happen in its entirely or not at all.
3. Critical section of a process should not be executed concurrently with the critical section of another process.
4. To avoid Race Condition, we must have the following:

   'if one process is in critical section, other competing process must be excluded to enter their critical sections, that is, a process must enter the critical section in a mutually exclusive way'.

   This is called *problem of mutual exclusion*.

   Region of code that updates or uses shared data to provide a consistent view of objects need to make sure an update is not in progress when reading the data.
5. Need to provide mutual exclusion for a critical section.

Requirements for Critical Section Problem

Mutual exclusion: No two contending processes should be simultaneously executing inside their critical section.

Bounded waiting: No process should have to wait forever to enter its critical section.

Progress: If no process is executing in its critical section and there exists some processes that wish to enter their critical sections, then only those processes that are not executing in the critical section can participate in the decision of which will enter its critical section next and this selection cannot be postponed indefinitely.

No Assumption: No assumption should be made about relative speed and properties of contenting processes.

Mutual Exclusion

1. Only one process at a time can be updating shared objects.
2. Successful use of concurrency among processes requires the ability to define critical sections and enforce mutual exclusion.
3. Mutual exclusion in the use of a shared resource is provided by making its access mutually exclusive among the processes that share the resources.

Any facility that provides mutual exclusion should meet the following requirements:

1. No assumption regarding the relative speed of the process.
2. A process is in its critical section for a finite time only.
3. Only one process allowed in the critical section.
4. Process requesting access to critical section should not wait indefinitely.
5. A process waiting to enter critical section cannot be blocking a process in critical section or any other process.

Mutual exclusion can be satisfied in one of the following ways:

1. ***Software approach:*** This approach leaves the mutual exclusion responsibility with the process that wish to execute concurrently. This approach is prone to high processing overhead and bugs.
2. ***Hardware support:*** Use special-purpose machine instructions. This approach involves less overhead.
3. ***Provide some level of support within the OS or a programming language:*** Such techniques are as follows:
   - Semaphores
   - Monitors
   - Message passing

Hardware support for mutual exclusion: Use one of the following techniques:

1. Interrupt disabling
2. Special machine instructions
   - Compare and swap instructions
   - Exchange instructions

Interrupt Disabling

To provide mutual exclusion, it is sufficient to prevent a process from being interrupted. A process can enforce mutual exclusion in the following way:

```
while(true)
{
/* disable interrupts */;
/* critical section */;
/* enable interrupts */;
/* remainder */;
}
```

Here, the critical section is not interrupted, so mutual exclusion is guaranteed.

Problems with Interrupt Disabling

The efficiency of execution could be noticeably degraded, because the processor is limited in its ability to interleave processes. Interrupt disabling does not work on in a multiprocessor architecture.

Special Machine Instructions

Two of the most commonly used special machine instructions are as follows:

1. Compare and swap
2. Exchange instruction

Compare and swap instructions It is defined as below:

```
int compare_and_swap(int *word, int
testval, int newval)
{
int oldval;
oldval = *word;
if(oldval == testval)
*word = newval;
return oldval;
}
```

- 'Access to a memory location excludes any other access to that same location.' On the basis of this principle, special machine instructions provide mutual exclusion.
- The above compare and swap instructions will check a memory location (*word) against a test value. If current value is test value, it is replaced with new value. Always the old value is returned.
- The below code provides mutual exclusion using compare and swap instructions:

```
/* Mutual Exclusion */
const int n = /* number of processes */
int S;
void P(int i)
{
while (true)
{
while (compare_and_swap(S, 0, 1)==1)
/* do nothing */;
/* critical section */;
S = 0;
/* remainder */;
}
}
void main( )
{
S = 0;
begin (P(1), P(2) .... P(n));
}
```

Here, a shared variable '*S*' is initialized to '0'. The only process that may enter its critical section is one that finds '*S*' equal to 0.

All other processes at enter their critical sections go into a busy waiting mode.

Busy waiting or *spin waiting* is a technique in which a process can do nothing but continue to execute an instruction or set of instructions that tests the appropriate variable to gain entrance.

When a process leaves, its critical section sets '*S*' to 0. Then the one of the waiting process will get access to enter its critical section.

Exchange instructions The exchange instructions can be defined as follows:

```
void exchange (int reg, int mem)
{
int temp;
temp = mem;
mem = reg;
reg = temp;
}
```

Mutual Exclusion Using Exchange Instructions

```
/* Mutual Exclusion */;
int const n = /* number of processes */;
int S;
void P(int i)
{
int ki = 1;
```

```
while (true)
{
do
exchange(ki, S);
while(ki! = 0);
/* critical section */;
S = 0;
/* remainder */;
}
}
void main( )
{
S = 0;
begin (P(1), P(2), ... P(n));
}
```

A shared variable '*S*' is initialized to '0'. Each process uses a local variable *ki* that is initialized to 1. The only process that may enter its critical section is one that finds '*S*' equal to 0. It excludes all other processes from critical section by setting '*S*' to 1. When a process leaves its critical section, it resets '*S*' to 0 allowing another process to its critical section.

Advantages of using machine instruction approach

1. Applicable to any number of processes on either a single processor or multiple processors, sharing the main memory.
2. Simple and easy to verify.
3. Used to support multiple critical sections.

Disadvantages

1. Busy waiting
2. Starvation is possible
3. Deadlock is possible

Other Mechanisms for Mutual Exclusion

Let us discuss OS and programming language mechanisms that are used to provide concurrency.

Semaphores

Semaphore is an integer value used for signalling among processes.

There are two types of semaphores as follows:

1. Binary semaphore
2. Counting (or) general semaphore

Counting or general semaphore

Three operations may be performed on a semaphore all of which are atomic:

- Initialize
- Decrement
- Increment

The working of a counting semaphore with its operations is defined as below:

1. The semaphore may be initialized to a non-negative integer value.
2. The semwait operation decrements the semaphore value. If the value becomes negative, then the process executing the semwait is blocked, otherwise the process continues execution.
3. The semsignal operation increments the semaphore value. If the resulting value is less than or equal to zero, then one of the processes blocked by a semwait operation, if any, is unblocked.

Example: Let the semaphore value $S = 3$.
If the semaphore value is positive, then that value gives the number of processes that can issue a wait and immediately continue to execute.

Let five processes P_1, P_2, P_3, P_4, P_5 are going to execute a critical section code based on the semaphore value $S = 3$.

$S = \boxed{3}$ ← Initially

↓ P_1 sem wait

$S = \boxed{2}$ ≥ 0, P_1 executes

↓ P_2 sem wait

$S = \boxed{1}$ ≥ 0, P_2 executes

↓ P_3 sem wait

$S = \boxed{0}$ ≥ 0, P_3 executes

↓ P_4 sem wait

$S = \boxed{-1}$ $\ngeq 0$, P_4 Blocked

↓ P_5 sem wait

$S = \boxed{-2}$ $\ngeq 0$, P_5 Blocked

Initially, the '*S*' value 3 means that at a time three processes can issue a 'wait' signal and continue execution.

Whenever S becomes 0, the next process which executes 'wait' operation will be blocked.

Here P_4 is blocked as it operates on the semaphore when $S = 0$.

If the semaphore value becomes negative, it specifies the number of processes waiting to be unblocked.

$S = -2$ means two processes are waiting to be unblocked.

Definition of semwait and semsignal operations

```
struct semaphore
{
int semvalue;
Queuetype Queue;
};
void semwait(semaphore S)
```

```
{
S.semvalue--;
if (S.semvalue < 0)
{
/* place the process in S. Queue */;
/* Block this process */;
}
}
void semsignal(semaphore S)
{
S.semvalue++;
if (S.semvalue < = 0)
{
/* remove a process from S. Queue */;
/* Place the process in ready queue */;
}
}
```

Advantages

1. Because the waiting processes will be permitted to enter their Critical Section in a FCFS order, so the requirement of bounded waiting is met.
2. CPU cycles are saved here as waiting process does not perform any busy waiting.

Disadvantages

1. Complex to implement, since it involves implementation of FCFS.
2. Context switching is more, so more overheads are involved.

Binary semaphore It is a semaphore that takes on only the values 0 and 1.

The operations performed on a binary semaphore are as follows:

1. A binary semaphore may be initialized to 0 or 1.
2. The semwaitB operation checks the semaphore value. If the value is 0, then the process executing the semwaitB is blocked. If the value is 1, then the value is changed to 0 and the process continues execution.
3. The semsignalB operation checks to see if any processes are blocked on this semaphore. If so, then a process blocked by a semwaitB operation is unblocked. If no processes are blocked, then the value of the semaphore is set to 1.

Definition of semwaitB and semsignalB

```
struct binary-semaphore
{
enum {zero, one} value;
Queue-type Queue;
};
void semwaitB(binary-semaphore S)
{
if (S.value = = one)
S.value = zero;
else
{
/* Place this process in S.Queue */;
/* Block this process */;
}
}
void semsignalB(binary-semaphore S)
{
if(S.Queue is empty)
S.value = one;
else
{
/* remove a process from S.Queue */;
/* Place process in ready list */;
}
}
```

Advantages

1. The implementation of binary semaphore is extremely simple.

Disadvantages

1. It does not meet the requirement of Bounded waiting.
2. A process, waiting to enter its Critical Section, will perform Busy waiting, thus wasting CPU cycles.

Notes:

1. Binary semaphores have the same expressive power as general semaphores.
2. *MUTEX*: It is similar to binary semaphore. The key difference between the two is that the process that locks the mutex must be the one to unlock it.
3. Both counting semaphores and binary semaphores use a queue to hold processes waiting on the semaphore. The order in which the processes removed from a Queue is FIFO, that is, the process that has been blocked the longest is released from Queue first.
4. A semaphore whose definition includes the order of removal of Blocked processes is referred as a strong semaphore otherwise it is a weak semaphore.
5. Strong semaphores guarantee freedom from starvation.

Mutual Exclusion Using Semaphores

```
const int n = /* number of processes */;
Semaphore S = 1;
void P(int i)
{
while (true)
{
Semwait(S);
/* critical section */;
Semsignal(S);
```

```
/* remainder */;
}
}
void main( )
{
begin (P(1), P(2),... P(n));
}
```

If no process is executing in Critical Section, then the semaphore values is 1. The first process that is executing 'wait' operation will decrement value to 0 and enter its critical section. The process which execute 'wait' operation while a cooperating process is executing in its critical section, will find the semaphore value to 0 and keep looping in the 'while-loop' of 'wait' operation. Spinning of a waiting process in the while-loop, the binary semaphores are also known as spin locks. When the process executing in the CS makes an exit (from CS), it will execute the 'signal' operation and increment the semaphore value to 1.

At a time, only one of the cooperating processes can enter critical section with the condition that the wait operation is executed automatically. Mutual Exclusion is satisfied.

Example: Consider the following figure (Figure 1) which shows the possible sequence of three processes using mutual exclusion with a semaphore, *S*. Processes *P*, *Q*, *R* accesses a shared resource protected by the semaphore *S*.

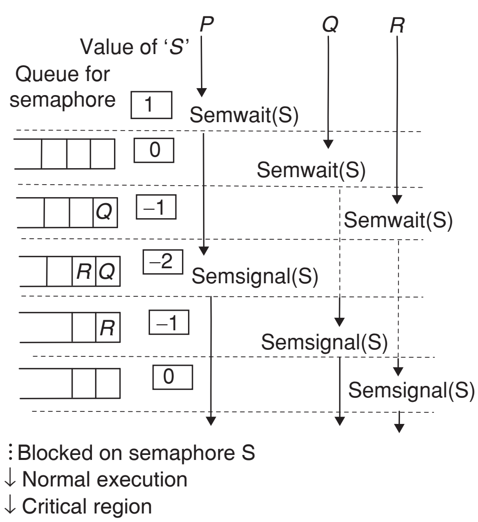


Figure 1 Mutual exclusion using semaphore.

Progress Using Semaphores

When no process is executing in the CS, the semaphore value will be 1. Then, one of the waiting process looping in the while loop of wait-operation will find the semaphore value of 1, exit from the while-loop, decrement the semaphore value to 0 and enter CS. Thus, if no process is executing in critical section and some are waiting to enter, then one of the waiting processes will enter its critical section immediately.

Bounded waiting using semaphores

Actually, one of the waiting processes will get entry into its CS when an operating process executing in its critical section exits. As this selection process is arbitrary, so a process waiting to enter its CS is likely to face starvation. So, the requirement of bounded waiting is not met.

Classical Problems of Synchronization

We will discuss three problems of synchronization:

1. Bounded buffer problem
2. Readers/writers problem
3. Dining philosophers problem

Producer–Consumer Problem

1. Producer inserts item in the buffer
2. Updates Insertion pointer
3. Consumer consumes items in the buffer
4. Updates removal pointer
5. Both update information about how full, how empty the buffer.
6. Prevents buffer overflow, prevents buffer underflow, proper synchronization.

Producer	Consumer
repeat	repeat
produce item *v*;	while (in < = out);
b[in] = *v*;	*w* = *b* [out];
in = in + 1;	out = out + 1;
forever;	consume *w*;
	forever;

Table 1 *Producer consumer problem solution using semaphores*

Producer	Consumer
repeat	repeat
produce item *v*;	while (in < = out);
SemwaitB(*S*);	SemsignalB(*S*);
b[in] = *v*;	*w* = *b*[out];
in = in + 1;	out = out + 1;
SemsignalB(S);	SemsignalB(S);
forever;	consume *w*;
	forever;

If producer is slow or late, then consumer will busy at the while statement.

Table 2 *Improved solution*

Producer	Consumer
repeat	repeat
produce item *v*;	Semwait(n);
Semwait(S);	Semwait(S);
b[in] = *v*;	*w* = *b*[out];
in = in + 1;	out = out + 1;
Semsignal(S);	Semsignal(S);
Semsignal(n);	consume *w*;
forever;	forever;

The initial value of n and S are $n = 0$, $S = 1$. (n is the number of items in the buffer).

Table 3 *Producer consumer bounded buffer problem*

Producer	Consumer
repeat	repeat
produce item v;	while(in == out)
while((in + 1) % n = = out)	no operation;
no operation;	$w = b$[out];
b[in] = v;	out = (out + 1) % n;
in = (in + 1) % n;	consume w;
forever;	forever;

The buffer size is enforced using another counting semaphore.

Table 4 *Producer consumer bounded buffer problem solution*

Producer	Consumers
Repeat	repeat
Produce item v;	Semwait(e);
Semwait(e);	Semwait(S);
Semwait(S);	$w = b$[out];
b[in] = v;	out = (out + 1) % n;
in = (in + 1) % n	Semsignal(S);
Semsignal(S);	Semsignal(e);
Semsignal(e);	consume w;
forever;	forever;

The initial value of buffer size, e is the size of the bounded buffer.

Observations on semaphores:

1. Semaphores are easy to use.
2. wait() and signal() are to be implemented as atomic operations.

Problems:

1. signal() and wait() may be exchanged by the programmer, this may result in deadlock or violation of mutual exclusion.

Readers/Writers Problem

1. A reader reads data.
2. A writer writes data.
3. Data is shared among a number of processes.
4. Multiple readers may read the data simultaneously, that is, concurrently.
5. Only one writer can write the data any time, that is, no reader should be present.
6. A reader and writer cannot access data simultaneously.
7. Locking table: Whether any two can be in the critical section simultaneously is shown in the table.

	Reader	Writer
Reader	OK	NO
Writer	NO	NO

Solution: Readers have priority; if a reader is in CS, any number of readers could enter irrespective of any writer waiting to enter critical section

Writer	Reader
`while(true)`	`while(true)`
`{`	`{`
`Semwait(S);`	`Semwait(x);`
`writeunit();`	`Num = Num + 1;`
`Semsignal(S);`	`if (Num = = 1)`
`}`	`Semwait(S);`
	`Semsignal(x);`
	`Readunit();`
	`Semwait(x);`
	`Num = Num - 1;`
	`if(Num == 0)`
	`Semsignal(S);`
	`Semsignal(x);`
	`}`

Semaphore 'S' is used to enforce mutual exclusion.

Semaphore 'x' is used to assure that 'Num' is updated properly.

Solution: If a writer wants critical section as soon as the critical section is available, writer enters it.

Dining Philosophers Problem

N philosophers are sitting around a dining table. There are N plates placed on the table such that each plate is in front of a philosopher and N forks placed between the plates. There is a bowl of Noodles placed at the centre of the table. Whenever a philosopher feels hungry, he tries to pick two forks which are shared with his nearest neighbour. If any of his neighbours happens to be eating at the time, the philosopher has to wait. Whenever a hungry philosopher gets two forks, he pours noodles into his plate. After he finishes, he places the chopsticks back onto the table and starts thinking. Now forks are available for neighbours.

Solution:

```
# define N 5 /* Number of philosophers*/
void philosopher(int i) /* philosopher number,
from 0 to 4*/
{
while (true)
{
think(); /* philosopher is thinking*/
take_fork(i) ; /*take left fork*/
take_fork ((i+ 1)% N); /* take right fork; %
is modulo operator*/
eat( );
put _ fork ( ); /* put left back on the table*/
put _ fork ((i + 1) % N); /* put right fork
back on the table */
```

Notes:

1. This solution leads to deadlock.
2. Everyone picks the left fork and indefinitely wait for right fork causing starvation.

Monitors

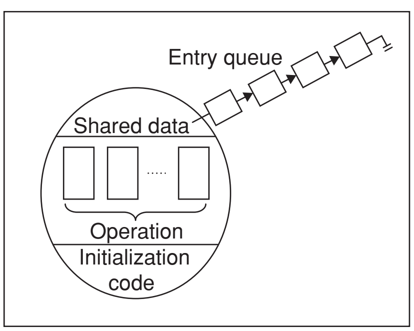
Monitor is a programming language construct that encapsulates variables, access procedures and initialization code within an abstract data type. The monitor's variable may only be accessed via its access procedures and only one process may be actively accessing the monitor at any one time. The access procedures are critical sections. A monitor may have a queue of processes that are waiting to access it.

1. If the data in a monitor represent some resource, then the monitor provides a mutual exclusion facility for accessing the resource.
2. A monitor supports synchronization by the use of condition variables that are contained within the monitor and accessible only within the monitor.
3. Operations on conditional variables:
   - cwait(c): Suspend execution of calling process on condition *c*. The monitor is now available for use by another process.
   - csignal(c): Resume execution of some process blocked after a wait on the same condition. If there are several such processes, choose one of them; if there is no such process, do nothing.

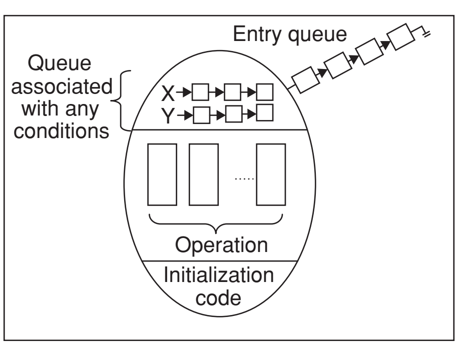
Monitor syntax is as follows:
Monitor monitor - name

```
{
shared variable declarations;
Procedure body P1 (...) {  }
Procedure body P2 (...) {  }
Procedure body Pn (...) {  }
initialization code {  }
}
```

Schematic view of a monitor:



Schematic View of a monitor with condition variables:



Message Passing

When processes interact with one another, two fundamental requirements must be satisfied:

1. Synchronization
2. Communication

One approach to provide both of these is *message passing*.

The primitive functions in message passing are send (destination, message) receive (source, message)

Design Characteristics of Message Systems for IPC and Synchronization

Synchronization There must be some synchronization existing between two processes to communicate with each other.

- ***Send:*** When a 'send' primitive is executed in a process, then the sender may
  - blocked or
  - non-blocked
  - until the message is received.
- ***Receive:*** When a process issues a 'Receive' primitive there are two possibilities:
  1. If a message has previously been sent, the message is received and execution continues.
  2. It there is no waiting message then either
     (i) The process is blocked until a message arrives or
     (ii) The process continues to execute, abandoning the attempt to receive.

Thus, both the sender and receiver may be in one of

1. Blocking send, blocking receive: Allows tight synchronization.
2. Non-blocking send, blocking receive:
   - Useful synchronization
   - Possibility of generating repeated messages
3. Non-blocking send, non-blocking receive: No need to wait

Addressing Two types of addressing methods:

1. Direct addressing
2. Indirect addressing

Direct Addressing

The send primitive includes a specific identifier of the destination process. The 'receive' primitive can be handled in one of two ways:

Explicit

The process must know ahead of time from which process a message is expected. Useful for cooperating concurrent processes.

Implicit

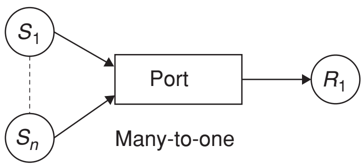
The 'source' parameter of 'receive' primitive possesses a value returned when the receive operation has been performed. Example, Printer server.

Indirect Addressing

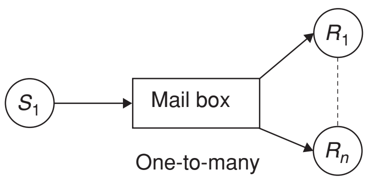
Messages are not sent directly from sender to receiver but rather are sent to a shared data structure consisting of queues that can temporarily hold messages. These queues are referred to as mailboxes The relationship between the sender and receiver is

- one-to-one
- one-to-many
- many-to-one
- many-to-many

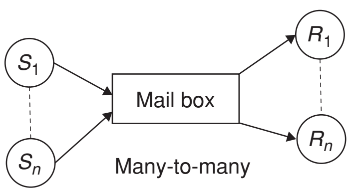
One-to-one



Many-to-one

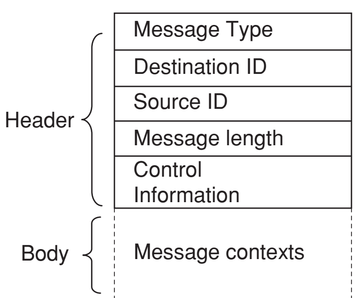


One-to-many



Many-to-many

Message format The general message format will be



Queuing discipline The queuing discipline may be

- FIFO
- Priority

Mutual Exclusion using message passing:

```
const int n = /* number of processes */;
void P(int i)
{
message msg;
while (true)
{
receive (box, msg);
/* critical section */;
send (box, msg);
/* remainder */;
}
}
void main( )
{
create mailbox(box);
send(box, null);
begin (P(1), P(2) ... P(n));
}
```

1. Here a set of concurrent processes that share a mailbox ‘box’, which can be used by all processes to send and receive.
2. The mailbox is initialized to contain a single message with null content. A process wishing to enter its critical section first attempts to receive a message.
3. If the mail box is empty, then the process is blocked.
4. Once a process acquired the message, it performs its critical section and then places the message back into the mailbox.
5. Hence, the message functions as a token that is passed from process to process.

Exercises

Practice Problems I

Directions for questions 1 to 17: Select the correct alternative from the given choices.

1. The value of a counting semaphore is 7. Then 15 wait operations and 10 signal operations were completed on this semaphore. The resulting value of semaphore is
(A) 5 (B) 7
(C) 2 (D) 0

2. At a particular time of computation, the value of counting semaphore is 7. Then 20 wait operations and ‘x’ signal operations were completed on this semaphore. If the final value of the semaphore is 5, what is x?
(A) 18 (B) 13
(C) 5 (D) 0

3. A process using a semaphore has a start value of 1 for its semaphore. Since the start of execution of the program, 12 signal operations were completed. How many wait operations have been completed so far if the current value of semaphore is 6?
(A) 1 (B) 5
(C) 7 (D) 11

4. It is found that a program has multiple critical sections. Choose correct statements from below:
(i) Multiple semaphores are needed for handling the situation.
(ii) A single semaphore that uncompresses all the critical section is sufficient and is also more efficient.
(iii) To get better control of the code, monitors need to be implemented.
(A) (i) and (ii) (B) (ii) and (iii)
(C) (i) and (iii) (D) (i), (ii), (iii)

5. Consider the below psuedocode:

```
semaphore S = 1;
semaphore E = 1;
if(thread_count++ <100)
spawnnewthread( );
wait(E);
 // critical section - begin
 ---------------
 ---------------
 // critical section - end
 signal(S);
```

Assume that above pseudocode gets called a hundred times, what is the count of semaphore E?
(A) 0 (B) 1
(C) –99 (D) –100

6. Consider the below code for a process i:

```
-------------
-------------
flag[i] = true;
if(turn == i and flag [i] == true)
/* critical section begin */
counter++;
/* critical section end */
turn = x;
-------------
-------------
```

If the value of a counter started, what would be the value of 'counter' count at the end of the program:
(A) Semaphore count
(B) Thread count
(C) Concurrency count
(D) Deadlock process count

7. Consider the below pseudocode:

```
function waitB(s)
{
if(s.value ==1)
s.value = 0;
else
place the process in the Queue;
}
function signalB(s)
{
s. value = 1;
}
```

What does the code most likely behave as
(A) general semaphore (B) weak semaphore
(C) binary semaphore (D) mutex

8. A shared variable x, initialized to 0 is operated on by four concurrent processes P, Q, R, S as follows:

```
P(x)                     Q(x)
{                        {
wait( );                 wait( );
read(x);                 read(x);
increment x by 1;        increment x by 1;
store(x);                store(x);
signal();                signal( );
}                        }
R(x)                     S(x)
{                        {
wait( );                 wait( );
read (x);                read(x);
decrement x by 2;        decrement x by 2;
store (x);               store(x);
signal( );               signal( );
}                        }
```

A counting semaphore 'N' is used by the processes whose value is initialized to 2. What is the maximum possible value of 'x' after all processes complete execution?
(A) –2 (B) –1
(C) 1 (D) 2

9. Consider the following code:

```
Program concurrency;
Var x: Integer (: = 0);
     y: Integer (: = 0);
Procedure threadA( ) ;
begin
x = 1; /*S1*/
y = y + x; /*S2*/
end;
Procedure threadB( ) ;
begin
y = 4;      /*S3*/
x = x + 5;  /*S4*/
end;
begin /*mainprogram*/
parbegin
threadA( );
threadB( );
parend;
    end.
```

Suppose a process has two concurrent threads: one thread executes statements S_1 and S_2, and the other thread executes statements S_3 and S_4. What are the maximum possible values of x and y when the code finishes execution? (All the statements S_1, S_2, S_3 and S_4 are atomic).

(A) $x = 6, y = 4$ (B) $x = 6, y = 5$
(C) $x = 1, y = 5$ (D) $x = 6, y = 10$

10. Consider the following program:

```
boolean lock[2];
int turn;
void P(int id)

{
while(true)
{
lock[id] = true;
while (turn! = id)
{
while (lock [1 - id])
/*do nothing*/
turn = id;
}
/*critical section*/
lock[id] = false;
/*reamainder*/
  }
}
void main( )
{
lock[0] = false;
lock[1] = false;
turn = 0;
parbegin (P(0), P(1));
}
```

Which of the following statements is correct for two processes executing this code?

(A) Given program provides mutual exclusion.
(B) Given program does not provide mutual exclusion.
(C) Given program provides mutual exclusion and also solves starvation problem.
(D) Given program provides mutual exclusion but does not prevent from starvation.

11. Consider two process P_0 and P_1 which share the following variables:

```
boolean flag [2]; /*initially false*/
int turn;
```

These two processes, $Pi(i = 0$ or $1)$, Pj $(j = 1$ or $0)$ execute the following code:

```
do
{
flag[i] = TRUE;
while(flag [j])
{
if (turn = = j)
{
flag[i] = false;
while (turn = = j);
flag [i] = TRUE;
}
}
// critical section
turn = j;
flag[i] = FALSE;
// remainder.
}
```

while(TRUE);

The code satisfies

(i) Mutual exclusion
(ii) Progress
(iii) Bounded waiting

(A) (i), (ii) only (B) (ii), (iii) only
(C) (i), (iii) only (D) (i), (ii), (iii)

12. Which of the following statement(s) is false?

(i) Spinlocks are not appropriate for single-processor systems.
(ii) Mailboxes may be used for synchronization.
(iii) Message passing and semaphores do not have equivalent functionality.

(A) (i) only (B) (iii) only
(C) (i), (iii) (D) (i), (ii), (iii)

13. Consider the following code:

```
signal (mutex);
............ .
Critical section
............ .
wait (mutex);
```

Here 'mutex' is a semaphore variable, which is initialized to 1. Then

(A) Mutual exclusion is provided
(B) Mutual exclusion violated, if several processes are simultaneously active in their critical section.
(C) Deadlock will occur
(D) Starvation is possible

14. Which of the following sequence of 'wait' and 'signal' operations leads to deadlock?
(Here 'mutex' is a semaphore variable initialized to 1.)

(A)
```
wait (mutex);
.........
Critical section
......... .
Signal (mutex);
```

(B)
```
wait (mutex);
………..
Critical section
……………
Wait (mutex);
```
(C)
```
Signal (mutex);
…………
Critical section
……….
Wait (mutex);
```
(D)
```
signal (mutex);
…………
Critical section
………….
Signal (mutex);
```

15. Which of the following situation arises if a process omits the wait(S) or the signal(S) on a semaphore variable '*S*' (Initially *S* = 1).
(i) Mutual exclusion violated
(ii) Deadlock will occur
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

16. consider the following shared data and code:
```
data:
int turn;
Boolean flag[2];
Code:
do
{
flag [i] = TRUE;
turn = j;
while (flag [j] && turn = = j);
//critical section
flag [i] = FALSE;
//remainder
}
while (TRUE);
```
Let two processes *Pi* (*i* = 0 or 1) and *Pj* (*j* = 1 or 0) use the shared data and executes the code. Then the code provides
(A) a solution to critical section problem
(B) mutual exclusion but not progress.
(C) progress but not mutual exclusion
(D) both mutual exclusion, progress but no bounded waiting.

17. Consider the following code that shows the structure of a process in an algorithm to solve the critical section problem for two processes.
var flag[2] of Boolean; /* initialized to false */
repeat
flag[*i*] = true;
while flag[*j*] do no – op;
//critical section
flag[*i*] = false;
// remainder
until false
Then which of the following statements is true?
(A) The algorithm satisfies all the requirements of critical section problem.
(B) The algorithm satisfies only mutual exclusion and progress.
(C) The algorithm only satisfies progress requirement.
(D) The algorithm does not satisfy critical section problem requirements.

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. The result of a computation depends upon the speed of the processes involved, is said to be:
(A) Cycle stealing (B) Race condition
(C) A time lock (D) A deadlock

2. A relation between processes such that each has some part which must not be executed, while the critical section of another is being executed is known as
(A) Mutual exclusion (B) Semaphore
(C) Multi-tasking D) Mutli-programming

3. Producer–consumer problem can be solved using
(A) Semaphores (B) Event counters
(C) Monitors (D) All of the above

4. To avoid the race condition, the number of processes allowed in critical section is
(A) 0 (B) 1
(C) 2 (D) 3

5. Mutual exclusion problem occurs between
(A) Two disjoint processes that unaware of each other
(B) Processes that share resources
(C) Processes directly aware of each other.
(D) Both (A) and (B)

6. Semaphores are used to solve the problem of
(A) Race condition (B) Multitasking
(C) Mutual exclusion (D) Both (A) and (C)

7. At a particular time, the value of a counting semaphore is 10. It will become 7 after
(A) 3 signal operations
(B) 3 wait operations
(C) 5 signal operations and 2 wait operations
(D) None of the above

8. Critical region is
(A) A part of the OS which is not allowed to be accessed by any process
(B) A set of instructions that accesses common shared resource, which exclude one another in time

(C) The portion of main memory, which can be accessed only by one process at a time
(D) Both (A) and (C)

9. Concurrent processes are:
(A) Processes that don't overlap in time
(B) Processes that overlap in time
(C) Processes that are executed
(D) Processes that are executed by a processor at the same time

10. Semaphore operations are atomic because they are implemented within the_______.
(A) Kernel (B) Shell
(C) User process (D) Normal process space

11. The programming language construct that provides equivalent functionality of a semaphore and better control is
(A) Signal (B) Monitor
(C) Mutex (D) Critical section.

12. What is the ideal way of emptying the queue of a strong semaphore?
(A) Random (B) LIFO
(C) FIFO (D) binary

13. What are the disadvantages of machine instruction approach?
(i) While a process is waiting for entering a critical section, process still consumes resources.
(ii) There could be starvation
(iii) There could be deadlocks
(A) (i), (ii) only (B) (ii), (iii) only
(C) (iii), (i) only (D) (i), (ii), (iii)

14. Select from below the advantages of Machine Instruction approach?
(i) Applicable to any number of processes either uniprocessor or multi-processor system
(ii) Simple and easy to verify
(iii) Supports multiple critical sections
(A) (i), (ii) only (B) (ii), (iii) only
(C) (iii), (i) only (D) (i), (ii), (iii)

15. Which of the below are requirements for mutual exclusion?
(i) Only one process is allowed into critical section.
(ii) A process remains inside its critical section for finite time only.
(iii) It must be possible for a process accessing critical section to be delayed indefinitely.
(iv) A process halting in critical section must do so without interfering with other processes.
(A) (i), (ii), (iii) (B) (ii), (i), (iv)
(C) (i), (ii), (iv) (D) (i), (ii), (iii), (iv)

Previous Years' Questions

1. Consider these two functions and two statements S_1 and S_2 about them: **[2006]**

`int work1 (int *a,int i, int j)` `{` `int x=a[i+2];` `a[j]=x+1;` `return a[i+2]-3;` `}`	`int work2 (int *a,int i, int j)` `{` `int t1=i+2;` `int t2=a[t1];` `a[j]=t2+1;` `return t2 - 3;` `}`

S1: The transformation from work1 to work2 is valid, that is,for any program state and input arguments, work 2 will compute the same output and have the same effect on program state as work 1
S2: All the transformations applied to work 1 to get work 2 will always improve the performance (i.e., reduce CPU time) of work 2 compared to work 1
(A) S_1 is false and S_2 is false
(B) S_1 is false and S_2 is true
(C) S_1 is true and S_2 is false
(D) S_1 is true and S_2 is true

2. The atomic fetch-and-set x, y instructions unconditionally sets the memory location x to 1 and fetches the old value of the of x in y without allowing any intervening access to the memory location x. Consider the following implementation of P and V functions on binary semaphore S.

```
void P (binary-semaphore *s) {
    unsigned y;
    unsigned *x = & (s → value);
    do {
          fetch-and-set x, y ;
          } while (y) ;
    }
void V (binary-semaphore *s) {
s → value = 0;
}
Which one of the following is true?
```
[2006]

(A) The implementation may not work if context switching is disabled in P
(B) Instead of using fetch-and-set, a pair of normal load/store can be used
(C) The implementation of V is wrong
(D) The code does not implement a binary semaphore

3. The P and V operations on counting semaphores, where s is a counting semaphore, are defined as follows:

$P(s)$: $s = s - 1$;
  if $s < 0$ then wait;
$V(s)$: $s = s + 1$;
if $s <= 0$ then wake up a process waiting on s;

Assume that P_b and V_b, the wait and signal operations on binary semaphores, are provided. Two binary semaphores x_b and y_b are used to implement the semaphore operations $P(s)$ and $V(s)$ as follows:

```
P(s) :        Pb (xb) ;
    s = s - 1;
    if (s < 0) {
       Vb (xb) ;
       Pb (yb) ;
    }
    else Vb (xb) ;
V(s) :        Pb (xb) ;
    s = s + 1;
    if (s<=0) Vb (yb) ;
       Vb (xb) ;
```

The initial values of x_b and y_b are respectively **[2008]**

(A) 0 and 0 (B) 0 and 1
(C) 1 and 0 (D) 1 and 1

4. Consider a system with four types of resources R_1 (3 units), R_2 (2 units), R_3 (3 units), R_4 (2 units). A non-pre-emptive resource allocation policy is used. At any given instance, a request is not entertained if it cannot be completely satisfied. Three processes P_1, P_2, P_3 request the resources as follows if executed independently. **[2009]**

Process P_1:	**Process P_2:**	**Process P_3:**
$t = 0$: requests 2 units of R_2	$t = 0$: requests 2 units of R_3	$t = 0$: requests 1 unit of R_4
$t = 1$: requests 1 unit of R_3	$t = 2$: requests 1 unit of R_4	$t = 2$: requests 2 units of R_1
$t = 3$: requests 2 units of R_1	$t = 4$: requests 1 unit of R_1	$t = 5$: releases 2 units of R_1
$t = 5$: releases 1 unit of R_2 and 1 unit of R_1.	$t = 6$: releases 1 unit of R_3	$t = 7$: requests 1 unit of R_2
$t = 7$: releases 1 unit of R_3	$t = 8$: Finishes	$t = 8$: requests 1 unit of R_3
$t = 8$: requests 2 units of R_4		$t = 9$: Finishes
$t = 10$: Finishes		

Which one of the following statements is true if all three processes run concurrently starting at time $t = 0$?

(A) All processes will finish without any deadlock
(B) Only P_1 and P_2 will be in deadlock.
(C) Only P_1 and P_3 will be in a deadlock.
(D) All three processes will be in deadlock.

5. The enter_CS() and leave_CS() functions to implement critical section of a process are realized using test-and-set instruction as follows: **[2009]**

```
void enter_CS(X)
{
    while (test-and-set(X)) ;
}
void (leave_CS(X))
{
    X=0;
}
```

In the above solution, X is a memory location associated with the CS and is initialized to 0. Now consider the following statements:

(i) The above solution to CS problem is deadlock-free.
(ii) The solution is starvation free.
(iii) The processes enter CS in FIFO order.
(iv) More than one process can enter CS at the same time.

Which of the above statements is true?

(A) (i) only
(B) (i) and (ii)
(C) (ii) and (iii)
(D) (iv) only

6. Consider the methods used by processes P_1 and P_2 for accessing their critical sections whenever needed, as given below. The initial values of shared Boolean variables S_1 and S_2 are randomly assigned. **[2010]**

Method used by P_1	**Method used by P_2**
while ($S_1 == S_2$);	while ($S_1 != S_2$);
Critical section	Critical section
$S_1 = S_2$;	$S_2 = \text{not}(S_1)$;

Which one of the following statements describes the properties achieved?

(A) Mutual exclusion but not progress
(B) Progress but not mutual exclusion
(C) Neither mutual exclusion nor progress
(D) Both mutual exclusion and progress

7. The following program consists of three concurrent processes and three binary semaphores. The semaphores are initialized as $S_0 = 1$, $S_1 = 0$, $S_2 = 0$.

Process P_0	**Process P_1**	**Process P_2**
`while (true) {`	`wait (S1);`	`wait (S2);`
`wait (S0);`	`Release (S0);`	`release (S0);`
`print '0'`		
`release (S1);`		
`release (S2);`		
`}`		

How many times will process P_0 print '0'? **[2010]**

(A) At least twice (B) Exactly twice
(C) Exactly thrice (D) Exactly once

8. Fetch _ And _Add(X, i) is an atomic Read- Modify-Write instruction that reads the value of memory location X, increments it by the value i, and returns the old value of X. It is used in the pseudocode shown below to implement a busy wait lock. L is an unsigned integer shared variable initialized to 0. The value of 0 corresponds to lock being available, while any non-zero value corresponds to the lock being not available. **[2012]**

```
AcquireLock(L) {
while(Fetch_And_Add(L, 1))
L = 1;
}
ReleaseLock (L) {
L = 0;
}
```

This implementation

(A) fails as L can overflow
(B) fails as L can take on a non-zero value when the lock is actually available
(C) works correctly but may starve some processes
(D) works correctly without starvation

9. A shared variable x, initialized to 0, is operated on by four concurrent processes W, X, Y, Z as follows. Each of the processes W and X reads x from memory, increments by one, stores it to memory, and then terminates. Each of the processes Y and Z reads x from memory, decrements by two, stores it to memory, and then terminates. Each process before reading x invokes the P operation (i.e., wait) on a counting semaphore S and invokes the V operation (i.e., signal) on the semaphore S after storing x to memory. Semaphore S is initialized to 2. What is the maximum possible value of x after all processes complete execution? **[2013]**

(A) –2 (B) –1
(C) 1 (D) 2

10. A certain computation generates two arrays 'a' and 'b' such that $a[i] = f(i)$ for $0 \le i < n$ and $b[i] = g(a[i])$ for $0 \le i < n$. Suppose this computation is decomposed into two concurrent processes X and Y such that X computes the array 'a' and Y computes the array 'b'. The processes employ two binary semaphores R and S, both initialized to zero. The array 'a' is shared by the two processes. The structures of the processes are shown below.

Process *X*:

```
private i;
for (i=0; i<n; i++) {
    a[i] = f(i);
    ExitX(R, S);
}
```

Process *Y*:

```
private i;
for (i=0; i<n; i++) {
EntryY(R, S);
    b[i] = g(a[i]);
}
```

Which one of the following represents the correct implementations of Exit$X$ and Entry$Y$? **[2013]**

(A)
```
ExitX(R, S) {
    P(R);
    V(S);
}
    EntryY(R,S){
    P(S);
    V(R);
}
```

(B)
```
ExitX(R, S) {
    V(R);
    V(S);
}
    EntryY(R,S){
    P(R);
    P(S);
}
```

(C)
```
ExitX (R, S){
    P(S);
    V(R);
}
    EntryY(R,S){
    V(S);
    P(R);
}
```

(D)
```
ExitX (R, S){
    V(R);
    P(S);
}
    EntryY(R,S){
    V(S);
    P(R);
}
```

11. Consider the procedure below for the *producer-consumer* problem which uses semaphores; **[2014]**

```
semaphore n = 0;
semaphore s = 1;
void producer ()
{
while (true)
{
produce ( )
semWait (s);
addToBuffer ();
semSignal (s);
semSignal (n);
}
}
void consumer ()
{
while (true)
{
semWait (s);
semWait (n);
remove FromBuffer ();
semSignal(s);
consume ( ) ;
}
}
```

Which one of the following is true?

(A) The producer will be able to add an item to the buffer, but the consumer can never consume it.
(B) The consumer will remove no more than one item from the buffer.
(C) Deadlock occurs if the consumer succeeds in acquiring semaphore s when the buffer is empty.
(D) The starting value for the semaphore n must be 1 and not 0 for deadlock free operation.

12. The following two function $P1$ and $P2$ that share a variable B with an initial value of 2 execute concurrently. **[2015]**

```
P1 ( ) {                     P2 ( )  {
   C = B – 1;                   D = 2 * B;
   B = 2 * C;                   B = D – 1;
}                               }
```

The number of distinct values that B can possibly take after the execution is _______

13. Two processes X and Y need to access a critical section. Consider the following synchronization construct used by both the processes **[2015]**

Process X	**Process Y**

`/* other code for process X */`
`while (true)`
`{`
`varP = true;`
`while(varQ == true)`
`{`
`/* Critical Section */`
`varP = false;`
`}`
`}`
`/* other code for process X */`
	`/* other code for process Y */ while (true)`
	`{`
	`varQ = true;`
	`while (varP == true)`
	`{`
	`/* Critical Section */`
	`varQ = false;`
	`}`
	`}`
	
	`/* other code for process Y */`

Here, var$P$ and var$Q$ are shared variables and both are initialized to false. Which one of the following statements is true?

(A) The proposed solution prevents deadlock but fails to guarantee mutual exclusion.
(B) The proposed solution guarantees mutual exclusion but fails to prevent deadlock.
(C) The proposed solution guarantees mutual exclusion and prevents deadlock.
(D) The proposed solution fails to prevent deadlock and fails to guarantee mutual exclusion.

14. Consider the following proposed solution for the critical section problem. There are n process: $P_0 \ldots P_{n-1}$. In the code, function pmax returns an integer not smaller than any of its arguments. For all i, $t[\,i\,]$ is initialized to zero. **[2016]**

do {
$c[\,i\,] = 1$; $t\,[\,i\,] = \text{pmax}\,(t[\,i\,], \ldots\ldots,$
$t[n-1]) + 1$; $c[\,i\,] = 0$;
for every $j \neq i$ in $(0, \ldots., n-1)$ {
while ($c\,[\,j\,]$);
while ($t[\,j\,]\,! = 0$ && $t[\,j\,] < = t[\,i\,]$);
}
Critical Section;
$t[\,i\,] = 0$;
Remainder Section;
} while (true);

Which one of the following is TRUE about the above solution?

(A) At most one process can be in the critical section at any time.
(B) The bounded wait condition is satisfied.
(C) The progress condition is satisfied.
(D) It cannot cause a deadlock.

15. Consider the following two - process synchronization Solution.

Process 0	Process 1
Entry: loop while (turn = = 1);	Entry: loop while (turn = = 0);
(Critical section)	(Critical section)
Exit: turn = 1;	Exist: turn = 0;

The shared variable *turn* is initialized to zero. Which one of the following is **TRUE**? **[2016]**

(A) This is a correct two - process synchronization Solution.
(B) This Solution violates mutual exclusion requirement.
(C) This Solution violates progress requirement.
(D) This Solution violates bounded wait requirement.

16. Consider a non-negative counting semaphore S. The operation $P(S)$ decrements S, and $V(S)$ increments S. During an execution, 20 $P(S)$ operations and 12 $V(S)$ operations are issued in some order. The largest initial value of S for which at least one $P(S)$ operation will remain blocked is _____ . **[2016]**

17. A multithreaded program P executes with x number of threads and uses y number of locks for ensuring mutual exclusion while operating on shared memory locations. All locks in the program are *non-reentrant*, i.e., if a thread holds a lock l, then it cannot re-acquire lock l without releasing it. If a thread is unable to acquire a lock, it blocks until the lock becomes available. The *minimum* value of x and the *minimum* value of y together for which execution of P can result in a deadlock are: **[2017]**

(A) $x = 1, y = 2$ (B) $x = 2, y = 1$
(C) $x = 2, y = 2$ (D) $x = 1, y = 1$

18. Consider the following solution to the producer-consumer synchronization problem. The shared buffer size is N. Three semaphores empty, full and mutex are defined with respective initial values of 0, N and 1. Semaphore empty denotes the number of available slots in the buffer, for the consumer to read from. Semaphore full denotes the number of available slots in the buffer, for the producer to write to. The placeholder variables, denoted by P, Q, R, and S, in the code below can be assigned either empty or full. The valid semaphore operations are: wait () and signal ().

Producer	Consumer

do {
  wait(P);
  wait (mutex);
  //Add item to
  buffer
  signal (mutex);
  signal (Q);
} while (1);
	do {
	  wait(R);
	  wait (mutex);
	  //Consume item
	  from buffer
	  signal (mutex);
	  signal (S);
	} while (1);

Which one of the following assignments to P, Q, R and S will yield the correct solution? **[2018]**

(A) P: full, Q: full, R: empty, S: empty
(B) P: empty, Q: empty, R: full, S: full
(C) P: full, Q: empty, R: empty, S: full
(D) P: empty, Q: full, R: full, S: empty

Answer Keys

Practice Problems 1

1. C **2.** A **3.** C **4.** D **5.** C **6.** C **7.** C **8.** D **9.** D **10.** B
11. D **12.** B **13.** B **14.** B **15.** C **16.** A **17.** D

Practice Problems 2

1. B **2.** A **3.** D **4.** B **5.** D **6.** C **7.** B **8.** B **9.** B **10.** A
11. B **12.** C **13.** D **14.** D **15.** C

Previous Years' Questions

1. D **2.** A **3.** C **4.** A **5.** A **6.** A **7.** A **8.** B **9.** D **10.** C
11. C **12.** 3 **13.** A **14.** A **15.** C **16.** 7 **17.** D **18.** C

Chapter 3

Deadlock and CPU Scheduling

LEARNING OBJECTIVES

- *Deadlock*
- *System model*
- *Bridge crossing*
- *Resources*
- *Resource allocation graph*
- *Methods of handling deadlocks*
- *Deadlock prevention*
- *Resource allocation denial (OR) banker's algorithm*
- *Deadlock detection*
- *Dining philosophers problem*
- *Scheduling algorithms*
- *Scheduling policies*
- *Round Robin scheduling*
- *Shortest remaining time*
- *Highest response ratio next*
- *Multilevel feedback queue scheduling*

Deadlock

It is a situation where a process or set of processes is blocked, waiting for some resource that is held by other waiting processes.

System Model

Let the resource types be R_1, R_2 ... R_m (like CPU cycles, memory space, input/output (I/O) devices, etc.). Each resource type R_i has W_i instances; each process utilizes a resource as follows:

Request A process, needing a resource, will request the operating system (OS) for assignment of the needed resource. Then the process waits, till operating system assigns it an instance of the requested resource.

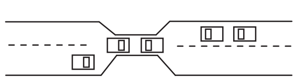
Assignment The OS will assign to the requesting process an instance of the requested resource, whenever, it is available. Then, the process comes out of its waiting state.

Use The process will use the assigned resource. In case, the resource is non-sharable, the process will have exclusive access to it.

Release After the process finished with the use assigned resource, it will return the resource to the system pool. The released resource can now be assigned to another waiting process.

Example:

Bridge crossing



Traffic is allowed only in one direction. Each section of a bridge can be viewed as a resource.

If a deadlock occurs, it can be resolved if one car backs up (preempt resources and rollback). Several cars may have to be backed up if a deadlock occurs.

Problem of starvation (infinite wait) is possible.

Resources

Types of resources:

1. Reusable resources
2. Consumable resources

Reusable resources These resources can be safely used by only one process at a time, and are not depleted by that use.

Examples: Processors, I/O channels, main and secondary memory, devices and files, etc.

Consider two processes P and Q that compete for exclusive access to a disk file D and tape drive T. Let their implementation is as shown below:

Table 1 *Process P*

Step	Action
P_0	Request (D)
P_1	Lock (D)
P_2	Request (T)
P_3	Lock (T)
P_4	Perform function
P_5	Unlock (D)
P_6	Unlock (T)

Table 2 *Process Q*

Step	Action
Q_0	Request (T)
Q_1	Lock (T)
Q_2	Request (D)
Q_3	Lock (D)
Q_4	Perform Function
Q_5	Unlock (T)
Q_6	Unlock (D)

P and Q are executing on a single processor in interleaved fashion. Then deadlock occurs if each process holds one resource and requests the other.

For example, deadlock occurs if the multiprogramming system interleaves the execution of the two processes as follows:

$$P_0, P_1, Q_0, Q_1, P_2, Q_2$$

One strategy to deal with this type of deadlocks is to impose system design constraints concerning the order in which resources can be requested.

Consumable resources A consumable resource is one that can be created and destroyed. There is no limit on the number of consumable resources of a particular type.

Examples: Interrupts, signals, messages, etc.
Consider the following pair of processes, in which each process attempts to receive a message from the other process and then send a message to the other process:

P_1	P_2
. . . .	. . . .
receive (P_2);	receive (P_1);
. . . .	. . . .
send (P_2, M_1);	send (P_1, M_2);

Deadlock occurs in above case, if the receive is blocking. There is no single effective strategy that can deal with all types of deadlocks.

Deadlock Characteristics

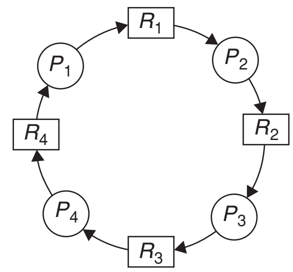
Deadlock is an undesirable state of the system. The following are the four conditions that must hold simultaneously for a deadlock to occur:

Mutual exclusion A resource can be used by only one process at a time. If another process requests for that resource then the requesting process must be delayed until the resource has been released.

Hold-and-wait Some processes must be holding some resources in a non-sharable mode and at the same time must be waiting to acquire some more resources, which are currently held by other processes in a non-sharable mode.

No pre-emption Resources granted to a process can be released back to the system only as a result of the voluntary action of that process, after the process has completed its task.

Circular wait Deadlocked processes are involved in a circular chain such that each process holds one or more resources being requested by the next process in the chain.



Resource Allocation Graph

A deadlock is described in terms of a directed graph called a system Resource Allocation Graph (RAG). It consists of two sets:

1. The set of vertices, V
2. The set of edges, E

The set of vertices is again divided into two categories.

The set of all active processes in the system is $P = \{P_1, P_2, \ldots P_n\}$ and the set of all different type of resources i.e., $R = \{R_1, R_2 \ldots R_m\}$

There are two types of edges in the RAG:

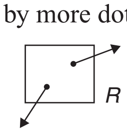
1. A directed edge from the process P_i to resources type R_j and is denoted by $P_i \to R_j$. It signifies that the ith process is requesting one unit of the resource type j. This edge is request edge.
2. A directed edge from the resource R_i to process P_j denoted by $R_i \to P_j$. It signifies that one unit of ith resource is held by the process j. This edge is also called as an *allocation edge/assignment edge*.

Notations Used in RAGs

We denote a process by a circle and each resource by a rectangle.

Process		P_i requests instance of R_j	R_j
Resource Type with four instances		P_i is holding an instance of R_j	R_j

However, if we have more number of instances of a resource type, then it is denoted by more dots.



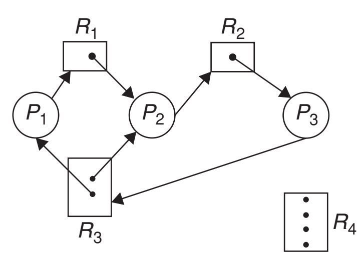


Figure 1 RAG with deadlock.

Notes:

1. If a cycle exists in the RAG, there may or may not be a deadlock.
2. Acyclic RAG implies no deadlock.
3. No deadlock implies acyclic RAG. This means that cycles can be there even if there is no deadlock.

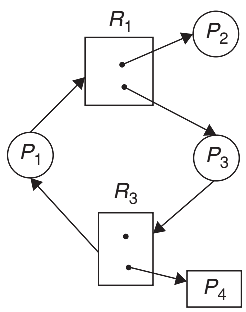


Figure 2 RAG with no deadlock but contains cycle.

Notes:

1. If graph contains no cycles, then no deadlock.
2. If graph contains a cycle

If there is only one instance per resource type, then deadlock occurs.

If there are several instances per resource type, deadlock may occur.

Methods of Handling Deadlocks

There are three approaches to deal with deadlocks. They are

1. Deadlock prevention
2. Deadlock avoidance
3. Deadlock detection

Deadlock Prevention

1. The strategy of deadlock prevention is to design a system in such a way that the possibility of deadlock is excluded.
2. Two classes of deadlock prevention are
   - Indirect method
   - Direct method

Indirect method Prevent the occurrence of one of three necessary conditions of deadlock i.e., mutual exclusion, No pre-emption and hold and wait.

Direct method Prevent the occurrence of circular wait.

Prevention Techniques

Mutual exclusion This is supported by the OS.

Hold and wait

1. This condition can be prevented by requiring that a process request all of its required resources at one time and blocking the process until all requests can be granted simultaneously.
2. But this prevention does not yield good results because
   - Long waiting time required
   - Not efficient use of allocated resources
   - A process may not know all the required resources in advance.

Advantages

1. Works well for processes that perform a single burst of activity.
2. No pre-emption necessary.

No pre-emption Prevention strategies for 'no pre-emption' are

1. If a process that is holding some resources, requests another resource that cannot be immediately allocated to it, then all resources currently being held are released and if necessary request them again together with the additional resources.
2. If a process requests a resource that is currently held by another process, the OS may pre-empt the second process and require it to release its resources. This technique works only when two processes do not have same priority.

Advantages

Convenient when applied to resources whose state can be saved and restored easily.

Disadvantage

Pre-empts more often than necessary.

Circular wait One way to ensure that this condition never holds is to impose a total ordering of all resource types and to require that each process requests resource in an increasing order of enumeration, i.e., if a process has been allocated resources of type R, then it may subsequently request only those resources of types following R in the ordering.

Advantages

1. Feasible to enforce via compile time checks.
2. No run-time computation required.

Disadvantages

1. Disallows incremental resources requests.

Note: The deadlock prevention strategies are conservative and undercommits resources.

Deadlock Avoidance

1. This approach allows the three necessary conditions of deadlock but makes judicious choices to assure that deadlock point is never reached.
2. Deadlock avoidance allows more concurrency than prevention.
3. A decision is made dynamically whether the current resource allocation request will, if granted, potentially lead to a deadlock.

4. It requires the knowledge of future process requests.
5. Two techniques to avoid deadlock:
   - Process Initiation Denial
   - Resource Allocation Denial

Process initiation denial In this technique, do not start a process if its demands might lead to deadlock.

Consider a system of 'n' processes and 'm' different types of resources. Let us define the following vectors and matrices:

Resources $R = (R_1, R_2, \dots R_m)$
R_1: amount of type 1 resources
R_2: amount of type 2 resources
Available $= V = (V_1, V_2, \dots V_m)$

'V' specifies total amount of each resource not allocated to any process.

$$\text{Claim } C = \begin{array}{c} P_1 \\ P_2 \\ P_n \end{array} \begin{bmatrix} C_{11} & C_{12} \cdots & C_{1m} \\ C_{21} & C_{22} \cdots & C_{2m} \\ C_{n1} & C_{n2} \cdots & C_{nm} \end{bmatrix}$$

C_{ij} = requirement of process i for resources j

$$\text{Allocation } A = \begin{array}{c} P_1 \\ P_2 \\ P_n \end{array} \begin{bmatrix} A_{11} & A_{12} \cdots & A_{1m} \\ A_{21} & A_{22} \cdots & A_{2m} \\ A_{n1} & A_{n2} \cdots & A_{nm} \end{bmatrix}$$

A_{ij} = Current allocation to process i of resource j.
The following relationships must hold:

1. All resources are either available or allocated, that is,

$$R_j = V_j + \sum_{i=1}^{n} A_{ij}, \ \forall j$$

2. No process can claim more than the total amount of resources in the system, that is,

$$C_{ij} \le R_j, \ \forall i, j$$

3. No process is allocated more resources of any type than the process originally claimed to need, that is,

$$A_{ij} \le C_{ij}, \ \forall i, j$$

With these properties satisfied, we can define a deadlock avoidance policy that refuses to start a new process if its resource requirements might lead to deadlock. Start a new process P_{n+1} only if

$$R_j \ge C_{(n+1)j} + \sum_{i=1}^{n} C_{ij}, \ \forall j$$

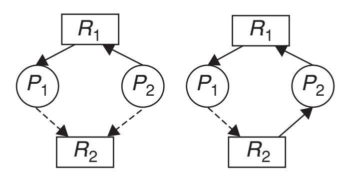
that is, a process is only started if the maximum claim of all current processes plus those of the new process can be met.

Resource allocation denial (OR) banker's algorithm Consider a system with a fixed number of processes and a fixed number of resources. At any time, a process may have zero or more resources allocated to it.

State: The state of a system reflects the current allocation of resources to processes.

Safe state

1. When a process requests an available resource, the system must decide if immediate allocation leaves the system in a safe state.
2. A state is safe if the system can allocate resources to each process in some order and still avoid deadlock.
3. More formally, a system is in safe state if there exists a safe sequence of all processes.
4. A sequence of processes $<P_1, P_2, \dots, P_n>$ is a safe sequence for the current allocation state, if for each P_i, the resource requests that P_i can still make can be satisfied by the currently available resources plus the resources held by all the P_j, with $j < i$.
5. When P_j is finished, P_i can obtain the needed resources, completed its designated task, return its allocated resources and terminates.
6. When P_i terminates, P_{i+1} can obtain its needed resources and so on.
7. If a system is in safe state, no deadlock occurs.
8. If a system is in unsafe state deadlock may occur.
9. Avoidance ensures that a system will never enter an unsafe state.



10. The dotted line in the above graph represents a claim edge, i.e., a process may request that resource sometime in the future.
11. A request can only be granted if it does not result in the formation of a cycle in the graph.
12. If P_2 request R_2, we cannot allocate it, since this would create a cycle.
13. A cycle indicates the system is in *unsafe state*.

Example 1: Consider the following state of a system consisting of three processes and two resources:

$$R = (R_1, R_2) = (5\ 3)$$
$$V = (V_1, V_2) = (2\ 1)$$

$C =$

	R_1	R_2
P_1	4	3
P_2	2	1
P_3	3	3

$A =$

	R_1	R_2
P_1	2	1
P_2	1	1
P_3	0	0

Is this is a safe state?

Solution:
To check whether the state is safe or not, identify whether any one of the three process can run to completion with the resources available, that is, $C_{ij} - A_{ij}$ ″ V_j, ∀j

$$
\begin{array}{cc} & \begin{array}{cc} R_1 & R_2 \end{array} \\ C - A = \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} & \begin{pmatrix} 2 & 2 \\ 1 & 0 \\ 3 & 3 \end{pmatrix} \end{array}
$$

We can identify that
[1 0] < [2 1].
∴ P_2 can execute first.
After P_2 execution it will release all its resources then
$V = (2\ 1) + (1\ 1) = (3\ 2)$.
Now P_1 can execute as (2 2) < (3 2).
After that P_1 can release its resources then
$V = (3\ 2) + (2\ 1) = (5\ 3)$
Now P_3 can execute and release the resources after completion.
Hence, the safe sequence is < P_2, P_1, P_3 >.

Example 2: Now suppose for the above system the allocation matrix

$$
A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \text{ and } V = (0\,0)
$$

Then no process can run to completion as no $C_{ij} - A_{ij}$ ″ V_j, ∀j.
Hence, the system is in unsafe state.

Detection Algorithm for Several Instances of a Resource Type

Safety algorithm

To find out whether or not a system is in a safe state.

Step I: Let 'work' and 'finish' be the two vectors of length m and n.
Initialize: Work = Available and Finish [i] = false;

Step II: Find i such that both:
(a) Finish [i] = false;
(b) Need ″ Work (Need = claim – Allocation)
If no such i exists, go to step 4.

Step III: Work = Work + Allocation
Finish [i] = true
Go to step 2

Step IV: If Finish [i] = true for all i, then the system is in safe state else it is in unsafe state. This algorithm takes $O(m \rightarrow n^2)$ operations to decide whether a state is safe.

Resource–Request algorithm

Let Request$_i$ be the request vector for process, P_i. If Request$_i$[j] = k, the process P_i wants k instances of resource type R_j. When a request for resources is made by process, P_i, the following actions are taken.

Step I: If Request$_i$ ″ Need$_i$, go to step 2. Else raise an exception (error) as the process has exceeded its maximum claim.

Step II: If Request$_i$ ″ Available, go to step 3. Else P_1 must wait, since the resources are not available.

Step IIII: Have the system pretend to have allocated the requested resources to process, P_i, by modifying the state as follows.
Available = Available – Request$_i$
Allocation = Allocation + Request$_i$
Need$_i$ = Need$_i$ – Request$_i$

If the resulting resource–allocation is safe, then the transaction is completed and process, P_i is allocated its resources. However, if the new state is unsafe then P_i must wait for Requesti and the old resource allocation state is restored.

Advantages of deadlock avoidance technique

1. Not necessary to pre-empt and rollback processes.
2. Less restrictive than deadlock prevention.

Disadvantages

1. Future resource requirement must be known in advance.
2. Processes can be blocked for long periods.
3. Exists fixed number of resources for allocation.

Deadlock Detection

1. This technique does not limit resource access or restrict process action.
2. Requested resources are granted to processes whenever possible.

Deadlock detection is used by employing an algorithm that tracks the circular waiting and killing one or more processes so that the deadlock is removed.

The system state is examined periodically to determine if a set of processes is deadlocked.

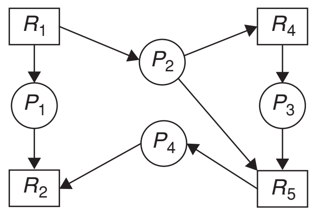
A deadlock is resolved by aborting and restarting a process, relinquishing all the resources that the process held.

For single instance of each resource type If in the RAG, every resource has only one instance (or single instance) then we define a deadlock detection algorithm that uses a variant of the RAG and is called a wait-for-graph.

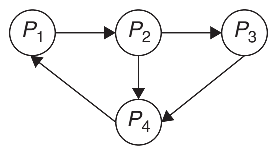
How can we get this graph from RAG? : We can get this by removing the nodes of type resource and collapsing the appropriate edges. Wait-for-graph has a cycle, then there is deadlock in the system.

To detect deadlocks, the system needs to maintain the wait-for-graph and to periodically invoke an algorithm. The complexity of this algorithm is $O(n^2)$ where n is the number of vertices in the graph.

Consider the RAG:



We draw the wait-for-graph by removing all nodes that represents resources and collapsing their edges.



Wait-for-graph

The system is in deadlock state.
Cycle – P_1, P_2, P_4, P_1
Cycle – P_1, P_2, P_3, P_4, P_1

Deadlock Detection Algorithm for Several Instances of Resource Type

Consider the Allocation matrix, A, Request matrix Q (Q_{ij} represents the amount of resource of type j requested by process i), Resource vector R and available vector V.

The algorithm proceeds by marking processes that are not deadlocked. Initially all processes are unmarked.

1. Mark each process that has a row in the Allocation matrix of all zeros.
2. Initialize a temporary vector W to equal the Available vector.
3. Find an index i such that process i is currently unmarked and the ith row of Q is less than or equal to W, that is, $Q_{ik} \le W_k$, for $1 \le k \le M$. If no such row is found, terminate the algorithm.
4. If such a row is found, mark process i and add the corresponding row of the allocation matrix to W. That is, set $W_k = W_k + A_{ik}$ for $1 \le k \le M$. Return to step 3.

A deadlock exists if and only if there are unmarked processes at the end of the algorithm. Each unmarked process is deadlocked.

Example 3: Let the

Request matrix Q =

	R_1	R_2	R_3
P_1	1	1	1
P_2	1	0	0
P_3	1	1	1
P_4	1	1	1

Allocation matrix A =

	R_1	R_2	R_3
P_1	1	1	0
P_2	1	0	0
P_3	0	1	1
P_4	0	1	1

Resource vector $R = (3\ 3\ 3)$
Available Vector $V = (1\ 0\ 1)$
Is deadlock existing in this system?

Solution:
$W = (1\ 0\ 1)$
The request of P_2 is less than W. So $W = W + (1\ 0\ 0) = (2\ 0\ 1)$
So mark P_2. No other unmarked process has a row Q that is less than or equal to W.
Terminate the algorithm.
$\therefore P_1, P_3, P_4$ are in deadlock.

Advantages

1. Never delays process initiation
2. Facilitates online handling

Disadvantages

1. Inherent pre-emption losses

Deadlock Recovery

The possible deadlock recovery strategies are as follows:

1. Abort all deadlocked processes.
2. Back up each deadlocked process to some previously defined checkpoint and restart all processes.
3. Successively abort deadlocked processes until deadlock no longer exists.
4. Successively pre-empt resources until deadlock no longer exists.

Dining Philosophers Problem

Consider the following solution for dining philosophers problem using semaphores:

```
Semaphore fork[5] = {1};
int i;
void philosopher(int i)
{
while (true)
{
think( );
wait(fork[i]);
wait(fork[(i + 1) mod 5]);
eat( );
signal(fork[(i + 1) mod 5]);
signal(fork[i]);
}
}
void main( )
{
Begin(Philosopher(0),Philosopher(1), Philosopher(2), Philosopher(3), Philosopher(4));
}
```

Here, each philosopher picks up first the fork on the left and then the fork on the right. After the philosopher is finished eating, the two forks are placed on the table. But this

solution leads to deadlock, if all of the philosophers are hungry at the same time, they all sit down, they all pick up the fork on their left and they all reach out for the other fork, which is not there.

A refined solution to dining philosophers problem which is deadlock free is shown below:

```
Semaphore fork[5] = {1};
Semaphore room = {4};

int i;
void philosopher(int i)
{
while(true)
{
think( );
wait(room);
wait(fork[i]);
wait(fork[(i + 1) mod 5]);
eat( );
signal(fork[(i + 1) mod 5]);
signal(fork[i]);
signal(room);
}
}
void main ( )
{
Begin
{
(Philosopher (0), Philosopher (1), Philosopher
(2), Philosopher (3), Philosopher (4));
}
}
```

This solution is free from deadlock and starvation.

CPU Scheduling

1. The objective of multi programmed OS is to maximize CPU utilization by having some process running at all times.
2. The objective of time shared OS is to switch the CPU among processes so frequently that the users can interact with each program while it is executing.
3. When there are more than one process ready to execute with the processor, a selection decision needs to be made to pick a process for execution from among the ready processes. This activity is called *process scheduling*.

Scheduling queue: It maintains information of all ready processes for CPU devices. It is maintained as a linked list.

Types of Scheduling Queue

1. ***Job queue:*** It consists of all processes in the system.
2. ***Ready queue:*** It consists of all processes that are residing in the main memory and are ready but waiting to execute on CPU.
3. ***Device queue:*** It consists of processes waiting for a particular I/O device. Each device has its own queue.

Process CPU–I/O Burst Cycle

The execution of process consists of CPU burst and I/O burst. The execution of process starts with CPU burst and I/O burst, which are executed alternatively.

The alternating sequence of CPU and I/O burst are shown below:

Read *a*
Inc *a*
Read *x* } CPU Burst

I/O waiting } I/O Burst
Dec *x*
Store *x* } CPU Burst

I/O waiting } I/O Burst
.
.
.
.

There should be proper balance between CPU bound process and I/O bound process in a schedule.

Scheduler

A process migrates between various scheduling queues throughout its lifetime. The process of selecting processes from the queues is carried out by scheduler.

Types of Processor Scheduling

There are three types of processor scheduling:

1. Long-term scheduling
2. Medium-term scheduling
3. Short-term scheduling

The following figure relates the scheduling functions to the process state transition diagram:

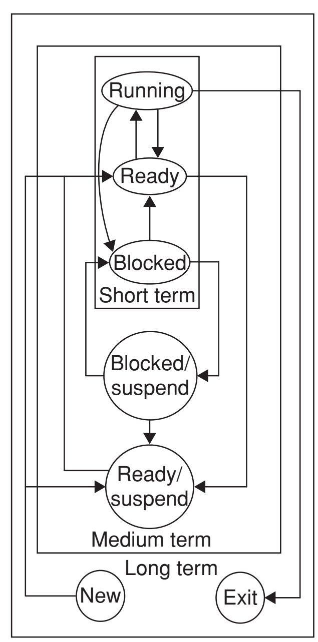


Figure 3 Levels of Scheduling

Long-term Scheduling

1. This is performed when a new process is created. This is also called *job scheduling*.
2. This is a decision whether to add a new process to the set of process that is currently active.
3. It controls the degree of multiprogramming.
4. The long-term scheduler creates processes from the queue when it can.
5. This involves two decisions:
   - The Scheduler must decide when the OS can take on one or more additional processes.
   - The scheduler must decide which job(s) to accept and turn into processes.

Medium-term Scheduling

1. It is a part of swapping function.
2. This is a decision whether to add a process to those that are at least partially in main memory and therefore avail for execution.

Short-term Scheduling

1. It is the decision regarding which ready process to be executed next.
2. This is also known as CPU scheduler.
3. This is invoked whenever an event occurs that may lead to the blocking of the current process or that may provide an opportunity to pre-empt a currently running process in favour of another.
4. Another term involved in short-term scheduling is *dispatcher* which is a module that gives control of the CPU to the process selected by short-term scheduler.

Examples: Clock interrupts, I/O interrupts, OS calls, etc.

Scheduling Algorithms

Scheduling Criteria

The commonly used scheduling criteria can be categorized along two dimensions:

1. User oriented versus system oriented.
2. Performance related versus others.

User-oriented, performance-related criteria

1. ***Turnaround time:*** It is the time taken to execute a process. It is calculated as the interval from the time of submission of a process to the time of completion.
   Turnaround time = waiting time + execution time + time spent in I/O + time spent to get into memory
2. ***Response time:*** Amount of time it takes from when a request was submitted until the first response is produced. It should be minimum.
3. ***Deadlines:*** When process completion deadlines can be specified, the scheduling discipline should subordinate other goals to that of maximizing the percentage of deadlines met.

Waiting time: It is the amount of time that a process spends in ready queue and doing I/O, and it should be minimum.

User-oriented, other criteria

Predictability: A given job should run in about the same amount of time and at about the same cost regardless of the load on the system.

System-oriented, performance-related criteria

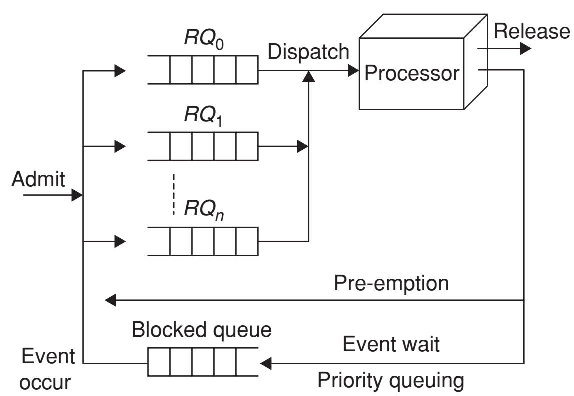
1. ***Throughput:*** The scheduling policy should attempt to maximize the number of processes completed per unit of time. This is a measure of how much work is being performed.
2. ***Processor (CPU) utilization:*** This is the percentage of time that the processor is busy. In real system, it should range from 40–90%.

System-oriented, other criteria

1. ***Fairness:*** No process should suffer from starvation.
2. ***Enforcing priorities:*** Should favour higher priority processes and use Aging technique in order to increase the priority of processes is that wait in the system for long time.
3. ***Balancing resources:*** Should keep the resources of the system busy.

Use of priorities

1. Each process is assigned a priority, and the scheduler will always choose a process of higher priority over one of lower priority.



Here RQ_0, RQ_1,⊃RQ_n are ready queues with priority (RQ_i) > priority (RQ_j) for $i < j$. The scheduler starts with RQ_0 processes, if it is empty choose a process from RQ_1 and so on.

Scheduling Policies

We will discuss the following scheduling algorithms:

1. FCFS
2. Round Robin
3. SPN
4. SRN
5. HRRN
6. Feedback

Before discussing these algorithms, let us discuss some basic concepts:

Selection function

1. Determines which process among ready process, is selected next for execution.
2. This may depend on priority, resource requirement, execution characteristics of process.
3. Execution characteristics of a process include
   w = waiting time
   e = execution time,
   $S = w + e$

Decision mode It is of the following two types:

1. Non-pre-emptive
2. Pre-emptive
   - In non-pre-emptive scheduling, if once the CPU has been allocated to a process, the process can keep the CPU until it releases it, either by terminating or switching to waiting state.
   - In pre-emptive scheduling, CPU can be taken away from a process during execution.

Comparison of non-pre-emptive and pre-emptive scheduling

	Non-pre-emptive Scheduling	Pre-emptive Scheduling
1.	In non-pre-emptive scheduling, if once a process has been allocated CPU then the CPU cannot be taken away from that process.	In pre-emptive scheduling, the CPU can be taken away before the completion of the process.
2.	No preference is given when a higher priority job comes.	It is useful when a higher priority job comes as here the CPU can be snatched from a lower priority process.
3.	The treatment of all processes is fairer.	The treatment of all processes is not fairer as CPU snatching is done either due to constraints or due to higher priority, process request for its execution.
4.	It is a cheaper scheduling method. First come first served is an example.	It is a costlier scheduling method. Round Robin is an example.

First come, first served scheduling (FCFS) The process that requests the CPU first is allocated the CPU first. It is non-pre-emptive scheduling and average waiting time is quite long.

Example 4: Consider the following processes:

Process	Cpu Burst Time (Millisecond)
P_1	20
P_2	5
P_3	3

Find the average waiting time.

Solution:
Suppose they are in the order P_1, P_2, P_3 at time 0. So, Gantt chart is

P_1	P_2	P_3

0 20 25 28

$$\text{Average waiting time} = \frac{(0+20+25)}{3}$$
$$= 15 \text{ ms}$$

If they arrived in order P_3, P_2, P_1 then, Gantt chart is

P_3	P_2	P_1

0 3 8 28

$$\text{Average waiting time} = \frac{0+3+8}{3} = \frac{11}{3}$$
$$= 3.67 \text{ ms}$$

Notes:

1. Throughput is not that much emphasized.
2. Response time may be high especially if there is a large variance in process execution times.
3. Minimum overhead required.
4. It penalizes short processes, also penalizes I/O bound processes.
5. There is no possibility of starvation.

Advantages

1. Simple and brutally fair.
2. It is suitable for batch systems.

Disadvantages

1. The average waiting time is not minimal.
2. Not suitable for time sharing systems like Unix.
3. ***Convoy effect:*** Short process behind long process results in lower CPU utilization.

Round Robin scheduling

1. It is designed for time sharing system.
2. Similar to FCFS with pre-emption added.
3. Each process gets a small central CPU time (a time slice) usually 10 – 100 ms.
4. After time slice has elapsed and added to the end of the ready queue.
5. The scheduler picks the first process from the ready queue, sets a timer to interrupt after one time quantum and then dispatches the process. One of the following happens.
6. The process may have a CPU burst of less than 1 time quantum. (or)
7. CPU burst time of the currently executing process is longer than one time quantum. In this case, the timer will go off, cause an interrupt, a context switch is then

executed and the process is put at the tail of the ready queue.
8. Average waiting time is quite long.

Performance of Round Robin scheduling

Let us assume that we have onlys one process of 10 time units.

Process time = 10	Quantum	Switch
0 10	12	0
0 6 10	6	1
0 1 2 3 4 5 6 7 8 9 10	1	9

Example 5: Consider the following processes, arrival times and CPU processing requirements with Round Robin scheduling algorithm.

Process	CPU time	Arrival time
A	8	0
B	1	1
C	2	3
D	1	4
E	5	2

What will be the mean turnaround time if time quantum is 4 msec?

Solution:
Plotting the Gantt chart

A	*B*	*E*	*C*	*D*	*A*	*E*

0 4 5 9 11 12 16 17

Turnaround time = Finish time – Arrival time
TAT of $A = 16 - 0 = 16$
$B = 5 - 1 = 4$
$C = 11 - 3 = 8$
$D = 12 - 4 = 8$
$E = 17 - 2 = 15$

$$\text{Mean Turnaround time } = \frac{16+4+8+8+15}{5}$$
$$= 10.2 \text{ msec}$$

1. If there are n processes in the ready queue and time quantum q, then each process gets $\frac{1}{n}$ of the CPU time in chunks of at most q time units at once.
2. No process waits more than $(n - 1)q$ time units until the next time quantum.
3. The performance of Round Robin depends on time slice. If it is larger it is same as FCFS. If q is very small overhead is too high as the number of context switches increases.

Notes:
1. Throughput is low if quantum is too small.
2. Provides good response time for short processes.
3. Minimum overhead.
4. All processes treated fairly.
5. No starvation.

Shortest process next (SPN)

1. This is a non-pre-emptive policy in which the process with the shortest expected processing time is selected next.
2. A short process will jump to the head of the queue past longer jobs.

Example 6: Consider the following process, such that all have arrived at time = 0

Process	Burst time
P_1	5
P_2	9
P_3	6
P_4	3

Find the average waiting time using SPN.

Solution: Gantt chart is

P_4	P_1	P_3	P_2

0 3 8 14 23

$$\text{Average waiting time } = \frac{0+3+8+14}{4} = 6.25 \text{ ms}$$

Notes:

1. Difficulty with this policy is that we need to know or at least estimate the required processing time of each process.
2. High throughput is possible.
3. Provides good response time for short processes.
4. High overhead.
5. It penalizes long processes.
6. There is a possibility of starvation.

Shortest remaining time (SRT)

1. It is a pre-emptive version of SPN.
2. Here the scheduler always chooses the process that has the shortest expected remaining processing time.
3. When a new process joins the ready queue, it may in fact have a shorter remaining time than the currently running process.
4. Accordingly, the scheduler may pre-empt the current process when a new process becomes ready.

Example 7: Consider the following process. Find the average waiting time for SRT.

Process	Arrival time	Burst time
P_1	0	8
P_2	1	4
P_3	2	9
P_4	3	5

Solution: Gantt chart

P_1	P_2	P_4	P_1	P_3

0 1 5 10 17 26

So average waiting time

$$\frac{(10-1)+(1-1)+(17-2)+(5-3)}{4}$$

$$\frac{9+0+15+2}{4}=\frac{26}{4}$$

$$=6.5 \text{ ms}$$

Here P_3 starts at time 17 but the arrival time was at 2. So waiting time of P_3 will be $(17-2)$.

Notes:

1. High throughput.
2. Provides good response time.
3. High overhead.
4. Penalizes long processes.
5. Starvation is possible.

Highest Response Ratio Next (HRRN)

This algorithm works on the principle the executes the job first which has the highest response ratio. We define response ratio as the ratio between turnaround time and response time.

$$\text{Response Ratio } = \frac{(W+S)}{S}, \text{ where}$$

W – Time spend waiting for the processor
S – Service time
This response ratio is also named as normalized turnaround time.

Example 8: Consider 5 processes with their arrival and service times:

Process	Arrival time	Service time
$P1$	0	3
$P2$	2	6
$P3$	4	4
$P4$	6	5
$P5$	8	2

What is the average turnaround time using HRRN technique?

Solution:
Gantt chart

P_1	P_2	P_3	P_5	P_4

0 3 9 13 15 20

Average turnaround time

$$=\frac{3+(6+1)+(4+5)+(5+9)+(2+5)}{5}=\frac{40}{5}=8 \text{ ms}$$

Notes:

1. It is a non-pre-emptive scheduling algorithm.
2. High throughput.
3. Provides good response time.
4. High overhead.
5. Good balance of any type processes.
6. No starvation.

Multilevel feedback queue scheduling

1. In multilevel queue scheduling algorithm, processes are permanently assigned to a queue on entry to the system. Processes cannot move between queues.
2. Processes can move between queues. If a process uses too much CPU time, it will be moved to a lower priority queue.
3. I/O bound and interactive processes are put into higher priority queue.
4. A process that waits too long in a lower priority queue may be moved to a higher priority queue. This form of aging prevents starvation.

In general, a multilevel feedback queue scheduler is defined by following parameters:

1. The number of queues.
2. The scheduling algorithm for each queue.
3. The method used to determine when to upgrade a process to a higher priority queue.
4. The method used to determine when to denote a process to a lower priority queue.
5. The method used to determine in which queue a process will enter when process needs service.

Notes:

1. Pre-emptive at time quantum.
2. Throughput is not that much emphasized.
3. Response time is not that much emphasized.
4. High overhead.
5. Favours I/O bound processes.
6. Starvation is possible.

Exercises

Practice Problem 1

Directions for questions 1 to 19: Select the correct alternative from the given choices.

1. Consider the following processes; find the average waiting time using non-pre-emptive priority scheduling?

Process	Arrival time (ms)	Burst time (ms)	Priority
P_0	0	10	5
P_1	1	6	4
P_2	3	2	2
P_3	5	4	0

(A) 2.36 ms (B) 0.31 ms
(C) 7.75 ms (D) 13.25 ms

2. Consider a set of five processes whose arrival time, CPU times needed are given below.

Process	CPU time (in m sec)	Arrival time (in msec)
P_1	10	5
P_2	5	2
P_3	3	0
P_4	20	4
P_5	2	3

If the CPU scheduling policy is SJF, find the average waiting time (with pre-emption).
(A) 4.8 ms (B) 5.6 ms
(C) 2.16 ms (D) 2.8 ms

3. Consider the following snapshot of a system:

	Allocation				Max				Available			
	A	*B*	*C*	*D*	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*
P_0	0	0	1	2	0	0	1	2	1	5	2	0
P_1	1	0	0	0	1	7	5	0				
P_2	1	3	5	4	2	3	5	6				
P_3	0	6	3	2	0	6	5	2				
P_4	0	0	1	4	0	6	5	6				

Which of the following is true?
(i) The system is in a safe state.
(ii) If a request from process *P*, arrives for (0, 4, 2, 0). Then the request can be granted.
(A) Only (i) (B) Only (ii)
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

4. In a Round Robin scheduling context switch time is 4 units, the average process running time before blocking is 6 units then CPU efficiency is
(A) 0.2 (B) 0.4
(C) 0.6 (D) 0.1

5. A short-term scheduler executes at least once every 20 msec. If it takes 2 msec to decide to execute a process for 2 msec, what is the percentage of CPU time wasted?
(A) 8% (B) 9%
(C) 10% (D) 11%

6. Consider a system which has n resources of the same type. The n resources are shared among three processes *A*, *B*, *C*, which have high demands of 3, 5, 6, respectively. For what value of n will deadlock not occur?
(A) 11 (B) 10
(C) 9 (D) 15

7. A comparative study of scheduling algorithm was performed, the average arrival time in the queue is 5 *m* sec and waiting time of the processes is 10 msec. What is the average queue length of the waiting processes?
(A) 50 (B) 60
(C) 70 (D) 80

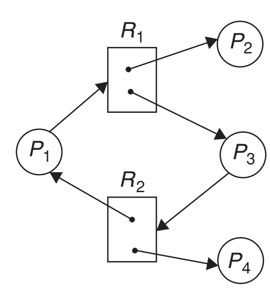
8. A CPU scheduling algorithm determines an order for the execution of its scheduled processes. Given five processes to be scheduled on one processor, how many possible different schedules are there?
(A) 50 (B) 100
(C) 120 (D) 150

9. Consider the following set of jobs (processes) along with their Arrival Time (AT), start time (ST) and Finish Time (FT). Find weighted turnaround time.

Job no.	AT	ST	FT
1	10.0	10.0	10.3
2	10.2	10.3	10.8
3	10.4	10.8	10.9
4	10.5	10.9	11.3
5	10.8	11.3	11.4

(A) 3.04 (B) 2.04
(C) 4.04 (D) 0.56

10. Is the following resource allocation graph in a deadlock state?



(A) Yes (B) No
(C) Not predictable (D) Insufficient data

11. Starvation of longer jobs happens in one of the following scheduling algorithm?
(A) Shortest run remaining time first
(B) Round Robin
(C) Highest response ratio next
(D) First-come first-served

12. Suppose n processes, $P_1 \ldots P_n$ share n identical resource units, which can be reserved and released one at a time. The maximum resource requirement of process P_i is S_i, where $S_i > 0$. Which one of the following is a sufficient condition for ensuring that deadlock does not occur?
(A) $+ i, S_i < m$ (B) $+ i, S_i < n$
(C) $\sum_{i=1}^{n} S_i < (m+n)$ (D) $\sum_{i=1}^{n} S_i < (m*n)$

13. A system with following processes and resources exists. Check the system for safe state and find the safe sequence of processes

	Allocation			Max			Available		
	X	Y	Z	X	Y	Z	X	Y	Z
P_0	0	1	0	7	5	3	3	3	2
P_1	2	0	0	3	2	2			
P_2	3	0	2	9	0	2			
P_3	2	1	1	2	2	2			
P_4	0	0	2	4	3	3			

(A) $< P_1, P_3, P_4, P_2, P_0 >$
(B) $< P_3, P_4, P_2, P_0, P_1 >$
(C) $< P_2, P_4, P_0, P_1, P_3 >$
(D) The system is in unsafe state.

14. Does the below statements be executed concurrently?
$S_1 : a = x + y$
$S_2 : b = z + 1$
(A) Yes (B) No
(C) Not predictable (D) None of the above

15. Let A, B, C be three jobs. Their arrival time and execution time are shown below. By applying monoprogramming and multiprogramming (use Round Robin with time slice 1 unit) approaches, calculate the amount of reduction in turnaround time?

Job	Arrival time	Execution time
A	1	2
B	2	6
C	3	1

(A) 3.33 (B) 4.33
(C) 5.33 (D) 2.33

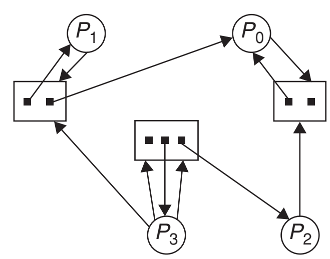
16. Consider a system with three processes A, B, C with 15 tape drivers. Process A has 4 tape drives but requires 14 tape drives.
Process B has 5 tape drives but requires 9 tape drives.
Process C has 3 tape drives but requires 7 tape drives.
Among the following processes which will enter the deadlock state?
(A) A, B only (B) A, B, C
(C) A, C only (D) B, C only

17. Assume that the following jobs are to be executed on a uniprocessor system:

Job id	CPU burst time
P	4
Q	1
R	8
S	1
T	2

The jobs are assumed to have arrived at 0, and in the order P, Q, R, S and T. Calculate the departure with time slice (completion time) for job P if scheduling is Round Robin with time slices of 1 unit (slice).
(A) 4 (B) 10
(C) 11 (D) 12

Common data for questions 18 and 19: Consider the following Resource Allocation Graph:



18. The system is in a deadlock state. This remark is:
(A) True (B) False
(C) Impossible to determine
(D) Unpredictable

19. Which one is a safe sequence?
(A) P_0, P_1, P_2, P_3 (B) P_1, P_0, P_2, P_3
(C) P_2, P_0, P_1, P_3 (D) Both (A) and (C)

Practice Problem 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Let there be five processes (P_1 to P_5) and three resource types A, B, C.
Resource type A has 10 instances,
Resource type B has 5 instances,
Resource type C has 7 instances.
Suppose that at time T_0, the following snapshot of the system has been taken.

	Allocation			Max			Available		
P_1	0	1	0	7	5	3	3	3	2
P_2	2	0	0	3	2	2			
P_3	3	0	2	9	0	2			
P_4	2	1	1	2	8	2			
P_5	0	0	2	4	3	3			

Which of the following statement is true?

(A) The system is in safe state.
(B) The system has process initiation denial problem.
(C) No process causes initiation denial
(D) Both (A) and (C)

2. Consider three CPU intensive processes, which require 20, 30 and 40 time units and arrive at times 0, 2 and 4, respectively. How many context switches are needed if the operating system implements a shortest remaining time first scheduling algorithm? Do not count the context switches at time zero and at the end.
(A) 0 (B) 1
(C) 2 (D) 3

3. Consider the set of processes P_1 to P_5 with the following CPU burst times. Find the average turnaround time using shortest remaining time first.

Process	CPU burst time	Arrival time
P_1	3	0
P_2	6	2
P_3	4	4
P_4	5	6
P_5	2	8

(A) 1.3 ms (B) 3.5 ms
(C) 5.8 ms (D) 7.2 ms

4. All processes are arriving at time 0, find the average waiting time.

Process	Burst time	Priority
P_1	10	3
P_2	1	1
P_3	2	3
P_4	1	4
P_5	5	2

(A) 8.2 ms (B) 4.1 ms
(C) 2.0 ms (D) 1.3 ms

5. Consider a set of three processes P_1, P_2 and P_3 with their priorities and arrival times as given below.

Process	Burst time	Priority	Arrival time
P_1	10	3	0
P_2	5	2	1
P_3	2	1(highest)	2

Find the average waiting time.
(A) 1 ms (B) 2 ms
(C) 3 ms (D) 4 ms

6. The portion of the process scheduler in an OS that dispatches processes is concerned with:
(A) assigning ready processes to the CPU
(B) activating suspended I/O bound processes
(C) temporarily suspending processes when the CPU load is too great.
(D) All the above

7. In a time-sharing OS, when the time slot given to a process is completed, the process goes from the running state to the
(A) blocked state (B) ready state
(C) suspended state (D) terminated state

8. On a system with n CPUs, what is the maximum number of processes that can be in the ready state?
(A) n processes
(B) No process can be in ready state
(C) There is no limit to the number of processes in the ready state
(D) None of the above

9. Consider a set of n tasks with known runtimes $r_1, r_2, \ldots, r_n$ to be run on a uniprocessor machine. Which of the following processor scheduling algorithms will result in the maximum throughput?
(A) Round Robin
(B) SJF
(C) Highest response ratio next
(D) First-come first-served

10. Match the following:

A	Critical region	I	Hoare's monitor
B	Wait/signal	II	Mutual exclusion
C	Working set	III	Principle of locality
D	Deadlock	IV	Circular wait

(A) A – II, B – I, C – III, D – IV
(B) A – I, B – II, C – III, D – IV
(C) A – II, B – I, C – IV, D – III
(D) A – I, B – II, C – IV, D – III

11. Consider three processes A, B, C to be scheduled as per SRT algorithm. A is known to be scheduled first and when A has been running for 7 units of time, C has arrived. C has run for 1 unit of time when B has arrived and completed running in 2 units of time, what could be the minimum time of executions for A and C?
(A) 11 and 4 (B) 11 and 3
(C) 12 and 3 (D) 12 and 4

12. Select the correct statements from below:
(i) SRT and SPN can cause starvation for larger processes.
(ii) FCFS can potentially block small processes in favour of much larger processes.
(iii) Round Robin algorithm gives fair treatment to all the processes.
(iv) FCFS is a pre-emptive algorithm.
(v) The throughput for Round Robin is high even for small time slices.
(A) (i), (ii), (iii) (B) (i), (ii), (iv)
(C) (iii), (iv), (v) (D) (i), (ii), (iii), (v)

13. Three processes share four resource units that can be reserved and released only one at a time. Each process needs a maximum of two units. Then
(A) there is a possibility of deadlock
(B) no deadlock will occur

(C) there will be a circular wait
(D) nothing can be predicted about dead lock.

14. N processes share M resource units that can be reserved and released only one at a time. The maximum need of each process does not exceed M and the sum of all maximum needs is less than $M + N$, then
(A) there is a possibility of deadlock
(B) there will be no deadlock
(C) circular wait exists
(D) nothing can be predicted about dead lock.

15. Which of the following scheduling algorithms could result in starvation?
(A) First-come, first-served
(B) Shortest job first
(C) Round Robin
(D) Highest response ratio next

Previous Years' Questions

1. Consider three processes (process id 0, 1, 2, respectively) with compute bursts 2, 4 and 8 time units. All processes arrive at time zero. Consider the longest remaining time first (LRTF) scheduling algorithm. In LRTF ties are broken by giving priority to the process with the lowest process id. The average turnaround time is: **[2006]**
(A) 13 units (B) 14 units
(C) 15 units (D) 16 units

2. Consider three processes, all arriving at zero, with total execution time of 10, 20 and 30 units, respectively. Each process spends the first 20% of execution time doing I/O, the next 70% of time doing computation, and the last 10% of time doing I/O again. The operating system uses a shortest remaining compute time first scheduling algorithm and schedules a new process either when the running process gets blocked on I/O or when the running process finishes its compute burst. Assume that all I/O operations can be overlapped as much as possible. For what percentage of time does the CPU remain idle? **[2006]**
(A) 0% (B) 10.6%
(C) 30.0% (D) 89.4%

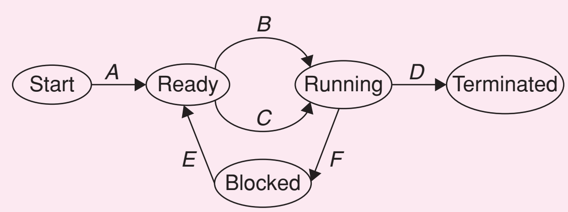
3. A single processor system has three resource types X, Y and Z, which are shared by three processes. There are five units of each resource type. Consider the following scenario, where the column *alloc* denotes the number of units of each resource type allocated to each process, and the column *request* denotes the number of units of each resource type requested by a process in order to complete execution. Which of these processes will finish *last*?

	alloc			request		
	X	Y	Z	X	Y	Z
P_0	1	2	1	1	0	3
P_1	2	0	1	0	1	2
P_2	2	2	1	1	2	0

(A) P_0 **[2007]**
(B) P_1
(C) P_2
(D) None of the above, since the system is in a deadlock.

4. Which of the following is *not* true of deadlock prevention and deadlock avoidance schemes? **[2008]**
(A) In deadlock prevention, the request for resources is always granted if the resulting state is safe
(B) In deadlock avoidance, the request for resources is always granted if the result state is safe
(C) Deadlock avoidance is less restrictive than deadlock prevention
(D) Deadlock avoidance requires knowledge of resource requirements *a priori*

5. In the following process state transition diagram for a uniprocessor system, assume that there are always some processes in the ready state: **[2009]**



Now consider the following statements:
I. If a process makes a transition D, it would result in another process making transition A immediately.
II. A process P_2 in blocked state can make transition E, while another process P_1 is in running state.
III. The OS uses pre-emptive scheduling.
IV. The OS uses non-pre-emptive scheduling.
Which of the above statements are true?
(A) I and II (B) I and III
(C) II and III (D) II and IV

6. Which of the following statements are true?
I. Shortest remaining time first scheduling may cause starvation
II. Pre-emptive scheduling may cause starvation
III. Round Robin is better than FCFS in terms of response time **[2010]**
(A) I only (B) I and III only
(C) II and III only (D) I, II and III

7. A system has n resources $R_0, \ldots, R_{n-1}$, and k processes $P_0, \ldots P_{k-1}$. The implementation of the resource request logic of each process P_i, is as follows:

```
if (i% 2 = = 0) {
if (i<n) request R_i;
if (i+2<n) request R_i+2;
}
else {
if (i<n) request R_n-i;
if (i+2<n) request R_n-i-2;
}
```

In which one of the following situations is a deadlock possible? **[2010]**

(A) $n = 40, k = 26$ (B) $n = 21, k = 12$
(C) $n = 20, k = 10$ (D) $n = 41, k = 19$

8. Consider the following table of arrival time and burst time for three processes P_0, P_1 and P_2. **[2011]**

Process	Arrival time	Burst time
P_0	0 ms	9 ms
P_1	1 ms	4 ms
P_2	2 ms	9 ms

The pre-emptive shortest job first scheduling algorithm is used. Scheduling is carried out only at arrival or completion of processes. What is the average waiting time for the three processes?

(A) 5.0 ms (B) 4.33 ms
(C) 6.33 ms (D) 7.33 ms

9. Consider the three processes, P_1, P_2 and P_3 as shown in the table. **[2012]**

Process	Arrival time	Time units required
P_1	0	5
P_2	1	7
P_3	3	4

The completion order of the three processes under the policies FCFS and RR_2 (Round Robin scheduling with CPU quantum of 2 time units) are

(A) **FCFS:** P_1, P_2, P_3 **RR2:** P_1, P_2, P_3
(B) **FCFS:** P_1, P_3, P_2 **RR2:** P_1, P_3, P_2
(C) **FCFS:** P_1, P_2, P_3 **RR2:** P_1, P_3, P_2
(D) **FCFS:** P_1, P_3, P_2 **RR2:** P_1, P_2, P_3

10. A scheduling algorithm assigns priority proportional to the waiting time of a process. Every process starts with priority zero (the lowest priority). The scheduler re-evaluates the process priorities every T time units and decides the next process to schedule. Which one of the following is TRUE if the processes have no I/O operations and all arrive at time zero? **[2013]**

(A) This algorithm is equivalent to the first-come-first-serve algorithm.
(B) This algorithm is equivalent to the Round Robin algorithm.
(C) This algorithm is equivalent to the shortest-job-first algorithm.
(D) This algorithm is equivalent to the shortest-remaining-time-first algorithm.

11. An operating system uses the *Banker's algorithm* for deadlock avoidance when managing the allocation of three resource types X, Y and Z to three processes P_0, P_1, and P_2. The table given below presents the current system state. Here, the *allocation* matrix shows the current number of resources of each type allocated to each process and the *Max* matrix shows the maximum number of resources of each type required by each process during its execution. **[2014]**

	Allocation			Max		
	X	*Y*	*Z*	*X*	*Y*	*Z*
P_0	0	0	1	8	4	3
P_1	3	2	0	6	2	0
P_2	2	1	1	3	3	3

There are three units of type X, two units of type Y and two units of type Z still available. The system is currently in a *safe* state. Consider the following independent requests for additional resources in the current state:

REQ1: P_0 requests 0 units of X, 0 units of Y and two units of Z

REQ 2: P_1 requests two units of X, 0 units of Y and 0 units of Z

Which one of the following is true?

(A) Only REQ1 can be permitted
(B) Only REQ2 can be permitted
(C) Both REQ1 and REQ2 can be permitted
(D) Neither REQ1 nor REQ2 can be permitted

12. Consider the following set of processes that need to be scheduled on a single CPU. All the times are given in milliseconds.

Process name	Arrival time	Execution time
A	0	6
B	3	2
C	5	4
D	7	6
E	10	3

Using the *shortest remaining time first* scheduling algorithm, the average process turnaround time (in msec) is ——. **[2014]**

13. Three processes A, B and C each execute a loop of 100 iterations. In each iteration of the loop, a process performs a single computation that requires t_c CPU milliseconds and then initiates a single I/O operation that lasts for t_{io} milliseconds. It is assumed that the computer where the processes execute has sufficient number of I/O devices and the OS of the computer assigns different I/O devices to each process. Also, the scheduling overhead of the OS is negligible. The processes have the following characteristics:

Process id	t_c	t_{io}
A	100 ms	500 ms
B	350 ms	500 ms
C	200 ms	500 ms

The processes *A*, *B* and *C* are started at times 0, 5 and 10 milliseconds, respectively, in a pure time sharing system (Round Robin scheduling) that uses a time slice of 50 milliseconds. The time in milliseconds at which process *C* would *complete* its first I/O operation is ——— **[2014]**

14. A system contains three programs and each requires three tape units for its operation. The minimum number of tape units which the system must have such that deadlocks never arise is ———. **[2014]**

15. An operating system uses *shortest remaining time first* scheduling algorithm for pre-emptive scheduling of processes. Consider the following set of processes with their arrival times and CPU burst times (in milliseconds):

Process	Arrival time	Burst time
P_1	0	12
P_2	2	4
P_3	3	6
P_4	8	5

The average waiting time (in milliseconds) of the processes is _____. **[2014]**

16. Consider a uniprocessor system executing three tasks T_1, T_2 and T_3, each of which is composed of an infinite sequence of jobs (or instances) which arrive periodically at intervals of 3, 7 and 20 milliseconds, respectively. The priority of each task is the inverse of its period, and the available tasks are scheduled in order of priority, with the highest priority task schedule first. Each instance of T_1, T_2 and T_3 requires an execution time of 1, 2 and 4 milliseconds, respectively. Given that all tasks initially arrive at the beginning of the 1st millisecond and task preemptions are allowed, the first instance of T_3 completes its execution at the end of ________ milliseconds. **[2015]**

17. A system has 6 identical resources and *N* processes competing for them. Each process can request atmost 2 resources. Which one of the following values of *N* could lead to a deadlock? **[2015]**

(A) 1 (B) 2
(C) 3 (D) 4

18. The maximum number of processes that can be in Ready state for a computer system with n CPUs is **[2015]**

(A) n (B) n^2
(C) 2^n (D) Independent of n

19. Consider the following policies for preventing deadlock in a system with mutually exclusive resources. **[2015]**

(1) Processes should acquire all their resources at the beginning of execution. If any resource is not available, all resources acquired so far are released.

(2) The resources are numbered uniquely, and processes are allowed to request for resources only in increasing resource numbers.

(3) The resources are numbered uniquely, and processes are allowed to request for resources only in decreasing resource numbers.

(4) The resources are numbered uniquely. A process is allowed to request only for a resource with resource number larger than its currently held resources.

Which of the above policies can be used for preventing deadlock?

(A) Any one of 1 and 3 but not 2 or 4
(B) Any one of 1, 3 and 4 but not 2
(C) Any one of 2 and 3 but not 1 or 4
(D) Any one of 1, 2, 3 and 4

20. For the processes listed in the following table, which of the following scheduling schemes will give the lowest average turnaround time? **[2015]**

Process	Arrival Time	Processing Time
A	0	3
B	1	6
C	4	4
D	6	2

(A) First Come First Serve
(B) Non-preemptive Shortest Job First
(C) Shortest Remaining Time
(D) Round Robin with Quantum value two

21. Consider an arbitrary set of CPU - bound processes with unequal CPU burst lengths submitted at the same time to a computer system. Which one of the following process scheduling algorithms would minimize the average waiting time in the ready queue? **[2016]**

(A) Shortest remaining time first
(B) Round-robin with time quantum less than the shortest CPU burst
(C) Uniform random
(D) Highest priority first with priority proportional to CPU burst length

22. Consider the following processes, with the arrival time and the length of the CPU burst given in milliseconds. The scheduling algorithm used is preemptive shortest remaining - time first.

Process	Arrival Time	Burst Time
P_1	0	10
P_2	3	6

P_3	7	1
P_4	8	3

The average turn around time of these processes is ____ milliseconds. **[2016]**

23. Consider the following CPU processes with arrival times (in milliseconds) and length of CPU bursts (in milliseconds) as given below :

Process	Arrival time	Burst time
P1	0	7
P2	3	3
P3	5	5
P4	6	2

If the pre-emptive shortest remaining time first scheduling algorithm is used to schedule the processes, then the average waiting time across all processes is milliseconds. **[2017]**

24. A system shares 9 tape drives. The current allocation and maximum requirement of tape drives for three processes are shown below:

Process	Current Allocation	Maximum Requirement
P1	3	7
P2	1	6
P3	3	5

Which of the following best describes current state of the system? **[2017]**

(A) Safe. Deadlocked
(B) Safe. Not Deadlocked
(C) Not Safe. Deadlocked
(D) Not Safe, Not Deadlocked

25. Consider the set of processes with arrival time (in milliseconds). CPU burst time (in milliseconds). and priority (0 is the highest priority) shown below. None of the processes have I/O burst time.

Process	Arrival Time	Burst Time	Priority
P_1	0	11	2
P_2	5	28	0
P_3	12	2	3
P_4	2	10	1
P_5	9	16	4

The average waiting time (in milliseconds) of all the processes using preemptive priority scheduling algorithm is________. **[2017]**

26. Consider a system with 3 processes that share 4 instances of the same resource type. Each process can request a maximum of K instances. Resource instances can be requested and released only one at a time. The largest value of K that will always avoid deadlock is ______. **[2018]**

27. In a system, there are three types of resources: E, F and G. Four processes P_0, P_1, P_2 and P_3 execute concurrently. At the outset, the processes have declared their maximum resource requirements using a matrix named Max as given below. For example, Max[P_2, F] is the maximum number of instances of F that P_2 would require. The number of instances of the resources allocated to the various processes at any given state is given by a matrix named Allocation.

Consider a state of the system with the Allocation matrix as shown below, and in which 3 instances of E and 3 instances of F are the only resources available.

Allocation			
	E	F	G
P_0	1	0	1
P_1	1	1	2
P_2	1	0	3
P_3	2	0	0

Max			
	E	F	G
P_0	4	3	1
P_1	2	1	4
P_2	1	3	3
P_3	5	4	1

From the perspective of deadlock avoidance, which one of the following is true? **[2018]**

(A) The system is in safe state.
(B) The system is not in safe state, but would be safe if one more instance of E were available.
(C) The system is not in safe state, but would be safe if one more instance of F were available.
(D) The system is not in safe state, but would be safe if one more instance of G were available.

Answer Keys

Exercises

Practice Problem 1

1. C	**2.** A	**3.** C	**4.** C	**5.** B	**6.** D	**7.** A	**8.** C	**9.** A	**10.** B
11. A	**12.** C	**13.** A	**14.** A	**15.** B	**16.** B	**17.** C	**18.** B	**19.** D	

Practice Problem 2

1. B	**2.** C	**3.** D	**4.** A	**5.** C	**6.** A	**7.** B	**8.** C	**9.** B	**10.** A
11. D	**12.** A	**13.** B	**14.** B	**15.** B					

Previous Years' Questions

1. A	**2.** B	**3.** C	**4.** A	**5.** C	**6.** D	**7.** B	**8.** A	**9.** C	**10.** B
11. B	**12.** 7.2	**13.** 1000	**14.** 7	**15.** 5.5	**16.** 12	**17.** none	**18.** D	**19.** D	**20.** C
21. A	**22.** 8.2 to 8.3		**23.** 3	**24.** B	**25.** 29	**26.** 2	**27.** A		

Chapter 4

Memory Management and Virtual Memory

LEARNING OBJECTIVES

- *Basic concepts*
- *Memory management requirements*
- *Relocation and memory mapping techniques*
- *Placement algorithm*
- *Dynamic partitioning*
- *Placement algorithm*
- *Buddy system*
- *Non-contiguous storage allocation methods*
- *Paging*
- *Segmentation*
- *Page table structure*
- *Hierarchical page table*
- *Inverted page table*
- *Address translation in a segmentation system*

Basic Concepts

Uniprogramming system Main memory is divided into two parts as follows:

1. Operating system (OS) part
2. Program part (which is currently being executed)

Multiprogramming system Here the user part of memory must be further subdivided to accommodate multiple processes.

The task of subdivision is carried out dynamically by the OS and is known as memory management.

Memory Hierarchy

The triangle in Figure 1 gives the hierarchy of memory. The memory hierarchy shows the performance issues.

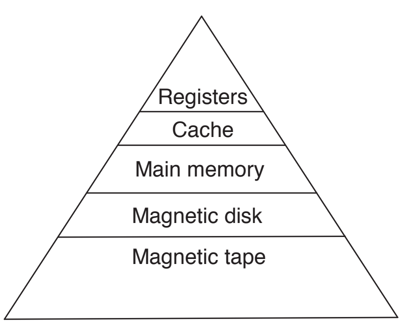


Figure 1 Memory hierarchy.

The memory hierarchy has different types of storage system in computers which are arranged in hierarchy, with respect to speed and cost.

If one moves down the hierarchy, access time increases, the cost per bit decreases, the memory capacity increases and memory access frequency by the processor decreases.

The registers, cache and main memory are volatile, whereas magnetic disc and magnetic tapes are non-volatile storage devices.

Memory Management Requirements

Memory management requirements are as follows:

1. Relocation
2. Protection
3. Sharing
4. Logical organization
5. Physical organization

Relocation

1. The role of relocation, the ability to execute processes independently from their physical location in memory, is central for memory management.
2. In a general purpose multiprogramming environment, a program cannot know in advance what processes will be running in memory when it is executed, nor how much memory the system has available for it, nor where it is located.
3. Hence program relocation is required such that a program must be compiled and linked in such a way that it can later be loaded starting from an unpredictable address in memory, an address that can even change during the execution of the process itself, if any swapping occurs.

4. The basic requirement for program relocation is that all the references to memory it makes during execution must not contain absolute (physical) address of memory cells, but must be generated relatively, that is, as a distance measured in number of contiguous memory words, from some known point.

Protection

Each process should be protected against unwanted interference by other processes, whether accidental or intentional.

Thus, programs in other processes should not be able to reference memory locations in a process for reading or writing purpose without permission.

Sharing

1. Any protection mechanism must have the flexibility to allow several processes to access the same portion of main memory.
2. Processes that are cooperating on some task may need to share access to the same data structure.
3. The memory management system must therefore allow controlled access to shared areas of memory without compromising essential protection.

Logical Organization

Main memory in a computer system is organized as a linear address space consisting of a sequence of bytes or words. But most of the programs are organized into modules.

If the OS and hardware can effectively deal with user programs and data in the form of modules, then there are some advantages.

1. Modules can be written and compiled independently.
2. Different degrees of protection can be given to different modules.
3. It is better to share modules among processes.

Physical Organization

Computer memory is organized in two levels:

1. Main memory
2. Secondary memory

The flow of information between these two modules is a major concern. If this is assigned to user, then there are some problems:

1. ***Overlaying*** may be possible. In overlaying concept, the various modules of a program can be assigned to the same region of memory, which causes wastage of programmer time.
2. The programmer does not know at the time of coding how much space will be available or where that space will be. So it must be handled by the system.

Address binding Addresses may be represented in different ways during the program execution:

1. Addresses in source program are generally symbolic.
2. A complier will typically bind these symbolic addresses to relocatable addresses.
3. The linkage editor or loader will in turn bind the relocatable addresses to absolute addresses.

So the binding of instructions and data to memory addresses can be done at any step along the way:

1. Compile time
2. Load time
3. Execution time

Compile time If you know at compile time where the process will reside in memory, then absolute code can be generated.

Load time If it is not known at compile time where the process will reside in memory, then the compiler must generate relocatable code.

Execution time If the process can be moved during its execution from one memory segment to another, then binding must be delayed until run time.

Logical versus Physical Address Apace

Logical address An address generated by the CPU is commonly referred to as a logical address.

Physical address An address seen by the memory unit, that is, one loaded into MAR is referred as a *physical address*.

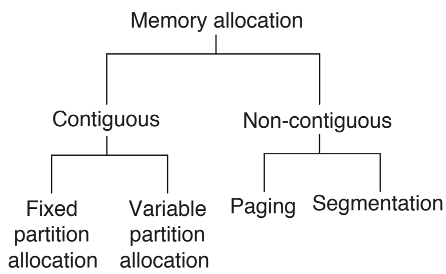
Notes:

1. Logical and physical addresses differ in execution time address-binding scheme.
2. Logical and physical addresses are same in compile time and load time address-binding schemes.
3. The run-time mapping from logical to physical address is done by a hardware device called the *memory management unit* (MMU).

Memory Mapping Techniques

The principle operation of memory management is to bring processes into main memory for execution by the processor. Let's now discuss various memory management techniques as follows:

1. Fixed partitioning
2. Dynamic partitioning
3. Simple paging
4. Simple segmentation
5. Virtual memory paging
6. Virtual memory segmentation.



Contiguous Storage Allocation

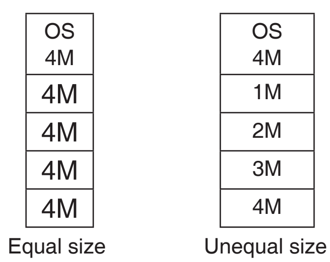
In this allocation, a memory resident program occupies a single contiguous block of memory.

Fixed/Static Partitioning

The main memory is divided into a number of static partitions at system generation time. Moreover, a process may be loaded into a partition of equal or greater size.

Partition size Two alternatives of fixed partition are as follows:

1. Equal-size partitions
2. Unequal-size partitions



Equal-size partitions: Any process whose size is less than or equal to the partition size can be loaded into any available partition.

Two problems with this technique are as follows:

1. A program may be too big to fit into a partition. Use overlaying to solve this problem.
2. Main memory utilization is extremely inefficient, as there is a possibility of internal fragmentation.

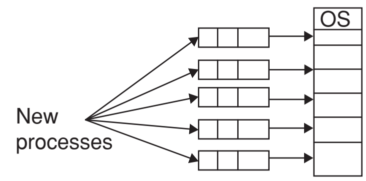
In *internal fragmentation*, there is a space wastage internal to a partition due to the fact that the block of data loaded is smaller than the partition.

Unequal-sized partition: Both the problems with equal-size partition can be lessened by using unequal-sized partitions.

Placement algorithm: With equal-size partitions, the placement of processes in memory is trivial. As all partitions are of equal size, it doesn't matter which partition is used.

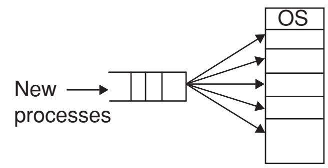
With Unequal-size partitions, there are two possible ways to assign processes to partitions:

1. Assign each process to the smallest partition within which it will fit.



- Figure shows one process queue for partition.
- Minimized internal fragmentation.
- Possibility of unused partitions.

2. Employ a single queue for all processes.



- When it is time to load a process into main memory, the smallest available partition that will hold the process is selected.

Advantages

1. Simple to implement.
2. Little OS overhead.

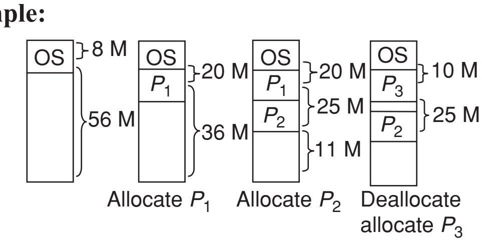
Disadvantages

1. Inefficient use of memory due to internal fragmentation.
2. Maximum number of active processes is fixed.

Dynamic Partitioning

With dynamic partitioning, the partitions are of variable length and number. When a process is brought into main memory, it is allocated exactly as much memory as it requires and no more.

Example:



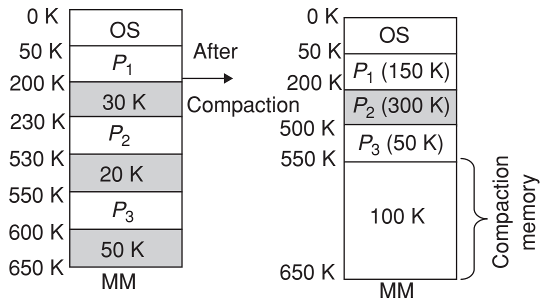
- This method starts out well, but eventually it leads to a situation in which there are a lot of small holes in memory.
- As time goes on, memory becomes more and more fragmented and memory utilization declines. This phenomenon is referred to as *external fragmentation*.

It indicates the memory that is external to all partitions becomes increasingly fragmented.

Compaction

Compaction is a technique by which the resident program are relocated in such a way that the small chunks of free

memory are made contiguous to each other and clubbed together into a single free partition that may be big enough to accommodate more programs.



It should be noted that compaction involves dynamic relocation of a program.

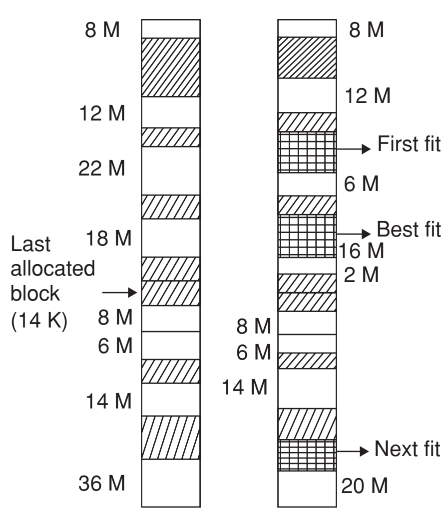
Placement algorithm

Memory compaction is a time-consuming process, and hence the OS uses some placement algorithms.

The three most common strategies to allocate free partitions to the new processes are as follows:

1. *First fit*: Allocate the first free partition, large enough to accommodate the process. IT executes faster.
2. *Best fit*: Allocate the smallest free partition that meets the requirement of the process. It achieves higher utilization of memory by searching smallest free partition.
3. *Worst fit*: Allocate the largest available partition to the newly entered process in the system.
4. *Next fit*: Start from current location in the list.

Example: Consider the following memory configuration after a number of placement and swapping out operations. The last block that was used was a 22 MB block from which a 14 MB partition was created. The figure (b) shows 16 MB allocation request.



(a) Before allocation (b) After allocation

Allocated block
Free block
Possible new allocation

Advantages of dynamic partitioning

1. Memory utilization is generally better as partitions are created dynamically.
2. No internal fragmentation as partitions are changed dynamically.
3. The process of merging adjacent holes to form a single larger hole is called coalescing.

Disadvantages

1. Lots of OS space, time, complex memory management algorithms are required.
2. Compaction time is very high.

Buddy system: Both fixed and dynamic partitioning schemes have drawbacks.

In Buddy system, memory blocks are available of size 2^K words, $L \le K \le U$, where,

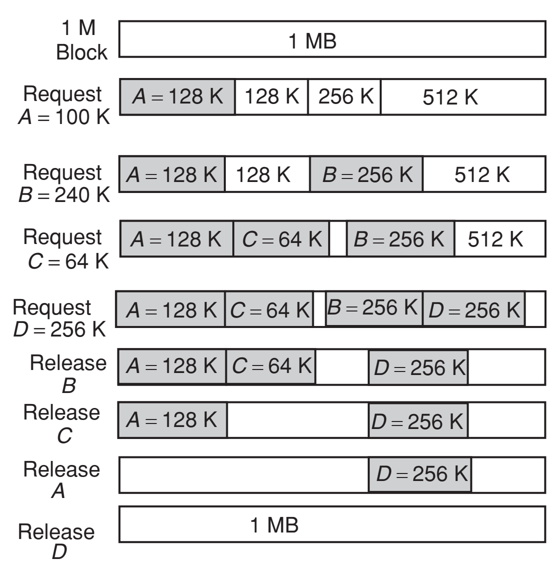
2^L = Smallest-size block that is allocated.

2^U = Largest-size block that is allocated.

Generally, 2^U is the size of the entire memory available for allocation. If a request of size 'S' such that $2^{U-1} < S \le 2^U$ is made, then the entire block is allocated. Otherwise the block is split into two equal buddies of size 2^{U-1}. If $2^{U-2} < S \le 2^{U-1}$, then the request is allocated to one of the two buddies. Otherwise, one of the buddies is split in half again. This process continues until the smallest block greater than or equal to 'S' is generated and allocated to the request.

1. At any time, the buddy system maintains a list of holes of each size 2^i.
2. A hole may be removed from the $(i + 1)$ list by splitting it in half to create two buddies of size 2^i in the 'i' list.
3. Whenever a pair of buddies on the i list both become unallocated, they are removed from the list and coalesced into a single block on the $(i + 1)$ list.

Example:



Non-contiguous Storage Allocation Methods

Paging

In simple paging, the main memory is divided into a number of equal-size frames. Each process is divided into a number of equal-size frames. The chunks of processes are referred as *pages*. A process is loaded by loading all of its pages into available, not necessarily contiguous frames.

Example: At a point in time, some of the frames in memory are in use and some are free. A list of free frames is maintained by the OS.

Consider four processes with their pages as displayed below:

Process P	Process Q	Process R	Process S
P.0	Q.0	R.0	S.0
P.1	Q.1	R.1	S.1
P.2	Q.2	R.2	S.2
P.3		R.3	S.3
			S.4

Let the main memory consist of 15 frames:
Main memory

		Load P
0		P.0
1		P.1
2		P.2
3		P.3
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		

	Load Q	Load R	Swap Q	Load S
0	P.0	P.0	P.0	P.0
1	P.1	P.1	P.1	P.1
2	P.2	P.2	P.2	P.2
3	P.3	P.3	P.3	P.3
4	Q.0	Q.0		S.0
5	Q.1	Q.1		S.1
6	Q.2	Q.2		S.2
7		R.0	R.0	R.0
8		R.1	R.1	R.1
9		R.2	R.2	R.2
10		R.3	R.3	R.3
11				S.3
12				S.4
13				
14				

1. The OS maintains a page table for each process.
2. The page table shows the frame location for each page of the process.
3. Within a program, each logical address consists of a page number and an offset with in the page.
4. Here a logical address is the location of a word relative to the beginning of the program; the processor translates that into a physical address.
5. For this, the processor must know the following details:

- *Logical address:* Consists page number and offset.
- *Page table:* Used to produce physical address (Frame number, offset).

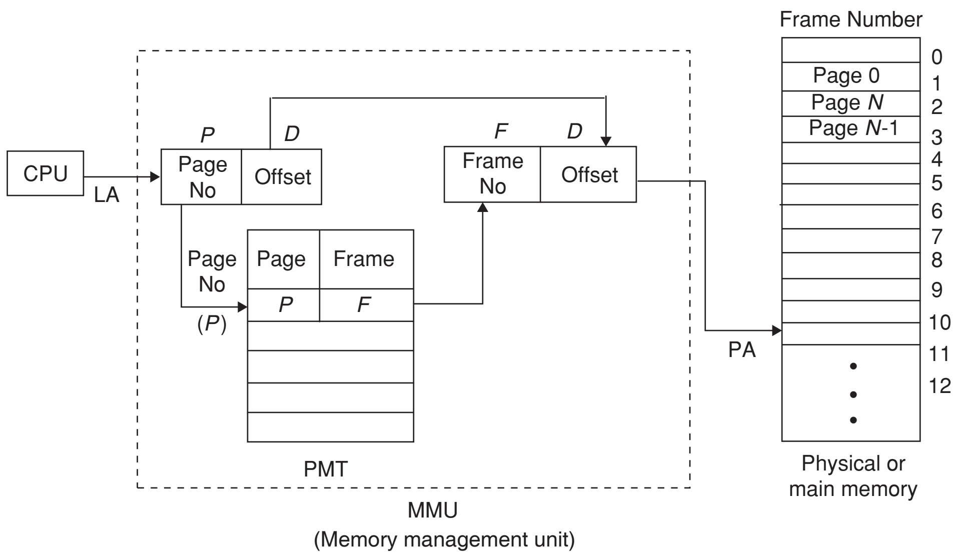
In the previous example, the page tables of each process will be:

	Process P page table	Process Q page table	Process R page table
0	0	–	7
1	1	–	8
2	2	–	9
3	3		10

	Process S page table
0	4
1	5
2	6
3	11
4	12

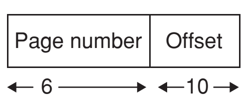
Free frame list.
13
14

Address mapping in paging



Example: Let 16-bit address is used by the processor and the page size is 1 KB.

Then number of pages in main memory $= \frac{2^{16}}{2^{10}} = 2^6$.
∴ Page number = 6-bits
Offset = 10-bits



For the relative address 1502 =
0000010111011110
The page number = 000001 = 1
and offset = 0111011110 = 478, that is, the physical location will be an offset $(478)_{10}$ on page 1.

Let the page 1 is present in frame 6. Then the physical address will be 0001100111011110.

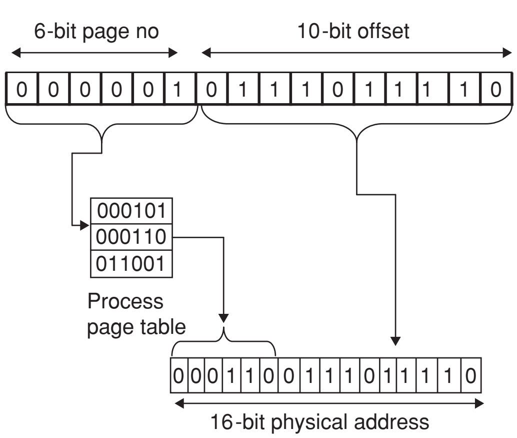


Figure 2 Paging.

Steps for address translation

1. Extract the page number from the logical address.
2. Use the page number as an index into the process page table to find the frame number.
3. The physical address will be constructed by appending the frame number to the offset.

Advantage

There is no external fragmentation.

Disadvantage

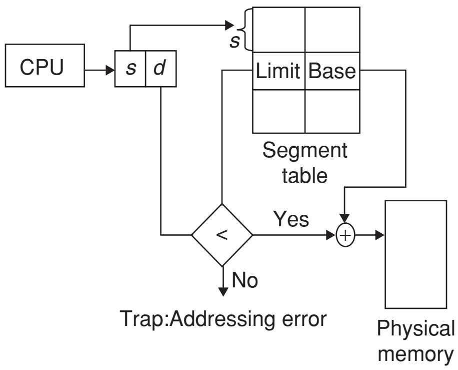
There is a small amount of internal fragmentation.

Segmentation

1. Each process is divided into a number of unequal-size segments.
2. A process is loaded by loading all of its segments into dynamic partitions that need not be contiguous.
3. The logical address using segmentation consists of two parts: *segment number* and an *offset*.
4. The principle inconvenience of segmentation is that the programmer must be aware of the maximum segment size limitation.
5. It makes use of a segment table for each process and a list of free blocks of main memory.
6. Each segment table entry would have to give the starting address in main memory of the corresponding segment. It also contains the length of the segment.
7. Steps for address translation:
   - Extract the segment number from the logical address.
   - Use the segment number as an index into the process segment table to find the starting physical address of the segment.

- Compare the offset to the length of the segment. If the offset is greater than or equal to the length, the address is invalid.
- The desired physical address is the sum of the starting physical address of the segment plus the offset.

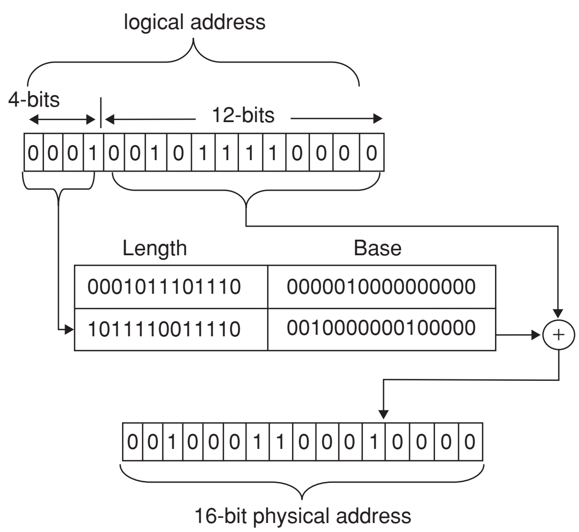
Hardware support for segmentation



Example: Consider the logical address 0001001011110000.

Let the segment number consists of 4-bits. Then segment number = 0001 = 1

Offset = 001011110000 = 752.



Advantages

1. No internal fragmentation
2. Improved memory utilization.
3. Reduced overhead compared to dynamic partitioning.

Disadvantage

1. External fragmentation.

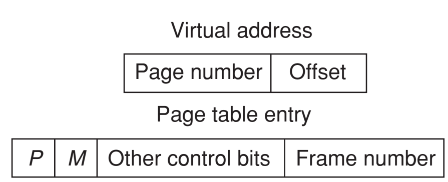
Virtual Memory

1. In simple paging/segmentation, it is not necessary that all of the pages or all of the segments of a process be in main memory during execution.
2. Suppose that it is time to bring new process into memory. The OS begins by bringing in only one or a few pages to include the initial program page and initial data page to which those instructions refer.
3. The portion of a process that is actually in main memory at any time is defined to be the resident set of the process.
4. If the processor encounters a logical address that is not in main memory, it generates an interrupt indicating a memory access fault.
5. Then the OS brings the required page to the main memory.
6. With virtual memory,
   - More processes may be maintained in main memory.
   - A process may be larger than all of main memory, then also it will be executed.
7. Virtual memory is a storage allocation scheme in which secondary memory can be addressed as though it were part of main memory.
8. ***Thrashing:*** When the OS brings one page in, it must throw another out. If it throws out a page just before it is used, then it will just have to go get that piece again almost immediately. Too much of this leads to a condition known as *thrashing*.

   If the system spends most of its time in swapping rather than executing instructions then that situation refers to thrashing.
9. Principle of locality suggests that a virtual memory scheme may work.
10. Virtual memory will be practical and effective if
    - There is a hardware support for paging/segmentation.
    - The OS includes software for managing the movement of pages/segments.

Paging with Virtual Memory

1. The main difference between paging and virtual memory paging is that in virtual memory paging concept, not all pages of a process need to be in main memory frames for the process to run. Pages may be read in as needed.
2. A page table is also needed for a virtual memory scheme based on paging. Also it is typical to associate a unique page table with each process.
3. The virtual address and page table entries for virtual memory paging are shown below:



P: Present bit. This bit specifies whether that particular page is present in main memory or not.

M: Modified bit. This bit indicates whether the contents of the corresponding page have been altered since the page was last loaded into main memory.

Page table structure

1. To read a word from memory, translate the virtual or logical address consisting of page number and offset into physical address consisting of frame number and offset, using a page table.
2. Page table must be stored in main memory to access it.

Hardware implementation for virtual memory paging

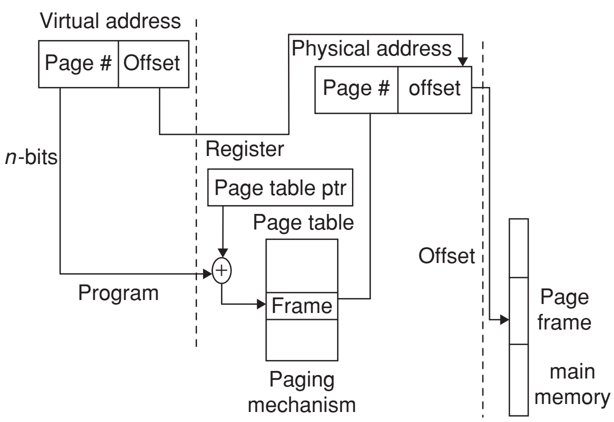
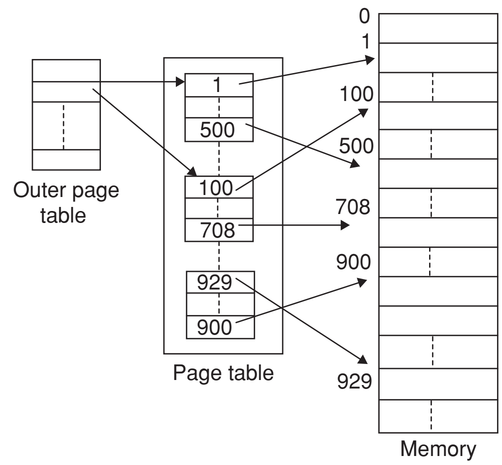


Figure 3 Address translation in paging system.

1. The amount of memory used by the page tables could be high.
2. To overcome this problem, most virtual memory schemes store page tables in virtual memory rather than real memory, that is, page tables are subject to paging just as other pages are.
3. When a process is running, at least a part of its page table must be in main memory, including the page table entry of the currently executing page.
4. Some processors make use of a two-level scheme to organize large page tables.

Hierarchical page table

1. If page table size is large, then use hierarchical page table.
2. The logical address space is broken up into multiple page tables.

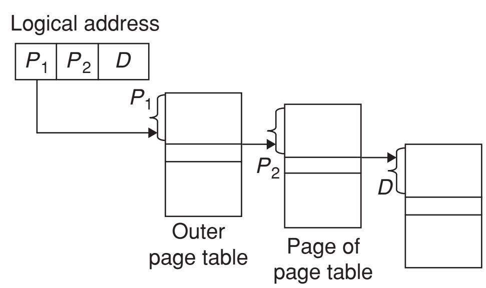


A logical address space (on 32-bit machine with 1 K page size) is divided into a page number consisting of 22-bits, page offset consisting of 10-bits. The page number is paged, the page number is divided into 12-bit page number and 10-bit page offset.

Page number		Page offset
P_1	P_2	D
12	10	10

Here, P_1 is an index into the outer page table, P_2 is the displacement within the page of the outer page table.

Address translation (diagrammatic)



Drawback: Page table size is proportional to that of the virtual address space.

Inverted page table

1. Here, the page number portion of a virtual address is mapped into a hash value, using simple hash function.
2. The hash value is a pointer to the inverted page table, which contains the page table entries.
3. There is one entry in the inverted page table for each real memory page frame rather than one per virtual page.
4. Thus, a fixed proportion of real memory is required for the tables.
5. One virtual address may map into the same hash table entry, so a chaining technique is used for managing the overflow.
6. The page table's structure is called *inverted*, because it indexes page table entries by frame number rather than by virtual page number.

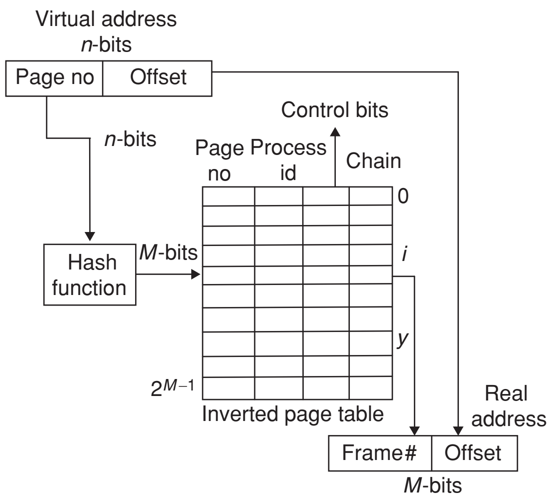


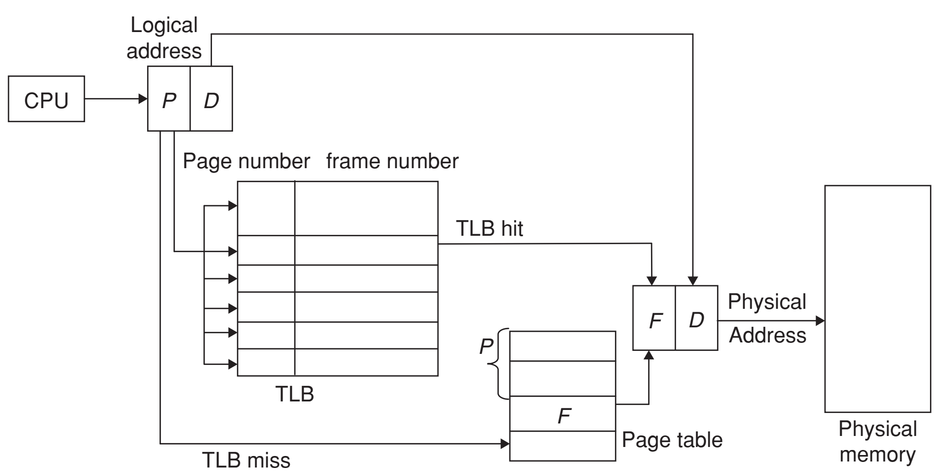
Figure 4 Inverted page table structure.

Translation look-a-side buffer (TLB)

1. The straight-forward virtual memory scheme would have the effect of doubling the memory access time.
2. To overcome this problem, most virtual memory schemes make use of a special high speed cache for page table entries, usually called TLB.
3. TLB contains the page table entries that have been most recently used.

Paging hardware with TLB

1. Given a virtual address, the processor will first examine the TLB. If the desired page table entry is present, that is, TLB hit, then the frame number is retrieved and real address is formed.
2. If there is a TLB miss, the processor uses the page number to index the process page table and examine the corresponding page table entry.
3. If present bit is set, then the page is in main memory and the processor can retrieve the frame number from the page table entry to form the real address.
4. The processor also updates the TLB to include this new page table entry.
5. If the page is not in main memory, then page fault is issued.
6. Then the OS will load the needed page and updates the page table.



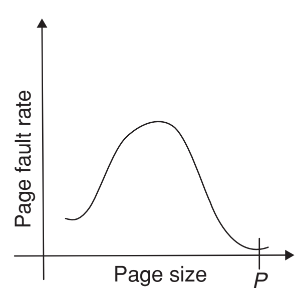
Organization of TLB

1. Each entry in TLB must include the page number and the complete page table entry.
2. The TLB may organized its entries either in
   - Direct mapping
   - Associative Mapping
3. The hardware must also consider the ways in which entries are organized and which entry to replace.

Note: The virtual memory mechanism must interact with the main memory cache system also.

Page size The factors to be considered for page size are as follows:

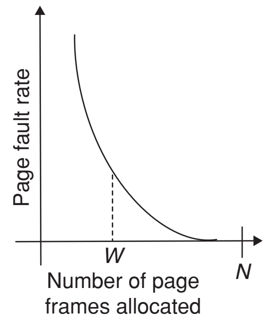
1. If page size is smaller: Then internal fragmentation is less. But it results in larger page tables. For large programs, the page fault rate increases.
2. Rotational secondary devices favour a larger page size for more efficient block transfer.



where P = size of entire process.

The above figure shows the relationship between page size and page fault rate.

The page fault rate is also determined by the number of frames allocated to a process. This relation is shown below.



where W = working set size
N = Total number of pages in process

Note: The design issue of page size is related to the size of physical main memory and program size.

Advantages of virtual memory paging

1. No external fragmentation
2. Higher degree of multiprogramming
3. Large virtual address space

Disadvantage

Overhead of complex memory management.

Segmentation with Virtual Memory

1. Memory consists of multiple segments.
2. Segments are of unequal and dynamic in size.
3. It simplifies the handling of growing data structures.
4. It allows programs to be altered and recompiled independently.
5. It lends itself to sharing among processes.
6. It lends itself to protection.
7. A unique segment table is associated with each process.
8. The virtual address and segment table entries are as shown below:

Virtual address

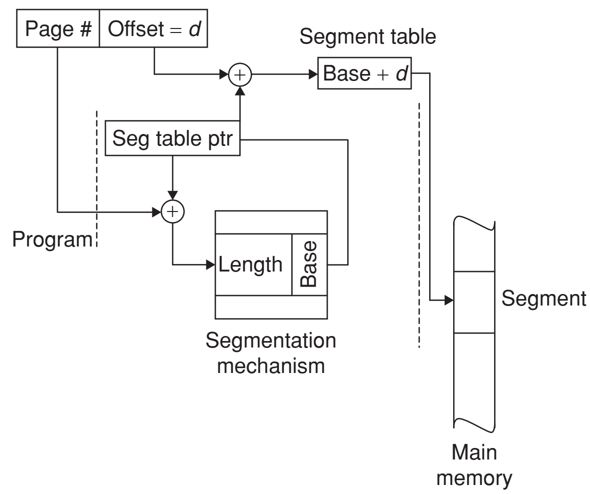
Segment number	Offset

Segment table entry

P	M	Other control bits	Length	Segment base

9. Only some of the segments of a process may be in main memory. To identify which segment is present in the main memory, use present bit P.
10. To know whether the segment is modified or not, use M-bit.

Address translation in a segmentation system (using virtual memory)



Advantages

1. No internal fragmentation
2. Higher degree of multiprogramming
3. Large virtual address space.
4. Protection and sharing support.

Disadvantage

1. Overhead of complex memory management.

Combined paging and segmentation

1. Here, the users address space is broken up into a number of segments by the programmer.
2. Each segment is, in turn, broken up into a number of fixed size pages, which are equal in length to a main memory frame.
3. If a segment has length less than that of a page, the segment occupies just one page.
4. The virtual address, segment table and page table entries are as shown below:

Virtual address

Segment number	Page number	Offset

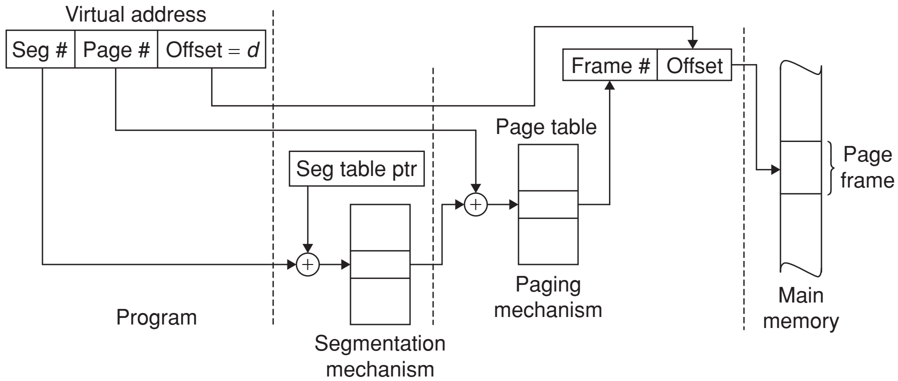
Segment table entry

Control bits	Length	Segment base

Page table entry

P	M	Other control bits	Frame number

Structure of combined segmentation/paging system



Notes:

1. Segmentation lends itself to the implementation of protection and sharing policies.
2. To achieve sharing, it is possible for a segment to be referenced in the segment tables of more than one process.

OS Software for Memory Management

We consider the following software policies for virtual memory:

1. Fetch policy:
   - Demand
   - Prepaging
2. Placement policy
3. Replacement policy:
   - Optimal
   - LRU
   - FIFO
   - Clock
4. Resident set management
   - Resident set size
   I. Fixed
   II. Variable
   - Replacement scope
   I. Local
   II. Global
5. Cleaning policy
   - Demand
   - Pre-cleaning
6. Load control
   - Degree of multi-programming

Fetch policy Determines when a page should be brought into main memory.

1. ***Demand paging:*** Here a page is brought into main memory only when a reference is made to a location on that page.
2. ***Prepaging:*** It is a technique that reduces the large number of page faults at process start up.
   - Prepaging is used to get before all or some of the pages a process will need, before they are referenced.
   - If prepaged pages are unused, I/O and memory would be wasted.
   - Assume 's' pages are prepaged and α of the pages are used.
   - Cost of $(s * \alpha)$ to save page faults greater or less than the cost of prepaging $s \times (1 - \alpha)$ unnecessary page. If α near zero $\Rightarrow$ prepaging is lost.

Placement Policy

1. Determines where in real memory a process piece is to reside.
2. In pure segmentation system, the policies like best fit, first fit, etc., are used.
3. For a system that uses either pure paging or paging combined with segmentation, placement is usually irrelevant.

Replacement policy This deals with the selection of a page in main memory to be replaced when a new page must be brought in. In a replacement policy, we have to consider the following:

1. How many page frames are to be allocated to each active process.
2. Whether the set of pages to be considered for replacement should be limited to those of the process that caused the page fault or encompass all the page frames in main memory.
3. Among the set of pages considered, which particular page should be selected for replacement.

Page Fault

1. Whenever a processor needs to execute a particular page and that page is not available in main memory, this situation is said to be *page fault*.

2. When the page fault occurs, the page replacement will be done.
3. 'Page Replacement' means select a victim page in the main memory, replace that page with the required page from the backing store (disk).
4. Some of the replacement algorithms are as follows:
   - FIFO
   - Optimal
   - LRU
   - Clock

FIFO (First-in-First-Out Algorithm)

1. Replace a page that is the oldest page of all the pages of the main memory.
2. Focuses on the length of time a page has been in memory rather than how much the page is being used.

Example:
Consider the reference string: 0 1 2 3 0 1 2 3 0 1 2 3 1

0	1	2	3	0	1
F	*F*	*F*	*F*	*F*	*F*
0	0	0	3	3	3
		1	1	0	0
		2	2	2	1

2	3	0	1	2	3	1
F	*F*	*F*	*F*	*F*	*F*	*H*
2	2	2	1	1	1	1*
0	3	3	3	2	2	2
1	1	0	0	0	3	3

Here the symbol '*F*' indicates page fault.

The number of page faults = 12

'*H*' indicates the page is already in the memory. The remaining pages are not present in memory that is why page fault occurs. In general, the more frames there are, the less page fault.

$$\text{Page fault rate} = \frac{\text{Number of page faults}}{\text{Number of bits in reference string}}$$

$$= \frac{12}{13} = 0.923 = 92.3\%.$$

Belady's Anomaly

Example: Consider the reference string: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

Number of frames = 4

F	*F*	*F*	*F*	*H*	*H*
1	1	1	1	1	1
	2	2	2	2	2
		3	3	3	3
			4	4	4
1	2	3	4	1	2

F	*F*	*F*	*F*	*F*	*F*
5	5	5	5	4	4
2	1	1	1	1	5
3	3	2	2	2	2
4	4	4	3	3	3
5	1	2	3	4	5

The number of page faults = 10

Consider the same reference string with three frames.

1	2	3	4	1	2
F	*F*	*F*	*F*	*F*	*F*
1	1	1	4	4	4
	2	2	2	1	1
		3	3	3	2

5	1	2	3	4	5
F	*H*	*H*	*F*	*F*	*H*
5	5	5	5	5	5
1	1	1	3	3	3
2	2	2	2	4	4

Here number of page faults = 9

Here, as the number of frames increases the page fault also increases. This is known as *Belady's anomaly*.

Optimal page replacement algorithm

1. Replace the page that will not be used for the longest period of time.

   Example: Consider the reference string: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5 using four frames.

F	*F*	*F*	*F*	*H*	*H*
1	1	1	1	1	1
	2	2	2	2	2
		3	3	3	3
			4	4	4
1	2	3	4	1	2

F	*H*	*H*	*H*	*F*	*H*
1	1	1	1	4	4
2	2	2	2	2	2
3	3	3	3	3	3
5	5	5	5	5	5
5	1	2	3	4	5

Number of page faults = 6

Disadvantage

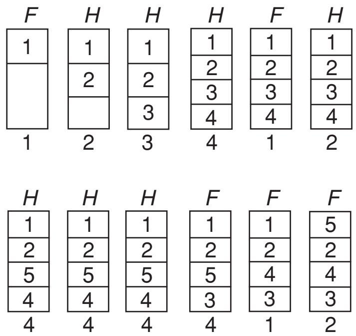
It requires future knowledge of reference string, so used for comparison studies.

LRU (least recently used) algorithm

1. Replace a page that has not been used for the longest period of time.
2. It looks backward in time rather than forward.
3. It associates with each page the time of that page last used.
4. Two methods of implementation:
   - Counters
   - Stack

   Example: Consider reference String: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

Number of frames = 4



Number of page faults = 8 (less than FIFO)

Approach to Implement LRU Replacement

LRU is a good page replacement policy, but the problem with these is how to determine the frame used for the last time.

This is implemented by using two approaches:

1. Using counters
2. Using stacks

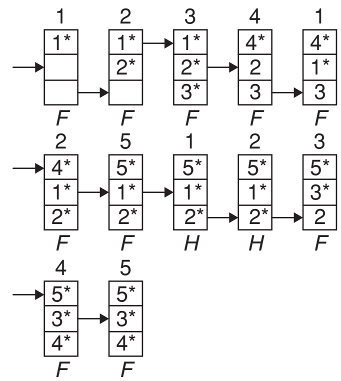
Using counters, LRU is implemented as follows: Every page entry has a counter, whenever a page is referenced, the clock value is copied into the counter. If the page has to be replaced, then it refers to the look up of the counter, whichever is having oldest time that is changed.

Using stack, LRU is implemented as follows: Form a doubly linked list of page numbers and keep it in stack whenever a page is referenced it is moved to the top of the stack that is top of the stack contains recently referenced page. Bottom of the stack will have least recently used one.

Clock replacement algorithm

1. The simplest form of clock policy requires the association of an additional bit with each frame, referred to as the use bit.
2. When a page is first loaded into frame in memory, the use bit for that frame is set to 1.
3. Whenever the page is subsequently referenced, its use bit is set to 1.
4. The set of frames that are candidates for replacement is considered to be a circular buffer, with which a pointer is associated.
5. When a page is replaced, the pointer is set to indicate the next frame in the buffer after the one just updated.
6. When it comes time to replace a page, the OS scans the buffer to find a frame with a use bit set to 0.
7. Each time it encounters a frame with a use bit of 1, it resets that bit to 0 and continues on.
8. If any of the frames in the buffer have a use bit of 0 at the beginning of this process, the first such frame encountered is chosen for replacement.
9. If all of the frames have a use bit of 1, then the pointer will make one complete cycle through the buffer, setting all the use bits to 0 and stop at its original position, replacing the page in that frame.

Example: Consider the reference string 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5, with three main memory frames:



∴ Number of misses = 9

Note: The clock algorithm was approximately closer in performance to LRU.

Effective memory access time

The percentage of times a page number is found in the associative registers is called the *hit ratio*. If we fail to find the page number in the associative registers, then we must first access memory for the page table and frame number, and then access the required byte in memory. To find the effective access time, we should weigh each case by its probability.

EMAT: Is given as $= p * s + (1 - p) * m$.

Where

p = Page fault rate

s = Page fault service time

m = Main memory access time

$(1 - p)$ = page hit ratio.

Frame locking Some of the frames in main memory may be locked. When a frame is locked, the page currently stored in that frame may not be replaced.

Page buffering To improve performance, a replaced page is not lost but rather is assigned to one of two lists:

1. The free page list if the page has not been modified or
2. The modified page list if it has modified.

Resident set management

Resident set size The OS must decide how many pages to bring in, that is, how much main memory to allocate to a particular process. Two policies are there:

1. Fixed allocation
2. Variable allocation

Fixed allocation This policy gives to a process a fixed number of frames in main memory within which to execute.

Variable allocation This policy allows the number of page frames allocated to a process to be varied over the life time of the process.

Replacement scope This is of two types as follows:

Local page replacement: When a process requests for a new page to be brought in and there are no free frames in the memory, we choose a frame allocated to only that process for replacement.

Global replacement: It allows a process to select a replacement frame from the set of all frames, even if that frame is currently allocated to some other processes. So, one process can take a frame from another.

Fixed allocation, local replacement

1. Number of frames allocated to a process is fixed.
2. Page to be replaced is chosen from among the frames allocated to that process.

Variable allocation, global scope

1. Page to be replaced is chosen from all available frames in main memory.
2. Size of resident set of processes varies.

Variable allocation, local scope

1. The number of frames allocated to a process may be changed from time to time.
2. Page to be replaced is chosen from among the frames allocated to that process.

Working set: This strategy is used to determine the resident set size and the timing of changes.

The working set with parameter Δ for a process at virtual time t, which we designated as $W(t, \Delta)$, is the set of pages of that process that have been referenced in the last Δ virtual time units.

Virtual time: Consider a sequence of memory references, $r(1)$, $r(2)$, … in which $r(i)$ is the page that contains the ith virtual address generated by a given process.

Time is measured in memory references; thus, $t = 1, 2, 3, \ldots$ measures the processes internal virtual time.

The variable 'Δ' is a window of virtual time over which the process is observed.

The working set size will be a non-decreasing function of the window size.

$$W(t, \Delta + 1) \supseteq W(t, \Delta).$$

For the sequence of page references 24, 15, 18, 23, 17, 15, 24, 18, 17, 17, 15. And window size = 2 then working set will be

{24, {24, 15,}, {15, 18}, {18, 23}, {23, 24}, {24, 17}, {17, 18}, {18, 24}, {18, 24}, {18, 17}, {17}, {17, 15}}

Page fault frequency (PFF) algorithm

1. The algorithm requires a use bit to be associated with each page in memory.
2. The bit is set to 1, when that page is accessed.
3. When a page fault occurs, the OS notes the virtual time since the last page fault for the process.
4. A threshold F is defined. If the amount of time since the last page fault is less than F, then a page added to resident set of the process.
5. Otherwise, discard all pages with a use bit of 0 and shrink the resident set according.

Note: PFF does not perform well during the transient periods when there is a shift to a new locality.

Variable interval sampled working set (VSWS)

1. The VSWS policy evaluates the working set of a process at sampling instances based on elapsed virtual time.
2. VSWS considers three parameters:
   M: The minimum duration of sampling interval
   L: The maximum duration of sampling interval
   Q: The number of page faults that are allowed to occur between sampling instances.
3. The policy works as following:
   - If virtual time since the last sampling instance reaches L, then suspend the process and scan the use bits.
   - If, prior to an elapsed virtual time of L, Q page faults occur,
     I. If the virtual time since the last sampling instance is less than M, then wait until the elapsed virtual time reaches M to suspend the process and scan the use bits.
     II. If the virtual time since the last sampling instance is greater than or equal to M, suspend the process and scan the use bits.

Cleaning policy It determines when a modified page should be written out to secondary memory. Two approaches are as follows:

1. Demand cleaning
2. Pre-cleaning

Demand cleaning A page is written out to secondary memory only when it has been selected for replacement.

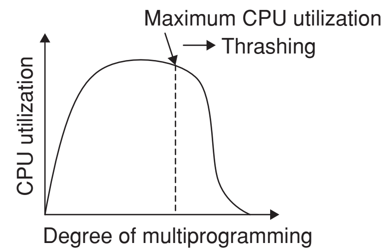
Pre-cleaning This policy writes modified pages before their page frames are needed so that pages can be written out in batches.

Load control It is concerned with determininig the number of processes that will be resident in main memory, which has been referred to as the multiprogramming level.

If too few processes are resident at any one time, it leads to swapping. If too many processes are present, thrashing will occur.

Cause of thrashing Consider the following scenario:

The OS monitors CPU utilization. If the utilization is too low, we increase the degree of multiprogramming by introducing a new process to the system. A global page replacement algorithm is used, which replaces pages with no regard to the process to which they belong. Now, say that a process enters a new phase in its execution and needs more frames. It starts faulting and taking frames away from other processes. These processes need those pages, however and so they also fault, taking frames from other processes. These faulting processes must use the paging device to swap pages in and out. As they queue up for paging device, the ready queue empties. As processes wait, for the paging device, CPU utilization decreases the CPU scheduler sees the decreasing CPU utilization; so it increases the degree of multiprogramming. The new process tries to get started by taking frames from running processes, causing more page faults, and a longer queue for paging device. As a result, CPU utilization drops even, further. The CPU scheduler tries to increase the degree of multiprogramming even more. Thrashing occurs, and the system throughput plunges. The page fault rate (FT) increases tremendously. Effective memory access time increases. No work is getting done because the processes are spending all their time in paging.



Thrashing

Exercises

Practice Problems 1

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Consider a 3-level memory hierarchy shown in the following table, with access times to memory:

Hierarchy level	Cache hit ratio	Page transfer time
M_1	0.55	0.003 ms
M_2	0.92	0.3 ms
M_3	–	1.0 ms

When a miss occurs, data is fetched from the next level. Calculate the average time required for a process to read one word from the memory system.

(A) 0.379 (B) 0.162
(C) 0.2798 (D) 0.172

2. Consider a memory system with FIFO page replacement algorithm policy. For an arbitrary page access pattern, increasing the number of page frames in main memory will

(A) Always decrease the number of page faults
(B) Always increase the number of page faults
(C) Sometimes increase the number of page faults
(D) Never effect the number of page faults

3. Consider the below page address stream generated by executing a program:

4 5 4 3 7 4

Assuming that LRU is used for page replacement and at most three frames are available in the memory for the process, find the number of page faults that can occur (initially all frames empty).

(A) 0 (B) 1
(C) 3 (D) 4

4. Consider a logical address space of four pages of 2048 words each mapped into a physical memory of 32 frames. How many bits in logical address?

(A) 12-bits (B) 14-bits
(C) 13-bits (D) 11-bits

5. The time taken to service a page fault is on average 10 ms and the memory access time is 20 μs. If the hit ratio is 70%, calculate the average access time.

(A) 3018 μs (B) 4014 μs
(C) 3014 μs (D) 4024 μs

6. Consider the following page trace:

4, 3, 2, 1, 4, 3, 5, 4, 3, 2, 1, 5

Number of frames for the Job $M = 4$. Then the page fault ratio using FIFO technique will be

(A) 63% (B) 75%
(C) 83% (D) 94%

7. The available main memory for loading pages is 64 MB with a frame size of 8 MB. If pages of size 6 MB, and 4 MB are loaded into memory, what is the percentage of the internal fragmentation resulted?

(A) 42.5 (B) 37.5
(C) 57.5 (D) 62.45

8. A demand paging system takes 50 time units to handle a page fault and 200 time units to replace a dirty page. Access time of memory is 2 time units. Probability of page fault and dirty page is P. Average access time is 4 time units. Then what is the value of P?

(A) 0.037 (B) 0.027
(C) 0.012 (D) 0.042

9. Assume that a total memory 20 KB is available with no partition. If Buddy system technique is used and there are total of four partitions to serve the request, the closest range of the requested size is

(A) 12.5 and 25 (B) 14.5 and 30
(C) 15.5 and 25 (D) 14.5 and 40

10. A system uses FIFO page replacement algorithm. It has three page frames with no pages loaded. First 50 pages are accessed in some order and the same pages are accessed in the reverse order. What is the number of page faults?
(A) 98 (B) 96
(C) 97 (D) 95

11. If 32-bit addressing is used for pages whose maximum size is 512 KB, what is the maximum number of pages that can be addressed?
(A) 4096 (B) 2048
(C) 8192 (D) 16384

12. Calculate the overhead due to page table if given the average process in bytes is 16-bytes, the page size is 32-bytes and the page entry is 2-bytes.
(A) 15 (B) 17
(C) 18 (D) 19

13. In a system with 32-bit virtual address and 1 KB page size, use of one-level page tables for virtual to physical address translation is not practical because of
(A) The large amount of internal fragmentation
(B) The large amount of external fragmentation
(C) The large memory overhead in maintaining page tables
(D) The large computation overhead in the translation process

14. If an instruction takes time 10 m sec if there is no page fault and time 20 m sec if there is a page fault, what is the effective instruction time if page fault occurs once every 5 instructions?
(A) 12.5 msec (B) 12 msec
(C) 14 msec (D) 15 msec

15. Let an instruction take 10 ms and page fault takes an additional 5 ms. If the average page fault occurs after 20 instructions, the effective instruction time will be
(A) 10 ms (B) 10.25 ms
(C) 0.25 ms (D) 10.75 ms

16. A 0.8 MB-sized memory is managed using variable partitions, while the rest of the memory is occupied by a 0.26 MB partition, 0.27 MB partition and 0.25 MB partitions in the order. Best-fit strategy is being adopted where would be a 0.18 MB allocation request is fulfilled?
(A) 0.26
(B) 0.27
(C) 0.25
(D) Request will be denied

Common data for questions 17 and 18: Suppose that the OS uses variable length partitions for memory management. At some particular time, the running process occupies a partition between physical addresses 20,000 and 40,000.

17. The values of base and limit register are respectively
(A) 20, 000, 40,000
(B) 20,000, 20,000
(C) 0, 10,000
(D) 0, 40,000

18. What physical address corresponds to a virtual address of 13,000?
(A) 13,000
(B) 43,000
(C) 33,000
(D) Out of range

19. Consider a page table where translation look ahead buffer is used. TLB hit ratio is 0.95 and generally takes 1 nanosecond to retrieve the frame number. If a miss is recorded by the TLB then an additional overhead of 10 nanoseconds should be taken into consideration, further cache and main memory reference takes 100 ns on average, what is the average memory fetch time using the TLB? Assume main memory accesses are always a success.
(A) 101 ns (B) 105 ns
(C) 200 ns (D) 205 ns

20. Consider a 1 MB process which is divided into five segments. Each segment is further divided into pages whose size is 4 KB. What is the maximum segments possible? Assume that the system is byte addressable.
(A) 64 (B) 128
(C) 256 (D) 512

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Consider a logical address space of 32 pages of 2048 words mapped into memory of 64 frames. Then the number of bits required for logical address are
(A) 16-bits (B) 17-bits
(C) 18-bits (D) 20-bits

2. In which of the page table techniques the logical address space is broken into multiple page table?
(A) Inverted Page Table
(B) Hierarchical Page Table
(C) Hashed Page Table
(D) None of the above

3. Consider a system with 70% hit ratio, 60 nanoseconds time to search the associative registers, 800 nanoseconds to access memory. What is the effective memory access time?
(A) 1200 nsec (B) 1100 nsec
(C) 1300 nsec (D) 2200 nsec

4. On a system using fixed partitions, all of size 2^8, the number of bits used by the limit register is
   (A) 128 (B) 256
   (C) 8 (D) 1024

5. Working set (t, k) at an instant of time, t, is the set of
   (A) k future references that the operating system will make
   (B) future references that the operating system will make in the next 'k' time units
   (C) k references with high frequency
   (D) pages that have been referenced in the last k time units.

6. Cache and interleaved memories are ways of speeding up memory access between CPUs and slower RAM. Which of the following memory models are best suited (i.e., improves performance the most) for which programs?
   (i) Cached memory is best suited for small loops.
   (ii) Interleaved memory is best suited for small loops.
   (iii) Interleaved memory is best suited for large loops.
   (iv) Cache memory is best suited for large sequential code.
   (A) (i) and (ii) are true
   (B) (i) and (iii) are true
   (C) (iv) and (ii) are true
   (D) (iv) and (iii) are true

7. A paging system with a page table in memory every reference to memory takes 100 ns. The TLB hit ratio is 85% and the time needed for searching TLB is almost negligible. What is the effective memory access time?
   (A) 115 ns (B) 135 ns
   (C) 145 ns (D) 125 ns

8. Consider the page sequence 4, 2, 1, 5, 3, 2, 1, 5, 0, 2, 5. If FIFO page replacement algorithm is used and frame size is 3, then the percentage of page fault is
   (A) 99% (B) 90.9%
   (C) 80.8% (D) 89.9%

9. If the page size is 32 KB, primary page table contains 4096 entries and the secondary page table contains 256 entries, then what is the size of logical address in bits?
   (A) 15-bits (B) 20-bits
   (C) 32-bits (D) 35-bits

10. If page size is 2 KB and logical address is 20-bit, then the number of entries in the page table is
    (A) 2048 B (B) 256 B
    (C) 512 B (D) 1 MB

11. Consider a paging system with the page table stored in memory. If a memory reference takes 200 ns, how long does a paged memory reference take?
    (A) 100 ns (B) 200 ns
    (C) 300 ns (D) 400 ns

12. Consider a logical address space of eight pages of 1024 words each mapped onto a physical memory of 32 frames. How many bits are there in the logical address and in the physical address?
    (A) 10, 18 (B) 13, 18
    (C) 13, 15 (D) 10, 5

13. For a paged system, TLB hit ratio is 0.9. Let the RAM access time 't' be 20 ns and the TLB access time 'T' be 100 ns. Then effective memory access (with TLB) will be
    (A) 120 ns (B) 200 ns
    (C) 130 ns (D) 150 ns

14. Assume that a user program is 100 K words and secondary storage device is a fixed hard disk with an average latency of 8 ms and a transfer rate of 2,50,000 words/second. Then find the swap time of a transfer of 100 K words to or from memory.
    (A) 816 ms (B) 408 ms
    (C) 204 ms (D) 8 ms

15. Consider the following segment table:

Segment	Limit	Base
0	1000	1400
1	400	6300
2	400	4300
3	1100	3200
4	1000	4700

    The physical address for a logical address which is in segment 2 with offset 253 is
    (A) 4553 (B) 6353
    (C) 6253 (D) 4453

16. Consider a process of size 2 MB. If the page size is 0.5 KB, what is the size of the page table (assuming that each page is mapped by a 32-bit size page table entry)?
    (A) 8 KB (B) 16 KB
    (C) 24 KB (D) 32 MB

17. A CPU generates 32-bit virtual address. The page size is 2 KB. The translation look-aside buffer (TLB) which can hold 256 page table entries and is two-way set associative mapping. The number of bits in the TLB tag is
    (A) 10-bits (B) 12-bits
    (C) 14-bits (D) 15-bits

18. Assume that a total memory M is available with no partitions made yet. If Buddy system strategy is being used and a total of n partitions have been made to serve the request. The closest range of the requested size is
    (A) $\frac{M}{2^n+1}$ and $\frac{M}{2^n}$ (B) $\frac{M}{n}$ and $\frac{M}{n-1}$
    (C) $\frac{M}{2^n}$ and $\frac{M}{2^{n-1}}$ (D) $\frac{M}{n+1}$ and $\frac{M}{n}$

19. The memory has four free blocks of sizes 2K, 6K, 20K, 4K. The request blocks are allocated according to best fit allocation method. The allocation requests are stored in queue as shown:

Reqest no.	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Reqest sizes	4 K	10 K	2 K	3 K	5 K	4 K	2 K
Usage time	1	4	2	6	3	1	8

The time at which request for P_7 will be completed is
(A) 10 unit time
(B) 14 unit time
(C) 20 unit time
(D) 15 unit time

20. A memory page containing a heavily used variable that was initialized very early and is in constant use is removed when ______ page replacement is used.
(A) LRU
(B) FIFO
(C) LFU
(D) Optimal

Previous Years' Questions

1. A processor uses 36-bit physical addresses and 32-bit virtual addresses, with a page frame size of 4 K bytes. Each page table entry is of size 4 bytes. A three-level page table is used for virtual to physical address translation, where the virtual address is used as follows: **[2008]**

- Bits 30–31 are used to index into the first level page table
- Bits 21–29 are used to index into the second level page table
- Bits 12–20 are used to index into the third level page table, and
- Bits 0–11 are used as offset within the page

The number of bits required for addressing the next level page table (or page frame) in the page table entry of the first, second and third level page tables are, respectively,
(A) 20, 20 and 20 (B) 24, 24 and 24
(C) 24, 24 and 20 (D) 25, 25 and 24

2. How many 32 K × 1 RAM chips are needed to provide a memory capacity of 256 K bytes? **[2009]**
(A) 8 (B) 32
(C) 64 (D) 128

3. In which one of the following page replacement policies, Belady's anomaly may occur? **[2009]**
(A) FIFO (B) Optimal
(C) LRU (D) MRU

4. The essential content(s) in each entry of a page table is/are **[2009]**
(A) Virtual page number
(B) Page frame number
(C) Both virtual page number and page frame number
(D) Access right information

5. A multilevel page table is preferred in comparison to a single-level page table for translating virtual address to physical address because **[2009]**
(A) It reduces the memory access time to read or write a memory location.
(B) It helps to reduce the size of page table needed to implement the virtual address space of a process.
(C) It is required by the translation look-aside buffer.
(D) It helps to reduce the number of page faults in page replacement algorithms.

6. A system uses FIFO policy for page replacement. It has four-page frames with no pages loaded to begin with. The system first accesses 100 distinct pages in some order and then accesses the same 100 pages but now in the reverse order. How many page faults will occur? **[2010]**
(A) 196 (B) 192
(C) 197 (D) 195

7. Let the page fault service time be 10 ms in a computer with average memory access time being 20 ns. If one page fault is generated for every 10^6 memory accesses, what is the effective access time for the memory? **[2011]**
(A) 21 ns (B) 30 ns
(C) 23 ns (D) 35 ns

8. Consider the virtual page reference string

1, 2, 3, 2, 4, 1, 3, 2, 4, 1

on a demand paged virtual memory system running on a computer system that has main memory size of three-page frames which are initially empty. Let LRU, FIFO and OPTIMAL denote the number of page faults under the corresponding page replacement policy. Then **[2012]**
(A) OPTIMAL< LRU < FIFO
(B) OPTIMAL < FIFO < LRU
(C) OPTIMAL = LRU
(D) OPTIMAL = FIFO

9. A RAM chip has a capacity of 1024 words of 8 bits each (1 K × 8). The number of 2 × 4 decoders with

enable line needed to construct a 16 K × 16 RAM from 1 K × 8 RAM is **[2013]**
(A) 4 (B) 5
(C) 6 (D) 7

Common data for Questions 10 and 11: A computer uses 46-bit virtual address, 32-bit physical address, and a three-level paged page table organization. The page table base register stores the base address of the first-level table (T_1), which occupies exactly one page. Each entry of T_1 stores the base address of a page of the second-level table (T_2). Each entry of T_2 stores the base address of a page of the third-level table (T_3). Each entry of T_3 stores a page table entry (PTE). The PTE is 32-bits in size. The processor used in the computer has a 1 MB 16-way set associative virtually indexed physically tagged cache. The cache block size is 64 bytes.

10. What is the size of a page in KB in this computer? **[2013]**
(A) 2 (B) 4
(C) 8 (D) 16

11. What is the minimum number of page colours needed to guarantee that no two synonyms map to different sets in the processor cache of this computer? **[2013]**
(A) 2 (B) 4
(C) 8 (D) 16

12. Assume that there are three page frames which are initially empty. If the page reference string is 1, 2, 3, 4, 2, 1, 5, 3, 2, 4, 6, the number of page faults using the *optimal replacement policy* is ———. **[2014]**

13. A computer has 20 physical page frames which contain pages numbered 101 through 120. Now a program accesses the pages numbered 1, 2, … 100 in that order, and repeats the access sequence THRICE. Which one of the following page replacement policies experiences the same number of page faults as the optimal page replacement policy for this program? **[2014]**
(A) Least-recently used (B) First-in-first-out
(C) Last-in-first-out (D) Most-recently-used

14. A system uses three page frames for storing process pages in main memory. It uses the Least Recently Used (LRU) page replacement policy. Assume that all the page frames are initially empty. What is the total number of page faults that will occur while processing the page reference string given below? **[2014]**
4, 7, 6, 1, 7, 6, 1, 2, 7, 2

15. Consider a paging hardware with a TLB. Assume that the entire page table and all the pages are in the physical memory. It takes 10 milliseconds to search the TLB and 80 milliseconds to access the physical memory. If the TLB hit ratio is 0.6, the effective memory access time (in milliseconds) is ——— **[2014]**

16. Consider a system with byte-addressable memory, 32-bit logical addresses, 4 kilobyte page size and page table entries of 4 bytes each. The size of the page table in the system in megabytes is _______. **[2015]**

17. Consider a main memory with five page frames and the following sequence of page references: 3, 8, 2, 3, 9, 1, 6, 3, 8, 9, 3, 6, 2, 1, 3. Which one of the following is true with respect to page replacement policies First In First Out (FIFO) and Least Recently Used (LRU)? **[2015]**
(A) Both incur the same number of page faults
(B) FIFO incurs 2 more page faults than LRU
(C) LRU incurs 2 more pages faults than FIFO
(D) FIFO incurs 1 more page faults than LRU

18. Consider six memory partitions of sizes 200 KB, 400 KB, 600 KB, 500 KB, 300 KB and 250 KB, where KB refers to kilobyte. These partitions need to be allotted to four processes of sizes 357 KB, 210 KB, 468 KB and 491 KB in that order. If the best fit algorithm is used, which partitions are NOT allotted to any process? **[2015]**
(A) 200 KB and 300 KB
(B) 200 KB and 250 KB
(C) 250 KB and 300 KB
(D) 300 KB and 400 KB

19. A computer system implements 8 kilobyte pages and a 32-bit physical address space. Each page table entry contains a valid bit, a dirty bit, three permission bits, and the translation. If the maximum size of the page table of a process is 24 megabytes, the length of the virtual address supported by the system is _______ bits. **[2015]**

20. Consider the following two *C* code segments. *Y* and *X* are one and two dimensional arrays of size *n* and $n \times n$ respectively, where $2 \leq n \leq 10$. Assume that in both code segments, elements of Y are initialized to 0 and each element $X[i]\ [j]$ of array *X* is initialized to $i + j$. Further assume that when stored in main memory all elements of *X* are in same main memory page frame.

Code segment 1: **[2015]**

```
    //initialize elements of Y to 0
//initialize elements X[i][j] of X to
i+j
for (i = 0; i < n; i++)
Y[i] += X[0] [i];
```

Code Segment 2:

```
    //initialize elements of Y to 0
```

```
    //initialize elements X[i] [j] of X
to i + j
    for (i = 0; i < n; i++)
    Y[i] += X[i] [0];
```

Which of the following statements is/are correct?

S_1: Final contents of array Y will be same in both code segments

S_2: Elements of array X accessed inside the for loop shown in code segment 1 are contiguous in main memory

S_3: Elements of array X accessed inside the for loop shown in code segment 2 are contiguous in main memory.

(A) Only S_2 is correct
(B) Only S_3 is correct
(C) Only S_1 and S_2 are correct
(D) Only S_1 and S_3 are correct

21. Consider a computer system with 40-bit virtual addressing and page size of sixteen kilobytes. If the computer system has a one-level page table per process and each page table entry requires 48 bits, then the size of the per-process table is ______ megabytes. **[2016]**

22. Consider a computer system with ten physical page frames. The system is provided with an access sequence ($a_1, a_2, \ldots, a_{20}, a_1, a_2, \ldots a_{20}$), where each ai is a distinct virtual page number. The difference in the number of page faults between the last-in-first-out page replacement policy and the optimal page replacement policy is ______. **[2016]**

23. In which one of the following page replacement algorithms it is possible for the page fault rate to increase even when the number of allocated frames increases? **[2016]**

(A) **LRU** (Least Recently Used)
(B) **OPT** (Optimal Page Replacement)
(C) **MRU** (Most Recently Used)
(D) **FIFO** (First In First Out)

24. Recall that Belady's anomaly is that the page-fault rate may *increase* as the number of allocated frames increases. Now, consider the following statements:

S1: *Random page replacement* algorithm (where a page chosen at random is replaced) suffers from Belady's anomaly

S2: *LRU page replacement* algorithm suffers from Belady's anomaly

Which of the following is CORRECT? **[2017]**

(A) S1 is true, S2 is true
(B) S1 is true, S2 is false
(C) S1 is false, S2 is true
(D) S1 is false, S2 is false

25. Consider a process executing on an operating system that uses demand paging. The average time for a memory access in the system is M units if the corresponding memory page is available in memory and D units if the memory access causes a page fault. It has been experimentally measured that the average time taken for a memory access in the process is X units.

Which one of the following is the correct expression for the page fault rate experienced by the process? **[2018]**

(A) $(D - M)/(X - M)$ (B) $(X - M/(D - M)$
(C) $(D - X/(D - M)$ (D) $(X - M/(D - X)$

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** B	**3.** D	**4.** C	**5.** C	**6.** C	**7.** B	**8.** A	**9.** A	**10.** C
11. C	**12.** B	**13.** C	**14.** B	**15.** B	**16.** C	**17.** B	**18.** C	**19.** A	**20.** C

Practice Problems 2

1. B	**2.** B	**3.** B	**4.** C	**5.** D	**6.** B	**7.** A	**8.** B	**9.** D	**10.** C
11. D	**12.** B	**13.** C	**14.** A	**15.** A	**16.** B	**17.** C	**18.** C	**19.** A	**20.** B

Previous Years' Questions

1. D	**2.** C	**3.** A	**4.** B	**5.** B	**6.** A	**7.** B	**8.** B	**9.** B	**10.** C
11. C	**12.** 7	**13.** D	**14.** 6	**15.** 122	**16.** 4	**17.** A	**18.** A	**19.** 36	**20.** C
21. 384	**22.** 1	**23.** D	**24.** B	**25.** B					

Chapter 5

File Systems, I/O Systems, Protection and Security

LEARNING OBJECTIVES

- *File systems*
- *File management systems*
- *File system architecture*
- *Device drivers*
- *Basic input/output supervisor*
- *Logical input/output*
- *Access methods*

File Systems

The file system consists of two distinct parts:

1. Collection of files
2. Directory structure

File A file is a named collection of related information that is recorded on secondary storage. The files must have

1. Long-term existence
2. Sharable between processes
3. Structure

File attributes A typical file attributes are

1. Name
2. Identifier
3. Type
4. Location
5. Size
6. Protection
7. Time, date and user identification

File operations The operations that are applied on files are

1. Creation
2. Deletion
3. Closing
4. Reading
5. Writing

File types A common technique for implementing file types is to include the type as part of the file name. The name is split into two parts:

1. A name
2. An extension (usually separated by a period character)

The type of a file may be

1. Executable (exe, com, bin)
2. Object (obj, o)
3. Source code (c, cc, java)
4. Batch (bat, sh)
5. Text (txt, doc)
6. Word processor (wp, text, doc)
7. Library (lib)
8. Print or view (ps, pdf, jpg)
9. Archive (zip, tar)
10. Multimedia (mpeg, mov, rm)

File structure The four common terms of file systems are

1. Field
   - Basic element of data
   - Contains a single value
   - Has a particular length and data type

2. Record
   - It is a collection of related fields
   - It is treated as a unit

3. File
   - It is a collection of similar records
   - Treated as a single entity
   - Has file names
   - Access to file may be restricted or unrestricted

4. Database
   - Collection of related data.
   - Relationship exists among elements.

File Management Systems

It is a set of system software that provides services to users and applications in the use of files. Objectives of file management systems are as follows:

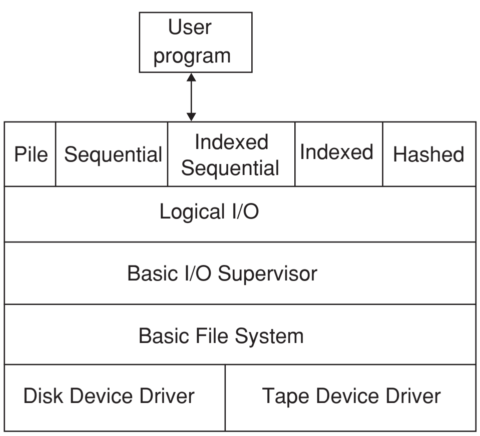
1. To meet the data management needs and requirements of the user including storage of data.
2. To guarantee, that the data in the file are valid.
3. To optimize performance (i.e., throughput, response time).
4. To provide Input/Output (I/O) support for a variety of storage device types.
5. To minimize or eliminate the potential for lost or destroyed data.
6. To provide a standardized set of I/O interface routines.
7. To provide I/O support for multiple users.

The minimal set of requirements from user's point of view for an interactive, general purpose, file system are as follows:

1. Ability to create, delete, read, write and modify files.
2. Controlled access to other users files.
3. Control the type of accesses to files.
4. Restructure the user's files.
5. Able to move data between files.
6. Ability to back up and recover user's files.
7. Able to access a file by name.

File System Architecture

The file system architecture is shown below:



Device Drivers

1. It is at the lower part.
2. Communicates directly with peripheral devices.
3. Initiative to start I/O operations on a device.
4. Processes the completion of an I/O request.

Basic File System

1. Mainly concerned with I/O.
2. Exchanges blocks of data.
3. Deals with placement of blocks.
4. Deals with buffering blocks in main memory.

Basic I/O Supervisor

1. Responsible for file initiation and termination.
2. Maintain control structures.
3. Selects the device while file I/O is to be performed.
4. Deals with scheduling access to optimize performance.
5. Part of the operating system (OS).

Logical I/O

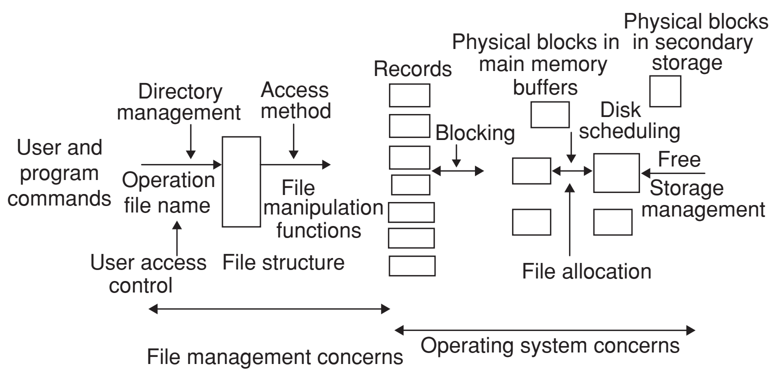
1. Enables users and application to access records.
2. Provides general purpose record I/O capability.
3. Maintains basic data about file.

Access Methods

1. They reflect different file structures.
2. Provide different ways to access and process data.
3. Access methods are as follows:
   - Sequential: read next, write next
   - Direct: read block n, write block n

File Management Functions

The functions of a file system is shown below:



File Organization and Access

File organization refers to the logical structuring of the records as determined by the way in which they are accessed. We choose a particular file organization based on

1. Short access time
2. Ease of update
3. Economy of storage
4. Simple maintenance
5. Reliability

We will discuss five types of file organizations:

1. The pile
2. The sequential file
3. The indexed sequential file
4. The indexed file
5. The direct or hashed file

The pile The pile organization is shown in Figure 1:

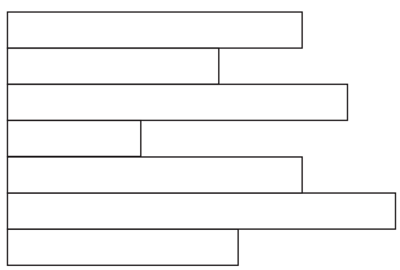


Figure 1 Pile file.

1. Data are collected in order they arrive.
2. The aim is to accumulate mass of data and save.
3. Records may have different fields.
4. There is no structure.
5. Record are accessed by exhaustive search.
6. There can be variable length records.
7. This type of files are encountered when data are collected and stored prior to processing or when data are not easy to organize.

The sequential file The sequential file is shown in Figure 2.

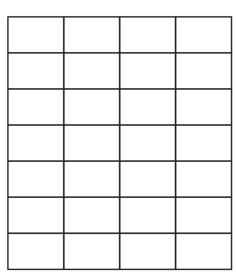


Figure 2 Sequential file.

1. Fixed format used for records.
2. Records are of same length.
3. All fields are of the same order and length.
4. One field is the key field. It uniquely identifies the record. Records are stored in a key sequence.
5. New records are placed in a log file or transaction file.
6. Batch update is performed to merge the log file with the master file.
7. Used in batch applications.
8. Not suitable for interactive applications.

Indexed sequential file

1. Index provides a look up capability to quickly reach the vicinity of the desired record.
2. It contains key field and a pointer to the main file.
3. Index is searched to find highest key value that is equal to or precedes the desired key value.
4. Search continues in the main file at the location indicated by the pointer.
5. New records are added to an overflow file.
6. Record in main file that precedes it is updated to contain a pointer to the new record.
7. The overflow is merged with main file during a batch update.
8. Multiple indexes for the same key can be set up to increase efficiency.

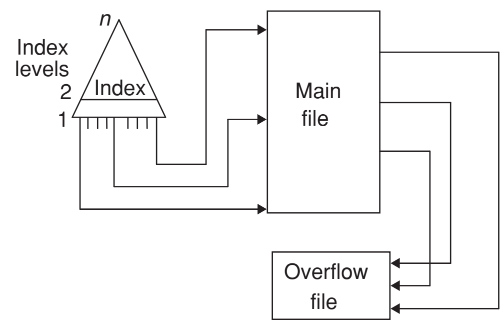
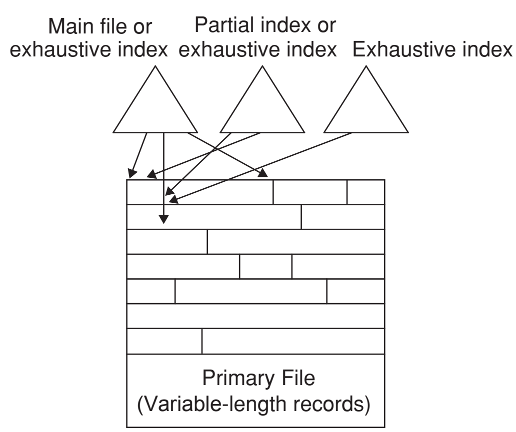


Figure 3 Indexed file.

9. Key field required for each record.
10. It uses multiple indexes for different key fields.
11. It may contain exhaustive index that contains one entry for every record in the main file or partial index.



12. Used in the applications where timeliness of information is critical and where data are rarely processed exhaustively.

Direct or Hashed file

1. This file has the capability to access any block of a known address.
2. Key field required in each record.
3. No concept of sequential ordering.

Figure 4 shows hashed file organization:

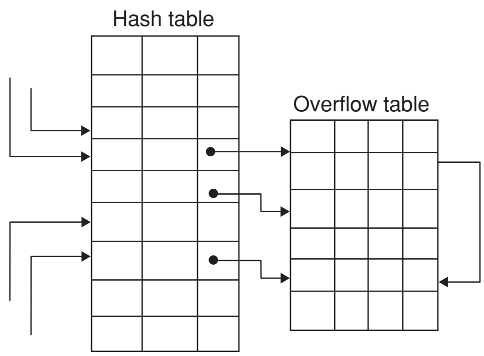


Figure 4 Hashed file organization.

File Directories

The collection of files is a file directory.

Contents

1. Contain information about files, such as attributes, location, and ownership.
2. Itself a file owned by the OS.
3. It provides mapping between file names and the file themselves.

Directory Structure

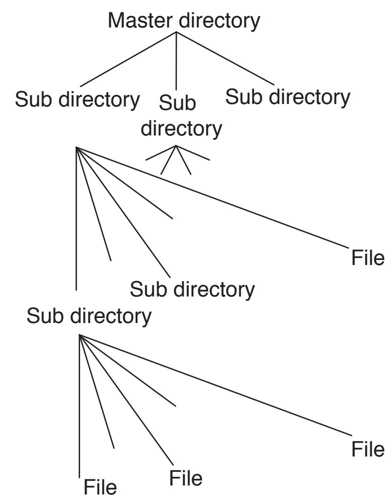
1. It consists of list of entries, one for each file.
2. Sequential file with the name of the file serving as the key.
3. Provides no help in organizing the files.
4. Not scalable (same name cannot be used for two different files).
5. As directory grows in size, searching is too time consuming.

Two-level Directory Structure

1. Contains one directory for each user and a master directory.
2. The master directory contains entry for each user
3. User directory is a simple list of files for that user.
4. File naming conflict is solved.

Hierarchical or Tree-structured Directory

1. Contains master directory with user directory underneath it.
2. Each user directory may have subdirectories and files as entries.



3. Files can be located by following a path from the root, or master directory down various branches which is called *pathname* for the file.
4. Current directory is called the *working directory*.
5. Files are referenced relative to the working directory.

Naming The use of a tree-structured directory minimizes the difficulty in assigning unique names. Any file in the system can be located by following a path from the root or master directory down various braches until the file is reached.

Path name The series of directory names, culminating in the file name itself, constitutes a path name for the file.

Example: Time/Gate/Exam/OS
The slash is used to delimit names in the sequence. The name of master directory is implicit, because all paths starts at that directory. Files can also be referenced from the working directory.

File sharing It has got two issues, such as:

1. Access rights
2. Management of simultaneous access

Access rights Users or groups of users are granted certain access rights to a file. A wide range of access rights has been used. The access rights may be

1. None
2. Knowledge
3. Execution
4. Reading
5. Appending
6. Updating
7. Changing protection
8. Deletion

These access rights can be specified to specific users or user groups or all users.

Record Blocking For I/O to be performed, records must be organized as blocks. Given the size of a block, there are three methods of blocking that can be used:

1. Fixed blocking
2. Variable length spanned blocking
3. Variable length unspanned blocking.

Fixed blocking

1. Fixed-length records (Figure 5) are used and an integral number of records are stored in a block.
2. Possibility of internal fragmentation.
3. Used for sequential files.

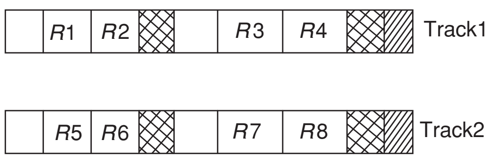


Figure 5 Fixed blocking.

Variable length spanned blocking Variable length records are used and are packed into blocks with no unused space. Some records can span two blocks. These do not limit the size of records.

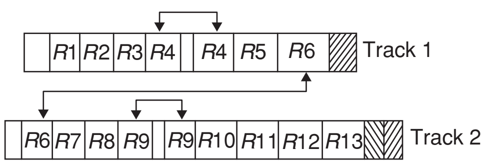


Figure 6 Variable blocking: spanned.

Variable length unspanned blocking Variable length records (Figure 7) are used, but spanning is not employed. There is wasted space in most blocks and limits record size.

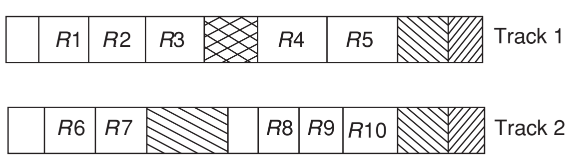
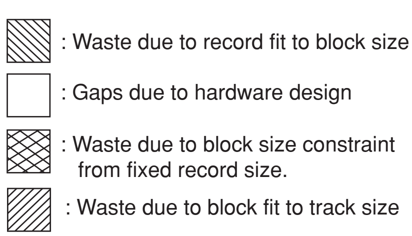


Figure 7 Variable blocking: unspanned.



Secondary Storage Management

1. Space (or blocks) must be allocated to files on disk.
2. Need to keep track of the space available (free blocks) for allocation to files.
3. Preallocation of blocks to files can be used to allocate space for files. For this, it needs to know the maximum size of the file at the time of creation.

File Allocation

Preallocation Versus Dynamic Allocation

1. A preallocation policy requires that the maximum size of a file be declared at the time of the file creation request. For many applications, it is difficult to estimate the file size.
   It is better to use dynamic allocation, which allocates space to a file in portions as needed.

Portion size The portion size which is allocated to a file may be

1. Variable, large contiguous portions.
2. Blocks

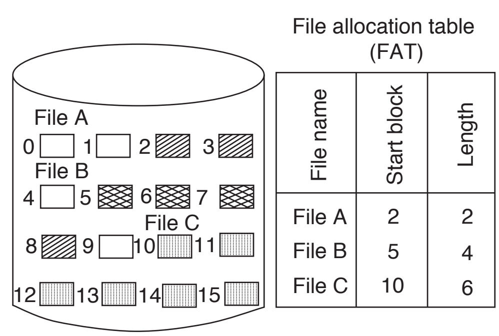
Some strategies for dealing with fragmentation of free space are as follows:

1. *First-fit*: Choose the first unused contiguous group of blocks of sufficient size.
2. *Best-fit*: Choose the smallest unused group that is of sufficient size.
3. *Nearest-fit*: Choose the unused group of sufficient size that is closest to the previous allocation.

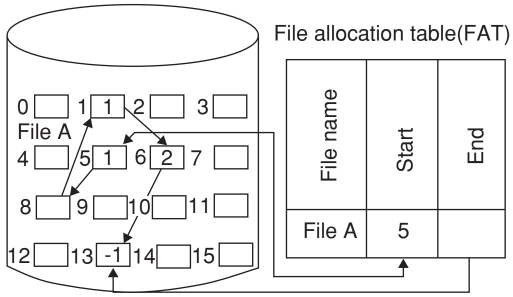
File allocation methods It has three methods as follows:

1. Contiguous allocation
2. Chained allocation
3. Indexed allocation

Contiguous allocation Here, a single set of blocks is allocated to a file at the time of creation. Only a single entry in the file allocation table is created consisting of starting block and length of the file. It exhibits external fragmentation and performs compaction.

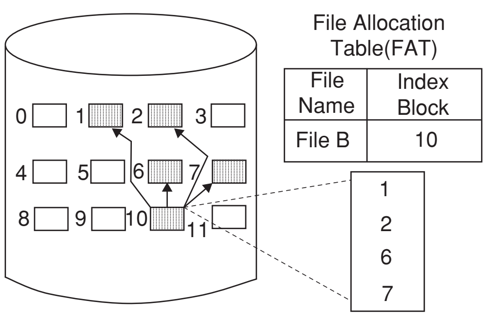


Linked or chained allocation The allocation is done on basis of individual block. Each block contains a pointer to the next block in the chain. Only single entry is created in the file allocation table consisting of starting block and length of file. There occurs no external fragmentation and it is best for sequential files. There is no accommodation of principle of locality. If block size is n, then only $n - 1$ units of data are stored and 1 unit stores the link information.

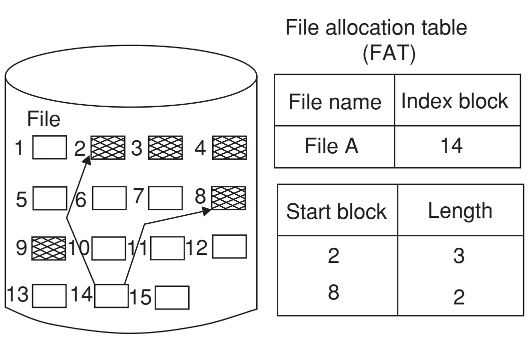


Indexed file allocation The file allocation table contains a separate one level index for each file. The index has one entry for each portion allocated to the file. The file allocation table contains block number for the index. If a file requires n blocks, then $n + 1$ blocks are used, where the first block contains index information (pointers to data blocks).

Index Allocation with Block Pointers



Indexed Allocation with Variable length Portion



Example 1: A direct access of file has fixed size 50 byte records. Assuming the first record is record 1, the first byte of record 10 will be at what logical location?

Solution:
Total records = 50 × 10 = 500
First record is record 1. This record is already read.
Logical location of first byte = 500 – 50 = 450
The correct logical location = 450 + 1 = 451.

Example 2: A sequential access file has fixed-size 32-byte records. Assuming that the first record is record 0, the first byte of record 20 will be at what location?

Solution: Since the first record is record 0, the first byte of record 20 will be at logical location = 32 × 20 = 640

Free Space Management

In addition to file allocation table, disk allocation table is also required to know what blocks on the disk are available. Some of the free space management techniques are as follows:

1. Bit tables
2. Chained free portions
3. Indexing
4. Free block list

Bit tables This method uses a vector containing, one bit for each block on the disk. Each entry of a '0' corresponds to a free block and each '1' corresponds to a block in use.

Advantage

1. Easy to find one or a contiguous group of free blocks.
2. Smaller in size.

The amount of memory required for a block bitmap will be

$$\frac{\text{Disk size in (bytes)}}{8 \times \text{file system block size}}$$

Chained free portion The free portions may be chained together by using a pointer and length value in each free portion. This method has negligible space overhead. This method is suitable for all file allocation methods. The disk will become quite fragmented, after some use. It is slower for individual block file creation and also for deletion.

Indexing It treats the free space as a file and uses an index table (same as in file allocation). The index should be on the basis of variable size portions rather than blocks.

Free block list Here, each block is assigned a number sequentially and the list of the numbers of all free blocks is maintained in a reserved portion of the disk.

Volumes

It is a collection of addressable sectors in a secondary memory that an OS or application can use for data storage. The sectors in a volume need not be consecutive on a physical storage device. (a single disk equals one volume).

Unix File Management

I-nodes (Index Node)

UNIX files are administered by the OS by means of i-node. An i-node (index node) is a control structure that contains the key information needed by the OS for a particular file.

The attributes of the file as well as its permissions and other control information are stored in the i-node. The

exact i-node structure varies from UNIX implementation to another. The FreeBSD i-node structure is shown in Figure 8.

File Allocation

1. It is done on a block basis.
2. Allocation is dynamic.
3. The blocks of a file on disk are not necessarily contiguous.
4. An indexed method is used to keep track of each file, with i-node includes a number of direct pointers and three indirect pointers.

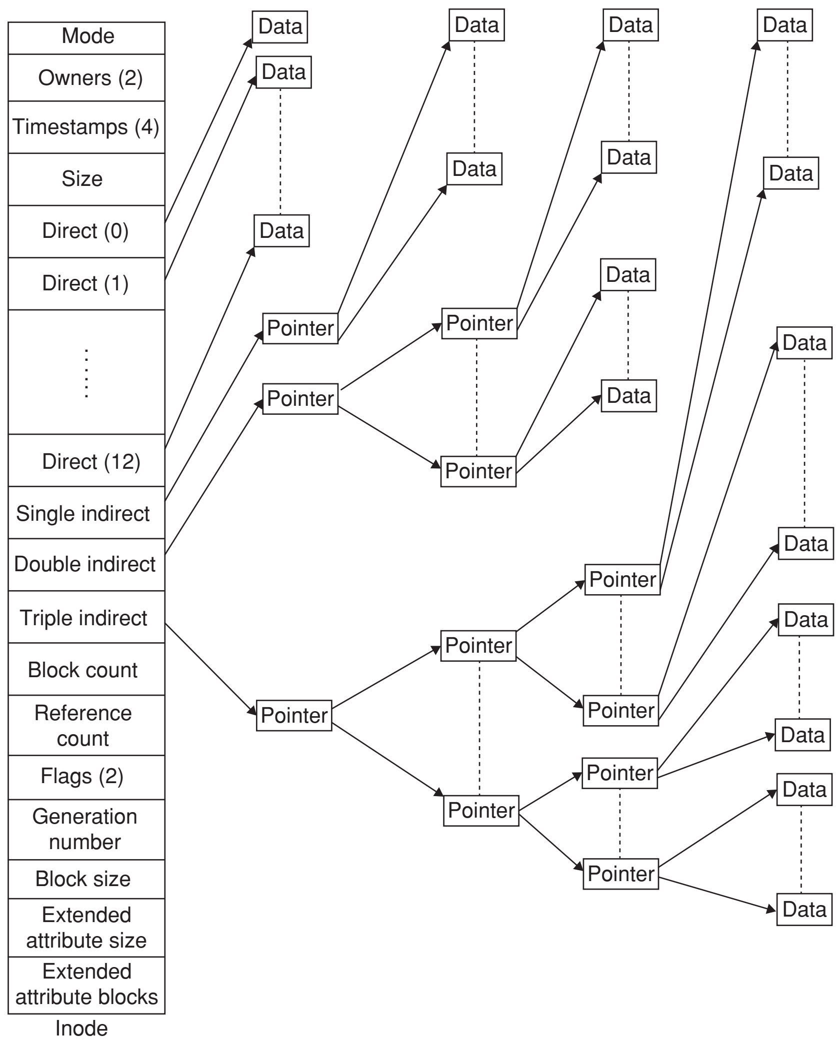


Figure 8 Structure of free BSD i-node and file.

5. The free BSD i-node includes 120 bytes of address information that is organized as fifteen 64-bit addresses.
6. The first 12 addresses point to the first 12 data blocks of the file.
7. If the file requires more than 12 data blocks, one or more levels of indirection are used as follows:
   - The thirteenth address in the i-node points to a block on disk that contains the next portion of the index. This is referred to as the *single indirect block*.
   - If the file contains more blocks, the fourteenth address in the i-node points to a double indirect block. Each block consists of single indirect blocks, each of which contains pointers to file blocks.
   - If the file contains still more blocks, the fifteenth address in the i-node points to a triple indirect block that is a third level of indexing. This block points to additional double indirect blocks.

The capacity of FreeBSD file with 4 kB block size is shown below:

Level	Number of Blocks	Number of Bytes
Direct	12	48 K
Single indirect	512	2 M
Double indirect	$512 \times 512 = 256$ K	1 G
Triple indirect	512×256K $= 128$ M	512 G

The total number of data blocks in a file depends on the capacity of the fixed-size blocks in the system. In FreeBSD, the minimum block size is 4 kB, and each block can hold a total of 512 block addresses. Thus, the maximum size of a file with this block size is over 500 GB.

Windows NT File System

The windows NT file system provides a combination of reliability, compatibility and performance, which are not available in the FAT file system.

1. It will quickly perform standard file operations, such as write, read and search.
2. It also performs file-system recovery on very large hard disks.
3. NTFS file system formatting on a volume results in the creation of several system files and the master file table (MST), which contains information about all the files and folders on the NTFS (Figure 9).

Partition boot sector	Master file table	System files	File area

Figure 9 NTFS after formatting.

4. It includes security features required for high-end personal computers and file servers.
5. It supports data access control and ownership privileges.
6. Folders shared on a Windows NT are assigned specific permissions.
7. It allows users to assign permissions to individual files.

I/O Systems

Design Objectives of OS

Two objectives of OS for I/O systems are as follows:

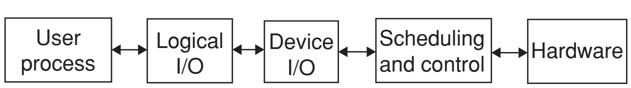
1. Efficiency
2. Generality

Logical Structure of the I/O Function

The three most important logical structures are as follows:

1. Local peripheral devices
2. Communication ports
3. File system

Local Peripheral Device



Logical I/O

Manages general I/O functions on behalf of user processes, allowing them to deal with the device in terms of a device identifier and simple commands, such as open, close, read, write.

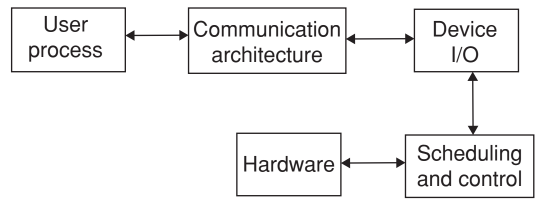
Device I/O

The requested operations and data are converted into appropriate sequences of I/O instructions, channel commands and controller orders.

Scheduling and Control

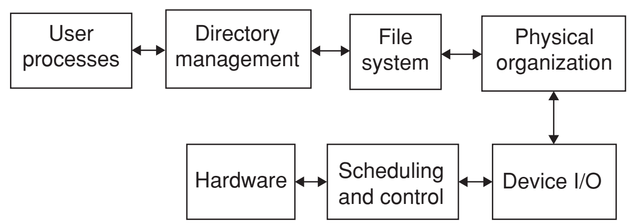
The actual queuing and scheduling of I/O operations occurs at this layer as well as the control of the operations. Interrupts are handled. I/O status is collected and reported.

Communication Port



Here, the communication architecture may itself consist of a number of layers.

File System



Directory management Here, the symbolic file names are converted to identifiers that either reference the file directly or indirectly through a file descriptor or index table.

File system This layer deals with the logical structure of files and with the operations that can be specified by users, such as open, close, read, write.

Physical organization Allocation of secondary storage space and main storage buffers is generally treated here.

I/O buffering In buffering, we perform input transfers in advance of requests being made and perform output transfers sometime after the request is made. We will discuss

1. Single buffering
2. Double buffering
3. Circular buffering

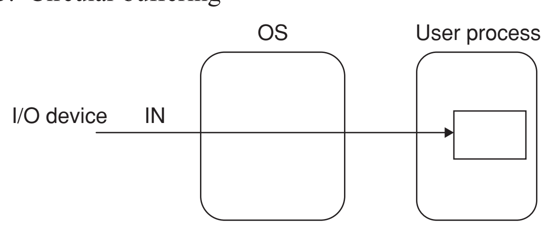


Figure 10 No buffering.

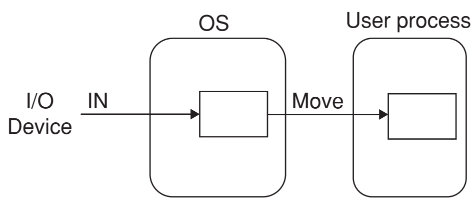


Figure 11 Single buffering.

- When a user process issues an I/O request, the OS assigns a buffer in the system portion of main memory to the operation.
- Provides a speed up compared to the lack of system buffering.
- For block oriented devices, input transfers are made to the system buffer. When the transfer is complete, the process moves the block into user space and immediately requests another block. This is same for block-oriented output.
- Suppose that T is the time required to input one block and that C is the computation time that intervenes between input requests.
  Without buffering, the execution time per block = $T + C$.
  With a single buffer, the execution time per block = max $[C, T] + M$,
  where M = Time required to move the data from the system buffer to user memory.
- For stream-oriented I/O, the single buffering scheme can be used in a line-get-a-time fashion or a byte-at-a-time fashion.

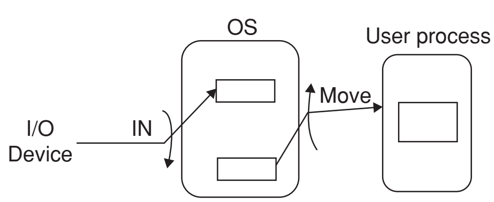


Figure 12 Double buffer

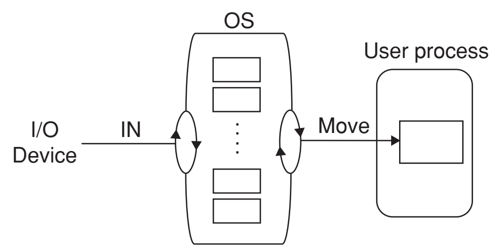
- An improvement over single buffer.
- A process now transfers data to one buffer while the OS empties the other. This is known as *double buffering* or *buffer swapping*.
- For block-oriented transfer, the estimated execution time is max $[C, T]$.
- The stream-oriented transfer may be line-at-a-time or byte-at-a-time.

Automatic and Explicit Buffering

An indefinite length queue is provided in automatic buffering. Sender never blocks to copy a message. No specifications are present for providing automatic buffering. One method is to reserve large amount of memory. In this method, most of the memory is wasted.

The size of the queue is provided in explicit buffering. Sender will block if the requested space is more than available. In this scheme, memory wastage is less likely to happen.

Circular Buffer



1. Here, more than two buffers are used. Each individual buffer is one unit of circular buffer.

Kernal I/O Subsystem

Kernel I/O provides I/O-related services. It is built on the hardware and device driver infrastructure.

One of the responsibilities of Kernal I/O subsystem is to protect itself from the erroneous process and malice users.

Services provided by the I/O subsystem are as follows:

1. Scheduling
2. Buffering
3. Caching
4. Spooling
5. Device reservation
6. Error handling

Disk Scheduling

Hard Disk Performance Parameters (Terminologies)

Seek time It is defined as the time required to move the disk arm to the required track. It consists of

1. Initial start-up time
2. The time taken to traverse the tracks that has to be crossed once the access arm is up to speed.

$$\text{Seek time, } T_s = m \times n + S$$

where T_s = Estimated seek time
n = Number of track traversed
m = Constant that depends on the disk drive
S = Start-up time

Rotational delay Time required to reach the desired sector by read/write head. Rotational speed ranges from 5400 to 10,000 rpm.

1. Floppy disks typically rotate between 300 and 600 rpm.
2. For 1,00,000 rpm, the average rotational delay will be 3 ms.

Transfer time The transfer time to or from the disk depends on the speed of the disk.

$$T = \frac{b}{rN}$$

where, T = Transfer time
b = Number of bytes to be transferred
N = Number of bytes on a track
r = Rotation speed in revolutions/second

Total average access time

$$T_a = T_s + \frac{1}{2r} + \frac{b}{rN}$$

$T_s \rightarrow$ average seek time

Hard Disk Scheduling Algorithms

Disk bandwidth and fast access time are to be considered. Bandwidth is the total number of bytes transferred divided by the total time between the first request for service and completion of the last transfer.

FCFS (First Come First Served) The disk in controller processes the I/O requests the order in which they arrive, thereby moving backwards and forwards across the surface of the disk to get the next requested location each time (Figure 10).

Example 3: A disk queue has the following requests to read tracks:

87, 170, 40, 150, 36, 72, 66, 15

Consider the disk head is initially at cylinder 60. Total head movement = (87 – 60) + (170 – 87) + (170 – 40) + (150 – 40) + (150 – 36) + (72 – 36) + (72 –66) + (66 – 15) = 27 + 83 + 130 + 110 + 114 + 36 + 6 + 51 = 557 cylinders

Average head movement $= \frac{557}{8} = 69.6$ cylinders

Advantage

Improved response time as a request gets response in fair amount of time.

Disadvantages

1. Involves a lot of random head movements and disk rotations.
2. Throughput is not efficient.
3. Used in small systems only where I/O efficiency is not very important.

Shortest Seek Time First (SSTF) When a disk operation finishes, choose the request that is closer to the current head position or choose the request that has minimum seek time from the current head position.

Example 4: Consider the following requests: 87, 170, 40, 150, 36, 72, 66, 15. Find the average head movement for SSTF.

The initial head position is say 60. Now, closest to the head position is the request at cylinder 66. Then, the closest to 66 is 72, closest request to 72 is 87, and so on. Total head movements = (66 – 60) + (72 – 66) + (87 – 72) + (87 – 40) + (40 – 36) + (36 – 15) + (150 – 15) + (170 – 150) = 6 + 6 + 15 + 47 + 4 + 21 + 135 + 20 = 254 cylinders

Average head movements $= \frac{254}{8} = 31.75$ cylinders

Advantages

1. It minimizes latency
2. Better throughput than FIFO method

Disadvantages

1. Starvation occurs if some process has to wait for long time until its requests are satisfied.
2. SSTF services requests for those tracks which are highly localized.

SCAN/elevator algorithm The disk head constantly moves from the most inner cylinder to the outer cylinder and then it changes its direction back towards the centre. As the head moves, if there is a request for the current disk position then it is satisfied.

1. It is known as *elevator algorithm* because it services all the request of going up and then reaching at the top, it goes downward.
2. It needs two information:
   - Direction of head movement
   - Last position of the disk head

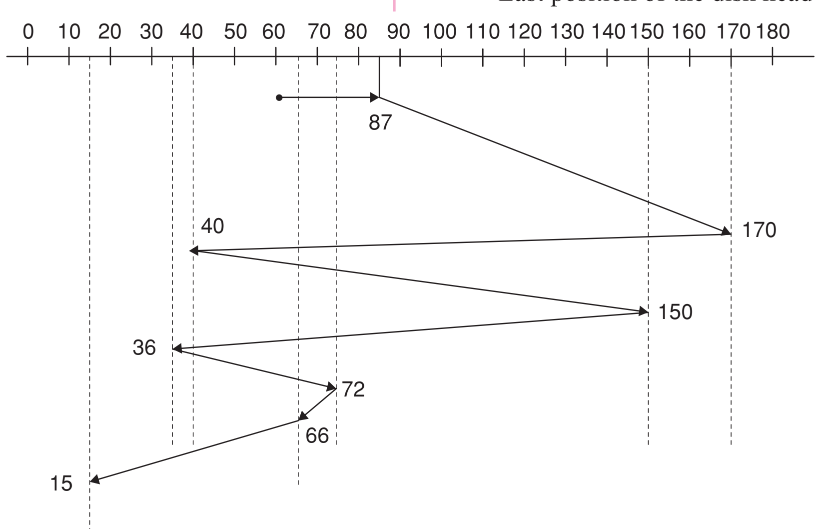


Figure 13 FCFS.

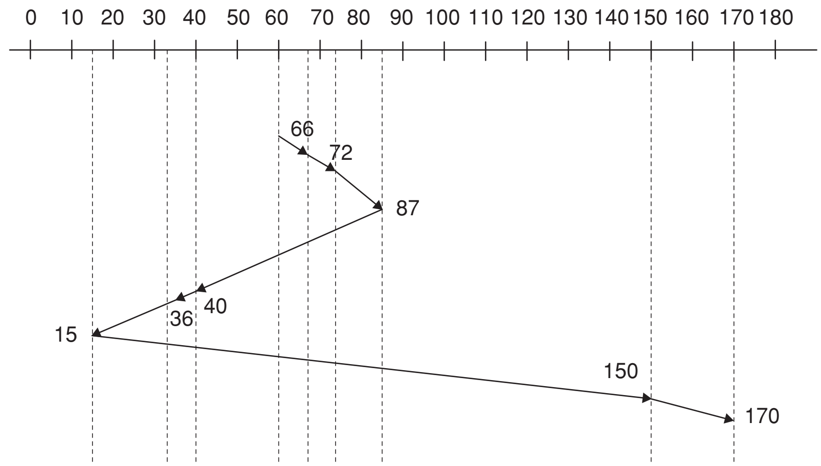


Figure 14 SSTF

Example 5: For the following track requests: 87, 170, 40, 150, 36, 72, 66, 15. (Initially head is at track 60 to the arm is moving outwards.

Total head movement = (66 – 60) + (72 – 66) + (87 – 72) + (150 – 87) + (170 – 150) + (180 – 170) + (180 – 40) + (40 – 36) + (36 – 15)

= 6 + 6 + 15 + 63 + 20 + 10 + 140 + 4 + 21

= 285 cylinders

Average head movement $= \frac{285}{8} = 35.6$ cylinders

Advantages

1. Throughput better than FIFO.
2. Basic for most scheduling algorithms.
3. Eliminates the discrimination.
4. No starvation.

Disadvantages

1. Because of the continuous scanning of disk from end to end, the outer tracks are visited less often than the mid-range tracks.
2. Disk arm keeps scanning between two extremes; this may result in wear and tear of the disk assembly.
3. Certain requests arriving ahead of the arm position would get immediate service but some other requests that arrive behind the arm position will have to wait for the arm to return back.

C–SCAN algorithm (one–way elevator algorithm) It treats the cylinder as a circular list. The head sweeps from the innermost cylinder to the outermost cylinder, satisfying the waiting requests in order of their locations. When it reaches the outermost cylinder, it sweeps back to the innermost cylinder without satisfying any requests and then starts again.

Example 6: Consider the cylinders requests:
87, 170, 40, 150, 36, 72, 66, 15 Starting cylinder = 60th (arm moving outwards)

Total head movement = (66 – 60) + (72 – 66) + (87 – 72) + (150 – 87) + (170 – 150) + (180 – 170) + (180 – 0) + (15 – 0) + (36 – 15) + (40 – 36)

= 6 + 6 + 15 + 63 + 20 + 10 + 180 + 15 + 21 + 4 = 340

Average head movement $= \frac{340}{8} = 35.6 = 42.5$

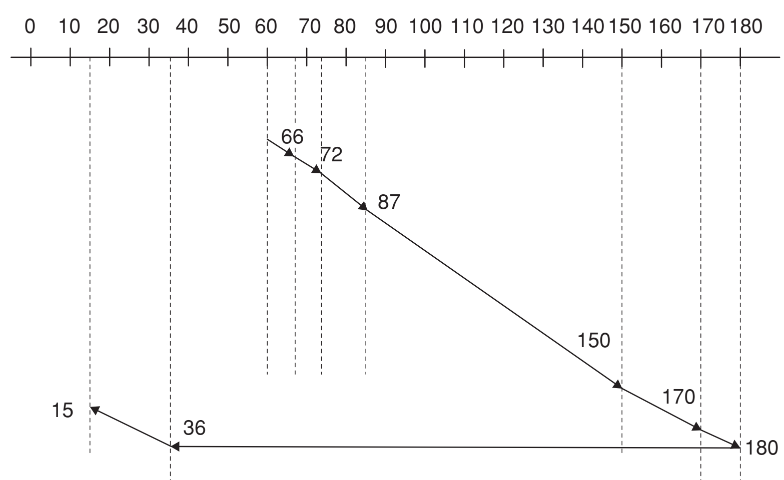


Figure 15 SSTF

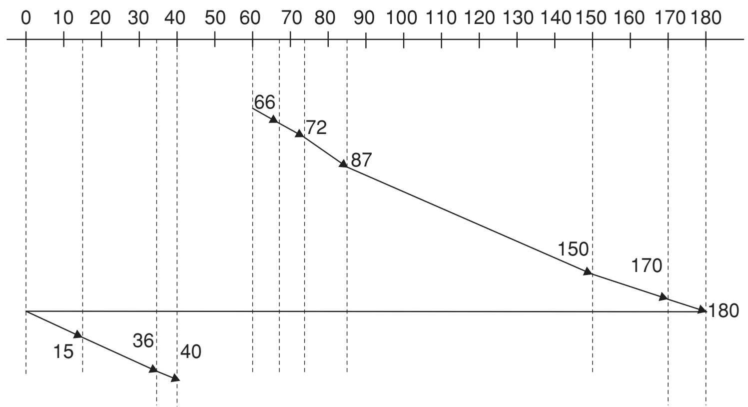


Figure 16 C–SCAN

Advantage

Lower service variability.

Disadvantages

1. An average head movement is more compared to SCAN algorithm.
2. Increase in the total seek time

LOOK/SEEK algorithm Look is similar to SCAN, but stops moving inwards (or) outwards when there are no more requests in that direction.

Example 7: Consider the following requests:
87, 170, 40, 150, 36, 72, 66, 15 initially head is at track 60 (moves outwards).

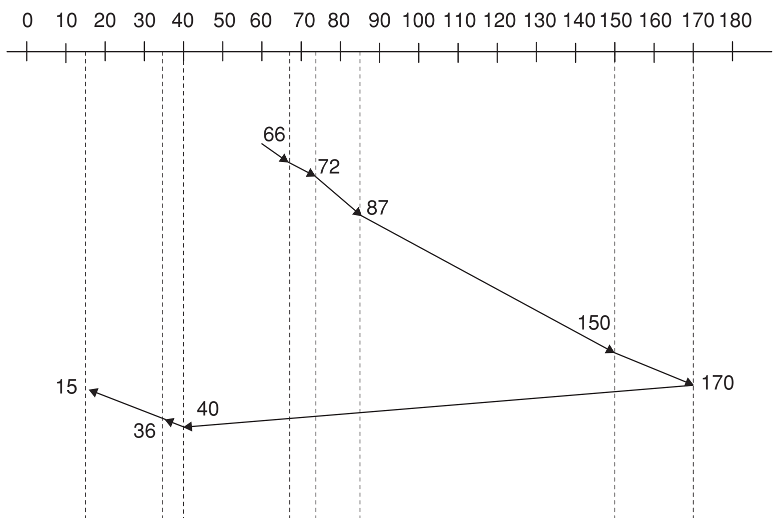


Figure 17 LOOK

$$\text{Average head movement} = \frac{275}{8} = 34.37$$

C–LOOK/C–SEEK algorithm The head moves inwards servicing requests until there are no more requests in that direction. Then it jumps to the outer most outstanding request.

Magnetic Disk and Factors that Determine Access Speed

The factors that determine the access speed are rotational speed, which is measured in revolutions per minute are seek time, the time taken to read or write the particular sector of the disk.

The other factors are sequential read, sequential write, random read and random write. These vary enormously, but for spinning disks one can expect 25 Mbps to 150 Mbps for sequential read and write it is about 3 Mbps to 50 Mbps for the random read and write.

RAID (Redundant Array of Independent Disks)

1. It is multiple disk database design.
2. It consists of seven levels, zero through six.
3. Characteristics of RAID Levels:
   - RAID is a set of physical disk drives viewed by the OS as a single logical drive.
   - Data are distributed across the physical drives of an array in a scheme known as striping.
   - Redundant disk capacity is used to store parity information, which guarantees data recoverability in case of a disk failure.

RAID Level 0:

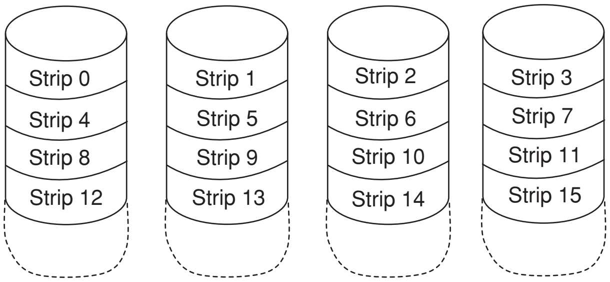


Figure 18 Non-redundant (RAID0)

1. It does not include redundancy.
2. N disks are required.
3. Data available in RAID level 0 is lower than single disk.
4. It has high data transfer capacity.
5. It has high I/O request rate.

RAID Level 1:

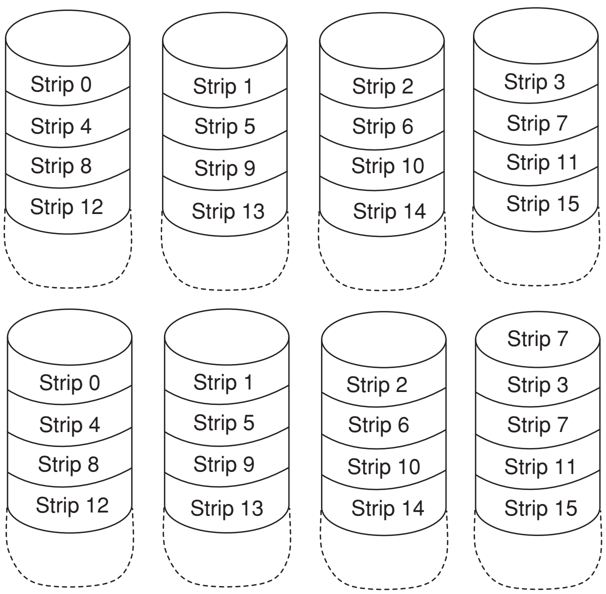
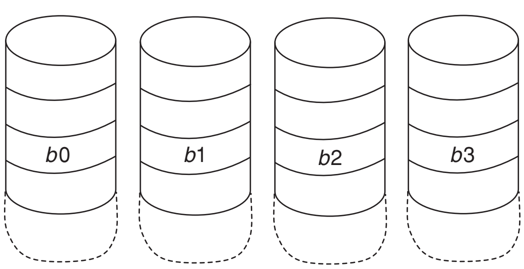


Figure 19 RIAD 1 (Mirrored)

1. Redundancy is achieved by the simple expedient of duplicating all the data.
2. 2N disks required.
3. Data availability is higher than RAID 2, 3, 4, or 5. But lower than RAID 6.
4. Recovery from failure is simple.
5. RAID1 costs more.

RAID Level 2:



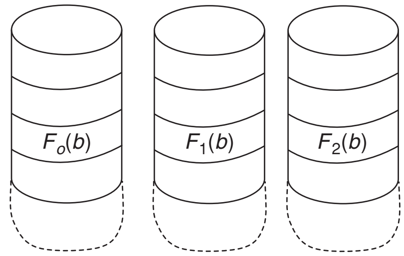


Figure 20 RAID 2 (Redundancy through Hamming code).

6. Here redundancy is achieved through hamming code.
7. $N + m$ disks required.

RAID Level 3:

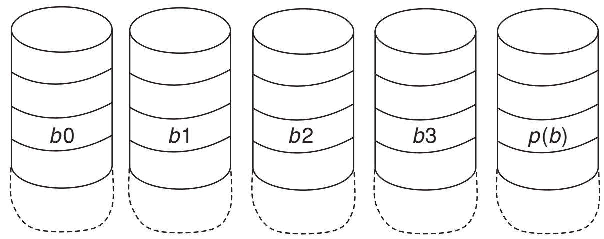


Figure 21 RAID 3 (bit-interleaved parity)

1. It has bit interleaved parity.
2. Provides parallel access.
3. $N + 1$ disks required.

RAID Level 4:

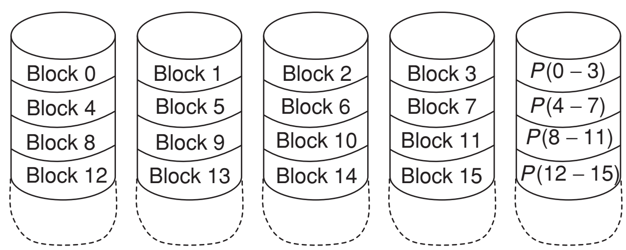


Figure 22 RAID 4 (Block-level parity)

1. $N + 1$ disks required.
2. Provides independent access.
3. Consider an array of five drives in which X_0 through X_3 contains data and X_4 is the parity disk.

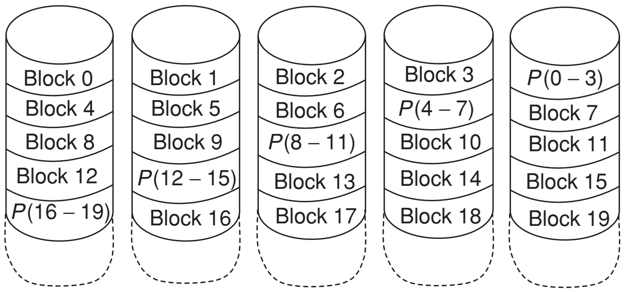
Suppose that a write is performed that only involves a strip on disk X_1. Initially for each bit i, we have the following relationship:

$$X_4(i) = X_3(i) \oplus X_2(i) \oplus X_1(i) \oplus X_0(i)$$

After the update, with potentially altered bits indicated by a prime symbol:

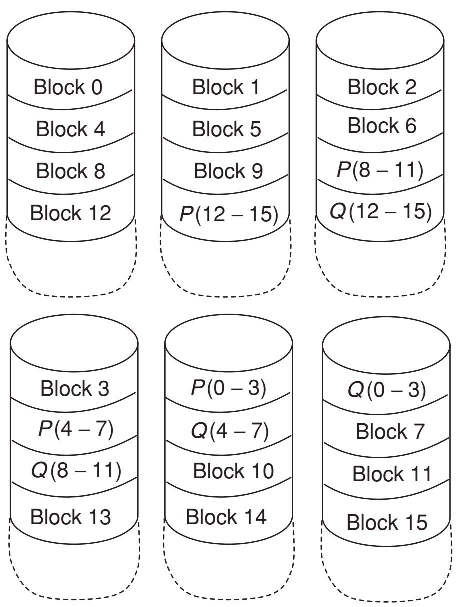
$$\begin{aligned} X_4'(\text{i}) &= X_3(i) \oplus X_2(i) \oplus X_1'(i) \oplus X_0(i) \\ &= X_3(i) \oplus X_2(i) \oplus X_1'(i) \oplus X_0(i) \oplus X_1(i) \oplus X_1(i) \\ &= X_4(i) \oplus X_1(i) \oplus X_1'(i) \end{aligned}$$

RAID Level 5:



1. It has block level distributed parity.
2. Provides independent access.
3. It has $N + 1$ disks.

RAID Level 6:

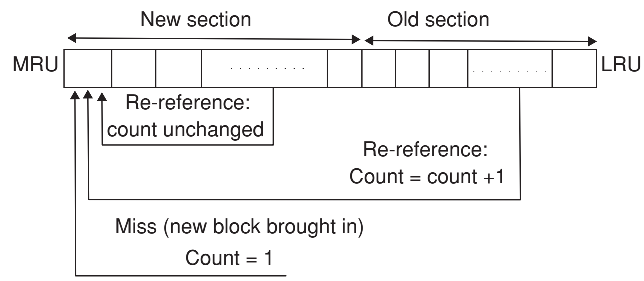


1. It has dual redundancy.
2. It provides independent access.
3. It has $N + 2$ disks.

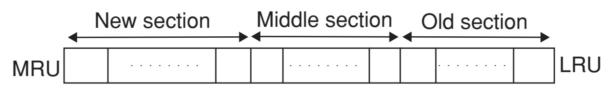
Disk Cache:

1. It is a buffer in main memory for disk sectors.
2. The cache contains a copy of some of the sectors on the disk.
3. Replacement policy:
   - LRU
   - LFU

But these two replacement policies lead to poor performance. So a new technique frequency-based replacement is proposed. Two alternatives of frequency-based technique are shown below:



(A) FIFO



(B) Use of three sections.

In FIFO, the blocks are logically organized in a stack; where the top part of the stack is the new section. When there is a cache hit, the referenced block is moved to top of the stack. If the block was already in the new section, its reference count is not incremented; otherwise it is incremented by 1.

In another technique, we divide the stack into three sections: New, middle and old. Here only blocks in the old section are eligible for replacement.

Protection and Security

System protection Protection refers to a mechanism for controlling the access of programs, processes or users to the resources defined by a computer system.

Principles of Protection

Programs, users and even systems are given just enough privileges to perform their tasks. This is the principle of *least privilege*.

Domain of protection A computer system is a collection of processes and objects. The objects may be software or hardware objects. The operations that are possible may depend on the object.

A process should be allowed to access only those resources for which it has authorization. At any time, a process should be able to access only those resources that it currently requires to complete its task. This requirement is referred as *need to know principle* and is useful in limiting the amount of damage, which a faulty process can cause in the system.

Domain structure Each domain defines a set of objects and the types of operations that may be invoked on each object.

The ability to execute an operation on an object is an *access right*. A domain is a collection of access rights, each of which is an ordered pair <obj-name, rights-set>

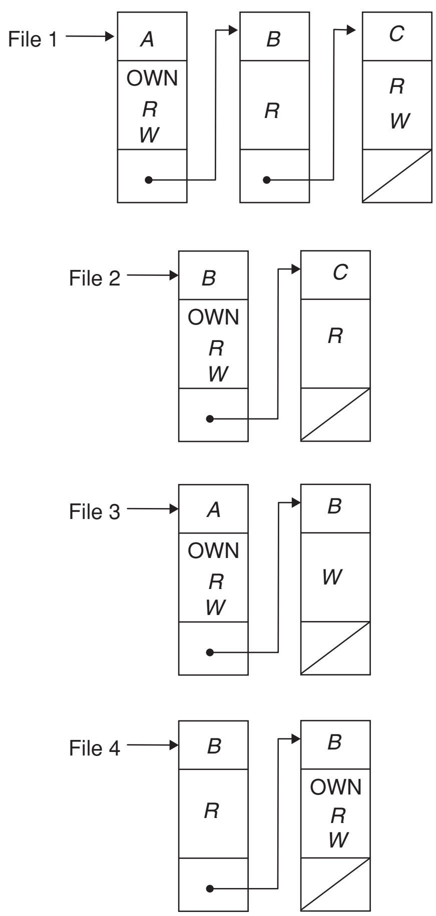
File System Security

1. A general model of access control for file management is an access matrix.
2. The basic elements of the model are as follows:
   - *Subject*: An entity capable of accessing objects.
   - *Object*: Anything to which access is controlled.
   - *Access right*: The way in which an object is accessed by a subject.

Access matrix format is shown below:

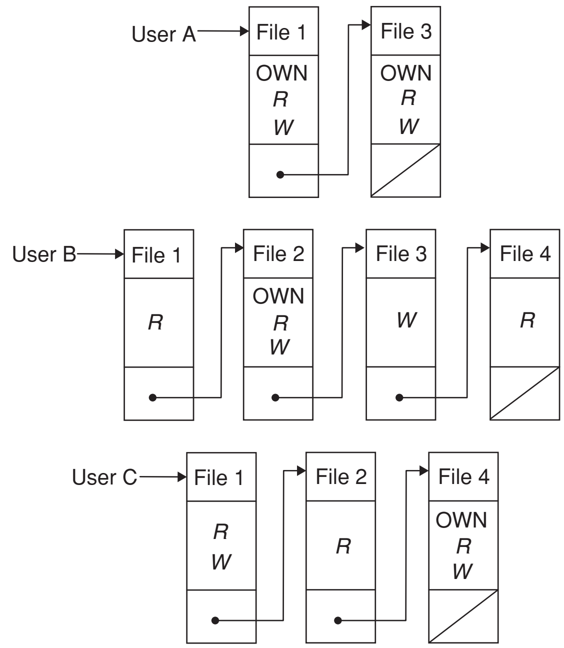
	File 1	File 2	File 3	File 4	Account 1	Account 2
User A	OWN R W		OWN R W		Inquiry credit	
User B	R	OWN R W	W	R	Inquiry debit	Inquiry credit
User C	R W	R		OWN R W		Inquiry Debit

The access matrix may be decomposed by columns, yielding access control lists. The access control list for above matrix is



Note: This allows users that are not explicitly listed as having special rights to have a default set of rights.

Decomposition by row yields. Capability tickets.



Note: These tickets would have to be held in a region of memory inaccessible to users.

System security This is the protection afforded to an automated information system in order to attain objectives of preserving the integrity, availability and confidentiality of information system resources.

Objectives of Computer Security

Confidentiality Preserving authorized restrictions on information access and disclosure, including means for protecting personal privacy and proprietary information.

Integrity Guarding against improper information modification or destruction, including ensuring information non-repudiation and authenticity.

Availability Ensuring timely and reliable access to and use of information.

Authenticity The property of being genuine and being able to be verified and trusted.

Accountability The security goal that generates the requirement for actions of an entity to be traced uniquely to that entity.

Threats, Attacks and Assets

Threats and attacks *Unauthorized disclosure* is the threat to confidentiality. The attacks of following this threat are as follows:

1. *Exposure*: Sensitive data are directly released to an unauthorized entity.
2. *Interception*: An unauthorized entity directly accesses sensitive data traveling between authorized sources and destinations.

3. *Inference*: A threat action where an unauthorized entity indirectly accesses sensitive data by reasoning from characteristics or by-products of communications.
4. *Intrusion*: An unauthorized entity gains access to sensitive data by circumventing a system's security protections.

Deception Threat to either system integrity or data integrity. Types of attacks that can result are as follows:

1. *Masquerade*: An unauthorized entity gains access to a system or performs a malicious act by posing as an authorized entity.
2. *Falsification*: False data deceive an authorized entity.
3. *Repudiation*: An entity deceives another by falsely denying responsibility for an act.

Disruption A circumstance or event that interrupts or prevents the correct operation of system services and functions. Attacks for this threat are as follows:

1. *Incapacitation*: Prevents or interrupts system operation by disabling a system component.
2. *Corruption*: Undesirably alters system operation by adversely modifying system functions or data.
3. *Obstruction*: A threat action that interrupts delivery of system service by hindering system operation.

Usurpation A circumstance or event that results in control of system services or functions by an unauthorized entity. Attacks with this threat are as follows:

1. *Misappropriation*: An entity assumes unauthorized logical or physical control of a system resource.
2. *Misuse*: Causes a system component to perform a function or service that is detrimental to system security.

Threats and assets The assets of a computer are as follows:
1. Hardware
2. Software
3. Data
4. Communication lines

Hardware A major threat to computer system hardware is the threat to availability (e.g., theft of CD-ROMS).

Software
1. A key threat to software is an attack on availability (e.g., deletion of software).
2. A threat to integrity.
3. A threat to confidentiality.

Data Threats to data are an attack on
1. Availability
2. Confidentiality
3. Integrity

Communication lines and networks Two types of attacks:
1. Passive attacks
2. Active attacks

Passive attacks
1. These are in the nature of monitoring of the transmissions.
2. Attackers obtain information that is being transmitted.
3. Two types of passive attacks:
   - Release of message contents.
   - Traffic analysis
4. These are very difficult to detect because they do not involve any alteration of the data.

Active attacks
1. These attacks involves some modification of the date stream or the creation of a false stream and can be subdivided into four categories:
   - Replay
   - Masquerade
   - Modification of messages
   - Denial of service
2. It is difficult to prevent active attacks absolutely.

Intruders Three types of intruders:

Masquerader An individual who is not authorized to use the computer and who penetrates a systems access controls to exploit a legitimate user's account. He is likely to be an outsider.

Misfeasor A legitimate user who accesses data, programs or resources for which such access is not authorized or who is authorized for such access but misuses his or her privileges (generally an insider).

Clandestine user An individual who seizes supervisory control of the system and uses this control to evade auditing and access controls or to suppress audit collection (either outsider or insider).

Hackers Those who hack into computers do so for the thrill of it or for status. Attackers often took for targets of opportunity and then share the information with others.

Criminals Organized group of hackers have become a widespread and common threat to internet based systems.

Malicious software overview The most sophisticated types of threats to computer systems are presented by programs that exploit vulnerabilities in computing systems. These threats are referred as malicious software (or) malware.

1. It is designed to cause damage to or use up the resources of a target computer.
2. There are two types of malicious software:
   - Those that need a host program.
     Example: Viruses, logic bombs.
   - Those that are independent.
     Example: Worms, bot programs.
3. We can also differentiate between two types of software threats:
   - That do not replicate. These programs are activated by a trigger.
     Example: Logic bombs, backdoors.

- Those that replicate.
  Example: Viruses, worms

Backdoor (trapdoor) It is a secret entry point into a program that allows someone who is aware of the backdoor to gain access without going through usual security access procedures.

Logic bomb It is a program inserted into software by an intruder. It lies dormant until a predefined condition is met; the program then triggers an unauthorized act.

Trojan Horse It is an apparently useful program, containing hidden code that, when invoked, performs some unwanted or harmful function.

Mobile code It is a software that can be shipped unchanged to a heterogeneous collection of platforms and execute with identical semantics.

Viruses A computer virus is a piece of software that can infect other programs by modifying them.

Nature of Viruses

A virus can do anything that other programs do. It attaches itself to another program and executes secretly when the host program is run. Three parts of computer virus are as follows:

1. Infection mechanism
2. Trigger
3. Payload

Phases of computer virus are

1. Dormant phase
2. Propagation phase
3. Triggering phase
4. Execution phase

Types of Virus

1. Encrypted virus
2. Stealth virus
3. Polymorphic virus
4. Metamorphic virus

Worms A worm is a program that can replicate itself and send copies from computer to computer across network connections.

Bots A bot is a program that secretly takes over another internet attached computer and then uses that computer to launch attacks that are difficult to trace to the bot's creator.

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Given a system using unspanned blocking and 100 byte blocks. A file contains records 30, 40, 55, 80, 30, 40. What percentage of space will be wasted in the blocks allocated for the file?
(A) 31.25% (B) 41.25%
(C) 51.25% (D) 62.15%

2. Disk requests come into the disk driver for cylinders 15, 25, 10, 2, 35, 9, 42 in that order. The disk head is currently positioned over cylinder 15. A seek takes 6 msec per cylinder moved. What is the total seek time using First Come First Served Algorithm?
(A) 750 msec (B) 650 msec
(C) 550 msec (D) 450 msec

3. A Java application needs to load 50 libraries. To load each library, one disk access is required. Seek time to access the location is 10 ms. Rotational speed is 6000 rpm. The total time needed to load all libraries is
(A) 0.65 sec (B) 0.75 sec
(C) 0.85 sec (D) 1 sec

4. A program has just read the 13th record in a sequential access file. If it wants to read the 10th record next, how many records must the program read to input the tenth record?
(A) 5 (B) 0
(C) 10 (D) 13

5. A disk is formatted into 40 sectors and 20 tracks. The disk rotates at 200 ms in one revolution. The time taken by the head to move from the centre to the rim is 10 ms. There are three different files stored on the disk:
File P : Sector 2, track 4
File Q : Sector 5, track 1
File R : Sector 6, track 2
Calculate the average latency time required for the three files.
(A) 22.55 ms (B) 32.22 ms
(C) 21.66 ms (D) 30.22 ms

6. Match the following

(a) RAID0	(1) Parallel access
(b) RAID1	(2) Striping
(c) RAID2	(3) Use hamming code
(d) RAID3	(4) Mirrored

(A) a – 2, b – 4, c – 3, d – 1
(B) a – 1, b – 2, c – 3, d – 4
(C) a – 3, b – 2, c – 4, d – 1
(D) a – 4, b – 1, c – 2, d – 3

7. The correct matching for the following pairs is

(A) Disk scheduling	(1) Round Robin
(B) Batch processing	(2) SCAN
(C) Time sharing	(3) LIFO
(D) Interrupt processing	(4) FIFO

(A) A – 3, B – 4, C – 2, D – 1
(B) A – 4, B – 3, C – 2, D – 1
(C) A – 2, B – 4, C – 1, D – 3
(D) A – 2, B – 1, C – 4, D – 3

8. A program P reads and processes 2000 consecutive records from a sequential file stored on device R without using any file system facilities. Given the following:
Size of each record = 3500 bytes
Access time of R = 20 ms
Data transfer rate of $R = 500 \times 10^3$ bytes/sec
CPU time to process each record = 5 m sec
What is the elapsed time of P if R contains unblocked records and P does not use buffering?
(A) 64 sec (B) 46 sec
(C) 34 sec (D) 17 sec

9. A disk has 19456 cylinders, 16 heads and 63 sectors per track. The disk spins at 5400 rpm. Seek time between adjacent tracks is 2 ms. Assuming the read/write head is already positioned at track 0, how long does it take to read the entire disk?
(A) 35 min (B) 68 min
(C) 58 min (D) 53 min

10. On a disk with 1000 cylinders, numbered 0 to 999, compute the number of tracks the disk arm must move to satisfy all the requests in the disk queue using SCAN algorithm. Assume the last request serviced was at track 345 and the head is moving towards 0. The queue in FIFO order contains requests for the following tracks:
123, 874, 692, 475, 105, 376
(A) 219 (B) 635
(C) 845 (D) 1219

Common data for questions 11 to 14: In the operation of a certain disk drive mechanism, a disk is formatted into 20 sectors and 10 tracks. The disk can be rotated either clockwise or anti-clockwise. The times required to perform certain operations are as follows:

I. Rotate the disk through one revolution = 200 ms
II. Move the disk head from the centre to the rim = 20 ms
III. Read and transmit one block of data = 0.3 ms

Three files are stored on the disk:
File A: 2 blocks at track 6
File B: 5 blocks at track 2
File C: 1 block at track 5

11. The disk head is initially at sector 0, track 0. If all three files A, B and C are to be read in the minimum amount of time, they should be read in the following order:
(A) A, B, C (B) A, C, B
(C) B, C, A (D) C, A, B

12. The disk head is initially at sector 0, track 0. The files are read in the order C, B, A. The total time to read the files is
(A) 143.9 ms (B) 100.4 ms
(C) 114.0 ms (D) 102.6 ms

13. The most nearly average latency time for the sequence CBA is:
(A) 1 ms (B) 7 ms
(C) 27 ms (D) 50 ms

14. The most nearly average seek time for CBA is
(A) 1 ms (B) 8 ms
(C) 30 ms (D) 50 ms

15. A CD has 150 tracks rotating at 3500 rpm. Average seek time for consecutive tracks is 0.1 ms, the disk is subjected to read data from the track numbers 89, 75, 112, 5. What is the total seek time if the requests are served unidirectionally (C-Scan) and the first request determines initial direction? Assume that the current position of the head is at track 100.
(A) 20.2 ms (B) 16.9 ms
(C) 21.3 ms (D) 14 ms

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. The strategy that allocates the smallest possible chunk of disk space that is sufficient to the file is
(A) Nearest fit (B) Best fit
(C) Worst fit (D) First fit

2. If a process of 200 kB is transferred from backing store to memory and average disk latency is 10 ms, then what would be the total swap time, if transfer ratio is 2 Mbps?
(A) 10 ms
(B) 20 ms
(C) 30 ms
(D) 40 ms

3. Let us assume that the user process is 10 MB in size and backing store is a standard hard disk with a transfer rate of 40 MB per second. Let the average latency is 8 millisecond. Find actual time transfer of the 10 MB process to or from main memory?
(A) 8 ms (B) 250 ms
(C) 258 ms (D) 516 ms

4. Disk scheduling involves deciding
(A) which disk should be accessed next
(B) the order in which disk access requests must be serviced.
(C) the physical location when files should be accessed in the disk
(D) disk access time and an unused space.

5. The root directory of a disk should be placed
   (A) at a fixed address in the memory
   (B) anywhere on the disk
   (C) at a fixed location on the system disk
   (D) at a location on floppy.
6. Direct access methods are not effectively supported by
   (A) Contiguous allocation
   (B) Linked allocation
   (C) Indexed allocation
   (D) Sequential allocation
7. In which of the following directory systems, it is possible to have multiple paths for a file, starting from the root directory?
   (A) Single-level directory
   (B) Two-level directory
   (C) Tree-structured directory
   (D) A cyclic graph directory
8. The most common system's security method is:
   (A) Passwords
   (B) Key card systems
   (C) Surveillance system
   (D) Lock system
9. Trojan Horse programs
   (A) are legitimate programs that allow unauthorized access.
   (B) are hacker programs that do not show up on the system
   (C) really do not work
   (D) are immediately discovered
10. Which of the following is a program that spreads throughout the network?
    (A) Trojan Horse (B) Virus
    (C) TSR (D) Worm
11. A program has just read the 15th record in a sequential access file. If it wants to read the 10th record next, how many records must the program read to input the tenth record?
    (A) 0 (B) 5
    (C) 4 (D) 10
12. Formatting of a floppy disk refers to
    (A) Arranging the data on the disk in contiguous fashion
    (B) Writing the directory
    (C) Erasing the system area
    (D) Writing identification information on all tracks
13. Sector interleaving in disks is done by
    (A) the disk manufacturer
    (B) the disk controller card
    (C) the operating system
    (D) the user
14. Disk I/O is done in terms of
    (A) Tracks
    (B) Blocks
    (C) Bits
    (D) Bytes
15. How many six-letter passwords can be constructed using lowercase letters and digits?
    (A) 26^6 (B) 10^6
    (C) 36^6 (D) 35^6

Previous Years' Questions

1. Consider a disk drive with the following specifications: **[2005]**
   16 surfaces, 512 tracks/surfaces, 512 sectors/track, 1 KB/sector, rotation speed 3000 rpm. The disk is operated in cycle stealing mode whereby whenever one 4 byte word is ready it is sent to memory; similarly, for writing, the disk interface reads a 4 byte word from the memory in each DMA cycle. Memory cycle time is 40 nsec. The maximum percentage of time that the CPU gets blocked during DMA operation is:
   (A) 10 (B) 25
   (C) 40 (D) 50
2. Consider a disk pack with 16 surfaces, 128 tracks per surface and 256 sectors per track. 512 bytes of data are stored in a bit serial manner in a sector. The capacity of the disk pack and the number of bits required to specify a particular sector in the disk are, respectively: **[2007]**
   (A) 256 Mbyte, 19 bits
   (B) 256 Mbyte, 28 bits
   (C) 512 Mbyte, 20 bits
   (D) 64 Gbyte, 28 bits
3. For a magnetic disk with concentric circular tracks, the seek latency is not linearly proportional to the seek distance due to **[2008]**
   (A) non-uniform distribution of requests
   (B) arm starting and stopping inertia
   (C) higher capacity of tracks on the periphery of the platter
   (D) use of unfair arm scheduling policies
4. Consider a file of 16384 records. Each record is 32 bytes long and its key field is of size 6 bytes. The file is ordered on a non-key field, and the file organization is unspanned. The file is stored in a file system with block size 1024 bytes, and the size of a block pointer is 10 bytes. If the secondary index is built on the key field of the file, and a multi-level index scheme is used to store the secondary index, the number of first-level and second-level blocks in the multi-level index are, respectively **[2008]**

(A) 8 and 0 (B) 128 and 6
(C) 256 and 4 (D) 512 and 5

5. Consider a disk system with 100 cylinders. The requests to access the cylinders occur in following sequence: **[2009]**

4, 34, 10, 7, 19, 73, 2, 15, 6, 20

Assuming that the head is currently at cylinder 50, what is the time taken to satisfy all requests if it takes 1ms to move from one cylinder to adjacent one and shortest seek time first policy is used?

(A) 95 ms (B) 119 ms
(C) 233 ms (D) 276 ms

Common data for questions 6 and 7: A hard disk has 63 sectors per track, 10 platters each with 2 recording surfaces and 1000 cylinders. The address of a sector is given as a triple $\langle c, h, s \rangle$, where c is the cylinder number, h is the surface number and s is the sector number. Thus, the 0th sector is addressed as $\langle 0, 0, 0 \rangle$, the 1st sector as $\langle 0, 0, 1 \rangle$, and so on

6. The address <400, 16, 29> corresponds to the sector number: **[2009]**

(A) 505035 (B) 505036
(C) 505037 (D) 505038

7. The address of the 1039th sector is **[2009]**

(A) $\langle 0, 15, 31 \rangle$ (B) $\langle 0, 16, 30 \rangle$
(C) $\langle 0, 16, 31 \rangle$ (D) $\langle 0, 17, 31 \rangle$

8. A file system with 300 GByte disk uses a file descriptor with 8 direct block addresses, 1 indirect block address and 1 doubly indirect block address. The size of each disk block is 128 bytes and the size of each disk block address is 8 bytes. The maximum possible file size in this file system is **[2012]**

(A) 3 Kbytes
(B) 35 Kbytes
(C) 280 Kbytes
(D) dependent on the size of the disk

9. Consider a hard disk with 16 recording surfaces (0–15) having 16384 cylinders (0–16383) and each cylinder contains 64 sectors (0–63). Data storage capacity in each sector is 512 bytes. Data are organized cylinder-wise and the addressing format is <cylinder no., surface no., sector no.>. A file of size 42797 KB is stored in the disk and the starting disk location of the file is <1200, 9, 40>. What is the cylinder number of the last sector of the file, if it is stored in a contiguous manner? **[2013]**

(A) 1281 (B) 1282
(C) 1283 (D) 1284

10. A FAT (File allocation table)-based file system is being used, and the total over head of each entry in the FAT is 4 bytes in size. Given a 100×10^6 bytes disk on which the file system is stored and data block size is 10^3 bytes, the maximum size of a file that can be stored on this disk in units of 10^6 bytes is ______? **[2014]**

11. Consider a disk pack with a seek time of 4 milliseconds and rotational speed of 10000 rotations per minute (RPM). It has 600 sectors per track and each sector can store 512 bytes of data. Consider a file stored in the disk. The file contains 2000 sectors. Assume that every sector access necessitates a seek, and the average rotational latency for accessing each sector is half of the time for one complete rotation. The total time (in milliseconds) needed to read the entire file is ________. **[2015]**

12. Suppose the following disk request sequence (track numbers) for a disk with 100 tracks is given: 45, 20, 90, 10, 50, 60, 80, 25, 70. Assume that the initial position of the R/W head is on track 50. The additional distance that will be traversed by the R/W head when the Shortest Seek Time First (SSTF) algorithm is used compared to the SCAN (Elevator) algorithm (assuming that SCAN algorithm moves towards 100 when it starts execution) is ______ tracks. **[2015]**

13. Consider a typical disk that rotates at 15000 rotations per minute (RPM) and has a transfer rate of 50×10^6 bytes/sec. If the average seek time of the disk is twice the average rotational delay and the controller's transfer time is 10 times the disk transfer time, the average time (in milliseconds) to read or write a 512-byte sector of the disk is ________ **[2015]**

14. Consider a disk queue with requests for I/O to blocks on cylinders 47, 38, 121, 191, 87, 11, 92, 10. The C-LOOK scheduling algorithm is used. The head is initially at cylinder number 63, moving towards larger cylinder numbers on its servicing pass. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is _____. **[2016]**

15. In a file allocation system, which of the following allocation scheme(s) can be used if no external fragmentation is allowed? **[2017]**

I. Contiguous
II. Linked
III. Indexed

(A) I and III only (B) II only
(C) III only (D) II and III only

16. Consider a storage disk with 4 platters (numbered as 0, 1, 2 and 3), 200 cylinders (numbered as 0, 1, ..., 199), and 256 sectors per track (numbered as 0, 1, ..., 255). The following 6 disk requests of the form [sector number, cylinder number, platter number] are

received by the disk controller at the same time:

[120, 72, 2], [180, 134, 1], [60, 20, 0], [212, 86, 3], [56, 116, 2], [118, 16, 1]

Currently the head is positioned at sector number 100 of cylinder 80, and is moving towards higher cylinder numbers. The average power dissipation in moving the head over 100 cylinders is 20 milliwatts and for reversing the direction of the head movement once is 15 milliwatts. Power dissipation associated with rotational latency and switching of head between different platters is negligible.

The total power consumption in milliwatts to satisfy all of the above disk requests using the Shortest Seek Time First disk scheduling algorithm is _________. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** A	**3.** B	**4.** C	**5.** C	**6.** A	**7.** C	**8.** A	**9.** C	**10.** D
11. C	**12.** B	**13.** C	**14.** B	**15.** A					

Practice Problems 2

1. B	**2.** C	**3.** D	**4.** B	**5.** C	**6.** D	**7.** C	**8.** A	**9.** A	**10.** D
11. D	**12.** D	**13.** C	**14.** B	**15.** C					

Previous Years' Questions

1. B	**2.** A	**3.** C	**4.** C	**5.** B	**6.** C	**7.** C	**8.** B	**9.** D	**10.** 99.6
11. 14020		**12.** 10	**13.** 6.1 to 6.2		**14.** 346	**15.** D	**16.** 85		

Test

Operating System

Time: 60 min.

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. A program is _____ entity, while a process is _____ entity.
   (A) Active, passive
   (B) Active, sometimes active
   (C) Passive, active
   (D) Both (B) and (C)
2. All the information associated with a specific process is contained in:
   (A) Process control block
   (B) Program control block
   (C) TLB
   (D) Heap
3. Kernel-level threads and user-level threads are supported, respectively, by _____.
   (A) Operating system and operating system
   (B) Operating system and user
   (C) User and user
   (D) None of these
4. Which of the following is false about user-level threads?
   (A) User-level threads are visible to the programmer and are unknown to the Kernel.
   (B) These are faster to create.
   (C) Kernel never interferes.
   (D) There is no effect of a system call () on process.
5. Which of the following interprocess communication models are implemented using system calls?
   (A) Shared memory
   (B) Message passing
   (C) Both (A) and (B)
   (D) Neither (A) nor (B)
6. Peterson's solution
   (i) is restricted to two processes
   (ii) share two data items turn and flag [i]
   (iii) mutual exclusion is achieved
   (iv) is a hardware solution

   Which of the above are true?
   (A) (i), (ii), (iii)
   (B) (ii), (iii), (iv)
   (C) (iv), (i), (ii)
   (D) (i), (ii), (iii), (iv)
7. Which of the following requires a mode switch from one thread to another?
   (A) One process multiple thread
   (B) User-level thread
   (C) Kernel-level threads
   (D) Both (B) and (C)
8. When a process is created, its state is
   (A) New
   (B) Ready
   (C) Block
   (D) Suspend
9. The data section of a process in memory contains
   (A) Local variables, function parameters
   (B) Return addresses
   (C) Global variables
   (D) None of the above
10. Which one of the following is true about process states?
    (i) A process which is running must have terminated as next state.
    (ii) From running state process can go to either waiting, ready or terminated state.
    (iii) Only one process can run at any instant.
    (iv) Ready process can go to waiting state.

    (A) (i), (ii), (iii)
    (B) (ii) and (iii) only
    (C) (i) and (iv) only
    (D) (ii), (iii), (iv)
11. Message passing model of inter process communication can be
    (A) Blocking only
    (B) Blocking and non-blocking
    (C) Synchronous and asynchronous
    (D) Both (B) and (C)
12. The definition of wait( ) is as follows:
    ```
    wait (S) {
    while (S <=0);
    S - - ;
    }
    ```
    The semicolon after while statement, signifies
    (A) Infinite looping
    (B) Blank statement
    (C) Depends on interpretation of compiler
    (D) No operation
13. To avoid race condition, the number of processes using the critical sections is/are:
    (A) 1 (B) 2
    (C) 3 (D) More than 3
14. The 'Critical Section' is the region in which
    (A) Any number of processes can enter without any permission
    (B) Only one process enters at a time and others wait for it.
    (C) Section is very critical
    (D) None of these

15. What does process control block contain?
(A) Process Identification
(B) Process state information
(C) Process control information
(D) All of the above

16. Match the following

(i) Multiprogramming	(x) Managing multiple processes executing on multiple computers
(ii) Multiprocessing	(y) Management of multiple processes within a uniprocessor system.
(iii) Distributed process Management	(z) Management of multiple processes within a multiprocessor.

(A) (i) –y (ii) –z (iii) –x
(B) (i) –z (ii) –x (iii) –y
(C) (i) –y (ii) –x (iii) –z
(D) Ambiguous

17. For '*n*' number of fork( ) system call, how many parent and child processes will be created?
(A) 1, $2^n - 1$, respectively
(B) 1, 2^n, respectively
(C) $2^n - 1$, 1, respectively
(D) n, $2n$, respectively

18. If the value of binary semaphore is initialized with 1 and three wait( ) operations are performed, how many processes are there in the block list?
(A) 1 (B) 0
(C) 3 (D) 2

19. A counting semaphore is initialized with the value 3. A list of '*P*' and '*V*' operations are performed on the semaphore as: 1*P*, 2*V*, 2*P*, 3*V*, 5*P*, 7*V*, 2*P*, 3*V*.
The final value of semaphore is?
(A) 5 (B) 8
(C) 7 (D) 6

20. The final value of semaphore after 10 '*P*' operations and 23 '*V*' operations is 1. What will be the initial value of this counting semaphore?
(A) –14 (B) –13
(C) –12 (D) –11

21. For a machine-instruction approach to enforce mutual exclusion, following are its properties:
(i) starvation and deadlock free
(ii) it is applicable to any number of processes.
(iii) it can be used to support multiple critical sections, each defined by its own variable.
(iv) it is simple, easy to verify and employed with busy waiting

Which of the above is false?
(A) (iv) only (B) (ii), (iii) only
(C) (i), (ii) only (D) (i) only

22. Consider the following code:

```
if (fork( ) == 0)
{
a = a + 5;
printf("%d,%d\n",a,&a);
}
else
{
a = a - 5;
printf("%d,%d\n",a,&a);
}
```

Let p, q be the values printed by the parent process, and s, t be the values printed by the child process. Which one of the following is true?
(A) $p = s + 10$ and $q = t$
(B) $p = s + 10$ and $q \uparrow t$
(C) $p + 10 = s$ and $q = t$
(D) $p + 10 = s$ and $q \uparrow t$

23. Consider the following statements with respect to user-level threads:
(i) Context switch is faster with kernel-supported threads.
(ii) For user-level threads, a system call can block the entire process.
(iii) Kernel-supported threads can be scheduled independently.
(iv) User-level threads are transparent to the Kernel.

Which of the above statements are true?
(A) (i), (iii) and (iv) only
(B) (ii) and (iii) only
(C) (i) and (iii) only
(D) (i) and (ii) only

24. Suppose there are '*n*' CPUs and '*m*' processes such that $m > n$. What will be the minimum and maximum number of ready, running and blocked process, respectively?
(A) 0, 0, 0 and m, n, m (B) 1, m, 1 and n, n, n
(C) m, 1, 0 and m, m, n (D) 0, 0, 0 and n, m, m

25. Consider the following signal semaphore code signal (semaphore *s)

```
{
s.value++;
if( ____(I)_____ )
{
    remove a process P from S.list;
    ____(II)_____
    }
}
```

Choose the suitable options for (I) and (II), respectively
(A) S.value = 0 and wakeup(P);
(B) S.value <= 0 and wakeup(P);
(C) S.value <0 and block( );
(D) S.value <= 0 and block( );

26. Consider the methods used by processes P_1 and P_2 for accessing their critical sections whenever needed. The initial values of shared Boolean variables S_1 and S_2 are randomly assigned.

Method used by P_1

While ($S_1 = = S_2$);
Critical section
$S_1 = S_2$;

Method used by P_2

While ($S_1 \,!= S_2$);
Critical section
$S_2 = !(S_1)$;

Which of the following statements describes the properties achieved?

(A) Mutual exclusion but not progress
(B) Progress only
(C) Bounded waiting, progress
(D) Mutual exclusion, progress, bounded waiting

27. Consider the following statements regarding spin locks:

(i) No context switch is required when a process wait on a lock
(ii) Spin locks are useful when locks are expected to be held for short times.
(iii) They are often employed on multiprocessor systems
(iv) Process 'spins' while waiting for a lock

Choose the correct option:

(A) (i), (ii), (iii), (iv) are true
(B) Only (i) and (ii) are true
(C) (iii) is false
(D) (ii) is true and (iv) is false

Common data for questions 28, 29 and 30: From the Readers-Writers problem, the data structure for reader process is:

```
semaphore mutex, wrt;
int readcount;
```

```
while(1)
{
wait(mutex);
readcount++;
if(readcount == 1)
wait(wrt);
signal(mutex);

_ _ _ _ _

_ _ _ _ _
wait(mutex);
readcount - -;
if(readcount ==0)
signal(wrt); signal(mutex);
}
```

mutex and wrt are initialized to 1 and readcount is initialized to 0.

28. Mutual exclusion for readers is attained by

(A) Wrt
(B) Mutex
(C) Readcount
(D) Both (A) and (B)

29. Which of the following semaphore or semaphores is used by the first or last reader that enters or exits the critical section?

(A) Wrt
(B) Mutex
(C) Readcount
(D) Both (A) and (B)

30. The readcount variable keeps track of how many processes are _______.

(A) Currently reading the object
(B) Currently writing the object
(C) Waiting in the queue
(D) Reading the shared data

Answer Keys

1. C	**2.** A	**3.** B	**4.** D	**5.** B	**6.** A	**7.** C	**8.** A	**9.** C	**10.** B
11. D	**12.** D	**13.** A	**14.** B	**15.** D	**16.** A	**17.** A	**18.** D	**19.** B	**20.** C
21. D	**22.** D	**23.** B	**24.** A	**25.** B	**26.** A	**27.** A	**28.** B	**29.** A	**30.** A

Operating Systems Test 1

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Which of the following service is provided by an Operating System?
   (i) Access to I/O Devices
   (ii) Controlled access to files
   (iii) System Access
   (iv) Accounting
   (A) (i), (ii) (B) (iii), (iv)
   (C) (i), (ii), (iii) (D) (i), (ii), (iii), (iv)
2. In which of the following modes, privileged instructions will be executed?
   (A) Kernel mode only
   (B) User mode only
   (C) Either kernel mode or user mode
   (D) execution of privileged instruction is independent of mode
3. In which of the following environment, the computer memory can hold three, four or more programs and a single CPU switches among all of them for execution?
   (A) Uniprogramming (B) Multiprogramming
   (C) Multiprocessing (D) Both (B) and (C)
4. What is the principle objective of Time sharing system?
   (A) Maximize processor use
   (B) Minimize response time
   (C) Maximize memory usage
   (D) Minimize CPU idle time
5. Which of the following will not be included in the Process Control Block of a process?
   (A) Process state (B) Program counter
   (C) Priority (D) None of the above
6. Which of the following best explains 'process spawning'?
   (A) The OS creates a process at the explicit request of another process.
   (B) The OS creates a process to perform a function on behalf of a user program.
   (C) A process is created when a user logs on to the system.
   (D) The OS creates a process as per its requirement.
7. For which of the following reason, a process will move from 'Running' state to 'Blocked' state?
   (A) Timeout (B) Dispatch
   (C) Suspend (D) Event wait
8. If a process is in secondary memory and is available for execution as soon as it is loaded into main memory, then it is in __________ state.
   (A) Ready (B) Blocked
   (C) Blocked/Suspend (D) Ready/Suspend
9. Which of the following will be a reason for Process Suspension?
   (A) Swapping
   (B) Interactive user request
   (C) Parent process request
   (D) All the above
10. Which of the following is TRUE?
    (A) The overhead involved in mode switch is more compared to process switch.
    (B) The overhead involved in process switch is more compared to mode switch.
    (C) The overhead involved in process switch is same as mode switch.
    (D) comparing the overhead involved in process switch and mode switch is irrelevant.
11. Viruses, Logic bombs and Backdoors are examples of
    (A) Parasitic malware
    (B) Self-replicated malware
    (C) Self-contained malware
    (D) Active malware
12. Which of the following details will be maintained by each thread, in a multithreaded environment?
    (i) Register state (ii) Priority
    (iii) Stack
    (A) (i), (ii) (B) (ii), (iii)
    (C) (i), (iii) (D) (i), (ii) and (iii)
13. Which of the following are advantages of using User-Level-Threads (ULT) over Kernel-Level-Threads?
    (i) There is no need of kernel mode privileges for thread switching.
    (ii) ULTs can run on any OS.
    (iii) A system call cannot block the entire process.
    (A) (i), (ii) (B) (i) only
    (C) (ii), (iii) (D) (i), (iii)
14. Consider a multiprogramming environment, in which two processes are running and one process is unaware of another process, then which of the following problems will occur?
    (A) Mutual exclusion (B) Deadlock
    (C) Starvation (D) All the above.
15. Which of the following will be (a) requirement(s) for mutual exclusion?
    (A) A process remains inside its critical section for a finite time only.
    (B) A process that halts in its non critical section must do so without interfering with other processes.
    (C) When no process is in a critical section, any process that requests entry to its critical section may be permitted to enter with some delay.
    (D) Both (A) and (B)

16. Identify the correct sequence of actions from the following, to read data with a Virtual address using TLB and cache (Assume TLB and cache has miss):
(i) Access the TLB
(ii) Read from cache
(iii) Access memory to get address
(iv) Access memory to read data.
(A) (i), (ii), (iii), (iv) (B) (i), (iii), (ii), (iv)
(C) (i), (ii), (iv), (iii) (D) (i), (iii), (iv), (ii)

17. A 2-Level page translation scheme has 4 K byte pages and 4 Byte page table entries. The virtual address has 32-bits. What is the number of bits required to access first level; second level page table entries and offset in a page respectively?
(A) 9, 9, 14 (B) 12, 12, 8
(C) 10, 10, 12 (D) 8, 12, 12

18. What is the effective access time for TLB with 90% hit rate, 1 ns to access TLB and 10 ns to access memory. (Assume a 2-level page table)
(A) 1 ns (B) 2 ns
(C) 3 ns (D) 4 ns

19. Consider the page reference string: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5. The number of page frames allocated is 3. Using FIFO, in which way, the number of page faults change by increasing the page frames allocated to 4.
(A) increases by two (B) decreases by two
(C) increases by one (D) decreases by one

20. A 512 KB file is stored contiguously on one disk track. Each track contains 1024 sectors, each sector is 512 bytes. Average seek time is 1 ms. And the rotational speed is 15000 rpm. What is the average access time to read the entire file?
(A) 2 ms (B) 4 ms
(C) 7 ms (D) 3 ms

21. Consider mapping a Virtual memory of 1 GB onto a physical memory organized into 256 page frames of 4 KB each. Assume that the smallest addressable unit is 1 byte. Then which of the following is TRUE?
I. The page table fit in the main memory
II. The frame table fit in a single page.
(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

22. Consider below program segment:
```
# include < stdio.h>
# include < unistd.h>
..................
int main ()
{
pid p;
p = fork ();
if (p = = 0)
{
fork ();
fork ();
for k ():
}
return 0;
}
```
Including the initial parent process, how many processes will be created?
(A) 7 (B) 15
(C) 8 (D) 9

23. Consider the following resource requests.
P_1 requests exclusive use of both R_1 and R_2
P_2 requests exclusive use of both R_2 and R_3
P_3 requests exclusive use of both R_3 and R_4
P_4 requests exclusive use of both R_4 and R_2
Resources are assigned in process request order. (P_1 is first). Which of the following is TRUE?
(A) Given system is in deadlock.
(B) There is a possibility of deadlock after P_1 finishes its execution.
(C) There is no deadlock in given system.
(D) After P_1, if P_2 executes then there is a possibility of deadlock.

24. Which of the following instructions can only be executed in kernel mode?
(i) Load Instruction
(ii) Modify PC register
(iii) Modify SP register
(iv) Modify the register that controls kernel/user mode.
(v) Direct access to I/O device.
(A) (i), (ii), (v) (B) (i), (iii), (iv)
(C) (iv), (v) (D) (ii), (iv)

Directions for questions 25 and 26: A computer with a 32-bit address uses a 2-level page table. The virtual address format is shown below:

1st-level page table	2nd-level page table	Off set

25. What is the page size?
(A) 1024 B (B) 2048 B
(C) 4096 B (D) 8192 B

26. How many pages are there?
(A) 2^9 (B) 2^{11}
(C) 2^{12} (D) 2^{20}

27. Suppose that a 32-bit virtual address is broken up into four fields p, q, r and s. The first three are used for a 3-level page table system. The fourth field, s is the offset. Then which of the following is TRUE?
(A) The number of pages depend on the total number of bits in p, q, r combined.
(B) The number of pages depend on the total number of bits in p, q, r and s combined.
(C) The number of pages depend on the split among the fields p, q, r.
(D) The number of pages depend on the split among the fields p, q, r and s.

28. A computer provides each process with 65, 536 bytes of address space. A particular program has a text size of 32768 bytes, a data size of 16386 bytes and a stack size of 15870 bytes. If a page may not contain parts of two different segments then which of the following is TRUE?
(A) The program will fit in the address space if the page size is 4096 bytes.
(B) The program will fit in the address space if the page size is 2048 bytes.
(C) The program will fit in the address space if the page size is 1024 bytes.
(D) The program will fit in the address space if the page size is 512 bytes.

29. The Newton–Raphson method is applied to compute a root of the equation $f(x) = x^4 - x^3 - x^2 - 21x + 18 = 0$. With $x_0 = 3.1$ as the initial solution, the method converges to an exact solution after how many iterations?
(A) 1 (B) 4
(C) 7 (D) 10

30. How many disk operations are needed to fetch the inode for the file Admin/OS/UNIX/File/src.p?
Assume that the inode for this root directory is memory but nothing else along the path is in memory. (Note: All directories fit in one disk block)
(A) 5 (B) 10
(C) 4 (D) 8

31. The beginning of a free space bitmap looks like below after the disk partition is first formatted:
1000 0000 0000 0000 ....
The system always searches for free blocks starting at the lowest numbered block. So after writing file *A*, which uses 5 blocks, the bit map looks like: 1111 1100 0000 0000 Then what will be the bit map after performing following
actions:
(i) File *B* of 6 blocks is written
(ii) File *A* is deleted
(iii) File *C* of 7 blocks is written
(iv) File *B* is deleted
(A) 1111 1111 0000 0000 ....
(B) 1111 1111 1111 1100 ......
(C) 1111 1100 0000 1100 ......
(D) 1000 0000 0111 1111 ....

32. Consider below features:
(i) This scheduler has more speed.
(ii) The scheduler has less control over the degree of multiprogramming.
(iii) The scheduler is minimal in time sharing systems.
Which scheduler has above features?
(A) Long-term scheduler
(B) Medium-term scheduler
(C) Short-term scheduler
(D) None of these

33. Consider a system consisting of four processes and single resource. The current status of the Claim and Allocation matrices are:

$$C = \begin{bmatrix} 4 \\ 3 \\ 10 \\ 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 \\ 2 \\ 4 \\ 2 \end{bmatrix}$$

What is the minimum number of units of the resource(s) needed to be available for this state to be safe?
(A) 0 (B) 1
(C) 2 (D) 3

Common Data Questions 34 and 35:
Consider two CPU scheduling algorithms for a single CPU: Round Robin scheduling and Shortest job First scheduling. Consider below five processes with arrival times and expected CPU time.

Process	Arrival time	Expected CPU Time
P1	0	15
P2	3	13
P3	5	9
P4	6	5
P5	18	8

34. What is the average waiting time using SJF?
(A) 5 (B) 10
(C) 13.6 (D) 15.4

35. What is the average waiting time using Round Robin scheduling with time quantum of 6 units?
(A) 36.2 (B) 18.4
(C) 12.6 (D) 23.4

Answer Keys

1. D	**2.** A	**3.** B	**4.** B	**5.** D	**6.** A	**7.** D	**8.** D	**9.** D	**10.** B
11. A	**12.** D	**13.** A	**14.** D	**15.** D	**16.** B	**17.** C	**18.** C	**19.** C	**20.** C
21. C	**22.** D	**23.** B	**24.** C	**25.** C	**26.** D	**27.** A	**28.** D	**29.** B	**30.** B
31. C	**32.** C	**33.** B	**34.** C	**35.** D					

Hints and Explanations

1. All those services are provided by OS. Choice (D)

2. Privileged instructions will be executed in kernel mode, in which protected areas of memory will be accessed. Choice (A)

3. In Uniprogramming, the memory holds only a single program. In multiprogram, the CPU executes more than one program in time interleaved fashion, which are present in memory. In multiprocessing, There will be more than one CPU executing more than one program. Choice (B)

4. The Time sharing system handles multiple interactive jobs, so that response time will be reduced. Choice (B)

5. PCB holds information about a process. Choice (D)

7. A process will be in blocked state if it is waiting for an event to occur. Choice (D)

10. Mode switch occurs without changing the state of process. Process switch requires some changes in its environment.

∴ Overhead of process switch is greater than mode switch. Choice (B)

11. In Parasitic malware, the fragments of programs do not exist independently Choice (A)

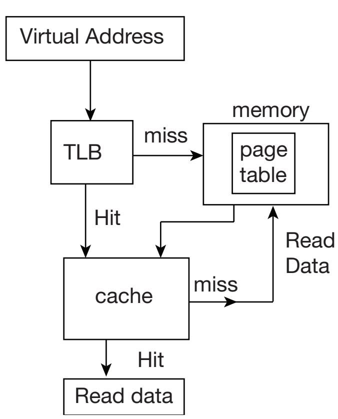
12. In multithreaded environment, each thread maintains a stack, register values, priority and other thread related state information. Choice (D)

13. In ULTs, the thread management will be done by thread library. So no need of kenel mode privileges for thread switching. On any OS, ULTs can run. A system call blocks entire process. Choice (A)

14. If processes are unaware of each other then competition exist between them. There is a possibility of Mutual exclusion, Deadlock and Starvation. Choice (D)

15. There will be no delay in permitting a process to enter its critical section if no process is in critical section. Choice (D)

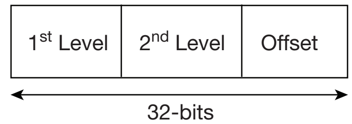
16.



So the sequence of actions will be (i), (iii), (ii), (iv) Choice (B)

17. Page size = 4 KB = 2^{12} B.

⇒ 12-bits required to identify a byte in a page.



Number of entries in 2nd level Page table $= \frac{2^{12}}{4} = 2^{10}$

∴ 10 bits required for 2nd level. Similarly for first level also 10-bits required. Choice (C)

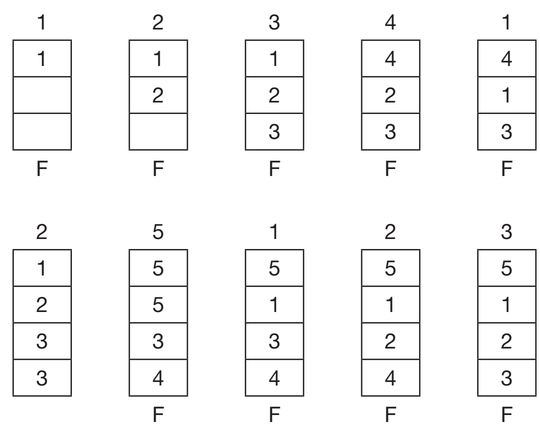
18. Effective TLB access time

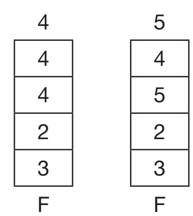
= hit ratio * TLB access time + (1-hit ratio) * (TLB access time + memory access time *2)

= 0.9 * 1 + (0.1) * (1 + 20) = 0.9 + 2.1 = 3 ns.

Choice (C)

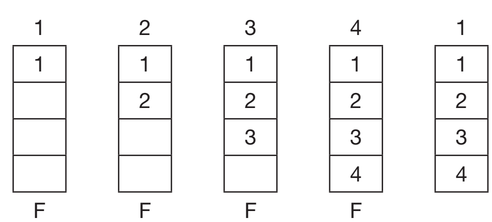
19. Initially, page frames allocated = 3





∴ Number of page faults = 9

Page frames Allocated = 4



2	5	1	2	3
1	5	5	5	5
2	2	1	1	1
3	3	3	2	2
4	4	4	4	3
	F	F	F	F

4	5
4	4
1	5
2	2
3	3
F	F

∴ Number of page faults = 10

∴ Number of page fault increases by one. Choice (C)

20. Seek time = 1 ms

Rotational speed = 15000 RPM.

15000 RPM = 15000/60 RPS = 250 rps

Rotational delay $= \dfrac{1}{2\times 250} = 2$ ms

Transfer time $= \dfrac{512\times 1024}{250\times 1024\times 512} = 4$ ms

∴ Total time = seek time + Rotational delay + Transfer time = 7 ms Choice (C)

21. Virtual memory = 1 GB = 2^{30} B

Page frames = 256

Page size = 4 KB

Physical memory = 256 * 4 K = 2^{20} B.

Number of pages $= \dfrac{2^{30}}{2^{12}} = 2^{18}$

Page table size will fit in main memory. (main memory = 2^{20} B, Page table size = 2^{18} * PTE,

PTE will be less than 4 B)

∴ I is true

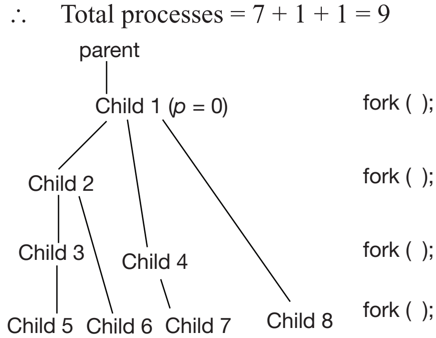
Frame table size = 256 * (8 + 18) = 6656 bits

This can fit in a page.

∴ It is true. Choice (C)

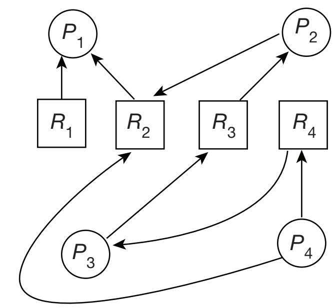
22. Fork () is used to create a child process *A* parent process returns the id of its children. A child process returns zero The first fork () call creates a child process. Only for child process $p = 0$, so child process only executes 3 fork() call statements, which creates 7 child processes.

∴ Total processes = 7 + 1 + 1 = 9



Choice (D)

23. The resource allocation graph of given system is shown below:



Initially P_1 completes and releases its resources. Now if P_4 acquires R_2, then P_2 waits for R_2, P_4 waits for R_4, P_3 waits for R_3. There is a circular wait. But if P_2 acquires R_2 then there will be no deadlock. Choice (B)

24. Load, modify *PC*, *SP* are not protected instructions. Modifying the register that controls kernel/user mode is a protected instruction. Otherwise any process can make itself run in kernel mode. I/O devices are only directly accessible by the *OS*. Choice (C)

25. Offset = 12 bits

Using 12-bits, we can access 2^{12} bytes. So the page size is 2^{12} = 4096 bytes. Choice (C)

26. Number of pages $= \dfrac{\text{Virtual memory}}{\text{Page size}}$

Number of pages $= \dfrac{2^{32}}{2^{12}} = 2^{20}$ Choice (D)

27. The number of pages depend on $p + q + r$. Choice (A)

28. Given total address space = 65536 bytes

Text size = 32768 bytes

Data size = 16386 bytes

Stack size = 15870 bytes

If page size = 4096

Total pages in memory $= \dfrac{65536}{4096} = 16$

Pages for text $= \dfrac{32768}{4096} = 8$

Pages for data $= \dfrac{16386}{4096} = 5$

Pages for stack $= \dfrac{15870}{4096} = 4$

Total pages required for process = 8 + 5 + 4 = 17

∴ Choice (A) is wrong.

Similarly if page size is 2048, the memory consists of 32 pages. If page size is 1024, the memory consists of 64 pages, but process requires 65 pages.

If page size = 512 bytes,

Memory consists of 128 pages and process requires 128 pages.

∴ Choice (D) is correct Choice (D)

29. The iterative formula for finding root of $f(x) = 0$ in Newton–Raphson method is

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

By using this formula, starting with $k = 0$ and $x_0 = 3.1$, it can be easily observed that $x_4 = 3$ (the exact root)

$\therefore$ After 4 iterations, the method converges to exact root. Choice (B)

30. As all directories fit in one disk block, then all required data on a directory is stored in the inode itself.

To open /Admin/OS/UNIX/file/src.p

We do following reads:

1. inode of /→ Already in memory
2. Directory for /
3. inode of /Admin
4. Directory of /Admin
5. inode of /Admin/OS
6. Directory of /Admin/OS
7. inode of /Admin/OS/UNIX
8. Directory of /Admin/ OS/UNIX/File
9. inode of /Admin/OS/UNIX/File
10. Directory of /Admin/ OS/UNIX/File
11. inode of /Admin/OS/UNIX/file /scr.p

$\therefore$ 10-disk read operations required. Choice (B)

31. Bitmap after writing A is

1111 1100 0000 0000 …..

File B (6 Blocks): $\underbrace{1111\ 11}_{A}\ \underbrace{11\ 1111}_{B}\ 0000$ .....

Delete A : 1000 0011 1111 0000 ……

File C (7 blocks): $\underbrace{1111\ 11}_{C}\ \underbrace{11\ 1111}_{B}\ \underbrace{1}_{C}100$ .....

Delete B: $\underbrace{1\,111\ 11}_{C}\ 000000\ \underbrace{1}_{C}10$ .....

$\therefore$ Final bitmap : 1111 1100 0000 1100 ... Choice (C)

32. A short-term scheduler selects the process which is ready to execute. It does not have that much of control over degree of multiprogramming Choice (C)

33.

$$\text{Allocation} = \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}\begin{bmatrix} 2 \\ 2 \\ 4 \\ 2 \end{bmatrix}$$

Total resources (R) = 10, Available (V) = 0

Need 1 more additional resource for execution of P_2.

$\Rightarrow$ $R = 11$, $V = 1$

Now p_2 can complete. And it releases its allocted resources.

$\Rightarrow$ $R = 11$, $V = 3$

p_1 requires 2 more resources. After execution it releases those 2 + allocated resources i.e., 4

$\Rightarrow$ $R = 11$, $V = 5$

Now p_4 can execute with the available and allocated resources.

$\Rightarrow$ $R = 11$, $V = 7$

Now p_3 can execute.

$\therefore$ One additional resource required.

Choice (B)

34. Shortest-job-First algorithm will select the shortest CPU burst time first.

P_1	P_4	P_5	P_3	P_2

0 15 20 28 37 50

Waiting time of $P_1 = 0$

Waiting time of $P_2 = 37 - 3 = 34$

Waiting time of $P_3 = 28 - 5 = 23$

Waiting time of $P_4 = 15 - 6 = 9$

Waiting time of $P_5 = 20 - 18 = 2$

$$\therefore \quad \text{Average waiting time } = \frac{(0+34+23+9+2)}{5}$$

$= 13.6$ units

Choice (C)

35. In Round Robin algorithm with time quantum 6 units, each job executed for 6 units and wait in waiting queue. The job with more waiting time will be executed first.

P_1	P_2	P_3	P_4	P_1	

0 6 12 18 23 29

P_2	P_5	P_3	P_1	P_5	

35 41 44 47 49

Waiting time of $P_2 = (6 - 3\ ) + (29 - 12) = 20$

Waiting time of $P_3 = (12 - 5\ ) + (41 - 18) = 30$

Waiting time of $P_4 = (18 - 6) = 12$

Waiting time of $P_5 = (\ 35 - 18) + (47 - 41) = 23$

$$\text{Average waiting time } = \frac{32+20+30+12+23}{5} = 23.4$$

Choice (D)

OPERATING SYSTEMS TEST 2

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices

1. Suppose we have variable logical records of length 10 bytes, 20 bytes and 30 bytes, while the physical block size in disk is 15 bytes. What is the maximum and minimum fragmentation seen in bytes?
(A) 10 and 5 (B) 10 and 0
(C) 5 and 0 (D) 5 and 5

2. Match the following:

List-A	List-B
1. Contiguous allocation policy	(i) Internal fragmentation
2. Chained list allocation	(ii) External Fragmentation
3. Indexed allocation	

(A) 1–(i), 2–(i), 3–(i) (B) 1–(i), 2–(i), 3–(ii)
(C) 1–(ii), 2–(i), 3–(i) (D) 1–(ii), 2–(ii), 3–(i)

3. Suppose that the scheduling algorithm favors those processes that have used the least processor time in the recent past. Then which of the following is TRUE?
(A) The algorithm favors I/O bound programs.
(B) The algorithm favors CPU bound programs.
(C) Fair treatment to all the programs.
(D) The algorithm favors longer programs.

4. Which of the following is not a necessary condition for deadlock?
(A) Mutual exclusion (B) Hold and wait
(C) Circular wait (D) Reentrancy

5. What will be the state of a process after it encounters an I/O instruction?
(A) Ready (B) Blocked
(C) Idle (D) Running

6. Which of the following is used by a program to request a service from an operating system's kernel that it does not normally have permission to run?
(A) Context switch (B) Threads
(C) System call (D) Service call

7. Which of the following is the reason for having threads within a process?
(i) Threads are lighter in weight than processes.
(ii) Threads have high performance when the processes have both I/O and CPU activity.
(iii) Threads are useful on systems with multiple CPU's.
(A) (i), (ii) (B) (ii), (iii)
(C) (i), (iii) (D) (i), (ii), (iii)

8. Which of the following is TRUE?
(A) No protection required between threads.
(B) Different threads in a process are always independent.
(C) All the threads cannot share open files, child processes.
(D) Each thread will maintain Accounting Information.

9. Which of the following is FALSE with respect to a monitor?
(A) Only one process can be active in a monitor at any instant.
(B) A monitor is a collection of procedures.
(C) Procedures declared outside the monitor can access monitor's internal data structure.
(D) Monitors can be used to provide mutual exclusion.

10. Which of the following is a reason for process scheduling?
(i) When a new process is created.
(ii) When a process exits.
(iii) When a process blocks.
(iv) When an interrupt occurs.
(A) (i), (ii) (B) (i), (iii), (iv)
(C) (ii), (iii) (D) (i), (ii), (iii), (iv)

11. Match the following:

List-A	List-B
1. First Come First Serve	I. High throughput
2. Shortest Process Next	II. Fair treatment of processes
3. Round Robin	III. Non-preemptive

(A) 1-II, 2-III, 3-II (B) 1-III, 2-I, 3-II
(C) 1-I, 2-III, 3-II (D) 1-I, 2-II, 3-III

12. To use process switching in hardware, instead of interrupts, the CPU needs to know about
(A) Process table data structure
(B) System calls
(C) CPU burst time
(D) PC only

13. Which of the following is FALSE?
(A) Throughput is the number of processes that complete their execution per unit time.
(B) Turnaround time is the amount of time required to execute a particular process.
(C) Waiting time is the amount of time, a process has been waiting in the ready queue.
(D) Response time is the amount of time taken to get the output.

14. Which of the following scheduling algorithm is also referred as preemptive version of FCFS?
(A) Shortest Job First
(B) Round Robin
(C) Shortest Remaining Time First
(D) None of these

15. Which of the following malware can result in pop-up ads or can redirect a browser to a commercial site?
(A) Spyware (B) Adware
(C) Zombie (D) Backdoor

16. In 48-bit machine, with 4 GB RAM and 8 KB page size, how many entries will be there in the page table if it is inverted?
(A) 2^{35} (B) 2^{20}
(C) 2^{19} (D) 2^{13}

17. Consider the following process and resource requirement of each process.

Process	Resource 1		Resource 2	
	Used	Max	Used	Max
P1	2	3	2	4
P2	2	4	2	3
P3	3	5	2	5

Assume that there are a total of 8 instances of resource type 1 and 7 instances of resource type 2. What is the state of this system?
(A) Can go to safe or unsafe state based on sequence.
(B) Safe state
(C) Unsafe state
(D) Deadlock state

18. Consider a system that has two CPU's and each CPU has two threads. Suppose three programs *P*1, *P*2 and *P*3 are started with run times 10, 15 and 25 ms respectively. What is the minimum time required to complete the execution of these programs?
(A) 25 ms (B) 35 ms
(C) 45 ms (D) 40 ms

19. A computer system has enough room to hold five programs in its main memory. These programs are blocked on I/O, half the time. What fraction of the CPU time is wasted?
(A) 96.8% (B) 32%
(C) 3.125% (D) 50%

20. A computer has 4 GB of RAM, of which the operating system occupies 256 MB. The processes are all 128 MB and have the same characteristics. If the goal is 95% CPU utilization, what is the maximum I/O wait that can be tolerated?
(A) 90.4% (B) 85.4%
(C) 72% (D) 50%

21. A file system with 4 KB blocks can access 64 GB worth of data through an i-node triple indirect block.
How many bits does the file system use for block pointers?
(A) 8 (B) 16
(C) 32 (D) 64

22. In a File Allocation table, each entry is of size 24-bits. For a 32 GB disk, what is the minimum size of a file allocation in this system?
(A) 1 KB (B) 2 KB
(C) 4 KB (D) 8 KB

23. On a system with 2^{64} bytes of memory and fixed partitions with a partition size of 2^{20} bytes, what is the minimum number of bits needed in an entry in the process table to record the partition to which a process has been allocated?
(A) 20-bits (B) 32-bits
(C) 36-bits (D) 44-bits

24. Which of the following features are required by an ideal CPU scheduling algorithm?
(i) Maximize the CPU utilization
(ii) Maximize the throughput
(iii) Minimize the turnaround time
(iv) Minimize the waiting time
(v) Minimize the response time
(A) (i), (iii), (v) (B) (ii), (iv), (v)
(C) (i), (ii), (iv), (v) (D) (i), (ii), (iii), (iv), (v)

25. Consider the following set of processes:

Process	Burst time	Priority
P1	10	3
P2	1	1
P3	2	3
P4	1	4
P5	5	2

The processes are assumed to have arrived in the order *P*1, *P*2, *P*3, *P*4 and *P*5 all at time 0. What is the average Turnaround time using Shortest Job First scheduling?
(A) 5 (B) 7
(C) 15 (D) 19

26. Consider the given concurrent processes:

```
Process P0:
while (true)
{
    while(turn = = 1);
    critical section
    turn = 1
    remainder section
}
Process P1:
    while (true)
{
        while (turn = = 0);
            critical section
        turn = 0
        remainder section
}
```

'turn' is a global Boolean variable, Which will take either 0 or 1. Which of the following is TRUE for above two processes?
(A) There is mutual exclusion and progress.
(B) There is progress but no mutual exclusion

(C) There is mutual exclusion but no progress.
(D) There is no mutual exclusion and no progress.

27. Consider 'n' concurrent processes P_1, P_2, ... P_n whose code is shown below:

```
var mutex : semaphore
mutex = 1;
process Pi:
    Repeat
        wait (mutex);
            critical section
        signal (mutex);
            Remainder section
        until false;
```

Then which of the following is TRUE?
(A) There is mutual exclusion.
(B) There is no mutual exclusion in the system.
(C) There is a deadlock in the system
(D) Both (B) and (C)

28. Which of the following correctly specifies the 'wait' operation of a counting semaphore?
(A) semvalue = semvalue – 1;
if semvalue < 0 then
Add this process to Blocked queue.
(B) semvalue = semvalue + 1 ;
if semvalue ≤ 0 then
Block the process;
(C) semvalue = semvalue – 1;
if semvalue ≤ 0 then
Block the process;
(D) semvalue = semvalue – 1;
if semvalue < 1 then Block the process

29. Identify the TRUE statements from the following:
(i) Logical address is generated by the CPU.
(ii) Physical address is the address seen by the memory unit.
(iii) The user program always deals with physical address.
(A) (i), (ii) (B) (i), (iii)
(C) (ii), (iii) (D) (i), (ii), (iii)

30. The correct sequence of steps required for page fault handling from the following is:
(i) Search in the page table.
(ii) Search for the page in the backing store.
(iii) Reset page table
(iv) Bring in missing page
(v) Restart instruction
(vi) The OS takes control on trap.
(A) (i), (ii), (iv), (v), (vi), (iii)
(B) (ii), (i), (v), (iv), (iii), (vi)
(C) (i), (vi), (ii), (iv), (v), (iii)
(D) (i), (vi), (ii), (iv), (iii), (v)

31. If the multiprogramming level increases rapidly then the processor utilization:
(A) always increases (B) always decreases
(C) decreases some times (D) doesn't effected

Common Data for Questions 32 and 33

Consider a swapping system in which memory consists of the following hole sizes in order: 11 KB, 5 KB, 21 KB, 19 KB, 8 KB, 10 KB, 13 KB and 16 KB.

32. Which hole is taken for successive segment requests 13 KB, 11 KB and 10 KB for First-fit?
(A) 13 KB, 11 KB, 10 KB (B) 21 KB, 11 KB, 19 KB
(C) 11 KB, 5 KB, 21 KB (D) 13 KB, 16 KB, 11 KB

33. Which hole is taken for successive segment requests 12 KB, 10 KB, 9 KB for Best-fit?
(A) 21 KB, 19 KB, 11 KB (B) 13 KB, 16 KB, 11 KB
(C) 13 KB, 10 KB, 11 KB (D) 13 KB, 11 KB, 10 KB

Common Data for Questions 34 and 35

A computer has four page frames. The time of loading, time of last access and modify (M) bits of each page are shown as below.

Page	Load Time	Last access time	M
0	149	198	1
1	255	280	0
2	85	293	0
3	129	285	1

34. Which page will FIFO algorithm replace next?
(A) page 0 (B) page 1
(C) page 2 (D) page 3

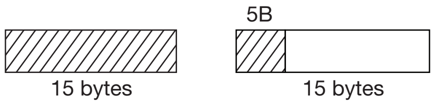
35. Which page will LRU algorithm replace next?
(A) page 0 (B) page 1
(C) page 2 (D) page 3

Answer Keys

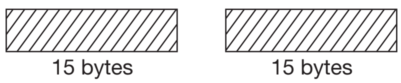
1. B	**2.** C	**3.** A	**4.** D	**5.** B	**6.** C	**7.** D	**8.** A	**9.** C	**10.** D
11. B	**12.** A	**13.** D	**14.** B	**15.** B	**16.** C	**17.** D	**18.** A	**19.** C	**20.** A
21. A	**22.** B	**23.** D	**24.** D	**25.** B	**26.** C	**27.** A	**28.** A	**29.** A	**30.** D
31. C	**32.** B	**33.** C	**34.** C	**35.** A					

Hints and Explanations

1. Record lengths are 10, 20, 30 Bytes
Physical block size = 15 bytes.
Maximum fragmentation occurs with 20 byte records.



$\therefore$ Maximum fragmentation = 10 bytes
Minimum fragmentation occurs with 30 byte records.



$\therefore$ Minimum fragmentation = 0 Bytes. Choice (B)

2. In Contiguous file Allocation, each file occupies a set of contiguous blocks on the disk. It suffers from external fragmentation. In linked and indexed allocation, there is internal fragmentation. Choice (C)

3. The algorithm favors I/O bound programs because of the relatively short CPU burst request by them. Choice (A)

4. The necessary and sufficient conditions for deadlock are Mutual exclusion, Hold-and-wait, No-preemption and Circular wait. Choice (D)

5. During I/O instruction execution, a process will be in 'Blocked' state. Choice (B)

6. A system call is used to get services of OS's kernel. Choice (C)

8. No protection is required between threads. (As it is impossible and not necessary). Threads are dependent on each other. Threads can share open files and child processes. Accounting information is maintained by process. Choice (A)

9. Procedures declared outside the monitor can't access monitor's internal data. Choice (C)

10. All the four are reasons for process scheduling. Choice (D)

11. FCFS, SPN both are non-preemptive. FCFS's throughput is not that much emphasized. It also penalizes short processes. SPN penalizes long processes. Its throughput is high. Round Robin is preemptive and it treats all processes fairly. Choice (B)

12. To use process switching, the CPU needs to know about process table and state. Choice (A)

13. Response time is the amount of time it takes from, when a request was submitted until the first response is produced. Choice (D)

14. Round Robin is the preemptive version of FCFS. Choice (B)

15. Advertising that is integrated into software is adware. Choice (B)

16. In inverted page table, the number of entries is the number of frames in the main memory.
Main memory capacity = 4 GB = 2^{32}B
Page size = frame size = 8 KB = 2^{13}B

$\therefore$ Number of entries $= \dfrac{2^{32}}{2^{13}} = 2^{19}$ Choice (C)

17. Total resources $(R) = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$

Total used resources $= \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

Available resources$(V) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$P1$ requires (1, 2) resources.
$P2$ requires (2, 1) resources.
$P3$ requires (2, 3) resources.
No process can execute with available resources.
So the system is in deadlock. Choice (D)

18. On one CPU, at a time only one program will be executed. At a time two programs will be in running state. If $P1$, $P2$ are executed on CPU1 and $P3$ executed on another CPU, the execution time will be 25 ms. Choice (A)

19. CPU utilization = $1 - p^n$
where n is the degree of multiprogramming and 'p' is the fraction of time a process spends waiting for I/O.
Here $n = 5$

$p = \dfrac{1}{2}$

$\therefore$ CPU time wasted $= \left(\dfrac{1}{2}\right)^5 = \dfrac{1}{32} 0.03125 = 3.125\%$

Choice (C)

20. RAM capacity = 4 GB = 2^{32} B
OS capacity = 256 MB = 2^{28}B
Remaining = $2^{32} - 2^{28} = 4026531840$
Process capacity = 128 MB = 2^{27}B
$\therefore$ Number of processes fit in remaining space

$= \dfrac{4026531840}{2^{27}} = 30$

30 processes can be placed in memory at a time.
Let 'P' is the probability that a process has an I/O.
If all the 30 processes are in I/O, the probability is P^{30}.
CPU idle percentage = 5%
By equating both,
$P^{30} = 0.05 \Rightarrow P = 0.904$
So we can tolerate processes with upto 90.4% I/O. Choice (A)

21. Data accessed using inode triple indirect block = 64 GB
Block size = 4 KB
$(\text{Number of blocks})^3 \times$ block size
= Data accessed using triple indirect blocks
$(\text{Number of blocks})^3 \times 4\text{ K} = 64\text{ G}$

$\Rightarrow \quad (\text{Number of blocks})^3 = \dfrac{2^{36}}{2^{12}}$

$\Rightarrow \quad$ Number of blocks $= 2^8$

$\therefore \quad$ Bits required for block pointer = 8-bits

Choice (A)

22. Disk capacity = 32 GB = 2^{35}B
Each entry size = 24-bits
Using 24-bits, we can access 2^{24} B.
$\therefore \quad$ Minimum size of a file allocation

$= \dfrac{2^{35}}{2^{24}} = 2^{11}\text{B} = 2\text{ KB}$ Choice (B)

23. Memory = 2^{64}B
Partition size = 2^{20}B

Number of partitions $= \dfrac{2^{64}}{2^{20}} = 2^{44}$

$\therefore \quad$ 44-bits required for an entry in the process table.

Choice (D)

24. All those features are required by an ideal algorithm.

Choice (D)

25. In SJF the job with smallest CPU burst will execute first. The Gantt chart for given processes is shown below:

P2	P4	P3	P5	P1

0 1 2 4 9 19

Turn Around Time (TAT) of $P1$ = 19 (waiting time + CPU burst)
TAT of $P2$ = 1
TAT of $P3$ = 4
TAT of $P4$ = 2
TAT of $P5$ = 9

Average TAT $= \dfrac{19 + 1 + 4 + 2 + 9}{5} = \dfrac{35}{5} = 7$ units

Choice (B)

26. Initially 'turn = 0' then P_0 only can enter the critical section and if P_1 tries to enter, it is not possible to enter critical section.
Only one process can enter critical section. So there is mutual exclusion. But there is no progress. ($\because$ A process cannot be able to enter into critical section even if no process is in critical section). Choice (C)

27. Only one process can enter the critical section at a time. So there is mutual exclusion. Choice (A)

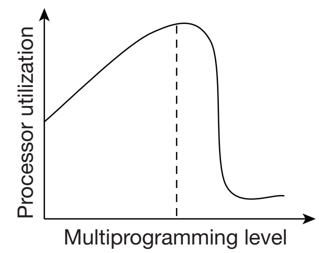
28. The wait operation decrements the semaphore value. If it is less than zero, block the process. Choice (A)

29. User program deals with logical address.

Choice (A)

30. Page fault occurs when the page for which CPU is searching is not in memory. Initially we search in the page table. If it is a trap, the OS takes the control. OS searches the backing store for the required page and places it in memory. Updates the page table and restarts the instruction. Choice (D)

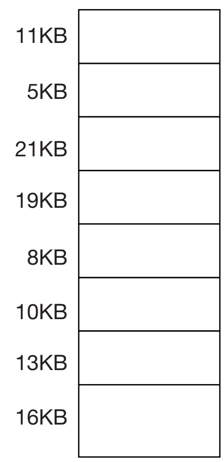
31. If the multiprogramming level increases from a small value, then the processor utilization rises.
But from a point onwards, the number of page faults rises dramatically and processor utilization collapses. The graph is shown below.



Choice (C)

Common Data for Questions 32 and 33

32. Memory system with given hole sizes is shown below:



In first-fit policy, choose the hole from beginning of the memory, using which given request is satisfied.
13 KB placed in 21 KB.
11 KB placed in 11 KB.
10 KB placed in 19 KB. Choice (B)

33. Best-fit policy searches for all holes in the memory and selects the smallest hole using which the memory request will be satisfied.
12 KB placed in 13 KB
10 KB placed in 10 KB
9 KB placed in 11 KB Choice (C)

34. FIFO will replace the page which entered first into the memory.
Load time of page 2 is least
$\therefore \quad$ FIFO replaces page 2. Choice (C)

35. LRU replaces the page whose access time is least.
Access time of page 0 is least. So LRU replaces page 0.
Choice (A)

Operating Systems Test 3

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Round Robin scheduling with large time slice behaves as:
   (A) FCFS
   (B) Priority based scheduling
   (C) Multi-level queue scheduling
   (D) Preemptive SJF

2. Priority inversion means:
   (A) Shortest Job waits for longest job
   (B) High priority process waits for low priority process
   (C) Longest job waits for shortest job
   (D) both (A) and (C).

3. Consider the following table with 4 processes:

Process	Arrived Time	Burst Time
P1	0	5
P2	1	4
P3	2	2
P4	3	3

   If Longest Remaining Time scheduling (Preemptive longest Job First) is used, then the average turnaround time is ______.
   (A) 11 (B) 12
   (C) 13 (D) 14

4. Consider the following table:

Process	Arrival Time	Burst Time
P0	0	4
P1	1	3
P2	1	3
P3	2	5

   If Longest Job First scheduling is used then the average waiting time is ______.
   (A) 5.00 (B) 5.25
   (C) 5.50 (D) 5.75

5. Consider the following table:

Process	Arrival Time	Turn around Time
P0	0	15
P1	0	2
P2	0	18
P3	0	20
P4	0	7

   If priority scheduling is used for scheduling, what is the burst time of process P_o?
   (A) 15 (B) 13
   (C) 8 (D) 3

6. As the time quantum increases for Round Robin scheduling, generally the average waiting time:
   (A) Increases
   (B) Decreases
   (C) Unchanged
   (D) Cannot be determined

7. Consider three processes $P0$, $P1$, $P2$ arrived at Time 0, with the burst times x, y, z respectively. $x < z < y$. What is the average waiting, if SJF is used for scheduling?
   (A) $\frac{x+y+z}{2}$ (B) $\frac{x+z}{3}$
   (C) $\frac{2x+z}{3}$ (D) $\frac{x+z+y}{3}$

8. Consider the following table:

Process	Arrival Time	Burst Time
P0	0	8
P1	1	4
P2	2	2

   What is average waiting time of processes which have taken more than one slot for completion, When SRTF is used for scheduling?
   (A) 2.66 (B) 3.0
   (C) 4.0 (D) 3.33

9. Match the following:

List 1		List 2	
(a)	Ready → Running	1.	Dispatching
(b)	Running → Waiting	2.	Preemption
(c)	Waiting → Ready	3	Completion
(d)	Running → Terminate	4.	I/O Request
(e)	Running → Ready	5	Event occurred

	a	b	c	d	e
(A)	1	2	3	4	5
(B)	1	4	5	3	2
(C)	1	4	2	3	5
(D)	1	2	5	4	3

10. Consider the following:

Process	Arrival Time	Burst Time
A	0	4
B	1	6
C	5	3
D	7	2

   What is the waiting time of process D, if FIFO scheduling is used?

(A) 3 (B) 2
(C) 6 (D) 12

11. Preemptive scheduling takes place when ______.
(I) process switches from Running to Ready
(II) process switches from Waiting to Ready
(III) process switches from Running to waiting
(IV) process terminates
(A) I, II (B) I, II, IV
(C) I, II, III (D) I, II, III and IV

12. Blocking and Non-blocking message passing is also known as:
(A) Synchronous and Asynchronous
(B) Direct and Indirect
(C) Limited Buffer and Zero buffer
(D) Pipes and FIFO

13. Number of child processes created for the following code segment is ______.

```
fork();
fork();
fork();
fork();
```

(A) 4 (B) 8
(C) 15 (D) 16

14. Which of the following statements are TRUE about threads?
I. Thread library provides support to both user and kernel level threads.
II. Threads improves the Responsiveness and Resource sharing.
III. Kernel level thread switching is faster than user level switching.
IV. User level thread maintenance is faster than kernel level threads.
(A) I and III (B) II and IV
(C) II and III (D) I, II and III

15. Match the following:

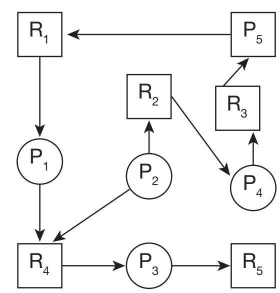
	List 1		List 2
P.	Starvation	1.	FCFS
Q.	Ageing	2.	Round Robin
R.	Context switching overhead	3.	Preemptive Priority
S.	Batch processing	4.	Highest Response Ratio next

	P	Q	R	S
(A)	1	4	2	3
(B)	4	1	3	2
(C)	3	4	2	1
(D)	1	4	3	2

16. Consider a system with four processes *A*, *B*, *C* and *D* and '*m*' instances of resource '*r*'. The resource requirements are 5, 7, 3 and 4 instances of resource '*r*' respectively. What is the minimum value of '*m*', hence system is dead lock free?
(A) 7 (B) 16
(C) 19 (D) 15

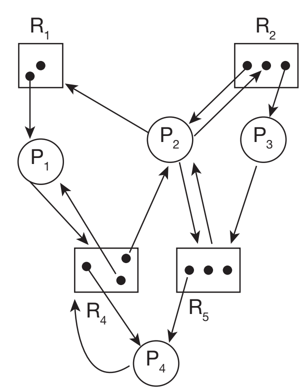
17. Which of the following system state may lead to deadlock? Let the system contains '*r*' instances of resources with '*n*' processes.
(Resource requests of each process represented in sets)
(A) $n = 5, r = 20, \{5, 5, 5, 5, 5\}$
(B) $n = 5, r = 20, \{5, 5, 4, 5, 5\}$
(C) $n = 6, r = 26, \{6, 6, 4, 3, 3, 2\}$
(D) $n = 6, r = 26, \{6, 6, 4, 3, 3, 3\}$

18. Consider the following Resource allocation Graph:



Which of the following cycle exist in it's equivalent wait-for-Graph?
(A) $P1 \rightarrow P2 \rightarrow P3 \rightarrow P4 \rightarrow P5 \rightarrow P1$
(B) $P1 \rightarrow P2 \rightarrow P3 \rightarrow P5 \rightarrow P1$
(C) $P1 \rightarrow P3 \rightarrow P4 \rightarrow P5 \rightarrow P1$
(D) $P2 \rightarrow P3 \rightarrow P4 \rightarrow P5 \rightarrow P1 \rightarrow P2$

19. Consider the following Resource allocation graph:



Which of the following dead lock cycle occurs in the given graph?
(A) $P_2 \rightarrow P_3 \rightarrow P_2$ (B) $P_2 \rightarrow P_1 \rightarrow P_4 \rightarrow P_2$
(C) Both (A) and (B) (D) None of the above

20. A counting semaphore has a value—a at a certain time, it represents:
(A) '*a*' number of processes waiting
(B) '*a*' number of process in critical section
(C) Either (A) or (B)
(D) None, negative values are not allowed on Counting semaphore

21. Semaphores ______.
(A) are process synchronization tools to avoid deadlock.

(B) are process synchronization tools to avoid race condition.
(C) uses Test And Set for synchronization.
(D) All the above

22. The system is running with 5 processes. Consider the following code segments for synchronization:

```
process 1:
while(1)
{
signal (mutex);
<Critical Section; >
signal (mutex);
}
Process i where i = 2,3,4,5.
while (1)
{
wait (mutex);
< Critical Section >
signal (mutex);
}
```

'mutex' is a binary semaphore.
Atmost how many processes can enter into the critical section?

(A) 1 (B) 2
(C) 3 (D) 5

23. Let $P0$ and $P1$ are two processes, each accesses two binary semaphores $s1$ and $s2$ to enter critical section. $s1$ and $s2$ are initialized to 1.

P_0:	P_1:
X;	W;
<critical section>	<critical section>
Y;	Z;

Consider the following code segments:

I.	wait ($s1$);	wait ($s2$);
II.	wait ($s2$);	wait ($s1$);
III.	signal ($s1$);	signal ($s2$);
IV.	signal ($s2$);	signal ($s1$);

Which of the following may lead to deadlock?

	X	Y	W	Z
(A)	I	III	II	IV
(B)	I	III	I	III
(C)	II	IV	II	IV
(D)	I	III	I	IV

24. Consider the following snapshot of a system:

	Max			Allocation		
Process	R_1	R_2	R_3	R_1	R_2	R_3
a	3	2	4	1	1	1
b	5	5	2	2	3	2
c	4	4	3	3	1	1
d	3	4	4	1	0	1

Available = {5, 3, 2}
Which of the following is not a safe sequence?

(A) *a b c d* (B) *b c d a*
(C) *b a c d* (D) *c a d b*

25. In the above system, if process 'A' requests for {0, 1, 2} resources and if the request is granted, then the system state is ______.

(A) safe state
(B) unsafe state
(C) deadlock
(D) either (B) or (C)

Answer Keys

1. A **2.** B **3.** A **4.** B **5.** C **6.** A **7.** C **8.** C **9.** B **10.** C
11. D **12.** A **13.** C **14.** B **15.** C **16.** B **17.** A **18.** C **19.** D **20.** A
21. B **22.** D **23.** A **24.** A **25.** A

Hints and Explanations

1. Choice (A)

2. Choice (B)

3.

P_1	P_1	P_2	P_1	P_2	P_4	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Process	AT	CT	TAT
P_1	0	11	11
P_2	1	12	11
P_3	2	13	11
P_4	3	14	11

$$\text{Average TAT } = \frac{11+11+11+11}{4} = 11$$ Choice (A)

4.

0 4 9 12 15

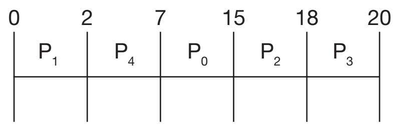
P_0	P_3	P_1	P_2

Process	AT	BT	CT	TAT	WT
P_0	0	4	4	4	0
P_1	1	3	12	11	8
P_2	1	3	15	14	11
P_3	2	5	9	7	2

Average $WT = \dfrac{(0+8+11+2)}{4} = \dfrac{21}{4} = 5.25$

Choice (B)

5. Gantt Chart:



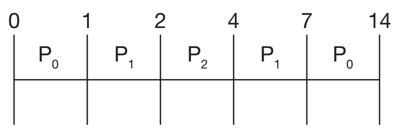
Waiting Time of P_0 is 7.

$BT = TAT - WT = 15 - 7 = 8$ Choice (C)

6. Choice (A)

7. Choice (C)

8.



Process	AT	TAT	WT
P_0	0	14	6
P_1	1	6	2
P_2	2	2	0

Processes P_0 and P_1 executed in multiple slots.

Hence average waiting time of (P_0 and P_1) $= \dfrac{6+2}{2} = 4$

Choice (C)

9. Choice (B)

10. Processes 'D' schedules at time 13.

Waiting Time = 13 – 7 (AT) = 6 Choice (C)

11. Choice (D)

12. Choice (A)

13. Choice (C)

14. Choice (B)

15. Choice (C)

16. **<u>Method 1:</u>**

Each process requirement is S_i for process 'i'.

i	–	A	B	C	D
S_i	–	5	7	3	4

Assume for each process i, $(S_i - 1)$ resources are allocated.

A	B	C	D	
4	6	2	3	allocated

This system state result in deadlock i.e., the system with ≤15 resources may lead to deadlock.

If atleast one extra resource available in this state, the system becomes deadlock free.

∴ 16 resources required.

<u>Method 2:</u>

$$\sum_{i=1}^{n} S_i < (m+n)$$

S_i – resources required for process 'i'.

m – number of resources in system

n – number of process in system.

$(5+7+3+4) < m+4 \Rightarrow m = 16$ Choice (B)

17. Use this formula to check deadlock state.

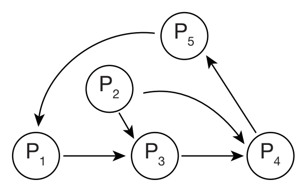
$$\sum_{i=1}^{n} S_i < (m+n)$$

S_i – Resources required for process 'i'.

m – Number of resources

n – Number of process Choice (A)

18. The wait-for-graph for given resource allocation graph is shown below:



Choice (C)

19. No dead lock exists in given graph. Choice (D)

20. Choice (A)

21. Choice (B)

22. Assume that process 2 entered critical section that results in wait of process 3, 4 and 5. But process 1 can enter. Then one more process can enter C.S. If process 1 leaves C.S., then again one more process can enter C.S. Similarly if process 1 enters and exits the C.S. again, 2 more processes can enter C.S. Choice (D)

23. Semaphores may be signalled in any order. But semaphores must be locked in same order. Choice (A)

24. Need of process 'A' is {2, 1, 3}, which is greater than available. Choice (A)

25. Process 'B' can execute in resultant state, which leads to completion of A, C and D. Choice (A)

Operating Systems Test 4

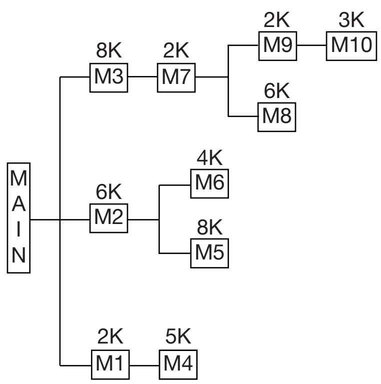
Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Suppose that, the time to service a page fault is 10 sec on average, memory access time is 1 m sec. If a page fault occurs for every 10000 references, what is the average memory access time?
(A) 1.9999 m sec (B) 1 m sec
(C) 9.999 m sec (D) 1.9999 μ sec

2. If an instruction takes 'n' micro seconds and page fault takes an additional 'm' micro seconds. If the average page fault occurs for every 'k' instructions, then effective access time is _____.
(A) $(n + m)/k$ (B) $m + n/k$
(C) $n + m/k$ (D) $n + m * k$

3. The overlay tree of the program is shown below:



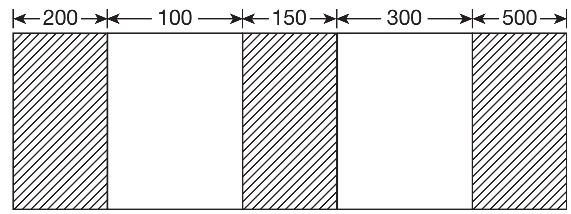
What will be the minimum partition size required to execute the program?
(A) 50K (B) 11K
(C) 8K (D) 20K

4. Dirty bit for a page table ________.
(i) helps to reduce number of page faults
(ii) helps to reduce page fault penalty
(iii) helps to avoid unnecessary writes on a paging device.
Which of the following is true?
(A) (i) (ii) (B) (i) (iii)
(C) (ii) (iii) (D) (i) (ii) and (iii)

5. A 2000 KB memory is managed using variable partitions but with no compaction. If, there exists two free partitions with sizes 400 KB and 250 KB. The smallest allocation request that could be denied is:
(A) 225K (B) 375K
(C) 425K (D) 650K

6. The address sequence generated during the execution of a program is given below:
681 351 357 421 499 099 118 129 654 454
There exists only one free frame, with the size 100 records per frame. If pure demand paging is used, what is the number of page faults?
(A) 10 (B) 9
(C) 8 (D) 7

7. Consider the following heap.



The hatched regions are not free. The sequence of requests for the blocks of size 250, 25, 75, 50 can be satisfied if we use:
(i) First fit (ii) Best fit
(iii) Worst fit
(A) (i) (ii) (B) (ii) (iii)
(C) (i) (iii) (D) (i) (ii) and (iii)

8. Consider a system with 128 MB physical memory and a 32-bit virtual address space. If the page size is 2 KB, what is the approximate page table size?
(A) 2 MB (B) 4 MB
(C) 8 MB (D) 16 MB

9. Match the following:

List–1		List–2	
1.	Virtual memory	p.	Spatial locality
2.	Shared memory	q.	Mutual exclusion
3.	Look-ahead buffer	r.	Temporal locality
4.	Look-aside buffer	s.	Address translation

	1	2	3	4
(A)	p	q	r	s
(B)	s	q	r	p
(C)	q	p	s	r
(D)	s	r	q	p

10. Let a memory has five free blocks $2k$, $4k$, $6k$, $8k$ and $20k$. These blocks are allocated using best fit strategy. The allocations requests are stored in a queue as shown below:

Job	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
Size	$2k$	$16k$	$4k$	$5k$	$7k$	$12k$	$8k$	$18k$
Time	4	9	3	7	6	2	8	10

The time at which the process '$P7$' will be completed is _______.
(A) 8 (B) 14
(C) 21 (D) 39

11. Consider a memory consisting of the following holes in the given order:
20K, 15K, 28K, 18K, 23K, 24K and 30K
For the request sequence 24K, 22K and 18K, the first fit allocates the Blocks _______.
(A) 24K, 23K, 18K (B) 28K, 23K, 20K
(C) 28K, 24K, 20K (D) 28K, 23K, 24K

12. Consider the following statements:
*S*1: Segmentation may suffer from external fragmentation.
*S*2: Paging may suffer from internal fragmentation.
*S*3: Segmentation may suffer from internal fragmentation.
*S*4: Paging may suffer from external fragmentation.
Which of the following are true?
(A) *S*1, *S*3 (B) *S*3, *S*4
(C) *S*1, *S*2 (D) *S*1, *S*2, *S*3, *S*4

13. Inverted Page Table contains,
(A) Page Number, Process ID, Offset.
(B) Page Number, Frame Number, Link Field
(C) Page Number, Process ID, Link field
(D) Page Number, Frame Number, Process ID

14. Consider a simple paging system with 4 GB main memory and 256 MB virtual memory. The page size is 1 KB. Based on this dat*a*, Match the following:

List-1		List-2	
1.	Bits in Logical Address	p.	22
2.	Bits in Frame Number	q.	18
3.	Entries in Page Table	r.	28
4.	Bits in Page Number	s.	2^{18}

	1	2	3	4
(A)	r	p	s	q
(B)	p	q	r	s
(C)	r	p	q	s
(D)	r	s	p	q

15. Consider the following page reference sequence:
2, 1, 5, 2, 2, 1, 3, 6, 4, 2, 5, 1, 3, 6
How many page faults will occur using LRU, FIFO and optimal respectively, if the memory contains 6 frames?
(A) 6 6 6 (B) 6 5 4
(C) 5 6 4 (D) 7 8 6

16. In a particular Unix OS, the *i*-node block of size 2K, contains 10 direct data block, addresses, one address for single indirect block. One for double indirect block, and one for Triple indirect block. Each block can contain 128 addresses. The approximate maximum size of the file in the file system is __________.
(A) 2 GB (B) 4 GB
(C) 8 GB (D) 16 GB

17. Consider a system with four files of sizes 11150 B, 4970 B, 5520 B and 10640 B. For storing these files on disk we can use either 100 B blocks or 200 B blocks but not both. For each block 4 B are used for book information. A disk block can store either book keeping information or file data, but not both.
What is the total number of blocks required for files using 100 B blocks and 200 B blocks respectively?
(A) 340 and 338 (B) 340 and 169
(C) 340 and 170 (D) 315 and 163

18. Hard disk transfers 20 MB/sec using DMA. The processor runs at 1.2 GHz takes 600 and 1800 clocks to initiate and terminate DMA transfer respectively. If the size of transfer is 10 KB, what is the percentage of processor time consumed for the transfer operation?
(A) 4% (B) 0.4%
(C) 1% (D) 10%

19. Consider a disk pack with 16 platters, each with 2 surfaces, 256 tracks per surface, 512 sectors per track and 1 KB sector size. The capacity of the disk and number bits required to address a sector uniquely in the disk are respectively:
(A) 8 GB, 32 bits (B) 4 GB, 22 bits
(C) 4 GB, 32 bits (D) 16 GB, 22 bits

20. Which of the following request set will cause the head to change its direction after servicing every request assuming that the head does not change direction if there is a tie in SSTF and all the requests arrive before the servicing starts?
(A) 1, 129, 160, 168, 171, 174, 191, 255
(B) 0, 128, 160, 168, 171, 175, 191, 255
(C) 0, 129, 159, 168, 171, 174, 191, 255
(D) 0, 128, 160, 168, 171, 175, 190, 255

21. What is the maximum cardinality of request set, so that head changes its direction for every request service, if there exists a total of 4096 tracks and head can start from any track?
(A) 10 (B) 11
(C) 12 (D) 13

22. Consider a disk with 100 cylinders. The per track seek time is 2 m sec. The requests to access the cylinders occurs in the following sequence:
5, 40, 15, 12, 24, 65, 3, 18, 10
Assuming that the head is currently at 55 cylinder, what is the time taken to satisfy all the requests using SSTF.
(A) 72 m sec (B) 144 m sec
(C) 204 m sec (D) 102 m sec

23. Consider a disk with 100 cylinders. The per track seek time given as 3 m sec. The requests to access the cylinders occurs in the following sequence:
25, 45, 65, 32, 75, 15, 50, 20
What is the total seek time using FCFS disk scheduling. Assume that the initial position of head is at track 0.
(A) 269 (B) 300
(C) 807 (D) 225

24. Consider a system with 3 memory frames and the following reference string over 8 pages: 1, 2, 3, 4, 2, 8, 1, 3, 5, 7, 6, 1, 2
What will be the final content of memory if FIFO replacement policy used.
(A) 6 1 2
(B) 1 2 6
(C) 1 6 2
(D) 2 1 6

25. For the above count what will be the number of page faults if LRU is used?
(A) 10 (B) 11
(C) 12 (D) 13

Answer Keys

1. A	**2.** C	**3.** D	**4.** C	**5.** C	**6.** D	**7.** C	**8.** B	**9.** A	**10.** B
11. B	**12.** C	**13.** C	**14.** A	**15.** A	**16.** B	**17.** B	**18.** B	**19.** B	**20.** B
21. C	**22.** B	**23.** C	**24.** A	**25.** C					

Hints and Explanations

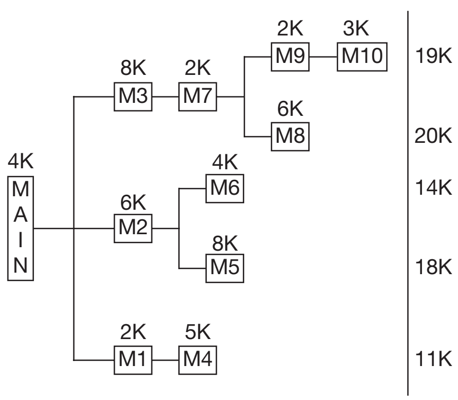
1. EAT = H*MAT + (1 – H)*($P.F.T$)
H = 0.9999 (1 page fault for every 10000)
MAT = 1 m sec
$P.F.T$ = 10 sec
EAT = 0.9999 * 1 m sec + (0.0001)* 10 sec.
= 0.9999 + 1 m sec
= 1.9999 m sec Choice (A)

2. EAT = H * MAT + (1 – H)*($P.F.T$ + MAT)

$$= \frac{k-1}{k} \times n + \frac{1}{k}(m+n)$$

$$= n + \frac{m}{k}$$ Choice (C)

3.



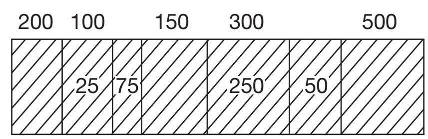
The maximum memory required when module $M8$ is under execution, which is 20K. Choice (D)

4. Choice (C)

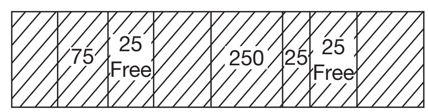
5. Choice (C)

6. 681 351 357 421 499 019 118 129 654 454
PF PF PH PF PH PF PF PH PF PF
$PF \rightarrow$ Page Fault, $PH \rightarrow$ Page Hit
Number of PFs = 7 Choice (D)

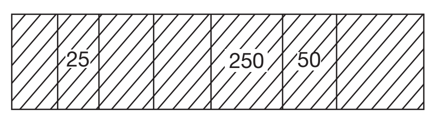
7. First Fit:



Best Fit:



50 cannot be allocated.
Worst Fit:



Choice (C)

8. Number of page $= \dfrac{\text{(Virtual memory size)}}{\text{(Page size)}}$

Number of frames $= \dfrac{\text{Main Memory size}}{\text{Page size}} = \dfrac{2^{27}}{2^{11}} = 2^{16}$

Page Table size = Number of pages * Number of bits to address frame
= 2^{21} * 16 bits
= 2^{21} * 2 B = 4 MB Choice (B)

9. Choice (A)

10.

2K	4K	6K	8K	20K
P1	P3	P4	P5	P2

On completion of $P5$, Block 4 will be allocated to $P7$.
Hence completion time of $P7$ is = 6 + 8 = 14
Choice (B)

11. Choice (B)

12. Choice (C)

13. Choice (C)

14. Virtual memory = 256 MB = 2^{28} B

Bits in logical address = 28

Number of frames $= \dfrac{4\text{ GB}}{1\text{ kB}} = \dfrac{2^{32}}{2^{10}} = 2^{22}$

Number of bits in frame Number = 22

Number of pages $= \dfrac{256\text{ MB}}{1\text{ kB}} = \dfrac{2^{28}}{2^{10}} = 2^{18}$

Number of entries in page table = 2^{18}

Number of bits in page number = 18 Choice (A)

15. Number of pages referred is 6, Number of frames also 6. Hence for any algorithm, number of page faults are 6. Choice (A)

16. Maximum file size

$$[\underset{\text{Inode}}{1} + \underset{\text{Direct}}{10} + \underset{\text{Single indirect}}{(1+128)} + \overbrace{(1+128+128\times128)}^{\text{Double indirect}}$$
$$+ \underbrace{(1+128+128\times128+128\times128\times128)]}_{\text{Triple indirect}}$$

$2k$ (Block size)

$\cong$ 4 GB Choice (B)

17.

File Size	100B Data Blocks	Book Keeping Blocks
11150	112	5
4970	50	2
5520	56	3
10640	107	5
Total	325	15

Total = 340

File Size	200B Data Blocks	Book Keeping Blocks
11150	56	2
4970	25	1
5520	28	1
10640	54	2
Total	163	6

Total = 169 Choice (B)

18. **Transfer Time:**

20 MB – 1 sec

10 KB – x

x*20 MB = 1 sec * 10 KB

x = ½ m sec = 500 µ sec

CPU Time:

1.2 GHz ⇒ 1.2 * 10^9 clocks/sec

= 1200 * 10^6 clocks/sec

CPU Time consumed = 600 + 1800

= 2400 clocks

1200 * 10^6 clocks – 1 sec

2400 clocks – y

y * 1200 × 10^6 = 1 sec * 2400

y = 2 µs

Percentage of CPU involvement

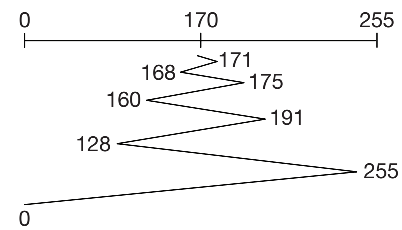
$= \dfrac{2\,\mu\text{s}}{500\,\mu\text{s}} \times 100 = 0.4\%$ Choice (B)

19. Disk Capacity = 16 * 2 * 256 * 512 * 1 KB = 4 GB

Number of sectors = 16 × 2 × 256 × 512 = 2^{22}

Choice (B)

20. Assume the head at track 170 (Based on choices). Then

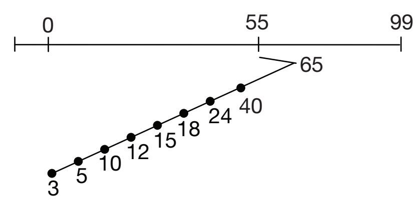


Choice (B)

21. 4096 = 2^{12} tracks

Hence cardinality = 12 Choice (C)

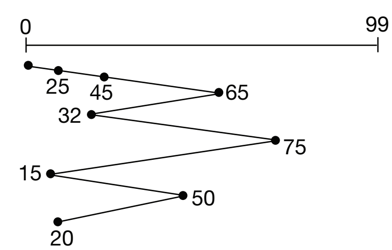
22.



Total tracks moved = 72

Total seek time = 72 × 2 = 144 m sec Choice (B)

23.



Total tracks moved = |0 – 25| + |25 – 45| + |45 – 65| + |65 – 32| + |32 – 75| + 75 – 12| + |15 – 50| + |50 – 20|

= 25 + 20 + 20 + 33 + 43 + 63 + 35 + 30 = 269

Total seek time = 269 * 3 = 807 Choice (C)

24. Choice (A)

25.

1, 2, 3, 4, 2, 8, 1, 3, 5, 7, 6, 1, 2

x x x x ✓ x x x x x x x x

~~1~~ ~~4~~ ~~1~~ ~~7~~ 2
~~2~~ ~~3~~ 6
~~3~~ ~~8~~ ~~5~~ 1

x ⇒ Page Fault

✓ ⇒ Page Hit Choice (C)

Chapter 1

Lexical Analysis and Parsing

LEARNING OBJECTIVES

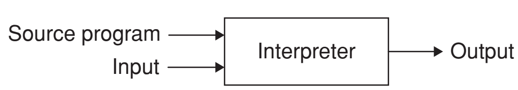
- *Language processing system*
- *Lexical analysis*
- *Syntax analysis*
- *Context free grammars and ambiguity*
- *Types of parsing*
- *Top down parsing*
- *Bottom up parsing*
- *Conflicts*
- *Operator precedence grammar*
- *LR parser*
- *Canonical LR parser(CLR)*

Language Processing System

Language Processors

Interpreter

It is a computer program that executes instructions written in a programming language. It either executes the source code directly or translates source code into some efficient intermediate representation and immediately executes this.



Example: Early versions of Lisp programming language, BASIC.

Translator

A software system that converts the source code from one form of the language to another form of language is called as translator. There are 2 types of translators namely (1) Compiler (2) Assembler.

Compiler converts source code of high level language into low level language.

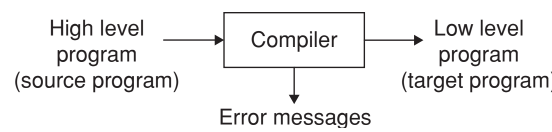
Assembler converts assembly language code into binary code.

Compilers

A compiler is a software that translates code written in high-level language (i.e., source language) into target language.

Example: source languages like C, Java,… etc. Compilers are user friendly.

The target language is like machine language, which is efficient for hardware.



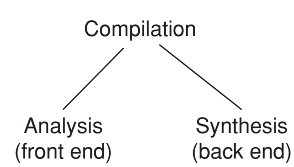
Passes

The number of iterations to scan the source code, till to get the executable code is called as a pass.

Compiler is two pass. Single pass requires more memory and multipass require less memory.

Analysis–synthesis model of compilation

There are two parts of compilation:

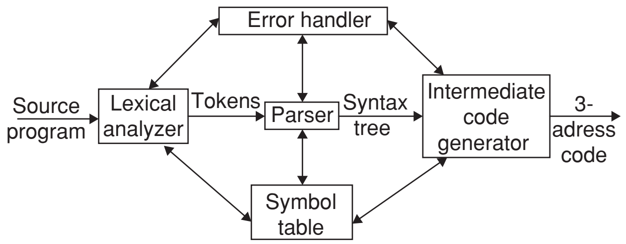


Analysis It breaks up the source program into pieces and creates an intermediate representation of the source program. This is more language specific.

Synthesis It constructs the desired target program from the intermediate representation. The target program will be more machine specific, dealing with registers and memory locations.

Front end vs back end of a compiler

The front end includes all analysis phases and intermediate code generator with part of code optimization.



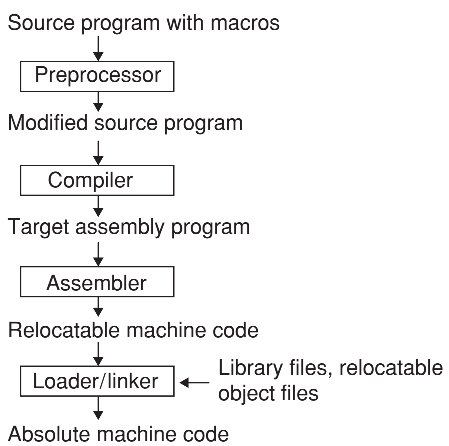
The back end includes code optimization and code generation phases. The back end synthesizes the target program from intermediate code.

Context of a compiler

In addition to a compiler, several other programs may be required to create an executable target program, like preprocessor to expand macros.

The target program created by a compiler may require further processing before it can be run.

The language processing system will be like this:



Phases

Compilation process is partitioned into some subproceses called phases.

In order to translate a high level code to a machine code, we need to go phase by phase, with each phase doing a particular task and parsing out its output for the next phase.

Lexical analysis or scanning

It is the first phase of a compiler. The lexical analyzer reads the stream of characters making up the source program and groups the characters into meaningful sequences called lexemes.

Example: Consider the statement: if $(a < b)$

In this sentence the tokens are if, $(a, <, b,)$.

Number of tokens = 6

Identifiers: a, b

Keywords: if

Operators: <, (,)

Syntax analyzer or Parser

- Tokens are grouped hierarchically into nested collections with collective meaning.
- A context free grammar (CFG) specifies the rules or productions for identifying constructs that are valid in a programming language. The output is a parse/syntax/derivation tree.

Example: Parse tree for –(id + id) using the following grammar:

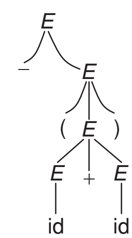
$E \to E + E$

$E \to E * E$

$E \to -E \qquad (G_1)$

$E \to (E)$

$E \to id$



Semantic analysis

- It checks the source program for semantic errors.
- Type checking is done in this phase, where the compiler checks that each operator has matching operands for semantic consistency with the language definition.
- Gathers the type information for the next phases.

Example 1: The bicycle rides the boy.

This statement has no meaning, but it is syntactically correct.

Example 2:

```
int a;
bool b;
char c;
c = a + b;
```

We cannot add integer with a Boolean variable and assign it to a character variable.

Intermediate code generation

The intermediate representation should have two important properties:

(i) It should be easy to produce.

(ii) Easy to translate into the target program

'Three address code' is one of the common forms of Intermediate code.

Three address code consists of a sequence of instructions, each of which has at most three operands.

Example:

$\mathtt{id_1 = id_2 + id_3 \times 10;}$

$\mathtt{t_1 := inttoreal(10)}$

$t_2 := id_3 \times t_1$

$t_3 := id_2 + t_2$

$id_1 = t_3$

Code optimization

The output of this phase will result in faster running machine code.

Example: For the above intermediate code the optimized code will be

$t_1 := id_3 \times 10.0$

$id_1 : = id_2 + t1$

In this we eliminated t_2 and t_3 registers.

Code generation

- In this phase, the target code is generated.
- Generally the target code can be either a relocatable machine code or an assembly code.
- Intermediate instructions are each translated into a sequence of machine instructions.
- Assignment of registers will also be done.

Example:

MOVF	id_3, R_2
MULF	≠ 60.0, R_2
MOVF	id_2, R_1
ADDF	R_2, R_1
MOVF	R_1, id_1

Symbol table management

A symbol table is a data structure containing a record for each variable name, with fields for the attributes of the name.

What is the use of a symbol table?

1. To record the identifiers used in the source program.
2. Its type and scope
3. If it is a procedure name then the number of arguments, types of arguments, the method of parsing (by reference) and the type returned.

Error detection and reporting

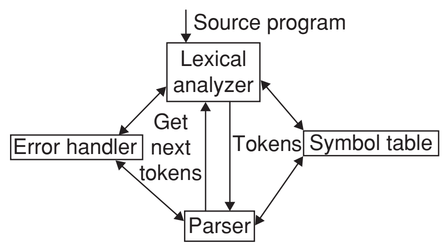
(i) Lexical phase can detect errors where the characters remaining in the input 'do not form any token'.
(ii) Errors of the type, 'violation of syntax' of the language are detected by syntax analysis.
(iii) Semantic phase tries to detect constructs that have the right syntactic structure but no meaning.

Example: adding two array names etc.

Lexical Analysis

Lexical Analysis is the first phase in compiler design. The main task of the lexical analyzer is to read the input characters of the source program, group them into lexemes, and produce as output a sequence of tokens for each lexeme in the source program. The stream of tokens is sent to the parser for syntax analysis.

There will be interaction with the symbol table as well.



Lexeme: Sequence of characters in the source program that matches the pattern for a token. It is the smallest logical unit of a program.

Example: 10, x, y, <, >, =

Tokens: These are the classes of similar lexemes.

Example: Operators: <, >, =
Identifiers: x, y
Constants: 10
Keywords: if, else, int

Operations performed by lexical analyzer

1. Identification of lexemes and spelling check
2. Stripping out comments and white space (blank, new line, tab etc).
3. Correlating error messages generated by the compiler with the source program.
4. If the source program uses a macro-preprocessor, the expansion of macros may also be performed by lexical analyzer.

Example 1: Take the following example from Fortran
DO 5 I = 1.25
Number of tokens = 5
The 1st lexeme is the keyword DO
Tokens are DO, 5, I, =, 1.25.

Example 2: An example from C program
for (int i = 1; i < = 10; i + +)
Here tokens are for, (, int, i, =, 1,;, i, < =, 10,;, i, ++,)
Number of tokens = 13

LEX compiler

Lexical analyzer divides the source code into tokens. To implement lexical analyzer we have two techniques namely hand code and the other one is LEX tool.

LEX is an automated tool which specifies lexical analyzer, from the rules given by the regular expression.

These rules are also called as pattern recognizing rules.

Syntax Analysis

This is the 2nd phase of the compiler, checks the syntax and constructs the syntax/parse tree.

Input of parser is token and output is a parse/ syntax tree.

Constructing parse tree

Construction of derivation tree for a given input string by using the production of grammar is called parse tree.

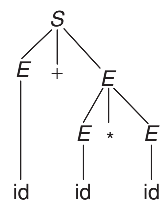
Consider the grammar

$S \to E + E/E * E$

$E \to id$

The parse tree for the string

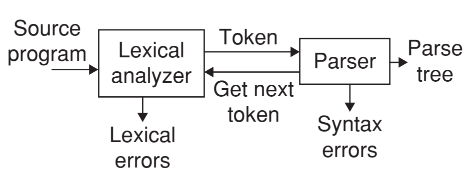
$\omega = id + id * id$ is



$\omega = id + id * id$

Role of the parser

1. Construct a parse tree.
2. Error reporting and correcting (or) recovery. A parser can be modeled by using CFG (Context Free Grammar) recognized by using pushdown automata/table driven parser.
3. CFG will only check the correctness of sentence with respect to syntax not the meaning.



How to construct a parse tree?

Parse tree's can be constructed in two ways.

(i) Top-down parser: It builds parse trees from the top (root) to the bottom (leaves).

(ii) Bottom-up parser: It starts from the leaves and works up to the root.

In both cases, the input to the parser is scanned from left to right, one symbol at a time.

Parser generator

Parser generator is a tool which creates a parser.

Example: compiler – compiler, YACC

The input of these parser generator is grammar we use and the output will be the parser code.

The parser generator is used for construction of the compilers front end.

Scope of declarations

Declaration scope refers to the certain program text portion, in which rules are defined by the language.

Within the defined scope, entity can access legally to declared entities.

The scope of declaration contains immediate scope always. Immediate scope is a region of declarative portion with enclosure of declaration immediately.

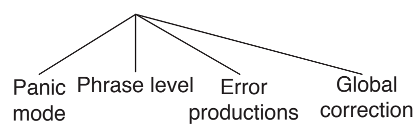
Scope starts at the beginning of declaration and scope continues till the end of declaration. Whereas in the over loadable declaration, the immediate scope will begin, when the callable entity profile was determined.

The visible part refers text portion of declaration, which is visible from outside.

Syntax Error Handling

1. Reports the presence of errors clearly and accurately.
2. Recovers from each error quickly.
3. It should not slow down the processing of correct programs.

Error Recovery Strategies



Panic mode On discovering an error, the parser discards input symbols one at a time until one of the synchronizing tokens is found.

Phrase level A parser may perform local correction on the remaining input. It may replace the prefix of the remaining input.

Error productions Parser can generate appropriate error messages to indicate the erroneous construct that has been recognized in the input.

Global corrections There are algorithms for choosing a minimal sequence of changes to obtain a globally least cost correction.

Context Free Grammars and Ambiguity

A grammar is a set of rules or productions which generates a collection of finite/infinite strings.

It is a 4-tuple defined as $G = (V, T, P, S)$

Where

V = set of variables

T = set of terminals

P = set of production rules

S = start symbol

Example: $S \to (S)/e$

$S \to (S)$ (1)

$S \to e$ (2)

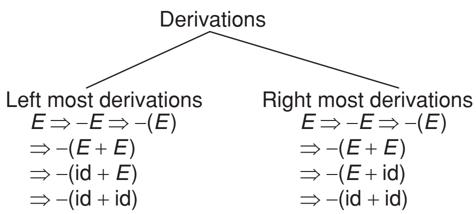
Here S is start symbol and the only variable.
(,), e is terminals.
(1) and (2) are production rules.

Sentential forms

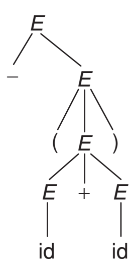
$s \overset{*}{\Rightarrow} \alpha$, Where α may contain non-terminals, then we say that α is a sentential form of G.

Sentence: A sentence is a sentential form with no non-terminals.

Example: $-(id + id)$ is a sentence of the grammar (G_1).



Right most derivations are also known as canonical derivations.



Ambiguity

A grammar that produces more than one parse tree for some sentence is said to be ambiguous.

Or

A grammar that produces more than one left most or more than one right most derivations is ambiguous.

For example consider the following grammar:

String → String + String/String – String /0/1/2/…/9

9 – 5 + 2 has two parse trees as shown below

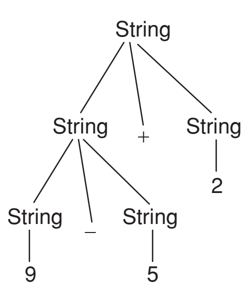


Figure 1 Leftmost derivation

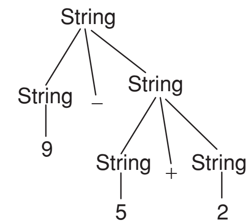


Figure 2 Rightmost derivation

- Ambiguity is problematic because the meaning of the program can be incorrect.
- Ambiguity can be handled in several ways
  1. Enforce associativity and precedence
  2. Rewrite the grammar by eliminating left recursion and left factoring.

Removal of ambiguity

The grammar is said to be ambiguous if there exists more than one derivation tree for the given input string.

The ambiguity of grammar is undecidable; ambiguity of a grammar can be eliminated by rewriting the grammar.

Example:

$E \to E + E/id\}$ → ambiguous grammar

$\left.\begin{array}{l} E \to E + T/T \\ T \to id \end{array}\right\}$ rewritten grammar (unambiguous grammar)

Left recursion

Left recursion can take the parser into infinite loop so we need to remove left recursion.

Elimination of left recursion

$A \to A\alpha/\beta$ is a left recursive.

It can be replaced by a non-recursive grammar:

$$A \to \beta A'$$
$$A' \to \alpha A'/\varepsilon$$

In general

$$A \to A\alpha_1/A\alpha_2/\ldots/A\alpha_m/\beta_1/\beta_2/\ldots/\beta_n$$

We can replace A productions by

$$A \to \beta_1 A'/\beta_2 A'/\text{–}\beta_n A'$$
$$A' \to \alpha_1 A'/\alpha_2 A'/\text{–}\alpha_m A'$$

Example 3: Eliminate left recursion from

$E \to E + T/T$

$T \to T * F/F$

$F \to (E)/\text{id}$

Solution $E \to E + T/T$ it is in the form

$A \to A\alpha/\beta$

So, we can write it as $E \to TE'$

$E' \to +TE'/\varepsilon$

Similarly other productions are written as

$$T \to FT'$$
$$T^1 \to \times FT'/\in$$
$$F \to (E)/\text{id}$$

Example 4 Eliminate left recursion from the grammar

$$S \to (L)/a$$
$$L \to L, S/b$$

Solution: $S \to (L)/a$

$L \to bL'$

$L' \to SL'/\in$

Left factoring

A grammar with common prefixes is called non-deterministic grammar. To make it deterministic we need to remove common prefixes. This process is called as Left Factoring.

The grammar: $A \to \alpha\beta_1/\alpha\beta_2$ can be transformed into

$A \to \alpha A'$

$A' \to \beta_1/\beta_2$

Example 5: What is the resultant grammar after left factoring the following grammar?

$S \to iEtS/iEtSeS/a$

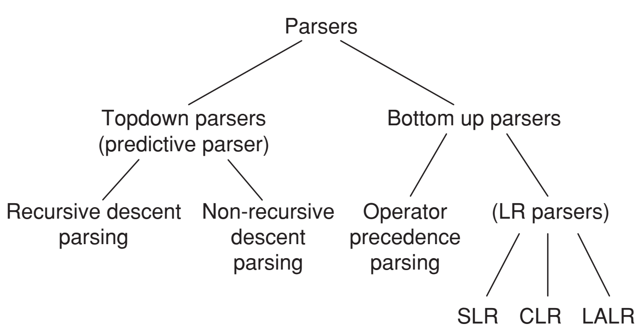
$E \to b$

Solution: $S \to iEtSS'/a$

$S' \to eS/\in$

$E \to b$

Types of Parsing



Topdown Parsing

A parse tree is constructed for the input starting from the root and creating the nodes of the parse tree in preorder. It simulates the left most derivation.

Backtracking Parsing

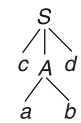
If we make a sequence of erroneous expansions and subsequently discover a mismatch we undo the effects and roll back the input pointer.

This method is also known as brute force parsing.

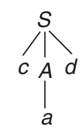
Example: $S \to cAd$

$A \to ab/a$

Let the string w = cad is to generate:



The string generated from the above parse tree is cabd.
but, w = cad, the third symbol is not matched.
So, report error and go back to A.
Now consider the other alternative for production A.



String generated 'cad' and $w = cad$. Now, it is successful.

In this we have used back tracking. It is costly and time consuming approach. Thus an outdated one.

Predictive Parsers

By eliminating left recursion and by left factoring the grammar, we can have parse tree without backtracking. To construct a predictive parser, we must know,

1. Current input symbol
2. Non-terminal which is to be expanded

A procedure is associated with each non-terminal of the grammar.

Recursive descent parsing

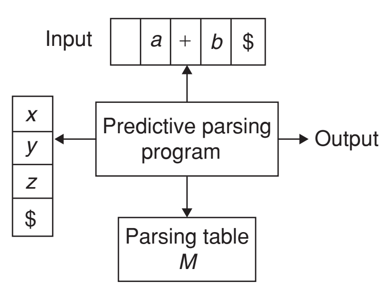
In recursive descent parsing, we execute a set of recursive procedures to process the input.

The sequence of procedures called implicitly, defines a parse tree for the input.

Non-recursive predictive parsing

(table driven parsing)

- It maintains a stack explicitly, rather than implicitly via recursive calls.
- A table driven predictive parser has
  - $\to$ An input buffer
  - $\to$ A stack
  - $\to$ A parsing table
  - $\to$ Output stream



Constructing a parsing table

To construct a parsing table, we have to learn about two functions:

1. FIRST ( )
2. FOLLOW ( )

FIRST(X) To compute FIRST(X) for all grammar symbols X, apply the following rules until no more terminals or ε can be added to any FIRST set.

1. If X is a terminal, then FIRST(X) is $\{X\}$.
2. If $X \rightarrow \varepsilon$ is a production, then add ε to FIRST(X).
3. If X is non-terminal and $X \rightarrow Y_1 Y_2 - Y_k$ is a production, then place 'a' in FIRST(X) if for some i, a is an FIRST (Y_i) and $\in$ is in all of FIRST(Y_1), …, FIRST(Y_{i-1}); that is, $Y_1, \ldots, Y_{i-1} \overset{*}{\Rightarrow} \in$. If $\in$ is in FIRST (Y_j) for all $j = 1, 2, \ldots, k$, then add $\in$ to FIRST(X). For example, everything in FIRST (Y_1) is surely in FIRST(X). If Y_1 does not derive $\in$, then add nothing more to FIRST(X), but if $Y_1 \overset{*}{\Rightarrow} \in$, then add FIRST ($Y_2$) and so on.

FOLLOW (A): To compute FOLLOW (A) for all non-terminals A, apply the following rules until nothing can be added to any FOLLOW set.

1. Place \$ in FOLLOW(S), where S is the start symbol and \$ is input right end marker.
2. If there is a production $A \rightarrow \alpha B \beta$, then everything in FIRST (β) except ε is placed in FOLLOW (B).
3. If there is a production $A \rightarrow \alpha B$ or a production $A \rightarrow \alpha B \beta$, where FIRST ($\beta$) contains ε, then everything in FOLLOW (A) is in FOLLOW (B).

Example: Consider the grammar

$E \rightarrow TE'$
$E' \rightarrow +TE'/\varepsilon$
$T \rightarrow FT'$
$T' \rightarrow *FT'/\varepsilon$
$F \rightarrow (E)/id$. Then
FIRST (E) = FIRST (T) = FIRST (F) = $\{(, id\}$
FIRST (E') = $\{^+, \varepsilon\}$
FIRST (T') = $\{^*, \varepsilon\}$
FOLLOW (E) = FOLLOW (E') = {), \$}
FOLLOW (T) = FOLLOW (T') = {+,), \$}
FOLLOW (F) = {*, +,), \$}

Steps for the construction of predictive parsing table

1. For each production $A \rightarrow \alpha$ of the grammar, do steps 2 and 3.
2. For each terminal a in FIRST (α), add $A \rightarrow \alpha$ to $M[A, a]$
3. If ε is in FIRST (α), add $A \rightarrow \alpha$ to $M[A, b]$ for each terminal b in FOLLOW (A). If ε is in FIRST (α) and \$ is in FOLLOW (A), add $A \rightarrow \alpha$ to $M[A, \$]$
4. Make each undefined entry of M be error.

By applying these rules to the above grammar, we will get the following parsing table.

	Input Symbol					
Non-terminal	**id**	**+**	*****	**(**	**)**	**\$**
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \varepsilon$	$E' \rightarrow \varepsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \varepsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \varepsilon$	$T' \rightarrow \varepsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

The parser is controlled by a program. The program consider x, the symbol on top of the stack and 'a' the current input symbol.

1. If $x = a = \$$, the parser halts and announces successful completion of parsing.
2. If $x = a \neq \$$, the parser pops x off the stack and advances the input pointer to the next input symbol.
3. If x is a non-terminal, the program consults entry $M[x, a]$ of the parsing table M. This entry will be either an x-production of the grammar or an error entry. If $M[x, a] = \{x \rightarrow UVW\}$, the parser replaces x on top of the stack by WVU with U on the top.

If $M[x, a]$ = error, the parser calls an error recovery routine.
For example, consider the moves made by predictive parser on input id + id $*$ id, which are shown below:

Matched	Stack	Input	Action
	E\$	id+id*id\$	
	TE′\$	id+id*id\$	Output E → TE′
	FT′E′\$	id+id*id\$	Output T → FT′
	idT′E′\$	id+id*id\$	Output F → id
id	T′E′\$	+id*id\$	Match id
id	E′\$	+id*id\$	Output T′→ ε
id	+TE′\$	+id*id\$	Output E′→ +TE′
id+	TE′\$	id*id\$	Match+
id+	FT′E′\$	id*id\$	Output T → FT′
id+	idT′E′\$	id*id\$	Output F → id
id+id	T′E′\$	*id\$	Match id
id+id	*FT′E′\$	*id\$	Output T′ → *FT′
id+id*	FT′E′\$	id\$	Match*
id+id*	idT′E′\$	id\$	OutputF → id
id+id*id	T′E′\$	\$	Match id
id+id*id	E′\$	\$	Output T′ → ε
id+id*id	\$	\$	Output E′ → ε

Bottom up Parsing

- This parsing constructs the parse tree for an input string beginning at the leaves and working up towards the root.
- General style of bottom-up parsing is shift-reduce parsing.

Shift–Reduce Parsing

Reduce a string to the start symbol of the grammar. It simulates the reverse of right most derivation.

In every step a particular substring is matched (in left right fashion) to the right side of some production and then it is substituted by the non-terminal in the left hand side of the production.

For example consider the grammar

$S \rightarrow aABe$

$A \rightarrow Abc/b$

$B \rightarrow d$

In bottomup parsing the string 'abbcde' is verified as

$$\left.\begin{array}{l} abbcde \\ aAbcde \\ aAde \\ aABe \\ S \end{array}\right\} \rightarrow \text{ reverse order}$$

Stack implementation of shift–reduce parser

The shift reduce parser consists of input buffer, Stack and parse table.

Input buffer consists of strings, with each cell containing only one input symbol.

Stack contains the grammar symbols, the grammar symbols are inserted using shift operation and they are reduced using reduce operation after obtaining handle from the collection of buffer symbols.

Parse table consists of 2 parts goto and action, which are constructed using terminal, non-terminals and compiler items.

Let us illustrate the above stack implementation.

$\rightarrow$ Let the grammar be

$S \rightarrow AA$

$A \rightarrow aA$

$A \rightarrow b$

Let the input string 'ω' be abab\$

ω = abab\$

Stack	Input String	Action
\$	*abab*\$	Shift
\$*a*	*bab*\$	Shift
\$*ab*	*ab*\$	Reduce ($A \rightarrow b$)
\$*aA*	*ab*\$	Reduce ($A \rightarrow aA$)
\$*A*	*ab*\$	Shift
\$*Aa*	*b*\$	Shift
\$*Aab*	\$	Reduce ($A \rightarrow b$)
\$*AaA*	\$	Reduce ($A \rightarrow aA$)
\$*AA*	\$	Reduce ($S \rightarrow AA$)
\$*S*	\$	Accept

Rightmost derivation

$S \Rightarrow aABe \Rightarrow aAde \Rightarrow aAbcde \Rightarrow abbcde$

For bottom up parsing, we are using right most derivation in reverse.

Handle of a string Substring that matches the RHS of some production and whose reduction to the non-terminal on the LHS is a step along the reverse of some rightmost derivation.

$$S \overset{rm}{\underset{*}{\Rightarrow}} \alpha Ar \Rightarrow \alpha\beta r$$

Right sentential forms of a unambiguous grammar have one unique handle.

Example: For grammar, $S \rightarrow aABe$

$A \rightarrow Abc/b$

$B \rightarrow d$

$S \Rightarrow \underline{aABe} \Rightarrow aA\underline{d}e \Rightarrow a\underline{Abc}de \Rightarrow a\underline{b}bcde$

Note: Handles are underlined.

Handle pruning The process of discovering a handle and reducing it to the appropriate left hand side is called handle pruning. Handle pruning forms the basis for a bottomup parsing.

To construct the rightmost derivation:

$$S = r_0 \Rightarrow r_1 \Rightarrow r_2 \_\_\_\_ \Rightarrow r_n = w$$

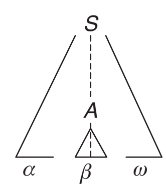
Apply the following simple algorithm:

For $i \leftarrow n$ to 1

Find the handle $A_i \rightarrow B_i$ in r_i

Replace B_i with A_i to generate r_{i-1}

Consider the cut of a parse tree of a certain right sentential form:



Here $A \rightarrow \beta$ is a handle for $\alpha\beta\omega$.

Shift reduce parsing with a stack There are 2 problems with this technique:

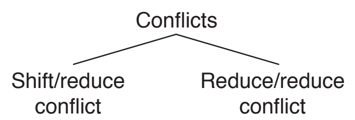
(i) To locate the handle
(ii) Decide which production to use

General construction using a stack

1. 'Shift' input symbols onto the stack until a handle is found on top of it.
2. 'Reduce' the handle to the corresponding non-terminal.
3. 'Accept' when the input is consumed and only the start symbol is on the stack.
4. Errors – call an error reporting/recovery routine.

Viable prefixes The set of prefixes of a right sentential form that can appear on the stack of a shift reduce parser are called viable prefixes.

Conflicts



Shift/reduce conflict

Example: stmt → if expr then stmt | if expr then stmt else stmt | any other statement

If exp then stmt is on the stack, in this case we can't tell whether it is a handle. i.e., 'shift/reduce' conflict.

Reduce/reduce conflict

Example: $S \to aA/bB$
$A \to c$
$B \to c$
W = ac it gives reduce/reduce conflict.

Operator Precedence Grammar

In operator grammar, no production rule can have:

- ε at the right side.
- two adjacent non-terminals at the right side.

Example 1: $E \to E + E\,/E - E/$ id is operator grammar.

Example 2: $\left.\begin{array}{l} E \to AB \\ A \to a \\ B \to b \end{array}\right\}$ not operator grammar

Example 3: $\underbrace{E \to E0E/\text{id}}_{\text{not operator grammar}}$

Precedence relation If
$a < b$ then b has higher precedence than a
$a = b$ then b has same precedence as a
$a > b$ then b has lower precedence than a

Common ways for determining the precedence relation between pair of terminals:

1. Traditional notations of associativity and precedence.

Example: × has higher precedence than + × .> + (or) + <. ×

2. First construct an unambiguous grammar for the language which reflects correct associativity and precedence in its parse tree.

Operator precedence relations from associativity and precedence

Let us use $ to mark end of each string. Define $ <. b and b ⋗ $ for all terminals b. Consider the grammar is:

$$E \to E + E/E \times E/\text{id}$$

Let the operator precedence table for this grammar is:

	id	+	×	$
id		⋗	⋗	⋗
+	⋖	⋗	⋖	⋗
×	⋖	⋗	⋗	⋗
$	⋖	⋖	⋖	accept

1. Scan the string from left until ⋗ is encountered
2. Then scan backwards (to left) over any = until ⋖ is encountered.
3. The handle contains everything to the left of the first ⋗ and to the right of the ⋖ is encountered.

After inserting precedence relation is

$id + id * id $ is

$ ⋖ id ⋗ + ⋖ id ⋗ * ⋖ id ⋗ $

Precedence functions Instead of storing the entire table of precedence relations table, we can encode it by precedence functions f and g, which map terminal symbols to integers:

1. $f(a) ⋖ f(b)$ whenever $a ⋖ b$
2. $f(a) ⋗ f(b)$ whenever $a \doteq b$
3. $f(a) > f(b)$ whenever $a ⋗ b$

Finding precedence functions for a table

1. Create symbols $f(a)$ and $g(a)$ for each 'a' that is a terminal or $.
2. Partition the created symbols into as many groups as possible in such away that $a = b$ then f (a) and g (b) are in the same group
3. Create a directed graph
   If $a < b$ then place an edge from $g(b)$ to $f(a)$
   If $a > b$ then place an edge from $f(a)$ to $g(b)$
4. If the graph constructed has a cycle then no precedence function exists.
   If there are no cycles, let $f(a)$ be the length of the longest path being at the group of $f(a)$.
   Let $g(a)$ be the length of the longest path from the group of $g(a)$.

Disadvantages of operator precedence parsing

- It can not handle unary minus.
- Difficult to decide which language is recognized by grammar.

Advantages

1. Simple
2. Powerful enough for expressions in programming language.

Error cases

1. No relation holds between the terminal on the top of stack and the next input symbol.
2. A handle is found, but there is no production with this handle as the right side.

Error recovery

1. Each empty entry is filled with a pointer to an error routine.
2. Based on the handle tries to recover from the situation.

To recover, we must modify (insert/change)

1. Stack or
2. Input or
3. Both

We must be careful that we don't get into an infinite loop.

LR Parsers

- In LR (K), L stands for Left to Right Scanning, R stands for Right most derivation, K stands for number of look ahead symbols.
- LR parsers are table-driven, much like the non-recursive LL parsers. A grammar which is used in construction of LR parser is LR grammar. For a grammar to be LR it is sufficient that a left-to-right shift-reduce parser be able to recognize handles of right-sentential forms when they appear on the top of the stack.
- The Time complexity for such parsers is $O(n^3)$
- LR parsers are faster than LL (1) parser.
- LR parsing is attractive because
  - The most general non-backtracking shift reduce parser.
  - The class of grammars that can be passed using LR methods is a proper superset of predictive parsers. LL (1) grammars $\subset$ LR (1) grammars.
  - LR parser can detect a syntactic error in the left to right scan of the input.
- LR parsers can be implemented in 3 ways:

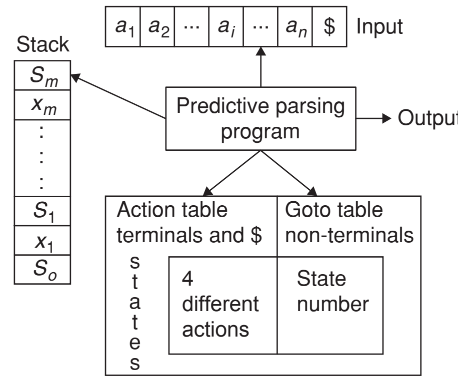
1. Simple LR (SLR): The easiest to implement but the least powerful of the three.
2. Canonical LR (CLR): most powerful and most expensive.
3. Look ahead LR (LALR): Intermediate between the remaining two. It works on most programming language grammars.

Disadvantages of LR parser

1. Detecting a handle is an overhead, parse generator is used.
2. The main problem is finding the handle on the stack and it was replaced with the non-terminal with the left hand side of the production.

The LR parsing algorithm

- It consists of an input, an output, a stack, a driver program and a parsing table that has two parts (action and goto).
- The driver/parser program is same for all these LR parsers, only the parsing table changes from parser to another.



Stack: To store the string of the form,

$S_o x_1 S_1 \ldots x_m S_m$ where

S_m: state

x_m: grammar symbol

Each state symbol summarizes the information contained in the stack below it.

Parsing table: Parsing table consists of two parts:

1. Action part
2. Goto part

ACTION Part:

Let, $S_m \rightarrow$ top of the stack
$a_i \rightarrow$ current symbol

Then action $[S_m, a_i]$ which can have one of four values:

1. Shift S, where S is a state
2. Reduce by a grammar production $A \rightarrow \beta$
3. Accept
4. Error

GOTO Part:

If goto $(S, A) = X$ where $S \rightarrow$ state, $A \rightarrow$ non-terminal, then GOTO maps state S and non-terminal A to state X.

Configuration

$$(S_o\, x_1 S_1\, x_2 S_2 - x_m S_m,\ a_i a_{i+1} - a_n \$)$$

The next move of the parser is based on action $[S_m, a_i]$

The configurations are as follows.

1. If action $[S_m, a_i]$ = shift S
$$(S_o\, x_1 S_1\, x_2 S_2 \text{---}\ x_m S_m,\ a_i a_{i+1} \text{---}\ a_n \$)$$
2. If action $[S_m, a_i]$ = reduce $A \rightarrow \beta$ then
$$(S_o\, x_1 S_1\, x_2 S_2 \text{---}\ x_{m-r} S_{m-r},\ AS,\ a_i a_{i+1} \text{---}\ a_n \$)$$
   Where S = goto $[S_{m-r}, A]$
3. If action $[S_m, a_i]$ = accept, parsing is complete.
4. If action $[S_m, a_i]$ = error, it calls an error recovery routine.

Example: Parsing table for the following grammar is shown below:

1. $E \rightarrow E + T$
2. $E \rightarrow T$

3. $T \rightarrow T * F$ 4. $T \rightarrow F$
5. $F \rightarrow (E)$ 6. $F \rightarrow \text{id}$

	Action						Goto		
State	**id**	**+**	**×**	**(**	**)**	**$**	***E***	***T***	***F***
0	S_5			S_4			1	2	3
1		S_6				*acc*			
2		r_2	S_7		r_2	r_2			
3		r_4	r_4		r_4	r_4			
4	S_5			S_4			8	2	3
5		r_6	r_6		r_6	r_6			
6	S_5			S_4				9	3
7	S_5			S_4					10
8		S_6			S_1				
9		r_1	S_7		r_1	r_1			
10		r_3	r_3		r_3	r_3			
11		r_5	r_5		r_5	r_5			

Moves of LR parser on input string id*id+id is shown below:

Stack	Input	Action
0	id * id + id$	Shift 5
0id 5	* id + id$	reduce 6 means reduce with 6th production $F \rightarrow$ id and goto [0, F] = 3
0F 3	* id + id$	reduce 4 i.e $T \rightarrow F$ goto [0, T] = 2
0T 2	* id + id$	Shift 7
0T2 * 7	id + id$	Shift 5
0T2 * 7 id 5	+ id$	reduce 6 i.e $F \rightarrow$ id goto [7, F] = 10
0T2 * 7 F 10	+ id$	reduce 3 i.e $T \rightarrow T * F$
0T 2	+ id$	goto [0, T] = 2
0E 1	+ id$	reduce 2 i.e $E \rightarrow T$ & goto [0, E] = 1
0E1 + 6	id$	Shift 6
0E1 + 6 id 5	$	Shift 5
0E1 + 6F 3	$	reduce 6 & goto [6, F] = 3
0E1 + 6T 9	$	reduce 4 & goto [6, T] = 9
0E1	$	reduce 1 & goto [0, E] = 1
0E1	$	accept

Constructing SLR parsing table

LR (0) item: LR (0) item of a grammar G is a production of G with a dot at some position of the right side of production.

Example: $A \rightarrow BCD$
Possible LR (0) items are

$A \rightarrow .BCD$
$A \rightarrow B.CD$
$A \rightarrow BC.D$
$A \rightarrow BCD.$

$A \rightarrow B.CD$ means we have seen an input string derivable from B and hope to see a string derivable from CD.

The LR (0) items are constructed as a DFA from grammar to recognize viable prefixes.

The items can be viewed as the states of NFA.

The LR (0) item (or) canonical LR (0) collection, provides the basis for constructing SLR parser.

To construct LR (0) items, define
(a) An augmented grammar
(b) closure and goto

Augmented grammar (G′) If G is a grammar with start symbol S, G' the augmented grammar for G, with new start symbol S' and production $S' \rightarrow S$.

Purpose of G' is to indicate when to stop parsing and announce acceptance of the input.

Closure operation Closure (I) includes

1. Intially, every item in I is added to closure (I)
2. If $A \rightarrow \alpha.B\beta$ is in closure (I) and $\beta \rightarrow \gamma$ is a production then add $B \rightarrow .\gamma$ to I.

Goto operation

Goto (I, x) is defined to be the closure of the set of all items [$A \rightarrow \alpha X.\beta$] such that [$A \rightarrow \alpha.X\beta$] is in I.

Items

Kernel items: $S' \rightarrow .S$ and all items whose dots are not at the left end

Non-kernel items: Which have their dots at the left end.

Construction of sets of Items

Procedure items (G')
Begin
C: = closure ({[$S' \rightarrow .S$]});
repeat
For each set of items I in C and each grammar symbol x Such that goto (I, x) is not empty and not in C do add goto (I, x) to C;
Until no more sets of items can be added to C, end;

Example: LR (0) items for the grammar

$E' \rightarrow E$
$E \rightarrow E + T/T$
$T \rightarrow T * F/F$
$F \rightarrow (E)/\text{id}$

is given below:
I_0: $E' \rightarrow .E$
$E \rightarrow .E + T$
$E \rightarrow .T$
$T \rightarrow .T * F$

$T \to .F$

$F \to .(E)$

$F \to .\text{id}$

I_1: got (I_0, E)

$E' \to E.$

$E \to E. + T$

I_2: goto (I_0, T)

$E \to T.$

$T \to T. * F$

I_3: goto (I_0, F)

$T \to F.$

I_4: goto $(I_0, ()$

$F \to (.E)$

$E \to .E + T$

$E \to .T$

$E \to .T * F$

$T \to .F$

$F \to .(E)$

$F \to .\text{id}$

I_5: goto (I_0, id)

$F \to \text{id}.$

I_6: got $(I_1, +)$

$E \to E+ .T$

$T \to .T * F$

$T \to .F$

$F \to .(E)$

$F \to .id$

I_7: goto $(I_2, *)$

$T \to T* .F$

$F \to .(E)$

$F \to .id$

I_8: goto (I_4, E)

$F \to (E.)$

I_9: goto (I_6, T)

$E \to E+ T.$

$T \to T.* F$

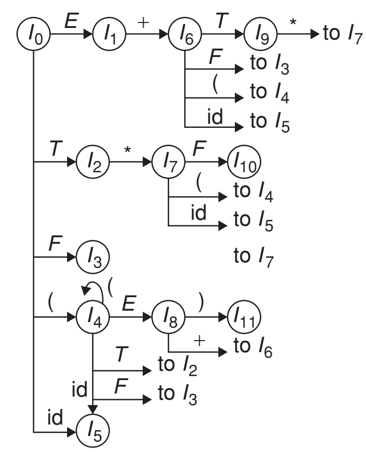
I_{10}: goto (I_7, F)

$T \to T* F.$

I_{11}: goto $(I_8,))$

$F \to (E).$

For viable prefixes construct the DFA as follows:



SLR parsing table construction

1. Construct the canonical collection of sets of LR (0) items for G'.
2. Create the parsing action table as follows:
   (a) If a is a terminal and $[A \to \alpha.a\beta]$ is in I_i, goto $(I_i, a) = I_j$ then action (i, a) to shift j. Here 'a' must be a terminal.
   (b) If $[A \to \alpha.]$ is in I_i, then set action $[i, a]$ to 'reduce $A \to \alpha$' for all a in FOLLOW (A);
   (c) If $[S' \to S.]$ is in I_i then set action $[i, \$]$ to 'accept'.
3. Create the parsing goto table for all non-terminals A, if goto $(I_i, A) = I_j$ then goto $[i, A] = j$.
4. All entries not defined by steps 2 and 3 are made errors.
5. Initial state of the parser contains $S' \to S$.
   The parsing table constructed using the above algorithm is known as SLR (1) table for G.

Note: Every SLR (1) grammar is unambiguous, but every unambiguous grammar is not a SLR grammar.

Example 6: Construct SLR parsing table for the following grammar:

1. $S \to L = R$
2. $S \to R$
3. $L \to * R$
4. $L \to \text{id}$
5. $R \to L$

Solution: For the construction of SLR parsing table, add $S' \to S$ production.

$S' \to S$

$S \to L = R$

$S \to R$

$L \to *R$

$L \to \text{id}$

$R \to L$

LR (0) items will be

I_0: $S' \to .S$

$S \to .L = R$

$S \to .R$

$L \to .{*}R$

$L \to .\text{id}$

$R \to .L$

I_1: goto (I_0, S)

$S' \to S.$

I_2: goto (I_0, L)

$S \to L. = R$

$R \to L.$

I_3: got (I_0, R)

$S \to R.$

I_4: goto (I_0, *)

$L \to {*}.R$

$R \to .L$

$L \to .{*}R$

$L \to .\text{id}$

I_5: goto(I_0, id)

$L \to \text{id}.$

I_6: goto(I_2, =)

$S \to L = .R$

$R \to .L$

$L \to .{*}R$

$L \to .\text{id}$

I_7: goto(I_4, R)

$L \to {*}R.$

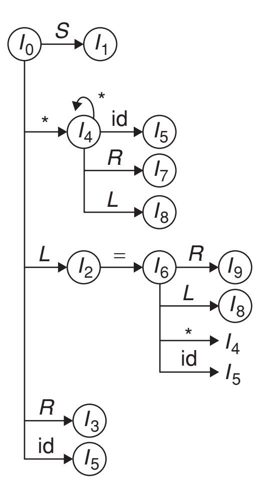
I_8: goto(I_4, L)

$R \to L.$

I_9: goto(I_6, R)

$S \to L = R.$

The DFA of LR(0) items will be



States	Action				Goto		
	=	*	id	$	S	L	R
0		S_4	S_5		1	2	3
1				acc			
2	S_6, r_5			r_5			
3							
4		S_4	S_5			8	7
5							
6		S_4	S_5			8	9
7							
8							
9							

FOLLOW (S) = {\$}
FOLLOW (L) = {=}
FOLLOW (R) = {\$, =}
For action [2, =] = S_6 and r_5
∴ Here we are getting shift – reduce conflict, so it is not SLR (1).

Canonical LR Parsing (CLR)

- To avoid some of invalid reductions, the states need to carry more information.
- Extra information input into a state by including a terminal symbol as a second component of an item.
- The general form of an item

  $[A \to \alpha.\beta, a]$
  Where $A \to \alpha\beta$ is a production.
  a is terminal/right end marker (\$). We will call it as LR (1) item.

LR (1) item

It is a combination of LR (0) items along with look ahead of the item. Here 1 refers to look ahead of the item.

Construction of the sets of LR (1) items Function closure (I):
Begin
Repeat
For each item $[A \to \alpha.B\beta, a]$ in I,
Each production $B \to .\gamma$ in G',
And each terminal b in FIRST (βa)
Such that $[B \to .\gamma, b]$ is not in I do
Add $[B \to .\gamma, b]$ to I;
End;
Until no more items can be added to I;

Example 7: Construct CLR parsing table for the following grammar:

$$S' \to S$$
$$S \to CC$$
$$C \to cC/d$$

Solution: The initial set of items is

I_0: $S' \to .S, \$$
$S \to .CC, \$$
$A \to \alpha.B\beta, a$

Here $A = S$, $\alpha = \in$, $B = C$, $\beta = C$ and $a = \$$
First (βa) is first ($C\$$) = first (C) = $\{c, d\}$
So, add items $[C \to .cC, c]$
$[C \to .cC, d]$

$\therefore$ Our first set I_0: $S' \to .S, \$$
$S \to .CC, \$$
$C \to$ .coca, c/d
$C \to .d, c/d.$

I_1: goto (I_0, X) if $X = S$
$S' \to S., \$$

I_2 : goto (I_0, C)
$S \to C.C, \$$
$C \to .cC, \$$
$C \to .d, \$$
I_3: goto (I_0, c)
$C \to c.C, c/d$
$C \to .cC, c/d$
$C \to .d\ c/d$

I_4: goto (I_0, d)
$C \to d., c/d$

I_5: goto (I_2, C)
$S \to CC., \$$

I_6: goto (I_2, c)
$C \to c.C; \$$
$C \to ..cC, \$$
$C \to ..d, \$$

I_7: goto (I_2, d)
$C \to d.\ \$$

I_8: goto (I_3, C)
$C \to cC., c/d$

I_9: goto (I_6, C)
$C \to cC., \$$

CLR table is:

	Action			Goto	
States	***c***	***1***	**$**	***S***	***C***
I_0	S_3	S_4		1	2
I_1			acc		
I_2	S_6	S_7			5
I_3	S_3	S_4			8
I_4	R_3	r_3			
I_5			r_1		
I_6	S_6	S_7			9
I_7			r_3		
I_8	R_2	r_2			
I_9			r_2		

Consider the string derivation '*dcd*':

$$S \Rightarrow CC \Rightarrow CcC \Rightarrow Ccd \Rightarrow dcd$$

Stack	Input	Action
0	*dcd*$	shift 4
0*d*4	*Cd*$	reduce 3 i.e. $C \to d$
0*C*2	*Cd*$	shift 6
0*C*2*C*6	*D*$	shift 7
0*C*2*C*6*d*7	$	reduce $C \to d$
0*C*2*C*6*C*9	$	reduce $C \to cC$
0*C*2*C*5	$	reduce $S \to CC$
0*S*1	$	

Example 8: Construct CLR parsing table for the grammar:
$S \to L = R$
$S \to R$
$L \to *R$
$L \to id$
$R \to L$

Solution: The canonical set of items is
I_0: $S' \to .S, \$$
$S \to .L = R, \$$
$S \to .R, \$$
$L \to .*\ R, =$ [first (= R$) = {=}]
$L \to$ .id, =
$R \to .L, \$$

I_1: got (I_0, S)
$S' \to S., \$$

I_2: goto (I_0, L)
$S \to L. = R, \$$
$R \to L., \$$

I_3: goto (I_0, R)
$S \to R., \$$

I_4: got (I_0, *)
$L \to *.\ R, =$
$R \to .L, =$
$L \to .*\ R, =$
$L \to$ .id, =
I_5: goto (I_0, id)
$L \to$ id.,=
I_6: goto (I_7, L)
$R \to L., \$$

I_7: goto (I_2, =)
$S \to L = .R,$
$R \to .L, \$$
$L \to .*R, \$$
$L \to$ .id, $

I_8: goto (I_4, R)
$L \to *R., =$

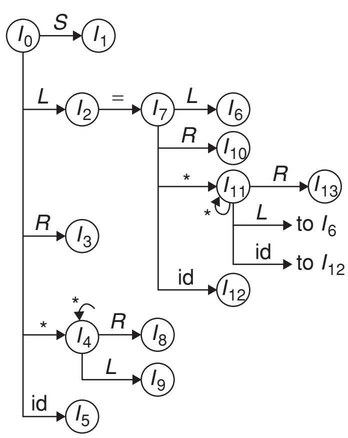
I_9: goto (I_4, L)
$R \rightarrow L., =$

I_{10}: got (I_7, R)
$S \rightarrow L = R., \$$

I_{11}: goto $(I_7, *)$
$L \rightarrow *.R, \$$
$R \rightarrow .L, \$$
$L \rightarrow .*R, \$$
$L \rightarrow .\text{id}, \$$

I_{12}: goto (I_7, id)
$L \rightarrow \text{id}. , \$$

I_{13}: goto (I_{11}, R)
$L \rightarrow *R., \$$



We have to construct CLR parsing table based on the above diagram.

In this, we are going to have 13 states

The shift –reduce conflict in the SLR parser is reduced here.

States	id	*	=	$	S	L	R
0	S_5	S_4			1	2	3
1				acc			
2			S_7	r_5			
3				r_2			
4	S_5	S_4				9	8
5			r_4				
6				r_5			
7	s_{12}	s_{11}				6	10
8			r_3				
9			r_5				
10				r_1			
11	S_{12}	S_{11}					13
12				r_4			
13				r_3			

Stack	Input
0	$Id = id\$$
$0id5$	$= id\$$
$0L2$	$= id\$$
$0L2 = 7$	$id\$$
0L2 = 7! d12	$
$0L2 = 7L6$	$
$0L2 = 7R10$	$
$0S1$ (accept)	$

Every SLR (1) grammar is LR (1) grammar.
CLR (1) will have 'more number of states' than SLR Parser.

LALR Parsing Table

- The tables obtained by it are considerably smaller than the canonical LR table.
- LALR stands for Lookahead LR.
- The number of states in SLR and LALR parsing tables for a grammar G are equal.
- But LALR parsers recognize more grammars than SLR.
- YACC creates a LALR parser for the given grammar.
- YACC stands for 'Yet another Compiler'.
- An easy, but space-consuming LALR table construction is explained below:
  1. Construct $C = \{I_0, I_1, –I_n\}$, the collection of sets of LR (1) items.
  2. Find all sets having the common core; replace these sets by their union
  3. Let $C' = \{J_O, J_1 \text{ --- } J_m\}$ be the resulting sets of LR (1) items. If there is a parsing action conflict then the grammar is not a LALR (1).
  4. Let k be the union of all sets of items having the same core. Then goto $(J, X) = k$
- If there are no parsing action conflicts then the grammar is said to LALR (1) grammar.
- The collection of items constructed is called LALR (1) collection.

Example 9: Construct LALR parsing table for the following grammar:

$S' \rightarrow S$
$S \rightarrow CC$
$C \rightarrow cC/d$

Solution: We already got LR (1) items and CLR parsing table for this grammar.
After merging I3 and I6 are replaced by I36.

I_{36}: $C \rightarrow c.C, c/d/\$$
$C \rightarrow .cC, c/d/\$$
$C \rightarrow .d, c/d/\$$

I_{47}: By merging I_4 and I_7
$C \rightarrow d.\ c/d/\$$

I_{89}: I_8 and I_9 are replaced by I_{89}
$C \rightarrow cC.,\ c/d/\$$

The LALR parsing table for this grammar is given below:

	Action			goto	
State	*c*	*d*	$	*S*	*C*
0	S_{36}	S_{47}		1	2
1			*acc*		
2	S_{36}	S_{47}			5
36	S_{36}	S_{47}			89
47	r_3	R_3	r_3		
5			r_1		
89	r_2	r_2	r_2		

Example: Consider the grammar:

$S' \rightarrow S$
$S \rightarrow aAd$
$S \rightarrow bBd$
$S \rightarrow aBe$
$S \rightarrow bAe$
$A \rightarrow c$
$B \rightarrow c$

Which generates strings acd, bcd, ace and bce

LR (1) items are

I_0: $S' \rightarrow .S,\ \$$
$S \rightarrow .aAd,\ \$$
$S \rightarrow .bBd,\ \$$
$S \rightarrow .aBe,\ \$$
$S \rightarrow .bAe,\ \$$

I_1: goto (I_0, S)
$S' \rightarrow S.,\ \$$

I_2: goto (I_0, a)
$S \rightarrow a.Ad,\ c$
$S \rightarrow a.Be,\ c$
$A \rightarrow .c,d$
$B \rightarrow .c,e$

I_3: goto (I_0, b)
$S \rightarrow b.Bd,\ c$
$S \rightarrow b.Ae,\ c$
$A \rightarrow .c,\ e$
$B \rightarrow .c,\ e$

I_4: goto (I_2, A)
$S \rightarrow aA.d,\ c$

I_5: goto (I_2, B)
$S \rightarrow aB.e,\ c$

I_6: goto (I_2, c)
$A \rightarrow c.,\ d$
$B \rightarrow c.,\ e$

I_7: goto (I_3, c)
$A \rightarrow c.,\ e$
$B \rightarrow c.,\ d$

I_8: goto (I_4, d)
$S \rightarrow aAd.,\ c$

I_9: goto (I_5, e)
$S \rightarrow aBe.,\ c$

If we union I_6 and I_7
$A \rightarrow c.,\ d/e$
$B \rightarrow c.,\ d/e$

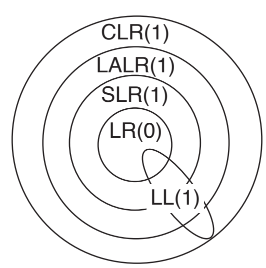
It generates reduce/reduce conflict.

Notes:

1. The merging of states with common cores can never produce a shift/reduce conflict, because shift action depends only on the core, not on the lookahead.
2. SLR and LALR tables for a grammar always have the same number of states (several hundreds) whereas CLR have thousands of states for the same grammar.

Comparison of parsing methods

Method	Item	Goto and Closures	Grammar it Applies to
SLR (1)	LR(0) item	Different from LR(1)	SLR (1) $\subset$ LR(1)
LR (1)	LR(1) item		LR(1) – Largest class of LR grammars
LALR(1)	LR(1) item	Same as LR(1)	LALR(1) $\subset$ LR(1)



Every LR (0) is SLR (1) but vice versa is not true.

Difference between SLR, LALR and CLR parsers

Differences among SLR, LALR and CLR are discussed below in terms of size, efficiency, time and space.

Table 1 *Comparison of parsing methods*

Sl. No.	Factors	SLR Parser	LALR Parser	CLR Parser
1	Size	Smaller	Smaller	Larger
2.	Method	It is based on FOLLOW function	This method is applicable to wider class than SLR	This is most powerful than SLR and LALR.
3.	Syntactic features	Less exposure compared to other LR parsers	Most of them are expressed	Less
4.	Error detection	Not immediate	Not immediate	Immediate
5.	Time and space complexity	Less time and space	More time and space complexity	More time and space complexity

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Consider the grammar
$S \to a$
$S \to ab$
The given grammar is:
(A) LR (1) only
(B) LL (1) only
(C) Both LR (1) and LL (1)
(D) LR (1) but not LL (1)

2. Which of the following is an unambiguous grammar, that is not LR (1)?
(A) $S \to$ Uab|Vac
$U \to d$
$V \to d$
(B) $S \to$ Uab/Vab/Vac
$U \to d$
$V \to d$
(C) $S \to AB$
$A \to a$
$B \to b$
(D) $S \to Ab$
$A \to a/c$

Common data for questions 3 and 4: Consider the grammar:

$S \to T; S/\in$
$T \to UR$
$U \to x/y/[S]$
$R \to .T/\in$

3. Which of the following are correct FIRST and FOLLOW sets for the above grammar?
(i) FIRST(S) = FIRST (T) = FIRST (U) = $\{x, y, [, \varepsilon\}$
(ii) FIRST (R) = $\{,\varepsilon\}$
(iii) FOLLOW (S) = {], \$}
(iv) FOLLOW (T) = Follow (R) = {;}
(v) FOLLOW (U) = {. ;}
(A) (i) and (ii) only
(B) (ii), (iii), (iv) and (v) only
(C) (ii), (iii) and (iv) only
(D) All the five

4. If an LL (1) parsing table is constructed for the above grammar, the parsing table entry for [$S \to$ [ ] is
(A) $S \to T; S$ (B) $S \to \in$
(C) $T \to UR$ (D) $U \to [S]$

Common data for questions 5 to 7: Consider the augmented grammar
$S \to X$
$X \to (X)/a$

5. If a DFA is constructed for the LR (1) items of the above grammar, then the number states present in it are:
(A) 8 (B) 9
(C) 7 (D) 10

6. Given grammar is
(A) Only LR (1)
(B) Only LL (1)
(C) Both LR (1) and LL (1)
(D) Neither LR (1) nor LL (1)

7. What is the number of shift-reduce steps for input (a)?
(A) 15 (B) 14
(C) 13 (D) 16

8. Consider the following two sets of LR (1) items of a grammar:

$X \to c.X, c/d$	$X \to c.X, \$$
$X \to .cX, c/d$	$X \to .cX, \$$
$X \to d, c/d$	$X \to .d, \$$

Which of the following statements related to merging of the two sets in the corresponding LALR parser is/are FALSE?
1. Cannot be merged since look ahead are different.
2. Can be merged but will result in $S-R$ conflict.
3. Can be merged but will result in $R-R$ conflict.
4. Cannot be merged since goto on c will lead to two different sets.

(A) 1 only (B) 2 only
(C) 1 and 4 only (D) 1, 2, 3 and 4

9. Which of the following grammar rules violate the requirements of an operator grammar?
(i) $A \to BcC$ (ii) $A \to dBC$
(iii) $A \to C/\in$ (iv) $A \to cBdC$

(A) (i) only (B) (i) and
(C) (ii) and (iii) only (D) (i) and (iv) only

10. The FIRST and FOLLOW sets for the grammar:

$$S \rightarrow SS + /SS^*/a$$

(A) First (S) = $\{a\}$
Follow (S) = $\{+, *, \$\}$
(B) First (S) = $\{+\}$
Follow (S) = $\{+, *, \$\}$
(C) First (S) = $\{a\}$
Follow (S) = $\{+, *\}$
(D) First (S) = $\{+, *\}$
Follow (S) = $\{+, *, \$\}$

11. A shift reduces parser carries out the actions specified within braces immediately after reducing with the corresponding rule of the grammar:
$S \rightarrow xxW$ [print '1']
$S \rightarrow y$ [print '2']
$W \rightarrow Sz$ [print '3']
What is the translation of '$x\ x\ x\ x\ y\ z\ z$'?
(A) 1231 (B) 1233
(C) 2131 (D) 2321

12. After constructing the predictive parsing table for the following grammar:
$Z \rightarrow d$
$Z \rightarrow XYZ$
$Y \rightarrow c/\in$
$X \rightarrow Y$
$X \rightarrow a$
The entry/entries for [Z, d] is/are
(A) $Z \rightarrow d$
(B) $Z \rightarrow XYZ$
(C) Both (A) and (B)
(D) $X \rightarrow Y$

13. The following grammar is
$S \rightarrow AaAb/BbBa$
$A \rightarrow \varepsilon$
$B \rightarrow \varepsilon$
(A) LL (1) (B) Not LL (1)
(C) Recursive (D) Ambiguous

14. Compute the FIRST (P) for the below grammar:
$P \rightarrow AQR$be/mn/DE
$A \rightarrow ab/\varepsilon$
$Q \rightarrow q_1q_2/\varepsilon$
$R \rightarrow r_1r_2/\varepsilon$
$D \rightarrow d$
$E \rightarrow e$
(A) $\{m, a\}$ (B) $\{m, a, q_1, r_1, b, d\}$
(C) $\{d, e\}$ (D) $\{m, n, a, b, d, e, q_1, r_1\}$

15. After constructing the LR(1) parsing table for the augmented grammar
$S' \rightarrow S$
$S \rightarrow BB$
$B \rightarrow aB/c$
What will be the action [I_3, a]?
(A) Accept (B) S_7
(C) r_2 (D) S_5

Practice Problems 2

Directions for questions 1 to 19: Select the correct alternative from the given choices.

1. Consider the grammar
$S \rightarrow aSb$
$S \rightarrow aS$
$S \rightarrow \varepsilon$
This grammar is ambiguous by generating which of the following string.
(A) aa (B) $\in$
(C) aaa (D) aab

2. To convert the grammar $E \rightarrow E + T$ into LL grammar
(A) use left factor
(B) CNF form
(C) eliminate left recursion
(D) Both (B) and (C)

3. Given the following expressions of a grammar
$E \rightarrow E \times F/F + E/F$
$F \rightarrow F$? F/id
Which of the following is true?
(A) $\times$ has higher precedence than +
(B) ? has higher precedence than $\times$
(C) + and? have same precedence
(D) + has higher precedence than *

4. The action of parsing the source program into the proper syntactic classes is known as
(A) Lexical analysis
(B) Syntax analysis
(C) Interpretation analysis
(D) Parsing

5. Which of the following is not a bottom up parser?
(A) LALR (B) Predictive parser
(C) CLR (D) SLR

6. A system program that combines separately compiled modules of a program into a form suitable for execution is
(A) Assembler.
(B) Linking loader.
(C) Cross compiler.
(D) None of these.

7. Resolution of externally defined symbols is performed by a
(A) Linker (B) Loader.
(C) Compiler. (D) Interpreter.

8. LR parsers are attractive because
(A) They can be constructed to recognize CFG corresponding to almost all programming constructs.
(B) There is no need of backtracking.
(C) Both (A) and (B).
(D) None of these

9. YACC builds up
(A) SLR parsing table
(B) Canonical LR parsing table
(C) LALR parsing table
(D) None of these

10. Language which have many types, but the type of every name and expression must be calculated at compile time are
(A) Strongly typed languages
(B) Weakly typed languages
(C) Loosely typed languages
(D) None of these

11. Consider the grammar shown below:
$S \to iEtSS'/a/b$
$S' \to eS/\varepsilon$
In the predictive parse table M, of this grammar, the entries $M[S', e]$ and $M[S', \$]$ respectively are
(A) $\{S' \to eS\}$ and $\{S' \to \in\}$
(B) $\{S' \to eS\}$ and $\{\ \}$
(C) $\{S' \to \in\}$ and $\{S' \to \in\}$
(D) $\{S' \to eS, S' \to \varepsilon\}\}$ and $\{S' \to \in\}$

12. Consider the grammar $S \to CC$, $C \to cC/d$.
The grammar is
(A) LL (1)
(B) SLR (1) but not LL (1)
(C) LALR (1) but not SLR (1)
(D) LR (1) but not LALR (1)

13. Consider the grammar
$E \to E + n/E - n/n$
For a sentence $n + n - n$, the handles in the right sentential form of the reduction are
(A) n, $E + n$ and $E + n - n$
(B) n, $E + n$ and $E + E - n$
(C) n, $n + n$ and $n + n - n$
(D) n, $E + n$ and $E - n$

14. A top down parser uses ___ derivation.
(A) Left most derivation
(B) Left most derivation in reverse
(C) Right most derivation
(D) Right most derivation in reverse

15. Which of the following statement is false?
(A) An unambiguous grammar has single leftmost derivation.
(B) An LL (1) parser is topdown.
(C) LALR is more powerful than SLR.
(D) An ambiguous grammar can never be LR (K) for any k.

16. Merging states with a common core may produce ___ conflicts in an LALR parser.
(A) Reduce – reduce
(B) Shift – reduce
(C) Both (A) and (B)
(D) None of these

17. LL (K) grammar
(A) Has to be CFG
(B) Has to be unambiguous
(C) Cannot have left recursion
(D) All of these

18. The I_0 state of the LR (0) items for the grammar
$S \to AS/b$
$A \to SA/a$.
(A) $S' \to .S$
$S \to .As$
$S \to .b$
$A \to .SA$
$A \to .a$
(B) $S \to .AS$
$S \to .b$
$A \to .SA$
$A \to .a$
(C) $S \to .AS$
$S \to .b$
(D) $S \to A$
$A \to .SA$
$A \to .a$

19. In the predictive parsing table for the grammar:
$S \to FR$
$R \to \times S/e$
$F \to \text{id}$
What will be the entry for $[S, \text{id}]$?
(A) $S \to FR$
(B) $F \to \text{id}$
(C) Both (A) and (B)
(D) None of these

Previous Years' Questions

1. Consider the grammar:
$S \to (S) \mid a$
Let the number of states in SLR (1), LR (1) and LALR (1) parsers for the grammar be n_1, n_2 and n_3 respectively. The following relationship holds good: **[2005]**
(A) $n_1 < n_2 < n_3$ (B) $n_1 = n_3 < n_2$
(C) $n_1 = n_2 = n_3$ (D) $n_1 \geq n_3 \geq n_2$

2. Consider the following grammar:
$S \to S * E$
$S \to E$
$E \to F + E$
$E \to F$
$F \to \text{id}$
Consider the following LR (0) items corresponding to the grammar above.
(i) $S \to S * .E$
(ii) $E \to F. + E$
(iii) $E \to F + .E$
Given the items above, which two of them will appear in the same set in the canonical sets-of items for the grammar? **[2006]**
(A) (i) and (ii) (B) (ii) and (iii)
(C) (i) and (iii) (D) None of the above

3. Consider the following statements about the context-free grammar
$G = \{S \to SS, S \to ab, S \to ba, S \to \in\}$
(i) G is ambiguous
(ii) G produces all strings with equal number of a's and b's
(iii) G can be accepted by a deterministic PDA.
Which combination below expresses all the true statements about G? **[2006]**
(A) (i) only (B) (i) and (iii) only
(C) (ii) and (iii) only (D) (i), (ii) and (iii)

4. Consider the following grammar:
$S \to FR$
$R \to *S|\varepsilon$
$F \to \text{id}$
In the predictive parser table, M, of the grammar the entries M[S, id] and M[R, \$] respectively. **[2006]**
(A) $\{S \to FR\}$ and $\{R \to \varepsilon\}$
(B) $\{S \to FR\}$ and $\{\ \}$
(C) $\{S \to FR\}$ and $\{R \to *S\}$
(D) $\{F \to \text{id}\}$ and $\{R \to \varepsilon\}$

5. Which one of the following grammars generates the language $L = \{a^i b^j | i \neq j\}$? **[2006]**
(A) $S \to AC|CB$
$C \to aCb|a|b$
$A \to aA|\in$
$B \to Bb|\in$
(B) $S \to aS|Sb|a|b$
(C) $S \to AC|CB$
$C \to aC|b|\in$
$A \to aA|\in$
$B \to Bb|\in$
(D) $S \to AC|CB$
$C \to aCb|\in$
$A \to aA|a$
$B \to Bb|b$

6. In the correct grammar above, what is the length of the derivation (number of steps starting from S) to generate the string $a^\ell b^m$ with $\ell \neq m$? **[2006]**
(A) max (l, m) + 2
(B) l + m + 2
(C) l + m + 3
(D) max (l, m) + 3

7. Which of the following problems is undecidable? **[2007]**
(A) Membership problem for CFGs.
(B) Ambiguity problem for CFGs.
(C) Finiteness problem for FSAs.
(D) Equivalence problem for FSAs.

8. Which one of the following is a top-down parser? **[2007]**
(A) Recursive descent parser.
(B) Operator precedence parser.
(C) An LR (k) parser.
(D) An LALR (k) parser.

9. Consider the grammar with non-terminals $N = \{S, C, \text{and } S_1\}$, terminals $T = \{a, b, i, t, e\}$ with S as the start symbol, and the following set of rules: **[2007]**
$S \to iCtSS_1/a$
$S_1 \to eS/\varepsilon$
$C \to b$
The grammar is NOT LL (1) because:
(A) It is left recursive
(B) It is right recursive
(C) It is ambiguous
(D) It is not context-free.

10. Consider the following two statements:
P: Every regular grammar is LL (1)
Q: Every regular set has a LR (1) grammar
Which of the following is TRUE? **[2007]**
(A) Both P and Q are true
(B) P is true and Q is false
(C) P is false and Q is true
(D) Both P and Q are false

Common data for questions 11 and 12: Consider the CFG with $\{S, A, B\}$ as the non-terminal alphabet, $\{a, b\}$ as the terminal alphabet, S as the start symbol and the following set of production rules:

$S \to aB$	$S \to bA$
$B \to b$	$A \to a$
$B \to bS$	$A \to aS$
$B \to aBB$	$S \to bAA$

11. Which of the following strings is generated by the grammar? **[2007]**
(A) *aaaabb* (B) *aabbbb*
(C) *aabbab* (D) *abbbba*

12. For the correct answer strings to Q.78, how many derivation trees are there? **[2007]**
(A) 1 (B) 2
(C) 3 (D) 4

13. Which of the following describes a handle (as applicable to LR-parsing) appropriately? **[2008]**
(A) It is the position in a sentential form where the next shift or reduce operation will occur.
(B) It is non-terminal whose production will be used for reduction in the next step.
(C) It is a production that may be used for reduction in a future step along with a position in the sentential form where the next shift or reduce operation will occur.
(D) It is the production *p* that will be used for reduction in the next step along with a position in the sentential form where the right hand side of the production may be found.

14. Which of the following statements are true?
(i) Every left-recursive grammar can be converted to a right-recursive grammar and vice-versa
(ii) All ε-productions can be removed from any context-free grammar by suitable transformations
(iii) The language generated by a context-free grammar all of whose productions are of the form $X \rightarrow w$ or $X \rightarrow wY$ (where, w is a string of terminals and Y is a non-terminal), is always regular
(iv) The derivation trees of strings generated by a context-free grammar in Chomsky Normal Form are always binary trees **[2008]**
(A) (i), (ii), (iii) and (iv) (B) (ii), (iii) and (iv) only
(C) (i), (iii) and (iv) only (D) (i), (ii) and (iv) only

15. An LALR (1) parser for a grammar *G* can have shift-reduce (*S*–*R*) conflicts if and only if **[2008]**
(A) The SLR (1) parser for *G* has *S*–*R* conflicts
(B) The LR (1) parser for *G* has *S*–*R* conflicts
(C) The LR (0) parser for *G* has *S*–*R* conflicts
(D) The LALR (1) parser for *G* has reduce-reduce conflicts

16. $S \rightarrow aSa \mid bSb \mid a \mid b$;
The language generated by the above grammar over the alphabet $\{a, b\}$ is the set of **[2009]**
(A) All palindromes.
(B) All odd length palindromes.
(C) Strings that begin and end with the same symbol.
(D) All even length palindromes.

17. Which data structure in a compiler is used for managing information about variables and their attributes? **[2010]**
(A) Abstract syntax tree
(B) Symbol table
(C) Semantic stack
(D) Parse table

18. The grammar $S \rightarrow aSa|bS|c$ is **[2010]**
(A) LL (1) but not LR (1)
(B) LR (1) but not LR (1)
(C) Both LL (1) and LR (1)
(D) Neither LL (1) nor LR (1)

19. The lexical analysis for a modern computer language such as Java needs the power of which one of the following machine models in a necessary and sufficient sense? **[2011]**
(A) Finite state automata
(B) Deterministic pushdown automata
(C) Non-deterministic pushdown automata
(D) Turing machine

Common data for questions 20 and 21: For the grammar below, a partial LL (1) parsing table is also presented along with the grammar. Entries that need to be filled are indicated as E_1, E_2, and E_3. Is the empty string, \$ indicates end of input, and, *I* separates alternate right hand side of productions

$S \rightarrow a\ A\ b\ B | b\ A\ a\ B |\ e$

$A \rightarrow S$

$B \rightarrow S$

	a	**b**	**$**
S	E_1	E_2	$S \rightarrow e$
A	$A \rightarrow S$	$A \rightarrow S$	Error
B	$B \rightarrow S$	$B \rightarrow S$	E_3

20. The FIRST and FOLLOW sets for the non-terminals A and B are **[2012]**
(A) FIRST $(A) = \{a, b, \varepsilon\}$ = FIRST (B)
FOLLOW $(A) = \{a, b\}$
FOLLOW $(B) = \{a, b, \$\}$
(B) FIRST $(A) = \{a, b, \$\}$
FIRST $(B) = \{a, b, \varepsilon\}$
FOLLOW $(A) = \{a, b\}$
FOLLOW $(B) = \{\$\}$
(C) FIRST $(A) = \{a, b, \varepsilon\}$ = FIRST (B)
FOLLOW $(A) = \{a, b\}$
FOLLOW $(B) = \varnothing$
(D) FIRST $(A) = \{a, b\}$ = FIRST (B)
FOLLOW $(A) = \{a, b\}$
FOLLOW $(B) = \{a, b\}$

21. The appropriate entries for E_1, E_2, and E_3 are **[2012]**
(A) E_1: $S \rightarrow a\ A\ b\ B,\ A \rightarrow S$
E_2: $S \rightarrow b\ A\ a\ B,\ B \rightarrow S$
E_3: $B \rightarrow S$

(B) E_1: $S \to a\,A\,b\,B$, $S \to \varepsilon$
E_2: $S \to b\,A\,a\,B$, $S \to \varepsilon$
E_3: $S \to \in$

(C) E_1: $S \to a\,A\,b\,B$, $S \to \varepsilon$
E_2: $S \to b\,A\,a\,B$, $S \to \varepsilon$
E_3: $B \to S$

(D) E_1: $A \to S$, $S \to \varepsilon$
E_2: $B \to S$, $S \to \varepsilon$
E_3: $B \to S$

22. What is the maximum number of reduce moves that can be taken by a bottom-up parser for a grammar with no epsilon-and unit-production (i.e., of type $A \to \in$ and $A \to a$) to parse a string with n tokens? **[2013]**

(A) $n/2$ (B) $n - 1$
(C) $2n - 1$ (D) 2^n

23. Which of the following is/are undecidable?

(i) G is a CFG. Is $L(G) = \phi$?
(ii) G is a CFG, Is $L(G) = \Sigma^*$?
(iii) M is a Turing machine. Is $L(M)$ regular?
(iv) A is a DFA and N is an NFA. Is $L(A) = L(N)$? **[2013]**

(A) (iii) only
(B) (iii) and (iv) only
(C) (i), (ii) and (iii) only
(D) (ii) and (iii) only

24. Consider the following two sets of LR (1) items of an LR (1) grammar. **[2013]**

$X \to c.X,\ c/d$	$X \to c.X,\ \$$
$X \to .cX,\ c/d$	$X \to .cX,\ \$$
$X \to .d,\ c/d$	$X \to .d,\ \$$

Which of the following statements related to merging of the two sets in the corresponding LALR parser is/are FALSE?

(i) Cannot be merged since look - ahead are different.
(ii) Can be merged but will result in S-R conflict.
(iii) Can be merged but will result in R-R conflict.
(iv) Cannot be merged since goto on c will lead to two different sets.

(A) (i) only (B) (ii) only
(C) (i) and (iv) only (D) (i), (ii), (iii) and (iv)

25. A canonical set of items is given below

$S \to L. > R$
$Q \to R.$

On input symbol < the sset has **[2014]**

(A) A shift–reduce conflict and a reduce–reduce conflict.
(B) A shift–reduce conflict but not a reduce–reduce conflict.
(C) A reduce–reduce conflict but not a shift reduce conflict.
(D) Neither a shift–reduce nor a reduce–reduce conflict.

26. Consider the grammar defined by the following production rules, with two operators * and +

$S \to T * P$
$T \to U | T * U$
$P \to Q + P | Q$
$Q \to Id$
$U \to Id$

Which one of the following is TRUE? **[2014]**

(A) + is left associative, while * is right associative
(B) + is right associative, while * is left associative
(C) Both + and * are right associative.
(D) Both + and * are left associative

27. Which one of the following problems is undecidable? **[2014]**

(A) Deciding if a given context -free grammar is ambiguous.
(B) Deciding if a given string is generated by a given context-free grammar.
(C) Deciding if the language generated by a given context-free grammar is empty.
(D) Deciding if the language generated by a given context free grammar is finite.

28. Which one of the following is TRUE at any valid state in shift-reduce parsing? **[2015]**

(A) Viable prefixes appear only at the bottom of the stack and not inside.
(B) Viable prefixes appear only at the top of the stack and not inside.
(C) The stack contains only a set of viable prefixes.
(D) The stack never contains viable prefixes.

29. Among simple LR (SLR), canonical LR, and look-ahead LR (LALR), which of the following pairs identify the method that is very easy to implement and the method that is the most powerful, in that order? **[2015]**

(A) SLR, LALR
(B) Canonical LR, LALR
(C) SLR, canonical LR
(D) LALR, canonical LR

30. Consider the following grammar G

$S \to F \mid H$
$F \to p \mid c$
$H \to d \mid c$

Where S, F and H are non-terminal symbols, p, d and c are terminal symbols. Which of the following statement(s) is/are correct? **[2015]**

S_1. LL(1) can parse all strings that are generated using grammar G

S_2. LR(1) can parse all strings that are generated using grammar G

(A) Only S_1 (B) Only S_2
(C) Both S_1 and S_2 (D) Neither S_1 nor S_2

31. Match the following: **[2016]**

(P) Lexical analysis (i) Leftmost derivation
(Q) Top down parsing (ii) Type checking
(R) Semantic analysis (iii) Regular expressions
(S) Runtime environments (iv) Activation records

(A) $P \leftrightarrow$ i, $Q \leftrightarrow$ ii, $R \leftrightarrow$ iv, $S \leftrightarrow$ iii
(B) $P \leftrightarrow$ iii, $Q \leftrightarrow$ i, $R \leftrightarrow$ ii, $S \leftrightarrow$ iv
(C) $P \leftrightarrow$ ii, $Q \leftrightarrow$ iii, $R \leftrightarrow$ i, $S \leftrightarrow$ iv
(D) $P \leftrightarrow$ iv, $Q \leftrightarrow$ i, $R \leftrightarrow$ ii, $S \leftrightarrow$ iii

32. A student wrote two context - free grammars **G1** and **G2** for generating a single C-like array declaration. The dimension of the array is at least one.

For example, int a [10] [3];

The grammars use D as the start symbol, and use six terminal symbols int; id [ ] num. **[2016]**

Grammar **G1**	Grammar **G2**
D → **int** L;	D → **int**L;
L → **id** [E	L → **id** E
E → **num**]	E → E [**num**]
E → **num**] [E	E → [**num**]

Which of the grammars correctly generate the declaration mentioned above?

(A) Both **G1** and **G2**
(B) Only **G1**
(C) Only **G2**
(D) Neither **G1** nor **G2**

33. Consider the following grammar:

$$\boxed{\begin{array}{l} P \to xQRS \\ Q \to yz \mid z \\ R \to w \mid \varepsilon \\ S \to y \end{array}}$$

What is FOLLOW (Q)? **[2017]**

(A) $\{R\}$ (B) $\{w\}$
(C) $\{w, y\}$ (D) $\{w, \$\}$

34. Which of the following statements about parser is/are CORRECT? **[2017]**

I. Canonical LR is more powerful than SLR.
II. SLR is more powerful than LALR.
III. SLR is more powerful than Canonical LR.

(A) I only (B) II only
(C) III only (D) I and III only

35. Consider the following expression grammar G :

E – > E – T | T
T – > T + F | F
F – > (E) | id

Which of the following grammars is not left recursive, but is equivalent to G? **[2017]**

(A) E – >E – T | T
T – > T + F | F
F – > (E) | id

(B) E – > TE
E′ – > –TE | ∈
T – > T + F | F
F – > (E) | id

(C) E – > TX
X – > –TX | ∈
T – > FY
Y – > + FY | ∈
F – > (E) | id

(D) E – > TX | (TX)
X – > –TX | +TX | ∈
T – > id

36. Which one of the following statements is FALSE? **[2018]**

(A) Context-free grammar can be used to specify both lexical and syntax rules.
(B) Type checking is done before parsing.
(C) High-level language programs can be translated to different Intermediate Representations.
(D) Arguments to a function can be passed using the program stack.

37. A lexical analyzer uses the following patterns to recognize three tokens T_1, T_2, and T_3 over the alphabet $\{a, b, c\}$.

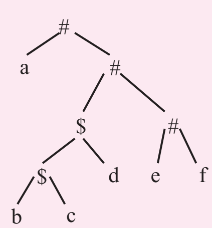
T_1: $a?(b|c)^*a$
T_2: $b?(a|c)^*b$
T_3: $c?(b|a)^*c$

Note that 'x?' means 0 or 1 occurrence of the symbol x. Note also that the analyzer outputs the token that matches the longest possible prefix.

If the string bbaacabc is processed by the analyzer, which one of the following is the sequence of tokens it outputs? **[2018]**

(A) $T_1T_2T_3$ (B) $T_1T_1T_3$
(C) $T_2T_1T_3$ (D) T_3T_3

38. Consider the following parse tree for the expression a#b\$c\$d#e#f, involving two binary operators \$ and #.



Which one of the following is correct for the given parse tree? **[2018]**

(A) \$ has higher precedence and is left associative; # is right associative
(B) # has higher precedence and is left associative; \$ is right associative
(C) \$ has higher precedence and is left associative; # is left associative
(D) # has higher precedence and is right associative; \$ is left associative

Answer Keys

Exercises

Practice Problems 1

1. D	**2.** A	**3.** B	**4.** A	**5.** D	**6.** C	**7.** C	**8.** D	**9.** C	**10.** A
11. C	**12.** C	**13.** A	**14.** B	**15.** D					

Practice Problems 2

1. D	**2.** C	**3.** B	**4.** A	**5.** B	**6.** B	**7.** A	**8.** C	**9.** C	**10.** A
11. D	**12.** A	**13.** D	**14.** A	**15.** D	**16.** A	**17.** C	**18.** A	**19.** A	

Previous Years' Questions

1. B	**2.** D	**3.** B	**4.** A	**5.** D	**6.** A	**7.** B	**8.** A	**9.** C	**10.** A
11. C	**12.** B	**13.** D	**14.** C	**15.** B	**16.** B	**17.** B	**18.** C	**19.** A	**20.** A
21. C	**22.** B	**23.** D	**24.** D	**25.** D	**26.** B	**27.** A	**28.** C	**29.** C	**30.** D
31. B	**32.** A	**33.** C	**34.** A	**35.** C	**36.** B	**37.** D	**38.** A		

Chapter 2

Syntax Directed Translation

LEARNING OBJECTIVES

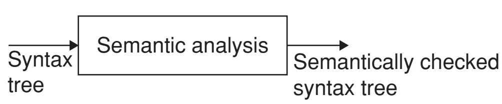
- *Syntax directed translation*
- *Syntax directed definition*
- *Dependency graph*
- *Constructing syntax trees for expressions*
- *Types of SDD's*
- *S-attributed definition*
- *L-attributed definitions*
- *Synthesized attributes on the parser*
- *Syntax directed translation schemes*
- *Bottom up evaluation of inherited attributes*

Syntax Directed Translation

To translate a programming language construct, a compiler may need to know the type of construct, the location of the first instruction, and the number of instructions generated…etc. So, we have to use the term 'attributes' associated with constructs.

An attribute may represent type, number of arguments, memory location, compatibility of variables used in a statement which cannot be represented by CFG alone.

So, we need to have one more phase to do this, i.e., 'semantic analysis' phase.



In this phase, for each production CFG, we will give some semantic rule.

Syntax directed translation scheme

A CFG in which a program fragment called output action (semantic action or semantic rule) is associated with each production is known as Syntax Directed Translation Scheme.

These semantic rules are used to

1. Generate intermediate code.
2. Put information into symbol table.
3. Perform type checking.
4. Issues error messages.

Notes:

1. Grammar symbols are associated with attributes.
2. Values of the attributes are evaluated by the semantic rules associated with production rules.

Notations for Associating Semantic Rules

There are two techniques to associate semantic rules:

Syntax directed definition (SDD) It is high level specification for translation. They hide the implementation details, i.e., the order in which translation takes place.

Attributes + CFG + Semantic rules = Syntax directed definition (SDD).

Translation schemes These schemes indicate the order in which semantic rules are to be evaluated. This is an input and output mapping.

Syntax Directed Definitions

A SDD is a generalization of a CFG in which each grammar symbol is associated with a set of attributes.

There are two types of set of attributes for a grammar symbol.

1. Synthesized attributes
2. Inherited attributes

Each production rule is associated with a set of semantic rules.

Semantic rules setup dependencies between attributes which can be represented by a dependency graph.

The dependency graph determines the evaluation order of these semantic rules.

Evaluation of a semantic rule defines the value of an attribute. But a semantic rule may also have some side effects such as printing a value.

Attribute grammar: An attribute grammar is a syntax directed definition in which the functions in semantic rules 'cannot have side effects'.

Annotated parse tree: A parse tree showing the values of attributes at each node is called an annotated parse tree.

The process of computing the attribute values at the nodes is called annotating (or decorating) of the parse tree.

In a SDD, each production $A \to \propto$ is associated with a set of semantic rules of the form:

$b = f(c_1, c_2, \ldots c_n)$ where

f : A function

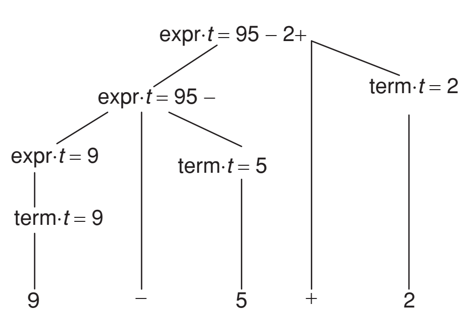
b can be one of the following:

b is a 'synthesized attribute' of A and $c_1, c_2, \ldots c_n$ are attributes of the grammar symbols in $A \to \propto$.

The value of a 'synthesized attribute' at a node is computed from the value of attributes at the children of that node in the parse tree.

Example:

Production	Semantic Rule
expr → expr1 + term	expr.t: = expr1.t||term.t||'+'
expr → expr1 – term	expr.t: = expr1.t||term.t||'–'
expr → term	expr.t: = term.t
term → 0	term.t: = '0'
term → 1	term.t: = '1'
⋮	⋮
term → 9	term.t: = '9'



b is an 'inherited attribute' of one of the grammar symbols on the right side of the production.

An 'inherited attribute' is one whose value at a node is defined in terms of attributes at the parent and/or siblings of that node. It is used for finding the context in which it appears.

Example: An inherited attribute distributes type information to the various identifiers in a declaration.

For the grammar

$D \to$ TL
$T \to$ int
$T \to$ real
$L \to L_1$, id
$L \to$ id

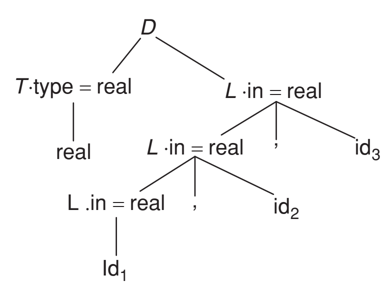
That is, The keyword int or real followed by a list of identifiers.

In this T has synthesized attribute type: T.type. L has an inherited attribute in L.in

Rules associated with L call for procedure add type to the type of each identifier to its entry in the symbol table.

Production	Semantic Rule
$D \to$ TL	L.in = T.type
$T \to$ int	T.type = integer
$T \to$ real	T.type = real
$L \to L_1$, id	addtype L_1.in = L.in(id.entry, L.in)
$L \to$ id	addtype (id.entry, L.in)

The annotated parse tree for the sentence real id_1, id_2, id_3 is shown below:



Synthesized Attribute

The value of a synthesized attribute at a node is computed from the value of attributes at the children of that node in a parse tree. Consider the following grammar:

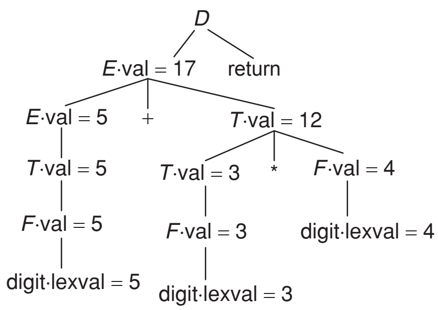
$L \to E_n$
$E \to E_1 + T$
$E \to T$
$T \to T_1 * F$
$T \to F$
$F \to (E)$
$F \to$ digit.

Let us consider synthesized attribute value with each of the non-terminals E, T and F.

Token digit has a synthesized attribute lexical supplied by lexical analyzer.

Production	Semantic Rule
$L \to E_n$	print (E.val)
$E \to E_1 + T$	E.val: = E_1.val + T.val
$E \to T$	E.val: = T_1.val
$T \to T_1 * F$	T.val: = T_1.val*F.val
$T \to F$	T.val: = F.val
$F \to (E)$	F.val: = E.val
$F \to$ digit	F.val: = digit.lexval

The Annotated parse tree for the expression 5 + 3 * 4 is shown below:



Example 1: Consider an example, which shows semantic rules for Infix to posfix translation:

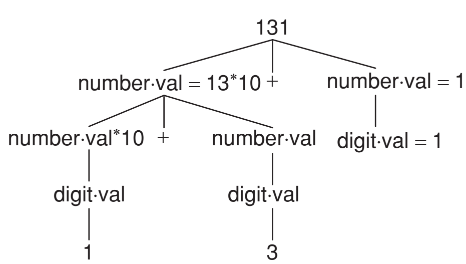
Production	Semantic Rules
expr → expr1 + term	expr.t: = expr1.t||term.t||'+'
expr → expr1 – term	expr.t: = expr1.t||term.t ||'–'
expr → term	expr.t: = term.t
term → 0	term.t: = '0'
⋮	⋮
term→ 9	term.t := '9'

Example 2: Write a SDD for the following grammar to determine number.val.

number → number digit
digit→ 0|1|...9

$$\left\{\begin{array}{l}\text{digit.val := '0'}\\ \text{digit.val := '1'}\\ \quad\vdots\\ \text{digit.val = '9'}\end{array}\right\}$$

number.val:=number.val * 10 + digit.val
Annotated tree for 131 is

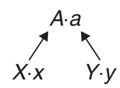


Dependency Graph

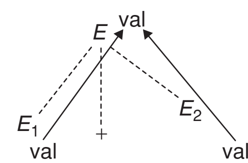
The interdependencies among the attributes at the nodes in a parse tree can be depicted by a directed graph called dependency graph.

- Synthesized attributes have edges pointing upwards.
- Inherited attributes have edges pointing downwards and/or sidewise.

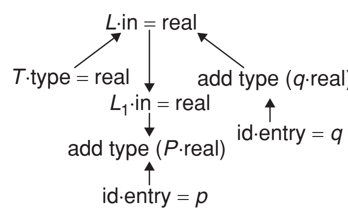
Example 1: $A.a := f(X.x,\ Y.y)$ is a semantic rule for $A \to XY$. For each semantic rule that consists of a procedure call:



Example 2:



Example 3: real p, q;



Evaluation order

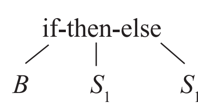
A topological sort of directed acyclic graph is an ordering $m_1, m_2, \ldots m_k$ of nodes of the graph S. t edges go from nodes earlier in the ordering to later nodes.

$m_i \to m_j$ means m_i appears before m_j in the ordering.

If $b: = f(c_1, c_2, \ldots, c_k)$, the dependent attributes $c_1, c_2, \ldots c_k$ are available at node before f is evaluated.

Abstract syntax tree

It is a condensed form of parse tree useful for representing language constructs.
Example



Constructing Syntax Trees for Expressions

Each node in a syntax tree can be implemented as a record with several fields.

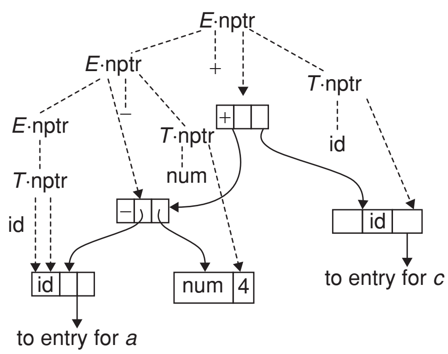
In the node for an operator, one field identifies the operator and the remaining fields contain pointers to the nodes for the operands.

1. mknode (op, left, right)
2. mkleaf (id, entry). Entry is a pointer to symbol table.
3. mkleaf (num, val)

Example:

Production	Semantic Rules
$E \to E_1 + T$	E.nptr := mknode ('+', E_1.nptr, T.nptr)
$E \to E_1 - T$	E.nptr := mknode ('–', E_1.nptr, T.nptr)
$E \to T$	E.nptr := T.nptr
$T \to (E)$	T.nptr := E.nptr
$T \to$ id	T.nptr := mkleaf(id, id.entry)
$T \to$ num	T.nptr := mkleaf(num, num.val)

Construction of a syntax tree for $a - 4 + c$



Types of SDD's

Syntax Directed definitions (SDD) are used to specify syntax directed translations. There are two types of SDD.

1. S-Attributed Definitions
2. L-Attributed Definitions.

S-attributed definitions

- Only synthesized attributes used in syntax direct definition.
- S-attributed grammars interact well with LR (K) parsers since the evaluation of attributes is bottom-up. They do not permit dependency graphs with cycles.

L-attributed definitions

- Both inherited and synthesized attribute are used.
- L-attributed grammar support the evaluation of attributes associated with a production body, dependency–graph edges can go from left to right only.
- Each S-attributed grammar is also a L-attributed grammar.
- L-attributed grammars can be incorporated conveniently in top down parsing.
- These grammars interact well with LL (K) parsers (both table driven and recursive descent).

Synthesized Attributes on the Parser Stack

A translator for an S-attributed definition often be implemented with LR parser generator. Here the stack is implemented by a pair of array state and val.

- Each state entry is pointed to a LR (1) parsing table.
- Each val[i] holds the value of the attributes associated with the node. For $A \to xyz$, the stack will be:

	State	Val
Top →	Z	$Z.z$
	Y	$Y.y$
	X	$X.x$

Example: Consider the following grammar:

$S \to E\,\$$	{print(E.val)}
$E \to E + E$	{E.val := E.val + E.val}
$E \to E * E$	{E.val := E.val * E.val}
$E \to (E)$	{E.val := E.val}
$E \to I$	{I.val := I.val * 10 + digit}
$I \to I$ digit	
$I \to$ digit	{I.val := digit}

Implementation

$S \to E\,\$$	print (val [top])
$E \to E + E$	val[ntop] := val[top] + val[top-2]
$E \to E * E$	val[ntop] := val[top] * val[top-2]
$E \to (E)$	val[ntop] := val[top-1]
$E \to I$	val[ntop] := val[top]
$I \to I$ digit	val[ntop] := 10*val[top] + digit
$I \to$ digit	val[ntop] := digit

L-attributed Definitions

A syntax directed definition is L-attributed if each inherited attribute of X_j, $1 \le j \le n$, on the right side of $A \to X_1 X_2 \ldots X_n$, depends only on

1. The attributes of symbols $X_1, X_2, \ldots, Xj_{-1}$ to the left of X_j in the production.
2. The inherited attributes of A.

Every S-attributed definition is L-attributed, because the above two rules apply only to the inherited attributes.

Syntax directed Translation Schemes

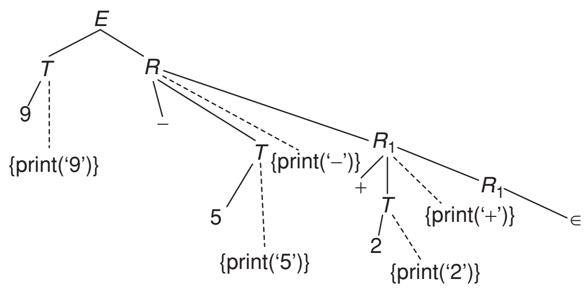
A translation scheme is a CFG in which attributes are associated with grammar symbols and semantic actions are enclosed between braces { } are inserted within the right sides of productions.

Example: $E \to TR$

$R \to$ op T {print (op.lexeme)} $R_1 | \in$

$T \to$ num {print (num.val)}

Using this, the parse tree for 9 – 5 + 2 is



If we have both inherited and synthesized attributes then we have to follow the following rules:

1. An inherited attribute for a symbol on the right side of a production must be computed in an action before that symbol.
2. An action must not refer to a synthesized attribute of a symbol on the right side of the action.
3. A synthesized attribute for the non–terminal on the left can only be computed after all attributes it references, have been computed.

Note: In the implementation of L-attributed definitions during predictive parsing, instead of syntax directed translations, we will work with translation schemes.

Eliminating left recursion from translation scheme

Consider following grammar, which has left recursion

$E \to E + T$ {print ('+') ;}

When transforming the grammar, treat the actions as if they were terminal symbols. After eliminating recursion from the above grammar.

$E \to TR$
$R \to +T$ {print ('+');} R
$R \to \in$

Bottom-up Evaluation of Inherited Attributes

- Using a bottom up translation scheme, we can implement any L-attributed definition based on LL (1) grammar.
- We can also implement some of L-attributed definitions based on LR (1) using bottom up translations scheme.
  - The semantic actions are evaluated during the reductions.
  - During the bottom up evaluation of S-attributed definitions, we have a parallel stack to hold synthesized attributes.

Where are we going to hold inherited attributes?

We will convert our grammar to an equivalent grammar to guarantee the following:

- All embedding semantic actions in our translation scheme will be moved to the end of the production rules.
- All inherited attributes will be copied into the synthesized attributes (may be new non-terminals).

Thus we will evaluate all semantic actions during reductions, and we find a place to store an inherited attribute. The steps are

1. Remove an embedding semantic action S_i, put new non-terminal M_i instead of that semantic action.
2. Put S_i into the end of a new production rule $M_i \to \in$.
3. Semantic action S_i will be evaluated when this new production rule is reduced.
4. Evaluation order of semantic rules is not changed. i.e., if

$$A \to \{S_1\}\ X_1\{S_2\}X_2 \ldots \{S_n\}X_n$$

After removing embedding semantic actions:

$A \to M_1X_1M_2X_2 \ldots M_nX_n$
$M_1 \to \in \{S_1\}$
$M_2 \to \in \{S_2\}$
⋮
$M_n \to \in\ \{S_n\}$

For example,

$E \to TR$
$R \to +T$ {print ('+')} R_1
$R \to \in$
$T \to$ id {print (id.name)}
⇓ remove embedding semantic actions
$E \to TR$
$R \to +TMR_1$
$R \to \in$
$T \to$ id {print (id.name)}
$M \to \in$ {print ('+')}

Translation with inherited attributes

Let us assume that every non-terminal A has an inherited attribute $A.i$ and every symbol X has a synthesized attribute $X.s$ in our grammar.

For every production rule $A \to X_1, X_2 \ldots X_n$, introduce new marker non-terminals

$M_1, M_2, \ldots M_n$ and replace this production rule with $A \to M_1X_1M_2X_2 \ldots M_nX_n$

The synthesized attribute of X_i will not be changed.

The inherited attribute of X_i will be copied into the synthesized attribute of M_i by the new semantic action added at the end of the new production rule

$$M_i \to \in$$

Now, the inherited attribute of X_i can be found in the synthesized attribute of M_i.

$A \to \{B.i = f_1(.\ .)\}\ B\ \{\ c.i = f_2(.\ .)\}\ c\ \{A.s = f_3(.\ .)\}$

⇓

$A \to \{M_1.i = f_1(.\ .)\}\ M_1\ \{B.i = M_1.s\}\ B\ \{M_2.i = f_2(.\ .)\}M_2$
$\{c.i = M_2.S\}\ c\ \{A.s = f_3\ (.\ .)\}$
$M_1 \to \in\ \{M_1.s = M_1.i\}$
$M_2 \to \in\ \{M_2.s = M_2.i\}$

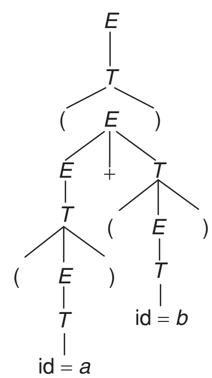
Exercises

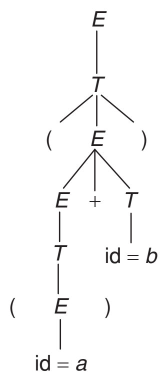
Practice Problems I

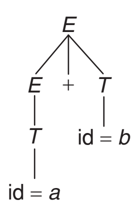
Directions for questions 1 to 13: Select the correct alternative from the given choices.

1. The annotated tree for input $((a) + (b))$, for the rules given below is

Production	Semantic Rule
$E \to E + T$	$ $ = mknode ('+', $1, $3)
$E \to E–T$	$ $ = mknode ('–', $1, $3)
$E \to T$	$ $ = $1;
$T \to (E)$	$ $ = $2;
$T \to$ id	$ $ = mkleaf (id, $1)
$T \to$ num	$ $ = mkleaf (num, $1)

(A)


(B)


(C)


(D) None of these

2. Let synthesized attribute val give the value of the binary number generated by S in the following grammar.
$S \to L\ L$
$S \to L$
$L \to LB$
$L \to B$
$B \to 0$
$B \to 1$
Input 101.101, S.val = 5.625
use synthesized attributes to determine S.val
Which of the following are true?
(A) $S \to L_1.L_2$ {S.val = L_1.val + L_2.val/ (2**L_2.bits)
|L {S.val = L.val; S.bits = L.bits}
(B) $L \to L_1\ B$ {L.val = L_1.val*2 + B.val;
L.bits = L_1.bits + 1}
|B {L.val = B.val; L.bits = 1}
(C) $B \to 0$ {B.val = 0}
|1 {B.val = 1}
(D) All of these

3. Which of the following productions with translation rules converts binary number representation into decimal.

(A)

Production	Semantic Rule
$B \to 0$	B.trans = 0
$B \to 1$	B.trans = 1
$B \to B_0$	B_1.trans = B_2.trans*2
$B \to B_1$	B_1.trans = B_2.trans * 2 + 1

(B)

Production	Semantic Rule
$B \to 0$	B.trans = 0
$B \to B_0$	B_1.trans = B_2.trans*4

(C)

Production	Semantic Rule
$B \to 1$	B.trans = 1
$B \to B_1$	B_1.trans = B_2.trans*2

(D) None of these

4. The grammar given below is

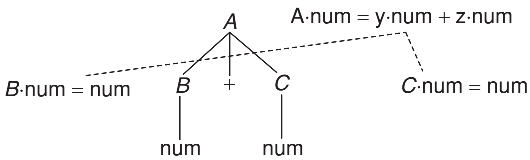
Production	Semantic Rule
$A \to$ LM	L.i := l(A. i)
	M.i := m(L.s)
	A.s := f(M.s)
$A \to$ QR	R.i := r(A.i)
	Q.i := q(R.s)
	A.s := f(Q.s)

(A) A L-attributed grammar
(B) Non-L-attributed grammar
(C) Data insufficient
(D) None of these

5. Consider the following syntax directed translation:
$S \to aS$ {$m := m + 3$; print (m);}
|bS {$m: = m$*2; print (m) ;}
|$\in$ {$m: = 0$;}
A shift reduce parser evaluate semantic action of a production whenever the production is reduced.
If the string is = $a\ a\ b\ a\ b\ b$ then which of the following is printed?
(A) 0 0 3 6 9 12
(B) 0 0 0 3 6 9 12
(C) 0 0 0 3 6 9 12 15
(D) 0 0 3 9 6 12

6. Which attribute can be evaluated by shift reduce parser that execute semantic actions only at reduce moves but never at shift moves?
(A) Synthesized attribute
(B) Inherited attribute
(C) Both (a) and (b)
(D) None of these

7. Consider the following annotated parse tree:



Which of the following is true for the given annotated tree?

(A) There is a specific order for evaluation of attribute on the parse tree.
(B) Any evaluation order that computes an attribute 'A' after all other attributes which 'A' depends on, is acceptable.
(C) Both (A) and (B)
(D) None of these.

Common data for questions 8 and 9: Consider the following grammar and syntax directed translation.

$E \to E + T$	E_1.val = E_2.val + T.val
$E \to T$	E.val = T.val
$T \to T^*P$	T_1.val = T_2.val * P.val * P.num
$T \to P$	T.val = P.val * P.num
$P \to (E)$	P.val = E.val
$P \to 0$	P.num = 1
	P.val = 2
$P \to 1$	P.num = 2
	P.val = 1

8. What is E.val for string 1*0?
(A) 8 (B) 6
(C) 4 (D) 12

9. What is the E.val for string 0 * 0 + 1?
(A) 8 (B) 6
(C) 4 (D) 12

10. Consider the following syntax directed definition:

Production	Semantic Rule
$S \to b$	$S.x = 0$ $S.y = 0$
$S \to S_1$ I	$S.x = S_1.x + I.dx$ $S.y = S_1.y + I.dy$
$I \to$ east	$I.dx = 1$ $I.dy = 0$
$I \to$ north	$I.dx = 0$ $I.dy = 1$
$I \to$ west	$I.dx = -1$ $I.dy = 0$
$I \to$ south	$I.dx = 0$ $I.dy = -1$

If Input = begin east south west north, after evaluating this sequence what will be the value of $S.x$ and $S.y$?
(A) (1, 0) (B) (2, 0)
(C) (–1, –1) (D) (0, 0)

11. What will be the values $s.x$, $s.y$ if input is 'begin west south west'?
(A) (–2, –1)
(B) (2, 1)
(C) (2, 2)
(D) (3, 1)

12. Consider the following grammar:

$S \to E$	S.val = E.val
	E.num = 1
$E \to E^*T$	E_1.val = 2 * E_2.val + 2 * T.val
	E_2.num = E_1.num + 1
	T.num = E_1.num + 1
$E \to T$	E.val = T.val
	T.num = E.num + 1
$T \to T + P$	T_1.val = T_2.val + P.val
	T_2.num = T_1.num + 1
	P.num = T_1.num + 1
$T \to P$	T.val = P.val
	P.num = T.num + 1
$P \to (E)$	P.val = E.val
$P \to i$	$\begin{Bmatrix} E.\text{num} = P.\text{num} \\ P.\text{val} = I \mid P.\text{num} \end{Bmatrix}$

Which attributes are inherited and which are synthesized in the above grammar?

(A) Num attribute is inherited attribute. Val attribute is synthesized attribute.
(B) Num is synthesized attribute. Val is inherited attribute.
(C) Num and val are inherited attributes.
(D) Num and value are synthesized attributes.

13. Consider the grammar with the following translation rules and E as the start symbol.

$E \to E_1 @ T$ {E.value = E_1.value*T.value}
 |T {E.value = T.value}
$T \to T_1$ and F {T.value = T_1.value + F.value}
 |F {T.value = F.value}
$F \to$ num {F.value = num.value}

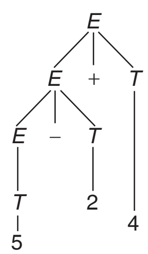
Compute E.value for the root of the parse tree for the expression: 2 @ 3 and 5 @ 6 and 4
(A) 200 (B) 180
(C) 160 (D) 40

Practice Problems 2

Directions for questions 1 to 10: select the correct alternative from the given choices.

1. Consider the following Tree:

Production	Meaning
$E \rightarrow E_1 + T$	$E.t = E_1.t{*}T.t$
$E \rightarrow E_1 - T$	$E.t = E_1.t + T.t$
$E \rightarrow T$	$E.t = T.t$
$t \rightarrow 0$	$T.t =$ ‘0’
$t \rightarrow 5$	$T.t =$ ‘5’
$t \rightarrow 2$	$T.t =$ ‘2’
$t \rightarrow 4$	$T.t =$ ‘4’



After evaluation of the tree the value at the root will be:
(A) 28 (B) 32
(C) 14 (D) 7

2. The value of an inherited attribute is computed from the values of attributes at the ________
(A) Sibling nodes (B) Parent of the node
(C) Children node (D) Both (A) and (B)

3. Consider an action translating expression:

expr → expr + term {print (‘+’)}
expr → expr – term {print (‘–’)}
expr → → term
term → 1 {print (‘1’)}
term → 2 {print (‘2’)}
term → 3 {print (‘3’)}

Which of the following is true regarding the above translation expression?
(A) Action translating expression represents infix notation.
(B) Action translating expression represents prefix notation.
(C) Action translating expression represents postfix notation.
(D) None of these

4. In the given problem, what will be the result after evaluating 9 – 5 + 2?
(A) + – 9 5 2 (B) 9 – 5 + 2
(C) 9 5 – 2+ (D) None of these

5. In a syntax directed translation, if the value of an attribute node is a function of the values of attributes of children, then it is called:
(A) Synthesized attribute (B) Inherited attribute
(C) Canonical attributes (D) None of these

6. Inherited attribute is a natural choice in:
(A) Keeping track of variable declaration
(B) Checking for the correct use of L-values and R-values.
(C) Both (A) and (B)
(D) None of these

7. Syntax directed translation scheme is desirable because
(A) It is based on the syntax
(B) Its description is independent of any implementation.
(C) It is easy to modify
(D) All of these

8. A context free grammar in which program fragments, called semantic actions are embedded within right side of the production is called,
(A) Syntax directed translation
(B) Translation schema
(C) Annotated parse tree
(D) None of these

9. A syntax directed definition specifies translation of construct in terms of:
(A) Memory associated with its syntactic component
(B) Execution time associated with its syntactic component
(C) Attributes associated with its syntactic component
(D) None of these

10. If an error is detected within a statement, the type assigned to the Statement is:
(A) Error type (B) Type expression
(C) Type error (D) Type constructor

Previous Years’ Questions

Common data for questions 1 (A) and 1 (B): Consider the following expression grammar. The semantic rules for expression evaluation are stated next to each grammar production: **[2005]**

$E \rightarrow$ number $\quad E$.val = number.val
$|E$ ‘+’ $E \quad E^{(1)}$.val = $E^{(2)}$.val + $E^{(3)}$.val
$|E \rightarrow E \quad E^{(1)}$.val = $E^{(2)}$.val × $E^{(3)}$.val

1. (A) The above grammar and the semantic rules are fed to a yacc tool (which is an LALR (1) parser generator) for parsing and evaluating arithmetic expressions. Which one of the following is true about the action of yacc for the given grammar?
(A) It detects recursion and eliminates recursion
(B) It detects reduce-reduce conflict, and resolves

(C) It detects shift-reduce conflict, and resolves the conflict in favor of a shift over a reduce action.
(D) It detects shift-reduce conflict, and resolves the conflict in favor of a reduce over a shift action.

(B) Assume the conflicts in Part (A) of this question are resolved and an LALR (1) parser is generated for parsing arithmetic expressions as per the given grammar. Consider an expression 3 × 2 + 1. What precedence and associativity properties does the generated parser realize?
(A) Equal precedence and left associativity; expression is evaluated to 7
(B) Equal precedence and right associativity; expression is evaluated to 9
(C) Precedence of '×' is higher than that of '+', and both operators are left associative; expression is evaluated to 7
(D) Precedence of '+' is higher than that of '×', and both operators are left associative; expression is evaluated to 9

2. In the context of abstract-syntax-tree (AST) and control-flow-graph (CFG), which one of the following is TRUE? **[2015]**
(A) In both AST and CFG, let node N_2 be the successor of node N_1. In the input program, the code corresponding to N_2 is present after the code corresponding to N_1.
(B) For any input program, neither AST nor CFG will contain a cycle.
(C) The maximum number of successors of a node in an AST and a CFG depends on the input program.
(D) Each node in AST and CFG corresponds to at most one statement in the input program.

3. Consider the following Syntax Directed Translation Scheme (SDTS), with non-terminals {S, A} and terminals {**a, b**}. **[2016]**

$S \to aA$	{ print 1 }
$S \to a$	{ print 2 }
$A \to Sb$	{ print 3 }

Using the above SDTS, the output printed by a bottom-up parser, for the input **aab** is:
(A) 1 3 2 (B) 2 2 3
(C) 2 3 1 (D) syntax error

4. Which one of the following grammars is free from *left recursion*? **[2016]**
(A) $S \to AB$
$A \to Aa/b$
$B \to c$
(B) $S \to Ab/Bb/c$
$A \to Bd/\varepsilon$
$B \to e$
(C) $S \to Aa/B$
$A \to Bb/Sc/\varepsilon$
$B \to d$
(D) $S \to Aa/Bb/c$
$A \to Bd/\varepsilon$
$B \to Ae/\varepsilon$

Answer Keys

Exercises

Practice Problems 1

1. A **2.** D **3.** A **4.** B **5.** A **6.** A **7.** B **8.** C **9.** B **10.** D
11. A **12.** A **13.** C

Practice Problems 2

1. A **2.** D **3.** C **4.** C **5.** A **6.** C **7.** D **8.** B **9.** C **10.** C

Previous Years' Questions

1. (a) C (b) B **2.** C **3.** C **4.** A

Chapter 3

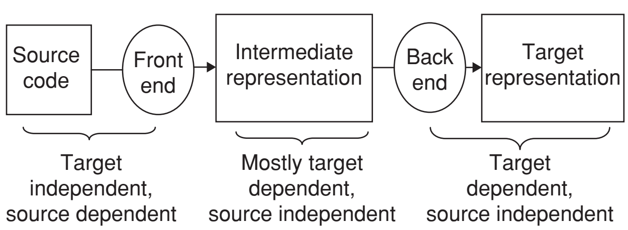
Intermediate Code Generation

LEARNING OBJECTIVES

- *Introduction*
- *Directed Acyclic Graphs (DAG)*
- *Three address code*
- *Symbol table operations*
- *Assignment statements*
- *Boolean expression*
- *Flow control of statements*
- *Procedure calls*
- *Code generation*
- *Next use information*
- *Run-time storage management*
- *DAG representations of basic blocks*
- *Peephole optimization*

Introduction

In the analysis–synthesis model, the front end translates a source program into an intermediate representation (IR). From IR the back end generates target code.



There are different types of intermediate representations:

- High level IR, i.e., AST (Abstract Syntax Tree)
- Medium level IR, i.e., Three address code
- Low level IR, i.e., DAG (Directed Acyclic Graph)
- Postfix Notation (Reverse Polish Notation, RPN).

In the previous sections already we have discussed about AST and RPN.

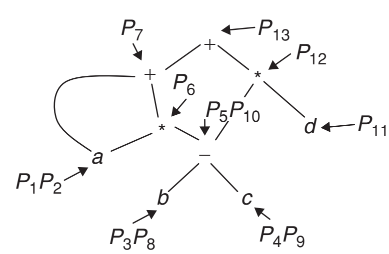
Benefits of Intermediate code generation: The benefits of ICG are

1. We can obtain an optimized code.
2. Compilers can be created for the different machines by attaching different backend to existing front end of each machine.
3. Compilers can be created for the different source languages.

Directed acyclic graphs for expression: (DAG)

- A DAG for an expression identifies the common sub expressions in the given expression.
- A node N in a DAG has more than one parent if N represents a common sub expression.
- DAG gives the compiler, important clues regarding the generation of efficient code to evaluate the expressions.

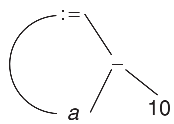
Example 1: DAG for $a + a*(b - c) + (b - c)*d$



P_1 = makeleaf (id, a)
P_2 = makeleaf (id, a) = P_1
P_3 = makeleaf (id, b)
P_4 = makeleaf (id, c)
P_5 = makenode (–, P_3, P_4)
P_6 = makenode (*, P_1, P_5)
P_7 = makenode (+, P_1, P_6)
P_8 = makeleaf (id, b) = P_3
P_9 = makeleaf (id, c) = P_4
P_{10} = makenode (–, P_8, P_9) = P_5

P_{11} = makeleaf (id, d)
P_{12} = makenode (*, P_{10}, P_{11})
P_{13} = makenode (+, P_7, P_{12})

Example 2: $a: = a - 10$



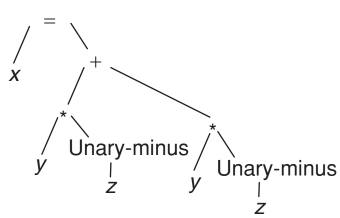
THREE-ADDRESS CODE

In three address codes, each statement usually contains 3 addresses, 2 for operands and 1 for the result.

Example: $-x = y$ OP z

- x, y, z are names, constants or complier generated temporaries,
- OP stands for any operator. Any arithmetic operator (or) Logical operator.

Example: Consider the statement $x = y * -z + y* -z$



The corresponding three address code will be like this:

Syntax Tree	DAG
$t_1 = -z$	$t_1 = -z$
$t_2 = y * t_1$	$t_2 = y * t_1$
$t_3 = -z$	$t_5 = t_2 + t_2$
$t_4 = y * t_3$	$X = t_5$
$t_5 = t_4 + t_2$	
$X = t_5$	

The postfix notation for syntax tree is: xyz unaryminus *yz unaryminus *+=.

- Three address code is a 'Linearized representation' of syntax tree.
- Basic data of all variables can be formulated as syntax directed translation. Add attributes whenever necessary.

Example: Consider below *SDD* with following specifications:
E might have E. place and E.code
E.place: the name that holds the value of E.
E.code: the sequence of intermediate code starts evaluating E.
Let Newtemp: returns a new temporary variable each time it is called.
New label: returns a new label.
Then the SDD to produce three–address code for expressions is given below:

Production	Semantic Rules
$S \rightarrow$ id ASN E	S. code = E.code \\ gen (ASN, id.place, E.place) E. Place = newtemp ();
$E \rightarrow E_1$ PLUS E_2	E. code = E_1. code || E_2. code || gen (PLUS, E. place, E_1. place, E_2. place); E. place = newtemp();
$E \rightarrow E_1$MUL E_2	E. code = E_1. code || E_2.code || gen (MUL, E. place, E_1. place, E_2. place); E. Place = Newtemp();
$E \rightarrow$ UMINUS E_1	E. code = E_1 code || gen (NEG, E. Place, E_1. place); E. code = E_1.code
$E \rightarrow$ LP E_1 RP	E. Place = E_1. Place
$E \rightarrow$ IDENT	E.place = id. place E. code = empty.list ();

Types of Three Address Statement

Assignment

- Binary assignment: $x: = y$ OP z Store the result of y OP z to x.
- Unary assignment: $x: =$ op y Store the result of unary operation on y to x.

Copy

- Simple Copy $x: = y$ Store y to x
- Indexed Copy $x: = y[i]$ Store the contents of $y[i]$ to x
- $x[i]:= y$ Store y to $(x + i)$th address.

Address and pointer manipulation

$x : = \&y$ Store address of y to x

$x : = *y$ Store the contents of y to x

$*x : = y$ Store y to location pointed by x .

Jump

- Unconditional jump:- goto L, jumps to L.
- Conditional:

```
if (x relop y)
goto L1;
else
```

```
{
goto L2;
}
```

Where relop is <, < =, >, > = , = or ≠.

Procedure call

Param x_1;

Param x_2;

.

.

.

Param x_n;

Call p, n, x;	Call procedure p with n parameters and store the result in x.
return x	Use x as result from procedure.

Declarations

- Global x, n_1, n_2: Declare a global variable named x at offset n_1 having n_2 bytes of space.
- Proc x, n_1, n_2: Declare a procedure x with n_1 bytes of parameter space and n_2 bytes of local variable space.
- Local x, m: Declare a local variable named x at offset m from the procedure frame.
- End: Declare the end of the current procedure.

Adaption for object oriented code

- $x = y$ field z: Lookup field named z within y, store address to x
- Class x, n_1, n_2: declare a class named x with n_1 bytes of class variables and n_2 bytes of class method pointers.
- Field x, n: Declare a field named x at offset n in the class frame.
- New x: Create a new instance of class name x.

Implementation of Three Address Statements

Three address statements can be implemented as records with fields for the operator and the operands. There are 3 types of representations:

1. Quadruples
2. Triples
3. Indirect triples

Quadruples

A quadruple has four fields: op, arg1, arg2 and result.

- Unary operators do not use arg2.
- Param use neither arg2 nor result.
- Jumps put the target label in result.
- The contents of the fields are pointers to the symbol table entries for the names represented by these fields.
- Easier to optimize and move code around.

Example 1: For the expression $x = y * - z + y * - z$, the quadruple representation is

	OP	Arg1	Arg2	Result
(0)	Uminus	z		t_1
(1)	*	y	t_1	t_2
(2)	Uminus	z		t_3
(3)	*	y	t_3	t_4
(4)	+	t_2	t_4	t_5
(5)	=	t_5		x

Example 2: Read (x)

	Op	Arg1	Arg2	Result
(0)	Param	x		
(1)	Call	READ	(x)	

Example 3: WRITE ($A*B$, x +5)

	OP	Arg1	Arg2	Result
(0)	*	A	B	t_1
(1)	+	x	5	t_2
(2)	Param	t_1		
(3)	Param	t_2		
(4)	Call	Write	2	

Triples

Triples have three fields: OP, arg1, arg2.

- Temporaries are not used and instead references to instructions are made.
- Triples are also known as two address code.
- Triples takes less space when compared with Quadruples.
- Optimization by moving code around is difficult.
- The DAG and triple representations of expressions are equivalent.
- For the expression $a = y* - z + y*-z$ the Triple representation is

	Op	Arg1	Arg2
(0)	Uminus	z	
(1)	*	y	(0)
(2)	Uminus	z	
(3)	*	y	(2)
(4)	+	(1)	(3)
(5)	=	a	(4)

Array – references

Example: For A [I]: = B, the quadruple representation is

	Op	Arg1	Arg2	Result
(0)	[ ] =	A	I	T_1
(1)	=	B		T_2

The same can be represented by Triple representation also.

[] = is called L-value, specifies the address to an element.

	Op	Arg1	Arg2
(0)	[ ] =	*A*	*I*
(1)	=	(0)	*B*

Example 2: $A := B[I]$

	Op	Arg1	Arg2
(0)	= [ ]	*B*	*I*
(1)	=	*A*	(0)

= [ ] is called r-value, specifies the value of an element.

Indirect Triples

- In indirect triples, pointers to triples will be there instead of triples.
- Optimization by moving code around is easy.
- Indirect triples takes less space when compared with Quadruples.
- Both indirect triples and Quadruples are almost equally efficient.

Example: Indirect Triple representation of 3-address code

	Statement
(0)	(14)
(1)	(15)
(2)	(16)
(3)	(17)
(4)	(18)
(5)	(19)

	Op	Arg1	Arg2
(14)	Uminus	*z*	
(15)	*	*y*	(14)
(16)	Uminus	*z*	
(17)	*	*y*	(16)
(18)	+	(15)	(17)
(19)	=	*x*	(18)

Symbol Table Operations

Treat symbol tables as objects.

- Mktable (previous);
  - create a new symbol table.
  - Link it to the symbol table previous.
- Enter (table, name, and type, offset)
  - insert a new identifier name with type and offset into table
  - Check for possible duplication.
- Add width (table, width);
  - increase the size of symbol table by width.
- Enterproc (table, name, new table)
  - Enter a procedure name into table.
  - The symbol table of name is new table.
- Lookup (name, table);
  - Check whether name is declared in the symbol table, if it is in the table then return the entry.

Example:

Declaration → $M_1 D$

$M_1 \rightarrow \in$ `{TOP (Offset): = 0 ;}`

$D \rightarrow D$ ID

$D \rightarrow$ id: T `{enter (top (tblptr), id.name, T.type top (offset)); top (offset): = top (offset) + T. width ;}`

$T \rightarrow$ integer `{T.type : = integer; T. width: = 4 :}`

$T \rightarrow$ double `{T.type: = double; T.width = 8 ;}`

$T \rightarrow * T_1$ `{T. type: = pointer (T. type); T.width = 4;}`

Need to remember the current offset before entering the block, and to restore it after the block is closed.

Example: Block → begin M4 Declarations statements end `{pop (tblptr); pop (offset) ;}`

$M_4 \rightarrow \in$ `{t: = mktable (top (tblptr); push (t, tblptr); push (top (offset), offset) ;`

Can also use the block number technique to avoid creating a new symbol table.

Field names in records

- A record declaration is treated as entering a block in terms of offset is concerned.
- Need to use a new symbol table.

Example: $T \rightarrow$ record M_5 D end

```
{T. type: = (top (tblptr));
 T. width = top (offset);
 pop (tblptr);
 pop (offset) ;}
```

$M_5 \rightarrow \in$

```
{t: = mktable (null);
push (t, tblptr);
push {(o, offset) ;}
```

Assignment Statements

Expressions can be of type integer, real, array and record. As part of translation of assignments into three address code, we show how names can be looked up in the symbol table and how elements of array can be accessed.

Code generation for assignment statements gen ([address # 1], [assignment], [address #2], operator, address # 3);

Variable accessing Depending on the type of [address # i], generate different codes.

Types of [address # i]:

- Local temp space
- Parameter
- Local variable
- Non-local variable
- Global variable
- Registers, constants,…

Error handling routine error – msg (error information);

The error messages can be written and stored in other file. Temp space management:

- This is used for generating code for expressions.
- newtemp (): allocates a temp space.
- freetemp (): free t if it is allocated in the temp space

Label management

- This is needed in generating branching statements.
- newlabel (): generate a label in the target code that has never been used.

Names in the symbol table

$S \to$ id: $= E$ {p: = lookup (id-name, top (tblptr));
If p is not null then gen (p, ":=", E.place);
Else error ("var undefined", id. Name) ;}
$E \to E_1 + E_2$ {E. place = newtemp ();
gen (E.place, ": = ", E_1.place, "+", E_2.Place); free temp (E1.pace); freetemp (E2. place) ;}
$E \to -E_1$ {E. place = newtemp ();
gen (E.place, ": =", "uminus", E_1.place);
Freetemp (E_1. place ;)}
$E \to (E_1)$ {E. place = E_1. place ;}
$E \to$ id {p: = lookup (id.name, top (tblptr);
If p ≠ null then E.place = p. place else error ("var undefined", id. name) ;}

Type conversions

Assume there are only two data types: integer, float.

For the expression,

$$E \to E_1 + E_2$$

If E_1. type = E_2. type then
generate no conversion code
E.type = E_1. type;

Else

E.type = float;
temp1 = newtemp ();
If E_1. type = integer then
gen (temp1,':=' int - to - float, E_1.place);
gen (*E*,':=' temp1, '+', E_2.place);

Else

gen (temp1,':=' int - to - float, E_2. place);
gen (*E*,':=' temp1, '+', E_1. place);
Free temp (temp1);

Addressing array elements

Let us assume

low: lower bound
w: element data width

Start_addr: starting address

1D Array: A[i]

- Start_addr + (i – low)* $w = i * w$ + (start_addr – low *w)
- The value called base, (start_addr – low * w) can be computed at compile time and then stored at the symbol table.

Example: array [–8 …100] of integer.

To declare [–8] [–7] … [100] integer array in Pascal.

2D Array A [i_1, i_2]

Row major order: row by row. A [i] means the ith row.

1st row A [1, 1]
A [1, 2]

2nd row A [2, 1]
A [2, 2]
A [i, j] = A [i] [j]

Column major: column by column.

A [1, 1] ¦ A [1, 2]
A [2, 1] ¦ A [2, 2]
1st Column 2nd column

Address for A [i_1, i_2]:

Start _ addr + ((i_1 – low$_1$) *n_2 + (i_2 – low$_2$))*w

Where low$_1$ and low$_2$ are the lower bounds of i_1 and i_2. n_2 is the number of values that i_2 can take. High$_2$ is the upper bound on the valve of i_2. n_2 = high$_2$ – low$_2$ + 1

We can rewrite address for A [i_1, i_2] as (($i_1 \times n_2$) + i_2) × w + (start _ addr - ((low$_1 \times n_2$) + low$_2$) × w). The value (start _ addr – low$_1 \times n_2 \times w$ – low$_2 \times w$) can be computed at compiler time and then stored in the symbol table.

Multi-Dimensional Array A [i_1, i_2,…i_k]

Address for A [i_1, i_2,…i_k]

$$= i_1 * \pi_{i=2}^{k} {}^{n_i} + i_2 * \pi_{i=3}^{k} {}^{n_i} + \cdots + i_k \Big) * w$$
$$+ \Big(start\_addr - low_1 * w * \pi_{i=2}^{k} ni$$
$$- low_2 * w * \pi_{i=3}^{k} {}^{n_i} - \cdots - low_k * w \Big)$$

It can be computed incrementally in grammar rules:

f (1) = i_1;
f (j) = f (j –1) * n_j + i_j;
f (k) is the value we wanted to compute.

Attributes needed in the translation scheme for addressing array elements:

Elegize: size of each element in the array

Array: a pointer to the symbol table entry containing information about the array declaration.

Ndim: the current dimension index

Base: base address of this array

Place: where a variable is stored.

Limit (array, n) = n_m is the number of elements in the mth coordinate.

Translation scheme for array elements

Consider the grammar

$S \to L := E$
$E \to L$

$L \rightarrow$ id

$L \rightarrow$ [Elist]

Elist$\rightarrow$ Elist$_1$, E

Elist$\rightarrow$ id [E]

$E \rightarrow$ id

$E \rightarrow E + E$

$E \rightarrow (E)$

- $S \rightarrow L{:}=E$ `{if L. offset = null then /* L is a simple id */ gen (L. place, ":=", E.place);`
```
Else
gen (L. place, "[", L. offset, "]",":=", E.place);
```
- $E \rightarrow E_1 + E_2$ `{E.place = newtemp ();`
```
gen (E. place, ":=", E1.place, "+", E2.place) ;}
```
- $E \rightarrow (E_1)$ `{E.place= E1.place}`
- $E \rightarrow L$ `{if L. offset = null then /* L is a simple id */ E.place:= L .place);`
```
Else begin
E.place:=newtemp();
gen (E.place, ":=",L.place, "[",L.offset, ']");
end }
```
- $L \rightarrow$ id `{P! = lookup (id.name, top (tblptr));`
```
If P ≠ null then
Begin
L.place: = P.place:
L.offset:= null;
End
Else
Error ("Var underfined", id. Name) ;}
```
- $L \rightarrow$ Elist `{L. offset: = newtemp ();`
```
gen (L. offset, ":=", Elist.elesize,
"*", Elist.place );
freetemp (Elist.place);
L.Place := Elist . base ;}
```
- Elist$\rightarrow$ Elist$_1$, E `{t: =newtemp (); m: = Elist1.ndim+1;`
```
gen (t, ":=" Elist1.place, "*", limit (Elist1.array, m));
Gen (t, ":=", t"+", E.place); freetemp (E.place);
Elist.array: = Elist.array;
Elist.place:= t; Elist.ndim:= m ;}
```
Elist $\rightarrow$ id [E `{Elist.Place:= E.place; Elist.ndim:=1;`
```
P! = lookup (id.name, top (tblptr)); check for id errors;
Elist.elesize:= P.size; Elist.base: = p.base;
Elist.array:= p.place ;}
```
- $E \rightarrow$ id `{P:= lookup (id,name, top (tblptr);`

```
Check for id errors; E. Place: = Populace ;}
```

Boolean Expressions

There are two choices for implementation of Boolean expressions:

1. Numerical representation
2. Flow of control

Numerical representation

Encode true and false values.
Numerically, 1:true 0: false.
Flow of control: Representing the value of a Boolean expression by a position reached in a program.

Short circuit code: Generate the code to evaluate a Boolean expression in such a way that it is not necessary for the code to evaluate the entire expression.

- If a_1 or a_2
  a_1 is true then a_2 is not evaluated.
- If a_1 and a_2
  a_1 is false then a_2 is not evaluated.

Numerical representation

$E \rightarrow$ id$_1$ relop id2

```
{B.place:= newtemp ();
gen ("if", id1.place, relop.op, id2.place,"goto", next stat +3);
gen (B.place,":=", "0");
gen ("goto", nextstat+2);
gen (B.place,":=", "1")'}
```

Example 1: Translate the statement (if $a < b$ or $c < d$ and $e < f$) without short circuit evaluation.

```
100: if a < b goto 103
101: t1:= 0
102: goto 104
103: t1:= 1 /* true */
104: if c < d goto 107
105: t2:= 0 /* false */
106: goto 108
107: t2:= 1
108: if e < f goto 111
109: t3:= 0
110: goto 112
111: t3 := 1
112: t4 := t2 and t3
113: t3:= t1 or t4
```

Flow of Control Statements

$B \rightarrow$ id$_1$ relop id$_2$

```
{
B.true: = newlabel ();
B.false:= newlabel ();
B.code:= gen ("if", id1. relop, id2, "goto",
```

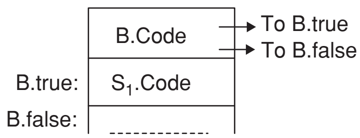
```
B.true, "else", "goto", B. false) ||
gen (B.true, ":")
}
```

$S \rightarrow$if B then S_1 S.code:= B.code || S_1 .code ||gen (B.false, ':')

|| is the code concatenation operator.

1. **If – then implementation:**

   $S \rightarrow$if B then $S1$ `{gen (Befalls," :");}`



2. **If – then – else**

   $P \rightarrow S$ `{S.next:= newlabel ();`

   `P.code:= S.code || gen (S.next," :")}`

   $S \rightarrow$ if B then S_1 else S_2 `{S`$_1$`.next:= S.next;`

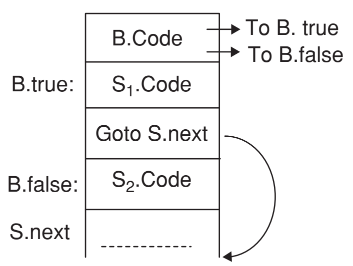
   `S`$_2$`.next:= S.next;`

   `Secede: = B.code || S`$_1$`.code ||.`

   `Gen ("goto" S.next) || B. false," :")`

   `||S`$_2$`.code}`

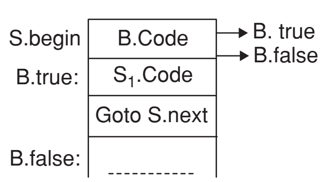
   Need to use inherited attributes of S to define the attributes of S_1 and S_2



3. **While loop:**

   $B \rightarrow$ id$_1$ relop id$_2$ B.true:= newlabel ();
   B.false:= newlabel ();
   B.code:=gen ('if', id.relop,
   id$_2$, 'goto', B.true 'else', 'goto', B. false) ||
   gen (B.true ':');

   $S \rightarrow$ while B do S_1 S.begin:= newlabel ();
   S.code:=gen (S.begin,':')||
   B.code||S1.code || gen
   ('goto', S.begin) || gen (B.false, ':');



4. **Switch/case statement:**

   The c - like syntax of switch case is

```
switch epr {
case V [1]: S [1]
.
.
.
case V [k]: S[k]
default: S[d]
        }
```

Translation sequence

- Evaluate the expression.
- Find which value in the list matches the value of the expression, match default only if there is no match.
- Execute the statement associated with the matched value.

How to find the matched value? The matched value can be found in the following ways:

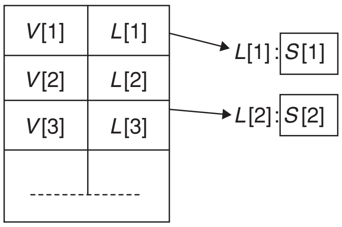
1. Sequential test
2. Lookup table
3. Hash table
4. Back patching

Two different translation schemes for sequential test are shown below:

1. Code to evaluate E into t
   Goto test
   $L[i]$: code for S [1]
   goto next

   $L[k]$: code for $S[k]$
   goto next
   $L[d]$: code for $S[d]$
   Go to next test:
   If $t = V$ [1]: goto L [1]
   .
   .
   .
   goto L[d]
   Next:
2. Can easily be converted into look up table
   If $t <> V[i]$ goto L [1]
   Code for S [1]
   goto next
   ....
   L [1]: if $t <> V$ [2] goto L [2]
   Code for S [2]
   Goto next
   L [k – 1]: if $t <> V$ [k] goto L[k]
   Code for S[k]
   Goto next
   .
   .
   .
   L[k]: code for S[d]
   Next:

Use a table and a loop to find the address to jump



3. **Hash table:** When there are more than two entries use a hash table to find the correct table entry.
4. **Back patching:**
   - Generate a series of branching statements with the targets of jumps temporarily left unspecified.
   - To determine label table: each entry contains a list of places that need to be back patched.
   - Can also be used to implement labels and gotos.

Procedure Calls

- Space must be allocated for the activation record of the called procedure.
- Arguments are evaluated and made available to the called procedure in a known place.
- Save current machine status.
- When a procedure returns:
  - Place returns value in a known place.
  - Restore activation record.

Example: $S \rightarrow$ call id (Elist)

```
{for each item P on the queue Elist.
Queue do gen ('PARAM', q);
gen ('call:', id.place) ;}
```

Elist $\rightarrow$ Elist, E {append E.place to the end of Elist.queue}

Elist $\rightarrow$ E {initialize Elist.queue to contain only E.place}

Use a queue to hold parameters, then generate codes for params.

Code for E_1, store in t_1

.

.

.

Code for E_k, store in t_k

PARAM $t1$

:

.

.

PARAM tk

Call P

Terminology:

Procedure declaration:

Parameters, formal parameters

Procedure call:

Arguments, actual parameters.

The values of a variable: $x = y$

r – value: value of the variable, i.e., on the right side of assignment. Ex: y, in above assignment.

l – value: The location/address of the variable, i.e., on the leftside of assignment. Ex: x, in above assignment.

There are different modes of parameter passing

1. call-by-value
2. call-by-reference
3. call-by-value-result (copy-restore)
4. call-by-name

Call by value

Calling procedure copies the r values of the arguments into the called procedure's Activation Record.

Changing a formal parameter has no effect on the actual parameter.

Example: void add (int C)

```
{
C = C+ 10;
printf ('\nc = %d', &C);
}
main ()
{
int a = 5;
printf ('a=%d', &a);
add (a);
printf ('\na = %d', &a);
}
```

In main a will not be affected by calling add (a)

It prints $a = 5$

$a = 5$

Only the value of C in add ( ) will be changed to 15.

Usage:

1. Used by PASCAL and C++ if we use non-var parameters.
2. The only thing used in C.

Advantages:

1. No aliasing.
2. Easier for static optimization analysis.
3. Faster execution because of no need for redirecting.

Call by reference

Calling procedure copies the l-values of the arguments into the called procedure's activation record. i.e., address will be passed to the called procedure.

- Changing formal parameter affects the corresponding actual parameter.
- It will have some side effects.

Example:

```
void add (int *c)
{
*c = *c + 10;
printf('\nc=%d', *c);
```

```
}
void main()
{
int a = 5;
printf ('\na = %d', a);
add (&a);
printf ('\na = %d', a);
```

output: a = 5
c = 15
a = 15

That is, here the actual parameter is also modified.

Advantages

1. Efficiency in passing large objects.
2. Only need to copy addresses.

Call-by-value-result

Equivalent to call-by-reference except when there is aliasing. That is, the program produces the same result, but not the same code will be generated.

Aliasing: Two expressions that have the same l-values are called aliases. They access the same location from different places.

Aliasing happens through pointer manipulation.

1. Call by reference with global variable as an argument.
2. Call by reference with the same expression as argument twice.

Example: test (x,y,x)

Advantages:

1. If there is no aliasing, we can implement it by using call – by – reference for large objects.
2. No implicit side effect if pointers are not passed.

Call by-name

used in Algol.

- Procedure body is substituted for the call in calling procedure.
- Each occurrence of a parameter in the called procedure is replaced with the corresponding argument.
- Similar to macro expansion.
- A parameter is not evaluated unless its value is needed during computation.

Example:

```
void show (int x)
{
for (int y = 0; y < 10; y++)
x++;
}
main ()
{
int j;
j = -1;
show (j);
}
Actually it will be like this
main ()
{
int j;
j = - 1;
For (in y= 0; y < 10; y ++)
x ++;
}
```

- Instead of passing values or address as arguments, a function is passed for each argument.
- These functions are called **thunks.**
- Each time a parameter is used, the thunk is called, then the address returned by the thunk is used.

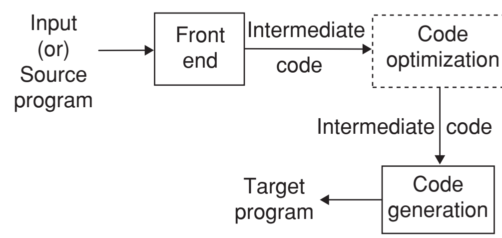
$y = 0$: use return value of thunk for y as the ℓ-value.

Advantages

- More efficient when passing parameters that are never used.
- This saves lot of time because evaluating unused parameter takes a longtime.

Code Generation

Code generation is the final phase of the compiler model.



The requirements imposed on a code generator are

1. Output code must be correct.
2. Output code must be of high quality.
3. Code generator should run efficiently.

Issues in the Design of a Code Generator

The generic issues in the design of code generators are

- Input to the code generator
- Target programs
- Memory Management
- Instruction selection
- Register Allocation
- Choice of Evaluation order

Input to the code generator

Intermediate representation with symbol table will be the input for the code generator.

- High Level Intermediate representation

Example: Abstract Syntax Tree (AST)

- Medium – level intermediate representation

Example: control flow graph of complex operations

- Low – Level Intermediate representation

Example: Quadruples, DAGS

- Code for abstract stack machine, i.e., postfix code.

Target programs

The output of the code generator is the target program. The output may take on a variety of forms:

1. Absolute machine language
2. Relocatable machine language
3. Assembly language

Absolute machine language

- Final memory area for a program is statically known.
- Hard coded addresses.
- Sufficient for very simple systems.

Advantages:

- Fast for small programs
- No separate compilation

Disadvantages: Can not call modules from other languages/ compliers.

Relocatable code It Needs

- Relocation table
- Relocating linker + loader (or) runtime relocation in Memory management Unit (MMU).

Advantage: More flexible.

Assembly language Generates assembly code and use an assembler tool to convert this to binary (object) code. It needs (i) assembler (ii) linker and loader.

Advantage: Easier to handle and closer to machine.

Memory management

Mapping names in the source program to addresses of data objects in runtime memory is done by the front end and the code generator.

- A name in a three address statement refers to a symbol entry for the name.
- Stack, heap, garbage collection is done here.

Instruction selection

Instruction selection depends on the factors like

- Uniformity
- Completeness of the instruction
- Instruction speed
- Machine idioms

- Choose set of instructions equivalent to intermediate representation code.
- Minimize execution time, used registers and code size.

Example: $x = y + z$ in three address statements:

MOV y, R_0 / * load y into R_0 * /

ADD z, R_0

MOV R_0, x /* store R_0 into x*/

Register allocation

- Instructions with register operands are faster. So, keep frequently used values in registers.
- Some registers are reserved.

Example: *SP*, *PC* … etc.
Minimize number of loads and stores.

Evaluation order

- The order of evaluation can affect the efficiency of the target code.
- Some orders require fewer registers to hold intermediate results.

Target Machine

Lets us assume, the target computer is

- Byte addressable with 4 bytes per word
- It has n general purpose registers $R_0, R_1, R_2, \ldots R_{n-1}$
- It has 2 address instructions of the form

  OP source, destination
  [cost: 1 + added]

Example: The op may be MOV, ADD, MUL.
Generally cost will be like this

Source	Destination	Cost
Register	Register	1
Register	Memory	2
Memory	Register	2
Memory	Memory	3

Addressing modes:

Mode	Form	Address	Cost
Absolute	M	M	2
Register	R	R	1
Indexed	C(R)	C+contents(R)	2
Indirect register	*R	Contents (R)	1
Indirect indexed	*C(R)	Contents (C+contents (R))	2

Example: $x: = y - z$

MOV y, R0 $\rightarrow$ cost = 2

SUB z, R0 $\rightarrow$ cost = 2

MOV R_0, $x \rightarrow$ cost = 2

6

Runtime Storage Management

Storage Organization

To run a compiled program, compiler will demand the operating system for the block of memory. This block of memory is called runtime storage.

This run time storage is subdivided into the generated target code, Data objects and Information which keeps track of procedure activations.

The fixed data (generated code) is stored at the statically determined area of the memory. The Target code is placed at the lower end of the memory.

The data objects are stored at the statically determined area as its size is known at the compile time. Compiler stores these data objects at statically determined area because these are compiled into target code. This static data area is placed on the top of the code area.

The runtime storage contains stack and the heap. Stack contains activation records and program counter, data object within this activation record are also stored in this stack with relevant information.

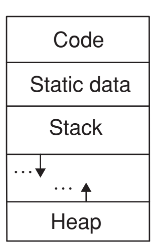
The heap area allocates the memory for the dynamic data (for example some data items are allocated under the program control)

The size of stack and heap will grow or shrink according to the program execution.

Activation Record

Information needed during an execution of a procedure is kept in a block of storage called an activation record.

- Storage for names local to the procedures appears in the activation record.
- Each execution of a procedure is referred as activation of the procedure.
- If the procedure is recursive, several of its activation might be alive at a given time.
- Runtime storage is subdivided into
  1. Generated target code area
  2. Data objects area
  3. Stack
  4. Heap



- Sizes of stack and heap can change during program execution.

For code generation there are two standard storage allocations:

1. **Static allocation:** The position of an activation record in memory is fixed at compile time.
2. **Stack allocation:** A new activation record is pushed on to the stack for each execution of the procedure.

The record is poped when the activation ends.

Control stack The control stack is used for managing active procedures, which means when a call occurs, the execution of activation is interrupted and status information of the stack is saved on the stack.

When control is returned from a call, the suspended activation is resumed after storing the values of relevant registers it also includes program counter which sets to point immediately after the call.

The size of stack is not fixed.

Scope of declarations Declaration scope refers to the certain program text portion, in which rules are defined by the language.

Within the defined scope, entity can access legally to declared entities.

The scope of declaration contains immediate scope always. Immediate scope is a region of declarative portion with enclosure of declaration immediately.

Scope starts at the beginning of declaration and scope continues till the end of declaration. Whereas in the over loadable declaration, the immediate scope will begin, when the callable entity profile was determined.

The visible part refers text portion of declaration, which is visible from outside.

Flow Graph

A flow graph is a graph representation of three address statement sequences.

- Useful for code generation algorithms.
- Nodes in the flow graph represents computations.
- Edges represent flow of control.

Basic Blocks

Basic blocks are sequences of consecutive statements in which flow of control enters at the beginning and leaves at the end without a halt or branching.

1. First determine the set of leaders
   - First statement is leader
   - Any target of goto is a leader
   - Any statement that follows a goto is a leader.
2. For each leader its basic block consists of the leader and all statements up to next leader.

Initial node: Block with first statement is leader.

Example: consider the following fragment of code that computes dot product of two vectors x and y of length 10.

begin

Prod: = 0;

i: = 1;
repeat
begin
Prod: = Prod + x [i] * y [i];
i: = i + 1;
end
until i < = 10;
end

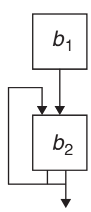
B_1

(1)	Prod : = 0
(2)	I: = 1

B_2

(3)	t_1:= 4*i
(4)	t_2: =x[t_1]
(5)	t_3: =4 * i
(6)	t_4: =y [t_3]
(7)	t_5: =t_2* t_4
(8)	t_6; =Prod + t_5
(9)	Prod := t_6
(10)	t_7: = i+1
(11)	i:= t_7
(12)	if i < = 10 goto (3)

∴The flow graph for this code will be



Here b_1 is the initial node/block.

- Once the basic blocks have been defined, a number of transformations can be applied to them to improve the quality of code.
    1. **Global:** Data flow analysis
    2. **Local:**
        - Structure preserving transformations
        - Algebraic transformations
- Basic blocks compute a set of expressions. These expressions are the values of the names live on exit from the block.
- Two basic blocks are equivalent if they compute the same set of expressions.

Structure preserving transformations:

1. *Common sub-expression elimination:*

$$\begin{array}{l} a := b + c \\ b := a - d \\ c := b + c \\ d := a - d \end{array} \Rightarrow \begin{array}{l} a := b + c \\ b := a - d \\ c := b + c \\ d := b \end{array}$$

2. *Dead code elimination:* Code that computes values for names that will be dead i.e., never subsequently used can be removed.
3. *Renaming of temporary variables*
4. *Interchange of two independent adjacent statements*

Algebraic Transformations

Algebraic identities represent other important class optimizations on basic blocks. For example, we may apply arithmetic identities, such as $x + 0 = 0 + x = x$,

$$x * 1 = 1 * x = x$$
$$x - 0 = x$$
$$x/1 = x$$

Next-Use Information

- Next-use info used in code generation and register allocation.
- Remove variables from registers if not used.
- Statement of the form $A = B$ or C defines A and uses B and C.
- Scan each basic block backwards.
- Assume all temporaries are dead or exit and all user variables are live or exit.

Algorithm to compute next use information

Suppose we are scanning
i: x: = y op z
in backward scan

- attach to i, information in symbol table about x, y, z.
- set x to not live and no next-use in symbol table
- set y and z to be live and next-use in symbol table.

Consider the following code:
1: $t_1 = a * a$
2: $t_2 = a * b$
3: $t_3 = 2 * t_2$
4: $t_4 = t_1 + t_2$
5: $t_5 = b * b$
6: $t_6 = t_4 + t_5$
7: $x = t_6$
Statements:
7: no temporary is live
6: t_6: use (7) t_4 t_5 not live
5: t_5: use (6)
4: t_4: use (6), t_1 t_3 not live
3: t_3: use (4) t_2 not live
2: t_2: use (3)
1: t_1: use (4)
Symbol Table:
t_1 dead use in 4

t_2 dead use in 3
t_3 dead use in 4
t_4 dead use in 6
t_5 dead use in 6
t_6 dead use in 7
The six temporaries in the basic block can be packed into two locations t_1 and t_2:
1: $t_1 = a * a$
2: $t_2 = a * b$
3: $t_2 = 2 * t_2$
4: $t_1 = t_1 + t_2$
5: $t_2 = b * b$
6: $t_1 = t_1 + t_2$
7: $x = t_1$

Code Generator

- Consider each statement
- Remember if operand is in a register
- Descriptors are used to keep track of register contents and address for names
- There are 2 types of descriptors
  1. Register Descriptor
  2. Address Descriptor

Register Descriptor

Keep track of what is currently in each register. Initially all registers are empty.

Address Descriptors

- Keep track of location where current value of the name can be found at runtime.
- The location might be a register, stack, memory address or a set of all these.

Issues in design of code generation The issues in the design of code generation are

1. Intermediate representation
2. Target code
3. Address mapping
4. Instruction set.

Intermediate Representation It is represented in post fix, 3-address code (or) quadruples and syntax tree (or) DAG.

Target Code The Target Code could be absolute code, relocatable machine code (or) assembly language code. Absolute code will execute immediately as it is having fixed address relocatable, requires linker and loader to get the code from appropriate location for the assembly code, assemblers are required to convert it into machine level code before execution.

Address mapping In this, mapping is defined between intermediate representations to target code address.

It is based on run time environment like static, stack or heap.

Instruction set It should provide a complete set in such a way that all its operations can be implemented.

Code Generation Algorithm

For each three address statement $x = y$ op z do

- Invoke a function getreg to determine location L where x must be stored. Usually L is a register.
- Consult address descriptor of y to determine y'. Prefer a register for y'. If value of y is not already in L generate MOV y', L.
- Generate
  OP z', L
  Again prefer a register for z. Update address descriptor of x to indicate x is in L. If L is a register update its descriptor to indicate that it contains x and remove x from all other register descriptors.
- If current value of y and/or z have no next use and are dead or exit from block and are in registers then change the register descriptor to indicate that it no longer contain y and /or z.

Function getreg

1. If y is in register and y is not live and has no next use after $x = y$ OP z then return register of y for L.
2. Failing (1) return an empty register.
3. Failing (2) if x has a next use in the block or OP requires register then get a register R, store its contents into M and use it.
4. Else select memory location x as L.

Example: $D: = (a - b) + (a - c) + (a - c)$

Stmt	Code Generated	reg desc	addr desc
$t = a - b$	MOV a, R_0 SUB b, R_0	R_0 contains t	t in R_0
$u = a - c$	MOV a, R_1 SUB c, R_1	R_0 contains t R_1 contains u	t in R_0 u in R_1
$v = t + u$	ADD R_1, R_0	R_0 contains v R_1 contains u	u in R_0 v in R_0
$d = v + u$	ADD R_1, R_0 MOV R_0,d	R_o contains d	d in R_0 d in R_0 and memory

Conditional Statements

Machines implement conditional jumps in 2 ways:

1. Based on the value of the designated register (R)
   Branch if values of R meets one of six conditions.
   (i) Negative (ii) Zero
   (iii) Positive (iv) Non-negative
   (v) Non-zero (vi) Non-positive

Example: Three address statement: if $x < y$ goto z
It can be implemented by subtracting y from x in R, then jump to z if value of R is negative.

2. Based on a set of **condition codes** to indicate whether last quantity computed or loaded into a location is negative (or) Zero (or) Positive.
   - compare instruction set codes without actually computing the value.

Example: CMP x, y
CJL Z.

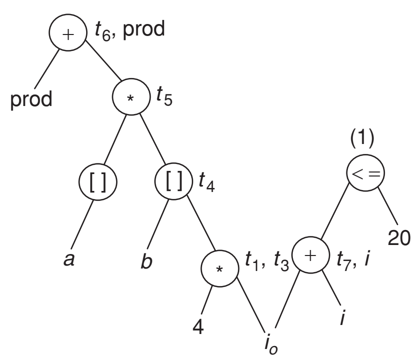
- Maintains a condition code descriptor, which tells the name that last sets the condition codes.

Example: $X: = y + z$
If $x < 0$ goto z
By
MOV y, R_o
ADD z, R_o
MOV R_o, x
CJN z.

DAG Representation of Basic Blocks

- DAGS are useful data structures for implementing transformations on basic blocks.
- Tells, how value computed by a statement is used in subsequent statements.
- It is a good way of determining common sub expressions.
- A DAG for a basic block has following labels on the nodes:
  - Leaves are labeled by unique identifiers, either variable names or constants.
  - Interior nodes are labeled by an operator symbol.
  - Nodes are also optionally given as a sequence of identifiers for labels.

Example: 1: $t_1 := 4 * i$
2: $t_2 := a\ [t_1]$
3: $t_3 := 4 * i$
4: $t_4 := b\ [t_3]$
5: $t_5 := t_2 * t_4$
6: $t_6 :=$ prod $+ t_5$
7: prod: $= t_6$
8: $t_7 := i + 1$
9: $i = t_7$
10: if $i < = 20$ got (1)

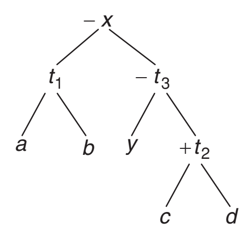


Code Generation from DAG:

$S_1 = 4 * i$	$S_1 = 4 * i$
$S_2 = \text{add(A)} - 4$	$S_2 = \text{add(A)} - 4$
$S_3 = S_2\ [S_1]$	$S_3 = S_2\ [S_1]$
$S_4 = 4 * i$	
$S_5 = \text{add(B)} - 4$	$S_5 = \text{add(B)} - 4$
$S_6 = S_5[S_4]$	$S_6 = S_5[S_4]$
$S_7 = S_3 * S_6$	$S_7 = S_3 * S_6$
$S_8 = \text{prod} + S_7$	$\text{prod} = \text{prod} + S_7$
$\text{prod} = S_8$	
$S_9 = I + 1$	
$I = S_9$	$I = I + 1$
if $I < = 20$ got (1)	if $I < = 20$ got (1)

Rearranging order of the code

Consider the following basic block
$t_1 := a + b$
$t_2 := c + d$
$t_3 := e - t_2$
$x = t_1 - t_3$ and its DAG



Three address code for the DAG:
(Assuming only two registers are available)
MOV a, R_o
ADD b, R_o
MOV c, R_1
MOV R_o, t_1 Register Spilling
MOV e, R_o Register Reloading
SUB R_1, R_o
MOV t_1, R_1
SUB R_o, R_1
MOV R_1, x

Rearranging the code as
$t_2 := c + d$
$t_3 := e - t_2$
$t_1 := a + b$
$x = t_1 - t_3$
The rearrangement gives the code:
MOV c, R_o
ADD d, R_o
MOV e, R_1
SUB R_o, R_1

MOV a, R_o
ADD b, R_o
SUB R_1, R_0
MOV R_1, x

Error detection and Recovery The errors that arise while compiling

1. Lexical errors
2. Syntactic errors
3. Semantic errors
4. Run-time errors

Lexical errors If the variable (or) constants are declared (or) defined, not according to the rules of language, special symbols are included which were not part of the language, etc is the lexical error.

Lexical analyzer is constructed based on pattern recognizing rules to form a token, when a source code is made into tokens and if these tokens are not according to rules then errors are generated.

Consider a c program statement
printf ('Hello World');

Main printf, (, ', Hello world,' ,),; are tokens.

Printf is not recognizable pattern, actually it should be printf. It generates an error.

Syntactic error These errors include semi colons, missing braces etc. which are according to language rules.

The parser reports the errors

Semantic errors This type of errors arises, when operation is performed over incompatible type of variables, double declaration, assigning values to undefined variables etc.

Runtime errors The Runtime errors are the one which are detected at runtime. These include pointers assigned with NULL values and accessing a variable which is out of its boundary, unlegible arithmetic operations etc.

After the detection of errors. The following recovery strategies should be implemented.

1. Panic mode recovery
2. Phrase level recovery
3. Error production
4. Global correction.

Peephole Optimization

- Target code often contains redundant instructions and suboptimal constructs.
- Improving the performance of the target program by examining a short sequence of target instructions (peephole) and replacing these instructions by a shorter or faster sequence is peephole optimization.
- The peephole is a small, moving window on the target program. Some well known peephole optimizations are

1. Eliminating redundant instructions
2. Eliminating unreachable code
3. Flow of control optimizations or Eliminating jumps over jumps
4. Algebraic simplifications
5. Strength reduction
6. Use of machine idioms

Elimination of Redundant Loads and stores

Example 1: (1) MOV R_o, a
(2) MOV a, R_o

We can delete instruction (2), because the value of a is already in R_0.

Example 2: Load x, R_0
Store R_0, x

If no modifications to R_0/x then store instruction can be deleted

Example 3: (1) Load x, R_0
(2) Store R_0, x

Example 4: (1) store R_0, x
(2) Load x, R_0

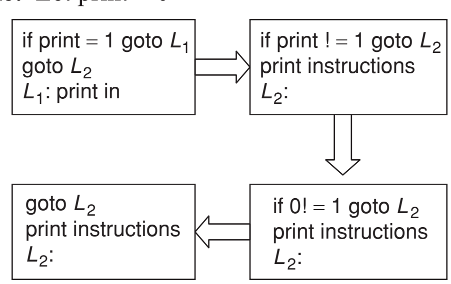
Second instruction can be deleted from both examples 3 and 4.

Example 5: Store R_0, x
Load x, R_0
Here load instruction can be deleted.

Eliminating Unreachable code
An unlabeled instruction immediately following and unconditional jump may be removed.

- May be produced due to debugging code introduced during development.
- May be due to updates in programs without considering the whole program segment.

Example: Let print = 0



In all of the above cases print instructions are unreachable.
∴ Print instructions can be eliminated.

Example: goto L_2
…
L_2:

Flow of control optimizations The unnecessary jumps can be eliminated.
Jumps like:
Jumps to jumps,
Jumps to conditional jumps,
Conditional jumps to jumps.

Example 1: we can replace the jump sequence

goto L_1

…

L_1: got L_2

By the sequence

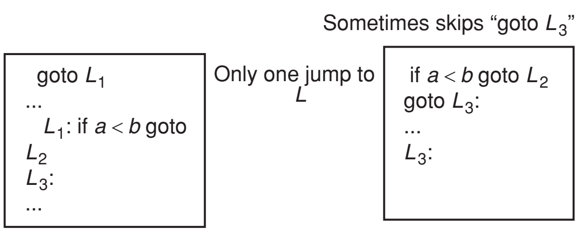
Got L_2

L_1: got $L_{2,}$

…

If there are no jumps to L_1 then it may be possible to eliminate the statement L_1: goto L_2.

Example 2:



Reduction in strength

- x^2 is cheaper to implement as $x * x$ than as a call to exponentiation routine.
- Replacement of multiplication by left shift.

Example: $x * 2^3 \Rightarrow x << 3$

- Replace division by right shift.

Example: $x >> 2$ (is $x/2^2$)

Use of machine Idioms

- Auto increment and auto decrement addressing modes can be used whenever possible.

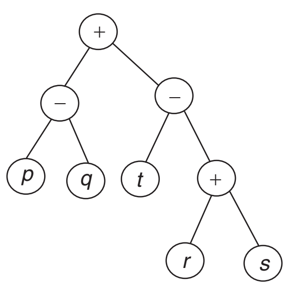
Example: replace add #1, R by INC R

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices

1. Consider the following expression tree on a machine with bad store architecture in which memory can be accessed only through load and store instructions. The variables p, q, r, s and t are initially stored in memory. The binary operators used in this expression tree can be evaluated by the machine only when the operands are in registers. The instructions produce result only in a register if no intermediate results can be stored in memory, what is the minimum number of registers needed to evaluate this expression?



(A) 2 (B) 9
(C) 5 (D) 3

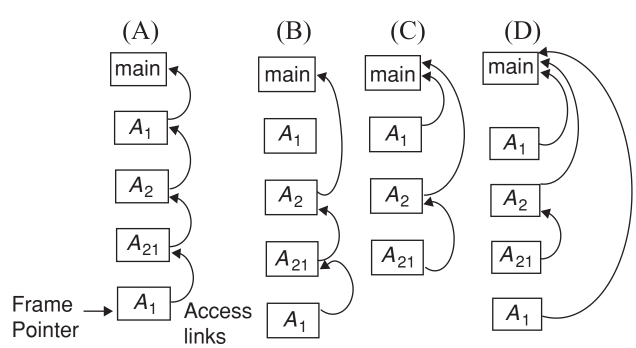
2. Consider the program given below with lexical scoping and nesting of procedures permitted.

Program main ( )

```
{
Var …
Procedure A1 ( )
{
Var …
call A2;
}
Procedure A2 ( )
{
Var..
Procedure A21 ( )
{
Var…
call A21 ( );
}
Call A1;
}
Call A1;
}
```

Consider the calling chain: main ( )→ A_1 ( ) → A_2 ( ) → A_{21} ( ) → A_1 ( ).

The correct set of activation records along with their access links is given by



3. Consider the program fragment:

```
sum = 0;
For (i = 1; i < = 20; i++)
sum = sum + a[i] +b[i];
```

How many instructions are there in the three-address code for this?

(A) 15 (B) 16
(C) 17 (D) 18

4. Suppose the instruction set of the processor has only two registers. The code optimization allowed is code motion. What is the minimum number of spills to memory in the complied code?

$c = a + b;$

$d = c*a;$

$e = c + a;$

$x = c*c;$

```
If (x > a)
{
y = a*a;
Else
{
d = d*d; e = e*e;
}
```

(A) 0 (B) 1
(C) 2 (D) 3

5. What is the minimum number of registers needed to compile the above problem's code segment without any spill to memory?

(A) 3 (B) 4
(C) 5 (D) 6

6. Convert the following expression into postfix notation:

$a = (-a + 2*b)/a$

(A) $aa - 2b$ *+a/= (B) $a - 2ba$ */+ =
(C) $a2b * a$/+ (D) $a2b - * a$/+

7. In the quadruple representation of the following program, how many temporaries are used?

int $a = 2$, $b = 8$, $c = 4$, d;

For ($j = 0$; $j< = 10$; j++)

$a = a * (j* (b/c));$

$d = a * (j* (b/c));$

(A) 4 (B) 7
(C) 8 (D) 10

8. Let $A = 2$, $B = 3$, $C = 4$ and $D = 5$, what is the final value of the prefix expression: + * $AB - CD$

(A) 5 (B) 10
(C) –10 (D) –5

9. Which of the following is a valid expression?

(A) $BC * D - +$ (B) $* ABC -$
(C) BBB ***– + (D) $-*/bc$

10. What is the final value of the postfix expression $B\ C\ D\ A\ D - + - +$ where $A = 2$, $B = 3$, $C = 4$, $D = 5$?

(A) 5 (B) 4
(C) 6 (D) 7

11. Consider the expression $x = (a + b)* -C/D$. In the quadruple representation of this expression in which instruction '/' operation is used?

(A) 3rd (B) 4th
(C) 5th (D) 8th

12. In the triple representation of $x = (a + b)*- c/d$, in which instruction $(a + b) * - c/d$ result will be assigned to x?

(A) 3rd (B) 4th
(C) 5th (D) 8th

13. Consider the three address code for the following program:

While ($A < C$ and $B > D$) do

If ($A = = 1$) then $C = C + 1$;

Else

While ($A < = D$) do

$A = A + 3$;

How many temporaries are used?

(A) 2 (B) 3
(C) 4 (D) 0

14. Code generation can be done by

(A) DAG (B) Labeled tree
(C) Both (A) and (B) (D) None of these

15. Live variables analysis is used as a technique for

(A) Code generation (B) Code optimization
(C) Type checking (D) Run time management

Practice Problems 2

Directions for questions 1 to 19: Select the correct alternative from the given choices

1. Match the correct code optimization technique to the corresponding code:

(i) $i = i * 1$ $j = 2 * i$	$\Rightarrow j = 2 * i$	(p) Reduction in strength
(ii) $A = B + C$ $D = 10 + B + C$	$\Rightarrow A = B + C$ $D = 10 + A$	(q) Machine Idioms
(iii) For $i = 1$ to 10 $A[i] = B + C$	$\Rightarrow$ for $i = 1$ to 10 $t = B + C$ $A[i] = t$;	(r) Common sub expression elimination.
(iv) $x = 2 * y \Rightarrow y << 2$;		(s) Code motion

(A) i – r, iii – s, iv – p, ii – q
(B) i – q, ii – r, iii – s, iv –p
(C) i – s, iii – p, iii – q, iv – r
(D) i – q, ii – p, iii – r, iv – s

2. What will be the optimized code for the following expression represented in DAG?

$a = q * -r + q * -r$

(A) $t_1 = -r$
$t_2 = q * t_1$
$t_3 = a * t_1$
$t_4 = t_2 + t_3$
$a = t_4$

(B) $t_1 = -r$
$t_2 = q * t_1$
$t_3 = t_2 + t_2$
$a = t_3$

(C) $t_1 = -r$
$t_2 = q$
$t_3 = t_1 * t_2$
$t_4 = t_3 + t_3$
$a = t_4$

(D) All of these

3. In static allocation, names are bound to storage at _______ time.

(A) Compile (B) Runtime
(C) Debugging (D) Both (A) and (B)

4. The actual parameters are evaluate d and their r-values are passed to the called procedure is known as

(A) call-by-reference
(B) call-by-name
(C) call-by-value
(D) copy-restore

5. If the expression $-(a + b) *(c + d) + (a + b + c)$ is translated into quadruple representation, then how many temporaries are required?

(A) 5 (B) 6
(C) 7 (D) 8

6. If the above expression is translated into triples representation, then how many instructions are there?

(A) 6 (B) 10
(C) 5 (D) 8

7. In the indirect triple representation for the expression $A = (E/F) * (C - D)$. The first pointer address refers to

(A) $C - D$
(B) E/F
(C) Both (A) and (B)
(D) $(E/F) * (C - D)$

8. For the given assembly language, what is the cost for it?

MOV b, a

ADD c, a

(A) 3 (B) 4
(C) 6 (D) 2

9. Consider the expression

((4 + 2 * 3 + 7) + 8 * 5). The polish postfix notation for this expression is

(A) 423* + 7 + 85*+ (B) 423* + 7 + 8 + 5*
(C) 42 + 37 + *85* + (D) 42 + 37 + 85** +

Common data for questions 10 to 15: Consider the following basic block, in which all variables are integers, and ** denotes exponentiation.

$a: = b + c$
$z: = a * * 2$
$x: = 0 * b$
$y: = b + c$
$w: = y * y$
$u: = x + 3$
$v: = u + w$

Assume that the only variables that are live at the exit of this block are v and z. In order, apply the following optimization to this basic block.

10. After applying algebraic simplification, how many instructions will be modified?

(A) 1 (B) 2
(C) 4 (D) 5

11. After applying common sub expression elimination to the above code. Which of the following are true?

(A) $a: = b + c$ (B) $y: = a$
(C) $z = a + a$ (D) None of these

12. Among the following instructions, which will be modified after applying copy propagation?

(A) $a: = b + c$ (B) $z: = a * a$
(C) $y: = a$ (D) $w: = y * y$

13. Which of the following is obtained after constant folding?

(A) $u: = 3$ (B) $v: = u + w$
(C) $x: = 0$ (D) Both (A) and (C)

14. In order to apply dead code elimination, what are the statements to be eliminated?

(A) $x = 0$
(B) $y = b + c$
(C) Both (A) and (B)
(D) None of these

15. How many instructions will be there after optimizing the above result further?

(A) 1 (B) 2
(C) 3 (D) 4

16. Consider the following program:

L_0: $e: = 0$
$b: = 1$
$d: = 2$
L_1: $a: = b + 2$
$c: = d + 5$
$e: = e + c$
f: $a*a$
If $f < c$ goto L_3
L_2: $e: = e + f$
goto L_4
L_3: $e: = e + 2$
L_4: $d: = d + 4$
$b: = b - 4$
If $b! = d$ goto 4
L_5:

How many blocks are there in the flow graph for the above code?

(A) 5
(B) 6
(C) 8
(D) 7

17. A basic block can be analyzed by

(A) Flow graph
(B) A graph with cycles
(C) DAG
(D) None of these

18. In call by value the actual parameters are evaluated. What type of values is passed to the called procedure?

(A) l-values
(B) r-values
(C) Text of actual parameters
(D) None of these

19. Which of the following is FALSE regarding a Block?

(A) The first statement is a leader.
(B) Any statement that is a target of conditional / unconditional goto is a leader.
(C) Immediately next statement of goto is a leader.
(D) The last statement is a leader.

Previous Years' Questions

1. The least number of temporary variables required to create a three-address code in static single assignment form for the expression $q + r/3 + s - t * 5 + u * v/w$ is ________ **[2015]**

2. Consider the intermediate code given below.

(1) $i = 1$
(2) $j = 1$
(3) $t_1 = 5 * i$
(4) $t_2 = t_1 + j$
(5) $t_3 = 4 * t_2$
(6) $t_4 = t_3$
(7) $a[t_4] = -1$
(8) $j = j + 1$
(9) if $j <= 5$ goto (3)
(10) $i = i + 1$
(11) if $i < 5$ goto (2)

The number of nodes and edges in the control-flow-graph constructed for the above code, respectively, are **[2015]**

(A) 5 and 7 (B) 6 and 7
(C) 5 and 5 (D) 7 and 8

3. Consider the following code segment. **[2016]**

$x = u - t;$
$y = x * v;$
$x = y + w;$
$y = t - z;$
$y = x * y;$

The minimum number of total variables required to convert the above code segment *to static single assignment form is* _____ .

4. What will be the output of the following pseudo-code when parameters are passed by reference and dynamic scoping is assumed? **[2016]**

$a = 3;$
void $n(x)$ { $x = x * a$; print (x);}
void $m(y)$ {$a = 1$; $a = y - a$; $n(a)$; print (a)}
void main( ) {$m(a)$;}

(A) 6,2 (B) 6,6
(C) 4,2 (D) 4,4

5. Consider the following intermediate program in three address code

$p = a - b$
$q = p * c$
$p = u * v$
$q = p + q$

Which one of the following corresponds to a *static single assignment* form of the above code? **[2017]**

(A) $p_1 = a - b$
$q_1 = p_1 * c$
$p_1 = u * v$
$q_1 = p_1 + q_1$

(B) $p_3 = a - b$
$q_4 = p_3 * c$
$p_4 = u * v$
$q_5 = p_4 + q_4$

(C) $p_1 = a - b$
$q_1 = p_2 * c$
$p_3 = u * v$
$q_2 = p_4 + q_3$

(D) $p_1 = a - b$
$q_1 = p * c$
$p_2 = u * v$
$q_2 = p + q$

Answer Keys

Exercises

Practice Problems 1

1. D **2.** D **3.** C **4.** C **5.** B **6.** A **7.** B **8.** A **9.** A **10.** A
11. B **12.** C **13.** A **14.** C **15.** B

Practice Problems 2

1. B **2.** B **3.** A **4.** B **5.** B **6.** A **7.** B **8.** C **9.** A **10.** A
11. B **12.** D **13.** A **14.** C **15.** C **16.** A **17.** C **18.** B **19.** D

Previous Years' Questions

1. 8 **2.** B **3.** 10 **4.** D **5.** B

Chapter 4

Code Optimization

LEARNING OBJECTIVES

- *Code optimization basics*
- *Principle sources of optimization*
- *Loop invariant code motion*
- *Strength reduction on induction variables*
- *Loops in flow graphs*
- *Pre-header*
- *Global data flow analysis*
- *Definition and usage of variables*
- *Use-definition (u-d) chaining*
- *Data flow equations*

Code Optimization Basics

The process of improving the intermediate code and the target code in terms of both speed and the amount of memory required for execution is known as code optimization.

Compilers that apply code–improving transformations are called optimizing compilers.

Properties of the transformations of an optimizing compiler are

1. A transformation must preserve the meaning of programs.
2. It must speed up programs by a measurable amount.
3. A transformation must be worth the effort.

Places for improvements

1. Source Code:
   User can – profile a program
   – change an algorithm
   – transform loops
2. Intermediate code can be improved by improving
   – Loops
   – Procedure calls
   – Address calculations
3. Target code can be improved by
   – Using registers
   – Selecting instructions
   – Peephole transformations

Optimizing compiler organization

This applies

- Control flow analysis
- Data flow analysis
- Transformations

Issues in design of code optimization The issues in the design of code optimization are

1. Target machine characteristics
2. Target CPU architecture
3. Functional units

Target machine Optimization is done, according to the target machine characteristics. Altering the machine description parameters, one can optimize single piece of compiler code.

Target CPU architecture The issues to be considered for the optimization with respect to CPU architecture

1. Number of CPU registers
2. RISC Instruction set
3. CISC instruction set
4. Pipelining

Functional units Based on number of functional units, optimization is done. So that instructions can be executed simultaneously.

Principle Sources of Optimization

Some code improving transformation is Local transformations and some are Global transformations.

Local Transformations can be performed by looking only at a statement in a basic block. Otherwise it is global transformation.

Function Preserving Transformations

These transformations improve the program without changing the function it computes. Some of these transformations are

1. Common sub expression elimination
2. Copy propagation
3. Dead-code elimination
4. Loop optimization
   - Code motion
   - Induction variable elimination
   - Reduction in strength

Common sub expression elimination The process of identifying common sub expressions and eliminating their computation multiple times is known as common sub expression elimination.

Example: Consider the following program segment:

```
int sum_n, sum_n2, sum_n3;
int sum (int n)
{
Sum_n = ((n)*(n+1))/2;
sum_n2 = ((n)*(n+1)*(2n+1))/6;
sum_n3 = (((n)*(n+1))/2)*(((n)*(n+1))/2;
}
```

Three Address code for the above input is

(0) Proc-begin sum
(1) t_0: = $n + 1$
(2) t_1: = $n * t_0$
(3) t_2: = $t_1/2$
(4) sum_$n = t_2$
(5) t_3: = $n + 1$
(6) t_4: = $n *$ t_3
(7) t_5: $2 * n$
(8) t_6: = $t_5 + 1$
(9) t_7: = $t_4 * t_6$
(10) t_8: = $t_7/6$
(11) sum_n_2: = t_8
(12) t_9: = $n + 1$
(13) t_{10}: $n * t_9$
(14) t_{11}: $t_{10}/2$
(15) t_{12}: = $n + 1$
(16) t_{13}: = $n * t_{12}$
(17) t_{14}: = $t_{13}/2$
(18) t_{15}: = $t_{11} * t_{14}$
(19) sum_n_3: = t_{15}
(20) label L_o
(21) Proc end sum

The computations made in quadruples

(1) – (3), (12) – (14), (15) – (17) are essentially same. That is, $((n)*(n + 1))/2$ is computed.

It is the common sub expression.

This common sub expression is computed four times in the above example.

It is possible to optimize the code to have common sub expressions computed only once and then reuse the computed values further.

∴ Optimized intermediate code will be

(0) proc-begin sum
(1) t_0: = $n + 1$
(2) t_1: = $n * t_0$
(3) sultan: = $t_1/2$
(4) t_5: = $2 * n$
(5) t_6: = $t_5 + 1$
(6) t_7: = $t_1 * t_6$
(7) sum_n2: = $t_7/6$
(8) sum_n3: sum_n * sum_n
(9) proc-end sum

Constant folding The constant expressions in the input source are evaluated and replaced by the equivalent values at the time of compilation.

For example 10*3, 6 + 101 are constant expressions and they are replaced by 30, 107 respectively.

Example: Consider the following ‘*C*’ code:

```
int arr1 [10];
int main ( )
{
    arr1 [0] = 3;
    arr1 [1] = 4;
}
```

Unoptimized three address code equivalent to the above ‘*C*’ code is

(0) proc-begin main
(1) t_0: = 0*4
(2) t_1: = &arr1
(3) t_1 [t_0]: = 3
(4) t_2: = 1*4
(5) t_3: = &arr1
(6) t_3 [t_2]: = 4
(7) Label L_0
(8) Proc – end main

In the above code, 0*4 is a constant expression its value = 0. 1*4 is a constant expression, its value = 4.

∴ After applying constant folding, optimized code will be

(0) proc-begin main
(1) t_0: = 0
(2) t_1: = &arr1
(3) t_1 [t_0]: = 3
(4) t_2: = 4

(5) t_3: = &arr1
(6) t_3 [t_2]: = 4
(7) label L_0
(8) proc – end main

Copy propagation In copy propagation, if there is an expression $x = y$ then use the variable 'y' instead of 'x'. This propagated in the statements following $x = y$.

Example: In the previous example, there are two copy statements.

(1) $t_0 = 0$
(2) $t_2 = 4$

After applying copy propagation, the optimized code will be

(0) proc-begin main
(1) t_0: = 0
(2) t_1: = &arr1
(3) t_1 [0]: = 3
(4) t_2: = 4
(5) t_3: = &arr1
(6) t_3 [4]: = 4
(7) Label L_0
(8) proc-end main

In the three address code shown above, quadruples (1) and (4) are no longer used in any of the following statements.

∴ (1) and (4) can be eliminated.

Three address code after dead store elimination

(0) proc-begin main
(1) t_1: = &arr1
(2) t_1 [0]: = 3
(3) t_3: = &arr1
(4) t_3 [4]: = 4
(5) Label L_0
(6) proc-end main

In the above example, we are propagating constant values. It is also known as constant propagation.

Variable propagation Propagating another variable instead of the existing one is known as variable propagation.

Example: int func(int a, int b, int c)

```
{
    int d, e, f;
    d = a;
    If (a > 10)
    {
       e = d + b;
    }
    Else
    {
       e = d + c;
    }
    f = d*e;
    return (f);
}
```

Three address code (unoptimized):

(0) proc-begin func
$\boxed{(1)\ d := a}$
(2) if a >10 goto L_0
(3) goto L_1
(4) label : L_0
$\boxed{(5)\ e := d + b}$
(6) goto L_2
(7) label : L_1
$\boxed{(8)\ e := d + c}$
(9) label : L_2
$\boxed{(10)\ f := d * e}$
(11) return f
(12) goto L_3
(13) label : L_3
(14) proc-end func

Three address code after variable (copy) propagation:

(0) proc-begin func
(1) d: = a
(2) If a >10 goto .L_0
(3) goto L_1
(4) label: L_0
(5) e: = $a + b$
(6) goto L_2
(7) label: L_1
(8) e: = $a + c$
(9) label: L_2
(10) f: = a*e
(11) return f
(12) goto L_3
(13) label: L_3
(14) proc-end func

After dead store elimination:
In the above code (1) d: = a is no more used
∴ Eliminate the dead store d: = a

(0) proc-begin func
(1) If a > 10 goto L_0
(2) goto L_1
(3) label: L_0
(4) e: = $a + b$
(5) goto L_2
(6) label: L_1
(7) e: $a + c$
(8) label: L_2
(9) f: = a*e
(10) return f
(11) goto L_3
(12) label: L_3
(13) proc-end func

Dead code elimination Eliminating the code that never gets executed by the program is known as Dead code elimination. It reduces the memory required by the program

Example: Consider the following Unoptimized Intermediate code:

(0) proc-begin func
(1) debug: = 0
(2) If debug = = 1 goto L_0
(3) goto L_1
(4) label: L_0
(5) param c
(6) param b
(7) param a
(8) param lc1
(9) call printf 16
(10) retrieve to
(11) label: L_1
(12) $t_1 := a + b$
(13) $t_2 := t_1 + c$
(14) $v_1 := t_2$
(15) Return v_1
(16) goto L_2
(17) label: L_2
(18) proc-end func

In copy propagation, debug is replaced with 0, wherever debug is used after that assignment.

∴ Statement 2 will be changed as

If 0 = = 1 goto L_0

0 = = 1, always returns false.

∴ The control cannot flow to label: L_0

This makes the statements (4) through (10) as dead code. (2) Can also be removed as part of dead code elimination. (1) Cannot be eliminated, because 'debug' is a global variable. The optimized code after elimination of dead code is shown below.

(0) proc-begin func
(1) debug: = 0
(2) goto L_1
(3) label: L_1
(4) $t_1 := a + b$
(5) $t_2 := t_1 + c$
(6) $v_1 := t_2$
(7) return v_1
(8) goto L_2
(9) label: L_2
(10) proc-end func

Algebraic transformations We can use algebraic identities to optimize the code further. For example

Additive Identity: $a + 0 = a$

Multiplicative Identity: $a*1 = a$

Multiplication with 0: $a*0 = 0$

Example: Consider the following code fragment:

```
struct mystruct
{
int a [20];
int b;
} xyz;
int func(int i)
{
xyz.a[i] = 34;
}
```

The Unoptimized three address code:

(0) proc-begin func
(1) $t_0 := \&xyz$
$\boxed{(2)\ \ t_1 := 0}$
(3) $t_2 := i*4$
(4) $t_1 := t_2 + t_1$
(5) $t_0\,[t_1] = 34$
(6) label: L_0
(7) proc-end func

Optimized code after copy propagation and dead code elimination is shown below:

The statement $t_1 := 0$ is eliminated.

(0) proc-being func
(1) $t_0 =: = \&xyz$
(2) $t_2 := i*4$
$\boxed{(3)\ t_1 := t_2 + 0}$
(4) $t_0\,[t_1]: = 34$
(5) label: L_0
(6) proc-end func

After applying additive identity:

(0) proc-begin func
(1) $t_0 := \&xyz$
(2) $t_2 := i*4$
$\boxed{(3)\ t_1 := t_2}$
(4) $t_0\,[t_1]: = 34$
(5) label: L_0
(6) proc-end func

After copy propagation and dead store elimination:

(0) proc-begin func
(1) $t_0 := \&xyz$
(2) $t_2 := i*4$
(3) $t_0\,[t_2]: = 34$
(4) label: L_0
(5) proc-end func

Strength reduction transformation This transformation replaces expensive operators by equivalent cheaper ones on the target machine.

For example $y: = x*2$ is replaced by $y: = x + x$ as addition is less expensive than multiplication.

Similarly
Replace $y: = x*32$ by $y: = x << 5$
Replace $y: = x/8$ by $y: = x >> 3$

Loop optimization We can optimize loops by

(1) Loop invariant code motion transformation.
(2) Strength reduction on induction variable transformation.

Loop invariant code motion

The statements within a loop that compute value, which do not vary throughout the life of the loop are called loop invariant statements.

Consider the following program fragment:

```
int a [100];
int func(int x, int y)
{
int i;
int n1, n2;
i = 0;
n1 = x*y;
n2 = x - y;
while (a[i] > (n1*n2))
i = i + 1;
return(i);
}
```

The Three Address code for above program is

(0) proc-begin func
(1) $i := 0$
(2) $n_1 := x*y$
(3) $n_2 := x - y$
(4) label : L_0
(5) $t_2 := i*4$
$\boxed{(6)\ t_3 := \&arr}$
(7) $t_4 := t_3[t_2]$
$\boxed{(8)\ t_5 := n_1 * n_2}$
(9) if $t_4 > t_5$ goto L_1
(10) goto L_2
(11) label : L_1
(12) $i := i + 1$
(13) goto L_0
(14) label : L_2
(15) return i
(16) goto L_3
(17) label : L_3
(18) proc-end func

In the above code statements (6) and (8) are invariant.

After loop invariant code motion transformation the code will be

(0) proc-begin func
(1) $i := 0$
(2) $n_1 := x*y$
(3) $n_2 := x\text{-}y$
(4) $t_3 := \&arr$
(5) $t_5 : n_1*n_2$
(6) label : L_0
(7) $t_2 := i*4$
(8) $t_4 := t_3[t_2]$
(9) if $t_4 > t_5$ goto L_1
(10) goto L_2
(11) label : L_1
(12) $i := i + 1$
(13) goto L_0
(14) label : L_2
(15) return i
(16) goto L_3
(17) label : L_3
(18) proc-end func

Strength reduction on induction variables

Induction variable: A variable that changes by a fixed quantity on each of the iterations of a loop is an induction variable.

Example: Consider the following code fragment:

```
int i;
int a[20];
int func( )
{
    while(i<20)
    {
    a[i] = 10;
    i = i + 1;
    }
}
```

The three-address code will be

(0) proc-begin func
(1) label : L_0
(2) if $i < 20$ goto L_1
(3) goto L_2
(4) label : L_1
(5) $t_0 := i*4$
(6) $t_1 := \&a$
(7) $t_1[t_0] := 10$
(8) $i := i + 1$
(9) goto L_0
(10) label : L_2
(11) label : L_3
(12) proc-end func

After reduction of strength the code will be
Here (5) $t_0 = i*4$ is moved out of the loop and (8) is followed by $t_0 = t_0 + 4$.

(0) proc-begin func
(0a) $t_0 := i*4$
(1) label : L_0
(2) if $i < 20$ goto L_1
(3) goto L_2
(4) label:L_1
(5)
(6) $t_1 := \&a$
(7) $t_1[t_0] := 10$
(8) $i := i + 1$
(8a) $t_0 := t_0 + 4$
(9) goto L_0
(10) label : L_2
(11) label : L_3
(12) proc-end func

Loops in Flow Graphs

Loops in the code are detected during the data flow analysis by using the concept called 'dominators' in the flow graph.

Dominators

A node d of a flow graph dominates node n, if every path from the initial node to 'n' goes through 'd'.

It is represented as d dom n.

Notes:

1. Each and every node dominates itself.
2. Entry of the loop dominates all nodes in the loop.

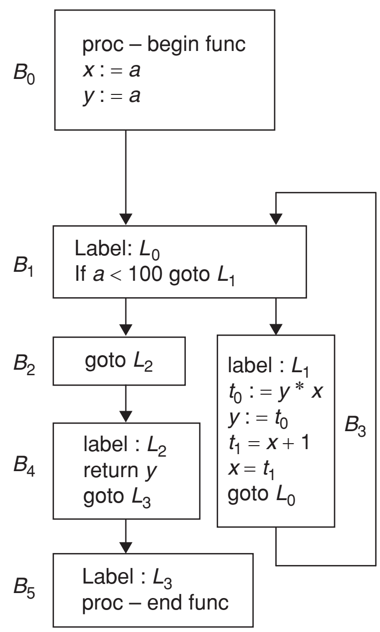
Example: Consider the following code fragment:

```
int func(int a)
{
int x, y;
x = a;
y = a;
While (a < 100)
{
y = y*x;
x = x+1;
}
return(y);
}
```

The Three Address code after local optimization will be

(0) proc-begin func
(1) $x := a$
(2) $y := a$
(3) label: L_0
(4) if $a < 100$ goto L_1
(5) goto L_2
(6) label: L_1
(7) $t_0 := y*x$
(8) $y := t_0$
(9) $t_1 := x + 1$
(10) $x := t_1$
(11) goto L_0
(12) label: L_2
(13) return y
(14) goto L_3
(15) label: L_3
(16) proc-end func

The Flow Graph for above code will be:



To reach B_2, it must pass through B_1

$\therefore$ B_1 dominates B_2. Also B_0 dominates B_2.

dominators $[B_1] = \{B_0, B1\}$ (or) dominators $[1] = \{0, 1\}$

The dominators for each of the nodes in the flow graph are

dominators [0] = {0}
dominators [1] = {0, 1}
dominators [2] = {0, 1, 2}
dominators [3] = {0, 1, 3}
dominators [4] = {0, 1, 2, 4}
dominators [5] = {0, 1, 2, 4, 5}

Edge

An edge in a flow graph represents a possible flow of control.

In the flow graph, B_0 to B_1 edge is represented as $0 \rightarrow 1$.

Head and tail: In the edge $a \rightarrow b$, the node b is called head and the node a is called as tail.

Back edges: There are some edges in which dominators [tail] contains the head.

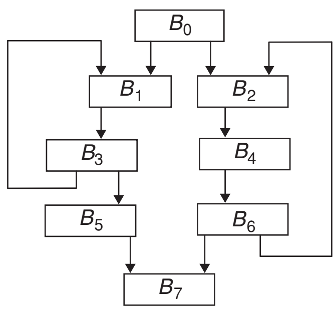
The presence of a back edge indicates the existence of a loop in a flow graph.

In the previous graph, $3 \rightarrow 1$ is a back edge.

Consider the following table:

Edge	Head	Tail	Dominators [head]	Dominators [tail]
$0 \rightarrow 1$	1	0	{0, 1}	{0}
$1 \rightarrow 2$	2	1	{0, 1, 2}	{0, 1}
$1 \rightarrow 3$	3	1	{0, 1, 3}	{0, 1}
$3 \rightarrow 1$	1	3	{0, 1}	{0, 1, 3}
$2 \rightarrow 4$	4	2	{0, 1, 2, 4}	{0, 1, 2}
$4 \rightarrow 5$	5	4	{0, 1, 2, 4, 5}	{0, 1, 2, 4}

Example: Consider below flow graph:



The dominators of each node are

dominators [0] = {0}
dominators [1] = {0, 1}
dominators [2] = {0, 2}
dominators [3] = {0, 1, 3}
dominators [4] = {0, 2, 4}
dominators [5] = {0, 1, 3, 5}
dominators [6] = {0, 2, 4, 6}
dominators [7] = {0, 7}

Edge	Head	Tail	Dominators [head]	Dominators [tail]
$0 \rightarrow 1$	1	0	{0, 1}	{0}
$0 \rightarrow 2$	2	0	{0, 2}	{0}
$1 \rightarrow 3$	3	1	{0, 1, 3}	{0, 1}
$3 \rightarrow 1$	1	3	{0, 1}	{0, 1, 3}
$3 \rightarrow 5$	5	3	{0, 1, 3, 5}	{0, 1, 3} Backedge
$5 \rightarrow 7$	7	5	{0, 7}	{0, 1, 3, 5}
$2 \rightarrow 4$	4	2	{0, 2, 4}	{0, 2}
$6 \rightarrow 2$	2	6	{0, 2}	{0, 2, 4, 6} Backedge
$4 \rightarrow 6$	6	4	{0, 2, 4, 6}	{0, 2, 4}
$6 \rightarrow 7$	7	6	{0, 7}	{0, 2, 4, 6}

Here $\{B_6, B_2, B_4\}$ form a loop (L_1), $\{B_3, B_1\}$ form another loop (L_2)

In a loop, the entry of the loop dominates all nodes in the loop.

Header of the loop The entry of the loop is also called as the header of the loop.

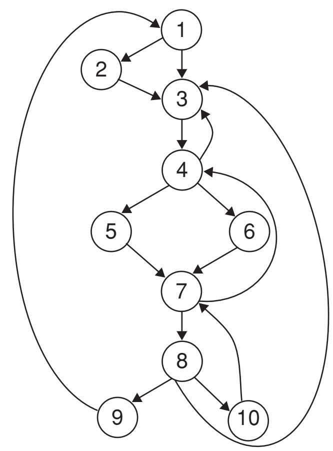
Loop exit block In loop L_1 can be exited from the basic block B_6. It is called loop exit block. The block B_3 is the loop exit block for the loop L_2. It is possible to have multiple exit blocks in a loop.

Dominator tree

A tree, which represents dominate information in the form of tree is a dominator tree. In this,

- The initial node is the root.
- Each node d dominates only its descendents in the tree.

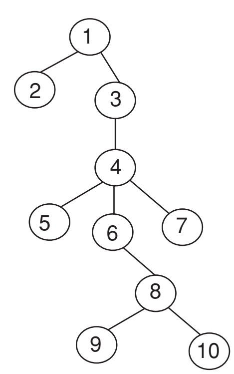
Consider the flow graph



The dominators of each node are

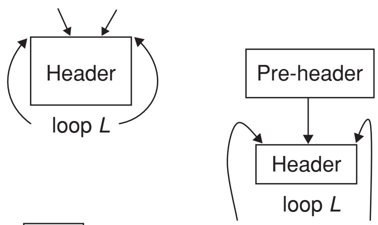
dominators [1] = {1}
dominators [2] = {1, 2}
dominators [3] = {1, 3}
dominators [4] = {1, 3, 4}
dominators [5] = {1, 3, 4, 5}
dominators [6] = {1, 3, 4, 6}
dominators [7] = {1, 3, 4, 7}
dominators [8] = {1, 3, 4, 7, 8}
dominators [9] = {1, 3, 4, 7, 8, 9}
dominators [10] = {1, 3, 4, 7, 8, 10}

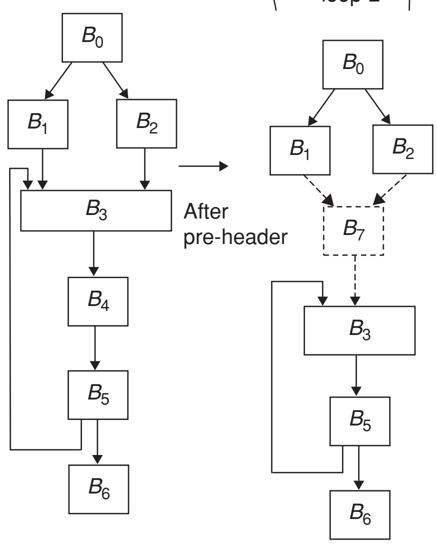
The dominator tree will be:



Pre-header

A pre-header is a basic block introduced during the loop optimization to hold the statements that are moved from within the loop. It is a predecessor to the header block.





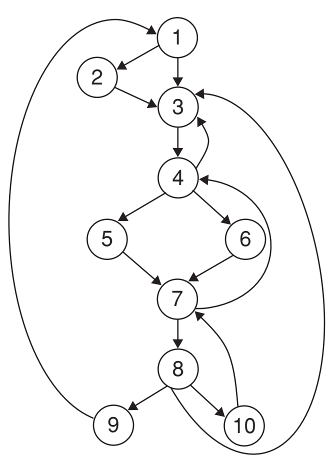
Reducible Flow Graphs

A flow graph G is reducible if and only if we can partition the edges into two disjoint groups:

(1) Forward edges
(2) Backward edges with the following properties.

(i) The forward edges form an acyclic graph in which every node can be reached from the initial node of G.
(ii) The back edges consist only of edges whose heads dominates their tails.

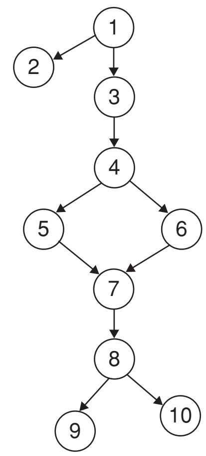
Example: Consider previous flow graph



In the above flow graph, there are five back edges

$$4 \to 3,\ 7 \to 4,\ 8 \to 3,\ 9 \to 1 \text{ and } 10 \to 7$$

Remove all backedges.
The remaining edges must be the forward edges.
The remaining graph is acyclic.



∴ It is reducible.

Global Dataflow Analysis

Point: A point is a place of reference that can be found at

1. Before the first statement in a basic block.
2. After the last statement in a basic block.
3. In between two adjacent statements within a basic block.

Example 1:

$a' = 10$
$b' = 20$
$c' = a * b$ $\quad B_1$

Here, In B_1 there are 4 points

Example 2:

B_1

• $P_1 - B_1$
proc-begin func
• $P_2 - B_1$
$v_3 = v_1 + v_2$
• $P_3 - B_1$
if $c > 100$ goto L_0
• $P_4 - B_1$

There is 4 point in the basic block B_1, given by $P_1 - B_1$, $P_2 - B_1$, $P_3 - B_1$ and $P_4 - B_1$.

Path: A path is a sequence of points in which the control can flow.

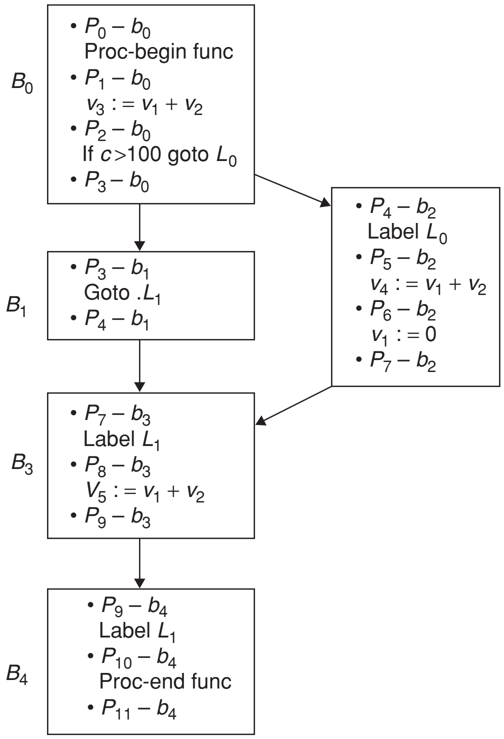
A path from P_1 to P_n is a sequence of points $P_1, P_2, \ldots, P_n$ such that for each i between 1 and n-1, either

(a) P_i is the point immediately preceding a statement and P_{i+1} is the point immediately following that statement in the same block.

(OR)

(b) P_i is the end of some block and P_{i+1} is the beginning of a successor block.

Example:



Path is between the points $P_0 - b_0$ and $P_6 - b_2$:

The sequence of points $P_0 - b_0$, $P_1 - b_0$, $P_2 - b_0$, $P_3 - b_0$, $P_4 - b_2$, $P_5 - b_2$ and $P_6 - b_2$.

Path between $P_3 - b_1$ and $P_6 - b_2$: There is no sequence of points.

Path between $P_0 - b_0$ and $P_7 - b_3$: There are two paths.

(1) Path 1 consists of the sequence of points, $P_0 - b_0$, $P_1 - b_0$, $P_2 - b_0$, $P_3 - b_0$, $P_3 - b_0$, $P_4 - b_1$ and $P_7 - b_3$.

(2) Path 2 consists of the sequence of points $P_0 - b_0$, $P_1 - b_0$, $P_2 - b_0$, $P_3 - b_0$, $P_4 - b_2$, $P_5 - b_2$, $P_6 - b_2$, $P_7 - b_2$ and $P_7 - b_3$

Definition and Usage of Variables

Definitions

It is either an assignment to the variable or reading of a value for the variable.

Use

Use of identifier x means any occurrence of x as an operand.

Example: Consider the statement

$$x = y + z;$$

In this statement some value is assigned to x. It defines x and used y and z values.

Global Data-Flow-Analysis

Data Flow Analysis (DFA) is a technique for gathering information about the possible set of values calculated at various points in a program.

- An example of a data-flow analysis is reaching definitions.
- A single way to perform data-flow analysis of program is to setup data flow equations for each node of the control flow graph.

Use definition (U-d) chaining

The use of a value is any point where that variable or constant is used in the right hand side of an assignment or is evaluating an expression.

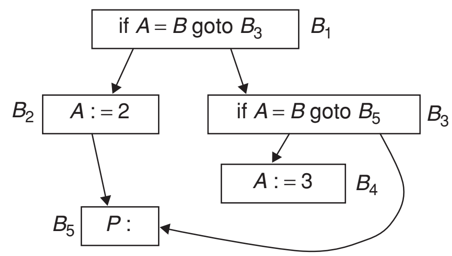
The definition of a value occurs implicitly at the beginning of the whole program for a variable.

A point is defined either prior to or immediately after a statement.

Reaching definitions

A definition of a variable A reaches a point P if there is a path in the flow graph from that definition to P, such that no other definitions of A appear on the path.

Example:



The definition A: = 3 can reach point p in B_5.

To determine the definitions that can reach a given program first assign distinct numbers to each definition, since it is associated with a unique quadruple.

- For each simple variable A, make a list of all definitions of A anywhere in the program.
- Compute two sets for each basic block B.

Gen [B] is the set of generated definitions within block B and that reach the end of the block.

1. Kill [B], which is the set of definitions outside of B that define identifiers that also have definitions within B.
2. IN [B], which are all definitions reaching the point just before B's first statement.

Once this is known, the definitions reaching any use of A within B are found by:

Let u be the statement being examined, which uses A.

1. If there are definitions of A within B before u, the last is the only one reaching u.
2. If there is no definition of A within B prior to u, those reaching u are in IN [B].

Data Flow Equations

1. For all blocks B,

$$\text{OUT }[B] = (\text{IN }[B] - \text{KILL }[B]) \text{ U GEN }[B]$$

A definition d, reaches the end of B if

(a) $d \in$ IN [B] and is not killed by B.
(or)
(b) d is generated in B and is not subsequently redefined here.

2. IN [B] = U OUT [P]
$\forall P$ preceding B
A definition reaches the beginning of B iff it reaches the end of one of its predecessors.

Computing U-d Chains

If a use of variable 'a' is preceded in its block by a definition of 'a', this is the only one reaching it.

If no such definition precedes its use, all definitions of 'a' in IN [B] are on its chain.

Uses of U-d Chains

1. If the only definition of 'a' reaching this statement involves a constant, we can substitute that constant for 'a'.
2. If no definitions of 'a' reaches this point, a warning can be given.
3. If a definition reaches nowhere, it can be eliminated. This is part of dead code elimination.

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Replacing the expression 2 * 3.14 by 6.28 is
(A) Constant folding
(B) Induction variable
(C) Strength reduction
(D) Code reduction

2. The expression (a*b)*c op … where 'op' is one of '+', '*' and '↑' (exponentiation) can be evaluated on CPU with a single register without storing the value of (a*b) if
(A) 'op' is '+' or '*'
(B) 'op' is '↑' or '+'
(C) 'op' is '↑' or '*'
(D) not possible to evaluate without storing

3. Machine independent code optimization can be applied to
(A) Source code
(B) Intermediate representation
(C) Runtime output
(D) Object code

4. In block B if S occurs in B and there is no subsequent assignment to y within B, then the copy statement $S : x = y$ is
(A) Generated (B) Killed
(C) Blocked (D) Dead

5. If E was previously computed and the value of variable in E have not changed since previous computation, then an occurrence of an expression E is
(A) Copy propagation
(B) Common sub expression
(C) Dead code
(D) Constant folding

6. In block B, if x or y is assigned there and s is not in B, then $s : x = y$ is
(A) Generated (B) Killed
(C) Blocked (D) Dead

7. Given the following code
$A = x + y$;
$B = x + y$;
Then the corresponding optimized code as
_____
_____
$C = x + y$;
_____
$A = C$;
_____
$B = C$;
When will be optimized code pose a problem?
(A) When C is undefined.
(B) When memory is consideration.
(C) C may not remain same after some statements.
(D) Both (A) and (C).

8. Can the loop invariant $X = A - B$ from the following code be moved out?
For i = 1 to 10
```
{
A = B * C;
X = A - B;
}
```
(A) No
(B) Yes
(C) $X = A - B$ is not invariant
(D) Data insufficient

9. If every path from the initial node goes through a particular node, then that node is said to be a
(A) Header (B) Dominator
(C) Parent (D) Descendant

Common data for questions 10 and 11: Consider the following statements of a block:

$a: = b + c$
$b: = a - d$
$c: = b + c$
$d: = a - d$

10. The above basic block contains, the value of b in 3rd statement is
(A) Same as b in 1st statement
(B) Different from b in 1st statement
(C) 0
(D) 1

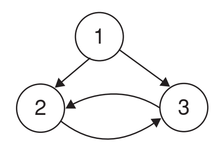
11. The above basic block contains
(A) Two common sub expression
(B) Only one common sub expression
(C) Dead code
(D) Temporary variable

12. Find the induction variable from the following code:
$A = -0.2;$
$B = A + 5.0;$
(A) A
(B) B
(C) Both A and B are induction variables
(D) No induction variables

13. The analysis that cannot be implemented by forward operating data flow equations mechanism is
(A) Interprocedural
(B) Procedural
(C) Live variable analysis
(D) Data

14. Which of the following consist of a definition, of a variable and all the uses, U, reachable from that definition without any other intervening definitions?
(A) Ud-chaining (B) Du-chaining
(C) Spanning (D) Searching

15. Consider the graph



The graph is
(A) Reducible graph
(B) Non-reducible graph
(C) Data insufficient
(D) None of these

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. In labeling algorithm, let n is a binary node and its children have L_1 and L_2, if $L_1 = L_2$ then LABEL (n):
(A) $L_1 - 1$ (B) $L_2 + 1$
(C) $L_1 + L_1$ (D) $L_1 + 1$

2. The input for the code generator is a:
(A) Tree at lexical level
(B) Tree at semantic level
(C) Sequence of assembly language instructions
(D) Sequence of machine idioms

3. In labeling algorithm, let n is a binary node and its children have i_1 and i_2, LABEL (n) if $i_1 \neq i_2$ is
(A) Max (i_1, i_2)
(B) $i_2 + 1$
(C) $i_2 - 1$
(D) $i_2 - i_1$

4. The following tries to keep frequently used value in a fixed register throughout a loop is:
(A) Usage counts
(B) Global register allocation
(C) Conditional statement
(D) Pointer assignment

5. Substitute y for x for copy statement $s : x = y$ if the following condition is met
(A) Statements s may be the only definition of x reaching u
(B) x is dead
(C) y is dead
(D) x and y are aliases

6. Consider the following code

```
for (i=0; i<m; i++)
{
for (j=0; j<m; j ++)
If (i%2)
{
a = a + (14*j+5*i);
b = b + (9 + 4*j);
}
}
```

Which of the following is false?
(A) There is a scope of common reduction in this code
(B) There is a scope of strength reduction in this code.
(C) There is scope of dead code elimination in this code
(D) Both (A) and (C)

7. S_1: In dominance tree, the initial node is the root.
S_2: Each node d dominates only its ancestors in the tree.
S_3: if $d \neq n$ and d dom n then d dom m.

Which of the statements is/are true?
(A) S_1, S_2 are true
(B) S_1, S_2 and S_3 are true

(C) Only S_3 is true
(D) Only S_1 is true

8. The specific task storage manager performs:
(A) Allocation/Deallocation of storage to programs
(B) Protection of storage area allocated to a program from illegal access by other programs in the system
(C) The status of each program
(D) Both (A) and (B)

9. Concept which can be used to identify loops is:
(A) Dominators
(B) Reducible graphs
(C) Depth first ordering
(D) All of these

10. A point cannot be found:
(A) Between two adjacent statements
(B) Before the first statement
(C) After the last statement
(D) Between any two statements

11. In the statement, $x = y*10 + z$; which is/are defined?
(A) x (B) y
(C) z (D) Both (B) and (C)

12. Consider the following program:

```
void main ( )
{
    int x, y;
    x = 3; y = 7;
    --------
    --------
    if (x<y)
    {
    int x;
    {
    int y;
    y = 9;
    ------
    x = 2*y;
    }
    -------
    -------
    x = x + y;
    printf ("%d", x);
    }
    ------
    printf ("%d", x);
}
```

The output is
(A) 3 – 25 (B) 25 – 3
(C) 3 – 3 (D) 25 – 25

13. The evaluation strategy which delays the evaluation of an expression until its value is needed and which avoids repeated evaluations is:
(A) Early evaluation (B) Late evaluation
(C) Lazy evaluation (D) Critical evaluation

14. If two or more expressions denote same memory address, then the expressions are:
(A) Aliases (B) Definitions
(C) Superiors (D) Inferiors

15. Operations that can be removed completely are called:
(A) Strength reduction
(B) Null sequences
(C) Constant folding
(D) None of these

Previous Years' Questions

1. In a compiler, keywords of a language are recognized during: **[2011]**
(A) parsing of the program
(B) the code generation
(C) the lexical analysis of the program
(D) dataflow analysis

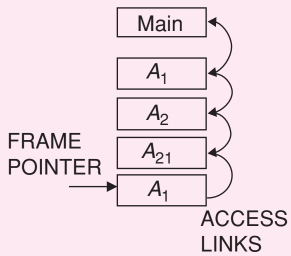
2. Consider the program given below, in a block structured pseudo-language with lexical scoping and nesting of procedures permitted. **[2012]**

Program main;
Var …
Procedure A_1;
Var …
Call A_2;
End A_1
Procedure A_2;
Var …
Procedure A_{21};
Var …
Call A_1;
End A_{21}
Call A_{21};
End A_2
Call A_1;
End main

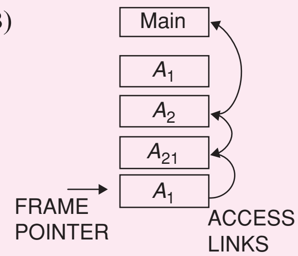
Consider the calling chain: Main $\rightarrow A_1 \rightarrow A_2 \rightarrow A_{21} \rightarrow A_1$

The correct set of activation records along with their access links is given by:

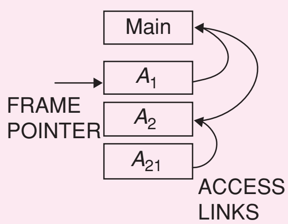
(A)



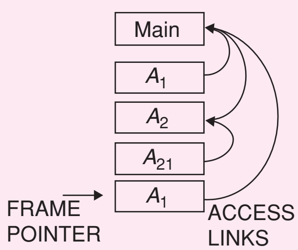
(B)



(C)



(D)



Common data for questions 3 and 4: The following code segment is executed on a processor which allows only register operands in its instructions. Each instruction can have atmost two source operands and one destination operand. Assume that all variables are dead after this code segment.

```
c = a + b;
d = c * a;
e = c + a;
x = c * c;
If (x > a) {
   y = a * a;
}
Else {
      d = d * d;
      e = e * e;
}
```

3. What is the minimum number of registers needed in the instruction set architecture of the processor to compile this code segment without any spill to memory? Do not apply any optimization other than optimizing register allocation. **[2013]**

(A) 3 (B) 4
(C) 5 (D) 6

4. Suppose the instruction set architecture of the processor has only two registers. The only allowed compiler optimization is code motion, which moves statements from one place to another while preserving correctness. What is the minimum number of spills to memory in the compiled code? **[2013]**

(A) 0 (B) 1
(C) 2 (D) 3

5. Which one of the following is NOT performed during compilation? **[2014]**

(A) Dynamic memory allocation
(B) Type checking
(C) Symbol table management
(D) Inline expansion

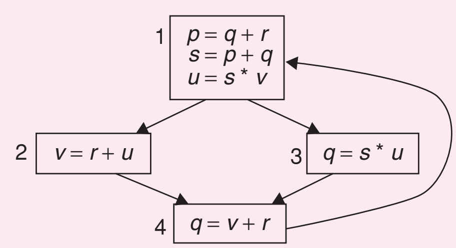
6. Which of the following statements are CORRECT? **[2014]**

(i) Static allocation of all data areas by a compiler makes it impossible to implement recursion.
(ii) Automatic garbage collection is essential to implement recursion.
(iii) Dynamic allocation of activation records is essential to implement recursion.
(iv) Both heap and stack are essential to implement recursion.

(A) (i) and (ii) only (B) (ii) and (iii) only
(C) (iii) and (iv) only (D) (i) and (iii) only

7. A variable x is said to be live at a statement S_i in a program if the following three conditions hold simultaneously: **[2015]**

1. There exists a statement S_j that uses x
2. There is a path from S_i to S_j in the flow graph corresponding to the program.
3. The path has no intervening assignment to x including at S_i and S_j.



The variables which are live both at the statement in basic block 2 and at the statement in basic block 3 of the above control flow graph are

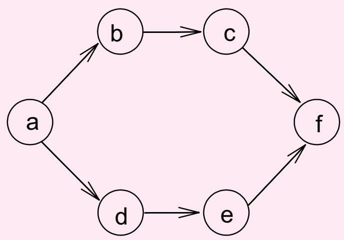
(A) p, s, u (B) r, s, u
(C) r, u (D) q, v

8. Match the following **[2015]**

P. Lexical analysis	1. Graph coloring
Q. Parsing	2. DFA minimization
R. Register allocation	3. Post-order traversal
S. Expression evaluation	4. Production tree

(A) P–2, Q–3, R–1, S–4 (B) P–2, Q–1, R–4, S–3
(C) P–2, Q–4, R–1, S–3 (D) P–2, Q–3, R–4, S–1

9. Consider the following directed graph:



The number of different topological orderings of the vertices of the graph is _______ . **[2016]**

10. Consider the following grammar:

stmt –> **if** expr **then** expr **else** expr; stmt | Ò
expr –> term **relop** term | term
term –> id | number
id –> **a** | **b** | **c**
number –> **[0 – 9]**

where **relop** is a relational operator (e.g., <, >,…), Ò refers to the empty statement, and **if, then, else** are terminals.

Consider a program *P* following the above grammar containing ten **if** terminals. The number of control flow paths in *P* is__________. For example, the program

if e_1 **then** e_2 **else** e_3

has 2 control flow paths, $e_1 \rightarrow e_2$ and $e_1 \rightarrow e_3$. **[2017]**

11. Consider the expression $(a—1) * (((b + c)/3) + d))$. Let X be the minimum number of registers required by an *optimal* code generation (without any register spill) algorithm for a load/store architecture, in which *(i) only load and store instruction can have memory operands and (ii) arithmetic instructions can have only register or immediate operands*. The value of X is _____. **[2017]**

12. Match the following according to input (from the left column) to the compiler phase (in the right column) that processes it: **[2017]**

(P) Syntax tree	(i) Code generator
(Q) Character stream	(ii) Syntax analyzer
(R) Intermediate representation	(iii) Semantic analyzer
(S) Token stream	(iv) Lexical analyzer

(A) P → (ii), Q → (iii), R → (iv), S → (i)
(B) P → (ii), Q → (i), R → (iii), S → (iv)
(C) P → (iii), Q → (iv), R → (i), S → (ii)
(B) P → (i), Q → (iv), R → (ii), S → (iii)

Answer Keys

Exercises

Practice Problems I

1. A **2.** C **3.** B **4.** A **5.** B **6.** B **7.** C **8.** B **9.** B **10.** B
11. B **12.** D **13.** C **14.** B **15.** B

Practice Problems 2

1. D **2.** B **3.** A **4.** B **5.** A **6.** D **7.** D **8.** D **9.** D **10.** D
11. A **12.** B **13.** C **14.** A **15.** B

Previous Years' Questions

1. C **2.** D **3.** B **4.** B **5.** A **6.** D **7.** C **8.** C **9.** 6 **10.** 1024
11. 2 **12.** C

Test

Compiler Design

Time: 45 min.

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. The most powerful parsing method is
(A) LALR (B) LR
(C) CLR (D) LL (1)

2. In which phase 'type checking' is done?
(A) Lexical analysis
(B) Code optimization
(C) Syntax analysis
(D) Semantic analysis

3. A shift reduces parser carries out the actions specified within braces immediately after reducing the corresponding rule of grammar, as below:
$S \to aaD$ {Print "1"}.
$S \to b$ {Print "2"}
$D \to Sc$ {Print "3"}
What is the translation of 'aaaabcc' using the syntax directed translation scheme described by the above rules?
(A) 33211 (B) 11233
(C) 11231 (D) 23131

4. $E \to TE'$
$E' \to +TE'/\in$
$T \to FT'$
$T' \to *FT'/\in$
$F \to (E)/id$
From above grammar, FOLLOW (E) is
(A) {), $} (B) {$, *)}
(C) {(, id} (D) {+,), $}

5. To eliminate backtracking, which one is used?
(A) Left Recursion
(B) Left Factoring
(C) Right Recursion
(D) Right Factoring

6. Consider the grammar
$T \to (T) \mid \in$
Let the number of states in SLR (1), LR (1) and LALR (1) parsers for the grammar be n_1, n_2 and n_3 respectively. Which relationship holds well?
(A) $n_1 = n_2 = n_3$
(B) $n_1 \geq n_3 \geq n_2$
(C) $n_1 = n_3 < n_2$
(D) $n_1 < n_2 < n_3$

7. If w is a string of terminals and A, B are two non-terminals then which of the following are left-linear grammars?
(A) $A \to wB/w$
(B) $A \to Bw/w$
(C) $A \to wB$
(D) None of the above

8. The grammar $E \to E * E/E + E/a$, is
(A) Ambiguous
(B) Unambiguous
(C) Will not depend on the given sentence
(D) None of these

9. Shift-reduce parsers are
(A) Bottom up parsers
(B) Top down parsers
(C) Both (A) and (B)
(D) None of these

10. Consider the following grammars:
I. $E \to TE'$
$E' \to +TE'/\in$
$T \to FT'$
$T' \to *FT'/\in$
$F \to (E)/id$
II. $S \to iCtSS' \mid a$
$S' \to eS \mid \in$
$C \to b$
Which of the following is true?
(A) II is LL (1) (B) I is LL (1)
(C) Both (A) and (B) (D) None of these

11. Consider the following grammar:
$S \to iCtSS'/a$
$S' \to eS/\in$
$C \to b$
First (S') is
(A) $\{i, a\}$ (B) $\{\$, e\}$
(C) $\{e, \in\}$ (D) $\{b\}$

12. From the above grammar Follow(S) is.
(A) $\{\$, e\}$ (B) $\{\$\}$
(C) $\{e\}$ (D) $\{\$,), e\}$

13. Find the LEADING (S) from the following grammar:
$S \to a \mid \wedge \mid (T)$
$T \to T, S / S$
(A) $\{a, \wedge, (\,\}$ (B) $\{, a,)\}$
(C) $\{, a, (\,\}$ (D) $\{, a, \wedge,)\}$

14. From above grammar find the TRAILING (T).
(A) $\{a,)\}$ (B) $\{a, \wedge,)\}$
(C) $\{),\}$ (D) $\{, a,)\}$

15. Which of the following remarks logically follows?
(A) FIRST $(\in) = \{\in\}$.
(B) If FOLLOW (A) contains $, then A may or may not be the start symbol.
(C) If $A \to w$, is a production in the given grammar G, then $FIRST_k$ (A) contains $FIRST_k$ (w).
(D) All of the above

16. Consider the following grammar:
$S \rightarrow AB$
$B \rightarrow ab$
$A \rightarrow aa$
$A \rightarrow a$
$B \rightarrow b.$
The grammar is
(A) Ambiguous
(B) Unambiguous
(C) Can't predictable
(D) None of these

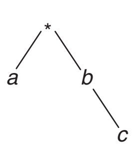
17. If a handle has been found but there is no production with this handle as a right side, then we discover
(A) Logical error
(B) Runtime error
(C) Syntactic error
(D) All of the above

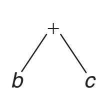
18. The function of syntax phase is
(A) To build a literal table
(B) To build an uniform symbol table
(C) To parse the tokens produced by lexical analyzer
(D) None of these

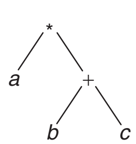
19. Which of the following are cousins of compilers?
(A) Pre-processor and Assembler
(B) Assembler and LEX
(C) Pre-processor and YACC
(D) LEX and YACC.

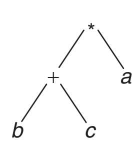
20. Error is detected in predictive parsing when ____ hold(s).
(i) 'a' on top of stack and next input symbol is 'b'.
(ii) When 'a' is on top of stack, 'a' is next input symbol and parsing table entry $M[A, a]$ is empty.
(A) Neither (i) nor (ii)
(B) Both (i) and (ii)
(C) only (i)
(D) only (ii)

21. Which one indicates abstract syntax tree (AST) of "$a * b + c$" with following grammar:
$E \rightarrow E * T/T$
$T \rightarrow T + F/F$
$F \rightarrow id$

(A) 

(B) 

(C) 

(D) 

22. The parse tree is constructed and then it is traversed and the semantic rules are evaluated in a particular order by a
(A) Recursive evaluator
(B) Bottom up translation
(C) Top down translation
(D) Phase tree method

23. The following grammar indicates
$S \rightarrow a\ \alpha\ b | b\ \alpha\ c | a\ b$
$S \rightarrow \alpha\ S\ | b$
$S \rightarrow \alpha\ b\ b/a\ b$
$S \rightarrow \alpha\ b\ d\ b/b$
(A) LR (0) grammar
(B) SLR grammar
(C) Regular grammar
(D) None of these

24. If the attributes of the child depends on the attributes of the parent node then it is ____ attribute.
(A) Inherited
(B) Directed
(C) Synthesised
(D) TAC

25. The semantic rule is evaluated and the intermediate code is generated when the production is expanded in ____
(A) Parse tree method
(B) Bottom up translation
(C) Top down translation
(D) Recursive evaluator model

26. Consider the grammar shown below:
$S \rightarrow CC$
$C \rightarrow cC/a$
The grammar is
(A) LL (1)
(B) SLR (1) But not LL (1)
(C) LALR (1) but not SLR (1)
(D) LR (1) but not LALR

27. The class of grammars for which we can construct predictive parsers looking k-symbols ahead in the input is called
(A) LR (k)
(B) CLR (k)
(C) LALR (k)
(D) LL (k)

28. A compiler is a program that
(A) Places programs into memory and prepares them for execution.
(B) Automates the translation of assembly language into machine language.
(C) Accepts a program written in a high level language and produces an object program.
(D) Appears to execute a source program as if it were machine language.

Common data for questions 29 and 30:

Consider the grammar
$E \rightarrow TE'$
$E' \rightarrow + TE' \mid \in$
$T \rightarrow FT'$
$T^{1} \rightarrow * FT' \mid \in$
$F \rightarrow (E) \mid id.$

29. Which one is FOLLOW (F)?
(A) {+,), $}
(B) {+, (,), *}
(C) {*,), $}
(D) {+, *,), $}

30. FIRST (E) will be as same as
(A) FIRST (T)
(B) FIRST (F)
(C) Both (A) and (B)
(D) None of these

Answers Keys

1. A	**2.** D	**3.** D	**4.** A	**5.** B	**6.** C	**7.** B	**8.** A	**9.** A	**10.** B
11. C	**12.** A	**13.** A	**14.** C	**15.** D	**16.** A	**17.** C	**18.** C	**19.** A	**20.** B
21. C	**22.** A	**23.** D	**24.** A	**25.** C	**26.** A	**27.** D	**28.** C	**29.** D	**30.** C

Compiler Design Test 1

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following implementation of Lexical analyzer is easy to modify and faster?
(i) Lexical-analyzer by hand
(ii) Lexical-analyzer generator
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

2. Which of the following tasks are performed by a Lexical analyzer?
(i) Identification of Lexemes
(ii) Stripping-out Comments
(iii) Stripping-out white spaces
(iv) Correlating error messages generated by the compiler with the source program.
(A) (i), (ii) (B) (i), (ii), (iii)
(C) (iii), (iv) (D) (i), (ii), (iii), (iv)

3. Which of the following statement is TRUE?
(i) Every LL(k) language is deterministic context-free language.
(ii) Every deterministic context free language is LL(k) language.
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

4. Which of the following grammar is LL(3) grammar but not LL(2) and not LL(1)?
(A) $S \rightarrow aS|b$ (B) $S \rightarrow aaS \mid ab \mid b$
(C) $S \rightarrow aaaS \mid aab \mid ab \mid b$ (D) None of the above

5. On errorneous input,
(A) LALR parser makes more moves than LR parser.
(B) LR parser makes more moves than LALR parser.
(C) LALR parser makes equal number of moves as an LR parser.
(D) Unpredictable

6. What kind of conflict does the following grammar leads to?
$S \rightarrow A$
$A \rightarrow xA \mid yA \mid y$
(A) Shift-reduce conflict
(B) Reduce-reduce conflict
(C) No conflict exist
(D) Both (A) and (B)

7. Which of the following statement(s) is/are TRUE?
(i) Synthesized attributes may be calculated only from attributes of children.
(ii) Inherited attributes may be calculated only from attributes of parents.
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

8. Which of the following action is not performed by an LL parser?
(i) Matching top of the parser stack with next input token.
(ii) Predicting a production and apply it in a derivation step.
(iii) Accept and terminate the parsing of a sequence of tokens.
(iv) Reporting an error message.
(A) (i), (iv) (B) (iii), (iv)
(C) (ii), (iii) (D) None of the above

9. Consider the following grammar:
$S \rightarrow AcB$
$A \rightarrow aA \mid \varepsilon$
$B \rightarrow bBS \mid \varepsilon$
For this grammar, FOLLOW(B) is
(A) $\{a, \$\}$ (B) $\{a, c\}$
(C) $\{a, c, \$\}$ (D) $\{c\}$

10. Which of the following statements is FALSE?
(A) Every LR(0) grammar is in SLR (1).
(B) Every SLR (1) grammar is in LR (0).
(C) Every SLR(1) grammar is a canonical LR(1) grammar.
(D) Every LR(1) grammar is not necessarily SLR(1).

11. Which of the following grammars for $\{a^n b^{n+k} | n, k \in N\}$ are in LL(1)?
(i) $S \rightarrow PQ$
$P \rightarrow aPb \mid \varepsilon$
$Q \rightarrow bQ \mid \varepsilon$
(ii) $S \rightarrow aSb| T$
$T \rightarrow bT| \varepsilon$
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

12. Based on which of the following parameters does a LR(k) parser performs reduction?
(i) The complete left context
(ii) The reducible phrase itself
(iii) The k-terminal symbols to its right
(A) (i), (iii) only (B) (ii), (iii) only
(C) (i), (ii),(iii) (D) (i) only

13. After removing left-recursion from the following grammar the resultant grammar will be:
$P \rightarrow P + Q \mid Q$
$Q \rightarrow \text{int} \mid (P)$
(A) $P \rightarrow Q + Q$
$Q \rightarrow \text{int} \mid (P)$
(B) $P \rightarrow QP^1$
$P^1 \rightarrow +QP^1 \mid \varepsilon$
$Q \rightarrow \text{int} \mid (P)$

(C) $P \rightarrow QP^1$
$P^1 \rightarrow +QP^1$
$Q \rightarrow \text{int} \mid (P)$

(D) $P \rightarrow Q + P^1$
$P^1 \rightarrow Q + P^1 \mid \varepsilon$
$Q \rightarrow \text{int} \mid (P)$

14. Consider the following grammar:
$S \rightarrow PQRt$
$P \rightarrow a \mid b \mid \varepsilon$
$Q \rightarrow c \mid d \mid \varepsilon$
$R \rightarrow e \mid f$
For this grammar, FIRST(S) is
(A) $\{a, b, c, d, e, f, \varepsilon\}$ (B) $\{a, b, c, d, e, f\}$
(C) $\{a, b, t\}$ (D) $\{a, b, c, d\}$

15. Consider the following grammar, G:
$S \rightarrow (L) \mid a$
$L \rightarrow L, S \mid S$
Which of the following is TRUE?
(i) G is not suitable to be parsed using recursive descent parsing.
(ii) FIRST (S) = FIRST (L) = $\{(, a\}$
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

16. Consider the following configuration:
$A \rightarrow X_1\, X_2 \ldots X_j, a$
Then which of the following is TRUE?
(i) With SLR(1) parsing we would reduce if the next token was any of those in FOLLOW (A).
(ii) With LR(1) parsing, we reduce only if the next token is exactly 'a'.
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

17. Which of the following is FALSE?
(A) An LALR(1) grammar is one which allows a LALR(1) parse table to be constructed with no more than one action per cell.
(B) LALR(1) grammar allow a wider set of structures than SLR(1) grammar.
(C) LALR(1) grammars do not handle some cases allowed in LR(1) grammars.
(D) None of these

18. Identify the correct statements from the following:
(i) LALR(1) is a subset of LR(1).
(ii) LALR(1) is a super set of SLR(1).
(iii) A grammar that is not LR(1) may LALR(1).
(A) (i), (ii) only (B) (ii), (iii) only
(C) (i), (iii) only (D) (i), (ii), (iii)

19. Based on the following grammar,
$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid id$
What is the closure $(\{T \rightarrow T \bullet * F\}$?
(A) $\{T \rightarrow T \bullet * F\}$
(B) $\{T \rightarrow T \bullet * F\ T \rightarrow \bullet F\}$
(C) $\{T \rightarrow T * \bullet F\ F \rightarrow \bullet (E)\ F \rightarrow \bullet\, id\}$
(D) $\{T \rightarrow \bullet\, T * F\}$

20. Consider the following grammar:
Stmts' $\rightarrow$ Stmts
Stmts $\rightarrow$ Stmt | Stmts; Stmt
Stmt $\rightarrow$ $Id = E$ | Print E
$E \rightarrow Id \mid E + E \mid (E)$
What does the closure set of the following rule will include?
Stmts' $\rightarrow \bullet$ Stmts
(A) Stmts' $\rightarrow \bullet$ Stmts
(B) Stmt's $\rightarrow \bullet$ Stmts
Stmts $\rightarrow \bullet$ Stmt
Stmts $\rightarrow \bullet$ Stmts ; Stmt
Stmt $\rightarrow \bullet$ $id = E$
Stmt $\rightarrow \bullet$ Print E
(C) Stmts' $\rightarrow \bullet$ Stmts
Stmts $\rightarrow \bullet$ Stmt
Stmts $\rightarrow \bullet$ Stmts ; Stmt
Stmt $\rightarrow \bullet$ $id = E$
Stmt $\rightarrow \bullet$ Print E
$E \rightarrow \bullet\, id$
$E \rightarrow \bullet\, E + E$
$E \rightarrow \bullet\, (E)$
(D) Stmts' $\rightarrow \bullet$ Stmts
Stmts $\rightarrow \bullet$ Stmt
Stmts $\rightarrow \bullet$ Stmts; Stmt

21. For the following grammar:
$A \rightarrow BCD$
$B \rightarrow Bb \mid \varepsilon$
$C \rightarrow cC \mid \varepsilon$
$D \rightarrow d$
What is FOLLOW (B)?
(A) $\{b, \$\}$ (B) $\{b, c\}$
(C) $\{b, d\}$ (D) $\{b, c, d\}$

22. Consider the following grammar, G:
$S \rightarrow AB$
$A \rightarrow aAa \mid \varepsilon$
$B \rightarrow bBb \mid \varepsilon$
Which of the following statements is TRUE?
(i) G is ambiguous.
(ii) G is not in LL(1).
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

23. Consider the following grammar:
$S \rightarrow a \mid AbC$
$A \rightarrow a$
$C \rightarrow A \mid c$
Which of the following is FALSE?
(A) FOLLOW(S) = $\{\$\}$
(B) FOLLOW(A) = $\{b, \$\}$
(C) FOLLOW(C) = $\{\$\}$
(D) The grammar is in SLR(1).

24. Which of the following is TRUE about LR-attributed grammar?

(i) Attributes can be evaluated in LR parsing
(ii) LR-Attribute grammars are a subset of L-attributed grammars
(iii) LR-attribute grammars are a subset of S-attributed grammars.

(A) (i), (ii) only (B) (ii), (iii) only
(C) (i), (iii) only (D) (i), (ii), (iii) only

25. Which of the following conditions must hold to call a grammar is in SLR(1)?

(i) For any production $A \to x.yz$ in the set, with terminal y, there is no complete item $B \to W$. In that set with 'y' in FOLLOW (B).
(ii) For any two complete productions $A \to X$ and $B \to Y$. In the set, the follow sets must be disjoint.

(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

Answer Keys

1. B	**2.** D	**3.** A	**4.** C	**5.** A	**6.** A	**7.** A	**8.** D	**9.** C	**10.** B
11. A	**12.** C	**13.** B	**14.** B	**15.** C	**16.** C	**17.** D	**18.** A	**19.** A	**20.** B
21. D	**22.** B	**23.** D	**24.** A	**25.** C					

Hints and Explanations

1. Lexical-analyzer generator makes the code easier to modify a lexical analyzer (Since we have to rewrite only the affected patterns).
It also speeds up the process of implementing the analyzer (Since the programmer specifies the software at the very high level of patterns and relies on generator to produce code). Choice (B)

3. LL(k) languages are Deterministic CFLs but there are Deterministic CFL's that are not LL(k). Choice (A)

4. $S \to aS/b$ is in LL(1)
($\because$ By looking single look-ahead, we can decide whether to shift or reduce).
$S \to aaS|ab|b$ is in LL(2).
$S \to aaaS|\ aab|\ ab|\ b$ is in LL(3). Choice (C)

5. On errorneous input, LALR parser makes more moves (i.e., reductions) than the LR parser. Choice (A)

6. Given grammar,
$S \to A$
$A \to xA \mid yA \mid y$
This grammar has shift-reduce conflict.
$A \to y\,.\,A$ || leads to shift
$A \to y.$ || leads of reduce
These productions leads to shift-reduce conflict.
Choice (A)

7. Synthesized attributes are calculated from attributes of children.
Inherited attributes are calculated from attributes of parents or siblings. Choice (A)

8. An LL parser perform the following actions:
(i) Match (ii) Predict
(iii) Accept (iv) Error Choice (D)

9. Given grammar,
$S \to AcB$
$A \to a\,A \mid \varepsilon$
$B \to bBS|\ \varepsilon$
FOLLOW (B) = FIRST (S) U FOLLOW (S)
FIRST (S) = {FIRST (A) – ε} ∪ {c} = {a, c}
Follow (S) = {\$}
∴ Follow (B) = {a, c, \$} Choice (C)

10. LR(0) ⊂ SLR (1) ⊂ CLR (1) Choice (B)

11. Given language:
$\{a^n\, b^{n+k} \mid n, k \in N\}$
(i) $S \to PQ$
$P \to aPb|\ \varepsilon$
$Q \to bQ|\ \varepsilon$
This is LL(1) (No ambiguity, no left recursion).
To derive 'ab':
$S \to PQ$
$\to aPbQ$
$\to ab$.
(ii) $S \to aSb \mid T$
$T \to bT \mid \varepsilon$
To derive 'ab' :
$S \to aSb$
$\to aTb$
$\to abTb$ ($T \to bT$)
(Back track)
(or)
$S \to aSb$
$\to aTb$
$\to ab(T \to \varepsilon)$
∴ (ii) not in LL(1) (as which production to use in some derivation is unknown). Choice (A)

12. All the three parameters are required during reduction in LR(k) parser. Choice (C)

13. Given grammar,
$P \to P + Q \mid Q$

$Q \rightarrow \text{int} \mid (P)$
To avoid left-recursion, replace recursive term with other terms and introduce a new variable as shown below.
$P \rightarrow P + Q \mid Q$ can be written as
$P \rightarrow QP^1$
$P^1 \rightarrow +QP^1 \mid \varepsilon$ Choice (B)

14. FIRST (S) = {FIRST (P) – ε} U {FIRST (Q) – ε} U {FIRST (R)}
$= \{a, b, c, d, e, f\}$. Choice (B)

15. Given grammar is not parsed using recursive descent parsing as the grammar is left recursive.
FIRST (S) = {(, a}
FIRST (L) = FIRST (S) = {(, a} Choice (C)

16. For the configuration,
$A \rightarrow X_1, X_2, \ldots X_j, a$
If 'a' is in FOLLOW(A) then reduce in SLR(1).
If the next token is exactly 'a' then reduce in LR(1).
Choice (C)

18. SLR(1) ⊂ LALR (1) ⊂ LR(1)
If a grammar is not in LR(1) then it is not in LALR(1).
(∵ Conflict exist in LALR also) Choice (A)

19. Given grammar
$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid id$
Closure ({$T \rightarrow T \bullet * F$}) will be {$T \rightarrow T \bullet * F$} only.
To calculate closure we add new productions to the set if the • is before a non-terminal.
(Here • is before '∗' which is a terminal)
Choice (A)

20. Given grammar,
Stmts' → Stmts
Stmts → Stmt | Stmts; Stmt
Stmt → id = E| Print E
$E \rightarrow id \mid E + E \mid (E)$
Closure ({Stmts' → • Stmts}),
will include the productions of Stmts (∵ • is before Stmts)
Stmts → • Stmt
Stmts → •Stmts; Stmt
Now production of 'Stmt' will be included (∵ • is before Stmt)
Stmt → •id = E
Stmt → • Print E Choice (B)

21. Follow (B) = {b} ∪ {FIRST (C) – ∈} ∪ {FIRST (D)}
$= \{b\} \cup \{c\} \cup \{d\}$
$= \{b, c, d\}$ Choice (D)

22. Given grammar G:
$S \rightarrow AB$
$A \rightarrow aAa\mid \varepsilon$
$B \rightarrow bBb \mid \varepsilon$
G is not ambiguous.
(∵ To derive any string only one possible method exists)
G is not in LL(1).
To derive aa.
$S \rightarrow AB$
$\rightarrow aA\ aB$
$\rightarrow aa\ A\ aaB$ (or) $S \rightarrow AB$
(Back track) $\rightarrow aA\ aB$
$\rightarrow aAa\ B$ $\rightarrow aaB$
$\rightarrow aa$ $\rightarrow aa$
∴ G is not in LL(1).
[OR]
To check whether G is in LL(1) or not:
Check whether for all $A \in N$ with $A \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \alpha_n$ being all A-Productions in G, the following holds:
(i) FIRST(α_1),………FIRST(α_n) are pairwise disjoint.
(ii) If ∈ ∈ FIRST(α_j) for some $j \in [1, n]$ then FOLLOW (A) η FIRST (α_i) = ϕ for all $1 \le i \le n, j \ne i$.
(i) is satisfied.
FIRST (aAa) = {a}
FIRST (ε) = {ε}
(ii) is failed.
FIRST ($aA\ a$) = {a}
FOLLOW (A) = {a, b, \$}
These are not disjoint.
∴ G is not in LL(1). Choice (B)

23. Given grammar,
$S \rightarrow a \mid AbC$
$A \rightarrow a$
$C \rightarrow A|c$
FOLLOW (S) = {\$}
FOLLOW (A) = {b} ∪ follow (C) = {b, \$}
FOLLOW (C) = FOLLOW (S) = {\$}
The grammar is not in SLR(1) , since there is reduce/reduce conflict with the productions.
$S \rightarrow a$.
$A \rightarrow a$. Choice (D)

24. LR-attribute grammar is a special type of attribute grammar. It allows the attributes to be evaluated on LR parsing.
LR-attribute grammar ⊂ L-attributed grammar.
S-attributed grammar ⊂ LR-attributed grammar.
Choice (A)

25. (i) avoids shift-reduce conflict.
(ii) avoids reduce-reduce conflict. Choice (C)

Compiler Design Test 2

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following issues are considered by the run-time environment of a compiler?
(i) Allocation of storage locations for the objects
(ii) Mechanisms to access variables
(iii) Linkages between procedures
(A) (i), (ii) only (B) (ii), (iii) only
(C) (iii) only (D) (i), (ii), (iii)

2. Program data objects (such as global constants) and data generated by the compiler (such as the information to support garbage collection) are placed in which of the following areas of run-time memory?
(A) Code area (B) Static area
(C) Heap area (D) Stack area

3. If an activation of procedure '*A*' calls procedure '*B*' then which of the following is TRUE?
(A) Activation of *B* must end before the activation of *A* can end.
(B) Activation of *A* must end before the activation of *B* can end.
(C) Activation of *A* must end before the activation of *B* can start.
(D) Activation of *B* must start after the activation of *A* can end.

4. Consider the activation of procedure *X* which calls procedure *Y*. the activation of *Y* aborts (i.e., it becomes impossible for execution to continue). Then which of the following is TRUE?
(A) *X* continues execution
(B) *X* ends simultaneously with *Y*
(C) *X* makes '*Y*' to execute
(D) None of these

5. Which of the following is true with respect to an "activation tree"?
(i) Root of the tree is the activation of "main" procedure.
(ii) Nodes of the tree corresponds to an activation.
(iii) At a node for an activation of procedure *P*, the children correspond to activations of the procedures called by this activation of *P*.
(A) (i), (ii), only (B) (ii) only
(C) (iii) only (D) (i), (ii), (iii)

6. The sequence of procedure calls of a program corresponds to which traversal of the activation tree?
(A) In order traversal (B) Pre order traversal
(C) Post order traversal (D) Level-order traversal

7. The sequence of returns of procedures corresponds to which traversal of the activation tree?
(A) In order traversal
(B) Pre order traversal
(C) Post order traversal
(D) Level-order traversal

8. Consider an activation tree '*T*'. Also assumes that the control lies within a particular activation of some procedure, corresponding to a node *N* of the activation tree.
Then the activations that are currently live (or open) are those that correspond to
(A) Node *N* only
(B) Node *N* and its ancestors only
(C) Node *N* and its children only
(D) Ancestors of node *N* only

9. Which of the following tasks is managed by a control stack (i.e., run-time stack)?
(A) Static data and functions
(B) Garbage collection
(C) Procedure calls and returns
(D) All of the above

10. Which of the following field of an activation record will point to the activation record of the caller?
(A) Returned values (B) Access link
(C) Temporaries (D) Control link

11. Consider the following statement :
"It is possible to allocate objects, arrays or other structures of unknown size on the stack".
Which of the following is TRUE with respect to given statement?
(i) This avoids the expense of garbage collection.
(ii) The array or the object needs to be local of a procedure.
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

12. Consider the languages which do not allow nested procedure declarations. Then which of the following is FALSE?
(i) Global variables are placed on stack.
(ii) Access links are used for accessing local variables of a procedure.
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

13. Consider the following procedure:
```
q( )
{
.
.
.
p ( )
```

```
{ .
 .
 .
.
r( )
{
 .
 .
 }
s( )
{ .
 .
 .
}
 }
 t( )
 {
 .
 .
 .
 }
 }
```

What is the nesting depth of procedure '*s*'?

(A) 2 (B) 3
(C) 4 (D) 5

14. Which of the following is attached to each activation record to use normal static scope rule for nested functions?

(A) Access link (B) Control link
(C) Process link (D) Data link

15. Consider a procedure '*P*' which is at the top of the stack with nesting depth 'n_p' and '*P*' needs to access '*x*', which is defined with in some procedure '*Q*' that surrounds '*P*' and has nesting depth 'n_q'. To find '*x*' initially top of the stack is checked and then how many times access links are followed?

(A) $n_q - n_p$ (B) $n_p - n_q$
(C) $n_q - n_p + 1$ (D) $n_p - n_q + 1$

16. Consider a procedure '*p*' which is passed to another procedure '*q*' as a parameters. And *q* calls its parameter. Then which of the following things are passed from caller to called?

(i) Name of the procedure parameter
(ii) Access link of the parameter

(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) None of these

17. Which of the following is FALSE with respect to reference counting?

(i) The reference count of a new object is set to 1.
(ii) The reference count of each object passed into a procedure is incremented.
(iii) Reference counting can be used to collect unreachable cyclic data structures.

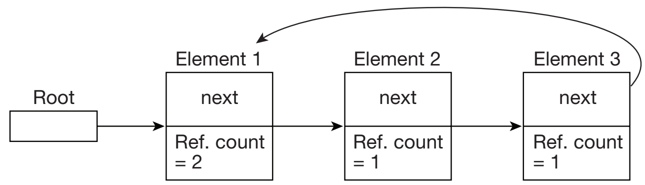
(A) (ii) only (B) (iii) only
(C) (ii), (iii) only (D) (i), (ii), (iii)

18. Which of the following statements is TRUE?

(i) If no references are found to an object then that object is considered as 'garbage'.
(ii) Reference counting garbage collector works in 'incremental' fashion.

(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

19. Consider the following figure:



Each object has its Reference count. If the 'Root' assigned 'NULL' value then which of the following is true?

(i) The reference counts of element 1, 2, 3 will be respectively 1, 1, 1.
(ii) The reference counting garbage collector can consider the elements as garbage.

(A) (i) only
(B) (ii) only
(C) Both (i) and (ii)
(D) Neither (i) nor (ii)

20. Which of the following task is not related to code generator?

(i) Instruction selection
(ii) Register Allocation
(iii) Instruction ordering

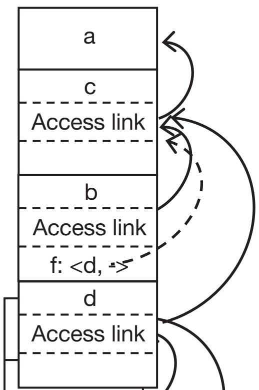
(A) (i), (ii) only (B) (ii), (iii) only
(C) (iii) only (D) None of these

21. Consider the following function:

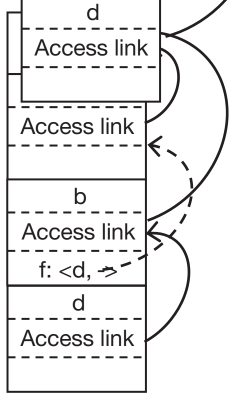
```
fun a(x) =
    Let
      fun b(f) =
      ….f….;
      fun c(y) =
        Let
      fun d(z) = ……
      in
      …..b(d)……….
      end
    in
    ……..c(1)…….
    end;
```

Here function '*a*' consists the functions '*b*' and '*c*' within it. Function '*b*' has function-valued parameter *f*, which is called by '*b*'. Function '*c*' defines within it self a function '*d*' and then '*c*' calls '*b*' with actual parameter '*d*'. Then which of the following correctly shows the access links between the functions?

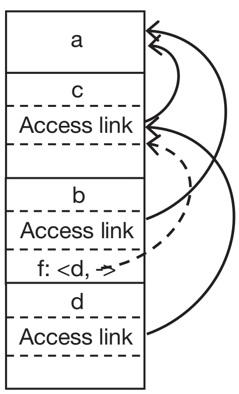
(A)



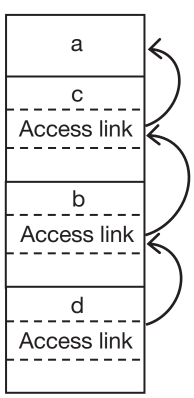
(B)



(C)



(D)



22. The number of non-leaf nodes present in the DAG representation of the following block is:

1. $x = y + z$
2. $y = x - w$
3. $z = y + z$
4. $w = x - w$

(A) 2 (B) 3
(C) 4 (D) 5

23. Consider the following block:

1. $p = q + r$;
2. $q = q + s$;
3. $r = r - s$;
4. $t = q + r$;

Which of the following is TRUE with respect to given block?

(i) DAG representation of given block can identify the common sub-expression of the block.
(ii) The DAG has 4 non-leaf nodes

(A) (i) only
(B) (ii) only
(C) Both (i) and (ii)
(D) Neither (i) nor (ii)

24. Consider the following expression:

$(c + d) - (-(f + g))$

To evaluate the given expression (All operands must be in registers). What is the minimum number of registers required?

(A) 1 (B) 2
(C) 3 (D) 4

25. Construct a syntax-directed translation scheme that takes strings of a's, b's and c's as input and produces as output the number of substrings in the input string that correspond to the pattern $a(a + b)^*c + (a + b)^*b$. For example, the input string '*abbcabcababc*' translation scheme outputs 3(*abbc*, *abc*, *ababc*).

(A) $S_1 \to S_2 a$ $\{S_1.n_1 = S_2..n_1 + 1;$
$S_1.n_2 = S_2.n_2;$
$S_1.n_3 = S_2.n_3;\}$
$S_1 \to S_2 b$ $\{S_1.\ n_1 = S_2.\ n_1 + 1;$
$S_1.n_2 = S_2.n_2;$
$S_1.n_3 = S_2.n_3 + S_2.n_2;\}$
$S_1 \to S_2 c$ $\{S_1.n_1 = 0;$
$S_1.n_2 = S_2.n_2;$
$S_1.\ n_3 = S_2.n_3;\}$
$S_1 \to a$ $\{S_1.n_1 = 1;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0;\}$
$S_1 \to b$ $\{S_1.n_1 = 0;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0\}$
$S_1 \to c$ $\{S_1.n_1 = 0;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0\}$

(n_1 = count of number of a's to the left of c
n_2 = count of number of a's to the right of 'c',
n_3 = total number of required substrings.)

(B) $S_1 \to S_2 a$ $\{S_1.n_1 = S_2.n_1 + 1;$
$S_1.\ n_2 = S_2.n_2;$
$S_1.n_3 = S_2.n_3;\}$
$S_1 \to S_2\ b$ $\{S_1.n_1 = S_2.n_1 + 1;$
$S_1.n_2 = S_2.n_2;$
$S_1.n_3 = S_2.n_3;\}$
$S_1 \to S_2\ c$ $\{S_1.n_1 = 0;$

$S_1.n_2 = S_2.n_2;$
$S_1.n_3 = S_2.n_3;\}$
$S_1 \rightarrow a\ \{S_1.n_1 = 1;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0;\}$
$S_1 \rightarrow b\ \{S_1.n_1 = 0;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0\}$
$S_1 \rightarrow c\ \{S_1.n_1 = 0;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0;\}$

(C) $S_1 \rightarrow S_2 a\ \{S_1.n_1 = S_2.n_1 + 1;$
$S_1.n_2 = S_2.n_2;$
$S_1.n_3 = S_2.n_3;\}$
$S_1 \rightarrow S_2 b\ \{S_1.n_1 = S_2.n_1;$
$S_1.n_2 = S_2.n_2;$
$S_1.n_3 = S_2.n_3;\}$
$S_1 \rightarrow S_2 c\ \{S_1.n_1 = 0;$
$S_1.n_2 = S_2.n_2;$
$S_1.n_3 = S_2.n_3;\}$
$S_1 \rightarrow a\ \{S_1.n_1 = 1;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0;\}$
$S_1 \rightarrow b\ \{S_1.n_1 = 0;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0;\}$
$S_1 \rightarrow c\ \{S_1.n_1 = 0;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0;\}$

(D) $S_1 \rightarrow S_2\ a\ \{S_1.n_1 = S_2.n_1 + 1;$
$S_1.n_2 = S_1.n_2;$
$S_1.n_3 = S_2.n_3;\}$
$S_1 \rightarrow S_2\ b\ \{S_1.n_1 = S_2.n_1;$
$S_1.n_2 = S_2.n_2;$
$S_1.n_3 = S_2.n_3;\}$
$S_1 \rightarrow S_2\ c\ \{S_1.n_1 = 0;$
$S_1.n_2 = S_2.n_2;$
$S_1.n_3 = S_2.n_3;\}$
$S_1 \rightarrow a\ \{S_1.n_1 = 0;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0;\}$
$S \rightarrow b\ \{S_1.n_1 = 0;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0;\}$
$S \rightarrow c\ \{S_1.n_1 = 0;$
$S_1.n_2 = 0;$
$S_1.n_3 = 0;\}$

Answer Keys

1. D	**2.** B	**3.** A	**4.** B	**5.** D	**6.** B	**7.** C	**8.** B	**9.** C	**10.** D
11. C	**12.** C	**13.** B	**14.** A	**15.** B	**16.** C	**17.** B	**18.** C	**19.** A	**20.** D
21. C	**22.** B	**23.** B	**24.** C	**25.** A					

Hints and Explanations

1. The run-time environment of a complier deals with the issues such as:
(1) Allocation of storage locations for the objects
(2) Linkages between procedures
(3) Mechanisms used by the target program to access variables
(4) Mechanisms for passing parameters etc.
Choice (D)

2. The size of some program data objects and data generated by the compiler can be known at compile time. So they can be placed in static area of run-time memory.
Choice (B)

3. Given, that *A* and *B* are procedures and

```
A ( )
{
.
.
B ( ) ;
.
.
.
}
```

Each time a procedure is called, space for its local variables is pushed on to a stack and when the procedure terminates, that space is popped off the stack.
In given scenario, Activation record of '*A*' created initially. Next *B*'s activation record is created and *B* must end before the termination of '*A*'. Choice (A)

4. If *Y* aborts then *X* also ends simultaneously with *Y*.
Choice (B)

5. Choice (D)

6. The preorder traversal of an activation tree corresponds to the sequence of procedure calls. Choice (B)

7. The sequence of returns corresponds to a post-order traversal of the activation tree. Choice (C)

8. The live nodes are *N* and its ancestors. ($\because$ After return of child nodes only parent nodes will terminate.)
Choice (B)

9. Control stack (run-time stack, manages procedure calls and returns). Choice (C)

10. Control link points to the activation record of the caller. Access link is used to locate data needed by the called procedure. Choice (D)

11. It is possible to allocate objects, whose size cannot be determined at compile time on a stack. This avoids the expense of garbage collecting their space.
But the stack can be used only for an object if it is local to a procedure. (∵ The local data will be inaccessible when the procedure returns). Choice (C)

12. Global variables are allocated in static storage.
Local variables are placed in stack (and accessed using top of the stack). Choice (C)

13. Nesting depth of a procedure is '1' if it is not nested within any other procedure.
If a procedure P is defined immediately with in a procedure Q then the nesting depth of $P = 2$
Here, in given code,
Nesting depth of 'q' = 1
Nesting depth of 'p = 2
Nesting depth of 'r' = 3
Nesting depth of 's' = 3
Nesting depth of 't' = 2 Choice (B)

14. Access links are attached to activation records to use static scope rules for nested procedures. Choice (A)

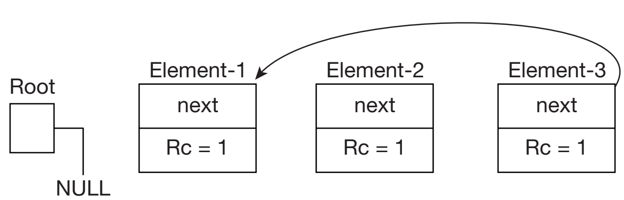
15. P is defined with in Q. Nesting depth of P is n_p. Nesting depth of 'Q' is n_q.
As P is inside Q,
$N_p \geq n_q$
We need to check $n_p - n_q + 1$ records to identify 'x'.
(one access is already given in problem). Choice (B)

16. The name of the procedure parameter and access link of the parameters needs to passed by caller. Choice (C)

17. Reference counting cannot collect unreachable, cyclic data structures. Choice (B)

18. An object is considered as garbage if it has no references.
References counting garbage collector works in an 'incremental' fashion. Choice (C)

19. The reference count of an object holds a value which specifies the number of objects referencing that object.
If root is made as "NULL : THEN:

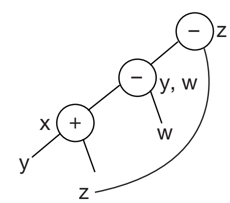


∴ (i) is correct
But References counting garbage collector cannot consider these elements as garbage (∴ their Reference counts is not zero).
∴ (ii) is false. Choice (A)

20. Choice (D)

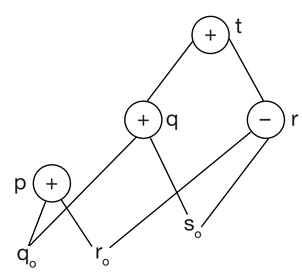
21. Let us see what will happen when 'a' is executed. First 'a' calls 'c', so we place an activation record for 'c' above that for 'a' on the stack. The access link for 'c' points to the record for 'a', since 'c' is defined immediately within 'a'. Then 'c' calls $b(d)$.
'c' knows about 'd', since 'd' is defined within 'c' and therefore 'c' passes a pointer to its own activation record as the access link. Choice (C)

22. Given block
$x = y + z$
$y = x - w$
$z = y + z$
$w = x - w$



Number of non-leaf nodes = 3 Choice (B)

23. Given block,
$p = q + r$
$q = q + s$
$r = r - s$
$t = q + r$
This block calculates '$q + r$' two times but both times The sum '$q + r$' is same. But the DAG does not consider this issue.
$(t = q + r = (q + s) + (r - s) = q + r)$
The DAG of given code will be



Number of non-leaf nodes = 4 Choice (B)

24. $R_0 \leftarrow c$
$R_1 \leftarrow d$
$R_0 \leftarrow R_0 + R_1$
$R_1 \leftarrow f$

$R_2 \leftarrow g$
$R_1 \leftarrow R_1 + R_2$
$R_1 \leftarrow -R_1$
$R_0 \leftarrow R_0 - R_1$
$\therefore$ 3 registers required.

Choice (C)

25. The CFG for given problem is

$$S \rightarrow Sa|Sb|Sc|a|b|c$$

This grammar parses any string in left-skewed manner. i.e., grammar is left-recursive. Lets take three synthesized attributes for the non-terminal symbol S, namely n_1, n_2, n_3. n_1 will capture the number of a's to the left of a given c, n_2 will count the number of a's to the right of given 'c' character and n_3 will accumulate the total number of substrings. Here we need to count the number of a's to the left of a 'c' character and to the right of that character so that we can add the value of n_1 to a running total for each occurrences of a 'b' character to the right of 'c' which recording the value of a's to the right of 'c' so that when we find a new 'c', we copy the value of the a's that were to the right of the first 'c' and which are now to the left of the second 'c'.
Hence, the resultant grammar is Choice (A).

Choice (A)

Theory of Computation and Compiler Design Test 3

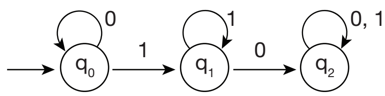
Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Which of the following is not a feature of 'Deterministic Finite Automata'?
   (A) Finite set of states
   (B) Finite set of input symbols
   (C) Any number of start states
   (D) A set of final states
2. Which language is accepted by an NFA?
   (A) Regular languages
   (B) Superset of Regular languages.
   (C) Proper subset of Regular languages.
   (D) Context free languages
3. Which of the following has the power to be in several states at once?
   (i) DFA (ii) NFA
   (A) (i) only (B) (ii) only
   (C) Both (i) and (ii) (D) Neither (i) nor (ii)
4. In which of the following terms DFA and NFA will differ?
   (i) Set of states (ii) Set of inputs
   (iii) Start state (iv) Set of final states
   (v) Transition function
   (A) (iii), (iv) (B) (iii), (iv), (v)
   (C) (i), (iii), (v) (D) (v) only
5. Which of the following is FALSE for a transition function of NFA?
   (A) It takes a state from a finite set of states as an argument.
   (B) It takes an input symbol from a finite set of inputs as an argument.
   (C) It returns a subset of states.
   (D) It returns a state.
6. Which of the following is FALSE?
   (A) A language L is accepted by some DFA if L is accepted by some NFA.
   (B) An NFA with n states will have an equivalent DFA with 2^n states.
   (C) NFA and DFA have equal expressing capability.
   (D) $\in$-NFA has more expressing capability than NFA.
7. What are the equivalent sets of ϕ^* and ϕ^0 respectively?
   (A) $\{\in\}, \{\in\}$ (B) $\{\in\}, \phi$
   (C) $\phi, \{\in\}$ (D) ϕ, ϕ
8. Let L be the language is defined over $L = \{0, 1\}$ then, L* =
   (A) $\{\in\}$
   (B) $\{0, 1\}$
   (C) $\{\in, 0, 1, 00, 11, 01, 10, ...\}$
   (D) $\{0, 1, 00, 11, 01, 10, ...\}$
9. While scanning the input string, all the constants stored in the following Data structure?
   (A) Symbol Table (B) Terminal Table
   (C) Numeric Table (D) Literal Table
10. Eliminating left recursion results
    (A) in converting a non LL(1) grammar to LL(1) grammar.
    (B) in nonretainment of left associativity.
    (C) in comparatively easy implementation
    (D) may fall in to infinite loop.
11. Which one of the following parser does not require intelligence to parse string?
    (A) RDP
    (B) Brute Force Technique
    (C) Table driven parser
    (D) Operator precedence parser
12. In the Parsing of a string 'w' using LL(1) parsing algorithm, Top of stack contains a terminal and Look ahead symbol is same as Top of stack, then which of the following action is performed?
    (A) successfull completion
    (B) pop of stack
    (C) increment input pointer
    (D) pop of stack and increment input pointer
13. Which of the following statement is false about LL(1)?
    (A) An ambiguous grammar is not LL(1)
    (B) Left factored grammar is not LL(1).
    (C) Left recursive grammar is not LL(1)
    (D) In a grammar 'G', if every non-Terminal if produces only one production then G is not in LL(1).
14. Which of the following statement is false?
    (A) Bottom up parsing uses Reverse Right most derivation.
    (B) Bottom up parsing uses canonical reduction sequence.
    (C) Top down parsing uses Left most derivation.
    (D) Top down parsing uses canonical Left sentential form.
15. Which of the following statement is false?
    (A) CLR is most widely used parser
    (B) Size of SLR(1), LALR(1) and CLR(1) parsers may or may not be equal.
    (C) Go to and shift action of LR(0), SLR(1) and LALR(1) must be equal.
    (D) LALR(1) parser is most widely used parser.
16. Consider the DFA which accepts L = {w/w is of the form $x10y$ for some strings x and y consisting of 0's and 1's only}.
    Which of the following strings are not accepted by given DFA?

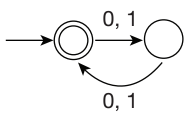
(i) 01 (ii) 11010
(iii) 100011 (iv) $\in$
(A) (i), (ii) (B) (i), (ii), (iv)
(C) (i), (iv) (D) (i), (ii), (iii), (iv)

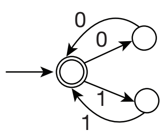
17. Consider below DFA:

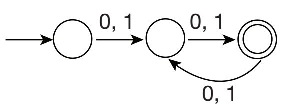


What is the language accepted by given DFA?
(A) All strings which ends with either '0' or '1' only.
(B) All strings which ends with '0' only.
(C) All strings which has the substring '10'.
(D) All strings which has the substring '110'.

18. Which of the following DFA accepts all the even length strings on {0, 1}?

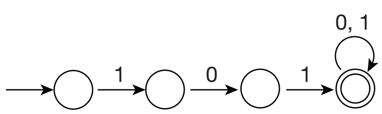
(A) 

(B) 

(C) 

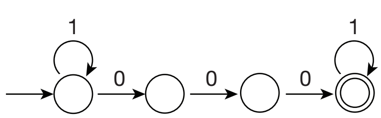
(D) All the above

19. Consider below DFA:



What is the language accepted by above DFA?
(A) All the strings which contain '101' as substring.
(B) All the strings which begin with '101'.
(C) All the strings which ends with either '0' or '1'.
(D) All the strings which do not contain the substring '00'.

20. Which of the following regular expression represents below DFA?



(A) 1*0001* (B) 1* + 000 + 1*
(C) 1* + 0 + 0 + 0 + 1* (D) (10001)*

21. What is the language accepted by the transition table of a DFA given below:

δ	A	B
Q_0	Q_1	Q_0
Q_1	Q_0	Q_1

(A) All the strings which do not have even length.
(B) All the strings which have at least one a.
(C) All the strings with even numbers of a's.
(D) All the strings with odd number of a's.

22. Consider below transition table of a DFA with some blanks:

δ	0	1
q_0	–	q_0
q_1	–	–
q_2	q_2	q_2

What are the missing transitions if the DFA accepts all the strings with '00' as substring?
(A) $q_0 \xrightarrow{0} q_1, q_1 \xrightarrow{0} q_2, q_1 \xrightarrow{0} q_0$
(B) $q_0 \xrightarrow{0} q_1, q_1 \xrightarrow{1} q_2$
(C) $q_0 \xrightarrow{0} q_1, q_1 \xrightarrow{0} q_2, q_0 \xrightarrow{1} q_0$
(D) $q_0 \xrightarrow{0} q_1, q_1 \xrightarrow{0} q_2$

23. Let R and S be two regular expressions then which of the following is not a regular expression?
(A) $R + S$ (B) RS
(C) R^* (D) None of the above

24. What is the regular expression for the language over {0, 1}, which accepts the set of all strings that begin with '110'?
(A) 110 (10)* (B) 110 (01)*
(C) 110 (1 + 0)* (D) 110^+ (1 + 0)*

25. Give a regular expression for the language over {0} which accepts the set of all strings of odd number of 0's?
(A) 0* (B) 0^+
(C) 0(00)* (D) 0 + (00)*

26. Lexical Analyzer uses which one of the following pattern (Lexer) to generate tokens?
(A) Regular expression + Priorities + Longest Matching Token rule.
(B) Regular Expression + priorities + Shortest Matching token rule.
(C) Regular Expression + longest Matching Token rule
(D) Regular Expression + Shortest Matching token rule.

27. Consider the following code segment. If C compiler compiles the code, what will be the response?

```
#include <stdio.h>
main( )
{
/* this is My first program*/
int a, b, c;
/* initializing variables
/* a = 10, b = 20 */ compute c value
   */
c = a + b;
printf("%d", c);
}
```

(A) No error, produces object code.
(B) Run time error but No compile time error.
(C) syntax error but No Lexical error.
(D) None of the above.

28. Find the Number of Tokens in following C code

```
main ( )
{
int a, b;
a = 1; b = 0;
if (!a! = b)
a << = 1;
else
b >> = 2;
}
```

(A) 36 (B) 38
(C) 39 (D) lexical error

29. Regular Expression for identifier is $L(L \cup N)^*$: Here L-Letter and N-number, and some keywords in a language are {int, float, main, double, ...}. Then a string "int" is given to scanner how many strings or sub strings satisfies the patterns of Lexical Analyzer?

(A) 3 (B) 4
(C) 1 (D) None of the above

30. Consider the following statements about function and Token, when it is recognized during scanning.
(i) it produces Token value.
(ii) put identifier in symbol table.
(iii) increment line number
(iv) get next line and input to scan
Which of the following is true?

(A) (i), (ii), (iv) (B) (ii) and (iv)
(C) (ii), (iii) and (iv) (D) (i), (ii), (iii), (iv)

31. Time complexity of a parser that works for any unambiguous grammar where 'n' is the length of the input is:

(A) $O(n)$ (B) $O(n^2)$
(C) $O(n^3)$ (D) $O(n \log n)$

32. While parsing a string $w = abcd$ using Bottom-up parsing, what are the possible strings or sub strings that can be considered?

(A) $\{a, bc, cd, d, bcd\}$ (B) $\{a, ab, abcd, bd\}$
(C) $\{b, c, ac, bd, dc\}$ (D) $\{d, dc, dcb, dcba\}$

33. Consider the following grammar:

$S \to \text{aABe}$

$A \to \frac{Abc}{b}$

$B \to d$

Which of the following is the correct sequence of handles to parse a string $w = abbcde$?

(A) $\{aABc, d, Abc, b\}$ (B) $\{b, d, Abc, aABc\}$
(C) $\{d, b, Abc, aABc\}$ (D) $\{b, Abc, d, aABc\}$

34. Which of the following statement is false?

(A) $\text{LALR}(1) \subseteq \text{LR}(1)$ (B) $\text{LL}(1) \subseteq \text{LALR}(1)$
(C) $\text{LL}(0) \subseteq \text{LR}(0)$ (D) $\text{LL}(K) \subseteq \text{LR}(K)$

35. Consider the following grammar:

$S \to Aa/b$

$A \to Bc/d/aA$

$B \to Sb/c/Bd$

Which one of the following statement is false?

(A) S has indirect Left recursion.
(B) All non terminals have indirect Left recursion.
(C) S, A have immediate Left Recursion.
(D) B has immediate and indirect left Recursion.

Answer Keys

1. C	**2.** A	**3.** B	**4.** D	**5.** D	**6.** D	**7.** A	**8.** C	**9.** D	**10.** C
11. B	**12.** D	**13.** D	**14.** D	**15.** A	**16.** C	**17.** C	**18.** A	**19.** B	**20.** A
21. D	**22.** C	**23.** D	**24.** C	**25.** C	**26.** A	**27.** D	**28.** A	**29.** B	**30.** D
31. C	**32.** A	**33.** D	**34.** B	**35.** B					

Hints and Explanations

1. A DFA will have finite states, inputs, transition function, a start state & a set of final states. Choice (C)

2. NFA's accept exactly the regular languages, same as DFA's. Choice (A)

3. NFA's have the power to be in several states at once (Because of the non determinism). Choice (B)

4. DFA & NFA will differ only in their transition function. Choice (D)

5. The transition function of NFA will take a state, an input symbol and returns a set of states. Choice (D)

6. DFA = NFA = $\in$ – NFA. Choice (D)

7. Choice (A)

8. L^* (kleen closure) of a language L represents the set of those strings that can be formed by taking any number of strings from L, possibly with repetitions and concatenating all of them. Choice (C)

9. Symbol Table mainly maintains record for identifiers, all constants and all type of literals are stored in Literal Table. Choice (D)

10. Left recursive grammars are easy to implement (compare to a grammar after eliminating Left Recursion). Choice (C)

11. Brute Force Technique 'blindly' substitutes first production; no intelligence is required. Choice (B)

12. In LL(1) parsing Algorithm 'a' is look ahead symbol, X is the top of the stack, then, if $a = X \neq \$$ pop of stack and increment input pointer. Choice (D)

13. A grammar is said to be LL(1) if each and every cell contains at most one production. If every non-terminal is deriving only one production there is no chance of occurrence of two productions in one cell.

Choice (D)

14. Top-down parsers uses LMD. Bottom up parser uses Reverse RMD. Reverse Right most derivation is also called canonical reduction sequence. Choice (D)

15. CLR(1) is most powerful parser.
LALR(1) is most widely used parser as it requires Less space compared to CLR(1) Choice (A)

16. 'L' must contain the substring '10'.
(i) is not accepted
(ii) accepted
(iii) accepted
(iv) not accepted

Choice (C)

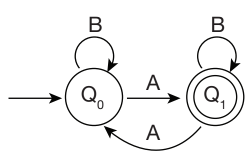
17. Given DFA accepts all the strings which have the substring '10'. The strings accepted are 10, 010, 110, ... Choice (C)

18. The even length strings on {0, 1} are ∈, 00, 11, 01, 10, 0011, 0101, 1100, ... only choice (A) accepts all these strings. Choice (A)

19. Given DFA accepts all the strings which begin with '101' only. It won't accept 0101; it accepts 10100. Choice (B)

20. Given DFA accepts all the strings with exactly three consecutive zeros. Choice (A)

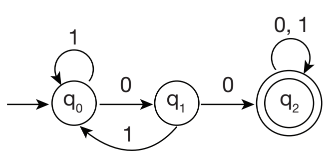
21. The DFA for given transition table is given below:



It can have any number of b's but the number of a's must be odd {a, aaa, $aaaaa$, ...}.

Choice (D)

22. The DFA for given partial transition table is:



If it accepts all the strings which have '00' as substring, $q_0 \to q_1$, $q_1 \to q_2$ must have '0' transition. After one '0' there is a possibility of 1's so put a '1' transition from q_1 to q_0. Choice (C)

23. $R + S$ contains union of $L(R)$ and $L(S)$. RS contains concatenation of $L(R)$ and $L(S)$. R^* is closure of R.

Choice (D)

24. The strings of the language L must start with '110' after that there may be any number of 0's and 1's, so the regular expression is 110 (1 + 0)*.

Choice (C)

25. 0* accepts zero or more number of 0's.
0^+ accepts one or more number of 0's.
0(00)* accepts 0, 000, 00000, ... i.e., odd number of zeros.
0 + (00)* accepts either 0 or 00 or 000, ...

Choice (C)

26. Ex : identifier L(LUT)*
int ab;
'a' is satisfying Regular Expression but it won't be treated as Lexeme. Longest matching string will be treated as Lexeme, 'ab' is Lexeme.

Choice (A)

27. There is only Lexical Error because of Nested comments. Choice (D)

28.
```
main/ (/) /
{/int/a/,/b/;/
a/=/1/;/b/=/0/;/
if/(/!/a/! =/b/)/
a/<</=/1/;/
else/
b/>>/=/2/;/
}/
```
∴ Total 36 Tokens. Choice (A)

29. Lexical Analysis scans the input string character by char acter and checks each and every substring matching with the pattern but using lexer definition it takes longest matching string as Lexeme.
i → L(LUN)*
in → L(LUN)*
int → L(LUN)*
keywords (priority high) Choice (B)

30. Whenever a Token is generated it produces a value. If it is an identifier a record is created in symbol table. A hidden token is created for every read.

Choice (D)

31. If string length is 'n' then any parser for unambiguous grammar takes $O(n^3)$ time. Choice (C)

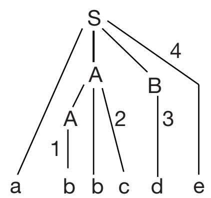
32. In bottom-up Parsing, when scanning a string for identifying handles, it considers all prefixes and suffixes of strings.

for *abcd*,

Prefixes	Suffixes
a	*d*
ab	*cd*
abc	*bcd*
abcd	*abcd*

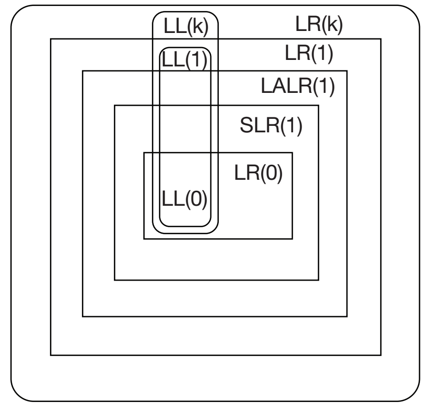
will be considered for handles Choice (A)

33.



$A \to b$
$A \to Abc$
$B \to d$
$S \to aABc$ Choice (D)

34. LL(1) $\subseteq$ LR(1)



Choice (B)

35. $S \Rightarrow Aa \Rightarrow Bca \Rightarrow Sbca$
$A \Rightarrow Bc \Rightarrow Sbc \Rightarrow Aabc$
$B \Rightarrow Bd$ (immediate)
$B \Rightarrow Sb \Rightarrow Aab \Rightarrow Bcab$ (indirect) Choice (B)

Theory of Computation and Compiler Design Test 4

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Which of the following is FALSE for regular expressions L, M and N?
(A) $L + M = M + L$
(B) $(L + M) + N = L + (M + N)$
(C) $(LM)\ N = (LM)\ N$
(D) $LM = ML$

2. Which of the following does not hold for two regular expressions R and S?
(A) $(R^*)^* = R^*$
(B) $(\in + R)^* = R^*$
(C) $(R^*S^*)^* = (R + S)^*$
(D) None of the above

3. Which of the following language is not regular?
(i) $(0 + 1)^*$
(ii) Palindromes over $\{0, 1\}$
(iii) $\{0^n10^n/n \geq 1\}$
(A) (i), (ii), (iii) (B) (i), (iii)
(C) (ii), (iii) (D) (iii) only

4. Identify the regular languages from the following:
(i) Set of strings of balanced parentheses
(ii) $\{0^n/n$ is a perfect square$\}$
(iii) $\{ww/w \in \{0, 1\}^*\}$
(A) (i), (ii) (B) (ii), (iii)
(C) (i), (iii) (D) None of these

5. Which of the following properties hold for regular languages?
(A) Complement (B) Difference
(C) Reversal (D) All the above

6. Which of the following is TRUE?
(A) Pushdown automata has same expressing power as Deterministic finite automata.
(B) NFA is an extension of pushdown automata.
(C) Pushdown automata is an extension of $\in$-NFA.
(D) PDA does not have any storage component.

7. Which type of language is accepted by DPDA?
(A) Regular languages only
(B) Context free languages only
(C) The class of languages between regular languages and context free languages.
(D) Recursively enumerable languages only

8. Which of the following statement is TRUE about Top down parsing and bottom up parsing?
(A) Bottom up parser is more powerful.
(B) Error detection in Top down parser is easy.
(C) Table size in Bottom up parser and Top down parser is approximately equal.
(D) Design complexity of Top down parser and Bottom up parser are same.

9. Limitation of L-Attributed Grammar is:
(A) uses only inherited Attributes.
(B) each inherited attribute inherits either from parent or Left side sibling
(C) each inherit attribute inherits either from parent or right side sibling
(D) each inherit attribute inherits either from parent or Left side sub tree.

10. Which of the following statement is false?
(A) It is always possible to rewrite a SDD to use only synthesized attributes.
(B) Evolution of L-Attributed grammar is BUP (Bottom up parsing)
(C) Evolution of S-Attribute grammar is BUP (Bottom up parsing)
(D) Evolution of L-Attributed grammar is Depth first. Traversal, left to right.

11. A grammar is said to be ‘attribute grammar’ if and only if it has
(A) L-Attribute definition
(B) S-Attribute definition
(C) SDT which does not have any side effects
(D) SDT which must have side effects

12. Which of the following statement is FALSE about L-Attributed definition?
(A) Traverse the parse tree in Breadth first manner from Left to Right.
(B) Evaluate inherited attributes if the node is visited for first time.
(C) Evaluate synthesized attributes if the node is visited for Last time.
(D) None of the above.

13. Which of the following is not an advantage of reverse polish notation?
(A) no rules of precedences
(B) no associativity
(C) no parenthesis
(D) not low level representation

14. A single tape Turing machine M has two states q_0 and q_1, of which q_0 is the starting state. The tape alphabet of M is $\{0, 1, B\}$ and its input alphabet is $\{0, 1\}$, the symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table.

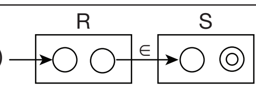
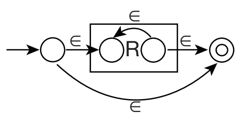
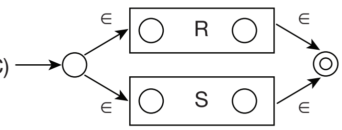
	0	1	B
q_0	q_1, 1, R	q_1, 0, R	q_1, B, L
q_1	q_0, 1, R	q_0, 0, R	Halt

Which of the following statements is TRUE about M?
(A) M accepts all strings that end with 0 only
(B) M accepts all strings that end with 1 only

(C) M accepts all strings of $(0 + 1)^+$ only.
(D) M accepts all strings of $(0 + 1)^*\ 01\ (0 + 1)^* + 1^*\ 0^*$

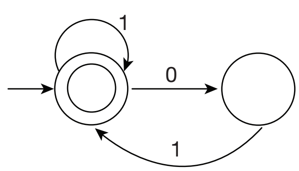
15. Which phase is most important for complex parts of any modern compiler?
(A) Syntax Analysis (B) Semantic Analysis
(C) Target code generator (D) Code optimizer

16. Match the following:

Regular expression	Finite Automata
(i) $R + S$	(A) 
(ii) RS	(B) 
(iii) R^*	(C) 

(A) (i)–(C), (ii)–(B), (iii)–(A)
(B) (i)–(A), (ii)–(B), (iii)–(C)
(C) (i)–(C), (ii)–(A), (iii)–(B)
(D) (i)–(B), (ii)–(A), (iii)–(C)

17. Consider below DFA:



What is the language accepted by this DFA?
(A) $1^* + (01)^*$ (B) $[1^*\ (01)^*]^*$
(C) $(1 + 01)^*$ (D) Both (B) and (C)

18. Consider the grammar:
$S \to AS/\in$
$A \to aa/ab/ba/bb$
What is the language generated by this grammar?
(A) Generates all the strings of the form $\{a^nb^n/n \geq 0\}$
(B) Generates all the strings of even length.
(C) Generates all the strings of equal number of a's and b's.
(D) Generates all the strings possible with $\{a, b\}^*$.

19. Which of the following is not Context-free language?
(A) $\{a^nb^nc^n/n \geq 1\}$ (B) $\{ww^R/w \in (0 + 1)^*\}$
(C) $\{wcw^R/w \in (0 + 1)^*\}$ (D) $\{a^nb^n/n \geq 1\}$

20. Which of the following are Deterministic context free language(s)?
(i) $\{0^n1^{m+n}\ 1^m/m, n \geq 0\}$
(ii) $\{0^n1^m0^n/n$ and m are arbitrary$\}$
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

21. Let L and M be two context free languages then which of the following is not context free?
(A) $L \cup M$ (B) $L\,M$
(C) L^* (D) $L \cap M$

22. Which of the following are undecidable?
(A) Is a given CFG, G ambiguous?
(B) Is the intersection of two CFL's empty?
(C) Are two CFL's the same.
(D) All the above

23. Which languages are accepted by Turing machines only?
(A) Regular languages
(B) Context free languages
(C) Recursively enumerable languages
(D) All the above

24. Which of the following languages are accepted by Turing machines?
(i) set of all strings of balanced parentheses
(ii) $\{0^n1^m0^n1^m/m, n \geq 1\}$
(iii) $\{ww/w \in \{0, 1\}^*\}$
(A) (i), (ii) (B) (ii), (iii)
(C) (i), (iii) (D) (i), (ii), (iii)

25. Which of the following is FALSE?
(A) A problem L is decidable if it is a recursive language.
(B) A problem L is undecidable if it is not a recursive language.
(C) If L is a recursive language then $\overline{L}$ is also recursive.
(D) If L is recursively enumerable language then it is also recursive language.

26. Consider the following grammar:
$S \to Sd/aA/aB$
$A \to ab/d$
$B \to acd/dde$
Parse a string $w = adde$ using Brute force technique, How many back tracks are required?
(A) 6 (B) 4
(C) 0 (D) None of the above

27. Consider the following Grammar:
$S \to Aa/bAc/Bc/bBa$
$A \to d/a/aB$
$B \to d/e$
Parse a string w = "bea" using Brute Force Technique, What is the maximum height of parse tree and how many back tracks are required?
(A) 2, 13 (B) 3, 16
(C) 3, 4 (D) 2, 4

28. In LL(1) parsing, while parsing a string 'w', the Top of the stack contains non terminal 'X' and input string look ahead symbol 'a' and LL(1) parsing Table contains $M[X, a]X \to aBc$

After performing appropriate action, how many symbols are there in stack and what is the Top of the stack?

(A) 4, a (B) 4, c
(C) 3, a (D) 3, c

29. Consider the following grammar:
$A \to X_1|X_2|\varepsilon$
here X_1 and X_2 are the productions of 'A'.
The grammar should be LL(1) grammar if and only if
(A) first(X_1) $\cap$ first (x_2) = ϕ
(B) first(X_1) = first(x_2)
(C) follow(A) $\cap$ first(x_1) $\neq \phi$
(D) first(X_1) $\cap$ first (x_2) $\neq \phi$

30. A non-terminal 'A' is deriving one production
$$A \to X_1\, X_2\, X_3,$$
here X_1, X_2, X_3 are non terminals, which have some productions. Which of the following statement always holds to find first(A)?
(A) first(A) = first(X_1) if ($X_1 \overset{*}{\Rightarrow} \in$)
(B) first(A) = first(X_1)$\cup$ first(X_2) $\cup$ first(X_3)
(C) first(A) = first(X_1)$\cup$ first(X_2) $\cup$ first(X_3) – {$\in$} if ($X_1 \overset{*}{\Rightarrow} \in$)
(D) first(A) = first(X_1) – {$\in$} $\cup$ first($X_2\, X_3$) if ($X_1 \overset{*}{\Rightarrow} \in$)

31. Consider the following Grammar:
S $\to$ AB/aAbAd
B $\to$ a/b/c/$\in$, find follow of the Non-terminal A?
(A) follow(A) = first(B) – {$\in$}
(B) follow(A) = first(B) $\cup$ {d}
(C) follow(A) = first(B) $\cup$ {\$} – {$\in$}
(D) follow(A) = first(B) $\cup$ {d, \$} – {$\in$}

32. While constructing LL(1) parsing Table, place production '$A \to \alpha$' in the following cells of M[X, a], where 'a' is [first (α) contains '$\in$' and terminals]:
(A) only first (α) – {$\in$}
(B) first (α) $\cup$ follow(A)
(C) only in follow(A) – {$\in$}
(D) first (α) $\cup$ follow(A) – {$\in$}

33. In LL(1) parsing Table, M[X, \$] contains production if and only if [here $X \to ABC$]
(A) first (X) contains {$\in$}
(B) first (A) contains {$\in$}
(C) first (AB) contains {$\in$}
(D) data is insufficient

34. Consider the following operator precedence Relational Table. For the string $W = id + id, \times id$, how many handles are possible to accept?

	id	+	*	$
id		•>	•>	•>
+	<•	•>	<•	•>
*	<•	•>	•>	•>
$	<•	<•	<•	

(A) 4 (B) 5
(C) 6 (D) Data insufficient

35. In the above precedence table, what is the associativity of '+' and '*'?
(A) + is right associative and * is left associative
(B) + and * Left associative
(C) + and * Right associative
(D) Data insufficient

Answer Keys

1. D	**2.** D	**3.** C	**4.** D	**5.** D	**6.** C	**7.** C	**8.** A	**9.** B	**10.** B
11. C	**12.** A	**13.** D	**14.** C	**15.** D	**16.** C	**17.** D	**18.** B	**19.** C	**20.** C
21. D	**22.** D	**23.** C	**24.** D	**25.** D	**26.** D	**27.** B	**28.** C	**29.** A	**30.** D
31. D	**32.** D	**33.** A	**34.** B	**35.** B					

Hints and Explanations

1. Commutative law for union, associative law for union and concatenation holds for regular expression. But commutative law for concatenation doesn't hold. Choice (D)

2. Choice (D)

3. Regular languages are accepted by FA. It has only limited storage, so it can't compare the front of a string with its back. Similarly it can't check the equality of 0's. Choice (C)

4. No language is regular (Not recognized by FA). Choice (D)

5. Choice (D)

6. PDA is an $\in$-NFA with the addition of a stack. Choice (C)

7. Choice (C)

8. Error detection in Top down parser is difficult compared to Bottom up parser. Choice (A)

9. S-Attribute grammar uses synthesized attributes. L-Attributed grammar uses both synthesized and inherited attributes. As it scans Left to right inherited attribute, it must inherit from parent or left hand side siblings. Choice (B)

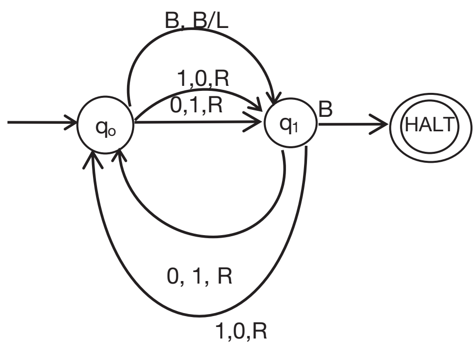
10. L-attributed grammar uses both synthesized and inherited attributes, it scans from Root node to leaf nodes in Depth first search, left to right manner. Choice (B)

11. A grammar is said to be attribute grammar if and only if it does not contain side effects like any action rule print something. Choice (C)

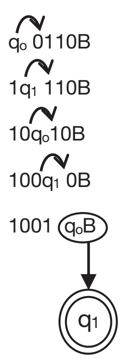
12. L-Attributed grammar Traverses parse tree in Depth first search manner, left to right. Choice (A)

13. Reverse polish notation, Abstract syntax tree, Direct acyclic graph are high level intermediate code forms, three address code is low level intermediate code form. Choice (D)

14.



For example, take string 0110 and check whether it is accepted
by TM or not



Final accepting state

$\therefore$ Any string of length 1 or more of $\{0, 1\}$ is accepted by given TM. Choice (C)

15. In early age of compilers, developers focused on syntax analysis and recently, on optimization, focused mainly on code optimization and risc. Choice (D)

16. Choice (C)

17. Given DFA accepts any number of 1's and (01'*s*) repeatedly. The regular expression is $(1 + 01)^*$ $= [1^* (01)^*]^*$. Choice (D)

18. $S \to AS/\in$

$A \to aa/ab/ba/bb$

It generates $\{a^n\ b^n\}$ but it generates $\{aa, aaaa, \ldots\}$ also. It also won't generate equal number of a's and b's. It won't generate single 'a'. It generates even length strings. Choice (B)

19. The CFL's are accepted by PDA's. The PDA has a stack storage it moves only towards right side of string. It can't check equality of a's b's and c's. But it can check $\{ww^R\}$, $\{wcw^R\}$, $\{a^n\ b^n\}$ patterns. Choice (C)

20. $\{0^n\ 1^{m+n}\ 0^m / m, n \in 0\}$

Push 0's whenever 1's are in input, pop 0's i.e., matching between 0's and 1's. After that push remaining 1's and pop them whenever 0's occur.

There is no non determinism. So it is DCFL.

Similarly we can check the equality of 0's in (ii) by pushing first set of 0's & pop them with last set of 0's. Choice (C)

21. CFL's are not closed under intersection. Choice (D)

22. Choice (D)

23. Recursively enumerable languages are only accepted by TM. Regular, CFL's are accepted by TM but they are also accepted by FA, PDA respectively. Choice (C)

24. A TM has infinite storage buffer, which can move in two directions (forward, backward) and also it has both read and write capabilities.

All the given three languages are accepted by TM's. Choice (D)

25. A language L is recursive if both L and $\overline{L}$ are recursively enumerable. Choice (D)

26. Fall into infinite Loop in Brute Force Technique, Blindly substitute the first productions.

$S \Rightarrow Sd$

$\Rightarrow\ Sdd$

$\Rightarrow\ Sddd$

$\Rightarrow\ Sdddd$

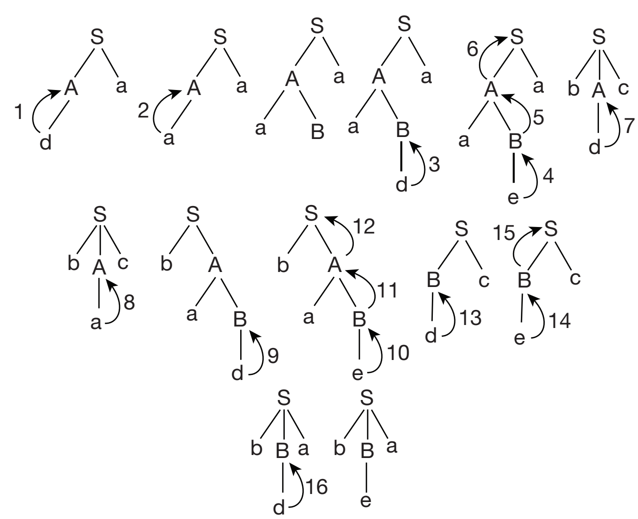
:

:

Infinite Loop

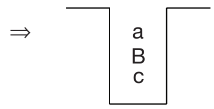
Choice (D)

27.



Choice (B)

28.



$M[X, a]$: $X \to aBc$

Replace X by aBc such that a is Top of the stack.

Choice (C)

29. In LL(1) parsing table construction, for a production '$A \to \alpha$'

Place the production in first (α) column and in 'A' row. $A \to X_1|X_{21}\ A \to X_1$, in first ($x_1$), and $A \to x_2$ in first (x_2). So if we have common symbol then it is a conflict in LL(1) parser. Choice (A)

30. first(A) = first ($X_1\ X_2\ X_3$) = first(X_1), if first (X_1) contains '$\in$' then we have to take first (X_2). If first (x_2) also contains '$\in$' then go for first (x_3) also. If first (x_3) contains '$\in$' then it will be added to 'A's first.

Choice (D)

31. follow(A) = $\{a, b, c, d, \$\}$

follow(A) = first (B) $\cup$ first(b) $\cup$ first (d)

first(B) contains '$\in$' then 'A' is followed by '$\in$'. If a non terminal followed by '$\in$' then take follow of Left hand side non terminal i.e., follow(A) = follow(S)

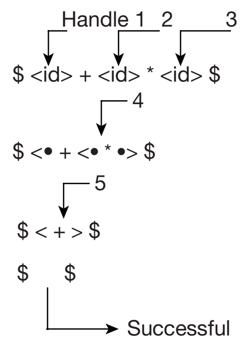
Choice (D)

32. In LL(1) parsing, place '$A \to \alpha$' in first (α) columns. If first (α) contains '$\in$' also place '$A \to \alpha$' in follow (A).

Choice (D)

33. \$ is in follow set only. A production '$A \to \alpha$' placed in every symbol follow set if and only if first(α) contains '$\in$'. So '$X \to ABC$' placed in '\$' column if and only if first (ABC) contains '$\in$'. Choice (A)

34.



Choice (B)

35. In operator precedence parsing table θ_1 and θ_2 are two Operators and have equal precedence then

$\theta_1 < \theta_2$ if Right associative

$\theta_1 > \theta_2$ if Left associative

So

$+ > +$, $* > *$

Left associativity

Choice (B)

Chapter 1

OSI Layers

LEARNING OBJECTIVES

- *Computer network*
- *LAN*
- *LAN topologies*
- *CSMA/CD*
- *WAN*
- *The OSI reference model*
- *LAN technologies*
- *Physical layer*
- *Data link layer*
- *Types of error*
- *MAC sub layer*
- *FDM/TDM*

Computer Network

Computer network is the collection of two or more computers that are interconnected with each other to perform data communication using the data communication protocol through communications media (wired or wireless) .So these computers can share information, data, programs, and use of hardware together. Data communications that can be done include text data, images, video and sound.

Or

A computer network, often simply referred, as a network is a collection of computers and devices interconnected by communication channels that facilitate communication and allow sharing of resources and information among interconnected devices. There are different networks:

1. LAN
2. MAN
3. WAN

LAN

A Local Area Network (LAN) is a network that is confined to a relatively small area. It is generally limited to a geographic area such as a lab, school, or building. LAN Computers rarely spans more than a mile apart.

In a typical LAN configuration, one computer is designated as the file server. It stores all the software that controls the network, as well as the software that can be shared by the computers attached to the network. Computers connected to the file server are called workstations. The workstations can be less powerful than the file server, and they may have additional software on their hard drives. On many LANs, cables are used to connect the network interface cards in each computer. However, one LAN can be connected to other LANs over any distance via telephone lines and radio waves.

Most LANs connect workstations and personal computers. Each node (individual computer) in a LAN has its own CPU which executes programs and it is also able to access data and devices anywhere on the LAN. This means that many users can share expensive devices, such as laser printers, as well as data. Users can also use LAN to communicate with each other, by sending e-mail or engaging in chat sessions.

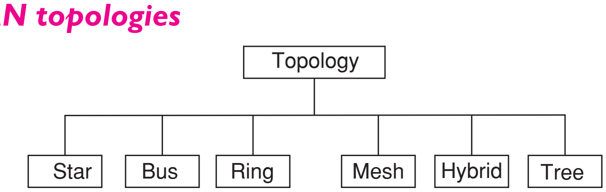
Three characteristic features of LAN

1. The size of a LAN network.
2. The topology of the local area network.
3. The technology used for transmission.

In simple LAN configuration, a single cable runs through the entire set up and the peripherals and computers are attached to the cable. Traditional LAN speeds are 10 Mbps to 100 Mbps. Modern LAN cables are capable of much higher data transfer per second.

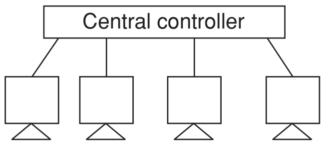
In case two or more systems need to use the LAN at the same time, then an arbitration mechanism is deployed to resolve the conflict. A first come first serve policy or a prioritized approach may be chosen.

LAN topologies



Star Topology Each device has a dedicated point-to-point link to a central controller called a hub. Most used LAN topology.

If one device wants to send data to another, it sends the data to the controller, which then relays the data to the other connected device as shown in the figure.



Each device needs only one link and one I/O port to connect it to any number of others.

Advantages

1. Robust, if one link fails, only that link is affected. All other links remain active.
2. As long as hub is working, it can monitor link problems and by pass defective links.

Disadvantages

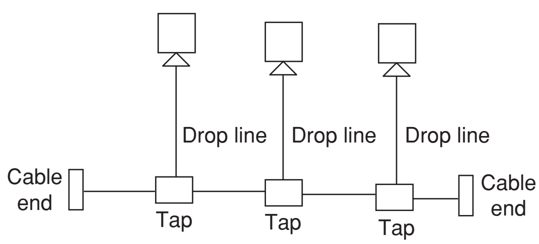
1. If hub goes down, the whole system dead.
2. More cabling is required in a star than ring or bus.

Bus Topology A bus topology is multipoint. One long cable acts as a backbone to link all the devices in a network.

Carrier Sense Multiple Access/Collision Detection (CSMA/CD) is the accessing technique used.

The traffic can go in either direction, i.e., it is bidirectional.

Nodes are connected to the bus cable by drop lines and taps as shown in the figure.



Advantages

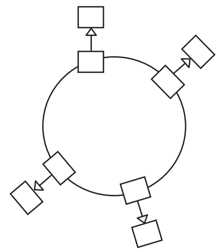
1. Ease of installation.
2. Require less cabling than mesh or star topologies.

Disadvantages

1. Difficult to add new devices.
2. A fault in the bus cable stops all transmission. The damaged area reflects signals back in the direction of origin, creating noise in both directions.

Ring Topology Each device has a dedicated point-to-point connection with only the two devices on either side of it.

Each device in the ring incorporates a repeater; when a device receives a signal intended for another device, its repeater regenerates the bits and passes them along.



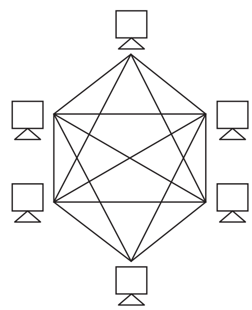
Advantages

1. Easy to install and reconfigure.
2. Fault isolation is simplified as it issues alarm which alerts the network operator to the problem and its location.

Disadvantages

1. A break in the ring can disable the entire network.
2. It is not relevant for higher-speed LANs.

Mesh topology Every station is interconnected to every other station as shown in the figure.



$n(n-1)/2$ (duplex mode) links are required for communication in both directions. Each device on the network must have $(n-1)$ I/O ports to be connected to the other $(n-1)$ stations.

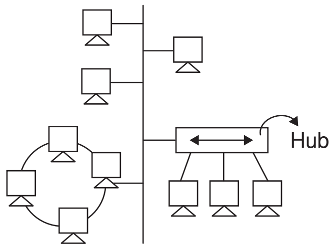
Advantages

1. The use of dedicated links guarantees that each connection can carry its own data load, thus eliminating traffic problems.
2. This topology is robust. If one link becomes unusable, it does not incapacitate the entire system.
3. There is advantage of security, only the intended recipient sees the message on the dedicated line.
4. Fault identification and fault isolation is easy because of point-to-point links.

Disadvantages

1. As the hardware(cables) required for connection is more, it is expensive.
2. Installation and reconnection are difficult.
3. The sheer bulk of the wiring can be greater than the available space.

Hybrid Topology More than one topology in a network.



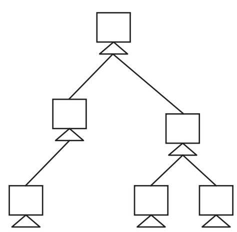
Advantages

1. Fault detection is easier.
2. We can add new stations without affecting the original architecture.

Disadvantages

1. As different topologies are combined so complexity of design increases. Very less practical implementation.
2. The hub which is used to connect different topologies is very costly. Moreover the cost of whole infrastructure is very high.

Tree Topology This topology uses the combination of star and bus topology.



Advantages

1. Expansion is easier; one can add new stations easily.
2. Errors can be easily detected.
3. Robust, if one link fails the remaining system is in communication.

Disadvantages

1. With the increase in the number of nodes, complexity and maintenance become difficult.

Examples: The most common type of local area network is an Ethernet LAN. The smallest home LAN can have exactly two computers; a large LAN can accommodate thousands of computers. Many LANs are divided into logical groups called subnets. An Internet Protocol (IP) 'Class A' LAN can in theory accommodate more than 16 million devices organized into subnets.

MAN

A metropolitan area network is a computer network that usually spans a city or a large campus. A MAN usually interconnects a number of local area networks (LANs) using a high-capacity backbone technology, such as fiber-optical links, and provides up-link services to wide area networks (or WAN) and the Internet.

WAN

Wide Area Networks (WANs) connect larger geographic areas, such as Florida, the United States, or the world. Dedicated transoceanic cabling or satellite uplinks may be used to connect this type of network.

A WAN is complicated; it uses multiplexers to connect local and metropolitan networks to global communications networks like the Internet. To users, however, a WAN will not appear to be much different than a LAN. As the term implies, a WAN spans a large physical distance. The Internet is the largest WAN, spanning the Earth.

A WAN is a geographically-dispersed collection of LANs. A network device called a router connects LAN to a WAN. In IP networking, the router maintains both a LAN address and a WAN address.

A WAN differs from a LAN in several important ways. Most WANs (like the Internet) are not owned by any one organization but rather exist under collective or distributed ownership and management. WANs tend to use technology like ATM, Frame Relay and X.25 for connectivity over the longer distances.

Residences typically employ one LAN and connect to the Internet WAN via an Internet Service Provider (ISP) using a broadband modem. The ISP provides a WAN IP address to the modem, and all of the computers on the home network use LAN (so-called private) IP addresses. All computers on the home LAN can communicate directly with each other but must go through a central gateway, typically a broadband router, to reach the ISP.

The OSI Reference Model

The concept of how a modern day network operates can be understood by dissecting it into seven layers. This seven layer model is known as the OSI Reference Model and defines how the vast majority of the digital networks on earth function. OSI is the acronym for Open Systems Interconnection. The important concept to realize about the OSI Reference Model is that it does not define a network standard, but rather provides guidelines for the creation of network standards.

Physical Layer

The first layer of a network is the Physical Layer. The Physical Layer is literally what its name implies: the physical infrastructure of a network.

This includes the cabling or other transmission medium and the network interface hardware placed inside computers

and other devices which enable them to connect to the transmission medium.

The purpose of the Physical Layer is to take binary information from higher layers, translate it into a transmission signal or frequency, transmit the information across the transmission medium, receive this information at the destination and finally translate it back into binary before passing it up to the higher layers.

Transmission signals or frequencies vary between network standards and can be as simple as pulses of electricity over copper wiring or as complex as flickers of light on optical lines or amplified radio frequency transmissions.

The information that enters and exits the Physical Layer must be bits; either 0s or 1s in binary. The higher layers are responsible for providing the Physical Layer with binary information. Since almost all information inside a computer is already digital, this is not difficult to achieve.

The Physical Layer does not examine the binary information nor does it validate it or make changes to it. The Physical Layer is simply intended to transport the binary information between higher layers located at points A and B.

Data Link Layer

The second layer in the OSI Model is the Data Link Layer, the only layer in the OSI model that specifically addresses both hardware and software.

The Data Link Layer receives information on its software side from higher layers, places this information inside ‘frames’, and finally gives this frame to the Physical Layer, Layer 1, for transmission as pure binary.

A frame essentially takes the information passed down from a higher layer and surrounds it with Physical Address information. This information is important for the Data Link Layer on the receiving end of the transmission.

When the frame, in binary form, arrives at the destination node, it is passed from the transmission medium to the Data Link Layer (Layer 2) by the Physical Layer (Layer 1).

The Data Link Layer on the receiving node checks the frame surrounding the information received to see if it’s Physical Address matches that of its own. If the Physical Address does not match, the frame and its encapsulated data is discarded. If the Physical Address is a match, then the information is removed from the frame and passed up to the next highest layer in the OSI Model.

The Physical Addressing system allows multiple nodes to be on the same network medium, but retain the ability to address only a specific node with a transmission.

The Physical Address used in the Data Link Layer’s Physical Addressing system is known as a MAC address and is embedded physically into the node’s Network Interface Card during manufacturing.

Every NIC’s MAC address is unique in order to prevent addressing conflicts. It is this relationship that causes the Data Link Layer to be known as the only layer that addresses both hardware and software.

In this layer the information on the network makes the move from the physical infrastructure of the network into the software realm. The remainders of the OSI reference model’s layers are entirely software.

Network Layer

OSI Layer 3 is known as the Network Layer. The purpose of the Network Layer is to direct network traffic to a destination node who’s Physical Address is not known. This is achieved through a system known as Logical Addressing.

Logical Addresses are software addresses assigned to a node at Layer 3 of the OSI Model. Since these addresses are able to be defined by software rather than being random and permanent like Physical Addresses, Logical Addresses are able to be hierarchical. This allows extremely large networks to be possible.

A smart device working at Layer 3 that handles network signals from each node directly rather than nodes just blindly repeating packets at Layer 1 until they happen to reach their destination. Such a device is known as a network router.

A network router sits in the center of a network with all nodes having a direct link to it rather than being linked to each other. This strategic position allows the router to intercept and direct all traffic on the network.

A routed network can be illustrated by a star formation, as shown in Diagram 1. On a routed network, Layer 3 packets are no longer broadcasted to all nodes, but rather received by the router and passed on only to the appropriate node. This is a valuable concept because it allows for the collision free-transport of packets across a network.

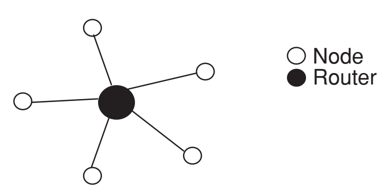


Figure 1

As being linked directly to all nodes in a local network, a router can be linked directly to other routers. This allows groups of nodes separated by distance to communicate with each other in a practical way.

It would not be practical to have nodes separated by a great distance all connected to a single router. The amount of cabling required would be immense and depending on the number of nodes involved, the router may not posses the required number of physical connections.

Routers can be chained in a line, or as shown in Diagram 2, can be connected by a central router. This concept is virtually infinitely scalable and is very efficient.

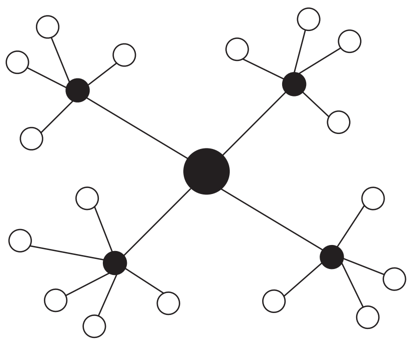


Figure 2

When a node starts a transmission, the OSI Layer 3 protocol takes the information passed down from higher layers and encapsulates it with the logical address of the destination node in a unit called a packet.

This packet, then passes through the remaining lower layer protocols, is transmitted over the network medium from the node to the router. This router reads the logical address that the packet contains and compares it to a list of physical addresses of nodes that are directly connected to it.

If the packet's destination address matches an entry in this list, the packet is transmitted directly on the line that leads straight to the destination node.

If the router does not know of a direct connection to the destination node, the packet is transmitted on a line leading directly to another router. This router then treats the packet much like the first router did upon receipt.

The packet's logical address is checked for matches against the list of logical addresses belonging to nodes directly connected to the router.

If the packet reaches a router with connections only to other routers, as shown in Diagram 2, the router uses the logical address's orderly numbering scheme to try and determine the closest router to the destination node and then transmits the packet to that router.

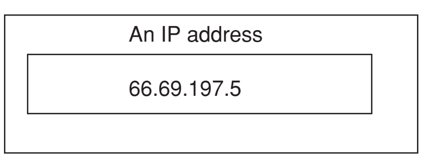


Figure 3

In IP, logical addresses look like four sets. Diagram 3 shows an example of an IP address. IP addresses are orderly on four levels, from left to right. The first section of the IP address refers to a top level router, or a router that is at the highest level of this particular branch of the network. In Diagram 3, the first number is 66. Therefore all IP addresses between 66.0.0.1 and 66.255.255.255 are managed by this router. Only one router is required in a routed network, but more may exist. A router may have a maximum of 255 nodes, which may be either ordinary nodes or other routers. This effectively means that each branch of a network, a group of nodes that have the first set of numbers in their IP address in common, could theoretically have over sixteen million end nodes.

Transport Layer

OSI Layer 4 is known as the Transport Layer, all information transferred is assumed to be at the correct destination node and is being passed up to Layer 4.

The Transport Layer is responsible for the reliability of the link between two end users and for dividing the data that is being transmitted by assigning port numbers to its Layer 4 packages, known as segments.

Ports can be thought of as virtual destination mailboxes or outlets. When information reaches a Layer 4 protocol, the segment is examined to determine the destination port of the data it contains. Once the port is determined, just as all of the past layers have done, the wrapper is discarded and the payload data passed up to the next layer's protocol.

Higher layer protocols that provide services such as email, web browsing, text chat, file transfer and more, each operate on their own unique Layer 4 port, allowing all of these protocols to be operated at once without interference.

On the reliability front, Transport Layer protocols are be capable of running a checksum on the payload data, which they carry. This allows the protocol to determine the integrity of incoming payload data. If this data has been corrupted, the Layer 4 protocol will request the segment to be retransmitted.

Session Layer

OSI Layer 5, known as the Session Layer, still serves a purpose in the OSI Reference Model. The Session Layer draws the outline for protocols that manage the combination and synchronization of data from two separate higher layers.

Layer 5 protocols are responsible for ensuring that the data is Synchronized and consistent before transmitted. A good example is the streaming of live multimedia audio and video, where perfect synchronization between video and audio is desired.

Presentation and Application Layers

The sixth and seventh layers in the OSI Reference Model are the Presentation Layer and the Application Layer. The primary purpose of these layers is to facilitate the movement of formatted information between applications interacting with end users on nodes.

Commonly used top layer protocols are HTTP (for the secure transfer of web page related files), File Transfer Protocol (FTP), Simple Mail Transfer Protocol (SMTP, used for sending email messages), and SSH (Secure Shell), used to secure remote shell access for a computer operating system.

OSI reference model concept

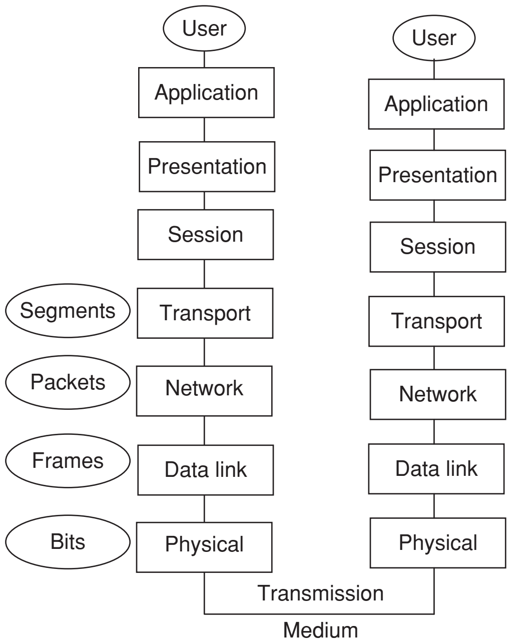


Figure 4

The OSI Reference Model exists not to make hard rules or to shape the industry, but to provide a logical, well-researched, and tested model after which the world's best communication protocol stacks are modeled. The TCP/IP stack is very well-known for being the driving force behind most of the internet, and represents the third (IP) and fourth (TCP) layers of the OSI Model. Every layer in the OSI Model is a reference for a protocol which must facilitate communication between both higher and lower layers. The 'U-shaped' example shown in Diagram 4 provides a visual concept of how two users may be linked on a given network in reference to the OSI Model. Data starts and ends with the user. From the Application Layer of the first user, it must travel down through layers 7 to 1, across the transmission medium, then back up to layers 1 to 7 to be presented at the Application Layer to the user on the end of the transmission. Diagram 4, shows an example of a path between two nodes. Protocols defined by this reference are dependent on the next lowest layer protocol. So, for example, one could not run an Application Layer protocol on a node without the presence of Layer 1 through 6, protocols also being utilized on the node.

LAN TECHNOLOGIES

IEEE standard for networking

IEEE standard project 802 is designed for the enter – connectivity between LAN's

IEEE 802 maps to physical and data link layer

Example: Ethernet, Token ring etc, the IEEE standards for the different groups are

802.1	–	Higher layer LAN Protocol	Active working grourp
802.3	–	Ethernet	
802.11	–	Wireless LAN	
802.15	–	WPAN	
802.16	–	Broad band wireless Access	
802.17	–	Resilient packet Ring	
802.18	–	Radio Regulatory TAG	
802.19	–	Co existence TAG	
802.20	–	Mobile Broad band wireless access	
802.21	–	Media independent Hand off	
802.2	–	Logical link control working group	In active or dis-banded working groups
802.4	–	Token Bus	
802.5	–	Token Ring	
802.7	–	Broad band area Network	
802.8	–	Fiber optic TAG	
802.9	–	Integrated service LAN	
802.10	–	Security working group	
802.12	–	Demand priority working group	
802.14	–	Cable modern working group	

Ethernet

We have

10 Mbps – Ethernet
100 Mbps – Fast ethernet
1 Gbps – Gigabit Ethernet
10 GE – 10 Gigabit Ethernet

Best suited for LAN because it is capable of handling high speed bandwidth.

- Ethernet medium:
  Thick wire – 10B5
  Thin wire – 10B2
  Twisted pair – 10BT, 100BT, 1000BT
  Fibre – 10BF, 100BF, 1000BF
  CAT 4 – 10 Mbps
  CAT 5 – 10/100 Mbps
  CAT 6 – 10/100/1000 Mbps
- Fundamental is CSMA/CD, Standard is 802.3.
- It defines two categories:
  1. Base band. 2. Broad band
- Baseband uses digital manchester encoding techniques.
- IP communication in ethernet is of 3 types:
  (i) Unicast
  (ii) multi cast
  (iii) broadcast
- When user sends data he puts destination and source address.
  (i) In unicast, only intended users responds, however all can get the signal (individual MAC).
  (ii) In multicast, group of users will get the data (group MAC).
  (iii) In broadcast, all users on Ethernet can see the data (all MAC).
- Every computer accepts 3 types of packets, to his own, to the group it belongs, to all.

CSMA/CD

CSMA (Carrier Sense Multiple Access)

CSMA protocols performance is better than ALOHA—Monitor the channel before and/or during data transmission.

1-Persistent Check whether the channel is free before transmitting the data. If busy, wait until it becomes free and then immediately start Re-transmitting.

Non-Persistent When the channel is busy, wait for a random period of time before trying again

If the waiting time is too long, the channel utilization decreases.

P-Persistent Used in slotted systems, If the channel is idle during the current slot, transmit with probability P, and defer until next slot with probability $(1 - P)$

Two or more computers can get connected on same physical medium. All computers can communicate whenever they feel like. Any computer want to communicate, it senses the medium, if medium is free and not used by anyone it captures the medium and puts its data on to the channel.

All computers listens to the sent data but only intended computer/system will respond. At this instance, sending computer is owner of the medium; no other system can be owner or can send the data. When two or more computers try to send data at same time by sensing the medium, collision occurs, which will be sensed by all the computers, then they keep integral wait unit of time for next transmission of data. Once the sending machine gets the corrupted collision message it retransmits using integral time.

MAC sublayer

The medium Access control (MAC) sub layer is the bottom half of the Data link layer. The upper half is commonly called logical link control (LLC) sublayer.

Frame format

Preamble	SFD	Destination address	Source address	Length or type	Data and padding	CRC
7 bytes	1 byte	6 bytes	6 bytes	2 bytes	46 to 1500 bytes	4 bytes

Preamble The first field of the 802.3 frame contains 7 bytes (56 bits) of alternating 0's and 1's that alerts the receiving system to the coming frame and enables it to synchronize its input timing.

Start frame delimiter (SFD) The second field (1 byte: 10101011) signals the beginning of the frame. The SFD warns the station or stations that, this is the last chance for synchronization. The last 2-bits is 11 and alerts the receiver that the next field is the destination address.

Destination address (DA) The DA field is 6 bytes and contains the physical address of the destination station to receive the packet.

Source address (SA) The SA field is also 6 bytes and contains the physical address of the sender of the packet.

Length field The original ethernet used this field as the type field to define the upper-layer protocol using the MAC frame. The IEEE standard uses it as the length field to define the number of bytes in the data field.

Data This field contains data encapsulated from the upper layer protocols. It is of minimum 46 bytes and a maximum of 1500 bytes.

Ethernet follows **binary exponential back off** algorithm to give waiting time for stations, which are involved in collisions. After collisions, waiting time for the stations will be K * 51.2 µ sec, where K is randomly picked up from 0 to $2^n - 1$, 'n' is the collision number. But after 10 collisions, the randomization internal is frozen at a maximum of 1023 slots.

If each station transmits during a contention slot with probability p, the probability A that some station acquires the channel in that slot is

$A = Kp\,(1 - p)^{k-1}$

A is maximized when $p = 1/k$, with $A \to 1/e$ as $K \to \infty$

The probability that the contention internal has exactly j slots in it is $A(1 - A)^{j-1}$, hence mean number of slots per contention is given by

$$\sum_{j=0}^{\infty} jA(1-A)^{j-1} = \frac{1}{A}$$

CRC The last field contains error detection information.

Frame length

Ethernet has imposed restrictions on both the minimum and maximum lengths of a frame, as shown in figure below:.

Minimum payload length: 46 bytes

Maximum payload length: 1500 bytes

Destination address	Source address	Length PDU	Data and padding	CRC
6 bytes	6 bytes	2 bytes		4 bytes

Minimum frame length: 512 bits or 64 bytes

Maximum frame length: 12,144 bits or 1518 bytes

Minimum length of frame is 512 bytes or 64 bytes. If we count 18 bytes of header and trailer, then minimum length of data from the upper layer is 64 – 18 = 46 bytes.

If the upper layer packet is less than 46, padding is added to make up the difference and used to find out collision. Maximum length of the frame is 1518 bytes. If we substract 18 bytes of header and trailer, the maximum length of the payload is 1500 bytes.

- The maximum length restriction has two reasons.
- First, memory was very expensive when ethernet was designed, a maximum length restriction helped to reduce the size of the buffer.
- Second, the maximum length restriction prevents one station from monopolizing the shared medium, blocking other stations that have data to send.

Since each slot has a duration $2T$, the mean contention internal, w, is $2T/A$. Assuming optional p, the mean number of contention slots is never more than e, so w is atmost $2Te$ = 5.4 T.

$$\text{Channel efficiency } = \frac{p}{p+2T/A}$$

$$= \frac{1}{1+2B\dfrac{Le}{cF}}$$

Where F = Frame length
B = Network bandwidth
L = Cable length
C = Speed of signal propagation
E = Contention slots per frame

802.5 TOKEN Ring

Here ring topology is used and devices are physically arranged to form a ring. A token is passed among stations. If a station wants to send data, it must wait and capture the token. Only the token holders are permitted to transmit frame. Token ring allows each station to send one frame per turn.

Frame formats

SD	AC	FC	Destination address	Source address	Data	CRC	ED	FS
1 byte	1 byte	1 byte	2-6 bytes	2-6 bytes	Up to 4500 bytes	4 bytes	1 byte	1 byte

SD	AC	ED

Token frame

SD	ED

Abort frame

802.5 Token ring uses differential Manchester digital signal encoding. It supports data rates upto 16 mbps. Tokens ring protocol specifies three types of frames: Data, token, and abort.

The token and abort frames are both truncated data frames.

Data frame

Start delimiter (SD) It is one byte long and is used to alert the receiving station for the arrival of a frame as well as to synchronize it's timing.

Access control (AC) It is one byte long and includes subfields. It has the format PPPTMRRR. First 3-bits are priority field. T denotes whether this is a data frame, token or an abort frame. Token bit is followed by monitor bit. The last 3 bits are the reservation field that can be set by stations wishing to resume access to ring.

Frame control This field is one byte long and contains two fields. The first is a one bit field used to indicate the type of information (whether it is a control information or data). The second uses the remaining seven bits of the byte and contains information used by the token ring logic.

Destination address (DA) The six byte DA field contains the physical address of the frame's next destination.

Source address (SA) The six byte SA field contains the physical address of the sending station.

Data contains LLC data unit Data contains 0 or more bytes, maximum size of the data depends upon taken holding time.

CRC The CRC field is 4 byte long and contains a CRC – 32 bit error detection sequence.

End delimiter (ED) ED is a second flag field of one byte and indicates the end of the sender's data and control information.

Frame status It is one byte long

A/C			A/C		

A: Addressed recognized bit
C: Copies bit

It can be set by the receiver to indicate that the frame has been read/copied etc.

When a frame arrives at the station with the destination address, the station turns A bits to 1. If station copies the frame to the station it also turns on the C bit. A station might fail to copy a frame due to lack of frame buffer or other reason.

When the sending station receives the frame, it examines the A and C bits.

Three combinations are possible:

1. $A = 0$, $C = 0$: destination not ready /present.
2. $A = 1$, $C = 0$: destination present byte frame not accepted.
3. $A = 1$, $C = 1$: destination present and frame copied.

Token frame

It includes only 3 fields: SD, AC and ED

1. The SD indicates, the frame is coming
2. The AC indicates that the frame is a token and includes priority and reservation fields. $T = 0$ for token in AC.
3. The ED indicates the end of the frame.

Abort frame

An abort frame contains no information at all just starting and ending delimiters. It can be generated by the sender to stop its own transmission. Each station has a priority code, as a frame passes by, a station waiting to transmit may reserve the next open token by entering its priority code in the Access control field (AC) of the token or data frame. A station with a higher priority may remove a lower priority reservation and replace it with its own. Among stations of equal priority, the process is first come, first served. Through this mechanism, the station holding the reservation gets the opportunity to transmit as soon as the token is free, whether or not it comes next physically on the ring.

Monitor station Several problems may appear to disrupt the operation of a token ring network. If the token is destroyed by noise there will be no token on the ring and no station can send data. To solve such a problem, one station on the ring is designated as a monitor. The monitor sets a time, each time the token passes. If the token does not appear in the allotted period of time, it is assumed to be lost and the monitor generates a new token and introduces it to the ring. The monitor detects the orphan frames, by setting the monitor bit in the access control byte.

As the frame passes, the monitor checks the status field. If the monitor bit is set, something is wrong since the frame has passed the monitor twice, so monitor discards it. The monitor then destroys the frame and puts a token on the ring. If monitor fails, the protocol ensures that another station is quickly selected as monitor. Every station has the capability of becoming the monitor. While the monitor is functioning properly, it alone is responsible for seeing that the ring operates correctly.

When station notices that either of its neighbors appears to be dead it transmits BEACON frame giving the address of the dead station.

Physical Layer

Physical layer is concerned with transmitting raw bits over a communication channel. The design issues have to do with making sure that when one side sends a 1-bit, it is received by the other side as 1-bit and not as 0 bit. In physical layer we deal with the communication medium used for transmission.

Types of Medium

Medium can be classified into two categories:

1. Guided Media: Guided media means that signals are guided by the presence of physical media i.e., signals are under control and remains in the physical wire. For example, copper wire.
2. Unguided Media: Unguided media means that there is no physical path for the signal to propagate. Unguided media has essentially electromagnetic waves. There is no control on flow of signal. For example, radio waves.

Transmission Media

In Guided transmission media, generally two kinds of materials are used.

1. Copper
   - Coaxial cable
   - Twisted pair
2. Optical Fiber

Coaxial cable

Coaxial cable consists of an inner conductor and an outer conductor which are separated by an insulator. The inner conductor is usually copper. The outer conductor is covered by a plastic jacket. It is named coaxial because the two conductors are coaxial. Typical diameter of coaxial cable lies between 0.4 inches to 1 inch.

Twisted pair

A twisted pair consists of two insulated copper wires, typically 1mm thick. The wires are twisted together in a helical form, the purpose of twisting is to reduce cross talk interference between several pairs. Twisted pair is much cheaper than coaxial cable but it is susceptible to noise and electromagnetic interference and attenuation is large.

Optical fiber

In optical Fiber light is used to send data. In general terms presence of light is taken as bit-1 and its absence as bit 0. Optical fiber consists of either glass or plastic core which is surrounded by cladding of the same material but of different refractive index. This cladding is surrounded by a plastic jacket which prevents optical fiber from electromagnetic interference and harshly environments. It uses the principle of total internal reflection to transfer data over optical fibers. Optical fiber is much better in bandwidth as compared to copper wire, since there is hardly any attenuation or electromagnetic interference in optical wires. Hence there is

less requirement to improve quality of signal, in long distance transmission. Disadvantage of optical fiber is that end points are fairly expensive.

Communication Links

In a network nodes are connected through links. The communication through links can be classified as

Simplex Communication can take place only in one direction.
Example: TV broadcasting.

Half duplex Communication can take place in one direction at a time. Suppose node A and B are connected then half duplex communication means that at a time data can flow from A to B or from B to A but not simultaneously.
Example: Two persons talking to each other such that when one speaks the other listens and vice versa, walkie-talkies, citizens band radios.

Full duplex Communication can take place simultaneously in both directions.
Example: telephone network.

Links can be further classified as:

Point-to-Point In this communication only two nodes are connected to each other. When a node sends a packet then it can be received only by the node on the other side and none else.

Multi-Point It is a kind of sharing communication in which signal can be received by all nodes. This is also called broadcast.

Digital Data to Digital Signals

A digital signal is sequence of discrete, discontinuous voltage pulses. Each pulse is a signal element. Encoding scheme is an important factor in knowing that how successfully the receiver interprets the incoming signal.

Encoding techniques

Following are several ways to map data bits to signal elements:

Non-return-to-zero (NRZ): NRZ codes share the property that voltage level is constant during a bit interval. High level voltage = bit 1 and low level voltage = bit 0. A problem arises when there is a long sequence of 0's and 1's and the voltage level is maintained at the same value for a long time.

This creates a problem on the receiving end because now, the clock synchronization is lost due to lack of any transitions and hence, it is difficult to determine the exact number of 0's and 1's in this sequence.

The two variations are as follows:

1. *NRZ–Level*: In NRZ–L encoding, the polarity of the signal changes only when the incoming signal changes from a '1' to a '0' or from a '0' to a '1'. NRZ – L method, looks just like the RZ method, except for the first input one data bit. This is because NRZ does not consider the first data bit to be a polarity change, where NRZ–L does.
2. *NRZ–Inverted*: Transition at the beginning of bit interval = bit 1 and no transition at the beginning of bit interval = bit 0 or vice versa. This technique is known as differential encoding.

Digital Data Communication Techniques

For two devices linked by a transmission medium to exchange data, a high degree of co-operation is required. Typically data is transmitted one bit at a time. The timing (rate, duration, spacing) of these bits be same for transmitter and receiver. There are two options for transmission of bits.

Parallel All bits of a byte are transferred simultaneously on separate parallel wires. Synchronization between multiple bits is required which becomes difficult over large distance. Parallel communication gives large bandwidth but expensive, possible only for devices which are close to each other.

Serial Bits transferred serially one after other. Serial communication gives less bandwidth but cheaper, suitable for transmission over long distances.

Manchester encoding

Manchester encoding is used in Ethernet (IEEE 802.3) it is a line code in which bit encoding has at least one transition and consumes the same time.

It ensures frequent line voltage transitions, which are directly proportional to clock rate

It is not dependent on data, so it will not carry any information.

Transmission Techniques

Asynchronous

Small blocks of bits (generally bytes) are sent at a time without any time relation between consecutive bytes. When no transmission occurs a default state is maintained corresponding to bit 1, due to arbitrary delay between consecutive bytes, the time occurrences of the clock pulses at the receiving end need to be synchronized for each byte. This is achieved by providing two extra bits, start and stop.

Start Bit It is prefixed to each byte and equals 0. Thus it ensures a transition from 1 to 0 at onset of transmission of byte. The leading edge of start bit is used as a reference for generating clock pulses at required sampling instants. Thus each onset of a byte results in resynchronization of receiver clock.

Stop Bit To ensure that transition from 1 to 0 is always present at beginning of a byte it is necessary that default state be 1, but there may be two bytes one immediately following the other and if last bit of first byte is 0, transition from 1 to 0 will not occur. Therefore a stop bit is suffixed to each byte equaling 1. It's duration is usually 1, 1.5, 2 bits. Asynchronous transmission is simple and cheap but requires an overhead of 3 bits i.e., for 7 bit code 2(start, stop bits) + 1 parity bit implying 30% overhead. However this percentage can be reduced by sending larger blocks of data but then timing errors between receiver and sender cannot be tolerated beyond [50/number. of bits in block]%. It will not only result in incorrect sampling but also misaligned bit count. i.e., a data bit can be mistaken for stop bit if receiver's clock is faster.

Synchronous

Larger blocks of bits are successfully transmitted. Blocks of data are either treated as sequence of bits or bytes. To prevent timing, drift clocks at two ends need to be synchronized. This can be done in two ways.

1. Provide a separate clock line between receiver and transmitter. (or)
2. Clocking information is embedded in data signal i.e., Biphase coding for digital signals.

Still another level of synchronization is required so that receiver determines beginning or end of block of data. Hence each block begins with a start code and end with a stop code. These are in general same, known as flag that is unique sequence of fixed number of bits. In addition some control characters encompass data within these flags. Data and control information is called a frame. Since any arbitrary bit pattern can be transmitted, there is no assurance that bit pattern for flag will not appear inside the frame, thus destroying frame stuffing.

Channel Allocation A large class of networks is built on broadcast channels, a number of stations will share the same channel, if one station sends, all other stations have to hear it.

Problem occurs when, 2 stations want to start data transmission at the same time, in this situation 2 fames collide.

To avoid frame collision, allocate the channel to one of the stations.

There are 3-strategies for channel allocation:

1. Let a station try to use the channel, and when the collision occurs, that is taken care of later.
2. Each station in turn is allowed to use the channel. This is applied in token-based systems. Only the station that has the token can use the channel.
3. Reserve the channel in prior, It is used in slotted systems. The problem is how to make a reservation.

Data Link Layer

Data link layer provides interface to the network layer, determines the number of bits of the physical layer to be grouped into frames, detects transmission error and regulates the flow of frames.

Functions of data link layer:

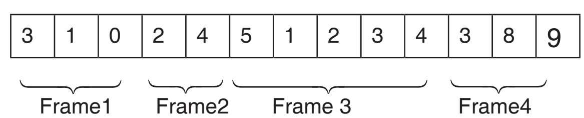
1. Framing
2. Physical addressing
3. Flow control
4. Error control
5. Access control

Various methods of Framing are

1. Time gaps
2. Character count
3. Starting and ending characters, with character stuffing
4. Starting and ending flags, with bit stuffing
5. Physical layer coding violations

Time gaps Framing is done by inserting time gaps between frames, very similar to the way of spacing between words in ordinary text. It is risky to count on timing to mark the start and end of each frame.

Character count It uses a field in the header to specify the number of characters in the frame. Thus at the destination by seeing the character count it knows how many characters follows and where the end of the frame exists.



Problem If count of any frame changes, destination will get out of synchronization and is unable to locate start of next frame.

Starting and ending characters, with character stuffing Each frame starts with the ASCII character sequence DLESTX and ends with the sequence DLEETX. If destination looses track of the frame boundaries, all it has to do is to look for DLESTX or DLEETX character

Starting and ending flags, with bit stuffing

Bit Stuffing: Suppose ou r flag bits are 01111110. So the transmitter will always insert an extra 0 bit after each occurrence of five 1s (except for flags). After detecting a starting flag the receiver monitors the bit stream. If pattern of five 1's appear, the sixth bit is examined and if it is 0 it is deleted; else if it is 1 and next bit is 0 the combination is accepted as a flag. Similarly byte stuffing is used for byte oriented transmition. Here we use an escape sequence to prefix a byte similar to flag and two escape sequences if byte itself is an escape sequence.

Has arbitrary number of bits and allows character codes with an arbitrary number of bits per character. Every frame begins and ends with a special bit pattern, 01111110, called a flag byte.

As soon as the sender's data link layer encounters five consecutive one's in the data, it stuffs a 0 bit into the outgoing bit stream.

Receiver de-shifts the 0 bit of the five consecutive incoming 1 bits, followed by a 0 bit.

If the user data is 01111110, transmitted as 011111010 but stored at receiver as 01111110.

Physical layer coding violations Applied to the networks in which the encoding on the physical medium contains some redundancy.

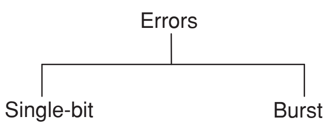
1 → high – low pair

0 → low – high pair

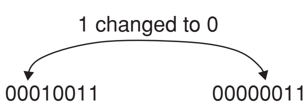
Here high-high, low-low not used for data.

Every data bit has a transition in the middle, thus easy for the receiver to locate the bit boundaries.

Types of Errors



Single bit error The term single bit error means that only one bit in the data unit has changed, it can either be from 1 to 0 or from 0 to 1.



Single bit error correction

A single bit error occurs when a bit changes in value from 0 to 1 (or) from 1 to 0 while storing (or) while performing read (or) write operation. If that error bit is identified, that can be corrected by complementing.

Hamming codes

In hamming codes, K parity bits are added to an n-bit data word, that forms a new word of $(n + k)$ bits. The bit positions are numbered in sequence from 1 to $n + k$. These positions numbered with powers of 2 are reserved for the parity bits; the remaining bits are the data bits.

Example: Consider the given 8-bit data word 11000100, we include four party bits with this word and arrange the bits as follows.

Bit position

1	2	3	4	5	6	7	8	9	10	11	12
$P1$	$P2$	1	$P4$	1	0	0	$P8$	0	1	0	0

The parity bits are in positions, 1, 2, 4, 8. Each parity bit is calculated as

$P1$ = XOR of bits (3, 5, 7, 9, 11) = $1 \oplus 1 \oplus 0 \oplus 0 \oplus 0 = 0$

$P2$ = XOR of bits (3, 6, 7, 10, 11) = 0

$P4$ = XOR of bits (5, 6, 7, 12) = 1

$P8$ = XOR of bits (9, 10, 11, 12) = 1

⇒ If there is odd number of 1s, XOR gives 0

⇒ If there is even number of 1s, XOR gives 1

The values $P1 = 0$, $P2 = 0$, $P4 = 1$, $P8 = 1$ are substituted in 12-bit composed word

Bit position

1	2	3	4	5	6	7	8	9	10	11	12
0	0	1	1	1	0	0	1	0	1	0	0

Check for errors:

$C1$ = XOR of bits (1, 3, 5, 7, 9, 11)

$C2$ = XOR of bits (2, 3, 6, 7, 10, 11)

$C4$ = XOR of bits (4, 5, 6, 7, 12)

$C8$ = XOR of bits (8, 9, 10, 11, 12)

Since the bits were written with even parity, the result $C = C8\ C4\ C2\ C1 = 0000$

∴ Indicates that no error has occurred.

- The code can be used with words of any length.

Burst Error The term burst means that two or more bits in the data unit have changed, either changed, from 1 to 0 or changed from 0 to 1.

Sent:

010011010000-sent bits corrupted by burst error

010001111000 Received

Parity bit

Parity bit is an error detecting code. This bit is added to data words depending on number of 1's in the data word; It could be even parity and odd parity.

n-bit data word is transformed to $(n + 1)$ bit code word with the addition of a bit. Even parity makes even number of 1's in a code word, similarly odd parity makes odd number of 1's in a code word.

Let us illustrate with example

Data word – 1 0 1 1 Parity bit

Code word – 1 0 1 1 1 parity bit (even parity)

Code word: 1 0 1 1 0 (odd parity)

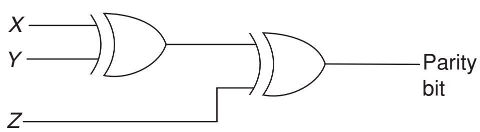
At the receiver side, when the code is received, the receiver checks the same as it is done by the generator. But here it adds all the bits which results in syndrome. If the syndrome is 0 then the number of 1's in code word is even, else number of 1's is odd.

Decision logic analyzer will decide, whether the code word is correct or not, based on syndrome value.

Parity bit generator

The parity bit generator for a 3-bit data word is given below.

The message is in the form of $X\,Y\,Z$



Parity bit generator

When the message is passed through the above circuit, the parity will be generated accordingly.

Error correction code

When data is transmitted form the source to destination, there is a chance of error introduction into the data. Error detection will detect the errors in data, while error correction will rebuild the original data,

Error correction code can be implemented in 2 ways

1. Forward error correction (FEC)
2. Automatic repeat request (ARQ)

In FEC, when sender is sending data, sender adds redundant data (encoded information) to the original data. At the receiver side this redundant data is used to recover the original data, when original data is tampered [error data].

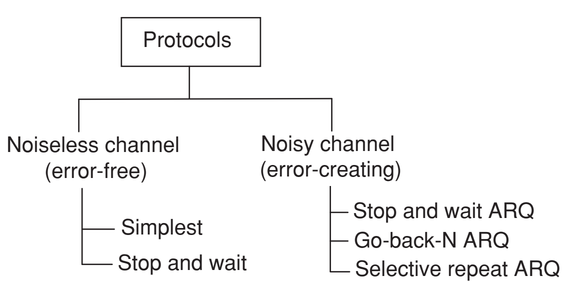
In ARQ, the receiver requests for the retransmission of the data packets, which are corrupted. Receiver will check the data using some error detection code.

Flow Control

It regulates the flow of frames so that slow receivers are not affected by the fast sender or vice versa.

It tell the sender how much data it should transmit before it waits for an acknowledgement from the receiver. Flow control refers to a set of procedures used to restrict the amount of data that the sender can send before waiting for acknowledgement.

Error control in the data link layer is often implemented simply. Any time an error is detected in an exchange, specified frames are retransmitted. This process is called automatic repeat request (ARQ).



All the protocols we discuss as unidirectional in the sense that data frames travels from sender to receiver. Although special frames called acknowledgment (ACK) and negative acknowledgment (NAK) can flow in the opposite direction for flow and error control purposes, data flow is in only one direction. In real life network, the data link protocols are implemented as bidirectional, data flow in both directions. In these protocols flow and error control information such as ACKs and NAKs are included in the data frames in a technique called piggybacking.

Stop and wait Sender sends one frame, stops until it receives confirmation from the receiver. Error correction in stop and wait ARQ is done by keeping a copy of the sent frame and retransmitting the frame when the timer expires.

Only 2 sequence numbers 0 and 1 are used.

Window size is 1.

No ACK for lost or damaged frames.

$$\text{Throughput} = \frac{\text{One packet}}{RTT}$$

$$\text{Utilization} = \frac{L}{L+BR}$$

L = packet length

B = Bandwidth

R = RTT

If $L < BR$, Efficiency > 50

$L > BR$, Efficiency = 50

$$\mu = \frac{1}{1+2a},\quad a = \frac{\text{propagation time}}{\text{Transmission time}}$$

Link utilization is low in stop and wait.

GBN protocol We can send several frames before receiving acknowledgements; we keep a copy of these frames with the acknowledgment.

- Sequence numbers ranges from $2^m - 1$.
- m – number of bits for sequence numbers.
- The sender window slides one or more slots when a valid acknowledgment arrives.
- It uses cumulative acknowledgement or piggy backing wherever possible to acknowledge the frames.
- It discards duplicate and out of order packets.
- Receiving window size is 1.
- If the sender receives a NAK, it resends all frames in the sender window.

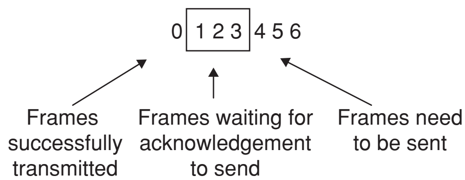
If a single packet is lost, damaged or acknowledgement is lost, it will resend all the packets.

$$\text{Link efficiency} = \frac{1-p}{1-p+p^w}$$

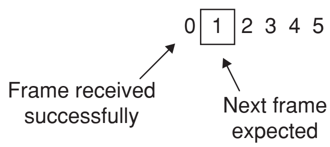
Where, p is the packet loss probability

w is the sender's window size.

Sender:



Receiver:



- If N is maximum sequence number, then sender window size = N, Receiver window size = 1.
- If N is the number of sequence number, sender window size = $N - 1$, Receiver window size = 1.

Selective repeat More efficient for noisy links but processing at the receiver is more complex. Receiver window size is same as of sender window size. Sender window maximum size is 2^{m-1}, receiver window maximum size is 2^{m-1}. Sender and receiver window must be at most one half of 2^m.

Receives out of order packets because receiver's window size is greater than 1.

It uses cumulative or independent or piggyback ACK whenever possible. If sender receives a NAK, it resends just the frame specified by the NAK.

If N is maximum sequence number,

$$\text{Sender window size } = \frac{N+1}{2},$$

$$\text{Receiver window size } = \frac{N+1}{2}$$

If N is the number of sequence numbers, sender window size $= \frac{N}{2}$, Receiver window size $= \frac{N}{2}$.

Medium Access Control Sublayer

Multiplexing

When two communicating nodes are connected through a media, it generally happens that bandwidth of media is several times greater than that of the communicating nodes. Transferring of a single signal at a time is both slow and expensive. The whole capacity of the link is not utilized in this case. This link can be further exploited by sending several signals combined into one. This combining of signals into one is called multiplexing.

Frequency Division Multiplexing (FDM)

This is possible in the case where transmission media has a bandwidth higher than the required bandwidth of signals to be transmitted. A number of signals can be transmitted at the same time. Each source is allotted a frequency range in which it can transfer it's signals, and a suitable frequency gap is given between two adjacent signals to avoid overlapping. This type of multiplexing is commonly seen in the cable TV networks.

Time Division Multiplexing (TDM)

This is possible when data transmission rate of the media is much higher than that of the data rate of the source. Multiple signals can be transmitted if each signal is allowed to be transmitted for a definite amount of time. These time slots are so small that all transmissions appear to be in parallel.

Synchronous TDM Time slots are pre. Assigned and are fixed. Each source is given it's time slot at every turn due to it. This turn may be once per cycle or several turns per cycle, if it has a high data transfer rate, or may be once in a number of cycles if it is slow. This slot is given even if the source is not ready with data. So this slot is transmitted empty.

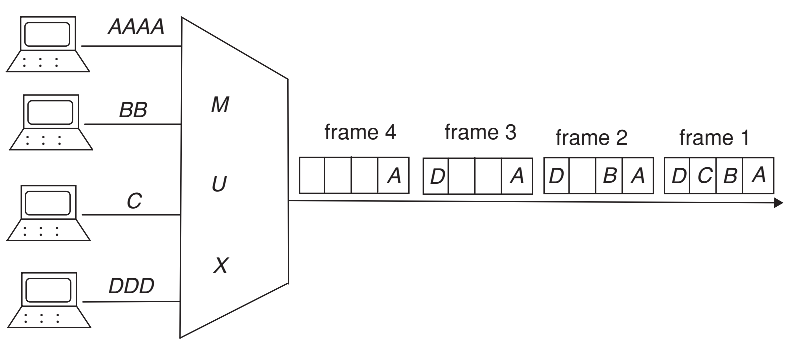


Figure 5 Synchronous TDM: Multiplexing process

Asynchronous TDM In this method, slots are not fixed. They are allotted dynamically depending on speed of sources and whether they are ready for transmission.

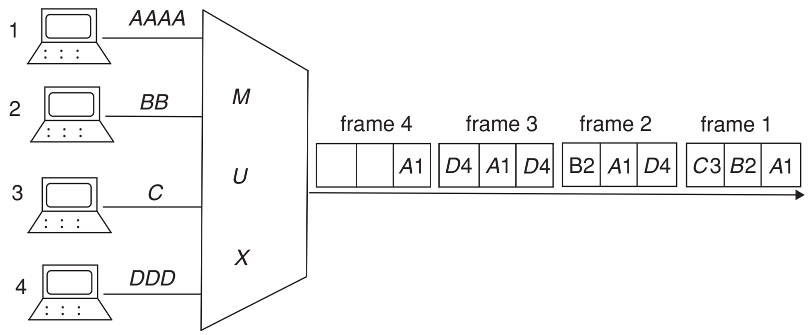


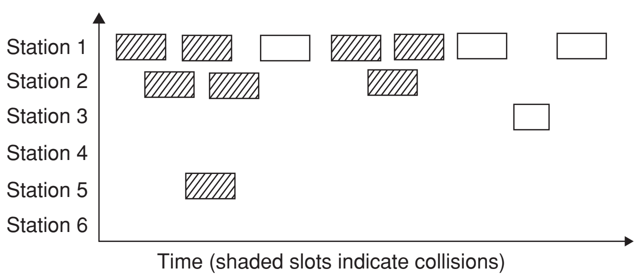
Figure 6 Asynchronous TDM

Aloha Protocols

The Aloha Protocol was designed to provide data transmission between computers on several islands using radio transmission.

Pure aloha

Pure Aloha is an unslotted, fully decentralized protocol. It is extremely simple and trivial to implement. The ground rule is 'when you want to talk, just talk!' So, a node which wants to transmit, will go ahead and sends the packet on its broadcast channel, with no consideration of who so ever to any body else is transmitting (or) (not).

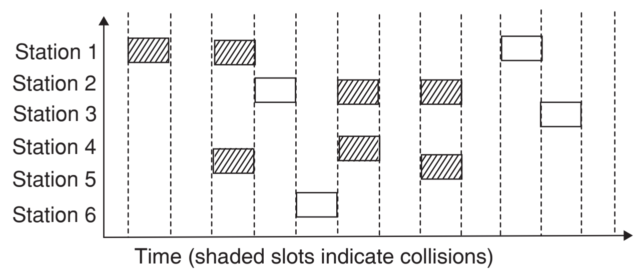


One serious drawback here is that, you don't know whether what you are sending, has been received properly or not. To resolve this in pure Aloha, when one node finishes speaking it expects an acknowledgement in a finite amount of time otherwise it simply retransmits the data. This scheme works well in small networks where the load is not high. But in large, load intensive networks where many nodes may want to transmit at the same time, this scheme fails miserably. This led to the development of slotted Aloha.

Slotted Aloha

This is quite similar to pure Aloha, differing only in the way transmissions take place. Instead of transmitting right at the demand time, the sender waits for some time. This delay is specified as follows–the timeline is divided into equal slots and then it is required that transmission should take place only at slot boundaries. To be more precise, the slotted Aloha makes the following assumptions.

- All frames consist of exactly L bits.
- Time is divided into slots of size L/K seconds. (i.e., a slot equals the time to transmit one frame)
- Nodes start to transmit frames only at the beginning of slots.
- The nodes are synchronized so that each node knows when the slots begin.
- If two or more frames collide in a slot, then all the nodes detect the collision event before slot ends.



In this, way the number of collisions that can possibly take place is reduced by a huge margin. And hence, the performance became much better compared to pure Aloha. Collisions may only take place with nodes that are ready to speak at the same time.

Virtual private network

Virtual Private Networking (VPN) Internet protocol security (IP sec) is one of the most complete, secure, standards-based protocol developed for transporting data.

A VPN is a shared network, where private data can be accessed only by the intended recipient.

The term VPN is used to describe a secure connection over the Internet.

VPN is also used to describe private networks such as Frame Relay and Asynchronous Transfer Mode (ATM).

The purpose of data security is that the data flowing across the network is protected by encryption technologies.

IP sec-based VPNs use encryption to provide data security, that increases the networks resistance to data tampering.

IP sec-based VPNs can be created over any type of IP Network, including Internet, ATM, Frame Relay, among all only Internet is inexpensive.

Uses of VPN

Intranets Intranets connect an organization's locations. These locations could be head quarters offices, branch offices, Employees home which is located in some Remote area.

This connectivity is used for e-mails, sharing files etc.

The cost of connecting remote home users is very expensive compared to Internet access technologies because of this organizations have moved their networks to the Internet.

Remote access It enables telecommuters and mobile workers to access e-mail and business applications.

A dial-up connection to an organizations modem pool is one method to access remote workers. It is expensive, because of long distance telephone and service costs.

IP sec

IP sec is an Internet Engineering Task Force (IETF) standard suite of protocols that provide data authentication, integrity, and confidentiality between 2 communication points across IP-Network.

It provides data security at the IP-packet level.

IP sec protects against possible security exposures by protecting data while in transit.

Features

IP sec was designed to provide the following security features when transferring packets across networks.

1. Authentication: Verifies that the packet received is actually from the correct sender or not.
2. Integrity: Ensures that the contents of packet did not change while transmitting data.
3. Confidentiality: Conceals the message content through encryption.

Components of IP sec

ESP: (Encapsulating security payload), It provides confidentiality, authentication and integrity.

AH: (Authentication Header) provides Authentication and Integrity.

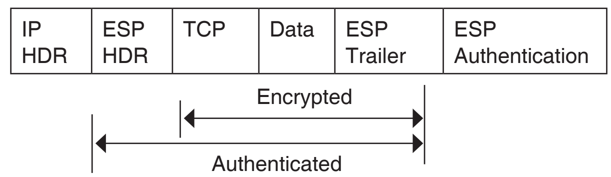
IKE: (Internet key Exchange) provides key management and security Association (SA) management.

ESP:

- Most importantly, it provides message content protection.
- IP sec provides an open frame work for implementing standard algorithms such as SHA and MD5.
- The algorithms IP sec uses produces a unique identifier for each packet, which is a data equivalent to a finger print.
- This Finger Print allows the device to determine whether a packet has been tampered with.
- Packets that are not authenticated are discarded and not delivered to the intended receiver
- ESP also provides all encryption services in IP sec.
- Encryption/decryption allows only the sender and the authorized receiver to read the data.
- The authentication performed by ESP is called ESP authentication.
- ESP provides authentication and integrity for the payload and not for the IP-header

IP HDR	TCP	Data

Figure 7 Original packet



The ESP Header is inserted into the packet between the IP-header and any subsequent packet contents.

- ESP encrypts the data, the payload is changed.
- ESP does not encrypt the ESP header, nor does it encrypt the ESP authentication.

AH:

Provides optional anti-replay protection, which protects against unauthorized retransmission of packets.

The authentication header is inserted into the packet between the IP-header and any sub sequent packet contents.

AH does not protect the data's confidentiality.

For added protection in certain cases, AH and ESP can be used together.

IP HDR	TCP	Data

Original packet

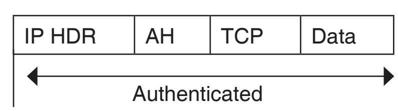


Figure 8 Packet with IP sec Authentication Header.

Exercises

Practice Problems I

Directions for questions 1 to 15 Select the correct alternative from the given choices.

1. Assume that, in a stop and wait ARQ system, the bandwidth of the line is 1 mbps, and 1 bit takes 20 ms to make a round trip. What is the bandwidth delay product utilization percentage of the link if we send 1000 bits?
(A) 1% (B) 5%
(C) 10% (D) 50%

2. A channel has a bit rate of 20 kbps and propagation delay of 100 msec. For what size does stop and wait gives an efficiency of 50%?
(A) 2000 bits (B) 3000 bits
(C) 4000 bits (D) 6000 bits

3. CSMA/CD LAN of 1 gbps is to be designed over 1 km cable without repeater. The minimum frame size that Data link layer should consider, if cable support signal speed of 20,000 km/sec

(A) 10 k bits (B) 20 k bits
(C) 30 k bits (D) 40 k bits

4. A 20 mbps satellite link has a propagation delay of 400 µs. The transmitter employs the 'go-back-n ARQ' scheme with n set to 10. Assuming each frame is 100 bytes long. What is the maximum data rate possible?
(A) 1 mbps (B) 2 mbps
(C) 5 mbps (D) 10 mbps

5. A satellite channel has capacity of B bits/sec, the frame size is of L bits, and round trip propagation time of R sec, uses stop and wait protocol, what is the channel utilization?
(A) $\frac{L}{L-BR}$ (B) $\frac{L}{L+BR}$
(C) $\frac{L}{B+R}$ (D) $\frac{L}{B-R}$

6. Find efficiency of the ring where data rate of link is 4 mbps, number of stations are 20 separated by 100 meters and bit delay in each station is 2.5 bits. (velocity of propagation = 2×10^8 m/s)
(A) 60 bits (B) 75 bits
(C) 90 bits (D) 120 bits

7. If you are designing sliding window protocol of 1 mbps which has one way delay of 1.25 seconds. Assuming each frame carries 1 kB of data, what is the minimum number of bits you need for the sequence number?
(A) 8 (B) 9
(C) 10 (D) 12

8. What are the sequence numbers of sender and receiver windows in Go-back-n and selective repeat if m-bits are used?
(A) 2^m–1,1, 2^{m-1}, 2^{m-1} (B) 2^m, 1, 2^{m-1}, 2^{m-1}
(C) 2^m, 2, 2^m, 2^m (D) 2^m–1,1, 2^m, 2^m

9. A 100 km long cable runs at 1.536 mbps. The propagation speed in the cable is 2/3 of speed of light. Number of bits fit in the cable would be?
(A) 428 bits (B) 526 bits
(C) 672 bits (D) 768 bits

10. If the bandwidth of the link is 256 mbps, Assume that sequence number field consists 32 bits. Find the wrap around time for sequence numbers?
(A) 128 sec (B) 256 sec
(C) 512 sec (D) 1024 sec

11. After a series of collisions a station has selected slot 984. In how many successive collisions, the station was a part of communication?
(A) 4 (B) 6
(C) 8 (D) 10

12. There are 10 stations in a LAN always having constant load and ready to transmit. During any particular contention slot each station transmits with a probability of 0.1. If the average frame takes 122 ms to transmit, what is the channel efficiency, if round trip time is 51.2 micro secs?
(A) 0.23 (B) 0.35
(C) 0.48 (D) 0.56

13. Which of the below are issues concerning data link layer?
(i) Ensures that the transmission facility is free of undetected transmission errors
(ii) Regulates the transmission rates so as to match the receiver's capabilities
(iii) Ensures the design of the line such that when a '1' bit is sent it is always received as '1' bit at receivers end.
(A) (i), (ii) (B) (ii), (iii)
(C) (iii), (i) (D) (i), (ii), (iii)

14. An Ethernet LAN has the capability of 100 Mbps. If Manchester encoding is used, what is the rate of signal change?
(A) 20 million times/sec
(B) 200 million times/sec
(C) 50 million times/sec
(D) 500 million times/sec

15. For 10 Mbps LAN it is found that 64 bytes is the minimum frame size to aid in collision detection. What should be the minimum frame size for a 100 Mbps LAN?
(A) 6.4 bytes (B) 64 bytes
(C) 640 bytes (D) 6400 bytes

Practice Problems 2

Directions for questions 1 to 15 Select the correct alternative from the given choices.

1. What is the probability of success for any arbitrary station among 'N' stations to transmit in CSMA/CD?
(A) $Np_s(1-p_s)^N$ (B) $(N-1)p_s\ (1-p_s)$
(C) $Np_s(1-p_s)^{N-1}$ (D) $Np_s(1-p_s)^N$

2. If 4-bits are used to represent sequence numbers for flow control. What are sender and receiver window sizes in Go-back-n and selective repeat?
(A) 16, 1, 8, 8 (B) 15, 1, 8, 8
(C) 15, 2, 8, 8 (D) 15, 1, 16, 8

3. If the available maximum sequence number is 13, compute sender and receiver window sizes in go-back-n and selective repeat?
(A) 4, 1, 4, 4 (B) 4, 1, 7, 7
(C) 13, 1, 7, 7 (D) 13, 1, 4, 4

4. In a gigabit ethernet LAN, the receiver couldn't empty the input buffer on some line for 1 millisecond. What is the maximum accumulation of frames possible neglecting propagation delays?

(A) 1024 frames (B) 2097 frames
(C) 4096 frames (D) 5120 frames

5. A Token ring LAN is using differential Manchester encoding. If the LAN speed is 10 Mbps. What is the baud rate?
(A) 10 M baud (B) 20 M baud
(C) 5 M baud (D) 100 M baud

6. Consider a 100-meter 10 mbps token ring containing 10 stations, each transmitting with equal priority. Each station can transmit 4 bytes before giving up the token. Token holding time per station is 10 ns. Also propagation speed is 200 m/s. Assume that the Ring monitor has created a new token, how long does it take for the token to come back to the Ring monitor if no station uses the token?
(A) 2.55 μsec (B) 3.64 μsec
(C) 4.65 μsec (D) 2.93 μsec

7. In the above question, if only 6 nodes including Ring monitor are active what is total propagation delay in μsec?
(A) 3.60 (B) 3.61
(C) 3.62 (D) 3.63

8. In the above case if bit regeneration time is 1 ns/bit. What is the regeneration overhead caused if a 4 kB token is taken by 1st node and if it uses to transmit 4B data to ring monitor.
(A) 412 ns (B) 544 ns
(C) 640 ns (D) 800 ns

9. Which of the below operation is applied to full-duplex mode operation of gigabit Ethernet?
(i) Traffic is allowed in both directions at any time.
(ii) CSMA/CD protocol is used.
(iii) Maximum length of cable segment used to connect stations is limited by CSMA/CD protocol.
(A) (i) and (ii) (B) (ii), (iii)
(C) (iii), (i) (D) (i), (ii), (iii)

10. Which of the below are not applied to Token Ring networks?
(i) Collisions
(ii) Limits on length of the cable segment
(iii) Time slots for transmission
(iv) Usage of repeaters
(A) (i), (ii) (B) (ii), (iii)
(C) (iii), (iv) (D) (i), (iv)

11. Select the correct statements from below (pertaining to Ethernet):
(i) Frame collisions don't occur at a repeater
(ii) Frame collisions can occur at the hub itself
(iii) Switch frames are never lost due to collisions
(iv) Entire bridge is a point of collisions
(A) (i), (ii), (iii) (B) (ii), (iii), (iv)
(C) (i), (iii), (iv) (D) (i), (ii), (iv)

12. Match the different layers with possible security methods in those layers.

(p) Data link layer	(i) user authentication
(q) Network layer	(ii) use firewalls
(r) Transport layer	(iii) encryption of connections
(s) Application layer	(iv) point to point encryption of data stream

(A) p – i, q – ii, r – iii, s – iv
(B) p – ii, q – iv, r – iii, s – i
(C) p – iv, q – ii, r – iii, s – i
(D) p – iii, q – iv, r – ii, s – i

13. The hamming distance between 001111 and 010011 is
(A) 1 (B) 2
(C) 3 (D) 4

14. Which of the following represents the polynomial $x^5 + x^4 + x^0$ using the CRC?
(A) 110000 (B) 110001
(C) 110010 (D) 110101

15. For a sliding window of size n–1 (n sequence number) there can be maximum of how many frames sent but unacknowledged?
(A) 0 (B) $n - 1$
(C) n (D) $n + 1$

Previous Years' Questions

1. The message 11001001 is to be transmitted using the CRC polynomial $x^3 + 1$ to protect it from errors. The message that should be transmitted is: **[2007]**
(A) 11001001000 (B) 11001001011
(C) 11001010 (D) 110010010011

2. The distance between two stations M and N is L kilometers. All frames are K bits long. The propagation delay per kilometer is t seconds. Let R bits/second be the channel capacity. Assuming that processing delay is negligible, the minimum number of bits for the sequence number field in a frame for maximum utilization, when the sliding window protocol is used, is: **[2007]**

(A) $\left\lceil \log_2 \frac{2LtR+2k}{k} \right\rceil$ (B) $\left\lceil \log_2 \frac{2LtR}{k} \right\rceil$

(C) $\left\lceil \log_2 \frac{2LtR+k}{k} \right\rceil$ (D) $\left\lceil \log_2 \frac{2LtR+k}{2k} \right\rceil$

3. Match the following:
(P) SMTP (1) Application layer
(Q) BGP (2) Transport layer

(R) TCP (3) Data link layer
(S) PPP (4) Network layer
(5) Physical layer

[2007]

(A) P–2 Q–1 R–3 S–5
(B) P–1 Q–4 R–2 S–3
(C) P–1 Q–4 R–2 S–5
(D) P–2 Q–4 R–1 S–3

4. A layer-4 firewall (a device that can look at all protocol headers up to the transport layer) CANNOT **[2011]**

(A) Block entire HTTP traffic during 9.00PM and 5.00PM
(B) Block all ICMP traffic
(C) Stop incoming traffic from a specific IP address but allow outgoing traffic to the same IP address
(D) Block TCP traffic from a specific user on a multi-user system during 9.00PM and 5.00AM.

5. If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads? **[2011]**

(A) 1/3 (B) 1/4
(C) 1/2 (D) 2/3

6. Which of the following transport layer protocols is used to support electronic mail? **[2012]**

(A) SMTP (B) IP
(C) TCP (D) UDP

7. Consider a source computer (S) transmitting a file of size 10^6 bits to a destination computer (D) over a network of two routers (R_1 and R_2) and three links (L_1, L_2 and L_3). L_1 connects S to R_1; L_2 connects R_1 to R_2; and L_3 connects R_2 to D. Let each link be of length 100 km. Assume signals travel over each link at a speed of 10^8 meters per second. Assume that the link bandwidth on each link is 1 Mbps. Let the file be broken down into 1000 packets each of size 1000 bits. Find the total sum of transmission and propagation delays in transmitting the file from S to D? **[2012]**

(A) 1005 ms (B) 1010 ms
(C) 3000 ms (D) 3003 ms

8. Determine the maximum length of the cable (in km) for transmitting data at a rate of 500 Mbps in an Ethernet LAN with frames of size 10,000 bits. Assume the signal speed in the cable to be 2,00,000 km/s. **[2013]**

(A) 1 (B) 2
(C) 2.5 (D) 5

9. Consider a token ring network with a length of 2 km having 10 stations including a monitoring station. The propagation speed of the signal is 2×10^8 m/s and the token transmission time is ignored. If each station is allowed to hold the token for 2 μsec, the minimum time for which the monitoring station should wait (in μsec) before assuming that the token is lost is ______ **[2014]**

10. Consider the store and forward packet switched network given below. Assume that the bandwidth of each link is 10^6 bytes/sec. A user on host A sends a file of size 10^3 bytes to host B through routers R_1 and R_2 in three different ways. In the first case a single packet containing the complete file is transmitted from A to B. In the second case, the file is spilt into 10 equal parts, and these packets are transmitted from A to B. In the third case, the file is spilt into 20 equal parts, and these packets are sent from A to B. Each packet contains 100 bytes of header information along with the user data. Consider only transmission time and ignore processing, queuing and propagation delays. Also assume that there are no errors during transmissions. Let T_1, T_2 and T_3 be the times taken to transmit the file in the first, second and third case respectively. Which one of the following is CORRECT? **[2014]**

(A) $T_1 < T_2 < T_3$
(B) $T_1 > T_2 > T_3$
(C) $T_2 = T_3$, $T_3 < T_1$
(D) $T_1 = T_3$, $T_3 > T_2$

11. In the following pairs of OSI protocol layer/ sub-layer and its functionality the INCORRECT pair is **[2014]**

(A) Network layer and Routing
(B) Data Link Layer and Bit synchornization
(C) Transport layer and End-to-end process communication
(D) Medium Access Control sub-layer and Channel sharing

12. A bit-stuffing based framing protocol uses an 8-bit delimiter pattern of 01111110. If the output bit-string after stuffing is 01111100101, then the input bit-string is **[2014]**

(A) 0111110100 (B) 0111110101
(C) 0111111101 (D) 011111111

13. Suppose that the stop-and-wait protocol is used on a link with a bit rate of 64 kilobits per second and 20 milliseconds propagation delay. Assume that the transmission time for the acknowledgement and the processing time at nodes are negligible. Then the minimum frame size in bytes to achieve a link utilization of at least 50% is _______ **[2015]**

14. A link has a transmission speed of 10^6 bits/sec. It uses data packets of size 1000 bytes each. Assume that the acknowledgement has negligible transmission delay, and that is propagation delay is the same as the data propagation delay. Also assume that the processing delays at nodes are negligible. The efficiency of the stop-and-wait protocol in this setup is exactly 25%. The value of the one-way propagation delay (in milliseconds) is _______ **[2015]**

15. Consider a CSMA/CD network that transmits data at a rate of 100 Mbps (10^8-bits per second) over a 1 km (kilometer) cable with no repeaters. If the minimum frame size required for this network is 1250 bytes, what is the signal speed (km/sec) in the cable? **[2015]**

(A) 8000 (B) 10000
(C) 16000 (D) 20000

16. Consider a LAN with four nodes S_1, S_2, S_3 and S_4. Time is divided into fixed-size slots, and a node can begin its transmission only at the beginning of a slot. A collision is said to have occurred if more than one node transmit in the same slot. The probabilities of generation of a frame in a time slot by S_1, S_2, S_3 and S_4 are 0.1, 0.2, 0.3 and 0.4, respectively. The probability of sending a frame in the first slot without any collision by any of these four stations is ______ **[2015]**

17. Consider a network connecting two systems located 8000 kilometers apart. The bandwidth of the network is 500×10^6-bits per second. The propagation speed of the media is 4×10^6 meters per second. It is needed to design a Go-Back-N sliding window protocol for this network. The average packet size is 10^7-bits. The network is to be used to its full capacity. Assume that processing delays at nodes are negligible. Then, the minimum size in bits of the sequence number field has to be _______ **[2015]**

18. Two hosts are connected via a packet switch with 10^7 bits per second links. Each link has a propagation delay of 20 microseconds. The switch begins forwarding a packet 35 microseconds after it receives the same. If 10000-bits of data are to be transmitted between the two hosts using a packet size of 5000-bits, the time elapsed between the transmission of the first bit of data and the reception of the last bit of the data in microseconds is _______ **[2015]**

19. A sender uses the Stop-and-Wait ARQ protocol for reliable transmission of frames. Frames are of size 1000 bytes and the transmission rate at the sender is 80 Kbps (1 Kbps = 1000 bits/second). Size of an acknowledgement is 100 bytes and the transmission rate at the receiver is 8 Kbps. The one-way propagation delay is 100 milliseconds. Assuming no frame is lost, the sender throughput is _____ bytes/second. **[2016]**

20. In an Ethernet local area network, which one of the following statements is **TRUE**? **[2016]**

(A) A station stops to sense the channel once it starts transmitting a frame.
(B) The purpose of the jamming signal is to pad the frames that are smaller than the minimum frame size.
(C) A station continues to transmit the packet even after the collision is detected
(D) The exponential back off mechanism reduces the probability of collision on retransmissions.

21. Consider a 128×10^3 bits/second satellite communication link with one way propagation delay of 150 milliseconds. Selective retransmission (repeat) protocol is used on this link to send data with a frame size of 1 kilobyte. Neglect the transmission time of acknowledgement. The minimum number of bits required for the sequence number field to achieve 100% utilization is ______. **[2016]**

22. A computer network uses polynomials over $GF(2)$ for error checking with 8 bits as information bits and uses $x^3 + x + 1$ as the generator polynomial to generate the check bits. In this network, the message 01011011 is transmitted as **[2017]**

(A) 01011011010 (B) 01011011011
(C) 01011011101 (D) 01011011100

23. The values of parameters for the Stop-and Wait ARQ protocol are as given below:

Bit rate of the transmission channel = 1Mbps.

Propagation delay from sender to receiver = 0.75 ms.

Time to process a frame = 0.25 ms.

Number of bytes in the information frame = 1980.

Number of bytes in the acknowledge frame = 20.

Number of overhead bytes in the information frame = 20.

Assume that there are no transmission errors. Then, the transmission efficiency (expressed in percentage) of the stop-and-wait ARQ protocol for the above parameters is ________ (correct to 2 decimal places) **[2017]**

24. Consider a binary code that consists of only four valid codewords as given below:

00000,01011,10101,11110

Let the minimum Hamming distance of the code be p and the maximum number of erroneous bits that can be corrected by the code be q. Then the values of p and q are **[2017]**

(A) p=3 and q=1
(B) p=3 and q=2
(C) p=4 and q=1
(D) p=4 and q=2

25. Consider two hosts X and Y connected by a single direct link of rate 10^6 bits/sec. The distance between the two hosts is 10.000 km and the propagation speed along the link is 2×10^8 m/sec. Host X sends a file of 50,000 bytes as one large message to host Y continuously. Let the transmission and propagation delays be p milliseconds and q milliseconds, respectively. Then the values of p and q are **[2017]**

(A) p=50 and q=100
(B) p=50 and q=400

(C) p=100 and q=50
(D) p=400 and q=50

26. Consider a simple communication system where multiple nodes are connected by a shared broadcast medium (like Ethernet or wireless). The nodes in the system use the following carrier-sense based medium access protocol. A node that receives a packet to transmit will carrier-sense the medium for 5 units of time. If the node does not detect any other transmission in this duration, it starts transmitting its packet in the next time unit. If the node detects another transmission, it waits until this other transmission finishes, and then begins to carrier-sense for 5 time units again. Once they start to transmit, nodes do not perform any collision detection and continue transmission even if a collision occurs. All transmissions last for 20 units of time. Assume that the transmission signal travels at the speed of 10 meters per unit time in the medium.

Assume that the system has two nodes P and Q, located at a distance d meters from each other. P starts transmitting a packet at time $t = 0$ after successfully completing its carrier-sense phase. Node Q has a packet to transmit at time $t = 0$ and begins to carrier-sense medium.

The maximum distance d (in meters, rounded to the closest integer) that allows Q to successfully avoid a collision between its proposed transmission and P's ongoing transmission is ______. **[2018]**

Answer Keys

Exercises

Practice Problems I

1. B **2.** C **3.** A **4.** D **5.** B **6.** C **7.** B **8.** A **9.** D **10.** A
11. D **12.** C **13.** A **14.** B **15.** C

Practice Problems I

1. C **2.** B **3.** C **4.** B **5.** B **6.** B **7.** A **8.** B **9.** D **10.** B
11. A **12.** D **13.** C **14.** B **15.** B

Previous Years' Questions

1. B **2.** C **3.** B **4.** A **5.** A **6.** C **7.** A **8.** B **9.** 28 to 30
10. D **11.** B **12.** B **13.** 160 **14.** 12 **15.** D **16.** 0.40 to 0.46 **17.** 8
18. 1575 **19.** 2500 **20.** D **21.** 4 **22.** C **23.** 87.3 **24.** A **25.** D **26.** 50

Chapter 2

Routing Algorithms

LEARNING OBJECTIVES

- *Routing algorithm basics*
- *Flooding*
- *Multipath routing*
- *Distance vector routing*
- *Link state routing*
- *Hierarchical routing*
- *Rip*
- *Ospf*
- *Congestion control techniques*
- *Traffic shaping*

Routing Algorithms Basics

The main function of network layer is routing packets from the source machine to the destination machine. The routing algorithms are part of the network layer software, responsible for deciding which output line an incoming packet should be transmitted on.

Routing algorithms can be grouped into two major classes: Non-adaptive and Adaptive.

1. Non-adaptive algorithms do not base their routing decisions on measurements or estimates of the current traffic and topology. Instead, the choice of the route to use is downloaded to the routers when the network is booted. This procedure is called static routing.
2. Adaptive algorithms, in contrast, change their routing decisions to reflect changes in the topology, and the traffic as well.

Store and Forward Packet Switching

In this Technique, the data packet will be stored at the node and it is forwarded to its next appropriate intermediate node. The next intermediate node will first store the packet in the buffer, based on the router decision, it selects an interface, and forwards to receiver.

The technique is most suitable for the networks with unsteady connectivity.

The length of the packet we take shows effect on the file transfer, if the data packet is small, in the store the forward, delay will be less at each node, but causes extra overhead with headers. So, the packet size selection should be done appropriately.

Flooding

Static algorithms, in which every incoming packet is sent out on every outgoing line except the one on which it is arrived. Header contains the hop count of each packet. Hop counter is decremented at each hop, with the packet being discarded when the counter reaches zero.

Another way for damming the flood is to keep track of which packets have been flooded, to avoid sending them out a second time. A variation of flooding that is slightly more practical is selective flooding. In this algorithm the routers do not send every incoming packet out on every line, only on those lines that are going approximately in the right direction.

Multipath Routing

Multipath routing is routing the packets from the source, on multiple paths to the destination. It is nothing but spreading the traffic.

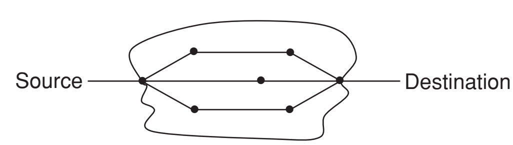


Figure 1 Multipath routing model

Single path routing causes QOS, throughput and delay problems, and multipath routing, improves network performance with sharing of available resources of network.

The components of multipath routing are

1. Multipath calculation algorithm
2. Multipath forwarding algorithm
3. End-Host protocol

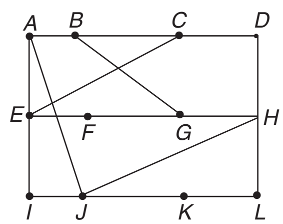
The algorithms specified above are based on Dijkstras shortest path algorithm they generate paths according to path characteristics and ensure path quality and path independence.

The end-host protocol uses the multipath (determined) effectively performance will be improved if end-users use the multiple paths effectively.

Distance Vector Routing

A dynamic routing algorithm, operates by having each router maintain a table (i.e., a vector) giving the best known distance to each destination and which line to use to get there. These tables are updated by exchanging information with the neighbors.

The Metric used might be number of hops, time delay in milliseconds, and total number of packets queued along the path or something similar.



(a)

New estimated — Delay from

	A	I	H	K	J	
A	0	24	20	21	8	A
B	12	36	31	28	20	A
C	25	18	19	36	28	I
D	40	27	8	24	20	H
E	14	7	30	22	17	I
F	23	20	19	40	30	I
G	18	31	6	31	18	H
H	17	20	0	19	12	H
I	21	0	14	22	10	I
J	9	11	7	10	0	-
K	24	22	22	0	6	K
L	29	33	9	9	15	k
	JA delay is 8	JI delay is 10	JH delay is 12	JK delay is 6	New routing table for J	

Vectors received from J's four neighbours.

(b)

Figure 2 (a) Subnet, (b) Delay vectors of J

Figure 2(a) 'shows a subnet. The first 4 columns of figure 2(b) shows the delay vectors received from the neighbors of router *J*. A claims to have a 12 m sec delay to *B*, a 25 m sec delay to *C*, a 40 m sec delay to D, etc.

Suppose that *J* has estimated its delay to its neighbour, *A*, *J*, *H* and *K* as 8, 10, 12 and 6 m sec, respectively.

Now *J* computes its new route to router *G*. It knows that it can get to *A* in 8 m sec, and *A* claims to be able to get to *G* in 18 m sec, so *J* knows it can count on a delay of 26 m sec to *G* if it forwards packets bound for *G* to *A*, similarly, it computes the delay to *G* via *I*, *H* and *K* as 41 (31 + 10), 18 (6 + 12) and 37 (31 + 6) m sec, respectively.

The best of these values is 18, so it makes an entry in its routing table that the delay to *G* is 18 m sec and that the route to use is via *H*.

Count to Infinity Problem

It reacts rapidly to good news, but leisurely to bad news. Actual network may be down but routers will exchange routes with one another.

Following measures are taken to avoid count-to-infinity problem:

1. **Hop limit:** Limit number of hops normally 0 hops directly connected, hop 16 is (0 – 15), 16 hops unreachable.
2. **Split horizon:** Never send information back in direction where it came from.
3. Route poisoning and poison reverse, hold on timer trigger.
4. As soon as network goes down, make metric of root infinity to resolve the immediate instability created because of routing updates from neighbor.
5. When router sends update with infinite metric to neighbor, neighbor will make it down.
6. Now routers will initiate hold on time to learn alternate paths and send update in direction where it came (Poison reverse) from.
7. Routers will incorporate final roots in routing table.

Link State Routing

The idea behind link state routing is simple and can be stated as five parts. Each router must do the following:

1. Discover its neighbors and learn their network addresses.
2. Measure the delay or cost to each of its neighbors.
3. Construct a packet telling all it has just learned.
4. Send this packet to all other routers.
5. Compute the shortest path to every other router.

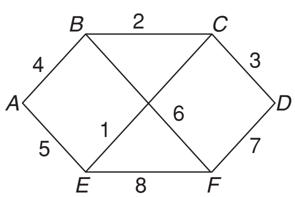
Learning about the neighbors When a router is booted, its first task is to learn who its neighbors are. It accomplishes this goal by sending a special HELLO packet on each point to point line. The router on the other end is expected to send back a reply telling who it is. These names must be globally unique.

Measuring the cost The link state routing algorithm requires each router to know, or at least have a reasonable estimate of, the delay to each of its neighbors.

The most direct way to determine this delay is to send over the line a special ECHO packet that the other side is required to send back immediately.

By measuring the round trip time and dividing it by two, the sending router can get a reasonable estimate of the delay. If two paths with same bandwidth exists and one path is heavily loaded then the path which is not heavily loaded is chosen. But this may oscillate in the choice of best path. So to avoid oscillation in the choice of best path, distribute the load over multiple lines with same known fraction going over each line.

Building link state packets Once the information needed for the exchange has been collected, the next step is, for each router to build a packet containing all the data. The packet starts with identity of the sender, followed by a sequence number and age, and a list of neighbors. For each neighbor, delay to that neighbor is given.



(a)

A	
Seq	
Age	
B	4
E	5

B	
Seq	
Age	
B	4
D	2
E	6

C	
Seq	
Age	
B	2
D	3
E	1

D	
Seq	
Age	
C	3
F	7

E	
Seq	
Age	
A	5
F	8
C	1

F	
Seq	
Age	
B	6
D	7
E	8

(b)

Figure 3 (a) Subnet5, (b) Link state packets for this subnet.

Distributing the link state packets As the packets are distributed and installed, the routers getting the routing packet first will change their routes.

Consequently, the different routers may be using different versions of the topology, which can lead to inconsistencies, loops, unreachable machines, and other problems. The fundamental idea is to use flooding to distribute the link state packets. To keep the flood in check, each packet contains a sequence number that is incremented for each new packet sent. Routers keep track of all the (source router, sequence) pairs they see.

When a new link state packet comes in, it is checked against the list of packets already seen. If it is new, it is forwarded on all lines except the one it arrived on. If it is a duplicate, it is discarded.

If a packet with a sequence number lower than the highest one seen so far ever arrives, it is rejected as being obsolete since the router has more recent data.

If a router ever crashes it will lose track of its sequence number. If its starts again at 0, the next packet will be rejected as a duplicate. Also due to bit error, packets may be rejected as obsolete. Solution to these problems is to include the age of each packet after the sequence number and decrement it once per second.

When the age hits zero, the information from that router is discarded.

Computing new routes Once a router has accumulated a full set of link state packets, it can construct the entire subnet graph because every link is represented. Every link is, in fact, represented twice, once for each direction. The two values can be averaged or used separately. Now dijkstra's algorithm can be run locally to construct the shortest path to all possible destinations.

Hierarchical Routing

Hierarchical Routing is mainly designed for large topologies. With increase in the topology there is proportionate increase in the routing tables, which consume more memory for maintaining tables and requires more bandwidth for the status reports.

In this routing, network topology is divided into hierarchies, these will reduce size of routing table. The node at each hierarchy will know about the nodes present in that level. It forwards the packet to its border router (at its level) if destination is not at its level. Hierarchical routing increases efficiency in routing, less traffic, reduction of table size in an order of about (log n).

RIP

1. It calculates best route based on hop count.
2. RIP cannot handle more than 15 hops, anything above 15 hops away is considered unsearchable by RIP. This fact is used by RIP to prevent routing loops.
3. RIP is a classful routing protocol.
4. Interval between route update advertisements: 30 sec. Time out/hold on times: 180 sec
5. RIP implements the split horizon, route isonning and hold down mechanisms to prevent looping.
6. It is a dynamic distance vector routing protocol.

OSPF

The open shortest path first is an adaptive routing protocol for IP networking. It uses a link state routing algorithm. OSPF keeps track of the state of all the various network connections between itself and a network it is trying to send

data to. OSPF selects the best route by finding the lowest cost paths to a destination. All router interfaces are given a cost. Its domain is an autonomous system.

Backbone routers Backbone routers have one or more interfaces in Area 0 (the backbone area).

Area border router (ABR) Routers that belong to multiple areas, and connect these areas to the backbone area are called ABR. It has interfaces in multiple areas.

Autonomous system boundary router (ASBR) If the router connects the OSPF autonomous system to another autonomous system, it is called an autonomous system boundary router (ASBF).

OSPF elects two or more routers to manage the link state advertisements.

Designated router (DR) Every OSPF will have a DR, a backup DR. The DR is the route to which all other routers within the area, send their link state advertisements.

OSPF areas

OSPF areas are used to impose a hierarchical structure to the flow of data over the network. A network using OSPF will always have atleast one area and if there is more than one area, one of the two areas must be the backbone area. Areas are used to group routers into manageable groups that exchange routing information locally, but summarize the routing information, when advertising the routes externally, ABR's are used to connect the areas.

CONGESTION CONTROL TECHNIQUES

Objective of congestion control technique is to limit queue lengths at the nodes, so as to avoid throughput collapse.

1. Send a control packet from a congested node to some or all source nodes to stop or slow the rate of transmission from source and thus limit the total number. of packets in the network.
2. Allow packet switching nodes to add congestion information to packets as they pass by. The packets carrying such information can go in both the directions i.e., opposite of the congestion and in the same direction of the congestion.

   Packets in the opposite direction of congestion quickly reache the source node which can reduce the flow of packets into the network.

   Packets going in the same direction as the congestion, reaches the destination. The destination asks the source to adjust the load by returning the signal back to the source in the packets.
3. Provides link delay information to other nodes. This information can be used to influence the rate at which new packets are produced. As these delays are influenced by the routing decision, they may vary too rapidly to use effectively for congestion control.

Congestion Control

Congestion control maintains the number of packets within the network below the level at which performance falls dramatically.

Every node has a queue of packets for each outgoing channel. If, rate at which packets arrive and queue up, exceeds the rate of packet transmission, then size of queue grows without bound and thus delay experienced by a packet goes to infinity.

When the packets arrive they are stored in the input buffer, of the corresponding link. The node examines each incoming packet to make a routing decision and then moves the packet to the appropriate output buffer. Packet queued up for output in output buffer is transmitted as soon as possible. When saturation point is reached, one can do any of the following:

1. Discard incoming packet for which there is no available buffer space.
2. Node should exercise some sort of flow control over its neighbors so that the traffic flow remains manageable.
3. Traffic shaping is about regulating the average rate of data transmission.

Leaky Bucket Algorithm

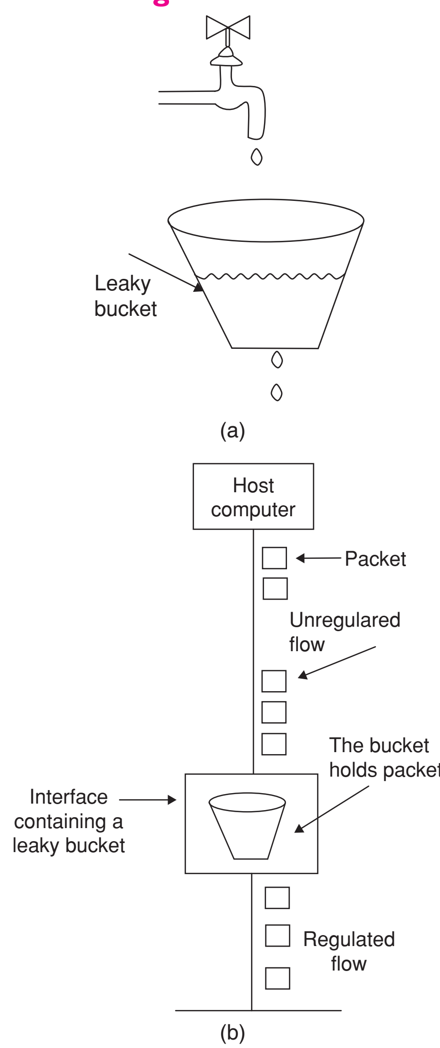
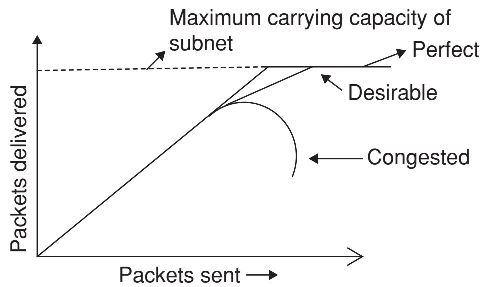


Figure 4 (a) A leaky bucket with water, (b) A leaky bucket with network

A leaky bucket is a bucket with a small hole. No matter at what rate water enters the bucket, the outflow is at constant rate, *S*, when there is any water in the bucket and zero when bucket is empty. Once the bucket is full, any additional water entering it, spills over the sides and it is lost. Each host is connected to the network by an interface containing a leaky bucket (i.e., a finite internal queue) congestion control algorithms.



When too many packets are present in the subnet, performance degrades. This situation is called congestion.

Causes of congestion

1. If all of a sudden, stream of packets are arriving on three or four input lines and all need same output line, a queue will build up.
2. Slow processor.
3. Low bandwidth line.

Token bucket

Tokens are added at a constant rate. For a packet to be transmitted, it must capture and destroy one token.

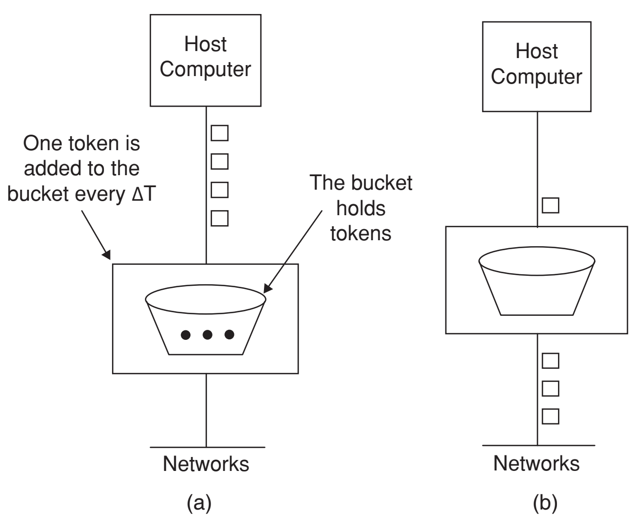


Figure 5 (a) shows that the bucket holds 3 tokens with 4 packets waiting to be transmitted, (b) shows that 3 packets have gotten through but the other one is stuck waiting for tokens to be generated.

Unlike leaky bucket, token bucket allows saving up to maximum size of bucket '*n*'.

The bursts of upto '*n*' packets can be sent at once, giving faster response to sudden bursts.

- Leaky bucket discards packets when the bucket is full, whereas token bucket throws away tokens when the bucket is full but never discards packets.
- Let Token bucket capacity be *c*(bits), token arrival rate ρ(bps), maximum output rate *M*(bps), and burst length *S*(*s*).
- During the burst length of *S*(*s*), tokens generated are ρS(bits), output burst contains a maximum of $C + \rho S$(bits)
- Output in a maximum burst of length *S*(*s*) is MS.
- $C + \rho S = MS \quad \text{(or)} \quad S = \dfrac{C}{M - P}$
- Token bucket still allows large bursts, even though the maximum burst length '*s*' can be regulated by selection of ρ and *M*.
- To reduce the peak rate, put a leaky bucket of a larger rate after the token bucket (To avoid discarding packets)

Traffic Shaping

1. One of the main causes of congestion is, that traffic is often burst.
2. If hosts could be made to transmit at uniform rate, congestion would be less.

This arrangement can be built into the network interface or simulated by the host OS. The host is allowed to put one packet per tick on the network.

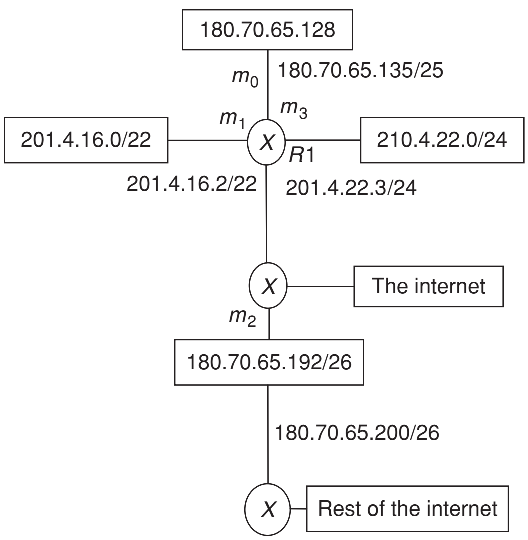
1. When the packets are all of the same size at every clock tick, one packet is transmitted.
2. When variable size packets are used.
   (i) At every tick, a counter is initialized to *n*. If the first packet on the queue has fewer bytes than the current value of the counter, it is transmitted and counter is decremented by that number of bytes.
   (ii) Additional packets may also be sent, as long as the counter is high enough.
   (iii) When the counter drops below the length of the next packet on the queue, transmission stops until the next tick, at that time the residual byte count is overwritten and lost.

Exercises

Practice Problems 1

Directions for questions 1 to 15: Select the correct alternative from the given choices.

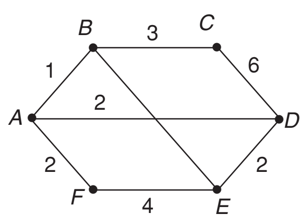
1. Consider below figure:



The network address, 180.70.65.130 goes through which of the following interface?

(A) m_0 (B) m_1
(C) m_2 (D) m_3

2. Consider below graphical representation of a subnet with each node denoting a router. If all the routers are booted at the same time, what is the number of link state packets that are generated having the cost/delay information?



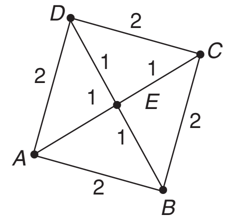
(A) 3 (B) 4
(C) 5 (D) 6

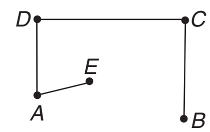
3. In a TCP connection it is found that burst size of 1024, 2048, 4096 have been transmitted while that of 8192 has resulted in a time out. The receiver has earlier set a window size of 4096. As per slow start algorithm which of the below statement is true?

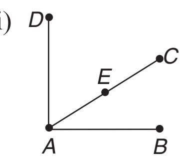
(i) Congestion window is set to 4096.
(ii) Maximum allowed burst size is 8192

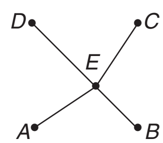
(A) (i) only
(B) (ii) only
(C) Both (i) and (ii)
(D) Neither (i) nor (ii)

4. From the below graph select the sink tree(s):



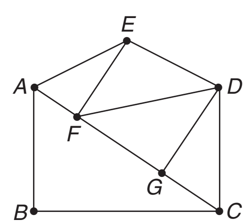
(i)


(ii)


(iii)


(A) (i), (ii) (B) (ii), (iii)
(C) (i), (iii) (D) (i), (ii), (iii)

5. Consider the below graph.

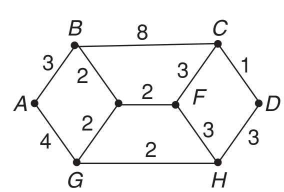


It is known that *D* is the optimal route from *A* to *C* and the optimal route from *A* to *C* has 3 hops. Which of the below statements is certainly true?

(i) *B* is not in the optimal route from *A* to *C*
(ii) *G* is not in the optimal route from *B* to *C*
(iii) Either *E* or *F* is in the optimal route from *A* to *C*
(iv) *ED*, *FD* are both optimal routes

(A) (i), (ii), (iii) (B) (ii), (iii), (iv)
(C) (i), (iii), (iv) (D) (i), (iv), (ii)

6. The shortest path using Dijkstra's algorithm after 3 iterations is



(A) A G (B) A B E
(C) A B C (D) A G H

7. There are totally 20 links among the routers of a subnet. How many rows are needed in all when link state packets

combined together, which are used to notify each other about cost/delay in transmitting data to immediate neighbours. Assume 1 row is needed for each neighbour?

(A) 10 (B) 20
(C) 40 (D) 80

8. Below are the link state packets generated by routers in a subnet. What is the shortest distance between *A* and *D*?

A	
Seq	
Age	
B	4
E	5

B	
Seq	
Age	
A	4
C	2
F	6

C	
Seq	
Age	
B	2
D	3
E	1

D	
Seq	
Age	
C	3
F	7

E	
Seq	
Age	
A	5
C	1
F	8

F	
Seq	
Age	
B	6
D	7
E	8

(A) 6 (B) 9
(C) 10 (D) 11

9. What are the advantages of reverse path forwarding over other broadcasting algorithms like spanning trees, multidestination routing, broadcasting, and flooding?

(i) Route does not need to know information regarding spanning tree structures
(ii) Uses destination tables for further forwarding
(iii) Does not need a halt mechanism to stop packets from further getting routed

(A) (i), (ii) (B) (ii), (iii)
(C) (i), (iii) (D) (i), (ii), (iii)

10. Which of the following specifies the correct sequence of steps to route packets to mobile hosts?

(i) Sender is given foreign agent's address
(ii) Packet is sent to mobile host's home address
(iii) Packet is tunneled to foreign agent
(iv) Subsequent packets are tunneled to the foreign agent

(A) (i), (ii), (iii), (iv)
(B) (ii), (iii), (iv), (i)
(C) (ii), (iii), (i), (iv)
(D) (iii), (iv), (i), (ii)

11. What are the different parts of congestion control by closed loop methods?

(i) Design the system in advance to make sure congestion doesn't occur in first place
(ii) Monitor the system to detect when and where congestion occurs
(iii) Pass congestion information to places where action can be taken
(iv) Adjust system operation to correct the problem

(A) (i), (ii), (iii)
(B) (ii), (iii), (iv)
(C) (iii), (iv), (i)
(D) (i), (ii), (iv)

12. In Selective flooding

(A) Packets are sent in all outgoing lines.
(B) Packets are sent in only on those lines that are approximately in the right direction.
(C) Both (A) and (B)
(D) None of these

13. There are 5 routers and 6 networks in an inter-networking, using link state routing, how many routing tables are there?

(A) 1 (B) 5
(C) 6 (D) 11

14. Congestion control for multicasting flows from multiple sources to multiple destinations, the solution that can handle this is

(A) RSVP (Resource reSerVation Protocol)
(B) Load shedding
(C) Both (A) and (B)
(D) None of these.

15. Which of the below are part of backward learning algorithm?

(i) As the bridge starts operating, a hash table to map source addresses to corresponding LANs is constructed.
(ii) It dynamically updates the hash tables when machines are connected and re connected to the LAN.
(iii) It encrypts the frames for security reasons.

(A) (i), (ii)
(B) (ii), (iii)
(C) (i), (iii)
(D) (i), (ii), (iii)

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. What does a routing algorithm perform?
(A) Decides if incoming packet should be further corrected for transmission errors
(B) Adds checksum bits to packets
(C) Encrypts the packets
(D) Decides the output line on which the incoming packet should be transmitted

2. What happens in session routing?
(A) User's session variables are managed by the network layer
(B) Route remains same throughout the user session
(C) Packets change their route for optimization sake during user session
(D) Provides special routes for important packets

3. What is the type of algorithm that changes their routing decision based on changes in topology and traffic?
(A) Adaptive routing
(B) Static routing
(C) Non-adaptive routing
(D) Network routing

4. Which of the below routing method always ensures the shortest path even though routers crash during course of routing?
(A) Dijkstra Routing
(B) Flooding
(C) Distance Vector Routing
(D) Link State Routing

5. What is the root cause for count-to-infinity problem?
(A) The routing tables are static and are not updated.
(B) The routing tables run out of space to accommodate more entries in table.
(C) When router *X* tells router *Y* that there is a path, it doesn't say if *Y* itself is in the path.
(D) When router *X* tells router *Y* that there is a path (to target route *Z*) it doesn't inform *Z* about the path.

6. In a strict sure security path ABCD, where A, B, C, D are routers, the maximum bandwidth is found to be 500 kbps, 700 kbps, 900 kbps, 300 kbps respectively. What is the effective bandwidth if no buffering is possible?
(A) 600 kbps (B) 900 kbps
(C) 300 kbps (D) 2400 kbps

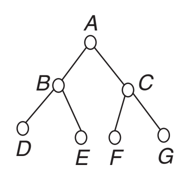
7. What is the characteristic of Distance Vector Routing?
(i) Time taken to reach other routers in the network is maintained in the routing tables.
(ii) Algorithm is susceptible to count-to-infinity problem.
(iii) The preferred outgoing line to be used for a particular destination is also stored in tables.
(A) (i), (ii) (B) (ii), (iii)
(C) (i), (iii) (D) (i), (ii), (iii)

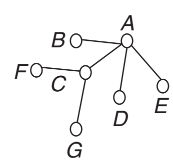
8. A subnet using link state algorithm has router, using link state packets with sequence of 16-bit fixed size. If a link state packet is sent every second, how long would it take before wrap around occurs. Assume starting sequence number is 0.
(A) 24.5 hours (B) 18.20 hours
(C) 17.5 hours (D) 16.4 hours

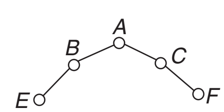
9. Which of the following are features of link state routing?
(i) In the first step discover all the routers in the subnet and find their network addresses.
(ii) Measure cost/delay to the neighbours.
(iii) Transmit the information as obtained in (ii) across the subnet.
(iv) Thus by pass the necessity for shortest path algorithm.
(A) (i), (ii) (B) (ii), (iii)
(C) (iii), (iv) (D) (i), (iv)

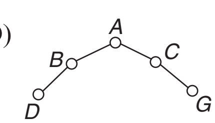
10. In multidestination routing,
(i) Each router makes new copies of the incoming packets.
(ii) It retains the same destination list in all copies.
(iii) It places them on appropriate outgoing lines.
(A) (i), (ii) (B) (ii), (iii)
(C) (iii), (i) (D) (i), (ii), (iii)

11. In a subnet which follows reverse path forwarding, routers *B* and *C* have received packets from *A* which have been further forwarded to *D* and *E* by *B* and to *F* and *G* by *C*. Of this *D*, *G* has always discarded the valid packets. Construct the preferred routing lines in the subnet.

(A)


(B)


(C)


(D)


12. Which of the following layers accept services from network layer and provides services to session layer?
(A) Data link layer
(B) Presentation layer
(C) Transport layer
(D) Physical layer.

13. Which of the below are different metrics for congestion?
(i) Packets discarded for lack of buffer space
(ii) Packets that are retransmitted
(iii) Average packet delay
(iv) Average queue length
(A) (i), (ii), (iii) (B) (ii), (iii), (iv)
(C) (iii), (iv), (i) (D) (i), (ii), (iii), (iv)

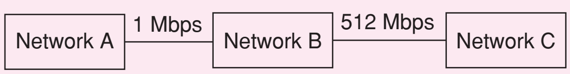
14. What are the ways to decrease congestion?
(i) Put spare routers to use
(ii) Increase bandwidth by routing on alternate lines
(iii) Increase the size of tables in the routers
(iv) Decrease the load
(A) (i), (ii), (iii) (B) (ii), (iii), (iv)
(C) (iii), (iv), (i) (D) (iv), (i), (ii)

15. The algorithm which tells the routers to maintain certain data structures in their memories for congestion control is
(A) Resource Reservation Protocol.
(B) Fair queuing algorithm.
(C) Token bucket algorithm.
(D) None of these

Previous Years' Questions

Common data for questions 1 and 2: Consider three IP networks, *A*, *B* and *C*. Host H_A in network *A* sends messages each containing 180 bytes of application data to a host H_C in network *C*. The TCP layer prefixes a 20 byte header to the message. This passes through an intermediate network *B*. The maximum packet size, including 20 byte IP header, in each network is:
A: 1000 bytes
B: 100 bytes
C: 1000 bytes

The networks *A* and *B* are connected through a 1 Mbps link, while *B* and *C* are connected by a 512 Kbps link (bps = bits per second)



1. Assuming that the packets are correctly delivered, How many bytes, including headers, are delivered to the IP layer at the destination for one application message, in the best case? Consider only data packets. **[2004]**
(A) 200 (B) 220
(C) 240 (D) 260

2. What is the rate at which the application data is transferred to host H_C? Ignore errors, acknowledgements, and other over heads. **[2004]**
(A) 325.5 kbps (B) 354.5 kbps
(C) 409.6 kbps (D) 512.0 kbps

3. In a packet switching network, packets routed from source to destination along a single path having two intermediate nodes. If the message size is 24 bytes and each packet contains a header of 3 bytes, then the optimum packet size is: **[2005]**
(A) 4 (B) 6
(C) 7 (D) 9

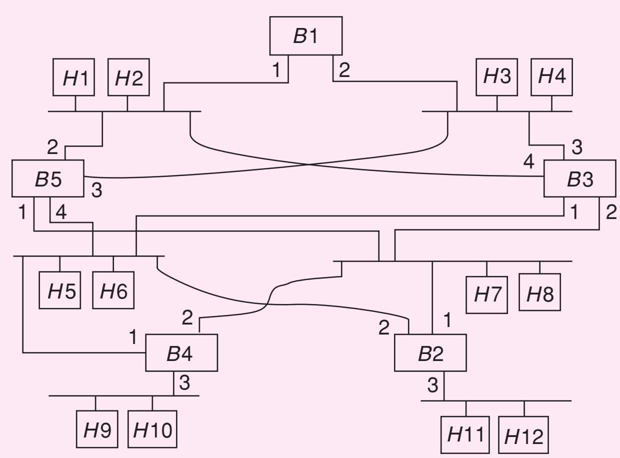
4. Suppose the round trip propagation delay for a 10 Mbps Ethernet having 48-bit jamming signal is 46.4 µs. The minimum frame size is: **[2005]**
(A) 94 (B) 416
(C) 464 (D) 512

5. Station *A* uses 32 byte packets to transmit messages to station *B* using a sliding window protocol. The round trip delay between *A* and *B* is 80 milliseconds and the bottleneck bandwidth on the path between *A* and *B* is 128 kbps. What is the optimal window size that *A* should use? **[2006]**
(A) 20 (B) 40
(C) 160 (D) 320

6. Station *A* needs to send a message consisting of 9 packets to station *B* using a sliding window (window size 3) and go-back-n error control strategy. All packets are ready and immediately available for transmission. If every 5th packet that *A* transmits gets lost (but no acks from *B* ever get lost), then what is the number of packets that *A* will transmit for sending the message to *B*? **[2006]**
(A) 12 (B) 14
(C) 16 (D) 18

Common data for questions 7 and 8: Consider the diagram shown, where a number of LANs are connected by (transparent) bridges. In order to avoid packets looping through circuits in the graph, the bridges organize themselves in a spanning tree. First, the root bridge is identified as the bridge with the least serial number. Next, the root sends out (one or more) data units to enable the setting up of shortest paths from the root bridge to each bridge.

Each bridge identifies a port (the root port) through which it will forward frames to the root bridge. Port conflicts are always resolved in favor of the port with the lower index value. When there is possibility of multiple bridges forwarding to the same LAN (But not through the root port), ties are broken as follows: bridges closest to the root get preference and between such bridges, the one with the lowest serial number is preferred.



7. For the given connection of LANs by bridges, which one of the following choices represents the depth first traversal of the spanning tree of bridges? **[2006]**
(A) B1, B5, B3, B4, B2 (B) B1, B3, B5, B2, B4
(C) B1, B5, B2, B3, B4 (D) B1, B3, B4, B5, B2

8. Consider the spanning tree for the previous question. let Host H1 send out a broadcast ping packet. Which of the following options represents the correct forwarding table on B3? **[2006]**

(A)

Hosts	Port
H1, H2, H3, H4	3
H5, H6, H9, H10	1
H7, H8, H11, H12	2

(B)

Hosts	Port
H1, H2	4
H3, H4	3
H5, H6	1
H7, H8, H10, H11, H12	2

(C)

Hosts	Port
H3, H4	3
H5, H6, H9, H10	1
H1, H2	4
H7, H8, H11, H12	2

(D)

Hosts	Port
H2, H2, H3, H4	3
H5, H7, H9, H10	1
H7, H8, 11, H12	4

9. In a token ring network the transmission speed is 10^7 bps and the propagation speed is 200 metres/μs. The 1-bit delay in this network is equivalent to: **[2007]**
(A) 500 metres of cable.
(B) 200 metres of cable.
(C) 20 metres of cable.
(D) 50 metres of cable.

10. In the slow start phase of the TCP congestion control algorithm, the size of the congestion window **[2008]**
(A) Does not increase
(B) Increases linearly
(C) Increases quadratically
(D) Increases exponentially

11. A computer on a 10 Mbps network is regulated by a token bucket. The token bucket is filled at a rate of 2 Mbps. It is initially filled to capacity with 16 Megabits. What is the maximum duration for which the computer can transmit at the full 10 Mbps? **[2008]**
(A) 1.6 seconds (B) 2 seconds
(C) 5 seconds (D) 8 seconds

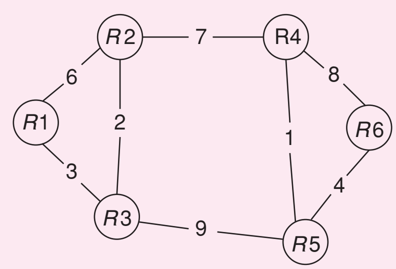
12. Let $G(x)$ be the generator polynomial used for CRC checking. What is the condition that should be satisfied by $G(x)$ to detect odd number of bits in error? **[2009]**
(A) $G(x)$ contains more than two terms
(B) $G(x)$ does not divide $1 + x^k$, for any k not exceeding the frame length
(C) $1 + x$ is a factor of $G(x)$
(D) $G(x)$ has an odd number of terms.

Common data for questions 13 and 14: Frames of 1000 bits are sent over a 10^6 bps duplex link between two hosts. The propagation time is 25 ms. Frames are to be transmitted into this link to maximally pack them in transit (within the link).

13. What is the minimum number of bits (l) that will be required to represent the sequence numbers distinctly? Assume that no time gap needs to be given between transmission of two frames. **[2009]**
(A) l = 2 (B) l = 3
(C) l = 4 (D) l = 5

14. Suppose that the sliding window protocol is used with the sender window size of 2^l, where l is the number of bits identified in the earlier part and acknowledgements are always piggy backed. After sending 2^l frames, what is the minimum time the sender will have to wait before starting transmission of the next frame? (Identify the closest choice ignoring the frame processing time.) **[2009]**
(A) 16 ms (B) 18 ms
(C) 20 ms (D) 22 ms

Common data for questions 15 and 16: Consider a network with 6 routers $R1$ to $R6$ connected with links having weights as shown in the following diagram



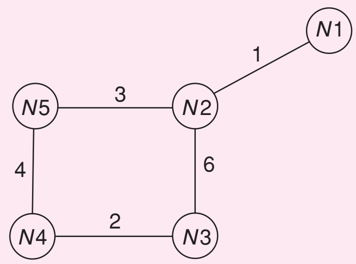
15. All the routers use the distance vector based routing algorithm to update their routing tables. Each router starts with its routing table initialized to contain an entry for each neighbour with the weight of the respective connecting link. After all the routing tables stabilize, how many links in the network will never be used for carrying any data? **[2010]**
(A) 4 (B) 3 (C) 2 (D) 1

16. Suppose the weights of all unused links in the previous question are changed to 2 and the distance

vector algorithm is used again until all routing tables stabilize. How many links will now remain unused? **[2010]**

(A) 0 (B) 1 (C) 2 (D) 3

Common data for questions 17 and 18: Consider a network with five nodes, *N*1 to *N*5 as shown below.



The network uses a distance vector routing protocol. Once the routes have stabilized, the distance vectors at different nodes are as following.

*N*1 : (0, 1, 7, 8, 4)
*N*2 : (1, 0, 6, 7, 3)
*N*3 : (7, 6, 0, 2, 6)
*N*4 : (8, 7, 2, 0, 4)
*N*5 : (4, 3, 6, 4, 0)

Each distance vector is the distance of the best known path at that instance to nodes, *N*1 to *N*5, where the distance to itself is 0. Also, all links are symmetric and the cost is identical in both directions. In each round, all nodes exchange their distance vectors with their respective neighbors. Then all nodes update their distance vectors. In between two rounds, any change in cost of a link will cause the two incident nodes to change only that entry in their distance vectors.

17. The cost of link *N*2-*N*3 reduces to 2 (in both directions). After the next round of updates, what will be the new distance vector at node, *N*3? **[2011]**

(A) (3, 2, 0, 2, 5) (B) (3, 2, 0, 2, 6)
(C) (7, 2, 0, 2, 5) (D) (7, 2, 0, 2, 6)

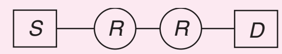
18. After the update in the previous question, the link *N*1-*N*2 goes down. *N*2 will reflect this change immediately in its distance vector as cost, ∞. After the NEXT ROUND of update, what will be the cost to *N*1 in the distance vector of *N*3? **[2011]**

(A) 3 (B) 9 (C) 10 (D) ∞

19. Consider an instance of TCP's Additive Increase Multiplicative Decrease (AIMD) algorithm where the window size at the start of the slow start phase is 2 MSS and the threshold at the start of the first transmission is 8 MSS. Assume that a timeout occurs during the fifth transmission. Find the congestion window size at the end of the tenth transmission. **[2012]**

(A) 8 MSS (B) 14 MSS
(C) 7 MSS (D) 12 MSS

20. Assume that source *S* and destination *D* are connected through two intermediate routers labeled *R*. Determine how many times each packet has to visit the network layer and the data link layer during a transmission from *S* to *D*. **[2013]**



(A) Network layer – 4 times and Data link layer – 4 times
(B) Network layer – 4 times and Data link layer – 3 times
(C) Network layer – 4 times and Data link layer – 6 times
(D) Network layer – 2 times and Data link layer – 6 times

21. Consider a selective repeat sliding window protocol that uses a frame size of 1 kB to send data on a 1.5 Mbps link with a one-way latency of 50 msec. To achieve a link utilization of 60%, the minimum number of bits required to represent the sequence number field is ______. **[2014]**

22. Consider the following three statements about link state and distance vector routing protocols, for a large network with 500 network nodes and 4000 links.

[*S*1] The computational overhead in link state protocols is higher than in distance vector protocols.
[*S*2] A distance vector protocol (with split horizon) avoids persistent routing loops, but not a link state protocol.
[*S*3] After a topology change, a link state protocol will converge faster than a distance vector protocol.

Which one of the following is correct about *S*1, *S*2 and *S*3? **[2014]**

(A) *S*1, *S*2 and *S*3 are all true
(B) *S*1, *S*2 and *S*3 are all false
(C) *S*1 and *S*2 are true, but *S*3 is false
(D) *S*1 and *S*3 are true, but *S*2 is false.

23. Let the size of congestion window of a TCP connection be 32 kB when a timeout occurs. The round trip time of the connection is 100 msec and the maximum segment size used is 2 kB. The time taken (in msec) by the TCP connection to get back to 32 kB congestion window is ______. **[2014]**

24. Which one of the following is TRUE about the interior gateway routing protocols-Routing information protocol (RIP) and Open Shortest Path First (OSPF)? **[2014]**

(A) RIP uses distance vector routing and OSPF uses link state routing
(B) OSPF uses distance vector routing and RIP uses link state routing
(C) Both RIP and OSPF use link state routing
(D) Both RIP and OSPF use distance vector routing

25. Consider the store and forward packet switched network given below. Assume that the bandwidth of each link is 10^6 bytes/sec. A user on host A sends a file of size 10^3 bytes to host B through routers R_1 and R_2 in three different ways. In the first case a single packet containing the complete file is transmitted from A to B. In the second case, the file is spilt into 10 equal parts, and these packets are transmitted from A to B. In the third case, the file is spilt into 20 equal parts, and these packets are sent from A to B. Each packet contains 100 bytes of header information along with the user data. Consider only transmission time and ignore processing, queuing and propagation delays. Also assume that there are no errors during transmissions. Let T_1, T_2 and T_3 be the times taken to transmit the file in the first, second and third case respectively. Which one of the following is CORRECT? **[2014]**

(A) $T_1 < T_2 < T_3$ (B) $T_1 > T_2 > T_3$
(C) $T_2 = T_3$, $T_3 < T_1$ (D) $T_1 = T_3$, $T_3 > T_2$

26. An IP machine Q has a path to another IP machine H via three IP routers R_1, R_2, and R_3.
$Q - R_1 - R_2 - R_3 - H$
H acts as an HTTP server, and Q connects to H via HTTP and downloads a file. Session layer encryption is used with DES as the shared key encryption protocol. Consider the following four pieces of information.
[*I*1] The URL of the file downloaded by Q
[*I*2] The TCP port numbers at Q and H
[*I*3] The IP addresses of Q and H
[*II*4] The link layer addresses of Q and H
Which of *I*1, *I*2, *I*3 and *I*4 can an intruder learn through sniffing at R_2 alone? **[2014]**
(A) Only *I*1 and *I*2 (B) Only *I*1
(C) Only *I*2 and *I*3 (D) Only *I*3 and *I*4

27. An IP router with a Maximum Transmission Unit (MTU) of 1500 bytes has received an IP packet of size 4404 bytes with an IP header of length 20 bytes. The values of the relevant fields in the header of the third IP fragment generated by the router for this packet are **[2014]**
(A) MF bit : 0, Datagram Length: 1444; Offset: 370
(B) MF bit: 1, Datagram Length : 1424; Offset: 185
(C) MF Bit: 1, Datagram Length: 1500; Offset: 370
(D) MF bit: 0, Datagram Length: 1424; Offset: 2960

28. Identify the correct order in which a server process must invoke the function calls accept, bind, listen, and recv according to UNIX socket API. **[2015]**
(A) listen, accept, bind, recv
(B) bind, listen, accept, recv
(C) bind, accept, listen, recv
(D) accept, listen, bind, recv

29. For a host machine that uses the token bucket algorithm for congestion control, the token bucket has a capacity of 1 megabyte and the maximum output rate is 20 megabytes per second. Token arrive at a rate to sustain output at a rate of 10 megabytes per second. The token bucket is currently full and the machine needs to send 12 megabytes of data. The minimum time required to transmit the data is _____ seconds. **[2016]**

30. Consider the following statements about the routing protocols. Routing Information Protocol (RIP) and Open Shortest Path First (OSPF) in an IPv4 network.
I: RIP uses distance vector routing
II: RIP packets are sent using UDP
III: OSPF packets are sent using TCP
IV: OSPF operation is based on link-state routing
Which of the statements above are CORRECT? **[2017]**
(A) I and IV only (B) I, II and III only
(C) I, II and IV only (D) II, III and IV only

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** D	**3.** A	**4.** B	**5.** A	**6.** A	**7.** C	**8.** B	**9.** C	**10.** C
11. B	**12.** B	**13.** B	**14.** A	**15.** A					

Practice Problems 2

1. D	**2.** B	**3.** A	**4.** B	**5.** C	**6.** C	**7.** D	**8.** B	**9.** B	**10.** C
11. C	**12.** C	**13.** D	**14.** D	**15.** A					

Previous Years' Questions

1. D	**2.** B	**3.** D	**4.** D	**5.** B	**6.** C	**7.** C	**8.** A	**9.** C	**10.** D
11. B	**12.** C	**13.** D	**14.** B	**15.** C	**16.** B	**17.** A	**18.** C	**19.**	**20.** C
21. 5	**22.** D	**23.** 1100 to 1300	**24.** A	**25.** D	**26.** C	**27.** A	**28.** B	**29.** 1.1	
30. C									

Chapter 3

TCP/UDP

LEARNING OBJECTIVES

- *Transport layer*
- *User Diagram Protocol (UDP)*
- *TCP/IP*
- *TCP/IP vs OSI reference model*
- *TCP state transition diagram*
- *TCP flow control*
- *Application layer*
- *ICMP, SMTP, POP3, IMAP 4, HTTP, FTP*
- *DNS*
- *Network devices*

Transport Layer

Real communication takes place between two applications programs i.e., processes. For this, process-to-process delivery is needed. A mechanism is required in order to deliver data from one of these processes running on the source host to the corresponding process running on the destination host.

The transport layer is responsible for process-to-process delivery.

Addressing in Transport Layer

Port addresses

- A transport layer address is a port number.
- The destination port number is needed for delivery and the source port number is needed for reply.
- The port numbers are 16-bit integers ranging from 0 to 65535.

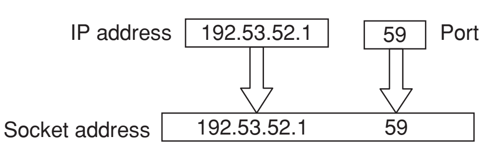
The IANA (Internet Assigned Number Authority) has divided the port numbers as:

- Well-known ports (0 to 1023)
- Registered ports (1024 to 49,151)
- Dynamic or private or ephemeral ports (49,152 to 65,535)

Socket address

Process to process delivery needs two identifiers, IP address and port address at each end to make a connection.

The combination of an IP address and a port number is socket address.



Protocols at transport layer

1. UDP
2. TCP
3. SCTP

User Datagram Protocol (UDP)

- UDP is connectionless protocol.
  - There is no mechanism for connection establishment or connection termination.
  - The packets may be delayed or lost or may arrive out of sequence, i.e., there is no acknowledgement.
  - Each user datagram sent by UDP is an independent program. Even if the user datagram's are coming from the same source program and going to the same destination process, there is no relationship between the different datagrams.

Thus, user datagrams can travel on a different path.

- Multicasting capability is embedded in UDP.
- It is a simple, unreliable transport protocol.
  - There is no flow control, no window mechanism.
  - There is no error control as well except for the checksum. The sender does not know if a message has been lost or duplicated. When the receiver detects an error through the checksum, the datagram is discarded silently.
- It is used in real-time applications.
  - The header length is fixed, of 8 bytes. Real time applications require a constant flow of data. Moreover, the unreliability (fast and less complex service) of UDP aids in real-time applications like voice over IP, online games etc.
- It encapsulates and decapsulates messages in an IP datagram.

User Datagram

UDP packets have other name called user datagrams. They have a fixed size header of 8 bytes. The datagram is divided into 4 fields.

Source port number (16-bits)	Destination port number (16-bits)
Total length (16-bits)	Checksum (16-bits)

Figure 1 User datagram header format

1. ***Source Port Number*** It is a 16-bit number used by the process running on the source host.
2. ***Destination Port Number*** It is also a 16 bit number used by the process running on the destination host.
3. ***Total length*** It is a 16-bit field, it defines the total length of the user datagram header and data. It can define a total length of 0 to 65535 bytes. A UDP packet is encapsulated in an IP packet.

UDP length = IP length – IP header's length

4. ***Checksum:*** It is optional field, if not available the field is filled with 1's. It is used to detect errors in user datagram (header plus data).

Protocols That Take UDP Services

Following are a few protocols that take the services of UDP:

1. Domain Name Service (port – 53): UDP is used to send small data. If the data is less than 512 bytes, then DNS uses UDP else it goes for TCP.
2. Trivial File Transfer Protocol (port – 69): TFTP is used to transfer simple and small files, it uses UDP service.
3. Routing Information protocol: It uses UDP service on port number 520 to update routers.
4. Simple Network Management Protocol (SNMP): The SNMP agent receives requests on UDP port 161 for management process.
5. Bootstrap protocol (BOOTP): For client (port 68) and for server (port – 67).

UDP Checksum Calculation

- The checksum includes a pseudo header, the UDP header and the data coming from the application layer.

32-bit source IP address		
32-bit destination IP address		
All 0's	8-bit protocol (17)	16-bit UDP total length

Figure 2 Pseudo header of UDP for checksum calculation

- The value of protocol field is 17. If this value is changed during transmission, the checksum calculation at the receiver will detect it and UDP drops the packet.
- If the checksum is not calculated, the field is filled with 0's. This means checksum calculation is optional.
- The calculated checksum can never be all 1's as this implies that the sum is all 0's. But this is impossible because for this the value of fields have to be 0's.

TCP/IP

TCP/IP is a network model which is used for the internet architecture, its main objectives are

- Connecting the multiple networks.
- Maintaining the intact connection between two machines, which are functioning.

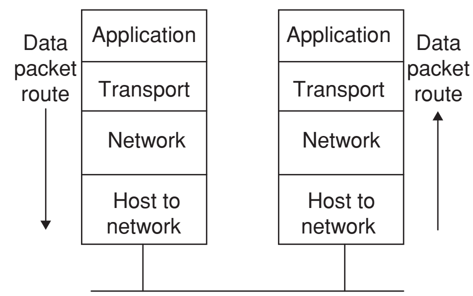


Figure 3 TCP/IP network protocol

TCP/IP vs OSI Reference Model

OSI	TCP/IP
(1) There are 7 layers	(1) There are 5 layers
(2) There is no definition for multicasting	(2) Multicasting is clearly defined
(3) Less flexibility	(3) Lot of flexibility
(4) Practically it is not suggestible as it is based on theoretical rules	(4) It is based on practical rules

- TCP stands for Transmission Control Protocol.
- It is connection-oriented protocol.
  - It creates a virtual connection between two TCPs to send data then data is transferred and at the end the connection is released.
  - There is acknowledgement mechanism for safe and sound arrival of data.
- It is a reliable transport protocol.
  - Uses flow and error control.
  - Slower and more complex service.
  - Duplicate segments are detected, lost segments are resent, the bytes are delivered to the end process in order.
- It is a stream-oriented protocol.

- Allows the sending process to deliver data as a stream of bytes and allows the receiving process to obtain data as a stream of bytes.
- TCP offers full-duplex service.
  - Data can flow in both directions at the same time.
  - Each TCP has a sending and receiving buffer.
  - It cannot be used in real time applications as the header length varies from 20-to-60 bytes, moreover it needs reliability.

TCP Header Format

- A packet in TCP is called a segment. The segment consists of a 20-to-60 bytes header.
- If there are no options, the header is of 20 bytes.

Source port address (16-bits)						Destination port address (16-bits)		
Sequence number (32-bits)								
Acknowledgement number (32-bits)								
HLEN (4-bits)	Reserved (6-bits)	URG	ACK	PSH	RST	SYN	FIN	Window size (16-bits)
Checksum (16-bits)					Urgent Pointer (16-bits)			
Options and Padding								

Figure 4 TCP header format

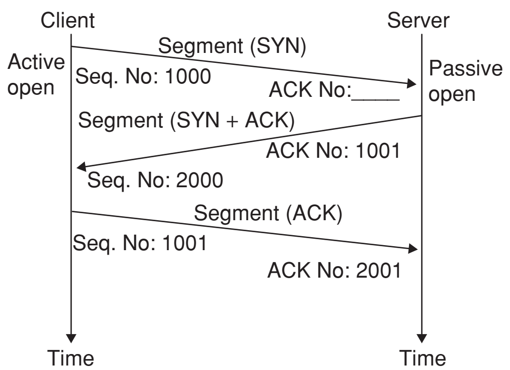
- If there are options, the header goes upto 60 bytes.
- ***Source Port addresses*** A 16-bit field that defines the port number of the application program in the host that is sending the segment.
- ***Destination Port address*** A 16-bit field that defines the port number of the application program in the host is receiving the segment.
- ***Sequence number*** A 32-bit field whose value defines the number of the first data byte contained in that segment. During connection establishment, a random number is generated to create an initial sequence number (ISN) which is usually different in each direction.
- ***Acknowledgement Number*** A 32-bit field whose value defines the number of the next byte, a party expects to receive. If the receiver of the segment has successfully received byte number x from the other party, it defines $x + 1$ as the acknowledgement number. The acknowledgement number is cumulative.
- ***HLEN(Header Length)*** This field is of 4-bit. The header length can be between 20 and 60 bytes. The value of this field can be between $5(5 \times 4 = 20)$ and $15(15 \times 4 = 60)$.
- ***Reserved*** This is a 6-bit field which is reserved for future use.
- ***Control*** This field contains 6 control flags. These are as follows.
  - URG: Urgent pointer. This flag is set when the value of urgent pointer field is valid.
  - ACK: Acknowledgement pointer. This flag is set when the value of acknowledgement field is valid. It is not set at the start of connection during 3-way handshake.
  - RST: Reset pointer. Used to reset the connection, reject an invalid segment or refuse an attempt to open a connection.
  - PSH: Push pointer. When a data is pushed the flag is set.
  - SYN: Synchronization pointer, used to synchronize sequence numbers during connection. If it is set to 1, then it is ISN. If set to 0, then it is the accumulated sequence number of the first data byte of the segment for the current session.
  - FIN: Finish Pointer. It is used to terminate a connection. It indicates that the sender is not interested in sending any more data.
- ***Window size*** The field size is of 16-bits and thus the maximum size of the window is 65,535 bytes. This field is determined by the receiver and thus referred to as the receiving window. The window size is variable.
- ***Checksum*** The inclusion of this 16-bits field is mandatory in TCP. The calculation of the checksum for TCP follows the same procedure as in UDP, only the value of protocol field in TCP is 6.
- ***Urgent pointer*** This 16-bit field, is valid only if the urgent flag is set. This field is used when the segment contains urgent data.
- ***Options and padding*** When the header length is greater than 5, option field is used to make the segment into the multiples of 32. Padding is used to ensure the ending of TCP header, it is composed to 32 zeros.

TCP Connection

- TCP is connection-oriented and the connection is virtual not physical.
- TCP operates at a higher level. TCP uses the services of IP to deliver individual segments to the receiver, but it controls the connection itself. Lost or corrupted segments are retransmitted.
- In TCP, connection-oriented transmission requires three phases:
  1. Connection establishment
  2. Data transfer
  3. Connection termination

Connection establishment

- The connection establishment in TCP is called three-way handshaking.
- The process starts with the server. The server program tells its TCP that it is ready to accept a connection. This is a request for a passive open.
- The client program issues a request for an active open. A client that wishes to connect to an open server tells its TCP that it needs to be connected to a particular server. Hence the TCP can start the three-way handshaking process as shown in the figure.



1. The first segment which is a SYN segment is identified by the randomly generated number and is assigned to a 1 byte dummy data indicating the sequence number.
2. Again from the server side a randomly generated number is assigned for the dummy data indicating the first byte.
3. A SYN segment cannot carry data, but it consumes one sequence number.

   A (SYN + ACK) segment cannot carry data, but consumes one sequence number.

   An ACK segment, if carrying no data, consumes no sequence number.
4. Initial Sequence Number (ISN) 1000 is sent from the client to server. Server receives the segment 1000 and is expecting segment 1001 as the next one.

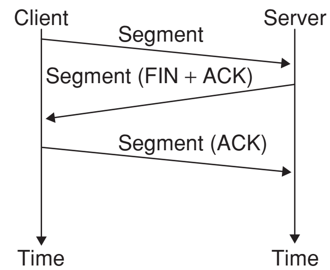
Data transfer

- After the connection is established, bidirectional data transfer can take place. Both the client and server can send data and acknowledgements.
- The data segments sent by the client have the PSH (push) flag set so that the server TCP knows to deliver data to the server process as soon as they are received.
- Sometimes the sending application program wants a piece of data to be read out of order by the receiving application program that means an application program needs to send urgent bytes then in this case the URG bit is set and the segment is send. The sending TCP creates a segment and inserts the urgent data at the beginning of the segment.

Connection termination

There are two options for connection termination.

Three-way handshaking

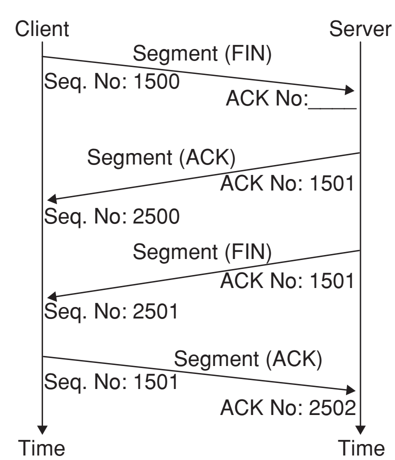


- The client process sends the first segment, a FIN segment in which the FIN flag is set. The FIN segment consumes one sequence number if it does not carry data.
- The server TCP sends the second segment, a FIN + ACK segment, to confirm the receipt of the FIN segment from the client and at the same time announce the closing of the connection in the other direction.

  The FIN + ACK segment consumes one sequence number if it does not carry data.
- The client sends the last ACK segment to the server. This segment contains acknowledgement number which is 1 plus, the sequence number received in the FIN segment from the server.

Four-way handshaking

- ***Half-close**:* In TCP, one end can stop sending data while still receiving data. This is half close.
- The client half-closes the connection by sending a FIN segment.
- The server accepts the half-close by sending the ACK segment. The data transfer from the client to the server stops.
- When the server has sent all the processed data, it sends a FIN segment, which is acknowledged by an ACK from the client.



TCP State Transition Diagram

The functionality of TCP connection setup, communication phase and termination phase can be easily depicted by the state transition diagram where the TCP will be only at one state at a time with respect to server or client.

A change in the state is only observed after receiving a request for change like ACK (acknowledgement).

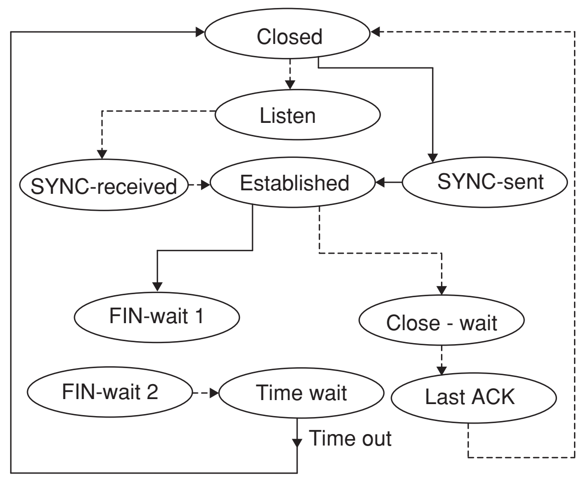


Figure 5 State transition diagram

Here, Solid line '–' is for client states, Break line '---' is for server states

State 'Closed' is common for both client and server. Initially the client and the server are in the closed state where no TCP connection is set. When an application request for a TCP connection then the client changes its state from closed to SYNC-sent state.

Client states

1. SYNC-sent — After the client sends a SYNC-sent and receives an ACK for the sent SYNC segment, it changes its state to ESTABLISHED STATE.
2. Established — In this state the client and the server exchange user data. After the requested application is completed, it sends a FIN segment and changes its state to FIN-wait 1.
3. FIN-wait 1 — FIN-wait 1 changes to FIN-wait 2 after receiving an ACK for sent FIN segment.
4. FIN-wait 2 — The client will remain in this state until it receives a FIN segment from the server. When the last ACK is sent by the client, the client changes its state to Time-wait.
5. Time-wait — A timer is set at this state for any delayed segment from the server which are removed or discarded at the client and after the timeout is reached, the client changes its state from present state to the closed state again.

Server states

1. Listen — This is a passive state where the server always listens for the SYNC request segment on different TCP ports.
2. SYN-received — After receiving the SYNC request from the client, the server acknowledges its state to the Established state.
3. Closed-wait — The server changes its state from Established to close-wait after receiving the finish segment from the client. In this state the server sends an ACK and finish segments. Afterwards it changes the state to last-ACK.
4. Last-ACK — In this state the server expects the last ACK segment from the client, as and when it receives the ACK segment it changes its state to again closed state.

TCP Congestion Control

- Deals with end-to-end delivery.
- Congestion handling in TCP is based on three phases:
  - Slow start
  - Congestion avoidance
  - Congestion detection

Slow start (exponential increase)

1. By default the receiver window size is initially set to 1.
2. In the first instance the transmitter receives an ACK for the window size indicating the receiver window size as 2 segments.
3. After 2 segments are sent it is acknowledged with 4 segments.
4. After 4 segments are sent it it acknowledged with 8 segments.
5. This is exponential growth and this growth continue until the window size reaches the threshold value.
6. If there is delayed ACKs, the increase in the size of the window is less than power of 2.

Congestion avoidance (additive increase)

1. To avoid the congestion before it happens, the exponential growth of slow start algorithm must be slowed down.
2. When the threshold is reached, then the additive phase begins. Here each time the whole window of segments is acknowledged, the size of congestion window is increased by 1.

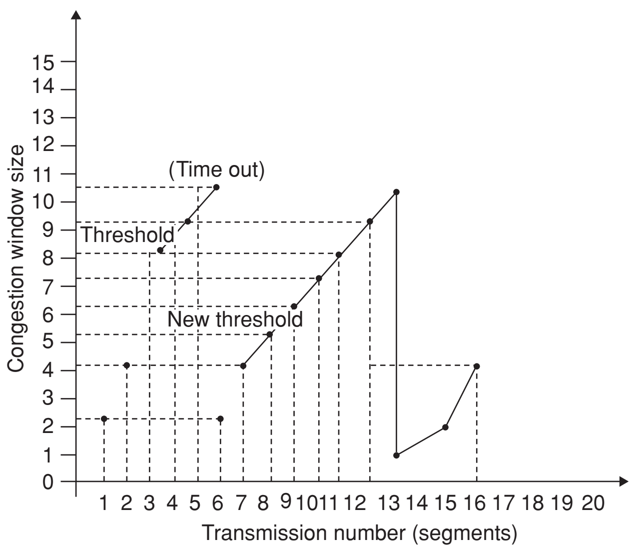
Congestion detection (multiplicative decrease)

1. If congestion occurs, the congestion window size must be decreased. The only way the sender can guess the congestion has occurred is by the need to retransmit a segment.
2. Retransmission can occur in two cases:
   (i) When a timer times out.
   (ii) When 3 ACKs are received.

3. In both the cases the size of threshold is dropped to one-half of the current window size and the window size is decreased to initial window size "1".
   This is multiplicative decrease.

Example: Let us take an example to explain the TCP congestion control.

Consider an instance of TCP additive increase, multiplicative decrease algorithm where the window size at the start of slow-start phase is 2 MSS (Maximum Segment Size) and threshold value is 8 MSS. The timeout occurs at the fifth transmission. Then what is the congestion window size at the end of the tenth transmission?



Window size is 2 MSS initially.

8 MSS is threshold value, after this there is only increase of 1-1 window size till timeout value which is 10.

The new threshold value becomes half of the value of current congestion window i.e., 5.

Timeout remains the same i.e., 10.

At 10th transmission the window size is 7.

After time-out, at 13th transmission window size = 1 and at 14th transmission window size = 2.

TCP Flow Control

- For flow control sliding window protocol is used.
- The window size is set by the receiver and is controlled by the receiver. The window size is not fixed (variable).
- The sliding window protocol in TCP looks like the Go-Back-N protocol because it does not use NAKs; it looks like Selective Repeat because the receiver holds the out-of-order segments until the missing ones arrive.
- A sliding window is used to make transmission more efficient as well as to control the flow of data so that the destination does not become overwhelmed with data. TCP sliding windows are byte-oriented.

TCP Error Control

- TCP provides reliability using error control.
- Error control includes mechanism for detecting corrupted segments, lost segments, out-of-order segments and duplicated segments.
- Error detection and correction in TCP is achieved through the use of three tools:
  - Checksum
  - Acknowledgment
  - Time-out

Checksum

Each segment includes a checksum field which is used to check for a corrupted segment. A 16-bit checksum is mandatory in every segment.

Acknowledgement

- There is no negative ACK in TCP.
- There is no ACK for the received ACK.
- Only the correctly received segments are acknowledged, if any segment if found to be corrupted through checksum such segments are not acknowledged.

Time-out

Different timers are deployed for error control.

1. ***Time-awaited timer:*** This timer is used to handle TCP termination process specially to handle duplicate finish segments.
   Its value is set to twice the life time of a segment.
2. ***Keep-Alive Timer:*** This timer is used to handle long idle TCP connections.
   By default its value is 2 hours, beyond which a probe (1 byte dummy data) is used for 10 consecutive times with a separation of 75 milliseconds. If there is no response beyond this, then the connection is terminated.
3. ***Persistence Timer:*** This timer is used to handle Zero(0) window size scenario.
   The sender sends 1 probe every 60 seconds until it receives a non-zero window size from where the communication resumes.
4. ***Retransmission Timer:*** This timer is used for handling any lost segments. Its value is twice the Round trip time, i.e., $2 \times$ RTT.
   RTT is time needed for a segment to reach a destination and for an acknowledgement to be received.

APPLICATION LAYER

An interface between the networks is called application. This section introduces two important concepts:

- Application Layer: The application layer of the OSI model provides the first step of getting data onto the network.

- Application Software: Applications are the software programs used by people to communicate over the network. Examples of application software, includes HTTP, FTP, e-mail, and others, used to explain the differences between these two concepts.

In the OSI model, information is passed from one layer to the next, starting at the application layer on the transmitting host and proceeding down the hierarchy to the physical layer, then passing over the communications channel to the destination host, where the information proceeds back up the hierarchy, ending at the application layer.

The following six steps explain the procedure:

1. People create the communication.
2. The application layer prepares human communication for transmission over the data network.
3. Software and hardware converts communication to digital format.
4. Application layer services initiate the data transfer.
5. Each layer plays its role. The OSI layers encapsulate data down the stack. Encapsulated data travels across the media to the destination. OSI layers at the destination unencapsulate the data.
6. The application layer receives data from the network and prepares it for human use.

The application layer, layer 7, is the top layer of both the OSI and TCP/IP models. Layer 7 provides the interface between the application you use to communicate and the underlying network over which your messages are transmitted. Application layer protocols are used to exchange data between programs, running on the source and destination hosts.

TCP/IP Application Layer Protocol

The most widely known TCP/IP application layer protocols are those that provide the exchange of user information. These protocols specify the format and control information necessary for many of the common internet communication functions. Among these, TCP/IP protocols are the following.

- Domain name system (DNS) is used to resolve internet names to IP addresses.
- Hypertext transfer protocol (HTTP) is used to transfer files that make up the web pages of the world wide web.
- Simple mail transfer protocol (SMTP) is used for the transfer of mail messages and attachments.
- Telnet, a terminal emulation protocol, is used to provide remote access to servers and networking devices.
- File transfer protocol (FTP) is used for interactive file transfers between systems.

Application Layer Services

Programs such as file transfer or network print spooling, might need the assistance of application layer services to use network resources. Although transparent to the user, these services have interface with the network and prepares the data for transfer. Different types of data whether it is text, graphics or video require different network services to ensure that it is properly prepared for processing by the functions occurring at the lower layers of OSI model. Application layer services establish an interface to the network and protocols provide the rules and formats that govern how data is treated, a single executable program can use all three components. For example, while discussing "Telnet", you could be referring to the Telnet application, the Telnet service, or the Telnet protocol.

Application Layer Protocol Functions

Both the source and destination devices use application layer protocols during a communication session. For the communications to be successful, the application layer protocols implemented on the source and destination host must match.

Protocols perform the following tasks

- Establish consistent rules for exchanging data between applications and services loaded on the participating devices.
- Specifies how data inside the messages is structured and the types of messages that are sent between source and destination. These messages can be requests for services, acknowledgements, data messages, status messages, or error messages.
- Defines message dialogues, ensuring that a message being sent is met by the expected response and that the correct services are invoked when data transfer occurs.

Applications and services can also use multiple protocols in the course of a single conversation. One protocol might specify how to establish the network connection and another might describe the process for the data transfer when the message is passed to the next lower layer.

A single application can employ many different supporting application layer services. Thus, what appears to the user as one request for a web page might, in fact, amount to dozens of individual requests. For each request, multiple processes can be executed. For example, the FTP requires a client to initiate a control process and a data stream process to a server. Additionally, servers typically have multiple clients requesting information at the same time, as shown in the figure below. For example, a Telnet server can have many clients requesting connections to it. These individual client requests must be handled simultaneously and separately for the network to succeed. The application layer processes and services rely on support from lower layer functions to successfully manage the multiple conversations.

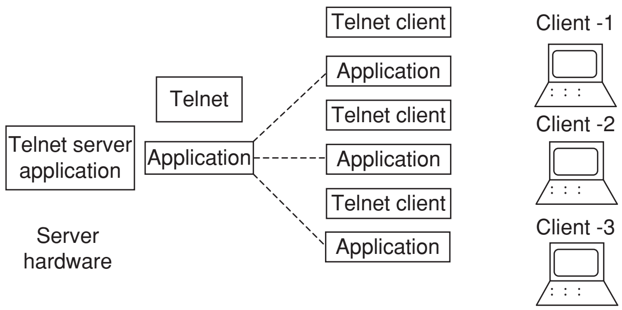


Figure 6 Multiple client's service Requests

Application Layer Protocols

The transport Layer uses an addressing scheme called a port number. Port numbers identify application layer services that are source and destination of data. Server programs generally use predefined port numbers that are commonly known by clients.

Some of these services are

- Domain Name System (DNS): TCP/UDP Port 53
- HTTP: TCP Port 80
- Simple Mail Transfer Protocol (SMTP): TCP Port 25
- Post office Protocol (POP): UDP Port 110
- Telnet: TCP Port 23
- DHCP: UDP Port 67
- FTP: TCP Ports 20 and 21

Internet Control Message Protocol (ICMP)

- Used by hosts and gateways to send notification of datagram problems back to the sender.
- Used for error reporting and query messages.
- Helpful in network debugging.
- Uses the services of TCP and UDP with the port number 7 as the ping command which is used for testing, this testing is done from a source which starts at the application layer and reaches network through transport layer.
- ICMP is encapsulated into an IP datagram and then transmitted into the network, if the protocol filed in the IP datagram is 1 then the IP datagram is said to be carrying ICMP message.

Types of messages

Error reporting

- **Destination Unreachable:** The packet is discarded due to the host not present in the network or the host is not responding to the request.
- **Source Quench:** The packet is discarded due to the congestion in the network.
- **Parameter Problem:** The packet is discarded due to the processing problem observing a change in the header format of the I/P datagram.
- **Time Exceeded:** The packet is discarded because the TTL value is decremented to zero(0).
- Redirection: Here the packet is not discarded but redirected to a network as the host doesn't belong to this network.

Query message

Router solicitation and router advertisement request and reply: Router solicitation is a request generated by the source requesting the router's presence in the network.

The response is a router advertisement generated by the router broadcasting its network id and its presence in the network.

Address mask request and reply: If by any means the node is unable to identify the network bits in its I/P address then this request is used by the source to a router requesting for the network id, the reply is also unicast in this scenario.

Time stamp echo request and reply: This is used to calculate the round trip time of a packet for network diagnose or debugging.

Echo request and reply: This is used to see the presence of a host or a router in the network. For example PING.

SMTP

- SMTP stands for simple mail transfer protocol.
- It uses the services of TCP on port number 25.
- It is a push protocol. Even when the destination is not interested to receive the message this push approach of the SMTP makes the receiver receive the message.
- Components of SMTP:

1. User Agent (UA) :
   (i) It provides Graphical User Interface access to the user.
   Example: Netscape navigation, Mozilla Firefox.
   It also provides command-driven access in early days.
   (ii) It handles the inbox transactions:
   (a) Composing messages: Helps the user compose the e-mail message to be sent out.
   (b) Reading messages: Helps to read incoming messages by checking the mail in the incoming mail box.
   (c) Replying to messages: Sends the message to the sender or recipients of the copy.
   (d) Forwarding messages: Sends the message to a third party.
   (e) Handling mailboxes: Two mailboxes, an inbox and an outbox are created by the user agent. The inbox keeps all the received e-mails until they are deleted by the user. The outbox keeps all the sent e-mails until the user deletes them.

2. Mail transfer agent (MTA): The actual mail is transferred using MTA.
3. Multipurpose Internet mail extension (MIME): By default SMTP uses ASCII format for transaction. But few languages like Japanese, German etc do not support ASCII format. Hence for carrying non-ASCII form of transactions MIME is used in congestion with SMTP. Thus, MIME is a set of software functions that transforms non-ASCII data (stream of bits) to ASCII data and vice-versa.
4. Mail access protocol (MAP): MAP is a pull approach where the emails of a client are retrieved from the mail server i.e., it is used to retrieve the clients emails from the mail server.

   Two protocol of MAP are

   (i) POP 3 (Post Office Protocol)

   (ii) IMAP4 (Internet MAP)

POP3

1. It is a pull protocol.
2. It uses the services of TCP on port number 110.
3. POP3 has several drawbacks and hence it is currently not in use.
   - A user cannot have different folders on the server.
   - A user cannot partially check the contents of the mail before downloading.
   - A user cannot search a mail with a keyword.
   - The user is not allowed to organize the mail on the server.

(4) Modes of POP3

   (i) Copy mode: The mails are copied from the mail server onto the client.

   (ii) Delete mode: The mails are transferred from the mail server to the client and deleted at the mail server. By default POP3 uses delete mode.

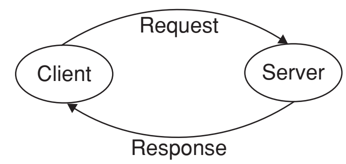
IMAP 4

To overcome the drawbacks of POP3, IMAP4 is in current use. It provides the following functions:

1. A user can create, delete or rename mail boxes on the mail server.
2. A user can create a hierarchy of mailboxes in a folder.
3. A user can partially download e-mail.
4. A user can check the e-mail header before downloading and can search the contents of the e-mail for any specific character prior to downloading.

HTTP

- HTTP stands for Hyper Text Transfer Protocol.
- It uses the services of TCP on well known port 80.
- It is a protocol mainly used to access data on the World Wide Web (www).
- HTTP functions as a combination of FTP and SMTP.
- It uses only one TCP connection, there is no separate control connection, only data is transferred between the client and the server.
- HTTP messages are read and interpreted by the HTTP server and HTTP client (browser).
- It works on two commands request and reply.
- It is a stateless protocol as it does not have any mapping from one transaction onto the other and treats a request and reply as a pair every time.



HTTP1.1 has several request types called methods:

1. GET: Requests a document from the server.
2. HEAD: Requests information about a document but not the document itself.
3. POST: Sends some information from the client to the server.
4. PUT: Sends a document from the server to the client.
5. TRACE: Echoes the incoming request.
6. CONNECT: Reserved.
7. OPTION: Inquires about available options.

- HTTP supports proxy servers. A proxy server is a computer that keeps copies of responses to recent requests. This reduces the load on the original server, decreases traffic and improves latency.
- HTTP Connections:

  (i) *Non-persistence:* In this connection approach for every request and reply (response) as a pair, a separate TCP connection is established every time. It suffers from slow start process. This was present in http version 1.0. Two RTTS are required to fetch each object.

  (ii) *Persistence:* Here a single TCP connection is set on which multiple request and response can be made. This is observed from http version 2.0 onwards (apache http server). For http/1.1 is default. Hence we have reduced network congestion and faster content delivery.

File Transfer Protocol (FTP)

- FTP uses the services of TCP.
- It needs two TCP connections:
  - Uses well-known port 21 for the control connection.
  - Uses well-known port 20 for the data connection.

- Mode of access:

  FTP(TCP) – requires username and password.
  TFTP(UDP) – requires no username and password.
- Types of files supported by FTP:

  i. ASCII: By default FTP follows ASCII mode for file transfer. It is composed of 7-bit + 1 parity bit.
  ii. EBCDIC: If any node supports EBCDIC then this type of technique is used for file transfer. BCDIC supports 8 bits data format and is used in IBM. There is no error control i.e., there is no parity bit.
  iii. Image file: If the file to be sent is very large then continuous streams of 0s and 1s are sent to the transport layer. This is image file. Here FTP does not care of code, it is done by the lower layers.

- **Transmission mode of FTP:** FTP can transfer a file across the data connection by using one of the following three transmission modes:

  i. Stream mode: This is the default mode. Data are delivered from FTP to TCP as a continuous stream of bytes.
  ii. Block mode: Data is delivered from FTP to TCP in blocks. Each block is preceded by a 3-byte header. The first byte is called the block descriptor, the next two bytes define the size of the block in bytes.
  iii. Compressed mode: If the file is big then the data is compressed. The compression method which is mostly used is run-length encoding. Consecutive appearances of a data unit are replaced by one occurrence and the number of repetitions. In a binary file, null characters are compressed.

DNS

- Stands for Domain Name System.
- The DNS is a client/server application that identifies each host on the Internet with a unique user-friendly name i.e., it is used to map an Uniform Resource Locator (URL) to an IP address.
- DNS can use the services of UDP or TCP using the well-known port 53.
- If the size of the response message is more than 512 bytes, it uses the TCP connection.
- When the size of the response message is less than 512 bytes, UDP connection is used. Even though the size of message is not known then also UDP can be used. The UDP server will truncate the message if the message size is more than 512 bytes.
- DNS organizes the namespace in a hierarchical structure to decentralize the responsibilities involved in naming.
- DNS can be pictured as an inverted hierarchical tree structure with one root node at the top and a maximum of 128 levels. Each node in the tree has a domain name.

For example, on the Internet, the domain names, such as http//www. cisco.com, are much easier for people to remember than 198.132.219.25. Also if, cisco decides to change the numeric address, it is transparent to the user, because the domain name will remain http//www.cisco.com. The new address will simply linked to the existing domain name and connectivity is maintained as shown in the figure.

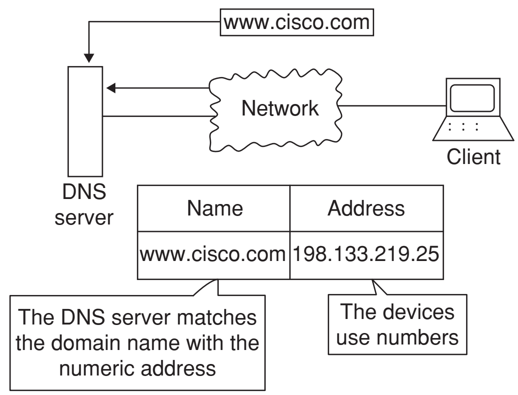


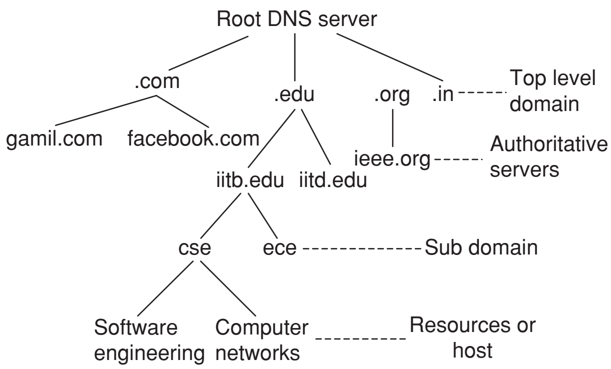
Figure 7 DNS addresses

When networks were small, it was a simple task to maintain the mapping between domain names and the addresses they represent. However, as networks began to grow and the number of devices increased, this manual system became unworkable. DNS was created for domain name to address resolution for these networks. DNS uses a distributed set of servers to resolve the names associated with these numbered addresses.

The DNS protocol defines an automated service that matches resource names with the required numeric network address. It includes the format for queries, responses, and data formats. DNS protocol communications use a single format called a message. This message format is used for all types of client queries and server responses, error messages, and the transfer of resource record information between servers. DNS is a client/server service, however, it differs from the other client/server services. Where as other services use a client that is an application (Web browser, e – mail, client, and so on) the DNS client runs as a service itself. The DNS client, sometimes called the DNS resolver, supports name resolution for the other network applications and other services that need it.

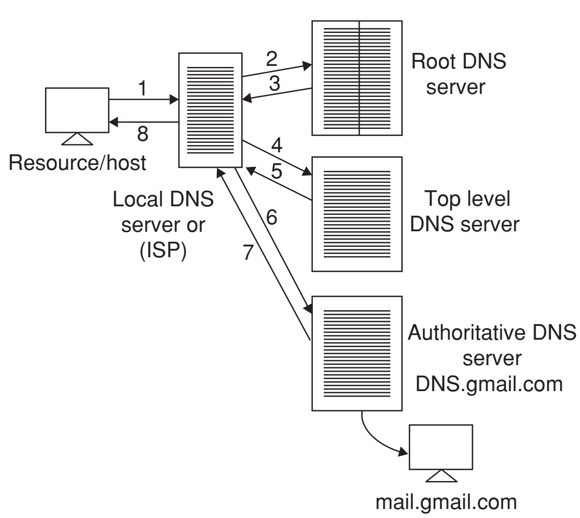
When configuring a network device, you generally provide one or more DNS server addresses that the DNS client can use for name resolution. Usually the Internet Service Provider (ISP) gives you the address to use for the DNS servers. When a user's application requests to connect to a remote device by name, the requesting DNS client queries one of these DNS servers to resolve the name to a numeric address.

- The domain name space consists of a tree of domain names. Each node or leaf in the tree has zero or more resource records, which holds information associated with the domain name. The tree sub-divides into zones beginning at the root zone. A DNS zone consists of a collection of connected nodes authoritatively served by an authoritative name server.



Components of DNS

1. Root DNS Server : Root name servers keep track of all the authoritative name servers of each of the top level domain (TLD) name servers.
2. Top Level Domain: It provides the information regarding the presence of different zone files like
   (i) based on geographical location (country domain): us—for United Statesm, in—for India
   (ii) based on general attributes (generic domain): com—used by commercial organization
   Example, gmail.com
   .edu—used by educational institutes
   .org—used by non-profit organizations
   Example, ieee.org
   .gov—used by government institutions
   Example, nasa.gov
   .mil—used by military organizations
   Example, army.mil
3. Zones: The TLD and the domains under TLD are divided into smaller units with the help of delegation. The domain is divided into small units, so that it can be managed easily. These small units are zones.
4. Authoritative DNS servers checks whether authoritative name servers are located in the DNS hierarchy.



Dns Resource Records (RR)

- Every domain, whether it is a TLD, subdomain or single host have a set of resource records associated with it in the DNS distributed data base.
- Resource Records provide the mapping of host name to IP address. When a query is made to the DNS server, the host or server. who sends that query receives a response which is nothing but the resource record associated with it.
- A Resource Record (RR) is a 5 tuple that contains (Name, Time to live, class, Type, Value)
  (i) Name: It is the domain name to which this RR belongs to. More than one resource records may exists for the same domain.
  (ii) Time to live: The TTL is measured in seconds and it is a 32-bit integer.
  (iii) Class: This field contains the value 'IN' which tells whether this record is used by internet or not.
  (iv) Type: Defines type of RR address, name service, canonical name.
  (v) Value: This field can be a number, ASCII strings or any domain.

Networking Devices

Repeater

In digital communication systems, a repeater is a device that receives a digital signal on an electromagnetic or optical transmission medium and regenerates the signal. Repeaters remove the unwanted noise in an incoming signal. Unlike an analog signal, the original digital signal, even if weak or distorted, can be clearly perceived and restored. With analog transmission, signals are re strengthened with *amplifiers* which unfortunately also amplify noise as well as information.

Hub

A hub is the central part of a wheel where the spokes come together. The term is familiar to frequent fliers who travel through airport "hubs" to make connecting flights from one point to another. In data communications, a hub is a place of convergence where data arrives from one or more directions and is forwarded out in one or more other directions. A hub usually includes a switch of some kind. (And a product that is called a "switch" could usually be considered a hub as well.)

Switch

In a telecommunications network, a switch is a device that channels incoming data from any of multiple input ports to the specific output port that will take the data towards its intended destination. In the traditional circuit-switched telephone network, one or more switches are used to set up a dedicated though temporary connection or circuit for an exchange between two or more parties.

In the open systems Interconnection (OSI) communications model, a switch performs the Layer 2 or Data-link layer

function. That is, it simply looks at each packet or data unit and determines from a physical address (the "MAC address") which device a data unit is intended for and switches it out towards that device. However, in wide area networks such as the Internet, the destination address requires a look-up in a routing table by a device known as a router. Some newer switches also perform routing functions (Layer 3 or the Network layer functions in OSI) and are sometimes called IP switches. On larger networks, the trip from one switch point to another in the network is called a hop. The time a switch takes to figure out where to forward a data unit is called its latency. The price paid for having the flexibility that switches provide in a network is this latency. In the simplest networks, a switch is not required for messages that are sent and received within the network. For example, a local area network may be organized in a token ring or bus arrangement in which each possible destination inspects each message and reads any message with its address.

Bridge

A bridge is a product that connects a local area network (LAN) to another local area network that uses the same protocol (for example, Ethernet or token ring). You can envision a bridge as being a device that decides whether a message from you to someone else is going to the local area network in your building or to someone on the local area network in the building across the street. A bridge examines each message on a LAN, passing those to be within the same LAN and forwarding those known to be on the other interconnected LAN (or LANs).

In bridging networks, computer or node addresses have no specific relationship to location. For this reason, messages are sent out to every address on the network and accepted only by the intended destination node. Bridges learn which addresses are on which network and develops a *learning table* so that subsequent messages can be forwarded to the right network.

Bridging networks are generally always interconnected local area networks since broadcasting every message to all possible destinations would flood a larger network with unnecessary traffic. For this reason, router networks such as the Internet uses a scheme that assigns addresses to nodes so that a message or packet can be forwarded only in one general direction rather than forwarded in all directions. A bridge works at the data-link (physical network) level of a network, copying a data frame from one network to the next network along the communications path. A bridge is sometimes combined with a router in a product called a brouter.

Routers

Routers operate on the Network layer, which is a higher level in the OSI conceptual model. Routers use a combination of software and hardware, but it is used to route data from its source to its destination. Routers actually have a sophisticated OS that allows them to configure various connection ports. You can setup a router to route data packets from different network protocol stacks, which include TCP/IP, IPX/SPX and AppleTalk.

Routers are also used to connect remote LANs together using different WAN technologies. But, when a router has become large, the large network is divided into logical segments called subnets. This division of the network is based on the addressing scheme related to a particular subnet is kept local. The router only forwards data that is meant for the subnets on the extended network.

Routers also help to decide how to forward data packets to their destination based on the routing table. The protocols built into the router's operating system is used to identify neighboring routers and their network addresses. This allows routers to build a routing table.

Brouter

A brouter is a network bridge and a router combined in a single product. A bridge is a device that connects one local area network (LAN) to another local area network that uses the same protocol (for example, Ethernet or token ring). If a data unit on one LAN is intended for a destination on an interconnected LAN, the bridge forwards the data unit to that LAN; otherwise, it passes it along the same LAN. A bridge usually offers only one path to a given interconnected LAN. A router connects a network to one or more other networks that are usually part of a wide area network and may offer a number of paths out to destinations on those networks. A router therefore needs to have more information than a bridge about the interconnected networks. It consults a routing table for this information. Since a given outgoing data unit from a computer may be intended for an address on the local network, on an interconnected LAN, or the wide area network, it makes sense to have a single unit that examines all data units and forwards them appropriately.

Gateway

A gateway is a network point that acts as an entrance to another network. On the Internet, a node or stopping point can be either a gateway node or a host (end-point) node.

In the network for an enterprise, a computer server acting as a gateway node is often also acting as a proxy server and a firewall server. A gateway is often associated with a router, which knows where to direct a given packet of data that arrives at the gateway, and a switch, which furnishes the actual path in and out of the gateway for a given packet.

Exercises

Practice Problems 1

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. If TCP RTT is currently 40 m/sec and the following acknowledgements come in after 26, 32 and 24 m/sec respectively. What is the new RTT estimate? $\alpha = 0.9$.
   (A) 32.69 (B) 24.31 (C) 36.55 (D) 42.23
2. If a TCP connection is transferring a file of 5000 bytes. The first byte is numbered 1001. What are the sequence numbers for each segment if data is sent in five segments, each carrying 1000 bytes?
   (A) 1001, 2001, 3001, 4001, 5001
   (B) 1000, 2000, 3000, 4000, 5000
   (C) 5000, 6000, 7000, 8000, 9000
   (D) 5001, 6001, 7001, 8001, 9001
3. Which of the below statements hold good with respect to routing done by a bridge?
   (i) they can route packets using IP addresses
   (ii) they use data link layer addresses to do routing
   (iii) the LAN route IPv4,IPv6,Apple Talk, ATM, OSI packets
   (A) (i), (ii) (B) (ii), (iii)
   (C) (i), (iii) (D) (i), (ii), (iii)
4. Match the following:

i	Repeaters	p	connects different nodes of a LAN
ii	Hub	q	amplifies the signal between segments
iii	Switch	r	connects different LANs
iv	Bridge		

   (A) i – q ii – r iii – p iv – r
   (B) i – r ii – p iii – q iv – q
   (C) i – q ii – p iii – p iv – r
   (D) i – p ii – p iii – q iv – r
5. Match the following.

i	Retransmission timer	p	goes off when a TCP connection is idle for a long time
ii	Keep-alive timer	q	goes off if sender and receiver are waiting for each other
iii	Persistence timer	r	goes off to trigger the delivery of a segment in case acknowledgement is not received for first attempt

   (A) i – r ii – q iii – p
   (B) i – q ii – r iii – p
   (C) i – p ii – r iii – q
   (D) i – p ii – q iii – r
6. In T/TCP (Transactional TCP) what does the packet that is sent by client, consist of?
   (i) SYN (ii) REQUEST (iii) FIN
   (A) (i), (ii) (B) (ii), (iii)
   (C) (i), (iii) (D) (i), (ii), (iii)
7. Assume TCP uses 32-bit sequence numbers and sequence numbers are given to each byte that gets transmitted. If data is transmitted at 1 Gbps. What is the wraparound time for sequence numbers?
   (A) 14.4 sec (B) 24.24 sec
   (C) 34.36 sec (D) 44.45 sec
8. What are the disadvantages of NAT?
   (i) NAT forms link between sender and receiver and then link can be broken irreparably during a connection.
   (ii) NAT violates architectural model of IP.
   (iii) NAT hacks source port field of TCP header which is of limited size.
   (iv) NAT alleviates IP shortage.
   (A) (i), (ii), (iii) (B) (ii), (iii), (iv)
   (C) (i), (iii), (iv) (D) (i), (ii), (iv)
9. What is the main protocol in the transport layer?
   (A) TCP (B) UDP
   (C) FTP (D) Both (A) and (B)
10. Number of bytes for header in UDP segment and TCP segment are
    (A) 8 bytes, 20 bytes (B) 16 bytes, 16 bytes
    (C) 32-bits, 20-bits (D) None of these
11. TCP maintains a variable RTT (Round trip time), for determining the time to reach destination and receiving acknowledgement, the formula for RTT is
    (A) RTT = RTT + D
    (B) RTT = 4RTT
    (C) RTT = αRTT + $(1 - \alpha)$ M ($\alpha = 7/8$)
    (D) None of these.
12. Maximum segment size is
    (A) The size of the segment without header.
    (B) The size of the segment with limit.
    (C) The transmission link capacity.
    (D) Less than maximum transfer unit.
13. What is meant by silly window syndrome that ruins TCP performance?
    (A) This occurs when sender sends data in large blocks and receiver receives in large blocks.
    (B) This occurs when sender sends data in large blocks and receiver receives in or reads one byte at a time.
    (C) Both (A) and (B)
    (D) None of these

Common data for questions 14 and 15: A TCP segment begins with a fixed-format, 20-byte header. The header is followed by reader options. After the options, upto 65,495 bytes of data may follow.

14. Number of one bit flags available in the TCP header are
    (A) 5 (B) 6
    (C) 2 (D) None of these
15. Which of the flags is used for establishing connections?
    (A) PSH (B) ACK (C) URG (D) SYN

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Which of the below TCP primitives block a port?
   (i) LISTEN (ii) CONNECT (iii) RECEIVE
   (A) (i), (ii) (B) (ii), (iii)
   (C) (i), (iii) (D) (i), (ii), (iii)
2. In the context of TCP sockets how is a symmetric DISCONNECT different from that of an asymmetric one?
   (i) In symmetric DISCONNECT each direction is closed separately.
   (ii) In asymmetric DISCONNECT each direction is closed separately.
   (iii) In asymmetric DISCONNECT transport user can release the connection
   (A) (i), (ii) (B) (ii), (iii)
   (C) (i), (iii) (D) (i), (ii), (iii)
3. When does RPC/UDP does not make a good combination?
   (i) When the caller and callee machines are separated by small network distance.
   (ii) When the parameters of the procedures are too huge in size.
   (iii) When the procedure requested cannot be repeated safely as needed.
   (A) (i), (ii) (B) (ii), (iii)
   (C) (i), (iii) (D) (i), (ii), (iii)
4. Which of the following statements below are true with reference to RTP (Real Time Transport Protocol)?
   (i) It multiplexes server real time data stream into a single stream of UDP packets.
   (ii) RTP has flow control, error control mechanism.
   (iii) RTP has no mechanism for retransmission.
   (A) (i), (ii) (B) (ii), (iii)
   (C) (i), (iii) (D) (i), (ii), (iii)
5. What does RTCP (real time transport control protocol) accomplish?
   (i) Provides feedback on delay, jitter etc to sources.
   (ii) Handles introstream synchronization.
   (iii) Provides a way to name the sources.
   (A) (i), (ii) (B) (ii), (iii)
   (C) (i), (iii) (D) (i), (ii), (iii)
6. Which of the following are applicable to TCP?
   (i) Breaks the data coming from upper layers into 64 kbyte size packets and transmits them.
   (ii) Manages the time out and re-uses them.
   (iii) Should reassemble the packets in correct order at receiving end.
   (iv) TCP supports multicasting.
   (A) (i), (ii), (iii) (B) (ii), (iii), (iv)
   (C) (i), (iii), (iv) (D) (i), (ii), (iv)
7. Which of the below statements about sockets is/are true?
   (i) For sender and receiver to avail TCP service sockets have to be created.
   (ii) Each socket is a 16 bit number local to that host.
   (iii) Sockets can involve themselves in one connection at a time.
   (iv) Ports below 1024 are reserved.
   (A) (i), (ii), (iii) (B) (ii), (iii), (iv)
   (C) (iii), (iv), (i) (D) (i), (ii), (iv)
8. What are the functions of application layer?
   (A) Mail service provides a basis for electronic mails forwarding and storage
   (B) File access transfer and management
   (C) Creates virtual terminal that allows us to log onto remote host
   (D) All the above
9. Which of the following application uses UDP?
   (A) Streaming a multimedia
   (B) Client–server interaction
   (C) Internet telephony
   (D) All the above
10. What are the reasons for choosing an UDP by an application?
    (A) No connection establishment
    (B) No connection state
    (C) Small packet header
    (D) All the above
11. TCP uses multiple timers to do its work, the timers are
    (A) Retransmission timer
    (B) Persistence timer
    (C) Keep alive timer
    (D) All the above
12. Which of the following is supported by TCP connections?
    (A) Full-duplex (B) Point-to-point
    (C) Multicasting (D) Both (A) and (B)
13. TCP connection is ________ stream.
    (A) Byte (B) Message
    (C) Packet (D) None of these.
14. If a sender wants to indicate that, it has no data for the receiver, one of the following bits is set.
    (A) PSH (B) RST
    (C) FIN (D) ACK
15. If the receiver host is responding by sending a primitive SYN (SEQ = y, ACK = $x + 1$) means
    (A) The receiver data sequence number is y.
    (B) It has received up to $x + 1$ bytes of data.
    (C) Both (A) and (B)
    (D) None of these

Previous Years' Questions

1. The transport layer protocols used for real time multimedia, file transfer, DNS and email respectively are **[2013]**

(A) TCP, UDP, UDP and TCP
(B) UDP, TCP, TCP and UDP
(C) UDP, TCP, UDP and TCP
(D) TCP, UDP, TCP and UDP

2. Which one of the following socket API functions converts an unconnected active TCP socket into a passive socket? **[2014]**

(A) Connect (B) Bind
(C) Listen (D) Accept

3. Suppose two hosts use a TCP connection to transfer a large file. Which of the following statements is/are FALSE with respect to the TCP connection? **[2015]**

I. If the sequence number of a segment is m, then the sequence number of the sub sequent segment is always $m + 1$.
II. If the estimated round trip time at any given point of time is t sec, the value of the retransmission timeout is always set to greater than or equal to t sec.
III. The size of the advertised window never changes during the course of the TCP connection.
IV. The number of unacknowledged bytes at the sender is always less than or equal to the advertised window.

(A) III only (B) I and III only
(C) I and IV only (D) II and IV only

4. In one of the pairs of protocols given below, both the protocols can use multiple TCP connections between the same client and the server. Which one is that? **[2015]**

(A) HTTP, FTP (B) HTTP, TELNET
(C) FTP, SMTP (D) HTTP, SMTP

5. Assume that the bandwidth for a TCP connection is 1048560 bits/sec. Let α be the value of RTT in milliseconds (rounded off to the nearest integer) after which the TCP window scale option is needed. Let β be the maximum possible window size with window scale option. Then the values of α and β are **[2015]**

(A) 63 milliseconds, 65535×2^{14}
(B) 63 milliseconds, 65535×2^{16}
(C) 500 milliseconds, 65535×2^{14}
(D) 500 milliseconds, 65535×2^{16}

6. Consider the following statements

1. TCP connections are full duplex
2. TCP has no option for selective acknowledgement
3. TCP connections are message streams

(A) Only 1 is correct
(B) Only 1 and 3 are correct
(C) Only 2 and 3 are correct
(D) All of 1, 2 and 3 are correct

7. Which one of the following protocols is **NOT** used to resolve one form of address to another one? **[2016]**

(A) DNS (B) ARP
(C) DHCP (D) RARP

8. Which of the following is/are example(s) of stateful application layer protocols? **[2016]**

(i) HTTP (ii) FTP
(iii) TCP (iv) POP3

(A) (i) and (ii) only
(B) (ii) and (iii) only
(C) (ii) and (iv) only
(D) (iv) only

9. Identify the correct sequence in which the following packets are transmitted on the network by a host when a browser requests a webpage from a remote server, assuming that the host has just been restarted. **[2016]**

(A) **HTTP GET** request, **DNS** query, **TCP SYN**
(B) **DNS** query, **HTTP GET** request, **TCP SYN**
(C) **DNS** query, **TCP SYN, HTTP GET** request
(D) **TCP SYN**, **DNS** query, **HTTP GET** request

10. Consider a TCP client and a TCP server running on two different machines. After completing data transfer, the TCP client calls **close** to terminate the connection and a FIN segment is sent to the TCP server. Server-side TCP responds by sending an ACK, which is received by the client-side TCP. As per the TCP connection state diagram (RFC 793), in which state does the client-side TCP connection wait for the FIN from the server-side TCP? **[2017]**

(A) LAST-ACK
(B) TIME-WAIT
(C) FIN-WAIT-1
(D) FIN-WAIT-2

11. Consider socket API on a Linux machine that supports connected UDP sockets. A connected UDP socket is a UDP socket on which **connect** function has already been called. Winch of the following statements is/are CORRECT? **[2017]**

I. A connected UDP socket can be used to communicate with multiple peers simultaneously.
II. A process can successfully call **connect** function again for an already connected UDP socket.

(A) I only
(B) II only
(C) Both I and II
(D) Neither I nor II

12. Consider the following statements regarding the slow start phase of the TCP congestion control algorithm. Note that cwnd stands for the TCP congestion window

and MSS denotes the Maximum Segment Size.

(i) The cwnd increases by 2 MSS on every successful acknowledgment.

(ii) The cwnd approximately doubles on every successful acknowledgement.

(iii) The cwnd increases by 1 MSS every round trip time.

(iv) The cwnd approximately doubles every round trip time.

Which one of the following is correct? **[2018]**

(A) Only (ii) and (iii) are true

(B) Only (i) and (iii) are true

(C) Only (iv) is true

(D) Only (i) and (iv) are true

13. Consider a long-lived TCP session with an end-to-end bandwidth of 1 Gbps (= 10^9 bits-per-second). The session starts with a sequence number of 1234. The minimum time (in seconds, rounded to the closest integer) before this sequence number can be used again is ________. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. C	**2.** A	**3.** B	**4.** D	**5.** A	**6.** D	**7.** C	**8.** A	**9.** D	**10.** A
11. C	**12.** D	**13.** B	**14.** B	**15.** D					

Practice Problems 2

1. C	**2.** C	**3.** B	**4.** C	**5.** D	**6.** A	**7.** D	**8.** D	**9.** D	**10.** D
11. D	**12.** D	**13.** A	**14.** C	**15.** C					

Previous Years' Questions

1. C	**2.** C	**3.** B	**4.** A	**5.** C	**6.** A	**7.** C	**8.** C	**9.** C	**10.** D
11. B	**12.** C	**13.** 34							

Chapter 5

Network Security

LEARNING OBJECTIVES

- ☞ *Network security basics*
- ☞ *Terminologies*
- ☞ *Cryptographic techniques*
- ☞ *Encryptions*
- ☞ *Types of keys*
- ☞ *Traditional cipher algorithms*
- ☞ *Substitution cipher*
- ☞ *Traditional cipher*
- ☞ *Symmetric key encryption*
- ☞ *Asymmetric key encryption*
- ☞ *Diffie-hellman*
- ☞ *Digital signatures and certificates*

Network Security Basics

It is necessary to define some fundamental terms relating to network security and are the elements used to measure the security of a network. These terms are used to measure the security of a network. To be considered sufficiently advanced along the spectrum of security, a system must adequately address identification, integrity, accountability, non-repudiation, authentication, availability, confidentiality each of which is defined in the following sections:

Identification

Identification is simply the process of identifying one's self to another entity or determining the identity of the individual or entity, with whom you are communicating.

Authentication

Authentication serves as proof that you are who you say you are or what you claim to be. Authentication is critical if there is to be any trust between parties. Authentication is required when communicating over a network or logging into a network. When communicating over a network you should ask yourself two questions.

1. With whom am I communicating?
2. Why do I believe this person or entity is who he claims to be?

Access Control (Authorization)

This refers to the ability to control the level of access that individuals or entities have to a network or system and how much information they can receive. Level of authorization basically determines what you're allowed to do once you are authenticated and allowed access to a network, system or some other resource such as data or information. Access control is the determination of the level of authorization to a system, network or information.

Availability

This refers to whether the network, system, hardware and software are reliable and can recover quickly and completely in the event of an interruption in service. Ideally, these elements should not be susceptible to denial of service attacks.

Confidentiality

This is also be called privacy or secracy to the protection of information from unauthorized disclosure. Usually achieved either by restricting access to the information or by encrypting the information so that it is not meaningful to unauthorized individuals or entities.

Integrity

This can be thought of as accuracy, this refers to the ability to protect information, data, or transmissions from unauthorized, uncontrolled, or accidental alterations.

Accountability

This refers to the ability to track or audit what an individual or entity is doing on a network or system.

Non-repudiation

The ability to prevent individuals or entities from denying (repudiating) that information, data or files were sent or received or that information or files were accessed or altered, when infact they were. This capability is crucial in e-commerce, without if an individual or

entity can deny that he, she or it is responsible for a transaction and that he, she or it is, therefore, not financially liable.

Threats

A threat is anything that can disrupt the operation, functioning, integrity, or availability of a network or system. This can take any form and can be malevolent, accidental, or simply an act of nature.

Vulnerabilities

A vulnerability is an inherent weakness in the design, configuration, implementation, or management of a network or system that renders it susceptible to a threat. Vulnerabilities are what make networks susceptible to information loss and downtime. Every network and system has some kind of vulnerability.

Attacks

An attack is a specific technique used to exploit a vulnerability. For example, a threat could be a denial of service. A vulnerability is in the design of the operating system, and an attack could be a 'Ping of death'. There are two general categories of attacks:

1. Passive
2. Active

Passive attacks These are very difficult to detect because there is no overt activity that can be monitored or detected.

Examples of passive attacks would be packet sniffing or traffic analysis.

These types of attacks are designed to monitor and record traffic on the network. They are usually employed for gathering information that can be used later in active attacks.

Active attacks These employ more overt actions on the network or system. As a result, they can be easier to detect, but at the same time they can be much more devastating to a network.

Examples of this type of attack would be a denial-of-service attack or active probing of systems and networks.

Viruses

A virus, a parasitic program that cannot function independently, is a program or code fragment that is self propagating. It is called a virus, because like its biological counterpart, it requires a 'host' to function. In the case of a computer virus the host is some other program to which the virus attaches itself. A virus is usually spread by executing an infected program or by sending an infected file to someone else, usually in the form of an e-mail attachment.

Worm

A worm is a self-contained and independent program that is usually designed to propagate or spawn itself on infected systems and to seek other systems via available networks. The difference between a virus and a Worm is that a virus is not an independent program.

Trojan horses

A trojan horse is a program or code fragment that hides inside a program and performs a disguised function. A trojan horse program hides within another program or disguises itself as a legitimate program. This can be accomplished by modifying the existing program or by simply replacing the existing program with a new one. The Trojan horse program functions much the same way as the legitimate program, but usually it also performs some other function, such a recording sensitive information or providing a trap door. An example would be a 'password grabber'.

Logic bombs

A logic bomb is a program or subsection of a program designed with malevolent intent. It is referred to as a logic bomb, because the program is triggered when certain logical conditions are met. This type of attack is almost always perpetrated by an insider with privileged access to the network. The perpetrator could be a programmer or a vendor that supplies software.

Denial of service (DOS)

Denial of service attacks are designed to shut down or render inoperable a system or network. The goal of the denial-of-service attack is not to gain access or information but to make a network or system unavailable for use by other users. It is called denial-of-service attack, because the end result is to deny legitimate users access to network services.

Protection against network threats

Network threats may cause a massive harm to the system, as the network users are increasing, there is a good chance to attack a system protection against threats should be done.

To protect system form virus and worms, a security suite should be installed.

Similarly, to protect a system from Trojan horse, internet security suite prevents from downloading Trojan horse.

SPAM filters should be used to stop SPAM, this is available within the mail servers by default.

A strong encryption should be used to protect against packet sniffers, so that packets become unreadable making packet sniffers useless.

Cryptographic Techniques

For the exchange of information and commerce to be secure on any network, a system or process must be put in place that satisfies requirements for confidentiality, access control, authentication, integrity, and non-repudiation. The key

to the securing information on a network is cryptography. Cryptography can be used as a tool to provide privacy.

Traditionally, cryptography conjures up thoughts of spies and secret codes. In reality, cryptography and encryption have found broad applications in society. Every time you use an ATM machine to get cash or a point-of-sale machine to make a purchase, you are using encryption.

Encryption

Encryption is the process of scrambling the contents of a file or message to make it unintelligible to anyone not in possession of the 'key' required to unscramble it.

A cryptosystem or algorithm is the process or procedure to turn plain text into crypto text. A crypto algorithm is also known as a 'cipher'. Theoretically, all algorithms can be broken by one method or another. However, an algorithm should not contain an inherent weakness that an attacker can easily exploit

Example: Below is an example of a cipher, to scramble a message with this cipher, simply match each letter in a message to the first row and convert it into the number or letter in the second row. To unscramble a message, match each letter or number in a message to the corresponding number or letter in the second row and convert it into the letter in the first row.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T

To illustrate how this works see the following where the cipher is used to scramble the message:

'Little green apples'

Cipher text: FCNNF5 AL55H 1JJF5M
Clear text: LITTLE GREEN APPLES

This cipher would not be effective at keeping a message secret for long. It does not comply with one of the qualities of a truly effective cipher. Ciphers usually fall into one to two categories:

1. Block Ciphers
2. Stream Cipher

Stream ciphers

Steam cipher algorithms process plaintext to produce a stream of cipher text. The cipher inputs the plaintext in a stream and outputs a steam of cipher text.

Example:
Plaintext: LET US TALK ONE TO ONE
Cipher text: F5N OM NLFE ITS NI ITS

Stream cipher have several weaknesses. The most crucial short coming of stream ciphers is the fact that patterns in the plain text can be reflected in the cipher text. Knowing that certain words repeat makes breaking the code easier. In addition, certain words in the English language appear with predictable regularity. Letters of the alphabet also appear in predictable regularity. The most commonly used letters of the alphabet in the English language are E, T, A, O, N and I. The least commonly used letters are J, K, X, Q and Z. The most common combination of letters in the English language is '*th*', As a result, if a code breaker is able to find a '*t*' in a code, it doesn't take long to find an '*h*'.

Block ciphers

Block ciphers differ from stream ciphers in that they encrypt and decrypt information in fixed size blocks rather than encrypting and decrypting each letter or word individually. A block cipher passes a block of data or plaintext through its algorithm to generate a block of cipher text. Another requirement of block cipher is that the cipher texts should contain no detectable pattern.

Types of keys

We deal with three types of keys in cryptography:

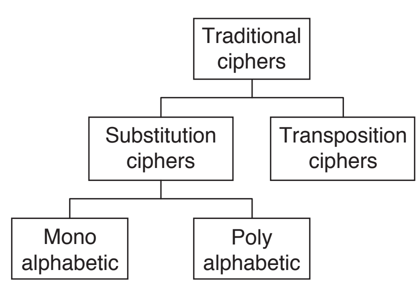
1. Secret key
2. Public key
3. Private Key

- The secret key, is the shared key used in symmetric-key cryptography.
- Public and Private keys are used in asymmetric-key cryptography.
- In symmetric-key cryptography, the same key locks and unlocks the box.
- In asymmetric-key cryptography, one key locks the box, but another key is needed to unlock it.

Traditional Cipher Algorithms

Traditional ciphers are character oriented, these ciphers can be divided into two broad categories:

1. Substitution ciphers
2. Transposition ciphers.



Substitution Cipher

A substitution cipher substitutes one symbol with another. If the symbols in the plain text are alphabetic characters, we replace one character with another. Substitution ciphers can be categorized as either mono-alphabetic or poly-alphabetic ciphers.

- In a mono-alphabetic cipher, a character or symbol in the plaintext is always changed to the same character or symbol in the cipher text regardless of its position in the text. For example if the algorithm says that character '*A*' in the plain text is changed to character '*E*', every character '*A*' is changed to character '*E*'.
- The relationship between characters in the plain text and the cipher text is a one-to-one relationship.
- In a poly-alphabetic cipher, each occurrence of a character can have a different substitute. The relationship between a character in the plain text to a character in the cipher text is a one-to-many relationship.
- To achieve this goal, we need to divide the text into groups of characters and use a set of keys.
- In substitution cipher, if 'a' becomes *D*, '*b*' becomes '*E*' then the word 'corrupt' becomes ETUUXSW, plain text will be given in lower case, and cipher text in upper case.
- A slight generalization of the ceasar cipher allows the cipher text alphabet to be shifted by '*K*' letters, instead of always '3'.
- The next improvement is to have each of the symbols in the plain text, say, the 26 letter for simplicity, map onto some other letter.

Example:

Plain Text	a	b	c	d	e	f	g	h	i	j
Cipher Text	L	N	O	B	R	M	S	U	V	Z

Plain text	k	l	m	n	o	p	q	r	s	t	u
Cipher Text	P	A	K	C	L	H	W	Q	X	Y	J

Plain Text	v	w	x	y	z
Cipher Text	E	F	D	G	J

Plain Text	corrupt
Cipher Text	OIQQJHY

- In this method, if a small cipher is given it can be broken easily. The basic attack takes advantage of the statistical properties of natural languages. For example, In English, '*e*' is the most common letter followed by *t*, *o*, *a*, *n*, *i* etc.
- The most common 2 letter combinations, are *th*, *in*, *er*, *re* and *an*.
- The most common three-letter combinations are are, the, ing, and, and ion.
- By making guesses at common letters, digrams and trigrams and knowing about likely patterns of vowels and consonants, the cryptanalyst builds up a tentative plaintext, letter by letter.

Transposition Ciphers

Substitution ciphers preserve the order of the plaintext symbols but disguise them.

Transposition ciphers, in contrast, reorder the letters but do not disguise them. Following figure depicts a common transposition cipher, the columnar transposition.

- The cipher is keyed by a word or phrase not containing any repeated letters.

Example: 'NETWORKS' is the key.
Plaintext: Transfer ten million dollars to my account.
What is the cipher text using transposition cipher?

Solution: Key: NETWORKS

N	E	T	W	O	R	K	S
3	1	7	8	4	5	2	6
T	r	a	n	s	f	e	r
t	e	n	m	i	l	l	i
o	n	d	o	l	l	a	r
s	t	o	m	y	a	c	c
o	u	n	t	a	b	c	d

The purpose of key is to number the columns, column 1 being under the key letter closest to the start of the alphabet, and so on.

The plain text is written horizontally in rows, padding is required to fill the matrix, if it is not complete'. The cipher text is read out by columns, starting with the column whose key letter is the lowest.

Plain text: Transfer ten million dollars to my account
Cipher Text: rentue laccttososilyafllabrircdandonnmomt.

Symmetric Key Encryption

Symmetric key, also referred to as private key or secret key, is based on a single key and algorithm being shared between the parties who are exchanging encrypted information. The same key both encrypts and decrypts messages.

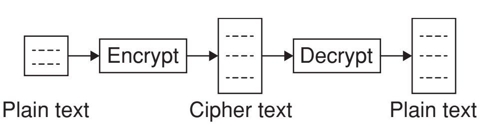


Figure 1 Symmetric key encryption

The strength of the scheme is largely dependent on the size of the key and on keeping it secret. Generally the larger the key, the more secure the scheme. In addition, symmetric key encryption is relatively fast. Private key cryptosystems are not well suited for spontaneous communication over open and unsecured networks. Symmetric key provides on process for authentication or non-repudiation.

Data Encryption Standard: (DES)

DES consists of an algorithm and a key. The key is a sequence of eight bytes, each containing eight bits for a 64 bit key. Since each byte contains one parity bit, the key is actually 56 bits in length. DES is widely used in automated teller machine (ATM) and point-of-sale (POS) networks, so if you use an ATM or debit card you are using DES.

Asymmetric Key Encryption

Asymmetric cryptography is also known as public key cryptography, public key cryptography uses two keys one is public key and the other is private key. The key names describe their function. One key is kept private, and the other key is made public. Knowing the public key doesn't reveal the private key. A message encrypted by the private key can only be decrypted by the corresponding public key. Conversely, a message encrypted by the public key can only be decrypted by the private key.

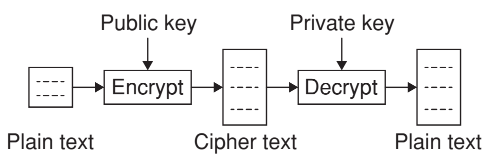


Figure 2 Asymmetric key encryption

With the aid of public key cryptography, it is possible to establish secure communications with any individual or entity when using a compatible software or hardware device.

There are three public key algorithms in wide use today:

1. Diffie–Hellman
2. RSA
3. Digital Signature Algorithm (DSA)

Diffie–Hellman

It was the first usable public key algorithm. Diffie–Hellman is based on the difficulty of computing discrete logarithms. It can be used to establish a shared secret key that can be used by two parties for symmetric encryption. Diffie–Hellman is often used for IPsec key management protocols. For spontaneous communications with Diffie–Hellman, two communicating entities would each generate a random number that is used as their private keys. They exchange public keys they each apply their private keys to the other's. public key to compute identical values (shared secret key). They then use the shared secret key to encrypt and exchange information.

Diffie–Hellman key exchange

The protocol that allows strangers to establish a shared secret key is called the Diffie–Hellman key exchange and works as follows:

- Ana and Brat have to agree on 2 large numbers, 'n' and 'g', where 'n' is a prime.
- $(n - 1)/2$ is also a prime and certain conditions apply to 'g'.
- These numbers may be public, so either one of them can just pick 'n' and 'g' and tell the other openly.
- Now Ana picks a large number (suppose 512-bit) 'x', and keeps it secret. Similarly Brat picks a large secret number, 'y'.
- Ana initiates the key exchange protocol by sending Brat a message containing (n, g, g^x mod n)
- Brat responds by sending Ana a message containing (g^y mod n)
- Now Ana raises the number Brat sent her to the xth power modulo 'n' to get $[(g^y \bmod n)^x \bmod n]$
- Brat performs a similar operation to get $[(g^x \bmod n)^y \bmod n]$, Both the calculations yield (g^{xy} mod n).

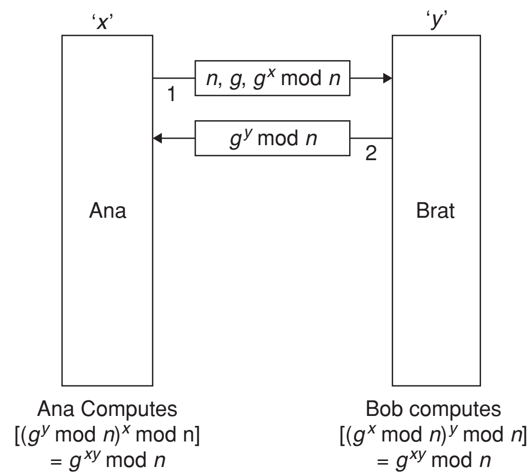


Figure 3 Diffie-Hellman key exchange

RSA (Rivest, Shamir, Adelman)

RSA multiplies large prime numbers together to generate keys. It's strength lies in the fact that it is extremely difficult to factor the product of large prime numbers. This algorithm is the one, most often associated with public key encryption. The RSA algorithm also provides digital signature capabilities.

Example:

- Select two large primes = p, q $p = 17$, $q = 11$
- $n = p \times q = 17 \times 11 = 187$
- calculate $\phi = (p - 1)\ (q - 1) = 16 \times 10 = 160$
- select e, such that LCD $(\phi, e) = 1$, $0 < e < \phi$ say, $e = 7$
- calculate d such that d mod $\phi = 1$
- $160\,k + 1 = 161, 321, 481, 641,$
- Check which of these is divisible by 7
- 161 is divisible by 7 giving $d = 161/7 = 23$
- Key 1 = {7, 187}, key 2 = {23, 187}

Digital Signatures

A digital signature allows a receiver to authenticate (to a limited extent) the identity of the sender and to verify the integrity of the message for the authentication process, you

must already know the senders public key, either from prior knowledge or from some trusted third party. Digital signatures are used to ensure message integrity and authentication. In its simplest from, a digital signature is created by using the senders private key to hash the entire contents of the message being sent to create a message digest. The recipient uses the sender's public key to verity the integrity of the message by recreating the message digest. By this process you ensure the integrity of the message and authenticate the sender.

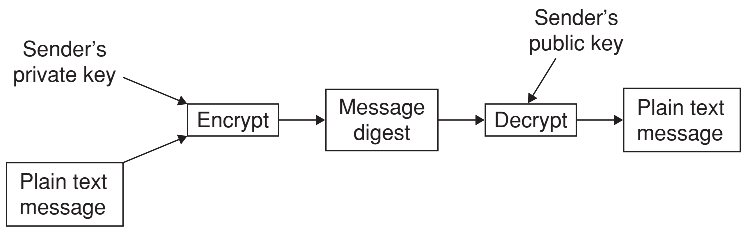


Figure 4 Digital signature

To sign a message, senders usually append their digital signature to the end of a message and encrypt it using the recipient's public key. Recipients decrypt the message using their own private key and verify the sender's identity and the message integrity by decrypting the sender's digital signature using the sender's pubic key. The strength of digital signatures are that they are almost impossible to counterfeit and they are easily verified.

Digital certificate

Digital signatures can be used to verify that a message has been delivered unaltered and to verify the identify of the sender by public key. The problem with authenticating a digital signature, however, is that you must be able to verify that a public key does in fact belong to the individual or entity that claims to have sent it and that the individual or entity is in fact who or what it claims to be.

A digital certificate issued by a certification authority (CA) utilizing a hierarchical public key infrastructure (PKI) can be used to authenticate a sender's identify for spontaneous, first–time contacts. Digital certificates provide a means for secure first time spontaneous communication. A digital certificate provides a high level of confidence in the identify of the individual.

A digital certificate is issued by a trusted/unknown third party (CA) to bind an individual or entity to a public key. The digital certificate is digitally signed by the CA with the CA's private key. This provides independent confirmation that an individual or entity is in fact who it claims to be. The CA issued digital certificates that certify for the identities of those to whom the certificates were issued.

Firewalls

Firewall is a control link between internet and organization intranet. It protects network premises from internet based attacks by providing single choke point. All the network traffic is forced to travel through this fire wall. Firewall allows only authorized traffic to pass through.

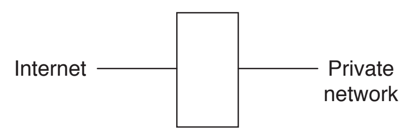
The different types of firewalls are:

1. Packet – filtering router
2. Application level gateways
3. Circuit level gateways
4. Bastion host

Packet filtering router

It filters packets with incoming and outgoing interfaces, and permits or denies certain services. It uses the information of transport layer like IP sources, ICMP message etc.

The drawbacks are IP address spoofing, tiny fragment attack and source routing attacks.



Application level gateway

It provides proxies for each service, when user requests service, it validates the request as legal one and return results to the user.

Application level gateway is more secure than the packet filter.

The drawback of this gateway is processing overhead at each connection.

Circuit-level gateway

It is application level gateway functionality for certain applications. It does not allow end-end TCP connection, rather it maintains two connections, one with the inner host and the other with the outer host. Once the connections are established TCP segment is allowed without examining contents. It only checks the incoming data.

Bastion host

It provides a platform for the application gateway (or) circuit level gateway, it is a critical strong point in network security.

An additional authentication is required for the user who want access to proxy services. Even proxy service authenticates itself before granting the access to user.

Only essential services are installed in the Bastion host which are decided by admin.

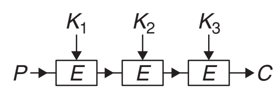
Exercises

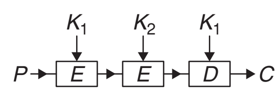
Practice Problems 1

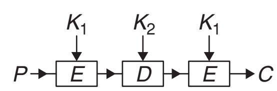
Directions for questions 1 to 15: Select the correct alternative from the given choices.

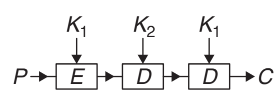
1. In an encryption scheme that uses RSA, values, for p and q are selected to be 5 and 7 respectively what could be the value of d?
   (A) 12 (B) 3 (C) 11 (D) 9
2. A person x is supposed to send a document with digitized signature to another person y using public key Cryptography. p is the message. D_x, D_y are private keys of x and y respectively. E_x, E_y are public keys of x, y respectively. Select the best possible sequence of events from below:
   (i) $D_x\ (p)$
   (ii) $D_y\ (p)$
   (iii) $E_y\ (D_x(p))$
   (iv) $D_y\ (D_x(p))$
   (v) $D_y(E_y(p))$
   (vi) $D_y(E_y(D_x(p))$
   (vii) $E_x(D_x(p))$
   (viii) $E_y(p)$
   (ix) $E_x(D_y(p))$
   (x) $D_x(E_y(p))$
   (A) (ii), (ix), (viii), (v) (B) (viii), (x), (v), (i)
   (C) (i), (iii), (v), (vii) (D) (vii), (v), (iii), (i)
3. Select correct statements about PGP:
   (i) Uses existing cryptographic algorithms that have been quite successful.
   (ii) Support text compression, digital ignatures.
   (iii) Takes plaintext as feed and generates base-64 text.
   (iv) No key management capability is rovided.
   (A) (i), (ii), (iii) (B) (ii), (iii), (iv)
   (C) (i), (iii), (iv) (D) (i), (ii), (iv)

Linked answer questions 4 and 5:

4. Using mono alphabetic substitution a string a b b a c a a b c d is transformed to one of the below strings. Select the most appropriate option:
   (A) p q q p r p p s r s (B) j t t x j j i t x t x
   (C) u s s u a u u s a b (D) d c c d b b b c b a
5. Using the mapping obtained above, encrypt the phrase 'bad cab' using same method: Assume space is not encrypted.
   (A) q p s r p q (B) t j z x j t
   (C) s u b a u s (D) c d a b d c
6. Select the correct statements with regard to packet filters of a firewall:
   (i) They are usually driven by a table with information in regards to acceptable sources and destinations.
   (ii) Default rules about what needs to be done in regards to packets coming from or going to other machines.
   (iii) Can block TCP ports.
   (A) (i), (ii) (B) (ii), (iii)
   (C) (i), (iii) (D) (i), (ii), (iii)
7. What is meant by non-repudiation in the area of digital signatures?
   (A) Receiver verifying the signature of the sender.
   (B) Receiver concocting the message.
   (C) Sender denying having signed digitally.
   (D) Receiver changing the contents after receiving the signed document.
8. Which of the following statements about DES is/are true?
   (i) DES is public key algorithm.
   (ii) DES has 19 distinct stages.
   (iii) In the 16 iterations of DES, different keys are used.
   (A) (i), (ii) (B) (ii), (iii)
   (C) (i), (iii) (D) (i), (ii), (iii)
9. Which of the below represents Triple encryption using DES? (P is the unencrypted input, 'C' is encrypted output, k_1, k_2, k_3 are keys used in encryption and decryption, E stands for encryption and D stands for decryption).
   (A)


   (B)


   (C)


   (D)


10. Which of the below statements are applied for cipher block chaining?
    (i) Each plaintext block is XOR'ed with previous block before encryption.
    (ii) Encryption is a mono alphabetic substitution cipher.
    (iii) Cipher block chaining can result in same plaintext blocks encrypted to different cipher text blocks.
    (A) (i), (ii) (B) (ii), (iii)
    (C) (i), (iii) (D) (i), (ii), (iii)
11. Which of the below statements are applied to RSA algorithm?
    (i) RSA is a relatively slow algorithm when encrypting large data.
    (ii) Mainly used where key is to be distributed.
    (iii) The strength of the algorithm lies in the fact that determining the key can take exceedingly long time by brute force.

(A) (i), (ii) (B) (ii), (iii)
(C) (i), (iii) (D) (i), (ii), (iii)

12. The security and usefulness of a digital signature depends on
(A) A public hash function
(B) A two-way hash function
(C) Protection of user's private key
(D) Protection of user's public key

13. Let '*M*' be the message to be encrypted, *E* be Encryption key and *N* be the product of two random prime numbers, then what is the cipher text using RSA algorithm?
(A) $C = E^m \bmod N$ (B) $C = M^E \bmod N$
(C) $C = N^E \bmod M$ (D) $C = E^N \bmod M$

14. Which of the following best describes the decryption in Triple DES?
(A) Plain text $= D_{K_1}(E_{K_2}(D_{k_1}(\text{ciper text})))$
(B) Plain text $= D_{K_1}(E_{K_2}(D_{k_3}(\text{ciper text})))$
(C) Plain text $= E_{K_1}(D_{K_2}(E_{k_1}(\text{ciper text})))$
(D) Plain text $= E_{K_1}(D_{K_2}(E_{k_1}(\text{ciper text})))$

15. In which cipher mode, all cipher blocks will be chained so that if one is modified the cipher text cannot be decrypted correctly?
(A) Electronic Code Book
(B) Cipher Block Chaining
(C) Cipher Feedback Mode
(D) Counter Mode

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. 'All algorithms must be public only the keys are secret' is
(A) Rijndael Principle
(B) Kerckhoff's principle
(C) Rivest shamir Adleman principle
(D) None of these

2. Pretty Good Privacy encrypts data by using a block cipher called
(A) RSA (B) MD5
(C) IDEA (D) DES

3. E-mail security package is related to
(A) Pretty Good Privacy
(B) DNS spoofing
(C) Secure Socket Layer
(D) Transport Layer Security

4. Which of the following protocols will be proxy, on an application firewall?
(A) IPX (B) FTP
(C) POP (D) SMS

5. A good recommendation is that if a private key is ____ or longer, the key is thought to be secure.
(A) 40 bits (B) 60 bits
(C) 70 bits (D) 80 bits

6. Which issue is related to server side security?
(A) Protection of the server from legitimate web access
(B) Security of the information stored on server
(C) Security of the customer's physical credit card
(D) Security of the customer's computer

7. Which of the following is not an active attack?
(A) Denial of service (B) Traffic Analysis
(C) Replay (D) Masquerade

8. Verifying the true identity of the sender of a message recipient is known as _____.
(A) Authentication (B) fabrication
(C) Cryptography (D) availability

9. In which of the following techniques, letters are arranged in a different order?
(A) Transposition
(B) Substitution
(C) Private key Encryption
(D) None of the above

10. In which type of attack, Algorithm, cipher text, chosen plaintext and cipher text are known?
(A) Cipher text only
(B) Known plain text
(C) Chosen cipher text
(D) Chosen text

11. In which type of ciphers the encryption depends on current state?
(A) Link cipher
(B) Block cipher
(C) Stream cipher
(D) Current cipher

12. Traffic Analysis can be counted using
(A) Encryption (B) Decryption
(C) Replay (D) Data padding

13. DES Algorithm is vulnerable to
(A) Masquerade attack
(B) Replay attack
(C) Denial of service
(D) Brute Force attack

14. What is the size of key in Triple DES?
(A) 168 bits (B) 112 bits
(C) 56 bits (D) Either (A) or (B) or (C)

15. Direct digital signature involves
(A) Source only
(B) Destination only
(C) Communicating parties, sender and receiver.
(D) Everyone including communicating parties.

Previous Years' Questions

1. Suppose that everyone in a group of N people wants to communicate secretly with the $N - 1$ others, using symmetric key cryptographic system. The communication between any two persons should not be decodable by the others in the group. The number of keys required in the system as a whole to satisfy the confidentiality requirement is **[2015]**
   (A) $2N$ (B) $N(N - 1)$
   (C) $N(N - 1)/2$ (D) $(N - 1)^2$

2. Consider that B wants to send a message m that is digitally signed to A. Let the pair of private and public keys for A and B be denoted by K_x^- and K_x^+ for $x = A, B$, respectively. Let $K_x(m)$ represent the operation of encrypting m with a key K_x and $H(m)$ represent the message digest. Which one of the following indicates the CORRECT way of sending the message m along with the digital signature to A? **[2016]**
   (A) $\{m, K_B^+ (H(m))\}$ (B) $\{m, K_B^- (H(m))\}$
   (C) $\{m, K_A^- (H(m))\}$ (D) $\{m, K_A^+ (m)\}$

3. Anarkali digitally signs a message and sends it to Salim. Verification of the signature by Salim requires **[2016]**
   (A) Anarkali's public key.
   (B) Salim's public key.
   (C) Salim's private key.
   (D) Anarkali's private key.

4. A sender S sends a message m to receiver R, which is digitally signed by S with its private key. In this scenario, one or more of the following security violations can take place.
   (I) S can launch a birthday attack to replace m with a fraudulent message.
   (II) A third party attacker can launch a birthday attack to replace m with a fraudulent message.
   (III) R can launch a birthday attack to replace m with a fraudulent message.

   Which of the following are possible security violations? **[2017]**
   (A) (I) and (II) only (B) (I) only
   (C) (II) only (D) (II) and (III) only

5. In a RSA cryptosystem, a participant A uses two prime numbers $p = 13$ and $q = 17$ to generate her public and private keys. If the public key of A is 35, then the private key of A is __________. **[2017]**

Answer Keys

Exercises

Practice Problems 1

1. C **2.** C **3.** A **4.** C **5.** C **6.** D **7.** C **8.** B **9.** C **10.** C
11. D **12.** C **13.** B **14.** B **15.** B

Practice Problems 2

1. B **2.** C **3.** A **4.** B **5.** C **6.** B **7.** B **8.** A **9.** A **10.** D
11. C **12.** D **13.** D **14.** D **15.** C

Previous Years' Questions

1. C **2.** B **3.** A **4.** B **5.** 11

Test

Computer Networks

Time: 60 min.

Directions for questions 1 to 30: Select the correct alternative from the given choices

1. What is the Hamming distance between 000, 011?
(A) 0 (B) 1
(C) 2 (D) 3

2. Consider the given data:

Dataword	Codeword
00	00000
01	01011
10	10101
11	11110

Find the minimum hamming distance?
(A) 2 (B) 3
(C) 4 (D) 5

3. In Go-back-n, what should be the Window size?
(A) 2^m (B) 2^{m-1}
(C) 2^{m-2} (D) 2^{2m}

4. If there are 16 sequence numbers, what are the sender and receiver window sizes in go-back-n and selective repeat respectively?
(A) (15, 1) (8, 8) (B) (14, 2) (8, 8)
(C) (15, 1) (7, 8) (D) (15, 1) (8, 7)

5. A code needs to be designed with 8 data bits and r check bits. What is the minimum value of r in order to correct single bit errors?
(A) 1 (B) 2
(C) 3 (D) 4

6. A code has hamming distance of 6. What is the maximum number of bit errors that can be corrected?
(A) 1 (B) 2
(C) 3 (D) 4

7. In the above case what is the number of errors that can be detected?
(A) 3 (B) 4
(C) 5 (D) 6

8. CRC is being used to do error detection and correction. The frame with data 101001001 needs to be sent and the generator polynomial being used is $x^4 + x + 1$. What is the final transmitted frame?
(A) 1010010011110 (B) 1010010010010
(C) 1010010011010 (D) 1010010010000

9. OSI model seven layer is based on which of the following principles:
(A) A layer should be created where a different level of abstraction is needed
(B) Each layer should perform a well defined function
(C) The layer boundaries should be chosen to minimize the information flow across the interfaces
(D) All the above

10. Which of the following is/are the tasks of physical layer?
(A) How to link two or more devices physically
(B) What type of data flow is needed between two devices
(C) Type of topology required
(D) All the above

11. The functions of the data link layer are
(A) It provides services to network layer and accepts services from physical layer
(B) It is responsible for error control and detection within the network.
(C) It regulates the amount of data that can be transmitted on one line
(D) All the above

12. Which one of the following layers deals with problems that arise when packet travels from one network to another?
(A) Transport layer (B) Physical layer
(C) Data link layer (D) Network layer

13. What is the main function of the network layer?
(A) Routing
(B) Congestion control
(C) Both (A) and (B)
(D) None of these.

14. Which layer ensures interoperability among the communicating devices, and also computers to communicate even if their internal representation is different?
(A) Session layer (B) Transport layer
(C) Presentation layer (D) Application layer

15. Which of the following is not a layer in TCP/IP reference model?
(A) Application layer (B) Transport layer
(C) Data link layer (D) Host to Network layer

16. Suppose we want to transmit a character 'C', the binary value is 1000011 if we pass through an even parity generator then the output is
(A) 10000110 (B) 10000111
(C) 1000011 (D) 1000010

17. What type of frames can be recognized by stop and wait protocols?
(A) Damaged frames
(B) Lost frames
(C) Lost of acknowledgement frames
(D) All the above

18. IEEE project 802 divides the data link layer into two sub layers. What is the upper sublayer?
(A) LLC (B) MAC
(C) PDU (D) HDLC

Common data for questions 19 and 20: When a data frame, arrives at the receiver, instead of sending an acknowledgement separately the receiver rests itself and waits until the network layer passes it the next packet. The acknowledgement is attached to the outgoing data frame.

19. The technique of temporarily delaying outgoing ACK so that they can be hooked onto the next outgoing data frame is called______
(A) Pipelining (B) Piggybacking
(C) Flooding (D) None

20. Which layer implements technique of piggybacking?
(A) Physical layer (B) Data link layer
(C) Transport layer (D) Session layer

21. What is the protocol used in one bit sliding window protocol?
(A) Unrestricted simplex
(B) Simplex stop and wait
(C) Simplex protocol for noisy channel
(D) Restricted duplex.

22. The technique of keeping the sender window appropriately in such a way, that it can continuously transmit frames for a time equal to the round trip time, so that acknowledgement of first frame will arrive just after transmitting the last frame, is called
(A) Flooding (B) Piggy backing
(C) Pipelining (D) Selective repeat

23. Pick the incorrect statement from the following
(A) Go-Back-N method requires more storage at the receiving side.
(B) Selective repeat involves complex logic than Go-back-N
(C) Go-back-N has better line utilization
(D) Selective repeat has better line utilization

24. In stop and wait flow control, to send 'n' data packets how many acknowledgements are needed.
(A) n (B) $2n$
(C) $n - 1$ (D) $n + 1$

25. In sliding window flow control, if the window size is 64 what is the range of sequence numbers?
(A) 0 to 63 (B) 0 to 64
(C) 1 to 63 (D) 1 to 64

26. In Go-Back-N Automatic Repeat Request (ARR), if frames 4, 5, 6 are received successfully, the receiver will send which ACK number to the sender?
(A) 5 (B) 6
(C) 7 (D) 4

27. Which of the following are the responsibilities of a token ring monitor station?
(A) Check to see that token is not lost
(B) Taking action when ring breaks
(C) Clearing the ring when garbled frames appear
(D) All the above

Common data for questions 28 and 29:

28. In ISO-OSI reference model the layer that provides necessary translation of different control codes, character set and graphic character and it ensures interoperability among communicating devices.
The above explanation is about which of the following layers?
(A) Session layer
(B) Data link layer
(C) Presentation layer
(D) Application layer

29. What are the other tasks performed by the above layer?
(A) Encryption and compression
(B) Token management and synchronization
(C) Error detection and error correction
(D) None of these

30. The layer that takes a raw transmission and transforms it into a line that appears free of undetected transmission errors and it takes care of traffic regulation to keep fast transmitter from drowning slow receiver. The layer that provides these services is
(A) Physical layer (B) Transport layer
(C) Data link layer (D) Application layer

Answer Keys

1. C	**2.** B	**3.** A	**4.** A	**5.** D	**6.** B	**7.** C	**8.** D	**9.** D	**10.** D
11. D	**12.** D	**13.** C	**14.** C	**15.** C	**16.** B	**17.** D	**18.** A	**19.** B	**20.** B
21. B	**22.** C	**23.** C	**24.** A	**25.** A	**26.** C	**27.** D	**28.** C	**29.** A	**30.** C

Computer Networks Test I

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Among the following network topologies, which has the highest reliability?
(A) Mesh topology (B) Bus topology
(C) Star topology (D) Ring topology

2. Which one of the following uses UDP as transport protocol?
(A) SMTP (B) HTTP
(C) POP (D) DNS

3. The transport layer protocol that is used to support electronic mail is?
(A) IP (B) UDP
(C) TCP (D) SMTP

4. Find the number of characters per sec (7 bits + 1 priority) that can be transmitted over 4800 bps line using synchronous transfer. (1 start & 1 stop bit)?
(A) 480 (B) 500
(C) 600 (D) 520

5. Find the number of cross points needed for 15 lines in a cross point switch which is full duplex in nature and there are no self connection?
(A) 225 (B) 105
(C) 30 (D) 60

6. What is the maximum data rate of a channel for a noiseless 6 KHz binary channel?
(A) 12000 bps (B) 12 bps
(C) 3000 bps (D) 800 bps

7. Which of the following statements are true about flooding?
I. It is a type of isolated routing
II. Selective flooding is a type in which the packets are sent to those lines that are going approximately in the right direction
III. It is a method in which every incoming packet is sent out on every outgoing line except the one on which it has arrived.
IV. Flooding does not always select the shortest path
(A) I and III (B) II and III
(C) I, II, III and IV (D) I, II and III

8. Which one of the following is not a standard RS232C signal?
(A) RTS (B) VDR
(C) DSR (D) CTS

9. What happens to the size of congestion window in the slow start phase of TCP congestion algorithm?
(A) Increases linearly
(B) Does not change
(C) Increase exponentially
(D) Increase quadratically

10. A company has a class B network and they wanted to subdivide the network to all the 32 departments. The subnet mask would be
(A) 255. 255. 0. 0 (B) 255. 255. 248. 0
(C) 255. 255. 32. 0 (D) 255. 255. 252. 0

11. Which one of the following is true about a layer–4 firewall (can look at all protocol headers upto the transport layer)?
(A) Cannot block all ICMP traffic
(B) Cannot block entire HTTP traffic during 9.00 pm and 5.00 am
(C) Cannot block TCP traffic from a specific user on a multi user system during 9.00 pm and 5.00 am
(D) Cannot stop incoming traffic from a specific IP address but allow outgoing traffic to the same IP address

12. What is the maximum window size for data transmission using the selective Repeat protocol with 8 bit frame sequence number?
(A) 128 (B) 64
(C) 256 (D) 255

13. Which one of the following is not a network layer protocol?
(A) ICMP (B) DHCP
(C) RARP (D) SDLC

14. Find the number of connections in a mesh network with 10 nodes?
(A) 45 (B) 100
(C) 101 (D) 55

15. Find the number of networks allowed under class B address of an IPV_4 network?
(A) 2^{22} (B) 2^{24}
(C) 2^{14} (D) 2^{16}

16. Station A sends data to station B over a full duplex link. Both A and B are using the sliding window protocol for flow control. The window size of sender and receiver is 6 packets each. Data packets (sent only from A to B) are all 700 bytes long and the transmission time for such a packet is 60 µs. Acknowledgement packets that are sent only from B to A are very small and require negligible transmission time. The propagation delay over the link is 120 µs. What is the maximum achievable throughput in this communication?
(A) 20×10^6 bps (B) 70×10^6 bps
(C) 50×10^6 bps (D) 60×10^6 bps

17. Which of the following shows the Hamming distance of following strings Respectively.
I. 1101101 and 1101100
II. 1110111 and 1111011
III. 1111011 and 0011001
(A) 2, 1, 3 (B) 1, 2, 3
(C) 3, 2, 1 (D) 1, 1, 2

18. Find the minimum frame size for a 20 Mbps Ethernet having 56 bit jamming signal and round trip propagation delay of 64 μs:
(A) 2560 (B) 640
(C) 1336 (D) 320

19. Find the maximum number of hosts per subnet on a class B network with a subnet mask of 255.255.252.0:
(A) 1024 (B) 2046
(C) 2048 (D) 1022

20. Match the following:

I.	Telnet	1.	Session layer
II.	DHCP	2.	Application layer
III.	RPC	3.	Data link layer
IV.	HDLC	4.	Network layer
		5.	Presentation layer

(A) I–2, II–1, III–5, IV–4
(B) I–2, II–4, III–1, IV–5
(C) I–2, II–1, III–5, IV–3
(D) I–2, II–4, III–1, IV–3

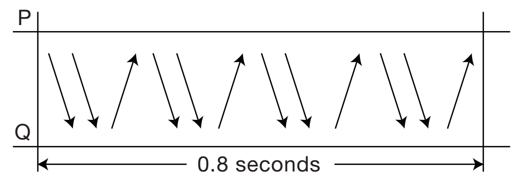
21. Consider the RSA public key cryptosystem with the private and public keys as (e, n) and (d, n) respectively, where $n = p * q$ and p and q are large primes. Besides 'n' is public, p and q are private. Let m be an integer such that $0 < m < n$ and $z = (p - 1)(q - 1)$, Then which of the following equations correctly represent RSA cryptosystem?
I. $m^1 = m^e \bmod z$
$m = (m^1)^d \bmod z$
II. $ed = 1 \bmod z$
III. $m^1 = m^e \bmod n$
$m = (m^1)^d \bmod n$
IV. $ed = 1 \bmod n$
(A) II and III (B) III and IV
(C) I and II (D) I and IV

22. Which of the following conditions must be satisfied by a polynomial generator used for CRC to detect odd number of errors?
I. It should be divisible by x
II. It should not be divisible by x
III. It should be divisible by $1 + x$
IV. It should not be divisible by $1 + x$
(A) I and IV (B) II and III
(C) II and IV (D) I and III

23. Consider an IPV_4 diagram, the M bit is 0, the value of header length is 20, the value of total length is 500 and the fragment offset value is 400. The sequence number of first and last bytes of payload along with the position of the datagram are respectively.
(A) 3200, 3619, First fragment
(B) 3200, 3619, Last fragment
(C) 3200, 3819, Last fragment
(D) 1800, 3819, Middle fragment

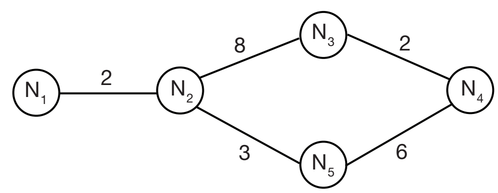
24. A 64 kbps pure ALOHA channel is to be shared among a number of stations, each sending 1k bit frames, on an average one frame every 10 seconds. Assume that sender can buffer frames to handle unsuccessful sendings.
What is the maximum number of stations this network can support? (A pure ALOHA channel has a maximum capacity of 18.4%).
(A) 111 (B) 117
(C) 122 (D) 124

25. The exchange of Ethernet frames between two computers P and Q is shown in the figure below. The connection uses 10 BaseT hub and each frame sent by computer P contains 1500 B of Ethernet payload data, while each frame sent by computer Q contains 100 B of Ethernet payload data. Find the average utilization of the media during this exchange?



(A) 5.68% (B) 10.24 %
(C) 1.27 % (D) 15.67 %

Linked Data Questions 26 and 27:
Consider a network with five nodes as shown in the figure.



The above network uses distance vector routing protocol. The distance vector is the distance of the best known path at that instance to nodes. The distance of a node to itself is zero. All the links are symmetric & the cost is same in both directions. Initially the distance vector at different nodes is
N_1 : (0, 2, 10, 11, 5)
N_2 : (2, 0, 8, 9, 3)
N_3 : (10, 8, 0, 2, 8)
N_4 : (11, 9, 2, 0, 6)
N_5 : (5, 3, 8, 6, 0)
In each round, all nodes exchange their distance vectors with their respective neighbours. Then all the nodes update their distance vectors. In between 2 rounds, any change in cost of a link will cause the two incident nodes to change only that entry in their distance vectors.

26. The cost of link $N_2 - N_3$ is reduced by 4 (in both directions). After the next round of updates what will be the new distance vector at node N_3?
(A) (6, 4, 0, 2, 7)
(B) (10, 4, 0, 2, 8)
(C) (10, 4, 0, 2, 7)
(D) (6, 4, 0, 2, 8)

27. After the update in the above question the link $N_1 - N_2$ goes down. N_2 will reflect this change immediately in its distance vector at cos t α. After the next round of update, what will be the cost to N_1 in the distance vector of N_3?
(A) 13 (B) ∞
(C) 11 (D) 6

28. The CIDR based IP address for an ISP is given as 252.246.128.0/21. The ISP wants to give half of this address to organization X and a quarter to organization Y, while retaining the remaining with itself. Which of the following is a valid allocation of addresses to X and Y?
(A) 252.246.128.0/20 and 252.246.128.0/21
(B) 252.246.136.0/20 and 252.246.128.0/21
(C) 252.246.128.0/22 and 252.246.136.0/23
(*D* 252.246.128.0/21 and 252.246.136.0/22

29. In Ethernet CSMA/CD, the special bit sequence transmitted by media access management for collision handling is called:
(A) jam (B) Preamble
(C) Postamble (D) None of the above

30. Which one of the following network routing method uses every possible path between the sender and receiver terminal?
(A) Directory routing (B) Random routing
(C) Message switching (D) Packet flooding

31. Which one of the following statements are correct?
(A) Baseband network uses analog technology
(B) Baseband network is Time Division Multiplexed
(C) In Broadband network, the carrier signals operate at low frequency
(D) Broadband network uses digital technology

32. How many number of bytes are there for a header in TCP segment and UDP segment respectively?
(A) 16 bytes and 8 bytes (B) 8 bytes and 16 bytes
(C) 8 bytes and 20 bytes (D) 20 bytes and 8 bytes

33. Which of the following statements about DES is/are true?
(i) DES is asymmetric key encryption
(ii) DES is symmetric key encryption
(iii) DES uses different keys in each of the 16 iterations
(A) Only I (B) Only II
(C) I and III (D) II and III

Common Data Questions 34 and 35:
Consider the routing table below.

Destination	Gateway	Mask	Flags	Interfaces
175.220.105.128	175.220.105.164	225.255.255.136	U	eth0
192.128.1.0	192.128.1.1	255.255.255.0	U	eth1
127.0.0.1	127.0.0.1	255.0.0.0	U	Loop back 0
Default	175.220.105.129	255.255.255.255	UG	eth0

34. How many local subnets is this machine attached to?
(A) 1 (B) 2
(C) 3 (D) 4

35. How many IP addresses can this machine reach to (excluding the loop back router)?
(A) 128 (B) 512
(C) 384 (D) 256

Answer Keys

1. A	**2.** D	**3.** C	**4.** C	**5.** B	**6.** A	**7.** D	**8.** B	**9.** C	**10.** B
11. C	**12.** A	**13.** D	**14.** A	**15.** C	**16.** B	**17.** B	**18.** C	**19.** D	**20.** D
21. A	**22.** B	**23.** B	**24.** B	**25.** C	**26.** A	**27.** A	**28.** C	**29.** A	**30.** D
31. B	**32.** D	**33.** D	**34.** B	**35.** C					

Hints and Explanations

4. $\frac{4800}{8}$ · Here start & stop bits are not considered as transfer is synchronous Choice (C)

5. $\frac{n(n-1)}{2} = \frac{15 \times 14}{2} = 105$ Choice (B)

6. $2H \log_2 v$ H → Band width
V → Discrete level
Here $H = 6$ – KHz and $V = 2$
$2 \times 6000 \log_2 2$ Choice (A)

12. $2^{(n-1)} = 2^{(8-1)} = 2^7$ Choice (A)

14. Number of connections $= \frac{n(n-1)}{2}$
n → no. of nodes Choice (A)

16. Maximum achievable throughput

$$= \frac{\text{Total size}}{\text{Total time}} = \frac{6 \times 700 \times 8}{(6 \times 60 + 120) \times 10^{-6}}$$

Total packet size = packet size × window size
Total time = Transmission time + propagation time

Choice (B)

17. I.

$$\begin{array}{l} 1101101 \\ 1101100 \\ \hline 0000001 \\ \hline \end{array}$$

Hamming distance is 1

II.

$$\begin{array}{l} 1110111 \\ 1111011 \\ \hline 0001100 \\ \hline \end{array}$$

Hamming distance is 2

III.

$$\begin{array}{l} 1111011 \\ 0011001 \\ \hline 1100010 \\ \hline \end{array}$$

Hamming distance is 3 Choice (B)

18. Minimum frame size
= 2 × propagation delay × Band width
= $64 \times 10^{-6} \times 20 \times 10^{6} = 1280$
[Round trip time = 2 × propagation delay]
$\Rightarrow$ 1280 + 56 = 1336 Choice (C)

19. 255.255.252.0 ↔ 11111111.11111111.

$$111111\underbrace{00.00000000}_{10 \text{ bits}}$$

No. of hosts per subnet = $2^{10} - 2 = 1024 - 2 = 1022$

Choice (D)

23. $M = 0$, means no more fragmentation i.e. the last fragment
Header Length = 20 × 4 = 80 bytes
Payload = 500 – 80 = 420 (0 – 419)
Fragment offset = 400 × 8 = 3200 bytes
Seq. No. of Ist fragment = 3200
Seq. No. of last fragment = 3200 + 419 = 3619

Choice (B)

24. Average data rate per station is 1,000 bits/10 sec or 100 bps.
A pure ALOHA channel has a maximum capacity of 18.4%
A 64 kbps channel would send
= 64,000 × 0.184
= 11, 776 bps
We can move 11,776 bps through this channel when managed by pure ALOHA and at its maximum throughput.

$\therefore \frac{11776}{100} = 117$ stations can be supported with 64 kbps pure ALOHA channel. Choice (B)

25. Number of frames from P to Q = 8
Ethernet MAC frame payload = 1500 = 1500 + 26
= 1526 bytes = 12208 bits
Number of frames from Q = 4
Frame payload = 100 B
100 + 26 = 126 bytes
126 × 8 = 1008 bits
Total utilized bandwidth = 12208 × 8 + 1008 × 4
= 101696 (achieved capacity) but actual capacity is 10 Mbps (10 Base T)

Utilization $= \frac{101696}{8 \times 10 \times 10^{6}} = 1.27\%$ Choice (C)

26. N_3: (6, 4, 0, 2, 7) Choice (A)

28. Total bit mask = 21
To allocate half of host to X need 10 bit i.e. 2^{10} host
$\therefore$ Mask for X = 22 bits

To allocate $\frac{1}{4}$ of host to Y need 9 bit i.e. 2^{9} host

Mask for Y (32 – 9) = 23 bits Choice (C)

35. 2^7 (in first n/w) + 2^8 (in second n/w)

Choice (C)

Computer Networks Test 2

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. In data link layer, the error detection is achieved by
   (A) Hamming code
   (B) Bit stuffing
   (C) Cyclic redundancy codes
   (D) Equalization

2. Which of the following statements are true about a carrier sense network, if the current status is "channel busy"?
   I. If the technique used is non-persistent then it results in randomized wait and sense
   II. If the technique used is non-persistent then it senses continuously
   III. If the technique used is 1-persistent then the channel is continually sensed
   IV. If the technique used is *p*-persistent then randomized retransmission is done
   (A) I and III (B) I, III and IV
   (C) II and III (D) II, III and IV

3. Which one of the following is a sample of client server application?
   (A) Network printing (B) E-mail
   (C) WWW (D) All the above

4. Find the number of characters per second (7 bits + 1 parity) that can be transmitted over a 8400 bps line using asynchronous transfer.
   (1 start & 1 stop bit)?
   (A) 820 (B) 420
   (C) 620 (D) 840

5. A terminal multiplexer has four 1200 bps terminals and '*n*' 600 bps terminals connected to it. The outgoing line is 9600 bps, what is the maximum value of '*n*'?
   (A) 16 (B) 4
   (C) 32 (D) 8

6. Which layer of the OSI model provides end to end connectivity between the hosts?
   (A) Session layer (B) Network layer
   (C) Transport layer (D) Data link layer

7. Which one of the following measures the number of lost or garbled messages as a fraction of the total sent in the sampling period?
   (A) Residual Error rate
   (B) Connection release failure probability
   (C) Connection establishment failure probability
   (D) Transfer failure probability

8. What is the use of Time-To-Live (TTL) field in the header fields of IP datagram?
   (A) To reduce delays
   (B) To prioritize packets
   (C) To prevent packet looping
   (D) To optimize throughput

9. The number of networks allowed under class C address of an IPv4 network is:
   (A) 2^{21} (B) 2^{24}
   (C) 2^{16} (D) 2^{8}

10. Which one of the following is true about an address resolution protocol (ARP)?
    (A) Used to find IP address that corresponds to a MAC address
    (B) Used to find MAC address that corresponds to an IP address
    (C) Does not perform the translation between IP addresses and MAC layer addresses
    (D) ARP is not a member of the TCP/IP.

11. The transport layer protocols used for DNS, WWW, VoIp and FTP respectively are:
    (A) UDP, TCP, TCP and UDP
    (B) TCP, UDP, UDP and TCP
    (C) TCP, UDP, TCP and UDP
    (D) UDP, TCP, UDP and TCP

12. In Ethernet when Manchester encoding is used, the baud rate is
    (A) Same as the bit rate (B) Half the bit rate
    (C) Twice the bit rate (D) None of these

13. Which one of the following is not a session layer protocol?
    (A) ADSP (B) L2TP
    (C) SDP (D) AFP

14. What is the maximum sending window size for data transmission using Go-Back *N* protocol with 8-bit frame sequence number?
    (A) 255 (B) 128
    (C) 8 (D) 256

15. A message is send from host *A* to host *B* using public key encryption mechanism. Which one of the following is true about the keys used at encryption and decryption?
    (A) Encryption: *A*'s private key and *B*'s private key
    Decryption: *A*'s public key and *B*'s public key
    (B) Encryption: *A*'s private key and *B*'s public key
    Decryption: *B*'s private key and *A*'s public key
    (C) Encryption: *A*'s public key and *B*'s private key
    Decryption: *A*'s private key and *B*'s public key
    (D) Encryption: *A*'s public key and *B*'s public key
    Decryption: *A*'s private key and *B*'s private key

16. Using 64 byte packets host *A* sends messages to host *B* (protocol is sliding window protocol). The round trip delay between *A* and *B* is 60 milliseconds and the bottleneck bandwidth on the path between *A* and *B* is 128 kbps. What is the optimal window size that *A* should use?

(A) 40 (B) 30
(C) 15 (D) 32

17. Consider a broadcast LAN of 8 km long, having a bandwidth of 10^7 bps. It uses CSMA/CD and the signal travels along the wire at 4×10^8 m/s. What is the minimum packet size that can be used on this network?

(A) 25 bytes (B) 400 bytes
(C) 50 bytes (D) 200 bytes

18. A company having a class *B* address has to be splitted into subnets with 5-bit subnet number. Find the maximum number of subnets and the maximum number of hosts in each subnet?

(A) Subnets: 32 and Hosts: 2046
(B) Subnets: 30 and Hosts: 2048
(C) Subnets: 32 and Hosts: 2048
(D) Subnets: 30 and Hosts: 2046

19. Consider a host system on a 20 Mbps network regulated by a token bucket. The arrival rate of token bucket is 4 Mbps. The token bucket is initially filled with 32 Megabits. Find the maximum duration for which the computer can transmit at the full 20 Mbps?

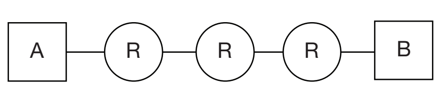
(A) 4 seconds (B) 1.5 seconds
(C) 1 seconds (D) 2 seconds

20. Match the following:

I.	Network Layer	1.	Hub
II.	Data Link Layer	2.	Switches
III.	Physical Layer	3.	Bridges
		4.	Routers
		5.	Repeater

(A) I–4, II–2 and 3, III–1 and 5
(B) I–3 and 4, II–1 and 2, III–5
(C) I–1 and 4, II–2 and 3, III–5
(D) I–4, II–1, 2 and 3, III–5

21. Consider 2 computers *A* and *B* connected through three intermediate routers (*R*) as shown in the figure



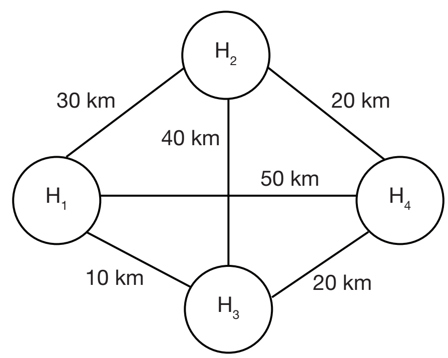
Determine how many times each packet has to visit the network layer and the data link layer during a transmission from *A* to *B*?

(A) Network layer – 8 times and
Data link layer – 8 times
(B) Network layer – 2 times and
Data link layer – 8 times
(C) Network layer– 5 times and
Data link layer – 8 times
(D) Network layer – 6 times and
Data link layer – 5 times

22. Consider an instance of TCP's Additive Increase Multiplicative Decrease (AIMD) algorithm where the size at the start of the slow start phase is 2 MSS and the threshold at the start of the first transmission is 8 MSS. Assume that a time-out occurs during 5^{th} transmission. Find the congestion window size at the end of the 8^{th} transmission?

(A) 6 MSS (B) 5 MSS
(C) 12 MSS (D) 10 MSS

23. Consider a network that consists of 4 hosts distributed as shown in the figure. Assume that the network uses CSMA/CD and signal travels at 4×10^5 km/sec. If the sender sends at 1 Mbps, what could be the minimum size of packet?



(A) 2500 bits (B) 250 bits
(C) 500 bits (D) 5000 bits

Linked Answer Questions 24 and 25

Consider three nodes *P*, *Q*, *R* connected in series. Node *P* is connected to Node *Q* via 2 Gbps link, 400 km length. Node *Q* is connected to Node *R* via 80 Mbps link, 20 km length. The links are full duplex. *A* large file is to be sent from node *P* to node *R*. The packets are 2048 bytes, and the velocity of propagation is 2×10^8 m/sec.

24. What is the round trip time?

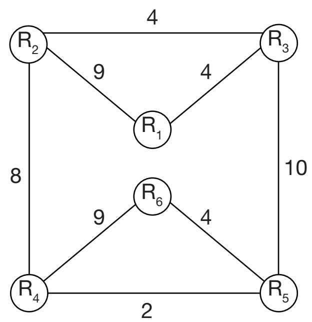
(A) 3.62 m sec
(B) 6.48 m sec
(C) 5.44 m sec
(D) 4.42 m sec

25. Find the optimal value of sender's window using end to end sliding window protocol?

(A) 24 packets (B) 26 packets
(C) 21 packets (D) 27 packets

Linked Answer Questions 26 and 27

Consider a network with six routers R_1 to R_6 connected with links having weights as shown in the following diagram



26. All the routers use the distance vector based routing algorithm to update their routing tables. Each router starts with its routing table initialized to contain an entry for each neighbour with the weight of the respective connecting link. After all the routing tables stabilize, how many links in the network will never be used for carrying any data?

(A) 2 (B) 3
(C) 1 (D) 4

27. The weight of all the unused links are changed to 2 and the distance vector algorithm is used again until all routing tables stabilize. How many links will now remain unused?

(A) 3 (B) 2
(C) 1 (D) 0

28. Two computers *X* and *Y* have IP addresses 20.126.5.109 and 20.126.5.80 respectively and they both use the same network mask *N*. Which of the following values of *N* given below should not be used if *A* and *B* should belong to the same network?

(A) 255.255.0.0 (B) 255.255.255.45
(C) 255.255.255.0 (D) 255.255.255.64

29. What will be the maximum number of hosts per subnet on a class A network with subnet mask of 255.255.224.0?

(A) 8192 (B) 4096
(C) 4094 (D) 8190

30. Which one of the following is true about adaptive or dynamic directory used in packet routing?

(A) Changes with each user session
(B) Changes within each user session
(C) Changes at system generation time only
(D) Both (A) and (B)

31. Choose the correct statement from the following statements?

(A) Initial setup is required for connectionless service
(B) Packet sequencing is not guaranteed in connection–oriented service
(C) In connection–oriented service, the destination address is to be specified only during the setup
(D) Initial setup is possible in connectionless service

32. Which one of the following statement is true about ICI (Interface Control Information)?

(A) It is a combination of Service Data Unit (SDU) and Protocol Control Information (PCI)
(B) It is a temporary parameter passed between *N* and *N* – 1 layers to involve service functions between two layers
(C) It is used to transfer user data from layer to layer
(D) It is used to exchange information by peer entities at different sites on the network to instruct an entity to perform a service function

33. Which one of the following statements is not correct?

(A) PGP encrypts data by using a block cipher MD5
(B) Masquerade is an active attack
(C) DES algorithm is vulnerable to Brute Force attack
(D) The key size of Triple DES can be 56 bits or 112 bits or 168 bits

34. Which one of the following statement is true?

(A) RIP is a distance vector routing protocol and OSPF is a link state routing protocol
(B) OSPF is a distance vector routing protocol and RIP is a link state routing protocol
(C) Both RIP and OSPF are distance vector routing protocols
(D) Both RIP and OSPF are link state routing protocols

35. Consider the path R_1–R_2–R_3–R_4, where R_1, R_2, R_3, R_4 are routers. The maximum bandwidth of each router, R_1 to R_4 are 600 Kbps, 500 Kbps, 800 Kbps and 400 Kbps respectively. What is the effective bandwidth if no buffering is possible?

(A) 800 Kbps
(B) 575 Kbps
(C) 400 Kbps
(D) 2300 Kbps

Answer Keys

1. C	**2.** A	**3.** D	**4.** D	**5.** D	**6.** C	**7.** A	**8.** C	**9.** A	**10.** B
11. D	**12.** C	**13.** D	**14.** A	**15.** B	**16.** C	**17.** C	**18.** D	**19.** D	**20.** A
21. C	**22.** B	**23.** B	**24.** D	**25.** C	**26.** A	**27.** C	**28.** B	**29.** D	**30.** B
31. C	**32.** B	**33.** A	**34.** A	**35.** C					

Hints and Explanations

4. $\dfrac{8400}{8+2} = 840$ Choice (D)

5. $4 \times 1200 + n \times 600 = 9600$

$\Rightarrow \quad n \times 600 = 9600 - 4800$

$\Rightarrow \quad n = \dfrac{4800}{600} = 8$ Choice (D)

12. Baud rate = 2 ∗ Bit rate Choice (C)

14. Sending window Size of Go Back = $2^n - 1$

$= 2^8 - 1 = 255$ Choice (A)

16. Window size $= \dfrac{R \times t}{\text{Packet size}}$

$= \dfrac{128 \times 10^3 \times 60 \times 10^{-3}}{64 \times 8} = 15$ Choice (C)

17. Minimum frame size = 2 × z × band width Minimum packet size

$= 2 \times \dfrac{8 \times 10^3}{4 \times 10^8} \times 10^7 = 400$ bits

In bytes, $\dfrac{400}{8} = 50$ bytes Choice (C)

18. Maximum number of subnets

$= 2^5 - 2 = 30$

Maximum number of hosts in each subnet

$= 2^{(16-5)} - 2 = 2^{11} - 2 = 2046$ Choice (D)

19. Data Arrival rate = 4 Mbps

Data transfer rate = 20 Mbps

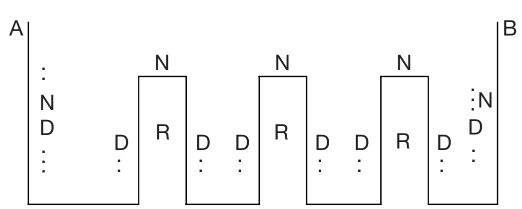
Initial capacity = 32 Mb

Net data rate = 20 – 4 = 16 Mbps

$\therefore$ Maximum duration $= \dfrac{32\text{ Mb}}{16\text{ Mbps}} = 2\,\text{sec}$

Choice (D)

21.



Choice (C)

22. Initial Threshold = 8 MSS

window size for: 1st transmission: 2 MSS

2nd transmission: 4 MSS

3rd transmission: 8 MSS

Threshold Reached, so increase linearly (according to AIMD):

4th transmission: 9 MSS

5th transmission: 10 MSS

Time out at 5th transmission:

New threshold $= \dfrac{10}{2} = 5$ MSS

6th transmission = 2 MSS

7th transmission = 4 MSS

Threshold reached (5 MSS)

8th transmission = 5 MSS Choice (B)

23. Minimum frame size = Transmission delay = RTT

Maximum distance between any 2 hosts = 50 km (between H_1 & H_4)

$\text{RTT} = \dfrac{(2 \times 50)}{4 \times 10^5} = 250\,\mu\,\text{sec}$

$\dfrac{L}{B} = \text{RTT}$, where B = 1 Mbps = 1×10^6 bps

$\therefore \quad L = \text{RTT} \times B$

$= 250 \times 10^{-6} \times 1 \times 10^6 = 250$ bits Choice (B)

24. RTT = Transmission delay + 2 × propagation Delay

Transmission delay $= \dfrac{\text{Number of bits}}{\text{Rate of transmission}}$

Propagation delay $= 2 \times \dfrac{\text{distance}}{\text{speed}}$

$= 2 \times \left(\dfrac{400 \times 10^3}{2 \times 10^8} + \dfrac{20 \times 10^3}{2 \times 10^8} \right)$

RTT = 4.42 m sec Choice (D)

25. Minimum bandwidth between P & R is 80 Mbps

Maximum end to end bandwidth = 80 Mbps

RTT = 4.42 m sec

Maximum bytes that can be transferred with in RTT

= 80 Mbps × 4.42 m sec

= 353600 bits = 44200 bytes

Number of packets $= \dfrac{44200}{2048} = 21$ packets Choice (C)

26. Shortest distances from R_1 to R_2, R_3, R_4, R_5, R_6

R_1(8, 4, 16, 14, 18)

The links used are R_1–R_3, R_3–R_2, R_2–R_4, R_3–R_5, R_5–R_6

R_2(4, 8, 10, 14)

The links used are R_2–R_3, R_2–R_4, R_4–R_5, R_5–R_6

R_3(12, 10, 14)

The links used are R_3–R_2, R_2–R_4, R_3–R_5, R_5–R_6

R_4(2, 6)

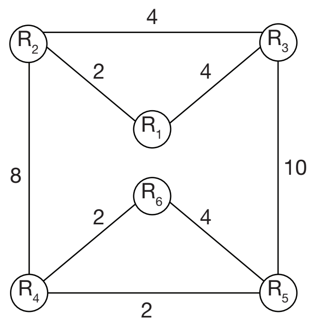
The links used are R_4–R_5, R_5–R_6

R_5(4)

The link used is R_5–R_6

$\therefore$ Unused links are R_1–R_2, R_4–R_6. Choice (A)

27. The network is shown below:



R_1 (2, 4, 10, 12, 12)

The links used are R_1–R_2, R_1–R_3, R_2–R_4, R_4–R_6, R_4–R_5

R_2 (4, 8, 10, 10)

The links used are R_1–R_3, R_2–R_4, R_4–R_5, R_4–R_6

R_3 (12, 10, 14)

The links used are R_3–R_2, R_2–R_4, R_3–R_5, R_4–R_6

R_4 (2, 2)

The links used are R_4–R_5, R_4–R_6

R_5 (4)

The links used are R_5–R_4, R_4–R_6

$\therefore$ Unused link is R_5–R_6 Choice (C)

28. IP of A = 20.126.5.109

$\downarrow$

01101101

IP of B = 20.126.5.80

$\downarrow$

01010000

perform AND operation between each mask with both the IP's of A and B. If the resultants are same then both belong to same network. Choice (B)

29. 224 – 11100000

Number of host bits = 32 – (8 + 8 + 3) = 13

$\therefore$ Total number of hosts per subnet = $2^{13} - 2$

= 8190 Choice (D)

35. As there is no buffering, the smallest bandwidth is the determining factor. Choice (C)

Computer Networks Test 3

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following layers of ISO/OSI model implements ARP (address Resolution protocol)?
(A) Data link layer (B) Network layer
(C) Transport layer (D) Application layer

2. Which of the following routing has more chances for getting TTL as zero?
(A) Source routing
(B) Router routing with default route
(C) Router routing with dynamic default route
(D) Routing will not affect the value of TTL

3. Which of these protocols doesnot accept the out of order packets?
(I) Go back N (II) selective repeat
(A) Only (I) (B) Only (II)
(C) Both (I) and (II) (D) None of the above

4. Which of the following is not the functionality of presentation layer?
(A) translation
(B) encryption
(C) compression
(D) synchronizes the interaction between the two parties

5. In 802.3 LAN the Bandwidth of the cable is 5 Mbps, velocity of propagation is 2.1×10^8 m/s, length of LAN is 2.1 km, what is the minimum packet size?
(A) 100 bits (B) 120 bits
(C) 210 bits (D) 115 bits

6. In 802.3 LAN the encoding technique used is:
(A) Manchester encoding
(B) defracted Manchester encoding
(C) Both (A) and (B)
(D) None of these

7. Match the following:

Column–I		Column–II	
(1)	Logical addressing	(i)	Data link layer
(2)	Physical addressing	(ii)	Network layer
(3)	Service Point addressing	(iii)	Physical Layer
		(iv)	Transport layer

(A) 1–(i), 2–(iv), 3–(iii)
(B) 1–(ii), 2–(iii), 3–(i)
(C) 1–(iii), 2–(i), 3–(iv)
(D) 1–(ii), 2–(i), 3–(iv)

8. CHECKSUM calculations for a packet is done by:
(A) data link layer (B) network layer
(C) transport layer (D) physical layer

9. Match the following:
(A) Dividing the transmitted bit stream into frames
(B) Determining which route the subnet has to use
(C) Fragment the incoming byte stream into discrete messages.
I. TRANSPORT LAYER
II. DATA LINK LAYER
III. NETWORK LAYER
(A) I–A, II–C, III–B (B) I–C, II–A, III–B
(C) I–A, II–B, III–C (D) I–B, II–C, III–A

10. Which of the following situations would not cause packet retransmission?
(A) when packet TTL becomes zero
(B) when there is congestion at the link
(C) when the buffer is full at the router
(D) None of the above

11. Consider a link with 10 Mbps bandwidth and 16 m sec round trip time, if frame size is 1 K bytes then how much percentage of link capacity is utilized?
(A) 9.5 (B) 7.8
(C) 5 (D) 12

12. What will be the length of the packet if the senders utilization is 50%, Band width of the link is 10 Mbps and the Round trip time is 4 msec?
(A) 40 KB (B) 8 KB
(C) 6 KB (D) 5 KB

13. What is the size of sender’s window for stop wand wait, Go-back N and selective repeat, when the available sequence numbers are 8.
(A) 1, 7 , 4 (B) 8, 8, 8
(C) 4, 4, 4 (D) 2, 4, 8

14. Host A sends 64 byte packets using sliding window protocol to host B. The round trip delay between A and B is 160 m sec Bandwidth of the link is 256 Kbps what will be the optimal window size that A should use:
(A) 40 (B) 80
(C) 120 (D) 160

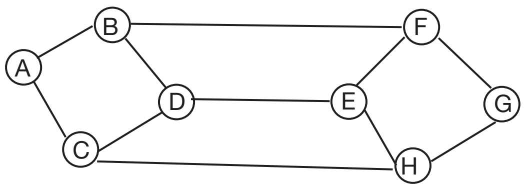
15. A computer on a 20 Mbps network is regulated by a token bucket algorithm, the token bucket is filled at a rate of 2 Mbps and it is initially filled with a capacity of 16 Mbps. The duration for which the computer can transmit, at the maximum regulated output rate (in seconds) is ________.
(A) 1.98 sec (B) 0.89 sec
(C) 2.5 sec (D) 3.0 sec

16. The message 100011001 is to be transmitted using the CRC polynomial $x^3 + x^2 + 1$ the message that should be transmitted is:
(A) 1000110010000 (B) 1000110010100
(C) 1000110011001 (D) 1000110011000

17. In 802.3, A and B are the only two stations each has a steady queue of frames to send, both A and B attempt to transmit a frame, collide and A wins the first back off race. At the end of this successful transmission by A both A and B attempt to transmit, collide and A again wins the back off race, at the end of this successful transmission by A, both A and B attempt to transmit, collide and A again wins the back off race. At the end of this successful transmission by A, both the stations attempt to transmit and collide. The probability that B wins the first back off race is:

(A) 0.625 (B) 0.0625
(C) 0.75 (D) 0.075

18. Consider the following figure:



For the above network, ignore the line weights, suppose flooding is used for the routing algorithm. If a packet is sent from 'D' to 'G', with a maximum hop count of 3, list all the routes it take and how many packets are sent in this flooding?

(A) 5 Routes, 11 packets
(B) 4 Routes, 11 packets
(C) 4 Routes, 3 packets
(D) 5 Routes, 5 packets

19. A network system with n-layers, Application generated message of length m-bytes. If each layer adds an h-byte header what fraction of network Bandwidth is filled with headers?

(A) $h/(n \times m)$ bytes
(B) $h \times n \times m$ bytes
(C) $(n \times h)/m$ bytes
(D) $m/(n \times h)$ bytes

20. Consider the following bit stream '0111101011111011110111' using bit stuffing framing method in the data link layer, how many 0s should be stuffed in the data using a flag pattern of '011110'?

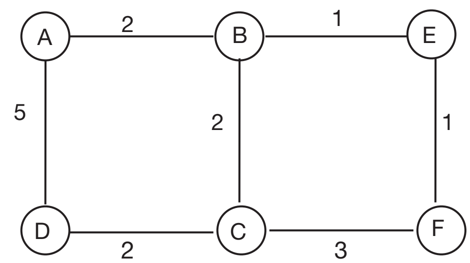
(A) 3 (B) 4
(C) 5 (D) 6

21. Match the following:

(a)	Distance vector routing	(I)	BGP
(b)	Link state routing	(II)	RIP
(c)	Path vector protocol	(III)	OSPF

(A) a–III, b–II, c–I
(B) a–II, b–III, c–I
(C) a–I, b–II, c–III
(D) a–I, b–II, c–III

22. Consider network:



At $t = 0$, the distance vector is

Information stored	Distance to reach node					
	A	B	C	D	E	F
A	0	2	∞	5	∞	∞
B	2	0	2	∞	1	∞
C	∞	2	0	2	∞	3
D	5	∞	2	0	∞	∞
E	0	1	∞	∞	0	3
F	∞	∞	3	∞	3	0

What would be the distance vector in next cycle?

(A)

Information stored at node	Distance to reach nodes					
	A	B	C	D	E	F
A	0	2	4	5	3	∞
B	2	0	2	4	1	2
C	4	2	0	2	3	3
D	5	4	2	0	∞	5
E	3	1	3	∞	0	3
F	∞	2	3	5	3	0

(B)

Information stored at node	Distance to reach node					
	A	B	C	D	E	F
A	0	2	4	5	3	∞
B	2	0	2	4	1	4
C	4	2	0	2	3	3
D	5	4	2	0	∞	5
E	3	1	3	∞	0	3
F	∞	2	3	5	3	0

(C)

Information stored at node	Distance to reach node					
	A	B	C	D	E	F
A	0	2	4	1	3	∞
B	2	0	2	4	1	2
C	4	2	0	2	3	3

D	5	1	2	0	∞	5
E	3	1	3	∞	0	3
F	∞	4	3	5	3	0

(D)

Information stored at node	Distance to reach node					
	A	**B**	**C**	**D**	**E**	**F**
A	∞	2	4	5	3	∞
B	2	0	2	4	1	4
C	4	2	0	2	3	3
D	5	4	2	0	∞	5
E	3	1	3	∞	0	3
F	∞	4	3	5	3	0

23. Which of the following statements are correct, when router malfunctions?
(i) each node computes only its own table
(ii) node can advertise incorrect link cost in link state algorithm
(iii) Each node's table is used by other in distance vector
(iv) Distance vector node can advertise in correct path cost
(A) (i) and (iii) are correct
(B) (i) and (iv) are correct
(C) (ii) and (iii) are correct
(D) All statements are correct

24. Which of the following is TRUE about routing information protocol (RIP)?
(a) RIP is based on distance vector routing
(b) RIP protocol is slow converging and instable
(c) the router forwards only the packets that have traveled the shortest path from the source to the route
(d) RIP uses Dijkstra algorithm to determine best path to a particular destination.
(A) Only (a) and (b) (B) Only (a) and (d)
(C) Only (b) and (c) (D) Only (c) and (d)

25. Which of the following is TRUE about open shortest path first protocol?
(a) OSPF is based on link state routing
(b) OSPF routing tables are calculated by using Dijkstra algorithm
(A) Only (a) is correct
(B) Only (b) correct
(C) (a) and (b) are correct
(D) (a) and (b) are false

Answer Keys

1. A	**2.** B	**3.** A	**4.** D	**5.** A	**6.** A	**7.** D	**8.** C	**9.** B	**10.** D
11. C	**12.** D	**13.** A	**14.** B	**15.** B	**16.** D	**17.** B	**18.** B	**19.** C	**20.** C
21. B	**22.** A	**23.** D	**24.** A	**25.** C					

Hints and Explanations

1. ARP (Address Resolution Protocol) maps IP addresses to hardware addresses. It is used by Internet protocol (IP) and used in Data link layer. Choice (A)

2. TTL (Time To Leave) is a field in a IP header which avoids packet going in to infinite loop. In default route routing there are more chances for getting a packet in to an infinite loop. Choice (B)

3. In Go back–N protocol, Receiver can't receive out of order packet, if it receives it sends NAK. In selective repeat, it can receive out of order packets and sorting algorithm is used for sorting the packets at the receiver end. Choice (A)

4. The functionalities of the presentation layer is
(a) Encoding
(b) Encryption
(c) Compression Choice (D)

5. Condition for minimum packet size in Ethernet:
Transmission delay = round trip delay

i.e., $2\times\frac{L}{B} = 2\times\frac{d}{v}$

$$2\times\frac{L}{5\times10^6} = 2\times\frac{2.1\times10^3\,\text{m}}{2.1\times10^8}$$

$L = 100$ bits. Choice (A)

6. In 802.3, Manchestar encoding is used for data encoding. In this, coding data bit is either high, low or of equal time. Choice (A)

7. (1) Logical addressing is implemented in Network layer
(2) Physical addressing is implemented in data link layer
(3) Service point addressing is implemented in Transport layer. Choice (D)

8. The end to end error control in a network is done by Transport layer by using checksum. Choice (C)

9. Data Link Layer divides bit stream into frames.
One of the main functionality of Network Layer is routing.
Fragmentation is done by Transport Layer.
Choice (B)

10. The packet gets discarded
When TTL becomes zero
When there is congestion at the link
When the buffer is full at the router Choice (D)

11. Throughput $= \dfrac{1 \text{ K bytes}}{16 \text{ m sec}}$

$= \dfrac{1 \times 10^3 \times 8 \text{ bits}}{16 \times 10^{-3} \text{ sec}}$

$= \dfrac{10^6}{2} \times \text{bps}$

% of link utilization

$$= \frac{\frac{10^6}{2} \times 100}{10 \times 10^6}$$

$$= \frac{10^6}{2 \times 10 \times 10^6} \times 100 = 5\%.$$

Choice (C)

12. When senders utilization is 50% then

$L = BR$

where L = length of the packet
B = Band width of link
R = Round trip time

$\therefore$ $L = 10 \times 10^6 \times 4 \times 10^{-3}$ bits
$= 10 \times 4 \times 10^3$ bits
$= 40 \times 10^3$ bits
$= 5$ KB. Choice (D)

13. When N is the available sequence numbers, then sender's window size of stop and wait, Go-back N and selective repeat are $1, N-1, \dfrac{N}{2}$. Choice (A)

14. R.T.T $= 160 \times 10^{-3}$ sec
B.W $= 256 \times 10^3$ bps
(Band width)
Length of the packet
$= 256 \times 160 \times 10^3 \times 10^{-3}$
$= 256 \times 160$

Window size $= \dfrac{256 \times 160}{64 \times 8} = 80.$ Choice (B)

15. Capacity of the bucket (C) = 16 Mbps
Maximum regulated output rate (M) = 20 Mbps
Token filling rate $\left(\int\right)$ = 2 Mbps
Time duration for incoming bursty traffic (t) = ?

$$C + \int t = Mt$$

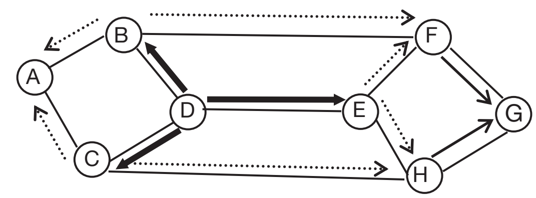
$$16 + 2t = 20t$$

$\Rightarrow$ $16 = 18t \Rightarrow t = 0.89$ seconds Choice (B)

16. Message = 100011001 divisor = 1101 and CRC remainder is 1000 the transmitted message is 1000110011000.
Choice (D)

17. A has two conditions to win first back off race (0, 1) & B has eight conditions (0, 1, 2, 3, 4, 5, 6 and 7) i.e., B has only one chance to win i.e., $\dfrac{1}{16} = 0.0625$.
Choice (B)

18. See the figure given below:



Hop 1 (→), Hop 2 (→), Hop 3 (→)
11 packets are sent in this flooding.
In Hop 1, 3 packets are sent (→)
In Hop 2, 6 packets are sent (→)
In Hop 3, 2 packets are sent (→) Choice (B)

19. Network with 'n' layers adds 'n' – h – byte headers to a packet of length 'm' then the fraction of network bandwidth filled with headers are $\left(\dfrac{n \times h}{m}\right)$ bytes.
Choice (C)

20. Data will be stuffed as
011101010111011100
↑ ↑ ↑
1110101110
↑ ↑
After every 3 consecutive 1s we will stuff '0' bit therefore total = 5. Choice (C)

21.
- Distance vector routing is implemented using RIP.
- Link state routing is implemented using OSPF.
- Path vector protocol includes BGP. Choice (B)

22. In distance vector routing, at every step shortest path are calculated to the all nodes from its neighbours.
Choice (A)

23. The functionalities in the above all statements are performed by a mal functioned router. Choice (D)

24. RIP uses the hop count for the packet routing and there is no compulsive that it should be shortest.
RIP uses Bellman–Ford algorithm for finding shortest path. Choice (A)

25. OSPF is based on link state routing.
Paths among the nodes are calculated using Dijkstra's algorithm. Choice (C)

Computer Networks Test 4

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following IP address belongs to class A network?
(A) 130. 140. 180. 210 (B) 127. 190. 120. 191
(C) 125. 150. 160. 189 (D) 132. 131. 139. 134

2. What will be the destination address if host A with IP address 156.180.190.16 wants to send data to all the members in network with network ID 19?
(A) 255.255.255.255 (B) 19.255.255.255
(C) 156.19.255.255 (D) 156.180.255.255

3. Which of the following work stations does not implement network layer?
(A) Switch (B) Brouter
(C) Gate way (D) Router

4. ______ layer responds when packet gets discarded due to its TTL (when TTL becomes 0)
(A) Application layer (B) Transport layer
(C) Network layer (D) Physical layer

5. Which of the following algorithm is not related to private key cryptography?
(A) DES (B) RSA
(C) AES (D) IDEA

6. What is the minimum positive integer 'p' such that 3^p mod 17 = 1?
(A) 5 (B) 8
(C) 12 (D) 16

7. X. 21 is:
(A) a method of determining which device has access to the transmission medium at any time
(B) a method for access control techniques for multiple access transmission media
(C) a very common bit oriented data link protocol
(D) a network access standard for connecting stations to a circuit switched networks

8. The network topology that supports bi-directional links between each possible node is:
(A) ring (B) star
(C) tree (D) mesh

9. Match the following:

Column–I		Column–II	
(a)	Logical addressing system	(i)	16 bit
(b)	Physical addressing system	(ii)	32 bit
(c)	Port addressing system	(iii)	48 bit

(A) a–(i) b–(ii) c–(iii) (B) a–(ii), b–(i), c–(iii)
(C) a–(ii), b–(iii), c–(i) (D) a–(iii), b–(i), c–(ii)

10. In TCP the flags syn = 0 and ack = 1 indicates:
(A) open connection packet
(B) open connection ack
(C) data packet
(D) reply packet

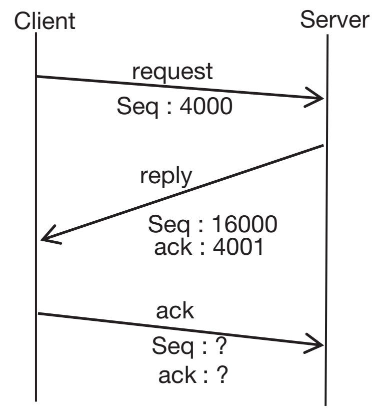
11. A user 'A' got an email containing multi media (requires high band width) if the band width is limited then:
(A) user can open the mail using SMTP
(B) user can download the message using the POP3
(C) user can partially download email using IMAP4
(D) user cannot open (or) download the email

12. MIME supports
(A) Text in character sets other than ASCII
(B) Non text attachments
(C) Message bodies with multiple parts
(D) All the above

13. For a link having bandwidth 10 Mbps, calculate the time to wrap the sequence numbers of TCP? (approximate value)
(A) 3436 minutes (B) 100 seconds
(C) 57 minutes (D) 10 minutes

14. Sender's window size in TCP is determined by
(i) Receiver window size
(ii) Congestion window size
(A) Both (i) and (ii)
(B) Only (i)
(C) Only (ii)
(D) None of the above

15. Consider the connection establishment in TCP using 3 way handshaking



What will be the values of sequence number (seq) and ack for the last hand shake?
(A) seq = arbitrary number ack = 16001
(B) seq = 4000 ack = arbitrary number
(C) seq = 4000 ack = 16001
(D) seq = 4001 ack = 16002

16. Which of the following can be used for only source IP in a packet header?
(A) 10.1.1.1 (B) 0.0.0.0
(C) 0.1.1.1 (D) 127.1.1.1

17. If WAN link is 2 Mbps and round trip time between source and destination is 300 m sec, what would be the optional TCP window size needed to fully utilize the line?
(A) 60,000 bits (B) 75,000 bytes
(C) 75,000 bits (D) 60,000 bytes

18. Device *A* IP address is 172.16.17.30/20 Device *B* IP address is 172.16.28.15/20, which one of the following statement is correct?
(A) Both devices are in the same subnet
(B) Devices are in different subnets
(C) Variable length subnet mask is used in the network
(D) Insufficient data

19. If the IP address is 130.45.34.56 and subnet mask is 255.255.240.0, then what is the subnet address?
(A) 130.45.0.0 (B) 130.45.34.0
(C) 130.45.32.0 (D) 130.45.32.56

20. If the IP address is 196.196.64.170 and subset mask is 255.255.255.240, what is the broadcasting address to the subnet?
(A) 196.196.64.255 (B) 255.255.255.255
(C) 196.196.240.170 (D) 196.196.64.175

21. Consider the following Networks 206.82.2.0, 206.82.3.0, 206.82.4.0 and 206.82.5.0; the Super net mask will be:
(A) 255.255.252.0 (B) 255.255.244.0
(C) 255.255.255.82 (D) 206.82.1.0/22

22. Consider the sub net mask of IP address 255.0.0.0 to which class of IP address this super net mask belongs to?
(A) class A (B) class B
(C) class C (D) None of these

Linked Answer Questions 23 and 24:
RSA algorithm is used by choosing two prime number $p = 3, q = 11$.

23. What are the encryption and decryption keys?
(A) 3, 7 (B) 2, 6
(C) 5, 7 (D) 8, 12

24. What will be the encrypted message for GATE (if $A = 1$, $B = 2$)?
(A) 1311426 (B) 2416513
(C) 10134261 (D) 1421331

25. Which of the following is TRUE for authentication and key management among two parties?
(A) 3 hand shake is necessary for the authentication with each handshake done with private key cryptography
(B) 3 handshake is necessary for the authentication and key management with each hand shake, done with public key cryptography
(C) 3 hand shake is necessary for the authentication and key management with 2 hand shakes one with public key cryptography and one with private key cryptography
(D) 3 hand shake is necessary for the authentication and key management with handshake as public key cryptography and 2 handshakes as private key cryptography

Answer Keys

1. C	**2.** B	**3.** A	**4.** C	**5.** B	**6.** D	**7.** D	**8.** D	**9.** C	**10.** C
11. C	**12.** D	**13.** C	**14.** A	**15.** C	**16.** B	**17.** B	**18.** A	**19.** C	**20.** D
21. B	**22.** B	**23.** A	**24.** A	**25.** C					

Hints and Explanations

1. Class *A* IP address range is 1–126 is in the first octet. In the given options, option (C) has the 125 in the first octet. So, 125.150.160, belongs to class *A* network. Choice (C)

2. Direct Broadcasting address for the network ID 19 will be 19.255.255.255. Direct Broadcasting address in the same network is 255.255.255.255. Choice (B)

3. Switch has only two layers, i.e. Physical layer and data link layer. Choice (A)

4. When packet gets discarded with TTL = 0, then the network layer sends the message to source, so that it can choose another route. Choice (C)

5. RSA is public key cryptography. Choice (B)

6. 3^{16} mod 17 = 1. Choice (D)

7. X.21 is a network access standard for connecting stations to a circuit switched networks. Choice (D)

8. In mesh topology, each node is connected to the other nodes with a link, so in Mesh topology it support bidirectional links. Mostly, Mesh topology is used in Internet. Choice (D)

9. Logical addressing system (IP address) takes a 32-bits.
Physical addressing system takes 48-bits.
Part addressing system takes 16-bits.
Choice (C)

10. When SYN = 0 and ACK = 1, it indicates the data packet in TCP. Choice (C)

11. User can download partially using IMAP4 protocol, when the network bandwidth is low. Choice (C)

12. MIME is used for the message having Multimedia objects. Choice (D)

13. TCP uses 32-bit sequence number, we have 2^{32} distinct numbers

Warp around time

$$= \frac{2^{32} \times 8}{10 \times 10^6}$$

= 57 minutes (approximately)

Choice (C)

14. Sender's window size is determined by using Receiver window size and congestion window size. Choice (A)

15. As the cluent received reply with sequence number 16000, it sends the ACK with sequence number 16001 and as it is not sending any data packet it sends with same sequence number.

i.e. sent in request i.e. 4000. Choice (C)

16. 0.0.0.0 Address can be used as only source address in DHCP. Choice (B)

17. In one sec we can send 2×10^6 bits

In 300×10^{-3} sec we can send

$= 300 \times 10^{-3} \times 2 \times 10^6$ bits

$= 600 \times 10^3$ bits

= 6,00,000 bits (or) 75000 bytes.

Choice (B)

18. Subnet mask for devices will be 255.255.240.0 as it has borrowed 4 bits from host ID, if AND operation is performed between subnet mask and IP address, then the result for both will be 172.16.16.0. Choice (A)

19. If we perform AND operation between IP and mask we will get subnetwork address

IP = 10000010.00101101.0010000.00111000

Subnet mask

11111111.11111111.11110000.00000000

130 . 45 . 32 .0. Choice (C)

20. Subnet mask = 255.255.255.240

11111111.11111111.11111111.11110000

IP = 196.196.64.10101010

From the above we can identify that if IP belongs to the subnet whose id is 160 with host as 10

The broadcast address will be196.196.64.175.

Choice (D)

21. the IP addresses are:

206.82.00000010.0

206.82.0000000.11.0

206.82.00000100.0

206.82.00000101.0

The super net mask is 255.255.244.0 Choice (B)

22. 255.0.0.0 can be a subnet for class *A*, supernet for class *B* and class *C*. Choice (B)

23. (A) $p = 3$ $q = 11$

$n = p \times q = 33$

$z = (p - 1)(q - 1) = 20$

$\gcd(d, z) = 1$

$\gcd(d, 20) = 1$

$\therefore \quad d = 7$

$(e \times 7) \bmod 20 = 1$

$\therefore \quad e = 3.$ Choice (A)

24. G A T E

7 1 20 5

Encrypted message

$= 7^3 \bmod 33 = 13$

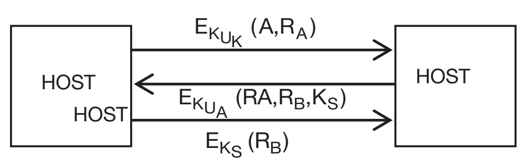
$1^3 \bmod 33 = 1$

$20^3 \bmod 33 = 14$

$5^3 \bmod 33 = 26$

Message is = 1311426. Choice (A)

25. $E_{k_{uB}}(A, R_A)$



R_A = random number generated by *A*

R_B = random number generated by *B*

K_S = session key

The first two handshakes are public key cryptography and last one is private key cryptography. Choice (C)

Chapter 1

Markup Languages

LEARNING OBJECTIVES

- *HTML*
- *Structure of an HTML document*
- *Tags*
- *Attributes*
- *External linking*
- *Hyperlinks*
- *Breaks*
- *Tables*
- *Images and color*
- *Frames*
- *CSS*
- *XML*

HyperText Markup Language (HTML)

HyperText markup language is a specialized markup language to create a web page. The language consists of ordinary text and special commands called tags.

HTML is not a formatting language. Rather it defines the parts of a document such as titles, headings, body text and block quotations. These parts are called elements. To define an element, tags are used. Tags give the browsers what they want to display on the web page.

Structure of an HTML Document

All HTML documents follow the same structure—a head which contains control information used by the browser and server and a body.

The body contains the content that displays on the screen and tags which control how that content is formatted by the browser.

```
<html>
    <head>
        <title> HTML document </title>
        </head>
        <body>
<h1> Largest heading </h1>
<p> A sample paragraph </p>
<hr>
</body>
</html>
```

- The entire document is surrounded by <html> ……. </html> which tell the software that it is now processing HTML.
- Format of our content should be according to the W3C recommendations.
- <head>……</head> and <body>…..</body> tags are compulsory in all HTML documents.
- Programming languages include a mechanism called the comment that lets developers write plain text inside their code files. Comment tags start <! ……….>. Each comment can contain as many lines of text as you like.
- If the comment runs over a number of lines, each must start and end with **- -** and must not contain **- -** within its body.

Example: <! ……..- -
- - ……. - -
- -……. - - >

Comments can be placed in either the head or body of the document.

Tags

Tags are instructions in HTML that are embedded directly into the text of an HTML document. Tags, their attributes and values are enclosed between angular brackets '<' '>'. Tags that come in pairs have a start tag and an end tag. The slash mark is used to denote the end tag. All the text with in the start tag and end tag is to be considered part of the element that the tag defines.

Tags are of two types—empty tag and container tag.

A formatted text document is composed of a set of elements such as paragraphs, headings and lists.

- A tag is a format name surrounded by angle brackets, end tags which switch a format off also contain a forward slash.
- Tags are delimited by angled brackets <h1>.

- Tags are not case sensitive. The following tags are equivalent:
  <HEAD>, <head> and <hEad>
- Styles must be switched off by an end tag.
- Some characters have to be replaced in the text by escape sequences if '<' was not escaped, the software would attempt to process anything that followed it as part of a tag.
- White space, tabs and new lines are ignored by the browser; they can be used to make the HTML source more readable without affecting the way the page is displayed.
- Multiple white spaces are replaced by a single space while new lines and tabs are treated as spaces.
- If a browser doesn't understand a tag it will ignore it.

Container tags

Tags specified in pairs, delimiting text that will have some type of formatting is called a container tag. A container along with a companion tag, encloses the text to be formatted. The effect of a container tag is applied only to the text they contain. A container tag is also called a paired tag since they always appear as a pair. The general form of a container tag can be represented as:

<tag>
Text to be formatted
</tag>

The '<tag>' is often called the opening tag and the '</tag>' is called the closing tag. The closing tag will always have a slash '/' to indicate the end of a tag. The opening tag activates the effect and the closing tag turns the effect off.

Example: <B> text </B>; the text will appear bold in the browser.

Empty tag

An empty tag is a single tag representing some formatting commands in HTML. It will not have a companion tag and are hence called stand alone or singular tag.

Example:
The tag will insert a line break at the specified position.

Attributes

HTML tags sometimes require additional information to be supplied to them. The additional information supplied to an HTML tag is known as attributes of a tag. Attributes are written immediately following the tag, separated by a space. Multiple attributes can be associated with a tag, also separated by a space.

Example: <FONT Face = "Arial" size = 12>
Welcome </FONT>
The face and size are attributes of the FONT tag.

- The document body encloses all the page formatting commands. The tags used to indicate the start and end of the main body of textual information are <BODY> and </BODY>. Page defaults like background colour, text colour, font size, etc. can be specified as attributes of the <BODY> tag.

Attributes of BODY tag

Attributes	Descriptions
Bg colour	Changes the default background colour to the colour specified with this tag. The colour can be specified by name or equivalent hexa decimal number. Example: Bg colour = RED
Text	Changes the body text colour from its default value to the colour specified with this attribute. Example: text = green
Background	Specifies the name of the 'GIF' file that will be used as the back ground of the document. This tiles up across the page to give a back ground. Background = "br. gif"

Text formatting tags

Tags	Descriptions
<P>	Paragraph break: The browser, moves onto a new line skipping one line between the previous line and the new line.
 	A line break is required when the text needs to start from a new line and not continues on the same line. It is an empty tag used to simply instruct the browser to start displaying the remaining text in a new line.
<CENTER>	Center tags are used to centre not only text but anything found between them, like texts, lists, rules, tables, etc.

HR element attributes

The HR element has no ending tag.

- **ALIGN:**
  ALIGN = "_____", sets the alignment of the line on the page to LEFT, RIGHT or CENTER
  The default is CENTER.
  The alignment has no purpose if the line width is 100%.
- **SIZE:**
  Sets the thickness or size of the line in pixels.
- **WIDTH:**
  Sets the width of the line across the page as a % (or) in pixels.

Linking

Links are elements in a web page which can be selected by clicking on it. Linking is one of the most important features of HTML. Link allows to connect a text or an image to another web page or section of a web page. A link will

be displayed in a special way in a browser. Links will be highlighted with colours or underlines to indicate that it is a hyperlink. The target of a hyperlink can be another web page, another location on the current page, an image or any other computer file available in the server. Links can be classified into two:

1. External links
2. Internal links

External linking

External linking refers to linking two documents. When a link in a web page is clicked a new document to which the hyperlink is linked will be opened. An external link points to another HTML document located anywhere in the www.

Internal linking

Internal linking refers to linking different sections of the same document. When a link is clicked a different section in the same document will be displayed in the browser window.

Text

The text on an HTML page can be altered in many ways. The actual font used can be changed to attempt to force the browser to use a specific font and the look of the text can be changed for emphasis.

- <base font size = "n">
  We can specify the minimum font size for basic text but not for headings. The size argument takes an integer from 1 to 7.
- <font size = " [+/-]"color = " # rrggbb"> The colour of the text is set with the colour argument. This takes a hex value which represents the amounts of red, green and blue in the chosen colour.

Example:

```
<html>
    <head >
        <title> changing font sizes </title >
        </head>
    <body>
<h1> Font sizes </h1>
<base font size = "3">
<p> Here is some text in size 3
<p> Here is some < font size = "7" >
                Larger </font>
<font size = "+4" > t </font >
<font size = "+3" > e </font >
<font size = "+2" > x </font >
<font size = "-1" > t </font >
</base font>
</body>
</html>
```

- Other Alternates are
   ……. → Bold
  <i> …….. </i> → Italic
  <strong > ……. </Strong > used as a form of emphasis
  < tt > …… </tt> mono spaced font
  _{……} subscript
  <sup>…….. </sup > super script

Break

Forces a line break within a passage of text where a paragraph is not desirable. On complex pages it is sometimes useful to put a <br> before and after tables, lists

- To display Escape sequences we need to use the following replacement sequences which always start with an ampersand '&' and are terminated with a semicolon.
  & amp; → &
  & nbsp; → (white space)
  & It ; → <
  & gt; → >
  & quot ; → "
  & copy ; → ©

Hyperlinks

The benefit of hypertext is that it lets us create links within a document.

- Links should be used within documents where they either add to the understanding of the work or can be used to reduce download times.
- It is better to have many links to medium sized documents containing about a screenful of information rather than forcing readers to download a single massive document.
   ……
  The link tag has 3 sections:
  1. The address of the referenced document
  2. A piece of text to display as the link
  3. Closing tag
- The link text can be formatted using any of the text formatting options. Hypertext references, the 'href' part of the tag, can be
  1. links to documents or services at other internet sites
  2. links to documents within the same website
  3. links to a specific part of either the current page or another page.

Example:

 Next page Links to another page in the same directory. The browser displays 'Next page' on the screen and highlights it so that readers know it is a hyperlink

Example:

 some site links to another website. This time some sight is displayed and highlighted.

Lists

One of the most effective ways of structuring a website or its contents is to use lists. For Example, a commercial website may use pictures of its products instead of text in hyperlinks. These can be built as nested lists to provide an interesting graphical interface to the site.

HTML provides 3 types of lists:

1. Basic bulleted list
2. A numbered list
3. A definition list

Each has a different use but generally the definition list is the most flexible of the three lists.

- **Ordered and unordered lists:**
  ….
  The ordered and unordered lists are each made up of sets of list items. Elements of a list may be formatted with any of the usual text formatting tags and may be images or hyperlinks.
  The closing tag is not part of HTML.
- <ul [type = " disc"/"square"/"circle"] [compact] > ….</ul]
  The basic unordered list has a bullet in front of each list item.
  Everything between the tags must be encapsulated within … tags.
- To minimize the amount of space that a list uses, we have to add the compact attribute
- <ol [type = "1" |"a"|"A"| "I" | "i"]
- [start = "n"][compact] > ….

An ordered list has a number instead of a bullet in front of each list item. Different numbering schemes can be specified depending upon preference.

A list can number from any value that you desire. The starting value is given by the "start" attribute. All items in an ordered list must be enclosed within ….. tags

Tables

Tables have two uses:

1. Structuring pieces of information
2. Structuring the whole web page
   - Alternatively we can structure a page using frames or images.
   - A table is a grid of information such as, we might have seen in a ledger or spreadsheet.
     Unlike a table from a spreadsheet the data items in an HTML table do not need to have any kind of relationship.
   - Most browsers struggle to process complex tables. The browsers are not optimized for tables and where tables are deeply nested on a page the browser may have difficulty displaying the page.
   - Web browsers have a layout engine which arranges the pieces before the web page is displayed.

- It is more difficult if table consists images where the size attribute of the image have not been set.
- <table [align = "center "/ "left" / "right"]
  [border [= "n"]]
  [cell padding = "n"] [width = "nn%"]
  [cell spacing = "n"]> ……. </table >
- Everything between <table>…..</table> tags will be part of table.
  These attributes control the formatting of the table as a whole, not that of the items in each cell.
- Tables can be aligned on the screen.
- A table can have a border, which includes a border between the cells. If the border attribute is not set, the table has no border.
- When the border attribute is set but a valid value is not given, a single pixel wide default border is drawn "cell padding" determines how much space is there between the contents of a cell and its border in pixels. Cell spacing sets the amount of white space between cells.
  - The "Width" attribute sets the amount of the screen that the table will use.
- <th [align = "left"/ "center"/ "right"]
  [valign = "top" / " center" / "bottom"]>…. <|tr>
  Each row of the table has to be delimited by these tags. The row can be aligned horizontally and vertically within the table.
- <th [align = "left " / "center" /"right"]
  [valign = "top" | "center" / "bottom"]
  [nowrap] [colspan = "n"] [rowspan = "n"] > …. <|th>
- These are table cells which are to be used for headings.
- The contents of the cell can be aligned vertically and horizontally within their row.
- If "nowrap" is set, the contents of the cell will not be automatically wrapped as the table is formatted for the screen.
- The "colspan" and "rowspan" attributes allow individual cells to be larger than a one by one grid.
- <td [align = "left"/ "center"/ "right"]
  [valign = " top"| "center"| "bottom"] [nowrap]
  [colspan = n] [rowspan = n] > …. </td>
  These describe the basic data cell.

Table elements

1. <caption> string </caption>
   This optional element is used to provide a string which describes the contents of the table. If used it must immediately follow the table element.
   - <thead> …… </thead>
     <tfoot> …… </tfoot>
     <tbody> …… </tbody>
     The rows in a table can be grouped into one of the three divisions.

- The idea is that the browsers will be able to scroll the tbody section of the table without moving either the thead or tfoot sections.
- When long tables extend over more than one page the information in thead and tfoot can be automatically replicated on each page.
- <colgroup [span = ‘n’] [width = ‘n’]> ….. </ col group>
Columns within a table can be logically grouped together. Each group of columns can be assigned a default width which will apply to all columns.
- The span indicates the number of columns in the group.
- <col [span = “n”] [width = “n”]> …. </col>
The attributes of individual columns are set using the “col” elements. The ‘span’ and ‘width’ attributes work in the same way as the ‘colgroup’ element.

Example:

```
<html>
<head>
   <title> A table </title>
      </head>
      <body>
<h1> A small table </h1>
<table align = “center” width = “75%”
                  border = “1”>
< caption> small table </caption >
< colgroup width = “30%” span = “2”>
</colgroup>
<colgroup span = “3” > </col group>
<thead>
<tr> <td colspan = “5”> The table header
</td> </tr>
   </thead>
   <tbody>
   <tr>
<td>First  </td>
<td> Second </td>
<td> Third </td>
<td> Fourth </td>
<td> Fifth </td>
   </tr>
   <tr>
<td>First </td>
<td> Second </td>
<td> Third </td>
<td> Fourth </td>
<td> Fifth </td>
   </tr>
   </tbody>
   </t foot>
   <tr> td co|span = “5”> Table Footer </td> </tr>
   </t foot>
   </table>
   </tbody>
   </html>
```

Images and Colour

- Colour can be used in a number of places on a web page; the background can be coloured, individual elements can be altered, and links which are already coloured can have their colours adjusted.
- To change the colours of links or the page background hexadecimal values are placed in the <body> tag.

```
<body bg color = “# nnnnnn” text = “# nnnnnn”
link = “#nnnnnn” vlink = “#nnnnnn”
alink = “#nnnnnn”>
```

- The ‘Vlink’ attribute sets the colour of links visited recently, ‘a link’ the colour of a currently active link.
- The six figure hexadecimal values must be enclosed in double quotes and preceded by a hash (#).
- The colours of page elements can be altered by using the colour modifier. To change the colour of an individual heading we can use

```
<h2 color = “#a b a b a b” > Heading < /h2> and within a
table the table headers could be coloured by:
<th bgcolor = “#a b a b a b”>
```

- Images: If we want high quality, good compression and lots of colours use JPG, GIFS are more common as they tend to be smaller files and can be animated.
- <body background = “URL”> ….. </body>
- Sets the background of your page to use the given image. Images are tiled (repeated) to fill the available space by default.
- If we want to use a single image across the width of a page make it 1281 pixels wide then it cannot be tiled horizontally.

```
<img src = “URL”| “name” height = “n” width = “n” [alt
= “string”] [align = top”| “center”/ “bottom”]
usemap = “URL”]>
```

Displays an inline image, that is an image which appears in the body of the text rather than on a page of its own or in a spawned viewer program.

- The height and width of the image, in pixels tell the browser how much space to allocate to an image when displaying a page.
- Some browsers use these to shrink/stretch images to fit.
- By default any text which follows an image will be aligned alongside its bottom edge. We can alter this so that the first line of text displays alongside the centre or top of the image.
- If we want a block of text shown next to an image we must use a table. To display an image without text, make it into paragraph.

```
<p align = “center”> <img src = “/mygif.gif”
alt =” mine”> </p>
```

The ‘usemap’ attribute is used in image mapping.

1. text message
2. <img src = “filename”>

The first case uses an ordinary hypertext link but the URL should point to the image file, giving its name and type.

- In the second case we are using an image as the link to another image. This can be useful if we want to display a page of thumbnail images and allow the reader to choose which ones to view full-size. This is one way of speeding up the loading times of graphically intensive sites.
- An **image map** is a large picture which has areas that the reader can click with a mouse.
  Each clickable area provides a hypertext link. The image map has 2 parts:
  1. Image
  2. Map

```
<img src = "URL" use map = "URL">
```

It tells the browser to display the source image and to map the second URL, the image map, onto it.

```
<area shape = "circle" | "rect"| "poly" | "default"
href = "URL" Coords = "string" alt = "string">
```

creates a clickable area on an image map. The 'alt' text in this case is displayed by the browser as an indicator for the reader of where the link goes.

- If we do not supply an 'alt', our image map is invalid and may not be displayed.
- The meaning of href should be clear, it is the destination of the link. The clickable area can have one of four shapes. Each shape is defined by coordinates, pairs of integers which give locations on the image in pixels.
- The default location does not require coordinates and is used to indicate what happens if the user clicks outside of the mapped areas.
  Each image map can have only one default
- A **'rect'** has four coordinates which are paired. The first pair defines the top left corner and the second pair the bottom right corner of the area.
- A **'circle'** is defined by its centre and its radius centre is given by a pair of values and the radius by a single value. This requires just three values in the coordinate string.
- A **'polygon'** is made from a set of coordinates with the last pair listed being joined to the first to complete the shape.
  The following example shows an image map with the mapping in the same file as the image link

```
<img src = "/mappicture.gif" usemap =" # main – map"
height = 30 width = 50>
<a name = "#main – map">
<map name = "main – map">
<area shape = "rect" href = "./images/ img1.jpg"
alt = "Image one" cords =" 0,0,25,25">
<area shape = "rect"  href = "./page1.html/"
alt = "page one" coords =" 26,26,50,50">
<area shape = default href  "./page26.html"
alt = "page 26">
</map>
</a>
```

Frames

If we want to represent a complex page structure and not confident about using a table to create it, then we can use frames. Frames are part of the HTML 4 specification. When we talk about frames what we refer to is a 'frameset' which is a special type of web page.

- The frameset page contains a set of references to HTML files, each of which is displayed inside a separate frame.
- All of the pages within a frameset are displayed inside the same browser window and can actually be made to appear to be a single page.
- Frame-based sites display more than one page at the same time, they can be complex to set up.
- Frame-based page is actually made from a set of documents, each displayed in its own frame. Each sub-documents can have its own scrollbars and can be loaded, reloaded, and printed.
- The tags that are needed are

```
<frameset [cols="%, %"] [rows = "%,%"> … </frame
set>
```

  - This tag determines how the screen will be divided between the various frames we can have as many frames either vertically or horizontally as we want.
  - Each frame has to be allocated a percentage of the screen.
  - We can also nest framesets so that individual rows or columns can themselves be broken up into frames.

```
<frame [name = "name"] src = "filename"
[scrolling = "yes"| "auto"|"no"]
[frame border = "zero" | "1"]>
```

  The src attribute works like an image source or a hyperlink address. It should point to a valid HTML file or image which can be displayed within the frame
- by setting the frame border attribute to "zero" stops it being displayed.
  <a href = " source" target = "n">
- to ensure that pages display in the correct frame we need to extend the basic address tag.

Example:

```
<html>
  <head>
    <title> TIME pvt. Limited  </title>
<frameset rows = "25% , 50%">
<frame name = "A" src = " ./ company. html">
< frame name = "B" src = " ./ orders html" scrolling = "no">
</frameset>
</html>
```

Forms

Forms are used to add an element of interactivity to a website. They are used to let the reader send information back to the server but can also be used to simplify navigation on complex websites.

```
<form action = "URL" method = "post" | "get" > … </form>
```

- A form can contain virtually all other markup tags but cannot be rested within another form..
- The action attribute specifies the name and location of a CGI script that will be used to process the data.
  Data can be sent in 2 ways:
  1. post
  2. get
- We should use 'get' to retrieve information from a server and 'post' to send information to a server. The choice of approach is made by the 'method' attribute.
- 'post' is secured than 'get'.
- 'post' is capable of sending a wide variety of character sets but 'get' can only return ASCII data.
- 'post' is used to get data written in non-English languages.

```
<input type = "text"| "password"| "checkbox" "radio"| "password"| "submit"| "reset" | "button"| "image"
name = "string" [value = "string"] [checked] [size = "n"] [maxlength = "n"] [src = "URL"]
[align = "top"| "bottom" | "middle" | "left"| "right"] >
```

Following are several types of input widgets:

- **Text** creates an input device up to size characters long and is able to accept up to max length characters as input.
- If value is set, that string will be used as the default text. These fields support only a single line of text. If we want to enter larger amount of text then use a "text area".
- **Password** works exactly like text but the input is not displayed to the screen. Each character is replaced by '*'. The password is not encoded but is sent to the server as plain text.
- **Radio** creates a radio button. These are always grouped; buttons within a group should have the same name but different values.
  The CGI script differentiates them by name + value.
- **Checkbox** produces a simple checkbox. It will be returned to the server as name = on if checked at submission.
- **Submit** creates a button which displays the value attribute as its text. It is used to send the data to the server.
- **Reset** also creates a button but this one is used to clear the form.
- **Image** can be used to place a picture on the page instead of a button.
- <select name = "string"> … </select>
  It often very useful to have a list of items from which the user can choose.
  The tag encloses a set of options and, when sent to the server, the name of the particular select tag and the name of the chosen option are returned.
- <option value = "string" [selected]> … </option> the 'select' statement will have several options from which the user can choose. The values will be displayed as the user moves through the list and the chosen one retuned to the server.
- <text area name = "string" rows = "n" cols = "n"> … </text area>
  creates a free format plain text area into which the user can enter anything. The area will be sized at 'rows' by cols but will support automatic scrolling.

Example:

```
<html>
  <head>
    <title> my company < /title>
    </head>
    <body>
<h2 align = "center"> Feedback form </h2>
<hr width = "60%">
<form action = "http:// www.My company.Com / cgi – bin/
feed back.cgi"
Method = "post">
<p align = "left" > name : <input type = "text"
Max length = "32" size = "16">
<p align = "left"> Email Address:
<input type = "text" max length = "32" size = "16">
<p align = "left"> Location:
<select name = "city" size = "1">
<option value = "Hyderabad" selected>
Hyderabad
<option value = "Chennai"> chennai
<option value = "Banglore"> Banglore
<option value = "Pune"> Pune
</select>
<p> comments:
<br> <textarea name = "comments" rows = "6" cols = "40">
</text area>
<P align = "center"> <input type = "submit" name = "feed-
back" value = "submit details">
</form>
<hr width = "60%">
</body>
</html>
```

Cascading Style Sheets

One of the most important aspects of HTML is the capability to separate presentation and content. HTML does not have the facilities that are needed to cope with this diversity, but style sheets provide them.

- A style is simply a set of formatting instructions that can be applied to a piece of text.
- There are 3 mechanisms by which we can apply styles to our HTML documents:
  1. The style can be defined within the basic HTML tag.
  2. Styles can be defined in the <head> section and applied to the whole document.
  3. Styles can be defined in external files called style sheets which can then be used in any document by including the style sheet via a URL.

Styles can be cascaded. This means that formats override any, which were defined or included earlier in the document. We may include an external style sheet which redefines the h_1 tag, then write an h_1 style in the head of your page before finally redefining h_1 in the body of page. The browser will use the last of these definitions when showing the content in the following example the <h_1> tag is redefined.

```
<html>
  <head>
    <title> simple style sheet </title>
    <style>
    <! - -
    h1{
      color : black;
      border : thin groove;
      text – align : center ;
      }
    - - >
  </style>
</head>
<body>
<h1> simple style sheet </h1>
</body>
</html>
```

Rules

A style rule has 2 parts, a selector and a set of declarations. The selector is used to create a link between the rule and the HTML tag.

The declaration has 2 parts, a property and a value.

- Selectors can be placed into classes so that a tag can be formatted in a variety of ways.
- Declarations must be separated using colons and terminated using semicolons.

  Selector {property: value; property: value …}

Classes

If we want to apply a style to some paragraphs we have to use classes.

Selector: classname {property : value ; property : value }
<selector class = classname>

- In the style sheet itself the rule is slightly modified by giving the style a unique name which is appended to the selector using a dot.
- In the HTML document when we want to use a named style the tag is extended by including

  Class = and the unique name

```
h1. f {
color :# a b a b a b ;
background – color : # d 9a b 29;
font – family : "Book Antiqua", Times, Serif;
border : thin groove # 9 a b a b a;
}
<h1 class = "f" > simple heading </h1>
```

Anonymous classes

Some times we want to apply a piece of formatting to many different elements within a page but not necessarily to the entire page.

- Cascading style sheets provides a way of defining styles within reusable classes.

Example:

```
<html>
  <head>
    <title> Anonymous classes </title>
    </style>
    <! - -
      f {
        color : # a b a b a b ;
        background – color :# d9a b29;
  font – family : "Book Anitqua", Times, serif;
  border : thin groove #9ab aba;
  - ->
</style>
</head>
<body>
<h1 class = "f"> A simple Heading </h1>
<P class = "f"> Appling the style f to a paragraph of text
</P>
</body>
</html>
```

Including style sheets:

```
< link rel = "stylesheet" href = "url"
Type = "text / css" media = "screen">
```

- The href is a hyperlink to your style sheet, 'rel' tells the browser what type of link you are using.
- We have to tell the browser what type of document we are including, the type statement gives the relevant MIME type.

Extensible Markup Language (XML)

Extensible markup language (XML) is a way to apply structure to a web page. It provides a standard open format and mechanisms for structuring a document so that it can be exchanged and manipulated. Like HTML, XML uses tags to 'markup' data content. Unlike HTML, in XML you define your own tags that meet the exact needs of your document. The custom tags make data easier to organize and search. XML will not change the way your web page look, but it will change the way the documents are read and the way documents are filed and stored.

XML is a markup language. The term 'markup' is used to identify anything put within a document which either adds or provides special meaning. A mark up language is the set of rules. It declares what constitutes markup in a document and defines exactly what the markup means. It also provides a description of document layout and logical structure.

There are three types of markup:

1. Stylistic – How a document is presented.
2. Structural – How the document is to be structured.
3. Semantic – Tells about the content of the data.

In XML, the only type of markup that we are concerned with is structural.

An XML document must begin with the XML declaration statement. This statement alerts the browser or other processing tools that the document contains XML tags.

The declaration is-

<? XML version = "1.0 "?>

This is the first line of an XML document.

Tags

Tags carry the smallest unit of meaning signifying structure, format or style of the data. They are always enclosed with angular brackets, '<' and '>'. Tags are case-sensitive. This means that the tags <fruit>, <Fruit>, <FRUIT> carry different meanings and cannot be used interchangeably. All the tags must be paired so that they have a start tag and an end tag. For example, <fruit> and </fruit>.

Elements

Tags combined with data form elements. Elements are the building block of a document. An element consists of a start tag, an end tag and the content between them:

<fruit> orange is a fruit </fruit>

Attributes

An attribute gives information about an element. Attributes are embedded in the element start tag. An attribute consists of an attribute name and attribute value. The attribute name precedes its value and they are separated by an equal sign. Also the attribute value is enclosed in quotes to delimit multiple attributes in the same element.

Example: <fruit number = "6"> orange </fruit>

Document Type Definition

- Well formed XML documents are those which have tags, elements and attributes in a correct nesting structure without really providing further definitions.
- Valid XML documents are documents which follow a more formal structure.
- The main difference between well-formed XML and valid XML is the document type definition.

The document type definition (DTD) is a set of rules that define the elements that may be used and where they may be applied in relation to each other.

XML Parser

The process of taking a file and breaking it into components is called parsing.

The components are defined by a grammar, the rules of the language. Although this may be implied by the file structure rather than formally specified.

XML is not a simple data structure and cannot be handled with regular expressions for parsing. There are 4 parameters which can be used to categorize parsers:

1. Validating
2. Non-validating
3. Stream-based
4. Tree-based

Validating parser A validating parser uses both an XML file and a DTD to check that the XML adheres to the rules of the application.

If the XML breaks the rules by straying from the DTD then the parser will create an error and stop processing the files.

Non-validating parser These parsers will only use the XML document and are quite content if it is well-formed.

Stream-based parser A stream-based parser must read the entire document each time that an operation is requested and send a message to the controlling application when specific events occur.

Example: SAX

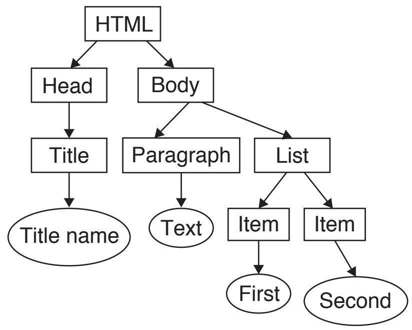
Tree-based parser This type of parser builds a static representation of the document which corresponds to the structure of the original XML.

Example: DOM

Document object model (DOM)

- The DOM is an application program interface (API) for XML documents.
- API is a set of data items and operations which can be used by developers of application programs.
- DOM API specifies the logical structure of XML documents and the ways in which they can be accessed and manipulated.
- DOM API is just a specification.
- DOM-Compliant applications include all the functionality needed to handle XML documents.
- They can build static documents, navigate and search through them, add new elements, delete elements and modify the content of existing elements
- The DOM views XML documents as trees, but this is very much a logical view of the document.
- Each node of the tree, each XML element, is modeled as an object.
- Each node encompasses both data and behavior and that the whole document can be seen as a single complex object.

- Sample DOM is shown below:



- DOM exposes the whole document to applications.

Namespaces

- A namespace is a way of keeping the names used by applications separate from each other.
- Within a particular namespace no duplicate names can exist.
- Applications may use many different namespaces at the same time.
- The implementation of namespaces is system dependent.
- XML developers can specify their own namespaces which can be used in many applications.
- A namespace can be included in the same way as a DTD.

Example:

```
<?xml version = “1.0”?>
<!DOCTYPE items SYSTEM “items.dtd “>
<!xml : namespace ns = “http : //URL/namespaces/jam”
prefix = “jam”>
<?!xml : namespace ns = “http : //URL/namespaces/bread
prefix = “bread”>
<items>
<item1>
<jam : name> kissan </jam : name>
</item1>
<item>
<bread : name> Roasted </bread : name>
</item>
</items>
```

Each item1 of items has a name element, But a namespace have been declared, so there is no chance of an application to get confused with the two names.

Attributes

- Attributes are important and useful when we are handling complexity.
- Some XML elements need to hold more than one piece of information.
- Some of these pieces are used to control the behaviour of the application. These are included as attributes.

Example:
<Quantity amount = “800” unit = “MC”> water </Quantity>
Here amount and unit are attributes of Quantity.

- The attributes of XML elements needs to be included in the DTD.
- Associated with the element declaration is an ATTLIST which may contain:
  - The name of the element
  - The name of each attribute
  - The data type of the attribute
  - Any value which will be used as a default if the attribute is omitted from the XML source.
  - Control information about the use of the element.

ATTLIST declaration

Example: <!ATTLIST Quantity amount CDATA # REQUIRED unit CDATA “g”>

The declaration shows an element with two attributes. The first one is ‘amount’, which is of CDATA type, means that it holds plain text which will not be passed through XML parser.

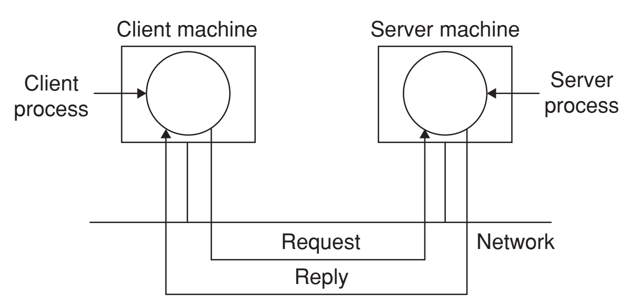
The attribute is REQUIRED which means that it must be included when the element is used. Failure of this will result in an error raised by the parser.

The second element ‘unit’ is optional which has a default value ‘g’. If the value of this attribute is omitted then default value will be used.

Client–Server Computing

Client–server networking grew when personal computers (PC’s) became the common alternative to older mainframe computers.

In client-server network, communication generally takes the form of a request message from the client to the server asking for some work to be done. The server does the work and sends back the reply, as shown below:

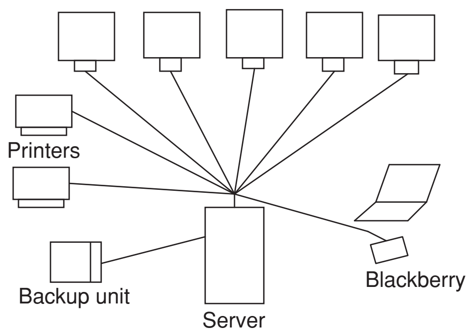


Usually, there are many clients using a small number of servers. Client devices are typically PC’s with network software applications installed, that request and receive information over the network. Mobile devices as well as desktop computers can both function as clients.

A server device typically stores files and databases including more complex applications like websites. Server devices often feature higher powered central processors, more memory and larger disk drives than clients. One server

generally supports numerous clients and multiple servers can be networked together in a pool to handle the increased processing load as the number of clients grows.

Client server network



Some of the most popular applications on the internet including email, FTP and web services follow client-server model. Each of these clients features a user interface (either graphic or text-based) and a client application that allows the user to connect to servers. In the case of email and FTP, users enter a computer name (or an IP Address) into the interface to setup connections to the server.

The client-server model was developed to allow more users to share access to database applications. Compared to the mainframe approach, client-server offers improved scalability because connections can be made as needed rather than being fixed. The client-server model also supports modular applications (software applications are divided into modules) that can make creation of software easier.

Example: Users accessing banking services from their computer uses a web browser client to send a request to a web server at a bank. That program may in turn forward the request to its own database client program that sends a request to a database server at another bank computer at bank to retrieve the account information. The balance is returned to the bank database client, which in turn serves it back to the web browser client displaying the results to the user.

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Which of the following statement is false?
(A) HTML is a markup language for hypertext.
(B) VML is used for freehand drawing in web page.
(C) WML is wireless markup language used for micro procure of mobiles and palmtops.
(D) None of these

2. The web browser request goes to the server in
(A) Hex form (B) ASCII form
(C) Binary form (D) Text form

3. The tag that contains information about the document including its title, scripts used, style definitions and documentation description is
(A) <HTML>, <\HTML>
(B) <HEAD>, <\HEAD>
(C) <BODY>, <\BODY>
(D) <TITLE>, <\TITLE>

4. Tag that is considered to be illegal in XML is
(A) <.document> (B) <document>
(C) <\document> (D) None of these

5. _____ specifies that a click in this area will not link anywhere.
(A) NOHREF (B) HREF = 0
(C) NULLHREF (D) HREF

6. A _______ specifies the layout for frames, including the locations and characteristics of the frame.
(A) frameset (B) border layout
(C) table (D) frame border

7. What is the correct syntax of the declaration, which defines the XML version?
(A) <xml version = "1.0"/>
(B) <?xml version = "1.0"?>
(C) <?xml version = "1.0"/>
(D) None of the above

8. Which of the following are predefined attributes?
(A) xml : lang (B) xml : space
(C) both (A) and (B) (D) None of these

9. Which of the following XML documents are well formed?
(A) <firstElement>
TIME Hyderabad
<secondElement>
Head Office
</secondElement>
</firstElement>
(B) <firstElement>
TIME Hyderabad
</firstElement>
Head Office
</secondElement>
<secondElement>
(C) <firstElement>
TIME Hyderabad
<secondElement>
Head Office
</firstElement>
</second Element
(D) </firstElement>
TIME Hyderabad
</secondElement>
Head Office
<secondElement>
<firstElement>

10. Which of the following XML fragments are well formed?
(A) <myExam mycity = “Hyderabad”/>
(B) <myExam mycity = ‘Hyderabad’/>
(C) <myExam mycity = “Hyderabad/”>
(D) <myExam mycity = ‘Hyderabad/>

11. In the following anchor tag <A HREF = “http://www.time4education.com”> time </A> which one is attribute?
(A) A (B) HREF
(C) time (D) http

12. A link to the document is like this:
– <A HREF =
http://www.time4education/document.html> document </A>
Then the link to proposal section will look like (from within same document)
(A) <A HREF = “#proposal”>
(B) <A HREF = “documents.html #proposal”>
(C) <A HREF = “→ proposal”>
(D) <A HREF = “~ proposal”>

13. DTD includes the specifications about the markup that can be used within the document, the specifications consists of all EXCEPT
(A) the browser name
(B) the size of element name
(C) entity declarations
(D) element declarations

14. Every website has a server process listening to TCP port 80 for incoming connections from clients (normally browsers). After a connection has been established the client sends one request and server sends one reply. Then the connection is released. The protocol that defines the legal requests and replies is called
(A) TFTP (B) FTP
(C) gopher (D) HTTP

15. Given below are several usages of the anchor tag in HTML:
I. < A HREF = “http://www.ebay.com/Html/Basic/Pageno.html”> Hello </A>
II. <A HREF = “/Basic /Pageno.html”> Hello </A>
III. <A HREF = “Pageno.html”>Hello</A>
IV. <A HREF = “Pageno.html # Hello“>Hello</A>
Which of the above are valid?
(A) I and II only (B) I, II, III and IV
(C) I and III only (D) I, II and III only

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Which of the following is not case sensitive?
(A) VML (B) XHTML
(C) XML (D) HTML

2. Which of the following statement is false?
(A) W3C stands for World Wide Web consortium.
(B) W3C implements the <layer> tag.
(C) W3C sets the HTML standards.
(D) None of these

3. Which of the following requires a closing tag?
(A) <H_1> (B) <ABBR>
(C) <B> (D) All of these

4. Which of the following does not require a closing tag?
(A) <B> (B) <FONT>
(C)
 (D) <ABR>

5. The smallest heading tag in HTML is
(A) <H_0> (B) <H_1>
(C) <H_6> (D) <H_8>

6. The largest size among the heading tags is
(A) H_6 (B) H_5
(C) H_7 (D) H_1

7. The ____ tag is effective for formatting program code or similar information, usually in a fixed font with ample space between words and lines.
(A) <Pre> (B) <Address>
(C) <Blockquote> (D) <Strong>

8. ____ sets the text characteristics for the document.
(A) <font>
(B) <size>
(C) <color>
(D) <basefont>

9. The tag used for creating a row in a HTML table is
(A) <TR> (B) <TD>
(C) <Table row> (D) <TH>

10. The SRC attribute is used to point to a ____ of the image.
(A) folder (B) file
(C) URL (D) pixel

11. How the position of files will be displayed in browser for the following code?

```
<frameset col’s = “50%, 50%”>
   <frameset rows = ”50%, 50%”>
      <frame src = “file1.html”>
      <frame src = “file3.html”>
   </frame set>
      <frame set rows = “50%, 50%”>
      <frame src = “file2.html”>
      <frame src = “file4.html”>
   </frameset?
</frameset>
```

(A)

50	50	
File 1	File 2	30
File 3	File 4	70

(B)

50	50	
File 1	File 3	30
File 2	File 4	70

(C)

70	30	
File 1	File 4	50
File 2	File 3	50

(D)

30	70	
File 1	File 4	50
File 3	File 2	50

12. 'We may have standalone attributes in XML'. This statement is
(A) True
(B) False
(C) True if it is well-formed
(D) True if it defined in DTD

13. Standalone is one of the possible attributes in the XML declaration. We can set this to ___ if the document does not refer to any external entity.
(A) Yes (B) No
(C) Not required to set it (D) None of these

14. Which of the following is not the difference between HTML and Java script?
(A) HTML is used to create web pages, java script is used to customize the web pages.
(B) HTML provides security where as java script doesn't provide security.
(C) HTML is more preferable by the clients or users where as java script is not more preferable by the users.
(D) HTML is less efficient than java script.

15. <HMTL> and </HMTL> tags indicates the beginning and ending of the document which are compulsory because these indicate that the software is _____.
(A) processing HTML (B) processing XML
(C) processing URL (D) deprocessing HTML

Previous Years' Questions

1. Match the following: **[2015]**

(P) Condition coverage	(i) Black-box testing
(Q) Equivalence class partitioning	(ii) System testing
(R) Volume testing	(iii) White-box testing
(S) Alpha testing	(iv) Performance testing

(A) P–ii, Q–iii, R–i, S–iv
(B) P–iii, Q–iv, R–ii, S–i
(C) P–iii, Q–i, R–iv, S–ii
(D) P–iii, Q–i, R–ii, S–iv

2. Which of the following statements is/are FALSE? **[2015]**

I. XML overcomes the limitations in HTML to support a structured way of organizing content.
II. XML specification is not case sensitive while HTML specification is case sensitive.
III. XML supports user defined tags while HTML uses pre-defined tags.
IV. XML tags need not be closed while HTML tags must be closed.

(A) II only (B) I only
(C) II and IV only (D) III and IV only

3. Consider the following C program segment.

```
while(first <= last)
{
   if (array[middle] < search)
            first = middle + 1;
   else if (array[middle] == search)
               found = TRUE;
            else last = middle - 1;
   middle = (first + last)/2;
}
if (first > last) notPresent = TRUE;
```

The cyclomatic complexity of the program segment is ______.

4. A software requirements specification (SRS) document should avoid discussing which one of the following? **[2015]**
(A) User interface issues
(B) Non-functional requirements
(C) Design specification
(D) Interfaces with third party software

5. Consider the basic COCOMO model where E is the effort applied in person-months, D is the development time in chronological months, KLOC is the estimated number of delivered lines of code (in thousands) and a_b, b_b, c_b, d_b have their usual meanings. The basic COCOMO equations are of the form **[2015]**
(A) $E = a_b(\text{KLOC})\exp(b_b)$, $D = c_b(E)\exp(d_b)$
(B) $D = a_b(\text{KLOC})\exp(b_b)$, $E = c_b(D)\exp(d_b)$
(C) $E = a_b\exp(b_b)$, $D = c_b(\text{KLOC})\exp(d_b)$
(D) $E = a_b\exp(d_b)$, $D = c_b(\text{KLOC})\exp(b_b)$

6. Which one of the following statements is NOT correct about HTTP cookies? **[2015]**
(A) A cookie is a piece of code that has the potential to compromise the security of an Internet user.
(B) A cookie gains entry to the user's work area through an HTTP header.
(C) A cookie has an expiry date and time.
(D) Cookies can be used to track the browsing pattern of a user at a particular site.

7. Which one of the following assertions concerning code inspection and code walkthrough is true? **[2015]**

(A) Code inspection is carried out once the code has been unit tested.
(B) Code inspection and code walkthrough are synonyms
(C) Adherence to coding standards is checked during code inspection
(D) Code walkthrough is usually carried out by independent test team

8. Consider a software project with the following information domain characteristics for calculation of function point metric.

Number of external inputs (I) = 30

Number of external outputs (O) = 60

Number of external inquiries (E) = 23

Number of files (F) = 08

Number of external interfaces (N) = 02

It is given that the complexity weighting factors for I, O, E, F and N are 4, 5, 4, 10 and 7, respectively. It is also given that, out of fourteen value adjustment factors that influence the development effort, four factors are not applicable, each of the other four factors have value 3, and each of the remaining factors have value 4. The computed value of function point metric is ______ **[2015]**

9. In a web server, ten WebPages are stored with the URLs of the form http://www.yourname.com/var.html; where, var is a different number from 1 to 10 for each Webpage. Suppose, the client stores the Webpage with var = 1 (say W_1) in local machine, edits and then tests. Rest of the WebPages remains on the web server. W_1 contains several relative URLs of the form 'var.html" referring to the other WebPages. Which one of the following statements needs to be added in W_1, so that all the relative URLs in W_1 refer to the appropriate WebPages on the web server? **[2015]**

(A) <a href: http://www.yourname.com/", href: "var.html">
(B) <base href: http://www.yourname.com/">
(C) <a href: http://www.yourname.com/">
(D) <base href: http://www.yourname.com/", range: "var.html">

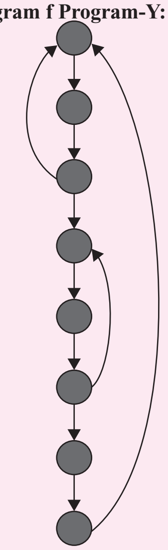
10. Consider a software program that is artificially seeded with 100 faults. While testing this program, 159 faults are detected, out of which 75 faults are from those artificially seeded faults. Assuming that both real and seeded faults are of same nature and have same distribution, the estimated number of undetected real faults is _________. **[2015]**

11. Consider three software items: Program-X, Control Flow Diagram of Program-Y and Control Flow Diagram of Program-Z as shown below **[2015]**

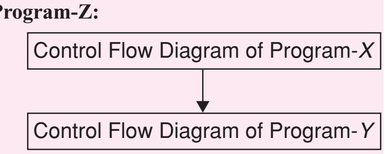
Program-X:

```
Sumcal (int maxint, int value)
{
int result=0, i=0;
if (value <0)
{
Value = -value;
}
While  (  (i<value)  AND  (result  <=
      maxint
{
i=i+1;
result = result + 1;
}
if (result <= maxint)
{
printf (result);
}
else
{
printf ("large")
}
printf ("end of program");
}
```

Control Flow Diagram f Program-Y:



Control Flow Diagram of Program-Z:



The value of McCabe's Cyclomatic complexity of program-X, Program-Y, and Program-Z respectively are

(A) 4, 4, 7 (B) 3, 4, 7
(C) 4, 4, 8 (D) 4, 3, 8

ANSWER KEYS

EXERCISES

Practice Problems 1

1. D **2.** B **3.** B **4.** A **5.** A **6.** A **7.** B **8.** C **9.** A **10.** A
11. B **12.** A **13.** A **14.** D **15.** D

Practice Problems 2

1. D **2.** B **3.** D **4.** C **5.** C **6.** D **7.** A **8.** D **9.** A **10.** C
11. A **12.** B **13.** A **14.** C **15.** A

Previous Years' Questions

1. C **2.** C **3.** 5 **4.** C **5.** A **6.** A **7.** C **8.** 612 to 613 **9.** B
10. 28 **11.** A

Test

Web Technologies

Time: 60 min.

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. The ______ tag contains information about the document including its title, scripts used, style definitions and documentation descriptions.
(A) <Head> (B) <Body>
(C) <HTML> (D) <Title>

2. Is it easier to process XML than HTML?
(A) Yes (B) No
(C) Sometimes (D) Can't say

3. Which of the following tags is the smallest heading tag?
(A) <H1> (B) <H6>
(C) <H0> (D) <H8>

4. In XML
(A) the internal DTD subset is read before the external DTD
(B) the external DTD subset is read before the internal DTD
(C) there is no external type of DTD
(D) there is no internal type of DTD

5. Attribute standalone = "no" should be included in XML declaration if a document
(A) is linked to an external XSL style sheet
(B) has external general references
(C) has processing instructions
(D) has an external DTD

6. Which of the following XML fragments are well-formed?
(A) <myEle myAtt = 'val1 = val2'/>
(B) <myEle myAtt = 'val1>val2'/>
(C) <myEle myAtt = 'val1>val2'/>
(D) None of the above

7. Parameter entities can appear in
(A) XML file (B) DTD file
(C) XSL file (D) Both (A) and (B)

8. The use of a DTD in XML document is
(A) required when validating XML document
(B) no longer necessary after the XML editor has been customized
(C) used to direct conversion using an XSLT processor
(D) a good guide to populating a template to be filled in when generating an XML document automatically.

9. To add the attribute named 'Branch' to the <Time> tag the syntax will be
(A) <Time attribute Branch = "DSNR">
(B) <Time Branch attribute = "DSNR">
(C) <Time Branch = "DSNR">
(D) None of these

10. The syntax for parameter entity is
(A) <!ELEMENT % NAME DEFINITION>
(B) <!ENTITY % NAME DEFINITION>
(C) <!ENTITY $ NAME DEFINITION>
(D) <ENTITY % NAME DEFINITION>

11. A schema can be named using the name attribute like
(A) <schema attribute = "schema1">
(B) <schema nameattribute = "sehema1">
(C) <schema nameattri = "schema1">
(D) <schema name = "schema1">

12. A schema describes
(i) grammer
(ii) vocabulary
(iii) structure
(iv) data type of XML document
(A) (i) and (ii) (B) (iii) and (iv)
(C) Both (A) and (B) (D) None of the above

13. The XML DOM object is
(A) an attribute (B) entity reference
(C) comment reference (D) comment data

14. The default model for COMPLEXTYPE, in XML schema for element is
(A) text only (B) element only
(C) no default type (D) Both (A) and (B)

15. To create a choice in XML schemas, we use
(A) <xsd: select> element
(B) <xsd: multi> element
(C) <xsd: choice> element
(D) <xsd: single> element

16. To bind the HTML element <INPUT> Type in text with the data source "dsoCustomer" we use
(A) <INPUT TYPE = "TEXT" DATAFIELD = "#dsoCustomer">
(B) <INPUT TYPE = "TEXT" DATASRC = "dsoCustomer">
(C) <INPUT TYPE = "TEXT" DATASRC = "#dsoCustomer">
(D) <INPUT TYPE = "TEXT" DATAFLD = "*dsoCustomer">

17. The attribute used to define a new namespace is
(A) XMLN Space (B) Xml Name Space
(C) Xmlns (D) XmlNs

18. Identify the most accurate statement about the application of XML:
(A) XML must be used to produce XML and HTML output.
(B) XML can not specify or contain presentation information.

(C) XML is used to describe hierarchically organized information.
(D) XML performs the conversion of information between different e-business applications.

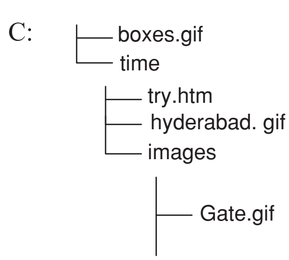
19. What is an XML namespace?
(A) A set of names applied to specific spaces within an XML document, such as the head and body.
(B) A set of names representing a specific XML vocabulary.
(C) Both (A) and (B)
(D) None of these

20. The standard model for network application is
(A) producer consumer model
(B) node-node model
(C) system-system model
(D) client-server model

21. Which is/are example of service(s) that a server can provide?
(A) Return the time-of-day to the client.
(B) Print a file for the client
(C) Execute a command for the client on the server's system
(D) All of the above

22. Print a file, read or write a file for client are handled in a
(A) iterative fashion
(B) concurrent fashion
(C) Both (A) and (B)
(D) None of these

Common data for questions 23 and 24: Consider working with the file try.htm and provided the following directory structure:



23. The tag specification to use boxes.gif file as background will be
(A) <Body BACKGROUND = "/boxes.gif">
(B) BODY BACKGROUND = "/boxes.gif
(C) <BODY BACKGROUND = "boxes.gif">
(D) <BODY BACKGROUND = "#boxes.gif">

24. The tag specification to use gate.gif file as back ground will be
(A) <BODY BACKGROUND = "../gate.gif">
(B) <BODY BACKGROUND = ".gate.gif">
(C) <BODY BACKGROUND = "images/gate.gif">
(D) None of these.

25. Consider below HTML code:
(1) <HTML>
(2) <FRAMESET ROWS = "30%, *">
(3) <FRAMESET COLS = "50%, 50%">
(4) <FRAME Src = "file1. html">
(5) <FRAME Src = "File2.html">
(6) </FRAMESET>
(7) <FRAMESET COLS = "50%, 50%">
(8) <FRAME Src = "File3.html">
(9) <FRAME Src = "File4.HTML">
(10) </FRAMESET>
(11) </FRAMESET>
(12) </HTML>
We can embed <BODY> tag at line number.
(A) 2 (B) 11
(C) Anywhere (D) No where

26. What is the purpose of <area> tag?
(A) It defines an area inside a table.
(B) It defines an area inside an image map.
(C) It defines the area of total HTML code.
(D) Both (A) and (B)

27. Which of the following tag is used to define a section in a document?
(A) <frame> (B)
(C) <div> (D) <wbr>

28. Which of the following is not a syntax rule of an XML document?
(A) All XML tags must have a closing tag.
(B) All XML tags are case-sensitive.
(C) XML elements must be properly nested.
(D) XML documents may or may not have a root tag.

29. XML documents follow which structure?
(A) Graph (B) Tree
(C) Stack (D) Queue

30. Which of the following is an advantage of client-server computing?
(A) Client-server computing provides cost-effective user interface.
(B) Client-server computing provides storage for data.
(C) Client-server computing provides application services.
(D) All the above

Answer Keys

1. A	**2.** A	**3.** B	**4.** A	**5.** D	**6.** C	**7.** B	**8.** A	**9.** C	**10.** B
11. D	**12.** C	**13.** B	**14.** B	**15.** C	**16.** C	**17.** C	**18.** C	**19.** B	**20.** D
21. D	**22.** B	**23.** A	**24.** C	**25.** D	**26.** B	**27.** C	**28.** D	**29.** B	**30.** D

Chapter 1

Process Life Cycle

LEARNING OBJECTIVES

- *Introduction*
- *Process vs program*
- *Software component and elements*
- *Information gathering*
- *Requirement analysis*
- *Feasibility study*
- *Data flow diagram*
- *Process specification*
- *Input/output design*
- *Software process life cycle*
- *Software process model*

Introduction

A system can be defined as an orderly grouping of interdependent components linked together according to a plan to achieve a specific objective.

Example: Telephone system, transportation system, accounting system, etc.

Process Versus Program

A software process gives all steps used to create a software application, from the customer's requirements to the finished product.

- The software process determines the organization and flexibility of the project.
- There are several different software processes and each describes their own solution to develop a valid software.
- Software programs are written programs or rules with associated documentation pertaining to the operation of a computer system.

Software Components and Elements

Software Component

It is a software element that can be independently deployed and composed without modification according to a composition standard.

- A component model implementation is the dedicated set of executable software elements required to support the execution of components.
- A component has clearly defined interfaces.
- An interface standard is the mandatory requirement enforced to enable software elements to directly interact with other software elements.
- An interface standard declares, when an interface comprises.

Standard An object or measure serving as a basis to which others should conform, by which the quality of others is judged.

Software Element

A sequence of abstract program statements that describe computations, which has to be performed by a machine.

Interface

It describes the behavior of a component that is obtained by considering only the interactions of that interface and by hiding all other interactions.

- An abstraction of the behavior consists of subset of the interactions of one component together with a set of constraints.

Interaction

It is defined as action between 2 or more software elements.

Composition It is a combination of 2 or more software components, the newly formed component, behaviour will be at a different level of abstraction.

The characteristics of new component is determined the components combined and the way in which they are combined.

Information Gathering

Complete and accurate information is essential in building computer-based systems. Information about the organization, the staff who uses the system and the workflow should be gathered.

Information about the organization's policies, goals, objectives and structure explains the kind of environment the computer-based system should produce.

Information about the people who run the present system, their job functions and information requirements, the relationships of their jobs to the existing system and the interpersonal network that holds the user groups together are required for determining the importance of the existing system for the organization and also for planning the proposed system.

Workflow focuses on what happens to the data through various points in a system and can be shown by a data flow diagram or a system flow chart.

Information can be gathered by studying documents, forms and files of existing system. Onsite observation of the system is also an effective method for gathering information. It is the process of recognizing and noting people, objects and occurrences to obtain information. Interview is one of the most often and oldest method for gathering information. Interview has the advantage of identifying relations or verifying information and also capture information face to face with the concerned person. Questionnaire is another method for gathering information and is an inexpensive mean for gathering data which can be tabulated and analyzed quickly. Visiting companies that have developed similar systems, reading journals and other computer related books, which specify how others have solved similar problems is also another means of information gathering.

Requirement Analysis

Requirement analysis results in specification of software's operational characteristics, indicates software's interface with other system elements and establishes constraints that the software must meet.

During requirement analysis, the primary focus should be on *what* not *how*. It should define what user interaction occurs in a particular circumstance, what objects does the system manipulate, what functions must the system perform, what behaviours does the system exhibit, what interfaces are defined and what constraints applied.

The requirement analysis model must achieve three primary objectives:

1. To describe what the customer requires.
2. To establish a basis for the creation of a software design
3. To define a set of requirements that can be validated once the software is built.

Requirement Negotiation

Requirement negotiation is required to have a win-win result. The customer should get product which satisfies most of his/her needs, and software team should develop a product within a budget, working in real-time environment and within deadlines.

Boehm has defined negotiation activities at the beginning of each software process iteration.

- Identify the key stake holder's system or subsystem.
- Determine the 'win conditions' of the stake holder.
- Negotiate the 'win conditions' of stake holder and establish them into win-win condition.

Requirement Elicitation

Requirement elicitation is gathering the requirements from stakeholders, customers, etc. The question and answer format is suitable for the first encounter with users and the remaining phases are replaced with requirement elicitation. As we can't get all the requirements by having questions and answers session, requirement elicitation practices should be implemented, which includes interviews, workshops, user scenarios, etc.

The approaches that are followed for eliciting the requirements are:

1. Collaborative requirement gathering
2. Quality function deployment
3. User scenarios
4. Elicitation work products

Functional Requirements

Functional requirements are primary actions that must take place in software in accepting and processing the input and in processing and generating the output.

Functional requirements capture the intended behaviour of the system, which could be expressed as service task (or) functions of the system.

These are core functionalities of the system. It also includes exact sequence of operations, input validation, mapping of outputs to inputs and error handling and recovery. These requirements are implemented in system design.

Non-functional Requirements

Non-functional requirements are expected requirements of a user. These define operational constraints based on the user characteristics.

Non-functional requirements are product, business and external-based. These requirements define how a software system has to be. It also defines the quality of product, type of reliability and usability of the system. Implementation requirements depend on organization, and delivery requirements are defined in the non-functional requirements.

Non-functional requirements are implemented in system-architecture.

Measuring Requirements

The requirements are the major component of project. The metrics for the requirement activities are:

1. Product size
2. Requirement quality
3. Requirement status
4. Requirement change (request for changes)
5. Effort

Product size refers to the count of functional and non-functional requirements. It tracks, whether these requirements are implemented as a function of time.

Requirement quality refers to the inspection of specification of requirements, counting the defects, missing of requirements incompleteness, ambiguities, etc.

Requirements status is monitoring of requirements over time, gives out project status. The status could be proposed, approved, implemented, verified deferred, deleted, rejected.

Requirement management handles the addition, modify and deletion of requirement, track the change of requirement which affects multiple requirements of different level.

Effort is the time taken to record requirements activities which includes development and management of requirements.

Types of Requirements

These are the services that a software system has to provide and constraints under which it must operate.

User requirements

- Written for customers.
- These statements will be given in natural language and diagrams of the services that the system provides and operational constraints.

System requirements

- Written as a contract between contractor and client.
- A structured document with detailed descriptions of the system services.

Software specification

- Written for developers.
- A detailed description of software that can act as basis for a design (or) implementation.

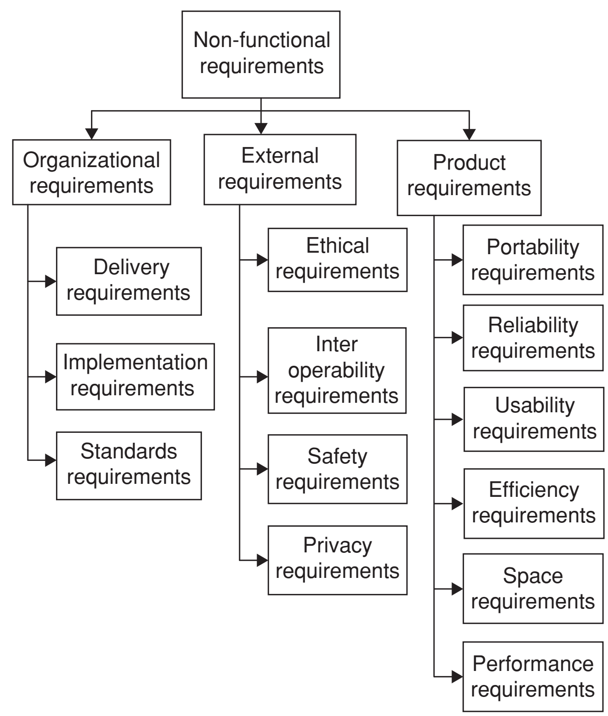
Functional requirements

The system should provide statements of services, how the system should react to particular inputs and how the system should behave in particular situations.

Non-functional requirements

Functions offered by the system such as timing constraints, constraints on the development process, standards, etc.

1. *Product requirements:* It specifies, that the delivered product must behave in a particular way.
   Example: Execution speed, reliability
2. *Organizational requirements:* These are consequences of organizational policies and procedures.
   Example: Process standards, implementation requirements
3. *External requirements:* These arise from factors which are external to the system and its development process.
   Example: Interoperability requirements



Domain requirements

These come from the application domain of the system that reflects the characteristics of the domain.

- These could be functional or non-functional.

Feasibility Analysis

Feasibility study is a test of a system proposal according to its workability, impact on the organization, ability to meet user needs, and effective use of resources.

The objective of a feasibility study is not to solve the problem but to acquire a sense of its scope. Costs and benefits are estimated with greater accuracy at this stage. Feasibility analysis helps to identify the best solution to the end user. The key considerations involved in feasibility analysis are:

1. Economic feasibility
2. Technical feasibility
3. Behavioral feasibility

Economic analysis is the most frequently used method for evaluating the effectiveness of a candidate system. Also known as cost/benefit analysis, economic analysis determines the benefits and savings that are expected from the candidate system and compares them with the costs. If benefits outweigh costs, then decision is made to design and implement the system, else alterations are made if it has a chance of being approved.

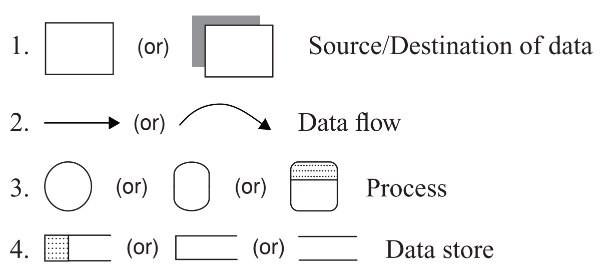
Technical feasibility is concerned with hardware and software requirements to implement the system. Technical analysis centres around the existing computer system (hardware, software, etc.) identifies, to what extent it can support the proposed addition. Additional hardware and software (OS, databases) requirements are identified and checks whether financial considerations/constraints can accommodate these technical enhancements.

Behavioural analysis makes an estimate of how strong a user staff is likely to react towards the development of a computerized system. Computer installations usually changes employee job status, and also there may be transfer, training period, etc. Thus the introduction of a new system requires special effort to educate, sell and train the staff on new ways of conducting business.

Data Flow Diagrams (DFD)

DFD also called bubble chart, clarifies system requirements and identifies major transformations that will become programs in system design. It functionally decomposes the requirements specification down to the lowest level of detail.

The four DFD symbols are:



The first symbol defines a source or destination of system data. The second symbol specifies data flow direction. It can be considered as a pipeline through which the information flows. The third symbol represents a process that transforms incoming data flows into outgoing data flows, and the fourth symbol is used to represent storage of data.

In short, DFD takes an input-process-output view of a system. That is, data objects flow into the software is transformed by processing elements and resultant data objects flow out of the software. Data objects are represented by labelled arrows and transformations are represented by circles (also called bubbles). The DFD is presented in a hierarchical fashion. That is, the first data flow model sometimes called level 0 DFD or context diagram represents the system as a whole. Subsequent data flow diagrams refine the context diagram, providing increase in detail with each subsequent level.

Process Specification (PSPEC)

The process specification (PSPEC) is used to describe all flow model processes that appear at the final level of refinement. The content of the process specification can include narrative text, a program design language (PDL) description of the process algorithm, mathematical equations, tables or UML activity diagrams.

By providing a PSPEC to accompany each transformation (bubble) in the flow model, a 'mini-spec' can be created that serves as a guide for design of the software component that will implement the transformation.

Input/Output Design

Input Design

The most common cause of errors in data processing is inaccurate input data. Errors occurred during data entry can be controlled by input design.

Input design is the process of converting user-originated inputs to a computer-based format.

Input data is collected and organized into groups of similar data. The goal of designing input data is to make data entry easy, logical and free from errors.

Source data is captured initially on original paper or a source document. A source document should be logical and easy to understand. Each area in the form should be clearly identified and should specify to the user what to write and where to write.

Source documents may enter into the system from punch cards, diskettes, optical character recognition (OCR) reader, Magnetic ink character recognition (MICR) reader, barcode reader, etc. Touch screen or voice input can be used for online data entry, for example, ATM.

There are three major approaches for entering data into the computer—menus, formatted forms and prompts.

A menu is a selection list that simplifies computer data access or entry. The user can choose what to enter from a list of options. Though a menu limits a user choice of responses, it reduces the chances of errors in data entry.

A formatted form is a preprinted form or a template that requests the user to enter data in appropriate locations (fill-in the blank type form). The form is displayed on the screen and the user can fill information by positioning the cursor in appropriate text boxes.

In prompt, the system displays one inquiry at a time, asking the user for a response, for example, asking for user-id and password.

Output Design

Computer output is the most important and direct source of information to the user. Efficient and intelligible output design will improve system's relationships with the user and helps in decision making.

The devices available for providing computer-based output are printer, CRT screen display, audio response (speaker), plotters, etc.

The task of output preparation is very critical, regaining skill and ability to align user requirements with the capabilities of the system in operation.

SOFTWARE PROCESS LIFE CYCLE

A software process can be defined as a frame work of the activities, actions and tasks that are required to build quality software.

All these activities, actions and tasks reside within a frame work or model that defines their relationship with the process and with one another.

A generic process frame work for software engineering encompasses five activities:

Communication Proper communication and collaboration with the customer is made in this activity to understand the objectives for the project and also to gather requirements that help to define software features and functions.

Planning This activity develops a software project plan which defines the software engineering work by specifying the technical tasks to be conducted, the risks that may occur, the resources that will require, the work products to produce and the work schedule.

Modelling A software engineer creates models to better understand software requirements and the design that will achieve those requirements.

Construction This activity combines code generation and testing required to uncover errors in the code.

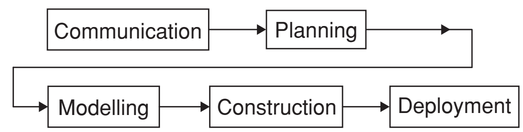
Deployment In this activity, the software (as a complete product or as a partial increment) is delivered to the customer. The customer evaluates the delivered product and provides feedback based on evaluation.

Another important aspect of the software process called process flow describes how the frame work activities and the actions and tasks that occur within each framework activity are organized with respect to sequence and time.

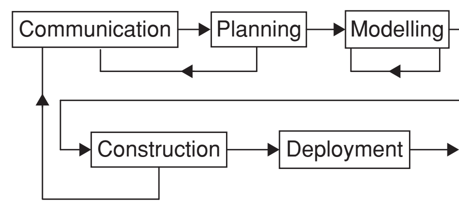
Process Quality and Improvement

Quality refers to characteristic or attribute of something. Process quality factors are portability, usability, reusability, correctness and maintainability. The process quality is the implementation of the following steps firstly initiates the process and design the solutions, implement these solutions with the impact demonstration.

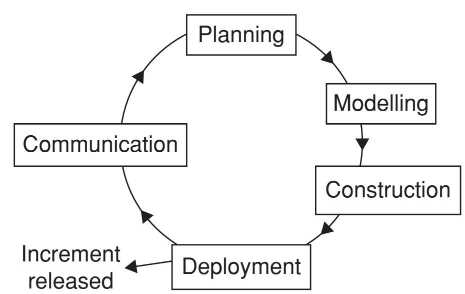
Linear process flow executes each of the five framework activities in sequence, beginning with communication and ends with deployment.



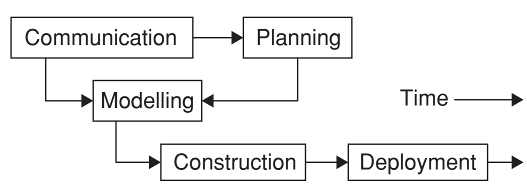
Iterative process flow repeats one or more of the activities before proceeding to the next activity.



Evolutionary process flow executes the activities in a circular manner. Each circuit through the five activities leads to a more complete version of the software.



Parallel process flow executes one or more activities in parallel with other activities.



The Unified Process

The unified process (UP) is an attempt to draw on the best features and characteristics of conventional software process models. It recognizes the importance of customer communication and streamlined methods for describing the customer's view of a system. It helps the architect focus on the right goals, such as understandability, reliance to future changes, and reuse. It suggests a process flow that is iterative and incremental, providing the evolutionary feel that is essential in modern software development.

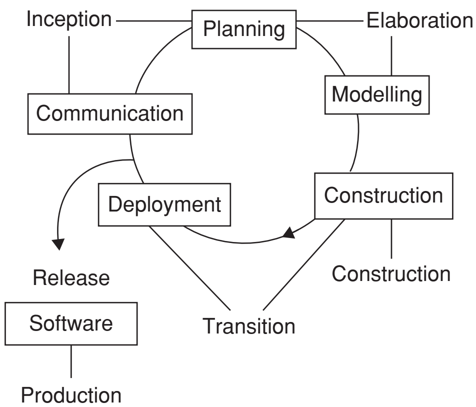


Figure 1 Phases of the unified process

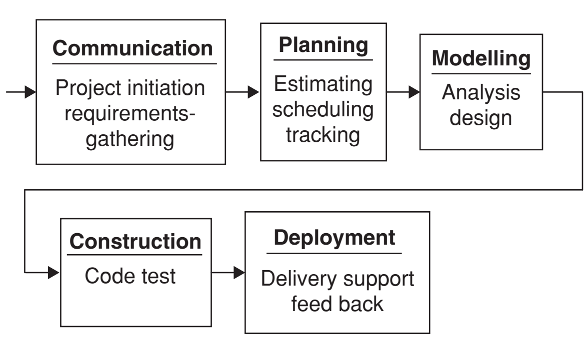
The unified process is an incremental model in which five phases are defined:

1. Inception phase: Encompasses both customer communication and planning activities and emphasizes the development and refinement of use cases as a primary model.
2. Elaboration Phase: Encompasses the customer communication and modelling activities focusing on the creation of analysis and design models with an emphasis on class definitions and architectural representations.
3. Construction phase: Refines and translates the design model into implemented software components.
4. Transition phase: Transfers the software from the developer to the end user for beta testing and acceptance.
5. Production phase: Ongoing monitoring and support are conducted. Defect reports and requests for changes are also submitted and revaluated.

Software Process Models

The Waterfall Model

The waterfall model also called classic life cycle, follows a systematic sequential approach to software development.



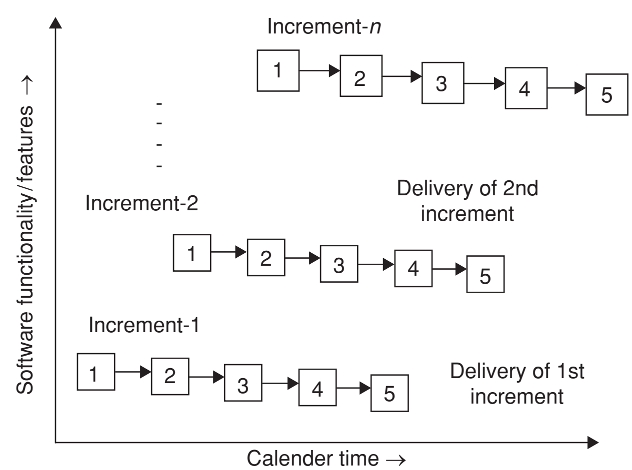
It begins with customer specification of requirements and progresses through planning, modelling, construction and deployment, culminating in on-going support of the completed software.

The waterfall model is the oldest paradigm for software engineering. The problems encountered when this model is applied are:

1. Real projects rarely follow the sequential flow that the model proposes.
2. This model requires the requirements explicitly which the customer cannot state all the requirements as it is difficult.
3. A working version of the program will not be available until late in the project time span. If a major blunder is undetected until the working program is reviewed, it can be disastrous.

Incremental Process Model

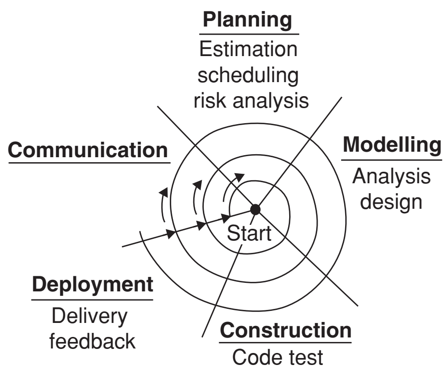
1. Communication
2. Planning
3. Modelling (analysis, design)
4. Construction (code, test)
5. Deployment (delivery, feedback)



Incremental development is particularly useful when staffing is unavailable for a complete implementation by the business deadline that has been established for the project.

Spiral Model

Spiral model is an evolutionary software process model. Using spiral model, software is developed in a series of evolutionary releases. During early iterations, the release might be a model or prototype. Later iterations produce more complete versions of the system.



The spiral development model is a risk-driven process model generator that is used to guide multi stakeholder concurrent engineering of software intensive systems.

The spiral model is a realistic approach to the development of large scale systems and software. It uses prototyping as a risk reduction mechanism but, more importantly, enables the developer to apply the prototyping approach at any stage in the evolution of the product. At demands a direct consideration of technical risks at all stages of the project.

Conceptual Modelling

Conceptual modelling refers to abstraction of a model which fits for the purpose. The purpose of this modelling is to make model valid credible, feasible and useful.

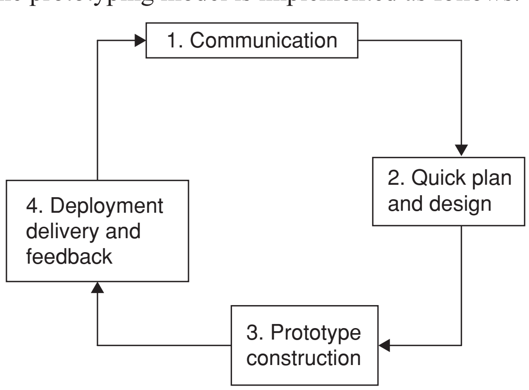
The main objective of conceptual modelling is improvising the understanding of an individual with respect to the system, an approach which will convey the system details among the stakeholders.

For the extraction of system specifications when a software is developed, some of the failures could occur in future due to lack of requirements [unclear requirements (or) changing requirements] This could be traced with the help of conceptual modelling.

Prototyping Model

Prototyping model is used when the user is not sure about the addition of requirements in the product. It is also implemented when the developer is not sure about the algorithm efficiency, operation system adaptability, etc. Prototyping paradigm provides the approaches.

The prototyping model is implemented as follows:



Prototyping model starts with communication in which software objectives is defined requirement, identification are done. In quick design, all the software aspects are represented quick design leads to prototype construction.

This prototype model is deployed, in which requirements are evaluated and refined by customer.

The iteration is done until customer gets satisfied with the needs at the same time developer will come to know what are the needs to be done.

Disadvantages

1. Developer may compromise at implementation, as prototyping works quickly. Un-ideal implementation issues may become an integral part of the system.
2. Customer just sees the working version of the software, he could not able to consider the quality of software and long-term maintenance.

Though there are some problems with prototyping, but it is effective paradigm for the software engineering when a software is developed using prototyping, both developer and customer should agree on the prototype.

It is more advantageous when the customer and user are not sure what they want it maintains a template of the older software.

Role of metrics and measurement in software development

The software attributes that were present in process, project and product levels are called measurement.

The metric refers to the attributes that are included in the project.

A software engineer gets the measurements and develops the metrics.

Measurement is done in two ways:

1. Direct measure
2. Indirect measure

Direct measure includes the lines of code, (least, moderate, worst) execution speed and size of the memory.

Indirect measure is done with the help of functional points. It measures quality, maintainability, efficiency and reliability.

Metric is used to control the cost, schedule, project quality. It means metric provides information for the control of process development.

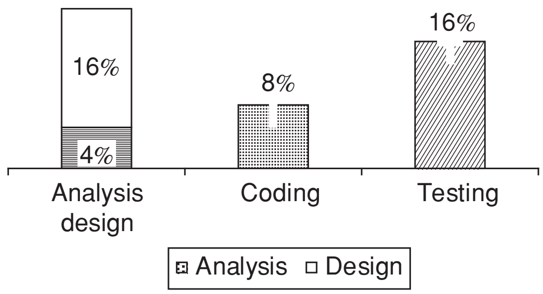
Effort distribution with phases

Software development is done in phases. It includes analysis, design, coding and testing.

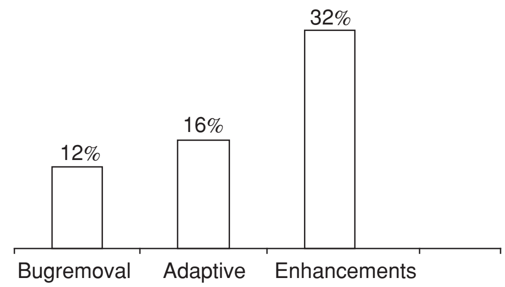
Design and testing plays major role in development, while coding is having least preference.

40% of the efforts were done on development and 60% of efforts are on the maintenance.

Distribution of the efforts on the development is shown below:



Maintenance includes removal of bug and corrective maintenance, adaptive maintenance and enhancement. Distribution of efforts in maintenance is shown below:



Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Which of the following statements is true?
   (A) The first step to the system study project is to announce the study project.
   (B) During the system study analysis determine manager's information needs by asking questions.
   (C) During the system study, flowcharts are drawn using general symbols.
   (D) All the above
2. Which of the following statement(s) is true regarding the spiral model of software development?
   (A) In the spiral model of software development, the primary determinant in selecting activities in each interaction is risk.
   (B) The spiral model is a risk driven process model generator that is to guide multi-stakeholder. Concurrent engineering of software intensive systems.
   (C) Using the spiral model, software is developed in a series of evolutionary releases.
   (D) All the above
3. Which of the following is a step in feasibility analysis?
   (A) Form a project team and appoint a project head.
   (B) Determine and evaluate performance and cost effectiveness of each candidate system.
   (C) Weigh system performance and cost data.
   (D) All the above
4. Which of the following statement(s) is true?
   (A) The risk driven nature of the spiral model allows it to accommodate any mixture of specification oriented or some other approach.
   (B) Each cycle of spiral is completed by review which covers all the products developed during that cycle, including plans for the next cycle.
   (C) Spiral model works for development as well as enhancement project.
   (D) All the above
5. Data flow diagram, regular expression and transition table can be combined to provide
   (A) decision table for functional specification of system software.
   (B) finite state automata for functional specification of system software.
   (C) event table for functional specification of system software.
   (D) None of these
6. Which of the following statements are true about software configuration management tool?
   (A) It keeps track of the schedule based on the mile stones reached.
   (B) It manages man power distribution by changing the structure of the project.
   (C) It maintains different versions of the configurable items.
   (D) All the above
7. The cost incurred on a project was ₹250,000 and benefits were ₹30,000 per month. The payback period using simple pay back method is
   (A) 8 months (B) 8.3 months
   (C) 12 months (D) 1.2 months
8. Which of the following phase has the maximum effort distribution?
   (A) Testing (B) Information gathering
   (C) Requirement analysis (D) Coding
9. Which of the following statement is true regarding cost benefit analysis?
   (A) It evaluates tangible and non-tangible factors.
   (B) It estimates the hardware and software costs.
   (C) It compares the cost with the benefits of introducing a computer-based system.
   (D) All the statements are true.
10. A project is considered economically feasible if the following factor holds good.
    (A) Return on investment (ROI)
    (B) Total cost of ownership (TCO)

(C) Gross domestic product (GDP)
(D) Net present value (NPV)

11. At the end of the feasibility study the system analyst
(A) meets the users for a discussion.
(B) gives system proposal to management.
(C) gives a feasibility report to management.
(D) gives a software requirement specification (SRS).

12. In a data flow diagram, data flows cannot take place between
(A) two data stores
(B) two external entities
(C) a data store and an external entity
(D) Both (A) and (B)

13. Consider the decision table shown below. It is

	R_1	R_2	R_3	R_4
C1	Y	Y	Y	N
C2	Y	N	Y	Y
C3			Y	Y
A1	X		X	
A2		X		X

(A) an ambiguous decision table.
(B) a complete decision table.
(C) an incomplete decision table.
(D) Both (A) and (B)

14. Which of the following requirement specifications can be validated?
(S1): If the system fails during any operation, there should not be any loss of data.
(S2): Checking the hardware compatibility.
(S3): Defining a data interface.
(S4): Specification of response time for various functions.
(A) S1 and S2 (B) S2, S3 and S4
(C) S1, S3 and S4 (D) S1 and S4

15. Which of the following are true?
(i) A DFD should have loops.
(ii) A DFD should not have crossing lines.
(iii) Leveled DFD is easier to understand.
(iv) Context diagrams are not used in DFDs.
(A) (ii) and (i) (B) (i) and (iv)
(C) (ii) and (iii) (D) (iii) and (iv)

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Questionnaire consists of
(A) Forms (B) Documents
(C) Qualitative data (D) Quantitative data

2. The method to obtain qualitative information is
(A) Background information
(B) Questionnaires
(C) Interviewing technique
(D) Journals and reports on similar systems

3. Which among the following is a functional requirement?
(A) Description of all input data and their sources
(B) Capacity requirements
(C) Operating system available on the system
(D) Maintaining a log of activities

4. The advantage of use case during requirement analysis phase is, it
(A) focuses on external behaviour only.
(B) focuses on internal behaviour only.
(C) focuses on additional behaviour.
(D) focuses on internal and external behaviour.

5. Operational feasibility refers to
(A) technology needed is available and if available whether it is usable
(B) the proposed solution can fit in with existing operations
(C) the money spent is recovered by savings
(D) superior quality of products

6. Software engineering is the application of
(A) Systematic approach of the development
(B) Quantifiable approach of the development
(C) Discipline approach of the development
(D) All of these

7. The data flow model of an application mainly shows:
(A) The underlying data and the relationship among them
(B) Processing requirement and the flow of data
(C) Decision and control information
(D) Communication network structure

8. DFD completeness is
(A) The process of discovering discrepancies between two or more sets of DFDs or discrepancies within a single DFD.
(B) The extent to which all necessary components of a data flow diagram have been included and fully decomposed.
(C) The conversation of inputs and outputs to a DFD process when that process is decomposed to a lower level.
(D) An iterative process of breaking the description of a system down into a finer and finer details, which creates a set of charts in which one on a given chart is explained in greater detail on another chart.

9. The requirement analysis is performed in
(A) System design phase
(B) System development phase
(C) System analysis phase
(D) System investigation phase

10. In data flow diagram, an originator or receiver of data is usually designed by
(A) square box (B) circle
(C) rectangle (D) arrow

11. A feasibility document should contain all the following except
(A) project name
(B) problem description
(C) feasible alternative
(D) data flow diagrams

12. SRS document is _______ between customers and developers.
(A) legal contract
(B) standard
(C) request proposal
(D) None of the above

13. According to Brooks, adding more people to an already late software project makes it
(A) late
(B) fast
(C) does not impact schedule
(D) None of the above

14. The following is a quality metric:
(A) Correctness
(B) Maintainability
(C) Usability
(D) All of the above

15. Feasibility study should focus on
(A) Technical feasibility
(B) Economic feasibility
(C) Operational feasibility
(D) All of the above

Previous Years' Questions

1. What is the appropriate pairing of items in the two columns listing various activities encountered in a software life cycle? **[2010]**

P	Requirements capture	1	Module development and integration
Q	Design	2	Domain analysis
R	Implementation	3	Structural and behavioural modelling
S	Maintenance	4	Performance tuning

(A) P–3, Q–2, R–4, S–1
(B) P–2, Q–3, R–1, S–4
(C) P–3, Q–2, R–1, S–4
(D) P–2, Q–3, R–4, S–1

2. Which one of the following is NOT desired in a good SRS document? **[2011]**
(A) Functional requirements
(B) Non-functional requirements
(C) Goals of implementation
(D) Algorithms for software implementation

Answer Keys

Exercises

Practice Problems 1

1. D **2.** D **3.** D **4.** D **5.** B **6.** C **7.** B **8.** A **9.** B **10.** A
11. C **12.** D **13.** C **14.** B **15.** C

Practice Problems 2

1. D **2.** C **3.** A **4.** A **5.** B **6.** D **7.** B **8.** B **9.** C **10.** A
11. D **12.** A **13.** A **14.** D **15.** D

Previous Years' Questions

1. B **2.** D

Chapter 2

Project Management and Maintenance

LEARNING OBJECTIVES

- *Project management*
- *Software design*
- *Modeling component level design*
- *SRS*
- *Software testing*
- *White-box testing*
- *Black box testing*
- *Implementation maintenance*
- *Software quality assurance*
- *Software Re-engineering*
- *COCOMO MODEL*

Project Management

Project management is a technique used to ensure successful completion of a project by the project managers.

The functions included in project management are:

- Estimating resource requirements
- Scheduling tasks and events
- Providing training and site preparation
- Selecting qualified staff and supervising their work
- Monitoring the projects program
- Documenting
- Periodic evaluation
- Contingency planning

Project management involves planning, organization and control projects. It uses tools and software packages for planning and managing projects.

Project planning involves plotting project activities against time frame.

Project Planning Tools

- Tools used during software planning
- Helps the top level managers to take critical decisions during planning stage

Gantt Charts

This activity scheduling method introduced in 1914 by Henry L. Gantt, uses horizontal bars to show the duration of actions or tasks. The left end marks the beginning of the task and the right end its finish. Earlier tasks appear in the upper left and later ones in the lower right.

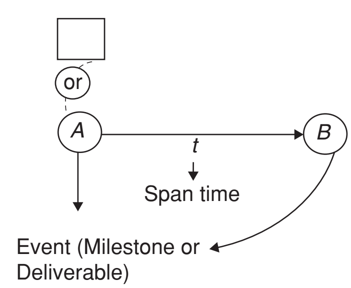
In real-life applications, an allowance for contingencies is provided. This is called **slack time.** Each project allows between 5 to 25 percent slack time for completion.

Program Evaluation and Review Technique (Pert)

Gantt charts do not show precedence relationships among the tasks and milestones of a project.

A PERT chart is a project management tool used to schedule, organize and coordinate tasks within a project.

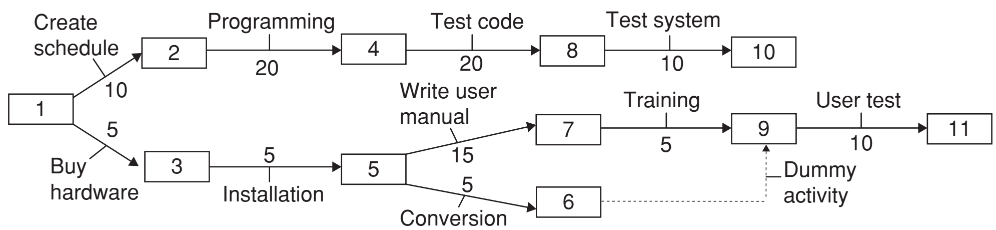
A PERT chart presents a graphic illustration of a project as a network diagram consisting of numbered nodes (either circles or rectangles) representing events, or milestones in the project linked by labelled vectors (directional lines) representing tasks in the project. The direction of the arrows on the lines indicates the sequence of tasks.



In the diagram, shown below the tasks between nodes 1, 2, 4, 8 and 10 must be completed in sequence and are called dependent or serial tasks. The tasks between nodes 1 and 2 and nodes 1 and 3 are not dependent on the completion of one to start the other and can be undertaken simultaneously. These tasks are called parallel or concurrent tasks. Tasks that must be completed in sequence but don't require resources or completion time are represented by dotted lines with arrows and are called dummy activates (Example: dashed arrow linking 6 and 9).

Numbers on the opposite sides of the vectors indicate the time allotted for the task.

The PERT chart is preferred over Gantt chart since it clearly illustrates task dependencies. But on complex projects, PERT chart may be much more difficult to interpret.



Thus in short,

Dependency diagrams can be defined as a formal notation to help in the construction and analysis of complex schedules. Dependency diagrams are drawn as a connected graph of nodes and arrows. Dependency diagrams consists of three elements:

- Event–A significant occurrence in the life of a project.
- Activity–Amount of work required to move from one event to the next.
- Span time–Actual calendar time required to complete an activity.

Software Design

Software design is the process in which requirements are translated into a blue print for constructing the software.

Once software requirements have been analyzed and modelled, software design is the last software engineering action within the modelling activity and sets the stage for construction (code generation and testing).

Architectural design defines the relationship between major structural elements of the software, the architectural styles and design patterns, that can be used to achieve the requirements defined for the system, and the constraints that affect the way in which architecture can be implemented.

The interface design describes how the software communicates with systems that interoperate with it, and with humans who use it.

The component-level design transforms structural elements of the software architecture into a procedural description of software components.

The major goals of the design process are:

- The design must implement all of the explicit requirements contained in the requirements model, and it must accommodate all the implicit requirements, desired by stakeholders.
- The design must be a readable, understandable guide for those who test and subsequently support the software.
- The design should provide a complete picture of the software, addressing the data, functional and behavioral domains from an implementation perspective.

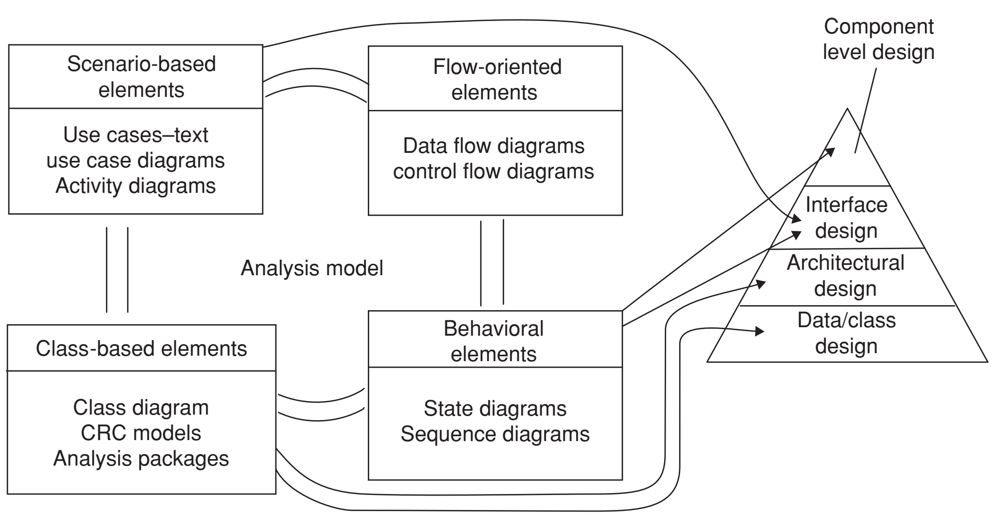


Figure 1 Design model.

Software design sits at the technical kernel of software engineering and is applied regardless of the software process model that is used. In the beginning, once the software requirements have been analyzed and modeled, software design is the last software engineering action within the modeling activity and sets the stage for construction (code generation and testing).

- The data/class design transforms analysis–class models into design class realizations and the requisite data structures required to implement the software.
- The architectural design defines the relationship between major structural elements of the software, the architectural styles and design patterns that can be used to achieve the requirements defined for the system.
- The interface design describes how the software communicates with systems that interoperate with it, and with humans who use it. An interface implies a flow of information (data/control) and a specific type of behavior.
- The component-level design transforms structural elements of the software architecture into a procedural description of software components.

Design Concepts

Important software design concepts:

Abstraction Many levels of abstraction can be posed while considering a modular solution to any problem. At highest level of abstraction, a solution is stated in broad terms and at lower levels, a more detailed description of the solution is provided. At the lowest level of abstraction, the solution is stated in a manner that can be directly implemented.

Architecture Architecture is the structure or organization of program components (modules), the manner in which these components interact, and the structure of data that are used by components.

Patterns The intent of each design pattern is to provide a description that enables a designer to determine:

1. whether the pattern is applicable to current work.
2. whether the pattern can be reused.
3. whether the pattern can serve as a guide for developing a similar, but functionally or structurally different pattern.

Separation of concerns Separation of concerns is a design concept that suggests that any complex problem can be more easily handled if it is subdivided into pieces that can be solved and/or optimized independently.

A concern is a feature or behaviour that is specified as part of the requirement model for the software.

Modularity common manifestation of separation of concerns. Software is divided into separately named and addressable components (modules) that are integrated to satisfy problem requirements.

Information hiding Modules should be specified and designed so that information (algorithm and data) contained within a module is inaccessible to other modules that have no need for such information.

Functional independence Software should be designed in such a way that each module addresses a specific subset of requirements and has a simple interface when viewed from other parts of the program structure.

Functional independence is achieved by developing modules, which can perform a single function.

Refinement Refinement is a process of elaboration, begins with a statement or function defined at a high level of abstraction and then elaborates the original statement, providing more and more details as each successive refinement (elaboration) occurs.

Refactoring Refactoring is the process of changing a software system in such a way that it does not alter the external behaviour of the code (design), yet improves its internal structure.

When software is refactored, the existing design is examined for redundancy, unused design elements, inefficient or unnecessary algorithms, poorly constructed or inappropriate data structures or any other design failures that can be corrected to yield a better design.

Modeling Component Level Design

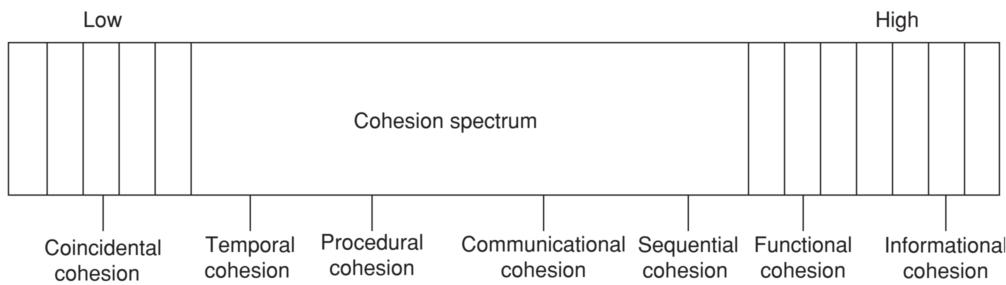
Component level design occurs after the first iteration of architectural design has been completed. At this stage the overall data and program structure of the software has been established.

Component A component is a modular building block for computer software.

Cohesion Cohesion implies that a component or class encapsulates only attributes and operations that are closely related to one another and to the class or component itself. Cohesion is a measure of internal relative strength of a module. It should be more. Different types of cohesion are:

1. **Coincidental cohesion:** If elements of a module are unrelated, then it is coincidental cohesive.
2. **Logical cohesion:** If elements of a module are related, then it is logical cohesion.
3. **Temporal cohesion:** If the elements of a module are elated and the elements are confined to initialization or time, it is temporal cohesion.
4. **Procedural cohesion:** If the elements are confined to one name and if they perform a set of operations, then the module is said to be procedural cohesive.
5. **Communicational cohesion:** If the elements in a module interact through data declared in it, then the module is said to be communicational cohesion.

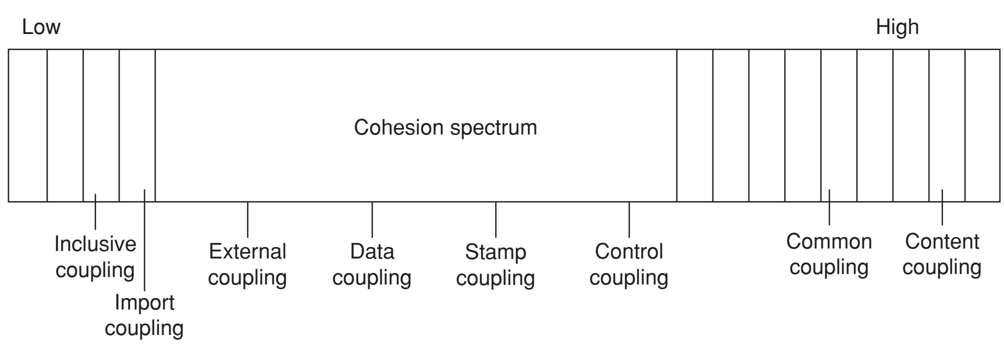
6. **Sequential cohesion:** If the elements are related and if they perform a set of operations in which the output of one operation is the input for another operation.
7. **Functional cohesion:** If the elements are related and if they are confined to one name and if they perform one and only one task, the module is functional cohesive.
8. **Informational cohesion:** If the elements of a module are confined to abstraction, it is informational cohesion.



Note: Cohesion metric should be high.

Coupling Coupling is a qualitative measure of the degree to which classes are connected to one another. As classes and components become more interdependent, coupling increases. In component-level design coupling is to be kept as low as possible. It includes:

1. **Procedural or routine call coupling:** A form of coupling in which modules interact nominally more or less they are almost independent.
2. **Low coupling:** Form of coupling in which modules interact minimally. In extreme case there is no coupling between them.
3. **Inclusion coupling:** A coupling in which source code of one module is included into another module.
4. **Import coupling:** A coupling in which one module is declared in another module for its functionality.
5. **External coupling:** A coupling in which modules interact with modules written by some third party, which may include specific hardware or software.
6. **Data coupling:** Occurs when operations pass long strings of data arguments.
7. **Stamp coupling:** Occurs when a class is declared as a type for an argument of an operation of another class.
8. **Control coupling:** Coupling in which one module controls the order of execution of other module by using flags.
9. **Common coupling:** If the components make use of a global variable, it can lead to uncontrolled error propagation and unforeseen side effects when changes are made.
10. **Content coupling:** Type of coupling in when one module refers to other module, in extreme case, it changes internal structure of other modules for its functionality.



Note: Coupling metric should be low.

Coding

Coding may be

1. The direct creation of programming language source code (e.g., Java, C).
2. The automatic generation of source code using an intermediate design like representation of the component to be built or
3. The automatic generation of executable code using a 'fourth generation programming language' (**e.g.,** VC++).

The principles that guide the coding task are closely aligned with programming style, programming languages and programming methods.

The fundamental principles are:

- Understand the problem you are trying to solve.
- Understand basic design principles and concepts.
- Pick a programming language that meets the needs of the software to be built and the environment in which it will operate.
- Select a programming environment that provides tools that will make the work easier.

Create a set of unit tests that will be applied once the component code is completed.

Characteristics of Good Srs

The characteristics of good SRS are

1. Correctness: The requirements specified in the software should meet, then the SRS is correct.
2. Unambiguous: The SRS is said to be unambiguous if every specified requirement can be interpreted in only one way.
3. Completed: The SRS is said to be complete, if and only if it has all significant requirements, definition of software responses to input data and labels and references to tables, figures and diagrams.
4. Consistent: The SRS is said to be consistent if the individual requirements are not defined in a conflict way and the SRS should be a high level document.
5. Stability: The SRS is said to be stable (or) ranked for the importance if each requirement has a preference. All the requirements may not have same importance; identify the requirements which are essential and requirements having least preference.
6. Verifiable: If each requirement is verifiable then the SRS is said to be verifiable.
7. Modifiable: The SRS is said to be modifiable, if the changes to the requirements can be made easily, consistent.
8. Traceable: Requirements should be clear so that each requirement can be referenced for enhancement, (or) future developments, which makes the SRS traceable.

Validation of SRS Validation of SRS is done to check whether the SRS is reflection of actual requirements and also to check the SRS documents is of good quality.

Testing

Testing is the process of executing a program with the intent of finding an error.

A good test case is one that has a high probability of finding an as-yet-undiscovered error. A successful test is one that uncovers an as-yet-undiscovered error.

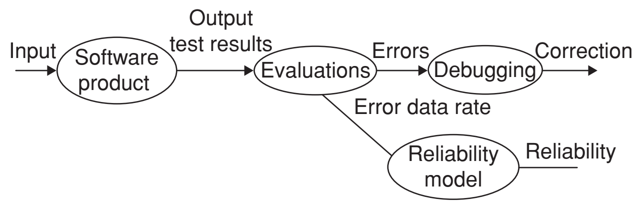
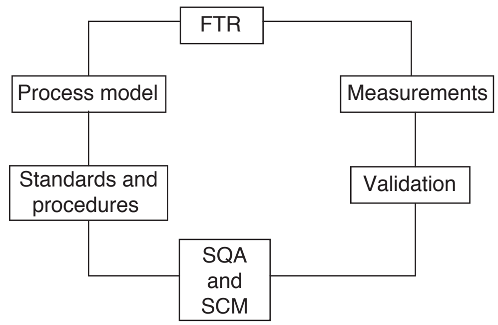


Figure 2 Formal technical review committee (FTR)



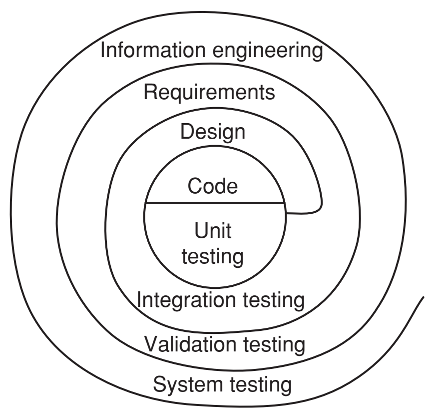


Figure 3 Verification

There are four software testing strategies:

1. Unit testing
2. Integration testing
3. Validation testing
4. System testing

Unit testing

Unit testing concentrates on each unit (e.g., class, component, etc). Unit test focuses on the internal processing logic and data structures within the boundaries of a component. Important control paths are tested to uncover errors within the boundary of the module, using component-level design description as a guide.

Integration testing

Integration testing focuses on design and construction of the software architecture. Integration testing is a systematic technique for constructing the software architecture while at the same time conducting tests to uncover errors associated with interfacing. The objective is to take unit-tested components and build a program structure that has been dictated by design.

- **Top-down integration** Modules are integrated by moving downward through the control hierarchy, beginning with the main control module (main program). Modules subordinate to the main control module are incorporated into the structure in either a depth-first or breadth-first manner.
- **Bottom-up Integration** begins construction and testing with the components at the lowest levels in the program structure.
- **Regression testing** in the context of an integration test strategy, regression testing is the re-execution of some subset of tests that have already been conducted to ensure that changes have not propagated unintended side effects.
- **Smoke testing** is an integration testing approach that is designed as a pacing mechanism for time-critical projects, allowing the software team to assess the project on a frequent basis.

Validation testing

Validation succeeds when software functions in a manner that can be reasonably expected by the customer.

In validation testing, the requirements established as part of requirements modeling are validated against the software that has been constructed.

Software validation is achieved through a series of tests that demonstrate conformity with requirements.

Alpha and beta testing can be used to uncover errors that occur only at the end user.

The alpha test is conducted at the developer's site by a representative group of end users. The software is used in a natural setting by end users in the presence of the developer and the developer records usage problems.

The beta test is conducted at one or more end user sites in the absence of developer. Therefore, beta test is a 'live' application of the software in an environment that cannot be controlled by the developer. The customer records all problems and reports to developer.

System testing

In system testing, the software and other system elements are tested as a whole.

System testing is a series of different tests whose primary purpose is to fully exercise the computer-based system. The types of system tests used for software-based systems are:

- **Recovery testing** is a system test that forces the software to fail in a variety of ways and verifies that recovery is properly performed.
- **Security testing** attempts to verify that protection mechanisms built into a system will protect it from improper penetration.
- **Stress testing** executes a system in a manner that demands resources in abnormal quantity, frequency or volume. A variation of stress testing called sensitivity testing attempts to uncover data combinations within valid input classes that may cause instability or improper processing.
- **Performance testing** is designed to test the run-time performance of software within the context of an integrated system.
- **Deployment testing** also called configuration testing exercises the software in each environment in which it is to operate. It also examines all installation procedures and specialized installation software that will be used by customers, and all documentation that will be used to introduce the software to end users.

Software Testing

The goal of testing is to find errors and a good test is one that has a high probability of finding an error.

The two ways of testing a software:

1. White-box testing (Internal testing)
2. Black-box testing (External testing)

White-box Testing

In white-box testing (also called glass-box testing) of software, tests are conducted to ensure that all internal operations are performed according to specifications and all internal components have been adequately exercised.

White-box testing methods should guarantee that:

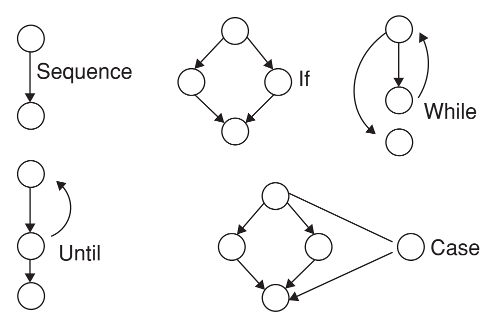
1. All independent paths, within a module are exercised at least once.
2. Exercise all logical decisions on their true or false sides.
3. Execute all loops at their boundaries and within their operational bounds and
4. Exercise internal data structures to ensure their validity.

Basis path testing

Basis path testing is a white-box testing technique. This method enables the test case designer to derive a logical complexity measure of a procedural design and uses this measure as a guide for defining a basis set of execution paths. Test cases derived are guaranteed to execute every statement in the program at least one time during testing.

Flow graphs can be used for better understanding the control flow and thus helps basis path testing to execute every statement in the program at least once.

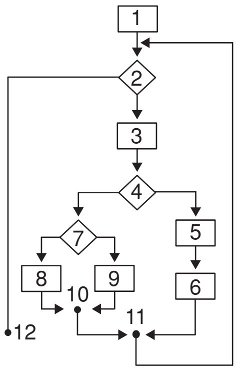
The flow graph symbols are:



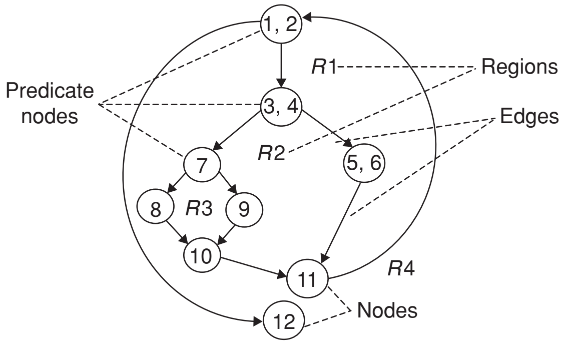
Each circle represents one or more non-branching PDL (Program Design Language) or source code statements.

Example:

Flowchart



Corresponding flow graph is



Each node that contains a condition is called a predicate node. Independent paths (any path through the program that introduces at least one new set of processing statements or a new condition) in the above example are:

Path 1: 1-2-12

Path 2: 1-2-3-4-5-6-11-2-12

Path 3: 1-2-3-4-7-8-10-11-2-12

Path 4: 1-2-3-4-7-9-10-11-2-12

Thus if tests can be designed to force execution of these paths (a basis set), every statement in the program will have been guaranteed to be executed at least one time, and every condition will have been executed on its true and false sides.

Cyclomatic complexity is a software metric that provides a quantitative measure of the logical complexity of a program. When used in the context of basis path testing method, the value computed for cyclomatic complexity defines the number of independent paths in the basis set of a program and provides an upper bound for the number of tests that must be conducted to ensure that all statements have been executed at least once.

Complexity is calculated in one of the three ways:

1. The number of regions of the flow graph corresponds to the cyclomatic complexity. (i.e., four Regions $R1$, $R2$, $R3$, $R4$ in the above case)
2. Cyclomatic complexity $V(G)$ for a flow graph G is defined as $V(G) = E - N + 2$, when E is the number of flow graph edges and N is the number of flow graph nodes (i.e., in the above case, there are 11 edges and 9 nodes. Thus $V(G) = 11 - 9 + 2 = 4$)
3. Cyclomatic complexity $V(G)$ for a flow graph G is also defined as $V(G) = P + 1$, where P is the number of predicate nodes contained in the flow Graph G. In the above flow graph, there are 3 predicate nodes.

$$\therefore \quad V(G) = 3 + 1 = 4$$

Control structure testing

Some of the variations on control structure testing to improve the quality of white-box testing are:

Condition testing

Condition testing is a test-case design method that exercises the logical conditions contained in a program module. This method focuses on testing each condition in the program to ensure that it does not contain errors.

Control Structure Testing

Condition testing

A simple condition is a Boolean variable or a relational expression, possibly preceded with one NOT (¬) operator. A compound condition is composed of two or more simple conditions, Boolean operators and parentheses. The possible types of elements in a condition include a Boolean operator, a Boolean variable, a pair of parentheses (surrounding a simple or compound Boolean condition), a relational operator, or an arithmetic expression.

Dataflow testing

This method selects test paths of a program according to the locations of definitions and use of variables in the program.

Loop testing

Loop testing is a white-box testing technique that focuses exclusively on the validity of loop constructs. Four classes of loop can be defined as:

Simple loops

The following set of tests can be applied to simple loops, where n is the maximum number of allowable passes through the loop.

1. Skip the loop entirely.
2. Only one pass through the loop.
3. Two passes through the loop
4. m passes through the loop where $m < n$
5. $n - 1$, n, $n + 1$ passes through the loop.

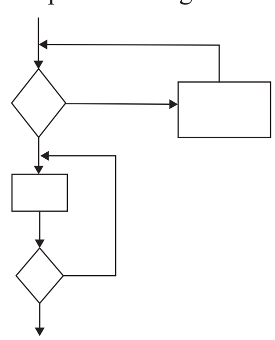


Figure 4 Simple loop.

Nested loops

Here the number of possible tests grows geometrically as the level of nesting increases. This results in an impractical number of tests.

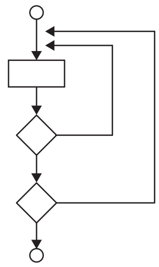


Figure 5 Nested loops.

Concatenated loops

Concatenated loops can be tested using approach of simple loops, if each of the loops is independent of the other. If two loops are concatenated and the loop counter for loop 1 is used as the initial value for loop 2, than the loops are not independent.

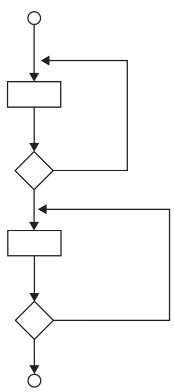


Figure 6 Concatenated loops.

Unstructured loops

Whenever possible, this class of loops should be redesigned to reflect the use of the structured programming constructs.

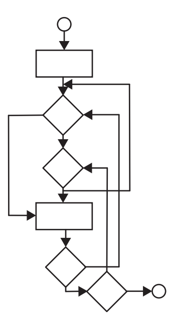


Figure 7 Unstructured loop.

Black-box Testing

Black-box testing, also called behavioral testing, focuses on the functional requirements of the software.

Black-box testing attempts to find errors in the following categories:

1. Incorrect or missing functions
2. Interface errors
3. Errors in data structures or external database access
4. Behaviour or performance errors and
5. Initialization and termination errors

By applying black-box techniques, we derive a set of test cases that satisfy the following criteria:

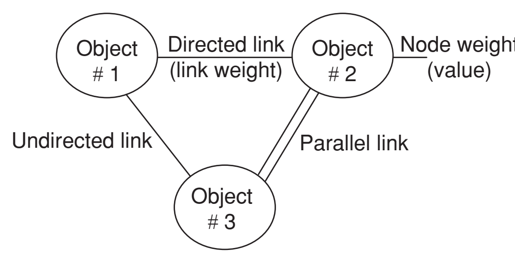
1. Test cases that reduce, by a count that is greater than one, the number of additional test cases that must be designed to achieve reasonable testing.
2. Test cases that tell something about the presence or absence of classes of errors, rather than an error associated only with the specific test at hand.

In graph-based black-box testing methods, software testing begins by creating a graph of important objects and their relationships and then devising a series of tests that will cover the graph so that each object and relationship is exercised and errors are uncovered.

Graph-based testing methods

To accomplish these steps, the software engineer begins by creating a graph – a collection of nodes that represent objects; links that represent the relationships between objects; node weights that describe the properties of a node and link weights that describe some characteristic of a link.

The symbolic representation of a graph is as shown in the figure.



- Nodes are represented as circles connected by links that take a number of different forms.
- A directed link indicates that a relationship moves in only one direction.
- A bidirectional link (symmetric link) implies that the relationship applies in both directions.
- Parallel links are used when a number of different relationships are established between graph nodes.

Equivalence partitioning

is a black-box testing method that divides the input domain of a program into classes of data from which test cases can be derived.

Equivalence partitioning strives to define a test case that uncovers classes of errors, thereby reducing the total number of test cases that must be developed.

Test case design for equivalence partitioning is based on an evaluation of equivalence classes for an input condition. An equivalence class represents a set of valid or invalid states for input conditions.

Boundary value analysis (BVA)

It is developed as a testing technique used to test bounding values since a greater number of error occurring at the boundaries of the input domain than at the centre.

Boundary value analysis is a test case design technique that complements equivalence partitioning. Rather than selecting any element of an equivalence class, BVA leads to the selection of test cases at the 'edges' of the class. BVA derives test cases from the input conditions as well as from the output domain.

Orthogonal array testing

The orthogonal array testing method is useful in finding region faults; an error category associated with faulty logic within a software component.

Orthogonal array testing can be applied to problems in which the input domain is relatively small.

When orthogonal array testing occurs, an Lg orthogonal array of test cases is created. This array has a 'balancing property', i.e., test cases are dispersed uniformly throughout the test doming.

Model-based testing (MBT)

It is a black-box testing technique that uses information contained in the requirements model as the basis for the generation of test cases.

White-box testing is usually performed at the early stages of testing process, while black-box testing tends to be applied during later stages of testing.

Implementation and Maintenance

System Implementation

Implementation is the process of converting a new or a revised system design into an operational one. Major aspects of implementation are conversion, post-implementation review and software maintenance.

There are three types of implementations:

1. Implementation of a computer system to replace a manual system.
2. Implementation of a new computer system to replace an existing one.
3. Implementation of a modified application to replace an existing one using the same computer.

Conversion

Conversion means changing from one system to another. The objective of conversion is to put the tested system into operation, while holding into costs, risks and personal irritation to a minimum.

It involves:

1. Creating computer-compatible files
2. Training the operating staff
3. Installing terminals and hardware

A very important aspect of conversion is not disrupting the functioning of the organization.

File conversion involves capturing data and creating a computer file from existing files.

Post implementation review

Every system requires periodic evaluation after implementation. A post-implementation review measures the system's performance against predefined requirements.

Unlike system testing, which determines where the system fails so that the necessary adjustments can be made, a post-implementation review determines how well the system continues to meet performance specifications. Post-implementation review is done after design and conversion are completed.

Software Project Estimation

Software is the most expensive element of virtually all computer-based systems. For complex, custom systems, a large cost estimation error can make the difference between profit and loss.

Software project estimation is a form of problem solving, and in most cases, the problem to be solved (i.e., developing a cast and effort estimate for a software project) is too complex to be considered in one piece. For this reason, we decompose the problem recharacterizing it as a set of smaller problems.

Problem-based estimation

Lines of code (LOC) and function point (FP) are used in two ways during software project estimation.

1. As an estimation variable to 'size' each element of the software.
2. As baseline metrics collected from past projects and used in conjunction with estimated variables to develop cost and effort projections.

The project planner begins by estimating a range of values of each information domain value. Using the historical data, the planner estimates an optimistic, most likely, and pessimistic size value for each function or count for each information domain value.

The expected value for the estimation variables is computed as

$$S = \frac{\text{optimistic} + 4 * \text{Most likely} + \text{pessimistic}}{6}$$

Empirical estimation models

An estimation model for computer software uses empirically derived formulas to predict effort as a function of LOC

or FP. The model should be tested by applying data collected from completed projects, plugging the data into the model and then comparing actual to predicted results.

Some of the LOC-oriented estimation models are

$E = 5.2 \times (\text{KLOC})^{0.91}$	Walston-Felix model
$E = 5.5 + 0.73 \times (\text{KLOC})^{1.16}$	Bailey-Basili model
$E = 3.2 \times (\text{KLOC})^{1.05}$	Boehm simple model
$E = 5.288 \times (\text{KLOC})^{1.047}$	Doty model for KLOC > 9

The software equation The software equation is a multi-variable model that assumes a specific distribution of effort over the life of a software development project.

$E = [\text{LOC} \times B^{0.333}/P]^3 \times (1/t^4)$

where

E = effort in person – months or person – years

t = project duration in months or years

B = Special spills factor

P = Productivity parameter that reflects overall process maturity and management practices, the extent to which good software engineering practices are used, the level of programming languages used, the state of software environment, the skills and experience of the software team, and the complexity of the application.

Note: B increases slowly as 'the need for integration, testing, quality assurance, and documentation and management skills grows'. For small programs KLOC = 5 to 15, $B = 0.16$.

For programs greater than 70 KLOC, $B = 0.39$

Putnam and Myers suggest a set of equations derived from the software equation.

Minimum development time is defined as

$t_{\min} = 8.14\ (\text{LOC}/P)^{0.43}$ in months for $t_{\min} > 6$ months

$E = 180\ \text{B}\ t^3$ in person – months for $E \geq 20$ person – months

Software Maintenance

Maintenance means restoring something to its original condition.

Maintenance is actually the implementation of the post-implementation review plan.

Maintenance is classified into corrective, adaptive or perfective maintenance.

Corrective maintenance repairs processing or performance failures or make changes because of previously uncorrected problems or false assumptions.

Adaptive maintenance means changing the program function.

Perfective maintenance enhances the performance or modify the programs to respond to the user's additional or changing needs.

About 50–80% of the total system development cost accounts for maintenance. Analysts and programmers spend far more time maintaining programs than they do writing them.

A manufacturer wants to minimize the variation among the products that are produced by maintaining the quality.

User satisfaction = compliant product + good quality + delivery within budget and schedule.

Software Quality Assurance (SQA)

Software Quality is defined as conformance to explicitly stated functional and performance requirements, explicitly documented development standards, and implicit characteristics that are points regarding quality is expected of all professionally developed software. In addition to the above definition some important

1. Software requirements are the foundation from which quality is measured. Lack of conformance to requirements is lack of quality.
2. If software conforms to its explicit requirements but fails to meet implicit requirements, software quality is suspect.

Activities performed by SQA group:

1. Prepares an SQA plan for a project.
2. Participates in the development of the project's software process description
3. Reviews software engineering activities to verify compliance with the defined software process.
4. Ensures that deviations in software work and work products are documented and handled according to a documented procedure.
5. Records any non-compliance and reports to senior management.

Software Reliability

Software reliability is defined as the probability of failure-free operation of a computer program in a specified environment for a specified time.

Measures of reliability and availability:

- A simple measure of reliability is mean-time-between-failure (MTBF).

  MTBF = MTTF + MTTR

  where

  MTTF = mean-time-to-failure

  MTTR = mean-time-to-repair

Although debugging (and related corrections) may be required as a consequence of failure, in many cases the software will work properly after a restart with no other change.

- In addition to a reliability measure, we must develop a measure of availability. Software availability is the probability that a program is operating according to requirements at a given point in time and is defined as

  Availability = [MTTF/(MTTF + MTTR)] × 100%

Software Safety

- Software safety is a software quality assurance activity that focuses on the identification and assessment of potential hazards that may affect software negatively and cause an entire system to fail.

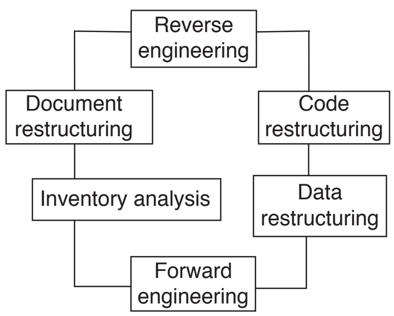
- Software safety examines the ways in which failures result in conditions that can lead to a mishap. That is the failures are evaluated in the context of an entire computer-based system and its environment.

Software Reengineering

Cost of redevelopment is very high compared to development.

The maintenance of existing software can account for over 60% of all effort expended by a development organization, and the percentage continues to rise as more software is produced.

A reengineering process model is shown below:



- Reengineering takes time, costs significant amount of money and absorbs resources that might be otherwise occupied on immediate concerns.
- Reengineering of information systems is an activity that will absorb information technology resources for many years.
- Inventory analysis : The inventory can be nothing more than a spreadsheet model containing information that provides a detailed description of every active application. It should be revisited on a regular cycle.
- Document restructuring : It creates a framework of documentation that is necessary for the long-term support of an application.
- Code restructuring : The source code is analyzed using a restructuring tool. The restricted code is reviewed and tested to ensure that no anomalies have been introduced.
- Data restructuring : It is a full-scale reengineering activity. Current data architecture is dissected and necessary data models are defined.
- Forward engineering : Also called renovation or reclamation, covers design information from existing software and uses this information to alter or reconstitute the existing system in an effort to improve its overall quality.
- Reverse engineering : It is the process of analyzing a program in an effort to extract data, architectural, and procedural design information.

The abstraction level of a reverse engineering process and the tools used to affect it refers to the sophistication of the design information that can be extracted from source code.

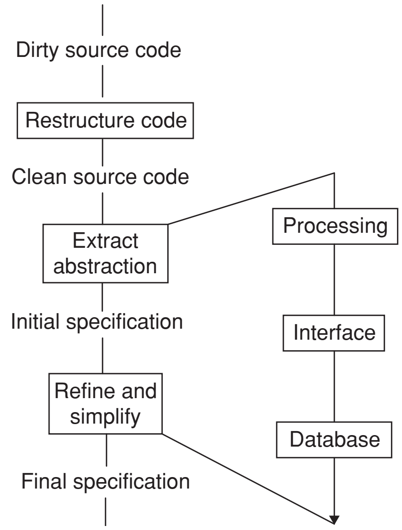


Figure 8 The reverse engineering process.

COCOMO Model

One of the famous model structures used to estimate the software effort is the constructive cost model, which is often called as COCOMO model. COCOMO was developed by Boehm. The model helps in defining the mathematical relationship between the software development time, the effort in man-months and the maintenance effort.

Basic COCOMO is defined as computers software development effort (and cost) as a function of program size. Program size is expressed in estimated thousand lines of code (KLOC) COCOMO is applied to three classes of software projects:

1. Organic projects
2. Semi-detached projects
3. Embedded projects

Organic projects

Organic projects are projects that are having small teams with good working experience with less than rigid requirements.

Semi-detached projects

Semi-detached projects are projects with medium teams having mixed working experience with a mix of rigid and less than rigid requirements

Embedded projects

Project that are developed within a set of tight constraints (hardware, software, operational…)

The general formula of the basic COCOMO model is

$E = a(s)^b$

where

$E \rightarrow$ Represents effort in person-months

$S \rightarrow$ Size of the software development in KLOC

‘a’ and ‘$\rightarrow$’ 5 Values dependent on the development mode

Development Mode	Value of *a*	Value of *b*
Organic	2.4	1.05
Semi-detached	3.0	1.12
Embedded	3.6	1.20

Development time $D = C(E)^d$

People required $(P) = \frac{E}{D}$[count]

Development Mode	Value of *c*	Value of *d*
Organic	2.5	0.38
Semi-detached	2.5	0.35
Embedded	2.5	0.32

For intermediate COCOMO model, the value of coefficient Q and the exponent b are given in the table below:

Development Mode	Value of *a*	Value of *b*
Organic	3.2	1.05
Semi-detached	3.0	1.12
Embedded	2.8	1.20

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

Common data for questions 1 and 2: Consider the following payroll program that prints a file of employees and a file of information (transaction file) for the current month and for each employee.

In addition, the program updates the employee file, and produces an earnings report, a deduction report and analysis report. The application is capable of interactive command to print an individually requested pay slip. It also processes a file containing details of payment. This program can give printout of pay slips when they are requested individually. The weight table is shown below:

	Simple	Average	Complex
No. of inputs	3	4	6
No. of outputs	4	5	7
No. of enquiries	3	4	6
No. of files	7	10	15
No. of interfaces	5	7	10

1. What is the unadjusted function point for the given payroll program?
(A) 60 (B) 62
(C) 68 (D) 72

2. From the above problem, find adjusted function point where $F4 = 4$, $F5 = 3$, $F12 = 2$, $F14 = 5$?
(A) 49 (B) 62
(C) 82 (D) 90

Common data for questions 3 and 4: The size estimated for software of a certain project is 45,000 lines of code. The average salary paid per engineer is ₹20,000 per month.

3. Calculate the effort required if the software is of organic type.
(A) 100 pm (B) 120 pm
(C) 130 pm (D) 140 pm

4. Calculate the cost required if the software is of semi-detached type.
(A) 113000 (B) 213000
(C) 315000 (D) 326515

5. A 40 KDSI embedded program for teleprocessing is to be developed. Estimate the time required for the project using basic COCOMO model.
(A) 12 pm (B) 14 pm
(C) 16 pm (D) 18 pm

6. Consider the following code:

```
begin
If (x ≤ 0) then x = 0 – x;
a = x;
end
```

Lata wants to test the program with test data. What are the sufficient values to execute both branches of the decision box?
(A) $x = 0, 4$ (B) $x = 0, -4$
(C) $x = 1, 4$ (D) $x = 0, -1$

7. What is the maintainability of a software with average number of days of repairing code is 10, adapting code is 20 and for enhancing code is 10?
(A) 6.3 (B) 12.5
(C) 32.6 (D) 40

8. Consider a Java program and the SLOC is given as 1000.

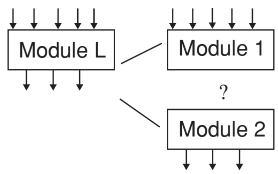
Class A

```
{
int x(int a);
int y(int b);
int z(int c);
}
```

What is the modularity?
(A) 0.001 (B) 0.002
(C) 0.003 (D) 0.004

9. Raj has written a program to add two numbers. Assuming a 32-bit representation for an integer, to exhaustively test his program, the number of test cases required are
(A) 2^8 (B) 2^{16}
(C) 2^{32} (D) 2^{64}

10. The module of the length 'L' is split up in two sub modules, module 1 and module 2, each of length $\frac{L}{2}$. How many links between the sub modules are allowed so that we maintain the value of information flow metric at same level?



(A) 2.4 (B) 3.6
(C) 4.8 (D) 1.2

11. The three estimates of the code size for a particular application for geometric analysis were most optimistic 4600, most likely 6900, most pessimistic is 8600. The value of estimated size that should be taken is
(A) 4600 (B) 6800
(C) 6900 (D) 8600

12. For an application of developing new operating system the KLOC is 34.5. What is the number of person-month (effort) best estimated using the intermediate COCOMO model?
(A) 126 (B) 130
(C) 158 (D) 196

13. For a real-time software systems the KLOC is 28.2. What is the effort in person–month calculated by using basic COCOMO model?
(A) 146 (B) 198
(C) 220 (D) 248

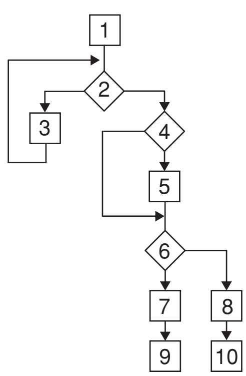
14. For inventory management system the KLOC is 25.5, what is the effort in person-month, using basic COCOMO model?
(A) 110 (B) 113
(C) 120 (D) 140

15. For the above, what is the estimated project duration in months?
(A) 6 (B) 8
(C) 10 (D) 13

Practice Problems 2

Directions for questions 1 to 16: Select the correct alternative from the given choices.

Common data for questions 1 and 2: Consider the below flow graph:



1. What is the number of paths to node 9?
(A) 2 (B) 3
(C) 4 (D) 5

2. What is the reachability measure?
(A) 1.8 (B) 2.8
(C) 2.4 (D) 2.1

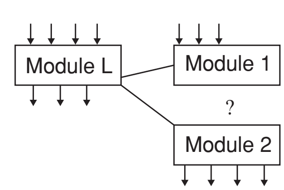
Common data for questions 3 and 4: For a software project the estimation is carried out by the Delphi method. Below table shows 5 experts with estimates:

Estimate	Pessimistic	Most likely	Optimistic
Expert 1	30	50	60
Expert 2	10	55	75
Expert 3	20	50	70
Expert 4	30	60	70
Expert 5	25	40	75

3. What is the average estimate?
(A) 48.3 (B) 49.4
(C) 50.8 (D) 56.7

4. What is the average variance?
(A) 5.0 (B) 6.7
(C) 7.8 (D) 8.3

5. The module of length L is split up into two sub modules (module-1 and module-2) each of length $\frac{L}{2}$. How many links between the sub modules exists so that we maintain the value of the information flow metric at the same level as found in the original module?



(A) 3 (B) 4
(C) 5 (D) 6

6. Constructive cost model is used to estimate
(A) Effort in man-month.
(B) Effort and schedule based on the size of the software.
(C) Size and duration based on the effort of the software.
(D) None of these

7. The theoretic concept that will be useful in software testing is
(A) Hamiltonian circuit
(B) Cyclomatic number
(C) Eulerian cycle
(D) None of these

8. Testing method that is normally used as the acceptance test for a software system is
(A) Regression testing
(B) Integration testing
(C) Unit testing
(D) None of these

9. Acceptance testing is
(A) The manner in which each component functions with other component of the system are tested.
(B) Running the system with given data by the actual user.
(C) The process of testing the changes in a new system or an existing system.
(D) None of these

10. Which of the following statements is true?
(A) Use of independent path testing criterion guarantees execution of each loop in a program under test more than once.
(B) Validation is the process of evaluating software at the end of the software development to ensure compliance with the software requirements.
(C) Statement coverage cannot guarantee execution of loops in a program under test.
(D) None of these

11. The size estimated for software of a certain project is 40,000 lines of code. The average salary paid per engineer is ₹15,000 per month. Calculate the cost required if the software is of organic type.
(A) 1,60,000
(B) 2,20,000
(C) 7,90,000
(D) 2,25,000

12. The size estimated for a software project is 35 Kloc. The average salary paid per engineer is ₹25,000 per month. Calculate the cost required if the software is of semi-detached type.
(A) 3,07,500
(B) 3,17,500
(C) 3,69,952
(D) 2,45,000

13. Which of the following statements is false?
(A) The cyclomatic complexity of a module is the number of decisions in the module plus one where a decision is effectively any conditional statement in the module.
(B) A direct flow of control in flow chart representing the lowest cyclomatic complexity.
(C) The reasonable limit of the cyclomatic complexity measure is 10.
(D) The cyclomatic complexity depends on the number of statements in the flowchart.

14. Which of the following is true regarding software testing?
(A) Software testing techniques are most effective if applied immediately after requirement specification.
(B) Software testing techniques are most effective if applied immediately after design.
(C) Software testing techniques are most effective if applied after coding.
(D) Software testing methods are most effective if applied after integration.

Common data for questions 15 and 16: A software project involves execution of 4 activities A_1, A_2, A_3, and A_4, of duration 11, 7, 8 and 3 days respectively. A_1 is the first one and needs to be completed before any other activity can commence. Activity A_2 and A_3 can be executed in parallel. Activity A_4 cannot commence until both A_2 and A_3 are completed.

15. Find the critical path of the above project.
(A) $A_1 - A_2 - A_4$
(B) $A_1 - A_3 - A_4$
(C) $A_1 - A_2 - A_3 - A_4$
(D) None of these

16. Find the slack time of the project.
(A) 0
(B) 1
(C) 12
(D) 13

Previous Years' Questions

1. The coupling between different modules of a software is categorized as follows:

I. Content coupling

II. Common coupling

III. Control coupling

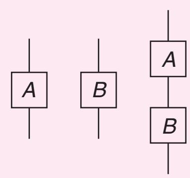
IV. Stamp coupling

V. Data coupling

Coupling between modules can be ranked in the order of strongest (least desirable) to weakest (most desirable) as follows: **[2009]**

(A) I-II-III-IV-V (B) V-IV-III-II-I
(C) I-III-V -II-IV (D) IV-II-V -III-I

2. The cyclomatic complexity of each of the modules *A* and *B* shown below is 10. What is the cyclomatic complexity of the sequential integration shown below? **[2010]**



(A) 19 (B) 21
(C) 20 (D) 10

3. A company needs to develop digital signal processing software for one of its newest inventions. The software is expected to have 40000 lines of code. The company needs to determine the effort in person-months needed to develop this software using the basic COCOMO model. The multiplicative factor for this model is given as 2.8 for the software development on embedded systems, while the exponentiation factor is given as 1.20. What is the estimated effort in person months? **[2011]**

(A) 234.25 (B) 932.50
(C) 287.80 (D) 122.40

4. The following is the comment written for a *C* function.

/ * This function computes the roots of a quadratic equation $ax \wedge 2 + bx + c = 0$. The function stores two real roots in * root 1 and * root 2 and returns the status of validity of roots. It handles four different kinds of cases.

(i) When coefficient '*a*' is zero irrespective of discriminant.

(ii) When discriminant is positive.

(iii) When discriminant is zero.

(iv) When discriminant is negative.

Only in case (ii) and (iii), the stored roots are valid. Otherwise 0 is stored in the roots. The function returns 0 when the roots are valid and −1 otherwise.

The function also ensures root1 > = root2

int get_QuadRoots (float *a*, float *b*, float *c*, float * root1, float * root 2); */

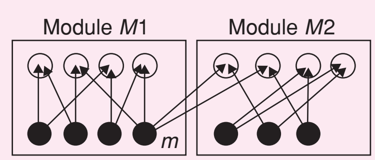
A software test engineer is assigned the job of doing black box testing. He comes up with the following test cases, many of which are redundant.

Test Case	Input Set			Expected Output Set		
	a	*b*	*c*	root1	root2	Return value
*T*1	0.0	0.0	7.0	0.0	0.0	–1
*T*2	0.0	1.0	3.0	0.0	0.0	–1
*T*3	1.0	2.0	1.0	–1.0	–1.0	0
*T*4	4.0	–12.0	9.0	1.5	1.5	0
*T*5	1.0	–2.0	–3.0	3.0	–1.0	0
*T*6	1.0	1.0	4.0	0.0	0.0	–1

Which one of the following options provide the set of non-redundant tests using equivalence class partitioning approach from input perspective for black-box testing? **[2011]**

(A) *T*1, *T*2, *T*3, *T*6 (B) *T*1, *T*3, *T*4, *T*5
(C) *T*2, *T*4, *T*5, *T*6 (D) *T*2, *T*3, *T*4, *T*5

5. The following figure represents access graphs of two modules *M*1 and *M*2. The filled circles represent methods and the unfilled circles represent attributes. If method *m* is moved to module *M*2 keeping the attributes where they are, what can we say about the average cohesion and coupling between modules in the system of two modules? **[2013]**



(A) There is no change
(B) Average cohesion goes up but coupling is reduced
(C) Average cohesion goes down and coupling also reduces.
(D) Average cohesion and coupling increase.

Common data for questions 6 and 7: The procedure given below is required to find and replace certain characters inside an input character string supplied in array *A*. The characters to be replaced are supplied in array 'oldc', while their respective replacement characters are supplied in array 'newc'. Array *A* has fixed length of five characters, while arrays 'oldc' and 'newc' contain three characters each.

However, the procedure is flawed.

```
void find_and_replace (char *A, char
*oldc, char *newc) {
for (int i = 0; i <5; i++)
for (int j=0; j<3; j++)
if (A[i] == oldc[j]) A[i] = newc[j];
}
```

The procedure is tested with the following four test cases.

1. oldc = “abc”, newc = “dab”
2. oldc = “cde”, newc = “bcd”
3. oldc = “bca”, newc = “cda”
4. oldc = “abc”, newc = “bac”

6. If array *A* is made to hold the string “abcde”, which of the above four test cases will be successful in exposing the flaw in this procedure? **[2013]**

(A) None (B) 2 only
(C) 3 and 4 only (D) 4 only

7. The tester now tests the program on all input strings of length five consisting of characters ‘*a*’, ‘*b*’, ‘*c*’, ‘*d*’ and ‘*e*’ with duplicates allowed. If the tester carries out this testing with the four test cases given above, how many test cases will be able to capture the flaw? **[2013]**

(A) Only one (B) Only two
(C) Only three (D) All four

8. In the context of modular software design, which one of the following combinations is desirable? **[2014]**

(A) High cohesion and high coupling
(B) High cohesion and low coupling
(C) Low cohesion and high coupling
(D) Low cohesion and low coupling

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** A	**3.** C	**4.** D	**5.** C	**6.** A	**7.** B	**8.** C	**9.** D	**10.** B
11. B	**12.** C	**13.** B	**14.** B	**15.** D					

Practice Problems 2

1. C	**2.** B	**3.** B	**4.** C	**5.** B	**6.** A	**7.** B	**8.** D	**9.** B	**10.** B
11. D	**12.** C	**13.** D	**14.** B	**15.** B	**16.** B				

Previous Years’ Questions

1. A	**2.** A	**3.** A	**4.** C	**5.** A	**6.** C	**7.** B	**8.** B

Test

Information System, Software Engineering

Time: 60 min.

Directions for questions 1 to 30: Select the correct alternative from the given choices

1. For a COCOMO model, organic projects are:
(A) Projects having small teams with good experience, working with less than rigid requirements.
(B) Projects having medium teams with mixed experience, working with more rigid requirements.
(C) Projects developed with a set of tight constraints.
(D) None of these

2. Which of the following statements is true?
(A) Basic COCOMO is good for quick estimate of software cost.
(B) COCOMO applies to three classes of software projects; organic, semi-detached and embedded.
(C) COCOMO does not account for differences in hardware constants, personal quality and experiences, etc.
(D) All the above

3. The first step in system analysis is
(A) software requirement analysis.
(B) software requirement specification.
(C) system design.
(D) information gathering.

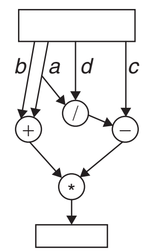
4. Questionnaire consists of
(A) qualitative data.
(B) quantitative data.
(C) Either (A) or (B)
(D) forms and documents.

5. The assessment of an intangible benefit is
(A) directly measurable.
(B) done by discussion amongst users of information system.
(C) irrelevant.
(D) done by discussion amongst the developers.

6. External entities in a DFD may be a
(A) source of input data only.
(B) destination of results only.
(C) source of input data and destination of results.
(D) data store.

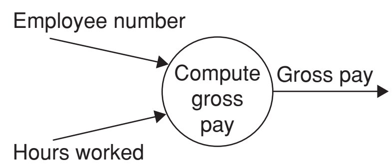
7. A context diagram
(A) is a DFD which gives an overview of the system.
(B) is a DFD that gives details of the system.
(C) is not used in DFDs.
(D) do not allow levelling of DFDs.

8. Consider the DFD below; derive an expression from the given data:



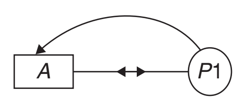
(A) $c - (a + b) * (a/d)$ (B) $a + b/d - (c * b)$
(C) $(a + b) * ((a/d) - c)$ (D) $(a + b) * (c + (a/d))$

9. Consider the following DFD:



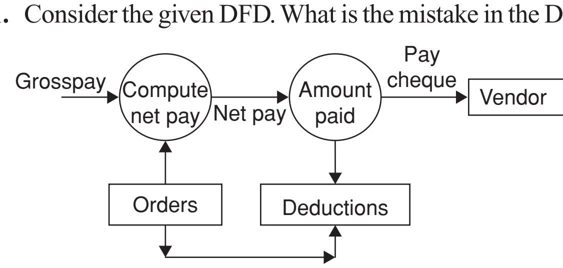
(A) It calculates the gross pay.
(B) The process is specified incorrectly.
(C) Insufficient data flow.
(D) Data flow diagrams are not used to specify these kind of computations.

10.



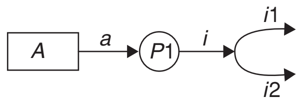
Which of the following is correct for above DFD?
(A) The given DFD is correct.
(B) A DFD cannot have arrows pointing in opposite directions.
(C) Data cannot flow from external entity to a data store.
(D) Data cannot flow from a data store to an external entity.

11. Consider the given DFD. What is the mistake in the DFD?



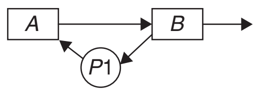
(A) A data flow cannot connect two processing steps.
(B) A data flow cannot connect two distinct data stores.
(C) Data stores cannot communicate with a process.
(D) Data flow cannot connect two distinct external entities.

12.



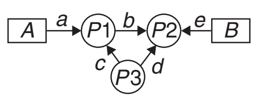
The mistake in above DFD is
(A) a data flow cannot be given two names.
(B) a data flow that has crossing lines.
(C) a DFD which forms loop.
(D) there are no mistakes in the DFD.

13.



Consider the above DFD. What is the mistake?
(A) The DFD forms a loop here.
(B) The DFD is correct.
(C) DFD does not allow communication among two external entities.
(D) DFD does not allow data flow among two data stores.

14. Consider the below DFD. What is the mistake?



(A) Process $P2$ is not designed properly.
(B) Process $P1$ is not designed properly.
(C) Process $P3$ is not designed properly.
(D) The external entities are not properly defined.

15. A good data flow diagram should have the following:
(A) A process which is a pure decision
(B) A DFD must be developed bottom up with higher levels giving more details
(C) Data flow should not act as signals to activate or initiate process
(D) All the above

16. The first phase of software development is
(A) Requirements Analysis
(B) Design
(C) Coding
(D) Testing

17. The lowest level of decomposition for a data flow diagram is
(A) primitive DFD
(B) unit DFD
(C) context DFD
(D) level 0 DFD

18. What is an important information while writing an SRS?
(A) Nature of SRS
(B) Characteristics of SRS
(C) Environment of SRS
(D) All of these

19. Which of the following is not an estimation metric for project size?
(A) LOC
(B) Function Point
(C) Feature Point
(D) None of the above

20. Human effort for developing a software project is measured in
(A) Dollars
(B) Person-Month
(C) Refects
(D) KLOC

21. Flight control software belongs to the following mode (as in basic COCOMO model):
(A) Organic mode
(B) Semi-detached mode
(C) Embedded mode
(D) None of the above

22. A transaction processing system with fixed requirements for terminal hardware and database software belongs to one of the following modes (in basic COCOMO model):
(A) Organic mode
(B) Semi-detached mode
(C) Embedded mode
(D) None of the above

23. In a software project, COCOMO is used to estimate
(A) effort and duration based on the size of the software.
(B) size, effort and duration based on the cost of the software.
(C) size and duration based on the effort of the software.
(D) effort and cost based on the duration of the software.

24. The maximum effort distribution in phases of software development is
(A) Requirement analysis
(B) Design phase
(C) Coding
(D) Testing

25. The minimum error distribution in the period of software development is in
(A) Requirement analysis
(B) Design phase
(C) Coding
(D) Testing

26. Basic Relation of COCOMO model is
(A) $E = (a * b)*(\text{KLOC})$
(B) $E = a * (\text{KLOC}^b)$
(C) $E = a * (\text{KDL})*b$
(D) $E = a/\text{KLOC}^b$

27. The extent to which the software can continue to operate correctly despite the input of invalid data is called as:
(A) Reliability
(B) Robustness
(C) Fault-tolerance
(D) Portability

28. Which of the following statement is false?

(A) The data flow diagram is presented in hierarchical fashion.

(B) Data flow modeling is a core modeling activity in structured analysis.

(C) Data flow diagram is formal part of UML.

(D) Data flow modeling depicts control flow.

29. For which of the following practices does requirements engineering provide appropriate mechanisms and tools?

(A) Analyzing need and validating the specification.

(B) Ambiguous specification of the solution.

(C) Risk Assessment.

(D) Implementing the system.

30. Which of the following is a common method of requirements elicitation?

(A) Transactional Analysis

(B) Observation

(C) Practical considerations

(D) Web accessibility

Answer Keys

1. A	**2.** D	**3.** D	**4.** C	**5.** B	**6.** C	**7.** A	**8.** C	**9.** C	**10.** B
11. B	**12.** A	**13.** C	**14.** A	**15.** C	**16.** A	**17.** A	**18.** D	**19.** D	**20.** B
21. C	**22.** B	**23.** B	**24.** D	**25.** A	**26.** B	**27.** B	**28.** C	**29.** A	**30.** B

PART - IV

- ↳ Mock Test - 1
- ↳ Mock Test - 2
- ↳ Mock Test - 3
- ↳ Mock Test - 4
- ↳ Mock Test - 5

Mock Test I

Number of Questions: 65 **Total Marks: 100**

Wrong answer for MCQ will result in negative marks, (–1/3) for 1 Mark Questions and (–2/3) for 2 Marks Question.

General Aptitude

Number of Questions: 10 **Section Marks: 15**

Direction for question 1: Fill in the blank with the suitable word/phrase:

1. There are ________candidates opting for Home Science today as a course of study at the college level.

(A) smaller (B) less
(C) fewer (D) lesser

Directions for questions 2 and 3: Select the correct alternative from the given choices.

2. The average weight of a class increases by 1 kg, when *A* joins the class. Later when *B* also joins, the average weight further increases by 1/2 kg. If the number of students now in the class is 14, the difference in the weights of *A* and *B* _______.

3. Every Saturday evening from 6 pm to 7 pm a game known as "FAMILY FORTUNES" is telecast on '*XTV*' channel. The mode of the game is as follows.

A table containing prices of different articles is present on the monitor. The anchor asks questions regarding the prices of different articles. If you are able to answer these questions correctly, the corresponding article is yours.

Be the lucky winner by answering the questions that follow the table given below:

2500	3000	1500	3500
2750	1750	3200	2800
2400	3600	4000	2200
1800	1200	1600	2250
3800	3400	3100	2000

A discount of 10% is offered on 'Ultra Microwave Oven' and in the above price table, the list price and the sale price of the above said article are adjacent to each other, not necessarily in the same order. What is its sale price?

(A) ₹4000 (B) ₹3600
(C) ₹1800 (D) ₹2000

Direction for question 4: Select the statement in which the underlined word is used correctly:

4. (A) These insects **adapted** themselves very easily to new environments.
(B) That woman has **adapted** a child from an orphanage.
(C) That Telugu family **immigrated** to Australia last year.
(D) People who have **emigrated** to the U.S. have had to deal with tougher labour laws.

Directions for question 5: Select the correct alternative from the given choices.

5. In a certain code language, if REPTILE is coded as 49 and CROCODILE is coded as 81, then how is ALLIGATOR coded in that language?

(A) 95 (B) 100
(C) 49 (D) 81

Direction for question 6: Out of the following four sentences, select the most suitable sentence with respect to grammar and usage:

6. (A) We took a month and a few days to get acclimated to our new teacher, who is from Baroda.
(B) We will take month and few days to get acclimated to our new teacher, who is from Baroda.
(C) We have taken month and a fewer days to get acclimated to our new teacher, who is from Baroda.
(D) We took a month few days to get acclimated to our new teacher, who is from Baroda.

Directions for question 7: Select the correct alternative from the given choices.

7. If $0 < x < 1$ and $1 + 3x + 5x^2 + 7x^3 + 9x^4 + \ldots \infty = 3$, then the value of x is ____.

(A) 1/2 (B) 1/3
(C) 1/4 (D) 1/5

Direction for question 8: In the following question, the first and the last sentences of a passage are in order and numbered 1 and 6. The rest of the passage is split into 4 parts and numbered as 2, 3, 4 and 5. These 4 parts are not arranged in the proper order. Read the sentences and arrange them in a logical sequence to make a passage and choose the correct sequence from the given order:

8. (1) A classic example of how the "get what you want by helping others get what they want" approach works is the result that a major automaker got when it came out with a new design.
(2) After all, the person turning the wrench knows more about the way it really works on the assembly line than the engineers who designed the wrench.
(3) Before making these changes, the management asked the employees who would actually be

building the new vehicles whether they had any ideas for making the assembly lines more effective.

(4) The workers had dozens of marvelous ideas.

(5) This design necessitated the construction of new plants and the retooling of existing ones.

(6) First, the employees explained that when they had to go down the steps into the pit to work on the underside of a car, they sometimes slipped and fell, injuring themselves.

(A) 5, 3, 4, 2 (B) 3, 4, 2, 5
(C) 2, 5, 4, 3 (D) 4, 3, 2, 5

Directions for questions 9 and 10: Select the correct alternative from the given choices.

9. In a parking lot six buses are parked in front of bus number 25 and fifteen buses are parked behind bus number 45. If six buses are parked between bus numbers 25 and 45, then how many buses are there in the parking lot?

(A) 29 (B) 26
(C) 15 (D) Cannot be determined

10. Jane Davis, founder of Get Into Reading, which has helped Clare Ross so much, discovered the healing power of books by accident. An English lecturer at Liverpool University, England, she also taught literature courses in her community. In the process she discovered that people derived consolation from great writers and the support network the group provided. So she set up Get Into Reading, which now has more than 135 groups.

Which of the statement(s) below is/are logically valid and can be inferred from the above statements?

(i) Reading gives multiple benefits, both unexpected and wholesome.

(ii) Reading helps everyone to set up an association like Get Into Reading.

(iii) Reading is just a waste of time which makes one hope for an unattainable goal.

(iv) Reading gives not only help from great writers but also support from other sources.

(A) (i) and (iii) (B) (i) and (iv)
(C) (ii) and (iii) (D) (ii) and (iv)

COMPUTER SCIENCE ENGINEERING

Number of Questions: 55 **Section Marks: 85**

Directions for questions 11 to 65: Select the correct alternative form the given choices

11. One of the eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \text{ is \_\_\_\_\_}$$

(A) $\begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$

(C) $\begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$ (D) $\begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix}$

12. The number of edges in a bipartite graph with 11 vertices should be at most _____

(A) 11 (B) 20
(C) 30 (D) 44

13. Which of the following is NOT a valid identity for any three arbitrary events A, B and C of a sample space?

(A) $P\left(\dfrac{\overline{A}}{B}\right) = 1 - P(A/B)$

(B) $P(A \cap B) = P(A)\, P(B/A)$

(C) $P((A \cup B)/C) = P(A/C) + P(B/C) - P((A \cap B)/C)$

(D) None of these

14. If the number of telephone calls coming to a telephone exchange follows Poisson distribution with an average of 3 calls per hour, then the probability that the number of telephone calls coming to the telephone exchange in one hour exceeding 3 is _____

(A) 0.3528 (B) 0.6472
(C) 0.2341 (D) 0.7659

15. If $f(x) = 2x$, $g(x) = 3x^2$ and $h(x) = 3$ and (fogoh) (x) exists, then the value of (fogoh) (3) is _____

16. Which of the following algorithm can be used to sort the numbers 1, 2, 3, ..., 99, in $O(n)$ time?

(A) Merge sort
(B) Quick sort
(C) Heap sort
(D) Radix sort

17. Consider the following statements:

I. The minimum number of edges in a connected graph which is not cyclic on 'n' vertices is $(n - 1)$.

II. A simple connected graph with 'n' nodes has maximum edges.

Which of the following is TRUE?

(A) Only I
(B) Only II
(C) Both I and II
(D) Neither I nor II

$$n\frac{(n-1)}{2}$$

18. A disk has the following parameters:
Number of tracks = 360
Number of blocks per track = 360
Disk rotates 600 RPM
What is the Latency?
(A) 5 seconds (B) 0.5 seconds
(C) 0.05 seconds (D) 0.005 seconds

19. Which of the following is TRUE?
I. A B-tree of order 3, constructed with *n*-elements which are in ascending order leads to maximum number of Node splits.
II. A B^+-tree of order 3, constructed with *n*-elements which are in descending order leads to maximum number of Node Splits.
(Assume value of '*n*' upto 10 elements)
(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

20. Which of the following is always TRUE?
I. If a program runs on a processor with a higher frequency, then it implies that the processor always executes more number of instructions per second than a low frequency processor.
II. If a processor executes more number of instructions per second then it implies that the processor always finishes a program faster than a processor which executes fewer instructions per second.
(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

21. A machine has 5-stage pipeline, which uses delay slots to handle control dependencies. If the dependencies are resolved in the 3^{rd} stage then the number of delay slots needed to ensure correct operation is ___________.

22. A language L consists of all binary strings beginning with a '1' such that when its value is converted to decimal, is divisible by '3'. Then which of the following is TRUE?
(A) L can be recognized by a Deterministic Finite Automata.
(B) L can be recognized by a Non-deterministic finite automata but not by deterministic finite automata.
(C) L can be recognized by a deterministic push-down automata but not by a finite automata.
(D) The language L cannot recognized by any push down automata.

23. Which of the following problem is decidable?
(A) A Context free language L is regular or not.
(B) Complement of a CFL is CFL or not.
(C) Finding whether a given CFL is empty or not.
(D) Finding whether the intersection of two CFLs is empty or not.

24. Which of the following is TRUE about socket programming?
I. The accept( ) socket call returns the non-negative file descriptor of the accepted socket (if there is no error).
II. The return value of receive( ) specifies the number of bytes read from or if the socket was closed or an error was encountered.
III. In socket programming close( ) and shut down( ) functions operate in same way.
(A) I only (B) I, II only
(C) I, II, III (D) II only

25. Which of the following is FALSE about the connecting device 'Repeater'?
(A) A Repeater is used to connect segments of a LAN.
(B) A Repeater can connect two LANs of different protocols.
(C) A Repeater forwards every frame as it has no filtering capability.
(D) A Repeater is a generator not an amplifier.

26. Using Caesar Cipher with key = 10, what is the encrypted message of "GATECS"?
(A) SCETAG (B) AGETSC
(C) WQJUSI (D) QKDOMC

27. The binding takes place at, which of the following?
(I) language design time
(II) compile time
(III) load time
(IV) link time (or) run time
(V) language implementation time
(A) (II), (III), (IV) (B) (II), (III), (IV), (V)
(C) (I), (II), (III), (IV) (D) All the above

28. Consider the following program:
```
var x;
function fun1( )
{
    var x1 = 20;
    document.write (∀x = ∀ + x);
}
function fun2( )
{
    var x ;
    x = 10;
    fun1( );
}
x = 5
fun2( );
```
Using static scoping what is the value of *x* printed when fun 2( ) is called?
(A) 5 (B) 10
(C) 20 (D) shows error

29. Number of binary trees that can be formed with 3 elements is ______

30. The root directory of the file system should be placed at
(A) Anywhere on the disk
(B) Fixed location on the disk

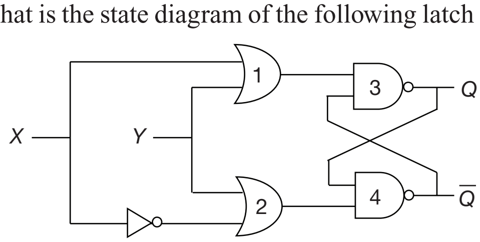
(C) Fixed location on the primary memory.
(D) None of the above

31. In a Round-Robin scheduling, if time Quantum and Process switch takes equal amount of time, then the CPU efficiency will be
(A) 75%
(B) 50%
(C) 65.5%
(D) Depending on the number of processes in the system

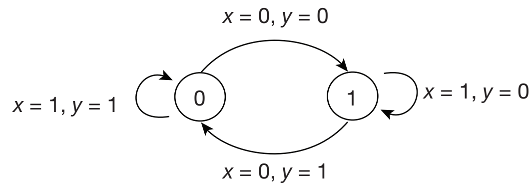
32. Which of the following statements is FALSE?
(A) Type incompatibilities between actual and formal parameters are detected at compile time.
(B) Array subscript out of range is detected at run time.
(C) Multiple declarations of a variable leads to compile time error.
(D) None of the above

33. Consider the following program
```
void fun( )
{
    int a, b, c, d;
:
:
:
            for ( ) - (I)
              {
                int b, c, d;
                    :
                    :
                      for ( ) - (II)
         {
           int c,d, x, y;
           ____ // line (I)
         }
              }
}
```
Which of the following variables are visible at line (I)?
(A) c, d, x, y of for (II); b, c, d of for (I); a, b, c, d of fun ( )
(B) c, d, x, y of for (II)
(C) c, d, x, y of for (II); b of for (I); a of fun ( )
(D) c, d, x, y of for (I); b of for (II); a, b, c, d of fun ( )

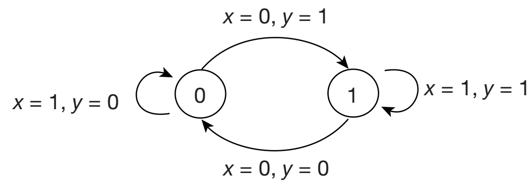
34. What is the state diagram of the following latch circuit?



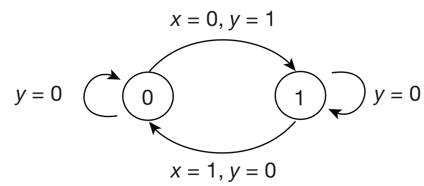
(A)



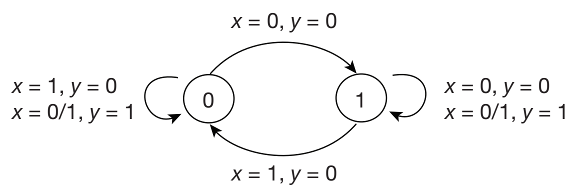
(B)



(C)



(D)



35. Consider the boolean function,
$F(a, b, c) = \pi M(2, 5)$
Then minimum SOP (Sum of Products) form of F is:
(A) $a^1 c + b^1 c^1 + ab$ (B) $a^1 b^1 + bc + ac^1$
(C) Both (A) and (B) (D) $a^1 b^1 + b^1 c^1 + bc + ab$

36. Consider the propositions:
S_1: If Ramu is in the classroom, then Lokesh will be in the playground
S_2: Ramu is in the classroom and S_3 : Lokesh is in the playground
Then which of the following is TRUE?
(A) S_2 logically follows from S_1 and S_3 but S_3 does not follow logically from S_1 and S_2
(B) S_2 does not follow logically from S_1 and S_3 but S_3 logically follows from S_1 and S_2
(C) S_2 does not follow logically from S_1 and S_3, and S_3 does not follow logically from S_1 and S_2
(D) S_2 logically follows from S_1 and S_3, and S_3 logically follows from S_1 and S_2

37. If $S_1 = \{0, 1, 2, 3, 4\}$, $S_2 = \{1, 2, 3, 4, 5\}$, $S_3 = \{0, 1, 2, 3, 4, 5\}$ and $S_4 = \{1, 2, 3, 4, 5, 6\}$ and $+_m$ and x_m denote the operations of addition modulo m and multiplication

modulo m respectively, then which of the following is NOT an abelian group?

(A) $(S_1, +_5)$ (B) (S_2, x_6)
(C) $(S_3, +_6)$ (D) (S_4, x_7)

38. A stationary value of a function $f(x)$ is a value of x, where $f^1(x) = 0$. The number of distinct stationary values of $f(x) = 8x^5 - 15x^4 + 10x^2$, where $f(x)$ has neither maximum nor minimum is ____

39. In a hostel, there are 5 triple rooms (each room can accommodate 3 students), 6 double rooms (each room can accommodate 2 students) and 3 single rooms (each room can accommodate 1 student). The number of ways in which 30 students can be accommodated in these 14 rooms is ____

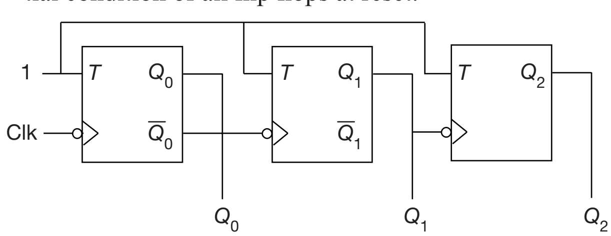
(A) $\dfrac{30!}{(3!)^5 \times (2!)^6 \times (1!)^3}$ (B) $\dfrac{30!}{(5!)^3 \times (2!)^6 \times (3!)^1}$

(C) $\dfrac{30!}{3! \times 2! \times 1!}$ (D) $\dfrac{30!}{5! \times 6! \times 3!}$

40. Which of the following statements is/are NOT TRUE?
(I) The product of two symmetric matrices A and B is symmetric if and only if $AB - BA = 0$
(II) The product of two skew-symmetric matrices A and B is skew-symmetric if and only if $AB + BA = 0$
(III) The eigenvectors corresponding to two distinct eigenvalues of a matrix are linearly independent
(IV) The eigenvectors corresponding to two distinct eigenvalues of a real symmetric matrix are orthogonal

(A) Only (I) (B) Only (II)
(C) Both (III) and (IV) (D) None of these

41. Analyze the following counter circuit, and find the Count sequence $Q_2Q_1Q_0$ (in decimal). Consider the initial condition of all flip flops at reset.



(A) 0, 7, 6, 5, 4, 3, 2, 1, 0….
(B) 0, 1, 2, 3, 4, 5, 6, 7, 0…..
(C) 0, 3, 2, 5, 4, 7, 6, 1, 0….
(D) 0, 6, 2, 5, 1, 7, 3, 4, 0….

42. Which of the following grammar is $LL(1)$, $LR(0)$ and SLR(1)?

(A) $S \to A\,B \mid B\,A$
$A \to b\,A^I$
$B \to a\,B^I$
$B^I \to a\,A\,B^I \mid \in$

(B) $E \to TE^I$
$E^I \to +\,TE^I \mid E$
$T \to E\,F \mid E\,a \mid b$
$F \to (E) \mid a$

(C) $E \to E\,F \mid e$
$F \to F + \mid f$
$T \to E\,T \mid g$

(D) $S \to a\,A\,B\,/\,B\,a \mid A\,b$
$A \to c$
$B \to c$

43. Consider the grammar:
$S \to i\,C\,t\,S\,S^1 \mid a$
$S^1 \to \text{e}\,S \mid \in$
$C \to b$
An $LL(1)$ Parsing table, M is constructed for this grammar and the grammar is not in $LL(1)$. Then which of the following entries in M, contains multiple entries?

(A) $M[S^1, t]$ (B) $M[S, i]$
(C) $M[S^1, e]$ (D) $M[S, a]$

44. Consider the following Syntax Directed Definition:

Production	Semantic Rule
(1) $L \to E_n$	L.val = E.val
(2) $E \to E_1 + T$	E.val = E_1. val + T.val
(3) $E \to T$	E.val = T.val
(4) $T \to T_1 * F$	T.val = T_1.val *F.val
(5) $T \to F$	T.val = F.val
(6) $F \to (E)$	F.val = E.val
(7) $F \to$ digit	F.val = digit.lexval

It evaluates the expressions, which are terminated by an end marker 'n' with operators + and *.
Then which of the following statement(s) is/are FALSE?
I. Given SDD is S-attributed grammar.
II. Given SDD is not L-attributed grammar.
III. Given SDD is an attribute grammar.

(A) I, III only (B) II only
(C) I,II only (D) III only

45. Consider a system which is using Round-Robin Scheduling Technique with context switch overhead of 2 ms, what is the value of time quantum(q) for $n = 6$ processes, so that each process must be guaranteed to get its chance (or) turn at the CPU exactly after 36 ms of time (in ms) is __________.

46. Consider the page reference string:
1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3, 2, 1, 2, 3, 5
If the number of page faults are 9 using LRU page replacement policy then the number of page frames required would be?

(A) 3 (B) 4
(C) 5 (D) 6

47. Consider a system with processes P_1, P_2. X is a Binary semaphore variable with an initial value of 1 and count is a variable with initial value of 1.
The processes P_1, P_2 executes routines $P1()$, $P2()$ for accessing critical section.

P1 ( )	P2 ( )
{	{
DOWN (X)	UP (X);
count + +;	count - -;
UP (X);	DOWN (X);
}	}

Consider the following statements ($S1$, $S2$, $S3$)
$S1$: The maximum value of count can be infinite.
$S2$: The system cannot guarantee progress.
$S3$: The system doesnot provide Mutual Exclusion.
Which of the above statements is/are TRUE?
(A) $S1$only (B) $S1$ and $S3$
(C) $S2$ and $S3$ (D) $S1$, $S2$, $S3$

48. Consider the function fun() which deletes the duplicate elements on a non-empty sorted single linked list.

```
typedef struct SL
{
    int data;
    struct  SL * next;
}SLL;
void fun(SLL *first)
{
    SLL *curr = first, *temp;
    if (!curr)
    return ;
    while (___I___)
    {
        temp = curr → next;
        if (curr → data = = temp →
    data)
        curr → next = temp → next;
        free(temp);
            (____II____)
}}
```

If first node address is passed to fun( ), Fill in the blanks.

	I	II
(A)	curr	temp = temp → next
(B)	curr	curr = curr → next
(C)	! temp	curr = curr → next
(D)	temp	temp = temp → next

49. Consider the following Tree traversals of a binary Tree:
Pre-order: A B C D E F G H J K L M P Q N
In order: C D E B G H F K L P Q M N J A
Construct a binary tree T from above traversals. The number of nodes with one child in the tree are ________

50. Consider the following program:

```
int fun (int * x)
{
    *x+ = 5;
    return (3 * (* x) - 1) ;
}
void main( )
{
int a = 8, b = 12, s1, s2;
s1 = (a/2) + fun(&a);
s2 = fun(&b) * (b/2);
}
```

Which of the following statement(s) is/are TRUE?
(A) The value of $s1$ will be same, irrespective of evaluation order.
(B) The value of $s2$ is will be irrespective of evaluation order.
(C) Both (A) and (B)
(D) None of the above

51. The first 32-bits of an IPv4 header in Hexa-decimal notation has a value 4500139CH. Then which of the following cannot be concluded by these 32-bits?
I. The header length is 20 Bytes.
II. The total length of the packet (both header and data) is 5000 Bytes.
III. The datagram is Last Fragment.
IV. The fragmentation offset field is 628.
(A) III, IV (B) II, III, IV
(C) II only (D) I, II, III, IV

52. The stations on a wireless ALOHA network are maximum of 1200 km apart. Signals propagate with a speed of 3×10^8 m/s. The maximum number of retransmission attempts are 15 and for the attempt number 2, by using Binary exponential Back-off algorithm, which of the following are possible Back-off time values?
(A) 0 ms, 1 ms
(B) 0 ms, 1 ms, 2 ms, 3 ms
(C) 0 ms, 4 ms, 8 ms, 12 ms
(D) 4 ms, 8 ms

53. Consider the below transition table of a PDA M_1:

	ε	0	1
$\to q_0$	$(q_1, \varepsilon, \$)$	–	–
q_1	$(q_2, 1, \varepsilon)$	$(q_1, \$, 0\$)$ M_1 M_2	$(q_1, \$, 1\$)$ M_3 M_4
q_2	–	–	–

Here q_0 is the start state and q_2 is the final state. The transition entries has the form (Q, A, B) where Q is the Target state, A is the top of the stack and B is the symbols to be written in place of top of stack.
$L(M_1)$ accepts the strings of the form,
{w/w contains more 1s than 0s}

Based on this information identify the correct transitions for M_1, M_2, M_3, M_4:

(A) $M1$: $(q_1, 1, \varepsilon)$, $M2$: Not required
$M3$: $(q_1, 0, \varepsilon)$, $M4$: Not required

(B) $M1$: $(q_1, 1, \varepsilon)$, $M2$: $(q_1, 0, 00)$
$M3$: $(q_1, 0, \varepsilon)$, $M4$: $(q_1, 1, 11)$

(C) $M1$: $(q_1, 1, 10)$, $M2$: $(q_1, 0, \varepsilon)$
$M3$: $(q_1, 0, 01)$, $M4$: $(q_1, 1, \varepsilon)$

(D) $M1$: $(q_1, 0, \varepsilon)$, $M2$: $(q_1, 1, \varepsilon)$
$M3$: $(q_1, 1, \varepsilon)$, $M4$: $(q_1, 0, \varepsilon)$

54. Consider the following languages:
$L_1 = \{x^m\, y^n\, z^p \mid (m = n \vee n = p) \wedge m + n + p \geq 10\}$
$L_2 = \{x^m\, y^n\, z^p \mid (m = n \vee n = p) \wedge m + n + p \leq 10\}$
Which of these languages is/are Regular?

(A) L_1 only (B) L_2 only
(C) Both L_1 and L_2 (D) Neither L_1 nor L_2

55. Consider the following languages:
$L_1 = \{a^i\, b^{2i} \mid i \in \{1, 2, \ldots\}\}$
$L_2 = \{a^i b^{i^2} \mid i \in \{1, 2, \ldots\}\}$
Which of the following statement is correct about L_1 and L_2?

(A) Both L_1 and L_2 are CFLs.
(B) L_1 is CFL and L_2 is recursive but not CFL.
(C) Complement of L_2 is CFL.
(D) L_1 is regular and L_2 is CFL.

56. Consider a branch instruction which has below format:

6 bits	5 bits	5 bits	
opcode	R_S	R_T	#immediate

The instruction has 6-bit branch type of opcode, 5-bit registers R_S, R_T and 16-bit immediate value.
A Branch instruction performs a modification of (PC + 4) (address of next instruction) if the condition is true. If the maximum range of jump instruction is $PC + 4 - X$ to $(PC + 4) + Y$. [$(PC + 4) - X$ and $(PC + 4) + Y$ are memory address locations]. Then the respective values of X and Y would be:

(A) –32768 and 32767 (B) 32768 and 32767
(C) 131072 and 131068 (D) 131068 and 131072

57. Consider the following code fragment:

Loop:	LOAD R_1, $0(R_2)$	$R_1 \leftarrow M[0 + R_2]$
	ADD R_1, R_1, #1	$R_1 \leftarrow R_1 + 1$.
	STORE R_1, $0(R_2)$	$M[0 + R_2] \leftarrow R_1$
	ADD R_2, R_2, #4	$R_2 \leftarrow R_2 + 4$.
	SUB R_4, R_3, R_2	$R_4 \leftarrow R_3 - R_2$.

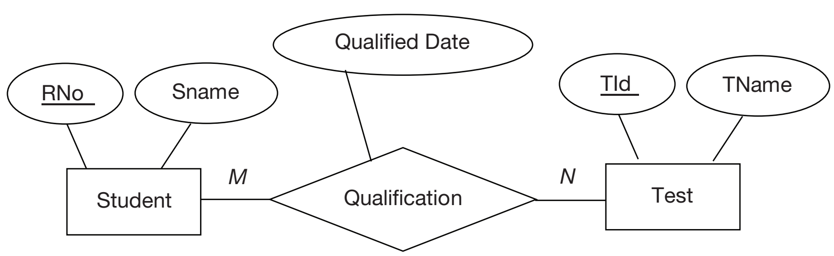
BNEZ R_4, loop Branch to loop if $R_1! = 0$
Let the initial value of R_3 is $R_2 + 400$.
The instruction sequence executed on a 5-stage RISC pipeline. (IF: Instruction Fetch, ID: Instruction decode and Register fetch, E.x., execute, MA: Memory Access, WB: Write Back) without any forwarding hardware, but register read and write can happen in the same clock cycle. Branch outcome is known in the EX stage and is handled by flushing the instructions which started execution before the EX stage of Branch instruction. If each stage takes 1 clock cycle, then the number of clock cycles required to execute the given loop will be ______.

58. A floating point number of 20-bit length has the following format:

1 bits	7 bits	
Sign	Excess – 64 Exponent	Mantissa

The decimal equivalent of the value $(5403E)_{16}$ is ______.

59. Consider the following Entity Relationship Diagram (ERD):



Which of the following possible relations will not hold if the above ERD is mapped into a relational model?

(A) Student (<u>RNo</u>, Sname)
(B) Qualification (<u>RNo</u>, <u>TID</u>, Qualified Date)
(C) Test (<u>TID</u>, <u>RNo</u>, TName)
(D) Test (<u>TID</u>, TName)

60. Consider the following relations:
Sailor (<u>sid</u>, SName, Rating, Age)
Reserves (<u>sid</u>, <u>bid</u>)
Boats (<u>bid</u>, bname, color)
Primary keys are shown with underline.
Query: Display the 'sids' of sailors who have reserved both black and white color boats.
Which of the following Relational algebra expression for the above query always gives correct result?

(A)
$$\pi_{sid}\left(\left(\left(\sigma^{(Boats)}_{color=\text{'black'}} \cap \sigma^{(Boats)}_{color=\text{'white'}}\right) \text{sailor}\right) \text{Reserves}\right)$$

(B) $\pi_{\text{sid}}\left(\sigma^{(\text{Sailor Reserves Boats})}_{\text{color='black'}} \cap \sigma^{(\text{Sailor Reserves Boats})}_{\text{color='white'}}\right)$

(C) $\pi_{\text{sid}}\left(\left(\left(\sigma^{(\text{Boats})}_{\text{color='black'}}\right) \text{Reserves}\right) \text{Sailor}\right) \cap$
$\pi_{\text{sid}}\left(\left(\left(\sigma^{(\text{Boats})}_{\text{color='white'}}\right) \text{Reserves}\right) \text{Sailor}\right)$

(D) All the above

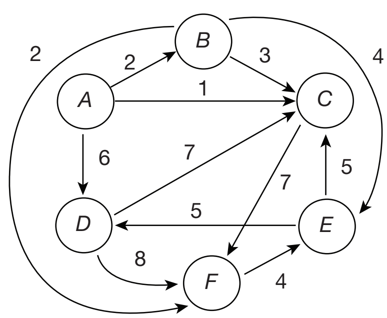
61. Consider the following sequence of elements:

I.	8	7	6	5	4	3	2	1
II.	8	1	2	7	3	6	4	5
III.	8	7	6	5	1	2	3	4

Which sequence(s) takes maximum number of swap operations if selection sort is implemented to arrange the elements in ascending order?

(A) only III
(B) II and III
(C) only II
(D) I and II

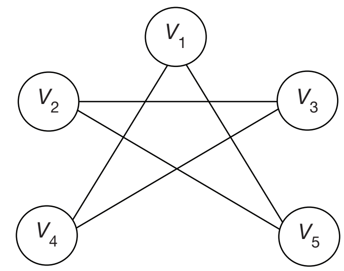
62. Let G be the Directed Weighted graph shown below:



What is the sequence of vertices identified by the Dijkstra's algorithm for single source shortest path, when the algorithm is started at vertex '*A*'?

(A) *ACBEDF*
(B) *ACFBED*
(C) *ACBFED*
(D) *ACBEFD*

63. A complete undirected, Weighted graph *G* is given on the vertex set $\{V_1, V_2, V_3, V_4, V_5\}$



Construct 2 minimum spanning trees, for the First spanning tree, the graph has weights for an edge (V_i, V_j) as $|i-j|$. For the second spanning tree, the graph has weights for an edge (V_i, V_j) as $|i+j|$.
What is the difference between Total weights of 2 Spanning Trees?

(A) 12 (B) 14
(C) 16 (D) 18

64. Consider the following relation schema:
Student (RNo, SName, age, grade)
Register (RNo, CNo, Day)
Course (CNo, CName, Duration)
Which of the following query CANNOT be expressed by using SELECT, FROM, JOIN, DIVIDE, WHERE clauses?

(A) For each course, display the CNo and number of students in that course.
(B) Display the names of students whose CNo = 201
(C) Display the Names and CNo of students who registered on a day = 'Monday'
(D) Display the Names of students whose course duration is 6 months.

65. Consider the Relation *R*(*ABCDE*) and the following functional dependencies:
$A \rightarrow BC$
$A \rightarrow D$
$D \rightarrow E$
Which decomposition is required for *R* and Is the decomposition lossless?

(A) 2NF and loss less (B) 3NF and lossy
(C) 3NF and loss less (D) BCNF and lossy

Answer Keys

1. C	**2.** 5	**3.** B	**4.** A	**5.** D	**6.** A	**7.** B	**8.** A	**9.** A	**10.** B
11. B	**12.** C	**13.** D	**14.** A	**15.** 54	**16.** D	**17.** C	**18.** C	**19.** C	**20.** D
21. 2	**22.** A	**23.** C	**24.** B	**25.** B	**26.** D	**27.** D	**28.** A	**29.** 30	**30.** B
31. B	**32.** D	**33.** C	**34.** D	**35.** C	**36.** B	**37.** B	**38.** 1	**39.** A	**40.** D
41. C	**42.** A	**43.** C	**44.** B	**45.** 4.8	**46.** C	**47.** B	**48.** B	**49.** 8	**50.** D
51. B	**52.** C	**53.** B	**54.** B	**55.** B	**56.** C	**57.** 1602	**58.** 15872	**59.** C	**60.** C
61. A	**62.** C	**63.** B	**64.** A	**65.** C					

Hints and Explanations

1. The grammatically correct choice is (C) "fewer". The reason is "fewer" is used when the noun is countable; "Less" is used for uncountable things, as illustrated in "there's less dust on the furniture today; there was less noise in the class, there is less milk in the fridge". "Smaller" and "lesser" are irrelevant. Choice (C)

2. Let the weight of A be a kg and that of B be b kg.
After A and B join, total number of students in the class is 14.
$\therefore$ Before A and B joined, the strength of the class was 12. If we assume that the average weight of the 12 students is n, then after A joins, it is $\frac{12n + a}{13}$

$$\frac{12n + a}{13} = n + 1,$$

$\Rightarrow$ $12n + a = 13n + 13$ —— (1)
$\Rightarrow$ $a = n + 13$
After B joined, the average increases by 1/2.

$$\therefore \quad \frac{12n + a + b}{14} = n + 1 + \frac{1}{2},$$

$\Rightarrow$ $12n + a + b = 14n + 14 + 7$ —— (2)
(2) – (1) gives $b = n + 8$ and $a = n + 13$
$\therefore$ $a - b = 5$. Ans: 5

3. Since 90% of 4000 = 3600 and both 3600 and 4000 are adjacent to each other the sale price of the article should be = ₹3600. Choice (B)

4. Sentences (B) to (D) are all wrong for several reasons. The grammatically correct sentence is (A). The verb "adapt" is rightly used and therefore it is syntactically correct. In sentence (B) the correct word is "adopt" not "adapt". Childless women or couples adopt others' child or children. In sentence (C) the appropriate word is "emigrated", not "immigrated". In sentence (D) the wrong word is "emigrated". The correct word is "immigrated". Choice (A)

5. The Number of letters in the word REPTILE is 7 and $7^2 = 49$. Similarly the number of letters in the word CROCODILE is 9 and $9^2 = 81$.
The number of letters in the word ALLIGATOR is 9 and $9^2 = 81$.
$\therefore$ 81 is the code for the word ALLIGATOR.
Choice (D)

6. The correct sentence with respect to grammar and usage is sentence (A). In sentences (B), (C) and (D) the article "*a*" is omitted before 'few' and that is what makes them wrong and unacceptable. Choice (A)

7. $S = 1 + 3x + 5x^2 + 7x^3 + 9x^4 + \ldots$ $\rightarrow$ (1)
$Sx = x + 3x^2 + 5x^3 + 7x^4 + \ldots$ $\rightarrow$ (2)
----
(1) – (2) $S \Rightarrow (1 - x) = 1 + 2x + 2x^2 + 2x^3 \ldots \infty$
$= 1 + 2x\,(1 + x + x^2 + \ldots \infty)$

$$= 1 + \frac{2x}{1 - x}$$

$$S(1 - x) = \frac{1 + x}{1 - x}$$

$$S = \frac{1 + x}{(1 - x)^2} = 3$$

$3x^2 - 7x + 2 = 0$
$(3x - 1)\,(x - 2) = 0$

$$\Rightarrow \quad x = \frac{1}{3} \text{ (or) } x = 2$$

But $|x| < 1 \Rightarrow x = \frac{1}{3}$ Choice (B)

8. Sentences (1) and (6) remain constant and unchanged while the following and preceding four sentences will be shuffled and rearranged in their proper and logical sequence. Sentence (1) says the passage illustrates how an approach was adopted by an automaker to arrive at a new design for an automobile. The new design entailed constructing new plants (5). In the second sentence (3) the management invited the employees' ideas. In the third sentence (4) the employees were forthcoming with their ideas. In the fourth sentence (2) the author agrees that the workers know better if the wrench works well or not. The logical sequence of the sentences is (A) 5, 3, 4, 2. Choice (A)

9. According to the given information the possible arrangement is as follows.
6 bus 25 6 bus 45 15
$\therefore$ The total number of buses in the parking lot is 29.
Choice (A)

10. The above short passage is exclusively about the varied and unforeseen benefits of reading. Though the benefits can be denied or disputed by some, they are nonetheless real and verifiable. The passage says reading provided healing power by accident to some people. Not only that, reading affords consolation and support to those who are sincerely devoted to reading. Belittling it is of no consequence. The answer choices are (i) and (iv), that is (B). Choice (B)

11. Given matrix is $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$

$$\text{i.e., } \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{bmatrix} = 0$$

$\Rightarrow$ $(1 - \lambda)\,(2 - \lambda)\,(3 - \lambda) = 0$

$\Rightarrow$ $\lambda = 1, 2, 3$

$\therefore$ The eigenvalues of A are 1, 2 and 3

If X is an eigenvector of A, corresponding to an eigenvalue λ, then $AX = \lambda X \rightarrow (1)$ for exactly one of $\lambda = 1, 2$ and 3.

Among the vectors given in the options, the vector $X = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$ given in option (B) will satisfy the condition $AX = \lambda X$ for $\lambda = 1$

The vector $\begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$ is an eigenvector of A

Choice (B)

12. We know that among all bipartite graphs with n vertices, the maximum number of edges occur in a complete bipartite graph.
Also, among all the complete bipartite graphs with n vertices, the complete bipartite graph $K_{\left(\frac{n-1}{2}\right),\left(\frac{n+1}{2}\right)}$ will have the maximum number of edges if n is odd.
Here $n = 11$

$\therefore$ The complete bipartite graph $K_{\left(\frac{11-1}{2}\right),\left(\frac{11+1}{2}\right)} = K_{5,6}$ will have the maximum number of edges with 11 vertices.

And the number of edges in $K_{5,6} = 5 \times 6 = 30$
Hence the number of edges in a bipartite graph
With 11 vertices = 30 Choice (C)

13. Standard Results Choice (D)

14. Average number of telephone calls coming to a telephone exchange $= \lambda = 3$ calls per hour

$\therefore$ Probability that the number of telephone calls coming to a telephone exchange exceeds 3

$= P(x > 3) = 1 - P(x \leq 3)$

$= 1 - [P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)]$

$$= 1 - \left[\frac{\lambda^0 e^{-\lambda}}{0!} + \frac{\lambda^1 e^{-\lambda}}{1!} + \frac{\lambda^2 e^{-\lambda}}{2!} + \frac{\lambda^3 e^{-\lambda}}{3!}\right]$$

$$= 1 - \left[e^{-3} + 3e^{-3} + \frac{9}{2}e^{-3} + \frac{9}{2}e^{-3}\right]$$

$= 1 - (13e^{-3}) = 0.3528$ Choice (A)

15. Given $f(x) = 2x$, $g(x) = 3x^2$ and $h(x) = 3$

$\therefore$ (fogoh) (3) $= f(g(h(3)) = f(g(3)) = f(27) = 54$

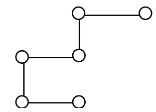
Ans : 54

16. The given numbers are 1, 2, 3, ..., 99.
Each key contains maximum 2-digits
Radix-sort is implemented 2-times (2-Iterations)
In each Iteration, we perform n-Enqueue operations and n-Dequeue operations, Total $(2n)$ operations.

2nd-Iteration also takes $(2n)$ operations
Let us assume, $k = 2$ (The number of digits in each key)

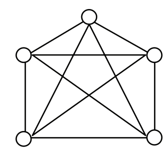
$\therefore$ Time complexity $O(k * n) = O(n)$
(k is very small) Choice (D)

17. I. Let us consider the following graph which is not a cyclic graph



Vertices (n) = 6
Edges (E) = 5
$E = n - 1$

II. Consider the following simple graph



Vertices (n) = 5
Edges (E) = 10
$E = \frac{(n-1)n}{2}$

Both statements are TRUE. Choice (C)

18. Latency or Rotational delay $= \frac{1}{2r}$

r = rotations per second
1 minute – 600 rotations

60 seconds – 600 rotations 1 second – r

$$r = \frac{600}{60} = 10\ \frac{1}{2r} = \frac{1}{2 \times 10} = \frac{1}{20} = 0.05 \text{ seconds}$$

Choice (C)

19. - B-tree construction on 10 elements, either ascending or descending order gives maximum number of Node splits.
- B+-tree, only on descending order of elements gives worst-case, that is maximum number of node splits. Choice (C)

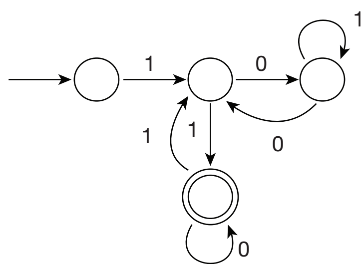
20. Statement I is FALSE. The lower frequency processor may have higher IPC than higher frequency processor.
II is also FALSE. Because the processor which executes more number of instructions does not imply that the processor always finishes a program faster.
(Θ Different processors requires different number of instructions to execute). Choice (D)

21. The Dependencies will be resolved in the 3rd stage. If the delay slot is 2 clock cycles then the pipeline ensures correct operation.

	1	2	3	4	5	6	7	8	
I_L			S_1	S_2	S_3	S_4	S_5		
I_{L+1}				S_1	S_2				
I_{L+2}					S_1				
I_T						S_1			

If the delay slot is 2 then in the 3rd stage the branch instruction will be evaluated and target instruction will be evaluated in the next cycle. Ans: 2

22. L consists of the binary strings
$\{11, 110, 1001, 1100, 1111, 10010, ...\}$
i.e., L consists of the numbers which are divisible by 3.
The DFA is shown below:



Choice (A)

23. Finding the emptiness of a CFL is decidable. Remaining three problems are undecidable. Choice (C)

24. Shutdown( ) will block the communication in both ways. Close( ) will destroy the socket. Choice (B)

25. A repeater does not connect two LANs. It connects segments of LANs. Choice (B)

26. Caesar Cipher is also referred as shift cipher.
In shift cipher, the encryption algorithm, "shift key characters down" i.e., towards the end of the alphabet. (with key as some number).
For GATECS with key = 10, the encrypted message is QKDOMC. Choice (D)

27. Binding can take place at language design time, language implementation time, compile time, load time and link time.

Choice (D)

28. According to static scoping, the value of 'x' will be 5, as fun1( ) will associate with global variable of 'x'.
Choice (A)

29. Number of binary tree structures with 3 elements is $\frac{{}^{6}C_3}{3+1} = 5$ (With 'n' element it is $\frac{{}^{2n}C_n}{(n+1)}$.)
Number of binary trees that can be formed with n nodes is $\left(\frac{{}^{2n}C_n}{n+1} \times n!\right) = 5 \times 3! = 30$ Ans: 30

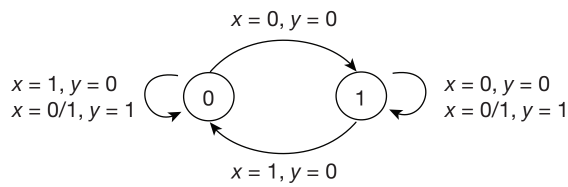
30. Root directory is placed at fixed location on the disk
Choice (B)

31. Efficiency $= \frac{\text{Process execution time}}{\text{Waiting time + Process execution time}}$.
As the switching of the processes and the execution of process takes same amount of time, the efficiency will be 50%. Choice (B)

34. When $y = 1$, the output of or gate is 1, so NAND gate outputs will be $\overline{1 \cdot \bar{Q}} = Q, \overline{1 \cdot Q} = \bar{Q}$ i.e., output Q, $\bar{Q}$ remains in same state as long as $Y = 1$, irrespective of X input.
When $Y = 0$, the circuit works as follows

Y	X	Output OR gate 1	Output OR gate 2	Output (Q) NAND gates	Output NAND gates ($\bar{Q}$)
0	0	0	1	1	0
0	1	1	0	0	1

So when $y = 0$, next state depends on X value.
If $x = 0$, next state = 1
if $x = 1$, next state = 0



Choice (D)

35. $F(a, b, c) = \pi M(2, 5)$
$= \Sigma_m(0, 1, 3, 4, 6, 7)$

a \ bc	00	01	11	10
0	1	1	1	
1	1		1	1

$= a^1c + b^1c^1 + ab$

a \ bc	00	01	11	10
0	1	1	1	
1	1		1	1

$= a^1b^1 + bc + ac^1$
So both (A) and (B) choices are true. Choice (C)

36. Let p : Ramu is in the class
And q : Lokesh is in the playground
$\therefore$ The propositions S_1, S_2 and S_3 can be symbolically represented as
$S_1 : p \to q$; $S_2 : p$ and $S_3 : q$
$\therefore$ $p, p \to q \Rightarrow q$ (By modus ponens)
$\therefore$ S_3 logically follows from S_1 and S_2
Also, when p is false and q is true, $p \to q$ is true i.e., when S_1 and S_3 are true, S_2 is false.
Hence S_2 does not logically follow from S_1 and S_3
Choice (B)

37. Among the options given, consider option (B), (S_2, X_6) where $S_2 = \{1, 2, 3, 4, 5\}$
For $3, 4 \in S_2$, $3\ X_6\ 4 = 0 \notin S_2$
$\therefore$ S_2 does not satisfy closure axiom under 'X_6'
Hence (S_2, X_6) is not an abelian group Choice (B)

38. Given $f(x) = 8x^5 - 15x^4 + 10x^2$

$f'(x) = 40x^4 - 60x^3 + 20x$

$f'(x) = 0 \Rightarrow 40x^4 - 60x^3 + 20x = 0$

$\Rightarrow \quad x(x-1)^2(2x+1) = 0$

$\Rightarrow \quad x = 0; x = 1, 1 \text{ and } x = \dfrac{-1}{2}$

$\therefore$ The stationary values of $f(x)$ are 0, 1 and $\dfrac{-1}{2}$

$f''(x) = 160x^3 - 180x^2 + 20$

At $x = 0$; $f^{II}(x) = 20 > 0$

$\therefore$ $f(x)$ has a minimum at $x = 0$

At $x = \dfrac{-1}{2}$, $f^{II}(x) = -45 < 0$

$\therefore$ $f(x)$ has a maximum at $x = \dfrac{-1}{2}$

At $x = 1$; $f^{II}(x) = 0$

$f'''(x) = 480x^2 - 360x$

At $x = 1$; $f^{III}(x) = 120 \neq 0$

$\therefore$ $f(x)$ has neither maximum nor minimum at $x = 1$

$\therefore$ The number of stationary values where $f(x)$ has neither maximum nor minimum = 1 Ans: 1

39. The number of ways of accommodating 30 students in 14 rooms, of which 5 are triple, 6 are double and 3 are single rooms = The number of ordered partitions of a set with 30 elements into 14 sets with first 5 sets containing 3 elements each, the next 6 sets containing 2 elements each and the last 3 sets containing one element each

$= P(30; 3, 3, 3, 3, 3, 2, 2, 2, 2, 2, 2, 1, 1, 1)$

$= \dfrac{30!}{(3!)^5 \times (2!)^6 \times (1!)^3}$ Choice (A)

40. Standard Results Choice (D)

41. From the given circuit, all flip flops are falling edge triggered flip flops and $T = 1$, so all flip flops will toggle their state when clk pulse occurs. clk input is connected to Q_0, so for every clk pulse Q_0 change its state.

$\bar{Q}_0$ is connected to negative edge triggered flip flop (Q_1)

So Q_1 will change (toggle) when $\bar{Q}_0$ changes from 1 to 0. (i.e., Q_0 changes from 0 to 1)

Q_1 is connected to next stage (Q_2), so whenever Q_1 has change from 1 to 0, then Q_2 will get toggle. we can summarize as below:

Q_0 – toggles for every clk pulse

Q_1 – toggles when Q_0 changes from 0 to 1

Q_2 – toggles when Q_1 changes from 1 to 0

Clk	Q_2	Q_1	Q_0
0	0	0	0
1	0	1	1
2	0	1	0
3	1	0	1
4	1	0	0
5	1	1	1
6	1	1	0
7	0	0	1
8	0	0	0

So sequence is 0, 3, 2, 5, 4, 7, 6, 1, 0….

Choice (C)

42. Option (A): it is LL(1), SLR(1) and LR(0)

Option (B): it is neither LL(1), LR(0) nor SLR(1)

Option (C): it is not LL(1), LR(0) but SLR(1)

Option (D): it is not LL(1), LR(0) but SLR(1)

Choice (A)

43. The parse table for the grammar is:

	a	b	e	i	t	$
S	$S \to a$			$S \to iCtSS^1$		
S^1			$S^1 \to \epsilon$ $S^1 \to eS$			$S^1 \to \epsilon$
C		$C \to b$				

$M[S^1, e]$ contains multiple entries

Choice (C)

44. Given SDD is an attribute grammar as it does not have any side effects.

It is also S-attributed as given SDD only involves synthesized attributes. (val and lexval are synthesized attributes).

It is also L-attributed as the attributes are synthesized.

Choice (B)

45. $q = \dfrac{t - ns}{(n-1)} = \dfrac{36 - 6 \times 2}{5} = \dfrac{24}{5} = 4.8$ Ans: 4.8

46. 1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3, 2, 1, 2, 3, 5

1
2
~~3~~ 6
~~4~~ 3
~~5~~ ~~7~~ 5

= 9 page faults Choice (C)

47. The maximum value of count is infinite only if $P1$ is in the system and executes $P1(\ )$ infinite times.

The solution guarantees progress among the processes. Both the processes can be in critical section at the same time if P_2 executes 1st instruction in $P2(\ )$ followed by process $P1$ executing $P1(\ \ )$. No mutual-exclusion is guaranteed.

Choice (B)

48. Choice (B)

49. **In-order:**

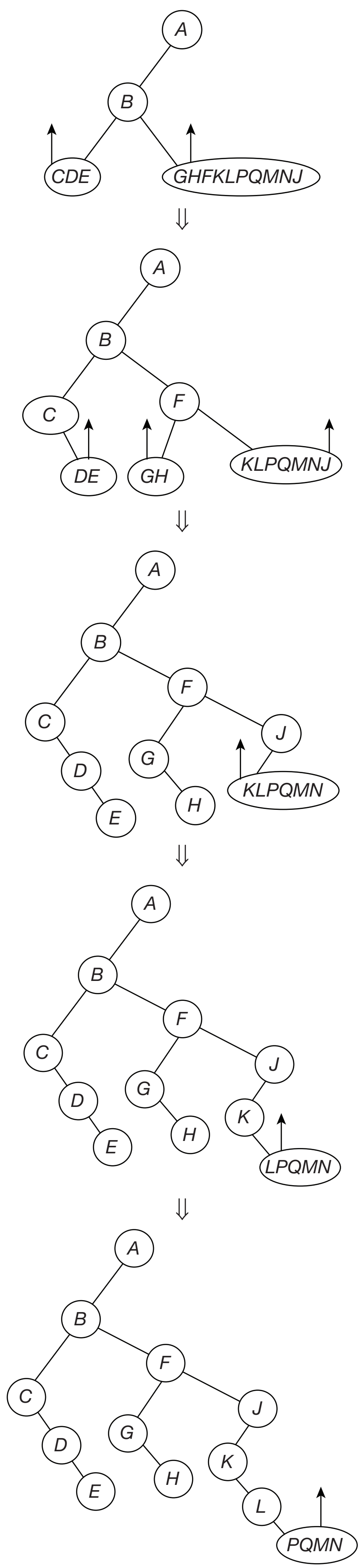
C D E B G H F K L P Q M N J (A)

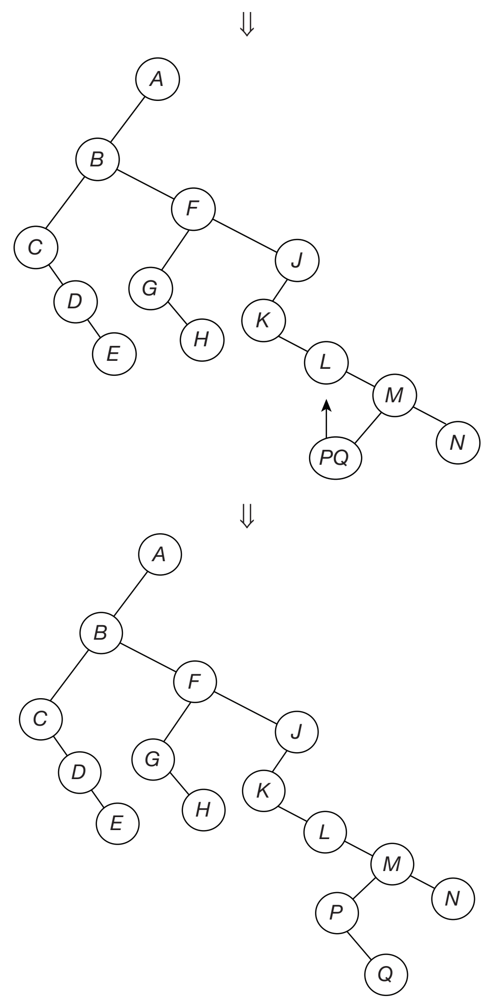
In order for the traversal is

C D E B G H F K L P Q M N J (A)

Pre-order: *A B C D E F G H J K L M P Q N*

C D E B G H F K L P Q M N J





The number of nodes with single child node is 8.

Ans: 8

50. The value of $s1$ and $s2$ will be 42 and 400, if the evaluation order is from left to right.
The value of $s1$ and $s2$ will be 44 and 300, if the evaluation order is from right to left.

Choice (D)

51. Given 32-bit of IPv4 header is
$(4500\ 139C)_{16}$
$= (0100\ 0101\ 0000\ 0000\ 0001\ 0011\ 1001\ 1100)_2$
First 4-bits represents version 4.
Next 4-bits represents header length (in multiple of 4).
$5 \times 4 = 20$ Bytes
Next 8-bits are for service field.
Next 16-bits represents Total length (Header + Data) 5020 bytes.
Fragmentation information is not included in these 32-bits.

Choice (B)

52. Distance between Stations = 1200 km
Signal propagation speed = 3×10^8 m/s

Maximum propagation time $= \dfrac{\text{Distance}}{\text{Speed}}$

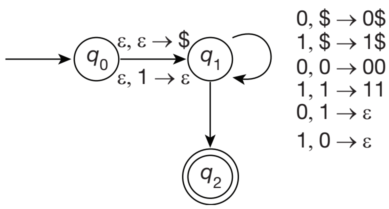
$= \dfrac{1200 \times 10^3}{3 \times 10^8} = 4$ msec

For number of attempts, $k = 2$, the range of values is $\{0, 1, 2, 3\}$.

T_B (Back-off time) = Random number in range of values * propagation time.

$\therefore$ Possible Back-off time values are 0 ms, 4 ms, 8 ms, 12 ms. Choice (C)

53. The PDA which accepts more number of 1's than 0's is shown below:



The PDA pushes a zero or one on to the stack and pops a zero or one with inputs one or zero respectively. It pushes 0's (or 1's) with Top of stack 0's (or 1's).

And finally it reaches final state if there is atleast one 1 remained on the stack. Choice (B)

54. L_1 is not regular. In L_1, sum of number of x, y, z are more than 10 and we have to check the equality of m, n or n, p, which is not possible with *FA*.

L_2 is regular. The sum of number of x, y, z is less than or equal to 10 and we can easily check the equality of m, n or n, p (as the number of combinations is Limited). i.e., we can have one x, one y and number $(z\text{'s}) \le 8$.

Two x's, Two y's and number $(z\text{'s}) \le 6$

Three x's, Three y's and number $(z\text{'s}) \le 4$ etc.

Choice (B)

55. $L_1 = \{a^i\, b^{2i} \mid i \in \{1, 2, ...\}\}$

L_1 is CFL but not regular. PDA can be designed, which pushes 2 a's for each a and pop an 'a' for each b.

$L_2 = \{a^i\, b^{i^2} \mid i \in \{1, 2, ...\}\}$

L_2 is not CFL. (Its complement is also not CFL). But we can design a Halting Turing machine to recognize strings of the form $a^i\, b^{i^2}$. Choice (B)

56. The immediate value has 16-bit length. For branch instructions, the immediate value takes signed 2's complement representation. The range of signed 2's complement numbers is

-2^{n-1} to $2^{n-1} - 1$ (n is number of bits)

Here $n = 16$

$\therefore$ $X = 2^{15} \times 4$

$Y = (2^{15} - 1) \times 4$

($\because$ Each instruction requires 4 locations).

$X = 131072$

$Y = 131068$ Choice (C)

57. Given code,

Loop: I_1: LOAD R_1, $0(R_2)$
I_2: ADD R_1, R_1 #1
I_3: STORE R_1, $0(R_2)$
I_4: ADD R_2, R_2, #4
I_5: SUB R_4, R_3, R_2
I_6: BNEZ R_4, Loop

Initially value of R_3 is R_2 +400. In every iteration R_2 is incremented by 4. Based on R_4 value the loop will get executed.

$R_4 = R_3 - R_2$

Both R_3 and R_2 will have equal value after $100 = \left(\dfrac{400}{4}\right)$ iterations.

The timing diagram for the 1st iteration of the loop is shown below:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
I_1	IF	ID	EX	MA	WB														
I_2		IF	Stall	Stall	ID	EX	MA	WB											
I_3					IF	Stall	Stall	ID	EX	MA	WB								
I_4								IF	ID	EX	MA	WB							
I_5									IF	Stall	Stall	ID	EX	MA	WB				
I_6												IF	Stall	Stall	ID	EX	MA	WB	
I_7																	IF	ID	

From the Timing diagram we can observe that 2 stages of two iterations are overlapped. So for 1^{st} 99 iterations it takes 16 clock cycles and 100^{th} iteration takes 18 clock cycles.

$\therefore$ Number of clock cycles for 100 iterations

$= 99 \times 16 + 18 = 1602$ Ans: 1602

58. Given number is $(5403E)_{16}$

= 0101 0100 0000 0011 1110

Sign = 0 = positive

excess – 64 exponent

= 101 0100 = 84

Exponent = 84 – 64 = 20

Mantissa = 0.0000 0011 1110 = $(0.015)_{10}$

$\therefore$ Given number = $(0.015)_{10} \times 2^{20} = 15872$

Ans: 15872

59. When we have M:N relation, A separate relation is taken to place all primary keys of participating entities.

Option (C)

Test (TID, RNo, TName)

If we include RNo in Test entity, there is a scope of lot of Redundant values.

Choice (C)

60. Lets Assume some tuples for all three tables:

Sailor

Sid	S. Name	Rating	age
11	Anu	7	30
12	Radha	8	20
13	Sita	6	40
14	Karan	7	20
15	Sita	7	30

Reserves

Sid	bid
11	201
11	203
12	201
13	203
14	202
15	201

Boats

bid	bname	color
201	A	black
202	B	blue
203	C	white
204	C	red

Option (A):

$\sigma^{(Boats)}_{color=\text{'black'}} \cap \sigma^{(Boats)}_{color=\text{'white'}}$

201	A	black

$\cap$

203	C	white

$= \phi$

Option (B)

Sid	Sname	Rating	age	bid	bname	color
11	Anu	7	30	201	A	black
12	Radha	8	20	201	A	black
15	Sita	7	30	201	A	black

Sid	Sname	Rating	age	bid	bname	color
11	Anu	7	30	203	C	white
13	Sita	6	40	203	C	white

Result = ϕ (ϕ values should be same under all the columns of 2-tables)

Option (C)

π_{sid} from First part of query

Sid
11
12
15

π_{sid} from second part of query

Sid
11
13

Sid
11
12
15

$\cap$

Sid
11
13

=

Sid
11

Choice (C)

61. I.

8 7 6 5 4 3 2 1 (1^{st} Swap)

1 7 6 5 4 3 2 8 (2^{nd} Swap)

2 6 5 4 3 7 8 (3^{rd} Swap)

3 5 4 6 7 8 (4^{th} Swap)

4 5 6 7 8

$\therefore$ 4-swap operations.

II.

8 1 2 7 3 6 4 5 (1^{st} Swap)

1 8 2 7 3 6 4 5 (2^{nd} Swap)

2 8 7 3 6 4 5 (3^{rd} Swap)

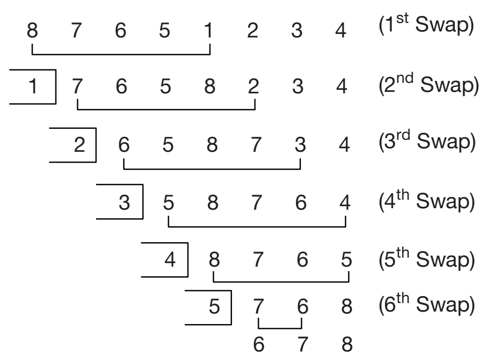
3 7 8 6 4 5 (4^{th} Swap)

4 8 6 7 5 (5^{th} Swap)

5 6 7 8

$\therefore$ 5 swap operations.

III.



∴ 6 swap operations Choice (A)

62. Solution set $S[A]$

I-Iteration:

$S[A]$ source = A
dist $[B] = 2$
dist $[C] = 1$
dist $[D] = 6$
dist $[E] = \infty$
dist $[F] = \infty$

II-Iteration:

S$[AC]$
dist $[B] = \min\{2, \infty\} = 2$
dist $[D] = \min\{6, \infty\} = 6$
dist $[E] = \min\{\infty, \infty] = \infty$
dist $[F] = \min\{\infty, 8\} = 8$

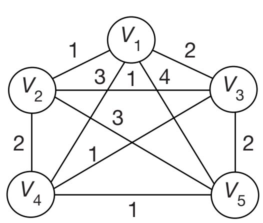
III – Iteration

$S[ACB]$
dist $[D] = \min\{6, \infty\} = 6$
dist $[E] = \min\{\infty, 6\} = 6$
dist $[F] = \min\{8, 4\} = 4$

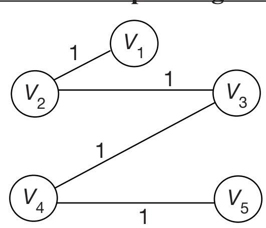
IV-Iteration

$S[ACBF]$
dist $[D] = \min\{6, \infty\} = 6$
dist $[E] = \min\{6, 8\} = 6$
Next we can take either D (or) E Choice (C)

63. Graph 1



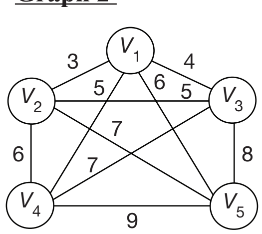
Minimum spanning Tree 1



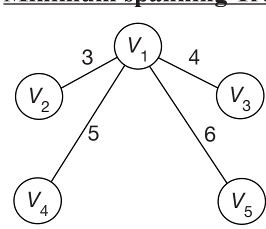
Total weight = 1 + 1 + 1 + 1 = 4

Total weight = 1 + 1 + 1 + 1 = 4

Graph 2



Minimum spanning Tree 2



Total weight = 3 + 4 + 5 + 6 = 18

Total weight = 3 + 4 + 5 + 6 = 18

∴ Difference between both Spanning Trees total weight is 18 – 4 = 14

Choice (B)

64. Option (A)

SELECT CNo, count (*)
FROM Register
WHERE
GROUPBY CNo

Option (B)

SELECT SName
FROM student JOIN Register ON(RNo)
WHERE CNo = 201

Option (D)

SELECT SName
FROM student JOIN Register ON(RNo),
Register JOIN Course ON(CNo)
WHERE Duration = 6 months

Option (C)

SELECT SName, CNo
FROM student JOIN Register ON(RNo)
WHERE Day = 'Monday'

Option (A) Cannot be expressed, because we need GROUPBY clause

Choice (A)

65. $A \rightarrow BC$
$A \rightarrow D$
$D \rightarrow E$

L	M	R
A	D	BC

$A^+ = \{ABCDE\}$
key = A
$A \rightarrow D$ Transitivity exists
$D \rightarrow E$ It violates 3NF
We need to perform 3NF – Decomposition.
The decomposed tables are $R_1(ABCD)$, $R_2(DE)$

Check for lossless decomposition:

	A	B	C	D	E
R_1 (ABCD)	*	*	*	*	*
R_2 (DE)				*	*

∴ It is loss less decomposition.

Choice (C)

Mock Test 2

Number of Questions: 65 **Total Marks: 100**

Wrong answer for MCQ will result in negative marks, (–1/3) for 1 Mark Questions and (–2/3) for 2 Marks Question.

General Aptitude

Number of Questions: 10 **Section Marks: 15**

Directions for question 1: Select the word most similar in meaning to the given word:

1. Risque:
   (A) Risky (B) Lascivious
   (C) Queasy (D) Pompous

***Directions for questions 2 and 3*:** Select the correct alternative from the given choices.

2. In a certain code language, GOOGLE is coded as HNPFMD. How is the word APPLE coded in that language?
   (A) BOGKE (B) BQOKF
   (C) BOQKF (D) None of these

3. The numbers a, b, c, d and e form a geometric progression. Which of the following also form a geometric progression?
   (i) a^2, b^2, c^2, d^2, e^2
   (ii) $a - 1, b + 2, c - 3, d + 4, e - 5$
   (iii) $3a, 3b, 3c, 3d, 3e$
   (A) Only (i) (B) Only (i) and (ii)
   (C) Only (iii) and (i) (D) None of these

***Directions for question 4*:** Select the most suitable one word substitute for the following expression:

4. A place where everything is perfect:
   (A) Heaven (B) Cosmos
   (C) Synagogue (D) Utopia

***Direction for question 5*:** Select the appropriate word/phrase out of the given options to complete the following sentence:

5. Neither the teacher nor the students __________ any clue as to who could have stolen the keys to the office treasury.
   (A) was (B) has
   (C) have (D) were

***Directions for question 6*:** Select the correct alternative from the given choices.

6. Consider a function $f(x) = 3 - |x|$, where $-2 \le x \le 2$. The minimum and maximum values of $f(x)$ are:
   (A) 0, 2 (B) 0, –2
   (C) 1, 3 (D) 0, –4

***Directions for question 7*:** In the following sentence certain parts are underlined and marked P, Q, and R. One of the parts may contain a certain error or may not be acceptable in standard written communication. Select the part containing the error. Choose D as your answer if there is no error.

7. <u>There are no machineries</u> for resolving
   (P)
   <u>these disputes and this has,</u> in no small
   (Q)
   measure, compounded <u>the present situation.</u>
   (R)
   (A) P (B) Q
   (C) R (D) No error

***Directions for question 8*:** Which one of the statements given below the passage is logically valid and can be inferred from the passage below?

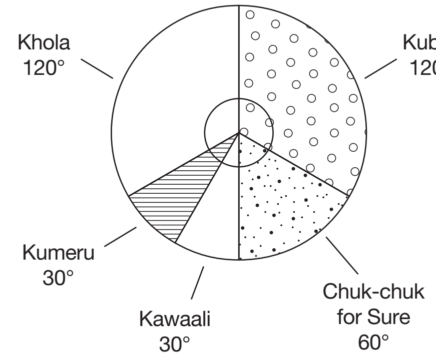
8. Napoleon Bonaparte was one of the world's youngest generals. At the age of 24 he was master of the art of war, a military general and a cruel dictator at heart. He had the magnetism of the great and he won devoted friendship from many. His glance, like Akbar's, was magnetic. He often said that he had won many battles with his eyes, not the sword. A strange statement for a man who had plunged Europe into war. And it appears, during his last years when he was imprisoned at St. Helena, he had a change of heart. Good thoughts came to him during the fading years of life, his painful period of exile. He was much chastened, and perhaps wrote to influence posterity in his favour. He wrote that the conquest of the spirit is greater than the conquest of the sword.
   (A) Napoleon Bonaparte was not the world's youngest general.
   (B) Napoleon Bonaparte was not the world's youngest dictator.
   (C) Napoleon Bonaparte believed that some day victories would be won without cannons and bayonets.
   (D) Napoleon Bonaparte was thoroughly irreligious but he encouraged religion.

***Directions for questions 9 and 10*:** Select the correct alternative from the given choices.

9. The chairman of a multinational corporation desires to appoint four of the five selected persons A, B, C, D and E to lead the four different domains of the organization, which are Operations, Marketing, Finance and R&D. C doesn't want to get hitched to one specific domain as he desires to have an exposure to all the four domains.

B is given the designation of Operations head. Neither *D* nor *A* is posted as Marketing heads.
Which of the following can be a valid assignment of heads to the domains?
(A) *A*–Marketing, *B*–Operations, *C*–Finance, *D*–R&D
(B) *A*–Finance, *B*–Operations, *D*–R&D, *E*–Marketing
(C) *A*–Operations, *B*–Finance, *C*–R&D, *D*–Marketing
(D) None of these

10. The pie chart below gives the breakup of market share by volume of five different fleet management companies in the year 2015. The proportion of male to female customers of each company is 5 : 1. If the total number of customers of the five companies in 2015 is 216000. The Number of customers of Chuk-chuk for Sure are females is _____.



Computer Science Engineering

Number of Questions: 55 **Section Marks: 85**

Directions for questions 11 to 65: Select the correct alternative from the given choices.

11. The third term in the Taylor's series expansion of the function $f(x) = 5x^2 + \cos x$ about $x = \pi$ is ______.

(A) 0 (B) $\frac{9}{2}(x-\pi)^2$
(C) $\frac{11}{2}(x-\pi)^2$ (D) $\frac{13}{6}(x-\pi)^3$

12. If *A* and *B* are two square matrices of same order such that $AB = A$, $BA = B$, then:
(A) both *A* and *B* are idempotent
(B) both *A* and *B* are involutory
(C) *A* is idempotent and *B* is involutory
(D) *A* is involutory and *B* is idempotent

13. If *X* is a continuous random variable with probability density function $f(x)$ given by:

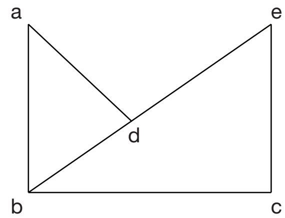
$$f(x) = \begin{cases} ax(1-x); & 0 \le x \le 1 \\ 0; & \text{otherwise} \end{cases}$$

Then the mean of *X* is ______.

14. The maximum possible number of reflexive relations that can be defined on a set $S = \{k, l, m, n\}$ is _________.

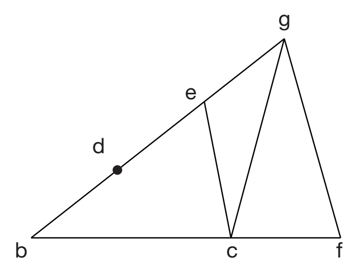
15. Consider an undirected graph G given below.
Which of the following is NOT an induced subgraph of the graph G?

(A)



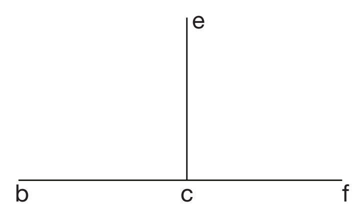
G_1

(B)



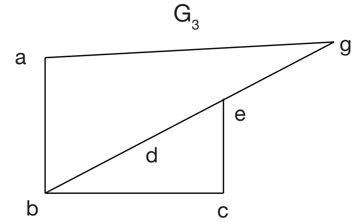
G_2

(C)



G_3

(D)



G_4

16. The minimum number of comparisons required to find the maximum element in a min-heap of *n* elements is:

(A) $\log_2 n$ (B) $\frac{n}{2}$
(C) $\left\lfloor \frac{n}{2} \right\rfloor - 1$ (D) $\left\lceil \frac{n}{2} \right\rceil - 1$

17. Five nodes labeled *P*, *Q*, *R*, *S*, *T* are used to construct a binary tree. The number of distinct binary trees that can be formed such that each of those in-order traversal gives *P Q R S T* is ________.

18. Which of the following mechanisms will interrupt the execution of a process?

(P) Interrupt (Q) Trap
(R) Supervisor call (S) Memory fault
(A) P, R (B) P, Q, R
(C) P, R, S (D) P, Q, R, S

19. Which of the following is affected from external fragmentation?

(A) Simple paging
(B) Virtual memory segmentation
(C) Virtual memory paging
(D) None of the above

20. Consider the following:

$E \rightarrow TE^I$
$E^I \rightarrow +TE^I | \in$
$T \rightarrow FT^I$
$T^I \rightarrow *FT^I | \in$
$F \rightarrow (E) |$ id

The follow set of F does not contain

(A) {id, (} (B) {+, $}
(C) {$, *} (D) {*,)}

21. Which of the following statement is TRUE?

(A) Preprocessing is required for every programming language before compiling a program.
(B) Symbolic names can be associated with the mnemonic instruction.
(C) Symbol table can be implemented using hash table.
(D) All of the above

22. Consider an AVL Tree with 'n' elements stored in an array [with array index starting from 1]. Now a Binary search is performed on some elements in AVL tree that is on the elements positioned on (2^i-1) places. Time complexity for searching an element on the above scenario will be:

(A) $O(\log(\log n))$
(B) $O(\log n)$
(C) $O(n)$
(D) Binary search cannot be implemented for above scenario.

23. Consider a Recurrence equation,

$T(n) = 3T(n-1) + O(n^2)$

Then the time complexity for $T(n)$ is:

(A) $O(n^2)$ B) $O(n^3)$
(C) $O(3^n \cdot n^3)$ (D) $O(3^n \cdot n^2)$

24. Consider a relation $R(X, Y, Z, P, Q)$ with functional dependencies:

$XYZ \rightarrow PQ$
$Q \rightarrow YZP$

The number of candidate key(s) for the relation R is ______.

25. Consider a relation,

Student (Roll-No, Name, mark1, mark2)

Which of the following query is valid on this relation?

(A) select SUM(Name) from Student
(B) select Average(mark1, mark2) from Student
(C) select Roll-No, Max (marks) from Student
(D) select SUM(mark1) from Student

26. Match the following:

Group–A		Group–B	
A	Binary search	1.	Divide and conquer
B	Traveling sales person problem	2.	Greedy approach
C	Merge sort	3.	Dynamic programming
D	Job sequencing problem	4.	Back tracking

(A) A–1, B–3, C–1, D–2
(B) A–4, B–2, C–4, D–4
(C) A–4, B–3, C–1, D–4
(D) A–1, B–2, C–1, D–2

27. Which of the following task is not related to Instruction cycle?

(i) Instruction fetch
(ii) Zero or more operand fetch
(iii) Zero or more operand stores
(iv) An interrupt check if interrupts are enabled

(A) (ii), (iii)
(B) (ii), (iv)
(C) (iv) only
(D) All the tasks are related to instruction cycle

28. A program which has 10000 instructions, run on a 20 MHz processor, with the following instruction mix and clock cycle count.

Instruction type	Instruction count	Clock cycles
I_1	4500	1
I_2	3200	2
I_3	1500	2
I_4	800	2

What will be the values of CPI and MIPS respectively?

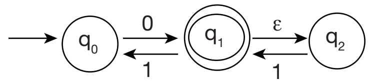
(A) 2 and 0.0775 (B) 1.55 and 12.9
(C) 1.75 and 12.9 (D) 1.55 and 0.0775

29. Which of the following operations is/are not internal processor operations of instruction cycle?

(i) Instruction fetch
(ii) Instruction address calculation
(iii) Instruction operation Decoding
(iv) Operand fetch
(v) Data operation
(vi) Operand address calculation
(vii) Operand store

(A) (i), (ii), (iii)
(B) (iv), (v), (vi), (vii)
(C) (i), (iv), (vii)
(D) (ii), (vi)

30. Consider the following ε-NFA:



If the ε-Transition needs to be removed, then how many new transitions will be added and what are they?

(A) 2, $q_1 \xrightarrow{1} q_2, q_1 \xrightarrow{1} q_1$
(B) 1, $q_1 \xrightarrow{1} q_2$
(C) 2, $q_1 \xrightarrow{1} q_1, q_0 \xrightarrow{0} q_2$
(D) 1, $q_0 \xrightarrow{0} q_2$

31. Which of the following statement is FALSE?

(A) Amongst the three languages Regular, context-free and recursive, Regular language is most specific.
(B) If P is recursively enumerable but not recursive then $\overline{P}$ is non-recursively enumerable.
(C) If C is a context free language then $\overline{C}$ is recursive.
(D) If R is a regular expression and T is a turning machine for the language then T may or may not halt on R.

32. Which of the following is FALSE about OSI layered model?

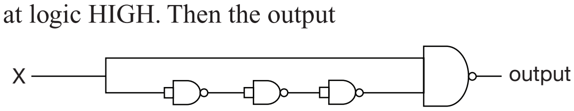
(i) Physical, Data link, Network and Transport layers are Host layers.
(ii) The layered architecture provides the modularity.
(iii) Transport layer is responsible for the communication between two applications running on different computers.

(A) (i), (ii) Only (B) (iii) only
(C) (i) only (D) (i), (iii) only

33. Consider below sequence of bits, arrived on a link:
0111 1110 1000 0111 1101 1001 1010 1000 0111 1101 1100 1111 1111 0111 1110.
If HDLC framing is used for the protocol then which of the following is not justified with the given frame format?

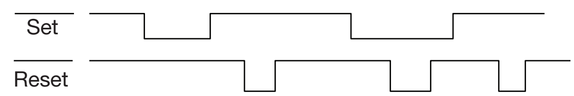
(A) The frame start and end bits are 0111 1110
(B) There are two stuffed bits in the frame
(C) There are 2 error bits in given frame
(D) The address field has 1000 0111

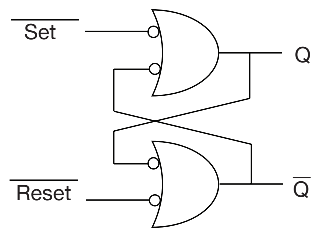
34. Consider the logic circuit with input signal X as shown in the figure. All the gates in the figure shown have identical non-zero delay. The signal X which was at logic LOW is switched to logic HIGH and maintained at logic HIGH. Then the output

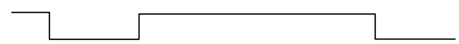


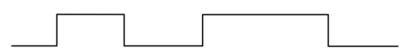
(A) Stays high throughout
(B) Stays low throughout
(C) pulses from LOW to HIGH to LOW
(D) pulses from HIGH to LOW to HIGH

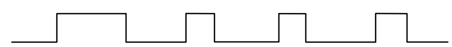
35. The wave forms $\overline{\text{Set}}$, $\overline{\text{Reset}}$ are applied to the inputs of the following latch, then the wave form of Q will look like? (Consider initial state as 0)

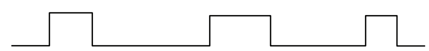




(A) 

(B) 

(C) 

(D) 

36. If 1, 4 and 5 are the eigen values of 3 × 3 matrix A, then the matrix $A^2 - 5A + 6I_3$ has ________.

(A) three distinct eigenvalues
(B) two distinct eigenvalues
(C) all the three eigenvalues are equal
(D) an eigenvalue zero.

37. If p, q and r are any three statement variables, then which of the following is NOT a tautology?

(A) $(p \wedge q) \rightarrow [(p \vee q) \vee (\neg p \wedge \neg q)]$
(B) $[p \wedge (q \vee r)] \vee [\neg p \vee (\neg q \wedge \neg r)]$
(C) $[(p \vee q) \rightarrow r] \leftrightarrow [\neg r \rightarrow \wedge (p \vee q)]$
(D) $[p \rightarrow (q \rightarrow r)] \wedge [\neg p \vee q \vee r]$

38. If a fair die is rolled thrice, then the expected value of the product of the numbers that appear on the die in the three rolls is ________.

(A) $\frac{7}{2}$ (B) $\frac{21}{2}$
(C) $\frac{343}{8}$ (D) $\frac{1}{2}$

39. The maximum value of the function $f(x) = (x - 1)^2 e^{-x}$ is _____.

40. Consider the 2 × 2 matrices $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and let $G = \{A, B, C, I\}$. Then the generators of the cyclic group G under matrix multiplication are _______.

(A) A and B (B) B and C
(C) C and I (D) I and A

41. Consider the below truth table:

X	*Y*	*Z*	*F(X, Y, Z)*
0	0	0	1
0	0	1	D
0	1	0	0
0	1	1	0
1	0	0	D
1	0	1	0
1	1	0	1
1	1	1	1

How many number of 2 input NOR gates are required to implement the function $F(X, Y, Z)$ (D-Stands for don't care condition)?

(A) 3 (B) 4
(C) 5 (D) 6

42. Consider a network topology with two nodes $N1$ and $N2$. $N1$ sends a file of 10 MB to $N2$ using sliding window protocol. The Data is sent using a frame of size 1 KB and the size of acknowledgement is 40 Bytes. The propagation delay is 100 ms. The link has a speed of 10 Mbps. $N1$'s sending window size is five frames and receiving window size is 10000 frames. Then the throughput of $N1$ (in KBPS) is ________.

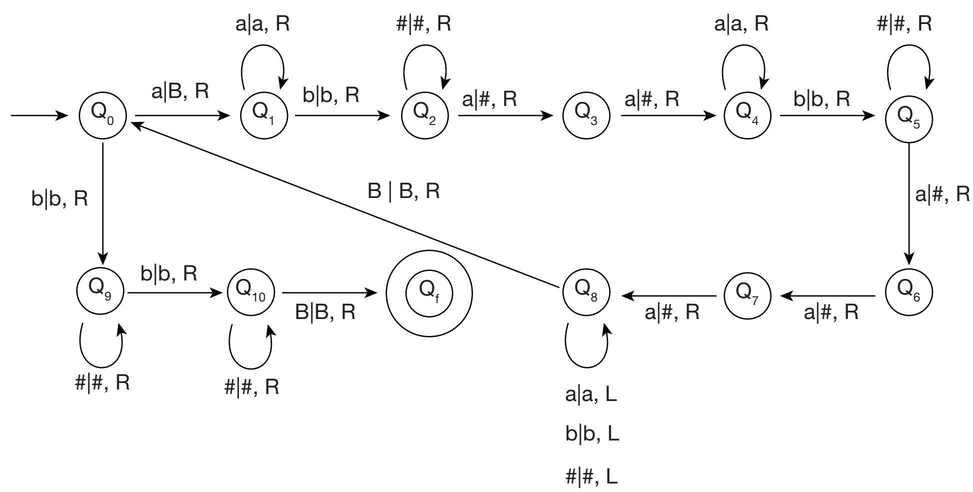
43. Consider a TCP socket connection between application processes $P1$ and $P2$ running on two different hosts. If $P1$ wants to send 10000 Bytes of data to $P2$ and the operating system started sending data from $P1$ to $P2$. Then which of the following is TRUE?

(i) If $P1$ calls close() on the socket before the OS finishes transmitting the data to $P2$ then also the OS continues the transmission of data to $P2$.
(ii) If $P1$ calls close() after the OS has transmitted all the data to $P2$ then OS sends TCP FIN packet to $P1$.
(iii) The process $P2$ can perform multiple receive() calls to receive the entire data from $P1$.
(iv) If $P1$'s connection is closed and $P2$ read all the data from $P1$ then next call of $P2$'s receive() returns an end of stream.

(A) (i), (iii) (B) (ii), (iii)
(C) (ii), (iii), (iv) (D) (i), (ii), (iii), (iv)

44. To transmit the message,
$x^{15} + x^{14} + x^{12} + x^{11} + x^8 + x^6 + x^4 + x^3 + 1$
Using the CRC generator $x^8 + x^6 + x^3 + 1$, the check sequence that will be added to the message to protect it from errors using CRC (in Binary) is _____.

45. Consider below Turing machine:



Each transition $X|Y, D$ specifies the input symbol X, symbol to be written Y and Direction of head movement D. Here $\Sigma = \{a, b\}$ and tape symbols are $\{a, b\ \#, B\}$. What is the language accepted by given Turing machine?

(A) $\{a^n\, b\, a^{n/2}\, b\, a^{n/3} \mid n \geq 0\}$
(B) Set of all strings over Σ such that (number of a's) % 6 = 0
(C) $\{a^n\, b\, a^{2n}\, b\, a^{3n} \mid n \geq 0\}$
(D) $\{a^n\, b\, a^{2n}\, b\, a^{3n} \mid n \text{ is an even number}\}$

46. Which of the following language is Regular?

(i) $\{w \mid w \in \{0, 1\}^*$ and w has prime number of 1's$\}$
(ii) $\{w \mid w \in \{0, 1\}^*$ and every odd position of w is 1$\}$
(iii) $\{w \mid w$ describes a valid Sudoku puzzle$\}$. Here $\Sigma = \{1, 2, 3, 4, 5, 6, 7, 8, 9, \text{Blank}\}$.
(iv) $\{w \in \Sigma^* \mid n_a(w) \bmod 4 = 1\}$.
Here $\Sigma = \{a, b\}$, and $n_a(w)$ is number of a's in the string 'w'.

(A) (i), (ii), (iii), (iv) (B) (ii), (iii), (iv)
(C) (ii), (iv) only (D) (i) only

47. Consider a standard pipeline with 5-Stages: Fetch (F), Decode (D), Execute (X),
Memory (M), Write back (W)
This 5-Stage pipeline is converted into a 4-stage pipeline by combining two stages.

Match list-A with list-B by identifying which improvement is obtained by combining which stages of the pipeline:

List-A		List-B	
1.	Combining D and X stages	P	Reduction in branch miss prediction penalty
2.	Combining X and M stages	Q	Reduction in Bypassing path
3.	Combining M and W stages	R	Elimination of load-to-use stall cycle
4.	Combining F and D stages	S	No need of branch target Buffer

(A) 1–P, 2–Q, 3–R, 4–S
(B) 1–S, 2–Q, 3–R, 4–P
(C) 1–P, 2–R, 3–Q, 4–S
(D) 1–S, 2–R, 3–Q, 4–P

48. During the design of a cache memory there are two kinds of choices available:
(i) A direct mapped cache with a hit latency of 3 clock cycles.
(ii) A set-associative cache with a hit latency of 4 clock cycles
In either case the miss penalty is 10 cycles.
If the set-associative cache is selected and it needs to provide lower average latency than direct mapped cache with 15% miss rate, then the miss rate of set-associative cache must be (less than in percentage) ________.

49. Consider a pipelined processor $P1$ with a cycle time of 10 ns. A Benchmark program executed on this processor, exhibited an average CPI of 1.2, the program has 10% branch instructions and branch prediction scheme accuracy is 90%. Every branch mis prediction costs 3 cycles delay on $P1$.
Now a new pipelined processor $P2$ is designed with cycle time of 9 ns and an increase in pipeline depth compared to $P1$. On $P2$, the penalty for branch misprediction is 5 cycles. If the benchmark program executed on $P2$, then the average CPI of $P2$ is ________.

50. Consider the following statements:
S_1: A Relation with 2 attributes does not have partial dependency and transitive dependency.
S_2: Query Retrieval performance will get reduced, if a relation is undergone with more number of normalizations.
S_3: BCNF mainly focuses on the primary key.
Which of the above statements is/are TRUE?
(A) Only S_1 (B) S_1 and S_2
(C) S_2 and S_3 (D) S_1, S_2, S_3

51. Construct a B-Tree of order 3 with the key values as given:
5, 10, 15, 20, 25, 30, 35, 40, 45, 50
The number of nodes at the last level is ________.

52. Consider a serial schedule (S)

T_1	T_2
Read (X)	
$X = X * 10$	
Write (X)	
Read (Y)	
$Y = Y + 100$	
Write (Y)	
	Read (X)
	$X = X - 1000$
	Write (X)
	Read (Y)
	$Y = Y * 20$
	Write (Y)

Consider the non-serial schedules S_1, S_2, S_3 for the above schedule 'S'.
$S1$:

T_1	T_2
Read (X)	
$X = X * 10$	
Write (X)	
	Read (X)
	$X = X - 1000$
	Write (X)
Read (Y)	
$Y = Y + 100$	
Write (Y)	
	Read (Y)
	$Y = Y * 20$
	Write (Y)

$S2$:

T_1	T_2
Read (X)	
$X = X * 10$	
	Read (X)
	$X = X - 1000$
Write (X)	
	Write (X)
Read (Y)	
$Y = Y * 20$	
	Read (Y)
	$Y = Y * 20$
Write (Y)	
	Write (Y)

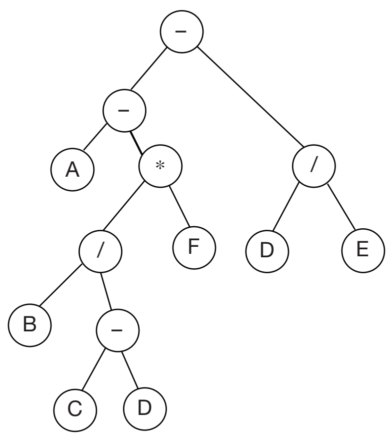
$S3$:

T_1	T_2
Read (X)	
$X = X * 10$	
Write (X)	
	Read (X)
	$X = X - 1000$
	Write (X)
Read (Y)	
$Y = Y + 100$	
	Read (Y)
	$Y = Y * 20$
Write (Y)	
	Write (Y)

Which of the above schedules (S_1, S_2, S_3) are view equivalent to serial schedule 'S'?

(A) Only S_1
(B) S_1 and S_2
(C) S_2 and S_3
(D) S_1, S_2, S_3

53. Consider the given expression tree:



Which of the following are correct Infix and Postfix expressions?

(A) $A - B/(C - D)*F - D/E$ and $ABCD/- F *- DE/-$
(B) $A/B - (C - D)*F - D/E$ and $ABCD -/F*- DE/-$
(C) $A - B/(C - D)*F - D/E$ and $ABCD -/F*- DE/-$
(D) $A - B/(C - D)*F/D - E$ and $ABCD -/F*- DE/-$

54. Consider an array containing 8 elements {36, 97, 44, 12, 58, 49, 11, 84}.
Implement three sorting algorithms on the above array: Bubble sort, Insertion sort, Selection sort.
Match the sorting algorithm with the intermediate result of an array after four passes or Iterations

Sorting algorithm	Intermediate result after 4-passes
(A) Bubble sort	(I) [11, 12, 36, 44, 49, 97, 58, 84]
(B) Selection sort	(II) [12, 36, 44, 97, 58, 49, 11, 84]
(C) Insertion sort	(III) [12, 36, 11, 44, 49, 58, 84, 97]
	(IV) [36, 11, 44, 12, 49, 58, 84, 97]
	(V) [11, 12, 36, 44, 58, 49, 97, 84]
	(VI) [12, 36, 44, 58, 97, 49, 11, 84]

(A) A–(I), B–(IV), C–(VI)
(B) A–(III), B–(IV), C–(VI)
(C) A–(I), B–(VI), C–(II)
(D) A–(III), B–(V), C–(II)

55. Number of labeled spanning trees of the complete bipartite graph $k_{3,3}$ is ________.

56. Consider the grammar:

$S \rightarrow abX$ {Print (0)}
$S \rightarrow c$ {Print (1)}
$X \rightarrow dS$ {Print (2)}

For this SDT construct, if the input string is "*a b d a b d c*", then the output is:

(A) 1 2 0 2 0
(B) 0 2 0 2 1
(C) 0 1 0 2 2
(D) 1 0 0 2 2

57. Consider the following grammar:

(P) $E \rightarrow T + E|T$
$T \rightarrow id$
(Q) $A \rightarrow (A)\,|a$
(R) $S \rightarrow Aba \mid Bb$
$A \rightarrow bAa \mid d$
$B \rightarrow d$

Which of the above grammar has both Shift-reduce and Reduce-Reduce Conflicts using LR(0) parsing algorithm?

(A) Only P
(B) Only Q
(C) Only R
(D) Both P, R

58. Match the following:

Group-A		Group-B	
P	Deadlock avoidance	1.	Cycle in the wait-for graph
Q	Deadlock prevention	2.	Safe state algorithm
R	Deadlock detection	3.	Allocate all the required resources before the start of process execution
S	Deadlock recovery	4.	Resource preemption
		5.	Bankers algorithm
		6.	Resource request algorithm

(A) P–2, Q–4, R–5, S–6
(B) P–5, Q–3, R–1, S–4
(C) P–5, Q–4, R–6, S–3
(D) P–2, Q–3, R–2, S–6

59. Which of the following statements is FALSE?

(A) Degree of multiprogramming in a system is independent of scheduling algorithm.
(B) Virtual memory size depends on the size of the address bus.
(C) Dynamic partition memory technique is affected with Internal fragmentation
(D) None of the above

60. A process has been allocated 4 page frames. Initially the page frames are empty. The process makes the following sequence of page references 1, 2, 3, 2, 1, 4, 6, 3, 4, 7, 3, 1, 2.
If optimal page replacement policy is used, then the number of page faults is ________.

61. Consider below table:

P–ID	Arrival time	Burst time	Priority
P_1	4	3	3
P_2	7	5	4(high)
P_3	1	6	1
P_4	2	5	2

If the processes are executed, using the scheduling algorithms–Round Robin with time slice 2 units (RR), preemptive shortest job first (P-SJF) Preemptive Priority scheduling (PP), then which of the following order specifies the correct order based on average response time (Ascending order)?

(A) RR, PP, P-SJF (B) P-SJF, RR, PP
(C) PP, RR, P-SJF (D) PR, P-SJF, PP

62. Consider the routine fun( ):

```
void fun(int x)
{
if (x > 0)
{
fun(x - 1);
printf("%d", x);
fun(x - 1);
}
}
```

When fun(4) is called, the number of function calls would be _________.

63. Suppose we have numbers between 1 and 1000 in a binary search tree and want to search for the number 364. Which of the following sequence could not be the sequence of nodes examined?

(A) 3, 253, 402, 399, 331, 345, 398, 364,
(B) 926, 203, 912, 241, 913, 246, 364
(C) 3, 400, 388, 220, 267, 383, 382, 380, 279, 364
(D) 936, 279, 346, 620, 347, 391, 359, 364

64. Consider a single linked list with '*n*' elements:

```
struct Node
{
struct Node *next;
int data;
} list;
Consider a routine fun():
struct Node *fun(list *head, int K)
{
struct node *P1, *P2;
P1 = P2 = head;
while ( I )
{
P2 = P2 → next;
if ( II )
return NULL;
K - -;
}
while ( III )
{
P1 = P1 → next;
P2 = P2 → next;
}
return P1;
}
```

Fill in the blanks, if fun ( ) gives the K^{th} node from the end of the list.

	I	II	III		I	II	III
(A)	P_1	P_2	K	(B)	$!P_2$	P_1	K
(C)	K	$!P_1$	P_2	(D)	$!P_2$	$P1$	K

65. A TCP machine is sending full windows of 65,535 bytes over a 1 Gbps channel that has 20 m sec one-way delay. What is the maximum throughput achievable?

(A) 1262645 bytes
(B) 1462625 bytes
(C) 1638375 bytes
(D) 1842625 bytes

Answer Keys

1. B	**2.** C	**3.** C	**4.** D	**5.** C	**6.** C	**7.** A	**8.** A	**9.** B	**10.** 6000
11. C	**12.** A	**13.** 0.5	**14.** 4,096	**15.** D	**16.** D	**17.** 42	**18.** D	**19.** B	**20.** A
21. C	**22.** A	**23.** D	**24.** 1	**25.** D	**26.** A	**27.** D	**28.** B	**29.** C	**30.** C
31. D	**32.** C	**33.** C	**34.** D	**35.** B	**36.** B	**37.** D	**38.** C	**39.** 0.18 to 0.20	
40. B	**41.** B	**42.** 25	**43.** D	**44.** 11010000		**45.** C	**46.** B	**47.** C	**48.** 5
49. 1.22	**50.** B	**51.** 5	**52.** A	**53.** C	**54.** D	**55.** 81	**56.** A	**57.** C	**58.** B
59. C	**60.** 7	**61.** C	**62.** 31	**63.** B	**64.** C	**65.** C			

HINTS AND EXPLANATIONS

1. The synonym of risque is lascivious or vulgar. Queasy means sickening and pompous means overdone or affected and have little to do with the headword. Choice (B)

2.

G	*O*	*O*	*G*	*L*	*E*
+1	–1	+1	–1	+1	–1
H	*N*	*P*	*F*	*M*	*D*

Similarly,

A	*P*	*P*	*L*	*E*
+1	–1	+1	–1	+1
B	*O*	*Q*	*K*	*F*

Choice (C)

3. The numbers a, b, c, d, e are in geometric progression.

$\therefore \quad \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \frac{e}{d}$ let each of these be k.

(i) $\frac{b^2}{a^2} = \frac{c^2}{b^2} = \frac{d^2}{c^2} = \frac{e^2}{d^2} = k^2$

$\therefore \quad a^2, b^2, c^2, d^2, e^2$ are in geometric progression.

(ii) The given terms need not be in geometric progression.

(iii) $\frac{3b}{3a} = \frac{3c}{3b} = \frac{3d}{3c} = \frac{3e}{3d} = k$

$3a$, $3b$, $3c$, $3d$, $3e$ are in geometric progression.
Only (i) and (iii) are in geometric progression.
Choice (C)

4. Utopia is a place where everything is perfect. Synagogue is a place of worship for Jews. Choice (D)

5. The pair conjunction "neither ... nor" always takes a plural verb with the plural subject being placed second. So "have" is apt. Choice (C)

6. $f(x) = 3 - |x|$ where $-2 \le x \le 2$,
$|a| = a$ when $a \ge 0$,
$= -a$ when $a < 0$,
$|x|$ ranges from 0 to 2.
$f(x)$ has the minimum and the maximum values when x has the maximum and the minimum values respectively.
Min $(f(x)) = 3 - 2 = 1$.
Max $(f(x)) = 3 - 0 = 3$. Choice (C)

7. The noun "machinery" is correct and it cannot be used in the plural with an "*s*". So the entire structure accompanying it has to be in the singular. Thus, "there is no machinery for ..." is apt. Choice (A)

8. Statement (B) is not true. The passage only states that he was a dictator "at heart", not a crowned and dreaded despotic ruler. Towards the end of the passage we are told of his beliefs regarding war. He came to believe that the conquest of the self was the greatest conquest. This DOES NOT mean he believed that men would shun violence and live peacefully. So (C) too is ruled out. (D) is ruled out as it is out of the text. Choice (A) is correct as per the first line of the passage. "... one of the ..." means not the only. Choice (A)

9.

Person	Designation
A	Head of R&D/Finance
B	Head of operations
C	Posted to work in all departments
D	Head of Finance/R&D
E	Head of Marketing

A valid assignment of heads to the domains can be.
A–Finance
B–Operations
D–R&D E–Marketing. Choice (B)

10. Total number of people using "Chuk–chuk for sure"

$= \frac{60}{360}(216000) = 36000$

Number of female customers of this company

$= \frac{1}{6}(36000) = 6000$ Ans: 6000

11. Given $f(x) = 5x^2 + \cos x$
The Taylor's series expansion of $f(x)$ about $x = a$ is:

$$f(x) = f(a) + 9x - a)f^1(a) + \frac{(x-a)^2}{2!}f^{11}(a) + \ldots\infty$$

$\therefore$ The third term is $\frac{(x-a)^2}{2!}f^{11}(a)$

Here $f(x) = 5x^2 + \cos x$ and $x = \pi$

$\Rightarrow \quad f^{11}(x) = 10 - \cos x$

$\therefore \quad f^{11}(a) = f^{11}(\pi) = 10 - \cos\pi = 11$

$\therefore$ The third term in the Taylor's series expansion of $f(x)$ about $x = \pi$ is $\frac{(x-\pi)^2}{2!} \times 11 = \frac{11}{2}(x-\pi)^2$

Choice (C)

12. Given, $AB = A \rightarrow (1)$
and $BA = B \rightarrow (2)$
From (1) $AB = A$
$\Rightarrow \quad (AB)A = A \times A$
$\Rightarrow \quad A(BA) = A^2$
$\Rightarrow \quad A(B) = A^2$ (from (2))
$\Rightarrow \quad A = A^2$ (from (1))
$\Rightarrow \quad A^2 = A \Rightarrow A$ is an idempotent matrix.
From (2), $BA = B$
$(BA)\,B = B \times B$
$\Rightarrow \quad B(AB) = B^2$
$\Rightarrow \quad B(A) = B^2$ (from (1))
$\Rightarrow \quad B = B^2$ (from (2))
$\Rightarrow \quad B^2 = B$
$\Rightarrow \quad B$ is an idempotent matrix. Choice (A)

13. Given the p.d.f of X is

$$f(x) = \begin{cases} ax(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

As $f(x)$ is a p.d.f of X, we have

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^1 a(x - x^2)\,dx = 1$$

$$\Rightarrow \quad a\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$$

$$\Rightarrow \quad a\left[\frac{1}{2} - \frac{1}{3}\right] = 1 \Rightarrow a\left(\frac{1}{6}\right) = 1$$

$$\Rightarrow \quad a = 6$$

$$\therefore \quad f(x) = \begin{cases} 6x(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

The mean of X is

$$E(X) = \int_{-\infty}^{\infty} xf(x)\,dx = \int_0^1 x(6x(1-x))\,dx$$

$$= 6\int_0^1 (x^2 - x^3)\,dx$$

$$= 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 6\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{6}{12}$$

$$\therefore \quad E(X) = \frac{1}{2}$$ Ans: 0.5

14. Given $S = \{k, l, m, n\}$

The number of ordered pairs in $S \times S = 16$

Of these 16 ordered pairs, the ordered pairs (k, k) (l, l) (m, m) and (n, n) must be present in every reflexive relation on S.

So, any reflexive relation on S contains these 4 ordered pairs along with any of the remaining 12 ordered pairs of $S \times S$.

$\therefore$ The maximum possible number of reflexive relations that can be defined on S

= The number of subsets of a set with 12 elements.

$= 2^{12} = 4{,}096$ Ans: 4,096

15. An induced subgraph H of a graph G is a subgraph of G that consists of all those edges of G which are present between every pair of vertices that are present in H.

Among the options given, graph G_4 is not an induced sub graph of G because in G there is an edge between the vertices a and d as well as an edge between c and g but those edges are not in G_4. Choice (D)

16. The maximum element in a min-heap will be resided in the leafs. The number of leaf's in heap of n-elements are $\left\lceil \frac{n}{2} \right\rceil$. So total number of comparisons required are $\left\lceil \frac{n}{2} \right\rceil - 1$. Choice (D)

17. The number of binary tree structures that can be formed with 'n' lables are $\left(\frac{{}^{2n}C_n}{n+1}\right)$ i.e., with 5 elements we can form $\left(\frac{{}^{10}C_5}{6}\right) = 42$ trees.

In these structures we can place the data elements, to get the in order traversal of the above order – PQRST.

Ans: 42

18. Choice (D)

19. Choice (B)

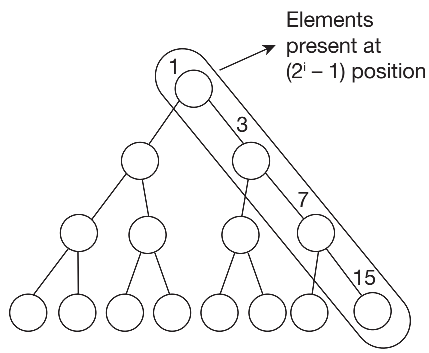
20. Follow (F) = {*, +, \$,)} Choice (A)

21. Preprocessing is optional. Symbolic names are associated with data or information.

Symbol table can be implemented using linear table, ordered list, tree and Hash table. Choice (C)

22. AVL Tree is a balanced binary search tree. The elements positioned at $(2^i - 1)$ places is nothing but right skewed tree elements [elements which are right child from root node onwards].

i.e., consider an example



Number of nodes in an AVL-Tree of 'n' nodes has $(\log n)$ elements for above scenario.

To search an element in $(\log n)$ elements using binary search, it takes $O(\log(\log n))$ time. Choice (A)

23. If a recurrence equation,

$T(n) = aT(n - b) + O(n^k)$, with $a > 0$, $b \ge 1$, $k \ge 0$.

The time complexity of $T(n)$ with $a > 1$, takes $O(n^k \cdot a^{n/2})$time.

So for the recurrence equation,

$T(n) = 3T(n-1) + O(n^2)$ takes

$O(n^2 \cdot 3^{n/2}) \gg O(3^n \cdot n^2)$. Choice (D)

24. The key is (XQ) Ans: 1

25. **<u>Option (A):</u>** Not valid

The Aggregate function SUM will work only on the numeric columns.

<u>Option (B):</u> Not valid

The Aggregate function will work on only single column and produce single column output.

<u>Option (C)</u> Not valid
If non-aggregate column associated with the aggregate column then that query must be associated with GROUP BY clause. Choice (D)

26. Choice (A)

27. An instruction cycle consists of instruction fetch, followed by 0 or more operand fetch, followed by 0 or more operand stores followed by an interrupt check. Choice (D)

28. $$\text{CPI} = \frac{4500\times1 + 3200\times2 + 1500\times2 + 800\times2}{10000}$$

$$= \frac{4500 + 6400 + 3000 + 1600}{10000} = \frac{15500}{10000} = 1.55$$

$$\text{MIPS} = \frac{\text{Clock rate}}{\text{CPI}\times10^6} = \frac{20\times10^6}{1.55\times10^6} = 12.9$$ Choice (B)

29. Instruction fetch, operand fetch, operand store requires some exchange between processor and either memory or I/O module. Remaining are done inside the processor. Choice (C)

30. To remove ε-Transition we have to add new transitions; we can reach q_2 from q_1 with 'ε' and from q_2 to q_1 with 1. i.e., q_1 will reach itself with input 1.
Similarly, q_0 can reach q_2 with input '0'. So a transition from q_0 to q_2 will be added. Choice (C)

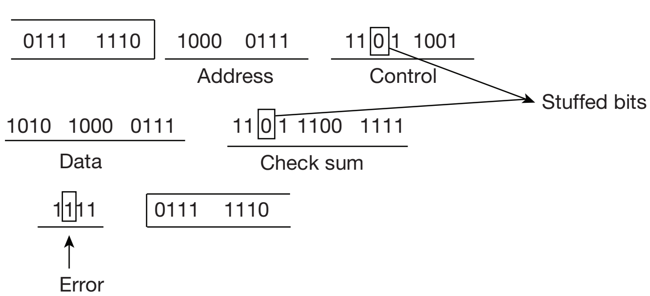
31. T must halt with Regular expression R. Choice (D)

32. The Host layers in OSI model are Transport, Session, Presentation and Application Layers. Choice (C)

33. The HDLC frame format is:

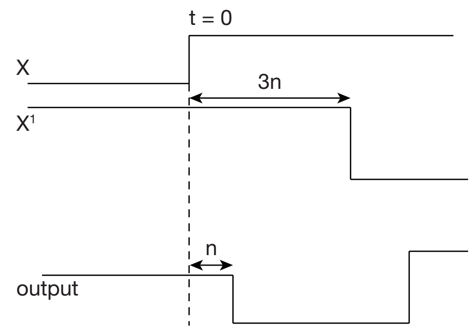
	8-bits	8-bits	≥0	16-bits	
0111 1110	Address	Control	Data	Check sum	0111 1110

The start and end of the frame are 0111 1110.
To differentiate start & end flags from data, a bit '1' is stuffed after 5 1's.



There is single error in given frame. Choice (C)

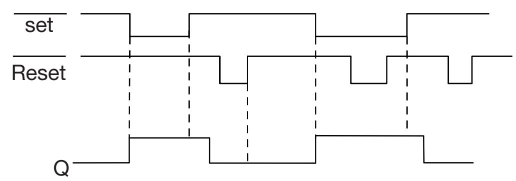
34. Consider the delay of each NAND gate is 'n' seconds. At $t = 0$, the logic LOW switched to logic HIGH
The wave forms will be as shown below.
X is input of NAND gate, other input will be X^1 which is complemented form of X, but delayed by $3n$. (n – delay of each NAND gate)



HIGH to LOW to HIGH Choice (D)

35. Given Latch is a $\bar{S}\bar{R}$ NAND gate latch.

$\overline{\text{Set}}$	$\overline{\text{Reset}}$	Q_{n+1}	Condition
0	0	X	($Q_{n+1} = 1, \bar{Q}_{n+1} = 1$)
0	1	1	Set
1	0	0	Reset
1	1	Q_n	Same State



Choice (B)

36. Given, 1, 4 and 5 are the eigenvalues of A.
∴ The eigenvalues of $A^2 - 5A + 6I_3$ are $1^2 - 5\times1 + 6 = 2$, $4^2 - 5\times4 + 6 = 2$ and $5^2 - 5\times5 + 6 = 6$.
Hence, the eigenvalue of $A^2 - 5A + 6I_3$ are 2, 2 and 6
∴ $A^2 - 5A + 6I_3$ has two distinct eigenvalues. Choice (B)

37. <u>Consider option (A):</u>
$(p \wedge q) \to [(p \vee q) \vee (\rceil p \wedge \rceil q)]$
$\Leftrightarrow (p \wedge q) \to [(p \vee q) \vee \rceil(p \vee q)]$
$(\because \rceil(A \vee B) \Leftrightarrow (\rceil A \wedge \rceil B)$
$\Leftrightarrow \rceil(p \wedge q) \vee T$
$(\because A \to B \Leftrightarrow \rceil A \vee B$ and $A \vee \rceil A \Leftrightarrow T$ = tautology)
$\Leftrightarrow T$ = Tautology ($\because A \vee T \Leftrightarrow T$)
<u>Consider (B):</u>
$[p \wedge (q \vee r)] \vee [\rceil p \vee (\rceil q \wedge \rceil r)]$
$\Leftrightarrow [p \wedge (q \vee r)] \vee [\rceil p \vee \rceil(q \vee r)]$
$(\because \rceil(A \vee B) \Leftrightarrow (\rceil A \wedge \rceil B))$
$\Leftrightarrow [p \wedge (q \vee r)] \vee \rceil[p \wedge (q \vee r)]$
$\Leftrightarrow T$ = Tautology ($\because A \vee \rceil A \Leftrightarrow T$)
<u>Consider (C):</u>
$[(p \vee q) \to r] \leftrightarrow [\rceil r \to \rceil(p \vee q)]$
$\Leftrightarrow [(p \vee q) \to r] \leftrightarrow [(p \vee q) \to r]$
$(\because A \to B \Leftrightarrow \rceil B \to \rceil A)$
$\Leftrightarrow T$ = Tautology ($\because A \leftrightarrow A \Leftrightarrow T$)

Consider (D):
$[p \to (q \to r)] \wedge [\rceil p \vee q \vee r]$
When p is true, q is true and r is false
$p \to (q \to r)$ is false.
$\therefore$ $[p \to (q \to r)] \wedge [\rceil p \vee q \vee r]$ is false.
Hence $[p \to (q \to r)] \wedge [\rceil p \vee q \vee r]$ is NOT a tautology. Choice (D)

38. Let X_1, X_2 and X_3 denote the number appeared on the die in the first, second and third rolls respectively.
$\therefore$ X_1, X_2 and X_3 take the values 1, 2, 3, 4, 5 and 6 with equal probability $\frac{1}{6}$.
$\therefore$ $E(X_1) = E(X_2) = E(X_3)$

$$= 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6} = \frac{7}{2}$$

$\therefore$ The expected value of the product of numbers that appear on the die when rolled thrice
$= E(X_1\, X_2\, X_3) = E(X_1) \cdot E(X_2) \cdot E(X_3)$

$$= \frac{7}{2}\times\frac{7}{2}\times\frac{7}{2} = \frac{343}{8}$$ Choice (C)

39. Given $f(x) = (x-1)^2\, e^{-x}$
$f'(x) = 2(x-1)e^{-x} - (x-1)^2\, e^{-x}$
$f'(x) = (x-1)\,(3-x)e^{-x} = (4x - x^2 - 3)e^{-x}$
$f'(x) = 0 \Rightarrow (x-1)\,(3-x)e^{-x} = 0$
$\Rightarrow$ $(x-1)\,(3-x) = 0$
$\Rightarrow$ $x = 1; x = 3$
And $f''(x) = (4-2x)e^{-x} - (4x - x^2 - 3)e^{-x}$
$= (x^2 - 6x + 7)\, e^{-x}$

At $x = 1, f''(x) = 2e^{-1} = \frac{2}{e} > 0$

At $x = 3; f''(x) = -2e^{-3} = \frac{-2}{e^3} < 0$

$\therefore$ $f(x)$ has a local maximum at $x = 3$
$\therefore$ The maximum value of

$f(x)$ at $x = 3$ is $f(3) = (3-1)^2\, e^{-3} = \frac{4}{e^3}$

$= 0.1991$

Ans: 0.18 to 0.20

40. Given $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

And $G = \{A, B, C, I\}$
As the binary operation is "matrix multiplication",

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity element of G

$\therefore$ I can't be a generator of G.

Consider $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\Rightarrow$ $S(A) = 2$
$\therefore$ A can't be a generator of G, because the order of the generator of a cyclic group is same as that of the order of G.
Also, one can observe that
$B^2 = A$, $B^3 = C$, and $B^4 = I$; $C^2 = A$, $C^3 = B$ and $C^4 = I$
Hence, B and C are the generators of the cyclic group G under matrix multiplication.

Choice (B)

41. $F(x, y, z) = \Sigma m(0, 6, 7) + \phi(1, 4)$

X \ YZ	00	01	11	10
0	1	D		
1	D		1	1

$F = \bar{X}\bar{Y} + XY = X \odot Y$
F is 2 input X NOR gate, so we need 4 NOR gates.

Choice (B)

42. Propagation delay = 100 ms
Round-trip delay = 200 ms
File size = 10 MB
Frame size = 1 KB

Frames required to send the file $= \frac{10\times10^6}{1\times10^3} = 10000$

Time to send 10000 frames

$= \frac{10000}{5} * 200$ ms

$= 2000 * 200$ ms $= 400$ seconds

Throughput of $A = \frac{\text{File Size}}{\text{Time Required to send file}}$

$= \frac{10\text{ MB}}{400\text{ s}} = 0.025$ MBps $= 25$ KBps Ans : 25

43. (i) Even if P_1 calls close(), the OS will transmit data to P_2.
(ii) The OS will send FIN (finish) to P_1
(iii) The receive () function will be called multiple times to receive the entire data.
(iv) P_2's receive will return a zero to indicate an end of stream. Choice (D)

44. Given message can be written as:
1101 1001 0101 1001
CRC generator is 1010 01001
We need to put 8 zeros next to the message and perform modulus division with given CRC generator.

```
                                  11  1001  1101  0100
              -----------------------------------------
101001001 )  1101  1001  0101  1001  1101  0100
             1010  0100  1
             -----------------------------------------
             0111  1101  11
              101  0010  01
             -----------------------------------------
              010  1111  100
               10  1001  001
             -----------------------------------------
               00  0110  1011
                0  0000  0000
             -----------------------------------------
                   0110  1011  1
                   0000  0000  0
             -----------------------------------------
                    110  1011  10
                    101  0010  01
             -----------------------------------------
                     11  1001  110
                     10  1001  001
             -----------------------------------------
                      1  0000  1111
                      1  0100  1001
             -----------------------------------------
                         0100  0110  0
                         0000  0000  0
             -----------------------------------------
                          100  0110  00
                          101  0010  01
             -----------------------------------------
                           01  0100  010
                           00  0000  000
             -----------------------------------------
                            1  0100  0100
                            1  0100  1001
             -----------------------------------------
                               0000  1101  0
                               0000  0000  0
             -----------------------------------------
                                000  1101  00
                                000  0000  00
             -----------------------------------------
                                 00  1101  000
                                 00  0000  000
             -----------------------------------------
                                  0  1101  0000
                                  0  0000  0000
             -----------------------------------------
                                     1101  0000
             -----------------------------------------
```

$\therefore$ The check sum that will be placed after the message is 11010000 Ans : 11010000.

45. Given turing machine writes a Blank in first 'a' position and traverses next a's, b's and #'s. Again replaces two a's with #'s and traverses next a's, b's and #'s; Again replaces 3 a's with #'s. So the language accepted is $\{a^n\, b\, a^{2n}\, b\, a^{3n} \mid n \geq 0\}$ Choice (C)

46. (i) Regular languages do not count prime number of 1's (by using its limited storage).
(ii) We can design a FA which accepts the strings such that every odd position is 1. So it is regular.
(iii) Sudoku is finite. So it is regular.
(iv) We can design a FA, in which number of a's is 1, 5, 9, 13 etc. so (iv) is regular. Choice (B)

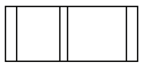
47. Combining D and X stages reduces miss prediction penalty, (Penalty reduce from 2 to 1). Combining X and M stages eliminates load-to-use stall cycle. (No stall to use a load value).
Combining M & W stages, will reduce By Pass Path.
By Combining F & D stages, BTB is not required to determine the branch instruction. Choice (C)

48. Average latency of direct mapped cache with 15% miss rate
$= (3 + 0.15 * 10) = 3 + 1.5 = 4.5$ cycles
Average latency of set associative cache with miss rate 'x' is $(4 + x * 10) = 4.5$
$x * 10 = 0.5$
$\Rightarrow \quad x = 0.05 = 5\%$ Ans: 5

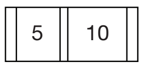
49. CPI on $P1 = 1.2$
CPI on $P2 = 1.2 + 0.1 * 0.1 * (5 - 3)$
$= 1.2 + 0.01 * 2$
$= 1.2 + 0.02 = 1.22$
($\therefore$ 10% of branch instructions, 10% mis-predictions and 5 cycles penalty). Ans: 1.22

50. BCNF mainly focuses on the candidate keys.
Choice (B)

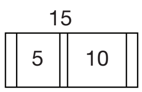
51.

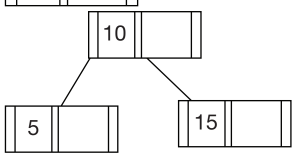


Insert: 5, 10

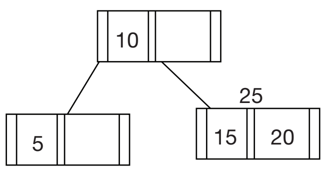


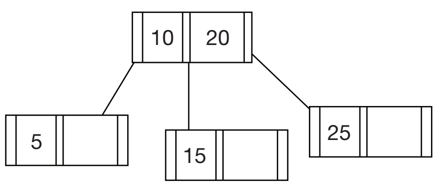
Insert : 15



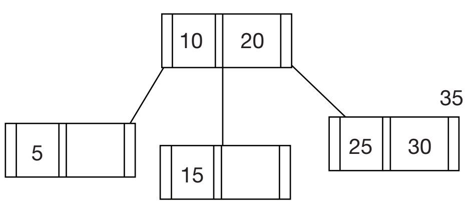


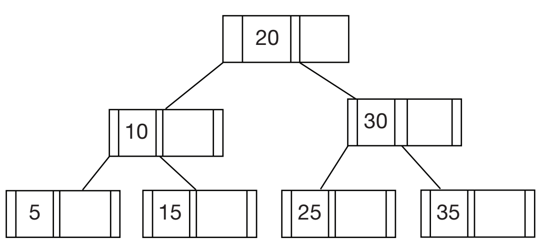
Insert: 20, 25



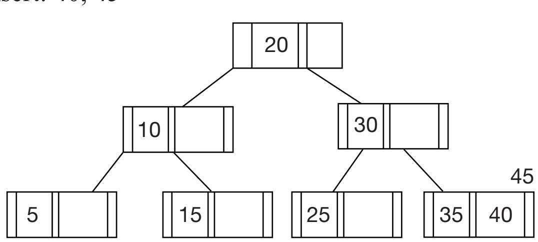


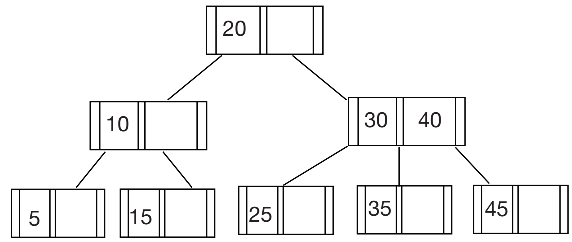
Insert: 30, 35



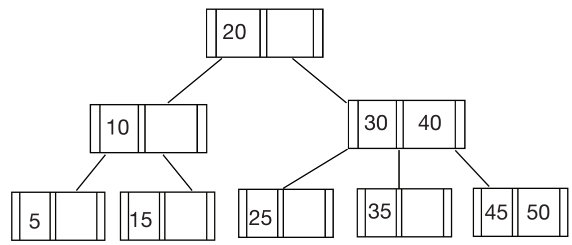


Insert: 40, 45





Insert: 50



Number of nodes at last level are 5. Ans: 5

52. Choice (A)

53. Post-fix order = Traverse left sub trees, Right sub tree and then Root
Infix order = Traverse left sub tree, Root and then Right sub tree Choice (C)

54. The input is 36, 97, 44, 12, 58, 49, 11, 84
<u>Bubble sort:</u>
<u>1st pass</u>
<u>36, 97</u> 44, 12, 58, 49, 11, 84
36, <u>97, 44</u>, 12, 58, 49, 11, 84
36, 44, <u>97, 12,</u> 58, 49, 11, 84
36, 44, 12, <u>97, 58,</u> 49, 11, 84
36, 44, 12, 58, <u>97, 49</u>, 11, 84
36, 44, 12, 58, 49, <u>97, 11</u>, 84
36, 44, 12, 58, 49, 11, <u>97, 84</u>
36, 44, 12, 58, 49, 11, 84, <u>97</u>
Similarly, 2^{nd} pass results
36, 12, 44, 49, 11, 58, <u>84, 97</u>
<u>3^{rd} pass results</u>
12, 36, 44, 11, 49, <u>58, 84, 97</u>
<u>4^{th} pass results</u>
12, 36, 11, 44, <u>49, 58, 84, 97</u>
<u>Selection sort:</u>

<u>36</u>, 97, 44, 12, 58, 49, 11, 84
1^{st} pass results in

Swap
11, 97, 44, <u>12</u>, 58, 49, 36, 84
2^{nd} pass results in

Swap
11, 12, <u>44</u>, 97, 58, 49, <u>36</u>, 84
3^{rd} pass results in

11, 12, 36, <u>97</u>, 58, 49, <u>44</u>, 84

4^{th} pass results in
11, 12, 36, 44, 58, 49, 97, 84

Insertion sort

1^{st} pass

$\boxed{36}$, 97, 44, 12, 58, 49, 11, 84

2^{nd} pass

$\boxed{36, 97}$, 44, 12, 58, 49, 11, 84

3^{rd} pass

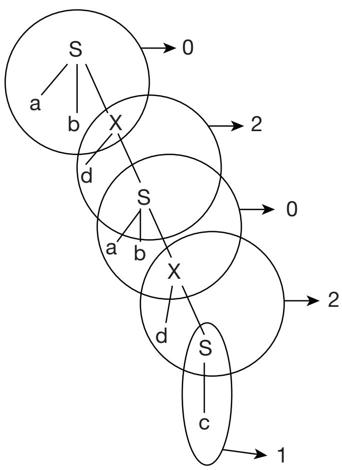
$\boxed{36, 44, 97}$, 12, 58, 49, 11, 84

4^{th} pass

$\boxed{12, 36, 44, 97}$, 58, 49, 11, 84 Choice (D)

55. Number of labeled spanning trees of the complete bipartite graph k_{mn} is $m^{n-1} \cdot n^{m-1}$.
For $k_{3,3}$, the number of labeled spanning trees will be $3^{3-1} \cdot 3^{3-1} = 81$. Ans: 81

56. Input string *a b d a b d c*



The output is 1 2 0 2 0 Choice (A)

57. (P) Has SR conflict
(Q) Q is a LR(0) grammar
(R) Has both RR and SR conflicts Choice (C)

58. Choice (B)

59. Choice (C)

60.

4	4	4	2
3	3	3	3
2	6	7	7
1	1	1	1
4 PF	1 PF	1 PF	1 PF

Ans.: 7

61. Preemptive SJF

//////	P_3	P_3	P_3	P_3	P_1	P_4	P_2

0 1 2 3 4 7 10 15 20

$$\text{Average response time} = \frac{7+15+1+10}{4} = 8.25$$

Preemptive priority scheduling:

//////	P_3	P_4	P_1	P_2

0 1 2 4 7

$$\text{Average response time} = \frac{4+7+1+2}{4} = 3.5$$

Round Robin Scheduling

//////	P_3	P_4	P_3	P_1	P_4	P_2	

0 1 3 5 7 9 11 13

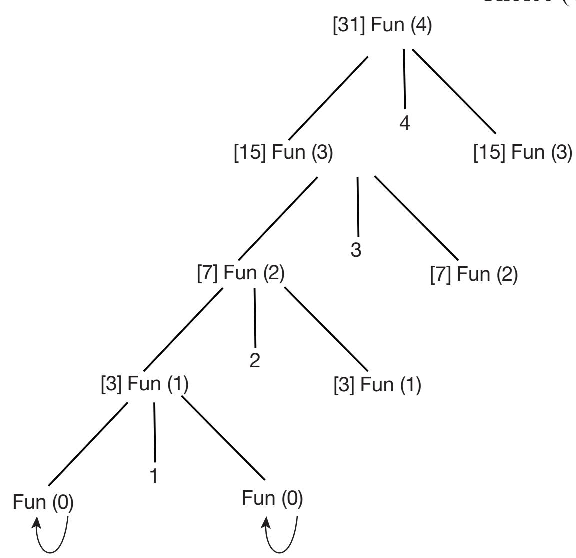
Ready Queue:

$\cancel{P_4}, \cancel{P_3}, \cancel{P_1}, \cancel{P_4}, P_2, P_3, P_1, P_4$

$$\text{Average response time} = \frac{7+11+1+3}{4} = 5.5$$

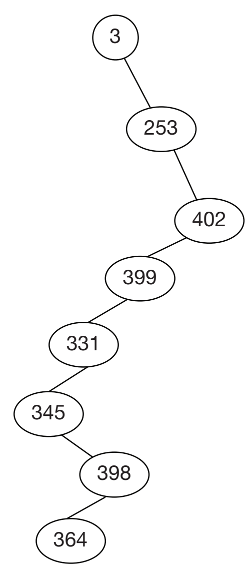
Choice (C)

62.

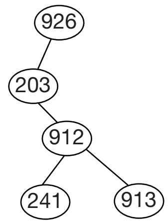


[*x*] specifies number of function calls of the functions.
Ans: 31

63. Option (A)



Option (B)



913 is violating node. Choice (B)

64. Choice (C)

65. One window can be sent for every 40 m sec

1 window ——— 40×10^{-3} sec

x ——————— 1 sec

$$x = \frac{1}{40 \times 10^{-3}} = \frac{10^3}{40}$$

$$= \frac{1000}{40} = 25$$

Per one second 25 windows can be sent, maximum rate of ($25 \times 65, 535$) bytes can be sent in a second. i.e., 1638375. Choice (C)

Mock Test 3

Number of Questions: 65 **Total Marks: 100**

Wrong answer for *MCQ will result in negative marks*, *(–1/3)* for *1 Mark Questions and (–2/3)* for *2 Marks Question.*

General Aptitude

Number of Questions: 10 **Section Marks: 15**

***Directions for question 1*:** Select the pair that best expresses a relationship similar to that expressed in the pair:

1. Road : Footpath
(A) Drawing room : Kitchen
(B) River : Riverbank
(C) Box : Lock
(D) Window : Shutter

***Directions for questions 2 and 3*:** Select the correct alternative from the given choices.

2. What is the total weight of 25 discs?
Statements:
I. Two-fifth of the weight of a disc is 13 kg.
II. The weights of no two discs are equal.
(A) Statement I alone is sufficient.
(B) Statement II alone is sufficient.
(C) Combining I and II sufficient.
(D) Both statements I and II together are not sufficient.

3. A function $f(x)$ is linear and has a value of 50 at $x = -4$, and a value of 6 at $x = 7$. The value of the function at $x = 8$ is ________.

***Directions for question* 4:** Fill in the blank with the correct idiom or phrase:

4. An upholder of the truth will never hesitate _________ .
(A) to let the grass grow under one's feet
(B) to see red
(C) to throw in the towel
(D) to call a spade a spade

***Directions for question 5*:** Select the correct alternative from the given choices.

5. The five corporate offices of *HUL* are located in five metros namely *A*, *B*, *C*, *D* and *E*. *E* is 5 km to the North-east of *A*, and is 2 kms to the South-east of *B*. *D* is 5 km to the North-east of *B*. *DE* = _______.
(A) 6.92 km (B) 29 km
(C) 47.27 km (D) 5.39 km

***Directions for question* 6:** Out of the four sentences, select the most suitable sentence with respect to grammar and usage:

6. (A) In the olden days, people used to worship the nature.
(B) In the olden days, people used to be worshipping nature.
(C) In the olden days, people worshipped nature.
(D) In the olden days, people used to be worshipping the nature.

Directions for question 7: Read the following paragraph and choose the correct statement:

7. One can understand, although one cannot excuse, a frightened person misbehaving, even though there was no real reason for his fright. But what amazed and angered India was the contemptuous justification of the deed when General Dyer, who had been responsible for the firing at Amritsar, and his subsequent barbarous neglect of the thousands of wounded. "That was none of my business", he had said. Some people in England and in the British government mildly criticized Dyer, but the general attitude of the British people was displayed in a debate at the House of Lords, in which praise was showered on him. All this fed the flame of wrath in India, and a great bitterness rose all over the country.
(A) General Dyer is an example of a frightened person misbehaving.
(B) The general attitude of the British people was displayed in the fact that the victims of the massacre received a fair trial.
(C) When the British government saw a great movement uprising in India, their fears grew.
(D) General Dyer's actions can neither be understood nor excused.

Directions for questions 8: The following question is based on a short argument, a set of statements, or a plan of action. For each question, select the best answer from the given choices.

8. The coolant Freon used in refrigerators was found to damage the ozone layer of the earth. Hence an urgent need was felt to substitute Freon with some other coolant which will not damage the ozone layer.
Which of the following can be a direct inference from the above statements?
(A) A coolant cheaper than Freon is available for use in the refrigerator.
(B) Coolants which do not have any damaging effects are available for use in the refrigerators.
(C) The ozone layer is on the verge of extinction.
(D) Preserving the ozone layer intact is essential for the inhabitants of the earth.

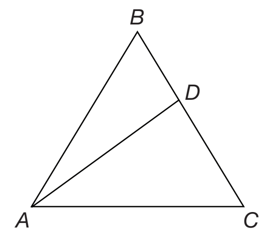
Directions for question 9: In the following question, the first and the last sentence of a passage are in order and numbered 1 and 6. The rest of the passage is split into 4 parts and numbered 2, 3, 4 and 5. These 4 parts are not arranged in the proper order. Read the sentences and arrange them in a logical sequence to make a passage and choose the correct sequence from the given options.

9. 1. Upon the same tree there are two birds of beautiful plumage, most friendly to each other.
2. This is the picture of the human soul.
3. One of the birds is eating fruits noisily while the other is sitting calm and silent without eating.
4. But the other one on top is calm and majestic.
5. The one on the lower branch is eating sweet and bitter fruits and is becoming sad and happy by turns.
6. Man is eating sweet and bitter fruits of this life, pursuing gold, sensory pleasures and the vanities of life so he is immersed in sorrow.

(A) 2, 4, 5, 3 (B) 3, 5, 4, 2
(C) 3, 4, 5, 2 (D) 5, 4, 3, 2

Directions for question 10: Select the correct alternative from the given choices.

10.



In triangle ABC, AD is the angle bisector of $\angle BAC$. $\angle CAD = 60°$ $AB = 10$ cm and $CA = 12$ cm. Find the length of AD.

(A) 5 cm (B) 5.45 cm
(C) 4.55 cm (D) 4.03 cm

Computer Science Engineering

Number of Questions: 55 **Section Marks: 85**

Directions for questions 11 to 65: Select the correct alternative from the given choices.

11. If 10 apples are to be distributed among Mahesh, Naresh and Ramesh, then the probability that Mahesh and Naresh together get exactly 7 apples is ______.

(A) $\left(\frac{2}{3}\right)^{10}$ (B) $15\times\left(\frac{2}{3}\right)^{10}$
(C) $5\times\left(\frac{2}{3}\right)^{10}$ (D) $3\times\left(\frac{2}{3}\right)^{10}$

12. If p and q are any two statement variables and T_o and F_o denote tautology and contradiction respectively, then which of the following is NOT equivalent to p?

(A) $p \wedge (p \vee q)$ (B) $p \wedge T_o$
(C) $p \wedge F_o$ (D) $p \vee (p \wedge q)$

13. The value of the definite integral $\int_1^5 \frac{\sqrt{x+5}}{\sqrt{x+5}+\sqrt{11-x}}\,dx$ is ______.

14. For a non-empty set L, if $(L, *, \oplus)$ is a lattice with $*$ and $\oplus$ as binary operations on L and if $a, b, c \in L$, then which of the following NEED NOT be true?

(A) $a * b = b * a$
(B) $a \oplus (b \oplus c) = (a \oplus b) \oplus c$
(C) $a * (a \oplus b) = a$
(D) $a \oplus (b * c) = (a \oplus b) * (a \oplus c)$

15. Consider two automobile companies Alpha Motors and Beta Motors that manufacture the bikes "Zigma" and "Harze" respectively. Zigma comes in three models, six colours, four engine sizes and two transmission types where as Harze comes in four models, five colours, three engine sizes and two transmission types. If Rakesh wants to buy a bike that is either a Zigma or a Harze, then different choices for him is the number of ______.

16. What is the number of distinct binary trees with 3 nodes (labelled as A, B, C) when traversed in post-order gives the sequence B, C, A?

(A) 2 (B) 3
(C) 4 (D) 5

17. What is the time complexity of the following Recurrence relation?

$$T(n) = \begin{cases} 2 & \text{if } n = 1 \\ 3T\left(\frac{n}{4}\right) & \text{if } n > 1 \end{cases}$$

(A) $\theta(n)$
(B) $\theta(\log n)$
(C) $\theta(n \log n)$
(D) $\theta(n^{3/4})$

18. Consider the following:

I. Multi-valued attributes must be represented by separate relations.
II. Composite attributes are represented only by their simple component attributes in the basic relational model.

Which of the following is TRUE?

(A) I is related to 1NF
(B) II is related to 2NF
(C) I and II are related to 1NF
(D) I is related to 2NF

19. A transaction T_1 updates item A and then fails before completion, so the system must change A back to its original value. Before it can do so, Transaction T_2 reads the temporary value of A, which will not be recorded permanently in the database because of failure of T_1. The problem described above is _______.
(A) The lost update problem
(B) Dirty Read problem
(C) The Incorrect summary problem
(D) Unrepeatable read problem

20. Consider a disk with following specifications:
Block size = 512 bytes
Inter block gap size = 64 bytes
Blocks per track = 20
Tracks per surface = 400
What is the total capacity of a track?
(A) 10240 bytes
(B) 1280 bytes
(C) 11520 bytes
(D) 11250 bytes

21. Consider an m-way set-associative cache with 's' number of slots (or blocks), with a block size of 'b' bytes. The main memory address has n-bits. Then which of the following gives the number of tag bits in the address?
(A) $n - \log(s/m) - \log(b/m)$
(B) $n - \log b$
(C) $n - \log s - \log b$
(D) $n - \log(s/m) - \log b$

22. The bus cycle time of a 32-bit micro-processor is same as that of a 16-bit microprocessor. The instructions and operands mix has 20% of 32-bit long instructions, 50% of 16-bit long and 30% of 8-bit long. The percentage of improvement achieved by fetching these instructions and operands with 32-bit microprocessor over 16-bit micro processor (specify two places after decimal point) is _______.

23. Consider the set of all words over the alphabet $\{a, b, c\}$ where the number of b's is not divisible by 3 or 5 and no 'c' appears after 'a'. This language is:
(A) Regular
(B) Context-free but not regular
(C) Recursive but not context-free
(D) Recursively enumerable but not Recursive

24. Let L_1 and L_2 are languages over an alphabet Σ such that $L_1 \subseteq L_2$. Then which of the following is always TRUE?
I. If L_2 is regular, then L_1 must be regular.
II. If L_1 is regular, then L_2 must be regular.
III. Either both L_1 and L_2 are regular or L_1, L_2 are Not Regular.
(A) I only (B) II only
(C) I, III only (D) None of these

25. Consider the CRC generator function:
$x^{10} + x^9 + x^6 + x^3 + 1$
The (maximum) number of bits in the resulting frame check sequence will be ______.

26. Consider an Ethernet wire which has a propagation speed of 2×10^8 m/s. If the size of network is 1000 m and data transmission rate is 100 Mbits/sec, then the minimum packet size needed to detect collision is ______.

27. Consider the code fragment:
```
i = 0 ;  j = 0 ;  k = 0 ;
for (x =1; x < = 100;  x = x + 1)
{
k = k + x ;
if ((x % 2) = = 0)
{
i = i + x;
}
else
{
j = j + x;
}
}
```
The value of 'k' at the end of the loop will be:
(A) $k < i + j$ (B) $k > i + j$
(C) $k = i + j$ (D) none of the above

28. Consider a complete graph 'G' of 8 vertices. Simple path in a graph is one in which no vertex is repeated. Let x, y, z be 3 distinct vertices in 'G'. The simple paths from x to y going through 'Z' will be ______.

29. Consider a lower triangular matrix $A[10][10]$ (array indices starts from 1), if the elements are stored in Row major order and each data element takes 1 byte of storage. If the base address is 1000, then the address of $A[8][3]$ is _______.

30. Consider the logical address space of 16 pages of 1024 bytes each, mapped onto a physical memory of 64 frames. How many bits are there in logical address?
(A) 4 (B) 10
(C) 14 (D) 6

31. Consider the page references 1, 3, 5, 6, 3, 1, 3, 5
Assume that main memory can accommodate 3 pages and the main memory already has the pages 1 and 3 with page 1 having been brought earlier than page 3. If LRU algorithm is used, then number of page faults that occur would be ______.

32. Consider the following three address code:
(1) $x = 1$ (2) $y = 1$
(3) if $x > i$ goto (9) (4) $y = y * x$
(5) $y = 10$ (6) $t = y * x$
(7) $x = t$ (8) goto (3)
(9) END
The number of basic blocks for above code is ______.

33. Which of the following statement is FALSE?

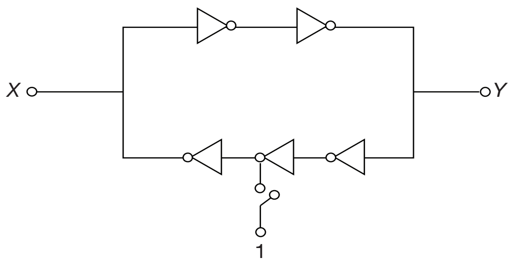
(A) In L-Attributed Syntax Directed Translations (SDT) attributes are evaluated using Depth first Search (Right to left) process.

(B) Every L-attribute can be converted into S-attribute.

(C) In S-attributed SDT, attributes are evaluated during Top down (or) Bottom up parsing.

(D) None of the above

34. In the circuit shown below, the switch is momentarily closed and then opened. Assuming the logic gates to have equal non-zero delay, at steady state, the logic states of X and Y are:



(A) X is latched to 0, Y is latched to 0

(B) X is latched to 0, Y toggles continuously

(C) X toggles continuously, Y is latched to 0

(D) X and Y both toggle continuously

35. A semi-conductor RAM has 16 bit address register and an 8 bit data register. The total number of bits in the memory is:

(A) 1,024 bits

(B) 4,096 bits

(C) 5,24,288 bits

(D) 10,48,576 bits

36. If $P = \begin{bmatrix} 2 & 131 & -243 & 566 \\ 0 & -2i & 174 & -237 \\ 0 & 0 & 2i & 0 \\ 0 & 0 & -713 & -2 \end{bmatrix}$ then which of the following is equal to $16P^{-1}$, where P^{-1} is the inverse of the matrix P?

(A) P^2 (B) $P^2 + 16P$

(C) P^3 (D) $P^3 + 16P^2 + P$

37. In a PSU (Public Sector Undertaking), if an employee is selected at random, then

(i) Probability that the employee has a Two Wheeler (TW) or a Four Wheeler (FW) is $\frac{7}{10}$

(ii) Probability that the employee has both a TW and a FW is $\frac{2}{5}$ and

(iii) Probability that the employee has a TW given that the employee has a FW is $\frac{2}{3}$.

Then the probability that the randomly selected employee has a TW is _______.

(A) $\frac{3}{5}$ (B) $\frac{1}{2}$

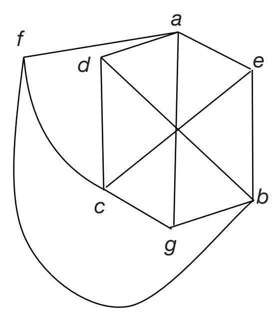
(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

38. In a men's hostel of a central university, out of 200 students who can speak atleast one of the three languages viz., Hindi, Punjabi and Gujarati, 120 students can speak Hindi, 35 students can speak both Hindi and Punjabi and 40 students can speak both Hindi and Gujarati but can't speak Punjabi. Then the number of students who can speak only Hindi is _______.

(A) 45 (B) 30

(C) 40 (D) 25

39. The chromatic number of the following graph G is _______.



G

40. Which of the following two statements is/are TRUE?

P: If $f(x)$ is continuous in $[a, b]$ then $f(x)$ assumes every value between $f(a)$ and $f(b)$

Q: If a function $f(x)$ assumes every value between $f(a)$ and $f(b)$, then $f(x)$ is continuous in $[a, b]$.

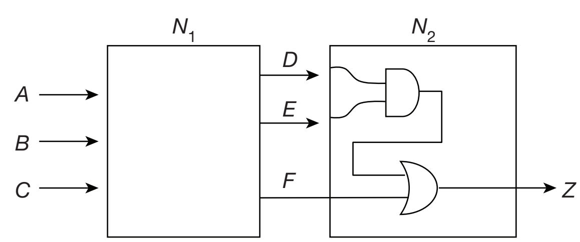
(A) P only

(B) Q only

(C) Both P and Q

(D) Neither P nor Q

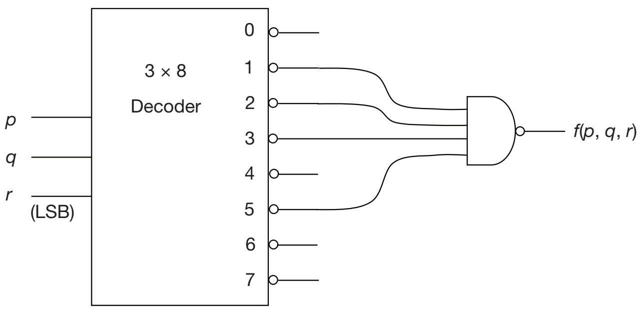
41. A combinational circuit is divided into two sub sections N_1 and N_2 as shown. The truth table of N_1 is given. Assume that the input combinations ABC = 101, and ABC = 001 never occur. Find the output expression for Z.



A	B	C	D	E	F
0	0	0	0	0	1
0	0	1	1	0	0
0	1	0	1	1	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	1	0	1
1	1	0	0	1	1
1	1	1	0	1	0

(A) $\overline{A}B + \overline{A}\overline{C} + B\overline{C}$ (B) $\overline{A} + \overline{B}C$
(C) $\overline{A} + B\overline{C}$ (D) $\overline{A} + \overline{B}C + B\overline{C}$

42. A 3 line to 8 line Decoder with active low outputs is used to implement a 3-variable Boolean function, as shown in the figure.
The simplified form of Boolean function $f(p, q, r)$ implemented in "sum of Products" form will be?



(A) $p^1q + pr^1$ (B) $pq + q^1r^1$
(C) $p^1q + q^1r + p^1r$ (D) $p^1q + q^1r$

43. Consider the following grammars:
$G1$:
$S \rightarrow (S)\ I\ a$
$G2$:
$X \rightarrow X(X)\ I\ x$
$G3$:
$X \rightarrow (Y)\ I\ x$
$Y \rightarrow Y, X\ I\ X$
$G4$:
$S \rightarrow Xx\ Xy\ I\ Yx\ Yy$
$X \rightarrow \in$
$Y \rightarrow \in$
$G5$:
$S \rightarrow Sx\ Sy\ I\ S\ y\ S\ x\ I\ \in$
Which of the above grammars are unambiguous?
(A) $G1, G2, G3$ (B) $G1, G4, G5$
(C) $G1, G3, G5$ (D) $G1, G2, G3, G5$

44. Consider the grammar G:
$S \rightarrow Xx$
$S \rightarrow yXz$
$S \rightarrow Yz$
$S \rightarrow yYx$
$X \rightarrow p$
$Y \rightarrow p$

Consider the statements:
(I) Grammar (G) is LL(1).
(II) Grammar G is SLR(1).
(III) Grammar G is CLR (1).
Which of the above statements is/are TRUE?
(A) only (I) (B) (II) and (III)
(C) only (III) (D) (I) and (III)

45. Consider two processes P_1 and P_2. There is a need to synchronize these processes using binary semaphores M_1, M_2, M_3.

Process P_1:	**Process P_2:**
$P(M1)$	A;
$P(M2)$	B;
$P(M3)$	C;

Process P_1: Process P_2:
$P(M1)$ A;
$P(M2)$ B;
$P(M3)$ C;
C.S /* critical section*I C.S
$V(M1)$ A;
$V(M2)$ B;
$V(M3)$ C;

$V(M1)$	A;
$V(M2)$	B;
$V(M3)$	C;

Number of possibilities to fill at A, B, C using binary semaphore in order to not to have Deadlock and synchronize among processes is _______.

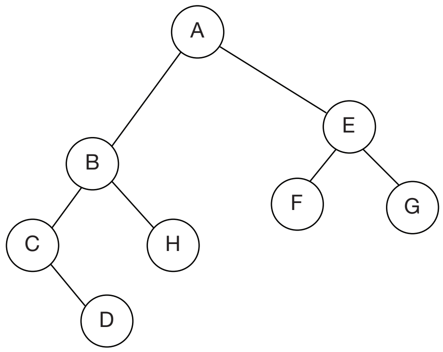
46. Consider a system in which a directory entry can store up to 32 disk block addresses for a file no larger than 32 blocks, the 32 addresses serve as the files index table. For files larger than 32 blocks, the addresses point to indirect blocks which in turn point to 512 file blocks each. Block size is 1024 bytes. The largest file size (in MB) is ______.

47. Consider the following:

Pid	Arrival time	Burst time
P_1	0	8
P_2	2	6
P_3	4	4
P_4	6	2

If Round Robin scheduling (with time slice = 3 units) preemptive SJF, and Highest Response Ratio Next (HRRN) were used. Which of the following specifies correct (ascending) order with respect to average turnaround time?
(A) RR, PSJF, HRRN
(B) P-SJF, HRRN, RR
(C) P-SJF, RR, HRRN
(D) RR, HRRN, P-SJF

48. Consider a Binary Tree



Consider the code fragment:

```
struct BTnode
{
struct BTnode *LC;
int data;
struct BTNode *RC;
}
void fun (struct BTnode *t)
{
if(t)
{
fun (t → LC)
printf ("%d", t →data):
fun (t → RC);
}
}
```

If the root node is passed as parameter to routine fun( ), the output is

(A) AEFGBCDH (B) FEGACDBH
(C) CDBHAFEG (D) FEGCDBHA

49. Consider a binary tree T on 200 vertices. let n_i be the number of vertices in T which have exactly i neighbours. Let $x = \sum_{i=1}^{200} i \cdot n_i$ then the value of x will be:

(A) 199 (B) 398
(C) $199 < x < 398$ (D) None of the above

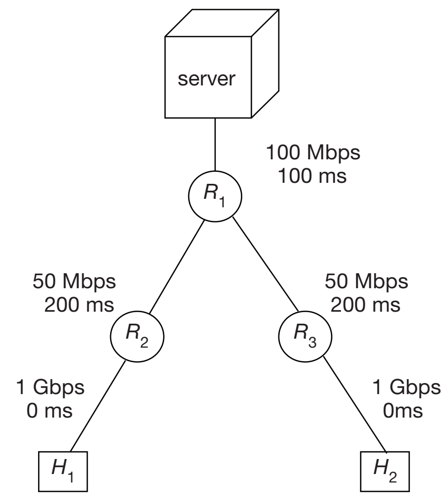
50. Consider the routine fun( ):

```
fun (x, y, k)
{
if (k = = 0)
return x + y;
else if ( y = = 0 && k = = 1)
return 0;
else if ( y = = 0 && k = = 2)
return 1;
else if ( y = = 0)
return x;
else
return fun (x, fun (x, y - 1, k), k
- 1);
}
```

The value returned when fun(2,3,3) called is ______.

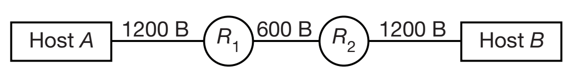
51. Suppose TCP uses Additive Increase Multiplicative Decrease (AIMD) Technique for its congestion control. The congestion window increases by 1MSS every time a batch of ACKs is received. Assume that the round-trip times are constant. If the initial window size is 5MSS, then the average throughput (in terms of MSS per RTT) for this TCP connection upto time 6RTT (MSS/RTT) is ______.

52. Consider below figure; in which a server is connected to a router, R_1 by a 100 Mbps link with a propagation delay of 100 ms. R_1 is connected to two routers R_2 and R_3 each over a 50 Mbps link with a propagation delay of 200 ms. The Routers R_2 and R_3 are connected to two hosts each with a 1 Gbps link and with no propagation delay.



All the packets in the network have 20,000 bits. If there are no caches, no queueing delays at the routers and if the packet processing delays at the routers and nodes are all zero then the end-to-end delay required by a client to receive a packet from server (in msec) is ______.

53. Consider below network:



Two Hosts A and B have a separate shared Ethernet with a Maximum Transfer rate (MTU) of 1200 Bytes. The two Hosts are connected with a point-to-point link, which has a MTU of 600 Bytes. The MTU includes TCP, IP headers and data.

If Host A passes 2000 Bytes to TCP of Host B. Then the number of bits delivered to network layer in Host B (in Bytes) is ______.

54. Consider the below languages which are defined over $\Sigma = \{0, 1\}$:

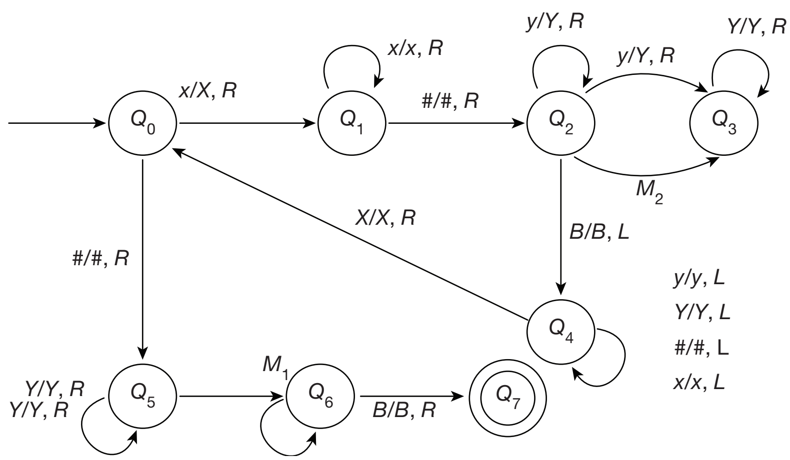
$L_1 = \{w/w$ contains an even number of 0's and an odd number of 1's$\}$

$L_2 = \{w/n_1(w) = k * n_0(w)$ for some $k \in N\}$

$L_3 = \{w/ \text{ I } n_0(w) - n_1(w) \text{ I mod } 3 = 0\}$
$L_4 = \{w/ \text{ I } n_0(w) - n_1(w) \text{ I mod } 3 = 1\}$
(here $n_0(w)$ is number of 0's in w, $n_1(w)$ is number of 1's in w)
Which of these languages are regular?
(A) I, III only (B) I, III, IV only
(C) I, II, IV only (D) I, II, III, IV

55. Consider a push down automata with the following instantaneous description:
$\delta(q_0, a, \in) = \{[q_0, A]\}$
$\delta(q_0, a, A) = \{[q, AA]\}$
$\delta(q_0, \in, \in) = \{[q_1, \in]\}$
$\delta(q_0, b, A) = \{[q_2, \in]\}$
$\delta(q_1, \in, A) = \{[q_1, \in]\}$
$\delta(q_2, b, A) = \{[q_2, \in]\}$
$\delta(q_2, \in, A) = \{[q_2, \in]\}$
Here q_0 is the initial state. Initially stack has empty symbol, $\in$.
Both states q_1, q_2, reach a final state with no input left and empty stack.
The stack symbol is $\{A\}$.
Input alphabet is $\{a, b\}$. Each δ transition has the form δ(current state, input, top of stack)
= {[Resultant state, X]}
X can be either the symbol which is written in-place of top of stack (if X is $\in$) or new item placed on top of stack.
What is the language accepted by the given PDA?
(A) $\{a^i\, b^j \text{ I } i! = j\}$
(B) $\{a^i\, b^j \text{ I } 0 \le i \le j \text{ or } 0 \le j \le i\}$
(C) $\{a^i\, b^j \text{ I } 0 \le i \le j\}$
(D) $\{a^i\, b^j \text{ I } 0 \le j \le i\}$

56. Consider the below Turing machine, M.



M is accepting the language $\{x^n\# \ y^{2^n} \text{ I } n \ge 0\}$
X, Y are symbols written in-place of x and y respectively. each transition has a form (a/b, D), where a is symbol read, b is symbol to be written and D is direction of head movement. Analyze the Turing machine and choose the missing transitions M_1 and M_2 for the correct functioning of M:
(A) $M_1 : y/y, L\ M_2 : y/y, L$
(B) $M_1 : y/y, R\ M_2 : y/y, R$
(C) $M_1 : y/y, L\ M_2 : y/y, R$
(D) $M_1 : y/Y, R\ M_2 : y/Y, L$

57. To increase the efficiency in program controlled I/O technique, the I/O software is written in such a way that the processor periodically checks the status of the device. If the device is not ready, the processor can jump to other tasks. After some time interval, the processor comes back to check status again.
Consider a device, keyboard with a single character buffer. On an average, characters are entered at a rate of 5 characters per second. The time interval between two consecutive key depressions can be 50 ms. Then at what frequency does the processor scans the keyboard?
(A) 200 ms (B) 150 ms
(C) 50 ms (D) Data insufficient

58. A 20 MHz processor has a cache which takes two clock cycles. Data access from main memory over the bus to the processor takes three clock cycles, with no wait states. The data is delivered to the processor in parallel with delivery to cache. If the cache has a hit ratio of 0.9 then the effective duration of memory access (in nano seconds) is ________

59. Consider the Relation R with attributes, Teacher, Course, Textbook, Project, Location, Hours .
For every course, Textbook there is only one Teacher.

For every Location, Hours, Teacher there is only one Course. There is only one Project for any Location and Hours.

P. Course, Textbook → Teacher
Q. Location, Hours, Teacher → Course
R. Project → Location, Hours, Teacher
S. Course → Location , Hours
W. Teacher → Course, Textbook

Which of the following functional dependencies can be inferred from the given statements?

(A) P, Q, R only (B) R, S only
(C) R, S, W only (D) P, R, S only

60. Given an undirected weighted graph $G = (V, E)$ with non-negative edge weights, we can compute a minimum cost spanning Tree $T = (V, E')$. We can also compute, for a given source vertex $S \in V$, the shortest paths from S to every other vertex in V. we now increase the weight of every edge in the graph by 1.

I. All the shortest paths from S to other vertices are unchanged.
II. T is still a minimum cost spanning Tree.

Which of the following is TRUE?

(A) I only (B) II only
(C) Both I and II (D) None of the above

61. Consider the following 2 transactions T_1 and T_2.

T_1:

$x = x - 50$
$y = y + 50$

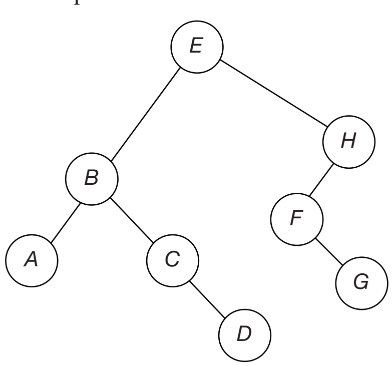
T_2:

$a = a + x$
$a = a + y$

Suppose we start with initial values of 100 for x, 200 for y, and 0 for a, and then T_1 and T_2 run concurrently, that is, at each step we either execute one statement from T_1 or one statement from T_2. Which of the following is not a possible final value of a?

(A) 200 (B) 250
(C) 300 (D) 350

62. In a binary search tree given below, finding node E required one comparison and finding node D requires four comparisons. What is the expected number of comparisons requires to find a node chosen at random?



(A) 3
(B) 2.25
(C) 3.25
(D) 2.75

63. Insert the characters of the string "s, m, o, w, g, v, e, k, y" into a Hash table of size 10. Hash function is $H(X)$ = (ASCII(X) − ASCII (C) + 1) mod 10

Hashing technique is Linear probing, while Hashing which characters, collision occurs?

(A) e, y, v
(B) w, e, k
(C) o, w, e
(D) w, e, y

64. Consider the following 2 tables.

R

X	Y
11	A
12	B
13	C

S

X	Y
11	A
12	B
16	D

The result of $R \cap S$, can be obtained by using which of the following operations (Rational algebra expression should contain same operator any number of times)?

(A) ∪ (union)
(B) − (set difference)
(C) ⋈ (natural JOIN)
(D) None of the above

65. Consider the following sequence of elements

I.	1	2	3	4	5	6	7	8
II.	1	8	2	6	3	7	4	5
III.	5	4	7	3	1	8	2	6

Assume that the last element of the set is used as pivot element in quick sort, to sort the given elements. Which sequence takes maximum time.

(A) I Only
(B) I and II
(C) I and III
(D) I, II and III

Answer Keys

1. B	**2.** D	**3.** 2	**4.** D	**5.** D	**6.** C	**7.** D	**8.** D	**9.** B	**10.** B
11. B	**12.** C	**13.** 2	**14.** D	**15.** 264	**16.** D	**17.** A	**18.** C	**19.** B	**20.** C
21. D	**22.** 16.67	**23.** A	**24.** D	**25.** 10	**26.** 1000	**27.** C	**28.** 1631	**29.** 1030	**30.** C
31. 4	**32.** 4	**33.** D	**34.** D	**35.** C	**36.** C	**37.** B	**38.** A	**39.** 2	**40.** A
41. C	**42.** D	**43.** A	**44.** C	**45.** 2	**46.** 16	**47.** B	**48.** C	**49.** B	**50.** 65536
51. 7.5	**52.** 300.62	**53.** 2140	**54.** A	**55.** D	**56.** B	**57.** C	**58.** 105	**59.** C	**60.** B
61. A	**62.** D	**63.** D	**64.** B	**65.** A					

Hints and Explanations

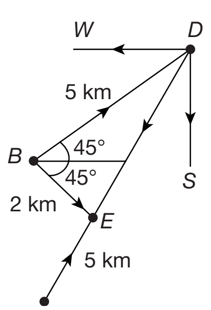
1. Choice (B) is correct. A footpath runs along the road on either side. Similarly, a riverbank runs along the river on either side. Choice (B)

2. I. 2/5th of the weight of a single disc is 13 kg. But we don't know if each disc has the same weight or not. I alone is not sufficient.
We do not know the weight of each disc.
∴ We cannot find the total weight of 25 discs.
II alone is not sufficient
I, II we still cannot answer the question.
Choice (D)

3. Let $f(x) = ax + b$ where a and b are constants.
$f(-4) = 50$ and $f(7) = 6$
$-4a + b = 50$ …… (1)
$7a + b = 6$ …… (2)
$-11a = 44$
$a = -4$
Substituting in (1), we get
$16 + b = 50 \Rightarrow b = 34$
∴ $f(x) = -4x + 34$.
When $x = 8$
$f(x) = -4(8) + 34 = -32 + 34 = 2$
∴ $f(8) = 2$. Ans: 2

4. The right choice is "to call a spade a spade" which means to speak very frankly and openly. None of the other options go as they are negative in connotation. "To see red" is to be afraid, "to throw in the towel" is to accept defeat and "to let the grass grow under one's feet" is to idle too long without any work. Choice (D)

5.



$\angle B = 90°$

$DE = \sqrt{DB^2 + BE^2}$

$= \sqrt{5^2 + 2^2}$ km $= \sqrt{29}$ km ≈ 5.39 km. Choice (D)

6. "Nature" is not preceded by "the" so choices (A) and (D) are ruled out. In (B) the tense is incorrect for a completed action. Choice (C) uses the simple past tense for a completed action and it is correct. Choice (C)

7. All the statements except (D) can be proved false by the passage itself. (A) is not what General Dyer was. Statement (B) too is a distortion of what is stated in the passage. The passage does not state that the victims got a fair trial. In fact, the trial was a travesty of justice and the public supported general Dyer's actions. Statement (C) is out of the scope of the text. Statement (D) is correct as understood from the first two lines of the passage. General Dyer was not a frightened person misbehaving so his actions can neither be understood nor excused. Choice (D)

8. Freon damages ozone layer. A need is felt to substitute Freon with some other coolant. This means that damage to ozone layer is harmful. Hence (D) is the correct answer.
As the cost is not the focus of the argument, (A) is wrong. (B) and (C) cannot be inferred. Choice (D)

9. Choice (B) is apt. The para, when rearranged, is the story of human life, metaphorically presented. 1 mentions two birds. In 3 both are described as "one" eating and the other not eating. 5 follows next as it tells as to what is being eaten, and more importantly, where it is sitting. 4 is a continuation of 5 as it tells the position of the other bird. So 5 and 4 is a definite pair. 2 is then concluding the analogy. 6 explains why it is the story of the human soul. Choice (B)

10. Area of triangle ABC = Area of ABD + Area of ADC
$\frac{1}{2}(AB)(AC)\sin\angle A = \frac{1}{2}(AB)(AD)\sin\angle BAD + \frac{1}{2}$
$(AD)(AC)\sin\angle DAC = (AB)(AC)\sin 120°$
$= (AB)(AD)\sin 60° + (AD)(AC)\sin 60°$

$$AD = \frac{(AB)(AC)\sin 120°}{AB\sin 60° + AC\sin 60°}$$

$= \dfrac{(AB)(AC)}{AB + AC} = \dfrac{60}{11} = 5.45$ cm. Choice (B)

11. Total number of ways of distributing 10 apples among three persons Mahesh, Naresh and Ramesh $= 3^{10}$
Mahesh and Naresh together has to get exactly 7 apples

$\Rightarrow$ Mahesh and Naresh together gets 7 apples and Ramesh gets 3 apples
The number of ways of selecting 7 apples from 10 to distribute to Mahesh and Naresh $= {}^{10}C_7$
The number of ways of distributing these 7 apples to Mahesh and Naresh $= 2^7$

$\therefore$ The total number of ways of distributing 10 apples among the three persons such that Mahesh and Naresh together get 7 apples $= {}^{10}C_7 \times 2^7$

$\therefore$ Required probability $= \dfrac{{}^{10}C_7 \times 2^7}{3^{10}} = \dfrac{\dfrac{10!}{3! \times 7!} \times 2!}{3^{10}}$

$= 15 \times \left(\dfrac{2}{3}\right)^{10}$ Choice (B)

12. The statement formulas given in options (A), (B) and (D) are all equivalent to p.
The statement formula given in option (C),
i.e., $p \wedge F_o \Leftrightarrow F_o$ = contradiction
and NOT equivalent to p. Choice (C)

13. We have $\displaystyle\int_1^5 \frac{\sqrt{x+5}}{\sqrt{x+5} + \sqrt{11-x}}\,dx$

$= \displaystyle\int_1^5 \frac{\sqrt{x+5}}{\sqrt{x+5} + \sqrt{(1+5-x)+5}}\,dx = \frac{5-1}{2}$

$(\because \displaystyle\int_a^b \frac{f(x)}{f(x) + f(a+b-x)}\,dx = \frac{b-a}{2}$ and here

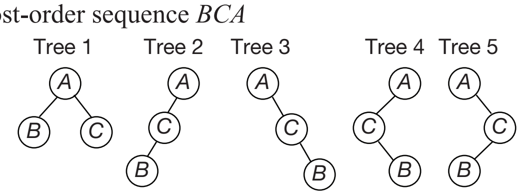
$f(x) = \sqrt{5+x};\ a = 1$ and $b = 5) = 2$ Ans: 2

14. We know that every lattice need not satisfy distributive laws. So, the relation given in option (D) need not be true. Choice (D)

15. The number of different choices available for Rakesh to buy a Zigma $= 3 \times 6 \times 4 \times 2 = 144$
The number of different choices available for Rakesh to buy a Harze $= 4 \times 5 \times 3 \times 2 = 120$

$\therefore$ The total number of different choices available for Rakesh to buy either a Zigma or a Harze $= 144 + 120 = 264$. Ans: 264

16. Post-order sequence *BCA*



All the five trees gives post order sequence *BCA*.
Choice (D)

17. $T(n) = 3T\left(\dfrac{n}{4}\right) + n$

$T(n) = aT\left(\dfrac{n}{b}\right) + f(n)$

$a = 3, b = 4, f(n) = n$

compare $f(n)$ Vs $n^{\log_b^a}$

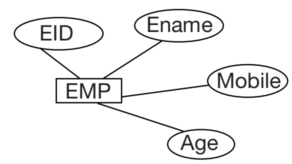
n Vs $n^{\log_4^3}$

n Vs $n^{0.792}$

$f(n) > n^{\log_b^a}$

Case 1 of master theorem
Time complexity $T(n) = \theta(f(n))) = \theta(n)$ Choice (A)

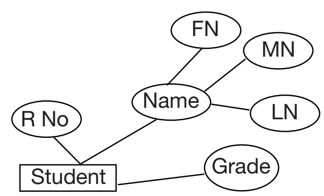
18. Example:



Mobile is a multi valued attribute, so keep mobile attribute in a separate relation (This is defined in 1*NF*)

EId	**Ename**	**Age**

EId	**Mobile**



Name $\rightarrow$ is a composite attribute
FN, *MN*, *LN* $\rightarrow$ are simple component attributes
In the relation only simple component attributes will appear (This is defined by 1NF)
Student

RNo	**FN**	**MN**	**LN**	**Grade**

Choice (C)

19. The value of item A that is read by T_2 is called dirty data because it has been created by a transaction that has not completed and committed yet, hence this problem is also known as the "dirty read problem".
Choice (B)

20. One track contains 20 blocks, means the same track also contains 20 inter block gaps.
Capacity of one track = 20 * 512 + 20 * 64
= 10240 + 1280 = 11520 byes Choice (C)

21. Main memory has n - bit address.

Tag	Set	Block

$\longleftarrow$ *n*-bits $\longrightarrow$

Block size = b bytes

$\Rightarrow$ Bits required for block offset = log b

The cache is m-way set associative.

Number of blocks = s

$\Rightarrow$ number of sets $= \dfrac{s}{m}$

Bits required for set field = log (s/m)

$\therefore$ Tag = n − log (s/m) – log b Choice (D)

22. Number of cycles required using 16-bit processor (for 100 instructions) = (2 * 20) + 1 * 50 + 1 * 30 = 120

Number of cycles required on 32-bit processor for 100 instructions = 100

$\therefore$ Percentage of improvement $= \dfrac{20}{120} * 100$

= 16.67% Ans:16.67

23. The words are defined over $\{a, b, c\}$ such that number of b's is not divisible by 3 or 5 and no 'c' appears after 'a'.

This can be written as

$(L_1 \cup L_2)L_3$ where

L_1 = { number of b's is not divisible by 3}

L_2 = { number of b's is not divisible by 5}

L_3 = { no 'c' appears after 'a'}

L_1, L_2 & L_3 are regular and regular languages are closed under union & concatenation.

$\therefore$ Given language is regular. Choice (A)

24. **Statement I:**

$L_2 = \{w/w \in \{a, b\}^*\}$

$L_1 = \{a^n b^n \text{ I } n \geq 0\}$

L_2 is regular but L_1 is not regular.

$\therefore$ I is not always true.

Statement II:

$L_1 = \phi$

$L_2 = \{a^n b^n \text{ I } n \geq 0\}$

L_1 is regular but L_2 is not regular.

Statement III is also not true always. Choice (D)

25. CRC generator has 11-bits, so the remainder cannot be more than 10.

$\therefore$ Maximum 10-bits present in resulting frame check sequence. Ans: 10

26. 1-way propagation time $= \dfrac{\text{distance}}{\text{speed}}$

$$= \frac{1000 \text{ m}}{2 \times 10^8 \text{ m/s}} = 0.5 \times 10^{-5} \text{ sec}$$

Minimum packet size required is = 2 * propagation time * Bandwidth

$= 2 * 0.5 \times 10^{-5} * 100 \times 10^6$

= 1000 bits. Ans: 1000

27. Choice (C)

28. Let 'V' be the set of vertices. A path between x to y through z is formed by a subset V' of V I $\{x, y, z\}$ and forming a permutation of $V' \cup \{z\}$ for each $i \leq 5$, there are $\binom{5}{i}$ subsets V' of size i, and $(i + 1)!$ permutations of $V' \cup \{z\}$

Thus the number of required paths is

$$\binom{5}{0}1! + \binom{5}{1}2! + \binom{5}{2}3! + \binom{5}{3}4! + \binom{5}{4}5! + \binom{5}{5}6!$$

= 1631

Ans: 1631

29. The address of $A[8][3]$ is $= B + \dfrac{i(i-1)}{2} + j - 1$

$$= 1000 + \frac{8 \times 7}{2} + 3 - 1 = 1030$$

Ans: 1030

30. Total virtual memory required = 16×1024

$= 2^4 \times 2^{10} = 2^{14}$

$\therefore$ 14 bits for logical address.

Choice (C)

31.

1, 3, 5, 6, 3, 1, 3, 5

~~1~~ ~~6~~ 5
3
~~5~~ 1

= 3 page faults

Ans: 4

32.

$$\text{B1}\begin{cases}(1) x = 1 \rightarrow \text{leader} \\ (2) y = 1\end{cases}$$

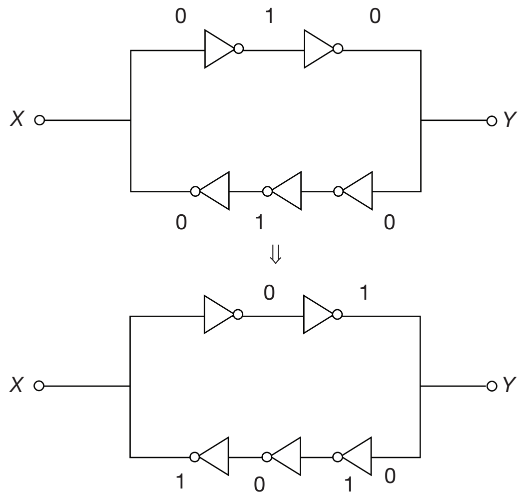
B2 {(3) if $x > i$ goto (9) $\rightarrow$ leader

$$\text{B3}\begin{cases}(4) y = y * x \\ (5) y = 10 \\ (6) t = y * x \\ (7) x = t \\ (8) \text{ goto } (3)\end{cases}$$

B4 {(9) END $\rightarrow$ leader

33. Choice (D)

34.



Choice (D)

35. Memory is specified as $(2^{Addr} \times \text{data})$ bits

So $2^{16} \times 8$ bits = 5, 24, 288 bits   Choice (C)

36. Given $P = \begin{bmatrix} 2 & 131 & -243 & 566 \\ 0 & -2i & 174 & -237 \\ 0 & 0 & 2i & 0 \\ 0 & 0 & -713 & -2 \end{bmatrix}$

The characteristic equation of P is $|P - \lambda I| = 0$

$\Rightarrow \begin{bmatrix} 2-\lambda & 131 & -243 & 566 \\ 0 & -2i-\lambda & 174 & -237 \\ 0 & 0 & 2i-\lambda & 0 \\ 0 & 0 & -713 & -2-\lambda \end{bmatrix} = 0$

$\Rightarrow (2-\lambda)(-2i-\lambda)(2i-\lambda)(-2-\lambda) = 0$

$\Rightarrow (2-\lambda)(2+\lambda)(2i+\lambda)(2i-\lambda) = 0$

$\Rightarrow (4-\lambda^2)(-4-\lambda^2) = 0$

$\Rightarrow -(4-\lambda^2)(4+\lambda^2) = 0$

$\Rightarrow 16-\lambda^4 = 0$

$\Rightarrow \lambda^4 - 16 = 0$

$\therefore$ The characteristic equation of P is $\lambda^4 - 16 = 0$

Hence by Cayley-Hamilton theorem, we have

$P^4 - 16I_4 = 0$ $\rightarrow (1)$

Where I_4 = Identity matrix of order 4.

Multiplying (1) on both sides with P^{-1}, we have

$P^{-1}(P^4 - 16\,I_4) = P^{-1} \times 0$

$\Rightarrow P^3 - 16\,P^{-1} = 0$

$\Rightarrow 16P^{-1} = P^3$ Choice (C)

37. Let A and B denote the events of a randomly selected employee having a Two Wheeler (TW) and a Four Wheeler (FW) respectively.

$\therefore \quad P(A \cup B) = \frac{7}{10}, P(A \cap B) = \frac{2}{5}$ and $P(A/B) = \frac{2}{3}$

We know that $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$\Rightarrow P(B) = \dfrac{P(A \cap B)}{P\left(\frac{A}{B}\right)} = \dfrac{\frac{2}{5}}{\frac{2}{3}}$

$\Rightarrow P(B) = \dfrac{3}{5}$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{7}{10} = P(A) + \frac{3}{5} - \frac{2}{5}$

$\Rightarrow P(A) = \frac{7}{10} - \frac{3}{5} + \frac{2}{5}$

$\Rightarrow P(A) = \frac{1}{2}$

Hence probability that a randomly selected employee has two wheeler (TW) $= \frac{1}{2}$.   Choice (B)

38. Let H, P and G denote the sets of students, who can speak Hindi, Punjabi and Gujarati respectively.

$\therefore$ $n(H) = 120$, $n(H \cap P) = 35$ and

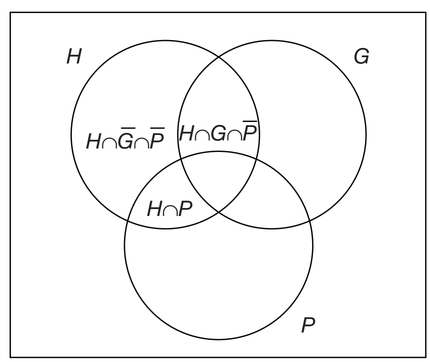
$n(H \cap G \cap \bar{P}) = 40$

We know that $n(H) = n(H \cap \bar{G} \cap \bar{P}) + n(H \cap P) + n(H \cap G \cap \bar{P})$

$\Rightarrow 120 = n(H \cap \bar{G} \cap \bar{P}) + 35 + 40$

$\Rightarrow n(H \cap \bar{G} \cap \bar{P}) = 45$

$\therefore$ The number of students who can speak only Hindi $= n(H \cap \bar{G} \cap \bar{P}) = 45$.



Choice (A)

39. In the given graph G, one can observe that the vertex set $\{a, b, c, d, e, f, g\}$ can be partitioned into two sets $M = \{a, b, c\}$ and $N = \{d, e, f, g\}$ such that any edge in G is between a vertex in M and a vertex in N.

Hence G is a bipartite graph (in fact, a complete bipartite graph).

$\therefore$ The chromatic number of $G = X(G) = 2$ Ans: 2

40. Statement P is nothing but the intermediate value theorem and hence is TRUE.

Counter Example for statement Q:

$$\text{Let } f(x) = \begin{cases} x; & \text{if } 0 \le x < 1 \\ x - 1; & \text{if } 1 \le x \le 3 \end{cases}$$

Then, $f(x)$ assumes every value between $f(0)$ (=0) and $f(3)$ (=2). But $f(x)$ is not continuous at $x = 1 \in [0, 3]$

$\therefore$ Statement Q is not TRUE. Choice (A)

41. From the given circuit $Z = DE + F$

We need expression of Z in terms of A, B, C

ABC = 101, 001 combinations do not occur.

A	B	C	D	E	F	Z = DE + F
0	0	0	0	0	1	1
0	0	1	1	0	0	X
0	1	0	1	1	0	1
0	1	1	1	1	1	1
1	0	0	0	0	0	0
1	0	1	1	0	1	X
1	1	0	0	1	1	1
1	1	1	0	1	0	0

$Z(A, B, C) = \Sigma\, m(0, 2, 3, 6) + \phi(1, 5)$

A \ BC	00	01	11	10
0	1	X	1	1
1		X		1

$Z = \bar{A} + B\bar{C}$ Choice (C)

42. The output of Decoder are active low outputs so

$f(p, q, r) = \overline{\bar{Y}_1, \bar{Y}_2, \bar{Y}_3, \bar{Y}_5} = Y_1 + Y_2 + Y_3 + Y_5$

$= \Sigma m(1, 2, 3, 5)$

p \ qr				
0		1	1	1
1		1		

$= p^1q + q^1r$ Choice (D)

43. Grammars $G1$, $G2$, $G3$ are unambiguous, while $G4$ and $G5$ are ambiguous. Choice (A)

44. The grammer is $LR(1)$. Choice (C)

45. At A we need to place $P(M1)$ to not have Deadlock, at B and C we can place either $P(M2)$ or $P(M3)$ which can be done in 2! ways. Ans: 2

46. The larger possible file will have $32 = 2^5$ indirect blocks. Since each indirect block can point to $512 = 2^9$ blocks the largest file has $512 * 32 = 2^{9+5}$ blocks. Given block size = 1024 bytes = 2^{10} B

The maximum file size is 2^{9+5+10} B

$= 2^{24}$ B = 16 MB Ans: 16

47. RR (T. S. = 3 units):

P_1	P_2	P_1	P_3	P_4	P_2	P_1	P_3	
0	3	6	9	12	14	17	19	20

Ready Queue: $\cancel{P_1} < \cancel{P_2}, \cancel{P_1}, \cancel{P_3}, \cancel{P_4}, \cancel{P_2}, \cancel{P_1}, P_3$

$$\text{Average Turnaround Time} = \frac{19 + 15 + 16 + 8}{4} = 14.5$$

P-SJF:

P_1	P_1	P_1	P_1	P_4	P_3	P_2	
0	2	4	6	8	10	14	20

$$\text{Average Turnaround Time} = \frac{8 + 18 + 10 + 4}{4} = 10$$

HRRN:

P_1	P_2	P_4	P_3	
0	8	14	16	20

At $t = 8$

Response Ratio of process P_2

$$(RR_2) = \frac{6 + 6}{6} = 2$$

$$RR_3 = \frac{4 + 4}{4} = 2$$

$$RR_4 = \frac{2 + 2}{2} = 2$$

At $t = 14$

$$RR_3 = \frac{10 + 4}{4} = 3.5$$

$$RR_4 = \frac{8 + 2}{2} = 5$$

$$\text{Avg. TAT} = \frac{8 + 12 + 10 + 16}{4} = 11.5$$

The order is P – SJF, HRRN, RR. Choice (B)

48. The fun ( ) prints the Inorder traversal of the tree. Choice (C)

49. Tree is connected graph. The value of x will be $(2 \times 200 - 2) = 398$ Choice (B)

50. fun $(x, y, 1)$ will evaluate to xy

fun $(x, y, 1) \Rightarrow$ fun $(x$, fun $(x, y - 1, 1), 0)$

$\Rightarrow$ fun $(x, x\,(y - 1), 0)$

$\Rightarrow$ $x + x\,(y - 1)$

$\Rightarrow$ xy

Similarly fun $(x, y, 2)$ will evaluate to x^y

This can be proved by induction

fun $(x, 0, 2) = 1 = x^0$

fun $(x, 1, 2) = x = x^1$

Similarly fun $(x, y - 1, 2) = x^{y-1}$

fun $(2, 3, 3)$ = fun $(2$, fun $(2, 2, 3), 2)$

= fun $(2, 16, 2)$ [$\because$ fun $(2, 2, 3)$ evaluate to 2^4)

$= 2^{16} = 65536$ Ans: 65536

51. In 1st RTT, 5MSS is sent.

In 2nd RTT, 6 MSS is sent.

In 3rd RTT, 7 MSS is sent.

In 4th RTT, 8 MSS is sent.

In 5th RTT, 9 MSS is sent.

In 6th RTT, 10 MSS is sent.

$\therefore$ Average throughput till 6^{th}

$$RTT = \frac{5+6+7+8+9+10}{6} = \frac{45}{6}$$

$= 7.5\ MSS/RTT$. Ans :7.5

52. Packet size = 20000 bits

Time to transmit the packet from sever to R_1 over 100 Mbps link is $\frac{20000}{100 \times 10^6} = 200$ μsec.

Time to transmit the packet from R_2 over 50 Mbps link is $\frac{20000}{50 \times 10^6} = 400$ μsec.

Time to transmit the packet from R_2 to $H1$ over 1 Gbps link is $\frac{20000}{1 \times 10^9} = 20$ μ sec.

$\therefore$ End-to-end delay = sum of propagation delays of the links + sum of link transmission times

= (100 + 200 msec) + (200 + 400 + 20) μsec

= 300 msec + 620 μsec

= 300 msec + 0.620 msec

= 300.62 msec Ans : 300.62

53. The MTU between Host A and R_1 is 1200 Bytes. The data to be transferred is 2000 Bytes. So 2 packets required and each with both TCP & IP headers.

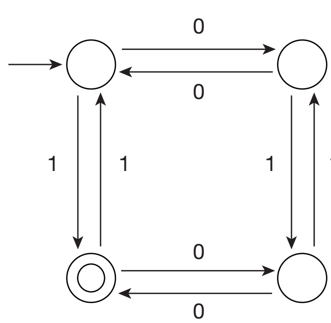
The link R_1 to R_2 has a MTU of 600 B so 1^{st} packet fragmented to 3 packets with separate IP headers.

Also the 2^{nd} packet fragmented to 2 packets with separate IP headers.

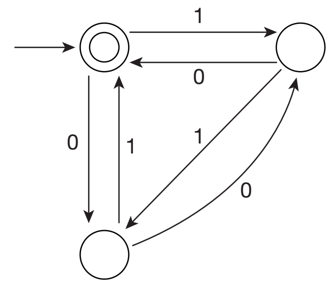
Host B receives 5 packets in which 2 have both TCP & IP headers, 3 have only IP header & 2000 Bytes of data.

$\therefore$ Data received by the Network layer of Host B is $2000 + 5 * 20 + 2 * 20 = 2140$ Bytes. Ans :2140

54. L_1 is regular. The DFA for L_1 is shown below:



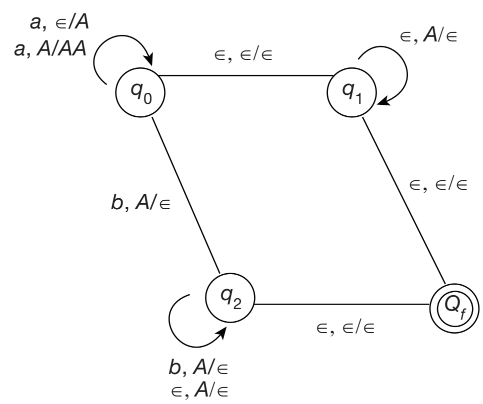
L_2 is not regular. For $k = 1$, we need to construct a DFA which accepts the strings with equal number of 0's and 1's. This is not possible to construct a DFA for this language. L_3 is regular. The DFA for L_3 is shown below.



L_4 is not regular. We can prove this using pumping lemma.

Choice (A)

55. Based on the given description, the PDA will be as below:



State q_0 writes 'A' for each 'a' in input. With no input it goes to q_1 to pop all the pushed A's, and reaches final state.

$q_{1,}$ is used to accept any number of a's.

q_0 goes to q_2 with input b and top of stack A and replaces A with $\in$.

q_2 pops A's for each b. There need to be A's on stack even after processing all b's. With no input it goes to final state.

From this we can observe that the PDA accepts strings of the form, $\{a^i b^j \text{ I } 0 \le j \le i\}$. Choice (D)

56. Given TM, M accepts the language,

$\{x^n \# y^{2^n} \text{ I } n \ge 0\}$

After analyzing the machine we can identify that the logic is to "Replace an 'x' with X and traverse till #. Now replace half of the (remaining) y's with Y". This needs to be repeated till a y is remained in the input and with that accept the string.

Example: for $n = 3$,

$x\,x\,x\,\#\,y\,y\,y\,y\,y\,y\,y\,y\,B$

$\Downarrow$

$X\,x\,x\,\#\,Y\,y\,Y\,y\,Y\,y\,Y\,y$

$\Downarrow$

$X\,X\,x\,\#\,Y\,Y\,Y\,y\,Y\,Y\,Y\,y$

$\Downarrow$

$X\,X\,X\,\#\,Y\,Y\,Y\,Y\,Y\,Y\,Y\,y$

We can replace that left out y with Y and accept the string. (i.e., for each x, strike-off half of y's and for zero x there will be one y).

Hence M_1 is y/y, R

M_2 is y/y, R. Choice (B)

57. The keyboard has single character buffer.

The time interval between two consecutive depressions is 50 ms.

To avoid overwriting of data the processor needs to check the keyboard once every 50 ms. Choice (C)

58. Processor has 20 MHz frequency.

Cycle time $= \dfrac{1}{20\times 10^{6}} = 0.05\times 10^{-6}$ sec.

$= 50 \times 10^{-9}$ sec $= 50n$ sec.

Cache access time = 2 cycles = 2 × 50 = 100n sec.

Main memory access time = 3 cycles = 3 × 50 = 150n sec.

Effective memory access time = 0.9 * 100 + 0.1 * 150

= 90 + 15 = 105n sec. Ans:105

59. Functional dependency $X \rightarrow Y$

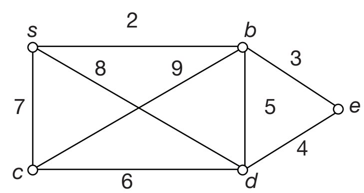
Unique values

R. Project → Location, Hours

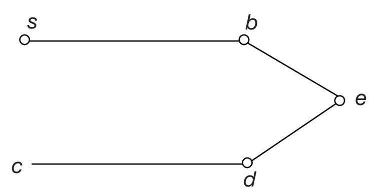
S. Course → Location, Hours, Teacher

W. Teacher → Course, Textbook Choice (C)

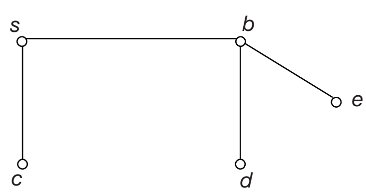
60. Consider the given graph.



Minimum spanning Tree:



Shortest path from s:



Solution set [s]

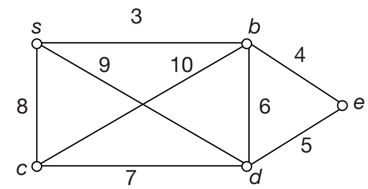
I. Distance [b] = 2
   Distance [c] = 7
   Distance [d] = 8
   Distance [e] = ∞

II. Solution set [sb]
   Distance [c] = min {7, 11} = 7
   Distance [d] = min {8, 7} = 7
   Distance [e] = min {∞, 5} = 5

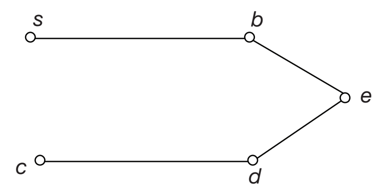
III. Solution set [sbe]
   Distance [c] = min {7, ∞} = 7
   Distance [d] = min {7, 9} = 7

IV. Solution set [$sbec$]
   Distance [d] = min {7, 13} = 7

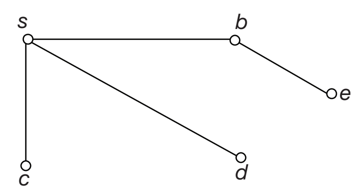
Consider the same graph, with edge weights increased by 1.



Minimum spanning Tree:



Shortest path tree:



Shortest path from node s:

I. Solution set [s]
   Distance [b] = 3
   Distance [c] = 8
   Distance [d] = 9
   Distance [e] = ∞

II. Solution set [sb]
   Distance [c] = min {8, 13} = 8
   Distance [d] = min {9, 9} = 9
   Distance [e] = min {∞, 7} = 7

III. Solution set [sbe]
   Distance [c] = min {8, ∞} = 8

Distance [d] = min {9, 12} = 9

IV. solution set [$sbec$]
   Distance [d] = min {9, 15} = 9

∴ Spanning Tree is same

∴ Shortest path tree may varies. Choice (B)

61. Lets assign a number to statements

	T_1:	T_2:
(1)	$x = x - 50$	$a = a + x$
(2)	$y = y + 50$	$a = a + y$

Sequence 1:
1, 3, 2, 4
$x = 50$
$a = 50$
$y = 250$
$\boxed{a = 300}$
Sequence 2:
3, 1, 2, 4
$a = 100$
$x = 50$
$y = 250$
$\boxed{a = 350}$
Sequence 3:
1, 3, 4, 2
$x = 50$
$a = 50$
$a = 250$
$y = 250$
∴ The value 200 is not possible for 'a'. Choice (A)

62. The number of comparisons required for,
Node $A = 3$
Node $B = 2$
Node $C = 3$
Node $D = 4$
Node $E = 1$
Node $F = 3$
Node $G = 4$
Node $H = 2$
The expected number of comparisons required to find a node at random

$$= \frac{3+2+3+4+1+3+4+2}{\text{Number of nodes}} = \frac{22}{8} = 2.75$$ Choice (D)

63. (1) ASCII (s) = 115
$H(115) = ((115 - 99) + 1) \bmod 10$
$= 17 \bmod 10 = 7$
(2) ASCII (m) = 109
$((109 - 99) + 1) \bmod 10$
11 and 10 = 1
(3) ASCII (o) = 111
$((111 - 99) + 1) \bmod 10$
13 and 10 = 3
(4) ASCII(w) = 119
$((119 - 99) + 1) \bmod 10$
21mod 10 = 1 (collision)
(5) ASCII (g) = 103
$((103 - 99) + 1) \bmod 10$
5 mod 10 = 5
(6) ASCII (v) = 118
$((118 - 99) + 1) \bmod 10$
20 mod 10 = 0
(7) ASCII (e) = 101
$((101 - 99) + 1) \bmod 10$
3 mod 10 = 3 (collision)
(8) ASCII (k) = 107
$((107 - 99) + 1) \bmod 10$
9 mod 10 = 9
(9) ASCII (y) = 121
$((121 - 99) + 1) \bmod 10$
23 mod 10 = 3 (collision)
∴ Collision occurred while hashing 'w', 'e', 'y'
Choice (D)

64. $R \cap S =$

X	Y
11	A
12	B

The same result can be obtained by $R - (R - S)$
$R - S$

X	Y
13	C

$R - (R - S)$

x	y
11	A
12	B
13	C

−

x	Y
13	C

=

x	y
11	A
12	B

Choice (B)

65.
I. 1 2 3 4 5 6 7 8
Quick sort gives worst case Time complexity (maximum time) on a sorted array, that is $O(n^2)$ Time.

II. 1 8 2 6 3 7 4 5 (pivot 5)
Swap

First Iteration: 1 5 2 6 3 7 4 8

1 5 2 4 3 7 6 8

1 3 2 4 $\boxed{5}$ 7 6 8

Second Iteration: 1 3 2 4 $\boxed{5}$ 7 6 8
Pivot = 4 Pivot = 8

1 3 2 $\boxed{4}$ $\boxed{5}$ 7 6 $\boxed{8}$

Third iteration $\boxed{1\ 3\ 2}$ $\boxed{4}$ $\boxed{5}$ $\boxed{7\ 6}$ $\boxed{8}$
Pivot = 2 Pivot = 6

1 5 2 4 3 7 6 8
∴ The time required is $O(n \log n)$
III. first iteration:
5 4 7 3 1 8 2 6 (pivot 6)
5 4 6 3 1 8 2 7

5 4 2 3 1 $\boxed{6}$ 8 7

Second iteration:
5 4 2 3 1 $\boxed{6}$ 8 7

Pivot 1 Pivot 7

1 4 2 3 5 $\boxed{6}$ $\boxed{8}$ $\boxed{7}$

$\boxed{1}$ 4 2 3 5 $\boxed{6}$ $\boxed{7}$ $\boxed{8}$

Third iteration:

4 2 3 5 (pivot 5)

4 2 3

Fourth iteration:

4 2 3 (pivot 3)

3 2 4

2 3 4

∴ The time required is $O(n \text{ long } n)$

∴ I – takes maximum time. Choice (A)

Mock Test 4

Number of Questions: 65 **Total Marks: 100**

Wrong answer for *MCQ will result in negative marks, (–1/3) for 1 Mark Questions and (–2/3) for 2 Marks Question.*

General Aptitude

Number of Questions: 10 **Section Marks: 15**

Directions for question 1: Choose the most appropriate word from the options given below to complete the following sentence.

1. Despite its known toxicity, lead continues to be used, to the great _______ of human and environmental health.
   (A) impediment (B) detriment
   (C) chagrin (D) solace

Directions for questions 2 and 3: Select the correct alternative form the given choices.

2. A 7-year old child assembles an object from lego blocks. The object is in the shape of a cylinder surmounted by a hemisphere of radius 7 cm. If the height of the cylinder is 7 cm, find the volume of the object (in cm^3).
   (A) $\frac{3\pi}{5}(7^2)$ (B) $\frac{5\pi}{3}(7^3)$
   (C) 7^3 (D) $7^3\pi$

3. In a certain code language, if Zoology is called Anthropology, Anthropology is called Ornithology, Ornithology is called Biology, Biology is called Cosmology, Cosmology is called Ecology, Ecology is called Etymology, then what is the study of human called in that language?
   (A) Biology (B) Ecology
   (C) Ornithology (D) Cosmology

Directions for question 4: Select the pair that best expresses a relationship similar to that expressed in the capitalized pair.

4. BIOLOGY: LIFE
   (A) Archaeology: Antiques (B) Astrology: Stars
   (C) Cosmetology: Beauty (D) Mythology: Myths

Directions for question 5: Fill in the blanks from the options given below:

5. _______ summer monsoon _______ been showing a weakening trend _______ the past century with decreasing rainfall over large regions of _______ Indian subcontinent.
   (A) The, has, over, the
   (B) The, has, over, no article
   (C) A, had, across, no article
   (D) The, has, in, the

Directions for questions 6 to 10: Select the correct alternative form the given choices.

6. a, b, c, d are distinct positive integers such that:
   $f(a, b, c, d) = \max(a, b, c, d)$
   $g(a, b, c, d) = \min(a, b, c, d)$
   $h(a, b, c, d)$ = remainder of $(c \times d/a \times b)$
   If $(c \times d) > (a \times b)$
   $h(a, b, c, d)$ = remainder of $(a \times b)/(c \times d)$
   if $(c \times d) < (a \times b)$
   Also, a function $fgh(a, b, c, d) = f(a, b, c, d) \times g(a, b, c, d) \times h(a, b, c, d)$
   the value of fg [h(12, 11, 8, 16), 17, 9, 16] _______

7. Textbooks of medicine say that there is no direct connection between the brain and the lymphatic system, yet a paper published in the journal *Nature* refers to the discovery of exactly such a connection.
   Which one of the statements given below is logically valid and can be inferred from the above sentence.
   (A) A paper published in the journal *Nature* corroborates the fact that there is no direct connection between the brain and the lymphatic system.
   (B) Textbooks of medicine declare that there is no direct connection between the brain and the lymphatic system and this is affirmed by a paper published in the journal *Nature*.
   (C) Texts of medicine proclaim that there is no direct connection between the brain and the lymphatic system but a paper published in the journal *Nature* says that such a connection is indeed there.
   (D) While textbooks of medicine maintain that there is no direct connection between the brain and the lymphatic system, a paper published in the journal *Nature* claims that the brain and the lymphatic system are connected indirectly.

8. The last decade has witnessed a slow but steady realisation within the Indian government that the threats of the future will come from cyberspace. Unfortunately, while the realisation exists, the Indian security establishment has not been jolted into action in the manner in which the Kargil War or the 26/11 terrorist attack on Mumbai galvanised the nation into adopting a series of corrective measures.
   Which of the statement(s) below is/are logically valid and can be inferred from the above passage?
   (i) Although the Indian government realizes that the threats of the future will be from cyberspace, it is yet to accord the seriousness which is due to this threat.
   (ii) Despite the fact that the Indian government realizes that the future threats will come from cyber-

space, it does not consider these threats as devious as other threats which spurred the government to adopt corrective measures.

(iii) The Indian government considers that the threats from cyberspace will not pose a serious threat to the security of the nation.

(iv) The Indian government fails to consider that the threats from cyberspace could pose a threat to the security of a nation.

(A) Only i (B) i and ii
(C) iii and iv (D) i, ii and iii

9. 30 students in class of *BV* School, wrote a test with 4 questions. For each question the number of students who answered correctly, incorrectly and did not attempt are tabulated below. The marks for each question are also listed in the table. There is no negative marking or partial marking.

Q.No.	Marks	Answered correctly	Answered incorrectly	Did not attempt
1	3	15	5	10
2	2	10	7	13
3	2	20	9	1
4	1	11	10	9

If the number of students who attempted all questions is 5, what is the maximum possible number of students who left at least 2 questions unattempted?

(A) 3 (B) 16
(C) 8 (D) None of these

10. The given statement is followed by some courses of action. Assuming the statement to be true, decide which of the given courses of action logically follows for pursuing.

Statement:

There have been steep increases in the government's expenditures on garbage collection, sorting, storage and recycling.

Courses of action:

(i) Government should encourage people to sort their garbage, by providing them separate bins for dry and wet garbage.

(ii) Once garbage is collected from households, to the extent possible it should be directed towards recycling, so that it can cut down on storage costs.

(iii) The routes of the collection vehicles should be planned in such a way that they are used in a most efficient manner.

(A) Only (i) and (ii) follow
(B) Only (ii) and (iii) follow
(C) Only (i) and (iii) follow
(D) All (i), (ii) and (iii) follow

COMPUTER SCIENCE ENGINEERING

Number of Questions: 55 **Section Marks: 85**

Directions for questions 11 to 65: Select the correct alternative from the given choices

11. A letter is selected at random from each of the two words 'FRACTION' and 'DECIMAL'. Probability that the selected letter in a word should NOT be in the other word is ______________

(A) 3/14 (B) 5/14
(C) 7/13 (D) 9/13

12. The value of $\lim_{x \to 3} \log_5[4x^3 + \sqrt{(3x^4 + 5x^2 + 1)}]$ is _______

13. The solution of the recurrence relation $a_n - a_{n-1} = n^2$; $n \geq 1$ with $a_0 = 2$ is ___________

(A) $\dfrac{n^3 + 6n^2 + 2n + 6}{6}$ (B) $\dfrac{2n^3 + 3n^2 + n + 12}{6}$

(C) $\dfrac{n^3 + 3n^2 + 2n + 6}{12}$ (D) $\dfrac{2n^3 + 3n^2 + 4n + 12}{12}$

14. If $S = \{1, \{2, 3\}, 4\}$, then the cardinality of $P(P(A))$ (where $P(S)$ stands for the power set of the set S) is ____________

15. If $P(x)$ and $Q(x, y)$ denote two predicate variables, then which of the following is NOT equivalent to the negation of $(\forall x)\ (P(x) \rightarrow Q(x, y))$?

(A) $\rceil(\forall x)\ (\rceil Q\ (x, y) \rightarrow \rceil P(x))$

(B) $(\exists x)\ \rceil(P(x) \rightarrow Q\ (x, y))$

(C) $(\exists x)\ (P(x) \wedge \rceil Q\ (x, y))$

(D) $(\forall x)\ (P(x) \wedge \rceil Q\ (x, y))$

16. Consider the below C program:

```
int main(void)
{
    int x =1023, c = 0 ;
    while (x ! = 0)
    {
    x = x & (x - 1);
    c = c + 1 ;
    }
    printf ("%d" , c);
}
```

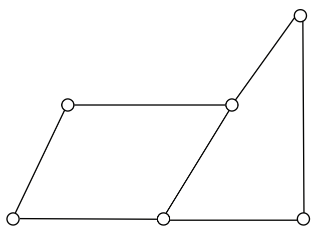
The above program prints _____.

17. Consider the adjacency matrix given below:

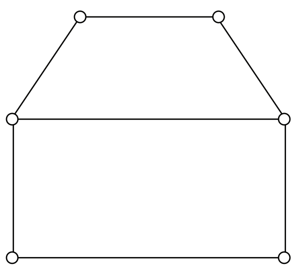
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

In Adjacency matrix, rows and columns are indexed by Vertices of graph. Entry is 1 if corresponding vertices are connected by an edge and is 0 otherwise. Which of the following graphs has the above adjacency matrix?

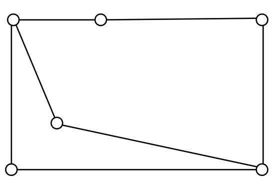
(i)



(ii)



(iii)



(A) (i) & (ii) (B) (i) & (iii)
(C) (ii) & (iii) (D) None of the above

18. An array A[- 32 ... 75] is stored in a memory whose starting address is 209. If each element takes 2 bytes. Then the location of A[50] is ______.

19. A direct access file has fixed size of 10 byte records. The logical location of first byte of record 9 will start at ______.

20. Consider a system with 6 processes ($P_1, P_2, ..., P_6$) with its resource distribution as given below:

Process-id	Allocated resources	Resources needed
P_1	2	2
P_2	3	3
P_3	4	3
P_4	2	2
P_5	2	2
P_6	2	3

The minimum number of processes should be killed to make the above system safe is:

(A) 1 (B) 2
(C) 3 (D) 4

21. Which of the following statement is FALSE?

(A) Every SLR(1) grammar is LALR(1).
(B) Operator precedence parser can parse both ambiguous and unambiguous grammar.
(C) Every operator precedence grammar is LR(0) grammar.
(D) None of the above.

22. Consider below code:

```
i = j = 0;
for (; ;)
{
j++;
if (i > x)
    break;
    i++;
}
```

The value of j, after the loop termination is:

(A) x (B) $x + 1$
(C) $x + 2$
(D) The loop doesnot terminate.

23. Assume that a new memory technology is developed, which is named as NextGenRAM. NextGenRAM is non-volatile. The access latency of NextGenRAM is twice that of an SRAM cell but same as that of a DRAM cell. The read/write capability of NextGenRAM is same as that of a DRAM energy. The cost of NextGenRAM is similar to DRAM, but the density of NextGenRAM is more than DRAM. Also the NextGenRAM cell stops functioning, after 2000 writes are performed to the cell. Then which of the following statement is TRUE?

I. There is an advantage of NextGenRAM over DRAM.
II. There is an advantage of NextGenRAM over SRAM.

(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

24. Which of the following statement is FALSE?

(A) If the pipeline depth increases, then the latency to process a single instruction also increases.
(B) Having a 32-bit wide data bus to memory is a micro-architecture level design choice.
(C) Having a predicted instruction execution is ISA level design choice.
(D) A 2-level global branch prediction is ISA (Instruction set Architecture) level Design choice.

25. Consider the following languages:
$L_1 = \{a^n b^n \mid n \in N, N \text{ is a set of natural numbers}\}$
$L_2 = \{b^n a^n \mid n \in N, N \text{ is a set of natural numbers}\}$
Then the language $L_1 \cap L_2$ is:
(A) Regular (B) non-regular
(C) CFL but not regular (D) not a CFL

26. Consider a set of Turing decidable languages D and a set of Turing recognizable languages R. Then which of the following is always TRUE?
(A) $D \equiv R$ (B) $D \subset R$
(C) $R \subset D$ (D) $D \not\subset R$

27. Which of the following IPv4 address is used by hosts when they are being booted?
(A) 0.0.0.0 (B) 127.0.0.0
(C) 0.0.0.1 (D) 255.255.255.255

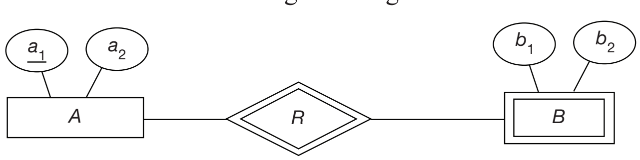
28. Which of the following statement(s) is/are TRUE?
I. A passive hub connects the wires coming from different branches.
II. A repeater can connect two LANs.
III. A Bridge can check the MAC address contained in the frame to forward or drop it.
(A) I, II only (B) II, III only
(C) I, III only (D) I, II, III

29. A Hash table T with 40 slots, stores 5000 elements, what is load factor ($\propto$)?
(A) 250 (B) 125
(C) 500 (D) 400

30. Consider the elements 1, 2, 3, 4, 5, 6, 7 construct a complete binary tree with the given elements in the given sequence. To convert the tree into MAX-HEAP what is the number of swaps required?
(A) 7 (B) 5
(C) 4 (D) 3

31. To calculate the value of a^n, by using Divide - and - conquer. Which of the following Recurrence relation shows the procedure?
(A) $T(n) = T(n-1) + T(1)$
(B) $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + C$
(C) $T(n) = T\left(\frac{n}{4}\right) + \log n$
(D) $T(n) = T\left(\frac{n}{2}\right) + C$

32. Consider the following ER-Diagram:



For a weak entity set to be meaningful, it must be associated with another entity set, that is called
(A) Neighbor set (B) Strong Entity set
(C) Weak Entity set (D) None of the above

33. What does the following SQL query return?
SELECT *
FROM Student, Department.
(A) It returns the Natural JOIN of student and department.
(B) It returns the CROSS PRODUCT of student and department.
(C) It returns the JOIN of student and department.
(D) It returns the UNION of student and department.

34. Which of the following is/are functionally complete set of Boolean operators?
(A) (NOT, XOR) (B) (AND, OR)
(C) (NOT, OR) (D) All the above

35. Sensors are used to monitor the pressure and the temperature of a chemical solution stored in a boiler. The circuitry for each sensor produces a HIGH voltage (Logic 1) when a specified maximum value is exceeded. An alarm requiring a Low voltage (Logic 0) input must be activated when either the pressure or the temperature is excessive. The circuit for this application uses _______ gate.
(A) NAND (B) NOR
(C) AND (D) XOR

36. The maximum volume of a cylindrical cistern with open top of surface area (Area of bottom and side) 108π square feet is __________
(A) 216π cubic feet (B) 432 cubic feet
(C) 864 cubic feet (D) 1728 cubic feet

37. If the eigenvalues of a 3×3 matrix A are -1, 2 and 5 then the determinant of the inverse of the matrix $A + 2I_3$, where I_3 is the identity matrix of order 3×3 is _____
(A) $\frac{-1}{10}$ (B) $\frac{1}{10}$
(C) $\frac{-1}{28}$ (D) $\frac{1}{28}$

38. In a city, 60% of the youngsters are engineering graduates and 40% of the youngsters are science graduates. Among the engineering graduates, 45% are self employed where as 30% of the science graduates are self employed. If an youngster is selected at random from that city, who is self employed, then the probability that the person is a science graduate is _____________

39. Which of the following statements is TRUE?
(A) The number of functions that can be defined from $A = \{2, 3, 5, 7, 9\}$ to $B = \{6, 4, 5, 3, 1\}$ is zero.
(B) The number of one-one functions that can be defined from $A = \{a, b, c, d, e\}$ to $B = \{3, 5\}$ is 32.
(C) The number of onto functions that can be defined from $A = \{p, q, r\}$ to $B = \{1, 2, 3, 4, 5\}$ is 60.
(D) The number of bijections that can be defined from $A = \{a, b, c, d, e\}$ to $B = \{p, q, r, s, t\}$ is 120.

40. Match each of the graphs given in Group-I with its chromatic number in Group-II

Group-I	**Group-II**
P: A cycle graph of order 5	1. 2
Q: A wheel graph of order 6	2. 3
R: A bipartite graph of order 7	3. 4
S: A tree of order 8	4. 5

(A) P–1, Q–2, R–3, S–4
(B) P–3, Q–1, R–1, S–2
(C) P–2, Q–3, R–1, S–1
(D) P–2, Q–3, R–4, S–1

41. Consider the code segment:
int fun (int x)
{if ($x <= 1$) return 1; $x = (x - 1)(x - 1) - 2 - x * x + 3 * x$;
fun (x); print$f$ (“%d”, x)}
What is the output when fun(6) is called?
(A) 5 5 5 5 5 (B) 1 1 1 1 1
(C) 5 4 3 2 1 (D) 1 2 3 4 5

42. Consider below process table:

Process	Arrival time	Burst time
P_1	1	6
P_2	2	10
P_3	4	14
P_4	8	12

If the Longest remaining time First scheduling (LRTF) algorithm is used then the completion time of process P_3 is ______.

43. Consider a system with physical address “F” bits, logical address “L” bits and the page size “P” Mb. The size of page table (in bits) is ______.
(A) $[F - 20 - \log_2 P][2^{L-(20+\log_2 P)}]$ bits
(B) $[P - 20 - \log_2 F][2^{P-(20+\log_2 F)}]$
(C) $[F - 20 - \log_2 F][2^{P-(20+\log_2 P)}]$
(D) Date Inadequate

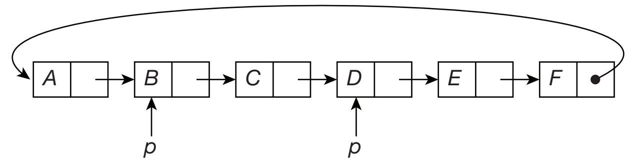
44. Consider a system with five processes P_1, P_2, P_3, P_4, P_5 with resources R_1, R_2, R_3, R_4, R_5:

	Allocated resources $R_1\ R_2\ R_3\ R_4\ R_5$	Maximum resources needed $R_1\ R_2\ R_3\ R_4\ R_5$
P_1	1 0 0 1 2	1 1 1 1 2
P_2	2 1 0 1 1	3 2 1 1 1
P_3	1 2 3 0 1	2 2 3 0 1
P_4	0 1 1 1 0	1 2 2 1 0
P_5	0 0 0 1 0	1 1 1 1 0

Which of the following resources availability makes above system deadlock free?

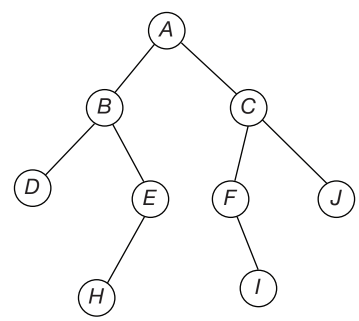
	R_1	R_2	R_3	R_4	R_5
(A)	1	0	0	0	0
(B)	0	0	0	1	0
(C)	0	0	0	0	1
(D)	0	0	1	0	0

45. Consider below linked list:



struct SL
{int data;struct SL *link;} ;
“p” and “q” are pointers pointing to the nodes as shown in above figure. Consider the code snippet:
struct node * temp;
$q = q \rightarrow$ link;
$q \rightarrow$ link $\rightarrow$ link $\rightarrow$ link $= p \rightarrow$ link $\rightarrow$ link;
$p \rightarrow$ link = NULL;
The above code results in:
(A) The list that remains same
(B) deletion of two elements.
(C) deletion of the node to which pointer “p” points.
(D) deletion of the node to which pointer “q” is pointing

46. Consider below binary tree:




```
Consider the routine fun ()
    struct BT
    {
    struct BT *left;
    int data;
    struct BT *right;
    };
struct BT* fun (struct BT * t)
    {
      if (t)
      {
        fun (t → left);
        printf ("%d", t → data);
        fun (t → right);
        printf("%d", t → data);
      }
    }
```

if root node is passed as a parameter to fun ( ), the output is:

(A) $D\,B\,H\,H\,E\,E\,B\,D\,A\,A\,C\,F\,F\,I\,I\,C\,J\,J$
(B) $A\,B\,B\,A\,D\,D\,E\,E\,H\,H\,C\,C\,F\,F\,I\,I\,J\,J$
(C) $D\,D\,B\,H\,H\,E\,E\,B\,A\,F\,I\,I\,F\,C\,J\,J\,C\,A$
(D) $D\,D\,B\,B\,H\,H\,E\,E\,A\,F\,F\,I\,I\,J\,J\,C\,C\,A$

47. Consider the grammar given below:
$A \rightarrow AA + | AA * | a$.
The grammar is:
(A) $LL(1)$
(B) $SLR(1)$ but not $LL(1)$
(C) $LALR(1)$ but not $SLR(1)$
(D) $LR(1)$ but not $LALR(1)$

48. Consider the following SDT:
$X \rightarrow a\,X$ {print "x"}
$X \rightarrow b\,X$ {print "y"}
$X \rightarrow a$ {print "z"}
$X \rightarrow b$ {print "z"}
If the input is $a\,a\,b\,b\,a\,a\,b$ then the output is:
(A) $z\,x\,y\,x\,x\,y\,x$ (B) $x\,y\,x\,y\,x\,x\,z$
(C) x $x\,y\,y\,x\,x\,z$ (D) $z\,x\,x\,y\,y\,x\,x$

49. Consider the following code segment (which is in a high-level language):

```
int a [1000];
int S1 = S2 = S3 = S4 = 0;
for (i = 0; i < 1000; i++)// loop 1
     a[i] = i + 1;
   for (i = 0;i < 1000; i++) //loop 2
   {
   if(i % 4 = = 0) // cond 1
     S1 + = a[i];
   else
     S2 + = a[i];
   if(i % 2 = = 0) // cond 2
     S3 + = a[i];
   else
     S4 + = a[i];
   }
```

What are the branch prediction accuracies of loop 2, cond 1, cond 2 using a 2-bit counter-based prediction scheme (Assume that initially Predictor has False)?
(A) 100%, 75%, 0% (B) 99.7%, 75%, 50%
(C) 99.7%, 25%, 50% (D) 100%, 25%, 0%

50. Consider a processor $P1$. The CPU has a one-level cache. The cache has 128 Bytes with a block size of 32 Bytes. The cache uses LRU replacement policy. The cache is direct mapped cache. A program is executed on this processor. The program tests only the memory read performance by issuing read requests to the cache. Initially the cache is empty. The cache accesses below blocks in the order from left to right.
A, B, A, H, B, G, H, H, A, E, H, D, H, G, C, C, G, C, A, B, H, D, E, C, C, B, A, D, E, F.
Each letter represents a unique cache block.
When the cache accessed for these blocks (as mentioned above), below cache misses occurred.
A, B, A, H, B, G, A, E, D, H, C, G, C, B, D, A, F.
By using this, identify which blocks are competing for same cache block?
I. A and B II. A and E
III. D and H IV. C and F
(A) I, II, III, IV (B) II, IV
(C) I, III (D) I, III, IV

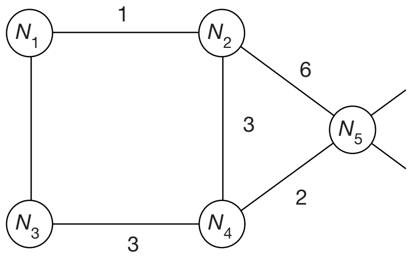
51. Consider below sequence of instructions which are executed on a 5-stage pipeline having the stages:
Instruction Fetch (IF), Instruction Decode and Register Fetch (ID), Execute (EX), Memory access (MA) and Write Back (WB).
ID can happen in First-phase of clock cycle. WB can happen in the second-phase of the clock cycle.
I_1: LOAD R_1, 50(R_6) $R_1 \leftarrow M[50 + R_6]$
I_2: ADD R_2, R_3, R_1 $R_2 \leftarrow R_3 + R_1$
I_3: ADD R_1, R_6, R_4 $R_1 \leftarrow R_6 + R_4$
I_4: STORE R_2, 20(R_4) $M[20 + R_4] \leftarrow R_2$
I_5: AND R_1, R_1 R_4 $R_1 \leftarrow R_1 \wedge R_4$
Each stage requires one clock cycle. If there is no forwarding to hazard detection then the number of stalls that occur during the execution of the program is ________.

52. Which of the following language(s) is/are regular?
L_1 = {ω/ω is the binary representation of a number greater than 3}
L_2 = {ω/ω is some string from L with exactly one character deleted}. (Here L is regular)
$L_3 = \{a^n\, a^n\, a^n \mid n \geq 0\}$
(A) L_1 and L_2 only (B) L_2 and L_3 only
(C) L_1 only (D) L_1, L_2, L_3

53. Consider the following grammar:
$S \rightarrow 0S1 \mid 1X \mid X0$
$X \rightarrow 1X \mid 0X \mid \varepsilon$
Which of the following specifies the language accepted by this grammar?
(A) {ω/ω contains unequal number of 0's and 1's}
(B) {ω/ω contains more number of 1's than 0's}
(C) Complement of the language $\{0^n\, 1^n \mid n \geq 0\}$
(D) {ω/ω contains more number of 0's than 1's}

54. Which of the following language is not context-free but decidable?
I. $\{a^n\, b^m\, a^n\, b^m \mid m, n >= 0\}$
II. {$\omega \in \{a, b\}^*$ | The length of ω is odd and first half contains all a's}
III. {$\omega \in \{a, b\}^*$ | the number of b's in ω is a multiple of the number of a's in ω}
IV. {$\omega \in \{a, b\}^*$ | the number of times 'ab' appears as a substring is equal to the number of times 'ba' appears as a substring}
(A) I, III only (B) II, IV only
(C) I, II, III (D) I, II, III, IV

55. Consider the network shown below:



Assume that each node knows the cost to reach its neighbours. By using Distance-vector algorithm, what could be the forwarding table at node N_5?

(A)

Destination	Next hop
N_1	N_2
N_2	N_2
N_3	N_4
N_4	N_4

(B)

Destination	Next hop
N_1	N_4
N_2	N_2
N_3	N_2
N_4	N_4

(C)

Destination	Next hop
N_1	N_4
N_2	N_4
N_3	N_4
N_4	N_4

(D)

Destination	Next hop
N_1	N_2
N_2	N_4
N_3	N_4
N_4	N_2

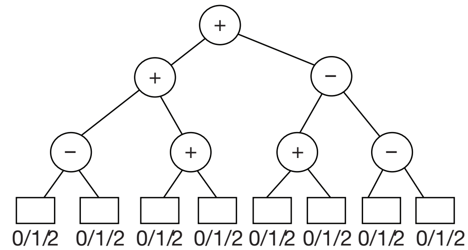
56. Which of the following IP addresses belong to the subnet 137.132.96.0/20?

(i) 137.132.100.0 (ii) 137.132.115.255
(iii) 137.132.102.255 (iv) 137.132.111.0

(A) (i), (ii), (iii), (iv) (B) (i), (iii), (iv)
(C) (iii), (iv) (D) (i), (iii)

57. Consider the Transmission of a 10000 bit frame on 10 Mbps channel with a propagation delay of 250 ms. The sender window size is 7 and probability of error in single frame is 0.001 then the channel utilization for go-back-N protocol (in percentage) is ________ .

58. Consider the given expression tree, Each leaf represents a numerical value, which can be 0 or 1 or 2. What is the maximum possible value of the tree?



(A) 9 (B) 11
(C) 10 (D) 12

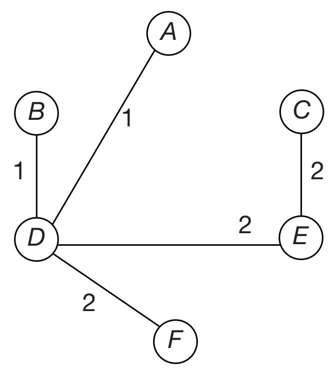
59. Consider the given Recurrence Relation:

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2/\log n$$

What is the time complexity of given recurrence relation?

(A) $\theta(n^2.\log n)$ (B) $\theta(n^3)$
(C) $\theta(n^2.\log.\log n)$ (D) $\theta(n.\log n)$

60. Consider the following weighted undirected graph G,



What is the total weight of minimum spanning tree T, such that the nodes A, B, C must be leaf nodes in the tree T?

(A) 8 (B) 5
(C) 7 (D) 6

61. Consider the following Relation:

Student

RNo	Name

Assume that there are 100 tuples in the student Relation. Both the columns individually contains unique values, then which of the following is correct?

(A) Student Relation is in 1 NF only
(B) Student Relation is in 1 NF, 2 NF only
(C) Student Relation is in 1 NF, 2 NF, 3 NF only
(D) Student Relation is in 1 NF, 2 NF, 3 NF, BCNF

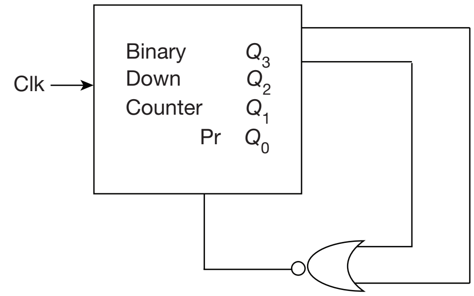
62. In a Relational database there are 3 relations:

Student (Sname)

Department (DName)
Register (Sname, DName)
Which of the following relational algebra expression returns the name of departments that have no students at all?
(A) Department – π_{Dname} (Register)
(B) π_{Dname} Department
(C) Department – π_{Dname} ((Student × Department) – Register)
(D) None of the above

63. Let $f(w, x, y, z) = \Sigma m(0, 2, 5, 6, 7, 8, 9, 10, 11, 13, 15)$. Which of the following expressions are not equivalent to f?
(P) $w^1x^1z^1 + w^1yz^1 + xz + wx^1$
(Q) $x^1z^1 + wz + xz + w^1x$
(R) $wz + xz + x^1z^1 + w^1yz^1$
(S) $x^1z^1 + xz + w^1xy + wx^1$
(A) P and Q (B) R only
(C) Q and S (D) Q only

64. A Binary down sequence, asynchronous counter with synchronous and active high present input is shown in following figure. With the decoding logic shown the counter works as a (consider initially all flip flops at preset state)



(A) Mod - 3 counter (B) Mod - 12 counter
(C) Mod - 13 counter (D) Mod - 14 counter

65. Plain Text: THIS TASK IS IMPOSSIBLE
Key: WANTED
Convert the above plain text to cipher text with key being WANTED, by using Transposition cipher.
(A) IISA SSSB TIIC HKOE AMBD TSPL
(B) AMBD HKOE TIIC IISA SSSB TSPL
(C) HKOE AMBD TIIC IISA SSSB TSPL
(D) TIIC HKOE AMBD IISA SSSB TSPL

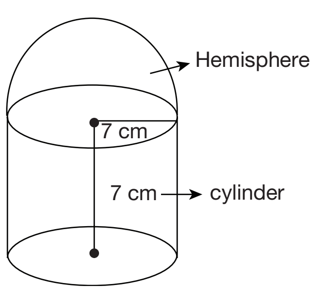
Answer Keys

1. B	**2.** B	**3.** C	**4.** D	**5.** A	**6.** 68	**7.** C	**8.** A	**9.** C	**10.** D
11. B	**12.** 3	**13.** B	**14.** 256	**15.** D	**16.** 10	**17.** A	**18.** 373	**19.** 81	**20.** A
21. C	**22.** C	**23.** C	**24.** D	**25.** A	**26.** B	**27.** A	**28.** C	**29.** B	**30.** C
31. D	**32.** B	**33.** B	**34.** C	**35.** B	**36.** A	**37.** D	**38.** 0.29 to 0.31		**39.** D
40. C	**41.** D	**42.** 42	**43.** A	**44.** A	**45.** B	**46.** C	**47.** B	**48.** D	**49.** B
50. C	**51.** 3	**52.** D	**53.** C	**54.** A	**55.** C	**56.** B	**57.** 1.38	**58.** D	**59.** C
60. A	**61.** D	**62.** A	**63.** D	**64.** C	**65.** C				

Hints and Explanations

1. 'Toxicity' gives the clue to the correct answer. Something which is toxic is detrimental (harmful). Hence it is logical to say that despite its known 'toxicity', lead continues to be used to the detriment of human and environmental health. The word solace (relief; comfort) runs contrary to what is stated in the sentence. The word chagrin (annoyance) is too mild to be used for something which is toxic. The word impediment (hindrance; obstacle) does not make sense here. Choice (B)

2.



Volume of the hemisphere $= \left(\frac{1}{2}\right)\left(\frac{4}{3}\right)\pi r^3$

$= \left(\frac{2}{3}\right)(\pi)(7^3)$

Volume of the cylinder $= \pi r^2 h = 7^3\pi$

Total volume $= (\pi)7^3\left[1 + \frac{2}{3}\right] = \frac{5\pi}{3}(7^3)$ Choice (B)

3. The study of man is called Anthropology. Form the point, "Anthropology is called ornithology", we can say that Ornithology is the word that is used to refer to Anthropology. Choice (C)

4. Biology is the study of life, Mythology is the study of myths. Archaeology is not the study of antiques, it is the specific study of historic or prehistoric peoples and their cultures by analysis of their artifacts, inscriptions, monuments etc, especially those that have been excavated. Astrology is the study that assumes and attempts

to interpret the influence of heavenly bodies on human affairs. Cosmetology is the art or profession of applying cosmetics. Only option D expresses a relationship which is similar to that expressed by the headwords. Choice (D)

5. The reference is to a specific monsoon hence, the definite article 'the' is more appropriate in the first blank. The present perfect continuous tense 'has been showing' is apt in the second blank because here, the reference is to a trend which started sometime in the past and is still continuing. In the third blank, the reference is to something which took place during a specific time period. This is best denoted using the preposition 'over'. The definite article is required in the fourth blank because the reference is to a particular subcontinent. Choice (A)

6. $h\,(12, 11, 8, 16) = 12 \times 11 > 8 \times 16$
$132 > 128$
Remainder when 132 is divided by 128 = 4.
$fg\,(h(12,11,8,16),17,9,16) = fg\,(4,17,9,16)$
$= f\,(4,17,9,16) \times g\,(4,17,9,16)$
$= 17 \times 4 = 68.$
Ans: 68

7. Options A and B run contrary to what is stated. Option D is incorrect because it is not mentioned that the brain and the lymphatic system are connected 'indirectly'. Only option C can be logically inferred from the given sentence. Choice (C)

8. Only statement (i) can be inferred from the given passage. It cannot be inferred from the passage that the government does not consider cyber threats as serious as other security threats. Options (iii) and (iv) are illogicall. Choice (A)

9. There are 33 instances of students leaving out a question (The sum of the 4 numbers in column 5) Exactly 5 students attempted all 4 questions. We distribute there 33 instances over all the remaining 25 students there would be 8 more instances. We can collect them to at the most 8 students.
∴ The maximum possible number of students who left out at least 2 questions is 8. Choice (C)

10. All I, II and III are appropriate and apt courses of action. Choice (D)

11. The number of ways of selecting one letter each from the words 'FRACTION' and 'DECIMAL' is $8 \times 7 = 56$. The selected letter in a word will not be in the other word only if the common letters are not selected.
The common letters in the two words are *A*, *C* and *I*.
The number of ways of selecting the letters from the words such that the selected letter in a word should not be in the other word = (The number of ways of selecting a letter from *F*, *R*, *T*, *O* and *N*) × (The number of ways of selecting a letter from *D*, *E*, *M* and *L*)
$= 5 \times 4 = 20$
∴ The required probability $= \dfrac{20}{56} = \dfrac{5}{14}$. Choice (B)

12. We have $\lim\limits_{x\to 3} \log_3[4x^3 + \sqrt{(3x^4 + 5x^2 + 1)}]$
$= \log_5\left(\lim\limits_{x\to 3}[4x^3 + \sqrt{(3x^4 + 5x^2 + 1)}]\right)$
$= \log_5(108 + \sqrt{289})$
$= \log_5 125$
$= \log_5 5^3$
$= 3$
Ans: 3

13. Given recurrence relation is:
$a_n - a_{n-1} = n^2;\ n \geq 1 \to (1)$
and $a_0 = 2$
$a_n - a_{n-1} = n^2$
$\Rightarrow\ a_n = a_{n-1} + n^2$
$\therefore\ a_1 = a_0 + 1^2$
$a_2 = a_1 + 2^2 = a_0 + 1^2 + 2^2$
$a_3 = a_2 + 3^2 = a_0 + 1^2 + 2^2 + 3^2$
$a_4 = a_3 + 4^2 = a_0 + 1^2 + 2^2 + 3^2 + 4^2$
$a_n = a_{n-1} + n^2 = a_0 + 1^2 + 2^2 + 3^2 + \ldots + n^2$
$= a_0 + \dfrac{n(n+1)(2n+1)}{6}$
$= 2 + \dfrac{n(2n^2 + 3n + 1)}{6}$
$= \dfrac{2n^3 + 3n^2 + n + 12}{6}$ Choice (B)

14. Given $A = \{1, \{2, 3\}, 4\}$
∴ The cardinality of the set $P(A)$ = Numbers of elements in $P(A) = 2^{n(A)} = 2^3 = 8$
∴ The cardinality of $P(P(A)) = 2^{n(P(A))} = 2^8 = 256$.
Ansr: 256

15. We have negation of $(\forall x)\,(P(x) \to Q(x, y))$
$\Leftrightarrow \rceil[(\forall x)\,(P(x) \to Q(x, y))]$
$\Leftrightarrow \rceil(\forall x)\,(\rceil Q(x, y) \to \rceil P(x))$
$(\because A(x) \to B(x) \Leftrightarrow \rceil B(x) \to \rceil A(x))$
which is same as option (A)
$\Leftrightarrow (\exists x)\,\rceil(\rceil(\rceil Q\,(x, y)) \vee \rceil P(x))$
$(\because \rceil(\forall x)\,A(x) \Leftrightarrow (\exists x)\,\rceil A\,(x)$
and $A(x) \to B(x) \Leftrightarrow \rceil A\,(x) \vee B(x))$
$\Leftrightarrow (\exists x)\,(\rceil Q(x, y)\,\rceil P(x))$
$(\because \rceil(\rceil A\,(x)) \Leftrightarrow A(x)$ and $\rceil(A\,(x) \vee B(x))$
$\Leftrightarrow (\rceil A\,(x) \wedge \rceil B(x)))$
$\Leftrightarrow (\exists x)\,(P(x) \wedge \rceil Q\,(x, y))$
$(\because A(x) \wedge B(x) \Leftrightarrow B(x) \wedge A\,(x))$
which is same as option (C)
Also, consider $\rceil((\forall x)\,(P(x) \to Q(x, y)))$
$\Leftrightarrow (\exists x)\,\rceil(P(x) \to Q(x, y))$
Which is same as option (B)
And the predicate formula given in option (D) is NOT equivalent to the negation of $(\forall x)\,(P(x) \to Q(x, y))$
Choice (D)

16. Given program prints number of 1's in the binary representation of given number (i.e., '*x*'). As x =1023 it prints 10. Ans: 10

17. Choice (A)

18. Address (A[5]) = 209 + (50 − (−32) × 2
= 209 + (82) × 2
= 373

Ans: 373

19. Record 9 will start at byte
((9−1) × 10) + 1
= 80 + 1 = 81

Ans: 81

20. Kill any 1 process to make the system safe.

Choice (A)

21. Choice (C)

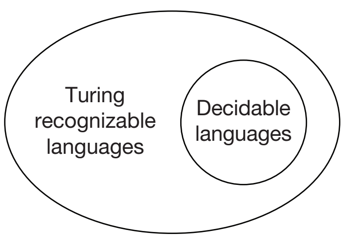
22. Choice (C)

23. The developed NextGenRAM has an advantage over DRAM, as it is non-volatile.
Also NextGenRAM has an advantage over SRAM, as it has lower cost than SRAM. Choice (C)

24. A 2-level global branch prediction is micro-architecture level Design choice. Choice (D)

25. Given, $L_1 = \{a^n\, b^n \mid n \in N\}$
$L_2 = \{b^n\, a^n \mid n \in N\}$
$\Rightarrow \quad L_1 \cap L_2 = \varnothing$
and $\varnothing$ is regular so $L_1 \cap L_2$ is regular. Choice (A)

26. Decidable languages are subset of Turing recognizable languages.



Choice (B)

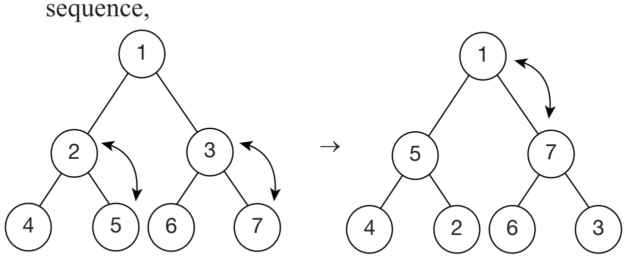
27. The address used by hosts during booting is 0.0.0.0. Choice (A)

28. A repeater can't connect two LANs. It is used to connect two segments of the same LAN. Choice (C)

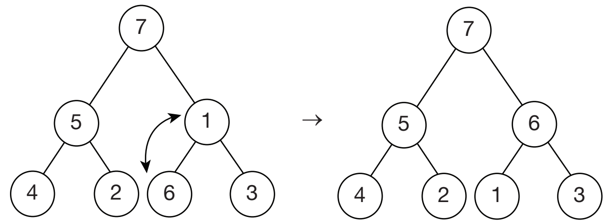
29. Load factor (α) $= \dfrac{\text{Number of elements}}{\text{Number of slots}} = \dfrac{5000}{40} = 125$

Choice (B)

30. Given elements 1, 2, 3, 4, 5, 6, 7
First construct a complete binary tree with the given sequence,



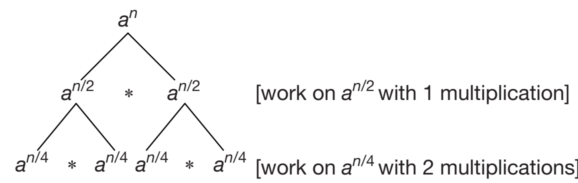
2 - Swaps 1 - Swap



1 - Swap

$\therefore$ 4-Swaps required to convert the tree into MAX-HEAP. Choice (C)

31.



At each level we consider only 'half' problem, so the recurrence relation would be

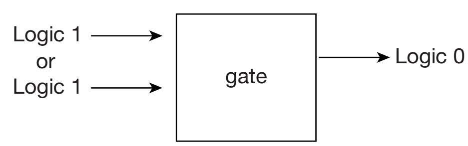
$T(n) = T\left(\dfrac{n}{2}\right) + C$ Choice (D)

32. B is weak entity set, it should be associated with the attributes of strong entity set that is A. Choice (B)

33. If no specific condition is given, the query will return cross product of student and department. Choice (B)

34. Functionally complete set are
(NOT, AND) and (NOT, OR) Choice (C)

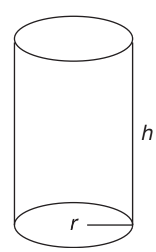
35. The boiler temperature or pressure when exceeds a specific maximum value, then alarm has to be activated. Pressure, temperature sensors produce HIGH voltage (Logic 1) when exceeds maximum value. Alarm requires low voltage (Logic 0) to get activated.



The above circuit can be implemented by NOR gate.

Choice (B)

36.



Let r and h be the radius and height of the cylindrical cistern with surface = 108π sq. feet

$\therefore \quad 2\pi rh + \pi r^2 = 108\pi$

$\Rightarrow \quad 2rh = 108 - r^2$

$\Rightarrow \quad h = \dfrac{108 - r^2}{2r} \rightarrow (1)$

Volume of the cylindrical cistern $= C = \pi r^2 h$

$= \pi r^2 \left(\dfrac{108 - r^2}{2r} \right)$

$\therefore \quad V = \dfrac{\pi}{2}(108r - r^3) \rightarrow (2)$

Let $f(r) = \dfrac{\pi}{2}(108r - r^3)$

$\therefore$ We have to find the maximum value of V.

$f'(r) = \dfrac{\pi}{2}(108 - 3r^2)$

$f'(r) = 0$

$\Rightarrow \quad \dfrac{\pi}{2}(108 - 3r^2) = 0$

$\Rightarrow \quad 108 - 3r^2 = 0$

$\Rightarrow \quad r^2 = 36 \quad \Rightarrow \quad r = 6$

And f'' (r) $= -3\pi r < 0$ for $r = 6$

$\therefore \quad f(r)$ is maximum at $r = 6$

Hence the maximum volume of the cistern

$= V$ at $x = 6$

$= \dfrac{\pi}{2}(108 \times 6 - 6^3)$

$= 216\pi$ cubic feet. Choice (A)

37. Given –1, 2 and 5 are the eigenvalues of A.

$\therefore \quad -1 + 2, 2 + 2$ and $5 + 2$ are the eigenvalues of $A + 2I_3$

i.e., 1, 4 and 7 are the eigenvalues of $A + 2I_3$

$\therefore$ Det $(A + 2I_3) = |A + 2I_3|$ = Product of the eigenvalues of $A + 2I_3$

$= 1 \times 4 \times 7 = 28$

$\Rightarrow$ Determinant of inverse of $A + 2I_3$

$= |(A + 2I_3)^{-1}| = \dfrac{1}{|A + 2I_3|} = \dfrac{1}{28}$ Choice (D)

38. Let B_1 and B_2 denote the events of selecting an engineering graduate and a science graduate respectively.

Let A be the event of selecting a youngster who is self employed.

$\therefore \quad P(B_1) = \dfrac{60}{100} = 0.6, P(B_2) = \dfrac{40}{100} = 0.4$

$P(A/B_1) = \dfrac{45}{100} = 0.45$ and $P(A/B_2) = \dfrac{30}{100} = 0.3$

If the youngster selected is found to be self employed, then the probability that the person is a science graduate

$$= P(B_2/A) = \frac{P(B_2) \cdot P\left(\frac{A}{B_2}\right)}{P(B_1) \cdot P\left(\frac{A}{B_1}\right) + P(B_2) \cdot P\left(\frac{A}{B_2}\right)}$$

(By Baye's Theorem)

$$= \frac{0.4 \times 0.3}{(0.6 \times 0.45) + (0.4 \times 0.3)} = \frac{12}{39} = 0.3077$$

Ans: 0.29 to 0.31

39. We know that, one can define a function from any finite non-empty set to any finite non-empty set.

One can define a one-one function from A to B, only if $n(A) \leq n(B)$.

One can define an onto function from A to B only if $n(A) \geq n(B)$.

Hence options (A), (B) and (C) are not true.

One can define a bijection from A to B, if $n(A) = n(B)$.

$\therefore$ The number of bijections from $A = \{a, b, c, d, e\}$ to $B = \{p, q, r, s, t\}$ is $5! = 120$. Choice (D)

40. We know that the chromatic number of a cycle graph of odd order is 3, a wheel graph of even order is 4 and a bipartite graph as well as a tree is 2.

$\therefore$ The correct matching is

$P - 2, Q - 2, R - 1$ and $S - 1$ Choice (C)

41. fun (6) prints 1 2 3 4 5. Choice (D)

42.

	P_1	P_2	P_3	P_4	P_3	P_4	P_3	P_4	P_2	P_3	P_4	P_2	P_3	P_4	P_2	P_3	P_4	P_1	P_2	P_3	P_4	

0 1 2 4 8 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27

In this way process P_3 completes 42 Ans: 42

43. Page size = ∀P" Mb

$= 2^{20+\log_x^P}$

Bits required for page $= (20 + \log_2 P)$ bits

Number of frames $= 2^{[F-(20+\log_2 P)]}$

Number of pages $= 2^{[L-(20+\log_2 P)]}$

Page table size $= [F - 20 - \log_2 P] * 2^{[L-(20+\log_2 P)]}$ bits

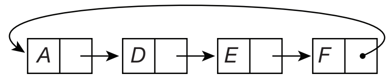
Choice (A)

44. Need matrix = max – Allocated

	R_1	R_2	R_3	R_4	R_5
P_1	0	1	1	0	1
P_2	1	1	1	0	0
P_3	1	0	0	0	0
P_4	1	1	1	0	0
P_5	1	1	1	0	0

Choice (A)

45. It deletes two elements. The resultant list will be

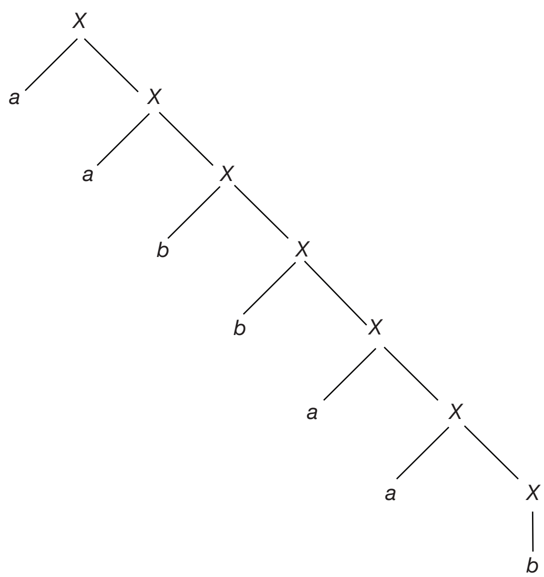


Choice (B)

46. Choice (C)

47. Given grammar is SLR(1) but not LL(1). Choice (B)

48.



Choice (D)

49. Using a 2-bit counter-based prediction scheme, the predictor will be initially False.
This scheme needs two consecutive wrong predictions to change its prediction.
For loop 2, first two predictions are F and F. And in 3rd prediction it predicts T. This prediction continues till $i = 1000$. But this is wrong prediction.

$\therefore$ In total 1001 predictions, 3 are wrong predictions.

$\therefore$ prediction accuracy for loop 2 is $\frac{998}{1001} \times 100 = 99.7\%$

For cond 1, which is tested for 1000 times, correct predictions using 2-bit predictor is 750.
Initially the predictor has Not-taken. (NT). Then the 2-bit predictor will not change to Taken, until two successive mis-predictions happen. But this will never happen.
The predictor mis-predicts 250 times. (1000/4 = 250).

$\therefore$ Cond 1 prediction accuracy $= \frac{750}{1000} \times 1000 = 75\%$.

For cond 2, which is tested for 1000 times, correct predictions using 2-bit predictor is 500.
Initially the predictor has Not-taken. Then the 2-bit predictor will not change to taken until two successive Mis-predictions happen. But this will never happen.

$\therefore$ The predictor Mis-predicts 500 times. (1000/2 = 500).

$\therefore$ Cond 2 prediction accuracy $= \frac{500}{1000} \times 100 = 50\%$.

Choice (B)

50. Required cache blocks by the program are:
A, *B*, *A*, *H*, *B*, *G*, *H*, *H*, *A*, *E*, *H*, *D*, *H*, *G*, *C*, *C*, *G*, *C*, *A*, *B*, *H*, *D*, *E*, *C*, *C*, *B*, *A*, *D*, *E*, *F*.
Cache has 128 Bytes.
Block size = 32 Bytes.

Blocks in cache $= \frac{128}{32} = 4$.

Misses occurred for the blocks:
A, *B*, *A*, *H*, *B*, *G*, *A*, *E*, *D*, *H*, *C*, *G*, *C*, *B*, *D*, *A*, *F*.
As there is a miss for *A* (in 3rd position), we can understand that both *A* and *B* are competing for same block location.
D and *H* are competing for same block (as after accessing *D*, we are getting a miss for *H*).
C and *G* are competing for same block (as after accessing *C*, we are getting a miss for *G*).
E and *F* are competing for the same block.

Choice (C)

51. The pipeline diagram during the execution of the program is shown below:

	1	2	3	4	5	6	7	8	9	10	11	12
I_1	IF	ID	EX	MA	WB							
I_2		IF	Stall	Stall	ID	EX	MA	WB				
I_3			IF			ID	EX	MA	WB			
I_4				IF			Stall	ID	EX	MA	WB	
I_5					IF				ID	EX	MA	WB

$\therefore$ 3-stalls occur during the execution.
I_2 stalled till R_1 value is available. I_4 stalled till R_2 is available.

Ans: 3

52. L_1 is regular and its regular expression is
$0^* \, 1 \, (0 + 1) \, (0 + 1) \, (0 + 1)^*$
L_2 is regular. We can check this by taking an example regular language.

If L contains even number of 1's then L_2 contains odd number of 1's (by deleting '1' from all the strings of L). L_3 is regular. It can be written as, $\{a^{3n} \mid n \geq 0\}$.
i.e., L_3 contains number of a's which is a multiple of a.

Choice (D)

53. Given grammar,
$S \rightarrow 0S1 \mid 1X \mid X0$
$X \rightarrow 1X \mid 0X \mid \varepsilon$
Let us derive some strings from given productions:

$S \rightarrow 0S1$
$\rightarrow 01X1$
$\rightarrow 011$
$n_0(\omega) < n_1(\omega)$

$S \rightarrow 0S1$
$\rightarrow 0\,X\,01$
$\rightarrow 001$
$n_0(\omega) > n_1(\omega)$

$S \rightarrow 0S1$
$\rightarrow 01X1$
$\rightarrow 010X1$
$\rightarrow 010X1$
$\rightarrow 0101 \Rightarrow n_0(\omega) = n_1(\omega)$

$S \rightarrow 1X$
$\rightarrow 11X$
$\rightarrow 11$

The strings can start with either 0 or 1. From these derivations, we can conclude that given grammar is complement of $\{0^n\, 1^n \mid n \geq 0\}$

Choice (C)

54. $\{a^n\, b^m\, a^n\, b^m \mid m, n >= 0\}$
This language is decidable but not context-free. The PDA can't check this kind of equality. (Let the PDA pushed n a's and m b's onto the stack but next it has to compare m b's with n a's. So this is not CFG).
$\{\omega \in \{a, b\}^* \mid$ the length of ω is odd and the first half is all a's$\}$:
We can design a FA which checks whether the length of a string is odd or not. So is CFG. But checking whether the first half is all a's or not is done by using a stack memory.
Hence this is CFL.
$\{\omega \in \{a, b\}^* \mid$ the number of b's in ω is a multiple of the number of a's in $\omega\}$:
This is not CFL. But is decidable. A non-deterministic TM is required to guess the multiple k such that the number of b's is k times the number of a's.
$\{\omega \in \{a, b\}^* \mid$ the number of times 'ab' appears as a substring is equal to the number of times 'ba' appears as a substring$\}$:
This is regular. The regular expression is given as $\varepsilon + a(a + bb^*\, a) + b(b + aa^*\, b)^*$.
Hence the correct choice is (A).

Choice (A)

55. Using Distance vector routing, the initial distance table entries are shown below:

Neighbours	N_1	N_2	N_3	N_4	N_5
N_2	∞	∞	∞	∞	∞
N_4	∞	∞	∞	∞	∞
N_5	∞	6	∞	2	0

Neighbours	N_1	N_2	N_3	N_4	N_5
N_2	1	0	∞	3	6
N_4	∞	3	3	0	2
N_5	7	5	5	2	0

Neighbours	N_1	N_2	N_3	N_4	N_5
N_2	1	0	3	3	5
N_4	4	3	3	0	2
N_5	6	5	5	2	0

Neighbours	N_1	N_2	N_3	N_4	N_5
N_2	1	0	3	3	5
N_4	4	3	3	0	2
N_5	6	5	5	2	0

For warding table of N_5:

Destination	Next hop
N_1	N_4
N_2	N_4
N_3	N_4
N_4	N_4

Choice (C)

56. Given subnet 137.132.96.0/20.
IP address is 137.132.96.0
Its Binary equivalent is
10001001.10000100.01100000.00000000
Subnet mask is /20. i.e., the IP address of mask is
11111111.11111111.11110000.00000000
So the address range using this mask is 137.132.96.0 to
10001001.10000100.01101111.11111111
i.e., from 137.132.96.0 to 137.132.111.255
only (i), (iii) and (iv) are in this range.

Choice (B)

57. Channel utilization for go-back-N protocol is

$$= \frac{N(1 - P)}{(1 + 2a)(1 - P + NP)}$$

N is window size.
P is probability of error in a frame

$$a = \frac{\text{Propagation delay}}{\text{Transmission delay}}$$

$$\text{Transmission delay} = \frac{10000}{10 \times 10^6} = 1 \text{ msec}$$

$$a = \frac{250}{1} = 250$$

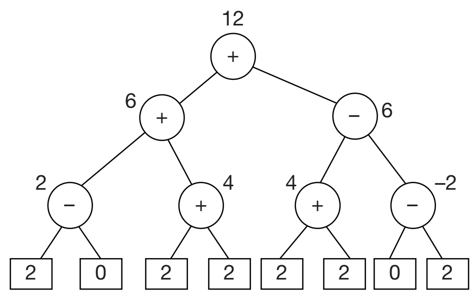
Channel utilization

$$= \frac{7 \times (1 - 0.001)}{(1 + 2 \times 250)(1 - 0.001 + 7 \times 0.001)} = \frac{6.993}{504.006}$$

$= 0.0138$

$\cong 1.38\%$ Ans: 1.38

58.



The maximum possible value of the tree is 12.

Choice (D)

59. $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n^2}{\log n} \Rightarrow 4T\left(\frac{n}{2}\right) + n^2 \log^{-1} n$

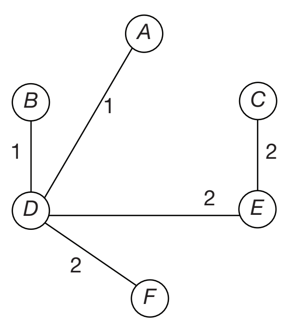
Compare the given Relation with

$T(n) = aT\left(\frac{n}{b}\right) + n^k \log^p n$

If $p = -1$, then according to master method time complexity is $T(n) = \theta(n^{\log_b^a} \cdot \log \cdot \log n)$

$= \theta(n^2 \cdot \log \cdot \log n)$ Choice (C)

60. Minimum spanning Tree T:



Leaf node:

The degree of node must be 1.

In the Tree T (A, B, C) nodes are leaf nodes.

Total weight = 2 + 2 + 2 + 1 + 1 = 8 Choice (A)

61. A Relation with only two attributes with unique values will be in 1 NF, 2 NF, 3 NF, BCNF.

Student

RNo	Name

Case 1:

It can have one FD

RNo → Name

It is in 1 NF, 2 NF, 3 NF, BCNF.

Case 2:

It can have one FD

Name → RNo

It is in 1 NF, 2 NF, 3 NF, BCNF.

Case 3:

It can have 2 FDs.

RNo → Name

Name → RNo

It is in 1 NF, 2 NF, 3 NF, BCNF Choice (D)

62. Assume some data:

Student

SName
A
B
C

Department

DName
Sales
Tax
Finance

Register

Sname	DName
A	Sales
B	Tax

Department π_{Dname} (Register)

DName
Sales
Tax
Finance

−

DName
Sales
Tax

=

DName
Finance

In the Finance department, there are no students.

Choice (A)

63. Minterms of P

$w^1x^1z^1 + w^1yz^1 + xz + wx^1$

00×0 0×10 ×1×1 10××

0, 2 2, 6 5,7,13,15 8,9,10,11

$\Sigma m(0, 2, 5, 6, 7, 8, 9, 10, 11, 13, 15)$

min terms of Q

$x^1z^1 + wz + xz + w^1x$

$x0x0$ $1xx1$ $x1x1$ $01x\,x$

0, 2, 8, 10 9, 11, 13, 15 5, 7, 13, 15 4, 5, 6, 7

$\Sigma m(0, 2, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15)$

which is not equal to f

Choice (D)

64.

Clk	Q_3	Q_2	Q_1	Q_0	Preset = $\overline{Q_3 + Q_2}$
0	1	1	1	1	0
1	1	1	1	0	0
2	1	1	0	1	0
:	:	:	:	:	:
:	:	:	:	:	:
10	0	1	0	1	0
11	0	1	0	0	0
12	0	0	1	1	1
13	1	1	1	1	0

Active high preset, so when output of NOR gate is 1, then preset activated, and output becomes 1111 in next clk pulse (synchronous input)

From the above table, we can understand that after 13 clk pulses the counter came to original state. So it is Mod 13 counter Choice (C)

65.

6	1	4	5	3	2
W	*A*	*N*	*T*	*E*	*D*
T	*H*	*I*	*S*	*T*	*A*
S	*K*	*I*	*S*	*I*	*M*
P	*O*	*S*	*S*	*I*	*B*
L	*E*	*A*	*B*	*C*	*D*

Give the numbering to the letters in the keyword. Plain text is written as it is. If some gaps are there, fill it with *A*, *B*, C, ..., *Z*

In the cipher text, the letters under Number 1 column would appear first then Number 2 column and so on

1. HKOE 2. AMBD
3. TIIC 4. IISA
5. SSSB 6. TSPL Choice (C)

Mock Test 5

Number of Questions: 65 **Total Marks: 100**

Wrong answer for MCQ will result in negative marks, (–1/3) for 1 Mark Questions and (–2/3) for 2 Marks Question.

General Aptitude

Number of Questions: 10 **Section Marks: 15**

Directions for question 1: Choose the most appropriate word from the options given below to complete the following sentence:

1. If I ___________ you I would not have taken the help of an outsider to solve my personal problems.
(A) was (B) were
(C) am (D) will be

Directions for questions 2 and 3: Select the correct alternative form the given choices.

2. Ram and Shyam started simultaneously from two different stations towards each other with speeds of x kmph and y kmph respectively. To cross each other, Ram travelled y times the distance travelled by Shyam. If the speed of Ram is 4 kmph, then the speed (in kmph) of Shyam is ______.

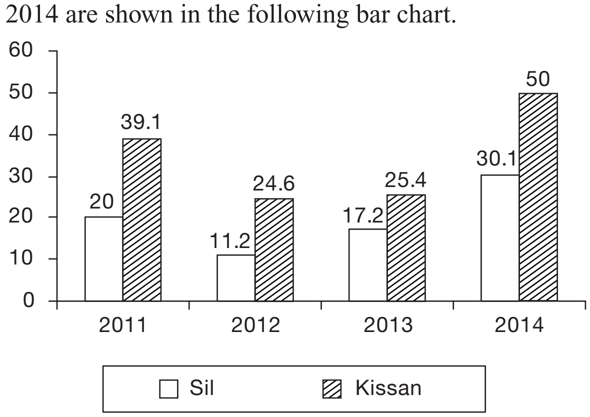
3. How is Khadar's wife's daughter's mother's daughter-in-law's husband's father related to Khadar?
(A) Grand-father (B) Father
(C) Father-in-law (D) Himself

Directions for question 4: Which one of the following combinations is incorrect?

4. (A) Beatific–Mundane
(B) Empirical–Experiential
(C) Gaunt–Emaciated
(D) Momentous–Critical

Directions for question 5: Select the correct alternative form the given choices.

5. The sales (in crores of ₹) of Kissan and Sil Mixed Fruit jams in Khaogali in each of the years from 2011 to 2014 are shown in the following bar chart.



The ratio of sales of Kissan to that of Sil is the highest in ______.
(A) 2012 (B) 2011
(C) 2013 (D) None of these

Directions for question 6: Select the alternative meaning of the underlined part of the sentence:

6. The government officials have promised the moon on the issue of regulation for industrial relations and so, have decided not to sign any new ventures.
(A) passed the buck
(B) broadened their horizons
(C) stood their ground
(D) heard something on the grapevine

Directions for question 7: The given statement is followed by some course of action. Assuming the statement to be true, decide the correct option:

7. Healthcare workers often reuse syringes or needles for multiple uses which increases the chance of infection and transmission of ailments, thus exposing people to a host of diseases from clinics, nursing homes and hospitals.
(i) Hospitals must encourage staff to incorporate smart disposal techniques.
(ii) Healthcare workers and patients must be made aware of WHO policy guidelines on safe injection practices.
(iii) Patients acquiring diseases from hospitals and nursing homes must be treated free of cost.
(iv) The government of India must make it mandatory for hospitals to switch from disposable syringes to Auto Disposable (AD) syringes.
(A) (i) and (iii) (B) (ii) and (iii)
(C) (i) and (ii) (D) (ii) and (iv)

Directions for questions 8 and 9: Select the correct alternative form the given choices.

8. Evaluate $\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5...}}}}$.

(A) $\dfrac{\sqrt{13}-1}{2}$ (B) $\dfrac{\sqrt{17}-1}{2}$

(C) $\dfrac{\sqrt{17}+1}{2}$ (D) $\sqrt{17}$

9. America had entered the world war since Japan had attacked Pearl Harbour.
Which one of the statements below is logically valid and can be inferred from the above sentence?

(A) Japan was feeling restless.
(B) America would not have entered the world war, if Japan would not have attacked Pearl Harbour.
(C) Japan and America are enemies.
(D) None of these

Directions for question 10: Out of the four sentences, select the most suitable sentence with respect to grammar and usage:

10. (A) Today's tip would have been sufficient to buy a full meal three years ago.
(B) Todays tip would pay for a full meal three years ago.
(C) Today's tip would be sufficient for a three- years-ago meal.
(D) A tip today would costed one a meal three years back.

Computer Science Engineering

Number of Questions: 55 **Section Marks: 85**

Directions for questions 11 to 65: Select the correct alternative from the given choices

11. The non-identity element (the element other than the identity element) which is the inverse of itself in the abelian group (G, X_7) with $G = \{1, 2, 3, 4, 5, 6\}$ under the binary operation of "multiplication module 7" is ______.

12. The value of the definite integral $\int_0^{\pi} \sin^8 x\, dx$ is______.

(A) $\frac{7^2.5^2.3^2.1^2}{8!}\pi$ (B) $\frac{8^2.6^2.4^2.2^2}{7!}\pi$

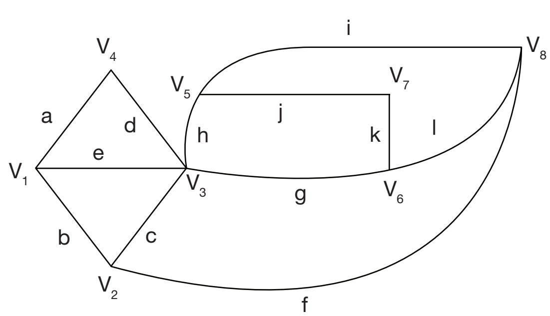
(C) $\frac{7^2.5^2 3^2.1^2}{8!}\cdot\frac{\pi}{2}$ (D) $\frac{8^2.6^2.4^2.2^2}{7!}\cdot\frac{\pi}{2}$

13. In the LU decomposition of a matrix $A = \begin{bmatrix} 3 & 1 & 2 \\ 6 & 4 & 5 \\ 9 & 7 & 11 \end{bmatrix}$ with each of the principal diagonal element of L being equal to 1, the matrix L is equal to _____.

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

14. Consider the graph G given below:



G

Which of the following sets of edges is NOT a perfect matching of G?
(A) $\{a, c, i, k\}$ (B) $\{d, e, f, j\}$
(C) $\{b, d, j, l\}$ (D) $\{a, f, g, j\}$

15. The number of non-negative integral solutions of the inequality $x_1 + x_2 + x_3 + x_4 < 13$ is ______.

16. Consider the grammar ('X' is start symbol)
$X \rightarrow Y Z a \mid Z$
$Y \rightarrow S Y \mid \in$
$Z \rightarrow c \mid \in$
$S \rightarrow s \mid b$
The follow(S) is:
(A) $\{c, s, b\}$
(B) $\{s, b, \$\}$
(C) $\{s, b\}$
(D) $\{s, b, c, a\}$

17. Consider the code:
int x, y;
$x = y + 15$;
Checking the type of variable while assigning in the Code is done during:
(A) Run time
(B) Load time
(C) Compile time
(D) Link time

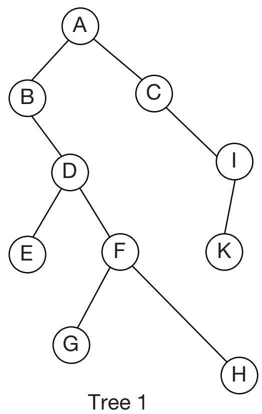
18. Consider a system with five processes and a single resource of multiple instances.

	Allocation	Maximum needed
P_1	2	4
P_2	2	3
P_3	4	10
P_4	3	8
P_5	1	6

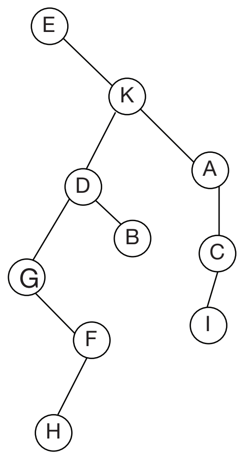
Then minimum number of resources need to be available, for the system to be in safe state is ______.

19. Consider a counting semaphore value as 25, if 33 down operations are performed followed by 40 up operations, then resultant value of semaphore is ______.

20. Consider below trees:



Tree 1



Tree 2

Which traversal of Tree 1 and Tree 2 will produce same sequence?

(A) Pre order, Post order (B) Post order, In order
(C) In order, Post order (D) Post order, Pre order

21. Number of possible permutations that can be obtained using stack if the input sequence is 1, 2, 3, 4, 5 (in the order) is ______.

22. The maximum number of elements in a heap of height 10 (Assume height of root node as 0) is ______.

23. Consider a branch predictor which uses a Branch History Table (BHT). Program Counter (PC) uses 8-bits to select the BHT entry and the history covers the last 8 branches and uses 2-bit predictor. The number of bits required for the storage of BHT is __________.

24. Consider a main memory which has 32-bit address. There is a 4-way set-associative cache. 5-bits of main memory address are used for set index and 4-bits required for Byte offset. The number of bytes required for cache data portion is __________.

25. Consider the regular expression:
$2^* (1 + \varepsilon) (01)^* (0 + \varepsilon)$,
over the alphabet $\Sigma = \{0, 1, 2\}$. Which of the following gives the language specified by given Regular expression?

(A) $\{w|$ the start and end symbols of w are not same$\}$
(B) $\{w|$ every 0 that is not right most symbol is immediately followed by a 1 and every 1 that is not rightmost symbol is immediately followed by a 0$\}$
(C) $\{w|$ There is an equal number of 0's and 1's in $w\}$
(D) $\{w|$ every 0 that is not right most symbol is immediately followed by a 1$\}$

26. Which of the following language(s) is/are closed under union, intersection and complementation?

(i) Regular language
(ii) Context-free language
(iii) Recursive language
(iv) Recursively enumerable language

(A) (i) only (B) (i), (iii) only
(C) (i), (iii), (iv) only (D) (i), (ii), (iii), (iv)

27. Consider 8 stations numbered 1 to 8, whose addresses are 10101, 01100, 11101, 00010, 00101, 01010, 11100, 10011 respectively. These 8 stations want to send data through a single channel using Binary countdown protocol. Then the station which starts transmission of data out of all 8 stations is __________.

28. Using RSA public key cryptography, if $p = 7$, $q = 11$ and $d = 7$ then which of the following is a valid 'e' value?

(A) 108 (B) 170
(C) 43 (D) 10

29. Which of the following algorithms sort 'n' integers having the range (1 to n^2), in ascending order in $O(n)$ time?

(A) Radix sort (B) Selection sort
(C) Merge sort (D) Quick sort

30. Which of the following is FALSE about Weak Entity?

I. A Weak Entity set has no primary keys unless attributes of the strong entity set on which it depends are included.
II. Weak entities can be deleted automatically when their strong entity is deleted.

(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

31. Which of the following is asymptotically smaller?

(A) $\log_2 (n!)$ (B) $\log_2 (\log n)$
(C) $\log (\log n^2)$ (D) $\log (\log_2 n!)$

32. $\pi_A (\sigma_B (R \times S))$ is equivalent to which of the following?

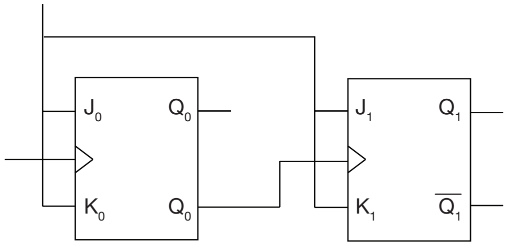
(A) Select A
From R, S
Where B

(B) Select DISTINCT(A)
From $R \times S$
Where B

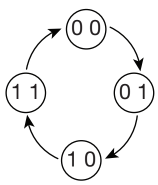
(C) Select DISTINCT(A)
From R, S
Where B

(D) All the above

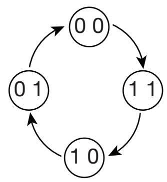
33. For the counter shown in figure, find the state diagram for the states $Q_1\, Q_0$?



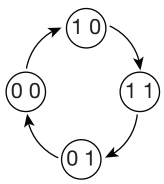
(A)



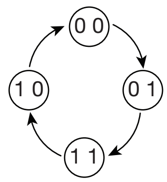
(B)



(C)



(D)



34. Consider the following Relation:
CREATE TABLE Authoring
(ArticleID INT REFERENCES Article(ID) ON DELETE SET NULL
AuthorID INT REFERENCES Author(ID) ON DELETE CASCADE)

I. If we delete a tuple from Article, some attributes of Authoring may have their values SET to NULL.
II. If we delete a tuple from Authoring, any tuples in Author referred to by this tuple are also deleted.

Which of the following is TRUE?
(A) I only (B) II only
(C) I and II (D) None of these

35. Consider the following schema.
Student (Roll-No, Name, Gender, Age, Marks, Address)
Among the given attributes Roll No, Name, Age are uniquely identified. Which of the following is NOT a super key?
(A) {Roll No, Name, Age, Gender, Marks}
(B) {Age, Gender, Marks, Address}
(C) {Gender, Marks, Address}
(D) {Gender, Marks, Address, Name}

36. If the system of linear equations
$2x_1 + 3x_2 + 5x_3 + 7x_4 = 0$
$-2x_2 + ax_3 = 0$
$3x_3 + 2x_4 = 0$
$6x_2 + bx_4 = 0$
has a non-trivial solution, then 'a' and 'b' are related by _____.
(A) $a + 2b = 0$ (B) $a - 2b = 0$
(C) $2a + b = 0$ (D) $2a - b = 0$

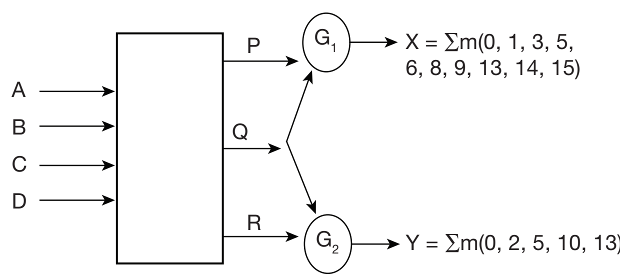
37. The coefficient of x^3 in the Maclaurin's series expansion of $(1 - x)^{5/2}$ is ______.

38. A fair die is rolled twice. Let X denote the number on the die in the first roll and let Y denote the number on the die in the second roll. Then the value of:
$P(X + Y = 6/X - Y = 2)$
is ______.
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$
(C) $\frac{1}{8}$ (D) $\frac{1}{16}$

39. Consider a relation $R = \{(x, y)/x, y \in Z^+$ and 'xy' is a perfect square$\}$ over the set of positive integers. Which of the following statements is/are TRUE about the relation R?
I. R is an equivalence relation.
II. R is a partial ordered relation.
(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

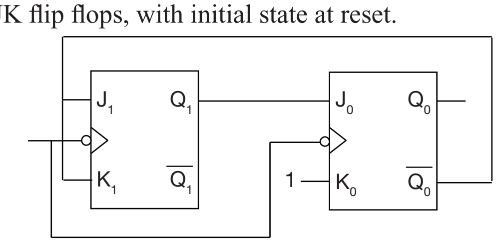
40. The dual of the statement formula "$p \to (\neg q \to r)$" is _____.
(A) $\neg p \to (q \to r)$ (B) $p \to (q \to \neg r)$
(C) $\wedge p \neg q \wedge r$ (D) $p \wedge q \wedge r$

41. Given Combinational network with four inputs A, B, C, D and three intermediate outputs P, Q, R and two outputs X and Y as shown in figure.



Assuming that G_1 is NAND gate, and G_2 is AND gate, find the smallest function 'Q'. (with minimum number of min terms) which makes it possible to produce X and Y?
(A) $\bar{A}\bar{C}D + A\bar{C}\bar{D} + \bar{A}B + B\bar{C}\bar{D}$
(B) $\bar{A}\bar{B} + \bar{A}C\bar{D} + \bar{A}C\bar{D} + \bar{B}C\bar{D}$
(C) $\bar{A}\bar{B} + \bar{A}C + B\bar{C}$
(D) $\bar{A}\bar{B}\bar{D} + \bar{A}BD + A\bar{B}C + B\bar{C}$

42. Consider the following synchronous counter with JK flip flops, with initial state at reset.



If the JK flip flops have to be replaced by D flip flops, for the same sequence then the D flip flop inputs D_1, D_0 are (corresponding to Q_1, Q_0 flip flops)

(A) $D_1 = Q_1 \oplus Q_0, D_0 = Q_1 \bar{Q}_0$

(B) $D_1 = Q_1 \odot Q_0, D_0 = Q_1 \bar{Q}_0$

(C) $D_1 = Q_1 + Q_0, D_0 = Q_1 \oplus Q_0$

(D) $D_1 = Q_1 \odot Q_0, D_0 = Q_1 + \bar{Q}_0$

43. Consider a word addressed memory hierarchy system with the following parameters:
Block size = 16 words
Main memory size = 64 blocks
Cache size = 8 blocks
The cache uses direct mapped technique. The tag values in the cache directory are:

Cache line number	Tag
0	000
1	101
2	100
3	010
4	101
5	010
6	100
7	001

Then which of the following main memory addresses will be a hit in cache?

(i) $(37A)_{16}$ (ii) $(22C)_{16}$
(iii) $(00C)_{16}$ (iv) $(1B9)_{16}$

(A) (i), (ii), (iii), (iv) (B) (ii), (iv)
(C) (i), (iii) (D) (ii), (iii)

44. Consider the following bit pattern:
1010 1101 0001 0000 0000 0000 0000 0000
Which of the following statements is/are correct?

(i) If the given bit pattern represents a 2's complement integer then its decimal equivalent is $(-1391460352)_{10}$.
(ii) If the given bit pattern represents an unsigned integer then its decimal equivalent is $(2804507955)_{10}$.
(iii) If the given bit pattern represents an IEEE 754 single precision floating point number then its decimal equivalent is $(-8.185 * 10^{-12})$

(A) (i) only (B) (i) and (iii)
(C) (ii) and (iii) (D) (i), (ii), (iii)

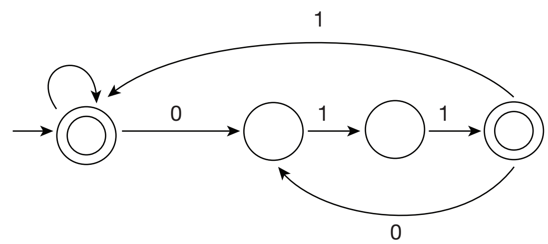
45. Which of the following statement is TRUE?

I. Assuming the same cache size and same block size, increasing set associativity of a cache reduces conflict misses.
II. Assuming the same set associativity and the same block size, increasing the size of a cache reduces compulsory misses.
III. Smaller caches have shorter hit time than larger caches.
IV. Increasing set associativity increases hit time.

(A) I, II, III (B) II, III, IV
(C) I, III, IV (D) I, II, IV

46. Construct a minimized DFA, M which accepts the binary strings w such that when you reverse w you get a binary integer that is divisible by 5.
If M has x states and y self-loops then the product of $x * y$ is ________.

47. Consider the following languages:
L_1 is the language described by 1* (0111*)*.
L_2 is the language of strings with atleast one 0 and atleast two 1's.
L_3 is the language of below finite automata:



L_4 is the language described by (0 + 1)* 01 (0 + 1)* 1.
Which of the following is TRUE?

(i) $L_1 = L_3$ (ii) $L_1 \subset L_4$ (iii) $L_4 \subset L_2$

(A) (i) only (B) (i), (iii) only
(C) (ii), (iii) only (D) (i), (ii), (iii)

48. Which of the following language is Decidable?

I. Checking whether a given natural number is prime or not.
II. {<M>| M is a DFA and $L(M) = \Sigma^*$}
III. Post correspondence problem (PCP).

(A) (I), (II) (B) (II), (III)
(C) (I), (III) (D) (I), (II), (III)

49. Consider sending a 2500 Byte datagram into a link which has a maximum transmission unit (MTU) of 700 Bytes. The datagram has an identification number 422. The number of fragments generated and their respective fragmentation offset values will be:

(A) 4; 0, 680, 1360, 2040
(B) 5; 0, 700, 1400, 2100, 2800
(C) 4; 0, 85, 170, 255
(D) 5; 0, 85, 170, 255, 340

50. Four equal-sized datagrams belonging to the same message leave for the destination one after another. These datagrams travel through different paths as given below:

Datagram	Path length	Visited switches
1	3000 km	1, 3, 5
2	10,800 km	1, 2, 5
3	13,000 km	1, 2, 3, 5
4	10,000 km	1, 4, 5

Assume that the delay for each switch is 2, 9, 23, 7 and 18 ms respectively. If the propagation speed is $2*10^8$ m, then the delays of the datagrams 1, 2, 3, 4, respectively is:

(A) 15 m sec, 54 m sec, 65 m sec, 50 m sec.
(B) 58 m sec, 83 m sec, 116 m sec, 77 m sec.

(C) 15 m sec, 83 m sec, 65 m sec, 77 m sec.
(D) 58 m sec, 54 m sec, 116 m sec, 50 m sec.

51. A multicast address for a group is 232.48.60.9. What is its equivalent 48-bit Ethernet address for a LAN using TCP/IP?
(A) 01:00:5E:7F:30:C0
(B) 01:1B:C2:43:03:C0
(C) 33:33:5E:30:3C:09
(D) 01:00:5E:30:3C:09

52. Quick sort algorithm is run on two inputs shown below to sort in ascending order:
(i) A sequence of 'n' even numbers, 2, 4, 6, 8… n.
(ii) A sequence of 'n' odd numbers, 1, 3, 5, 7…n.
Let A_1 and A_2 be the number of comparisons made for the inputs (i) and (ii) respectively, then:
(A) $A_1 > A_2$
(B) $A_1 < A_2$
(C) $A_1 = A_2$
(D) Cannot be determined

53. A binary search tree is used to locate the number 86. Few probe sequences are given below:
I. 102, 78, 98, 87, 90, 88, 86
II. 200, 190, 198, 76, 84, 77, 85, 86
III. 140, 139, 110, 120, 109, 86
IV. 100, 96, 92, 90, 83, 84, 86
Which of the following probe sequence(s) is/are possible to locate '86'?
(A) I and II (B) I and III
(C) III and IV (D) IV only

54. Given the alphabets A, B, C, D, E, F, G and H with the probabilities
$\frac{2}{40}, \frac{2}{40}, \frac{3}{40}, \frac{4}{40}, \frac{6}{40}, \frac{6}{40}, \frac{13}{40}$ respectively.
The average Huffman code size in bits per symbol is ________.
(A) $\frac{99}{40}$ (B) $\frac{101}{40}$
(C) $\frac{111}{40}$ (D) $\frac{121}{40}$

55. The following relation schema can be used to register information on the repayments on loans.
Repayment (Borrower-Id, name, address, loan-amount, request-date)
A borrower is identified with a unique borrower-id, and has only one address and name. Borrowers can have multiple simultaneous loans, but they always have different request-dates.
What is the key for Repayment?
(A) Borrower-Id
(B) Borrower-Id, request-date
(C) Borrower-Id, loan-amount
(D) request-date, loan-amount

56. Consider the SQL query given below:
DELETE
FROM Loan A
WHERE loan-amount = (SELECT SUM (repayment - amount))
FROM Loan-Payment B
WHERE B.Customer-Id = A.Customer-Id AND B.request-date = A.request-date
The above query
(A) Deletes all information of customers who have requested loan amount on same day.
(B) Deletes all information on ended loans, where the total repaid amount equals the lend amount.
(C) Deletes all information of customers whose have requested the same amount.
(D) None of the above

57. Match the following:

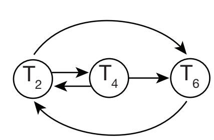
Set–1	Set–2
(P) Belady's anomaly	(a) Round Robin scheduling
(Q) Banker's algorithm	(b) Deadlock avoidance
(R) Time sharing system	(c) Deadlock prevention
(S) Simple Paging	(d) FIFO page replacement policy
	(e) Internal fragmentation
	(f) External fragmentation
	(g) Priority scheduling

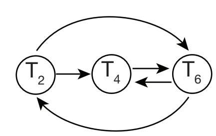
(A) P – g, Q – c, R – g, S – f
(B) P – d, Q – b, R – a, S – e
(C) P – d, Q – b, R – g, S – f
(D) P – d, Q – c, R – a, S – e

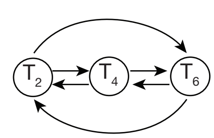
58. Consider the following transaction schedule:

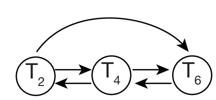
T_2	T_4	T_6
	R(Q)	
W(Q)		
		W(Q)
	R(Q)	
W(Q)		
	W(Q)	

Which of the following is the precedence graph for the above schedule?

(A)


(B)


(C)


(D)


59. Consider the following code:

```
procedure main
    procedure A
        procedure B
            procedure C
              ¦
              end
        end
    end

    procedure D
        ¦
        ¦
        end
end
```

The nesting depth of procedures A, B, C, D is:
(A) 1, 2, 3, 4 (B) 1, 2, 3, 1
(C) 2, 3, 4, 2 (D) 2, 3, 4, 5

60. Consider the grammar:
$G_1 : X \rightarrow X\,y \mid x$
$G_2 : X \rightarrow Y\,X\,y \mid x$
$Y \rightarrow \in$
Which of the following is True?
(A) Only G_1 is LR (1)
(B) Only G_2 is LR (1)
(C) Both G_1 and G_2 are LR (1)
(D) None of the above

61. Consider an AVL tree with root node as "a". Inorder predecessor of root as b, Inorder successor of root as c, the left child of root as d and the right child of root as e. Then which of the following relation is TRUE.
(A) $d < b < a < c < e$ (B) $a < b < c < d < e$
(C) $b < d < a < e < c$ (D) $e < c < a < b < d$

62. Consider the routine fun():
void fun(int x)
{if (x > = 2){fun (x/2); fun (x/2); printf ("#");}}
Number of times the printf() executed when fun(16) is called is ______.

63. Consider the following process table:

	Arrival time	Burst time
P_1	1	4
P_2	2	8
P_3	3	5
P_4	4	6

If Round Robin scheduling (with time slice = 2 units) is used to schedule above processes, then the number of context switches (don't consider start and end context switches) is ________.

64. Consider the postfix expression:
$a\ b + c * d\ e\ f\ g$ ^ ^ - /
consider the following statements:
$S1$: +, − has high precedence over ^ .
$S2$: ^ has high precedence over *, / and ^ associates from left to right.
$S3$: / has high precedence over *
Which of the above statements are TRUE?
(A) $S1$, $S2$ (B) $S1$, $S3$
(C) only $S3$ (D) None of the above

65. Consider a system with 1 GB physical memory and 64-bit virtual address space if the page size is 1 MB then the size of the page table is (size in Tb) ______.

Answer Keys

1. B	**2.** 2	**3.** D	**4.** A	**5.** A	**6.** C	**7.** D	**8.** C	**9.** B	**10.** A
11. 6	**12.** A	**13.** A	**14.** B	**15.** 1820	**16.** D	**17.** C	**18.** 1	**19.** 32	**20.** B
21. 42	**22.** 2047	**23.** 2048	**24.** 2048	**25.** B	**26.** B	**27.** 3	**28.** C	**29.** A	**30.** D
31. B	**32.** C	**33.** A	**34.** A	**35.** C	**36.** D	**37.** –0.32 to – 0.31	**38.** B	**39.** A	
40. C	**41.** D	**42.** B	**43.** D	**44.** B	**45.** C	**46.** 10	**47.** B	**48.** A	**49.** C
50. B	**51.** D	**52.** C	**53.** D	**54.** C	**55.** B	**56.** B	**57.** B	**58.** C	**59.** C
60. A	**61.** A	**62.** 15	**63.** 11	**64.** D	**65.** 160				

Hints and Explanations

1. The given statement is a hypothetical one. An unreal situation is presented here so the verb "were" is apt. Choice (B)

2. Let t hours be the time taken to cross each other.
Then, distance covered by Ram, to meet the other $= xt$ km → (1)
The distance covered by Shyam, to meet the other $= yt$ km → (2)
But, as per data, (1) is y times (2).
Hence, $xt = (y)\,(yt)$;
$\Rightarrow x = y^2$.
It is given that $x = 4$; hence $y = 2$. Ans: 2

3. Khadar's wife's daughter is Khadar's daughter whose mother is Khadar's wife. Khadar's wife's daughter-in-law is Khadar's daughter-in-law. Her husband's father is Khadar himself. Choice (D)

4. Except (A) all the other combinations have a synonymous relationship. "Empirical" is that which can be practically proved while "emaciated" is lean and weak. Momentous means significant. In (A) both the words are antonyms. Beatific means sublime while mundane is common or coarse. Choice (A)

5. The ratio of sales of Kissan to Sil is the highest in the year 2012 and this highest ratio equals 2.19.

Choice (A)

6. The right idiom to fit the bill is "stood their ground", which means to stick to one's stand on one's decision. To "pass the buck" is to shrug off responsibility, "broaden one's horizons" is to enlarge one's range of activities and world and "to hear something on the grapevine" is to get to know something via rumours. To "promise somebody the moon" is to promise somebody something that is impossible to deliver. Choice (C)

7. Statement (i) is about disposal techniques which is not the point of discussion or the source of the problem. Similarly (iii) is not the point of discussion which actually finds a solution to the problem. The possible solutions are offered in (ii) and (iv). It is necessary to create awareness among public and staff to incorporate safe injection practices and make extensive use of AD syringes. Choice (D)

8. Let $x = \sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5\ldots}}}}$

We can see that $x > \sqrt{5} (\sqrt{5} \approx 2.25)$

Choice (1): $\dfrac{\sqrt{13}-1}{2} \approx \dfrac{3.6-1}{2} \approx 1.3$

Choice (2): $\dfrac{\sqrt{17}-1}{7} \approx \dfrac{4.2-1}{2} \approx 1.6$

$\therefore (x^2-5)^2 = 5-x$ (1)

Now consider $x = \dfrac{\sqrt{17}+1}{2}$ (2)

$\therefore \quad 5-x = \dfrac{9-\sqrt{17}}{2}$

$(2) \Rightarrow x^2 = \dfrac{18+2\sqrt{17}}{4} = \dfrac{9+\sqrt{17}}{2}$

$\therefore \quad x^2-5 = \dfrac{\sqrt{17}-1}{2}$

$\therefore \quad (x^2-5)^2 = \dfrac{18-2\sqrt{17}}{4} = \dfrac{9-\sqrt{17}}{2}$

$\therefore \quad x = \dfrac{\sqrt{17}+1}{2}$ satisfies (1) Choice (C)

9. The sentence which is logically valid and can be inferred from the given sentence is:
America would not have entered the world war if Japan would not have attacked the Pearl Harbour.
Japan's attack on pearl Harbour is cited as the reason for the America entering. Choice (B)

10. Statement (A) is grammatically correct and clearly brings out the intended meaning that a tip today would be enough to buy a meal three years ago. Choice (B) is ungrammatical as "today's" does not use an apostrophe. In (C) "three-years-ago meal" distorts the meaning. (D) uses "would costed" which is ungrammatical. Choice (A)

11. In the abelian group (G, x_7) with $G = \{1, 2, 3, 4, 5, 6\}$ 1 is the identity element.

$\therefore$ If 'a' is the element in G which is the inverse of itself, then $a x_7 a = 1$.

$\Rightarrow$ The remainder when $a \times a$ is divided by 7 should be 1

$\Rightarrow$ $(a \times a) - 1$ should be a multiple of 7

$\Rightarrow$ $a^2 - 1 = 7k$ for some possitive integer k

And among the elements in G, the only element that satisfies this condition is 6.

$\therefore$ $a = 6$

$\therefore$ $6 \in G$ is the inverse of itself. Ans: 6

12. We have $\int_0^{\pi} \sin^8 x\,dx = 2\int_0^{\frac{\pi}{2}} \sin^8 x\,dx$

$$\left(\because \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)\,dx;\ \text{if } f(2a-x) = f(a)\right)$$

$$= 2\left[\frac{8-1}{8}\cdot\frac{8-3}{8-2}\cdot\frac{8-5}{8-4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}\right]$$

$$\left(\because \int_0^{\frac{\pi}{2}} \sin^n x dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\ldots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} \text{ if } n \text{ is even}\right)$$

$$= 2\left[\frac{7}{8}:\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}\right]$$

$$= \frac{7^2}{8.7}\cdot\frac{5^2}{6.5}\cdot\frac{3^2}{4.3}\cdot\frac{1^2}{2.1}\pi = \frac{7^2\cdot 5^2\cdot 3^2\cdot 1^2}{8!}\pi.$$ Choice (A)

13. Given $A = \begin{bmatrix} 3 & 1 & 2 \\ 6 & 4 & 5 \\ 9 & 7 & 11 \end{bmatrix}$

As the principal diagonal elements of L are equal to 1 in the LU decomposition of A, we have

$A = LU$ (1)

where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ and

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$\therefore$ From (1)

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 4 & 5 \\ 9 & 7 & 11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{12} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 4 & 5 \\ 9 & 7 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

Comparing the corresponding elements on both sides

We have

$u_{11} = 3$; $u_{12} = 1$ and $u_{13} = 2$

$l_{21}u_{11} = 6$

$\Rightarrow \quad l_{21} = \frac{6}{3} = 2$

$l_{21}u_{12} + u_{22} = 4$

$\Rightarrow \quad u_{22} = 4 - 2 \times 1 = 2$

$l_{31}u_{11} = 9$

$\Rightarrow \quad l_{31} = \frac{9}{3} = 3$

$l_{31}u_{12} + l_{32}u_{22} = 7$

$\Rightarrow \quad 3 \times 1 + l_{32} \times 2 = 7$

$\Rightarrow \quad l_{32} = \frac{7-3}{2} = 2$

$\therefore \quad L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$. Choice (A)

14. A matching of a graph G is a set of edges in G such that no two edges of that set are incident on the same vertex. A perfect matching (Also known as a complete matching) of a graph G is a matching in which every vertex of G is incident on exactly one of the edges of the set of edges in perfect matching.

Among the sets of edges given in options, the set given in option (B), viz $\{d, e, f, j\}$ is NOT a perfect matching because both the edges e and d are incident on the same vertex V_3 and also, the vertex V_6 is incident on none of the edges given in the set. Choice (B)

15. We know that the number of non-negative integral solutions of $x_1 + x_2 + x_3 + ... + x_n \le r$.

= The number of non negative integral solutions of $x_1 + x_2 + x_3 + ... + x_n + x_{n+1} = r$.

$\therefore$ The number of non negative integral solutions of $x_1 + x_2 + x_3 + x_4 < 13$

The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 \le 12$

= The number of non-negative integral solutions of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 12$ is

$= C(5 - 1 + 12, 12)$

($\because$ The number of non-negative integral solutions of $x_1 + x_2 + ... + x_k = s$ is $C(k - 1 + s, s)$)

$= C(16, 12)$

$= {}^{16}C_{12} = 1{,}820$ Ans: 1,820

18. Need matrix:

Process	need
P_1	2
P_2	1
P_3	6
P_4	5
P_5	5

Resources required to be available is 1. Ans : 1

19. Counting semaphore value = 25

33 down operations results semaphore value to be 25-33 = −8. and 40 UP operations results semaphore value to be –8 + 40 = 32 Ans : 32

20. Choice (B)

21. No. of possible permutations that can be obtained with 'n' numbers with 1, 2, 3, ... n (in that order) using stack is

$\frac{2^n C_n}{n+1} = \frac{^{10}C_5}{5+1} = 42 = 42$ Ans: 42

22. The maximum number of elements in a heap of height h is $2^{h+1} - 1$

Here $h = 10$,

So number of elements $= 2^{10+1} - 1$

$= 2048 - 1 = 2047$ Ans: 2047

23. PC uses 8-bits to access BHT.

$\Rightarrow$ There will be 2^8 entries. Each entry requires 8-bits for storing history. (for 8 branches)

$\therefore$ Total bits required for BHT $= 8 \times 2^8 = 2048$ bits.

Ans: 2048

24. Cache is 4-way set-associative. i.e., each set has 4 blocks.

Given byte offset is 4-bits, so block size $= 2^4$ bytes.

5-bits required for set index, so the number of sets in cache $= 2^5$

$\therefore$ Data portion in cache = number of sets × lines in set × line size

$= 2^5 \times 4 \times 2^4 = 2048$ Bytes Ans: 2048

25. Given regular expression accepts the strings in which every 0 that is not last symbol is immediately followed by a 1 and every 1 that is not last symbol is immediately followed by a 0.

The strings accepted are

$\{\varepsilon, 1, 0, 01, 10, 0101, ...\}$ Choice (B)

26. Regular and Recursive languages are closed under union, intersection and complementation CFL is not closed under intersection, complement.

Recursively enumerable languages are not closed under complementation. Choice (B)

27. Given 8 stations,

	Bit time 0	1	2	3	4
1 0 1 0 1	1	0	-	-	-
0 1 1 0 0	0	-	-	-	-
1 1 1 0 1	1	1	1	0	1
0 0 0 1 0	0	-	-	-	-
0 0 1 0 1	0	0	-	-	-
0 1 0 1 0	0	-	-	-	-
1 1 1 0 0	1	1	1	0	0
1 0 0 1 1	1	0	-	-	-
Result	1	1	1	0	1

3^{rd} station will transmit data first. (Higher numbered station has higher priority). Ans: 3

28. Given p = 7, $q = 11$

$\Rightarrow \quad z = (p-1) * (q-1) = 6 * 10 = 60$

Based on RSA,

$(e * d) \bmod z = 1$

if $e = 108 \quad \Rightarrow (108 * 7)\% \ 60 \neq 1$

if $e = 170 \quad \Rightarrow (170 * 7)\% \ 60 \neq 1$

if $e = 43 \quad \Rightarrow (43 * 7)\% \ 60 = 1$

$\therefore$ valid e value is 43. Choice (C)

29. Radix sort:

It will sort the keys, based on the digits in a key. Choice (A)

30. Both the given statements are TRUE about Weak Entity. Choice (D)

31. Assume a very large value for 'n', then the sequence will be $\log(\log n) \leq \log(\log n^2) \leq \log(\log_2 n!) \leq \log_2(n!)$ Choice (B)

32. π-eliminates duplicates from the result, Cartesian product is represented with (,) operator in SQL. Choice (C)

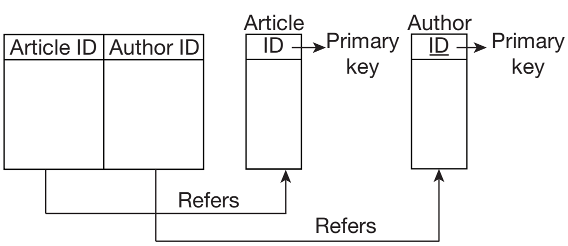
33. Given circuit is a ripple counter, $\bar{Q}_0$ is connected to rising edge Clk pulse, so it is UP counter.

Clk	Q_1	Q_0
0	0	0
1	0	1
2	1	0
3	1	1
4	0	0

Choice (A)

34. Table:

Authoring



—table that does not affect other tables, because the Authoring table is not referred by any table.

—If any deletion is performed in tables, Article and Author, that will affect the contents of Authoring table, because Authoring table refers to both the tables Author and Article.

I. TRUE II. FALSE Choice (A)

35. The uniquely identified attributes are candidate keys, among available candidate keys one attribute can be chosen as primary key. Super key must contain atleast one candidate key along with other attributes option (C) has no candidate key. Choice (C)

36. Given system of linear equations is:

$$\left.\begin{aligned} 2x_1 + 3x_2 + 5x_3 + 7x_4 &= 0 \\ -2x_2 + ax_3 &= 0 \\ 3x_3 + 2x_4 &= 0 \\ 6x_2 + bx_4 &= 0 \end{aligned}\right\} \quad (1)$$

(1) can be written in matrix form as $AX = O$

$$\text{Where } A = \begin{bmatrix} 2 & 3 & 5 & 7 \\ 0 & -2 & a & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 6 & 0 & b \end{bmatrix};\ X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \text{ and } O = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Given that (1) has a non-trivial solution

$\Rightarrow \quad \text{Det}(A) = 0$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 5 & 7 \\ 0 & -2 & a & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 6 & 0 & b \end{vmatrix} = 0$$

$$\Rightarrow \quad 2\begin{vmatrix} -2 & a & 0 \\ 0 & 3 & 2 \\ 6 & 0 & b \end{vmatrix} = 0$$

$$\Rightarrow \quad 2\left(-2\begin{vmatrix} 3 & 2 \\ 0 & b \end{vmatrix} + 6\begin{vmatrix} a & 0 \\ 3 & 2 \end{vmatrix}\right) = 0$$

$\Rightarrow \quad 2(-6b + 12a) = 0$

$\Rightarrow \quad 2a - b = 0.$ Choice (D)

37. Let $f(x) = (1-x)^{5/2}$

The coefficient of x^3 in the Maclaurin's series expansion of $f(x) = \dfrac{f'''(0)}{3!}$

$$f(x) = (1-x)^{5/2} \quad \Rightarrow \quad f'(x) = \frac{-5}{2}(1-x)^{3/2}$$

$$\Rightarrow \quad f''(x) = \frac{5}{2} \times \frac{3}{2}(1-x)^{1/2} \text{ and}$$

$$f'''(x) = \frac{-5}{2} \times \frac{3}{2} \times \frac{1}{2}(1-x)^{-\frac{1}{2}}$$

$$\therefore f'''(0) = \frac{-15}{8}$$

The coefficient of x^3 in the Maclaurin's series expansion of $(1-x)^{5/2} = \dfrac{\left(\dfrac{-15}{8}\right)}{3!} = \dfrac{-5}{16} = -0.3125$

Ans: –0.32 to –0.31

38. Given that X and Y denote the numbers shown up on the die in the first roll and the second roll respectively

$\therefore \quad P(X+Y=6/X-Y=2)$

$$= \frac{P[(X+Y=6)\cap(X-Y=2)]}{P(X-Y=2)}$$

$$= \frac{P(X=4, Y=2)}{P\left[\begin{array}{l}(X=3,Y=1)\cup(X=4,Y=2)\cup\\ (X=5,Y=3)\cup(X=6,Y=4)\end{array}\right]}$$

$$= \frac{P(X=4,Y=2)}{\begin{array}{l}P(X=3,Y=1)+P(X=4,Y=2)\\ +P(X=5,Y=3)+P(X=6,Y=4)\end{array}}$$

$$= \frac{P(X=4)\,P(Y=2)}{\begin{array}{l}P(X=3)\,P(Y=1)+P(X=4)\,P(Y=2)\\ +P(X=5)\,P(Y=3)+P(X=6)P(Y=4)\end{array}}$$

(QX and Y are independent random variables)

$$= \frac{\frac{1}{6}\times\frac{1}{6}}{\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}} = \frac{1}{4}.$$ Choice (B)

39. Given relation is $R = \{(x, y)/x, y \in Z^+$ and 'xy' is a perfect square$\}$

For any $a \in Z^+$, $a.\,a = a^2$ is a perfect square

$\therefore \quad (a, a) \in R, \forall\, a \in Z^+$

$\therefore \quad R$ is reflexive (1)

Consider $(a, b) \in R$

$\Rightarrow \quad a\,b$ is a perfect square

$\Rightarrow \quad b\,a$ is also a perfect square

$\Rightarrow \quad (b, a) \in R$

$\therefore \quad R$ is symmetric (2)

Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow \quad a\,b$ is a perfect square and $b\,c$ is a perfect square

Let a $b = k^2$ for some $k \in Z^+$ and $bc = l^2$ for some $l \in Z^+$

$$ab = k^2 \Rightarrow a = \frac{k^2}{b} \text{ and } bc = l^2 \Rightarrow c = \frac{l^2}{b}$$

$$\text{Consider } ac = \left(\frac{k^2}{b}\right)\left(\frac{l^2}{b}\right) = \frac{k^2 l^2}{b^2}$$

$$\therefore \quad ac = \left(\frac{kl}{b}\right)^2$$

$\Rightarrow \quad ac$ is also a perfect square

$\Rightarrow \quad (a, c) \in R$

$\therefore \quad R$ is transitive (3)

Hence from (1), (2) and (3), R is an equivalence relation

$\therefore \quad$ (I) is TRUE

Consider $(2, 18) \in R$

Clearly, $(18, 2) \in R$

But $2 \neq 18$

$\therefore \quad R$ is NOT anti-symmetric

Hence R is NOT a partial ordered relation

$\therefore \quad$ (II) is NOT TRUE

$\therefore \quad$ Only (I) is TRUE. Choice (A)

40. Let $F(p, q, r) \Leftrightarrow p \to (\rceil q \to r)$ (1)

$\Leftrightarrow \rceil p \vee (\rceil(\rceil q) \vee r)$

$(\because A \to B \Leftrightarrow \rceil A \vee B)$

$\Leftrightarrow \rceil p \vee q \vee r\ (\because \rceil(\rceil A) \Leftrightarrow A)$

The dual of $F(p, q, r)$ is $\rceil p \wedge q \wedge r$. Choice (C)

41. $X = \overline{P \cdot Q}$ $(\because G_1$ is NAND gate)

$= \oplus m\,(0, 1, 3, 5, 6, 8, 9, 13, 14, 15)$

$P.Q = \oplus m\,(2, 4, 7, 10, 11, 12)$

$Y = Q \cdot R = \oplus m\,(0, 2, 5, 10, 13)$ $(\because G_2$ is AND gate)

From above two equations, $Q = \oplus m\,(0, 2, 4, 5, 7, 10, 11, 12, 13)$

AB \ CD	00	01	11	10
00	1			1
01	1	1	1	
11	1	1		
10			1	1

$\therefore \quad Q = \bar{A}\bar{B}\bar{D} + \bar{A}BD + A\bar{B}C + B\bar{C}$ Choice (D)

42. Here $J_1 = K_1 = \bar{Q}_0,\ J_0 = Q_1, K_0 = 1$

If JK flipflop has to be replaced with D flip flop then

$D = J\,K$ characteristic equation $= J\bar{Q} + \bar{K}Q$

So, $D_1 = J_1\bar{Q}_1 + \bar{K}_1 Q_1$

But here $J_1 = \bar{Q}_0, K_1 = \bar{Q}_0$ (as per connections given)

$D_1 = \bar{Q}_0 \cdot \bar{Q}_1 + \bar{\bar{Q}}_0 Q_1 = \bar{Q}_0\bar{Q}_1 + Q_0 Q_1 = Q_0 \odot Q_1$

Similarly $D_0 = J_0\bar{Q}_0 + \bar{K}_0 Q_0$

$J_0 = Q_1, K_0 = 1$

So $\quad D_0 = Q_1 \cdot \bar{Q}_0 + 0 \cdot Q_0$

$D_0 = Q_1\,\bar{Q}_0$

(or) Find the sequence of given counter, and design the same sequence counter with D-flip flops. Choice (B)

43. Given, Block size = 16 words

Main memory size = 64 Blocks

Cache size = 8 Blocks

Main memory size = 64 × 16 words = 2^{10} words

In direct mapped system,

Tag	Line	Word

10-bits

10-bits

Word field size = 4

Line field size = 3 $(\because$ 8 blocks in cache)

$\Rightarrow \quad$ Tag = 3 bits

(i) 37A: 0011 0 | 111|1010

Line 7 but Tag mis-match

$\Rightarrow$ (i) is a miss

(ii) 22C:0010 0 | 010 | 1100

Line 2 and Tag matched

$\Rightarrow$ (ii) is a Hit

(iii) 00C:0000 0 | 000 | 1100

Line 0 and Tag matched

$\Rightarrow$ (iii) is a Hit.

(iv) $(1B9)_{16}$: 0001 1 | 011 | 1001

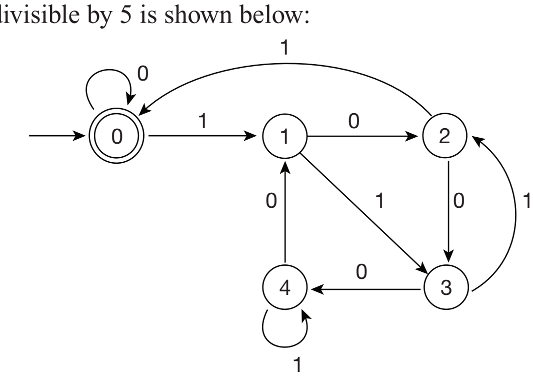
Line 3 but tag mis-match

$\Rightarrow$ (iv) is a miss. Choice (D)

44. (i) assumes, given bit pattern represents a 2's complement integer.
For a 2's complement number, if the MSB is 1, take 2's complement to the magnitude to get correct binary number and then convert to decimal.
1|010 1101 0001 0000 0000 0000 0000 0000
$\Downarrow$ 1's complement
101 0010 1110 1111 1111 1111 1111 1111
$\Downarrow$ 2's complement
101 0010 1111 0000 0000 0000 0000 0000
$\Downarrow$ Decimal
– 1391460352
$\therefore$ (i) is correct
(ii) assumes that given bit pattern represents an unsigned integer. To get its decimal equivalent all the bits in the given number are considered.
1010 1101 0001 0000 0000 0000 0000 0000
$\Downarrow$ Decimal
2903506944
$\therefore$ (ii) is incorrect.
(iii) assumes that given bit pattern represents a single precision floating point number.
1 | 010 1101 0 | 001 0000 0000 0000 0000 0000
Sign = –
Exponent = Biased exponent –127
= 90 – 127 = –37
Mantissa = 1.001
$\therefore$ Given number is equivalent to $(-1.001) \times 2^{-37}$
$= -8.185 \times 10^{-12}$
$\therefore$ (iii) is correct Choice (B)

45. Conflict misses can be reduced by increasing set-associativity. Compulsory misses will not be reduced by increasing the size of cache without increasing block size. Smaller caches have shorter hit time than larger cache. By increasing associativity of a cache the hit time also increases as the Tag bits need to be compared with all the blocks of the set. Choice (C)

46. The DFA which accepts the binary strings ω such that when you reverse ω, the resultant binary integer is divisible by 5 is shown below:



This accepts the strings,
{101, 0101, 1111, 00101, 10011, ...}
It has $x = 5$ states and $y = 2$ self loops.
$\therefore x * y = 5 * 2 = 10$ Ans: 10

47. L_1 is described by 1* (0111*)*
L_3 is the finite automata for the language L_1.
L_1 is not subset of L_4. L_1 has all 1's which is not accepted by L_4.
L_4 has one zero and two 1's. L_2 has atleast one 0 and atleast two 1's. So $L_4 \subset L_2$. Choice (B)

48. PCP is an undecidable problem. Choice (A)

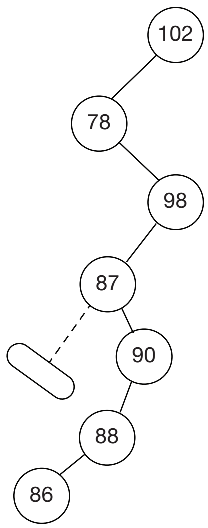
49. Size of data field in each fragment = 680 Bytes
(20 Bytes is for header)
Number of fragments required $= \left\lceil \frac{2500 - 20}{680} \right\rceil = 4$
Each fragment will have an identification number of 422.
The offsets of the 4 fragments will be 0, $\frac{680}{8} = 85, \frac{1360}{8} = 170, \frac{2040}{8} = 255.$
(The fragmentation offset is a multiple of 8 Bytes).
Choice (C)

50. Delay for each datagram is (time taken to reach destination + delays at visited switches).
For Datagram 1,
arrival time $= \frac{3000 \text{ km}}{2 \times 10^8} = 15$ m sec
Delay at switches 1, 3, 5 is
2 + 23 + 18 = 43 msec
$\therefore$ Delay for Datagram 1 is 15 + 43 = 58 msec
Delay for Datagram 2 is
$\frac{10,800 \times 10^3}{2 \times 10^8}$ + (2 + 9 + 18) msec
= 83 m sec
Delay for Datagram 3 is
$\frac{13000 \times 10^3}{2 \times 10^8}$ + (1 + 9 + 23 + 18)
= 65 m sec + 51 m sec
= 116 m sec
Delay for Datagram 4 is
$\frac{10000 \times 10^3}{2 \times 10^8}$ + (2 + 7 + 18)
= 50 m sec + 27 m sec = 77 m sec Choice (B)

51. Given IPv4 multicast address is 232.48.60.9.
Its binary equivalent is
1110 1000.00110000.00111100.00001001
First 4-bits represents, multicasting.
Remaining 28-bits gives the group ID.
48-bit Ethernet address for given multicast IPv4 address has the range
01:00:5*E*:00:00:00 – 01:00:5*E*:7*F*:*FF*:*FF*
In which last 23-bits are replaced with low 23-bits of the multicast IPv4 address.
Last 23-bits of given address are
011 0000.0011 1100.0000 1001
$\therefore$ Required 48-bit address is
01:00:5*E*:0011 0000:0011 1100:0000 1001
01:00:5*E*:30:3*C*:09 Choice (D)

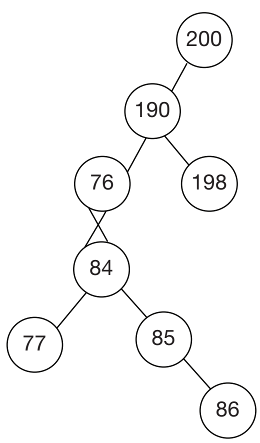
52. Quick Sort Algorithm gives worst case (maximum comparisons). Time Complexity is ($O(n^2)$) if the elements are already in ascending order.
Both (i) and (ii) are in ascending order.
$\therefore A_1 = A_2$ Choice (C)

53. **I.**



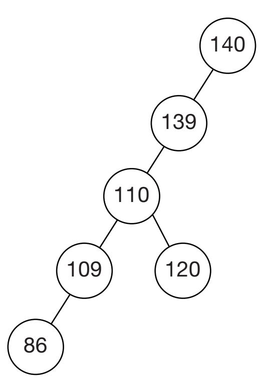
The search should have taken left path from element (87).

II.



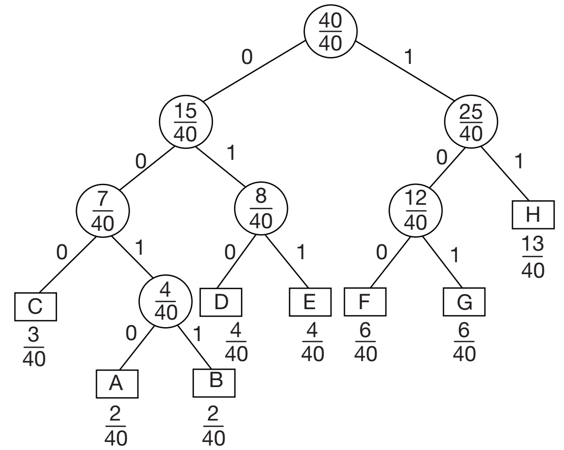
The search should be continuous (It should not change the paths).

III.



$\therefore$ Search is not continuous.
$\therefore$ Correct search Choice (D)

54. **Huffman-coding:**



Codes:
$A = 0010$ (4-bits)
$B = 0011$ (4-bits)
$C = 000$ (3-bits)
$D = 010$ (3-bits)
$E = 011$ (3-bits)
$F = 100$ (3-bits)
$G = 101$ (3-bits)
$H = 11$ (2-bits)
The average Huffman code size in bits per symbol is

$$\frac{2}{40}\times 4 + \frac{2}{40}\times 4 + \frac{3}{40}\times 3 + \frac{4}{40}\times 3 + \frac{4}{40}\times 3 + \frac{6}{40}\times 3 + \frac{6}{40}\times 3 + \frac{13}{40}\times 2$$

$$= \frac{8}{40} + \frac{8}{40} + \frac{9}{40} + \frac{12}{40} + \frac{12}{40} + \frac{18}{40} + \frac{18}{40} + \frac{26}{40}$$

$$= \frac{111}{40}$$

Choice (C)

55. From the data, The functional dependencies are
Borrower-id $\rightarrow$ name, address ($B \rightarrow NA$)
Borrower-id, request-date $\rightarrow$ loan-amount ($BR \rightarrow L$)
key:

L	M	R
BR		NAL

$BR^+ = \{BRNAL\}$
key = Borrower - Id, request - date Choice (B)

56. Lets assume some data.
Loan-payment B

Customer-Id	Repayment amount	Request date
C_1	1000	23/1/15
C_2	2000	28/2/15
C_3	4000	20/3/15
C_4	6000	19/1/15

C_1	2000	6/7/15
C_2	2000	8/8/15

Loan A

Customer-Id	Request date	Loan-amount
C_1	23/1/15	3000
C_2	28/2/15	4000
C_3	20/3/15	8000
C_4	19/1/15	7000

Sub query result:
Sum (repayment-amount)

C_1	3000
C_2	4000
C_3	4000
C_4	6000

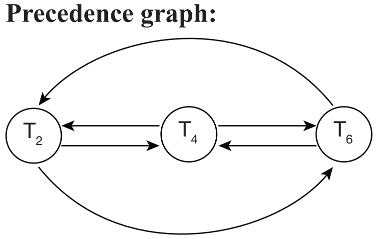
Loan amount = (select sum (repayment-amount))
C_1 3000 = C_1 3000
C_2 4000 = C_2 4000
C_3 8000 ≠ C_3 4000
C_4 7000 ≠ C_4 6000
For customers C_1, C_2 the repayment of loan is ended.
Choice (B)

57. Choice (B)

58.

	T_2	T_4	T_6
1		R(Q)	
2	W(Q)		
3			W(Q)
4		R(Q)	
5	W(Q)		
6		W(Q)	

Precedence graph:

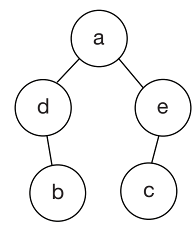


Conflicts
$T_4 \to T_2$ (1 to 2)
$T_2 \to T_6$ (2 to 3)
$T_4 \to T_6$ (1 to 3)
$T_6 \to T_4$ (3 to 4)
$T_2 \to T_4$ (2 to 4)
$T_6 \to T_2$ (3 to 5) Choice (C)

59. Nesting depth is calculated as follows
(1) The nesting depth of main program is 1.
(2) Add 1 to depth each time when a new procedure begins.
(3) Subtract 1 from depth each time when you exit from a nested procedure. Choice (C)

60. Choice (A)

61.



From this, $d < b < a < c < e$. Choice (A)

62. It prints 15 times Ans : 15

63.

P_1	P_2	P_3	P_1	P_4	P_2	P_3	P_4	P_2	P_3	P_4	P_2

Ready Queue: P_1, P_2, P_3, P_1, P_4, P_2, P_3, P_4, P_2, P_3, P_4, P_2, P_4 Ans: 11

64. The infix expression for the given postfix expression is
$a + b * c/d - e \wedge f \wedge g$
^ has high precedence (associated from Right to left).
+ / has least precedence. Choice (D)

65. Page size is 1 MB so requires 20 bits.
$d = 20$ $f + d = 30$
$p + d = 64$ $f = 30 - 20$
$p = 64 - 20$ $f = 10$
$p = 44$
page table size
$= 2^{44} \times 10$
$= 5 \times 2^{45}$
= 160 Tb
Ans : 160

PART - V

↳ Previous Year Paper
GATE CS/IT 2022 - 2007

GATE 2022 General Aptitude (GA)

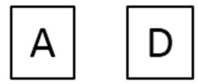
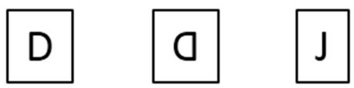
Q.1 – Q.5 Carry ONE mark each.

Q.1	The __________ is too high for it to be considered __________.
(A)	fair / fare
(B)	faer / fair
(C)	fare / fare
(D)	fare / fair

Q.2	A function $y(x)$ is defined in the interval $[0, 1]$ on the x-axis as $$y(x) = \begin{cases} 2 & \text{if } 0 \le x < \frac{1}{3} \\ 3 & \text{if } \frac{1}{3} \le x < \frac{3}{4} \\ 1 & \text{if } \frac{3}{4} \le x \le 1 \end{cases}$$ Which one of the following is the area under the curve for the interval $[0, 1]$ on the x-axis?
(A)	$\frac{5}{6}$
(B)	$\frac{6}{5}$
(C)	$\frac{13}{6}$
(D)	$\frac{6}{13}$

Q.3	Let r be a root of the equation $x^2 + 2x + 6 = 0$. Then the value of the expression $(r+2)(r+3)(r+4)(r+5)$ is
(A)	51
(B)	-51
(C)	126
(D)	-126

Q.4	Given below are four statements. Statement 1: All students are inquisitive. Statement 2: Some students are inquisitive. Statement 3: No student is inquisitive. Statement 4: Some students are not inquisitive. From the given four statements, find the two statements that **CANNOT BE TRUE** simultaneously, assuming that there is at least one student in the class.
(A)	Statement 1 and Statement 3
(B)	Statement 1 and Statement 2
(C)	Statement 2 and Statement 4
(D)	Statement 3 and Statement 4

Q.5	A palindrome is a word that reads the same forwards and backwards. In a game of words, a player has the following two plates painted with letters.  From the additional plates given in the options, which one of the combinations of additional plates would allow the player to construct a five-letter palindrome. The player should use all the five plates exactly once. The plates can be rotated in their plane.
(A)	
(B)	
(C)	
(D)	

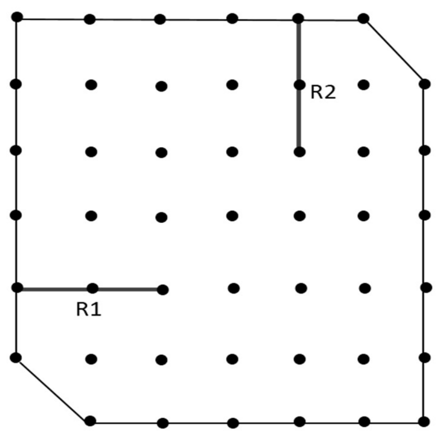
Q. 6 – Q. 10 Carry TWO marks each.

Q.6	Some people believe that "what gets measured, improves". Some others believe that "what gets measured, gets gamed". One possible reason for the difference in the beliefs is the work culture in organizations. In organizations with good work culture, metrics help improve outcomes. However, the same metrics are counterproductive in organizations with poor work culture. Which one of the following is the CORRECT logical inference based on the information in the above passage?
(A)	Metrics are useful in organizations with poor work culture
(B)	Metrics are useful in organizations with good work culture
(C)	Metrics are always counterproductive in organizations with good work culture
(D)	Metrics are never useful in organizations with good work culture

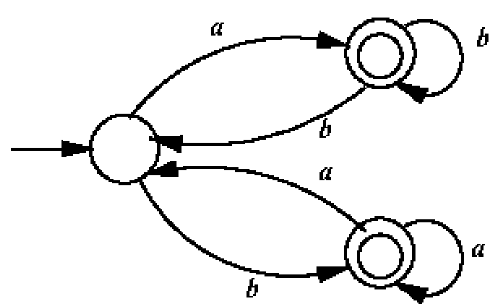
Q.7	In a recently conducted national entrance test, boys constituted 65% of those who appeared for the test. Girls constituted the remaining candidates and they accounted for 60% of the qualified candidates. Which one of the following is the correct logical inference based on the information provided in the above passage?
(A)	Equal number of boys and girls qualified
(B)	Equal number of boys and girls appeared for the test
(C)	The number of boys who appeared for the test is less than the number of girls who appeared
(D)	The number of boys who qualified the test is less than the number of girls who qualified

Q.8	A box contains five balls of same size and shape. Three of them are green coloured balls and two of them are orange coloured balls. Balls are drawn from the box one at a time. If a green ball is drawn, it is not replaced. If an orange ball is drawn, it is replaced with another orange ball. First ball is drawn. What is the probability of getting an orange ball in the next draw?
(A)	$\frac{1}{2}$
(B)	$\frac{8}{25}$
(C)	$\frac{19}{50}$
(D)	$\frac{23}{50}$

Q.9	The corners and mid-points of the sides of a triangle are named using the distinct letters P, Q, R, S, T and U, but not necessarily in the same order. Consider the following statements: • The line joining P and R is parallel to the line joining Q and S. • P is placed on the side opposite to the corner T. • S and U cannot be placed on the same side. Which one of the following statements is correct based on the above information?
(A)	P cannot be placed at a corner
(B)	S cannot be placed at a corner
(C)	U cannot be placed at a mid-point
(D)	R cannot be placed at a corner

Q.10	A plot of land must be divided between four families. They want their individual plots to be similar in shape, not necessarily equal in area. The land has equally spaced poles, marked as dots in the below figure. Two ropes, R1 and R2, are already present and cannot be moved. What is the least number of **additional** straight ropes needed to create the desired plots? A single rope can pass through three poles that are aligned in a straight line. 
(A)	2
(B)	4
(C)	5
(D)	3

Q.11 – Q.22 Multiple Choice Questions (MCQ), carry ONE mark each.

Q.11	Which one of the following statements is TRUE for all positive functions $f(n)$?
(A)	$f(n^2) = \theta(f(n)^2)$, when $f(n)$ is a polynomial
(B)	$f(n^2) = o(f(n)^2)$
(C)	$f(n^2) = O(f(n)^2)$, when $f(n)$ is an exponential function
(D)	$f(n^2) = \Omega(f(n)^2)$
Q.12	Which one of the following regular expressions correctly represents the language of the finite automaton given below?
	
(A)	$ab^*bab^* + ba^*aba^*$
(B)	$(ab^*b)^*ab^* + (ba^*a)^*ba^*$
(C)	$(ab^*b + ba^*a)^*(a^* + b^*)$
(D)	$(ba^*a + ab^*b)^*(ab^* + ba^*)$

Q.13	Which one of the following statements is TRUE?
(A)	The $LALR(1)$ parser for a grammar G cannot have reduce-reduce conflict if the $LR(1)$ parser for G does not have reduce-reduce conflict.
(B)	Symbol table is accessed only during the lexical analysis phase.
(C)	Data flow analysis is necessary for run-time memory management.
(D)	$LR(1)$ parsing is sufficient for deterministic context-free languages.
Q.14	In a relational data model, which one of the following statements is TRUE?
(A)	A relation with only two attributes is always in BCNF.
(B)	If all attributes of a relation are prime attributes, then the relation is in BCNF.
(C)	Every relation has at least one non-prime attribute.
(D)	BCNF decompositions preserve functional dependencies.

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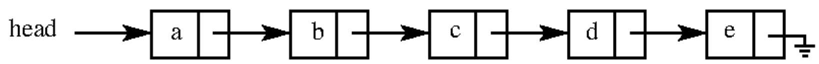
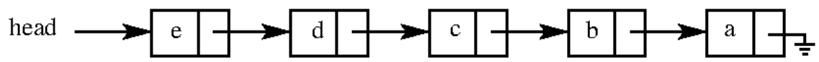
Q.15 Consider the problem of reversing a singly linked list. To take an example, given the linked list below,



the reversed linked list should look like



Which one of the following statements is TRUE about the time complexity of algorithms that solve the above problem in $O(1)$ space?

(A) The best algorithm for the problem takes $\theta(n)$ time in the worst case.

(B) The best algorithm for the problem takes $\theta(n \log n)$ time in the worst case.

(C) The best algorithm for the problem takes $\theta(n^2)$ time in the worst case.

(D) It is not possible to reverse a singly linked list in $O(1)$ space.

Q.16 Suppose we are given n keys, m hash table slots, and two simple uniform hash functions h_1 and h_2. Further suppose our hashing scheme uses h_1 for the odd keys and h_2 for the even keys. What is the expected number of keys in a slot?

(A) $\frac{m}{n}$

(B) $\frac{n}{m}$

(C) $\frac{2n}{m}$

(D) $\frac{n}{2m}$

Q.17	Which one of the following facilitates transfer of bulk data from hard disk to main memory with the highest throughput?
(A)	DMA based I/O transfer
(B)	Interrupt driven I/O transfer
(C)	Polling based I/O transfer
(D)	Programmed I/O transfer
Q.18	Let R1 and R2 be two 4-bit registers that store numbers in 2's complement form. For the operation R1+R2, which one of the following values of R1 and R2 gives an arithmetic overflow?
(A)	R1 = 1011 and R2 = 1110
(B)	R1 = 1100 and R2 = 1010
(C)	R1 = 0011 and R2 = 0100
(D)	R1 = 1001 and R2 = 1111

Q.19 Consider the following threads, T_1, T_2, and T_3 executing on a single processor, synchronized using three binary semaphore variables, S_1, S_2, and S_3, operated upon using standard `wait()` and `signal()`. The threads can be context switched in any order and at any time.

T1	T2	T3

`while(true){`
`wait(S3);`
`print("C");`
`signal(S2); }`
	`while(true){`
	`wait(S1);`
	`print("B");`
	`signal(S3); }`
		`while(true){`
		`wait(S2);`
		`print("A");`
		`signal(S1); }`

Which initialization of the semaphores would print the sequence `BCABCABCA`….?

(A) $S_1 = 1; S_2 = 1; S_3 = 1$

(B) $S_1 = 1; S_2 = 1; S_3 = 0$

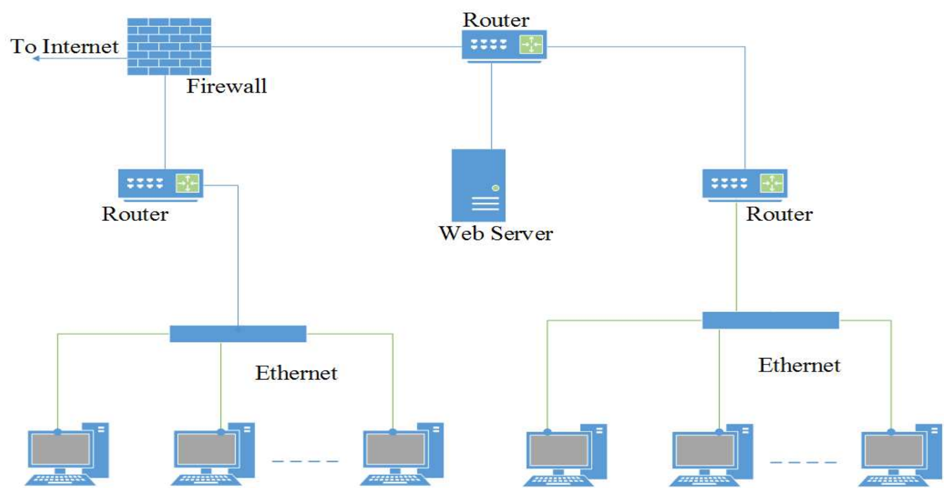
(C) $S_1 = 1; S_2 = 0; S_3 = 0$

(D) $S_1 = 0; S_2 = 1; S_3 = 1$

Q.20	Consider the following two statements with respect to the matrices $A_{m\times n}$, $B_{n\times m}$, $C_{n\times n}$ and $D_{n\times n}$. Statement 1: $tr(AB) = tr(BA)$ Statement 2: $tr(CD) = tr(DC)$ where $tr()$ represents the trace of a matrix. Which one of the following holds?
(A)	Statement 1 is correct and Statement 2 is wrong.
(B)	Statement 1 is wrong and Statement 2 is correct.
(C)	Both Statement 1 and Statement 2 are correct.
(D)	Both Statement 1 and Statement 2 are wrong.

Q.21	What is printed by the following ANSI C program?
	`#include<stdio.h>`
	`int main(int argc, char *argv[])`
	`{`
	`    int x = 1, z[2] = {10, 11};`
	`    int *p = NULL;`
	`    p = &x;`
	`    *p = 10;`
	`    p = &z[1];`
	`    *(&z[0] + 1) += 3;`
	`    printf("%d, %d, %d\n", x, z[0], z[1]);`
	`    return 0;`
	`}`

(A)	1, 10, 11
(B)	1, 10, 14
(C)	10, 14, 11
(D)	10, 10, 14

Q.22	Consider an enterprise network with two Ethernet segments, a web server and a firewall, connected via three routers as shown below.
	 What is the number of subnets inside the enterprise network?
(A)	3
(B)	12
(C)	6
(D)	8

Q.23 – Q.27 Multiple Select Questions (MSQ), carry ONE mark each.

Q.23	Which of the following statements is/are TRUE?
(A)	Every subset of a recursively enumerable language is recursive.
(B)	If a language L and its complement $\overline{L}$ are both recursively enumerable, then L must be recursive.
(C)	Complement of a context-free language must be recursive.
(D)	If L_1 and L_2 are regular, then $L_1 \cap L_2$ must be deterministic context-free.
Q.24	Let WB and WT be two set associative cache organizations that use LRU algorithm for cache block replacement. WB is a write back cache and WT is a write through cache. Which of the following statements is/are FALSE?
(A)	Each cache block in WB and WT has a dirty bit.
(B)	Every write hit in WB leads to a data transfer from cache to main memory.
(C)	Eviction of a block from WT will not lead to data transfer from cache to main memory.
(D)	A read miss in WB will never lead to eviction of a dirty block from WB.

Q.25	Consider the following three relations in a relational database. $Employee(\underline{eId}, Name),\ Brand(\underline{bId}, bName),\ Own(\underline{eId}, \underline{bId})$ Which of the following relational algebra expressions return the set of *eId*s who own all the brands?
(A)	$\Pi_{eId}(\Pi_{eId,bId}(Own) / \Pi_{bId}(Brand))$
(B)	$\Pi_{eId}(Own) - \Pi_{eId}\left((\Pi_{eId}(Own) \times \Pi_{bId}(Brand)) - \Pi_{eId,bId}(Own)\right)$
(C)	$\Pi_{eId}(\Pi_{eId,bId}(Own) / \Pi_{bId}(Own))$
(D)	$\Pi_{eId}\left((\Pi_{eId}(Own) \times \Pi_{bId}(Own)) / \Pi_{bId}(Brand)\right)$
Q.26	Which of the following statements is/are TRUE with respect to deadlocks?
(A)	Circular wait is a necessary condition for the formation of deadlock.
(B)	In a system where each resource has more than one instance, a cycle in its wait-for graph indicates the presence of a deadlock.
(C)	If the current allocation of resources to processes leads the system to unsafe state, then deadlock will necessarily occur.
(D)	In the resource-allocation graph of a system, if every edge is an assignment edge, then the system is not in deadlock state.

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Q.27	Which of the following statements is/are TRUE for a group G ?
(A)	If for all $x, y \in G$, $(xy)^2 = x^2 y^2$, then G is commutative.
(B)	If for all $x \in G$, $x^2 = 1$, then G is commutative. Here, 1 is the identity element of G.
(C)	If the order of G is 2, then G is commutative.
(D)	If G is commutative, then a subgroup of G need not be commutative.

Q.28 – Q.35 Numerical Answer Type (NAT), carry ONE mark each.

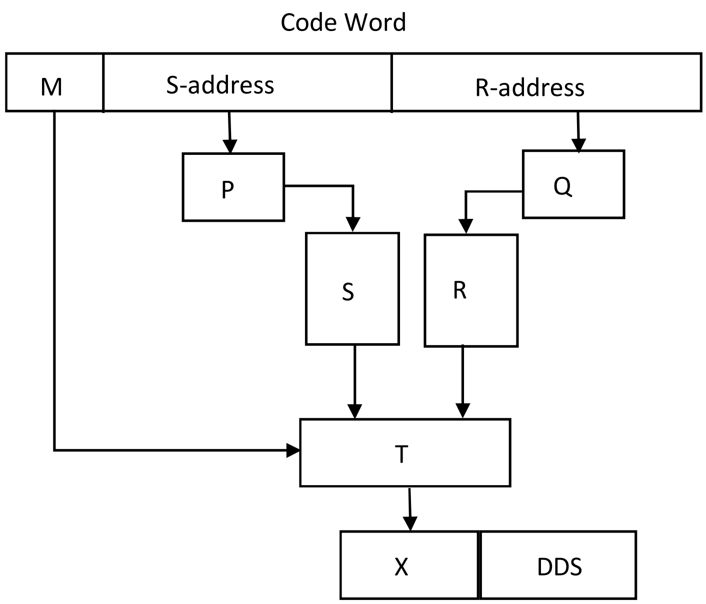
Q.28	Suppose a binary search tree with 1000 distinct elements is also a complete binary tree. The tree is stored using the array representation of binary heap trees. Assuming that the array indices start with 0, the 3rd largest element of the tree is stored at index______________.
Q.29	Consider the augmented grammar with $\{+, *, (,), id\}$ as the set of terminals. $S' \rightarrow S$ $S \rightarrow S + R \mid R$ $R \rightarrow R * P \mid P$ $P \rightarrow (S) \mid id$ If I_0 is the set of two $LR(0)$ items $\{[S' \rightarrow S.], [S \rightarrow S. + R]\}$, then $goto(closure(I_0), +)$ contains exactly __________ items.
Q.30	Consider a simple undirected graph of 10 vertices. If the graph is disconnected, then the maximum number of edges it can have is _____________.
Q.31	Consider a relation $R(A, B, C, D, E)$ with the following three functional dependencies. $AB \rightarrow C;\ \ BC \rightarrow D;\ \ C \rightarrow E;$ The number of superkeys in the relation R is ______________.
Q.32	The number of arrangements of six identical balls in three identical bins is______.

Q.33	A cache memory that has a hit rate of 0.8 has an access latency 10 ns and miss penalty 100 ns. An optimization is done on the cache to reduce the miss rate. However, the optimization results in an increase of cache access latency to 15 ns, whereas the miss penalty is not affected. The minimum hit rate (*rounded off to two decimal places*) needed after the optimization such that it should not increase the average memory access time is _____________.
Q.34	The value of the following limit is _____________. $$\lim_{x \to 0+} \frac{\sqrt{x}}{1-e^{2\sqrt{x}}}$$
Q.35	Consider the resolution of the domain name `www.gate.org.in` by a DNS resolver. Assume that no resource records are cached anywhere across the DNS servers and that iterative query mechanism is used in the resolution. The number of DNS query-response pairs involved in completely resolving the domain name is_____________.

Q.36 – Q.45 Multiple Choice Questions (MCQ), carry TWO marks each.

Q.36	Which one of the following is the closed form for the generating function of the sequence $\{a_n\}_{n\geq 0}$ defined below? $$a_n = \begin{cases} n+1, & n \text{ is odd} \\ 1, & \text{otherwise} \end{cases}$$
(A)	$\frac{x(1+x^2)}{(1-x^2)^2}+\frac{1}{1-x}$
(B)	$\frac{x(3-x^2)}{(1-x^2)^2}+\frac{1}{1-x}$
(C)	$\frac{2x}{(1-x^2)^2}+\frac{1}{1-x}$
(D)	$\frac{x}{(1-x^2)^2}+\frac{1}{1-x}$
Q.37	Consider a simple undirected unweighted graph with at least three vertices. If A is the adjacency matrix of the graph, then the number of 3-cycles in the graph is given by the trace of
(A)	A^3
(B)	A^3 divided by 2
(C)	A^3 divided by 3
(D)	A^3 divided by 6

Q.38	Which one of the following statements is FALSE?
(A)	The TLB performs an associative search in parallel on all its valid entries using page number of incoming virtual address.
(B)	If the virtual address of a word given by CPU has a TLB hit, but the subsequent search for the word results in a cache miss, then the word will always be present in the main memory.
(C)	The memory access time using a given inverted page table is always same for all incoming virtual addresses.
(D)	In a system that uses hashed page tables, if two distinct virtual addresses V1 and V2 map to the same value while hashing, then the memory access time of these addresses will not be the same.
Q.39	Let $R_i(z)$ and $W_i(z)$ denote read and write operations on a data element z by a transaction T_i , respectively. Consider the schedule ***S*** with four transactions. ***S***: $R_4(x)R_2(x)R_3(x)R_1(y)W_1(y)W_2(x)W_3(y)R_4(y)$ Which one of the following serial schedules is conflict equivalent to ***S***?
(A)	$T_1 \rightarrow T_3 \rightarrow T_4 \rightarrow T_2$
(B)	$T_1 \rightarrow T_4 \rightarrow T_3 \rightarrow T_2$
(C)	$T_4 \rightarrow T_1 \rightarrow T_3 \rightarrow T_2$
(D)	$T_3 \rightarrow T_1 \rightarrow T_4 \rightarrow T_2$

Q.40	Consider a digital display system (DDS) shown in the figure that displays the contents of register X. A 16-bit code word is used to load a word in X, either from S or from R. S is a 1024-word memory segment and R is a 32-word register file. Based on the value of mode bit M, T selects an input word to load in X. P and Q interface with the corresponding bits in the code word to choose the addressed word. Which one of the following represents the functionality of P, Q, and T?
	
(A)	P is 10:1 multiplexer; Q is 5:1 multiplexer; T is 2:1 multiplexer
(B)	P is 10:2^{10} decoder; Q is 5:2^{5} decoder; T is 2:1 encoder
(C)	P is 10:2^{10} decoder; Q is 5:2^{5} decoder; T is 2:1 multiplexer
(D)	P is 1:10 de-multiplexer; Q is 1:5 de-multiplexer; T is 2:1 multiplexer

Q.41	Consider three floating point numbers A, B and C stored in registers R_A, R_B and R_C, respectively as per IEEE-754 single precision floating point format. The 32-bit content stored in these registers (in hexadecimal form) are as follows. R_A= 0xC1400000 | R_B = 0x42100000 | R_C = 0x41400000 Which one of the following is FALSE?
(A)	$A+C=0$
(B)	$C=A+B$
(C)	$B=3C$
(D)	$(B-C)>0$
Q.42	Consider four processes P, Q, R, and S scheduled on a CPU as per round robin algorithm with a time quantum of 4 units. The processes arrive in the order P, Q, R, S, all at time t = 0. There is exactly one context switch from S to Q, exactly one context switch from R to Q, and exactly two context switches from Q to R. There is no context switch from S to P. Switching to a ready process after the termination of another process is also considered a context switch. Which one of the following is NOT possible as CPU burst time (in time units) of these processes?
(A)	P = 4, Q = 10, R = 6, S = 2
(B)	P = 2, Q = 9, R = 5, S = 1
(C)	P = 4, Q = 12, R = 5, S = 4
(D)	P = 3, Q = 7, R = 7, S = 3

Q.43	What is printed by the following ANSI C program?

```
#include<stdio.h>
int main(int argc, char *argv[])
{
          int a[3][3][3] =
          {{1, 2, 3, 4, 5, 6, 7, 8, 9},
           {10, 11, 12, 13, 14, 15, 16, 17, 18},
           {19, 20, 21, 22, 23, 24, 25, 26, 27}};
     int i = 0, j = 0, k = 0;
     for( i = 0; i < 3; i++ ){
          for(k = 0; k < 3; k++ )
               printf("%d ", a[i][j][k]);
          printf("\n");
     }
     return 0;
}
```

(A)
```
1 2 3
10 11 12
19 20 21
```

(B)
```
1 4 7
10 13 16
19 22 25
```

(C)
```
1 2 3
4 5 6
7 8 9
```

(D)
```
1 2 3
13 14 15
25 26 27
```

Q.44 What is printed by the following ANSI C program?

```
#include<stdio.h>
int main(int argc, char *argv[]){
    char a = 'P';
    char b = 'x';
    char c = (a & b) + '*';
    char d = (a | b) - '-';
    char e = (a ^ b) + '+';
    printf("%c %c %c\n", c, d, e);
    return 0;
}
```

ASCII encoding for relevant characters is given below

A	B	C	...	Z
65	66	67	...	90

a	b	c	...	z
97	98	99	...	122

*	+	–
42	43	45

(A) `z K S`

(B) `122 75 83`

(C) `* - +`

(D) `P x +`

Q.45	Consider solving the following system of simultaneous equations using LU decomposition. $$\begin{aligned} x_1 + x_2 - 2x_3 &= 4 \\ x_1 + 3x_2 - x_3 &= 7 \\ 2x_1 + x_2 - 5x_3 &= 7 \end{aligned}$$ where L and U are denoted as $$L = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$ Which one of the following is the correct combination of values for L_{32}, U_{33}, and x_1?
(A)	$L_{32} = 2,\ U_{33} = -\frac{1}{2},\ x_1 = -1$
(B)	$L_{32} = 2,\ U_{33} = 2,\ x_1 = -1$
(C)	$L_{32} = -\frac{1}{2},\ U_{33} = 2,\ x_1 = 0$
(D)	$L_{32} = -\frac{1}{2},\ U_{33} = -\frac{1}{2},\ x_1 = 0$

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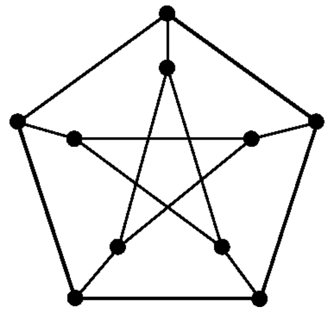
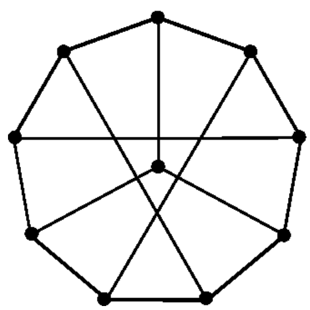
Q.46 – Q.555 Multiple Select Questions (MSQ), carry TWO marks each.

Q.46	Which of the following is/are undecidable?
(A)	Given two Turing machines M_1 and M_2, decide if $L(M_1) = L(M_2)$.
(B)	Given a Turing machine M, decide if $L(M)$ is regular.
(C)	Given a Turing machine M, decide if M accepts all strings.
(D)	Given a Turing machine M, decide if M takes more than 1073 steps on every string.
Q.47	Consider the following languages: $L_1 = \{a^n w a^n \mid w \in \{a,b\}^*\}$ $L_2 = \{w x w^R \mid w, x \in \{a,b\}^*, \lvert w\rvert, \lvert x\rvert > 0\}$ Note that w^R is the reversal of the string w. Which of the following is/are TRUE?
(A)	L_1 and L_2 are regular.
(B)	L_1 and L_2 are context-free.
(C)	L_1 is regular and L_2 is context-free.
(D)	L_1 and L_2 are context-free but not regular.

Q.48	Consider the following languages: $L_1 = \{ww \mid w \in \{a,b\}^*\}$ $L_2 = \{a^n b^n c^m \mid m,n \geq 0\}$ $L_3 = \{a^m b^n c^n \mid m,n \geq 0\}$ Which of the following statements is/are FALSE?
(A)	L_1 is not context-free but L_2 and L_3 are deterministic context-free.
(B)	Neither L_1 nor L_2 is context-free.
(C)	L_2, L_3 and $L_2 \cap L_3$ all are context-free.
(D)	Neither L_1 nor its complement is context-free.
Q.49	Consider a simple undirected weighted graph G, all of whose edge weights are distinct. Which of the following statements about the minimum spanning trees of G is/are TRUE?
(A)	The edge with the second smallest weight is always part of any minimum spanning tree of G.
(B)	One or both of the edges with the third smallest and the fourth smallest weights are part of any minimum spanning tree of G.
(C)	Suppose $S \subseteq V$ be such that $S \neq \phi$ and $S \neq V$. Consider the edge with the minimum weight such that one of its vertices is in S and the other in $V \setminus S$. Such an edge will always be part of any minimum spanning tree of G.
(D)	G can have multiple minimum spanning trees.

Q.50	The following simple undirected graph is referred to as the Peterson graph.  Which of the following statements is/are TRUE?
(A)	The chromatic number of the graph is 3.
(B)	The graph has a Hamiltonian path.
(C)	The following graph is isomorphic to the Peterson graph. 
(D)	The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)

Q.51	Consider the following recurrence: $$\begin{aligned} f(1) &= 1; \\ f(2n) &= 2f(n)-1, \quad \text{for } n \geq 1; \\ f(2n+1) &= 2f(n)+1, \quad \text{for } n \geq 1. \end{aligned}$$ Then, which of the following statements is/are TRUE?
(A)	$f(2^n - 1) = 2^n - 1$
(B)	$f(2^n) = 1$
(C)	$f(5 \cdot 2^n) = 2^{n+1} + 1$
(D)	$f(2^n + 1) = 2^n + 1$
Q.52	Which of the properties hold for the adjacency matrix A of a simple undirected unweighted graph having n vertices?
(A)	The diagonal entries of A^2 are the degrees of the vertices of the graph.
(B)	If the graph is connected, then none of the entries of $A^{n-1} + I_n$ can be zero.
(C)	If the sum of all the elements of A is at most $2(n-1)$, then the graph must be acyclic.
(D)	If there is at least a 1 in each of A's rows and columns, then the graph must be connected.

Q.53	Which of the following is/are the eigenvector(s) for the matrix given below? $$\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$$
(A)	$\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$
(B)	$\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$
(C)	$\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$
(D)	$\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

Q.54	Consider a system with 2 KB direct mapped data cache with a block size of 64 bytes. The system has a physical address space of 64 KB and a word length of 16 bits. During the execution of a program, four data words P, Q, R, and S are accessed in that order 10 times (*i.e.,* PQRSPQRS…). Hence, there are 40 accesses to data cache altogether. Assume that the data cache is initially empty and no other data words are accessed by the program. The addresses of the first bytes of P, Q, R, and S are 0xA248, 0xC28A, 0xCA8A, and 0xA262, respectively. For the execution of the above program, which of the following statements is/are TRUE with respect to the data cache?
(A)	Every access to S is a hit.
(B)	Once P is brought to the cache it is never evicted.
(C)	At the end of the execution only R and S reside in the cache.
(D)	Every access to R evicts Q from the cache.

Q.55 Consider routing table of an organization's router shown below:

Subnet Number	Subnet Mask	Next Hop
12.20.164.0	255.255.252.0	R1
12.20.170.0	255.255.254.0	R2
12.20.168.0	255.255.254.0	Interface 0
12.20.166.0	255.255.254.0	Interface 1
default		R3

Which of the following prefixes in CIDR notation can be collectively used to correctly aggregate all of the subnets in the routing table?

(A) 12.20.164.0/20

(B) 12.20.164.0/22

(C) 12.20.164.0/21

(D) 12.20.168.0/22

Q.56 – Q.65 Numerical Answer Type (NAT), carry TWO marks each.

Q.56 Consider the relational database with the following four schemas and their respective instances.

`Student(sNo, sName, dNo)  Dept(dNo, dName)`

`Course(cNo, cName, dNo)  Register(sNo, cNo)`

Student

sNo	sName	dNo
S01	James	D01
S02	Rocky	D01
S03	Jackson	D02
S04	Jane	D01
S05	Milli	D02

Dept

dNo	dName
D01	CSE
D02	EEE

Course

cNo	cName	dNo
C11	DS	D01
C12	OS	D01
C21	DE	D02
C22	PT	D02
C23	CV	D03

Register

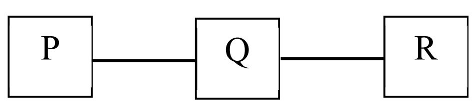
sNo	cNo
S01	C11
S01	C12
S02	C11
S03	C21
S03	C22
S03	C23
S04	C11
S04	C12
S05	C11
S05	C21

SQL Query:

```
SELECT * FROM Student AS S WHERE NOT EXIST
 (SELECT cNo FROM Course WHERE dNo = "D01"
     EXCEPT
  SELECT cNo FROM Register WHERE sNo = S.sNo)
```

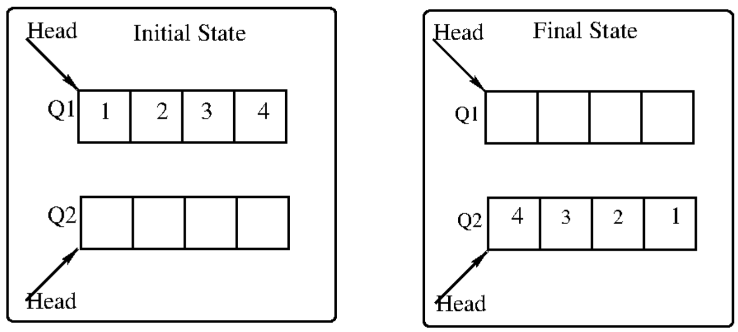
The number of rows returned by the above SQL query is____________.

Q.57 Consider a network with three routers P, Q, R shown in the figure below. All the links have cost of unity.



The routers exchange distance vector routing information and have converged on the routing tables, after which the link Q−R fails. Assume that P and Q send out routing updates at random times, each at the same average rate. The probability of a routing loop formation (*rounded off to one decimal place*) between P and Q, leading to count-to-infinity problem, is____________.

Q.58	Let $G(V,E)$ be a directed graph, where $V = \{1,2,3,4,5\}$ is the set of vertices and E is the set of directed edges, as defined by the following adjacency matrix A. $$A[i][j] = \begin{cases} 1, & 1 \le j \le i \le 5 \\ 0, & otherwise \end{cases}$$ $A[i][j] = 1$ indicates a directed edge from node i to node j. A *directed spanning tree* of G, rooted at $r \in V$, is defined as a subgraph T of G such that the undirected version of T is a tree, and T contains a directed path from r to every other vertex in V. The number of such directed spanning trees rooted at vertex 5 is_____________.
Q.59	Consider a 100 Mbps link between an earth station (sender) and a satellite (receiver) at an altitude of 2100 km. The signal propagates at a speed of 3×10^8 m/s. The time taken (in milliseconds, *rounded off to two decimal places*) for the receiver to completely receive a packet of 1000 bytes transmitted by the sender is_________.
Q.60	Consider the data transfer using TCP over a 1 Gbps link. Assuming that the maximum segment lifetime (MSL) is set to 60 seconds, the minimum number of bits required for the sequence number field of the TCP header, to prevent the sequence number space from wrapping around during the MSL is____________.
Q.61	A processor X_1 operating at 2 GHz has a standard 5-stage RISC instruction pipeline having a base CPI (cycles per instruction) of one without any pipeline hazards. For a given program P that has 30% branch instructions, control hazards incur 2 cycles stall for every branch. A new version of the processor X_2 operating at same clock frequency has an additional branch predictor unit (BPU) that completely eliminates stalls for correctly predicted branches. There is neither any savings nor any additional stalls for wrong predictions. There are no structural hazards and data hazards for X_1 and X_2. If the BPU has a prediction accuracy of 80%, the speed up (*rounded off to two decimal places*) obtained by X_2 over X_1 in executing P is____________.

Q.62	Consider the queues Q_1 containing four elements and Q_2 containing none (shown as the Initial State in the figure). The only operations allowed on these two queues are `Enqueue(Q,element)` and `Dequeue(Q)`. The minimum number of `Enqueue` operations on Q_1 required to place the elements of Q_1 in Q_2 in reverse order (shown as the Final State in the figure) without using any additional storage is____________.
	
Q.63	Consider two files systems A and B, that use contiguous allocation and linked allocation, respectively. A file of size 100 blocks is already stored in A and also in B. Now, consider inserting a new block in the middle of the file (between 50th and 51st block), whose data is already available in the memory. Assume that there are enough free blocks at the end of the file and that the file control blocks are already in memory. Let the number of disk accesses required to insert a block in the middle of the file in A and B are n_A and n_B, respectively, then the value of $n_A + n_B$ is__________.
Q.64	Consider a demand paging system with four page frames (initially empty) and LRU page replacement policy. For the following page reference string $$7,2,7,3,2,5,3,4,6,7,7,1,5,6,1$$ the page fault rate, defined as the ratio of number of page faults to the number of memory accesses (*rounded off to one decimal place*) is__________.

Q.65	Consider the following grammar along with translation rules. $S \rightarrow S_1 \# T \quad \{S_{\bullet val} = S_{1\bullet val} * T_{\bullet val}\}$ $S \rightarrow T \quad \{S_{\bullet val} = T_{\bullet val}\}$ $T \rightarrow T_1 \% R \quad \{T_{\bullet val} = T_{1\bullet val} \div R_{\bullet val}\}$ $T \rightarrow R \quad \{T_{\bullet val} = R_{\bullet val}\}$ $R \rightarrow id \quad \{R_{\bullet val} = id_{\bullet val}\}$ Here $\#$ and $\%$ are operators and *id* is a token that represents an integer and $id_{\bullet val}$ represents the corresponding integer value. The set of non-terminals is $\{S, T, R, P\}$ and a subscripted non-terminal indicates an instance of the non-terminal. Using this translation scheme, the computed value of $S_{\bullet val}$ for root of the parse tree for the expression $20\#10\%5\#8\%2\%2$ is _____________.

Q. No.	Session	Question Type	Subject Name	Key/Range	Mark
1	1	MCQ	GA	D	1
2	1	MCQ	GA	C	1
3	1	MCQ	GA	D	1
4	1	MCQ	GA	A	1
5	1	MCQ	GA	B	1
6	1	MCQ	GA	B	2
7	1	MCQ	GA	D	2
8	1	MCQ	GA	D	2
9	1	MCQ	GA	B	2
10	1	MCQ	GA	D	2
11	1	MCQ	CS	A	1
12	1	MCQ	CS	D	1
13	1	MCQ	CS	D	1
14	1	MCQ	CS	A	1
15	1	MCQ	CS	A	1
16	1	MCQ	CS	B	1
17	1	MCQ	CS	A	1
18	1	MCQ	CS	B	1
19	1	MCQ	CS	C	1
20	1	MCQ	CS	C	1
21	1	MCQ	CS	D	1
22	1	MCQ	CS	C	1
23	1	MSQ	CS	B,C,D	1
24	1	MSQ	CS	A,B,D	1
25	1	MSQ	CS	A,B	1
26	1	MSQ	CS	A,D	1
27	1	MSQ	CS	A,B,C	1
28	1	NAT	CS	509 to 509	1
29	1	NAT	CS	5 to 5	1
30	1	NAT	CS	36 to 36	1
31	1	NAT	CS	8 to 8	1
32	1	NAT	CS	7 to 7	1
33	1	NAT	CS	0.85 to 0.85	1
34	1	NAT	CS	-0.5 to -0.5	1
35	1	NAT	CS	4 to 4	1
36	1	MCQ	CS	A	2
37	1	MCQ	CS	D	2
38	1	MCQ	CS	C	2
39	1	MCQ	CS	A	2
40	1	MCQ	CS	C	2
41	1	MCQ	CS	B	2
42	1	MCQ	CS	D	2
43	1	MCQ	CS	A	2
44	1	MCQ	CS	A	2

45	1	MCQ	CS	D	2
46	1	MSQ	CS	A,B,C	2
47	1	MSQ	CS	A,B,C	2
48	1	MSQ	CS	B,C,D	2
49	1	MSQ	CS	A,B,C	2
50	1	MSQ	CS	A,B,C	2
51	1	MSQ	CS	A,B,C	2
52	1	MSQ	CS	A	2
53	1	MSQ	CS	A,C,D	2
54	1	MSQ	CS	A,B,D	2
55	1	MSQ	CS	B,D	2
56	1	NAT	CS	2 to 2	2
57	1	NAT	CS	0.5 to 0.5	2
58	1	NAT	CS	24 to 24	2
59	1	NAT	CS	7.07 to 7.09	2
60	1	NAT	CS	33 to 33	2
61	1	NAT	CS	1.42 to 1.45	2
62	1	NAT	CS	0 to 0	2
63	1	NAT	CS	153 to 153	2
64	1	NAT	CS	0.6 to 0.6	2
65	1	NAT	CS	80 to 80	2

SECTION - A GENERAL APTITUDE

Q.1 Given below are two statements I and II and two conclusions I and II:

Statement:

I. All bacteria are microorganisms.

II. All pathogens are microorganisms.

Conclusions:

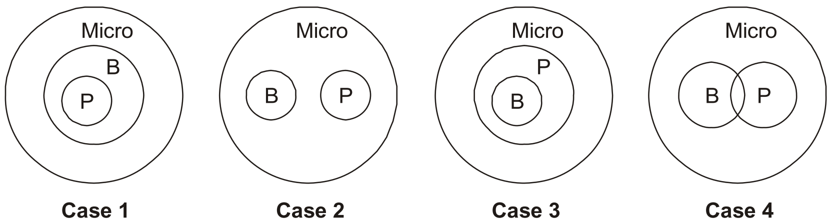
I. Some pathogens are bacteria.

II. All pathogens are not bacteria.

Based on the above statements and conclusions, which one of the following options is logically CORRECT?

(a) Only conclusion I is correct
(b) Either conclusion I or II is correct.
(c) Only conclusion II is correct
(d) Neither conclusion I nor II is correct.

Ans. (d)



None of the two conclusions will satisfy all the 4 cases.

End of Solution

Q.2 There are five bags each containing identical sets of ten distinct chocolates. One chocolate is picked from each bag.

The probability that at least two chocolates are identical is ________.

(a) 0.6976
(b) 0.3024
(c) 0.4235
(d) 0.8125

Ans. (a)

10 distinct chocolates, 5 bags

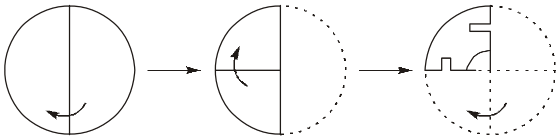
$$P(\text{atleast } 2) = 1 - P(\text{exactly } 1)$$

$$P(\text{exactly } 1) = P(\text{all different}) = \frac{10\times9\times8\times7\times6}{10^5}$$

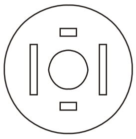
$$P(\text{atleast } 2) = 1 - \frac{10\times9\times8\times7\times6}{10^5}$$

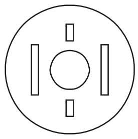
$$= 1 - 0.3024 = 0.6976$$

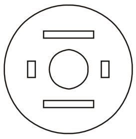
End of Solution

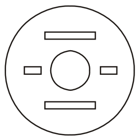
Q.3 

A circular sheet of paper is folded along the lines in the directions shown. The paper, after being punched in the filial folded state as shown and unfolded in the reverse order of folding, will look like ________.

(a) 

(b) 

(c) 

(d) 

Ans. (c)

End of Solution

Q.4 We have 2 rectangular sheets of paper, M and N, of dimensions 6 cm × 1 cm each. Sheet M is rolled to form an open cylinder by bringing the short edges of the sheet together. Sheet N is cut into equal square patches and assembled to form the largest possible closed cube. Assuming the ends of the cylinder are closed, the ratio of the volume of the cylinder to that of the cube is ________.

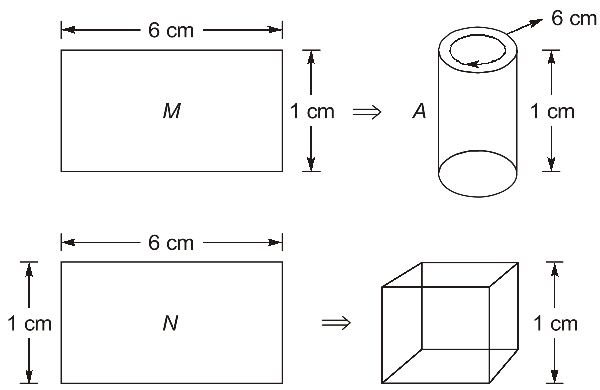
(a) $\frac{9}{\pi}$

(b) 3π

(c) $\frac{\pi}{2}$

(d) $\frac{3}{\pi}$

Ans. (a)



Volume of cylinder $= \pi r^2 h$

Now, $2\pi r = 6$ (figure A)

$$r = \frac{3}{\pi}$$

$$\text{Volume of cylinder} = \pi \times \frac{3}{\pi} \times \frac{3}{\pi} \times 1 = \frac{9}{\pi}$$

$$\text{Volume of cube} = (1)^3$$

$$\text{Ratio} = \frac{\frac{9}{\pi}}{1} = \frac{9}{\pi}$$

End of Solution

Q.5 Consider the following sentences:

(i) Everybody in the class is prepared for the exam.

(ii) Babu invited Danish to his home because he enjoys playing chess,

Which of the following is the CORRECT observation about the above two sentences?

(a) (i) is grammatically incorrect and (ii) is unambiguous

(b) (i) is grammatically correct and (ii) is ambiguous

(c) (i) is grammatically incorrect and (ii) is ambiguous

(d) (i) is grammatically correct and (ii) is unambiguous

Ans. (b)

Everybody is singular and takes singular verb is, first statement is correct and the purpose of invitation is unclear so second statement is vague and that sounds ambiguous.

End of Solution

Q.6 Some people suggest anti-obesity measures (AOM) such as displaying calorie information in restaurant menus. Such measures sidestep addressing the core problems that cause obesity: poverty and income inequality.

Which one of the following statements summarizes the passage?

(a) The proposed AOM addresses the care problems that cause obesity.

(b) If obesity reduces, poverty will naturally reduce, since obesity causes poverty.

(c) AOM are addressing the problem superficially.

(d) AOM are addressing the core problems and are likely to succeed.

Ans. (c)

Superficially is the deciding key word which means apparently/seemingly.

End of Solution

Q.7 ________ is to *surgery* as *writer* is to ________

Which one of the following options maintains a similar logical relation in the above sentence?

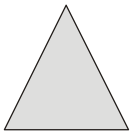
(a) Plan, outline
(b) Hospital, library
(c) Doctor, book
(d) Medicine, grammar

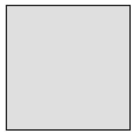
Ans. (c)

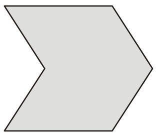
Doctor performs surgery as writer writes book.

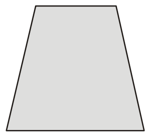
End of Solution

Q.8 A polygon is convex if, for every pair of points. P and Q belonging to the polygon, the line segment PQ lies completely inside or on the polygon.
Which one of the following is NOT a convex polygon?

(a) 

(b) 

(c) 

(d) 

Ans. **(c)**

End of Solution

Q.9

Items	Cost (₹)	Profit%	Marked price (₹)
P	5400	–	5860
Q	–	25	10000

Details of prices of two items P and Q are presented in the above table. The ratio of cost of item P to cost of item Q is 3 : 4. Discount is calculated as the difference between the marked price and the selling price. The profit percentage is calculated as the ratio of the difference between selling price and cost, to the cost

$$\left(\text{Profit\%} = \frac{\text{Selling price} - \text{Cost}}{\text{Cost}} \times 100\right).$$

The discount on item Q, as a percentage of its marked price, is ________.

(a) 25 (b) 10

(c) 5 (d) 12.5

Ans. **(b)**

$\Rightarrow$ $CP_P = 5400, \quad MP_P = 5860$

$$\frac{5400 \times 4}{5} = CP_Q$$

$$CP_Q = 7200$$

$$SP_Q = ?$$

$$\text{Profit }\% = \frac{\text{Profit}}{\text{CP}} \times 100 \text{ [for Q]}$$

$$25 = \left(\frac{SP - 7200}{7200}\right)100$$

$$SP_Q = 9000$$

$$\text{Discount Q} = MP_Q - SP_Q = 1000$$

$$\text{Discount \%} = \frac{1000}{10000} \times 100 = 10\%$$

End of Solution

Q.10 The ratio of boys to girls in a class is 7 to 3.
Among the options below, an acceptable value for the total number of students in the class is:

(a) 37
(b) 50
(c) 21
(d) 73

Ans. **(b)**

$$\text{Boys} = 7x, \quad \text{Girls} = 3x$$

Total number of students = $10x$

Now, $7x$ and $3x$ should be integers.

Hence, x should be integer.

So $10x$ should be an integer.

Only 50 satisfies the above constraints.

End of Solution

■■■■

SECTION - B TECHNICAL

Q.1 Consider the following C code segment:

$a = b + c;$
$e = a + 1;$
$d = b + c;$
$f = d + 1;$
$g = e + f;$

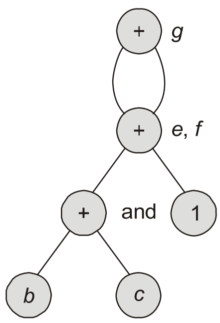
In a compiler, this code segment is represented internally as a directed acyclic graph (DAG). The number of nodes in the DAG is ________.

Ans. (6)

$a = b + c;$
$e = a + 1;$
$d = b + c;$
$f = d + 1;$
$g = e + f;$

Using common sub-expression elimination this code will become.

$a = b + c$
$e = a + 1$
$d = a$
$f = e$
$g = e + e$



Number of nodes = 6

End of Solution

Q.2 Consider the two statements:

S_1 : There exist random variables X and Y such that

$(E[X - E(X)) (Y - E(Y))]^2) > Var[X] Var[Y])$

S_2 : For all random variables X and Y

$Cov[X, Y] = E[|X - E[X]||Y - E[Y]|]$

Which one of the following choices is correct?

(a) Both S_1 and S_2 are false.
(b) Both S_1 and S_2 are true.
(c) S_1 is true, but S_2 is false.
(d) S_1 is false, but S_2 is true.

Ans. **(a)**

S_2 : $\text{Cov}(x, y) = E\{|x - \bar{x}||y - \bar{y}|\}$ is false

Case-I: If $x > \bar{x}$ and $y > \bar{y}$ then above is true.

Case-II: If $x < \bar{x}$ and $y < \bar{y}$ then above is true.

Case-III: If $x > \bar{x}$ but $y < \bar{y}$ then above is false.

Case-IV: If $x < \bar{x}$ but $y > \bar{y}$ then above is false.

$\because$ Given expression is not always true. So we can conclude that it is false.

S_1 : It is obviously false.

$\because$ True statement is $[E\{(x - \bar{x})(y - \bar{y})\}]^2 < \text{var}(x) \cdot \text{Var}(y)$

So both S_1 and S_2 are false.

End of Solution

Q.3 Consider the following two statements:

S_1 : Destination MAC address of an ARP reply is a broadcast address.

S_2 : Destination MAC address of an ARP request is a broadcast address.

Which one of the following choices is correct?

(a) S_1 is false and S_2 is true.
(b) Both S_1 and S_2 are false.
(c) Both S_1 and S_2 are true,
(d) S_1 is true and S_2 is false.

Ans. **(a)**

ARP request is broadcasting.

ARP reply is unicasting.

End of Solution

Q.4 Three processes arrive at time zero with CPU bursts of 16, 20 and 10 milliseconds. If the scheduler has prior knowledge about the length of the CPU bursts, the minimum achievable average waiting time for these three processes in a non-preemptive scheduler (round to nearest integer) is ________ milliseconds.

Ans. (12)

WT	CT	P. No.	AT	BT
0	10	A	0	16
26	46	B	0	20
10	26	C	0	10

C	A	B

0 10 26 46

$$\text{Average writing time} = \frac{0+26+10}{3}$$

$$= \frac{36}{3} = 12$$

End of Solution

Q.5 Consider the following matrix:

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

The largest eigenvalue of the above matrix is ________.

Ans. (3)

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -\lambda & 1 & 1 & 1 \\ 1 & -\lambda & 1 & 1 \\ 1 & 1 & -\lambda & 1 \\ 1 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} -\lambda+3 & 1 & 1 & 1 \\ -\lambda+3 & -\lambda & 1 & 1 \\ -\lambda+3 & 1 & -\lambda & 1 \\ -\lambda+3 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

$C_1 \leftarrow C_1 + C_2 + C_3 + C_4$

$$\Rightarrow \qquad (-\lambda+3)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -\lambda & 1 & 1 \\ 1 & 1 & -\lambda & 1 \\ 1 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

$R_2 \leftarrow R_2 - R_1,\ R_3 \leftarrow R_3 - R_1,\ R_4 \leftarrow R_4 - R_1$

$$\Rightarrow \quad (-\lambda+3)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -\lambda-1 & 0 & 0 \\ 0 & 0 & -\lambda-1 & 0 \\ 0 & 0 & 0 & -\lambda-1 \end{vmatrix} = 0$$

or $\qquad (-\lambda + 3)\ (-\lambda - 1)^3 = 0$

$\Rightarrow \qquad \lambda = 3,\ -1,\ -1,\ -1,$

So maximum eigen value is $\lambda = 3$.

End of Solution

Q.6 Consider the following grammar (that admits a series of declarations, followed by expressions) and the associated syntax directed translation (SDT) actions, given as pseudo-code:

$P \rightarrow D^*\ E^*$

$D \rightarrow$ int ID {record that ID.lexeme is of type int}

$D \rightarrow$ bool ID {record that ID.lexeme is of type bool}

$E \rightarrow E_1 + E_2$ {check that E_1.type = E_2.type = int; set E.type := int}

$E \rightarrow\ !E_1$ {check that E_1.type = bool; set E.type := bool}

$E \rightarrow$ ID {set E.type := int}

With respect to the above grammar, which one of the following choices is correct?

(a) The actions can be used to type-check syntactically correct integer variable declarations and integer expressions.

(a) The actions can be used to type-check syntactically correct boolean variable declarations and boolean expressions.

(c) The actions will lead to an infinite loop.

(d) The actions can be used to correctly type-check any syntactically correct program.

Ans. (a)

1. $P \rightarrow D^*\ E^*$
2. $D \rightarrow$ int ID {record that ID.lexeme is of type int}
3. $D \rightarrow$ bool ID {record that ID.lexeme is of type bool}
4. $E \rightarrow E_1 + E_2$ {check that E_1.type = E_2.type = int; set E.type = int}
5. $E \rightarrow\ !E_1$ {check that E_1.type = bool; set E.type = bool}
6. $E \rightarrow$ ID {set E.type = int}

Rules 2 and 3 are used for entry into the symbol table. Rule 4 is used for type checking of the integer expression. But, in rule 6 only int type is set.

Hence answer is option (a).

End of Solution

Q.7 Define R_n to be the maximum amount earned by cutting a rod of length n meters into one or more pieces of integer length and selling them. For $i > 0$, let $p[i]$ denote the selling price of a rod whose length is i meters. Consider the array of prices:

$$p[1] = 1,\ p[2] = 5,\ p[3] = 8,\ p[4] = 9,\ p[5] = 10,\ p[6] = 17,\ p[7] = 18$$

Which of the following statements is/are correct about R_7?

(a) $R_7 = 19$

(b) $R_7 = 18$

(c) R_7 is achieved by three different solutions.

(d) R_7 cannot be achieved by a solution consisting of three pieces.

Ans. (b, c)

R_7 : Maximum amount earned by cutting rod of length '7' into 1, 2, 3, 4, 5, 6, 7 pieces (whichever way is maximum)

$$\mathbf{p[7] = 18}$$
$$\mathbf{p[6] + p[1] = 17 + 1 = 18}$$

(Also, R_7 is achieved by 3 diff. solution)

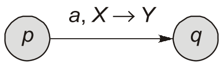
$$p[5] + p[2] = 10 + 5 = 15$$
$$p[4] + p[3] = 9 + 8 = 17$$
$$p[5] + p[1] + p[1] = 10 + 1 + 1 = 12$$
$$p[4] + p[2] + p[1] = 9 + 5 + 1 = 15$$
$$p[4] + p[1] + p[1] + p[1] = 9 + 3(1) = 12$$
$$p[3] + p[4] = 17$$
$$p[3] + p[3] + p[1] = 8 + 8 + 1 = 17$$
$$\mathbf{p[3] + p[2] + p[2] = 8 + 5 + 5 = 18}$$

(R_7 is achieved by 3 diff. pieces)

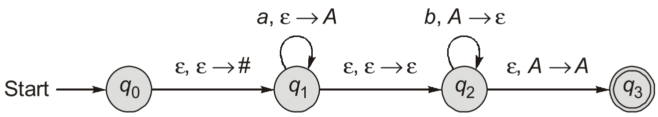
$$p[3] + 4\,p[1] = 8 + 4 = 12$$
$$p[2] + p[5] = 15$$
$$p[2] + p[4] + p[1] = 5 + 9 + 1 = 15$$
$$p[2] + p[3] + p[2] = 18$$
$$p[2] + 5\,p[15] = 5 + 5 = 10$$
$$p[2] + p[2] + 3\,p[1] = 5 + 5 + 3 = 13$$

End of Solution

Q.8 In a pushdown automaton $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$, a transition of the form,



where $p, q, \in Q$, $a \in \Sigma \cup \{\varepsilon\}$, and $X, Y \in \Gamma \cup \{\varepsilon\}$, represents $(q, Y) \in \delta(p, a, X)$
Consider the following pushdown automaton over the input alphabet $\Sigma\{a, b\}$ and stack alphabet $\Gamma = \{\#, A\}$.



The number of strings of length 100 accepted by the above pushdown automaton is ________.

Ans. **(50)**

The machine is pushing a "A" for every 'a' in input and popping a "A" for every 'b' in input which follows a's. No a's are allowed after the b's and also at end of input, the stack must have at least one "A" for string to be accepted, which means number of a's > number of b's.

So language accepted is

$$L = \{a^m b^n \mid m > n \geq 0\}$$

Now strings of length 100 which satisfy above condition are $\{a^{100}, a^{99}b, a^{98}b^2, \ldots\ldots, a^{51}b^{48}\}$.
Number of such strings is therefore 50.

End of Solution

Q.9 Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free?

(a) $L_1 \cdot L_2$
(b) $L_1 \cup L_2$
(c) $L_1 - L_2$
(d) $L_1 \cap L_2$

Ans. **(c)**

$L_1 \rightarrow$ Reg
$L_2 \rightarrow$ Context-free

(a) $L_1 \cdot L_2 = \text{Reg} \cdot \text{CFL} = \text{CFL} \cdot \text{CFL} = \text{CFL}$

(b) $L_1 \cup L_2 = \text{Reg} \cup \text{CFL} = \text{CFL}$

(c) $L_1 - L_2 = L_1 \cap \overline{L_2} = \text{Reg} \cap \overline{\text{CFL}} = \text{Reg} \cap \overline{\text{CSL}}$

$= \text{Reg} \cap \text{CSL} = \text{CSL}$

(d) $L_1 \cap L_2 = \text{Reg} \cap \text{CFL} = \text{CFL}$

So (c) is not necessarily CFL.

End of Solution

Q.10 A TCP server application is programmed to listen on port number P on host S. A TCP client is connected to the TCP server over the network.

Consider that while the TCP connection was active, the server machine S crashed and rebooted. Assume that the client does not use the TCP keepalive timer.

Which of the following behaviors is/are possible?

(a) If the client sends a packet after the server reboot, it will receive a FIN segment.
(b) If the client was waiting to receive a packet, it may wait indefinitely.
(c) The TCP server application on S can listen on P after reboot.
(d) If the client sends a packet after the server reboot, it will receive a RST segment.

Ans. **(b, c, d)**

(a) False: The situation resolves itself when client tries to send data to server over the dead connection, and server replies with an RST packet (not FIN), causing client to finally to close the connection forcibly.

FIN is used to close TCP connections gracefully in each direction (normal close of connection), while TCP RST is used in a scenario where TCP connections cannot recover from errors and the connection needs to reset forcibly.

(b) True: Since broken connections can only be detected by sending data, the receiving side will wait forever. This scenario is called a "half-open connection" because one side realizes the connection was lost but the other side believes it is still active.

(c) True: Yes, a TCP Server can listen to same port number even after reboot. For example, the SMTP service application usually listens on TCP port 25 for incoming requests. So, even after reboot the port 25 is assigned to SMTP.

(d) True: The situation resolves itself when client tries to send data to server over the dead connection, and server replies with an RST packet (not FIN).

End of Solution

Q.11 There are 6 jobs with distinct difficulty levels, and 3 computers with distinct processing speeds. Each job is assigned to a computer such that:

- The fastest computer gets the toughest job and the slowest computer gets the easiest job.
- Every computer gets at least one job.

The number of ways in which this can be done is ________.

Ans. **(65)**

Let computers be A, B and C

Toughest job assigned to fastest computer (Say, A) is 1 way.

Easiest job assigned to shortest computer (Say, B) is 1 way.

Remaining 4 jobs to be assigned to 3 computers so that the computer C gets at least one job, since A and B already assigned a job.

Number of ways 4 jobs assigned to 3 computers = 3^4.

Number of ways 4 jobs assigned to 3 computers, so that computer C does not get any job = 2^4.

Required number of ways $= 3^4 - 2^4 = 81 - 16 = 65$ ways

End of Solution

Q.12 Consider two hosts P and Q connected through a router R. The maximum transfer unit (MTU) value of the link between P and R is 1500 bytes, and between R and Q is 820 bytes.

A TCP segment of size 1400 bytes was transferred from P to Q through R, with IP identification value as 0x1234. Assume that the IP header size is 20 bytes. Further, the packet is allowed to be fragmented, i.e., Don't Fragment (DF) flag in the IP header is not set by P.

Which of the following statements is/are correct?

(a) Two fragments are created at R and the IP datagram size carrying the second fragment is 620 bytes.

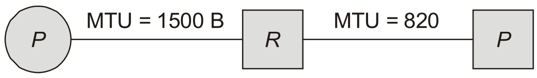
(b) If the second fragment is lost, P is required to resend the whole TCP segment.

(c) TCP destination port can be determined by analysing only the second fragment.

(d) If the second fragment is lost, R will resend the fragment with the IP identification value 0x1234

Ans. **(a, b)**

Data = 1400 B



For First Link – PR – There will be no fragment

For Second Link – RQ

First Fragment = 800(Data) + 20(Header) = 820 B

Second Fragment = 1400 – 800 = 600 + 20(H) = 620 B

End of Solution

Q.13 For a Turing machine M, $\langle M \rangle$ denotes an encoding of M. Consider the following two languages:

$L_1 = \{\langle M \rangle \mid M \text{ takes more than 2021 steps on all inputs}\}$

$L_2 = \{\langle M \rangle \mid M \text{ takes more than 2021 steps on some input}\}$

Which one of the following options is correct?

(a) Both L_1 and L_2 are decidable.

(b) L_1 is undecidable and L_2 is decidable.

(c) Both L_1 and L_2 are undecidable.

(d) L_1 is decidable and L_2 is undecidable.

Ans. **(a)**

$L_1 = \{\langle M \rangle \mid M \text{ takes more than 2021 steps on all inputs}\}$

$L_2 = \{\langle M \rangle \mid M \text{ takes more than 2021 steps on some input}\}$

A Turing machine reads at most 2021 bits of input while making 2021 steps. So the halting behaviour is completely determined by the first 2021 bits of input. Now the number of strings with 2021 bits is finite and so generate all of them and in finite amount of time we can check if the given TM, M halts on any of these strings.

For L_1, the algorithm will be as follows,

Does not halts on all of these strings → Yes

Halts on at least one of these strings → No

For L_2, the algorithm will be as follows,

does not halt on at least one of these strings $\rightarrow$ Yes

Halts on all of these strings $\rightarrow$ No

So, both L_1 and L_2 are decidable.

End of Solution

Q.14 In an undirected connected planar graph G, there are eight vertices and five faces. The number of edges in G is ________.

Ans. **(11)**

In a connected planar graph

$$r = e - n + 2$$

Here, $n = 8, \quad r = 5$

$\therefore$ $5 = e - 8 + 2$

$$e = 11$$

End of Solution

Q.15 Consider the following array:

23	32	45	69	72	73	89	97

Which algorithm out of the following options uses the least number of comparisons (among the array elements) to sort the above array in ascending order?

(a) Quicksort using the last element as pivot
(b) Selection sort
(c) Mergesort
(d) Insertion sort

Ans. **(d)**

Given array already sorted.

$\therefore$ Insertion sort takes least number of comparisons $\theta(n)$.

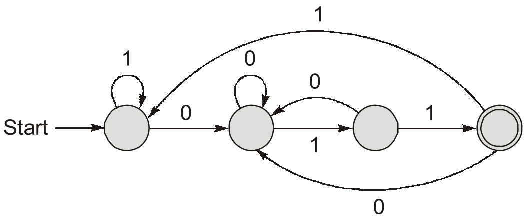
Since, for a number which is to be inserted in the already sorted array, only 1 comparison will be required.

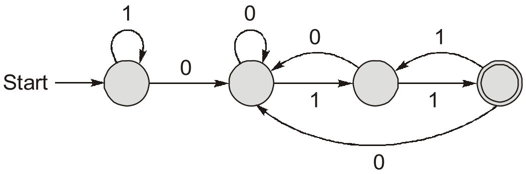
End of Solution

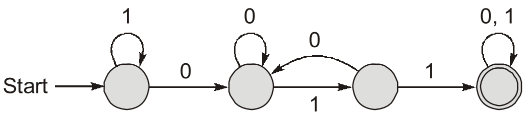
Q.16 Consider the following language:

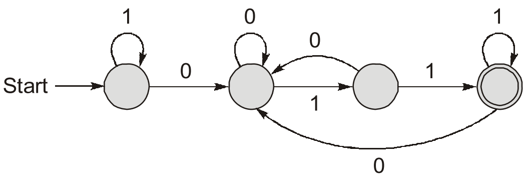
$L = \{w \in \{0,1\}^* \mid w \text{ ends with the substring } 011\}$

Which one of the following deterministic finite automata accepts L?

(a) 

(b) 

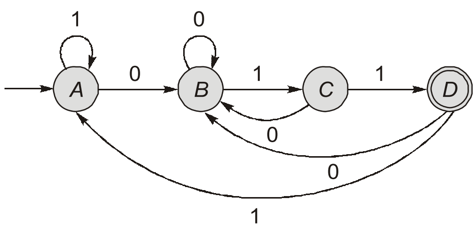
(c) 

(d) 

Ans. **(a)**

Since 4 states are required in minimal DFA for this language "Ending with 011", and since all given DFA's have 4 states, the answer must be same as minimal DFA.

The minimal DFA for this language is given below:



Which is same as option (a).

End of Solution

Q.17 Let p and q be two propositions. Consider the following two formulae in prepositional logic.

S_1 : $(\neg p \wedge (p \vee q)) \rightarrow p$
S_2 : $q \rightarrow (\neg p \wedge (p \vee q))$

Which one of the following choices is correct?

(a) Neither S_1 nor S_2 is a tautology.
(b) Both S_1 and S_2 are tautologies.
(c) S_1 is a tautology but S_2 is not a tautology.
(d) S_1 is not a tautology but S_2 is a tautology.

Ans. (c)

S_1 : $(\neg p \wedge (p \vee q)) \rightarrow p$
S_2 : $q \rightarrow (\neg p \wedge (p \vee q))$

$$
\begin{aligned}
S_1 &: [p'(p+q)] \rightarrow p \\
&\equiv (p'p + p'q) \rightarrow q \\
&\equiv p'q \rightarrow q \\
&\equiv (p'q)' + q \\
&\equiv p + q' + q \\
&\equiv p + 1 \\
&\equiv 1 \text{ (tautology)}
\end{aligned}
$$

$$
\begin{aligned}
S_2 &: q \rightarrow (p'(p+q)) \\
&\equiv q \rightarrow (p'p + p'q) \\
&\equiv q \rightarrow p'q \\
&\equiv q' + p'q \\
&\equiv (q' + p')(q' + q) \\
&\equiv q' + p' \text{ (contigency) (not a tautology)}
\end{aligned}
$$

So, option (c) S_1 is a tautology and S_2 is not a tautology is correct.

End of Solution

Q.18 Suppose a database system crashes again while recovering from a previous crash. Assume checkpointing is not done by the database either during the transactions or during recovery.

Which of the following statements is/are correct?

(a) The same undo and redo list will be used while recovering again.

(b) The system cannot recover any further.

(c) The database will become inconsistent.

(d) All the transactions that are already undone and redone will not be recovered again.

Ans. (a)

End of Solution

Q.19 A relation R is said to be circular if aRb and bRc together imply cRa.

Which of the following options is/are correct?

(a) If a relation S is reflexive and circular, then S is an equivalence relation.

(b) If a relation S is circular and symmetric, then S is an equivalence relation.

(c) If a relation S is transitive and circular, then S is an equivalence relation.

(d) If a relation S is reflexive and symmetric, then S is an equivalence relation.

Ans. (a)

Let S be reflexive and circular,

Let us checking symmetry:

Symmetry:

Let xSy

Now since S is reflexive ySy true.

So xSy and ySy is true

Now by circular property we get, ySx

So $$xSy \Rightarrow ySx$$

So S is symmetric.

Transitive:

Let xSy and ySz

Now by circular property we get zSx and by symmetry property proved above, we get

$$zSx \Rightarrow xSz$$

So $$xSy \text{ and } ySz \Rightarrow xSz$$

So S is transitive.

So S is reflexive, symmetric and transitive and hence an equivalence relation.

So option (a) is true.

Option (b): Let S be circular and symmetric.

Let S be defined on set {1, 2, 3}

Now empty relation is circular and symmetric but not reflexive. So S need not be an equivalence relation.

So option (b) is false.

Option (c): Let S be transitive and circular.

Let S be defined on the set {1, 2, 3}

Now empty relation again satisfies transitive and circular but is not reflexive. So S need not be an equivalence relation.

So option (c) is false.

Option (d): Reflexive and symmetric need not be transitive for example on {1, 2, 3}.

$$S = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$$

is reflexive and symmetric. But it is not transitive because (1, 2) and (2, 3) belong to S but (1, 3) does not.

So option (d) is false.

End of Solution

Q.20 Let P be an array containing n integers. Let t be the lowest upper bound on the number of comparisons of the array elements, required to find the minimum and maximum values in an arbitrary array of n elements. Which one of the following choices is correct?

(a) $t > n$ and $t \le 3\left\lceil \frac{n}{2} \right\rceil$

(b) $t > 3\left\lceil \frac{n}{2} \right\rceil$ and $t \le 2n-2$

(c) $t > 2n - 2$

(d) $t > \lceil \log_2(n) \rceil$ and $t \le n$

Ans. **(b)**

$t > 3\left\lceil \frac{n}{2} \right\rceil$ and $t \le 2n-2$

Using straight max-min algo, WC number of comparisons = $2n - 2$.

Using divide and conquer min-max algo, WC number of comparisons = $\frac{3n}{2} - 2$.

[Note: But the official answer key given by IIT-Bombay is option (a)]

End of Solution

Q.21 In the context of operating systems, which of the following statements is/are correct with respect to paging?

(a) Paging incurs memory overheads.

(b) Multi-level paging is necessary to support pages of different sizes.

(c) Page size has no impact on internal fragmentation.

(d) Paging helps solve the issue of external fragmentation.

Ans. **(a, d)**

End of Solution

Q.22 The lifetime of a component of a certain type is a random variable whose probability density function is exponentially distributed with parameter 2. For a randomly picked component of this type, the probability that its lifetime exceeds the expected lifetime (rounded to 2 decimal places) is ________.

Ans. **(0.367)**

Let, t = {lifetime of component} and $\mu = 2$

$$\text{Expected lifetime} = \frac{1}{\mu} \text{ and } f(t) = \begin{cases} \mu e^{-\mu t}, & t > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$P\left(t > \frac{1}{\mu}\right) = \int_{\frac{1}{\mu}}^{\infty} f(t)dt = \int_{\frac{1}{\mu}}^{\infty} \mu e^{-\mu t} dt = -\left[e^{-\mu t}\right]_{\frac{1}{2}}^{\infty} = -\left[0 - e^{-1}\right]$$

$$= \frac{1}{e} = \frac{1}{2.713} = 0.367$$

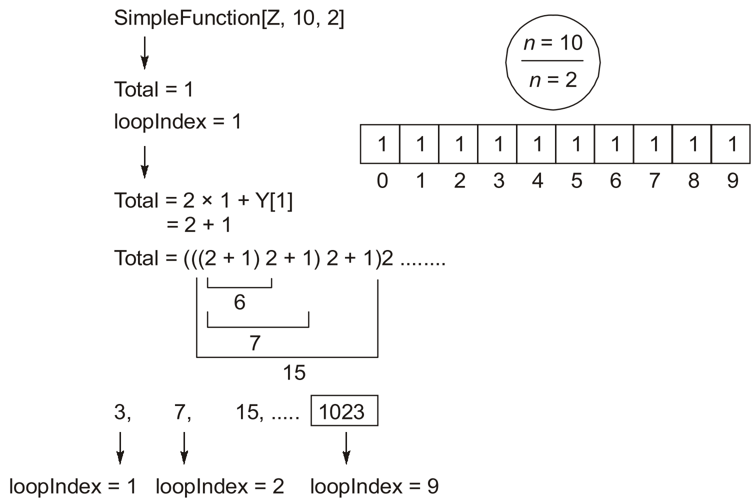
End of Solution

Q.23 Consider the following ANSI C function:

```
int SimpleFunction (int Y[ ], int n, int x)
{
    int total = Y[0], loopIndex;
    for (loopIndex = 1; loopIndex <= n – 1; loopIndex++)
    total = x × total + Y[loopIndex];
    return total;
}
```

Let Z be an array of 10 elements with Z[i] = 1, for all i such that $0 \le i \le 9$. The value returned by SimpleFunction (Z, 10, 2) is ________.

Ans. **(1023)**



End of Solution

Q.24 Consider a linear list based directory implementation in a file system. Each directory is a list of nodes, where each node contains the file name along with the file metadata, such as the list of pointers to the data blocks. Consider a given directory foo.

Which of the following operations will necessarily require a full scan of foo for successful completion?

(a) Opening of an existing file in foo
(b) Renaming of an existing file in foo
(c) Creation of a new file in foo
(d) Deletion of an existing file from foo

Ans. (a, b, d)

[Note: But the official answer key given by IIT-Bombay is option (b, c)]

End of Solution

Q.25 Let $G = (V, E)$ be an undirected unweighted connected graph. The diameter of G is defined as

$$\text{diam}(G) = \max_{u,v \in V} \{\text{the length of shortest path between } u \text{ and } v\}$$

Let M be the adjacency matrix of G.

Define graph G_2 on the same set of vertices with adjacency matrix N, where

$$N_{ij} = \begin{cases} 1 & \text{if } M_{ij} > 0 \text{ or } P_{ij} > 0, \text{ where } P = M^2 \\ 0 & \text{Otherwise} \end{cases}$$

Which one of the following statements is true?

(a) $\lceil \text{diam}(G)/2 \rceil < \text{diam}(G_2) < \text{diam}(G)$

(b) $\text{diam}(G_2) \le \lceil \text{diam}(G)/2 \rceil$

(c) $\text{diam}(G) < \text{diam}(G_2) \le 2\ \text{diam}(G)$

(d) $\text{diam}(G_2) = \text{diam}(G)$

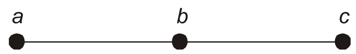
Ans. (b)

$$N_{ij} = \begin{cases} 1 & \text{if } M_{ij} > 0 \text{ or } P_{ij} > 0, \text{ where } P = M^2 \\ 0 & \end{cases}$$

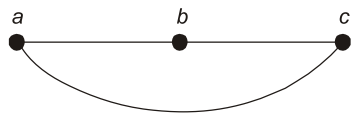
Means G_2 will have not only all the edges of in G, but also will have edges connecting vertices in G which have a path of length 2, since M^2 will have all edges between u and v if there is a path of length 2 between shown in G.

Option (a) $\lceil \text{diam}(G)/2 \rceil < \text{diam}(G_2) < \text{diam}(G)$

Let G be the graph shown below with dia(G) = 2



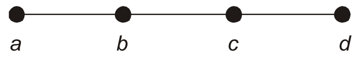
Now G_2 will be



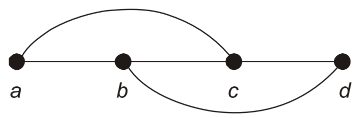
Since (a, c) has a path of length 2 in G, G_2 will have an edge connecting a and c.
Now diameter of G_2 = 1
This violates option (a) condition that dia(G_2) > dia(G)/2.
So option (a) false.

Option (b) $\text{diam}(G_2) \le \lceil \text{diam}(G)/2 \rceil$

Consider graph below with dia(G) = 3



Now G_2 will connect all paths of length 2 with edges and will be



Now $\qquad \text{dia}(G_2) = 2$
But $\qquad \text{dia}(G) = 3$

$$\text{dia}(G_2) \le \lceil \text{dia}(G)/2 \rceil$$

$$2 \le \lceil 1.5 \rceil$$

$$2 \le 2$$

Satisfied.
Option (b) is correct.
Option (c) diam(G) < diam(G_2) ≤ 2 diam(G)
Taking previous option example where dia(G) = 3 and dia(G_2) = 2

$$3 \le 2 \le 6$$

Is false.
So option (c) is false.
Option (d) diam(G_2) = diam(G)
Taking previous example where dia(G) = 3 and dia(G_2) = 2

$$2 = 3 \text{ is false}$$

So option (d) is false.
So correct option is option (b).

End of Solution

Q.26 Consider the following context-free grammar where the set of terminals is $\{a, b, c, d, f\}$

$S \to daT \mid Rf$
$T \to aS \mid baT \mid \epsilon$
$R \to caTR \mid \epsilon$

The following is a partially-filled LL(1) parsing table.

	a	b	c	d	f	$
S			(1)	$S \to daT$	(2)	
T	$T \to aS$	$T \to baT$	(3)		$T \to \epsilon$	(4)
R			$R \to caTR$		$R \to \epsilon$	

Which one of the following choices represents the correct combination for the numbered cells in the parsing table ("blank" denotes that the corresponding cell is empty)?

(a) (1) $S \to Rf$ (2) blank (3) blank (4) $T \to \epsilon$

(b) (1) blank (2) $S \to Rf$ (3) $T \to \epsilon$ (4) $T \to \epsilon$

(c) (1) blank (2) $S \to Rf$ (3) blank (4) blank

(d) (1) $S \to Rf$ (2) $S \to Rf$ (3) $T \to \epsilon$ (4) $T \to \epsilon$

Ans. **(d)**

$S \to daT \mid Rf$
$T \to aS \mid baT \mid \epsilon$
$R \to caTR \mid \epsilon$

First (S) = $\{d, c, f\}$ Follow (S) = $\{c, f, \$\}$
First (T) = $\{a, b, \epsilon\}$ Follow (T) = $\{c, f, \$\}$
First (R) = $\{c, \epsilon\}$ Follow (R) = $\{f\}$

	a	b	c	d	f	$
S			(1) $S \to Rf$	$S \to daT$	(2) $S \to Rf$	
T	$T \to aS$	$T \to baT$	(3) $T \to \epsilon$		$T \to \epsilon$	(4) $T \to \epsilon$
R			$R \to caTR$		$R \to \epsilon$	

(1) $S \to Rf$ (2) $S \to Rf$ (3) $T \to \epsilon$ (4) $T \to \epsilon$

Hence option (d) is answer.

End of Solution

Q.27 Let $\langle M \rangle$ denote an encoding of an automaton M. Suppose that $\Sigma\{0, 1\}$. Which of the following languages is/are NOT recursive?

(a) $L = \{\langle M \rangle | M$ is a DFA such that $L(M) = \Sigma^*\}$

(b) $L = \{\langle M \rangle | M$ is a PDA such that $L(M) = \Sigma^*\}$

(c) $L = \{\langle M \rangle | M$ is a PDA such that $L(M) = \phi\}$

(d) $L = \{\langle M \rangle | M$ is a DFA such that $L(M) = \phi\}$

Ans. **(b)**

Option (a): Completeness problem for regular $\rightarrow$ Decidable
$\rightarrow$ Recursive

Option (b): Completeness problem for CFL's $\rightarrow$ Undecidable
$\rightarrow$ Not recursive

Option (c): Emptiness problem for CFL's $\rightarrow$ Decidable
$\rightarrow$ Recursive

Option (d): Emptiness problem for Regular $\rightarrow$ Decidable
$\rightarrow$ Recursive

So only option (b) is not recursive.

End of Solution

Q.28 Consider the following instruction sequence where registers R1, R2 and R3 are general purpose and MEMORY[X] denotes the content at the memory location X.

Instruction	Semantics	Instruction Size (bytes)
MOV R1, (5000)	R1 $\leftarrow$ MEMORY [5000]	4
MOV R2, (R3)	R2 $\leftarrow$ MEMORY [R3]	4
ADD R2, R1	R2 $\leftarrow$ R1 + R2	2
MOV (R3), R2	MEMORY [R3] $\leftarrow$ R2	4
INC R3	R3 $\leftarrow$ R3 + 1	2
DEC R1	R1 $\leftarrow$ R1 – 1	2
BNZ 1004	Branch if not zero to the given absolute address	2
HALT	Stop	1

Assume that the content of the memory location 5000 is 10 and the content of the register R3 is 3000. The content of each of the memory locations from 3000 to 3010 is 50. The instruction sequence starts from the memory location 1000. All the numbers are in decimal format. Assume that the memory is byte addressable.

After the execution of the program, the content of memory location 3010 is ________.

Ans. **(50)**

Program execution 10 memory cells information is accessed for reading & writing starting from 3000 location.

So, 3000 to 3009 cells are accessed for read and write.

$\therefore$ No change in [3010] cell.

So, it contain 50 only.

End of Solution

Q.29 An *articulation point* in a connected graph is a vertex such that removing the vertex and its incident edges disconnects the graph into two or more connected components.

Let T be a DFS tree obtained by doing DFS in a connected undirected graph G.

Which of the following options is/are correct?

(a) Root of T is an articulation point in G if and only if it has 2 or more children.

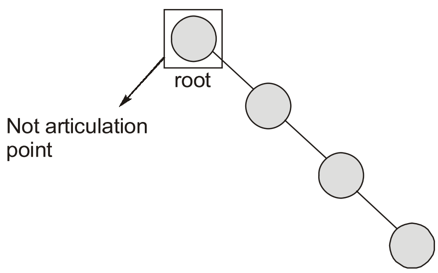
(b) A leaf of T can be an articulation point in G.

(c) Root of T can never be an articulation point in G.

(d) If u is an articulation point in G such that x is an ancestor of u in T and y is a descendent of u in T, then all paths from x to y in G must pass through u.

Ans. **(a)**

(a) True

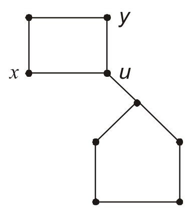


We need at least 2 children so that root is articulation point.

(b) False: This can never happen. Leaf will always have degree = 1.

(c) False: Check option (a) for more information.

(d) False: Below is the reasoning to show how this is false.



If u is articulation point, then removing u generates 2 connected components, now there might be a case when x and y will belong to either one of the connected component and hence a path will exist between them without passing through u.

Option (a) is correct.

End of Solution

Q.30 Consider the relation $R(P, Q, S, T, X, Y, Z, W)$ with the following functional dependencies

$$PQ \to X;\ P \to YX;\ Q \to Y;\ Y \to ZW$$

Consider the decomposition of the relation R into the constituent relations according to the following two decomposition schemes.

D_1 : $R = [(P, Q, S, T); (P, T, X); (Q, Y); (Y, Z, W)]$

D_2 : $R = [(P, Q, S); \{T, X); (Q, Y): (Y, Z, W)]$

Which one of the following options is correct?

(a) D_1 is a lossy decomposition, but D_2 is a lossless decomposition.
(b) Both D_1 and D_2 are lossless decompositions.
(c) Both D_1 and D_2 are lossy decompositions.
(d) D_1 is a lossless decomposition, but D_2 is a lossy decomposition.

Ans. **(d)**

- D_1: $R_1(P, Q, S, T)$ $R_2(P, T, X)$ $R_3(Q, Y)$ $R_4(Y, Z, W)$

  $R_1R_2(P, Q, S, T, X)$
  $PT \to X$
  [common attributes is a key of R_2 so lossless R_{12}]

  $R_{34}(Q, Y, Z, W)$
  $Y \to ZW$
  [common attributes is a key of R_4 so lossless R_{34}]

  $R_{12}(P, Q, S, T, X)$ $R_{34}(Q, Y, Z, W)$

  $Q^+ = \{Q, Y, Z, W\}$

  $R_{1234}(P, Q, S, T, X, Y, Z, W)$
  $Q \to YZW$
  [common attributes is a key of R_{34} overall lossless decomposition]

- D_2: $R_1(P, Q, S)$ $R_2(T, X)$ $R_3(Q, Y)$ $R_4(Y, Z, W)$

  $R_1(T, X)$ $R_1(P, Q, S)$ $R_{34}(Q, Y, Z, W)$

  $Y \to ZW$ [lossless R_{34}]

  $R_2(T, X)$ $R_{134}(P, Q, S, Y, Z, W)$ $Q \to YZW$ [lossless R_{134}]

  $\underbrace{R_2(T, X) \quad R_{134}(P, Q, S, Y, Z, W)}_{\text{No common attribute} \Rightarrow \text{Lossy}}$

D_1 is a lossless decomposition, but D_2 is a lossy decomposition.

End of Solution

Q.31 A five-stage pipeline has stage delays of 150, 120, 150, 160 and 140 nanoseconds. The registers that are used between the pipeline stages have a delay of 5 nanoseconds each.

The total time to execute 100 independent instructions on this pipeline, assuming there are no pipeline stalls, is ________ nanoseconds.

Ans. (17160)

$$K = 5$$
$$t_p = \text{max (Stage delay + Buffer delay)} = 165 \text{ ns}$$
$$n = 100 \text{ (finite)}$$
$$\downarrow$$
$$ET_{\text{pipe}} = (K + n - 1)t_p$$
$$= (5 + 100 - 1)\ 165 \text{ ns} = 17160 \text{ ns}$$

End of Solution

Q.32 Consider the sliding window flow-control protocol operating between a sender and a receiver over a full-duplex error-free link. Assume the following:

- The time taken for processing the data frame by the receiver is negligible.
- The time taken for processing the acknowledgement frame by the sender is negligible.
- The sender has infinite number of frames available for transmission.
- The size of the data frame is 2000 bits and the size of the acknowledgement frame is 10 bits.
- The link data rate in each direction is 1 Mbps (= 10^6 bits per second).
- One way propagation delay of the link is 100 milliseconds.

The minimum value of the sender's window size in terms of the number of frames, (rounded to the nearest integer) needed to achieve a link utilization of 50% is ________.

Ans. (51)

$$T_t(\text{packet}) = \frac{L}{B.W}$$

$$\Rightarrow \quad \frac{2000 \text{ bits}}{10^6 \text{ bps}} = 2 \times 10^{-3} \text{ sec} = 2 \text{ millisec}$$

$$T_t(\text{Ack}) = \frac{L}{B.W}$$

$$\Rightarrow \quad \frac{10 \text{ bits}}{10^6 \text{ bps}} = 10^{-5} \text{ sec} = 10^{-2} \text{ millisec} = 0.01 \text{ millisec}$$

$$T_p = 100 \text{ millisec}$$
$$\text{Total time} = T_t(\text{packet}) + T_p + T_t(\text{Ack})$$
$$= 2 + 100 + 0.01 = 102.01 \text{ millisec}$$

$$\text{Efficiency} = 50\% = \frac{1}{2}$$

$$\text{Efficiency} = \frac{\text{Useful time}}{\text{Total time}}$$

$$\frac{1}{2} = \frac{n \times T_t}{\text{Total time}}$$

$$\Rightarrow \quad 2 \times n = 102.01$$

$$\Rightarrow \quad n = \frac{102.01}{2} \Rightarrow 51.005$$

For minimum we have to take ceil, hence size of window = 51.

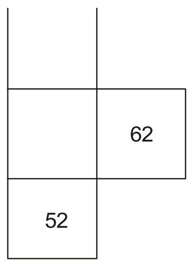
End of Solution

Q.33 Consider the following sequence of operations on an empty stack.

push(54); push(52); pop( ); push(55); push(62); s = pop( ):

Consider the following sequence of operations on an empty queue.
enqueue(21); enqueue(24); dequeue( ); enqueue(28); enqneue(32); q = dequeue( ):
The value of $s + q$ is ________.

Ans. **(86)**



S = 62
R = 24

43	24	57

S + R = 86

End of Solution

Q.34 Let the representation of a number in base 3 be 210. What is the hexadecimal representation of the number?

(a) 528 (b) 21
(c) 15 (d) D2

Ans. **(c)**

$$(210)_3 = 3^2 \times 2 + 3^1 \times 1 + 3^0 \times 0$$
$$= 18 + 3 + 0$$
$$= (21)_{10}$$

$$(21)_{10} = 16 \,\underline{|\, 21} \quad = (15)_{16}$$
$$1 \rightarrow 5$$

End of Solution

Q.35 A relation $r(A, B)$ in a relational database has 1200 tuples. The attribute A has integer values ranging from 6 to 20, and the attribute B has integer values ranging from 1 to 20. Assume that the attributes A and B are independently distributed.

The estimated number of tuples in the output of $\sigma_{(A > 10) \vee (B = 18)}(r)$ is ________.

Ans. **(820)**

- There are 10 distinct integer for $(A > 10)$ out of 15.
- There are 1 distinct integer for $(B = 18)$ out of 20.

$$P(A > 10) = \frac{10}{15} = \frac{2}{3}$$

$$P(B = 18) = \frac{1}{20}$$

$$P((A > 10) \wedge (B = 18)) = \frac{2}{3} \times \frac{1}{20} = \frac{1}{30}$$

Now,

$$P((A > 10) \vee (B = 18)) = P(A > 10) + P(B = 18) - P((A > 10) \wedge (B = 18))$$

$$= \frac{2}{3} + \frac{1}{20} - \frac{1}{30} = \frac{40+3-2}{60} = \frac{41}{60}$$

$$\text{Estimated number of tuples} = \frac{41}{60} \times 1200 = 820 \text{ tuples}$$

End of Solution

Q.36 Assume that a 12-bit Hamming codeword consisting of 8-bit data and 4 check bits is $d_8\ d_7\ d_6\ d_5\ c_8\ d_4\ d_4\ d_3\ d_2\ c_4\ d_1\ c_2\ c_1$, where the data bits and the check bits are given in the following tables:

Data bits							
d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1
1	1	0	x	0	1	0	1

Check bits			
c_8	c_4	c_2	c_1
y	0	1	0

Which one of the following choices gives the correct values of x and y?

(a) x is 0 and y is 0
(b) x is 0 and y is 1
(c) x is 1 and y is 0
(d) x is 1 and y is 1

Ans. (a)

1	2	3	4	5	6	7	8
1	0	1	0	x	0	1	1

c_1	c_2	c_4	c_8
0	1	0	y

c_1 = 1 3 5 7 9 11
0 1 0 0 x 1 $\xleftrightarrow{x=0}$

1	2	3	4	5	6	7	8	9	10	11	12
0	1	1	0	0	1	0	y	x	0	1	1

c_8 = 8 9 10 11 12
0 0 0 1 1 $\xleftrightarrow{y=0}$

End of Solution

Q.37 Let $r_i(z)$ and $w_i(z)$ denote read and write operations respectively on a data item z by a transaction T_i. Consider the following two schedules:

S_1 : $r_1(x), r_1(y), r_2(x), r_2(y), w_2(y), w_1(x)$

S_2 : $r_1(x), r_2(x), r_2(y), w_2(y), r_1(y), w_1(x)$

Which one of the following options is correct?

(a) S_1 is not conflict serializable, and S_2 is conflict serializable.
(b) Neither S_1 nor S_2 is conflict serializable.
(c) Both S_1 and S_2 are conflict serializable.
(d) S_1 is conflict serializable, and S_2 is not conflict serializable.

Ans. (a)

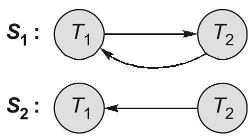
RW RW

$S_1 : r_1(x)\ r_1(y)\ r_2(x)\ r_2(y)\ w_2(y)\ w_1(x)$

$S_2 : r_1(x)\ r_2(x)\ r_2(y)\ w_2(y)\ r_1(y)\ w_1(x)$

WR

RW



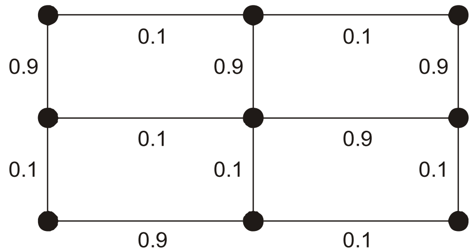
Cycle exists in precedence graph
∴ Not conflict serializable.

No cycle
∴ Conflict serializable.

S_1 is not conflict serializable, and S_2 is conflict serializable.

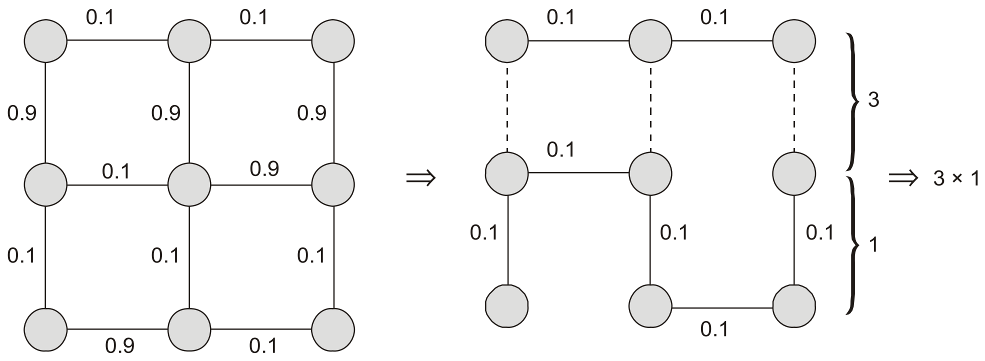
End of Solution

Q.38 Consider the following undirected graph with edge weights as shown:



The number of minimum-weight spanning trees of the graph is ________.

Ans. (3)

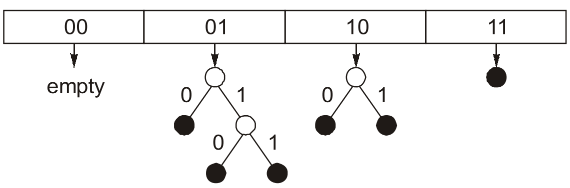


End of Solution

Q.39 Consider a dynamic hashing approach for 4-bit integer keys:

1. There is a main hash table of size 4.
2. The 2 least significant bits of a key is used to index into the main hash table,
3. Initially, the main hash table entries are empty.
4. Thereafter, when more keys are hashed into it, to resolve collisions, the set of all keys corresponding to a main hash table entry is organized as a binary tree that grows on demand.
5. First, the 3rd least significant bit is used to divide the keys into left and right subtrees.
6. To resolve more collisions, each node of the binary tree is further sub-divided into left and right subtrees based on the 4th least significant bit.
7. A split is done only if it is needed, i.e., only when there is a collision.

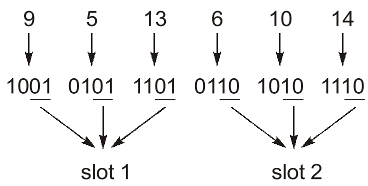
Consider the following state of the hash table.



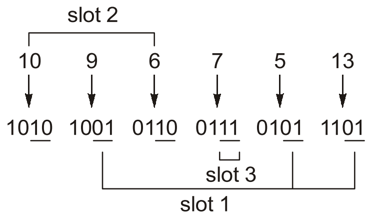
Which of the following sequences of key insertions can cause the above state of the hash table (assume the keys are in decimal notation)?

(a) 9, 5, 13, 6, 10, 14
(b) 9, 5, 10, 6, 7, 1
(c) 10, 9, 6, 7, 5, 13
(d) 5, 9: 4, 13, 10, 7

Ans. (c)



No entry for 11 in this option and 10 → has 3 entries but 2 are required.
So option (a) is wrong.
Similarly, (b), (d) options can also be proved wrong. Option (c) is correct.
Let's check option (c).



End of Solution

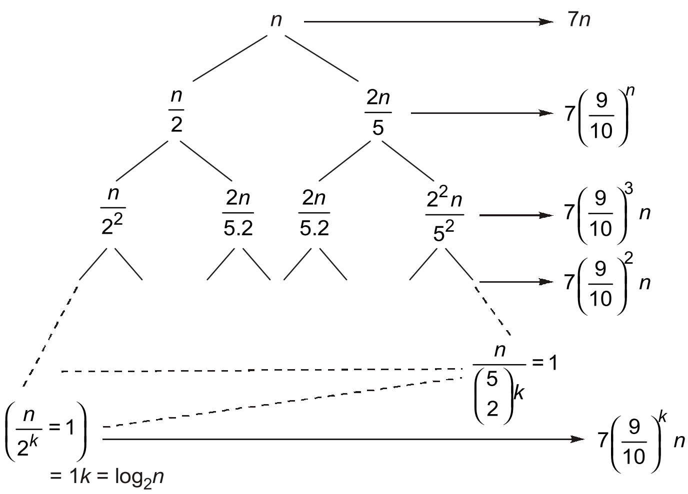
Q.40 Consider the following recurrence relation:

$$T(n) = \begin{cases} T\left(\frac{n}{2}\right) + T\left(\frac{2n}{5}\right) + 7n & \text{if } n > 0 \\ 1 & \text{if } n = 0 \end{cases}$$

Which one of the following options is correct?

(a) $T(n) = \theta(n \log n)$
(b) $T(n) = \theta(n^{5/2})$
(c) $T(n) = \theta(n)$
(d) $T(n) = \theta((\log n)^{5/2})$

Ans. (c)



$$T(n) = 7n\left[1+\frac{9}{10}+\left(\frac{9}{10}\right)^2+\left(\frac{9}{10}\right)^3+\ldots+\left(\frac{9}{10}\right)^k\right]$$

$$= \frac{7n\left[1-\left(\frac{9}{10}\right)^{k+1}\right]}{1-\frac{9}{10}} = \frac{7n}{\frac{1}{10}}\left[1-\left(\frac{9}{10}\right)^{\log_2 n+1}\right]$$

Decreasing term

$\therefore$ $$T(n) = \theta(n)$$

End of Solution

Q.41 Consider a computer system with a byte-addressable primary memory of size 2^{32} bytes. Assume the computer system has a direct-mapped cache of size 32 KB (1 KB = 2^{10} bytes), and each cache block is of size 64 bytes.
The size of the tag field is ________ bits.

Ans. **(17)**

$$\text{MM size} = 2^{32}\text{ B}$$

$$\text{Addressable size} = \log_2 2^{32} = 32\text{ bit}$$

Direct mapped

$$\text{CM size} = 32\text{ KB}$$
$$\text{Block size} = 64\text{ B}$$

$\therefore$ $$\text{Number of lines} = \frac{32\text{ K}}{64} \Rightarrow \frac{2^{15}}{2^6} = 2^9$$

Address format:

32 bit

Tag	LO	WO
17 bit	$\log_2 2^9$ → 9 bit	$\log_2 64 = 6$ bit

End of Solution

Q.42 Let G be a group of order 6, and H be a subgroup of G such that $1 < |H| < 6$. Which one of the following options is correct?

(a) Both G and H are always cyclic.
(b) G is always cyclic, but H may not be cyclic.
(c) G may not be cyclic, but H is always cyclic.
(d) Both G and H may not be cyclic.

Ans. (c)

$$|G| = 6$$

H is subgroup, so by Lagrange's theorem

$$|H| = 1, 2, 3 \text{ or } 6 \text{ (Divisor's of 6)}$$

Now it is given that $1 < |H| < 6$

or $|H| = 2$ or 3

Since 2 and 3 are both prime and since every group of prime order is cyclic, H is surely cyclic.

But order of $|G| = 6$ which is not prime.

So G may or may not be cyclic.

So G may not be cyclic, but H is always cyclic.

Option (c) is correct.

End of Solution

Q.43 The following relation records the age of 500 employees of a company, where empNo (indicating the employee number) is the key:

empAge(<u>empNo</u>, age)

Consider the following relational algebra expression:

$$\Pi_{\text{empNo}} (\text{empAge} \bowtie_{(\text{age} > \text{age1})} \rho_{\text{empNo1, age1}} (\text{empAge}))$$

What does the above expression generate?

(a) Employee numbers of all employees whose age is not the minimum.
(b) Employee numbers of all employees whose age is the minimum.
(c) Employee numbers of only those employees whose age is the maximum.
(d) Employee numbers of only those employees whose age is more than the age of exactly one other employee.

Ans. (a)

empAge(<u>empNo</u>, age)

$\Pi_{\text{empNo}} (\text{empAge} \bowtie_{(\text{age} > \text{age1})} \rho_{\text{empNo1, age1}} (\text{empAge}))$

Retrieve empNo values of emptAge those are having 'age' greater than some age.

$$\Downarrow$$

Retrieve Employee Number (empNo) of all employee whose age is greater than some employee' age.

$$\Downarrow$$

Employee number of all employees whose age is not the minimum. [Since it is greater than atleast 1 age].

End of Solution

Q.44 Consider the following expression:

$$\lim_{x \to -3} \frac{\sqrt{2x+22}-4}{x+3}$$

The value of the above expression (rounded to 2 decimal places) is _______.

Ans. (0.25)

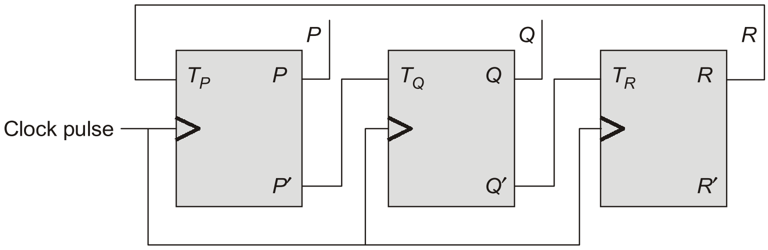
$$\lim_{x \to -3}\left(\frac{\sqrt{2x+22}-4}{x+3}\right)\left(\frac{0}{0}\text{ form}\right), \text{ so apply LHospital's rule}$$

$$= \lim_{x \to -3}\left[\frac{\frac{2}{2\sqrt{2x+22}}}{1}\right] = \lim_{x \to -3}\frac{1}{\sqrt{2x+22}}$$

$$= \frac{1}{\sqrt{16}} = \frac{1}{4} = 0.25$$

End of Solution

Q.45 Consider a 3-bit counter, designed using T flip-flops, as shown below:



Assuming the initial state of the counter given by *PQR* as 000. What are the next three states?

(a) 011, 101, 111
(b) 001, 010, 000
(c) 001, 010, 111
(d) 011, 101, 000

Ans. (d)

Clock	Present state			Flip-flop inputs			Next state		
	P	Q	R	$T_P = R$	$T_Q = \overline{P}$	$T_R = \overline{Q}$	P^+	Q^+	R^+
1	0	0	0	0	1	1	0	1	1
2	0	1	1	1	1	0	1	0	1
3	1	0	1	1	0	1	0	0	0

$\therefore$ Next three states of counter are 011, 101, 000
Hence option (d) is answer.

End of Solution

Q.46 Consider the following statements:

S_1 : Every SLR(1) grammar is unambiguous but there are certain unambiguous grammars that are not SLR(1).

S_2 : For any context-free grammar, there is a parser that takes at most $O(n^3)$ time to parse a string of length n.

Which one of the following options is correct?

(a) S_1 is true and S_2 is true
(b) S_1 is false and S_2 is false
(c) S_1 is true and S_2 is false
(d) S_1 is false and S_2 is true

Ans. (a)

S_1 : Every SLR(1) is unambiguous but every unambiguous in not SLR(1). So S_1 is true.

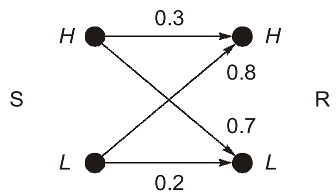
S_2 : Using CYK algorithm which takes at most $O(n^3)$ time to parse a string, where n is the length of the string. So S_2 is true.

Both S_1 is true and S_2 is true

End of Solution

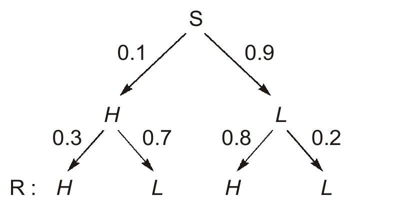
Q.47 A sender (S) transmits a signal, which can be one of the two kinds: H and L with probabilities 0.1 and 0.9 respectively, to a receiver (R).

In the graph below, the weight of edge (u, v) is the probability of receiving v when u is transmitted, where $u, v \in \{H, L\}$. For example, the probability that the received signal is L given the transmitted signal was H, is 0.7.



If the received signal is H, the probability that the transmitted signal was H (rounded to 2 decimal places) is ________.

Ans. (0.04)



$$P\left(\frac{H_S}{H_R}\right) = \frac{P(H_S \cap H_R)}{P(H_R)}$$

$$= \frac{0.1 \times 0.3}{0.1 \times 0.3 + 0.9 \times 0.8} = \frac{1}{25} = 0.04$$

End of Solution

Q.48 Consider the following ANSI C program:

```
#include <stdio.h>
int main( )
{
    int i, j, count;
    count = 0;
    i = 0;
    for (j = –3; j <= 3; j++)
    {
        if ((j >= 0) && (i++))
            count = count + j;
    }
    count = count + i;
    printf("%d", count);
    return 0;
}
```

Which one of the following options is correct?

(a) The program will compile successfully and output 8 when executed.

(b) The program will compile successfully and output 13 when executed.

(c) The program will not compile successfully.

(d) The program will compile successfully and output 10 when executed

Ans. **(d)**

$j = -3,\ j = -2,\ j = -1 \quad \Rightarrow$ Short circuiting in if((j >= 0) && (i++))

$j = 0;\ i = 0$ used but i is 1

$j = 1;\ i = 1$ used but i is 2

count = 0 + 1 = 1

$j = 2;\ i = 2$ used but i is 3

count = 1 + 2

$j = 3;\ i = 3$ used but i is 4

count = 3 + 3 = 6

count = 6 + 4 = 10 corresponding to count = count + i

Option (d) is correct.

End of Solution

Q.49 Consider the following pseudocode, where S is a semaphore initialized to 5 in line#2 and counter is a shared variable initialized to 0 in line#1. Assume that the increment operation in linc#7 is not atomic.

```
1.  int counter = 0;
2.  Semaphore S = init(5);
3.  void parop(void)
4.  {
5.       wait(S);
6.       wait(S);
7.       counter++;
8.       signal(S);
9.       signal(S);
10. }
```

If five threads execute the function parop concurrently, which of the following program behavior(s) is/are possible?

(a) The value of counter is 1 after all the threads successfully complete the execution of parop.

(b) The value of counter is 0 after all the threads successfully complete the execution of parop.

(c) There is a deadlock involving all the threads.

(d) The value of counter is 5 after all the threads successfully complete the execution of parop.

Ans. (a, c, d)

(a) True: (P_1) first process came executed wait(S); wait(S) counter++; was partially executed and the process preempted. Other process came updated counter to 4.
Now P_1 came and wrote counter to 1. So (a) is correct.

(b) Not possible at least counter will be 1.

(c) True: All 5 processes executed first wait(S) and are blocked.

(d) True: Each process executed the code sequentially and counter was finally updated to 5.

Option (a), (c) and (d) are correct.

End of Solution

Q.50 Which of the following standard C library functions will always invoke a system call when executed from a single-threaded process in a UNIX/Linux operating system?

(a) exit
(b) strlen
(c) sleep
(d) malloc

Ans. (a, c)

End of Solution

Q.51 Consider the following three functions:

$$f_1 = 10^n,\ f_2 = n^{\log n},\ f_3 = n^{\sqrt{n}}$$

Which one of the following options arranges the functions in the increasing order of asymptotic growth rate?

(a) f_2, f_3, f_1
(b) f_2, f_1, f_3
(c) f_1, f_2, f_3
(d) f_3, f_2, f_1

Ans. **(a)**

$f_2 : n^{\log n}$	$f_3 : n^{\sqrt{n}}$
$\log (n^{\log n})$	$\log\left(n^{\sqrt{n}}\right)$
$= (\log n) \cdot (\log n)$	$= \sqrt{n} \cdot \log n$

$\therefore$ $n^{\log n} = o\left(n^{\sqrt{n}}\right)$ $(f_2 < f_3)$

$f_1 = 10^n \leftarrow$ exponential function

$\therefore$ $f_1 > f_3 > f_2$

In increasing order of asymptotic growth rate.

End of Solution

Q.52 Consider the following statements:

S_1 : The sequence of procedure calls corresponds to a preorder traversal of the activation tree.

S_2 : The sequence of procedure returns corresponds to a postorder traversal of the activation tree.

Which one of the following options is correct?

(a) S_1 is false and S_2 is false
(b) S_2 is true and S_2 is false
(c) S_1 is true and S_2 is true
(d) S_1 is false and S_2 is true

Ans. **(c)**

End of Solution

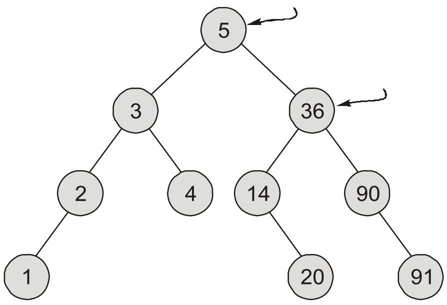
Q.53 A binary search tree T contains n distinct elements. What is the time complexity of picking an element in T that is smaller than the maximum element in T?

(a) $\theta(n)$ (b) $\theta(n \log n)$

(c) $\theta(\log n)$ (d) $\theta(1)$

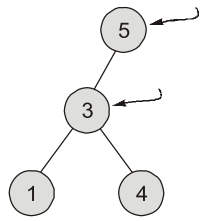
Ans. (d)

(i)



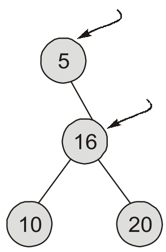
$\Rightarrow$ 5 is definitely lesser than max element

(ii)



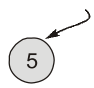
$\Rightarrow$ 3 is definitely lesser than max element

(iii)



$\Rightarrow$ 5 is definitely lesser than max element

(iv)



No such element

In any case, we can find one element that is less in just 2 seeks $\Rightarrow \theta(1)$.

End of Solution

Q.54 Consider the following Boolean expression:

$$F = (X + Y + Z)(\bar{X} + Y)(\bar{Y} + Z)$$

Which of the following Boolean expressions is/are equivalent to F?

(a) $X\bar{Y} + Y\bar{Z} + \bar{X}\bar{Y}\bar{Z}$ (b) $(\bar{X} + \bar{Y} + \bar{Z})(X + \bar{Y})(Y + \bar{Z})$

(c) $(X + \bar{Z})(\bar{Y} + \bar{Z})$ (d) $X\bar{Y} + \bar{Z}$

Ans. (a, c, d)

$$F = (X+Y+Z)(\overline{X}+Y)(\overline{Y}+Z)$$

$$\overline{F} = \overline{(X+Y+Z)(\overline{X}+Y)(\overline{Y}+Z)}$$

$$= \overline{(X+Y+Z)}+\overline{(\overline{X}+Y)}+\overline{(\overline{Y}+Z)} \quad \text{Using Demorgan's theorem}$$

$$= \overline{X}\,\overline{Y}\,\overline{Z}+X\overline{Y}+Y\overline{Z}$$

Option (a) is matching.

$$\overline{F} = \overline{Y}(\overline{X}\,\overline{Z}+X)+Y\overline{Z}$$

$$= \overline{Y}[(\underbrace{\cancel{X}+\overline{X}}_{1})]\,(X+\overline{Z})]+Y\overline{Z}$$

$$= \overline{Y}[X+\overline{Z}]+Y\overline{Z}$$

$$= X\overline{Y}+\overline{Y}\,\overline{Z}+Y\overline{Z}$$

$$= X\overline{Y}+\overline{Z}(\underbrace{\cancel{\overline{Y}}+Y}_{1})$$

$$= X\overline{Y}+\overline{Z}$$

Option (d) is matching.

$$X\overline{Y}+\overline{Z} = (X+\overline{Z})+(\overline{Y}+\overline{Z}) \quad \text{using distributing property.}$$

So option (c) is matching.

$\therefore$ So answer is (a), (c), (d).

End of Solution

Q.55 Consider the following representation of a number in IEEE 754 single-precision floating point format with a bias of 127.

S : 1 $\quad$ E : 10000001 $\quad$ F : 11110000000000000000000

Here S, E and F denote the sign, exponent and fraction components of the floating point representation.

The decimal value corresponding to the above representation (rounded to 2 decimal places) is __________.

Ans. (–7.75)

Value:

$(-1)^S\,(1.M) \times 2^{BE\text{-}Bias}$

$(-1)^1\,(1.11100\ldots) \times 2^{10000001\,-\,127}$

$-(1.1111) \times 2^{129\,-\,127}$

-1.1111×2^2

-111.11

$(-7.75)_{10}$

End of Solution

■■■■

SECTION - A GENERAL APTITUDE

Q.1 If $\left(x-\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2=x+2$, then the value of x is :

(a) 2
(b) 8
(c) 4
(d) 6

Ans. (c)

$$\left(x-\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2 = x + 2$$

Using $a^2 - b^2 = (a + b)(a - b)$

$$\left(x-\frac{1}{2}+x-\frac{3}{2}\right)\left(x-\frac{1}{2}-x+\frac{3}{2}\right) = x + 2$$

$$(2x - 2) = x + 2$$

$$2x - x = 2 + 2$$

$$x = 4$$

End of Solution

Q.2 Listening to music during exercise improves exercise performance and reduces discomfort. Scientists researched whether listening to music while studying can help students learn better and the results were inconclusive. Students who needed external stimulation for studying fared worse while students who did not need any external stimulation benefited from music.

Which one of the following statements is the CORRECT inference of the above passage?

(a) Listening to music has a clear positive effect on learning in all students. Music has a positive effect only in some students who exercise.
(b) Listening to music has a clear positive effect on physical exercise. Music has a positive effect on learning only in some students.
(c) Listening to music has a clear positive effect both on physical exercise and on learning.
(d) Listening to music has no effect on learning and a positive effect on physical exercise.

Ans. (b)

"Only in some students" is the key in option (b) and that matches well with the given informations in the passage.

End of Solution

Q.3 Pen : Write : : Knife : _____

Which one of the following options maintains a similar logical relation in the above?

(a) Sharp (b) Cut

(c) Vegetables (d) Blunt

Ans. (b)

Pen is used to write and knife is used to cut.

End of Solution

Q.4 Gauri said that she can play keyboard ________ her sister.

(a) as worse as (b) as better as

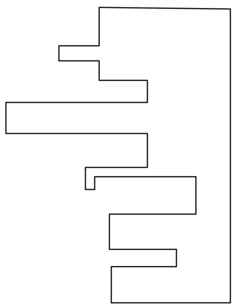
(c) as well as (d) as nicest as

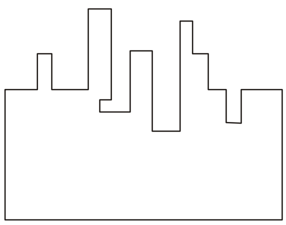
Ans. (c)

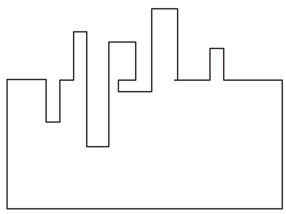
(as – as) and (so – as) are used in positive degree of comparison only. Worse, better are comparative degree and nicest is superlative degree, which are not fit between as–as and so–as.

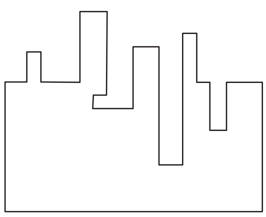
End of Solution

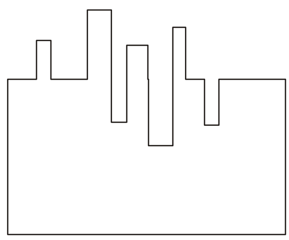
Q.5 What will be the remaining part to make it a rectangle?



(a) 

(b) 

(c) 

(d) 

Ans. (c)

Option (c) is flipped.

End of Solution

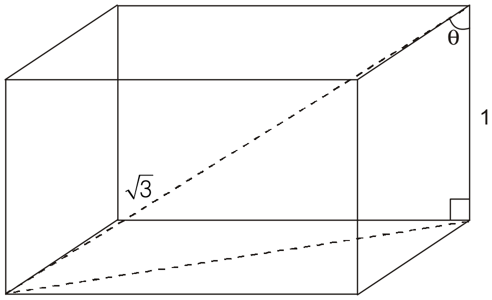
Q.6 If θ is the angle, in degrees, between the longest diagonal of the cube and any one of the edges of the cube, then $\cos\theta$ = ____.

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Ans. (a)

Angle of longest diagonal of cube with an edge of cube.



$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{\sqrt{3}}$$

End of Solution

Q.7 Six students P, Q, R, S, T and U, with distinct heights, compare their heights and make the following observations :

Observation I : S is taller than R.

Observation II : Q is the shortest of all.

Observation III : U is taller than only one student.

Observation IV : T is taller than S but is not the tallest.

The number of students that are taller than R is the same as the number of students shorter than _____.

(a) R (b) P

(c) S (d) T

Ans. (c)

$S > R$ ...(i)

Q is shortest and U is taller than only one. ...(ii)

$T > S$ ...(iii)

Hence, possible order is : $P > T > S > R > U > Q$.

$\because$ Number of students taller than R = 3.

$\therefore$ Number of students shorter than S = 3.

Hence, option (c) is the correct answer.

End of Solution

Q.8 The number of student in three classes is in the ratio 3 : 13 : 6. If 18 students are added to each class, the ratio changes to 15 : 35 : 21.
The total number of students in all the three classes in the beginning was:

(a) 88 (b) 110
(c) 66 (d) 22

Ans. (a)

3 : 13 : 6

Let $3k + 13k + 6k = n$

Now $+18 + 18 + 18$

$15 : 35 : 21$ ...(1)

$15y + 35y + 21y = 22k + 54$

$71y = 22k + 54$

Put value of k and satisfy

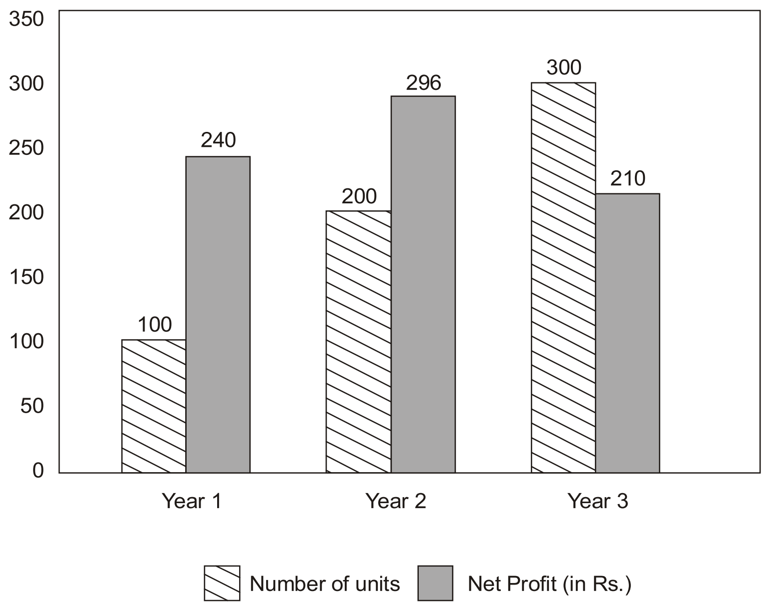
Here for $k = 4$

On putting $k = 4$ in equation (1)

$n = 88$

End of Solution

Q.9 The number of units of a product sold in three different years and the respective net profits are presented in the figure above. The cost/unit in year 3 was Rs. 1, which was half the cost/unit in year 2. The cost/unit in year 3 was one-third of the cost/unit in year 1. Taxes were paid on the selling price at 10%, 13% and 15% respectively for the three years. Net profit is calculated as the difference between the selling price and the sum of cost and taxes paid in that year.



The ratio of the selling price in Year 2 to the selling price in Year 3 is ________.

(a) 3 : 4 (b) 1 : 2
(c) 1 : 1 (d) 4 : 3

Ans. (d)

Cost/unit in year 3 = Rs. 1

Cost/unit in year 2 = Rs. 2

Cost/unit in year 1 = Rs. 3

Net Profit = S.P. – (Cost + Taxes)

In year 2, $296 = \text{S.P.} - (2 \times 200 + 0.13\ \text{S.P.})$

$\text{S.P.} = 800$

Selling price in year 2 = Rs. 800

In year 3, $210 = \text{S.P.} - (300 \times 1 + 0.15\ \text{S.P.})$

$210 = \text{S.P.} - 300 - 0.15\ \text{S.P.}$

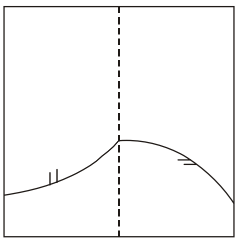
$$\text{S.P.} = \frac{210+300}{1-0.15}$$

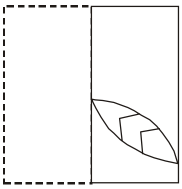
Selling price in year 3 = Rs. 600

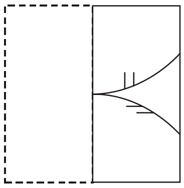
Hence, Required ratio = 800 : 600 = 4 : 3

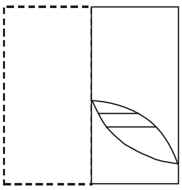
End of Solution

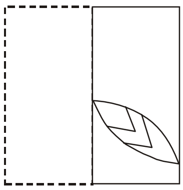
Q.10 A transparent square sheet shown above is folded along the dotted line. The folded sheet will look like _____.



(a) 

(b) 

(c) 

(d) 

Ans. (a)

End of Solution

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SECTION - B TECHNICAL

Q.1 Consider a computer system with DMA support. The DMA module is transferring one 8-bit character in one CPU cycle from a device to memory through cycle stealing at regular intervals. Consider a 2 MHz processor. If 0.5% processor cycles are used for DMA, the data transfer rate of the device is _____ bits per second.

Ans. (80000)

$$y = 0.5\ \mu\text{sec (Transfer Time)}$$
$$x = \text{Preparation time}$$
$$\%\text{ time CPU blocked} = \left(\frac{y}{x+y}\right)\times 100$$
$$0.5 = \left(\frac{0.5\ \mu\text{sec}}{x+0.5\ \mu\text{sec}}\right)\times 100$$
$$0.005x + 0.0025 = 0.5$$
$$0.005x = 0.4975$$
$$x = 99.5\ \mu\text{sec}$$

(99.5 μsec + 0.5 μsec) Total time ———————— 8 bit

1 sec ———————— ?

$$= \frac{8\text{ bit}}{100\ \mu\text{sec}} = 80000\text{ bits/sec}$$

End of Solution

Q.2 Let $L \subseteq \{0, 1\}^*$ be an arbitrary regular language accepted by a minimal DFA with k states. Which one of the following languages must necessarily be accepted by a minimal DFA with k states?

(a) $\{0, 1\}^* - L$
(b) $L \cup \{01\}$
(c) $L.L$
(d) $L - \{01\}$

Ans. (a)

If L is accepted by a min DFA with k states, by exchanging final and non-final states, we can make a minimal DFA with k states which accepts $\{0, 1\}^* - L = \overline{L}$.

So, option (a) is true.

End of Solution

Q.3. Consider a computer system with multiple shared resource types, with one instance per resource type. Each instance can be owned by only one process at a time. Owning and freeing of resources are done by holding a global lock (L). The following scheme is used to own a resource instance:

```
function OwnResource (Resource R)
    Acquire lock L // a global lock
    if R is available then
        Acquire R
        Release lock L
    else
        if R is owned by another process P then
            Terminate P, after releasing all resources owned by P
            Acquire R
            Restart P
            Release lock L
        end if
    end if
end function
```

Which of the following choice(s) about the above scheme is/are correct?

(a) The scheme violates the mutual exclusion property.

(b) The scheme ensures that deadlocks will not occur.

(c) The scheme may lead to live-lock.

(d) The scheme may lead to starvation.

Ans. **(b, c, d)**

- Mutual exclusion is not violated.
- Also, there will be no deadlock because of forceful preemption of resources.
- This may lead to starvation if the process is keeps on coming and preempting each other like P_1 is preempted by P_2 and P_2 is preempted by P_3.
- Live-lock is also possible due to continuous preemption of resources.

For option (c) consider two processes P_1 and P_2 now P_1 enter the code acquires lock and resource.

Now P_2 enters the else part kills P_1 and acquire R and restart P_1.

Now P_1 again acquire lock and kills the process P_2 this continues creating a live lock scenario but there is ambiguity in the code since “Release R” is not written any where so ambiguity is regarding how the process will release Resource R. According to the code, the only way to release the resource is by getting killed.

End of Solution

Q.4 Consider the three-way handshake mechanism followed during TCP connection establishment between hosts P and Q. Let X and Y be two random 32-bit starting sequence numbers chosen by P and Q respectively. Suppose P sends a TCP connection request message to Q with a TCP segment having SYN bit = 1. SEQ number = X, and ACK bit = 0. Suppose Q accepts the connection request. Which one of the following choices represents the information present in the TCP segment header that is sent by Q to P?

(a) SYN bit = 1, SEQ number = Y, ACK bit = 1, ACK number = X + 1, FIN bit = 0
(b) SYN bit = 1, SEQ number = Y, ACK bit = 1, ACK number = X, FIN bit = 0
(c) SYN bit = 0, SEQ number = X + 1, ACK bit = 0, ACK number = Y, FIN bit = 1
(d) SYN bit = 1, SEQ number = X + 1, ACK bit = 0, ACK number = Y, FIN bit = 0

Ans. **(a)**

Q will send the SYN bit = 1 to the connection establishment.
Q seq number will be Y different from X.
ACK bit = 1 because sending the ACK.
ACK number = X + 1 (Next seq number id)
FIN bit = 0 (Because establishing the connection)

End of Solution

Q.5 For a given biased coin, the probability that the outcome of a toss is a head is 0.4. This coin is tossed 1,000 times. Let X denote the random variable whose value is the number of times that head appeared in these 1,000 tosses. The standard deviation of X (rounded to 2 decimal places) is ________.

Ans. **(15.49)**

$$n = 1000,\ p = 0.4,\ q = 0.6$$

It is a binomially distributed random variable.

So,
$$\text{S.D.} = \sqrt{npq} = \sqrt{1000 \times 0.4 \times 0.6} = \sqrt{240}$$
$$= 15.49$$

End of Solution

Q.6 Consider the following statements S_1 and S_2 about the relational data model :

S_1 : A relation scheme can have at most one foreign key.

S_2 : A foreign key in a relation scheme R cannot be used to refer to tuples of R.

(a) Both S_1 and S_2 are false.
(b) S_1 is true and S_2 is false.
(c) S_1 is false and S_2 is true.
(d) Both S_1 and S_2 are true.

Ans. **(a)**

S_1 : A relation scheme can have at most one foreign key.

e.g.

$$\begin{tikzcd} E_1(\underline{AB}) & R(\underline{AC}) \arrow[l, bend right, "f_k"'] \arrow[r, bend left, "f_k"] & E_2(\underline{CD}) \end{tikzcd}$$

More than 1 f_k also possible.

S_2 : A foreign key in a relational scheme R_1 cannot be used to refer to tuples of R.

f_k

Emp (<u>Eid</u>	ename	Sup)
e_1	A	E_1
e_2	A	E_1
e_3	A	E_2

e.g. Self referential relationship.

So, both S_1 and S_2 are false.

End of Solution

Q.7 The format of the single-precision floating-point representation of a real number as per the IEEE 754 standard is as follows:

sign	exponent	mantissa

Which one of the following choices is correct with respect to the smallest normalized positive number represented using the standard?

(a) exponent = 00000000 and maintissa = 00000000000000000000001
(b) exponent = 00000001 and maintissa = 00000000000000000000000
(c) exponent = 00000001 and maintissa = 00000000000000000000001
(d) exponent = 00000000 and maintissa = 00000000000000000000000

Ans. **(b)**

$$\left\{\begin{array}{c} \text{All 0's BE} \Rightarrow \text{Used for "0"} \\ \text{All 1's BE} \Rightarrow \text{Used for } (+\infty \text{ and } -\infty) \end{array}\right\}$$

End of Solution

Q.8 For a string w, we define w^R to be the reverse of w. For example, if w = 01101 then w^R = 10110. Which of the following languages is/are context-free?

(a) $\{wxw^RxR \mid w, x \in \{0, 1\}^*\}$ (b) $\{wxw^R \mid w, x \in \{0, 1\}^*\}$

(c) $\{^{ww^R}xx^R \mid w, x \in \{0, 1\}^*\}$ (d) $\{wxxw^Rw^R \mid w, x \in \{0, 1\}^*\}$

Ans. **(b, c, d)**

Option (a) : $\{wxw^Rx^R \mid w_1x \in \{0, 1\}^*\}$

By putting w as "$\in$" we will get $\{xx^R \mid x \in \{0, 1\}^*\}$ which still has string matching. So, this will not be regular. Similarly, by putting x as $\in$ it will be $\{ww^R \mid w \in \{0, 1\}^*\}$ which still has string matching and will not become regular.

So, we need to do string matching but alternate order string matching is not possible in PDA. So, it is a CSL. Option (a) is a CSL but not CFL.

Option (b) : $L = \{wxw^R \mid w, x \in \{0, 1\}^*\}$

By putting w as "$\in$" we get $\{x \mid x \in \{0, 1\}^*\} = (0 + 1)^*$

Since a subset of L is $(0 + 1)^*$, L itself must be $(0 + 1)^*$ which is regular and hence CFL. Option (b) is a CFL.

Option (c) : $\{ww^Rxx^R \mid w, x \in \{0, 1\}^*\}$

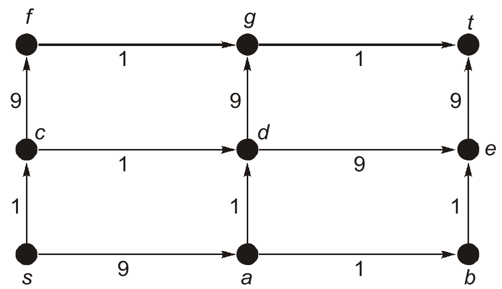
Here by putting x or w as $\in$, we cannot remove string matching. So, it is not regular. But it is CFL since in a NPDA we can push w, pop for w^R match it and then push x and pop for x^R and match it again and so this language is a CFL.

Option (d) : $\{wxx^Rw^R \mid w, x \in \{0, 1\}^*\}$

Here, also by putting w or x as $\in$, we cannot make it regular. NPDA can do this, push both w and x and then x^R pop and w^R pop and match. By push, push, pop, pop this can be accepted by NPDA. So, option (d) is CFL.

End of Solution

Q.9 In a directed acyclic graph with a source vertex s, the quality-score of a directed path is defined to be the product of the weights of the edges on the path. Further, for a vertex v other than s, the quality-score of v is defined to be the maximum among the quality-scores of all the paths from s to v. The quality-score of s is assumed to be 1.



The sum of the quality-scores of all the vertices in the graph shown above is ________.

Ans. **(929)**

Modify Dijkstra's Algo to get longest path in terms of quality scores (Use Max heap)

Algo:

1. Choose vertex u which is maximum quality score value.
   V is set of adjacency of u.
   for (each vertex (v))
   { if ($q[v] < q[u]$*cost (u, v)]
     {
       $q[v] = q[u]$ * cost(u, v)
       prev[v] = 4
2. Repeat (1) for each vertex exactly once.

Quality score (q):

1	0	0	0	0	0	0	0	0
1	9	9	1	9	81	9	81	729

Initially, let $q[v]$ = 0 or $-\infty$ (take smallest possible value)

Prev. :

–1	s	a	s	a	d	c	d	e

$S \Rightarrow \{a, c\}$ $\quad q(a) = 9,\ q(c) = 1$

$a \Rightarrow \{d, b\}$ $\quad q(d) = |ad|*q(a) = 1*9 = 9,\ q(b) = |ab|*q(a) = 1*9 = 9$

$b \Rightarrow \{e\}$ $\quad q(e) = |be|*q(b) = 1*9 = 9$

$d \Rightarrow \{e, g\}$ $\quad q(e) = |de|*q(d) = 9*9 = 81,\ q(g) = |dg|*q(d) = 9*9 = 81$

$e \Rightarrow \{t\}$ $\quad q(t) = |et|*q(e) = 9*81 = 729$

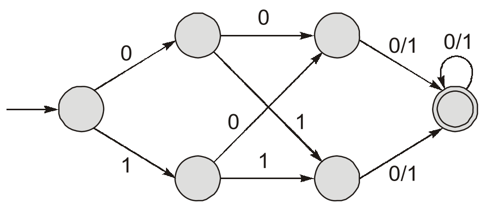
$g \Rightarrow \{t\}$ $\quad$ t already relaxed.

$c \Rightarrow \{f, d\}$ $\quad q(f) = |cf|*q(c) = 9*1 = 9$

$f \Rightarrow \{g\}$ $\quad$ g already relaxed

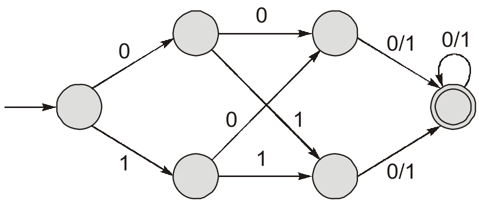
End of Solution

Q.10 Consider the following deterministic finite automation (DFA).



The number of strings of length 8 accepted by the above automation is ________.

Ans. **(256)**



The regular expression for $L(M)$ is $0(0+1)(0+1)(0+1)^* + 1(0+1)(0+1)(0+1)^*$

$$= (0+1)(0+1)(0+1)(0+1)^*$$

So, all strings of length ≥ 3 accepted.

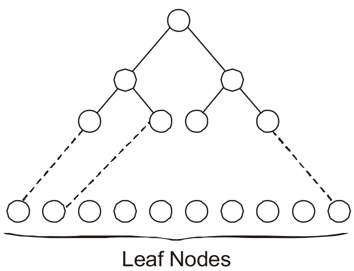
Therefore number of strings of length 8 is $2^8 = 256$.

End of Solution

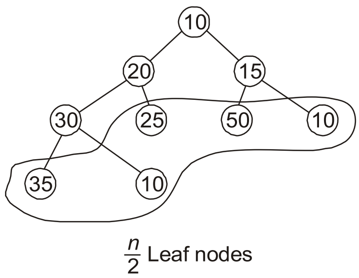
Q.11 Let H be a binary min-heap consisting of n elements implemented as an array. What is the worst case time complexity of an optimal algorithm to find the maximum element in H?

(a) $\Theta(\log n)$
(b) $\Theta(1)$
(c) $\Theta(n \log n)$
(d) $\Theta(n)$

Ans. **(d)**



OR



Maximum element is present somewhere in the leaf nodes.

$\therefore$ Find max in all leaf nodes from $\left(\left[\frac{n}{2}\right]+1\right)$.......$a[n]$

$\Rightarrow$ Number of comparisons $= \frac{n}{2} - 1 = \Theta(n)$

End of Solution

Q.12 In the context of compilers, which of the following is/are NOT an intermediate representation of the source program?

(a) Symbol table
(b) Three address code
(c) Control Flow Graph (CFG)
(d) Abstract Syntax Tree (AST)

Ans. (a)

Symbol table is a data structure which is used for storing the information about the variables. So, option (a) is correct.

There are three major categories of intermediate code representation.

Structural, linear and hybrid.

And CFG comes under the structural intermediate code representation.

End of Solution

Q.13 Consider the following two statements about regular languages:

S_1 : Every infinite regular language contains an undecidable language as a subset.

S_2 : Every finite language is regular.

Which one of the following choices is correct?

(a) Both S_1 and S_2 are true.
(b) Only S_2 is true.
(c) Only S_1 is true.
(d) Neither S_1 nor S_2 is true.

Ans. (a)

S_1 : Every infinite regular language contains an undecidable language as a subset.

S_2 : Every finite language is regular.

Clearly, S_2 is true, since for finite language, we can design FA by brute force, with a finite number of states.

Since, any language can be subset of an infinite language (No infinite language is closed under subset operation).

So, an infinite regular language can have any type of language as a subset including undecidable (non-REC) languages.

So, S_1 is also true. So, both S_1 and S_2 are true.

So, option (a) is correct.

End of Solution

Q.14 In an examination, a student can choose the order in which two questions (QuesA and QuesB) must be attempted.

- If the first question is answered wrong, the student gets zero marks.
- If the first questions is answered correctly and the second question is not answered correctly, the student gets the marks only for the first question.
- If both the questions are answered correctly, the student gets the sum of the marks of the two questions.

The following table shows the probability of correctly answering a question and the marks of the question respectively.

Question	Probability of answering correctly	Marks
QuesA	0.8	10
QuesB	0.5	20

Assuming that the student always wants to maximize her expected marks in the examination, in which order should she attempt the questions and what is the expected marks for that order (assume that the questions are independent)?

(a) First QuesA and then QuesB. Expected marks 14.
(b) First QuesB and then QuesA. Expected marks 22.
(c) First QuesB and then QuesA. Expected marks 14.
(d) First QuesA and then QuesB. Expected marks 16.

Ans. **(d)**

$X \rightarrow$ Random variable which represents total marks record.
$P(x) \rightarrow$ Probability of getting those marks.

X $\longrightarrow$ 0	10	20	30
$P(x) \longrightarrow$ 0.2 × 0.5	0.8 × 0.5	0.5 × 0.2	0.8 × 0.5
11	11	11	11
0.1	0.4	0.1	0.4
$\Sigma P(x) = 1$			

Now, if QuestionA is attempted first and it is correct.

Case-I:

$$E(x) = \Sigma(x) \cdot P(x)$$
$$= 0.4 \times 10 + 0.4 \times 30$$
$$= 4 + 12 = 16$$

Case-II:

If QuestionB is attempted first and is correct.

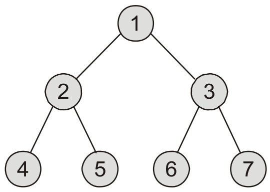
$$E(x) = \Sigma(x) \cdot P(x)$$
$$= 0.1\ (20) + 0.4\ (30)$$
$$= 2 + 12 = 14$$

So case-I is giving maximum expected marks.
Hence option (d) is correct.

End of Solution

Q.15 Consider a complete binary tree with 7 nodes. Let A denote the set of first 3 elements obtained by performing Breadth-First Search (BFS) starting from the root. Let B denote the set of first 3 elements obtained by performing Depth-First Search (DFS) starting from the root. The value of $|A - B|$ is ________.

Ans. **(1)**



Using BFS, $A = \{1, 2, 3\}$

Using DFS, $B = \{1, 2, 4\}$

$|A - B|$ = Number of elements which are in A but not in B is only element $\{3\}$

So only 1 element present.

End of Solution

Q.16 A data file consisting of 1,50,000 student-records is stored on a hard disk with block size of 4096 bytes. The data file is sorted on the primary key RollNo. The size of a record pointer for this disk is 7 bytes. Each student-record has a candidate key attributed called ANum of size 12 bytes. Suppose an index file with records consisting of two fields, ANum value and the record pointer to the corresponding student record, is built and stored on the same disk. Assume that the records of data file and index file are not split across disk blocks. The number of blocks in the index file is ________.

Ans. **(698)**

Index entries = Number of DB file record ($\because$ Dense index)

Block factor for Index,

$$(\text{Block factor})_{\text{Index}} = \left\lfloor \frac{4096}{19} \right\rfloor \text{ entries/block} = 215$$

$$\therefore \text{Number of Index blocks} = \left\lceil \frac{1{,}50{,}000}{215} \right\rceil \text{ blocks}$$

$$= \lceil 697.67 \rceil = 698 \text{ index blocks}$$

End of Solution

Q.17 Consider the following sets, where $n \geq 2$:

S_1 : Set of all $n \times n$ matrices with entries from the set $\{a, b, c\}$

S_2 : Set of all functions from the set $\{0, 1, 2, ..., n^2 - 1\}$ to the set $\{0, 1, 2\}$

Which of the following choice(s) is/are correct?

(a) There exists a surjection from S_1 to S_2.

(b) There does not exist a bijection from S_1 to S_2.

(c) There does not exist an injection from S_1 to S_2.

(d) There exists a bijection from S_1 to S_2.

Ans. **(a, d)**

$$|S_1| = 3^{n^2}$$

Since each of the n^2 entries in $n \times n$ matrix can be fills in 3 ways.

$$|S_2| = 3^{n^2}$$

Since $|\{0, 1, 2\}| = 3$ and $|\{0, 1, 2, ... n^2 - 1\}| = n^2)$

Now the theorem says A bijection $f_{A \to B}$ exists iff $|A| = |B|$.

Here $|S_1| = |S_2|$

So, there has to be a bijection from S_1 to S_2. So, option (d) is correct.

If bijection exists surely surjection also exists. So, option (a) is also correct.

Option (b) and (c) are incorrect.

End of Solution

Q.18 Suppose the following functional dependencies hold on a relation U with attributes P, Q, R, S and T:

$$P \to QR$$
$$RS \to T$$

Which of the following functional dependencies can be inferred from the above functional dependencies?

(a) $PS \to T$

(b) $P \to R$

(c) $R \to T$

(d) $PS \to Q$

Ans. **(a, b, d)**

1. $PS \to T$ $(PS)^+ = \{P, S, Q, R, T\}$
2. $P \to R$ $(P)^+ = \{P, Q, R\}$
3. $R \to T$ $(R)^+ = \{R\}$
4. $PS \to Q$ $(PS)^+ = \{P, S, Q, R, T\}$

End of Solution

Q.19 Which of the following statement(s) is/are correct in the context of CPU scheduling?

(a) The goal is to only maximize CPU utilization and minimize throughput.

(b) Turnaround time includes waiting time.

(c) Round-robin policy can be used even when the CPU time required by each of the processes is not known apriori.

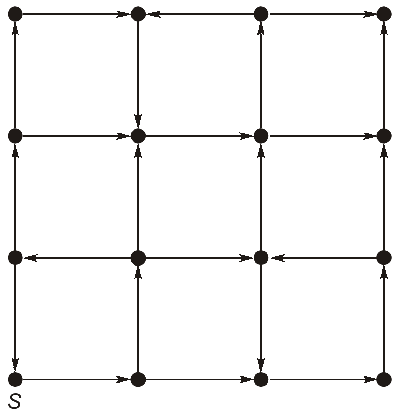
(d) Implementing preemptive scheduling needs hardware support.

Ans. **(b, c, d)**

- Goal is to maximize CPU utilization and maximize the throughput. So, statement (a) is false.
- Statement (b) is true, because turnaround time = completion time – arrival time and waiting time is included in this.
- Statement (c) is true because using time quantum, we can run the processes even if burst time is not known initially in round-robin.
- True for example, round robin scheduling requires hardware support which is timer.

End of Solution

Q.20 Consider the following directed graph:



Which of the following is/are correct about the graph?

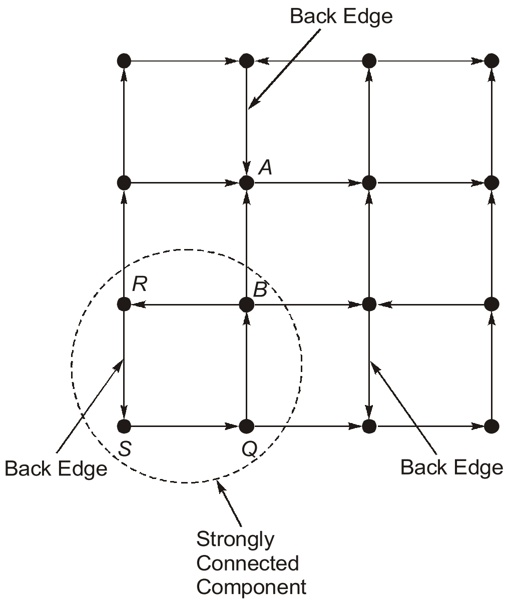
(a) The graph does not have a topological order.

(b) A depth-first traversal starting at vertex *S* classifies three directed edges as back edges.

(c) For each pair of vertices *u* and *v*, there is a directed path from *u* to *v*.

(d) The graph does not have a strongly connected component.

Ans. **(a, b)**



- As we can see there is cycle in given DAG. So, topological order is not possible.
- Statement (b) is also true.
- There is no path from A to B so statement (c) is false.
- In SQBR, it is strongly connected component. So, statement (d) is false.

End of Solution

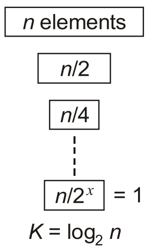
Q.21 What is the worst-case number of arithmetic operations performed by recursive binary search on a sorted array of size n?

(a) $\Theta(n)$ (b) $\Theta(n^2)$

(c) $\Theta\left(\sqrt{n}\right)$ (d) $\Theta(\log_2(n))$

Ans. **(d)**

Worst case is when the element not present in the sorted array.



Worst case occurrence relation is

$$T(n) = \begin{cases} T\left(\frac{n}{2}\right)+1, & n > 1 \\ 1, & n \leq 1 \end{cases}$$

$\therefore$ $\Theta(\log_2(n))$

End of Solution

Q.22 Consider a network using the pure ALOHA medium access control protocol, where each frame is of length 1,000 bits. The channel transmission rate is 1 Mbps (= 10^6 bits per second). The aggregate number of transmissions across all the nodes (including new frame transmissions and retransmitted frames due to collisions) is modelled as a Poisson process with a rate of 1,000 frames per second, Throughput is defined as the average number of frames successfully transmitted per second. The throughput of the network (rounded to the nearest integer) is ________.

Ans. **(135) [130 - 140]**

$$1 \text{ frames takes} = Tt = \frac{L}{B.W.}$$

$$\Rightarrow \quad \frac{1000}{10^6} = 1 \text{ millisec}$$

$$1000 \text{ frame } Tt = 1000 \times 1 \text{ millisec} = 1 \text{ sec}$$

In 1 sec, 1000 frames sends, which is 1 millisec per frame.

So, $G = 1$

Efficiency of Pure Aloha $(\eta) = G \times e^{-2G}$

where G = Number of requests per time slot willing to transmit

e = Mathematical constant approximately equal to 2.718

So, $\eta = 1 \times 2.718^{(-2 \times 1)} = 0.1353$

Therefore, in 1 sec 1000 frames = 0.1353 × 1000 = 135.3 (closest integer) ⇒ 135

Throughput ⇒ 135

End of Solution

Q.23 For a statement S in a program, in the context of liveness analysis, the following sets are defined :

$USE(S)$: the set of variables used in S

$IN(S)$: the set of variables that are live at the entry of S

$OUT(S)$: the set of variables that are live at the exit of S

Consider a basic block that consists of two statements, S_1 followed by S_2. Which one of the following statements is correct?

(a) $OUT(S_1) = USE(S_1) \cup IN(S_2)$

(b) $OUT(S_1) = IN(S_2) \cup OUT(S_2)$

(c) $OUT(S_1) = IN(S_2)$

(d) $OUT(S_1) = IN(S_1) \cup USE(S_1)$

Ans. **(c)**

In live variable analysis at any node, the set of variables live at just after the block are evaluated using the formula.

$$\text{OUT} = \cup \text{ IN (Successor nodes)}$$

So, the correct option is

$$\text{OUT } (S_1) = \text{IN } (S_2)$$

So, the correct option is (c).

End of Solution

Q.24 Let S be the following schedule of operations of three transactions T_1, T_2 and T_3 in a relational database system:

$$R_2(Y),\ R_1(X),\ R_3(Z),\ R_1(Y),\ W_1(X),\ R_2(Z),\ W_2(Y),\ R_3(X),\ W_3(Z)$$

Consider the statements P and Q below:

P : S is conflict-serializable.

Q : If T_3 commits before T_1 finishes, then S is recoverable.

Which one of the following choices is correct?

(a) P is true and Q is false.

(b) Both P and Q are false.

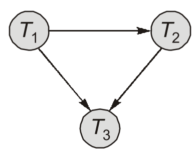
(c) Both P and Q are true.

(d) P is false and Q is true.

Ans. **(a)**

No cycle in precedence graph.

$\therefore$ Conflict serializable.



T_1	T_2	T_3
⋮	⋮	⋮
$W_1(X)$		
	$R_2(Z)$	
	$W_2(Y)$	
		$R_3(X)$
		$W_3(Z)$

T_2 is doing dirty read of updated X by T_1.

So, recoverability only possible if T_3 commits after commit/RB of T_1.

So, option (a) is correct.

End of Solution

Q.25 Consider the following ANSI C program:

```
#include <stdio.h>
#include <stdlib.h>
struct Node{
        int value;
        struct Node *next;};
int main( ) {
    struct Node *boxE, *head, *boxN; int index = 0;
    boxE = head = (struct Node *) malloc(sizeof(struct Node));
    head → value = index;
    for (index = 1; index <= 3; index++) {
        boxN = (struct Node *) malloc(sizeof(struct Node));
        boxE → next = boxN;
        boxN → value = index;
        boxE = boxN; }
    for (index = 0; index <= 3; index++) {
        printf("Value at index %d is %d\n", index, head → value);
        head = head → next;
        printf("Value at index %d is %d\n", index+1, head → value); } }
```

Which one of the statements below is correct about the program?

(a) Upon execution, the program creates a linked-list of five nodes.

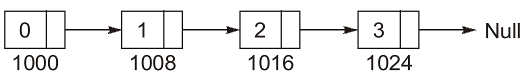
(b) It has a missing return which will be reported as an error by the compiler.

(c) It dereferences an uninitialized pointer that may result in a run-time error.

(d) Upon execution, the program goes into an infinite loop.

Ans. **(c)**

As we can see in the loop, i runs from 1 to 3. So, four nodes will be created because one node is already created with value 0.



When index = 3, then head value = 3

Head = Head → Next **(Now head will point to NULL)**

Head → Value [which is in print. So it will generate Run time Error].

So, option (c) is correct.

End of Solution

Q.26 If x and y are two decimal digits and $(0.1101)_2 = (0.8xy5)_{10}$, the decimal value of $x + y$ is __________.

Ans. **(3)**

$$(0.1101)_2 = 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$$

$$= 0.5 + 0.25 + 0 + 0.0625 = (0.8125)_{10}$$

$$(0.8125)_{10} = (0.8xy5)_{10}$$

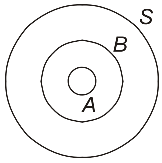
$$\therefore \quad x = 1,\ y = 2,\ x + y = 1 + 2 = 3$$

End of Solution

Q.27 Let S be a set consisting of 10 elements. The number of tuples of the form (A, B) such that A and B are subsets of S and $A \subseteq B$ is ________.

Ans. **(59049)**

The Venn diagram for this is



Now every element x in S has only 3 options. It can be $x \in A$ or $x \in B - A$ or $x \in S - B$. So the number of ways to choose A and B such that $A \subseteq B \subseteq S$ is 3^{10}.

End of Solution

Q.28 Consider the following ANSI C program:

```
int main( ) {
    Integer x;
    return 0;
}
```

Which one of the following phases in a seven-phase C compiler will throw an error?

(a) Syntax analyzer
(b) Machine dependent optimizer
(c) Semantic analyzer
(d) Lexical analyzer

Ans. **(*)**

[**Note:** Official answer key given by **IIT-Bombay** is option (c)]

End of Solution

Q.29 Assume a two-level inclusive cache hierarchy. L1 and L2, where L2 is the larger of the two. Consider the following statements.

S_1 : Read misses in a write through L1 cache do not result in writebacks of dirty lines to the L2.

S_2 : Write allocate policy must be used in conjunction with write through caches and no-write allocate policy is "used with writeback caches.

Which of the following statements is correct?

(a) S_1 is true and S_2 is true.
(b) S_1 is true and S_2 is false.
(c) S_1 is false and S_2 is true.
(d) S_1 is false and S_2 is false.

Ans. **(b)**

End of Solution

Q.30 A bag has r red balls and b black balls. All balls are identical except for their colours. In a trial, a ball is randomly drawn from the bag, its colour is noted and the ball is placed back into the bag along with another ball of the same colour. Note that the number of balls in the bag will increase by one, after the trial. A sequence of four such trials is conducted. Which one of the following choices gives the probability of drawing a red ball in the fourth trial?

(a) $\frac{r}{r+b}$

(b) $\frac{r}{r+b+3}$

(c) $\left(\frac{r}{r+b}\right)\left(\frac{r+1}{r+b+1}\right)\left(\frac{r+2}{r+b+2}\right)\left(\frac{r+3}{r+b+3}\right)$

(d) $\frac{r+3}{r+b+3}$

Ans. **(a)**

There are 10 favourable ways to calculate the probability of red ball in 4th trial.

(RFR)R ≈ R or (BRR)R ≈ 1 way or (RRR)R ≈ 3 ways or (BBR)R ≈ 3 ways

$$P(RRRR) = \frac{r}{r+b} \times \frac{r+1}{r+1+b} \times \frac{r+2}{r+2+b} \times \frac{+2}{r+3+b} \quad ...(1)$$

$$P(BBBR) = \frac{b}{r+b} \times \frac{b+1}{r+b+1} \times \frac{b+2}{r+b+2} \times \frac{r}{r+b+3} \quad ...(2)$$

$$P(RRBR) = \frac{3!}{2!} \times \frac{r}{r+b} \times \frac{r+1}{r+b+1} \times \frac{b}{r+b+2} \times \frac{r+2}{r+b+3} \quad ...(3)$$

$$P(BBRR) = \frac{3!}{2!} \times \frac{b}{r+b} \times \frac{b+1}{r+b+1} \times \frac{r}{r+b+2} \times \frac{r+1}{r+b+3} \quad ...(4)$$

Required probability = (1) + (2) + (3) + (4)

$$= \frac{r(r+1)(r+2)(r+3)+b(b+1)(b+2)r+3r(r+1)b(r+2)+3b(b+1)r(r+1)}{(r+b)(r+b+1)(r+b+2)(r+b+3)}$$

On solving it we get,

$$= \frac{r(r+1+b)}{(r+b)(r+b+1)} = \frac{r}{r+b}$$

End of Solution

Q.31 Consider the following multi-threaded code segment (in a mix of C and pseudo-code), invoked by two processes P_1 and P_2, and each of the processes spawns two threads T_1 and T_2:

int x = 0; // global
Lock L1; // global
main( ) {
  create a thread to execute foo( ); // Thread T_1
  create a thread to execute foo( ); // Thread T_2
  wait for the two threads to finish execution;
  print (x);}
foo( ) {
  int y = 0;
  Acquire L_1;
  $x = x + 1$;
  $y = y + 1$;
  Release L1;
  print (y); }

Which of the following statement(s) is/are correct?

(a) At least one of P_1 and P_2 will print the value of x as 4,
(b) Both P_1 and P_2 will print the value of x as 2.
(c) Both T_1 and T_2, in both the processes, will print the value of y as 1.
(d) At least one of the threads will print the value of y as 2.

Ans. **(b, c)**

P_1 and P_2 can spawn two threads T_1 and T_2.
int x = 0; //global
Lock L_1; //global
main( ) {
  foo( ); //Thread T_1
  foo( ); //Thread T_2
  print(x);
}
foo( ) {
a. int y = 0;
b. Acquire L_1;
c. $x = x + 1$;
d. $y = y + 1$;
  Release L_1;
  print(y);
}

- Let P_1 executed T_1 and in foo( ).

(a) $P_1 - T_1$ x $\boxed{\cancel{0}}$ 1
(b) $P_1 - T_1$

(c) $x = x + 1$ $(P_1 - T_1)$

.... Preempt T_1 of P_1.

Similarly, perform thread T_2 of P_1 then $x = 2$.

Now, if we similarly perform both threads of P_2 then x will be maximum 2.

Note: It is not mentioned about IPC or concurrent mechanism so the process will not shared the global variable. Each process will have its on own address space so the maximum value of x by each process can be 2 only.

Note: But as we know every foo call will have its own copy of variable y so y can not be more than 1 in any case.

End of Solution

Q.32 Suppose we want to design a synchronous circuit that processes a string of 0's and 1's. Given a string, it produces another string by replacing the first 1 in any subsequence of consecutive 1's by a 0. Consider the following example.

Input sequence : 00100011000011100

Output sequence : 00000001000001100

A Mealy Machine is a state machine where both the next state and the output are functions of the present state and the current input.

The above mentioned circuit can be designed as a two-state Mealy machine. The states in the Mealy machine can be represented using Boolean values 0 and 1. We denote the current state, the next state, the next incoming bit, and the output bit of the Mealy machine by the variables s, t, b and y respectively.

Assume the initial state of the Mealy machine is 0.

What are the Boolean expressions corresponding to t and y in terms of s and b?

(a) $t = b$

$y = s\bar{b}$

(b) $t = s + b$

$y = sb$

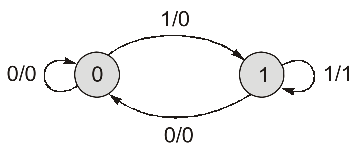
(c) $t = s + b$

$y = s\bar{b}$

(d) $t = b$

$y = sb$

Ans. (d)



PS	Next State t,	O/P y,
s	$b = 0$	$b = 1$
0	0,0	1,0
1	0,0	1,1

$t = \bar{s}b + sb = b$

$y = sb$

End of Solution

Q.33 Consider the following ANSI C code segment:

```
z = x + 3 + y → f1 + y → f2;
for (i = 0; i < 200; i = i + 2) {
    if (z > i) {
        p = p + x + 3;
        q = q + y → f1;
    } else {
        p = p + y → f2;
        q = q + x + 3;
    }
}
```

Assume that the variable y points to a struct (allocated on the heap) containing two fields $f1$ and $f2$, and the local variables x, y, z, p, q, and i are allotted registers. Common sub-expression elimination (CSE) optimization is applied on the code. The number of addition and dereference operations (of the form $y \rightarrow f1$ or $y \rightarrow f2$) in the optimized code, respectively, are :

(a) 303 and 2
(b) 403 and 102
(c) 203 and 2
(d) 303 and 102

Ans. (a)

$t_1 = y \rightarrow f_1$ (1 dereference)
$t_2 = y \rightarrow f_2$ (1 dereference)
$t_3 = x + 3$ (1 add)
$z = t_3 + t_1 + t_2$ (2 additions)

```
For (i = 0; i < 200; i+ = 2)
{
    if (z > i)
    {
        p = p + t3
        q = q + t1 (2 add)
    }
    else
    {
        p = p + p2
        q = q + t3 (2 add)
    }
}
```

If else condition $\Rightarrow$
Either if is executed (or) else is executed.
$\Rightarrow$ At any iteration 2 addition operations will be executed.

So, in loop the iterations are $\left(\dfrac{200}{2}\right) = 100$ times.

$\therefore$ In loop the number of additions $= 100 \times 2 = 200$ additions

$\therefore$ Total additions = 200 + 1 + 2 + 100 loop additions (inside for loop)
= 303

and 2 dereferences.

$\therefore$ Correct answer is 303 and 2 which is option (a).

End of Solution

Q.34 For two n-dimensional real vectors P and Q, the operation $s(P, Q)$ is defined as follows:

$$s(P, Q) = \sum_{i=1}^{n}(P[i].Q[i])$$

Let $\mathcal{L}$ be a set of 10-dimensional non-zero real vectors such that for every pair of distinct vectors $P, Q \in \mathcal{L}$, $s(P, Q) = 0$. What is the maximum cardinality possible for the set $\mathcal{L}$?

(a) 10
(b) 9
(c) 100
(d) 11

Ans. **(a)**

$\mathcal{L}$ is the set of 10-dimensional orthogonal vectors. So cardinality of $\mathcal{L} \leq 10$.

i.e., Maximum cardinality of $\mathcal{L}$ = 10.

So, option (a) is correct.

End of Solution

Q.35 The relation scheme given below is used to store information about the employees of a company, where empId is the key and deptId indicates the department to which the employee is assigned. Each employee is assigned to exactly one department.

emp(<u>empId</u>, name, gender, salary, deptId)

Consider the following SQL query:

```
select deptId, count(*)
from emp
where gender = “female” and salary > (select avg(salary) from emp)
group by deptId;
```

The above query gives, for each department in the company, the number of female employees whose salary is greater than the average salary of

(a) female employees in the department.
(b) employees in the department.
(c) female employees in the company.
(d) employees in the company.

Ans. (d)

emp	empId	Name	Gender	Salary	deptid
21K>20K	e_1	*X*	Female	210K	CS
	e_2	*Y*	Male	19K	CS
25K>20K	e_3	*XZ*	Female	25K	EC
	e_4	*YZ*	Male	14K	EC
21K>20K	e_5	*a*	Female	21K	CS

Average salary of all employees in the company.

O/P

deptid	count(*)
CS	2
EC	1

For each department, number of female employees whose salary is greater than average salary of employees in the company.

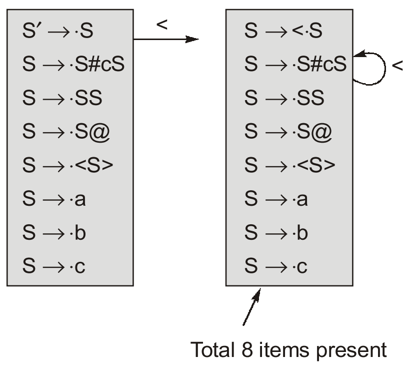
End of Solution

Q.36 Consider the following augmented grammar with {#, @, <, >, a, b, c} as the set of terminals.

S′ → S
S → S # cS
S → SS
S → S@
S → <S>
S → a
S → b
S → c

Let I_0 = CLOSURE($S' \rightarrow \bullet S$}). The number of items in the set GOTO(GOTO(I_0, <), <) is ________.

Ans. (8)



End of Solution

Q.37 Consider the following ANSI C function:

```
int SomeFunction(int x, int y)
{
    if ((x == 1)||(y == 1)) return 1;
    if (x == y) return x;
    if (x > y) return SomeFunction(x – y, y);
    if (y > x) return SomeFunction(x, y – x);
}
```

The value returned by SomeFunction(15, 255) is ________.

Ans. **(15)**

This function will keep on subtracting till both x and y becomes equal that is 15.

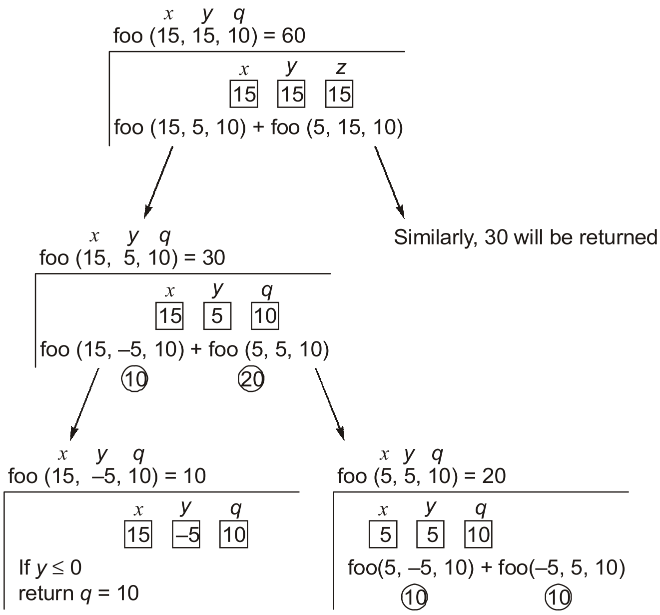
End of Solution

Q.38 Consider the following ANSI C program:

```
#include <stdio.h>
int foo(int x, int y, int q)
{
    if ((x <= 0) || (y <= 0))
        return q;
    if (x <= 0)
        return foo(x, y – q, q)
    if (y <= 0)
        return foo(x – q, y, q);
    return foo(x, y – q, q) + foo(x – q, y, q);
}
int main( )
{
    int r = foo(15, 15, 10);
    printf("%d", r);
    return 0;
}
```

The output of the program upon execution is _________.

Ans. (60)



End of Solution

Q.39 Consider a Boolean function $f(w, x, y, z)$ such that

$$f(w, 0, 0, z) = 1$$
$$f(1, x, 1, z) = x + z$$
$$f(w, 1, y, z) = wz + y$$

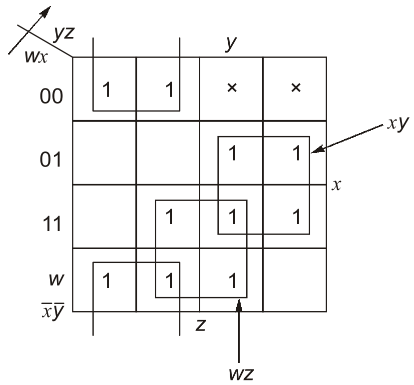
The number of literals in the minimal sum-of-products expression of f is ________.

Ans. (6)

$$f(w, 0\ 0, z) = 1 \quad ...(1)$$
$$f(1, x, 1, z) = x + z \quad ...(2)$$
$$f(w, 1, y, z) = wz + y \quad ...(3)$$

	wxyz	Eqn. 1	Eqn. 2	Eqn. 3	*f*
0	0000	1			1
1	0001	1			1
2	0010				×
3	0011				×
4	0100			0	0
5	0101			0	0
6	0110			1	1
7	0111			1	1
8	1000	1			1
9	1001	1			1
10	1010		0		0
11	1011		1		1
12	1100			0	0
13	1101			1	1
14	1110		1	1	1
15	1111		1	1	1

$$f(w, x, y, z) = \Sigma m(0, 1, 6, 7, 8, 9, 11, 13, 14, 15) + d(2, 3)$$



$\therefore$ $$f = \bar{x}\bar{y} + xy + wz$$

The number of literals in the minimal SOP expression is 6.

End of Solution

Q.40 Consider the cyclic redundancy check (CRC) based error detecting scheme having the generator polynomial $X^3 + X + 1$. Suppose the message $m_4m_3m_2m_1m_0 = 11000$ is to be transmitted. Check bits $c_2c_1c_0$ are appended at the end of the message by the transmitter using the above CRC scheme, The transmitted bit string is denoted by $m_4m_3m_2m_1m_0c_2c_1c_0$. The value of the checkbit sequence $c_2c_1c_0$ is

(a) 110 (b) 111
(c) 100 (d) 101

Ans. (c)

$$x^3 + x + 1 = 1011$$

```
1011 | 11000000 |
       1011
        1110
        1011
         1010
         1011
          100
```

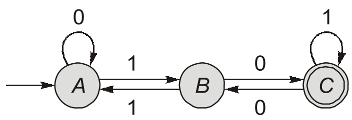
End of Solution

Q.41 Which of the following regular expressions represent(s) the set of all binary numbers that are divisible by three? Assume that the string e is divisible by three.

(a) (0*(1(01*0)*1)*)* (b) (0 +1(01*0)* 1)*
(c) (0 + 11 + 10(1 + 00)*01)* (d) (0 + 11 + 11(1 + 00)*00)*

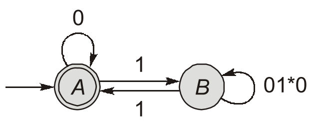
Ans. (a, b, c)

The DFA for accepting all binary strings divisible by 3 is given below:



where A is residue 0 state, B is residue 1 state and C is residue 2 state.

From this we get by deleting (c)



We get option (b) is correct (0 + 1(01*0)*1)*

Now option (a) = option (b)

because $(r^*s^*)^* = (r + s)^*$

So, option (a) is also correct.

Option (c) can be obtained by resolving the loop between B and C on "C" instead of on "B".

Option (c) is also correct.

Also note that whatever string option (b) can derive, option (c) also can derive. No counter example possible.

So, option (c) is also correct.

Option (d) (0 + 11 + 11(1 + 00)*00)* cannot derive “1001” which is accepted by machine.
So, option (d) is incorrect.
So, options (a), (b) and (c) are correct.

End of Solution

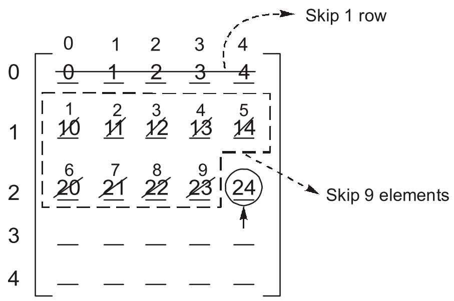
Q.42 Consider the following ANSI C program:

```
#include <stdio.h>
int main( ) {
    int arr[4] [5] ;
    int i, j;
    for (i = 0; i < 4; i++) {
        for (j = 0; j < 5; j++) {
            arr[i] [j] = 10 * i + j;
        }
    }
    printf("%d", * (arr[i] + 9));
    return 0;
}
```

What is the output of the above program?

(a) 24 (b) 20
(c) 14 (d) 30

Ans. **(a)**



int a[4][5]

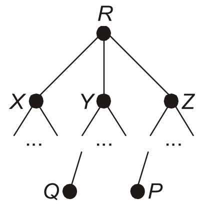
[*(a + 1) + 9]

It means skip one row and then skip 9 elements.

So, the resultant value is 24.

End of Solution

Q.43 Consider a computer network using the distance vector routing algorithm in its network layer. The partial topology of the network is as shown below.



The objective is to find the shortest-cost path from the router R to routers P and Q. Assume that R does not initially know the shortest routes to P and Q. Assume that R has three neighbouring routers denoted as X, Y and Z. During one iteration, R measures its distance to its neighbours X, Y and Z as 3, 2 and 5, respectively. Router R gets routing vectors from its neighbours that indicate that the distance to router P from routers X, Y and Z are 7, 6 and 5, respectively. The routing vector also indicates that the distance to router Q from routers X, Y and Z are 4, 6 and 8, respectively. Which of the following statement(s) is/are correct with respect to the new routing table of R, after updation during this iteration?

(a) The distance from R to Q will be stored as 7.
(b) The distance from R to P will be stored as 10.
(c) The next hop router for a packet from R to Q is Z.
(d) The next hop router for a packet from R to P is Y.

Ans. **(a, d)**

Given R gets the distance vector (3, 2, 5)

After the one iteration distance vector from X to P, Y to P, and Z to P is (7, 6, 5) respectively.

The distance vector from R to P via XYZ is $(3 + 7, 2 + 6, 5 + 5) = (10, 8, 10)$

So, take minimum distance from R to P which is 8 via Y.

After the iteration distance vector from X to Q, Y to Q, Z to Q is (4, 6, 8) respectively.

The distance vector from R to Q via XYZ is $(3 + 4, 2 + 6, 5 + 8) = (7, 8, 13)$.

So, take minimum distance from R to Q which is 7 via X.

End of Solution

Q.44 Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?

(a) $L_1 \cap \overline{L_2}$

(b) $\overline{\overline{L_1} \cup \overline{L_2}}$

(c) $L_1 \cup (L_2 \cup \overline{L_2})$

(d) $(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2)$

Ans. **(b, c, d)**

$L_1 \to$ Regular

$L_2 \to$ CFL

1. $$L_1 \cap \overline{L_2} = \text{Reg} \cap \overline{\text{CFL}} = \text{Reg} \cap \overline{\text{CSL}}$$
$$= \text{Reg} \cap \text{CSL} = \text{CSL} \qquad \text{(need not be CFL)}$$

2. $$\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2 = \text{Reg} \cap \text{CFL} = \text{CFL}$$

3. $$L_1 \cup (L_2 \cup \overline{L_2}) = L_1 \cup \Sigma^* = \Sigma^* = \text{Regular and hence CFL}$$

4. $$(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2) = (L_1 \cup \overline{L_1}) \cap L_2$$
$$= \Sigma^* \cap L_2$$
$$= L_2 = \text{CFL}$$

End of Solution

Q.45 For constants $a \geq 1$ and $b > 1$, consider the following recurrence defined on the non-negative integers:

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$$

Which one of the following options is correct about the recurrence $T(n)$?

(a) If $f(n)$ is $\Theta(n^{\log_b(a)})$, then $T(n)$ is $\Theta(n^{\log_b(a)})$.

(b) If $f(n)$ is $O(n^{\log_b(a) - e})$ for some $e > 0$, then $T(n)$ is $\Theta(n^{\log_b(a)})$.

(c) If $f(n)$ is $\dfrac{n}{\log_2(n)}$, then $T(n)$ is $\Theta(\log_2(n))$.

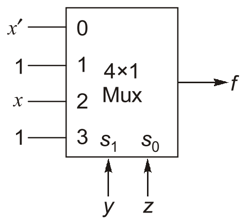
(d) If $f(n)$ is $n \log_2(n)$, then $T(n)$ is $\Theta(n \log_2(n))$.

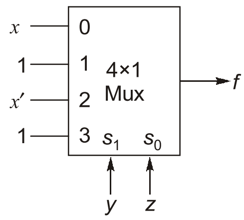
Ans. **(b)**

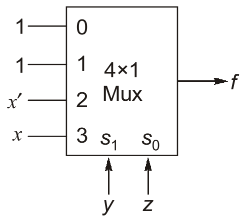
According to Standard Master Theorem, only option (b) is correct.

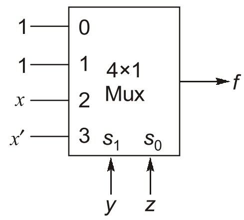
End of Solution

Q.46 Which one of the following circuits implements the Boolean function given below?
$f(x, y, z) = m_0 + m_1 + m_3 + m_4 + m_5 + m_6$, where m_i is the i^{th} minterm.

(a)


(b)


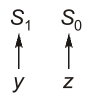
(c)


(d)


Ans. **(d)**

$$\Sigma(x, y, z) = m_0 + m_1 + m_3 + m_4 + m_5 + m_6$$
$$= \Sigma m(0, 1, 3, 4, 5, 6)$$

As per the given options, variable y is connected to multiplexer select input S_1 and z is connected to select input S_0.



	I_0 00	I_1 01	I_2 10	I_3 11
x 0	0	1	2	3
x 1	4	5	6	7
	1	1	x	$\bar{x}$

So, the correct option is (d).

End of Solution

Q.47 Consider the string abbccddeee. Each letter in the string must be assigned a binary code satisfying the following properties:

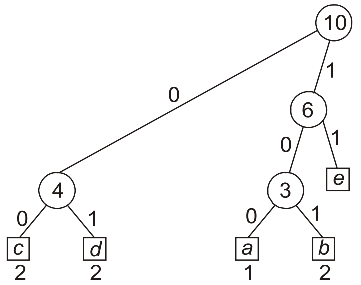
1. For any two letters, the code assigned to one letter must not be a prefix of the code assigned to the other letter.
2. For any two letters of the same frequency, the letter which occurs earlier in the dictionary order is assigned a code whose length is at most the length of the code assigned to the other letter.

Among the set of all binary code assignments which satisfy the above two properties, what is the minimum length of the encoded string?

(a) 23 (b) 25
(c) 21 (d) 30

Ans. (a)

Letters used in string :	*a*	*b*	*c*	*d*	*e*
Frequencies	1	2	2	2	3



Letters used in string:

∴ Minimum length of encoded string

= Sum frequency count using Huffman coding

= 1 * 3 + 2 * 3 + 2 * 2 + 2 * 2 + 3 * 2

= 3 + 6 + 4 + 4 + 6 = 23

End of Solution

Q.48 If the numerical value of a 2-byte unsigned integer on a little endian computer is 255 more than that on a big endian computer, which of the following choices represent(s) the unsigned integer on a little endian computer?

(a) 0x6665 (b) 0x0001

(c) 0x0100 (d) 0x4243

Ans. (a, c)

Option (a): In little endian 0x6665 on converting it to decimal = 26213

In big endian it will be 6566 on converting it to decimal = 25958

Now little endian – big endian = 26231 – 25958 = 255 which is correct.

Option (b): In little endian 0x0001 which is 1 in decimal in big endian 0x0100 which is greater than little endian in decimal. So this is incorrect option.

Option (c): Little endian = 0x0100 which is 256 in decimal and big endian = 0x0001 which is 1 in decimal an difference will be 255 hence this is also correct option.

Option (d): Little endian = 0x4243 and big endian = 0x4342, big endian value is greater than little endian so this is incorrect.

End of Solution

Q.49 Consider a pipelined processor with 5 stages. Instruction Fetch (IF), Instruction Decode (ID), Execute (EX), Memory Access (MEM), and Write Back (WB). Each stage of the pipeline, except the EX stage, takes one cycle. Assume that the ID stage merely decodes the instruction and the register read is performed in the EX stage. The EX stage takes one cycle for ADD instruction and two cycles for MUL instruction. Ignore pipeline register latencies. Consider the following sequence of 8 instructions:

ADD, MUL, ADD, MUL, ADD, MUL, ADD, MUL

Assume that every MUL instruction is data-dependent on the ADD instruction just before it and every ADD instruction (except the first ADD) is data-dependent on the MUL instruction just before it. The Speedup is defined as follows:

$$\text{Speedup} = \frac{\text{Execution time without operand forwarding}}{\text{Execution time with operand forwarding}}$$

The Speedup achieved in executing the given instruction sequence on the pipelined processor (rounded to 2 decimal places) is ________.

Ans. **(1.87) (1.87 - 1.88)**

With operand forwarding:

8 Instructions + 4 MUL Instruction × 1 Extra Cycle in Ex-stage

$\Rightarrow \quad n = 12$ (finite)

$\Rightarrow \quad K = 5$

$$ET_{\text{Pipe}} = (K + n - 1) \text{ Cycles}$$
$$= (5 + 12 - 1)$$
$$= 16 \text{ cycles}$$

Without operand forwarding :

- 8 Instructions + 4 MUL Instruction × 2 Stalls at ID stage for ADD O/P
  + 3 ADD Instruction × 3 Stalls at ID stage for MUL O/P
  + 1 MUL Instruction × 1 Extra Cycle in Ex-Stage (Last Instruction)

$\Rightarrow \quad n = 26$

$\Rightarrow \quad K = 5$

$$ET_{\text{Pipe}} = (K + n - 1) \text{ Cycles}$$
$$= (5 + 26 - 1) \text{ Cycles}$$
$$= 30 \text{ Cycles}$$

$$\therefore \quad S = \frac{30}{16} = 1.875$$

End of Solution

Q.50 Choose the correct choice(s) regarding the following propositional logic assertion S:

$$S : ((P \wedge Q) \to R) \to ((P \wedge Q) \to (Q \to R))$$

(a) S is a tautology.

(b) S is neither a tautology nor a contradiction.

(c) The antecedent of S is logically equivalent to the consequent of S.

(d) S is a contradiction.

Ans. **(a, c)**

$S : ((P \wedge Q) \to R) \to ((P \wedge Q) \to (Q \to R))$

$$\begin{aligned}
&\equiv (pq \to r) \to (pq \to (q \to r)) \\
&\equiv [(pq)' + r] \to [(pq)' + (q' + r)] \\
&\equiv [(pq)' + r]' + [(pq)' + q' + r] \\
&\equiv [pq \cdot r'] + [p' + q' + q' + r] \\
&\equiv pqr' + p' + q' + r \\
&\equiv (p + p')(qr' + p') + q' + r \\
&\equiv qr' + p' + q' + r \\
&\equiv (q + q')(r' + q') + p' + r \\
&\equiv r' + q' + p' + r \equiv r' + r + q' + p' \\
&\equiv 1 + q' + p' \equiv 1 \text{ (Tautology)}
\end{aligned}$$

So, S is a tautology.

So, option (a) is true.

Option (b) and (d) are false.

Option (c) antecedent of S is

$$\begin{aligned}
pq \to r &\equiv (pq)' + r \\
&\equiv p' + q' + r
\end{aligned}$$

The consequent of S is $pq \to (q \to r)$

$$\begin{aligned}
&\equiv (pq)' + q' + r \\
&\equiv p' + q' + q' + r \\
&\equiv p' + q' + r
\end{aligned}$$

So, Antecedent of $S \equiv$ Consequent of S

So, option (c) is also true.

End of Solution

Q.51 Suppose that P is a 4×5 matrix such that every solution of the equation $Px = 0$ is a scalar multiple of $[2 \ \ 5 \ \ 4 \ \ 3 \ \ 1]^T$. The rank of P is ________.

Ans. **(4)**

$P_{4 \times 5} \Rightarrow$ Number of unknowns (n) = 5 in $PX = 0$ ...(1)

Also it is given that Nullity $PX = 0$ is one, i.e., $N(P) = 1$.

Hence, Nullity = Number of unknowns – Rank

$$1 = 5 - \rho(P)$$

or $\rho(P) = 5 - 1 = 4$

End of Solution

Q.52 Consider a three-level page table to translate a 39-bit virtual address to a physical address as shown below:

←—— 39-bit virtual address ——→

Level 1 offset	Level 2 offset	Level 3 offset	Page offset
9 bits	9 bits	9 bits	12 bits

The page size is 4 KB (1 KB = 2^{10} bytes) and page table entry size at every level is 8 bytes. A process *P* is currently using 2 GB (1 GB = 2^{30} bytes) virtual memory which is mapped to 2 GB of physical memory. The minimum amount of memory required for the page table of P across all levels is ________ KB.

Ans. **(4108)**

←—— 39-bit virtual address ——→

Level 1 offset	Level 2 offset	Level 3 offset	Page offset
9 bits	9 bits	9 bits	12 bits

Since process P is using 2 GB physical memory and page size 2^{12} bytes.

So, Number of pages $= \dfrac{2^{31}}{2^{12}} = 2^{19}$

There are 2^9 entries per page table in 3^{rd} level.

So, we need $= \dfrac{2^{19}}{2^9} = 2^{10}$ page tables in 3^{rd} level

Now, it means 2^{10} entries in 2^{nd} level so number of page tables in 2^{nd} level

$$= \frac{2^{10}}{2^9} = 2 \text{ page table}$$

Now we have 2 entries in 1^{st} level hence we need only 1 page table in 1^{st} level.
Total number of page table $= 2^{10} + 2 + 1 = 1027$ per table
Each page table has 2^9 entries and each entry size is 8 bytes.
So, total size of page table in bytes $= 1027 \times 2^9 \times 8$ bytes $= 4108$ KB

End of Solution

Q.53 Suppose that $f : R \rightarrow R$ is a continuous function on the interval [–3, 3] and a differentiable function in the interval (–3, 3) such that for every x in the interval. $f'(x) \leq 2$. If $f(-3) = 7$, then then $f(3)$ is at most ________.

Ans. (19)

$$f'(x) \leq 2,\ f(-3) = 7$$

$$f'(x) = \frac{f(3)-7}{3-(-3)} \qquad \text{[Using Lagrange]}$$

$$f'(x) = \frac{f(3)-7}{6}$$

$$6f'(x) = f(3) - 7$$

$$f(3) = 6f'(x) + 7$$

Given max value of $f'(x)$ is 2

So, $f(3) = 6 \times 2 + 7$

$$f(3) = 19$$

End of Solution

Q.54 Let G be a connected undirected weighted graph. Consider the following two statements.

S_1 : There exists a minimum weight edge in G which is present in every minimum spanning tree of G.

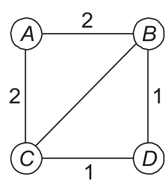
S_2 : If every edge in G has distinct weight, then G has a unique minimum spanning tree.

Which one of the following options is correct?

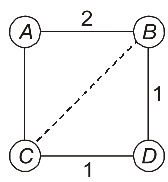
(a) S_1 is false and S_2 is true.
(b) Both S_1 and S_2 are false.
(c) S_1 is true and S_2 is false.
(d) Both S_1 and and S_2 are true.

Ans. (a)

S_1 : Consider the graph:



One of the possible MSTS can be:



But the minimum weight edge BC in G is not present. So, the S_1 is a false statement.

S_2 : In any undirected graph G, distinct edge weights means Unique MST.

So, the S_1 is false and S_2 is true.

Option (a) is correct.

End of Solution

Q.55 Consider a set-associative cache of size 2 KB (1 KB = 2^{10} bytes) with cache block size of 64 bytes. Assume that the cache is byte-addressable and a 32-bit address is used for accessing the cache. If the width of the tag field is 22 bits, the associativity of the ache is _________.

Ans. **(2)**

Set Associative Map

$$\text{CM Site} = 2 \text{ KB}$$
$$\text{Block Size} = 64 \text{ B}$$
$$\text{Number of lines} = \frac{2^{11}}{2^6} \Rightarrow 2^5(32)$$
$$\text{MM Adder} = 32 \text{ bit}$$
$$\text{Tag field size} = 22 \text{ bits}$$

Set associative CM adder format

32-bit

tag	S_o	w_o
22-bit	4-bit	$\log_2 64$ = 6 bit

$$\text{Set offset } (S_o) = 4 \text{ bit}$$
$$\therefore \quad \text{Number of sets } (S) = 2^4(16)$$
$$\text{Number of Sets } (S) = \frac{N}{P\text{-way}}$$
$$16 = \frac{32}{P\text{-way}}$$
$$\text{P-way} = \frac{32}{16} = 2$$

End of Solution

■■■■

COMPUTER SCIENCE & INFORMATION TECHNOLOGY

Q. No. 1 to 25 Carry One Mark Each

1. Which one of the following kinds of derivation is used by LR parsers?

(A) Rightmost

(B) Rightmost in reverse

(C) Leftmost

(D) Leftmost in reverse

Answer: (B)

2. Consider the following C program:

```
#include <stdio.h>
int main(){
int arr[]={1,2,3,4,5,6,7,8,9,0,1,2,5}, *ip=arr+4;
printf("%d \n", ip[1]);
return 0;
}
```

The number that will be displayed on execution of the program is _________.

Answer: (6)

3. For $\Sigma = \{a,b\}$, let us consider the regular language $L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$. Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L ?

(A) 5 (B) 24 (C) 9 (D) 3

Answer: (B)

4. Let U = {1, 2, …, n}. Let $A = \{(x, X) \mid x \in X, X \subseteq U\}$. Consider the following two statements on $|A|$.

I. $|A| = n2^{n-1}$

II. $|A| = \Sigma_{k=1}^{n} k\binom{n}{k}$

Which of the above statements is/are TRUE?

(A) Only II (B) Only I (C) Neither I nor II (C) Both I and II

Answer: (D)

5. A certain processor uses a fully associative cache of size 16 kB. The cache block size is 16 bytes Assume that the main memory is byte addressable and uses a 32-bit address. How many bits are required for the *Tag* and the *Index* fields respectively in the addresses generated by the processor?

(A) 28 bits and 4 bits

(B) 24 bits and 4 bits

(C) 24 bits and 0 bits

(D) 28 bits and 0 bits

Answer: (D)

6. Consider the grammar given below:

$$S \rightarrow Aa$$
$$A \rightarrow BD$$
$$B \rightarrow b \mid \varepsilon$$
$$D \rightarrow d \mid \varepsilon$$

Let a, b, d, and $ be indexed as follows:

a	b	d	$
3	2	1	0

Compute the FOLLOW set of the non-terminal B and write the index values for the symbols in the FOLLOW set in the descending order. (For example, if the FOLLOW set is {a, b. d, $}, then the Key should be 3210)

Answer: (31)

7. Let X be a square matrix. Consider the following two statements on X.

I. X is invertible

II. Determine of X is non-zero.

Which one of the following is TRUE?

(A) I implies II; II does not imply I

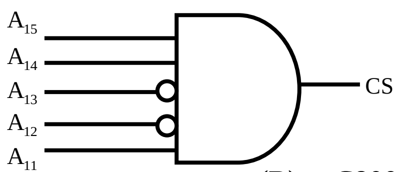
(B) I does not imply II; II does not imply I

(C) I and II are equivalent statements

(D) II implies I; I does not imply II

Answer: (C)

8. The chip select logic for a certain DRAM chip in a memory system design is shown below Assume that the memory system has 16 address lines denoted by A15 to A0. What is the range of addresses (in hexadecimal) of the memory system that can get enabled by the chip select (CS) signal?



(A) C800 to CFFF

(B) C800 to C8FF

(C) DA00 to DFFF

(D) CA00 to CAFF

Answer: (A)

9. Consider a sequence of 14 elements: A = {-5, -10, 6, 3, -1, -2, 13, 4, -9, -1, 4, 12, -3, 0]. The subsequence sum $S(i,j)=\sum_{k=i}^{j} A[k]$. Determine the maximum of $S(i,j)$, where $0 \le i \le j < 14$. (Divide and conquer approach may be used.)

Answer: (29)

10. An array of 25 distinct elements is to be sorted using quicksort. Assume that the pivot element is chosen uniformly at random. The probability that the pivot element gets placed in the worst possible location in the first round of partitioning (rounded off to 2 decimal places) is

Answer: (0.08)

11. The value of 3^{51} mod 5 is ________.

Answer: **(2)**

12. Consider the concurrent processes P1, P2 and P3 as shown below, which access a shared variable D that has been initialization to 100.

P1	P2	P3
:	:	:
:	:	:
D= D +20	D = D –50	D = D +10
:	:	:
:	:	:

The processes are executed on a uniprocessor system running a time-shared operating system. If the minimum and maximum possible values of D after the three processes have completed execution are X and Y respectively, then the value of Y–X is _________.

Answer: **(80)**

13. Compute $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

(A) 108/7

(B) 1

(C) 53/12

(D) Limit does not exist

Answer: **(A)**

14. Consider the following C program:

```
#include <stdio.h>
int jumble(int x, int y){
x=2*x+y;
```

```
return x;}
int main(){
int x=2, y=5;
y= jumble(y,x);
x= jumble(y,x);
printf("%d \n", x);
return 0;
}
```

The value printed by the program is _________.

Answer: (26)

15. Let G be an arbitrary group. Consider the following relations on G:

$R_1 : \forall a, b \in G, aR_1b$ if and only if $\exists g \in G$ such that $a = g^{-1}bg$

$R_2 : \forall a, b \in G, aR_2b$ if and only if $a = b^{-1}$

Which of the above is/are equivalence relation/relations?

(A) Neither R_1 nor R_2 (B) R_2 only

(C) R_1 only (D) R_1 and R_2

Answer: (C)

16. Consider the following two statements about database transaction schedules:

I. Strict two-phase locking protocol generates conflict serializable schedules that are also recoverable.

II. Timestamp-ordering concurrency control protocol with Thomas' Write Rule can generate view serializable schedules that are not conflict serializable.

Which of the above statements is/are TRUE?

(A) I only (B) II only (C) Neither I or II (D) Both I and II

Answer: (D)

17. Let G be an undirected complete graph on n vertices, where n > 2. Then, the number of different Hamiltonian cycles in G is equal to

(A) n! (B) $\frac{(n-1)!}{2}$ (C) 1 (D) (n–1)!

Answer: (B& C)

18. Which of the following protocol pairs can be used to send and retrieve e-mails (in that order)?

(A) SMTP, MIME (B) IMAP, POP3 (C) IMAP, SMTP (D) SMTP, POP3

Answer: (D)

19. The *following C program is executed on a Unix/Linux system:*

```
# include <unistd.h>
int main ( )
{
int i;
for (i = 0; i< 10; i++)
if (i % 2 = = 0) fork ( );
return 0;
}
```

The total number of child processes created is ________.

Answer: (31)

20. Consider Z = X – Y, where X, Y and Z are all in sign-magnitude form. X and Y are each represented in n bits. To avoid overflow, the representation of Z would require a minimum of:

(A) n bits (B) n + 1 bits (C) n + 2 bits (D) n – 1 bits

Answer: (B)

21. Which one of the following is NOT a valid identity?

(A) $(x \oplus Y) \oplus z = x \oplus (y \oplus z)$

(B) $x \oplus y = (xy + x'y')'$

(C) $(x + y) \oplus z = x \oplus (y + z)$

(D) $x \oplus y = x + y, \text{if } xy = 0$

Answer: (C)

22. Which one of the following statements is NOT correct about the B+ tree data structure used for creating an index of a relational database table?

(A) Key values in each node are kept in sorted order

(B) B+ Tree is a height-balanced tree

(C) Each leaf node has a pointer to the next leaf node

(D) Non-leaf nodes have pointers to data records

Answer: (D)

23. If L is a regular language over $\Sigma = \{a, b\}$, which one of the following languages is NOT regular?

(A) $\{ww^R \mid w \in L\}$

(B) $Prefix(L) = \{x \in \Sigma^* \mid \exists y \in \Sigma^* \text{ such that } xy \in L]$

(C) $L.L^R = \{xy \mid x \in L, y^R \in L\}$

(D) $Suffix(L) = \{y \in \Sigma^* \mid \exists x \in \Sigma^* \text{such that } xy \in L]$

Answer: (A)

24. In 16-bit 2's complement representation, the decimal number –28 is:

(A) 1000 0000 1110 0100

(B) 0000 0000 1110 0100

(C) 1111 1111 0001 1100

(D) 1111 1111 1110 0100

Answer: (D)

25. Two numbers are chosen independently and uniformly at random from the set $\{1, 2\ldots 13\}$. The probability (rounded off to 3 decimal places) that their 4-bit (unsigned) binary representations have the same most significant bit is __________.

Answer: (0.503)

Q. No. 26 to 55 Carry Two Marks Each

26. Consider the following relations P(X, Y, Z), Q(X, Y, T) and R(Y, V)

P		
X	**Y**	**Z**
X1	Y1	Z1
X1	Y1	Z2
X2	Y2	Z2
X2	Y4	Z4

Q		
X	**Y**	**T**
X2	Y1	2
X1	Y2	5
X1	Y1	6
X3	Y3	1

R	
Y	**V**
Y1	V1
Y3	V2
Y2	V3
Y2	V2

How many tuples will be returned by the following relational algebra query?

$\Pi_x\left(\sigma_{(P,Y=R,Y\wedge R.V=V2)}(P\times R)\right)-\Pi_x\left(\sigma_{(Q.Y=R.Y\wedge Q.T>2)}(Q\times R)\right)$. Answer ___________.

Answer: (1)

27. Which one of the following languages over $\Sigma=\{a,b\}$ is NOT a context free?

(A) $\{ww^R \mid w\in \{a,b\}^*\}$

(B) $\{wa^nw^Rb^n \mid w\in\{a,b\}^*, n\ \geq 0\}$

(C) $\{a^nb^i \mid i\in\ \{n,3n,5n\}, n\ \geq\ 0\}$

(D) $\{wa^nb^nw^R \mid w\in\{a,b\}^*, n\ \geq 0\}$

Answer: (B)

28. Consider the following c-program

```
#include <stdio.h>
int r(){
static int num=7;
return num --;}
int main(){
for (r();r();r())
printf("%d",r());
return 0;}
```

Which one of the following values will be displayed on execution of the programs?

(A) 52 (B) 630 (C) 41 (D) 63

Answer: (A)

29. There are n unsorted arrays: $A_1, A_2,....., A_n$. Assume that n is odd. Each of $A_1, A_2,....., A_n$ contains n distinct elements. There are no common elements between any two arrays. The worst-case time complexity of computing the median of the medians of $A_1, A_2,....., A_n$ is

(A) $O(n)$ (B) $O(n\log n)$ (C) $O(n^2)$ (D) $\Omega(n^2 \log n)$

Answer: (C)

30. A relational database contains two tables Student and Performance as shown below:

Student	
Roll_no.	**Student name**
1	Amit
2	Priya
3	Vinit
4	Rohan
5	Smita

Performance		
Roll_no.	**Subject_code**	**Marks**
1	A	86
1	B	95
1	C	90
2	A	89
2	C	92
3	C	80

The primary key of the student table is Roll_no. For the performance table, the columns Roll_no and Subject_code together form the primary key. Consider the SQL query given below:

Select S. Student_name, sum (P. Marks)

FROM Student S, Performance P

WHERE P. Marks >84

GROUP BY S.Student_name;

The number of rows returned by the above SQL query is________.

Answer: (5)

31. Consider the following C program

```
#include <stdio.h>
int main(){
float sum = 0.0, j = 1.0, i = 2.0;
while (i/j > 0.0625){
j= j+j;
sum = sum + i/j;
printf("%f\n", sum);
}
return 0;}
```

The number of times the variable sum will be printed, when the above program is executed, is _______.

Answer: (5)

32. Let the set of functional dependencies $F = \{QR \rightarrow S, R \rightarrow P, S \rightarrow Q\}$ hold on a relation schema X=(PQRS). X is not in BCNF. Suppose X is decomposed into two schemas Y and Z, where Y=(PR) and Z = (QRS).

Consider the two statements given below.

I. Both Y and Z are in BCNF

II. Decomposition of X into Y and Z is dependency preserving and lossless

Which of the above statements is/are correct?

(A) II only (B) Both I and II

(C) Neither I nor II (D) I only

Answer: (A)

33. The index node (inode) of a Unix-like file system has 12 direct, one single-indirect and one double-indirect pointers. The disk block size is 4 kB, and the disk block address is 32-bits long. The maximum possible file size is (rounded off to 1decimal place)______GB.

Answer: (4)

34. In a RSA cryptosystem, the value of the public modulus parameter n is 3007. If it is also known that $\phi(n) = 2880$, where $\phi()$ denotes Euler's Totient function, then the prime factor of n which is greater than 50 is_______.

Answer: (97)

35. Assume that in a certain computer, the virtual addresses are 64 bits long and the physical addresses are 48 bits long. The memory is word addressable. The page size is 8 kB and the word size is 4 bytes. The Translation Look-aside Buffer (TLB) in the address translation path has 128 valid entries. At most how many distinct virtual addresses can be translated without any TLB miss ?

(A) 256×2^{10} (B) 16×2^{10} (C) 4×2^{20} (D) 8×2^{20}

Answer: (A)

36. Consider the following grammar and the semantic actions to support the inherited type declaration attributes. Let X_1 ,X_2, X_3, X_4, X_5, and X_6be the placeholders for the non-terminals D, T, L or L_1 in the following table:

Production rule	Semantic action
$D \rightarrow TL$	X_1 .type = X_2.type
$T \rightarrow$ int	T.type = int
$T \rightarrow$ float	T.type = float

$L \rightarrow L1$, id	X_3.type = X_4.type addType(id.entry, X_5.type)
$L \rightarrow$ id	addType(id.entry, X_6.*typt)*

Which one of the following are the appropriate choices for X_1, X_2, X_3 and X_4?

(A) $X_1 = T$, $X_2 = L$, $X_3 = T$, $X_4 = L_1$

(B) $X_1 = L$, $X_2 = L$, $X_3 = L_1$, $X_4 = T$

(C) $X_1 = T$, $X_2 = L_1$, $X_3 = L_1$, $X_4 = L_1$

(D) $X_1 = L$, $X_2 = T$, $X_3 = L_1$, $X_4 = L$

Answer: (D)

37. Consider the following matrix

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

The absolute value of the product of Eigen values of R is_______.

Answer: (12)

38. Consider the following C function.

```
void convert(int n){
    if(n<0)
    printf(%d",n);
    else {
    convert(n/2);
    printf("%d",n%2);
    }
}
```

Which one of the following will happen when the function `convert` is called with any positive integer n as argument?

(A) It will not print anything and will not terminate

(B) It will print the binary representation of n and terminate

(C) It will print the binary representation of n in the reverse order and terminate

(D) It will print the binary representation of n but will not terminate

Answer: (A)

39. Suppose Y is distributed uniformly in the open interval (1,6). The probability that the polynomial $3x^2+6xY+3Y+6$ has only real roots is ____________. (rounded off to 1 decimal place).

Answer: (0.8)

40. Let Σ be the set of all bijections from $\{1,\ldots,5\}$ to $\{1,\ldots,5\}$, where id denotes the identity function, i.e. id(j) = j, $\forall$j. let $\circ$ denote composition on functions. For a string

$x = x_1\, x_2 \ldots x_n \in \Sigma^n, n \geq 0$, let $\pi(x) = x_1 \circ x_2 \circ \ldots \circ x_n$.

Consider the language $L = \{x \in \Sigma^* \mid \pi(x) = id\}$. The minimum number of states in any DFA accepting L is______.

Answer: (120)

41. Let *T* be a full binary tree with 8 leaves. (A full binary' tree has every level full.) Suppose two leaves *a* and *b* of *T* are chosen uniformly and independently at random. The expected value of the distance between *a* and *b* in *T* (i.e., the number of edges in the unique path between *a* and *b)* is (rounded off to 2 decimal places) _________

Answer: (4.25)

42. Consider the following statements:

I. The smallest element in a max-heap is always at a leaf node

II. The second largest element in a max-heap is always a child of the root node

III. A max-heap can be constructed from a binary search tree m $\theta(n)$ time

IV. A binary search tree can be constructed from a max-heap in $\theta(n)$ time

Which of the above statements are TRUE?

(A) I, II and III (B) I, III and IV

(C) II, III and IV (D) I, II and IV

Answer: (A)

43. Consider three machines M, N, and P with IP address 100.10.5.2, 100.10.5.5, and 100.10.5.6. respectively. The subnet mask is set to 255.255.255.252 for all the three machines. Which one of the following is true?

(A) M, N, and P all belong to the same subnet

(B) Only M and N belong to the same subnet

(C) M, N, and P belong to three different subnets

(D) Only N and P belong to the same subnet

Answer: (D)

44. Consider the following sets:

S1. Set of all recursively enumerable languages over the alphabet {0,1}

S2. Set of all syntactically valid C programs

S3. Set of all languages over the alphabet {0,1}

S4. Set of all non-regular languages over the alphabet {0,1}

Which of the above sets are uncountable?

(A) S1 andS4 (B) S3 and S4 (C) S1 and S2 (D) S2andS3

Answer: (B)

45. Let *G* be any connected, weighted, undirected graph.

I. *G* has a unique minimum spanning tree, if no two edges of *G* have the same weight.

II. *G* has a unique minimum spanning tree, if, for every cut of G, there is a unique minimum-weight edge crossing the cut.

Which of the above two statements is/are TRUE?

(A) I only

(B) II only

(C) Neither I nor II

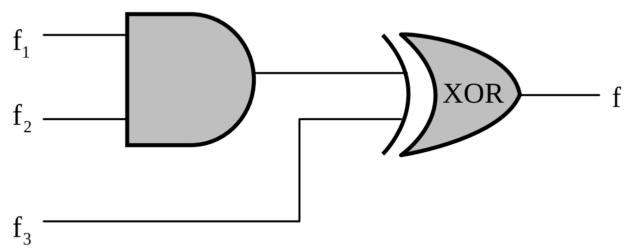
(D) Both I and II

Answer: (D)

46. Consider three 4-variable functions f_1, f_2, and f_3, which are expressed in sum-of-minterms as

$f_1 = \Sigma(0,2,5,8,14)$, $f_2\Sigma(2,3,6,8,14,15)$, $f_3 = \Sigma(2,7,11,14)$

For the following circuit with one AND gate and one XOR gate, the output function f can be expressed as:



(A) $\Sigma(7,8,11)$

(B) $\Sigma(2,14)$

(C) $\Sigma(2,7,8,11,14)$

(D) $\Sigma(0,2,3,5,6,7,8,11,14,15)$

Answer: (A)

47. Consider the following snapshot of a system running n concurrent processes. Process i is holding X_i instances of a resource R, $1 \le i \le n$. Assume that all instances of R are currently in use. Further, for all i, process i can place a request for at most Y_i additional instances of R while holding the X_i instances it already has. Of the n processes, there are exactly two processes p and q such that $Y_p = Y_q = 0$. which one of the following conditions guarantees that no other process apart from p and q can complete execution ?

(A) $X_p + X_q < \text{Min}\{Y_K \mid 1 \le K \le n, k \ne p, k \ne q\}$

(B) $\text{Min}(X_{p,} X_q) \le \text{Max}\{Y_k \mid 1 \le k \le n, k \ne p, k \ne q\}$

(C) $X_p + X_q < \text{Max}\{Y_k \mid 1 \le k \le n, k \ne p\ k \ne q\}$

(D) $\text{Min}(X_p, X_q) \ge \text{Min}\{Y_k \mid 1 \le k \le n, k \ne p, k \ne q\}$

Answer: (A)

48. A certain processor deploys a single-level cache. The cache block size is 8 words and the word size is 4 bytes. The memory system uses a 60-MHz clock. To service a cache miss, the memory controller first takes 1 cycle to accept the starting address of the block, it then takes 3 cycles to fetch all the eight words of the block, and finally transmits the words of the requested block at the rate of 1 word per cycle. The maximum bandwidth for the memory system when the program running on the processor issues a series of read operations is _____ $\times\ 10^6$ bytes/sec.

Answer: (160)

49. Consider the augmented grammar given below:

$$S' \rightarrow S$$
$$S \rightarrow < L > \ | \ id$$
$$L \rightarrow L, S \ | \ S$$

Let I_0 = CLOSURE ({[$S' \rightarrow \bullet S$]}). The number of items in the set GOTO (I_o,<) is:________

Answer: (5)

50. Consider the following four processes with arrival times (in milliseconds) and their length of CPU bursts (in milliseconds) as shown below:

Process	**P1**	**P2**	**P3**	**P4**
Arrival time	0	1	3	4
CPU burst time	3	1	3	Z

These processes are run on a single processor using preemptive shortest remaining time first scheduling algorithm. If the average waiting time of the processes is 1 millisecond, then the value of Z is ________.

Answer: (2)

51. Consider that 15 machines need to be connected in a LAN using 8-port Ethernet switches. Assume that these switches do not have any separate uplink port. The minimum number of switches needed is_________.

Answer: (3)

52. Consider the first order predicate formula ϕ :

$$\forall x\left[\left(\forall z\, z \mid x \Rightarrow \left((z=x) \vee (z=1)\right)\right) \Rightarrow \exists w\,(w > x) \wedge \left(\forall z\, z \mid w \Rightarrow \left((w=z) \vee (z=1)\right)\right)\right]$$

Here $'a \mid b'$ denotes that 'a divides b' where a and b are integers. Consider the following sets:

S1	$\{1, 2, 3, ..., 100\}$
S2	Set of all positive integers
S3	Set of all integers

Which of the above sets satisfy ϕ ?

(A) S1 and S3 (B) S1 and S2 (C) S2 and S3 (D) S1, S2 and S3

Answer: (C)

53. Consider the following C program

```
#include <stdio.h>
int main(){
int a[] = {2, 4, 6, 8, 10};
int i, sum = 0, *b = a + 4;
for (i = 0; i< 5; i+d+ )
sum = sum + (*b - i) - *(b - i);
printf("%d\n",sum);
return 0;
}
```

The output of the above C-program is _______.

Answer: (10)

54. Suppose that in an IP-over Ethernet network, a machine X wishes to find the MAC address of another machine Y in its subnet. Which one of the following techniques can be used for this?

(A) X sends an ARP request packet to the local gateway's MAC address which then finds the MAC address of Y and sends to X

(B) X sends an ARP request packet with broadcast IP address in its local subnet

(C) X sends an ARP request packet to the local gateway's IP address which then finds MAC address of Y and sends to X

(D) X sends an ARP request packet with broadcast MAC address in its local subnet

Answer: (D)

55. What is the minimum number of 2-input NOR gates required to implement a 4-variable function expressed in sum-of minterms form as $f = \Sigma(0,1,5,7,8,10,13,15)$? Assume that all the inputs and their complements are available.

Answer: (3)

Computer Science and Information Technology

Q. No. 1 – 25 Carry One Mark Each

1. Let $\oplus$ and $\odot$ denote the Exclusive OR and Exclusive NOR operations, respectively. Which of the following is NOT CORRECT?

(A) $\overline{P \oplus Q} = P \odot Q$

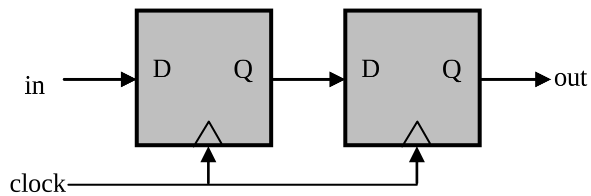
(B) $\overline{P} \oplus Q = P \odot Q$

(C) $\overline{P} \oplus \overline{Q} = P \oplus Q$

(D) $(P \oplus \overline{P}) \oplus Q = (P \odot \overline{P}) \odot \overline{Q}$

Answer: (D)

2. Consider the sequential circuit shown in the figure, where both flip-flops used are positive edge-triggered D flip-flops.



The number of states in the state transition diagram of this circuit that have a transition back to the same state on some value of "in" is ________.

Answer: (2)

3. Consider the following C program

```
#include<stdio.h>
struct Ournode{
    char x,y,z;
};
int main(){
    struct Ournode p =  {‘1’, '0', ‘a’ +2};
    struct Ournode *q = &p;
    printf("%c,  %c",   *((char*)q+1),   *((char*)q+2));
```

```
    return 0;
}
```

The output of this program is:

(A) 0, c (B) 0, a+2 (C) 0, a+2 (D) 0, c

Answer: (A)

4. In an Entity-Relationship (ER) model, suppose *R* is a many-to-one relationship from entity set *E1* to entity set *E2*. Assume that *E1*&*E2* participate totally in *R* and that the cardinality of *E1* is greater than the cardinality of *E2*.

Which one of the following is true about *R*?

(A) Every entity in *E1* is associated with exactly one entity in *E2*

(B) Some entity in *E1* is associated with more than one entity in *E2*

(C) Every entity in *E2* is associated with exactly one entity in *E1*

(D) Every entity in *E2* is associated with at most one entity in *E1*

Answer: (A)

5. The following are some events that occur after a device controller issues an interrupt while process *L* is under execution.

(P) The processor pushes the process status of *L* onto the control stack

(Q) The processor finishes the execution of the current instruction.

(R) The processor executes the interrupt service routine.

(S) The processor pops the process status of *L* from the control stack.

(T) The processor loads the new PC value based on the interrupt.

Which one of the following is the correct order in which the events above occur?

(A) QPTRS (B) PTRSQ (C) TRPQS (D) QTPRS

Answer: (A)

6. Let N be an NFA with n states. Let k be the number of states of a minimal DFA equivalent to N. Which one of the following is necessarily true?

(A) $k \geq 2^n$ (B) $k \geq n$ (C) $k \leq n^2$ (D) $k \leq 2^n$

Answer: (D)

7. Consider the following C program:

```
#include <stdio.h>
int counter = 0;
int calc(int a, int b){
    int c;
    counter++;
    if (b==3) return (a*a*a);
    else {
    c = calc(a,  b/3) ;
    return(c*c*c);
    }
}
int main(){
    calc (4,81);
    printf ("%d",  counter);
}
```

The output of this program is______

Answer: (4)

8. Consider the following processor design characteristics.

I. Register-to-register arithmetic operations only

II. Fixed-length instruction format

III. Hardwired control unit

Which of the characteristics above are used in the design of a RISC processor?

(A) I and II only (B) II and III only

(C) I and III only (D) I, II and III

Answer: (B)

9. The postorder traversal of a binary tree is 8, 9, 6, 7, 4, 5, 2, 3, 1. The inorder traversal of the same tree is 8, 6, 9, 4, 7, 2, 5, 1, 3. The height of a tree is the length of the longest path from the root to any leaf. The height of the binary tree above is_______

Answer: (4)

10. Consider a long-lived TCP session with an end-to-end bandwidth of 1 Gbps(=10^9 bits-per-second). The session starts with a sequence number of 1234. The minimum time (in seconds,rounded to the closet integer) before this sequence number can be used again is ___.

Answer: (34)

11. Consider a matrix $A = uv^T$ where $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Note that v^T denotes the transpose of v. The largest eigen value of A is _______.

Answer: (3)

12. Consider the following statements regarding the slow start phase of the TCP congestion control algorithm. Note that *cwnd* stands for the TCP congestion window and MSS denotes the Maximum Segment Size.

(i) The *cwnd* increases by 2 MSS on every successful acknowledgment

(ii) The *cwnd* approximately doubles on every successful acknowledgment

(iii) The*cwnd* increases by 1 MSS on every round trip time

(iv) The*cwnd* approximately doubles every round trip time

Which one of the following is CORRECT?

(A) Only (ii) and (iii) are true (B) Only (i) and (iii) are true

(B) Only (iv) is true (D) Only (i) and (iv) are true

Answer: (C)

13. Which one of the following statements is **FALSE**?

(A) Context-free grammar can be used to specify both lexical and syntax rules.

(B) Type checking is done before parsing.

(C) High-level Language programs can be translated to different Intermediate Representations.

(D) Arguments to a function can be passed using the program stack.

Answer: (B)

14. Match the following:

	Field		**Length in bits**
P.	UDP Header's Port Number	**I.**	48
Q.	Ethernet MAC Address	**II.**	8
R.	IPv6 Next Header	**III.**	32
S.	TCP Header's Sequence Number	**IV.**	16

(A) P-III, Q-IV, R-II, S-I (B) P-II, Q-I, R-IV, S-III

(C) P-IV, Q-I, R-II, S-III (D) P-IV, Q-I, R-III, S-II

Answer: (C)

15. Consider the following two tables and four queries in SQL.

Book (isbn, bname), Stock (isbn, copies)

Query 1:
```
SELECT B.isbn, S.copies
FROM Book B INNER JOIN Stock S
ON B.isbn=S.isbn:
```

Query 2:
```
SELECT B.isbn, S.copies
FROM Book B LEFT OUTER JOIN Stock S
ON B.isbn=S.isbn:
```

Query 3:
```
SELECT B, isbn, S.copies
FROM Book B RIGHT OUTER JOIN Stock S
ON B.isbn=S.isbn:
```

Query 4: SELECT B, isbn, S.copies

FROM Book B FULL OUTER JOIN Stock S

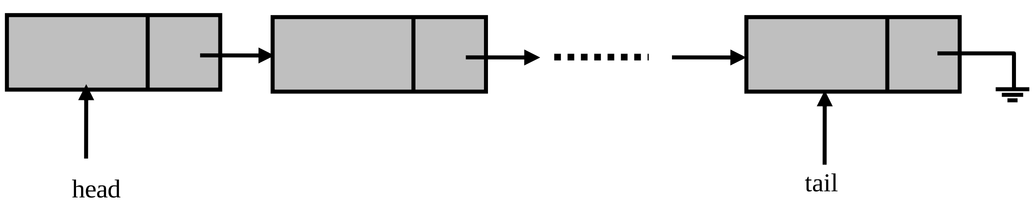
ON B.isbn=S.isbn:

Which one of the queries above is certain tohave an output that is a superset of the outputs of the other three queries?

(A) Query 1 (B) Query 2 (C) Query 3 (D) Query 4

Answer: (D)

16. A queue is implemented using a non-circular singly linked list. The queue has a head pointer and a tail pointer, as shown in the figure. Let *n* denote the number of nodes in the queue. Let `enqueue` be implemented by inserting a new node at the head and `dequeue` be implemented by deletion of a node from the tail.



Which one of the following is the time complexity of the most time-efficient implementation of *enqueue* and *dequeue*, respectively, for this data structure?

(A) $\theta(1), \theta(1)$ (B) $\theta(1), \theta(n)$ (C) $\theta(n), \theta(1)$ (D) $\theta(n), \theta(n)$

Answer: (B)

17. Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is____.

Answer: (42)

18. The set of all recursively enumerable languages is

(A) closed under complementation.

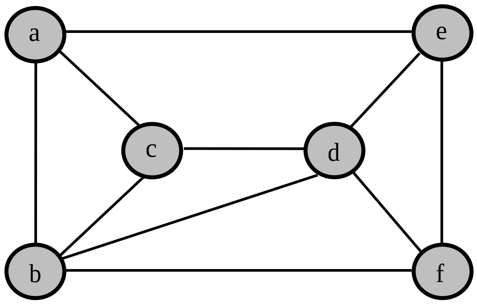
(B) closed under intersection.

(C) a subset of the set of all recursive languages.

(D) an uncountable set.

Answer: (B)

19. The chromatic number of the following graph is ______.



Answer: (3)

20. A 32-bit wide main memory unit with a capacity of 1 GB is built using 256M × 4-bit DRAM chips. The number of rows of memory cells in the DRAM chip is 2^{14}. The time taken to perform one refresh operation is 50 nanoseconds. The refresh period is 2 milliseconds. The percentage (rounded to the closest integer) of the time available for performing the memory read/write operations in the main memory unit is________

Answer: (60%)

21. Which one of the following is a closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n+3$ for all n = 0, 1, 2…?

(A) $\frac{3}{(1-x)^2}$ (B) $\frac{3x}{(1-x)^2}$ (C) $\frac{2-x}{(1-x)^2}$ (D) $\frac{3-x}{(1-x)^2}$

Answer: (D)

22. Consider a system with 3 processes that share 4 instances of the same resource type. Each process can request a maximum of *K* instances. Resource instances can be requested and released only one at a time. The largest value of *K* that will always avoid deadlock is _____.

Answer: (2)

23. Consider a process executing on an operating system that uses demand paging. The average time for a memory access in the system is *M* units if the corresponding memory page is available in memory and *D* units if the memory access causes a page fault. It has been experimentally measured that the average time taken for a memory access in the process is *X* units.

Which one of the following is the correct expression for the page fault rate experienced by the process?

(A) $(D-M)/(X-M)$ (B) $(X-M)/(D-M)$

(C) $(D-X)/(D-M)$ (D) $(X-M)/(D-X)$

Answer: (B)

24. Two people P and Q decide to independently roll two identical dice, each with 6 faces numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dicerepeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. Theprobability (rounded to 3 decimal places) that one of them wins on the third trial is ______.

Answer: (0.023)

25. The value of $\int_0^{\pi/4} x\cos\left(x^2\right)dx$ correct to three decimal places (assuming that $\pi = 3.14$) is _____.

Answer: (0.29)

26. Consider Guwahati *(G)* and Delhi *(D)* whose temperatures can be classified as high*(H)*, medium *(M)* and low *(L)*. Let $P(H_G)$ denote the probability that Guwahati has high temperature. Similarly $P(M_G)$ *and* $P(L_G)$ denotes the probability of Guwahati having medium and low temperatures respectively. Similarly, we use $P(H_D), P(M_D)$ *and* $P(L_D)$ for Delhi.

The following table gives the conditional probabilities for Delhi's temperatures given Guwahati's temperature

	H_D	M_D	L_D
H_G	0.40	0.48	0.12
M_G	0.10	0.65	0.25
L_G	0.01	0.50	0.49

Consider the first row in the table above. The first entry denotes that if Guwahati has high temperature (H_G) then probability of Delhi also having a high temperature (H_D) is 0.40 i.e., $P(H_D/H_G) = 0.40$. Similarly, the next two entries are $P(M_D/H_G)$=0.48 and $P(L_D/H_G) = 0.12$ Similarly for the other rows.

If it is known that $P(H_G)=0.2, P(M_G)=0.5\ and P(L_G)=0.3,$ then the probability (correct to two decimal places) that Guwahati has high temperature given that Delhi has high temperature is ________.

Answer: (0.6)

27. Assume that multiplying a matrix $G_1$of dimension $p \times q$ with another matrix G_2 of dimension $q \times r$ requires pqrscalar multiplications. Computing the product of n matrices $G_1G_2G_3...G_n$can be done by parenthesizing in different ways. Define $G_i\ G_{i+1}$as an explicitly computed pair for a given paranthesization if they are directly multiplied. For example, in the matrix multiplication chain $G_1G_2G_3G_4G_5G_6$using parenthesization$(G_1(G_2G_3))(G_4(G_5G_6)))$. *G2G3* and *G5G6* are the only explicitly computed pairs.

Consider a matrix multiplication chain $F_1F_2F_3F_4F_5$, where matrices F_1, F_2, F_3, $F_4$and $F_5$are of dimensions 2× 25, 25×3. 3×16. 16×1 and 1×1000. respectively, In the parenthesization of $F_1F_2F_3F_4F_5$that minimizes the total number of scalar multiplications, the explicitly computed pairs is are

(A) $F_1F_2$and F_3F_4only
(B) F_2F_3 only
(C) F_3F_4only
(D) $F_1F_2$and F_4F_5only

Answer: (C)

28. The instruction pipeline of a RISC processor has the following stages: Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Write back (WB). The IF, ID, OF and WB stages take 1 clock cycle each for every instruction. Consider a sequence of 100 instructions. In the PO stage. 40 instructions take 3 clock cycles each. 35 instructions take 2 clock cycles each, and the remaining 25 instructions take 1 clock cycle each. Assume that there are no data hazards and no control hazards. The number of clock cycles required for completion of execution of the sequence of instructions is______.

Answer: (219)

29. Consider the following solution to the producer-consumer synchronization problem. The shared buffer size is *N*. Three semaphores *empty*. *full* and *mutex* are defined with respective initial values of 0. *N* and 1. Semaphore *empty* denotes the number of available slots in thebuffer, for the consumer to read from. Semaphore *full* denotes the number of available slots in the buffer. for the producer to write to. The placeholder variables. denoted by `P, Q, R,and S.` in the code below can be assigned either *empty* or *full*. The valid semaphore operations arc: `wait ()` and `signal().`

Producer:	Consumer:

`do {`
`wait(P);`
`wait(mutex);`
`//Add item to buffer`
`signal (mutex);`
`signal (Q);`
`}while(1);`
	`do {`
	`wait(R);`
	`wait(mutex);`
	`//Consume item to buffer`
	`signal (mutex);`
	`signal (S);`
	`}while(1) ;`

Which one of the following assignments to `P`, `Q`, `R` and `S` will yield the correct solution?

(A) `P`:full, `Q`: full, `R`: empty, `S`: empty

(B) `P`:empty, `Q`: empty, `R`: full, `S`: full

(C) `P`:full, `Q`: empty, `R`: empty, `S`: full

(D) `P`:empty, `Q`: full, `R`: full, `S`: empty

Answer: (C)

30. A lexical analyzer uses the following patterns to recognize three tokens T_1. T_2. and T_3 over the alphabet $\{a, b, c\}$.

$T1$: $a?(b|c)*a$

T_2: $b?(a|c)*b$

T_3: $c?(b|a)*c$

Note that 'x? means 0 or 1 occurrence of the symbol *x* Note also that the analyzer outputs the token that matches the longest possible prefix.

It the string *bbaacabc*is processed by the analyzer, which one of the following is the sequence of tokens it outputs?

(A) $T_1T_2T_3$ (B) $T_1T_1T_3$ (C) $T_2T_1T_3$ (D) T_3T_3

Answer: (D)

31. Consider a matix P whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Consider the following statements:

(I) P does not have an inverse

(II) P has a repeated eigenvalue

(III) P cannot be diagonalized

Which one of the following options is CORRECT?

(A) Only I and III are necessarily true (B) Only II is necessarily true

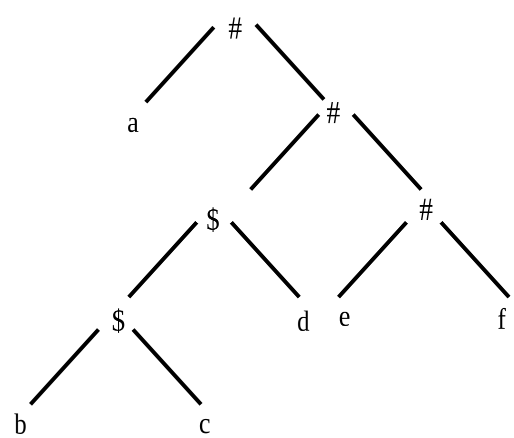
(C) Only I and II are necessarily true (D) Only II and III are necessarily true

Answer: (D)

32. A processor has 16 integer registers (R0, R1….R15) and 64 floating point registers (F0,Fl…….F63). It uses a 2-byte instruction format. There are four categories of instructions: Type-1, Type-2, Type-3, and Type-4. Type-1 category consists of four instructions, each with 3 integer register operands (3Rs). Type-2 category consists of eight instructions, each with 2 floating point register operands (2Fs). Type-3 category consists of fourteen instructions, each with one integer register operand and one floating point register operand (1R+1F). Type-4 category consists of X instructions, each with a floating point register operand (IF).The maximum value of N is___________.

Answer: (32)

33. Consider the following parse tree for the expression a#bcd#e#f. involving two binary operators $ and #.



Which one of the following is CORRECT for the given parse tree?

(A) $ has higher precedence and is left associative; # is right associative

(B) # has higher precedence and is left associative; $ is right associative

(C) $ has higher precedence and is left associative; # is left associative

(D) # has higher precedence and is right associative; $ is left associative

Answer: (A)

34. Consider the minterm list form of a Boolean function *F* given below:

$$F(P,Q,R,S)=\sum m(0,2,5,7,9,11)+d(3,8,10,12.14)$$

Here, m denotes a minterm and d denotes a don't care term. The number of essential prime implicants of the function F is _______

Answer: (3)

35. Consider the relations $r(A,B)$ and $s(B,C)$, where s.B is a primary key and r.B is a foreign key referencing s. B. Consider the query

$$Q: r \bowtie (\sigma_{B<5}(s))$$

Let LOJ denote the natural left outer-join operation. Assume that r and s contain no null values.

Which one of the following queries is NOT equivalent to Q?

(A) $\sigma_{B<5}(r \bowtie s)$

(B) $\sigma_{B<5}(r \text{LOJ} s)$

(C) $r\,\text{LOJ}(\sigma_{B<5}(s))$

(D) $\sigma_{B<5}(r)\,\text{LOJ}\,s$

Answer: (C)

36. Consider the following C code. Assume that unsigned long int type length is 64 bits.

```
unsigned long int fun(unsigned long int n){
unsigned long int i,   j = 0,   sum = 0;
    for   (i = n;  i> 1;   i = i/2)
                j++;
    for( ;   j > 1;    j = j/2)   sum++;
    return(sum);
    }
```

The value returned when we call fun with the input 2^{40} is

(A) 4 (B) 5 (C) 6 (D) 40

Answer: (B)

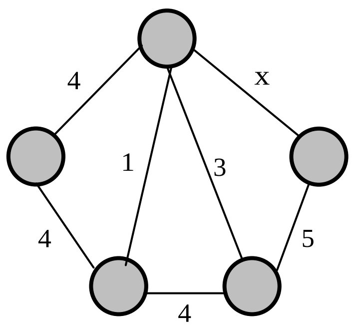
37. The size of the physical address space of a processor is 2^P bytes. The word length is 2^W bytes. The capacity of cache memory is 2^N bytes. The size of each cache block is 2^M words. For a K-way set-associative cache memory, the length (in number of bits) of the tag field is

(A) $P - N - \log_2 K$ (B) $P - N + \log_2 K$

(C) $P - N - M - W - \log_2 K$ (D) $P - N - M - W + \log_2 K$

Answer: (B)

38. Consider the following undirected graph G:



Choose a value for x that will maximize the number of minimum weight (MWSTs) of G. The number of MWSTs of G for this value of x is____________.

Answer: (4)

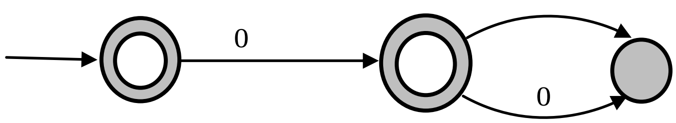
39. Given a language L, define L^i as follows:

$$L^0 = \{\varepsilon\}$$

$$L^i = L^{i-1}.L \text{ for all } i>0$$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$.

Consider the language L_1(over alphabet 0) accepted by the following automaton.



The order *of* $L_1$is

Answer: (2)

40. Let *N* be the set of natural number.

Consider the following sets

P: Set of Rational numbers (positive and negative)

Q: Set of functions from {0, 1} to N

R: Set of functions from N to {0, 1}

S: Set of finite subset of N.

Which of the sets above are countable?

(A) *Q* and *S* only (B) *P* and *S* only (C) *P* and *R* only (D) *P*, *Q* and *S* only

Answer: (D)

41. Consider a storage disk with 4 platters (numbered as 0.1. 2 and 3). 200 cylinders (numbered as 0. 1. .... 199). and 256 sectors per track (numbered as 0. 1. ... 255). The following 6 disk requests of the form [sector number. cylinder number. platter number] are received by the disk controller at the same time:

[120, 72, 2], [180, 134, 1], [60, 20, 0], [212, 86, 3], [56, 116, 2],[118, 16, 1]

Currently the head is positioned at sector number 100 of cylinder 80. and is moving: towardshigher cylinder numbers. The average power dissipation in moving the head over 100 cylinders is 20 milliwatts and for reversing the direction of the head movement once is 15milliwatts. Power dissipation associated with rotational latency and switching of head between different planers is negligible.

The total power consumption in milliwatts to satisfy all of the above disk requests using the Shortest Seek Time First disk scheduling algorithm is ________.

Answer: (85)

42. Consider the following program written in pseudo-code. Assume that *x* and *y* are integers.

```
Count(x,y)    {
if (y != 1){
    if (x!= 1) {
    printf ("*");
```

```
        Count (x/2, y);
        }
        else {
        y = y-1;
        Count(1024,  y) ;
        }
        }
}
```

The number of times that the `print` statement is executed by the call `Count(1024, 1024)` is____.

Answer: (10230)

43. Consider the following problems. $L(G)$denotes the language generated by a grammar G. $L(M)$denotes the language accepted by a machine M.

(I) For an unrestricted grammar G and a string w whether $w \in L(G)$

(II) Given a Turing machine M, whether $L(M)$is regular

(III) Given two grammars $G_1$and G_2. whether $L(G_1)=L(G_2)$

(IV) Given an NFA N. whether there is a deterministic PDA P such that N and P accept the same language.

Which one of the following statements is correct?

(A) Only I and II are undecidable

(B) Only III is undecidable

(C) Only II and IV are undecidable

(D) Only I. II and III are undecidable

Answer: (D)

44. Consider an IP packet with a length of 4.500 bytes that includes a 20-byte IPv4 header anda 40-byte TCP header. The packet is forwarded to an lPv4 router that supports a MaximumTransmission Unit (MTU) of 600 bytes. Assume that the length of the IP header in all the outgoing frag1ments of this packet is 20 bytes. Assume that the fragmentation offset valuestored in the first fragment is 0.

The fragmentation offset value stored in the third fragment is ____________.

Answer: (144)

45. Consider the first-order logic sentence

$$\phi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y \ \psi(s,t,u,v,w,x,y)$$

Where $\psi(s,t,u,v,w,x,y)$ is a quantifier-free first-order logic formula using only predicate symbols and possibly equality, but no function symbols. Suppose φ has a model with a universe containing 7 elements.

Which one of the following statements is necessarily true?

(A) There exists at least one model of φ with universe of size less than or equal to 3.

(B) There exists no model of φ with universe of size less than or equal to 3.

(C) There exists no model of φ with universe of size greater than 7.

(D) Every model of φ has a universe of size equal to 7.

Answer: (A)

46. Let G be a simple undirected graph. Let T_D be a depth first search tree of G. Let $T_B$be a breadth first search tree of G. Consider the following statements.

(I) No edge of G is a cross edge with respect to T_D(A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D.).

(II) For every edge (u, v) of G, if u is at depth $i$and v is at depth j in T_B, then $|i - j| = 1$.

Which of the statements above must necessarily be true?

(A) I only (C) Both I and II (B) II only (D) Neither I nor II

Answer: (A)

47. In a system. there are three types of resources: E, F and G. Four processes P_0. P_1, $P_2$and P_3execute concurrently. At the outset. the processes have declared their maximum resourcerequirements using a matrix named Max as given below. For example. Max(P1,Fl is the maximum nwnber of instances of F that P_2would require. The number of instances of theresources allocated to the various processes at any given state is given by a matrixnamedAllocation.

Consider a state of the system with the Allocation matrix as shown below, and in which 3instances of *E* and 3 instances of Fare the only resources available.

Allocation			
	E	F	G
P_0	1	0	1
P_1	1	1	2
P_2	1	0	3
P_3	2	0	0

Max			
	E	F	G
P_0	4	3	1
P_1	2	1	4
P_2	1	3	3
P_3	5	4	1

From the perspective of deadlock avoidance. which one of the following is true?

(A) The system is in *safe* state.

(B) The system is not in *safe* state. but would be *safe* if one more instance of *E* were available

(C) The system is not in *safe* state. but would be *safe* if one more instance of *F* were available

(D) The system is not in *safe* state. but would be *safe* if one more instance of *G* were available

Answer: (A)

48. Let *G* be a graph with 100! Vertices, with each vertex labelled by a distinct permutation ofthe numbers 1,2, ..., 100. There is an edge between vertices u and *v* if and only if the label of *u* can be obtained by swapping two adjacent numbers in the label of *v*. Let *y* denote the degree of a vertex in *G*. and *z* denote the number of connected components in *G*.

Then $y + 10z =$ __________.

Answer: (109)

49. Consider a simple communication system where multiple nodes arc connected by a sharedbroadcast medium (like Ethernet or wireless). The nodes in the system use the following carrier-sense based medium access protocol. A node that receives a packet to transmit will carrier-sense the medium for *5* units of time. If the node does not detect any other transmission in this duration it starts transmitting its packet in the next time unit. If the nodedetects another transmission it waits until this other transmission finishes, and then begins to carrier-sense for *5* time units again. Once they start to transmit nodes do not perform

any collision detection and continue transmission even if a collision occurs. All transmission last for20 units of time. Assume that the transmission signal travels at the speed of 10 meters per unit time in the medium.

Assume that the system has two nodes P and Q. located at a distance d meters from each other. P starts transmitting a packet at time $t=O$ after successfully completing its carrier-sense phase. Node Q has a packet to transmit at time $r=O$ and begins to carrier-sense the medium.

The maximum distance d (in meters. rounded to the closest integer) that allows Q to successfully avoid a collision between its proposed transmission and P's ongoing transmission is ___________.

Answer: (50)

50. Consider the following languages:

I. $\{a^m b^n c^p d^q \mid m+p = n+q, \text{ where } m, n, p, q \geq 0\}$

II. $\{a^m b^n c^p d^q \mid m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$

III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q, \text{ where } m, n, p, q \geq 0\}$

IV. $\{a^m b^n c^p d^q \mid mn = p + q, \text{ where } m, n, p, q \geq 0\}$

Which of the languages above are context-free?

(A) I and IV only

(B) I and II only

(C) II and III only

(D) II and IV only

Answer: (B)

51. Consider the following four relational schemas. For each schema all non-trivial functionaldependencies are listed. The underlined attributes are the respective primary keys.

Schema I: Registration (<u>rollno</u>, courses)
Field 'courses' is a set-valued attribute
containing the set of courses a student
hasregistered for:
Non-trivial functional dependency:
Rollno $\rightarrow$ courses

Schema II: Registration (<u>rollno, courseid,</u> email)
Non-trivial functional dependency:
rollno, courseid $\rightarrow$ email
email $\rightarrow$ rollno

Schema III: Registration (<u>rollno, courseid,</u> marks, grade)
Non-trivial functional dependencies:
rollno, courseid → marks, grade
marks → grade

Schema IV: Registration (<u>rollno, courseid,</u> credit)
Non-trivial functional dependencies:
rollno, courseid → credit
courseid → credit

Answer: (B)

52. Consider the following C program:

```
#include<stdio.h>
void fun1(char *s1, char *s2){
    char *tmp;
    tmp = s1;
    s1 = s2;
    s2 = tmp;
}
void fun2(char **s1, char **s2){
    char *tmp;
    tmp = *s1;

    *s1 = *s2;
    *s2 = tmp;
}
int main (){
    char *str1 = "Hi",   *str2 = "Bye";
    fun1(str1,  str2);      printf("%s %s ",  str1, str2);
    fun2(&str1,  &str2); printf("%s %s",  str1,  str2);
    return 0;
}
```

The output of the program above is

(A) `Hi Bye Bye Hi`

(B) `Hi Bye Hi Bye`

(C) `Bye Hi Hi Bye`

(D) `Bye Hi Bye Hi`

Answer: (A)

53. The number of possible min-heaps containing each value from {1,2,3,4, 5, 6, 7} exactly once is ________.

Answer: (80)

54. Consider the weights and values of items listed below. Note that there is only one unit of each item.

Item number	Weight (in Kgs)	Value (in Rupees)
1	10	60
2	7	28
3	4	20
4	2	24

The task is to pick a subset of these items such that their total weight is no more than 11 Kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt}. A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted V_{greedy}

The value of V_{opt}-V_{greedy} is____________.

Answer: (16)

55. Consider the unsigned 8-bit fixed point binary number representation below.

$b_7\ b_6\ b_5\ b_4\ b_3.b_2\ b_1\ b_0$

Where the position of the binary point is between b_3 and b_2. Assume b_7 is the most significant bit. Some of the decimal numbers listed below **cannot** be represented **exactly** in the above representation:

(i) 31.500

(ii) 0.875

(iii) 12.100

(iv) 3.001

Which one of the following statements is CORRECT?

(A) None of (i). (ii). (iii). (iv) can be exactly represented

(B) Only (ii) cannot be exactly represented

(C) Only (iii) and (iv) cannot be exactly represented

(D) Only (i) and (ii) cannot be exactly represented

Answer: (C)

Computer Science Engineering

Q. No. 1 – 25 Carry One Mark Each

1. Let X be a Gaussian random variable mean 0 and variance σ^2 .Let Y=max(X, 0) where max (a,b) is the maximum of a and b. The median of Y is ____________.

Answer: (0)

2. Consider the Karnaugh map given below, where x represents "don't care" and blank represents 0.

dc \ ba	00	01	11	10
00		x	x	
01	1			x
11	1			1
10		x	x	

Assume for all inputs (a,b, c, d) the respective complements $\left(\bar{a}, \bar{b}, \bar{c}, \bar{d}\right)$ are also available. The above logic is implemented 2-input NOR gates only. The minimum number of gates required is ____________.

Answer: (1)

3. The statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statements below?

I. $p \Rightarrow q$

II. $q \Rightarrow p$

III. $(\neg q) \vee p$

IV. $(\neg p) \vee q$

(A) I only

(B) I and IV only

(C) II only

(D) II and III only

Answer: (D)

4. Consider the following table:

Algorithms		Design Paradigms	
P.	Kruskal	i.	Divide and Conquer
Q.	Quicksort	ii.	Greedy
R.	Floyd-Warshall	iii.	Dynamic Programming

Match the algorithms to the design paradigms they are based on.

(A) P-(ii), Q-(iii),R-(i)

(B) P-(iii), Q-(i), R-(ii)

(C) P-(ii), Q-(i), R-(iii)

(D) P-(i), Q-(ii), R-(iii)

Answer: (C)

5. A sender S sends a message m to receiver R, which is digitally signed by S with its private key. In this scenario, one or more of the following security violations can take place.

I. S can launch a birthday attack to replace m with a fraudulent message.

II. A third party attacker can launch a birthday attack to replace m with a fraudulent message.

III. R can launch a birthday attack to replace m with a fraudulent message.

Which of the following are possible security violations?

(A) I and II only

(B) I only

(C) II only

(D) II and III only

Answer: (B)

6. Consider the following grammar.

$P \rightarrow xQRS$

$Q \rightarrow yz|z$

$R \rightarrow w|\in$

$S \rightarrow y$

What is FOLLOW (Q) ?

(A) {R}

(B) {w}

(C) {w, y}

(D) {w, \$}

Answer: (C)

7. Consider the language L given by the regular expression $(a+b)^* b(a+b)$ over the alphabet {a,b}. The smallest number of states needed in a deterministic finite-state automation (DFA) accepting L is ___________ .

Answer: (4)

8. Consider a two-level cache hierarchy with L1 and L2 caches. An application incurs 1.4 memory accesses per instruction on average. For this application, the miss rate of L1 cache 0.1, the L2 cache experiences, on average, 7 misses per 1000 instructions. The miss rate of L2expressed correct to two decimal places is ___________.

Answer: (0.05)

9. Consider the following CPU processes with arrival times (in milliseconds) and length of CPU burst (in milliseconds) as given below:

Process	Arrival time	Burst time
P1	0	7
P2	3	3
P3	5	5
P4	6	2

If the pre-emptive shortest remaining time first scheduling algorithm is used to schedule the processes, then the average waiting time across all processes is ________ milliseconds.

Answer: (3)

10. Threads of a process share

(A) global variable but not heap.

(B) heap but not global variables.

(C) neither global variables nor heap.

(D) Both heap and global variables.

Answer: (D)

11. Let $c_1 \ldots\ldots c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i = 0$ where a_i are column vectors in R^n.

Consider the set of linear equations $Ax = b$

where $A = [a_1 \ldots\ldots a_n]$ and $b = \sum_{i=1}^{n} a_i$ .The set of equations has

(A) a unique solution at $x = J_n$ where J_n denotes a n-dimensional vector of all 1

(B) no solution

(C) infinitely many solutions

(D) finitely many solutions

Answer: (C)

12. Consider the C code fragment given below.

```
typedef struct node
{
int data;
node* next ;
} node;
void join (node* m, node* n)    {
node* p=n ;
while (p->next ! =NULL){
p = p ->next ;
}
p-> next = m;
}
```

Assuming that m and n point to valid NULL- terminated linked lists, invocation of join will

(A) append list m to the end of list n for all inputs.

(B) either cause a null pointer dereference or append list m to the end of list n.

(C) cause a null pointer dereference for all inputs.

(D) append list n to the end of list m for all inputs.

Answer: (B)

13. The n-bit fixed-point representation of an unsigned real number real X uses *f* bits for the fraction part. Let $i = n - f$. The range of decimal values for X in this representation is

(A) 2^{-f} to 2^{i}

(B) 2^{-f} to $\left(2^{i} - 2^{-f}\right)$

(C) 0 to 2^{i}

(D) 0 to $\left(2^{i} - 2^{-f}\right)$

Answer: (D)

14. Consider the following intermediate program in three address code

```
p=a-b
q=p*c
p=u*v
q=p+q
```

Which one of the following corresponds to *a static single assignment* form of the above code ?

(A)
$p_1 = a - b$
$q_1 = p_1 * c$
$p_1 = u * v$
$q_1 = p_1 + q_1$

(B)
$p_3 = a - b$
$q_4 = p_3 * c$
$p_4 = u * v$
$q_5 = p_4 + q_4$

(C)
$p_1 = a - b$
$q_1 = p_2 * c$
$p_3 = u * v$
$q_2 = p_4 + q_3$

(D)
$p_1 = a - b$
$q_1 = p * c$
$p_2 = u * v$
$q_2 = p + q$

Answer: (B)

15. Consider the C struct defined below:

```
struct data {
    int marks [100] ;
    char grade;
    int cnumber;
};
struct data student;
```

The base address of student is available in register R1. The field student.grade can be accessed efficiently using

(A) Post-increment addressing mode. (R1)+

(B) Pre-decrement addressing mode, -(R1)

(C) Register direct addressing mode, R1

(D) Index addressing mode, X(R1), where X is an offset represented in 2's complement 16-bit representation.

Answer: (D)

16. Consider a TCP client and a TCP server running on two different machines. After completing data transfer, the TCP client calls **close** to terminate the connectional and a FIN segment is sent to the TCP server. Server-side TCP responds by sending an ACK which is received by the client-side TCP. As per the TCP connections state diagram (RFC 793), in which state does the client-side TCP connection wait for the FIN from the sever-side TCP?

(A) LAST-ACK (B) TIME-WAIT (C) FIN-WAIT-1 (D) FIN-WAIT-2

Answer: (D)

17. Consider the following context-free grammar over the alphabet $\Sigma = \{a, b, c\}$ with S as the start symbol.

$$S \rightarrow abScT \mid abcT$$
$$T \rightarrow bT \mid b$$

Which one of the following represents the language generated by the above grammar ?

(A) $\left\{(ab)^n (cb)^n \mid n\ {}^{3}\ 1\right\}$

(B) $\left\{(ab)^n cb^{m_1} cb^{m_2} ... cb^{m_n} \mid n, m_1, m_2, ... m_n\ {}^{3}\ 1\right\}$

(C) $\left\{(ab)^n \left(cb^m\right)^n \mid m, n\ {}^{3}\ 1\right\}$

(D) $\left\{(ab)^n \left(cb^n\right)^m \mid m, n\ {}^{3}\ 1\right\}$

Answer: (B)

18. Consider the first-order logic sentence $F: \forall z\left(\exists y R(x, y)\right)$. Assuming non-empty logical domains, which of the sentences below are *implied* by F?

I. $\exists y\left(\exists x R(x, y)\right)$

II. $\exists y\left(\forall x R(x, y)\right)$

III. $\forall y\left(\exists x R(x, y)\right)$

IV. $\neg\exists x\left(\forall y \neg R(x, y)\right)$

(A) IV only (B) I and IV only

(C) II only (D) II and III only

Answer: (B)

19. When two 8-bit numbers $A_7....A_0$ and $B_7.....B_0$ in 2's complement representation (with A_0 and B_0 as the least significant bits) are added using a **ripple-carry adder**, the sum bits obtained are $S_7....S_0$ and the carry bits are $C_7....C_0$. An overflow is said to have occurred if

(A) the carry bit C_7 is 1

(B) all the carry bits $(C_7....C_0)$ are 1

(C) $(A_7 B_7.\overline{S_7} + \overline{A_7}.\overline{B_7}.S_7)$ is 1

(D) $(A_0 .B_0.\overline{S_0} + \overline{A_0}.\overline{B_0}.S_0)$ is 1

Answer: (C)

20. Consider a database that has the relation schema EMP (EmpId, EmpName, and DeptName). An instance of the schema EMP and a SQL query on it are given below:

EMP		
EmpId	**EmpName**	**DeptName**
1.	XYA	AA
2.	XYB	AA
3.	XYC	AA
4.	XYD	AA
5.	XYE	AB
6.	XYF	AB
7.	XYG	AB
8.	XYH	AC
9.	XYI	AC
10	XYJ	AC
11.	XYK	AD
12.	XYL	AD
13.	XYM	AE

```
SELECT AVG(EC.Num)
FROM EC
WHERE(DeptName, Num)IN
        (SELECT DeptName,COUNT(EmpId)AS
                    EC(DeptName, Num)
        FROM EMP
        GROUP BY DeptName)
```

The output of executing the SQL query is _____.

Answer: (2.6)

21. The following functional dependencies hold true for the relational schema R{V,W,X,Y,Z}:

$V \rightarrow W$
$VW \rightarrow X$
$Y \rightarrow VX$
$Y \rightarrow Z$

Which of the following is irreducible equivalent for this set of functional dependencies ?

(A) $V \rightarrow W$, $V \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow Z$

(B) $V \rightarrow W$, $W \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow Z$

(C) $V \rightarrow W$, $V \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$

(D) $V \rightarrow W$, $W \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$

Answer: (A)

22. Consider the following functions from positive integers to real numbers:

$$10, \sqrt{n}, n, \log_2 n, \frac{100}{n}$$

The CORRECT arrangement of the above functions in increasing order of asymptotic complexity is:

(A) $\log_2 n, \frac{100}{n}, 10, \sqrt{n}, n$

(B) $\frac{100}{n}, 10, \log_2 n, \sqrt{n}, n$

(C) $10, \frac{100}{n}, \sqrt{n}, \log_2 n, n$

(D) $\frac{100}{n}, \log_2 n, 10, \sqrt{n}, n$

Answer: (B)

23. Let T be a tree with 10 vertices. The sum of the degrees of all the vertices in T is _________.

Answer: (18)

24. Let T be a binary search tree with 15 nodes. The minimum and maximum possible heights of T are :

Note: *The height of a tree with a single node is 0.*

(A) 4 and 15 respectively

(B) 3 and 14 respectively

(C) 4 and 14 respectively

(D) 3 and 15 respectively

Answer: (B)

25. Consider the following C code:

```
# include <stdio.h>
int * assignval (int *x, int val)  {
    *x = val;
    return x;
}
void main ( ){
    int  * x= malloc (sizeof (int));
    if (NULL = = x) return;
    x = assignval (x,0);
    if(x)   {
    x=(int*) malloc (sizeof (int));
    if (NULL = = x) return;
    x = assignval (x, 10);
}
printf("%d\n", *x);
free (x);   }
```

The code suffers from which one of the following problems:

(A) compiler error as the return of malloc is not typecast appropriately.

(B) compiler error because the comparison should be made as x==NULL and not as shown.

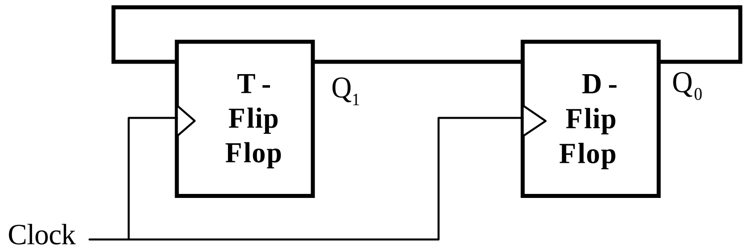
(C) compiles successfully but execution may result in dangling pointer.

(D) compiles successfully but execution may result in memory leak.

Answer: (D)

Q. No. 26 – 55 Carry Two Marks Each

26. Consider a combination of T and D flip-flops connected as shown below. The output of the D flip-flop is connected to the input of the T flip-flop and the output of the T Flip-flop is connected to the input of the D Flip-flop.



Initially, both Q_0 and Q_1 are set to 1 (before the 1st clock cycle). The outputs

(A) Q_1Q_0 after the 3rd cycle are 11 and after the 4th cycle are 00 respectively

(B) Q_1Q_0 after the 3rd cycle are 11 and after the 4th cycle are 01 respectively

(C) Q_1Q_0 after the 3rd cycle are 00 and after the 4th cycle are 11 respectively

(D) Q_1Q_0 after the 3rd cycle are 01 and after the 4th cycle are 01 respectively

Answer: (B)

27. The number of integers between 1 and 500 (both inclusive) that are divisible by 3 or 5 or 7 is ______________.

Answer: (271)

28. Consider a RISC machine where each instruction is exactly 4 bytes long. Conditional and unconditional branch instructions use PC- relative addressing mode with Offset specified in bytes to the target location of the branch instruction. Further the Offset is always with respect to the address of the next instruction in the program sequence. Consider the following instruction sequence.

Instruction No.	**Instruction**
i:	add R2, R3, R4
i +1:	sub R5, R6, R7
i + 2:	cmp R1, R9, R10
i + 3	beq R1, Offset

If the target of the branch instruction is i, then the decimal value of the Offset is __________.

Answer: **(–16)**

29. Consider the C functions foo and bar given below:

```
int foo (int val ) {
     int x = 0;
     while (val> 0)  {
     x = x + foo ( val --);
}
     return val ;
}
int bar (int val )  {
int x = 0;
while (val> 0) {
     x = x + bar (val – 1) ;
     }
return val ;
}
```

Invocations of foo (3) and bar (3) will result in:

(A) Return of 6 and 6 respectively.

(B) Infinite loop and abnormal termination respectively.

(C) Abnormal termination and infinite loop respectively.

(D) Both terminating abnormally

Answer: **(C)**

30. In a RSA cryptosystem a participant A uses two prime numbers p = 13 and q =17 to generate her public and private keys. If the public key of A is 35. Then the private key of A is __________.

Answer: **(11)**

31. Let A be an array of 31 numbers consisting of sequence of 0's followed by a sequence of 1's. The problem is to find the smallest index *i* that A[*i*] is 1 by probing the minimum numbers of locations in A. The *worst case* number of probes performed by an *optimal* algorithm is ______________.

Answer: (5)

32. If G is grammar with productions

$S \rightarrow SaS \mid aSb \mid bSa \mid SS \mid \hat{I}$

where S is the start variable, then which one of the following is not generated by G?

(A) abab (B) aaab (C) abbaa (D) babba

Answer: (D)

33. The value of $\lim_{x \to 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

(A) is 0 (B) is –1 (C) is 1 (D) does not exist

Answer: (C)

34. Instructions execution in a processor is divided into 5 stages. Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Execute (EX), and Write Back (WB), These stages take **5,4,20, 10 and 3 nanoseconds (ns)** respectively. A pipelined implementation of the processor requires buffering between each pair of consecutive stages with a delay of 2ns. Two pipelined implementations of the processor are contemplated.

(i) a naïve pipeline implementation (NP) with 5 stages and

(ii) an efficient pipeline (EP) where the OF stage id divided into stages OF1 and OF2 with execution times of 12 ns and 8 ns respectively.

The speedup (correct to two decimals places) achieved by EP over NP in executing 20 independent instructions with no hazards is _________________.

Answer: (1.508)

35. Consider a database that has the relation schemas EMP(EmpId, EmpName, DepId). And DEPT(DeptName, DeptId). Note that the DeptId can be permitted to be NULL in the relation EMP. Consider the following queries on the database expressed in tuple relational calculus.

(I) $\{t \mid \exists u \in EMP(t[EmpName] = u[EmpName] \wedge \forall v \in DEPT(t[DeptId] \neq v[DeptId]))\}$

(II) $\{t \mid \exists u \in EMP(t[EmpName] = u[EmpName] \wedge \exists v \in DEPT(t[DeptId] \neq v[DeptId]))\}$

(III) $\{t \mid \exists u \in EMP(t[EmpName] = u[EmpName] \wedge \exists v \in DEPT(t[DeptId] \neq v[DeptId]))\}$

Which of the above queries are safe?

(A) (I) and (II) only (B) (I) and (III) only

(C) (II) and (III) only (D) (I), (II) and (III)

Answer: (D)

36. Recall that Belady's anomaly is that the pages-fault rate may increase as the number of allocated frames increases. Now consider the following statements:

S_1: *Random page replacement* algorithm (where a page chosen at random is replaced) suffers from Belady's anomaly

S_2: *LRU page replacement* algorithm suffers from Belady's anomaly

Which of the following is CORRECT ?

(A) S_1 is true, S_2 is true (B) S_1 is true, S_2 is false

(C) S_1 is false , S_2 is true (D) S_1 is false, S_2 is false

Answer: (B)

37. The output of executing the following C program is ________.

```
# include <stdio.h>
int total (int v) {
while (v)   {
        count + = v & 1;
        v>> = 1;
}
        return count;
}
void main ( ) {
static int x = 0;
inti = 5;
for (; i> 0; i--)  {
        x=x + total (i) ;
```

```
}
printf ("%d\n", x) ;
}
```

Answer: **(23)**

38. Consider the following C program.

```
#include <stdio.h>
#include<string.h>
void printlength (char *s, char *t) {
     unsigned int c = 0;
     int len = ((strlen(s) - strlen (t)) > c) ?strlen(s): strlen(t);
     printf ("%d\n", len);
}
void main (  ) {
     char *x = "abc";
     char *y ="defgh";
     printlength (x,y);
```

Recall that strlen is defined in string.h as returning a value of type size_t, which is an unsigned int. The output of the program is _____________.

Answer: **(3)**

39. Consider the following languages over the alphabet $\sum=\{a,b,c\}$

Let $L_1=\{a^n b^n c^m \mid m,n\geq 0\}$ and $L_2=\{a^m b^n c^n \mid m,n\geq 0\}$

Which of the following are context-free languages ?

I. $L_1 \cup L_2$

II. $L_1 \cap L_2$

(A) I only (B) II only (C) I and II (D) Neither I nor II

Answer: **(A)**

40. Consider a 2-way set associative cache with 256 blocks and uses LRU replacement, Initially the cache is empty. Conflict misses are those misses which occur due the contention of multiple blocks for the same cache set. Compulsory misses occur due to first time access to the block. The following sequence of accesses to memory blocks.

(0,128,256,128,0,128,256,128,1,129,257,129,1,129,257,129)

is repeated 10 times. The number of *conflict misses* experienced by the cache is ___________.

Answer: (76)

41. Let u and v be two vectors in $\mathbf{R}^2$ whose Euclidean norms satisfy $\|u\| = 2\|v\|$. What is the value of α such that $w = u + \alpha v$ bisects the angle between u and v ?

(A) 2 (B) 1/2 (C) 1 (D) -1/2

Answer: (A)

42. Consider the following grammar:

```
stmt → ifexpr then else expr;stmt|0
expr → termrelopterm|term
term → id|number
if → a|b|c
number → [0-9]
```

Where **relop** is a relational operate (e.g<,>,….) ---O refers to the empty statement, and **if** , **then, else** are terminals.

Consider a program P following the above grammar containing ten **if** terminals. The number of control flows paths in P is _____________. For example the program

if e_1 **then** e_2 **else** e_3

has 2 controls flow paths $e_1 \rightarrow e_2$ and $e_1 \rightarrow e_3$

Answer: (1024)

43. In a database system, unique time stamps are assigned to each transaction using Lamport's logical clock . Let $TS(T_1)$ and $TS(T_2)$ be the timestamps of transactions T_1 and T_2 respectively. Besides, T_1 holds a lock on the resource R, and T_2 has requested a conflicting lock on the same resource R. The following

algorithm is used to prevent deadlocks in the database system assuming that a killed transaction is restarted with the same timestamp.

```
if TS(T2) < TS(T1) then
   T1 is killed
else T2 waits.
```

Assume any transactions that is not killed terminates eventually. Which of the following is TRUE about the database system that uses the above algorithm to prevent deadlocks?

(A) The database system is both deadlock-free and starvation- free.

(B) The database system is deadlock- free, but not starvation-free.

(C) The database system is starvation-free but not deadlock- free.

(D) The database system is neither deadlock- free nor starvation-free.

Answer: (A)

44. Let A and B be infinite alphabets and let # be a symbol outside both A and B. Let *f* be a total functional from A^* to B^*. We say *f* is *computable* if there exists a Turning machine M which given an input x in A^*, always halts with *f(x)* on its tape. Let L_f denote the language $\{x\#f(x) | x \in A^*\}$ .Which of the following statements is true:

(A) *f* if computable if and only if L_f is recursive.

(B) *f* is computable if and only L_f recursively enumerable.

(C) If *f* is computable then L_f is recursive, but not conversely.

(D) If *f* is computable then L_f is recursively enumerable, but not conversely.

Answer: (A)

45. Consider the expression $(a-1)^*(((b+c)/3)+d))$. Let X be the minimum number of registers required by an optimal code generation (without any register spill) algorithm for a load/store architecture in which

(i) only loads and store instructions can have memory operands and

(ii) arithmetic instructions can have only register or immediate operands.

The value of X is _________.

Answer: (2)

46. Let G = (V, E) be any connected undirected edge-weighted graph. The weights of the edges in E are positive and distinct. Consider the following statements:

I. Minimum spanning tree of G is always unique.

II. Shortest path between any two vertices of G is always unique.

Which of the above statements is/are necessarily true?

(A) I only (B) II only

(C) Both I and II (D) Neither I nor II

Answer: (A)

47. A multithreaded program P executes with x number of threads and uses y number of locks for ensuring mutual exclusion while operating on shared memory locations. All locks in the program are *non-reentrant*, i.e., if a thread holds a lock *l*, then it cannot re-acquire lock *l* without releasing it. If a thread is unable to acquire a lock, it blocks until the lock becomes available. The *minimum* value of x and the *minimum* value of y together for which execution of P can result in a deadlock are:

(A) x = 1, y = 2 (B) x =2, y=1 (C) x = 2,y=2 (D) x = 1, y = 1

Answer: (D)

48. The values of parameters for the Stop-and – Wait ARQ protocol are as given below:

Bit rate of the transmission channel = 1Mbps

Propagation delay from sender to receiver = 0.75 ms

Time to process a frame = 0.25ms

Number of bytes in the information frame =1980

Number of bytes in the acknowledge frame = 20

Number of overhead bytes in the information frame = 20

Assume that there are no transmission errors. Then the transmission efficiency (expressed in percentage) of the Stop-and – Wait ARQ protocol for the above parameters is _________(correct to 2 decimal places)

Answer: (89.33)

49. A computer network uses polynomials over *GF(2)* for error checking with 8 bits as information bits and uses $x^3 + x + 1$ as the generator polynomial to generate the check bits. In this network, the message 01011011 is transmitted as

(A) 01011011010 (B) 01011011011

(C) 01011011101 (D) 01011011100

Answer: (C)

50. Let p, q, and r be propositions and the expression $(p \to q) \to r$ be a contradiction. Then, the expression $(r \to p) \to q$ is

(A) a tautology

(B) a contradiction

(C) always TRUE when p is FALSE

(D) always TRUE when q is TRUE

Answer: (D)

51. A cache memory unit with capacity of N words and block size of B words is to be designed. If it is designed as a direct mapped cache, the length of the TAG field is 10 bits. If the cache unit is now designed as a 16-way set-associative cache, the length of the TAG field is ______bits.

Answer: (14)

52. Consider the following two functions.

```
void fun1 (int  n)
{
    if (n = =0 ) return;
    printf ("%d" , n);
    fun2 (n - 2);
    printf ("%d" , n);
}
```

```
void fun2 (int  n)
{
    if (n = = 0) return ;
    printf ("%d" , n);
    fun1(++n) ;
    printf ("%d" , n);
}
```

The output printed when fun1 (5) is called is

(A) 53423122233445 (B) 53423120112233

(C) 53423122132435 (D) 53423120213243

Answer: (A)

53. Consider a database that has the relation schema CR (StudentName, CourseName). An instance of the schema CR is as given below.

CR	
Student Name	**Course Name**
SA	CA
SA	CB
SA	CC
SB	CB
SB	CC
SC	CA
SC	CB
SC	CC
SD	CA
SD	CB
SD	CC
SD	CD
SE	CD
SE	CA
SE	CB
SF	CA
SF	CB
SF	CC

The following query is made on the database.

$$T1 \leftarrow \pi_{\text{CourseName}}\left(\sigma_{\text{StudentName='SA'}}\left(\text{CR}\right)\right)$$
$$T2 \leftarrow \text{CR} \div T1$$

The number of rows in T2 is ____________.

Answer: (4)

54. Let A be $n \times n$ real valued square symmetric matrix of rank 2 with $\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}^{2} = 50.$ Consider the following statements.

(I) One eigen value must be in [-5, 5]

(II) The eigen value with the largest magnitude must be strictly greater than 5.

Which of the above statements about eigen values of A is/are necessarily CORRECT?

(A) Both (I) and (II)

(B) (I) only

(C) (II) only

(D) Neither (I) nor (II)

Answer: (B)

55. Consider the context-free grammars over the alphabet {a,b,c} given below. S and T are non-terminals

$$G_1 : S \to aSb \mid T, T \to cT \mid \epsilon$$
$$G_2 : S \to bSa \mid T, T \to cT \mid \epsilon$$

The language $L(G_1) \cap L(G_2)$ is

(A) Finite.

(B) Not finite but regular.

(C) Context-free but not regular.

(D) Recursive but not context-free.

Answer: (B)

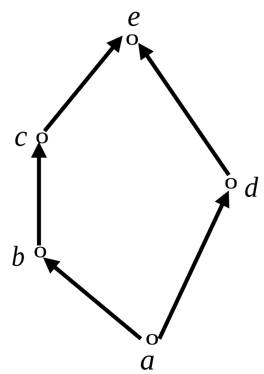
COMPUTER SCIENCE ENGINEERING

Q. No. 1 – 25 Carry One Mark Each

1. Consider the set $X=\{a,b,c,d,e\}$ under the partial ordering

$R=\{(a,a),(a,b),(a,c),(a,d),(a,e),(b,b),(b,c),(b,e),(c,c),(c,e),(d,d),(d,e),(e,e)\}$.

The Hasse diagram of the partial order (X, R) is shown below.



The minimum number of ordered pairs that need to be added to R to make (X, R) a lattice is ________.

Answer: (0)

2. Which of the following statements about parser is/are CORRECT?

I. Canonical LR is more powerful than SLR.

II. SLR is more powerful than LALR

III. SLR is more powerful than Canonical LR.

(A) I only (B) II only (C) III only (D) II and III only

Answer: (A)

3. Match the following:

P.	static char var;	**i.**	Sequence of memory locations to store addresses
Q.	m= malloc (10); m = NULL;	**ii.**	A variable located in data section of memory
R.	char * ptr [10]	**iii.**	Request to allocate a CPU register to store data
S.	register int var1;	**iv.**	A lost memory which cannot be freed

(A) P-(ii), Q-(iv), R-(i), S-(iii) (B) P-(ii), Q-(i), R-(iv), S-(iii)

(C) P-(ii), Q-(iv), R-(iii), S-(i) (D) P-(iii), Q-(iv), R-(i), S-(ii)

Answer: (A)

4. Let L_1, L_2 be any two context free languages and R be any regular language. Then which of the following is/are CORRECT ?

I. $L_1 \cup L_2$ is context – free **II.** $\overline{L_1}$ is context – free

III. $L_1 - R$ is context – free **IV.** $L_1 \cap L_2$ is context – free

(A) I, II and IV only (B) I and III only

(C) II and IV only (D) I only

Answer: (B)

5. G is undirected graph with *n* vertices and 25 edges such that each vertex of G has degree at least 3. Then the maximum possible value of *n* is ___________.

Answer: (16)

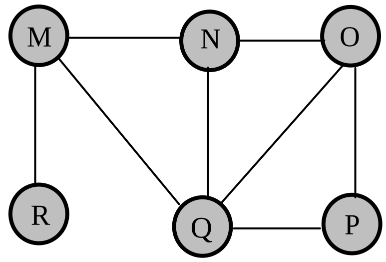
6. Let p, q, r denote the statements *"It is raining ," It is cold", and " It is pleasant,"* respectively. Then the statement *"It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold"* is represented by

(A) $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$ (B) $(\neg p \wedge r) \wedge ((p \wedge q) \rightarrow \neg r)$

(C) $(\neg p \wedge r) \vee ((p \wedge q) \rightarrow \neg r)$ (D) $(\neg p \wedge r) \vee (r \rightarrow (p \wedge q))$

Answer: (A)

7. The Breadth First Search (BFS) algorithm has been implemented using the queue data structure. Which one of the following is a possible order of visiting the nodes in the graph below?



(A) MNOPQR (B) NQMPOR (C) QMNROP (D) POQNMR

Answer: (D)

8. Let $P=\begin{bmatrix}1 & 1 & -1\\ 2 & -3 & 4\\ 3 & -2 & 3\end{bmatrix}$ and $Q=\begin{bmatrix}-1 & -2 & -1\\ 6 & 12 & 6\\ 5 & 10 & 5\end{bmatrix}$ be two matrices.

Then the rank of P +Q is _____________.

Answer: (2)

9. Consider socket API on a Linux machine that supports connected UDP sockets. A connected UDP socket is a UDP socket on which **connect** function has already been called. Which of the following statements is/are CORRECT ?

I. A connected UDP socket can be used to communicate with multiple peers simultaneously.

II. A process can successfully call **connect** function again for an already connected UDP socket.

(A) I only (B) II only (C) Both I and II (D) Neither I nor IIs

Answer: (B)

10. The minimum possible number of states of a deterministic automaton that accepts the regular language

$L=\{w_1aw_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1|=2, |w_2|\geq 3\}$ is ____________.

Answer: (8)

11. Consider the following tables T1 and T2.

P	**Q**
2	2
3	8
7	3
5	8
6	9
8	5
9	8

R	**S**
2	2
8	3
3	2
9	7
5	7
7	2

In table T1, **P** is the primary key and **Q** is the foreign key referencing **R** in table T2 with on-delete cascade and on-update cascade. In table T2, **R** is the primary key and **S** is the foreign key referencing **P** in table T1 on-delete set NULL and on-update cascade. In order to delete record $\langle 3,8 \rangle$ from table T1, the number of additional records that need to be deleted from table T1 is ______________.

Answer: (0)

12. Which of the following is/are shared by all the threads in a process ?

I. Program counter

II. Stack

III. Address space

IV. Registers

(A) I and II only (B) III only (C) IV only (D) III and IV only

Answer: (B)

13. A circular queue has been implemented using a single linked list where each node consists of a value and a single pointer pointing to the next node. We maintain exactly two external pointers **FRONT** and **REAR** pointing to the front node and the rear node of the queue, respectively. Which of the following statements is/are **CORRECT** for such a circular queue, so that insertion and deletion operation can be performed in O (1) time ?

I. Next pointer of front node points to the rear node.

II. Next pointer of rear node points to the front node.

(A) I only (B) II only (C) Both I and II (D) Neither I nor II

Answer: (B)

14. Given the following binary number in 32-bit (single precision) IEEE-754 format:

00111110011011010000000000000000

The decimal value closest to this floating- point number is

(A) 1.45×10^{1} (B) 1.45×10^{-1} (C) 2.27×10^{-1} (D) 2.27×10^{1}

Answer: (C)

15. An ER model of a database consists of entity types A and B. These are connected by a relationship R which does not have its own attribute. Under which one of the following conditions, can the relational table for R be merged with that of A?

(A) Relationship R is one-to-many and the participation of A in R is total

(B) Relationship R is one-to-many and the participation of A in R is partial

(C) Relationship R is many-to one and the participation of A in R is total

(D) Relationship R is many-to one and the participation of A in R is partial

Answer: (C)

16. Match the algorithms with their time complexities:

Algorithm		**Time complexity**	
P.	Towers of Hanoi with n disks	i.	$\theta(n^2)$
Q.	Binary search given n sorted numbers	ii.	$\theta(n \log n)$
R.	Heap sort given n numbers at the worst case	iii.	$\theta(2^n)$
S.	Addition of two $n \times n$ matrices	iv.	$\theta(\log n)$

(A) P-(iii),Q-(iv), R-(i), S-(ii) (B) P-(iv),Q-(iii), R-(i), S-(ii)

(C) P-(iii),Q-(iv), R-(ii), S-(i) (D) P-(iv),Q-(iii), R-(ii), S-(i)

Answer: (C)

17. Match the following according to input (from the left column) to the complier phase (in the right column) that processes it.

Column-1		Column-2	
P.	Syntax tree	i.	Code generator
Q.	Character stream	ii.	Syntax analyzer
R.	Intermediate representation	iii.	Semantic analyzer
S.	Token stream	iv.	Lexical analyzer

(A) P-(ii), Q-(iii), R-(iv), S-(i)
(B) P-(ii), Q-(i), R-(iii), S-(iv)
(C) P-(iii), Q-(iv), R-(i), S-(ii)
(D) P-(i), Q-(iv), R-(ii), S-(iii)

Answer: (C)

18. Consider the following statements about the routing protocols, Routing Information Protocol (RIP) and Open Shortest Path First (OSPF) in an IPv4 network.

I. RIP uses distance vector routing

II. RIP packets are sent using UDP

III. OSPF packets are sent using TCP

IV. OSPF operation is based on link-state routing

Which of the statements above are CORRECT?

(A) I and IV only
(B) I, II and III only
(C) I, II and IV only
(D) II, III and IV only

Answer: (C)

19. If $f(x) = R\sin\left(\frac{\pi x}{2}\right) + S, f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2R}{\pi}$, then the constants R and S are, respectively

(A) $\frac{2}{\pi}$ and $\frac{16}{\pi}$
(B) $\frac{2}{\pi}$ and 0
(C) $\frac{4}{\pi}$ and 0
(D) $\frac{4}{\pi}$ and $\frac{16}{\pi}$

Answer: (C)

20. In a file allocation system, which of the following allocation schemes(s) can be used if no external fragmentation is allowed?

I. Contiguous **II.** Linked **III.** Indexed

(A) I and III only (B) II only (C) III only (D) II and III only

Answer: (D)

21. Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base b. The solutions of this equation in the same base b are x = 5 and x = 6. Then b = ___________.

Answer: (8)

22. Identify the language generated by the following grammar, where S is start variable.

$S \to XY$

$X \to aX | a$

$Y \to a\,Yb | \in$

(A) $\{a^m b^n | m \geq n, n > 0\}$

(B) $\{a^m b^n | m \geq n, n \geq 0\}$

(C) $\{a^m b^n | m > n, n \geq 0\}$

(D) $\{a^m b^n | m > n, n > 0\}$

Answer: (C)

23. The representation of the value of a 16-bit unsigned integer X in hexadecimal number system is BCA9. The representation of the value of X in octal number system is

(A) 571244 (B) 736251 (C) 571247 (D) 136251

Answer: (D)

24. Consider the following function implemented in C:

```
void print xy (int x, int y) {
int *ptr ;
x = 0;
ptr = &x;
```

```
y = * ptr;
* ptr = l;
printf ("%d, %d," x,y);
}
```

The output of invoking print xy (l, l) is

(A) 0,0 (B) 0,1 (C) 1,0 (D) 1,1

Answer: (C)

25. The maximum number of IPv4 router addresses that can be listed in the record route (RR) option field of an IPv4 header is __________.

Answer: (9)

Q. No. 26 – 55 Carry Two Marks Each

26. Consider a binary code that consists of only four valid code words as given below:

00000,01011,10101,11110

Let the minimum Hamming distance of the code be p and the maximum number of erroneous bits that can be corrected by the code be q. Then the values of p and q are

(A) p = 3 and q = 1 (B) p = 3 and q = 2

(C) p = 4 and q = 1 (D) p = 4 and q = 2

Answer: (A)

27. A system shares 9 tape drives. The current allocation and maximum requirement of tape drives for three processes are shown below:

Process	Current Allocation	Maximum Requirement
P1	3	7
P2	1	6
P3	3	5

Which of the following best describes current state of the system ?

(A) Safe, Deadlocked (B) Safe, Not Deadlocked

(C) Not Safe, Deadlocked (D) Not Safe, Not deadlocked

Answer: (B)

28. Two transactions T_1 and T_2 are given as:

$$T_1 : r_1(X)w_1(X)r_1(Y)w_1(Y)$$
$$T_2 : r_2(Y)w_2(Y)r_2(Z)w_2(Z)$$

where $r_i(V)$ denotes a read operation by transaction T_i on a variable V and $w_i(V)$ denotes a *write* operations by transaction T_i on a variable V. The total number of conflict serializable schedules that can be formed by T_1 and T_2 is ______________.

Answer: (54)

29. If w, x, y, z are Boolean variables, then which one of the following is INCORRECT ?

(A) $wx + w(x+y) + x(x+y) = x + wy$

(B) $\overline{w\bar{x}\left(y+\bar{z}\right)} + \overline{w}x = \overline{w} + x + \bar{y}z$

(C) $(w\bar{x}(y + x\bar{z}) + \overline{w}\overline{x})y = x\bar{y}$

(D) $(w+y)(wxy+wyz) = wxy + wyz$

Answer: (C)

30. Consider the following C Program.

```
# include <stdio.h>
#include<string.h>
#int main ( ){
    char*  c = "GATECSIT2017";
    char*  p = c;
printf("%d", (int) strlen (c+2[p]-6[p]-1));
return 0;
}
```

The output of the program is ______________.

Answer: (2)

31. P and Q are considering to apply for a job. The probability that P applies for the job is $\frac{1}{4}$. The probability that P applies for the job given that Q applies for the job is $\frac{1}{2}$, and the probability that Q applies for the job given that P applies for the job $\frac{1}{3}$. Then the probability that P does not apply for the job given that Q does not apply for the job is

(A) $\frac{4}{5}$ (B) $\frac{5}{6}$ (C) $\frac{7}{8}$ (D) $\frac{11}{12}$

Answer: (A)

32. If the characteristics polynomial of 3×3 matrix M over R (the set of real numbers) is $\lambda^3-4\lambda^2+a\lambda+30, a\in R,$ and one eigen value of M is 2, then the largest among the absolute values of the eigen values of M is ________.

Answer: (5)

33. Consider the following expression grammar G:

$E\rightarrow E-T\,|\,T$
$T\rightarrow T+F\,|\,F$
$F\rightarrow (E)\,|\,id$

Which of the following grammars is not left recursive, but is equivalent to G?

(A)
$E\rightarrow E-T\,|\,T$
$T\rightarrow T+F\,|\,F$
$F\rightarrow (E)\,|\,id$

(B)
$E\rightarrow TE'$
$E'\rightarrow -TE'\,|\in$
$T\rightarrow T+F\,|\,F$
$F\rightarrow (E)\,|\,id$

(C)
$E\rightarrow TX$
$X\rightarrow -TX\,|\in$
$T\rightarrow FY$
$Y\rightarrow +FY\,|\in$
$F\rightarrow (E)\,|\,id$

(D)
$E\rightarrow TX\,|\,(TX)$
$X\rightarrow -TX\,|+TX\,|\in$
$T\rightarrow id$

Answer: (C)

34. In a two-level cache system, the access times of L_1 and L_2 caches are 1 and 8 clock cycles, respectively. The miss penalty from L_2 cache to main memory is 18 clock cycles . The miss rate of L_1 cache is twice that of L_2. The average memory access time (AMAT) of this cache system is 2 cycles. This miss rates of L_1 and L_2 respectively are :

(A) 0.111 and 0.056 (B) 0.056 and 0.111

(C) 0.0892 and 0.1784 (D) 0.1784 and 0.0892

Answer: (A)

35. Consider two hosts X and Y, connected by a single direct link of rate 10^6 bits/sec . The distance between the two hosts is 10,000km and the propagation speed along the link is 2×10^8 m/sec . Host X sends a file of 50,000 bytes as one large message to host Y continuously. Let the transmission and propagation delays be *p* milliseconds and *q* milliseconds, respectively . Then the values of *p* and *q* are

(A) p = 50 and q = 100 (B) p = 50 and q = 400

(C) p = 100 and q = 50 (D) p = 400 and q = 50

Answer: (D)

36. Consider the recurrence function

$$T(n) = \begin{cases} 2T(\sqrt{n}) + 1, & n > 2 \\ 2, & 0 < n \le 2 \end{cases}$$

Then T(n) in terms of θ notation is

(A) $\theta(\log\log n)$ (B) $\theta(\log n)$ (C) $\theta(\sqrt{n})$ (D) $\theta(n)$

Answer: (B)

37. If a random variable X has a Poisson distribution with mean 5, then the expectation

$E\left[(X+2)^2\right]$ equals _______________.

Answer: (54)

38. Consider the following C function

```
int fun (int n) {
     int i, j;
      for (i = 1; i< = n; i++)     {
                for (j = 1 ; j < n ; j+=i)     {
    printf ("%d %d ,i, j ) ;
                 }
    }     }
```

Time complexity of fun in terms of θ notation is

(A) $\theta\left(n\sqrt{n}\right)$ (B) $\theta\left(n^2\right)$ (C) $\theta\left(n\log n\right)$ (D) $\theta\left(n^2\log n\right)$

Answer: (C)

39. The pre-order transversal of a binary search tree is given by 12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20. Then the post-order traversal of this tree is:

(A) 2,6,7,8,9,10,12,15,16,17,19,20

(B) 2,7,6,10,9,8,15,17,20,19,16,12

(C) 7,2,6,8,9,10,20,17,19,15,16,12

(D) 7,6,2,10,9,8,15,16,17,20,19,12

Answer: (B)

40. Consider the C program fragment below which is meant to divide x by y using repeated subtractions. The variables x, y, q and r are all unsigned int.

```
while (r >= y)    {
r = r - y;
q = q +1;
}
```

Which of the following conditions on the variables x, y, q and r before the execution of the fragment will ensure that the loop terminates in a state satisfying the condition x = = (y*q+r)?

(A) (q = = r) && (r = =0)

(B) (x > 0) && (r = =x) && (y > 0)

(C) (q = = 0) && (r = = x) && (y > 0)

(D) (q = = 0) && (y > 0)

Answer: (C)

41. A message is made up entirely of characters from the set X= {P,Q,R,S,T}. The table of probabilities for each of the characters is shown below:

Character	Probability
P	0.22
Q	0.34
R	0.17
S	0.19
T	0.08
Total	1.00

If a message of 100 characters over X is encoded using Huffman coding, then the expected length of the encoded message in bits is_____

Answer: (225)

42. The next state table of a 2-bit saturating up-counter is given below.

Q_1	Q_0	Q_1^+	Q_0^+
0	0	0	1
0	1	1	0
1	0	1	1
1	1	1	1

The counter is built as a synchronous sequential circuit using T flip-flops. The expression for T_1 and T_0 are

(A) $T_1 = Q_1Q_0$, $T_0 = \overline{Q}_1\overline{Q}_0$

(B) $T_1 = \overline{Q}_1Q_0$, $T_0 = \overline{Q}_1 + \overline{Q}_0$

(C) $T_1 = Q_1 + Q_0$, $T_0 = \overline{Q}_1 + \overline{Q}_0$

(D) $T_1 = Q_1Q_0$, $T_0 = \overline{Q}_1 + \overline{Q}_0$

Answer: (B)

43. Consider the set of processes with arrival time (in milliseconds). CPU burst time (in milliseconds), and priority (0 is the highest priority) shown below. None of the processes have I/O burst time.

Process	Arrival Time	Burst Time	Priority
P_1	0	11	2
P_2	5	28	0
P_3	12	2	3
P_4	2	10	1
P_5	9	16	4

The average waiting time (in milliseconds) of all the processes using preemptive priority scheduling algorithm is __________

Answer: (29)

44. For any discrete random variable X, with probability mass function

$P(X=j)=p_j, p_j \geq 0, j \in \{0,.....N\}$ and $\sum_{j=0}^{N} p_j = 1$, define the polynomial function

$g_x(z)=\sum_{j=0}^{N} p_j z^j$ For a certain discrete random variable Y, there exists a scalar $\beta \in [0,1]$ such that

$g_Y(z)=(1-\beta+\beta z)^N$. The expectation of Y is

(A) $N\beta(1-\beta)$

(B) $N\beta$

(C) $N(1-\beta)$

(D) Not expressible in terms of N and β alone

Answer: (B)

45. The read access times and the hit ratios for different caches in a memory hierarchy are as given below.

Cache	Read access time (in nanoseconds)	Hit ratio
I-cache	2	0.8
D-cache	2	0.9
L2-cache	8	0.9

The read access time of main memory is 90 nanoseconds. Assume that the caches use the referred word-first read policy and the write back policy. Assume that all the caches are direct mapped caches. Assume that the dirty bit is always 0 for all the blocks in the caches. In execution of a program, 60% of memory reads are for instruction fetch and 40% are for memory operand fetch. The average read access time in nanoseconds (up to 2 decimal places) is__________.

Answer: (4.72)

46. If the ordinary generating function of a sequence $\{a_n\}_{n=0}^{\infty}$ is $\dfrac{1+z}{(1-z)^3}$, then $a_3 - a_0$ is equal to ______.

Answer: (15)

47. Consider the following snippet of a C program. Assume that swap (&x, &y) exchanges the contents of x and y.

```
int main ( )    {
int array[]={3,5,1,4,6,2};
```

```
int done =0 ;
inti ;
while (done = = 0)   {
     done = 1;
     for (i = 0; i<=4; i ++)    {
     if (array [i] < array [i +1])    {
     swap (& array [i], &array [i+1]);
          done = 0;
     }
}
for (i = 5 ;i> =1; i --)    {
if (array [i] >array [ i-1])    {
swap ( & array [i] , &array [i-1]);
done = 0;
}
     }
}
printf( " %d " , array [3] );
}
```

The output of the program is ____________.

Answer: **(3)**

48. Consider the following C program.

```
# include <stdio.h>
int main ( ){
     int m = 10;
     int n, n1;
     n = ++m;
     n1 = m++;
n--;
     --n1;
     n -=n1;
printf ("%d", n) ;
     return 0;
```

}

The output of the program is ______________.

Answer: **(0)**

49. Consider the following database table named *top _scorer*.

top _scorer.

Player	Country	Goals
Klose	Germany	16
Ronald	Brazil	15
G Muller	Germany	14
Fontaine	France	13
Pele	Brazil	12
Klinsmann	Germany	11
Kocsis	Hungary	11
Batistuta	Argentina	10
Cubillas	Peru	10
Lato	Poland	10
Lineker	England	10
T Miller	Germany	10
Rahn	Germany	10

Consider the following SQL query:

```
SELECTta.playerFROM top _scorer AS ta
WHERE ta.goals> ALL (SELECT tb. goals
```

```
                                        FROM top _ scorer AS tb
                                        WHERE tb.country = 'Spain')
AND ta.goals> ANY ( SELECT tc. goals
                         FROM top_ scorer AS tc
                         WHERE tc.country = 'Germany')
```

The number of tuples returned by the above SQL query is ____________.

Answer: (7)

50. Given f (w,x,y,z) $=\sum_m$ (0,1,2,3,7,8,10) $+\sum_d$ (5,6,11,15), where *d* represents the don't care condition in Karnaugh maps. Which of the following is a minimum product-of-sums (POS) form of f(w,x,y,z) ?

(A) $f=(\overline{w}+\overline{z})(\overline{x}+z)$

(B) $f=(\overline{w}+z)(x+z)$

(C) $f=(w+z)(\overline{x}+z)$

(D) $f=(w+\overline{z})(\overline{x}+z)$

Answer: (A)

51. In a B^+ tree, if the search –key value is 8 bytes long, the block size is 512bytes and the block pointer size is 2 bytes, then maximum order of the B^+ tree is ________________.

Answer: (52)

52. Let L(R) be the language represented by regular expression R. Let L(G) be the language generated by a context free grammar G. Let L (M) be the language accepted by a Turning machine M. Which of the following decision problems are undecidable ?

I. Given a regular expression R and a string w, is $w \in L(R)$?

II. Given a context-free grammar G, $L(G)=\varnothing$?

III. Given a context-free grammar G, is $L(G)=\Sigma^*$ for some alphabet Σ ?

IV. Given a Turning machine M and a string w, is $w \in L(M)$?

(A) I and IV only (B) II and III only (C) II, III and IV only (D) III and IV only

Answer: (D)

53. Consider a machine with a byte addressable main memory of 2^{32} bytes divided into blocks of size 32 bytes. Assume that a direct mapped cache having 512 cache lines is used with this machine. The size of the tag field in bits is _______.

Answer: (18)

54. Let δ denote that transition function and $\hat{\delta}$ denote the extended transition function of the $\in -$NFA whose transition table is given below:

δ	$\in$	a	b
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	$\varnothing$	$\varnothing$
q_3	$\varnothing$	$\varnothing$	$\{q_2\}$

Then $\hat{\delta}(q_2, aba)$ is

(A) $\varnothing$ (B) $\{q_0, q_1, q_3\}$ (C) $\{q_0, q_1, q_2\}$ (D) $\{q_0, q_2, q_3\}$

Answer: (C)

55. Consider the following languages.

$L_1 = \{a^p \mid p \text{ is a prime number}\}$

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$

$L_3 = \{a^n b^n c^{2n} \geq 0\}$

$L_4 = \{a^n b^n \mid n \geq 1\}$

Which of the following are CORRECT ?

I. L_1 is context-free but not regular.

II. L_2 is not context-free.

III. L_3 is not context-free but recursive.

IV. L_4 is deterministic context-free.

(A) I ,II and IV only (B) II and III only (C) I and IV only (D) III and IV only

Answer: (D)

GENERAL APTITUDE

Q. No. 1 – 5 Carry One Mark Each

1. Out of the following four sentences, select the most suitable sentence with respect to grammar and usage.

(A) I will not leave the place until the minister does not meet me.

(B) I will not leave the place until the minister doesn't meet me.

(C) I will not leave the place until the minister meet me.

(D) I will not leave the place until the minister meets me.

Answer: (D)

2. A rewording of something written or spoken is a ___________.

(A) paraphrase (B) paradox (C) paradigm (D) paraffin

Answer: (A)

3. Archimseedes said, "Give me a lever long enough and a fulcrum on which to place it, and I will move the world."

The sentence above is an example of a ________ statement.

(A) figurative (B) collateral (C) literal (D) figurine

Answer: (A)

4. If 'relftaga' means carefree, 'otaga' means careful and 'fertaga' means careless, which of the following could mean 'aftercare'?

(A) zentaga (B) tagafer (C) tagazen (D) relffer

Answer: (C)

5. A cube is built using 64 cubic blocks of side one unit. After it is built, one cubic block is removed from every corner of the cube. The resulting surface area of the body (in square units) after the removal is.

(A) 56 (B) 64 (C) 72 (D) 96

Answer: (D)

Q. No. 6 – 10 Carry Two Marks Each

6. A shaving set company sells 4 different types of razors, Elegance, Smooth, Soft and Executive. Elegance sells at Rs. 48, Smooth at Rs. 63, Soft at Rs. 78 and Executive at Rs. 173 per piece. The table below shows the numbers of each razor sold in each quarter of a year.

Quarter/Product	Elegance	Smooth	Soft	Executive
Q1	27300	20009	17602	9999
Q2	25222	19392	18445	8942
Q3	28976	22429	19544	10234
Q4	21012	18229	16595	10109

Which product contributes the greatest fraction to the revenue of the company in that year?

(A) Elegance (B) Executive (C) Smooth (D) Soft

Answer: (B)

7. Indian currency notes show the denomination indicated in at least seventeen languages. If this is not an indication of the nation's diversity, nothing else is.

Which of the following can be logically inferred from the above sentences?

(A) India is a country of exactly seventeen languages.

(B) Linguistic pluralism is the only indicator of a nation's diversity.

(C) Indian currency notes have sufficient space for all the Indian languages.

(D) Linguistic pluralism is strong evidence of India's diversity.

Answer: (D)

8. Consider the following statements relating to the level of poker play of four players P, Q, R and S.

I. P always beats Q

II. R always beats S

III. S loses to P only sometimes

IV. R always loses to Q

Which of the following can be logically inferred from the above statements?

(i) P is likely to beat all the three other players

(ii) S is the absolute worst player in the set

(A) (i) only (B) (ii) only (C) (i) and (ii) (D) neither (i) nor (ii)

Answer: (D)

9. If $f(x) = 2x^7 + 3x - 5$, which of the following is a factor of f(x)?

(A) (x^3+8) (B) (x-1) (C) (2x-5) (D) (x+1)

Answer: (B)

10. In a process, the number of cycles to failure decreases exponentially with an increase in load. At a load of 80 units, it takes 100 cycles for failure. When the load is halved, it takes 10000 cycles for failure. The load for which the failure will happen in 5000 cycles is ________.

(A) 40.00 (B) 46.02 (C) 60.01 (D) 92.02

Answer: (B)

Computer Science Engineering

Q. No. 1 – 25 Carry One Mark Each

1. Let p, q, r, s represent the following propositions.

p: $x \in \{8, 9, 10, 11, 12\}$

q: x is a composite number

r: x is a perfect square

s: x is a prime number

The integer $x \geq 2$ which satisfies $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is _________.

Answer: (11)

2. Let a_n be the number of n-bit strings that do NOT contain two consecutive 1s. Which one of the following is the recurrence relation for a_n?

(A) $a_n = a_{n-1} + 2a_{n-2}$

(B) $a_n = a_{n-1} + 2a_{n-2}$

(C) $a_n = a_{n-1} + 2a_{n-2}$

(D) $a_n = a_{n-1} + 2a_{n-2}$

Answer: (B)

3. $\lim\limits_{x \to 4} \frac{\sin(x-4)}{x-4} =$__________.

Answer: (1)

4. A probability density function on the interval [a, 1] is given by $1/x^2$ and outside this interval the value of the function is zero. The value of a is___________.

Answer: (0.5)

5. Two eigen values of a 3 × 3 real matrix P are $\left(2+\sqrt{-1}\right)$ and 3. The determinant of P is _________.

Answer: (15)

6. Consider the Boolean operator # with the following properties:

$x \# 0 = x$, $x \# 1 = \overline{x}$, $x \# x = 0$ and $x \# \overline{x} = 1$. Then $x \# y$ is equivalent to

(A) $x\overline{y} + \overline{x}y$ (B) $x\overline{y} + \overline{x}\,\overline{y}$ (C) $\overline{x}y + xy$ (D) $xy + \overline{x}\,\overline{y}$

Answer: (A)

7. The 16-bit 2's complement representation of an integer is 1111 1111 1111 0101; its decimal representation is _________.

Answer: (–11)

8. We want to design a synchronous counter that counts the sequence 0-1-0-2-0-3 and then repeats. The minimum number of J-K flip-flops required to implement this counter is ________.

Answer: (3 - 4)

9. A processor can support a maximum memory of 4 GB, where the memory is word-addressable (a word consists of two bytes). The size of the address bus of the processor is at least __________ bits.

Answer: (31)

10. A queue is implemented using an array such that ENQUEUE and DEQUEUE operations are performed efficiently. Which one of the following statements is **CORRECT** (n refers to the number of items in the queue)?

(A) Both operations can be performed in $O(1)$ time

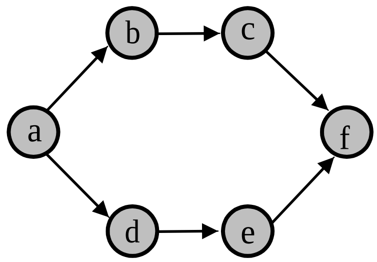
(B) At most one operation can be performed in $O(1)$ time but the worst case time for the other operation will be $\Omega(n)$

(C) The worst case time complexity for both operations will be $\Omega(n)$

(D) Worst case time complexity for both operations will be $\Omega(\log n)$

Answer: (A)

11. Consider the following directed graph:



The number of different topological orderings of the vertices of the graph is __________.

Answer: (6)

12. Consider the following C program.

```
void f(int, short);
void main()  {
int i = 100;
short s = 12;
short *p  = &s;
____________;    // call to f()
}
```

Which one of the following expressions, when placed in the blank above, will NOT result in a type checking error?

(A) `f(s,*s)` (B) `i = f(i,s)` (C) `f(i,*s)` (D) `f(i,*p)`

Answer: (D)

13. The worst case running times of Insertion sort, Merge sort and Quick sort, respectively, are:

(A) $\Theta(n \log n)$, $\Theta(n \log n)$, and $\Theta(n^2)$

(B) $\Theta(n^2)$, $\Theta(n^2)$, and $\Theta(n \log n)$

(C) $\Theta(n^2)$, $\Theta(n \log n)$, and $\Theta(n \log n)$

(D) $\Theta(n^2)$, $\Theta(n \log n)$, and $\Theta(n^2)$

Answer: (D)

14. Let G be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are TRUE?

P: Minimum spanning tree of G does not change

Q: Shortest path between any pair of vertices does not change

(A) P only (B) Q only (C) Neither P nor Q (D) Both P and Q

Answer: (A)

15. Consider the following C program.

```
#include<stdio.h>
void mystery(int *ptra, int *ptrb)
{
int *temp;
temp = ptrb;
ptrb = ptra;
ptra = temp;
}
int main() {
int a=2016, b=0, c=4, d=42;
mystery(&a, &b);
if (a < c)
mystery(&c, &a);
mystery(&a, &d);
printf("%d\n", a);
}
```

The output of the program is ______.

Answer: (2016)

16. Which of the following languages is generated by the given grammar?

$S \rightarrow aS|bS|\varepsilon$

(A) $\{a^n b^m \mid n, m \geq 0\}$

(B) $\{w \in \{a, b\}^* \mid w \text{ has equal number of } a\text{'s and } b\text{'s}\}$

(C) $\{a^n \mid n \geq 0\} \cup \{b^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$

(D) $\{a, b\}*$

Answer: (D)

17. Which of the following decision problems are undecidable?

I. Given NFAs N_1 and N_2, is $L(N_1) \cap L(N_2) = \Phi$?

II. Given a CFG $G = (N, \Sigma, P, S)$ and a string $x \in \Sigma*$, does $x \in L(G)$?

III. Given CFGs G1 and G2, is L(G1) = L(G2)?

IV. Given a TM M, is $L(M) = \Phi$?

(A) I and IV only (B) II and III only (C) III and IV only (D) II and IV only

Answer: (C)

18. Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0s and two consecutive 1s?

(A) $(0 + 1)*0011(0 + 1)* + (0 + 1)*1100(0 + 1)*$

(B) $(0 + 1)*(00(0 + 1)*11 + 11(0 + 1)*00)(0 + 1)*$

(C) $(0 + 1)*00(0 + 1)* + (0 + 1)*11(0 + 1)*$

(D) $00(0 + 1)*11 + 11(0 + 1)*00$

Answer: (B)

19. Consider the following code segment.

```
x = u - t;
y = x * v;
x = y + w;
y = t - z;
y = x  *  y;
```

The minimum number of total variables required to convert the above code segment to static single assignment form is _________.

Answer: (10)

20. Consider an arbitrary set of CPU-bound processes with unequal CPU burst lengths submitted at the same time to a computer system. Which one of the following process scheduling algorithms would minimize the average waiting time in the ready queue?

(A) Shortest remaining time first

(B) Round-robin with time quantum less than the shortest CPU burst

(C) Uniform random

(D) Highest priority first with priority proportional to CPU burst length

Answer: (A)

21. Which of the following is NOT a super key in a relational schema with attributes V, W, X, Y, Z and primary key V Y ?

(A) V XY Z (B) V W X Z (C) V W XY (D) V W XY Z

Answer: (B)

22. Which one of the following is NOT a part of the ACID properties of database transactions?

(A) Atomicity (B) Consistency (C) Isolation (D) Deadlock-freedom

Answer: (D)

23. A database of research articles in a journal uses the following schema. (VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE)

The primary key is (VOLUME, NUMBER, STARTPAGE, ENDPAGE) and the following functional dependencies exist in the schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) → TITLE

(VOLUME, NUMBER → YEAR

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) → PRICE

The database is redesigned to use the following schemas.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, PRICE) (VOLUME, NUMBER, YEAR)

Which is the weakest normal form that the new database satisfies, but the old one does not?

(A) 1NF (B) 2NF (C) 3NF (D) BCNF

Answer: (B)

24. Which one of the following protocols is NOT used to resolve one form of address to another one?

(A) DNS (B) ARP (C) DHCP (D) RARP

Answer: (C)

25. Which of the following is/are example(s) of stateful application layer protocols?

(i) HTTP (ii) FTP (iii) TCP (iv) POP3

(A) (i) and (ii) only

(B) (ii) and (iii) only

(C) (ii) and (iv) only

(D) (iv) only

Answer: (C)

Q. No. 26 – 55 Carry Two Marks Each

26. The coefficient of x^{12} in $(x^3 + x^4 + x^5 + x^6 + ...)^3$ is ____________.

Answer: **(10)**

27. Consider the recurrence relation $a_1 = 8, a_n = 6n^2 + 2n + a_{n-1}$. Let $a_{99} = K \times 10^4$. The value of K is __________.

Answer: **(198)**

28. A function $f : \mathbb{N}^+ \to \mathbb{N}^+$ defined on the set of positive integers N^+, satisfies the following properties

f (n) = f (n/2) if n is even

f (n) = f (n + 5) if n is odd

Let R = {i | ∃ j: f (j) = i} be the set of distinct values that f takes. The maximum possible size of R is ____________.

Answer: **(2)**

29. Consider the following experiment.

Step 1. Flip a fair coin twice.

Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.

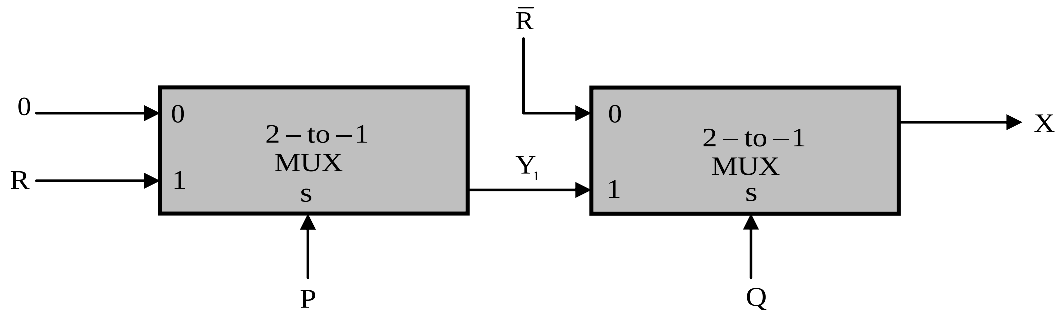
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is (up to two decimal places) __________.

Answer: **(0.33)**

30. Consider the two cascaded 2-to-1 multiplexers as shown in the figure.



The minimal sum of products form of the output X is

(A) $\bar{P}\bar{Q} + PQR$ (B) $\bar{P}Q + QR$ (C) $PQ + \bar{P}\bar{Q}R$ (D) $\bar{Q}\bar{R} + PQR$

Answer: (D)

31. The size of the data count register of a DMA controller is 16 bits. The processor needs to transfer a file of 29,154 kilobytes from disk to main memory. The memory is byte addressable. The minimum number of times the DMA controller needs to get the control of the system bus from the processor to transfer the file from the disk to main memory is __________.

Answer: (456)

32. The stage delays in a 4-stage pipeline are 800, 500, 400 and 300 picoseconds. The first stage (with delay 800 picoseconds) is replaced with a functionally equivalent design involving two stages with respective delays 600 and 350 picoseconds. The throughput increase of the pipeline is _________ percent.

Answer: (33.33)

33. Consider a carry look ahead adder for adding two n-bit integers, built using gates of fan-in at most two. The time to perform addition using this adder is

(A) $\Theta(1)$ (B) $\Theta(\log(n))$ (C) $\Theta(\sqrt{n})$ (D) $\Theta(n)$

Answer: (B)

34. The following function computes the maximum value contained in an integer array p[ ] of size n (n >= 1).

```
int max(int *p, int n) {
int a=0, b=n-1;
while ( __________ )     {
if (p[a] <= p[b]) {  a = a+1; }
else                    {  b  = b-1; }
}
return p[a];
}
```

The missing loop condition is

(A) `a != n` (B) `b != 0` (C) `b > (a + 1)` (D) `b != a`

Answer: (D)

35. What will be the output of the following C program?

```
void count(int n){
static int d=1;
printf("%d  ", n); printf("%d  ", d); d++;
if(n>1) count(n-1);
printf("%d  ", d);
}
void main(){
count(3);
}
```

(A) 3 1 2 2 1 3 4 4 4 (B) 3 1 2 1 1 1 2 2 2

(C) 3 1 2 2 1 3 4 (D) 3 1 2 1 1 1 2

Answer: (A)

36. What will be the output of the following pseudo-code when parameters are passed by reference and dynamic scoping is assumed?

```
a=3;
void n(x) {x  = x  *  a; print(x);}
void m(y) {a = 1; a = y  - a; n(a); print(a);}
void main() {m(a);}
```

(A) 6, 2 (B) 6, 6 (C) 4, 2 (D) 4, 4

Answer: (D)

37. An operator delete (i) for a binary heap data structure is to be designed to delete the item in the i-th node. Assume that the heap is implemented in an array and i refers to the i-th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?

(A) O(1)

(B) O(d) but not O(1)

(C) O(2d) but not O(d)

(D) O(d 2d) but not O(2d)

Answer: (B)

38. Consider the weighted undirected graph with 4 vertices, where the weight of edge {i, j} is given by the entry W_{ij} in the matrix W.

$$W = \begin{bmatrix} 0 & 2 & 8 & 5 \\ 2 & 0 & 5 & 8 \\ 8 & 5 & 0 & x \\ 5 & 8 & x & 0 \end{bmatrix}$$

The largest possible integer value of x, for which at least one shortest path between some pair of vertices will contain the edge with weight x is __________.

Answer: (12)

39. Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being 1, 2, 3, 4, 5, and 6. The maximum possible weight that a minimum weight spanning tree of G can have is ___________.

Answer: (7)

40. G = (V, E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE?

I. If e is the lightest edge of some cycle in G, then every MST of G includes e

II. If e is the heaviest edge of some cycle in G, then every MST of G excludes e

(A) I only (B) II only (C) both I and II (D) neither I nor II

Answer: (B)

41. Let Q denote a queue containing sixteen numbers and S be an empty stack. Head(Q) returns the element at the head of the queue Q without removing it from Q. Similarly Top(S) returns the element at the top of S without removing it from S. Consider the algorithm given below.

```
while Q is not Empty do
    if S is Empty OR Top(S) ≤ Head(Q) then
        x := Dequeue(Q);
        Push(S, x);
    else
        x := Pop(S);
        Enqueue(Q, x);
    end
end
```

The maximum possible number of iterations of the while loop in the algorithm is __________.

Answer: (256)

42. Consider the following context-free grammars:

G1: $S \rightarrow aS|B,\ B \rightarrow b|bB$

G2: $S \rightarrow aA|bB,\ A \rightarrow aA|B|\varepsilon,\ B \rightarrow bB|\varepsilon$

Which one of the following pairs of languages is generated by G1 and G2, respectively?

(A) $\{a^mb^n|m > 0 \text{ or } n > 0\}$ and $\{a^mb^n|m > 0 \text{ and } n > 0\}$

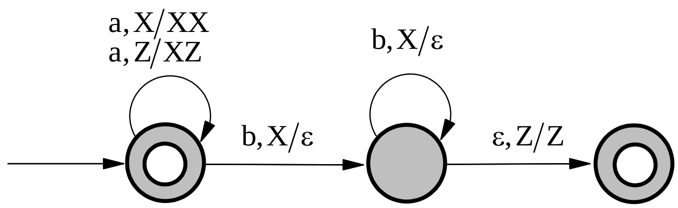
(B) $\{a^mb^n|m > 0 \text{ and } n > 0\}$ and $\{a^mb^n|m > 0 \text{ or } n \leq 0\}$

(C) $\{a^mb^n|m \geq 0 \text{ or } n > 0\}$ and $\{a^mb^n|m > 0 \text{ and } n > 0\}$

(D) $\{a^mb^n|m \geq 0 \text{ or } n > 0\}$ and $\{a^mb^n|m > 0 \text{ or } n > 0\}$

Answer: (D)

43. Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{X, Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.



Which one of the following is TRUE?

(A) $L = \{a^nb^n|n \geq 0\}$ and is not accepted by any finite automata

(B) $L = \{a^n|n \geq 0\} \cup \{a^nb^n|n \geq 0\}$ and is not accepted by any deterministic PDA

(C) L is not accepted by any Turing machine that halts on every input

(D) $L = \{a^n|n \geq 0\} \cup \{a^nb^n|n \geq 0\}$ and is deterministic context-free

Answer: (D)

44. Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that Y reduces to W, and Z reduces to X (reduction means the standard many-one reduction). Which one of the following statements is TRUE?

(A) W can be recursively enumerable and Z is recursive.

(B) W can be recursive and Z is recursively enumerable.

(C) W is not recursively enumerable and Z is recursive.

(D) W is not recursively enumerable and Z is not recursive.

Answer: (C)

45. The attributes of three arithmetic operators in some programming language are given below.

Operator	**Precedence**	**Associativity**	**Arity**
$+$	High	Left	Binary
$-$	Medium	Right	Binary
$*$	Low	Left	Binary

The value of the expression $2 - 5 + 1 - 7 * 3$ in this language is .

Answer: (9)

46. Consider the following Syntax Directed Translation Scheme (SDTS), with non-terminals {S, A} and terminals {a, b}.

$$S \rightarrow aA \ \{\text{print } 1\}$$
$$S \rightarrow a \quad \{\text{print } 2\}$$
$$A \rightarrow Sb \ \{\text{print } 3\}$$

Using the above SDTS, the output printed by a bottom-up parser, for the input aab is:

(A) 1 3 2 (B) 2 2 3 (C) 2 3 1 (D) syntax error

Answer: (C)

47. Consider a computer system with 40-bit virtual addressing and page size of sixteen kilobytes. If the computer system has a one-level page table per process and each page table entry requires 48 bits, then the size of the per-process page table is _________ megabytes.

Answer: (384)

48. Consider a disk queue with requests for I/O to blocks on cylinders 47, 38, 121, 191, 87, 11, 92, 10. The C-LOOK scheduling algorithm is used. The head is initially at cylinder number 63, moving towards larger cylinder numbers on its servicing pass. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is __________.

Answer: (346)

49. Consider a computer system with ten physical page frames. The system is provided with an access sequence ($a_1, a_2, \ldots, a_{20}, a_1, a_2, \ldots, a_{20}$), where each a_i is a distinct virtual page number. The difference in the number of page faults between the last-in-first-out page replacement policy and the optimal page replacement policy is __________.

Answer: (1)

50. Consider the following proposed solution for the critical section problem. There are n processes: $P_0 \ldots P_{n-1}$. In the code, function pmax returns an integer not smaller than any of its arguments. For all i, t[i] is initialized to zero.

```
Code for Pi:
do  {
c[i]=1; t[i] = pmax(t[0],. . .,t[n-1])+1;  c[i]=0;
for every j = i in {0,. . .,n-1}  {
while (c[j]);
    while (t[j] != 0  &&  t[j]<=t[i]);
    }
    Critical Section;
    t[i]=0;
    Remainder Section;
}   while (true);
```

Which one of the following is TRUE about the above solution?

(A) At most one process can be in the critical section at any time

(B) The bounded wait condition is satisfied

(C) The progress condition is satisfied

(D) It cannot cause a deadlock

Answer: (A)

51. Consider the following two phase locking protocol. Suppose a transaction T accesses (for read or write operations), a certain set of objects $\{O_1, \ldots, O_k\}$. This is done in the following manner:

Step 1: T acquires exclusive locks to O1, . . . , Ok in increasing order of their addresses.

Step 2: The required operations are performed.

Step 3: All locks are released.

This protocol will

(A) guarantee serializability and deadlock-freedom

(B) guarantee neither serializability nor deadlock-freedom

(C) guarantee serializability but not deadlock-freedom

(D) guarantee deadlock-freedom but not serializability

Answer: (A)

52. Consider that B wants to send a message m that is digitally signed to A. Let the pair of private and public keys for A and B be denoted by K_x^- and K_x^+ for x = A, B, respectively. Let $K_x(m)$ represent the operation of encrypting m with a key Kx and H (m) represent the message digest. Which one of the following indicates the CORRECT way of sending the message m along with the digital signature to A?

(A) $\{m, K_B^+(H(m))\}$ (B) $\{m, K_B^-(H(m))\}$

(C) $\{m, K_A^-(H(m))\}$ (D) $\{m, K_A^+(m)\}$

Answer: (B)

53. An IP datagram of size 1000 bytes arrives at a router. The router has to forward this packet on a link whose MTU (maximum transmission unit) is 100 bytes. Assume that the size of the IP header is 20 bytes.

The number of fragments that the IP datagram will be divided into for transmission is ________.

Answer: (13)

54. For a host machine that uses the token bucket algorithm for congestion control, the token bucket has a capacity of 1 megabyte and the maximum output rate is 20 megabytes per second. Tokens arrive at a rate to sustain output at a rate of 10 megabytes per second. The token bucket is currently full and the machine needs to send 12 megabytes of data. The minimum time required to transmit the data is _______ seconds.

Answer: (1.1)

55. A sender uses the Stop-and-Wait ARQ protocol for reliable transmission of frames. Frames are of size 1000 bytes and the transmission rate at the sender is 80 Kbps (1Kbps = 1000 bits/second). Size of an acknowledgement is 100 bytes and the transmission rate at the receiver is 8 Kbps. The one-way propagation delay is 100 milliseconds.

Assuming no frame is lost, the sender throughput is __________ bytes/second.

Answer: (2500)

GENERAL APTITUDE

Q. No. 1 – 5 Carry One Mark Each

1. The man who is now Municipal Commissioner worked as _____________.

(A) the security guard at a university

(B) a security guard at the university

(C) a security guard at university

(D) the security guard at the university

Answer: (B)

2. Nobody knows how the Indian cricket team is going to <u>cope with</u> the difficult and seamer-friendly wickets in Australia.

Choose the option which is closest in meaning to the underlined phase in the above sentence.

(A) put up with (B) put in with (C) put down to (D) put up against

Answer: (A)

3. Find the odd one in the following group of words.

Mock, deride, praise, jeer

(A) mock (B) deride (C) praise (D) jeer

Answer: (C)

4. Pick the odd one from the following options.

(A) CADBE (B) JHKIL (C) XVYWZ (D) ONPMQ

Answer: (D)

5. In a quadratic function, the value of the product of the roots (α, β) is 4. Find the value of $\dfrac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}}$

(A) n^4 (B) 4^n (C) 2^{2n-1} (D) 4^{n-1}

Answer: (B)

Q. No. 6 – 10 Carry Two Marks Each

6. Among 150 faculty members in an institute, 55 are connected with each other through Facebook and 85 are connected through WhatsApp. 30 faculty members do not have Facebook or WhatsApp accounts. The number of faculty members connected only through Facebook accounts is ___________.

(A) 35 (B) 45 (C) 65 (D) 90

Answer: (A)

7. Computers were invented for performing only high-end useful computations. However, it is no understatement that they have taken over our world today. The internet, for example, is ubiquitous. Many believe that the internet itself is an unintended consequence of the original invention with the advent of mobile computing on our phones, a whole new dimension is now enabled. One is left wondering if all these developments are good or more importantly, required.

Which of the statement(s) below is/are logically valid and can be inferred from the above paragraph?

(i) The author believes that computers are not good for us

(ii) Mobile computers and the internet are both intended inventions

(A) (i) only (B) (ii) only

(C) both (i) and (ii) (D) neither (i) nor (ii)

Answer: (D)

8. All hill-stations have a lake. Ooty has two lakes.

Which of the statement(s) below is/are logically valid and can be inferred from the above sentences?

(i) Ooty is not a hill-station

(ii) No hill-station can have more than one lake.

(A) (i) Only (B) (ii) Only

(C) Both (i) and (ii) (D) neither (i) nor (ii)

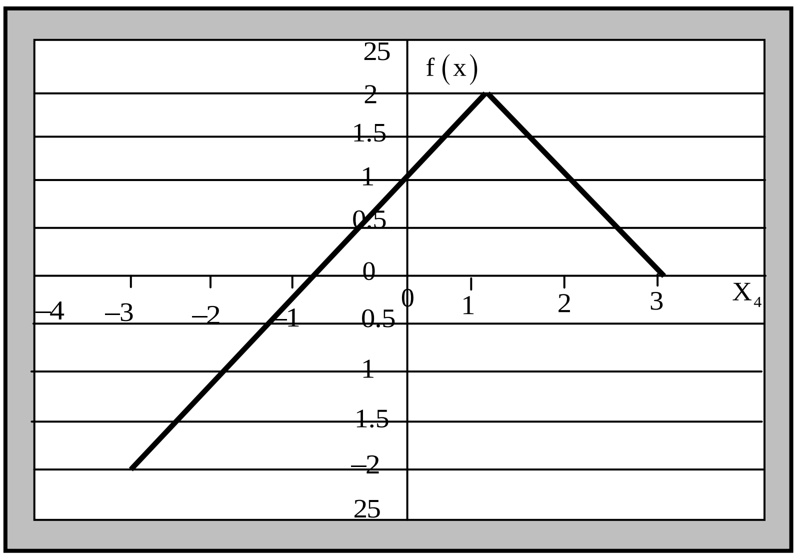
Answer: (D)

9. In a 2×4 rectangle grid shown below, each cell is a rectangle. How many rectangles can be observed in the grid?

(A) 21 (B) 27 (C) 30 (D) 36

Answer: (C)

10. Choose the correct expression for f(x) given in the graph.



(A) $f(x) = 1 - |x-1|$

(B) $f(x) = 1 + |x-1|$

(C) $f(x) = 2 - |x-1|$

(D) $f(x) = 2 + |x-1|$

Answer: (C)

COMPUTER SCIENCE ENGINEERING

Q. No. 1 – 25 Carry One Mark Each

1. Consider the following expressions:

(i) false **(ii)** Q **(iii)** true

(iv) $P \vee Q$ **(v)** $\neg Q \vee P$

The number of expressions given above that are logically implied by $P \wedge (P \Rightarrow Q)$ is________.

Answer: (4)

2. Let f (x) be a polynomial and g(x) = f '(x) be its derivative. If the degree of (f (x) + f (−x)) is 10, then the degree of (g(x) − g(−x)) is __________.

Answer: (9)

3. The minimum number of colours that is sufficient to vertex-colour any planar graph is _________.

Answer: (4)

4. Consider the systems, each consisting of m linear equations in n variables.

I. If m < n, then all such systems have a solution

II. If m > n, then none of these systems has a solution

III. If m = n, then there exists a system which has a solution

Which one of the following is CORRECT?

(A) I, II and III are true (B) Only II and III are true

(C) Only III is true (D) None of them is true

Answer: (C)

5. Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is __________.

Answer: (0.55)

6. Suppose that the eigen values of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is _________.

Answer: (0.125)

7. Consider an eight-bit ripple-carry adder for computing the sum of A and B, where A and B are integers represented in 2's complement form. If the decimal value of A is one, the decimal value of B that leads to the longest latency for the sum to stabilize is __________.

Answer: (-1)

8. Let, $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$ where x_1, x_2, x_3, x_4 are Boolean variables, and $\oplus$ is the XOR operator.

Which one of the following must always be TRUE?

(A) $x_1x_2x_3x_4 = 0$

(B) $x_1x_3 + x_2 = 0$

(C) $\bar{x}_1 \oplus \bar{x}_3 = \bar{x}_2 \oplus \bar{x}_4$

(D) $x_1 + x_2 + x_3 + x_4 = 0$

Answer: (C)

9. Let X be the number of distinct 16-bit integers in 2's complement representation. Let Y be the number of distinct 16-bit integers in sign magnitude representation.

Then $X - Y$ is __________.

Answer: (1)

10. A processor has 40 distinct instructions and 24 general purpose registers. A 32-bit instruction word has an opcode, two register operands and an immediate operand. The number of bits available for the immediate operand field is _________.

Answer: (16)

11. Breadth First Search (BFS) is started on a binary tree beginning from the root vertex. There is a vertex t at a distance four from the root. If t is the n-th vertex in this BFS traversal, then the maximum possible value of n is _________.

Answer: (31)

12. The value printed by the following program is __________.

```
void f(int* p, int m){
    m = m + 5;
    *p = *p + m;
    return;
}
void main(){
    int i=5, j=10;
    f(&i, j);
    printf("%d", i+j);
}
```

Answer: (30)

13. Assume that the algorithms considered here sort the input sequences in ascending order. If the input is already in ascending order, which of the following are TRUE?

I. Quick sort runs in $\Theta(n^2)$ time

II. Bubble sort runs in $\Theta(n^2)$ time

III. Mergesort runs in $\Theta(n)$ time

IV. Insertion sort runs in $\Theta(n)$ time

(A) I and II only
(B) I and III only
(C) II and IV only
(D) I and IV only

Answer: (D)

14. The Floyd-Warshall algorithm for all-pair shortest paths computation is based on

(A) Greedy paradigm
(B) Divide-and-Conquer paradigm.
(C) Dynamic Programming paradigm.
(D) Neither Greedy nor Divide-and-Conquer nor Dynamic Programming paradigm

Answer: (C)

15. N items are stored in a sorted doubly linked list. For a delete operation, a pointer is provided to the record to be deleted. For a decrease-key operation, a pointer is provided to the record on which the operation is to be performed.

An algorithm performs the following operations on the list in this order: $\Theta(N)$ delete, $O(\log N)$ insert, $O(\log N)$ find, and $\Theta(N)$ decrease-key. What is the time complexity of all these operations put together?

(A) $O(\log^2 N)$ (B) $O(N)$ (C) $O(N^2)$ (D) $\Theta(N^2 \log N)$

Answer: (C)

16. The number of states in the minimum sized DFA that accepts the language defined by the regular expression

$(0 + 1)*(0 + 1)(0 + 1)*$ is _________.

Answer: (2)

17. Language L_1 is defined by the grammar: $S_1 \rightarrow aS_1b|\varepsilon$

Language L_2 is defined by the grammar: $S_2 \rightarrow abS_2|\varepsilon$

Consider the following statements:

P: L_1 is regular

Q: L_2 is regular

Which one of the following is TRUE?

(A) Both P and Q are true (B) P is true and Q is false

(C) P is false and Q is true (D) Both P and Q are false

Answer: (C)

18. Consider the following types of languages: L_1: Regular, L_2: Context-free, L_3: Recursive, L_4: Recursively enumerable. Which of the following is/are TRUE?

I. $\bar{L}_3 \cup L_4$ is recursively enumerable

II. $\bar{L}_2 \cup L_3$ is recursive

III. $L_1^* \cap L_2$ is context-free

IV. $L_1 \cup \bar{L}_2$ is context-free

(A) I only (B) I and III only (C) I and IV only (D) I, II and III only

Answer: (D)

19. Match the following:

(P)	Lexical analysis	(i)	Leftmost derivation
(Q)	Top down parsing	(ii)	Type checking
(R)	Semantic analysis	(iii)	Regular expressions
(S)	Runtime environments	(iv)	Activation records

(A) P ↔ i, Q ↔ ii, R ↔ iv, S ↔ iii
(B) P ↔ iii, Q ↔ i, R ↔ ii, S ↔ iv
(C) P ↔ ii, Q ↔ iii, R ↔ i, S ↔ iv
(D) P ↔ iv, Q ↔ i, R ↔ ii, S ↔ iii

Answer: (B)

20. In which one of the following page replacement algorithms it is possible for the page fault rate to increase even when the number of allocated frames increases?

(A) LRU (Least Recently Used)
(B) OPT (Optimal Page Replacement)
(C) MRU (Most Recently Used)
(D) FIFO (First In First Out)

Answer: (D)

21. B+ Trees are considered BALANCED because

(A) the lengths of the paths from the root to all leaf nodes are all equal.
(B) the lengths of the paths from the root to all leaf nodes differ from each other by at most 1.
(C) the number of children of any two non-leaf sibling nodes differ by at most 1.
(D) the number of records in any two leaf nodes differ by at most 1.

Answer: (A)

22. Suppose a database schedule S involves transactions $T_1, \ldots, T_n$. Construct the precedence graph of S with vertices representing the transactions and edges representing the conflicts. If S is serializable, which one of the following orderings of the vertices of the precedence graph is guaranteed to yield a serial schedule?

(A) Topological order
(B) Depth-first order
(C) Breadth-first order
(D) Ascending order of transaction indices

Answer: (A)

23. Anarkali digitally signs a message and sends it to Salim. Verification of the signature by Salim requires

(A) Anarkali's public key (B) Salim's public key

(C) Salim's private key (D) Anarkali's private key

Answer: (A)

24. In an Ethernet local area network, which one of the following statements is TRUE?

(A) A station stops to sense the channel once it starts transmitting a frame.

(B) The purpose of the jamming signal is to pad the frames that are smaller than the minimum frame size.

(C) A station continues to transmit the packet even after the collision is detected.

(D) The exponential backoff mechanism reduces the probability of collision on retransmissions.

Answer: (D)

25. Identify the correct sequence in which the following packets are transmitted on the network by a host when a browser requests a webpage from a remote server, assuming that the host has just been restarted.

(A) HTTP GET request, DNS query, TCP SYN

(B) DNS query, HTTP GET request, TCP SYN

(C) DNS query, TCP SYN, HTTP GET request

(D) TCP SYN, DNS query, HTTP GET request

Answer: (C)

Q. No. 26 – 55 Carry Two Marks Each

26. A binary relation R on N × N is defined as follows: (a, b)R(c, d) if $a \le c$ or $b \le d$. Consider the following propositions:

P: R is reflexive

Q: R is transitive

Which one of the following statements is TRUE?

(A) Both P and Q are true (B) P is true and Q is false

(C) P is false and Q is true (D) Both P and Q are false

Answer: (B)

27. Which one of the following well-formed formulae in predicate calculus is NOT valid?

(A) $(\forall x\, p(x) \Rightarrow \forall x\, q(x)) \Rightarrow (\exists x \neg p(x) \vee \forall x\, q(x))$

(B) $(\exists x\, p(x) \vee \exists x\, q(x)) \Rightarrow \exists x\, (p(x) \vee q(x))$

(C) $\exists x\, (p(x) \wedge q(x)) \Rightarrow (\exists x\, p(x) \wedge \exists x\, q(x))$

(D) $\forall x\, (p(x) \vee q(x)) \Rightarrow (\forall x\, p(x) \vee \forall x\, q(x))$

Answer: (D)

28. Consider a set U of 23 different compounds in a Chemistry lab. There is a subset S of U of 9 compounds, each of which reacts with exactly 3 compounds of U. Consider the following statements:

I. Each compound in U \ S reacts with an odd number of compounds.

II. At least one compound in U \ S reacts with an odd number of compounds.

III. Each compound in U \ S reacts with an even number of compounds.

Which one of the above statements is ALWAYS TRUE?

(A) Only I (B) Only II (C) Only III (D) None of these

Answer: (B)

29. The value of the expression 13^{99}(mod 17), in the range 0 to 16, is __________.

Answer: (4)

30. Suppose the functions F and G can be computed in 5 and 3 nanoseconds by functional units U_F and U_G, respectively. Given two instances of U_F and two instances of U_G, it is required to implement the computation F $(G(X_i))$ for $1 \leq i \leq 10$. Ignoring all other delays, the minimum time required to complete this computation is _________ nanoseconds.

Answer: (28)

31. Consider a processor with 64 registers and an instruction set of size twelve. Each instruction has five distinct fields, namely, opcode, two source register identifiers, one destination register identifier, and a twelve-bit immediate value. Each instruction must be stored in memory in a byte-aligned fashion. If a program has 100 instructions, the amount of memory (in bytes) consumed by the program text is _________.

Answer: (500)

32. The width of the physical address on a machine is 40 bits. The width of the tag field in a 512 KB 8-way set associative cache is _________ bits.

Answer: (24)

33. Consider a 3 GHz (gigahertz) processor with a three-stage pipeline and stage latencies τ_1, τ_2, and τ_3 such that $\tau_1 = 3\tau_2/4 = 2\tau_3$. If the longest pipeline stage is split into two pipeline stages of equal latency, the new frequency is ________ GHz, ignoring delays in the pipeline registers.

Answer: (4)

34. A complete binary min-heap is made by including each integer in [1, 1023] exactly once.

The depth of a node in the heap is the length of the path from the root of the heap to that node.

Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is _________.

Answer: (8)

35. The following function computes XY for positive integers X and Y.

```
int exp(int X,  int Y)
{
int res = 1, a = X,  b  = Y;
while ( b  != 0  ){
if ( b%2 == 0) {  a = a*a; b  = b/2; }
else           {  res = res*a; b  = b-1; }
}
return res;
}
```

Which one of the following conditions is TRUE before every iteration of the loop?

(A) $X^Y = a^b$ (B) $(res * a)^Y = (res * X)^b$

(C) $X^Y = res * a^b$ (D) $X^Y = (res * a)^b$

Answer: (C)

36. Consider the following New-order strategy for traversing a binary tree:

- Visit the root;
- Visit the right subtree using New-order;
- Visit the left subtree using New-order;

The New-order traversal of the expression tree corresponding to the reverse polish expression

3 4 * 5 − 2 ˆ 6 7 * 1 + − is given by:

(A) + − 1 6 7 * 2 ˆ 5 − 3 4 *

(B) − + 1 * 6 7 ˆ 2 − 5 * 3 4

(C) − + 1 * 7 6 ˆ 2 − 5 * 4 3

(D) 1 7 6 * + 2 5 4 3 * − ˆ −

Answer: (C)

37. Consider the following program:

```
int f(int *p, int n)
    {
```

```
    if (n <= 1) return 0;
    else return max(f(p+1,n-1),p[0]-p[1]);
}
int main()
{
    int a[ ] = {3,5,2,6,4};
    printf("%d", f(a,5));
    }
```

Note: max(x,y) returns the maximum of x and y.

The value printed by this program is __________.

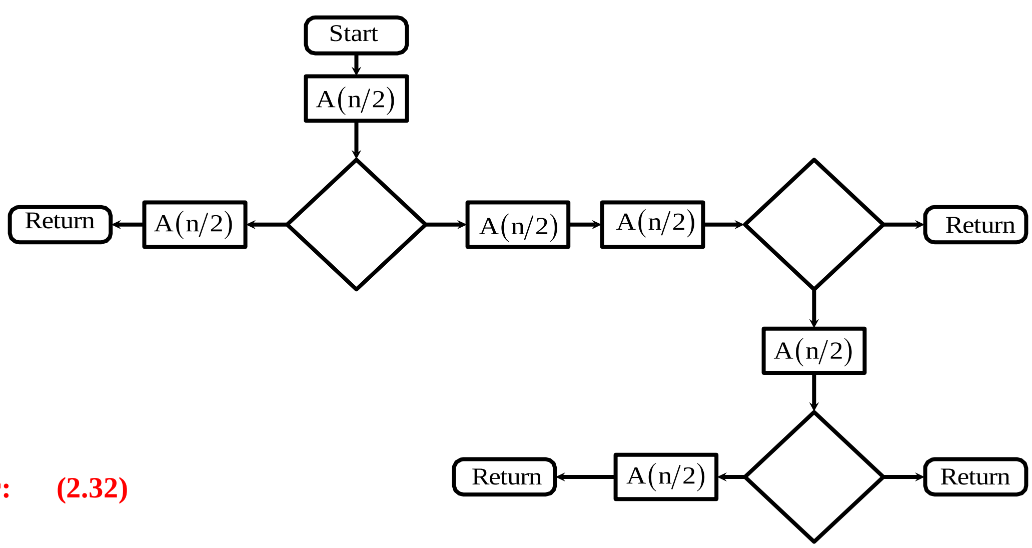
Answer: (3)

38. Let A_1, A_2, A_3, and A_4 be four matrices of dimensions 10×5, 5×20, 20×10, and 10×5, respectively. The minimum number of scalar multiplications required to find the product $A_1 A_2 A_3 A_4$ using the basic matrix multiplication method is __________.

Answer: (1500)

39. The given diagram shows the flowchart for a recursive function A(n). Assume that all statements, except for the recursive calls, have O(1) time complexity. If the worst case time complexity of this function is $O(n^{\alpha})$, then the least possible value (accurate up to two decimal positions) of α is _________.

Flowchart for Recursive Function A(n)



Answer: (2.32)

40. The number of ways in which the numbers 1, 2, 3, 4, 5, 6, 7 can be inserted in an empty binary search tree, such that the resulting tree has height 6, is _________.

Note: The height of a tree with a single node is 0.

Answer: (64)

41. In an adjacency list representation of an undirected simple graph G = (V, E), each edge (u, v) has two adjacency list entries: [v] in the adjacency list of u, and [u] in the adjacency list of v. These are called twins of each other. A twin pointer is a pointer from an adjacency list entry to its twin. If |E | = m and |V | = n, and the memory size is not a constraint, what is the time complexity of the most efficient algorithm to set the twin pointer in each entry in each adjacency list?

(A) $\Theta(n^2)$ (B) $\Theta(n+m)$ (C) $\Theta(m^2)$ (D) $\Theta(n^4)$

Answer: (B)

42. Consider the following two statements:

I. If all states of an NFA are accepting states then the language accepted by the NFA is Σ^*.

II. There exists a regular language A such that for all languages B, $A \cap B$ is regular. Which one of the following is CORRECT?

(A) Only I is true (B) Only II is true

(C) Both I and II are true (D) Both I and II are false

Answer: (B)

43. Consider the following languages:

$L_1 = \{a^n b^m c^{n+m}: m, n \geq 1\}$

$L_2 = \{a^n b^n c^{2n}: n \geq 1\}$

Which one of the following is TRUE?

(A) Both L_1 and L_2 are context-free.

(B) L_1 is context-free while L_2 is not context-free.

(C) L_2 is context-free while L_1 is not context-free.

(D) Neither L_1 nor L_2 is context-free.

Answer: (B)

44. Consider the following languages.

$L_1 = \{\langle M \rangle \mid M$ takes at least 2016 steps on some input$\}$,

$L_2 = \{\langle M \rangle \mid M$ takes at least 2016 steps on all inputs$\}$ and

$L_3 = \{\langle M \rangle \mid M$ accepts $\varepsilon\}$,

where for each Turing machine M, $\langle M \rangle$ denotes a specific encoding of M. Which one of the following is TRUE?

(A) L_1 is recursive and L_2, L_3 are not recursive

(B) L_2 is recursive and L_1, L_3 are not recursive

(C) L_1, L_2 are recursive and L_3 is not recursive

(D) L_1, L_2, L_3 are recursive

Answer: (C)

45. Which one of the following grammars is free from left recursion?

(A)
$S \to AB$
$A \to Aa \mid b$
$B \to c$

(B)
$S \to Ab \mid Bb \mid c$
$A \to Bd \mid \varepsilon$
$B \to e$

(C)
$S \to Aa \mid B$
$A \to Bb \mid Sc \mid \varepsilon$

(D)
$S \to Aa \mid Bb \mid c$
$A \to Bd \mid \varepsilon$
$B \to Ae \mid \varepsilon$

Answer: (B)

46. A student wrote two context-free grammars G1 and G2 for generating a single C-like array declaration. The dimension of the array is at least one. For example,

```
int a[10][3];
```

The grammars use D as the start symbol, and use six terminal symbols int ; id [ ] num.

Grammar G1	**Grammar G2**
`D → int L;`	`D → int L;`
`L → id [E`	`L → id E`
`E → num]`	`E → E [num]`
`E → num] [E`	`E → [num]`

Which of the grammars correctly generate the declaration mentioned above?

(A) Both G1 and G2 (B) Only G1

(C) Only G2 (D) Neither G1 nor G2

Answer: (A)

47. Consider the following processes, with the arrival time and the length of the CPU burst given in milliseconds. The scheduling algorithm used is preemptive shortest remaining-time first.

Process	Arrival Time	Burst Time
P_1	0	10
P_2	3	6
P_3	7	1
P_4	8	3

The average turnaround time of these processes is _________ milliseconds.

Answer: (8.25)

48. Consider the following two-process synchronization solution

Process 0

```
Entry: loop while (turn == 1);
     (critical section)
Exit:  turn = 1;
```

Process 1

```
Entry: loop while (turn == 0);
     (critical section)
Exit:  turn = 0;
```

The shared variable turn is initialized to zero. Which one of the following is TRUE?

(A) This is a correct two-process synchronization solution.

(B) This solution violates mutual exclusion requirement.

(C) This solution violates progress requirement.

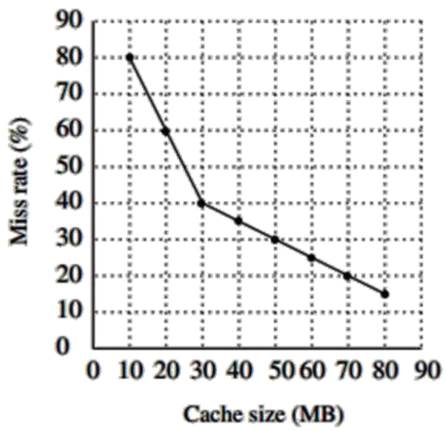
(D) This solution violates bounded wait requirement.

Answer: (C)

49. Consider a non-negative counting semaphore S. The operation P(S) decrements S, and V (S) increments S. During an execution, 20 P(S) operations and 12 V (S) operations are issued in some order. The largest initial value of S for which at least one P(S) operation will remain blocked is ___________.

Answer: **(7)**

50. A file system uses an in-memory cache to cache disk blocks. The miss rate of the cache is shown in the figure. The latency to read a block from the cache is 1ms and to read a block from the disk is 10ms. Assume that the cost of checking whether a block exists in the cache is negligible. Available cache sizes are in multiples of 10 MB.



The smallest cache size required to ensure an average read latency of less than 6 ms is _________ MB.

Answer: **(30)**

51. Consider the following database schedule with two transactions, T_1 and T_2.

$$S = r_2(X); r_1(X); r_2(Y); w_1(X); r_1(Y); w_2(X); a_1; a_2$$

where $r_i(Z)$ denotes a read operation by transaction T_i on a variable Z, $w_i(Z)$ denotes a write operation by T_i on a variable Z and a_i denotes an abort by transaction Ti.

Which one of the following statements about the above schedule is TRUE?

(A) S is non-recoverable

(B) S is recoverable, but has a cascading abort

(C) S does not have a cascading abort

(D) S is strict

Answer: **(C)**

52. Consider the following database table named water_schemes:

Water_schemes		
Scheme_no	**District_name**	**Capacity**
1	Ajmeer	20
1	Bikaner	10
2	Bikaner	10
3	Bikaner	20
1	Churu	10
2	Churu	20
1	Dungargarh	10

The number of tuples returned by the following SQL query is ___________.

```
with  total(name, capacity) as
     select  district_name, sum(capacity)
     from  water_schemes
     group by  district_name
with  total_avg(capacity) as select  avg(capacity)
     from  total
select  name
     from  total, total_avg
     where  total.capacity ≥  total_avg.capacity
```

Answer: (2)

53. A network has a data transmission bandwidth of 20×10^6 bits per second. It uses CSMA/CD in the MAC layer. The maximum signal propagation time from one node to another node is 40 microseconds. The minimum size of a frame in the network is __________ bytes.

Answer: (200)

54. For the IEEE 802.11 MAC protocol for wireless communication, which of the following statements is/are TRUE?

I. At least three non-overlapping channels are available for transmissions.

II. The RTS-CTS mechanism is used for collision detection.

III. Unicast frames are ACKed.

(A) All I, II, and III (B) I and III only

(C) II and III only (D) II only

Answer: (B)

55. Consider a 128×10^3 bits/second satellite communication link with one way propagation delay of 150 milliseconds. Selective retransmission (repeat) protocol is used on this link to send data with a frame size of 1 kilobyte. Neglect the transmission time of acknowledgement. The minimum number of bits required for the sequence number field to achieve 100% utilization is _________.

Answer: (4)

Q. No. 1 – 5 Carry One Mark Each

1. Which of the following options is the closest in meaning to the phrase underlined in the sentence below?
It is fascinating to see life forms <u>cope with</u> varied environmental conditions.
(A) Adopt to (B) Adapt to (C) Adept in (D) Accept with

Answer: (B)

2. Choose the most appropriate word from the options given below to complete the following sentence.
He could not understand the judges awarding her the first prize, because he thought that her performance was quite ___________________.
(A) Superb (B) Medium (C) Mediocre (D) Exhilarating

Answer: (C)

3. In a press meet on the recent scam, the minister said, "The buck stops here". What did the minister convey by the statement?
(A) He wants all the money (B) He will return the money
(C) He will assume final responsibility (D) He will resist all enquiries

Answer: (C)

4. If $(z+1/z)^2 = 98$, compute $\left(z^2+1/z^2\right)$

Answer: (96)

Exp: Expanding

$$z^2+\frac{1}{z^2}+2.z.\frac{1}{z}=98 \Rightarrow z^2+\frac{1}{z^2}=96$$

5. The roots of $ax^2+bx+c=0$ are real and positive a, b and c are real. Then $ax^2+b|x|+c=0$ has
(A) No roots (B) 2 real roots (C) 3 real roots (D) 4 real roots

Answer: (D)

Exp: $ax^2+bx+c=0$
for roots to be real & +ve
$b^2-4ac>0$
This will have 2 real positive roots.
$ax^2+b|x|+c=0$
This can be written as;
ax^2+bx+c
$\text{Discri min ant} = b^2-4ac>0$
ax^2-bx+c
$(-b)^2-4ac$
$\Rightarrow b^2-4ac$
Is also >0. This will have real roots
$\Rightarrow$ This will have 4 real roots.

Q.No. 6 – 10 Carry One Mark Each

6. The Palghat Gap (or Palakkad Gap), a region about 30 km wide in the southern part of the Western Ghats in India, is lower than the hilly terrain to its north and south. The exact reasons for the formation of this gap are not clear. It results in the neighbouring regions of Tamil Nadu getting more rainfall from the South West monsoon and the neighbouring regions of Kerala having higher summer temperatures.

What can be inferred from this passage?

(A) The Palghat gap is caused by high rainfall and high temperatures in southern Tamil Nadu and Kerala

(B) The regions in Tamil Nadu and Kerala that are near the Palghat Gap are low–lying

(C) The low terrain of the Palghat Gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala

(D) Higher summer temperatures result in higher rainfall near the Palghat Gap area

Answer: (C)

7. Geneticists say that they are very close to confirming the genetic roots of psychiatric illnesses such as depression and schizophrenia, and consequently, that doctors will be able to eradicate these diseases through early identification and gene therapy.

On which of the following assumptions does the statement above rely?

(A) Strategies are now available for eliminating psychiatric illnesses

(B) Certain psychiatric illnesses have a genetic basis

(C) All human diseases can be traced back to genes and how they are expressed

(D) In the future, genetics will become the only relevant field for identifying psychiatric illnesses

Answer: (B)

8. Round–trip tickets to a tourist destination are eligible for a discount of 10% on the total fare. In addition, groups of 4 or more get a discount of 5% on the total fare. If the one way single person fare is Rs 100, a group of 5 tourists purchasing round–trip tickets will be charged Rs ____________

Answer: (850)

Exp: One way force =100

Two way fare per person=200

5 persons=1000/-

Total discount applicable=10+5=15%

$\text{Discount amount} = \frac{15}{100} \times 1000 = 150$

Amount to be paid=1000-150=850

9. In a survey, 300 respondents were asked whether they own a vehicle or not. If yes, they were further asked to mention whether they own a car or scooter or both. Their responses are tabulated below. What percent of respondents do not own a scooter?

		Men	Women
Own vehicle	Car	40	34
	Scooter	30	20
	Both	60	46
Do not own vehicle		20	50

Answer: (48)

Exp: Total respondents=300

Those who don't have scooter

$\Rightarrow$ Men= 40+20=60

$$\text{Women} = 34 + 50 = \frac{84}{144}$$

$$\% = \frac{144}{300} \times 100 = 48\%$$

10. When a point inside of a tetrahedron (a solid with four triangular surfaces) is connected by straight lines to its corners, how many (new) internal planes are created with these lines? ________________

Answer: (6)

Q. No. 1 – 25 Carry One Mark Each

1. Consider the statement:

“Not all that glitters is gold”

Predicate glitters (x) is true if x glitters and predicate gold (x) is true if x is gold. Which one of the following logical formulae represents the above statement?

(A) $\forall x;\ \text{glitters}(x) \Rightarrow \neg \text{gold}(x)$ (B) $\forall x;\ \text{gold}(x) \Rightarrow \text{glitters}(x)$

(C) $\exists x;\ \text{gold}(x) \wedge \neg \text{glitters}(x)$ (D) $\exists x;\ \text{glitters}(x) \wedge \neg \text{gold}(x)$

Answer: (D)

Exp: It means “It is false that every glitter is gold” or “some glitters are not gold”.

Then we can say “atleast one glitter object is not gold”.

2. Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is ________ .

Answer: (0.25)

Exp: The smaller sticks, therefore, will range in length from almost 0 meters up to a maximum of 0.5 meters, with each length equally possible.

Thus, the average length will be about 0.25 meters, or about a quarter of the stick.

3. Let G=(V,E) be a directed graph where *V* is the set of vertices and *E* the set of edges. Then which one of the following graphs has the same strongly connected components as *G?*

(A) $G_1 = (V, E_1)$ where $E_1 = \{(u,v) \mid (u,v) \notin E\}$

(B) $G_2 = (V, E_2)$ where $E_2 = \{(u,v) \mid (v,u) \notin E\}$

(C) $G_3 = (V, E_3)$ where $E_3 = \{(u,v) \mid \text{there is a path of length} \le 2 \text{ from } u \text{ to } v \text{ in } E\}$

(D) $G_4 = (V_4, E)$ where V_4 is the set of vertices in G which are not isolated

Answer: (B)

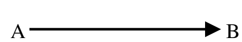
Exp: Take an example for Graph G

. .

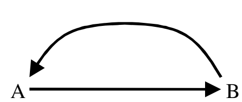
A B

Then option A and D will be eliminated.

Let G is below graph



Then G_3 is a graph with below structure



In G the numbers of strongly connected components are 2 where as in G_3 it is only one.

4 Consider the following system of equations:

$$3x + 2y = 1$$
$$4x + 7z = 1$$
$$x + y + z = 3$$
$$x - 2y + 7z = 0$$

The number of solutions for this system is __________________

Answer: (1)

Exp: $3x+2y=1$

$4x+7z=1$

$x+y+z = 3$

$x-2y+7z=0$

Augmented matrix is $\begin{bmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 7 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & -2 & 7 & 0 \end{bmatrix}$

$R_1 \leftrightarrow R_3 \begin{bmatrix} 1 & 1 & 1 & 3 \\ 4 & 0 & 7 & 1 \\ 3 & 2 & 0 & 1 \\ 1 & -2 & 7 & 0 \end{bmatrix}$

$R_2 \to R_2 - 4R_1\ R_3 \to R_3 - 3R_1,\ R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 3 & -11 \\ 0 & -1 & -3 & -8 \\ 0 & -3 & 6 & -3 \end{bmatrix}$$

$\begin{matrix} R_3 \to 4R_3 - R_2 \\ \\ R_4 \to 4R_4 - 3R_2 \end{matrix} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 3 & -11 \\ 0 & 0 & -15 & -21 \\ 0 & 0 & 15 & 21 \end{bmatrix}$

$R_4 \to R_4 + R_3 \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 3 & -11 \\ 0 & 0 & -15 & -21 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\rho(A:B) = \rho(A) = 3 = \text{no. of variables}$

$\therefore$ Unique solution exists

5. The value of the dot product of the eigenvectors corresponding to any pair of different eigen values of a 4-by-4 symmetric positive definite matrix is ______________.

Answer: (0)

Exp: (The eigen vectors corresponding to distinct eigen values of real symmetric matrix are orthogonal)

6. Let the function

$$f(\theta)=\begin{vmatrix} \sin\theta & \cos\theta & \tan\theta \\ \sin\left(\pi/6\right) & \cos\left(\pi/6\right) & \tan\left(\pi/6\right) \\ \sin\left(\pi/3\right) & \cos\left(\pi/3\right) & \tan\left(\pi/3\right) \end{vmatrix}$$

Where $\theta=\in\left[\frac{\pi}{6},\frac{\pi}{2}\right]$ and $f'(\theta)$ denote the derivative of f with respect to θ. Which of the following statement is / are TRUE?

(I) There exists $\theta\in\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ such that $f'(\theta)=0$.

(II) There exists $\theta\in\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ such that $f'(\theta)\neq 0$.

(A) I only (B) II only (C) Both I and II (D) Neither I nor II

Answer: (C)

Exp: (By Mean value theorem)

7. Consider the following Boolean expression for *F*:

$F(P,Q,R,S)=PQ+\overline{P}QR+\overline{P}Q\overline{R}S$

The minimal sum-of products form of F is

(A) $PQ+QR+QS$ (B) $P+Q+R+S$

(C) $\overline{P}+\overline{Q}+\overline{R}+\overline{S}$ (D) $\overline{P}R+\overline{P}\overline{R}S+P$

Answer: (A)

Exp: $PQ+\overline{P}QR+\overline{P}Q\overline{R}S$

$=PQ+\overline{P}Q\left(R+\overline{R}S\right)$

$=PQ+\overline{P}Q\left(\left(R+\overline{R}\right)(R+S)\right)\left[\because A+BC=(A+B)(A+C)\right]$

$=PQ+\overline{P}Q(R+S)\left[\because R+\overline{R}=1\right]$

$=Q\left(P+\overline{P}(R+S)\right)$

$=Q\left(\left(P+\overline{P}\right)(P+R+S)\right)\left[\because A+BC=(A+B)(A+C)\right]$

$=Q(P+R+S) \quad \because\left[P+\overline{P}=1\right]$

$=PQ+QR+QS$

8. The base (or radix) of the number system such that the following equation holds is____________.

$$\frac{312}{20} = 13.1$$

Answer: (5)

Exp: Let ‘x’ be the base or radix of the number system

$$\frac{2\times x^{o} + 1\times x + 3\times x^{2}}{0\times x^{o} + 2\times x} = 3\times x^{o} + 1\times x + 1\times x^{-1}$$

$$\Rightarrow \frac{2 + x + 3x^{2}}{2x} = 3 + x + \frac{1}{x}$$

$$\Rightarrow \frac{3x^{2} + x + 2}{2x} = \frac{3x + x^{2} + 1}{x}$$

$$\Rightarrow 3x^{2} + x + 2 = 6x + 2x^{2} + 2$$

$$\Rightarrow x^{2} - 5x = 0$$

$$\Rightarrow x(x-5) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 5$$

As base or radix of a number system cannot be zero, here $x = 5$

9. A machine has a 32-bit architecture, with 1-word long instructions. It has 64 registers, each of which is 32 bits long. It needs to support 45 instructions, which have an immediate operand in addition to two register operands. Assuming that the immediate operand is an unsigned integer, the maximum value of the immediate operand is ____________.

Answer: (16383)

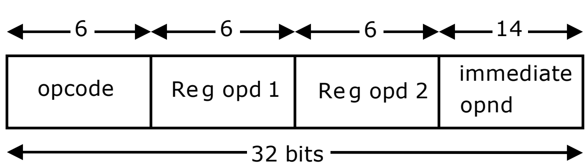
Exp: $1 \text{ Word} = 32 \text{ bits}$

Each instruction has 32 bits

To support 45 instructions, opcode must contain 6-bits

Register operand1 requires 6 bits, since the total registers are 64.

Register operand 2 also requires 6 bits



14-bits are left over for immediate Operand Using 14-bits, we can give maximum 16383, Since $2^{14} = 16384$ (from 0 to 16383)

10. Consider the following program in C language:

```
# include < stdio.h >
main( )
{
int  i;
int * pi = &i;
scanf ("%d", pi);
printf ("%d\n", i+5);
}
```

Which one of the following statements is **TRUE**?

(A) Compilation fails.

(B) Execution results in a run-time error.

(C) On execution, the value printed is **5** more than the address of variable **i**.

(D) On execution, the value printed is **5** more than the integer value entered.

Answer: (D)

Exp: pi contains the address of i. So scanf("%d",pi) places the value entered in console into variable i.e So printf("%d\n",i+5),prints 5 more than the value entered in console.

11. Let G be a graph with n vertices and m edges. What is the tightest upper bound on the running time of Depth First Search on G, when G is represented as an adjacency matrix?

(A) $\theta(n)$ (B) $\theta(n+m)$ (C) $\theta(n^2)$ (D) $\theta(m^2)$

Answer: (C)

Exp: DFS visits each vertex once and as it visits each vertex, we need to find all of its neighbours to figure out where to search next. Finding all its neighbours in an adjacency matrix requires O(V) time, so overall the running time will be $O(V^2)$.

12. Consider rooted n node binary tree represented using pointers. The best upper bound on the time required to determine the number of sub trees having exactly 4 nodes is $0(n^a \log^b n)$. Then the value of a + 10b is________

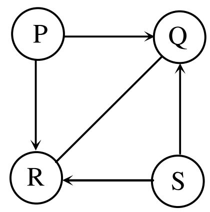
Answer: 1

Exp:
```
int print_subtrees_size_4(node  *n)
{
int size=0;
if(node==null)
return 0;
size=print_subtrees_size_4(node->left)+print_subtrees_size_4(node->right)+1;
 if(size==4)
printf("this is a subtree of size 4");
return size;
}
```

The above function on taking input the root of a binary tree prints all the subtrees of size 4 in O(n) time

so a=1 , b=0 and then a+10b=1

13. Consider the directed graph given below.



Which one of the following is **TRUE?**

(A) The graph does not have any topological ordering

(B) Both PQRS and SRQP are topological orderings

(C) Both PSRQ and SPRQ are topological orderings.

(D) PSRQ is the only topological ordering.

Answer: (C)

Exp: Topological ordering of a directed graph is a linear ordering of its vertices such that for every directed edge uv from vertex u to vertex v, u comes before v in the ordering. Topological ordering is possible iff graph has no directed cycles.

(A) As the given graph doesn't contain any directed cycles, it has at least one topological ordering. So option (A) is false

(B) PQRS cannot be topological ordering because S should come before R in the ordering as there is a directed edge from S to R.

SRQP cannot be topological ordering, because P should come before Q in the ordering as there is a directed edge from P to Q

(C) PSRQ and SPRQ are topological orderings as both of then satisfy the above mentioned topological ordering conditions.

(D) PSRQ is not the only one topological ordering as SPRQ is other possibility

14. Let P be a quick sort program to sort numbers in ascending order using the first element as the pivot. Let t_1 and t_2 and t_2 be the number of comparisons made by P for the inputs [1 2 3 4 5] and [4 1 5 3 2] respectively. Which one of the following holds?

(A) $t_1 = 5$ (B) $t_1 < t_2$ (C) $t_1 > t_2$ (D) $t_1 = t_2$

Answer: (C)

Exp: Partition algorithm for quick sort

$\text{Partition}(A, P, q) // A[P, \ldots\ldots q]$

$x \leftarrow A[P] // \text{pivot} = A[P]$

$i \leftarrow P$

$\text{for } j = P + 1 \text{ to } q$

$\quad \text{do if } A[j] \leq x$

$\qquad \text{then } i \leftarrow i + 1$

$\qquad \text{exchange } A[i] \leftrightarrow A[j]$

$\text{exchange} A[P] \leftrightarrow A[i]$

return i [returning where pivot element is there after partitioning]

Recursively call the above algorithm for the two sub arrays [elements before and after pivot element] to complete the sorting.

$x = \text{pivot}$

1 2 3 4 5
i j

$2 \le 1$? NO

1 2 3 4 5
i j

$3 \le 1$? NO

1 2 3 4 5
i j

$4 \le 1$? NO

1 2 3 4 5
i j

$5 \le 1$? NO

exchange $A[P]$ & $A[i]$

1 2 3 4 5

Pivot $\ge$Pivot

Call recursively
for this subarray

$\nearrow$ $\text{Pivot} = x = A[B]$

1 2 3 4 5
i j

$3 \le 2$? NO

1 2 3 4 5
i j

$4 \le 2$? NO

1 2 3 4 5
i j

$5 \le 2$? NO

exchange $A[P]$ & $A[J]$

1 2 3 4 5

Pivot $\ge$Pivot

Call recursively
for this

$\nearrow$ $x = \text{Pivot} = A[P]$

1 2 3 4 5
i j

$4 \le 3$? NO

1 2 3 4 5
i j

$5 \le 3$? NO

exchange $A[P]$ & $A[i]$

1 2 3 4 5

Pivot $\ge$Pivot

$\nearrow$ $x = \text{Pivot}$

1 2 3 4 5
i j

$5 \le 4$? NO

exchange $A[P]$ & $A[i]$

1 2 3 4 5

Pivot $\ge$Pivot

$\therefore$ Total 10 comparisons

$\nearrow x = \text{pivot} = A[P]$

4 1 5 3 2

i j $\quad 1 \le 4$? Yes

$i \leftarrow i+1$ exchange $A[i]$ & $A[j]$ & increment j

4 1 5 3 2

i j $\quad 5 \le 4$? NO

4 1 5 3 2

i j $\quad 3 \le 4$? Yes

$i \leftarrow i+1$ exchange $A[i]$ & $A[j]$ & increment j

4 1 3 5 2

i J $\quad 2 \le 4$? Yes

$i \leftarrow i+1$

4 1 3 2 5

i j

exchange $A[P]$ & $A[i]$

2 1 3 4 5

$\le$Pivot Pivot $\ge$Pivot

$x = \text{pivot} = A(P)$

2 1 3 |4| 5

i j $\quad 1 \le 2$? yes

2 1 3 |4| 5

i j $\quad 3 \le 2$? NO

exchange $A[P]$ & $A[i]$

1 2 3 4 5

Pivot

$\therefore$ 6 comparisons

15. Which one of the following is **TRUE**?

(A) The language $L = \{a^n b^n \mid n \ge 0\}$ is regular.

(B) The language $L = \{a^n \mid n \text{ is prime}\}$ is regular.

(C) The language $L = \{ w \mid w$ has $3k+1$ b's for some $k \in N$ with $\Sigma = \{a,b\}\}$ is regular.

(D) The language $L = \{ww \mid w \in \Sigma^*$ with $\Sigma = \{0,1\}\}$ is regular.

Answer: (C)

Exp: (A) $L = \{a^n b^n \mid n \ge 0\}$ is a CFL but not regular, it requires memory for the representation

(B) $L = \{a^n \mid n \text{ is prime}\}$ is neither regular nor CFL

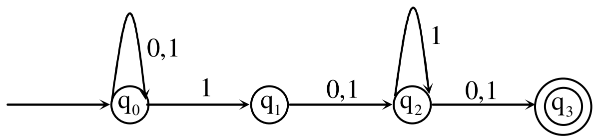
(C) $L = \{w \mid w \text{ has } 3K+1 \text{ b's for some } k \in N \text{ with}\}$

$\Sigma = \{a,b\}$

is a regular language, since the total count of b's are multiple of 3 plus one. The regular expression is $a^*ba^*(a^*ba^*ba^*ba^*)^* + (a^*ba^*ba^*ba^*)^*a^*ba^*$

(D) $L = \{ww \mid w \in \Sigma^* \text{ with } \Sigma = \{0,1\}\}$ is neither regular nor CFL

16. Consider the finite automaton in the following figure.



What is the set of reachable states for the input string 0011?

(A) $\{q_0, q_1, q_2\}$ (B) $\{q_0, q_1\}$ (C) $\{q_0, q_1, q_2, q_3\}$ (D) $\{q_3\}$

Answer: (A)

Exp:
$$\begin{aligned}\delta(q_0, 0011) &= \delta(q_0, 011)\\ &= \delta(q_0, 11)\\ &= \delta(\{q_0, q_1\}, 1)\\ &= \delta(q_0, 1) \cup \delta(q_1, 1)\\ &= \{q_0, q_1\} \cup \{q_2\}\\ &= \{q_0, q_1, q_2\}\end{aligned}$$

17. Which one of the following is **FALSE**?

(A) A basic block is a sequence of instructions where control enters the sequence at the beginning and exits at the end.

(B) Available expression analysis can be used for common subexpression elimination.

(C) Live variable analysis can be used for dead code elimination

(D) $x = 4 * 5 \Rightarrow x = 20$ is an example of common subexpression elimination

Answer: (D)

Exp: $x = 4 \times 5 \Rightarrow x = 20$ is not an example of common sub-expression but it is constant folding. In constant folding expression consisting of constants will be replaced by their final value at compile time, rather than doing the calculation in run-time.

18. Match the following

(1) Waterfall model	(a) Specifications can be developed
(2) Evolutionary model	(b) Requirements compromises are inevitable
(3) Component based software	(c) Explicit recognition of risk
(4) Spiral development	(d) Inflexible partitioning of the project into stages

(A) 1-a, 2-b, 3-c, 4-d (B) 1-d, 2-a, 3-b, 4-c

(C) 1-d, 2-b, 3-a, 4-c (D) 1-c, 2-a, 3-b, 4-d

Answer: (B)

Exp: The main drawback of the waterfall model is the difficulty of accommodating change after the process is underway. One phase has to be complete before moving onto the next phase. Inflexible partitioning of the project into distinct stages in waterfall model makes it difficult to respond to changing customer requirements.

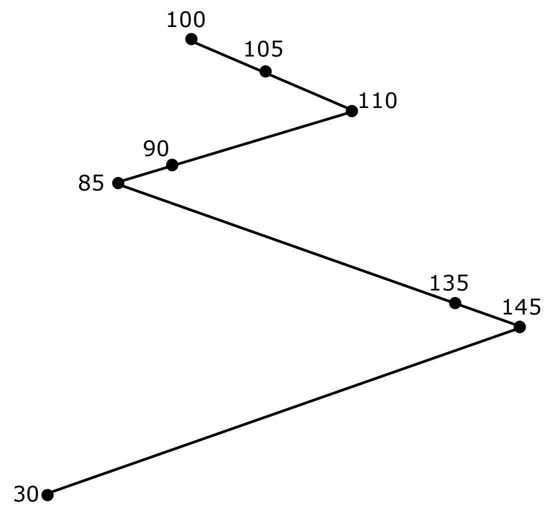
Evolutionary software models are iterative. They are characterized in manner that enables the software engineers to develop increasingly more complete version of software.

In Spiral model, Development can be divided in to smaller parts and more risky parts can be developed earlier which helps better risk management.

19. Suppose a disk has 201 cylinders, numbered from 0 to 200. At some time the disk arm is at cylinder 100, and there is a queue of disk access requests for cylinders 30, 85, 90, 100, 105, 110, 135 and 145. If Shortest-Seek Time First (SSTF) is being used for scheduling the disk access, the request for cylinder 90 is serviced after servicing ____________ number of requests.

Answer: (3)

Exp:



Request for cylinder is served after serving 3 requests (100,105 and 110)

20. Which one of the following is **FALSE**?

(A) User level threads are not scheduled by the kernel.

(B) When a user level thread is blocked, all other threads of its process are blocked.

(C) Context switching between user level threads is faster than context switching between kernel level threads.

(D) Kernel level threads cannot share the code segment.

Answer: (D)

Exp: User threads are supported above the kernel and a managed without kernel support. The thread function library to implement user level threads usually runs on top of the system in user mode. Thus these threads with in a process are invisible to the operating system. Since the kernel is unaware of the existence of such threads; when one user level thread is blocked in the kernel all other threads of its process are blocked. So options (A) and (B) are true

(C) The OS is aware of kernel level threads. Kernel threads are scheduled by the OS's scheduling algorithms and require a "lightweight" context switch to switch between (that is, registers, PC and SP must be changed, but the memory context remains the same among kernel threads in the (same process). User level threads are much faster to switch between as there is not context switch

(D) False

Kernel level threads within the same process share code section, data section and other operating system resources such as open files and signals.

21. Consider the relation scheme R = (E, F, G, H, I, J, K, L, M, N) and the set of functional dependencies $\{\{E,F\} \to \{G\}, \{F\} \to \{I,J\}, \{E,H\} \to \{K,L\} \to (M)$, $\{K\} \to \{M\}, \}$ $\{L\} \to \{N\}$ on R. What is the key for R?

(A) {E, F} (B) {E, F, H} (C) {E, F, H, K, L} (D) {E}

Answer: (B)

Exp: $R(EFGHI, JKLMN)$

$F = \{$

$EF \to G$

$F \to IJ$

$EH \to KL$

$K \to M$

$L \to N$

$\}$

$(EF)^+ = EFGIJ$, $E \,\&\, F$ Together functionally derive GIJ and if we observe given FDs, H can't be determined by any other attributes. So H must be part of all the (candidate) keys. H along with E determines K and L, K & L functionally determine M and N respectively.

$\therefore (EFH)^+ = EFGIJHKLMN$

$\therefore$ EFH is the only candidate for key.

22. Given the following statements:

S1: A foreign key declaration can always be replaced by an equivalent check assertion in SQL

S2: Given the table R(a,b,c) where a and b together form the primary key, the following is a valid table definition.

CREATE TABLE S (

a INTEGER,

d INTEGER,

e INTEGER,

PRIMARY KEY (d),

FOREIGN KEY (a) references R)

Which one of the following statements is **CORRECT**?

(A) S1 is TRUE and S2 is a FALSE (B) Both S1 and S2 are TRUE

(C) S1 is FALSE and S2 is a TRUE (D) Both S1 and S2 are FALSE

Answer: (D)

Exp: S_1: Manager (Name, DeptID)

Department (DeptName, Deptid)

In given relation Manager DeptID is a foreign key referencing Deptid (P.K) of relation Department.

Let's declare the foreign key by an equivalent check assertion as follows:-

```
CREATE TABLE Manager (
    Name  Varchar (10)
    DeptID  INT (6)  check (DeptID IN (  select Deptid  from Department)),
    PRIMARY KEY (Name)
    );
```

The above use of check assertion is good to declare the foreign key as far as insertion is considered for relation manager (will not insert any tuple in Manager containing such DeptID value which is not present in any tuple of Department).

But the above declaration will fail to implement changes done in Department relation in terms of deletion & updation. For an instance if a deptid present in Department gets deleted, then respective reference in Manager should also be deleted.

$\therefore S_1$ is false.

S_2: The given table definition is not valid due to invalid foreign key declaration. Attribute a is declared as foreign key which is a single valued attribute and it is referencing the primary key (ab) of relation R (a, b, c), which is a composite key.

A single value attribute cannot refer a composite key.

4 S_2 is false.

23. Consider the following three statements about link state and distance vector routing protocols, for a large network with 500 network nodes and 4000 links

[S1] The computational overhead in link state protocols is higher than in distance vector protocols.

[S2] A distance vector protocol (with split horizon) avoids persistent routing loops, but not a link state protocol.

[S3] After a topology change, a link state protocol will converge faster than a distance vector protocol.

Which one of the following is correct about S1, S2, and S3?

(A) S1, S2, and S3 are all true

(B) S1, S2, and S3 are all false.

(C) S1 and S2 are true, but S3 is false

(D) S1 and S3 are true, but S2 is false.

Answer: (D)

Exp: Statement S1

The Distance Vector routing protocols rely on the information from their directly connected neighbours in order to calculate and accumulate route information. Distance Vector routing protocols require very little overhead as compared to Link State routing protocols as measured by memory and processor power while the Link State routing protocols do not rely solely on the information from the neighbours or adjacent router in order to calculate route information. Instead, Link State routing protocols have a system of databases that they use in order to calculate the best route to destinations in the network. This is TRUE

Statement S3

Distance Vector exchanges the routing updates periodically whether the topology is change or not, this will maximize the convergence time which increases the chance of routing loops while the Link State routing protocols send triggered change based updates when there is a topology change. After initial flood, pass small event based triggered link state updates to all other routers. This will minimize the convergence time that's why there is no chance of routing loops. This is TRUE**.**

24. Which one of the following are used to generate a message digest by the network security protocols?

(P) RSA (Q) SHA-1 (R) DES (S) MD5

(A) P and R only (B) Q and R only (C) Q and S only (D) R and S only

Answer: (C)

Exp: RSA and DES are for Encryption where MD5 and SHA – 1 are used to generate Message Digest.

25. Identify the correct order in which the following actions take place in an interaction between a web browser and a web server.

1. The web browser requests a webpage using HTTP.
2. The web browser establishes a TCP connection with the web server.
3. The web server sends the requested webpage using HTTP.
4. The web browser resolves the domain name using DNS.

(A) 4,2,1,3 (B) 1,2,3,4 (C) 4,1,2,3 (D) 2,4,1,3

Answer: (A)

Exp: First of all the browser must now know what IP to connect to. For this purpose browser takes help of Domain name system (DNS) servers which are used for resolving hostnames to IP addresses. As browser is an HTTP client and as HTTP is based on the TCP/IP protocols, first it establishes a TCP connection with the web server and requests a webpage using HTTP, and then the web server sends the requested webpage using HTTP. Hence the order is 4,2,1,3

Q.No. 26 – 55 Carry Two Marks

26. Consider a token ring network with a length of 2km having 10 stations including a monitoring station. The propagation speed of the signal is 2×10^8 m/s and the token transmission time is ignored. If each station is allowed to hold the token for 2 μsec, the minimum time for which the monitoring station should wait (**in μsec**) before assuming that the token is lost is _______.

Answer: (28μs to 30μs)

Exp: Given Length (d) = 2 Km

No. of Stations (m) = 10

Propagation Speed (v) = 2×10^8 m/s

THT = 2μs

So, Max. TRT = T_P in the Ring + No. of Active Stations * THT

$$= 10 \times 10^{-6} + 10 \times 2 \times 10^{-6}$$

$$= 30\ \mu s$$

27. Let the size of congestion window of a TCP connection be 32 KB when a timeout occurs. The round trip time of the connection is 100 msec and the maximum segment size used is 2kB. The time taken (in msec) by the TCP connection to get back to 32KB congestion window is ______

Answer: (1100 to 1300)

Exp: Given that at the time of Time Out, Congestion Window Size is 32KB and RTT = 100ms

When Time Out occurs, for the next round of Slow Start, Threshold = (size of Cwnd) / 2

It means Threshold = 16KB

Slow Start

2KB

1RTT

4KB

2RTT

8KB

3RTT

16KB ----------- Threshold reaches. So Additive Increase Starts

4RTT

18KB

5RTT

20KB

6RTT

22KB

7RTT

24KB

8RTT

26KB

9RTT

28KB

10RTT

30KB

11RTT

32KB

So, Total no. of RTTs = 11 → 11 * 100 = 1100

28. Consider a selective repeat sliding window protocol that uses a frame size of 1 KB to send data on a 1.5 Mbps link with a one-way latency of 50 msec. To achieve a link utilization of 60%, the minimum number of bits required to represent the sequence number field is ________.

Answer: (5)

Exp: Given $L = 1KB$

$B = 1.5Mbps$

$T_p = 50ms$

$\eta = 60\%$

Efficiency formula for SR protocol is

$$\eta = \frac{W}{1+2a} \Rightarrow \frac{60}{100} = \frac{W}{1+2a}\left(\because a = \frac{T_p}{T_x}\right)$$

$$T_x = \frac{L}{B} = \frac{8\times10^3}{1.5\times10^6} = 5.3ms$$

$$a = \frac{T_p}{T_x} = \frac{50}{5.3} = \frac{500}{53} = 9.43$$

$$\Rightarrow \frac{60}{100} = \frac{W}{19.86} \Rightarrow W = 11.9 \approx 12$$

$$\Rightarrow W = 2^{n-1} = 12 \Rightarrow 2^n = 24 \Rightarrow 2^n = 24 \approx 2^5 \Rightarrow \boxed{n = 5}$$

29. Consider the following four schedules due to three transactions (indicted by the subscript) using read and write on a data item x, denoted r (x) and w (x) respectively. Which one of them is conflict serializable?

(A) $r_1(x)\,;\,r_2(x)\,;\,w_1(x)\,;\,r_3(x)\,;\,w_2(x)$

(B) $r_2(x)\,;\,r_1(x)\,;\,w_2(x)\,;\,r_3(x)\,;\,w_1(x)$

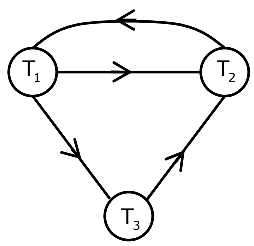
(C) $r_3(x)\,;\,r_2(x)\,;\,r_1(x)\,;\,w_2(x)\,;\,w_1(x)$

(D) $r_2(x)\,;\,w_2(x)\,;\,r_3(x)\,;\,r_1(x)\,;\,w_1(x)$

Answer: (D)

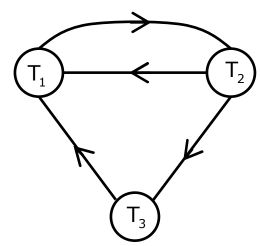
Exp: If there is a cycle in precedence graph, then the schedule is not conflict serializable.

(A)



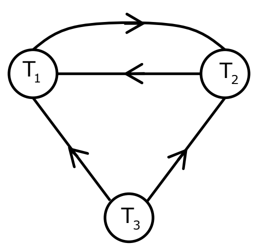
Not conflict serializable

(B)



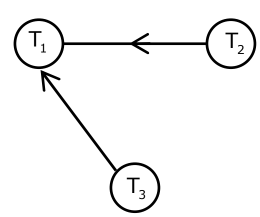
not conflict serializable

(C)



Not conflict serializable

(D)



it is conflict equivalent to

$T_2 \rightarrow T_3 \rightarrow T_1$ &

$T_3 \rightarrow T_2 \rightarrow T_1$

30. Given the following two statements:

S1: Every table with two single-valued attributes is in 1NF, 2NF, 3NF and BCNF

S2 : $AB \to C,\ D \to E,\ E \to C$ is a minimal cover for the set of functional dependencies $AB \to C,\ D \to E,\ AB \to E, E \to C$

Which one of the following is **CORRECT?**

(A) S1 is TRUE and S2 is FALSE.
(B) Both S1 and S2 are TRUE.
(C) S1 is FALSE and S2 is TRUE.
(D) Both S1 and S2 are FALSE.

Answer: (A)

Exp: S_1 : True

Consider any table R with two attributes $R(A.B)$

The possible FD sets are

$F_1 = \{$
$A \to B$
$\}$
Key : A and is in BCNF

$F_2 = \{$
$B \to A$
$\}$
Key : B and is in BCNF

$F_3 = \{$
$A \to B$
$B \to A$
$\}$
Key : A & B It is in BCNF

$F_4 = \{$
$\}$
Key : AB and is in BCNF

If a table is in BCNF it is also in 1NF, 2NF and 3NF also

S_2 : False

First FD set cannot cover second FD set because in second FD set AB can functionally derive E but that is not happing in first FD set.

31. An operating system uses the *Banker's algorithm* for deadlock avoidance when managing the allocation of three resource types X, Y, and Z to three processes P0, P1, and P2. The table given below presents the current system state. Here, the *Allocation* matrix shows the current number of resources of each type allocated to each process and the *Max* matrix shows the maximum number of resources of each type required by each process during its execution.

	Allocation			Max		
	X	Y	Z	X	Y	Z
P0	0	0	1	8	4	3
P1	3	2	0	6	2	0
P2	2	1	1	3	3	3

There are 3 units of type X, 2 units of type Y and 2 units of type Z still available. The system is currently in a **safe** state. Consider the following independent requests for additional resources in the current state:

REQ1: P0 requests 0 units of X, 0 units of Y and 2 units of Z

REQ2: P1 requests 2 units of X, 0 units of Y and 0 units of Z

Which one of the following is **TRUE**?

(A) Only REQ1 can be permitted.

(B) Only REQ2 can be permitted.

(C) Both REQ1 and REQ2 can be permitted.

(D) Neither REQ1 nor REQ2 can be permitted.

Answer: (B)

Exp: REQ1

Once P_0 is allocated with $(0,0,2)$, the status of the system will be as follows

Allocation	Max	Need	Available
X Y Z	X Y Z	X Y Z	X Y Z
0 0 3	8 4 3	8 4 0	3 2 0
3 2 0	6 2 0	3 0 0	
2 1 1	3 3 3	1 2 2	

With available $(3,2,0)$ only P_1 can be served. Once P_1 is executed, available will be $(6,4,0)$, with $(6,4,0)$ we can't serve either P_0 or P_2. Hence there is no safe sequence. Hence REQ1 can't be permitted.

REQ2

Once P_1 is allocated with $(2,0,0)$, the status of the system will be as follows

	Allocated	Max	Need	Available
	X Y Z	X Y Z	X Y Z	X Y Z
P_0	0 0 1	8 4 3	8 4 2	1 2 2
P_1	5 2 0	6 2 0	1 0 0	
P_2	2 1 1	3 3 3	1 2 2	

With available $(1,2,2)$, we can serve either P_1 or P_2.

If we serve P_1 then the safe sequence is $\langle P_1, P_2, P_0 \rangle$. If we serve P_2 then the safe sequence is $\langle P_2, P_1, P_0 \rangle$. As true is at least one safe sequence we can permit REQ2.

32. Consider the following set of processes that need to be scheduled on a single CPU. All the times are given in milliseconds

Process Name	**Arrival Time**	**Execution Time**
A	0	6
B	3	2
C	5	4
D	7	6
E	10	3

Using the *shortest remaining time first* scheduling algorithm, the average process turnaround time (in msec) is ______________________.

Answer: (7.2)

Exp:

A	B	A	C	E	D

0 3 5 8 12 15 21

$$\text{Average turn around time} = \frac{(8-0)+(5-3)+(12-5)+(21-7)+(15-10)}{5}$$

$$= \frac{36}{5} \Rightarrow 7.2\,\text{ms}$$

33. Assume that there are 3 page frames which are initially empty. If the page reference string 1, 2, 3, 4, 2, 1, 5, 3, 2, 4, 6, the number of page faults using the optimal replacement policy is __________

Answer: (7)

Exp:

1	2	3	4	2	1	5	3	2	4	
		3	4	4	4	4	4	4	4	4
	2	2	2	2	2	2	2	2	2	6
1	1	1	1	1	1	5	3	3	3	3
F	F	F	F	H	H	F	F	H	H	F

7 page faults

34. A canonical set of items is given below

$$s \to L. > R$$
$$Q \to R.$$

On input symbol $<$ the set has

(A) a shift-reduce conflict and a reduce-reduce conflict.

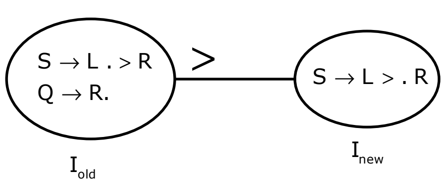
(B) a shift-reduce conflict but not a reduce-reduce conflict.

(C) a reduce-reduce conflict but not a shift-reduce conflict.

(D) neither a shift-reduce nor a reduce-reduce conflict.

Answer: (D)

Exp:



From above diagram, we can see that there is no shift- reduce or reduce-reduce conflict.

35. Let L be a language and $\overline{L}$ be its complement. Which of the following is NOT a viable possibility?

(A) Neither L nor $\overline{L}$ is recursively enumerable (r.e.).

(B) One of L and $\overline{L}$ is r.e. but not recursive; the other is not r.e.

(C) Both L and $\overline{L}$ are r.e. but not recursive.

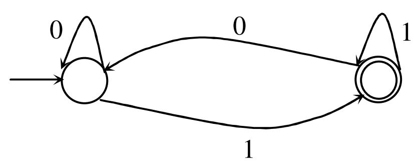
(D) Both L and $\overline{L}$ are recursive.

Answer: (C)

Exp: Recursive languages are closed under complement.

If a language L is recursive enumerable but not recursive then its complement is not a recursive enumerable, so both L and $\overline{L}$ are recursive enumerable but not recursive is not a viable possibility.

36. Which of the regular expressions given below represent the following DFA?



I) 0*1(1+00*1)*

II) 0*1*1+11*0*1

III) (0+1)*1

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II, and III

Answer: (B)

Exp: Given DFA will accept all the strings over $\varepsilon = \{0,1\}$ which are ending with 1.

$0^*1(1+00^*1)^*$ and $(0+1)^*1$, are the regular expressions for ending with 1.

37. There are 5 bags labelled 1 to 5. All the coins in a given bag have the same weight. Some bags have coins of weight 10 gm, others have coins of weight 11 gm. I pick 1, 2, 4, 8, 16 coins respectively from bags 1 to 5. Their total weight comes out to 323 gm. Then the product of the labels of the bags having 11 gm coins is ___.

Answer: 12

Exp: Let the weight of coins in the respective bags (1 through 5) be a,b,c,d and e-each of which can take one of two values namely 10 or 11 (gm).

Now, the given information on total weight can be expressed as the following equation:

1.a+2.b+4.c+8.d+16.e = 323

$\Rightarrow$ a must be odd $\Rightarrow$ a = 11

The equation then becomes:

11+2.b+4.c+8.d+16.e = 323

$\Rightarrow$ 2.b+4.c+8.d+16.e = 312

$\Rightarrow$ b+2.c+4.d+8.e = 156

$\Rightarrow$ b must be even $\Rightarrow$ b = 10

The equation then becomes:

10+2.c+4.d+8.e = 156

$\Rightarrow$ 2.c+4.d+8.e = 146

$\Rightarrow$ c+2.d+4.e = 73

$\Rightarrow$ c must be odd $\Rightarrow$ c = 11

The equation now becomes:

11+2.d+4.e = 73

$\Rightarrow$ 2.d+4.e = 62

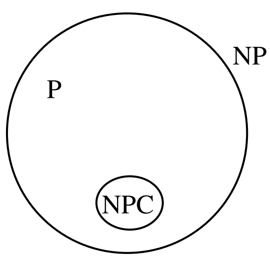
$\Rightarrow$ d+2.e = 31

$\Rightarrow$ e = 11 and e = 10

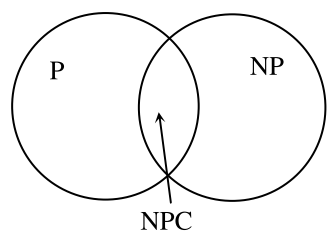
Therefore, bags labelled 1, 3 and 4 contain 11 gm coins $\Rightarrow$ Required Product = 1*3*4* = 12.

38. Suppose a polynomial time algorithm is discovered that correctly computes the largest clique in a given graph. In this scenario, which one of the following represents the correct Venn diagram of the complexity classes P, NP and NP Complete (NPC)?

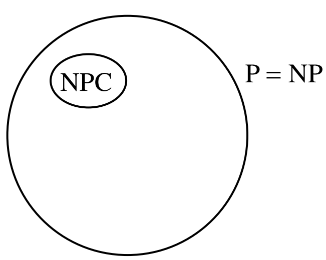
(A)



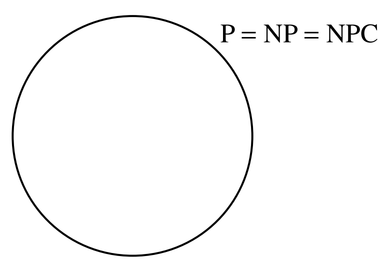
(B)



(B)



(D)



Answer: (D)

Exp: The most important open question in complexity theory is whether the **P = NP** , which asks whether polynomial time algorithms actually exist for **NP**-complete and all **NP** problems (since a problem "C" is in NP-complete, iff C is in NP and every problem in NP is reducible to C in polynomial time). In the given question it is given that some polynomial time algorithm exists which computes the largest clique problem in the given graph which is known NP-complete problem. Hence P=NP=NP-Complete.

39. The minimum number of comparisons required to find the minimum and the maximum of 100 numbers is _________________.

Answer: (148)

Exp: From the list of given n numbers [say n is even],

Pick up first two elements, compare them

assign $\text{Current}-\min = \min \text{ of two numbers}$

$\text{Current}-\max = \max \text{ of two numbers}$

From the remaining $n-2$ numbers, take pairs wise and follow this process given below.

1. Compare two elements

   Assign min = min of two numbers

   max = max of two numbers

2. Compare min and current - min

   Assign $\text{current}-\min = \min\{\text{current}-\min, \min\}$

3. Compare max and current - max

   Assign $\text{current}-\max = \max\{\text{current}-\max, \max\}$

Repeat above procedure for all the remaining pairs of numbers. We can observe that each of pair requires 3 comparisons

1. for finding min and max
2. For updating current – min
3. for updating current – max

But for initial pair we need only one comparison not 3.

$$\therefore \text{total number of comparisons} = \frac{3(n-2)}{2} + 1 = \frac{3n}{2} - 3 + 1 = \frac{3n}{2} - 2$$

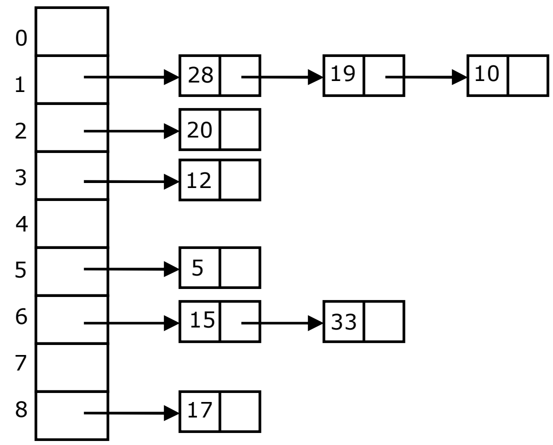
Here $n = 100,$ so number of comparisons = 148.

40. Consider a hash table with 9 slots. The hash function is $h(k) = k \bmod 9.$ The collisions are resolved by chaining. The following 9 keys are inserted in the order: 5, 28, 19, 15, 20, 33, 12, 17, 10. The maximum, minimum, and average chain lengths in the hash table, respectively, are

(A) 3, 0, and 1 (B) 3, 3, and 3 (C) 4, 0, and 1 (D) 3, 0, and 2

Answer: (A)

Exp:



$\therefore$ Maximum & minimum chain lengths are 3 & 0 respectively

$$\text{Average chain length} = \frac{0+3+1+1+0+1+2+0+1}{9} = 1$$

41. Consider the following C function in which **size** is the number of elements in the array **E**:

```
int MyX(int *E, unsigned int size)
{
 int Y = 0;
 int Z;
 int i, j, k;

for(i = 0; i < size; i++)
   Y = Y + E[i];
for(i = 0; i < size; i++)
  for(j = i; j < size; j++)
  {
Z = 0;
for(k = i; k <= j; k++)
Z = Z + E[k];
if (Z > Y)
  Y = Z;
}
  return Y;
}
```

The value returned by the function **MyX** is the

(A) maximum possible sum of elements in any sub-array of array **E**.

(B) maximum element in any sub-array of array **E**.

(C) sum of the maximum elements in all possible sub-arrays of array **E**.

(D) the sum of all the elements in the array **E**.

Answer: (A)

Exp: int Myx (int * E, unsinged int sixe)

```
{
 int Y = 0;
int z;
int i, j, k;
for (i = 0; i < size; i++)     } Calculates sum of the elements
    Y = Y + E[i]               } of the array E and stores it in Y
for (i = 0; i < size; i++)             }
  for (j = i; j < size; j++)           }
   {                                   } calculates the sum of elements of
      z = 0;                           }      all posible subarrays of E
      for (k = i; k <= j; k++)         }
          z = z + E[k];                }
      if (z > Y) → Checks whether sum of elements of each subarray
          Y = z;   is greaer than the sum of elements of array if so, that sum
       }            is assigned to Y, if not 'Y' will be the sum of elements of
return Y;           complete array
}
```

Ultimately returns the maximum possible sum of elements in any sub array of given array E.

42. Consider the following pseudo code. What is the total number of multiplications to be performed?

D= 2

for i = 1 to n do

for j = i to n do

 for k = j + 1 to n do

 D = D * 3

(A) Half of the product of the 3 consecutive integers

(B) One-third of the product of the 3 consecutive integers.

(C) One-sixth of the product of the 3 consecutive integers.

(D) None of the above.

Answer: (C)

Exp: $i=1,\ j=1,\ k=2 \text{ to } n \Rightarrow n-1 \text{ times}$

$$\left.\begin{array}{l} i=1,\ j=2,\ k=3 \text{ to } n \Rightarrow n-2 \text{ times} \\ i=1,\ j=3,\ k=4 \text{ to } n \Rightarrow n-3 \text{ times} \\ \ \vdots \\ i=1,\ j=n-2,\ k=n-1 \text{ to } n \Rightarrow 2 \text{ times} \end{array}\right\} \sum(n-1) \text{ times}$$

$i=1,\ j=n-1,\ k=n \text{ to } n \Rightarrow 1 \text{ time}$

$$\left.\begin{array}{l} i=2, j=2, k=3 \text{ to } n \Rightarrow n-2 \text{ times} \\ i=2, j=3, k=4 \text{ to } n \Rightarrow n-3 \text{ times} \\ \ \vdots \quad \vdots \\ \ \vdots \quad \vdots \\ i=2, j=n-1, k=n \text{ to } n \Rightarrow 1 \text{ time} \end{array}\right\} \sum(n-2) \text{ times}$$

$\vdots$

$\vdots$

$i=n-1, j=n-1, k=n \text{ to } n \Rightarrow 1 \text{ time } \} \sum 1 \text{ times}$

$\therefore$ Total number of multiplications

$$\Rightarrow \sum 1 + \sum 2 + \sum 3 + \ldots\ldots \sum(n-1)$$

$$= \underset{S_1}{\underset{\downarrow}{1}} + \underset{S_2}{\underset{\downarrow}{(1+2)}} + \underset{S_3}{\underset{\downarrow}{(1+2+3)}} + \ldots + \underset{S_{n-1}}{\underset{\downarrow}{(1+2+3+..n-1)}}$$

$$= \sum_{i=1}^{n-1} S_i = \sum \frac{n(n-1)}{2}$$

$$= \frac{1}{2}\sum n^2 - \frac{1}{2}\sum n$$

$$= \frac{1}{2}\frac{n(n+1)(2n+1)}{6} - \frac{1}{2}\frac{n(n+1)}{2} = \frac{(n-1)(n)(n+1)}{6}$$

43. Consider a 6-stage instruction pipeline, where all stages are perfectly balanced. Assume that there is no cycle-time overhead of pipelining. When an application is executing on this 6-stage pipeline, the speedup achieved with respect to non-pipelined execution if 25% of the instructions incur 2 pipeline stall cycles is ______________________.

Answer: (4)

Exp: For 6 stages, non- pipelining takes 6 cycles.

There were 2 stall cycles for pipelining for 25% of the instructions

$$\text{So pipe line time} = \left(1 + \frac{25}{100}2\right)$$

$$= \frac{3}{2} = 1.5$$

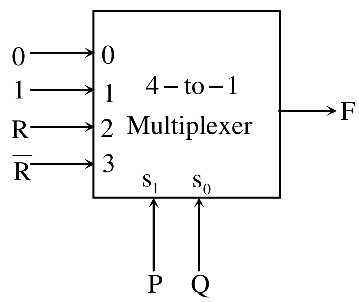
$$\text{Speed up} = \frac{\text{Non} - \text{pipeline time}}{\text{Pipeline time}} = \frac{6}{1.5} = 4$$

44. An access sequence of cache block addresses is of length N and contains n unique block addresses. The number of unique block addresses between two consecutive accesses to the same block address is bounded above K. What is the miss ratio if the access sequence is passed through a cache of associativity $A \geq k$ exercising least-recently-used replacement policy?

(A) n/N (B) l/N (C) 1/A (D) k/n

Answer: (A)

45. Consider the 4-to-1 multiplexer with two lines S_1 and S_0 given below.



The minimal sum of-products form of the Boolean expression for the output F of the multiplexer is

(A) $\bar{P}Q + Q\bar{R} + P\bar{Q}R$ (B) $\bar{P}Q + \bar{P}Q\bar{R} + PQ\bar{R} + P\bar{Q}R$

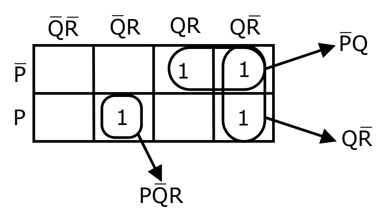
(C) $\bar{P}QR + \bar{P}Q\bar{R} + Q\bar{R} + P\bar{Q}R$ (D) $PQ\bar{R}$

Answer: (A)

Exp: $\bar{P}\bar{Q}.O + \bar{P}Q.1 + P\bar{Q}.R + PQ\bar{R}$

$= \bar{P}Q + P\bar{Q}.R + PQ\bar{R}$

Hence the minimized expression is $\bar{P}Q + Q\bar{R} + P\bar{Q}R$



46. The function f(x) = x sin x satisfies the following equation. f"(x) + f(x) +tcosx = 0. The value of t is______.

Answer: -2

Exp: Given $f''(x) + f(x) + t\cos x = 0$

and $f(x) = x\sin x$

$f'(x) = x\cos x + \sin x$

$f''(x) = x(-\sin x) + \cos x + \cos x$

$= 2\cos x - x\sin x$

$= 2\cos x - f(x)$

$\therefore 2\cos x - f(x) + f(x) + t\cos x = 0$

$\Rightarrow 2\cos x = -t\cos x \Rightarrow t = -2$

47. A function f(x) is continuous the interval [0,2]. It is known that f(0) = f(2) = -1 and f(1) = 1. Which one of the following statements must be true?

(A) There exists a y in the interval (0,1) such that f(y) = f(y + 1)
(B) For every y in the interval (0,1), f(y) = f(2 - y)
(C) The maximum value of the function in the interval (0.2) is 1
(D) There exists a y in the interval (0,1) such that f(y) =f(2 – y)

Answer: (A)

Exp: Define g(x) =f(x)-f(x+1) in [0,1]. g(0) is negative and g(1) is positive. By intermediate value theorem there is y€(0,1) such that g(y)=0

That is f(y) =f(y+1).

Thus answer is (a)

48. For fair six-sided dice are rolled. The probability that the sum of the results being 22is $X/1296$. The value of X is _________

Answer: (10)

Exp: 22 occurred in following ways

6 6 6 4 $\rightarrow$ 4ways

6 6 5 5 $\rightarrow$ 6ways

Required probability = $\dfrac{6+4}{2296} = \dfrac{10}{2296} \Rightarrow x = 10$

49. A pennant is a sequence of numbers, each number being 1 or 2. An n-pennant is a sequence of numbers with sum equal to n. For example, (1,1,2) is a 4-pennant. The set of all possible 1-pennants is {(1)}, the set of all possible 2-pennants is {(2), (1,1)}and the set of all 3-pennants is {(2,1), (1,1,1), (1,2)}. Note that the pennant (1,2) is not the same as the pennant (2,1). The number of 10- pennants is ______________.

Answer: (89)

Exp: No twos: 1111111111$\Rightarrow$1 pennant

Single two: 211111111$\Rightarrow$9!/8!1! = 9 pennants

Two twos: 22111111$\Rightarrow$8!/6!.2! = 28

Three twos: 2221111$\Rightarrow$7!/3!.4! = 35

Four twos: 222211$\Rightarrow$6!/4!.2! = 15

Five twos: 22222$\Rightarrow$1

Total = 89 pennants.

50. Let S denote the set of all functions $f : \{0,1\}^4 \rightarrow \{0,1\}$. Denote by N the number of functions from S to the set {0,1}. The value of $\log_2 \log_2 N$ is______.

Answer: (16)

Exp: The number of functions from A to B where size of A = |A| and size of B = |B| is $|B|^{|A|}$

$\{0,1\}^4 = \{0,1\}X\{0,1\}X\{0,1\}X\{0,1\} = 16$

$|S| = 2^{16}$

$N = 2^{|S|}$

$\log\log N = \log\log 2^{|S|} = \log|S| = \log 2^{16} = 16$

51. Consider an undirected graph G where self-loops are not allowed. The vertex set of G is {i, f): 1≤ i ≤ 12, 1 ≤ j ≤ 12}. There is an edge between (a,b) and (c,d) if $|a-c|\leq 1$ and $|b-d|\leq 1$ The number of edges in the graph is _____________.

Answer: (506)

Exp: The graph formed by the description contains 4 vertices of degree 3 and 40verices of degree 5 and 100 vertices of degree 8.

According to sum of the degrees theorem 4*3+40*5+100*8 = 2|E|

|E| = 1012/2 = 506

52. An ordered n-tuple (d_1,d_2,…,d_n) with $d_1 \geq d_2 \geq .... \geq d_n$ is called graphic if there exists a simple undirected graph with n vertices having degrees $d_1, d_2, ..., d_n$ respectively. Which of the following 6-tuples is NOT graphic?

(A) (1, 1, 1, 1, 1, 1)
(B) (2, 2, 2, 2, 2, 2)
(C) (3, 3, 3, 1, 0, 0)
(D) (3, 2, 1, 1, 1, 0)

Answer: (C)

Exp: According to havel-hakimi theorem

(1,1,1,1,1,1) is graphic iff<1,1,1,1,0> is graphic

(0,1,1,1,1) is graphic iff (0,1,1,0) is graphic

(0,0,1,1) is graphic iff (0,0,0) is graphic

Since (0,0,0) is graphic (1,1,1,1,1,1) is also graphic.

(The process is always finding maximum degree and removing it from degree sequence, subtract 1 from each degree for d times from right to left where d is maximum degree)

(2,2,2,2,2,2) is graphic iff (2,2,22-1,2-1) = (2,2,2,1,1) is graphic.

(1,1,2,2,2) is graphic iff (1,1,1,1) is graphic.

(1,1,1,1) is graphic iff (0,1,1)

(0,1,1) is graphic iff (0,0) is graphic.

Since (0,0) is graphic (2,2,2,2,2,2) is also graphic.

Consider option C now.

(3,3,3,1,0,0)→(0,0,1,3,3,3) is graphic iff (0,0,0,2,2) is graphic.

Note that before applying the havel-hakimi step degree sequence should be in non-increasing order.

(0,0,0,2,2) is graphic iff (0,0,-1,1) is graphic.

Since (0,0,-1,1) is not graphic (3,3,3,1,0,0) is also not graphic.

53. Which one of the following propositional logic formulas is TRUE when exactly two of p, q, and r are **TRUE**?

(A) $((p \leftrightarrow q) \wedge r) V (p \wedge q \wedge \sim r)$
(B) $(\sim (p \leftrightarrow q) \wedge r) V (p \wedge q \wedge \sim r)$
(C) $((p \rightarrow q) \wedge r) V (p \wedge q \wedge \sim r)$
(D) $(\sim (p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

Answer: (B)

Exp: P = T q=F and r=T

Option A will become false.

Option C will become false.

Option D is always false.

54. Given the following schema:

employees(emp-id, first-name, last-name, hire-date,

dept-id, salary)

departments(dept-id, dept-name, manager-id, location-id)

You want to display the last names and hire dates of all latest hires in their respective departments in the location ID 1700. You issue the following query:

SQL>SELECT last-name, hire-date

FROM employees

WHERE (dept-id, hire-date) IN

(SELECT dept-id, MAX(hire-date)

FROM employees JOIN departments USING(dept-id)

WHERE location-id = 1700

GROUP BY dept-id);

What is the outcome?

(A) It executes but does not give the correct result.

(B) It executes and gives the correct result.

(C) It generates an error because of pairwise comparison.

(D) It generates an error because the GROUP BY clause cannot be used with table joins in a sub-query.

Answer: (B)

Exp: In the inner sub query, "employees" and "departments" tables are joined by "using" clause (first Cartesian product of those two tables will be done and then and wherever these is a match on the dept-ids that tuple will be filtered). After this, the tuples of the resultant table will be filtered by using the condition "location-id=1700" and then will grouped on dept-id(all the tuples having equal values under dept-id will come into one group). After grouping, the columns dept-id in location-id 1700 and maximum of hire dates in that respective dept-id will be selected. Format of the tuples in the resultant table will be dept-id in location-id 1700 along with the latest hire date in the respective dept (two columns). Outer query takes each tuple from "employees" table and it will check whether dept-id and hire-date pair for this tuple is contained in the table given by inner sub query. If this is the case it will display the last-name of respective employee

IN operator compares one or multiple expressions on the left side of the operator to a set of one or more values on the right side of the operator. When using multiple expressions (like 2 columns - pair wise comparison), the number and data types of expressions in the list must match on both sides of the operator.

55. Consider two processors P_1 and P_2 executing the same instruction set. Assume that under identical conditions, for the same input, a program running on P_2 takes 25% less time but incurs 20% more CPI (clock cycles per instruction) as compared to the program running on P_1. If the clock frequency of P_1 is 1GHz, then the clock frequency of P_2 (in GHz) is _________.

Answer: (1.6)

Exp: 1 cycle time for $p_1 = \frac{10^9}{1GH} = 1 n.s$

Assume p_1 takes 5 cycles for a program then p_2 takes 20% more, means, 6 cycles.

p_2 Takes 25% less time, means, if p_1 takes 5 n.s, then p_2 takes 3.75 n.s.

Assume p_2 clock frequency is x GHz.

p_2 Taken 6 cycles, so $\frac{6 \times 10^9}{x\ GH} = 3.75,\ x = 1.6$

Q. No. 1 – 5 Carry One Mark Each

1. Choose the most appropriate phrase from the options given below to complete the following sentence.

   India is a post-colonial country because

   (A) it was a former British colony

   (B) Indian Information Technology professionals have colonized the world

   (C) India does not follow any colonial practices

   (D) India has helped other countries gain freedom

Answer: (A)

2. Who ___________ was coming to see us this evening?

   (A) you said (B) did you say (C) did you say that (D) had you said

Answer: (B)

3. Match the columns.

Column 1	Column 2
(1) eradicate	(P) misrepresent
(2) distort	(Q) soak completely
(3) saturate	(R) use
(4) utilize	(S) destroy utterly

   (A) 1:S, 2:P, 3:Q, 4:R (B) 1:P, 2:Q, 3:R, 4:S

   (C) 1:Q, 2:R, 3:S, 4:P (D) 1:S, 2:P, 3:R, 4:Q

Answer: (A)

4. What is the average of all multiples of 10 from 2 to 198?

   (A) 90 (B) 100 (C) 110 (D) 120

Answer: (B)

Exp:

$$\left.\begin{array}{l} 10+190 \rightarrow \\ 20-180 \rightarrow \\ : \\ : \\ 90-110 \\ 100 \end{array}\right\} \begin{array}{l} 200 \\ \\ 9 \end{array} \quad \Rightarrow \frac{[(200)\times 9+100]}{19} = \frac{1900}{19} = 100$$

5. The value of $\sqrt{12+\sqrt{12+\sqrt{12+....}}}$ is

   (A) 3.464 (B) 3.932 (C) 4.000 (D) 4.444

Answer: (C)

Exp: $\text{let} = \sqrt{12+\sqrt{12+\sqrt{12+....}}} = y$

$\Rightarrow \sqrt{12+y} = y$

$\Rightarrow 12+y = y^2$

$\Rightarrow (y-4)(y+3) = 0$

$\Rightarrow y = 4,\ y = -3$

Q. No. 6 – 10 Carry Two Marks Each

6. The old city of Koenigsberg, which had a German majority population before World War 2, is now called Kaliningrad. After the events of the war, Kaliningrad is now a Russian territory and has a predominantly Russian population. It is bordered by the Baltic Sea on the north and the countries of Poland to the south and west and Lithuania to the east respectively. Which of the statements below can be inferred from this passage?

(A) Kaliningrad was historically Russian in its ethnic make up

(B) Kaliningrad is a part of Russia despite it not being contiguous with the rest of Russia

(C) Koenigsberg was renamed Kaliningrad, as that was its original Russian name

(D) Poland and Lithuania are on the route from Kaliningrad to the rest of Russia

Answer: (B)

7. The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in north India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region.

Which one of the following statements, if true, does not contradict this conclusion?

(A) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent

(B) More cases of dengue are now reported because of an increase in the Municipal Office's administrative efficiency

(C) Many more cases of dengue are being diagnosed this year since the introduction of a new and effective diagnostic test

(D) The number of people with malarial fever (also contracted from mosquito bites) has increased this year

Answer: (D)

8. If x is real and $\left|x^2-2x+3\right| = 11$, then possible values of $\left|-x^3+x^2-x\right|$ include

(A) 2, 4 (B) 2, 14 (C) 4, 52 (D) 14, 52

Answer: (D)

Exp: $x^2 - 2x + 3 = 11$

$\Rightarrow (x-4)(x+2) = 0 \Rightarrow x = 4, x = -2$

Values of $\left|-x^3 + x^2 - x\right|$

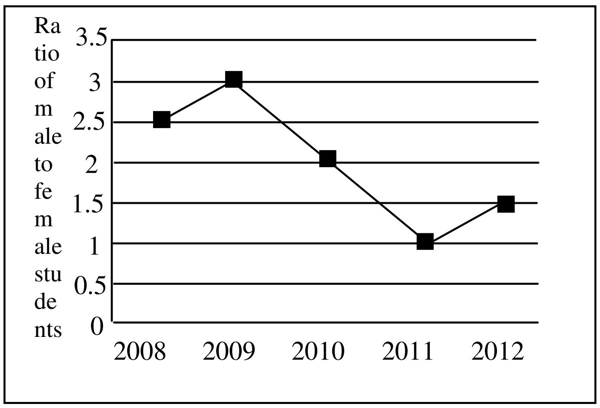
For $x = 4$

Value $= 52$

for $x = -2$

Value $= 14$

9. The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what percent did the number of male students increase in 2009?



Answer: (140)

Exp: $\frac{m}{f} = 3$ $\quad$ $\frac{m}{f} = 2.5$ m=2.5f

$\frac{m'}{2f} = 3$

$m' = 6f$

$= \frac{m' - m}{m}$

$\% \uparrow = \frac{3.5f}{2.5f} \times 100$

$= \frac{7}{8} = 1.4$

$\boxed{\% \uparrow = 140\%}$

10. At what time between 6 *a.m.* and 7 *a.m* will the minute hand and hour hand of a clock make an angle closest to 60°?

(A) 6: 22 *a. m.* (B) 6:27 *a.m.* (C) 6: 38 *a.m.* (D) 6:45 *a.m.*

Answer: (A)

Exp: Angle by minute's hand

$60 \text{ min} \rightarrow 360^{o}$

$1 \text{min} \rightarrow \frac{360}{60} = 6^{o}$

$8 \text{min} \rightarrow 48^{o}$

$\text{Angle} \rightarrow 48^{o}$ with number '6'

Angle by hours hand

$60 \text{ min} = 30^{o}$

$22 \text{ min} \rightarrow \frac{30}{60} \times 22$

$= 11$

Total Angle=48+11=59°.

Q. No. 1 – 25 Carry One Mark Each

1. The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p Then 100p= _____________.

Answer: (11.85 - 11.95)

Exp: p= P [at least three computers are working]

= P (3 or 4 computers working)

$$=\frac{\left(4_{C_3}\right)\times\left(6_{C_1}\right)}{10_{C_4}}+\frac{4_{C_4}}{10_{C_4}}=\frac{5}{42}$$

$$\Rightarrow 100\,p = 11.9.$$

2. Each of the nine words in the sentence "**The quick brown fox jumps over the lazy og**" is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The *expected* length of the word drawn is _____________. (The answer should be rounded to one decimal place.

Answer: (3.8889)

Exp: Given words are

THE, QUICK, BROWN, FOX, JUMPS, OVER, THE, LAXY, DOG

LET X be the random variable such that X =length of the word

The Length of the words THE, FOX, THE, DOG is 3

The Length of the words OVER, LAXY is 4

The length of the words QUICK, BROWN, JUMPS, is 5

The corresponding probabilities are given below

x	3	4	5
P(X)	4/9	2/9	3/9

Expected length of the word= $\sum xp(x) = 3\left(\frac{4}{9}\right)+4\left(\frac{2}{9}\right)+5\left(\frac{3}{9}\right)=3.8889$

3. The maximum number of edges in a bipartite graph on 12 vertices is _________________________.

Answer: (36)

Exp: The number of edges in a bipartite graph on n-vertices is atmost $\frac{n^2}{4}$

The maximum number of edges in a bipartite graph on 12 –vertices is $\frac{n^2}{4}=\frac{12\times 12}{4}=36$

4. If the matrix A is such that

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

Then the determinant of A is equal to ________.

Answer: (0)

Exp: $A = \begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & 20 \\ 7 & 63 & 35 \end{bmatrix}$

$\Rightarrow |A| = 0 \;\; (\because R_2 = -2R_1).$

5. A non-zero polynomial f(x) of degree 3 has roots at x = 1,x = 2 and x = 3. Which one of the following must be TRUE?

(A) f(0) f(4) < 0

(B) f(0) f(4) > 0

(C) f(0) + f(4) > 0

(D) f(0) + f(4) < 0

Answer: (A)

Exp: Since, the roots of f(x) = 0 i.e., x = 1, 2, 3 lies between 0 and 4 and f(x) is of degree 3

$\therefore$ f(0) and f(4) are of opposite signs

$\Rightarrow$ f(0).f(4)<0.

6. The dual of a Boolean function $F\left(x_1, x_2, ..., x_{n,} +, ., '\right)$, written as F^D, is the same expression as that of F with + and swapped. F is said to be self-dual if F = F^D. The number of self-dual functions with n Boolean variables is

(A) 2^n

(B) 2^{n-1}

(C) 2^{2^n}

(D) $2^{2^{n-1}}$

Answer: (D)

Exp: A function F is self dual if it has equal number of minterms and maxterms, also mutually exclusive terms should not be included.

The number of mutually exclusive terms (pair wise) is $\frac{2^n}{2} = 2^{n-1}$

Number of functions possible by taking any of the one term from the above mentioned mutually exclusive pair is $= 2^{2^{n-1}}$.

7. Let $k = 2^n$. A circuit is built by giving the output of an n-bit binary counter as input to an n-to-2^n bit decoder. This circuit is equivalent to a

(A) k-bit binary up counter.

(B) k-bit binary down counter.

(C) k-bit ring counter.

(D) k-bit Johnson counter.

Answer: (C)

Exp: In case of decoder output, single output will be 1 and remaining will be zero at a time. The output that is high will give the count of the ring counter at that time.

8. Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is _____ .

Answer: (3)

Exp : $(123)_5 = (x8)_y$

Converting both sides to decimal:

$\Rightarrow 25+10+3 = xy+8$

$\Rightarrow xy+8 = 38 \Rightarrow xy = 30$

$\Rightarrow x = 1, y = 30$

or , $x = 2, y = 15$ or, $x = 3,\ y = 10$

$\therefore$ Total number of solutions: 3

9. A 4-way set-associative cache memory unit with a capacity of 16 KB is built using a block size of 8 words. The word length is 32 bits. The size of the physical address space is 4 GB. The number of bits for the TAG field is _____

Answer: (20)

Exp: Physical address size = 32 bits

Cache size $= 16\text{k bytes} = 2^{14}$ Bytes

block size $= 8$ words $= 8\times 4$ Byte $= 32$ Bytes

$(\text{where each word} = \text{b Bytes})$

No. of blocks $= \dfrac{2^{14}}{2^5} = 2^9$

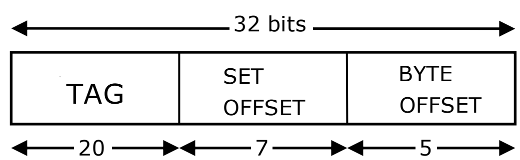
block offset $= 9^{\text{bits}}$

No. of sets $= \dfrac{2^9}{4} = 2^7$

set ofset = 7 bits

Byte offset $= 8\times 4$ Bytes $= 32$ Byte $= 2^5 = 5$ bits

TAG $= 32-(7+5) = 20$ bits



10. Consider the function func shown below:

```
int func(int num) {
int count = 0;
    while (num) {
    count++;
     num>>= 1;
}
return (count);
}
```

The value returned by func(435)is __________.

Answer: (9)

Exp:
```
int func (int num)
{
int count = 0;
while (num)                //After each right shift, checks whether the num value is not zero//
    {
    count ++;
    num >>= 1;    //shifts all bits of num one slot to the right//
    }
return(count);
}
```

Initially   num = 110110011, count = 0

count = 1; num = 101100110 after 1st right shift

count =2; num = 011001100 after 2nd right shift

:

:

Count = 9; num = 000000000 after 9th right shift.

After nine right shifts, num = 0; and while loop terminates count = 9 will be returned.

11. Suppose n and p are unsigned int variables in a C program. We wish to set p to ${}^{n}C_{3}$. If n is large, which one of the following statements is most likely to set p correctly?

(A) p = n * (n – 1) * (n-2) / 6;

(B) p = n * (n – 1) / 2 * (n-2) / 3;

(C) p = n * (n – 1) / 3 * (n-2) / 2;

(D) p = n * (n – 1) / 2 * (n-2) / 6.0;

Answer: (B)

Exp: $P = n_{C_3} = \dfrac{n(n-1)(n-2)}{6}$

It we multiply n, (n-1), (n-2) at once, it might go beyond the range of unsigned integer (resulting overflow). So options (A) and (D) are rested out. If n is even or odd n×(n-1)/2 will always result in integer value (no possibility of truncation, so more accuracy) where as incase of n*(n-1)/3, its not certain to get integer always (truncation possible, so less accuracy).

$$P = \underbrace{n*(n-1)/2}_{p_1} * \underbrace{(n-2)/3}_{p_2}$$

As P_1 will be having no error, resultant p will be more accurate.

$$P = \underbrace{n*(n-1)/3}_{p_1} * \underbrace{(n-2)/2}_{p_2}$$

As there is a possibility of truncation in P_1, there will be less accuracy in final result of P.

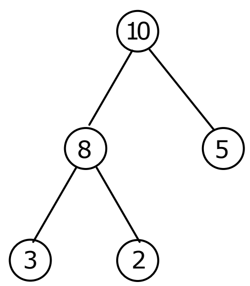
12. A priority queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is: 10, 8, 5, 3, 2. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:

(A) 10, 8, 7, 3, 2, 1, 5 (B) 10, 8, 7, 2, 3, 1, 5

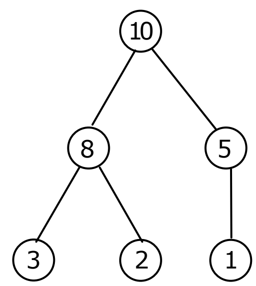
(C) 10, 8, 7, 1, 2, 3, 5 (D) 10, 8, 7, 5, 3, 2, 1

Answer: (A)

Exp: Initial max-heap is

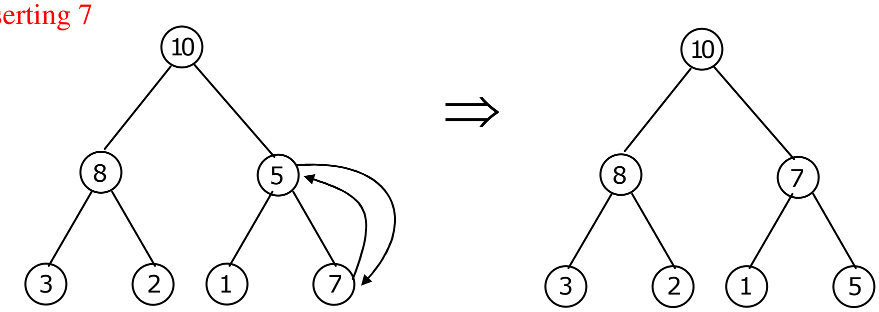


after inserting 1



Heapification is not required as it satisfies max-heap property

After inserting 7



Hence level order traversal is 10, 8, 7, 3, 2, 1, 5

13. Which one of the following correctly determines the solution of the recurrence relation with $T(1) = 1$?

$$T(n) = 2T\left(\frac{n}{2}\right) + \log n$$

(A) $\theta(n)$ (B) $\theta(n\log n)$ (C) $\theta(n^2)$ (D) $\theta(\log n)$

Answer: (A)

Exp: By Master's theorem case (i) $T(n)$ is $O(n)$

Here $a = 2, \quad b = 2, \quad f(n) = \log n$

$\log_b^a = \log_2^2 = 1$

we can choose $\in > 0$, in such a way that

$f(n) = O\left(n\log_b^{a-\in}\right)$; i.e., $\log n = O\left(n\log_b^{a-\in}\right)$

By master theorem, If $f(n) = O\left(n\log_b^{a-\in}\right)$ for some $\in > 0$, then

$T(n) = \theta\left(n\log_b^a\right) = \theta(n)$

14. Consider the tree arcs of a BFS traversal from a source node **W** in an unweighted, connected, undirected graph. The tree **T** formed by the tree arcs is a data structure for computing

(A) the shortest path between every pair of vertices.

(B) the shortest path from **W** to every vertex in the graph.

(C) the shortest paths from **W** to only those nodes that are leaves of **T**.

(D) the longest path in the graph.

Answer: (B)

Exp: One of the application of BFS algorithm is to find the shortest path between nodes u and v.

But in the given question the BFS algorithm starts from the source vertex w and we can find the shortest path from w to every vertex of the graph

15. If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, Consider

(I) $L_1.L_2$ is a regular language

(II) $L_1.L_2 = \{a^n b^n \mid n \geq 0\}$

Which one of the following is CORRECT?

(A) Only (I) (B) Only (II)

(C) Both (I) and (II) (D) Neither (I) nor (II)

Answer: (A)

Exp: L1.L2 is also regular since regular languages are closed under concatenation.

But L1.L2 is not $\{a^n b^n \mid n \geq 0\}$ because both the variable is independent in both languages.

16. Let $A \leq_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B. Which one of the following is FALSE?

(A) If $A \leq_m B$ and B is recursive then A is recursive.

(B) If $A \leq_m B$ and A is undecidable then B is undecidable.

(C) If $A \leq_m B$ and B is recursively enumerable then A is recursively enumerable.

(D) If $A \leq_m B$ and B is not recursively enumerable then A is not recursively enumerable.

Answer: (D)

Exp: A language A is mapping reducible to a language B, if there is a computable function $f: \varepsilon^* \rightarrow \varepsilon^*$ where for every $w, w \in A \Leftrightarrow f(w) \in B$

If $A \leq mB$ and B is Turing recognizable then A is Turing recognizable.

If $A \leq mB$ and B is not recursively enumerable then A is not recursively enumerable

17. Consider the grammar defined by the following production rules, with two operators □ and +

$S \to T * P$

$T \to U \mid T * U$

$P \to Q + P \mid Q$

$Q \to Id$

$U \to Id$

Which one of the following is TRUE?

(A) + is left associative, while * is right associative

(B) + is right associative, while * is left associative

(C) Both + and * are right associative

(D) Both + and * are left associative

Answer: (B)

Exp: $S \to T \times P$

$T \to U \mid T \times U$

$P \to Q + P \mid Q$

$Q \to Id$

$U \to Id$

As the production rule $T \to T \times U$ is defined as left recursive rule, so * is left associate operator.

As the production rule $P \to Q + P$ is defined as right recursive rule, so + is right associative operator.

18. Which one of the following is **NOT** performed during compilation?

(A) Dynamic memory allocation (B) Type checking

(C) Symbol table management (D) Inline expansion

Answer: (A)

Exp: Symbol table management is done during compitation to store and retriew the information about tokens. Type checking is one of the check performed during semantic analysis of compitation.

Inline expansion is a compiler optimization that replaces a function call by the bodyof respective function.

Dynamic memory allocation is when an executing pregoram request the oprating system to give it a block of main memory, so ti is performed during sum time not during complete time.

Option (A) is answer

19. Which one of the following is TRUE?

(A) The requirements document also describes how the requirements that are listed in the document are implemented efficiently.

(B) Consistency and completeness of functional requirements are always achieved in practice.

(C) Prototyping is a method of requirements validation.

(D) Requirements review is carried out to find the errors in system design.

Answer: (C)

20. A FAT (file allocation table) based file system is being used and the total overhead of each entry in the FAT is 4 bytes in size. Given a 100 x 10^6 bytes disk on which the file system is stored and data block size is 10^3 bytes, the maximum size of a file that can be stored on this disk in units of 10^6 bytes is ____________.

Answer: (99.55 to 99.65)

Exp: Number of entries in the FAT = Disk Capacity/Block size = $10^8/10^3 = 10^5$

- ⇨ Total space consumed by FAT = $10^5 * 4$ B = $0.4 * 10^6$ B
- ⇨ Maximum size of file that can be stored = $100 * 10^6 - 0.4 * 10^6 = 99.6 * 10^6$ B
- ⇨ Answer: 99.6

21. The maximum number of super keys for the relation schema R (E, F, G, H) with E as the key is __________.

Answer: (8)

Exp: The maximum number of super keys for the relation schema $R(E,F,G,H)$ with E as the key is $2^3 = 8$ as any subset of non key attributes along with key attribute will form the super key of R.

As we have 3 nonkey all (F, G and H) so subsets will be 2^3

22. Given an instance of the STUDENTS relation as shown below:

Student D	StudentName	Student Email	Student Age	CPI
2345	Shankar	Shankar@math	X	9.4
1287	Swati	swati@ee	19	9.5
7853	Shankar	Shankar@cse	19	9.4
9876	Swati	swati@mech	18	9.3
8765	Ganesh	ganesh@civil	19	8.7

For (StudentName, StudentAge) to be a key for this instance, the value X should NOT be equal to____________.

Answer: (19)

Exp: For (Student Name, student age) to be a key for given instance of STUDENTS relation, the pair value should not get repeated in any two tuples p and q (uniqueness in forced by the definition of key)

Tuple	Student Name	Student Age
P	Shankar	$\otimes \rightarrow$ should not be 19
Q	Shankar	19

23. Which one of the following is TRUE about the interior gateway routing protocols – Routing Information Protocol (RIP) and Open Shortest Path First (OSPF)?

(A) RILP uses distance vector routing and OSPF uses link state routing

(B) OSPF uses distance vector routing and RIP uses link state routing

(C) Both RIP and OSPF use link state routing

(D) Both RIP and OSPF use distance vector routing

Answer: (A)

Exp: RIP Uses Distance Vector Routing and OSPF uses Link State Routing.

24. Which one of the following socket API functions converts an unconnected active TCP socket into a passive socket?

(A) connect (B) bind (C) listen (D) accept

Answer: (C)

Exp:

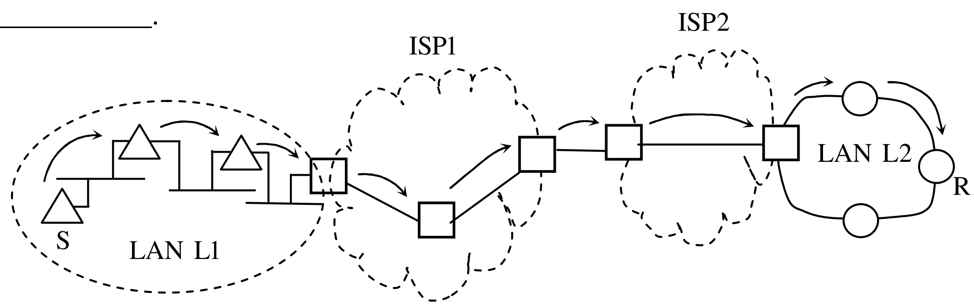
(a) The **connect function** is used by a TCP client to establish a connection with a TCP server.

(b) The **bind function** assigns a local protocol address to a socket. With the Internet protocols, the protocol address is the combination of either a 32-bit IPv4 address or a 128-bit IPv6 address, along with a 16-bit TCP or UDP port number.

(c) The **listen function** converts an unconnected socket into a passive socket, indicating that the kernel should accept incoming connection requests directed to this socket.

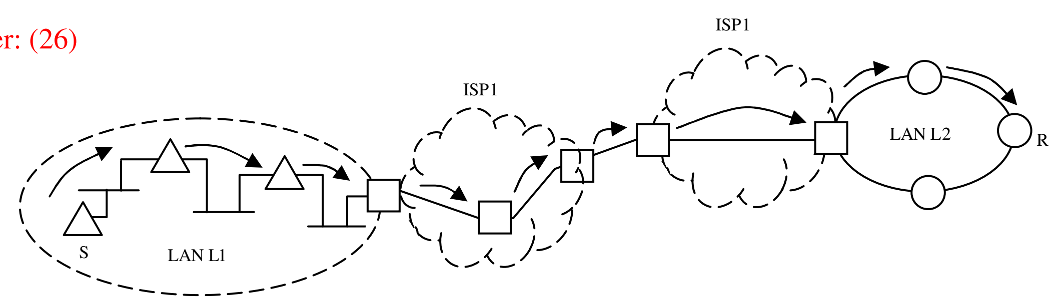
(d) The **accept function** is called by a TCP server to return the next completed connection from the front of the completed connection queue. If the completed connection queue is empty, the process is put to sleep (assuming the default of a blocking socket).

25. In the diagram shown below, L1 is an Ethernet LAN and L2 is a Token-Ring LAN. An IP packet originates from sender S and traverses to R, as shown. The links within each ISP and across the two ISPs, are all point-to-point' optical links. The initial value of the TTL field is 32. The maximum possible value of the TTL field when R receives the datagram is ____________.



Answer: (26)

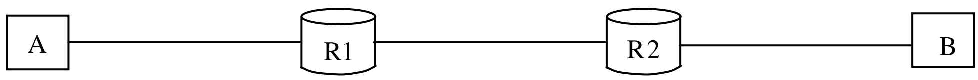
Exp:



The TTL field is set by the sender of the datagram, and reduced by every router on the route to its destination. So, there are 5visits at 5 routers and one visit at receiver R in above figure which leads 32 – 6 = 26.

Q. No. 26 – 55 Carry Two Marks Each

26. Consider the store and forward packet switched network given below. Assume that the bandwidth of each link is 10^6 bytes / sec. A user on host A sends a file of size 10^3 bytes to host B through routers R1 and R2 in three different ways. In the first case a single packet containing the complete file is transmitted from A to B. In the second case, the file is split into 10 equal parts, and these packets are transmitted from A to B. In the third case, the file is split into 20 equal parts and these packets are sent from A to B. Each packet contains 100 bytes of header information along with the user data. Consider only transmission time and ignore processing, queuing and propagation delays. Also assume that there are no errors during transmission. Let T1, T2 and T3 be the times taken to transmit the file in the first, second and third case respectively. Which one of the following is CORRECT?



(A) T1 < T2 < T3 (B) T1 > T2 > T3

(C) T2 = T3, T3 < T1 (D) T1 = T3, T3 > T2

Answer: (D)

Exp: Given Bandwidth = 10^6 bytes/sec

$L=10^3$ bytes

Case: 1

$L = 1000$ bytes

Header size $= 100$ bytes

Total Frame size $= 1000 + 100 = 1100$ bytes

$$\therefore T_x = \frac{1100 \times 8}{10^6 \times 8} = 1100\ \mu s$$

So, $T_1 = 3300\ \mu s$

Case: 2

$L = 100$ bytes

Header size $= 100$ bytes

Total Frame size $= 100 + 100 = 200$ bytes

$$\therefore T_x = \frac{200 \times 8}{10^6 \times 8} = 200\ \mu s \text{ for 1 packet}$$

For 10 packets $\Rightarrow Tx = 2000\mu s$

So, $T_2 = 2000 + 200 + 200 = 2400\ \mu s$

Case: 3

$L = 50$ bytes

Header size $= 100$ bytes

Total Frame size $= 50 + 100 = 150$ bytes

$\therefore T_x = \dfrac{150 \times 8}{10^6 \times 8} = 150\,\mu s$ for 1 packet

For 20 packets $\Rightarrow Tx = 3000\mu s$

So, $T_3 = 3000 + 150 + 150 = 3300\,\mu s$

$\therefore\ T_1 = T_3$

$T_3 > T_2$

27. An IP machine Q has a path to another IP machine H via three IP routers R1, R2, and R3.

Q—R1—R2—R3—H

H acts as an HTTP server, and Q connects to H via HTTP and downloads a file. Session layer encryption is used, with DES as the shared key encryption protocol. Consider the following four pieces of information:

[I1] The URL of the file downloaded by Q

[I2] The TCP port numbers at Q and H

[I3] The IP addresses of Q and H

[I4] The link layer addresses of Q and H

Which of I1, I2, I3, and I4 can an intruder learn through sniffing at R2 alone?

(A) Only I1 and I2 (B) Only I1 (C) Only I2 and I3 (D) Only I3 and I4

Answer: (C)

Exp: An Intruder can't learn [I1] through sniffing at R2 because URLs and Download are functioned at Application layer of OSI Model.

An Intruder can learn [I2] through sniffing at R2 because Port Numbers are encapsulated in the payload field of IP Datagram.

An Intruder can learn [I3] through sniffing at R2 because IP Addresses and Routers are functioned at network layer of OSI Model.

An Intruder can't learn [I4] through sniffing at R2 because it is related to Data Link Layer of OSI Model.

28. A graphical HTML browser resident at a network client machine *Q* accesses a static HTML webpage from a HTTP server *S*. The static HTML page has exactly one static embedded image which is also at *S*. Assuming no caching, which one of the following is correct about the HTML webpage loading (including the embedded image)?

(A) Q needs to send at least 2 HTTP requests to S, each necessarily in a separate TCP connection to server S

(B) Q needs to send at least 2 HTTP requests to S, but a single TCP connection to server S is sufficient

(C) A single HTTP request from Q to S is sufficient, and a single TCP connection between Q and S is necessary for this

(D) A single HTTP request from Q to S is sufficient, and this is possible without any TCP connection between Q and S

Answer: (B)

29. Consider the following schedule S of transactions T1, T2, T3, T4:

<table>
<tr><th>T1</th><th>T2</th><th>T3</th><th>T4</th></tr>
<tr><td rowspan="4">Writes [x]
Commit</td><td>Reads[X]</td><td></td><td></td></tr>
<tr><td></td><td>Writes[X]
Commit</td><td></td></tr>
<tr><td>Writes[Y]
Reads [Z]
Commit</td><td></td><td></td></tr>
<tr><td></td><td></td><td>Reads[X]
Reads[Y]
Commit</td></tr>
</table>

Which one of the following statements is CORRECT?

(A) S is conflict-serializable but not recoverable

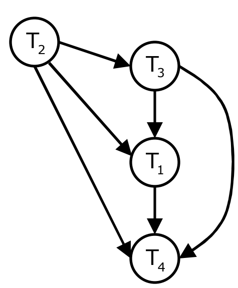
(B) S is conflict-serializable but is recoverable

(C) S is both conflict-serializable and recoverable

(D) S is neither conflict-serializable nor is it recoverable

Answer: (C)

Exp: The precedence graph of schedule s is a follows.



$\Rightarrow$ No cycle, so S is conflict serializable

In the schedule S of transactions T_1, T_2, T_3 and T_4 for each pair of transaction T_i and T_j, such that T_j reads a data item previously written by T_i the commit operation of T_j appears after the commit operation of T_i hence the schedule is recoverable schedule.

30. Consider a join (relation algebra) between relations r(R)and s(S) using the nested loop method. There are 3 buffers each of size equal to disk block size, out of which one buffer is reserved for intermediate results. Assuming size(r(R))<size(s(S)), the join will have fewer number of disk block accesses if

(A) relation r(R) is in the outer loop.

(B) relation s(S) is in the outer loop.

(C) join selection factor between r(R) and s(S) is more than 0.5.

(D) join selection factor between r(R) and s(S) is less than 0.5.

Answer: (A)

Exp: A join between r(R) and s (S) using nested loop method will be as follows.

For each tuple r in R do

For each tuple s in S do

If r and s satisfy the join condition then output the tuple <r,s>

This algorithm will involve $n_r*b_s+b_r$ block transfers and n_r+b_r seeks, where b_r and b_s are number of blocks in relations R and S respectively and n_r is number of tuple in relation R. Now to have less block accesses, n_r should be less and it is already given that |R|<|S|. Relation r(R) should be in the outer loop to have fewer number of disk block accesses.

31. Consider the procedure below for the *Producer-Consumer* problem which uses semaphores:

Semaphore n = 0;

Semaphore s = 1;

```
Void producer ( )                    Void consumer( )
{                                      {
    while (true)                       while(true)
    {                                  {
    Produce ( );                       semWait (s) ;
    SemWait (s);                       semWait (n) ;
    addToBuffer ( );                 removeFromBuffer ( ) ;
    semSignal (s);                     semsignal (s);
    semSignal (n);                     consume ( ) ;
    }                                  }
}                                        }
```

Which one of the following is TRUE?

(A) The producer will be able to add an item to the buffer, but the consumer can never consume it.

(B) The consumer will remove no more than one item from the buffer.

(C) Deadlock occurs if the consumer succeeds in acquiring semaphore **s** when the buffer is empty.

(D) The starting value for the semaphore **n** must be 1 and not 0 for deadlock-free operation.

Answer: (C)

Exp: (A) The producer will be able to add an item to the buffer, but the consumer can never consume if given statement is false, because once producer produces an item and places in buffer, the next turn to execute can be given to consumer ( ). [The value of s = 1 and n=1]. So consumer will be definitely able to consume it by performing successful down operations on s and n.

(B) The consumer will remove no more than one item from the buffer.

Given statement is false as if p( ) produces and adds to buffer, (c) will consume the added item, this sequence of alteration (p and c) will always make consumer to remove items from buffer.

This statement would have been true if it was said that the consumer will remove no more than one item from the buffer one after the other. (at a time).

(C) Dead lock occurs if the consumer succeeds in acquiring semaphore's' when the buffer is empty. Given statement is true as when buffer is empty initially if consumer gets the turn to execute as follows:-

$S = 1, \quad n = 0;$

consumer( )
{
  while (True)
  {
    $p(s); [s = \cancel{1}0]$
    $p(s); [m = 0;$ it blocks consumer$]$
    }
}

producer( )
{
  while (True)
  {
    producer( );
    $p(s); [s = 0;$ it blocks $p(\ )]$
    }
}

So from the above execution both producer and consumer goes in block state waiting for each other to wake them up and hence dead lock occurs.

(D) Even if the starting value for the semaphore 'n' becomes 1, there will be invalid execution of consumer on buffer empty condition, which should not happen. So statement is false.

32. Three processes A, B and C each execute a loop of 100 iterations. In each iteration of the loop, a process performs a single computation that requires t_c CPU milliseconds and then initiates a single I/O operation that lasts for t_{io} milliseconds. It is assumed that the computer where the processes execute has sufficient number of I/O devices and the OS of the computer assigns different I/O devices to each process. Also, the scheduling overhead of the OS is negligible. The processes have the following characteristics:

Process id	t_c	t_{io}
A	100ms	500ms
B	350ms	500ms
C	200ms	500ms

The processes A, B, and C are started at times 0, 5 and 10 milliseconds respectively, in a pure time sharing system (round robin scheduling) that uses a time slice of 50 milliseconds. The time in milliseconds at which process C would ***complete*** its first I/O operation is ___________.

Answer: (1000)

Exp:

Process id	t_c	t_{io}	A.T	TQ = 50ms
A	100 ms	500 ms	0 ms	
B	350 ms	500 ms	5 ms	
C	200 ms	500 ms	10 ms	

The Gantt chart for Round robin algorithm for the first iteration execution for each of the 3 processes is as follows:

R .Q ~~A~~ ~~B~~ ~~C~~ A

A	B	C	A	B	C	B	C	B	C	B

0 50 100 150 200 250 300 350 400 450 500 650

A goes for I/o operation

C goes for I/o operation

After finishing t_c CPU ms at time 500ms, C goes for I/O operation, that needs 500ms more, so the time at which process C would complete its first I/O operations is 500+500 = 1000ms

33. A computer has twenty physical page frames which contain pages numbered 101 through 120. Now a program accesses the pages numbered 1, 2, …, 100 in that order, and repeats the access sequence THRICE. Which one of the following page replacement policies experiences the same number of page faults as the optimal page replacement policy for this program?

(A) Least-recently-used (B) First-in-first-out

(C) Last-in-first-out (D) Most-recently-used

Answer: (D)

Exp: Page reference string for the program will be:-

1, 2, 3, 4, ------------100, 1, 2, 3, 4, ------------100, 1, 2, 3, 4, ------------100,

The current status of 20 frames shows page numbers from 101 to 120.

Implementation of optimal page replacement policy for above given page reference string would be as follows:

0	101	1	40	41	80	81	20	21	60	61	100	1	40	41	80	81
1	102	2	39	42	79	82										
2	103	3	38	43	78	83										
3	104	4	37	44	77	84										
4	105	5	36	45	76	85										
5	106	6	35	46	75	86										
6	107	7	34	47	74	87										
7	108	8	33	48	73	88										
8	109	9	32	49	72	89										
9	110	10	31	50	71	90										
10	111	11	30	51	70	91										
11	112	12	29	52	69	92										
12	113	13	28	53	68	93										
13	114	14	27	54	67	94										
14	115	15	26	55	66	95										
15	116	16	25	56	65	96										
16	117	17	24	57	64	97										
17	118	18	23	58	63	98										
18	119	19	22	59	62	99										
19	120	20	21	60	61	100	1	40	41	80	81	20	21	60	61	100
		↓ 20PF	↓ 20PF	↓ 20PF	↓ 20PF	↓ 20PF	↓ 20PF	↓ 20PF	↓ 20PF	↓ 20PF	↓ 20PF					
		1st time access					2nd time access					3rd time access				

So there would be 300 page faults in total (each access 100 page faults).

Also it is visible that every time a replacement is done for the page which is most recently referred as it will be least recently referred in future. So for the given page reference string optimal page replacement policy is working same as most recently used policy and thus number of page faults will be same in both of them.

34. For a C program accessing **X[i] [j] [k]**, the following intermediate code is generated by a compiler. Assume that the size of an **integer** is 32 bits and the size of a **character** is 8 bits.

t0 = i * 1024

t1 = j * 32

t2 = k * 4

t3 = t1 + t0

t4 = t3 + t2

t5 = X[t4]

Which one of the following statements about the source code for the C program is **CORRECT**?

(A) X is declared as "int X[32] [32] [8]".

(B) X is declared as "int X[4] [1024] [32]".

(C) X is declared as "char X[4] [32] [8]".

(D) X is declared as "char X[32] [16] [2]".

Answer: (A)

Exp: It is given that Size of int is 4B and of char is 1B. The memory is byte addressable.

Let the array be declared as Type X[A][B][C] (where Type = int/char and A,B,C are natural numbers).

From t0 = i*1024, we conclude that B*C*(size of Type) = 1024.

From t1 = j*32, we conclude that C*(size of Type) = 32.

From t2 = k*4, we conclude that size of Type = 4.

⇨ Type = int, and

⇨ C = 8, and

⇨ B = 32.

35. Let < M > be the encoding of a Turing machine as a string over $\Sigma = \{0,1\}$. Let L = {<M> |M is a Turning machine that accepts a string of length 2014}. Then, L is

(A) decidable and recursively enumerable

(B) undecidable but recursively enumerable

(C) undecidable and not recursively enumerable

(D) decidable but not recursively enumerable

Answer: (B)

Exp: The language accepted by the Turing machine is recursively enumerable. If is undecidable as the Turing machine may halt or it may loop for the strings whose length is not equal to 2014.

36. Let $L_1 = w \in \{0,1\}^* \mid w$ has at least as many occurrences of (110)'s as (011)'s}. Let $L_2 = \{w \in \{0,1\}^* \mid$ w has at least as many occurrence of (000)'s as (111)'s}. Which one of the following is TRUE?

(A) L_1 is regular but not L_2

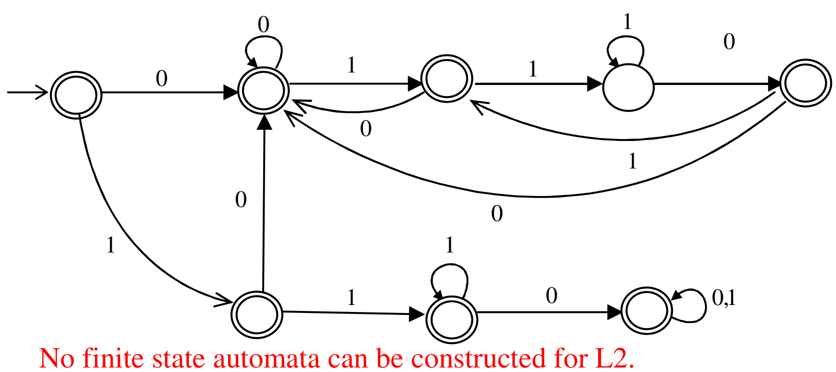
(B) L_2 is regular but not L_1

(C) Both L_1 and L_2 are regular

(D) Neither L_1 nor L_2 are regular

Answer: (A)

Exp: The automaton for L1 is as follows:



No finite state automata can be constructed for L2.

37. Consider two strings $A = "q\ pqrr"$ and $B = "pqprqrp"$. Let x be the length of the longest common subsequence (not necessarily contiguous) between A and B and let y be the number of such longest common subsequences between A and B. Then $x + 10y =$ ___.

Answer: (34)

Exp: Given is

A = "qpqrr" B = "pqprqrp"

The longest common subsequence (not necessarily contiguous) between A and B is having 4 as the length, so x=4 and such common subsequences are as follows:

(1) qpqr

(2) pqrr

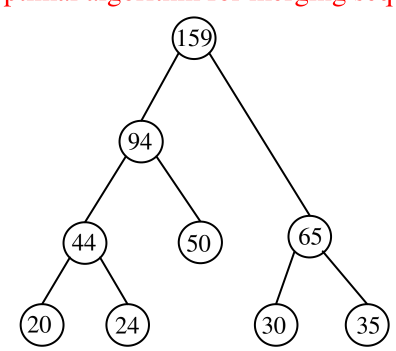
(3) qprr

So y = 3 (the number of longest common subsequences) hence x+10y = 4+10*3 = 34.

38. Suppose P, Q, R, S, T are sorted sequences having lengths 20, 24, 30, 35, 50 respectively. They are to be merged into a single sequence by merging together two sequences at a time. The number of comparisons that will be needed in the worst case by the optimal algorithm for doing this is ____.

Answer: (358)

Exp: The implementation of optimal algorithm for merging sequences is as follows.

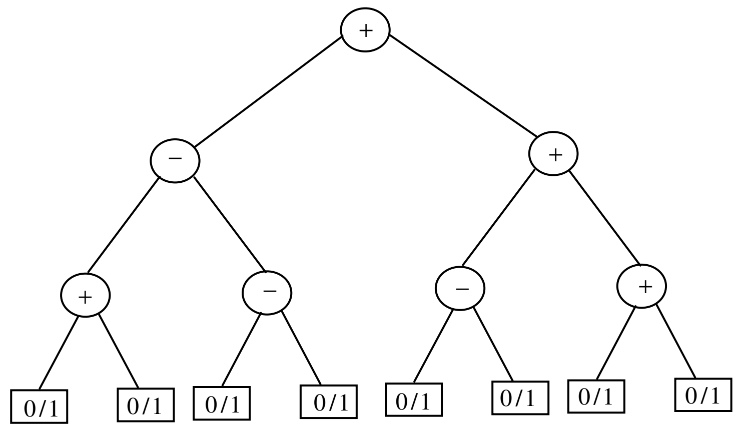


In the above implementation, total number of comparisons is

(44-1)+(94-1)+(65-1)+(159-1) = 358

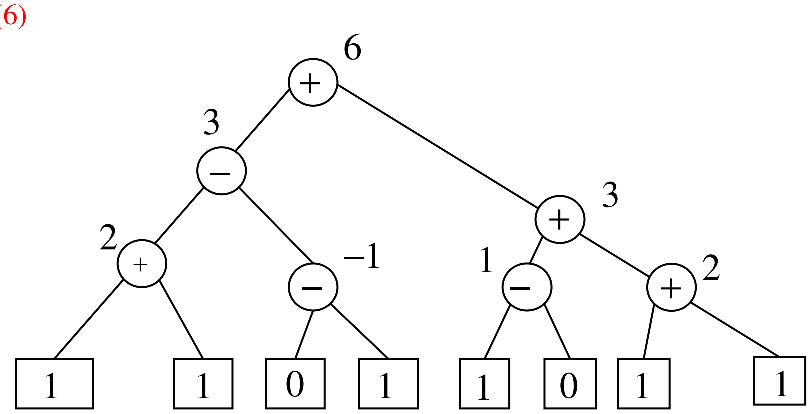
Hint: The number of comparisons for merging two sorted sequences of length m and n is m+n-1.

39. Consider the expression tree shown. Each leaf represents a numerical value, which can either be 0 or 1. Over all possible choices of the values at the leaves, the maximum possible value of the expression represented by the tree is ___.



Answer: (6)

Exp:



So as per the above tree where leaves have been given the values, the maximum possible value of the expression represented by the tree is 6.

40. Consider the following function

```
double  f (double x) {
if ( abs (x*x – 3) <  0. 01) return x;
else return f (x / 2 + 1.5/x);
}
```

Give a value q (to 2 decimals) such that f (q) will return q:______

Answer: (1.72 to 1.74)

Exp: If condition given in function definition should be 'TRUE', for $f(q)$ to return value q.

The condition is as follows:

$\text{if } (\text{abs}(x \times x - 3) < 0.01) \text{ return } x;$

The above condition will be true when x=1.73.

41. Suppose a stack implementation supports an instruction REVERSE, which reverses the order of elements on the stack, in addition to the PUSH and POP instructions. Which one of the following statements is TRUE with respect to this modified stack?

(A) A queue cannot be implemented using this stack.

(B) A queue can be implemented where ENQUEUE takes a single instruction and DEQUEUE takes a sequence of two instructions.

(C) A queue can be implemented where ENQUEUE takes a sequence of three instructions and DEQUEUE takes a single instruction.

(D) A queue can be implemented where both ENQUEUE and DEQUEUE take a single instruction each.

Answer: (C)

Exp: Option (a) is false because queue can be implemented by using the modified stack as by reversing the stack. LIFO will become FIFO.
Implementation of ENQUEUE & DEQUEUE takes four sequence of instructions as follows:

1. Enqueue: Reverse, Push, Revesre
   Dequeue: POP
   (OR)
2. Enqueue: Push
   Dequeue: Reverse, POP, Reverse

42. Consider the C function given below

```
int f(int j)
{
static int i = 50;
int k;
if (i == j)
  {
printf("something");
k = f(i);
return 0;
}
else return 0;
}
```

Which one of the following is **TRUE**?

(A) The function returns 0 for all values of j.

(B) The function prints the string something for all values of j.

(C) The function returns 0 when j = 50.

(D) The function will exhaust the runtime stack or run into an infinite loop when j = 50.

Answer: (D)

Exp: For any value of 'j' other than 50 the function will return 0, for j=50, then condition (i==j) will be true, it will print "something" and function will be called recursively with same value till the run time stack overflows.

43. In designing a computer's cache system, the cache block (or cache line) size is an important Parameter. Which one of the following statements is correct in this context?

(A) A smaller block size implies better spatial locality

(B) A smaller block size implies a smaller cache tag and hence lower cache tag overhead

(C) A smaller block size implies a larger cache tag and hence lower cache hit time

(D) A smaller block size incurs a lower cache miss penalty

Answer: (D)

Exp: When a cache block size is smaller, it could accommodate more number of blocks, it improves the hit ratio for cache, so the miss penalty for cache will be lowered.

44. If the associativity of a processor cache is doubled while keeping the capacity and block size unchanged, which one of the following is guaranteed to be NOT affected?

(A) Width of tag comparator

(B) Width of set index decoder

(C) Width of way selection multiplexor

(D) Width of processor to main memory data bus

Answer: (D)

Exp: When associativity is doubled, then the set offset will be effected, accordingly, the number of bits used for TAG comparator be effected.

Width of set index decoder also will be effected when set offset is changed.

Width of wag selection multiplexer wil be effected when the block offset is changed.

With of processor to main memory data bus is guaranteed to be NOT effected.

45. The value of a *float* type variable is represented using the single-precision 32-bit floating point format of IEEE-754 standard that uses 1 bit for sign, 8 bits for biased exponent and 23 bits for mantissa. A *float* type variable X is assigned the decimal value of -14.25. The representation of X in hexadecimal notation is

(A) C1640000H (B) 416C0000H (C) 41640000H (D) C16C0000H

Answer: (A)

Exp: (A)

S	E	M
1 bit	8 bits	23 bits

$(14.25)_{10} = (1110.01)_2$

1110.01×2^0

$= 1.11001 \times 2^3$

$S: 1$

$E: (3+127)_{10} = (10000010)_2$

$M: 11001000-----$

Value stored:

$\underbrace{1100}_{[C}\ \underbrace{0001}_{1}\ \underbrace{0110}_{6}\ \underbrace{0100}_{4}\ 00---$

$0\ 0\ 0\ 0\ ]_H$

46. In the Newton-Raphson method, an initial guess of $x_0 = 2$ is made and the sequence $x_0, x_1, x_2 \ldots$ is obtained for the function

$$0.75x^3 - 2x^2 - 2x + 4 = 0$$

Consider the statements

(I) $x_3 = 0$.

(II) The method converges to a solution in a finite number of iterations.

Which of the following is TRUE?

(A) Only I (B) Only II (C) Both I and II (D) Neither I nor II

Answer: (A)

Exp: $f(x) = 0.75\,x^3 - 2x^2 - 2x + 4;\ f'(x) = 2.25x^2 - 4x - 2$

$x_0 = 2,\ f_0 = -2;\ f_0' = -1$

$\therefore x_1 = x_0 - \dfrac{f_0}{f_0'} = 0$

$\Rightarrow f_1 = 4;\ f_1' = -2$

$\therefore x_2 = x_1 - \dfrac{f_1}{f_1'} = 2$

$\Rightarrow f_2 = -2,\ f_2' = -1$

$\therefore x_3 = x_2 - \dfrac{f_2}{f_2'} = 0$

Also, root does not lies between 0 and 1.

So, the method diverges if $x_0 = 2$

$\therefore$ only (I) is true.

47. The product of the non-zero eigenvalues of the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

is_______.

Answer: (6)

Exp: Let $A = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$

Let $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$ be eigen vector

By the definition of eigen vector, $AX = \lambda X$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \lambda\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$$

$x_1 + x_5 = \lambda x_5$

$x_2 + x_3 + x_4 = \lambda x_2$

$x_2 + x_3 + x_4 = \lambda x_3$

$x_2 + x_3 + x_4 = \lambda x_4$

$x_1 + x_5 = \lambda x_4 \Rightarrow x_1 + x_5 = \lambda x_5 = \lambda x_4$

and $x_2 + x_3 + x_4 = \lambda x_2 = \lambda x_3 = \lambda x_4$

(1) If $\lambda \neq 0$ say $x_1 = x_5 = a$

$x_2 = x_3 = x_4 = b$

$\Rightarrow x_1 + x_5 = \lambda a \Rightarrow 2a = \lambda a \Rightarrow \lambda = 2$

$x_2 + x_3 + x_4 = \lambda a \Rightarrow 3a = \lambda a \Rightarrow \lambda = 3$

(2) if $\lambda = 0 \Rightarrow$ eigen value =0

$\therefore$ Three distinct eigen values are 0,2,3 product of non zero eigen values $=2\times3=6$

48. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is _______ .

Answer: (0.259 to 0.261)

Exp: Let A = divisible by 2, B = divisible by 3 and C = divisible by 5, then

$n(A) = 50$, $n(B) = 33$, $n(C) = 20$

$n(A\cap B) = 16$, $n(B\cap C) = 6$, $n(A\cap C) = 10$

$n(A\cap B\cap C) = 3$

$P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C) = 74/100$

$\therefore$ Required probability is $P\left(\overline{A} \cap \overline{B} \cap \overline{C}\right) = 1 - P\left(A \cup B \cup C\right) = 0.26.$

49. The number of distinct positive integral factors of 2014 is ___________________________

Answer: (8)

Exp: $2014 = 2\times19\times53$ i.e., product of prime factors

$\therefore$ Number of distinct positive integral factors of 2014 is $(2)\times(2)\times(2) = 8$.

50. Consider the following relation on subsets of the set S of integers between 1 and 2014. For two distinct subsets U and V of S we say $U < V$ if the minimum element in the symmetric difference of the two sets is in U.

Consider the following two statements:

S1: There is a subset of S that is larger than every other subset.
S2: There is a subset of S that is smaller than every other subset.

Which one of the following is CORRECT?

(A) Both S1 and S2 are true
(B) S1 is true and S2 is false
(C) S2 is true and S1 is false
(D) Neither S1 nor S2 is true

Answer: (A)

Exp: From given data S1 is true ,since null set is larger than every other set ,and S2 is true since the universal set $\{1,2,...,2014\}$ is smaller than every other set.

Both s1 and s2 are true.

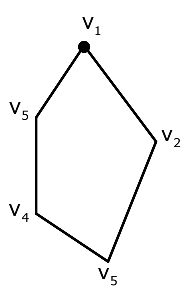
51. A cycle on *n* vertices is isomorphic to its complement. The value of *n* is _____.

Answer: (5)

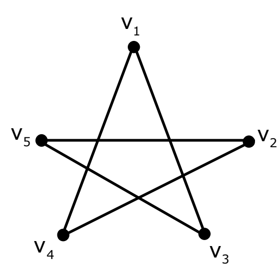
Exp: The number of edges in C_n is n where as the number of edges in $C_n^{'}$ is $n_{c_2} - n$

Cycle graph C_n and its complement $C_n^{'}$ have different number of edges if $n \neq 5$

Consider a cycle on five vertices C_5



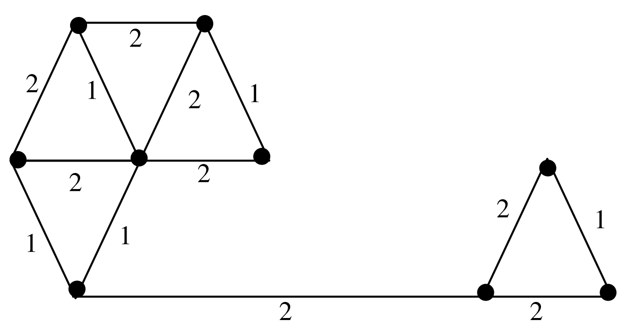
C_5



$\left(\text{complement of } C_5\right) = C_5^{'}$

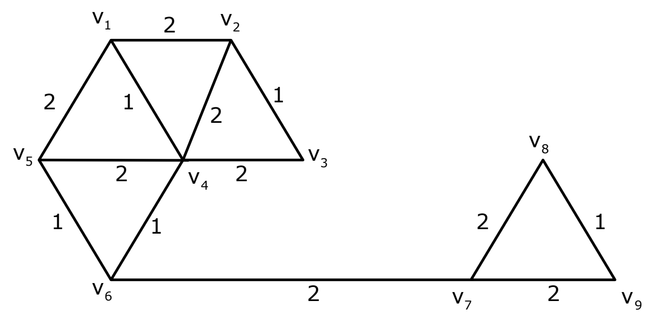
C_5 and $C_5^{'}$ are isomorphic
$\therefore n = 5$

52. The number of distinct minimum spanning trees for the weighted graph below is



Answer: (6)

Exp: Consider the connected weighted graph (Application of Kruskal's Algorithm)



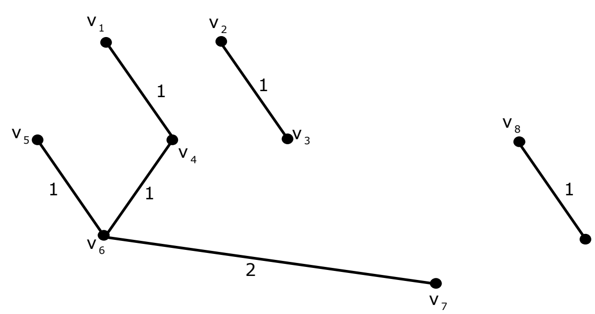
Sort the edges by increasing edges costs (weights)

$[\{v_1, v_4\}, \{v_2, v_3\}, \{v_4, v_6\}, \{v_5, v_6\}, \{v_8, v_9\} (\text{Cost} - 1)$

$[\overbrace{\{v_1, v_2\}, \{v_2, v_4\}, \{v_4, v_3\}}^{3}, \{\cancel{v_4, v_5}\}, \{\cancel{v_1, v_5}\}$

$\{\cancel{v_6, v_7}\}, \underbrace{\{v_7, v_8\}, \{v_7, v_9\}}_{2} (\text{cost} - 2)]$

Selection of edges of cost-1, will not form a cycle, so



Selection of $\{v_4, v_5\}$ and $\{v_1, v_5\}$ forms a cycle, so we will not consider the edges. The edge $\{v_6, v_7\}$ can be choosen because of connectedness.

Selection of $\{v_1, v_2\}$ and $\{v_7, v_8\}$ forms one minimum spanning tree.

Selection of $\{v_1, v_2\}$ and $\{v_7, v_9\}$ forms one minimum spanning tree.

Selection of $\{v_2, v_4\}$ and $\{v_7, v_8\}$ forms one minimum spanning tree.

Selection of $\{v_2, v_4\}$ and $\{v_7, v_9\}$ forms one minimum spanning tree.

$$\left.\begin{array}{l}\text{Selection of } \{v_3, v_4\} \text{ and } \{v_7, v_8\} \\ \text{Selection of } \{v_3, v_4\} \text{ and } \{v_7, v_9\}\end{array}\right\} \text{forms two minimum spanning trees}$$

$\therefore$ There are 6 distinct minimum spanning trees.

53 Which one of the following Boolean expressions is NOT a tautology?

$(A)\ ((a \to b) \wedge (b \to c)) \to (a \to c)$

$(B)\ (a \leftrightarrow c) \to (\sim b \to (a \wedge c))$

$(C)\ (a \wedge b \wedge c) \to (c \wedge a)$

$(D)\ a \to (b \to a)$

Answer: (B)

Exp: (D) $a \to (b \to a)(\Rightarrow \neg a \vee (b \to a))(\Rightarrow \neg a \vee (\neg b \vee a)) \Leftrightarrow T(\text{tautology})$

(C) $(a \wedge b \wedge c) \to (c \vee a) \Leftrightarrow \neg(a \wedge b \wedge c) \vee (c \vee a)$

$\Leftrightarrow (\neg a \vee \neg b \vee \neg c) \vee (c \vee a)$

$\Leftrightarrow (\neg a \vee a) \vee P\ (\text{where P is disjunction of literals})$

$\Leftrightarrow T \vee P \Rightarrow T$

(a) $\begin{array}{l} a \to b \\ \underline{b \to c} \\ \therefore a \to c \end{array}$ by hypothetical syllogism

$((a \to b) \wedge (b \to c)) \to (a \to c)$ is a tautology

Answer is B which is not tautology.

54 SQL allows duplicate tuples in relations, and correspondingly defines the multiplicity of tuples in the result of joins. Which one of the following queries always gives the same answer as the nested query shown below:

Select * from R where a in (select S. a from S)

(A) Select R. * from R, S where R. a=S. a

(B) Select distinct R. * from R,S where R. a=S. a

(C) Select R. * from R, (select distinct a from S) as S1 where R. a=S1.a

(D) Select R. * from R, S where R.a = S. a and is unique R

Answer: (C)

Exp: Consider the following instances of R & S

a	b	c
1	2	3
1	2	3
3	4	5
3	4	5

a d e

1	2	3
2	3	4
3	4	6
3	4	6

O/P of given nested query is

a	b	c
1	2	3
1	2	3
3	4	5
3	4	5

(A) O/P:- multiplicity of tuples is disturbed

a	b	c
1	2	3
1	2	3
3	4	5
3	4	5
3	4	5
3	4	5

(B) O/P:-

a	b	c
1	2	3
3	4	5

(C) O/P:-

Multiplicity of tuples is maintained

a	b	c
1	2	3
1	2	3
3	4	5
3	4	5

Multiplicity of duplicate tuples will be distributed when there is a match between R.a and S.a and for that match S.a's value is repeated.

55 Consider a main memory system that consists of 8 memory modules attached to the system bus, which is one word wide. When a write request is made, the bus is occupied for 100 nanoseconds (ns) by the data, address, and control signals. During the same 100 ns, and for 500 ns thereafter, the addressed memory module executes one cycle accepting and storing the data. The (internal) operation of different memory modules may overlap in time, but only one request can be on the bus at any time. The maximum number of stores (of one word each) that can be initiated in 1 millisecond is ____________

Answer: (10000)

Exp: Each write request, the bus is occupied for 100 n.s

Storing of data requires 100 n.s.

$$\text{In } 100 \text{ n.s} - 1 \text{ store}$$

$$\frac{100}{10^6} \text{ n.s} = 1 \text{ store}$$

$$1 \text{ m.s} = \frac{10^6}{100} \text{stores}$$

$$= 10000 \text{ stores}$$

Q. No. 1 – 5 Carry One Mark Each

1. While trying to collect an envelope from under the table, Mr. X fell down and was losing consciousness.
   (I: While trying to collect; II: from under the table; III: Mr. X fell down and; IV: was losing consciousness.)

   Which one of the above underlined parts of the sentence is NOT appropriate?

   (A) I (B) II (C) III (D) IV

Answer: (D)

2. If she ______________ how to calibrate the instrument, she ______________ done the experiment.

   (A) knows, will have
   (B) knew, had
   (C) had known, could have
   (D) should have known, would have

Answer: (C)

3. Choose the word that is opposite in meaning to the word "coherent".

   (A) sticky (B) well-connected (C) rambling (D) friendly

Answer: (C)

4. Which number does not belong in the series below?

   2, 5, 10, 17, 26, 37, 50, 64

   (A) 17 (B) 37 (C) 64 (D) 26

Answer: (C)

5. The table below has question-wise data on the performance of students in an examination. The marks for each question are also listed. There is no negative or partial marking in the examination.

Q.No	Marks	Answered Correctly	Answered Wrongly	Not Attempted
1	2	21	17	6
2	3	15	27	2
3	2	23	18	3

What is the average of the marks obtained by the class in the examination?

(A) 1.34 (B) 1.74 (C) 3.02 (D) 3.91

Answer: (C)

Exp: Total question

$44\times2=88$

$44\times3=132$

$$\frac{144}{132} = \frac{88}{308}$$

Total marks obtained= $(21\times2) + (15\times3) + (23\times2) = 133$

Total Number of students=44

$$\text{Average} = \frac{133}{44} = 3.02$$

Q. No. 6 – 10 Carry One Mark Each

6. A dance programme is scheduled for 10.00 a.m. Some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is

(A) Students should come at 9.00 a.m. and parents should come at 10.00 a.m.

(B) Participating students should come at 9.00 a.m. accompanied by a parent, and other parents and students should come by 10.00 a.m.

(C) Students who are not participating should come by 10.00 a.m. and they should not bring their parents. Participating students should come at 9.00 a.m.

(D) Participating students should come before 9.00 a.m. Parents who accompany them should come at 9.00 a.m. All others should come at 10.00 a.m.

Answer: (B)

7. By the beginning of the 20th century, several hypotheses were being proposed, suggesting a paradigm shift in our understanding of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun.

Which of the following inference(s) may be drawn from the above passage?

(i) Our understanding of the universe changes based on the positions of stars

(ii) Paradigm shifts usually occur at the beginning of centuries

(iii) Stars are important objects in the universe

(iv) Experimental evidence was important in confirming this paradigm shift

(A) (i), (ii) and (iv) (B) (iii) only (C) (i) and (iv) (D) (iv) only

Answer: (D)

8. The Gross Domestic Product (GDP) in Rupees grew at 7% during 2012-2013. For international comparison, the GDP is compared in US Dollars (USD) after conversion based on the market exchange rate. During the period 2012-2013 the exchange rate for the USD increased from Rs. 50/ USD to Rs. 60/ USD. India's GDP in USD during the period 2012-2013

(A) increased by 5 % (B) decreased by 13%

(C) decreased by 20% (D) decreased by 11%

Answer: (D)

Exp: Per 100 Rs final value 107 Rs

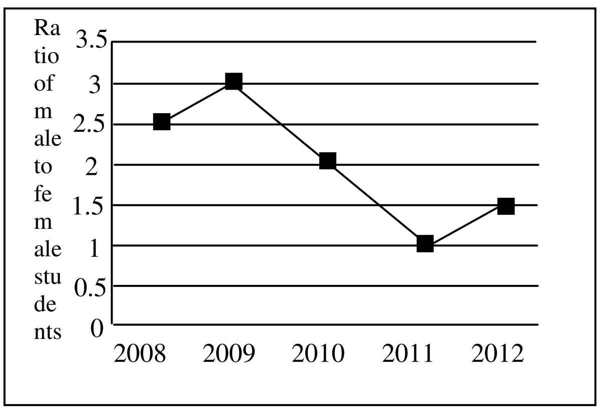
$\Rightarrow$ Per $\frac{100}{50}$ Dollars final value $\frac{107}{60}$

for 100 dollars____?

$= \frac{100\times 50}{100}\times\frac{107}{60} = 89.16$

Decreased by 11%.

9. he ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students in 2011 and 2012 is equal, what is the ratio of male students in 2012 to male students in 2011?



(A) 1:1 (B) 2:1 (C) 1.5:1 (D) 2.5:1

Answer: (C)

Exp: Take number of female students in 2011=100

$\therefore$ Number of male in 2011=100

No. of female in 2012=100

No. of male in 2012=150

$\text{Ratio} = \frac{150}{100}$ = 1.5: 1

10. Consider the equation: $(7526)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y.

(A) 1634 (B) 1737 (C) 3142 (D) 3162

Answer: (C)

Q. No. 1 – 25 Carry One Mark Each

1. Consider the following statements:

   P: Good mobile phones are not cheap

   Q: Cheap mobile phones are not good

   L: P implies Q

   M: Q implies P

   N: P is equivalent to Q

   Which one of the following about L, M, and N is **CORRECT**?

   (A) Only L is TRUE. (B) Only M is TRUE.

   (C) Only N is TRUE. (D) L, M and N are TRUE.

Answer: (D)

Exp: g : mobile is good c : mobile is cheap

P : Good mobile phones are not cheap $\cong g \to \neg c \cong (\neg g \vee \neg c)$ $[\because a \to b \equiv \neg a \vee b]$

Q : Cheap mobile phones are not good $\cong c \to \neg g \cong (\neg c \vee \neg g)$

$\therefore$ Both P and Q are equivalent which means P and Q imply each other

2. Let X and Y be finite sets and $f : X \to Y$ be a function. Which one of the following statements is TRUE?

   (A) For any subsets A and B of X, $|f(A \cup B)| = |f(A)| + |f(B)|$

   (B) For any subsets A and B of X, $f(A \cap B) = f(A) \cap f(B)$

   (C) For any subsets A and B of X, $|f(A \cap B)| = \min\{|f(A)|, |f(B)|\}$

   (D) For any subsets S and T of Y, $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

Answer: (D)

Exp: $f : X \to Y$ defined by $f(a) = 1, f(b) = 1, f(c) = 2$ where

$X = \{a, b, c\}\ Y = \{1, 2\}$

Let $A = \{a, c\}, B = \{b, c\}$ be subsets of X

then $|f(A \cup B)| = 2$; $|f(A)| = 2$; $|f(B)| = 2$

$f(A \cap B) = \{2\}$; $f(A) = \{1, 2\}$; $f(B) = \{1, 2\}$

$f(A) \cap f(B) = \{1, 2\}$

$|f(A \cap B)| = 1$

$\therefore$ Options (A), (B), (C) are not true

Hence, option (D) is true

3. Let G be a group with 15 elements. Let L be a subgroup of G. It is known that $L \neq G$ and that the size of L is at least 4. The size of L is _______.

Answer: (5)

Exp: Order of subgroup divides order of group (Lagrange's theorem).

3, 5 and 15 can be the order of subgroup. As subgroup has atleast 4 elements and it is not equal to the given group, order of subgroup can't be 3 and 15. Hence it is 5.

4. Which one of the following statements is TRUE about every $n \times n$ matrix with only real eigenvalues?

(A) If the trace of the matrix is positive and the determinant of the matrix is negative, at least one of its eigenvalues is negative.

(B) If the trace of the matrix is positive, all its eigenvalues are positive.

(C) If the determinant of the matrix is positive, all its eigenvalues are positive.

(D) If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

Answer: (A)

Exp: If the trace of the matrix is positive and the determinant of the matrix is negative then atleast one of its eigen values is negative.

Since determinant = product of eigen values.

5. If V_1 and V_2 are 4-dimensional subspaces of a 6-dimensional vector space V, then the smallest possible dimension of $V_1 \cap V_2$ is _______.

Answer: (2)

Exp: Let the basis of 6-dimensional vector space be {e1, e2, e3,e4, e5, e6}. In order for *V1* ∩ *V2* to have smallest possible dimension V1 and V2 could be, say, {e1, e2, e3,e4} and {e3, e4, e5, e6} respectively. The basis of *V1* ∩ *V2* would then be {e3, e4}. => Smallest possible dimension = 2.

6. If $\int_0^{2\pi} |x \sin x| \; dx = k\pi$, then the value of k is equal to _______.

Answer: (4)

Exp: $$\int_0^{2\pi} |x\sin x|dx = K\pi \Rightarrow \int_0^{\pi} x\sin x\,dx + \int_{\pi}^{2\pi} -(x\sin x)dx = K\pi \left[\begin{array}{l}\because |\sin x| = -\sin x \\ \pi < x < 2\pi\end{array}\right]$$

$$\Rightarrow x(-\cos x) - 1(-\sin x)\Big|_0^{\pi} - (-x\cos x + \sin x)\Big|_{\pi}^{2\pi} = K\pi$$

$$\Rightarrow (-\pi\cos\pi + \sin\pi) - 0 - \left[(-2\pi\cos 2\pi + \sin 2\pi) - (-\pi\cos\pi + \sin\pi)\right] = K\pi$$

$$\Rightarrow \pi + 0 - \left[-2\pi + 0 - (\pi + 0)\right] = K\pi \Rightarrow 4\pi = K\pi \Rightarrow k = 4$$

7. Consider the following minterm expression for F.

$F(P,Q,R,S) = \sum 0, 2, 5, 7, 8, 10, 13, 15$

The minterms 2, 7, 8 and 13 are ‘do not care terms. The minimal sum of-products form for F is

(A) $Q\bar{S}+\bar{Q}S$

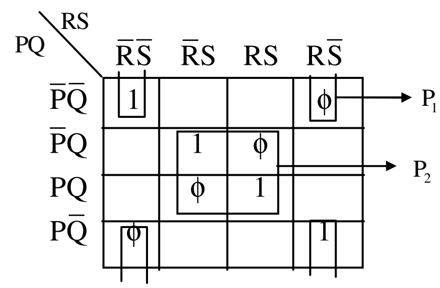
(B) $\bar{Q}\bar{S}+QS$

(C) $\bar{Q}\,\bar{R}\,\bar{S}+\bar{Q}\,R\,\bar{S}+Q\bar{R}S+QRS$

(B) $\bar{P}\bar{Q}\bar{S}+\bar{P}QS+PQS+P\bar{Q}\bar{S}$

Answer: (B)

Exp: The K-map for the function F is as follows:-



$P_1=\bar{Q}\bar{S}$ and $P_2=QS$

$\therefore F(P,Q,R,S)=P_1+P_2$

$=\bar{Q}\bar{S}+QS$

8. Consider the following combinational function block involving four Boolean variables x, y, a, b where x, a, b are inputs and y is the output.

```
f (x,  y,  a,  b)
{
    if (x  is  1) y = a;
    else y  =  b;
}
```

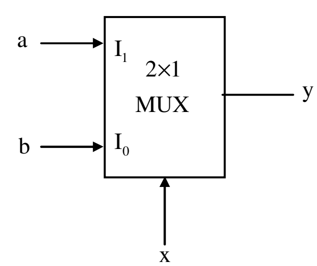
Which one of the following digital logic blocks is the most suitable for implementing this function?

(A) Full adder (B) Priority encoder (C) Multiplexor (D) Flip-flop

Answer: (C)

Exp: $y=\bar{x}b+xa$

‘x’ is working as selection line, where the two input lines are ‘a’ and ‘b’, so the function $F(x,y,a,b)$ can be implemented using (2×1) multiplexer as follows:



9. Consider the following processors (ns stands for nanoseconds). Assume that the pipeline registers have zero latency.

P1: Four-stage pipeline with stage latencies 1 ns, 2 ns, 2 ns, 1 ns.

P2: Four-stage pipeline with stage latencies 1 ns, 1.5 ns, 1.5 ns, 1.5 ns.

P3: Five-stage pipeline with stage latencies 0.5 ns, 1 ns, 1 ns, 0.6 ns, 1 ns.

P4: Five-stage pipeline with stage latencies 0.5 ns, 0.5 ns, 1 ns, 1 ns, 1.1 ns.

Which processor has the highest peak clock frequency?

(A) P1 (B) P2 (C) P3 (D) P4

Answer: (C)

Exp: Clock period (CP) = max stage delay + overhead

So $CP_{P1} = Max(1,2,2,1) = 2ns$

$CP_{P2} = Max(1, 1.5, 1.5, 1.5) = 1.5 ns$

$CP_{P3} = Max(0.5, 1,1,0.6,1) = 1ns$

$CP_{P4} = Max(0.5,0.5,1,1,1.1) = 1.1ns$

As frequency $\alpha \frac{1}{C.P}$, so least clock period will give the highest peak clock frequency.

So, $f_{p3} = \frac{1}{1ns} = 1GHz$

10. Let A be a square matrix size $n \times n$. Consider the following pseudocode. What is the expected output?

```
C = 100;
for i = 1 to n do
for j = 1 to n do
{
  Temp = A[ i ] [ j ] + C ;
  A [ i ] [ j ] = A [ j ] [ i ] ;
  A [ j ] [ i ] = Temp – C ;
}
for i = 1 to n do
 for j = 1 to n do
  output (A[ i ] [ j ]);
```

(A) The matrix A itself

(B) Transpose of the matrix A

(C) Adding 100 to the upper diagonal elements and subtracting 100 from lower diagonal elements of A

(D) None of these

Answer: (A)

Exp: In the computation of given pseudo code for each row and column of Matrix A, each upper triangular element will be interchanged by its mirror image in the lower triangular and after that the same lower triangular element will be again re-interchanged by its mirror image in the upper triangular, resulting the final computed Matrix A same as input Matrix A.

11. The minimum number of arithmetic operations required to evaluate the polynomial $P(X) = X^5 + 4X^3 + 6x + 5$ for a given value of X, using only one temporary variable is _____.

Answer: (7)

Exp: $P(x)=x^5+4x^3+6x+5$ can be re written as follows

$P(x)=x^3(x^2+4)+6x+5$

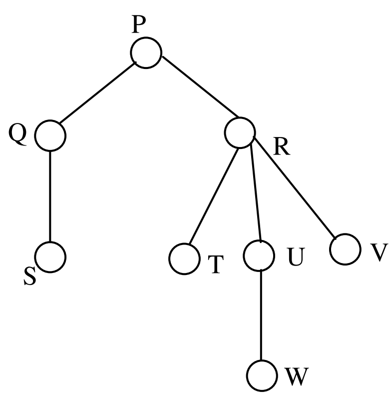
Now using only one temporary variable t and any number of data transfer as well as memory related operation the polynomial can be evaluated as follows

1. $t=x*x$ [Evaluate x^2 and store in memory]
2. $t=t+4$ [Evaluate (x^2+4) and store in memory]
3. $t=x^2$ [Retinue x^2 from memory]
4. $t=t*.X$ [Evaluate x^3 and store in memory]
5. $t=t*(x^2+4)$ [Evaluate $x^3(x^2+4)$ and store in memory]
6. $t=6*x$ [Evaluate 6x and store in memory]
7. $t=t+5$ [Evaluate $(6x+5)$ and store in memory]
8. $t=t+x^3(x^2+4)$ [Retrieve $x^2(x^2+4)$ from memory and evaluate $\{x^3(x^2+4)+6x+5\}$

In the above 8 steps of evaluation, the total number of arithmetic operations required and 7 [4 Multiplications, 3 Additions]

So answer is 7 arithmetic operations.

12. Consider the following rooted tree with the vertex labelled P as the root



The order in which the nodes are visited during an in-order traversal of the tree is

(A) SQPTRWUV (B) SQPTUWRV

(C) SQPTWUVR (D) SQPTRUWV

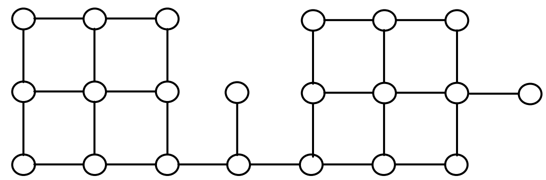
Answer: (A)

Exp: The In order Traversal of Ternary Tree is done as follows:

Left $\rightarrow$ Root $\rightarrow$ Middle $\rightarrow$ Right

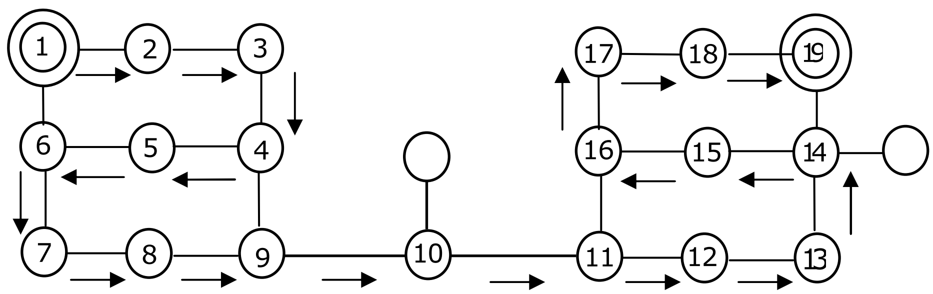
So the nodes are visited in SQPTRWUV order.

13. Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is _________.



Answer: 19

Exp:



Suppose, we start DFS at vertex numbered as 1 and continue calling recursive function for DFS on subsequent nodes numbered in ascending order.

The recursive calling sequence is shown as marked line in the above diagram which shows maximum possible recursion depth including the initial call is 19.

14. You have an array of n elements. Suppose you implement quick sort by always choosing the central element of the array as the pivot. Then the tightest upper bound for the worst case performance is

(A) $O(n^2)$ (B) $O(n\log n)$ (C) $\theta(n\log n)$ (D) $O(n^2)$

Answer: (A)

Exp: The Worst case time complexity of quick sort is O (n^2). This will happen when the elements of the input array are already in order (ascending or descending), irrespective of position of pivot element in array.

15. The length of the shortest string NOT in the language $\left(\text{over } \Sigma = \{a, b\}\right)$ of the following regular expression is _________. a*b* (ba)* a*

Answer: (3)

Exp: R.E= $a^*b^*(ba)^*a^*$

Length 0 is present as it accepts $\in$ all length 1 strings are present (a, b) also aa, ab, ba, bb are present, But 'bab' is not present. So it is 3

16. Let Σ be a finite non-empty alphabet and let $2\Sigma^*$ be the power set of Σ^*. Which one of the following is **TRUE**?

(A) Both $2\Sigma^*$ and Σ^* are countable

(B) $2\Sigma^*$ is countable Σ^* is uncountable

(C) $2\Sigma^*$ is uncountable and Σ^* is countable

(D) Both $2\Sigma^*$ and Σ^* are uncountable

Answer: (C)

Exp: 2^{ε^*} is the power set of ε^*

ε^* is countabily infinite.

The power set of countabily infinite set is uncountable.

So 2^{ε^*} is uncountable, and ε^* is countable.

17. One of the purposes of using intermediate code in compilers is to

(A) make parsing and semantic analysis simpler.

(B) improve error recovery and error reporting

(C) increase the chances of reusing the machine-independent code optimizer in other compliers.

(D) improve the register allocation.

Answer: (C)

Exp: Intermediate code is machine independent code which makes it easy to retarget the compiler to generate code for newer and different processors.

18. Which of the following statements are CORRECT?

1) Static allocation of all data areas by a compiler makes it impossible to implement recursion.

2) Automatic garbage collection is essential to implement recursion.

3) Dynamic allocation of activation records is essential to implement recursion.

4) Both heap and stack are essential to implement recursion.

(A) 1 and 2 only (B) 2 and 3 only (C) 3 and 4 only (D) 1 and 3 only

Answer: (D)

Exp: To implement recursion, activation record should be implemented by providing dynamic memory allocation. This dynamic allocation is done from runtime stack. Heap is essential to allocate memory for data structures at run-time, not for recursion.

So statement 1 and 3 are correction.

19. In the context of modular software design, which one of the following combinations is desirable?

(A) High cohesion and high coupling (B) High cohesion and low coupling

(C) Low cohesion and high coupling (D) Low cohesion and low coupling

Answer: (B)

Exp: Cohesion is a measure of internal strength within a module, whereas coupling is a measure of inter dependency among the modules. So in the context of modular software design there should be high cohesion and low coupling.

20. A system uses 3 page frames for storing process pages in main memory. It uses the Least Recently Used (LRU) page replacement policy. Assume that all the page frames are initially empty. What is the total number of page faults that will occur while processing the page reference string given below?

4, 7, 6, 1, 7, 6, 1, 2, 7, 2

Answer: (6)

Exp: Page reference string is
4, 7, 6, 1, 7, 6, 1, 2, 7, 2
Implementing LRU using 3 page frames
Total page faults = 6

$\cancel{4}$ 1
$\cancel{2}$ 2
$\cancel{6}$ 7

21. What is the optimized version of the relation algebra expression $\pi_{A1}(\pi_{A2}(\sigma_{F1}(r)))$,, where A1, A2 are sets of attributes in with $A_1 \subset A_2$ and F1, F2 are Boolean expressions based on the attributes in r?

(A) $\pi_{A_1}(\sigma(F1 \wedge F2)(r))$
(B) $\pi_{A_1}(\sigma(F1 \vee F2)(r))$
(C) $\pi_{A_2}(\sigma(F1 \wedge F2)(r))$
(D) $\pi_{A2}(\sigma(F1 \vee F2)(r))$

Answer: (A)

Exp: π is used to select a subset of attributes and σ_p is used to select subset of tuples matching the predicate P.

$$\Pi_{A_1}\left(\Pi_{A_2}\left(\sigma_{F_1}\left(\sigma_{F_2}(r)\right)\right)\right) = \Pi_{A_1}\left(\Pi_{A_2}\left(\sigma(F_1 \wedge F_2)(r)\right)\right)$$

and as $A_1 \subset A_2$, so final relation will be displaying values for attributes present in set A.

$$\Pi_{A_1}\left(\Pi_{A_2}\left(\sigma(F_1 \wedge F_2)(r)\right)\right) = \Pi_{A_1}\left(\sigma(F_1 \wedge F_2)(r)\right)$$

22. A prime attribute of a relation scheme R is an attribute that appears

(A) in all candidate keys of R.
(B) in some candidate key of R.
(C) in a foreign keys of R.
(D) only in the primary key of R.

Answer: (B)

Exp: A prime attribute or key attribute of a relation scheme R is an attribute that appears in any of the candidate key of R, remaining attributes are known as non-prime or non-key tribute

23. In the following pairs of OSI protocol layer/sub-layer and its functionality, the **INCORRECT** pair is

(A) Network layer and Routing
(B) Data Link Layer and Bit synchronization
(C) Transport layer and End-to-end process communication
(D) Medium Access Control sub-layer and Channel sharing

Answer: (B)

Exp: (a) One of the main functionality of Network Layer is Routing. So Option (a) is CORRECT.

(b) Bit Synchronization is always handled by Physical Layer of OSI model but not Data Link Layer. So

Option (b) is INCORRECT.

(c) End – to – End Process Communication is handled by Transport Layer. So Option (c) is CORRECT.

(d) MAC sub layer have 3 types of protocols (Random, Controlled and Channelized Access).

24. A bit-stuffing based framing protocol uses an 8-bit delimiter pattern of 01111110. If the output bit-string after stuffing is 01111100101, then the input bit-string is

(A) 0111110100 (B) 0111110101

(C) 0111111101 (D) 0111111111

Answer: (B)

Exp: Given 8 – bit delimiter pattern of 01111110.

Output Bit string after stuffing is $011111\underline{0}0101$

$\Downarrow$

StuffedBit

Now, Input String is 0111110101

25. Host A (on TCP/IP v4 network A) sends an IP datagram D to host B (also on TCP/IP V4 network B). Assume that no error occurred during the transmission of D. When D reaches B, which of the following IP header field(s) may be different from that of the original datagram D?

(i) TTL (ii) Checksum (iii) Fragment Offset

(A) (i) only (B) (i) and (ii) only

(C) (ii) and (iii) only (D) (i), (ii) and (iii)

Answer: (D)

Exp: While an IP Datagram is transferring from one host to another host, TTL, Checksum and Fragmentation Offset will be changed.

Q. No. 26 – 55 Carry Two Marks Each

26. An IP router implementing Classless Inter-domain routing (CIDR) receives a packet with address 131.23.151.76. The router's routing table has the following entries:

Prefix	Output Interface Identifier
131.16.00/12	3
131.28.0.0/14	5
131.19.0.0/16	2
131.22.0.0/15	1

The identifier of the output interface on which this packet will be forwarded is _____.

Answer: (1)

Exp: Given address 131.23.151.76.coming to the first field of given routing table

$\Rightarrow$ 131.16.0.0/12

131.0001 0111.151.76

131.0001 0000.0.0 ($\because$ given mask bits = 12)

$\Rightarrow$ 131.16.0.0 Matched

Coming to the 2nd field of given Routing table

$\Rightarrow$ 131.28.0.0/14

131.0001 0111.151.76

131.0001 0100.0.0 ($\because$ given mask bits = 14)

$\Rightarrow$ 131.20.0.0 Not matched.

Coming to the 3rd field of given Routing table

Error! Not a valid link. 131.19.0.0/16

131.0001 0111.151.76

131.0001 0111.0.0 ($\because$ given mask bits = 16)

$\Rightarrow$ 131.23.0.0 Not matched

Coming to the 4th field of given Routing table

$\Rightarrow$ 131.22.0.0/15

131.0001 0111.151.76

131.0001 0110.0.0 ($\because$ given mask bits = 15)

$\Rightarrow$ 131.22.0.0 Matched

We are getting 1st and 4th entries are matched so among them we have to picked up the longest mask bit , so output interface identifier is 1.

27. Every host in an IPv4 network has a 1-second resolution real-time clock with battery backup. Each host needs to generate up to 1000 unique identifiers per second. Assume that each host has a globally unique IPv4 address. Design a 50-bit globally unique ID for this purpose. After what period (in seconds) will the identifiers generated by a host wrap around?

Answer: (256)

Exp: Given that each host has a globally unique IPv4 Address and we have to design 50 – bit unique Id. So, 50 – bit in the sense (32 + 18). So, It is clearly showing that IP Address (32 – bit) followed by 18 bits.

1000 unique Ids => 1Sec

2^{18} unique Ids => 2^{18} / 1000 = 2^{8} = 256

28. An IP router with a Maximum Transmission Unit (MTU) of 1500 bytes has received an IP packet of size 4404 bytes with an IP header of length 20 bytes. The values of the relevant fields in the header of the third IP fragment generated by the router for this packet are

(A) MF bit: 0, Datagram Length: 1444; Offset: 370

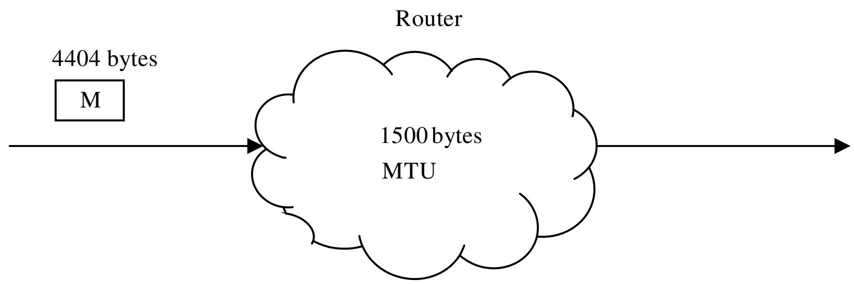
(B) MF bit: 1, Datagram Length: 1424; Offset: 185

(C) MF bit: 1, Datagram Length: 1500; Offset: 370

(D) MF bit: 0, Datagram Length: 1424; Offset: 2960

Answer: (A)

Exp:



DataLength	1480	1480	1444
	+	+	
HeaderLength	20	20	
MF	1	1	0

29. Consider the transactions T1, T2, and T3 and the schedules S1 and S2 given below.

T1 : r1 (X) ; r1 (z) ; w1 (X) ; w1 (z)

T2 : r2 (X) ; r2 (z) ; w2 (z)

T3 : r3 (X) ; r3 (X) ; w3 (Y)

S1: r1(X); r3(Y); r3(X); r2(Y); r2(Z); w3(Y); w2(Z); r1(Z); w1(X); w1(Z)

S2: r1(X); r3(Y); r2(Y); r3(X); r1(Z); r2(Z); w3(Y); w1(X); w2(Z); w1(Z)

Which one of the following statements about the schedules is **TRUE**?

(A) Only S1 is conflict-serializable.

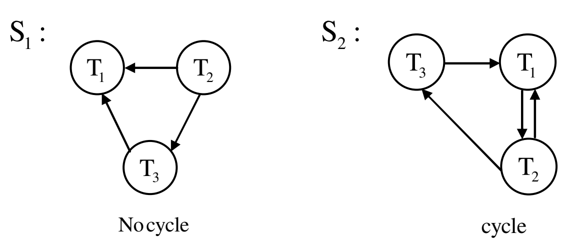
(B) Only S2 is conflict-serializable.

(C) Both S1 and S2 are conflict-serializable.

(D) Neither S1 nor S2 is conflict-serializable.

Ans: (A)

Exp: Precedence graph for S_1 & S_2 are as follows



$\therefore$ Only S_1 is conflict serializable.

30. Consider the relational schema given below, where eId of the relation dependentis a foreign key referring to empId of the relation employee. Assume that every employee has at least one associated dependent in the dependent relation.

employee $\left(\underline{empId}, empName, empAge\right)$

dependent $(\underline{depId, eId}, depName, depAge)$

Consider the following relational algebra query

$$\Pi_{empId}\,(employee) - \Pi_{empId}\,(employee \rhd\!\lhd (empId = eID) \wedge (empAge \le depAge) dependent)$$

The above query evaluates to the set of empIds of employees whose age is greater than that of

(A) some dependent. (B) all dependents.

(C) some of his/her dependents. (D) all of his/her dependents

Answer: (D)

Exp:

$$\underbrace{\prod_{empId}(employee) - \overbrace{\prod_{empId}\left(employee \bowtie_{(empId = eID) \wedge (empAge \le depAge)} dependent\right)}^{A}}_{B}$$

Part A of the above given relational algebra query will give the set of empIds of those employees whose age is less than or equal to the age of some of his/her dependents.

Now when set of empIds of all employees minus set of empIds obtained from part A is done, then we get the set of empIds of employees whose age is greater than that of all of his/her dependents.

31. A system contains three programs and each requires three tape units for its operation. The minimum number of tape units which the system must have such that deadlocks never arise is __________.

Answer: (7)

Exp:

	Maximum	Allocate	Need	Available
P_1	-3	2	1	1
P_2	-3	2	1	
P_3	-3	2	1	

With the above given data, after allocating 2 units of tape to each process, with 1 available unit any of the 3 process can be satisfied in such a way, that No dead lock will be there.

So answer is 7 tape units.

Q.32 An operating system uses *shortest remaining time first* scheduling algorithm for pre-emptive scheduling of processes. Consider the following set of processes with their arrival times and CPU burst times (in milliseconds):

Process	Arrival Time	Burst Time
P1	0	12
P2	2	4
P3	3	6
P4	8	5

The average waiting time (in milliseconds) of the processes is __________.

Answer: (5.5)

Exp: The Gantt chart for SRTF scheduling algorithm is as follows:

P_1	P_2	P_3	P_4	P_1

0 2 6 12 17 27

$$\text{Average waiting time} = \frac{15+0+3+4}{4} = \frac{22}{4} = 5.5$$

33. Consider a paging hardware with a TLB. Assume that the entire page table and all the pages are in the physical memory. It takes 10 milliseconds to search the TLB and 80 milliseconds to access the physical memory. If the TLB hit ratio is 0.6, the effective memory access time (in milliseconds) is __________.

Answer: (122)

Exp: $T_{ave} = H_1 \times (T_{TLB} + T_M) + (1 - H_1) \times (T_{TLB} + 2 \times T_M)$

T_{TLB} = time to search in TLB = 10ms

T_M=time to access physical memory = 30ms

H_1 = TLB hit ratio = 0.6

$T_{ave} = 0.6 \times (10+80) + (1-0.6)(10+2\times 80)$

$T_{ave} = 0.6 \times 90\,\text{ms} + 0.4(170)\,\text{ms}$

$T_{ave} = 54\,\text{ms} + 68\,\text{ms} = 122\,\text{ms}$

34. Consider the basic block given below.

$a = b + c$

$c = a + d$

$d = b + c$

$e = d - b$

$a = e + b$

The minimum number of nodes and edges present in the DAG representation of the above basic block respectively are

(A) 6 and 6 (B) 8 and 10 (C) 9 and 12 (D) 4 and 4

Answer: (A)

Exp:

The given basic block can be rewritten as

$$\begin{array}{lcl} a = b + c & & a = b + c \\ c = a + d & & c = b + c + d \\ d = b + c & \Rightarrow & d = b + b + c + d = 2b + c + d \\ e = d - b & & e = \cancel{b} + b + c + d - \cancel{b} = b + c + d \\ a = e + b & & a = b + b + c + d = 2b + c + d \end{array}$$

From above simplification it is visible that e is same as c and final value of a is same as d. So the final basic block can be written as follows:

$a = b + c$

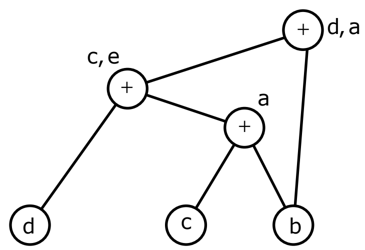
$c = a + d$

$d = 2b + c + d$

$e = c$

$a = d$

The DAG generated for the above basic block in as



Maximum number of modes and edges in above DAG is $(6,6)$

35. Which one of the following problems is undecidable?

(A) Deciding if a given context-free grammar is ambiguous.

(B) Deciding if a given string is generated by a given context-free grammar.

(C) Deciding if the language generated by a given context-free grammar is empty.

(D) Deciding if the language generated by a given context-free grammar is finite.

Answer: (A)

Exp: There were algorithms to find the membership of CFG (using CYK algorithm) and finiteness of CFG (using CNF graph) and emptiness. But there is no algorithm for ambiguity of CFG, so it is undecidable.

36. Consider the following languages over the alphabet $\Sigma = \{0,1,c\}$

$L_1 = \{0^n 1^n \mid n \geq 0\}$

$L_2 = \{wcw^r \mid w \in \{0,1\}^*\}$

$L_3 = \{ww^r \mid w \in \{0,1\}^*\}$

Here w^r is the reverse of the string w. Which of these languages are deterministic Context-free languages?

(A) None of the languages

(B) Only L_1

(C) Only L_1 and L_2

(D) All the three languages

Answer: (C)

Exp: For the languages L_1 and L_2 we can have deterministic push down automata, so they are DCFL's, but for L_3 only non-deterministic PDA possible. So the language L_3 is not a deterministic CFL.

37. Suppose you want to move from 0 to 100 on the number line. In each step, you either move right by a unit distance or you take a shortcut. A shortcut is simply a pre-specified pair of integers i,j with

$i < j$. Given a shortcut i, j if you are at position i on the number line, you may directly move to j. Suppose T(k) denotes the smallest number of steps needed to move from k to 100. suppose further that there is at most 1 shortcut involving any number, and in particular from 9 there is a shortcut to 15. Let y and z be such that T(9) = 1 + min(T(y), T(z)). Then the value of the product yz is_______.

Answer: (150)

Exp: By definition, T(9) = Dist. From 9 to 100

As given, T(9) = 1+min (T(y), T()z) = 1+min (Dist. from y to 100, Dist. From z to 100)

$\Rightarrow$1=Dist. from 9 to y/Dist. From 9 to z

$\Rightarrow$ There are only two such values-one is the simple one step on number line i.e. 10, and the other is the shortcut associated with 9 i.e. 15.

$\Rightarrow$ Therefore, y and z are 10 and 15 (in any order)

$\Rightarrow$ Product yz = 150. Answer

38. Consider the decision problem 2CNFSAT defined as follows:

$\{\ \phi \mid \phi$ is a satisfiable propositional formula in CNF with at most two literal per clause$\}$

For example, $\phi = (x_1 \vee x_2) \wedge (x_1 \vee \overline{x_3}) \wedge (x_2 \vee x_4)$ is a Boolean formula and it is in 2CNFSAT.

The decision problem 2CNFSAT is

(A) NP-Complete.

(B) solvable in polynomial time by reduction to directed graph reachability.

(C) solvable in constant time since any input instance is satisfiable.

(D) NP-hard, but not NP-complete.

Answer: (B)

Exp: 2 SAT is in P. This we can prove by reducing 2 SAT to directed graph reachability problem which is known to be in P.

Procedure for reducing 2 SAT to reachability problem:

1. Let φ be CNF with clauses of length 2 and let P be the set of propositional variables(literals) in φ
2. Build a graph G=(V,E) with $V = P \cup \{\neg p \mid p \in P\}$ and $(x, y) \in E$ iff there is a clause in φ that is equivalent to $x \to y$ (all the clauses are converted to equivalent implications and the graph built is called as implication graph)
3. Observe that φ is unsatisfiable iff there is a $p \in P$ such that there is both a path from p to to $\neg p$ and from $\neg p$ to p in G.

   This condition can be tested by running the reachability algorithm several times.

39. Suppose we have a balanced binary search tree T holding n numbers. We are given two numbers L and H and wish to sum up all the numbers in T that lie between L and H. Suppose there are m such numbers in T. If the tightest upper bound on the time to compute the sum is $O(n^a \log^b n + m^c \log^d n)$, the value of a + 10b + 100c + 1000d is _______.

Answer: (110)

Exp: It takes (log n) time to determine numbers n_1 and n_2 in balanced binary search tree T such that

1. n_1 is the smallest number greater than or equal to L and there is no predecessor n'_1 of n_1 such that n'_1 is equal to n_1.
2. n_2 is the largest number less than or equal to H and there is no successor of n'_2 of n_2 such that is equal to n_2.

Since there are m elements between n_1 and n_2, it takes 'm' time to add all elements between n_1 and n_2.

So time complexity is O (log n + m)

So the given expression becomes O (n^o log' n + m' $\log^o$ n)

And a+ 10b+ 100c+1000d = 0 + 10*1 + 100*1 + 1000*1 = 10 + 100 = 110

Because a = 0, b = 1, c = 1 and d =0

40. Consider a hash table with 100 slots. Collisions are resolved using chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?

(A) $(97 \times 97 \times 97)/100^3$

(B) $(99 \times 98 \times 97)/100^3$

(C) $(97 \times 96 \times 95)/100^3$

(D) $(97 \times 96 \times 95)/(3! \times 100^3)$

Answer: (A)

Exp:**A**

P(First insertion in such a way that first 3 slots are unfilled) $= \frac{97C_1}{100C_1} = \frac{97}{100}$

B

P(second insertion in such a way that first 3 slots are unfilled) $= \frac{97C_1}{100C_1} = \frac{97}{100}$ [$\because$ chaining is used to resolve collision, so second insertion can be done at same index as first index]

C

P(Third insertion in such a way that first 3 slots are unfilled) $= \frac{97C_1}{100C_1} = \frac{97}{100}$ [$\because$ Third insertion can be done at same index as first or second index]

So Total prob. $P(A) \times P(B) \times P(C)$

$$= \frac{97}{100} \times \frac{97}{100} \times \frac{97}{100} = \frac{(97 \times 97 \times 97)}{100^3}$$

41. Consider the pseudocode given below. The function Dosomething () takes as argument a pointer to the root of an arbitrary tree represented by the leftMostChild-rightSibling representation. Each node of the tree is of type treeNode.

```
typedef struct treeNode* treeptr;
Struct treeNode
{
    Treeptr leftMostchild, rightSibiling;
};
Int Dosomething (treeptr tree)
{
    int value =0;
    if (tree ! = NULL)   {
      If  (tree -> leftMostchild = = NULL)
else
    value = Dosomething (tree->leftMostchild);
value = value + Dosometing (tree->rightsibiling);
}
return (value);
}
```

When the pointer to the root of a tree is passed as the argument to **DoSomething**, the value returned by the function corresponds to the

(A) number of internal nodes in the tree.

(B) height of the tree.

(C) number of nodes without a right sibling in the tree.

(D) number of leaf nodes in the tree.

Answer: (D)

Exp: The key to solving such questions is to understand or detect where/by what condition the value (or the counter) is getting incremented each time.

Here, that condition is if (tree→leftMostchild == Null)

⇒ Which means if there is no left most child of the tree (or the sub-tree or the current node-as called in recursion)

⇒ Which means there is no child to that particular node (since if there is no left most child, there is no child at all).

⇒ Which means the node under consideration is a leaf node.

⇒ The function recursively counts, and adds to value, whenever a leaf node is encountered.

⇒ The function returns the number of leaf nodes in the tree.

42. Consider the C function given below. Assume that the array listA contains n (> 0) elements, sored in ascending order.

```
int ProcessArray (int * listA, int x, int n)
```

```
{
 Int 1, j, k;
i = 0;
j = n – 1;
do {
    k = (i + j) /2;
  if (x < = listA [k])
      j = k – 1;
If (listA [k] < = x)
    i = k+1;
}while (1 < = j);
If (listA [k] = = x)
    return (k) ;
else
   return -1;
}
```

Which one of the following statements about the function **ProcessArray is CORRECT**?

(A) It will run into an infinite loop when x is not in listA.

(B) It is an implementation of binary search

(C) It will always find the maximum element in listA.

(D) It will return – 1 even when x is present in listA.

Answer: (B)

Exp: By the logic of the algorithm it is clear that it is an attempted implementation of Binary Search. So option C is clearly eliminated. Let us now check for options A and D.

A good way to do this is to create small dummy examples (arrays) and implement the algorithm as it is. One may make any array of choice. Running iterations of the algorithm would indicate that the loop exits when the x is not present. So option A is wrong. Also, when x is present, the correct index is indeed returned. D is also wrong. Correct answer is B. It is a correct implementation of Binary Search.

43. An instruction pipeline has five stages, namely, instruction fetch (IF), instruction decode and register fetch (ID/RF), instruction execution (EX), memory access (MEM), and register write back (WB) with stage latencies 1 ns, 2.2 ns, 2 ns, 1 ns, and 0.75 ns, respectively (ns stands for nanoseconds). To gain in terms of frequency, the designers have decided to split the ID/RF stage into three stages (ID, RF1, RF2) each of latency 2.2/3 ns. Also, the EX stage is split into two stages (EX1, EX2) each of latency 1 ns. The new design has a total of eight pipeline stages. A program has 20% branch instructions which execute in the EX stage and produce the next instruction pointer at the end of the EX stage in the old design and at the end of the EX2 stage in the new design. The IF stage stalls after fetching a branch instruction until the next instruction pointer is computed. All instructions other than the branch instruction have an average CPI of one in both the designs. The execution times of this program on the old and the new design are *P* and *Q* nanoseconds, respectively. The value of *P*/*Q* is __________

Answer: (1.54)

Exp:

	No. of stages	Stall cycle	Stall frequency	Clock period	Avg. access time
Old design	5	2	20%	2.2ns	P
New design	8	5	20%	1 ns	Q

$$P = \left[80\%(1\text{clock}) + 20\%\left(\underset{\text{completion}}{1} + \underset{\text{stall clock}}{2}\right)\right] \times T_{c-p}$$

$$P = (.8 + .6) \times 2.2\,\text{ns} = 3.08\,\text{ns}$$

$$Q = \left[80\%(1\text{clock}) + 20\%\left(\underset{\text{completion}}{1} + \underset{\text{stall clock}}{5}\right)\right] \times T_{c-p}$$

$$P = (.8 + .12) \times 1\text{ns} = 2\,\text{ns}$$

So the value of $\frac{P}{Q} = \frac{3.08\text{ns}}{2\,\text{ns}} = 1.54$

44. The memory access time is 1 nanosecond for a read operation with a hit in cache, 5 nanoseconds for a read operation with a miss in cache, 2 nanoseconds for a write operation with a hit in cache and 10 nanoseconds for a write operation with a miss in cache. Execution of a sequence of instructions involves 100 instruction fetch operations, 60 memory operand read operations and 40 memory operand write operations. The cache hit-ratio is 0.9. The average memory access time (in nanoseconds) in executing the sequence of instructions is ___________.

Answer: (1.68)

Exp: $\text{Total instruction} = \begin{matrix}\text{100 instruction}\\ \text{fetch operation}\end{matrix} + \begin{matrix}\text{60 memory}\\ \text{operand read}\\ \text{operation}\end{matrix} + \begin{matrix}\text{40 memory}\\ \text{operand write op}\end{matrix}$

$= 200 \text{ instructions (operations)}$

Time taken for fetching 100 instructions (equivalent to read)

$= 90 * 1\,\text{ns} + 10 * 5\text{ns} = 140\,\text{ns}$

Memory operand Read operations $= 90\%(60) * 1\,\text{ns} + 10\%(60) \times 5\,\text{ns}$

$= 54\,\text{ns} + 30\,\text{ns} = 84\,\text{ms}$

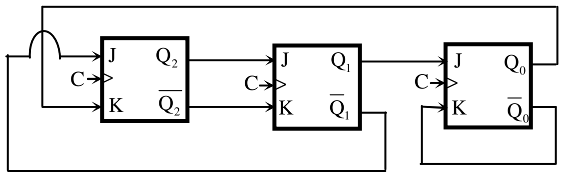
Memory operands write operation time $= 90\%(40) * 2\,\text{ns} + 10\%(40) * 10\,\text{ns}$

$= 72\,\text{ns} + 40\,\text{ns} = 112\,\text{ns}$

Total time taken for executing 200 instructions $= 140 + 84 + 112 = 336\,\text{ns}$

$\therefore$ Average memory access time $= \frac{336\text{ ns}}{200} = 1.68\,\text{ns}$

45.



The above synchronous sequential circuit built using JK flip-flops is initialized with $Q_2Q_1Q_0 = 000$. The state sequence for this circuit for the next 3 clock cycles is

(A) 001, 010, 011 (B) 111, 110, 101

(C) 100, 110, 111 (D) 100, 011, 001

Answer: (C)

Exp:

P.S.			FF inputs						N.S.		
Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0	Q_2^+	Q_1^+	Q_0^+
			$(\bar{Q}_1)$	(Q_0)	(Q_2)	$(\bar{Q}_2)$	(Q_1)	$(\bar{Q}_0)$			
0	0	0	1	0	0	1	0	1	1	0	0
1	0	0	1	0	1	0	0	1	1	1	0
1	1	0	0	0	1	0	1	1	1	1	1

46. With respect to the numerical evaluation of the definite integral, $K = \int_a^b x^2 dx$, where a and b are given, which of the following statements is/are TRUE?

(I) The value of K obtained using the trapezoidal rule is always greater than or equal to the exact value of the definite integral.

(II) The value of K obtained using the Simpson's rule is always equal to the exact value of the definite integral.

(A) I only (B) II only (C) Both I and II (D) Neither I nor II

Answer: (C)

Exp: $\int_a^b x^2\, dx$

let $a = 0,\ b = 1$

let $n = 4$

$\Rightarrow h = \dfrac{b-a}{n} = \dfrac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$y = x^2$	0	0.625	0.25	0.5625	1
	y_0	y_1	y_2	y_3	y_4

I. By Trapezoidal rule

$$\int_0^1 x^2\,dx = \frac{h}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$$

$$= \frac{0.25}{2}\left[(0+1)+2(0.0625+0.25+0.5625)\right] = 0.34375$$

II. By Simpson's $\frac{1}{3}$ rule

$$\int_0^1 x^2 dx = \frac{h}{3}\left[(y_0+y_4)+2(y_2)+4(y_1+y_3)\right]$$

$$= \frac{0.25}{3}\left[(0+1)+2(0.25)+4(0.0625+0.5625)\right] = \frac{1}{3}$$

Exact value $\int_0^1 x^2\,dx = \left.\frac{x^3}{3}\right|_0^1 = \frac{1}{3}$

47. The value of the integral given below is

$$\int_0^\pi x^2 \cos x\,dx$$

(A) -2π (B) π (C) $-\pi$ (D) 2π

Answer: (A)

Exp: $\int_0^\pi x^2 \cos x\,dx = x^2(\sin x) - 2x(-\cos x) + 2(-\sin x)\Big|_0^\pi$

$$= \left(\pi^2 \sin\pi + 2\pi\cos\pi - 2\sin\pi\right) - (0+0+0) = -2\pi$$

48. Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If $P(.)$ denotes the probability of the event, the maximum value of P(A)P(B) is ______

Answer: (0.25)

Exp: Given

$A \cup B = S$

$\Rightarrow P(A \cup B) = P(S) = 1$

$\Rightarrow P(A) + P(B) = 1$ ($\because$ A & B are mutually exclusive)

$\Rightarrow P(B) = 1 - P(A)$

Maximum value of $P(A)P(B) = ?$

Maximum value of P (A) [1- P (A)] =?

Let $P(A) = X$

Let $f(x) = x(1-x) = x - x^2$

$$\text{for } f(x) \text{ maximum} \Rightarrow f'(x)=0 \Rightarrow 1-2x=0 \Rightarrow x=\frac{1}{2}$$

$$f''(x)=-2;\quad f''\left(\frac{1}{2}\right)<0$$

$\therefore f(x)$ has maximum

At $x=\frac{1}{2}$ and maximum value

$$=f\left(\frac{1}{2}\right)=\frac{1}{2}\left(1-\frac{1}{2}\right)=\frac{1}{2}.\frac{1}{2}=\frac{1}{4}=0.25$$

49. Consider the set of all functions $f:\{0,1,...,2014\}\to\{0,1...,2014\}$ such that $f(f(i))=i$, for $0\le i\le 2014$. Consider the following statements.

P. For each such function it must be the case that for every i, f(i) = i,

Q. For each such function it must be the case that for some i,f(i) = i,

R. Each such function must be onto.

Which one of the following is CORRECT?

(A) P, Q and R are true
(B) Only Q and R are true
(C) Only P and Q are true
(D) Only R is true

Answer: (B)

Exp: Let us consider a function (counter example) as

$f(0)=1, f(1)=0, f(2)=3, f(3)=2, \ldots\ldots, f(2012)=2013,$

$f(2013)=2012$ and $f(2014)=2014$

Clearly $f(f(i))=i$ for $0\le i\le 2014$

Here $f(i)\ne i$ for every i and $f(i)=i$ for some i

Also f is onto

Hence, only Q and R are true

50. There are two elements x,y in a group (G,*) such that every element in the group can be written as a product of some number of x's and y's in some order. It is known that

$$x*x=y*y=x*y*x*y=y*x*y*x=e$$

where e is the identity element. The maximum number of elements in such a group is ________________.

Answer: (4)

Exp: $x\times x=e \quad \Rightarrow x$ is its own inverse

$y \times y = e \quad \Rightarrow$ y is its own inverse

$(x \times y) \times (x \times y) = e \Rightarrow (x \times y)$ is its own inverse

$(y \times x) \times (y \times x) = e \Rightarrow (y \times x)$ is its own inverse

also $x \times x \times e = e \times e$ can be rewritten as follows

$x \times y \times y \times x = e \times y \times y \times e = e [\because y \times y = e]$

$(x \times y) \times (y \times x) = e$ shows that $(x \times y)$ and $(y \times x)$

Are each other's inverse and we already know that

$(x \times y)$ and $(y \times x)$ are inverse of its own.

As per $(G, *)$ to be group any element should have

only one inverse element (unique)

This process $x \times y = y \times x$ (is one element)

So the elements of such group are 4 which are $\{x, y, e, x \times y\}$

51. If G is a forest with *n* vertices and *k* connected components, how many edges does *G* have?

(A) $\lfloor n/k \rfloor$ (B) $\lceil n/k \rceil$ (C) $n - k$ (D) $n - k + 1$

Answer: (C)

Exp: Let $n_1, n_2, \ldots, n_k$ be the number of vertices respectively in K connected components of a forest G, then $n_1 - 1, n_2 - 1, \ldots, n_k - 1$ be the number of edges respectively in K connected components and $n_1 + n_2 + \ldots + n_k = n$ (number of vertices in G)

Hence, number of edges in G = number of edges in K connected components

$= (n_1 - 1) + (n_2 - 1) + \ldots + (n_k - 1) = n - k$

52. Let δ denote the minimum degree of a vertex in a graph. For all planar graphs on n vertices with $\delta \geq 3$, which one of the following is **TRUE**?

(A) In any planar embedding, the number of faces is at least $\frac{n}{2} + 2$

(B) In any planar embedding, the number of faces is less than $\frac{n}{2} + 2$

(C) There is a planar embedding in which the number of faces is less than $\frac{n}{2} + 2$

(D) There is a planar embedding in which the number of faces is at most $\frac{n}{\delta + 1}$

Answer: (A)

Exp: We know that $v+r=e+2 \Rightarrow e=n+r-2 \quad ...(1)$

Where $V=n(\text{number of vertices}); r=\text{number of faces and}$

$e=\text{number of edges}$

Given, $\delta \geq 3$ then $3n \leq 2e$

$$\Rightarrow e \geq \frac{3n}{2}$$

$$\Rightarrow n+r-2 \geq \frac{3n}{2} (\text{using}(1))$$

$$\Rightarrow r \geq \frac{3n}{2} - n + 2 \Rightarrow r \geq \frac{n}{2} + 2$$

$\therefore$ Number of faces is atleast $\frac{n}{2}+2$

53. The CORECT formula for the sentence, "not all rainy days are cold" is

(A) $\forall d\ (\text{Rainy}(d) \wedge \sim \text{Cold}(d))$

(B) $\forall d\ (\sim \text{Rainy}(d) \rightarrow \text{Cold}(d))$

(C) $\exists d\ (\sim \text{Rainy}(d) \rightarrow \text{Cold}(d))$

(D) $\exists d\ (\text{Rainy}(d) \wedge \sim \text{Cold}(d))$

Answer: (D)

Exp: Given statement is $\sim \forall d[r(d) \rightarrow c(d)]$

$$\equiv \sim \forall d[\sim r(d) \vee c(d)]$$
$$\equiv \exists d[r(d) \wedge \sim c(d)]$$

(Since $p \rightarrow q \equiv \sim p \vee q$ and let r(d) be rainy day, c(d) be cold day)

54. Consider the following relational schema:

Employee (<u>empId</u>, empName, empDept)

Customer (<u>custId</u>, custName, salesRepId, rating)

SalesRepId is a foreign key referring to empId of the employee relation. Assume that each employee makes a sale to at least one customer. What does the following query return?

```
SELECT empName
FROM employee E
WHERE NOT EXISTS (SELECT custId
                  FROM customer C
                  WHERE C. salesRepId = E. empId
                  AND C. rating < > 'GOOD')
```

(A) Names of all the employees with at least one of their customers having a 'GOOD' rating.

(B) Names of all the employees with at most one of their customers having a 'GOOD' rating.

(C) Names of all the employees with none of their customers having a 'GOOD' rating.

(D) Names of all the employees with all their customers having a 'GOOD' rating.

Answer: (D)

Exp: The outer query will return the value (names of employees) for a tuple in relation E, only if inner query for that tuple will return no tuple (usage of NOT EXISTS).

The inner query will run for every tuple of outer query. It selects cust-id for an employee e, if rating of customer is NOT good. Such an employee should not be selected in the output of outer query.

So the query will return the names of all those employees whose all customers have GOOD rating.

55. Let $\oplus$ denote the Exclusive OR (XOR) operation. Let '1' and '0' denote the binary constants. Consider the following Boolean expression for F over two variables P and Q.

$$F(P,Q) = ((1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0))$$

The equivalent expression for F is

(A) $P + Q$ (B) $\overline{P + Q}$ (C) $P \oplus Q$ (D) $\overline{P \oplus Q}$

Answer: (D)

Exp:
$$F(P,Q) = ((1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0))$$
$$= \left(\bar{P} \oplus \left(P\bar{Q} + \bar{P}Q\right)\right) \oplus \left(\left(P\bar{Q} + \bar{P}Q\right) \oplus Q\right)$$
$$= \left[\bar{P}\left(PQ + \bar{P}\,\bar{Q}\right) + P\left(P\bar{Q} + \bar{P}Q\right)\right] \oplus \left[\left(PQ + \bar{P}\bar{Q}\right)Q + \left(P\bar{Q} + \bar{P}Q\right)\bar{Q}\right]$$
$$= \left(\bar{P}\bar{Q} + P\bar{Q}\right) \oplus \left(PQ + P\bar{Q}\right) \qquad = \bar{Q} \oplus P \qquad = PQ + \bar{P}\bar{Q} = \overline{P \oplus Q}$$

Q. No. 1 – 25 Carry One Mark Each

1. Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is ½. What is the expected number of unordered cycles of length three?

(A) 1/8 (B) 1 (C) 7 (D) 8

Answer:-(C)

Exp:- $P(\text{edge}) = \frac{1}{2}$

Number of ways we can choose the vertices out of 8 is 8_{c_3}

(Three edges in each cycle)

Expected number of unordered cycles of length 3 $= 8_{C_3} \times \left(\frac{1}{2}\right)^3 = 7$

2. Which of the following statements is/are **TRUE** for undirected graphs?

P: Number of odd degree vertices is even.

Q: Sum of degrees of all vertices is even.

(A) P only (B) Q only

(C) Both P and Q (D) Neither P nor Q

Answer:- (C)

Exp:- Q: Sum of degrees of all vertices $= 2 \times (\text{number of edges})$

3. Function f is known at the following points:

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0
f(x)	0	0.09	0.36	0.81	1.44	2.25	3.24	4.41	5.76	7.29	9.00

The value of $\int_0^3 f(x)\,dx$ computed using the trapezoidal rule is

(A) 8.983 (B) 9.003 (C) 9.017 (D) 9.045

Answer:- (D)

Exp:- $\int_0^3 f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_{10}) + 2\left(f(x_1) + f(x_2) + \ldots + f(x_9)\right)\right]$

$= \frac{0.3}{2}\left[9.00 + 2(25.65)\right] = 9.045$

4. Which one of the following functions is continuous at x = 3 ?

(A) $f(x)=\begin{cases} 2, \text{ if } x=3 \\ x-1, \text{ if } x>3 \\ \dfrac{x+3}{3}, \text{ if } x<3 \end{cases}$

(B) $f(x)=\begin{cases} 4, & \text{if } x=3 \\ 8-x & \text{if } x\neq 3 \end{cases}$

(C) $f(x)=\begin{cases} x+3, & \text{if } x\leq 3 \\ x-4 & \text{if } x>3 \end{cases}$

(D) $f(x)=\dfrac{1}{x^3-27}, \quad \text{if } x\neq 3$

Answer:-(A)

Exp:- $\lim\limits_{x\to 3+} f(x)=\lim\limits_{x\to 3+}(x-1)=2=f(3)$

$\lim\limits_{x\to 3-} f(x)=\lim\limits_{x\to 3-}\left(\dfrac{x+3}{3}\right)=2=f(3)$

$\therefore f(x)$ is continuous at $x=3$

5. Which one of the following expressions does **NOT** represent exclusive NOR of x and y?

(A) $xy+x'y'$ (B) $x\oplus y'$ (C) $x'\oplus y$ (D) $x'\oplus y'$

Answer: -(D)

Exp:- (A) $x\odot y=xy+\overline{x}\,\overline{y}$

(B) $x\oplus \overline{y}=x\overline{\overline{y}}+\overline{x}\,\overline{y}=xy+\overline{x}\,\overline{y}=x\odot y$

(C) $\overline{x}\oplus y=\left(\overline{x}\right)\overline{y}+\overline{\overline{x}}\,y=\overline{x}\,\overline{y}+xy=x\odot y$

(D) $\overline{x}\oplus\overline{y}=\left(\overline{x}\right)y+x\overline{y}=x\oplus y$

6. In a k-way set associative cache, the cache is divided into v sets, each of which consists of k lines. The lines of a set are placed in sequence one after another. The lines in set s are sequenced before the lines in set (s+1). The main memory blocks are numbered 0 onwards. The main memory block numbered j must be mapped to any one of the cache lines from

(A) $(j \bmod v)*k$ to $(j \bmod v)*k+(k-1)$

(B) $(j \bmod v)$ to $(j \bmod v)+(k-1)$

(C) $(j \bmod k)$ to $(j \bmod k)+(v-1)$

(D) $(j \bmod k)*v$ to $(j \bmod k)*v+(v-1)$

Answer: -(A)

Exp:- Position of main memory block in the cache (set) = (main memory block number) MOD (number of sets in the cache).

As the lines in the set are placed in sequence, we can have the lines from 0 to (K – 1) in each set.

Number of sets = v, main memory block number = j

First line of cache = (j mod v)*k; last line of cache = (j mod v)*k + (k – 1)

7. What is the time complexity of Bellman-Ford single-source shortest path algorithm on a complete graph of n vertices?

(A) $\Theta(n^2)$ (B) $\Theta(n^2 \log n)$ (C) $\Theta(n^3)$ (D) $\Theta(n^3 \log n)$

Answer:-(C)

Exp:- Bellman-ford time complexity: $\Theta(|V| \times |E|)$

For complete graph: $|E| = \frac{n(n-1)}{2}$

$|V| = n$

$\therefore \Theta\left(n \times \frac{n(n-1)}{2}\right) = \Theta(n^3)$

8. Which of the following statements are **TRUE**?

(1) The problem of determining whether there exists a cycle in an undirected graph is in P.

(2) The problem of determining whether there exists a cycle in an undirected graph is in NP.

(3) If a problem A is NP-Complete, there exists a non-deterministic polynomial time algorithm to solve A.

(A) 1,2 and 3 (B) 1 and 2 only (C) 2 and 3 only (D) 1 and 3 only

Answer: -(A)

Exp:- 1. Cycle detection using DFS: $O(V+E) = O(V^2)$ and it is polynomial problem

2. Every P-problem is NP$\left(\text{since } P \subset NP\right)$

3. $NP - \text{complete} \in NP$

Hence, NP-complete can be solved in non-deterministic polynomial time

9. Which of the following statements is/are **FALSE**?

(1) For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine.

(2) Turing recognizable languages are closed under union and complementation.

(3) Turing decidable languages are closed under intersection and complementation

(4) Turing recognizable languages are closed under union and intersection.

(A) 1 and 4 only (B) 1 and 3 only (C) 2 only (D) 3 only

Answer: -(C)

Exp:- (1) NTM $\cong$ DTM

(2) RELs are closed under union & but not complementation

(3) Turing decidable languages are recursive and recursive languages are closed under intersection and complementation

(4) RELs are closed under union & intersection but not under complementation

10. Three concurrent processes X, Y, and Z execute three different code segments that access and update certain shared variables. Process X executes the P operation (i.e., wait) on semaphores a, b and c; process Y executes the P operation on semaphores b, c and d; process Z executes the P operation on semaphores c, d, and a before entering the respective code segments. After completing the execution of its code segment, each process invokes the V operation (i.e., signal) on its three semaphores. All semaphores are binary semaphores initialized to one. Which one of the following represents a deadlock-free order of invoking the P operations by the processes?

(A) $X: P(a)P(b)P(c) \quad Y: P(b)P(c)P(d) \quad Z: P(c)P(d)P(a)$

(B) $X: P(b)P(a)P(c) \quad Y: P(b)P(c)P(d) \quad Z: P(a)P(c)P(d)$

(C) $X: P(b)P(a)P(c) \quad Y: P(c)P(b)P(d) \quad Z: P(a)P(c)P(d)$

(D) $X: P(a)P(b)P(c) \quad Y: P(c)P(b)P(d) \quad Z: P(c)P(d)P(a)$

Answer:-(B)

Exp:- Suppose X performs P(b) and preempts, Y gets chance, but cannot do its first wait i.e., P(b), so waits for X, now Z gets the chance and performs P(a) and preempts, next X gets chance. X cannot continue as wait on ‘a’ is done by Z already, so X waits for Z. At this time Z can continue its operations as down on c and d. Once Z finishes, X can do its operations and so Y. In any of execution order of X, Y, Z one process can continue and finish, such that waiting is not circular. In options (A),(C) and (D) we can easily find circular wait, thus deadlock

11. An index is clustered, if

(A) it is on a set of fields that form a candidate key

(B) it is on a set of fields that include the primary key

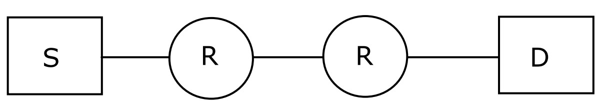
(C) the data records of the file are organized in the same order as the data entries of the index

(D) the data records of the file are organized not in the same order as the data entries of the index

Answer:-(C)

Exp:- Clustered index is built on ordering non key field and hence if the index is clustered then the data records of the file are organized in the same order as the data entries of the index.

12. Assume that source S and destination D are connected through two intermediate routers labeled R. Determine how many times each packet has to visit the network layer and the data link layer during a transmission from S to D.



(A) Network layer – 4 times and Data link layer-4 times

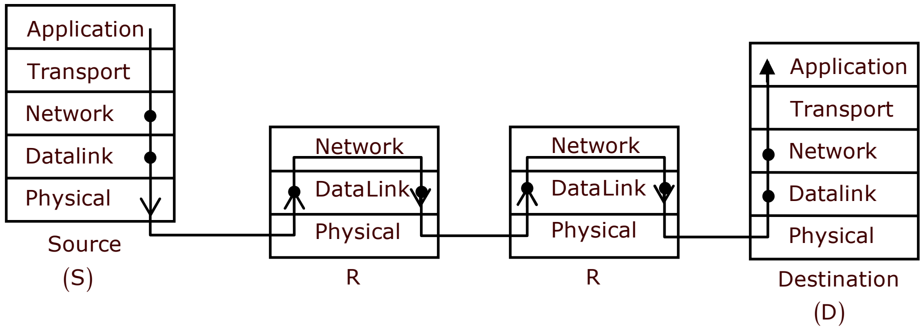
(B) Network layer – 4 times and Data link layer-3 times

(C) Network layer – 4 times and Data link layer-6 times

(D) Network layer – 2 times and Data link layer-6 times

Answer:-(C)

Exp:-



From above given diagram, its early visible that packet will visit network layer 4 times, once at each node [S, R, R, D] and packet will visit Data Link layer 6 times. One time at S and one time at D, then two times for each intermediate router R as data link layer is used for link to link communication.

Once at packet reaches R and goes up from physical –DL-Network and second time when packet coming out of router in order Network – DL- Physical

13. The transport layer protocols used for real time multimedia, file transfer, DNS and email, respectively are

(A) TCP, UDP, UDP and TCP
(B) UDP, TCP, TCP and UDP
(C) UDP, TCP, UDP and TCP
(D) TCP, UDP, TCP and UDP

Answer:- (C)

Exp:- Real time multimedia needs connectionless service, so under lying transport layer protocol used is UDP

File transfer rums over TCP protocol with port no-21

DNS runs over UDP protocol within port no-53

Email needs SMTP protocol which runs over TCP protocol within port no – 25

14. Using public key cryptography, X adds a digital signature σ to message M, encrypts <M, σ>, and sends it to Y, where it is decrypted. Which one of the following sequences of keys is used for the operations?

(A) Encryption: X's private key followed by Y's private key; Decryption: X's public key followed by Y's public key

(B) Encryption: X's private key followed by Y's public key; Decryption: X's public key followed by Y's private key

(C) Encryption: X's public key followed by Y's private key; Decryption: Y's public key followed by X's private key

(D) Encryption: X's private key followed by Y's public key; Decryption: Y's private key followed by X's public key

Answer:-(D)

Exp:-

$$\text{X} \longrightarrow \text{Y}$$

X – public
X – private

Y – public
Y – private

$$\text{Encryption}\begin{cases}\text{Source has to encrypt with its private key for}\\ \text{forming Digital signature for Authentication.}\\ \text{source has to encrypt the } \langle M, \sigma\rangle \text{ with Y's}\\ \text{public key to send it confidentially}\end{cases}$$

$$\text{Decryption}\begin{cases}\text{Destination Y has to decrypt first}\\ \text{with its private key, then decrypt}\\ \text{using source public key}\end{cases}$$

15. Match the problem domains in **Group I** with the solution technologies in **Group II**.

Group I	**Group II**
(p) Services oriented computing	(1) Interoperability
(q) Heterogeneous communicating systems	(2) BPMN
(R) Information representation	(3) Publish-find bind
(S) Process description	(4) XML

(A) P – 1, Q – 2, R – 3, S – 4
(B) P – 3, Q – 4, R – 2, S – 1
(C) P – 3, Q – 1, R – 4, S – 2
(D) P – 4, Q – 3, R – 2, S – 1

Answer:-(C)

16. A scheduling algorithm assigns priority proportional to the waiting time of a process. Every process starts with priority zero(the lowest priority). The scheduler re-evaluates the process priorities every T time units and decides the next process to schedule. Which one of the following is TRUE if the processes have no I/O operations and all arrive at time zero?

(A) This algorithm is equivalent to the first-come-first-serve algorithm

(B) This algorithm is equivalent to the round-robin algorithm

(C) This algorithm is equivalent to the shortest-job-first algorithm

(D) This algorithm is equivalent to the shortest-remaining-time-first algorithm

Answer:-(B)

Exp:- The given scheduling definition takes two parameters, one is dynamically assigned process priority and the other is 'T' time unit to re-evaluate the process priorities.

This dynamically assigned priority will be deciding processes order in ready queue of round robin algorithm whose time quantum is same as 'T' time units. As all the processes are arriving at the same time, they will be given same priority but soon after first 'T' time burst remaining processes will get higher priorities

17. What is the maximum number of reduce moves that can be taken by a bottom-up parser for a grammar with no epsilon- and unit-production $(\text{i.e., of type } A \to \in \text{ and } A \to a)$ to parse a string with n tokens?

(A) n/2 (B) n-1 (C) 2n-1 (D) 2^n

Answer: -(B)

Exp:- To have maximum number of reduce moves, all the productions will be of the type $A \to \alpha\beta$ (where α and β could be terminals or non-terminals). Consider the following illustration then:

$$\left.\begin{array}{l} \text{Input String}: a_1\, a_2\, a_3 \ldots\ldots\ldots a_{n-2}\, \underbrace{a_{n-1} a_n} \\ \qquad\qquad\qquad\qquad\qquad\quad \uparrow \\ \qquad\qquad a_1 a_2 a_3 \ldots\ldots\ldots\ldots a\, \underbrace{_{n-2} A} \\ \qquad\qquad\qquad\qquad\qquad \uparrow \\ \qquad\qquad a_1 a_2 a_3 \ldots\ldots\ldots\ldots\ldots\ldots A \\ \qquad\qquad\qquad\qquad \vdots \\ \qquad\qquad \underbrace{a_1\, a_2}_{A} \end{array}\right\} n-1 \text{ moves}$$

18. Consider the languages $L_1 = \Phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$?

(A) $\{\in\}$ (B) Φ (C) a^* (D) $\{\varepsilon, a\}$

Answer: -(A)

Exp:- Concatenation of empty language with any language will give the empty language and $L_1{}^* = \Phi^* = \in$. Hence $L_1 L_2^* \cup L_1^* = \{\in\}$

19. Which one of the following is the tightest upper bound that represents the time complexity of inserting an object into a binary search tree of n nodes?

(A) O(1) (B) O(log n) (C) O(n) (D) O(n log n)

Answer:-(C)

Exp:- For skewed binary search tree on n nodes, the tightest upper bound to insert a node is O(n)

20. Which one of the following is the tightest upper bound that represents the number of swaps required to sort n numbers using selection sort?

(A) O(log n) (B) O(n) (C) O(n log n) (D) $O(n^2)$

Answer:-(B)

Exp:- The maximum number of swaps that takes place in selection sort on n numbers is n

21. In the following truth table, V = 1 if and only if the input is valid.

Inputs				Outputs		
D_0	D_1	D_2	D_3	X_0	X_1	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

What function does the truth table represent?

(A) Priority encoder (B) Decoder

(C) Multiplexer (D) Demultiplexer

Answer: -(A)

Exp:- 4 to 2 priority encoder.

22. The smallest integer than can be represented by an 8-bit number in 2's complement form is

(A) -256 (B) -128 (C) -127 (D) 0

Answer: -(B)

Exp:- $-2^{8-1} = -128$. Range is $-2^{(n-1)}$ to $+2^{(n-1)}-1$

23. Which one of the following does **NOT** equal $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$?

(A) $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$ (B) $\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$

(C) $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$ (D) $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

Answer:- (A)

Exp:- If matrix B is obtained from matrix A by replacing the 1^{th} row by itself plus k times the m^{th} row, for $1 \neq m$ then det(B)=det(A). With this property given matrix is equal to the matrices given in options (B),(C) and (D).

24. Suppose p is number of cars per minute passing through a certain road junction between 5 PM and 6PM, and p has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

(A) $8/(2e^3)$ (B) $9/(2e^3)$ (C) $17/(2e^3)$ (D) $26/(2e^3)$

Answer:-(C)

Exp:- $P(p<3)=P(p=0)+P(p=1)+P(p=2)$

$$=\frac{e^{-\lambda}\lambda^0}{0!}+\frac{e^{-\lambda}\lambda^1}{1!}+\frac{e^{-\lambda}\lambda^2}{2!}\ (\text{where } \lambda=3)$$

$$=e^{-3}+e^{-3}\times 3+\frac{e^{-3}\times 9}{2}$$

$$=e^{-3}\left(1+3+\frac{9}{2}\right)=\frac{17}{2e^3}$$

25. A binary operation $\oplus$ on a set of integers is defined as $x\oplus y=x^2+y^2$. Which one of the following statements is **TRUE** about $\oplus$?

(A) Commutative but not associative

(B) Both commutative and associative

(C) Associative but not commutative

(D) Neither commutative nor associative

Answer:- (A)

Exp:- $x\oplus y=x^2+y^2=y^2+x^2=y\oplus x$

$\therefore$ commutative

Not associative, since, for example

$(1\oplus 2)\oplus 3\neq 1\oplus(2\oplus 3)$

Q. No. 26 – 51 Carry Two Marks Each

26. Which one of the following is **NOT** logically equivalent to $\neg\exists x(\forall y(\alpha)\wedge\forall z(\beta))$?

(A) $\forall x(\exists z(\neg\beta)\rightarrow\forall y(\alpha))$

(B) $\forall x(\forall z(\beta)\rightarrow\exists y(\neg\alpha))$

(C) $\forall x(\forall y(\alpha)\rightarrow\exists z(\neg\beta))$

(D) $\forall x(\exists y(\neg\alpha)\rightarrow\exists z(\neg\beta))$

Answer: -(A) and (D) [marks to all]

Exp:- $\neg\exists x(\forall y(\alpha)\wedge\forall z(\beta))$

$\equiv\forall x[\forall y(\alpha)\rightarrow\exists z(\neg\beta)]$ option "C" $[\because\neg(p\wedge q)\equiv p\Rightarrow\neg q]$

$\equiv\forall x[\forall z(\beta)\rightarrow\exists y(\neg\alpha)]$ option "B" $[\because p\Rightarrow q\equiv\neg q\Rightarrow\neg p]$

27. A RAM chip has a capacity of 1024 words of 8 bits each $(1K\times 8)$. The number of 2×4 decoders with enable line needed to construct a $16K\times 16$ RAM from $1K\times 8$ RAM is

(A) 4 (B) 5 (C) 6 (D) 7

Answer: -(B)

Exp:- RAM chip size = $1k\times8[1024 \text{ words of } 8 \text{ bits each}]$

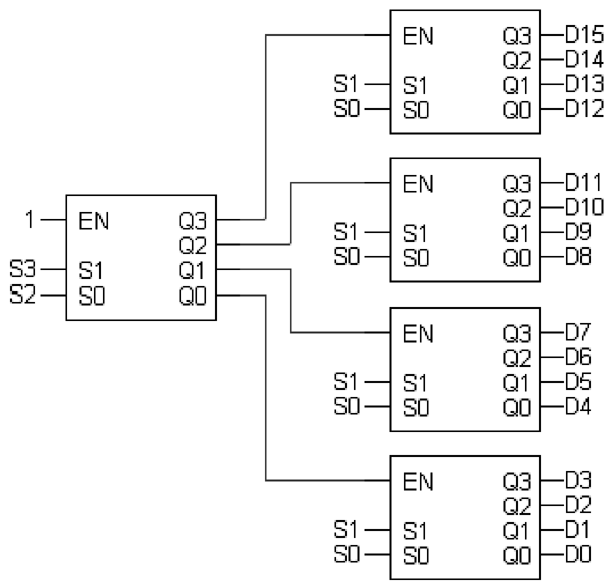
RAM to construct $=16k\times16$

Number of chips required = $\frac{16k\times16}{1k\times8}=16\times2$ [16 chips vertically with each having 2 chips horizontally]

So to select one chip out of 16 vertical chips, we need 4 x 16 decoder.

Available decoder is – 2 x 4 decoder

To be constructed is 4 x 16 decoder



So we need 5, 2 x 4 decoder in total to construct 4 x 16 decoder.

28. Consider an instruction pipeline with five stages without any branch prediction: Fetch Instruction (FI), Decode Instruction (DI), Fetch Operand (FO), Execute Instruction (EI) and Write Operand (WO). The stage delays for FI, DI, FO, EI and WO are 5 ns, 7 ns, 10 ns, 8 ns and 6 ns, respectively. There are intermediate storage buffers after each stage and the delay of each buffer is 1 ns. A program consisting of 12 instructions $I_1, I_2, I_3, ...I_{12}$ is executed in this pipelined processor. Instruction I_4 is the only branch instruction and its branch target is I_9. If the branch is taken during the execution of this program, the time (in ns) needed to complete the program is

(A) 132 (B) 165 (C) 176 (D) 328

Answer: - (B)

Exp:- Clock period=Maximum stage delay+ overhead (Buffer) =10+1=11 ns

Assume FI-1, DI-2, FO-3, EI-4, WO-5

$$
\begin{array}{lllllllllllllllll}
I_1: & 1 & 2 & 3 & 4 & 5 \\
I_2: & - & 1 & 2 & 3 & 4 & 5 \\
I_3: & - & - & 1 & 2 & 3 & 4 & 5 \\
I_4: & - & - & - & 1 & 2 & 3 & 4 & 5 \\
I_5: & - & - & - & - & 1 & 2 & 3 & 4 & 5 \\
I_6: & - & - & - & - & - & 1 & 2 & 3 & 4 & 5 \\
I_7: & - & - & - & - & - & - & 1 & 2 & 3 & 4 & 5 \\
I_8: & - & - & - & - & - & - & - & 1 & 2 & 3 & 4 & 5 \\
I_9: & - & - & - & - & - & - & - & - & 1 & 2 & 3 & 4 & 5 \\
I_{10}: & - & - & - & - & - & - & - & - & - & 1 & 2 & 3 & 4 & 5 \\
I_{11} & - & - & - & - & - & - & - & - & - & - & 1 & 2 & 3 & 4 & 5 \\
I_{12} & - & - & - & - & - & - & - & - & - & - & - & 1 & 2 & 3 & 4 & 5
\end{array}
$$

So number of clocks required to complete the program is = 15 clocks and time taken is = 15 ×11 ns=165 ns.

29. Consider the following operation along with Enqueue and Dequeue operations on queues, where k is a global parameter

```
MultiDequeue(Q) {
    m = k
    while (Q is not empty) and (m > 0) {
        Dequeue (Q)
        m = m − 1
    }
}
```

What is the worst case time complexity of a sequence of n queue operations on an initially empty queue?

(A) $\Theta(n)$ (B) $\Theta(n+k)$ (C) $\Theta(nk)$ (D) $\Theta(n^2)$

Answer:- (A)

Exp:- Initially the queue is empty and we have to perform n operations.

i) One option is to perform all Enqueue operations i.e. n Enqueue operations. Complexity will be $\theta(n)$

or

ii) We can perform a mix of Enqueue and Dequeue operations. It can be Enqueue for first n/2 times and then Dequeue for next n/2, or Enqueue and Dequeue alternately, or any permutation of Enqueues and Dequeues totaling 'n' times. Complexity will be $\theta(n)$

or

iii) We can perform Enqueues and MultiDequeues. A general pattern could be as follows:

Enqueue Enqueue … (ktimes) MultiDequeue Enqueue Enqueue … (ktimes) MultiDequeue … Up to total n

---- k items enqueued -----k items deleted----k items enqueued----k items deleted -- and so on.

The number of times this k-Enqueues, MutiDequeue cycle is performed = $n/_{k+1}$

So, Complexity will be k times Enqueue + 1 MultiDequeue) $\times$ $n/_{k+1}$

Which is $\theta\left(2k \times n/_{k+1}\right) = \theta(n)$

or

iv) We can just perform n MultiDequeues (or n Dequeues for that matter):

Each time the while condition is false (empty queue), condition is checked just once for each of the 'n' operations. So θ(n).

30. The preorder traversal sequence of a binary search tree is 30, 20, 10, 15, 25, 23, 39, 35, 42. Which one of the following is the postorder traversal sequence of the same tree?

(A) 10,20,15,23,25,35,42,39,30

(B) 15,10,25,23,20,42,35,39,30

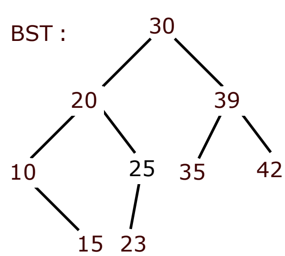
(C) 15,20,10,23,25,42,35,39,30

(D) 15,10,23,25,20,35,42,39,30

Answer:-(D)

Exp:- Preorder : 30,20,10,15,25,23,39,35,42

Inorder : 10,15,20,23,25,30,35,39,42



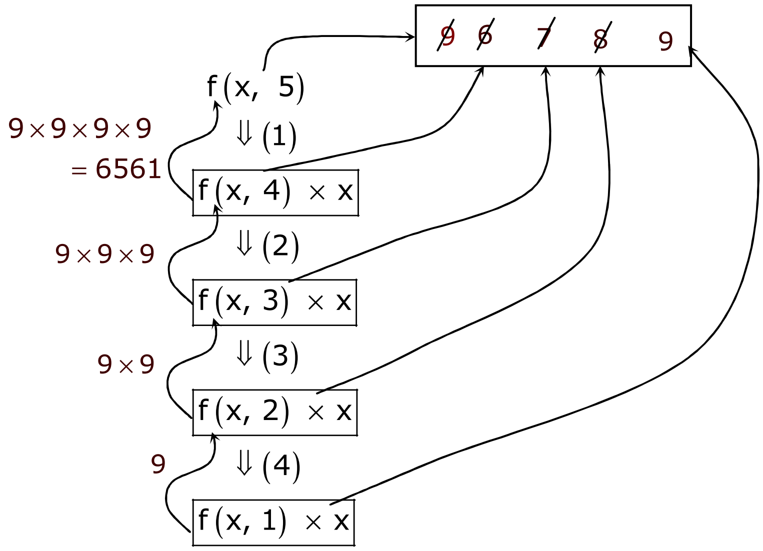
31. What is the return value of $f(p,p)$ if the value of p is initialized to 5 before the call? Note that the first parameter is passed by reference, whereas the second parameter is passed by value.

```
int f (int & x, int c) {
   c = c - 1;
   if (c == 0) return 1;
   x = x + 1;
   return f (x,c) * x;
}
```

(A) 3024 (B) 6561 (C) 55440 (D) 161051

Answer:-(B)

Exp:-



32. Which of the following is/are undecidable?

1. G is a CFG. Is $L(G) = \Phi$?
2. G is a CFG. IS $L(G) = \Sigma^*$?
3. M is a Turning machine. Is L(M) regular?
4. A is a DFA and N is a NFA. Is $L(A) = L(N)$?

(A) 3 only (B) 3 and 4 only (C) 1, 2 and 3 only (D) 2 and 3 only

Answer: -(D)

Exp:- There is an algorithm to check whether the given CFG is empty, finite or infinite and also to convert NFA to DFA hence 1 and 4 are decidable

33. Consider the following two sets of LR(1) items of an LR(1) grammar

$$
\begin{array}{ll}
X \to c.X, c/d & X \to c.X, \$ \\
X \to .cX, c/d & X \to .cX, \$ \\
X \to .d, c/d & X \to .d, \$
\end{array}
$$

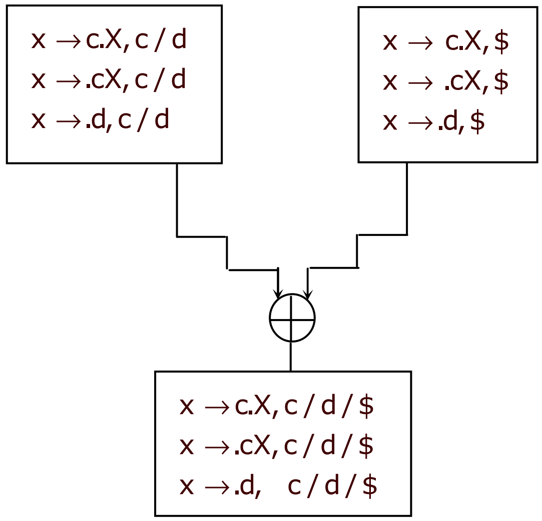
Which of the following statements related to merging of the two sets in the corresponding LALR parser is/are **FALSE**?

1. Cannot be merged since look aheads are different
2. Can be merged but will result in S–R conflict
3. Can be merged but will result in R–R conflict
4. Cannot be merged since goto on c will lead to two different sets

(A) 1 only (B) 2 only (C) 1 and 4 only (D) 1, 2, 3 and 4

Answer:-(D)

Exp:-



1. Merging of two states depends on core part (production rule with dot operator), not on look aheads.
2. The two states are not containing Reduce item ,So after merging, the merged state can not contain any S-R conflict
3. As there is no Reduce item in any of the state, so can't have R-R conflict.
4. Merging of stats does not depend on further goto on any terminal.

   So all statements are false.

34. A certain computation generates two arrays a and b such that $a[i] = f(i)$ for $0 \le i < n$ and $b[i] = g(a[i])$ for $0 \le i < n$. Suppose this computation is decomposed into two concurrent processes X and Y such that X computes the array a and Y computes the array b. The processes employ two binary semaphores R and S, both initialized to zero. The array a is shared by the two processes. The structures of the processes are shown below.

Process X;	Process Y;
private i;	private i;
$\text{for}(i = 0; i < n; i++)\{$	$\text{for}(i = 0; i < n; i++)\{$
$a[i] = f(i);$	$\text{EntryY}(R, S);$
$\text{ExitX}(R, S);$	$b[i] = g(a[i]);$
}	}

Which one of the following represents the **CORRECT** implementations of ExitX and EntryY?

(A) $\text{ExitX}(R,\ S)\{$
$P(R);$
$V(S);$
}
$\text{EntryY}(R,\ S)\{$
$P(S);$
$V(R);$
}

(B) $\text{ExitX}(R,\ S)\{$
$V(R);$
$V(S);$
}
$\text{EntryY}(R,\ S)\{$
$P(R);$
$P(S);$
}

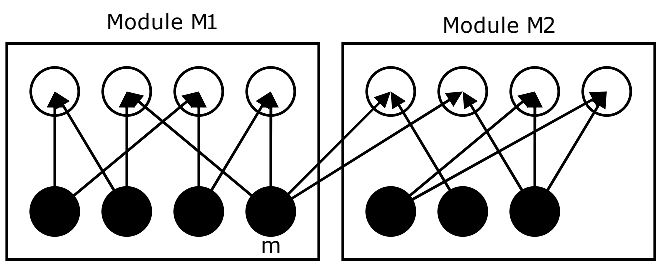
(C) ExitX(R, S) {
    P(S);
    V(R);
}
EntryY(R, S) {
    V(S);
    P(R);
}

(D) ExitX(R, S) {
    V(R);
    P(S);
}
EntryY(R, S) {
    V(S);
    P(R);
}

Answer:-(C)

Exp:- For computing both the array a[] and b[], first element a[i] should be computed using which b[i] can be computed. So process X and Y should run in strict alteration manner, starting with X. This requirement meets with implementation of ExitX and EntryY given in option C.

35. The following figure represents access graphs of two modules M1 and M2. The filled circles represent methods and the unfilled circles represent attributes. IF method m is moved to module M2 keeping the attributes where they are, what can we say about the average cohesion and coupling between modules in the system of two modules?



(A) There is no change

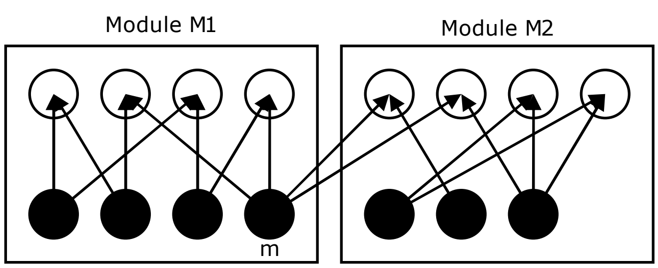
(B) Average cohesion goes up but coupling is reduced

(C) Average cohesion goes down and coupling also reduces

(D) Average cohesion and coupling increase

Answer:-(A)

Exp:-

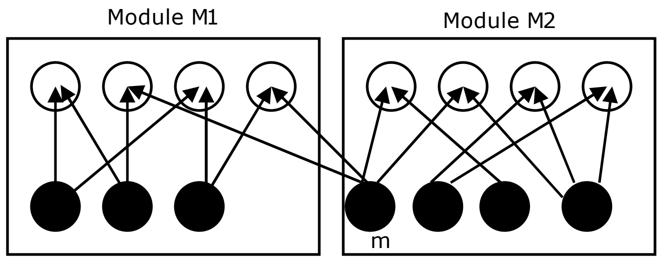


$$\text{Coupling} = \frac{\text{number of external links}}{\text{number of modules}} = \frac{2}{2}$$

$$\text{Cohesion of a module} = \frac{\text{number of internal links}}{\text{number of methods}}$$

$$\text{Cohesion of } M_1 = \frac{8}{4}; \text{ Cohesion of } M_2 = \frac{6}{3}; \quad \text{Average cohesion=2}$$

After moving method m to M2, graph will become



$\text{Coupling} = \frac{2}{2}$

$\text{Cohesion of } M_1 = \frac{6}{3}; \text{ Cohesion of } M_2 = \frac{8}{4};$ Average cohesion=2

$\therefore$ answer is no change

36. In an IPv4 datagram, the M bit is 0, the value of HLEN is 10, the value of total length is 400 and the fragment offset value is 300. The position of the datagram, the sequence numbers of the first and the last bytes of the payload, respectively are

(A) Last fragment, 2400 and 2789
(B) First fragment, 2400 and 2759
(C) Last fragment, 2400 and 2759
(D) Middle fragment, 300 and 689

Answer:-(C)

Exp:- M= 0 – Means there is no fragment after this, i.e. Last fragment
HLEN=10 - So header length is 4×10=40, as 4 is constant scale factor
Total Length = 400(40 Byte Header + 360 Byte Payload)
Fragment Offset = 300, that means 300×8 Byte = 2400 bytes are before this last fragment
So the position of datagram is last fragment
Sequence number of First Byte of Payload = 2400 (as 0 to 2399 Sequence no are used)
Sequence number of Last Byte of Payload = 2400+360-1=2759

37. Determine the maximum length of cable (in km) for transmitting data at a rate of 500 Mbps in an Ethernet LAN with frames of size 10,000 bits. Assume the signal speed in the cable to be 2,00,000 km/s

(A) 1
(B) 2
(C) 2.5
(D) 5

Answer:-(B)

Exp:- $500\times10^6 \text{ bits} ------1 \text{ sec}$

$$\therefore 10^4 \quad \text{bits} ------\frac{5\times10^8}{10^4} = \frac{10^4}{5\times10^8}\text{sec} = \frac{1}{5\times10^4}\text{sec}$$

$1 \text{ sec} ------2\times10^5 \text{ km}$

$$\therefore \frac{1}{5\times10^4}\text{sec} -----\frac{2\times10^5}{5\times10^4} = 4\,\text{km}$$

$$\therefore \text{Maximum length of cable} = \frac{4}{2} = 2 \text{ km}$$

38. Consider the following relational schema.

Students(rollno: integer, sname: string)

Courses(courseno: integer, cname: string)

Registration(rollno: integer, courseno; integer, percent: real)

Which of the following queries are equivalent to this query in English?

"Find the distinct names of all students who score more than 90% in the course numbered 107"

(I) SELECT DISTINCT S.sname
FROM Students as S, Registration as R
WHERE R.rollno=S.rollno AND R.Courseno=107 AND R.percent>90

(II) $\Pi_{sname}\left(\sigma_{courseno=107} \wedge percent > 90\left(Registration \quad Students\right)\right)$

(III) $\{T \mid \exists S \in Students, \exists R \in Registration\ (S.rollno = R.rollno \wedge$
$R.courseno = 107 \wedge R.percent > 90 \wedge T.sname = S.name)\}$

(IV) $\left\{< S_N > \mid \exists S_R \exists R_P \left(< S_R, S_N > \in Students \wedge < S_R, 107, R_P > \in Registration \wedge R_P > 90\right)\right\}$

(A) I, II, III and IV
(B) I, II and III only
(C) I, II and IV only
(D) II, III and IV only

Answer:- (A)

Exp:- Four queries given in SQL, RA, TRC and DRC in four statements respectively retrieve the required information.

39. A shared variable x, initialized to zero, is operated on by four concurrent processes W, X, Y, Z as follows. Each of the processes W and X reads x from memory, increments by one, stores it to memory, and then terminates. Each of the processes Y and Z reads x from memory, decrements by two, stores it to memory, and then terminates. Each process before reading x invokes the P operation (i.e., wait) on a counting semaphore S and invokes the V operation (i.e., signal) on the semaphore S after storing x to memory. Semaphore S is initialized to two. What is the maximum possible value of x after all processes complete execution?

(A) –2
(B) –1
(C) 1
(D) 2

Answer:-(D)

Exp:-

	W	X	Y	Z
1	R(x)	R(x)	R(x)	R(x)
2	x++	x++	x=x-2;	x=x-2;
3	w(x)	w(x)	w(x)	w(x)

R(x) is to read x from memory, w(x) is to store x in memory

(I) $w_1\left(x\boxed{0}\right)$ [W is Preempted]

(II) $Y_1, Y_2, Y_3\left(x\boxed{-2}\right)$ [Y is completed]

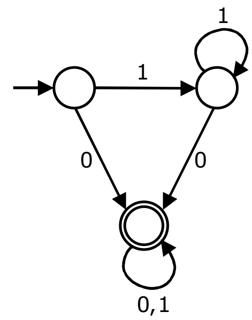
(III) $Z_1, Z_2, Z_3 \left(x \boxed{-4}\right)$ [Z is completed]

(IV) $W_2, W_3 \left(x \boxed{1}\right)$ [It increments local copy of x and stores & W is completed]

(V) $X_1, X_2, X_3 \left(x \boxed{2}\right)$ [X is completed]

Maximum value of x = 2

40. Consider the DFA given below.



Which of the following are FALSE?

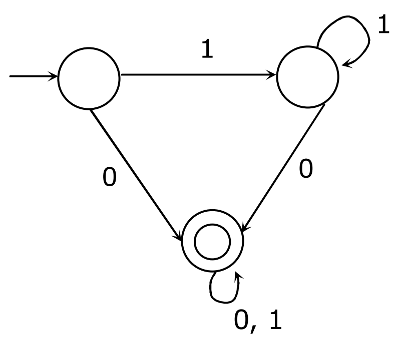
1. Complement of L(A) is context–free
2. $L(A) = L\left((11*0+0)(0+1)*0*1*\right)$
3. For the language accepted by A, A is the minimal DFA
4. A accepts all strings over {0, 1} of length at least 2

(A) 1 and 3 only (B) 2 and 4 only (C) 2 and 3 only (D) 3 and 4 only

Answer: - (D)

Exp:-

A :



(1) L(A) is regular, its complement is also regular and if it is regular it is also context free.

(2) $L(A) = (11*0+0)(0+1)*0*1* = 1*0\,(0+1)*$

Language has all strings where each string contains ‘0’.

(3) A is not minimal, it can be constructed with 2 states

(4) Language has all strings, where each string contains ‘0’. (atleast length one)

41. Consider the following languages

$L_1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$

$L_2 = \{0^p 1^q 0^r \mid p, q, r \geq 0,\ p \neq r\}$

Which one of the following statements is FALSE?

(A) L_2 is context–free

(B) $L_1 \cap L_2$ is context–free

(C) Complement of L_2 is recursive

(D) Complement of L_1 is context–free but not regular

Answer: -(D)

Exp:- $L_1 = \{0^P 1^q 0^r \mid p, q, r \geq 0\}$ is regular

$L_2 = \{0^P 1^q 0^r \mid p, q, r \geq 0,\ p \neq r\}$ is CFL

(A) L_2 is CFL (True)

(B) $L_1 \cap L_2 =$ CFL (True)

(C) L_2 complement is recursive (True)

(D) L_1 complement is CFL but not regular (False) as L_1 is regular $\overline{L_1}$ is regular

42. Consider the following function

```
int unknown(int n){
        int i, j, k = 0;
        for (i = n / 2; i <= n; i ++)
          for (j = 2; j <= n; j = j * 2)
          k = k + n / 2;
        return (k);
}
```

The return value of the function is

(A) $\Theta(n^2)$ (B) $\Theta(n^2 \log n)$ (C) $\Theta(n^3)$ (D) $\Theta(n^3 \log n)$

Answer:- (B)

Exp:- $i = \left(\frac{n}{2}, \frac{n}{2}+1, \frac{n}{2}+2, \text{-----} n\right)$

Repeats $\frac{n}{2}$ to $n = \left(\frac{n}{2}+1\right)$ times $\boxed{\left.\begin{cases} J = (2, 2^2, 2^3, 2^4, \text{-----} n) \\ k = k + \frac{n}{2} \end{cases}\right\} k = \Theta(n \log n)}$

$$k = \frac{n}{2} + \frac{n}{2} + \text{----} \log n \text{ times} = \frac{n}{2} \log n$$

$$= \frac{n}{2}\log n + \frac{n}{2}\log n + \frac{n}{2}\log n ----\left(\frac{n}{2}+1\right)\text{times}$$

$$= \left(\frac{n}{2}+1\right).\frac{n}{2}\log n$$

$$= \Theta\left(n^2 \log n\right)$$

43. The number of elements that can be sorted in $\Theta(\log n)$ time using heap sort is

(A) $\Theta(1)$ (B) $\Theta\left(\sqrt{\log n}\right)$ (C) $\Theta\left(\frac{\log n}{\log \log n}\right)$ (D) $\Theta(\log n)$

Answer:-(A)

Exp:- After constructing a max-heap in the heap sort , the time to extract maximum element and then heapifying the heap takes $\Theta(\log n)$ time by which we could say that $\Theta(\log n)$ time is required to correctly place an element in sorted array. If $\Theta(\log n)$ time is taken to sort using heap sort, then number of elements that can be sorted is constant which is $\Theta(1)$

44. Consider a hard disk with 16 recording surfaces $(0-15)$ having 16384 cylinders $(0-16383)$ and each cylinder contains 64 sectors $(0-63)$. Data storage capacity in each sector is 512 bytes. Data are organized cylinder–wise and the addressing format is <cylinder no., sector no.>. A file of size 42797 KB is stored in the disk and the starting disk location of the file is <1200, 9, 40>. What is the cylinder number of the last sector of the file, if it is stored in a contiguous manner?

(A) 1281 (B) 1282 (C) 1283 (D) 1284

Answer: -(D)

Exp:- $42797\text{ KB} \equiv \frac{42797 \times 1024}{512} = 85594 \text{ sectors}$

Starting is $\langle 1200,9,40 \rangle$ contains total $24 + (6 \times 64) = 408 \text{ sectors}$

Next, 1201, --------, 1283 cylinders contains total $1024 \times 83 = 84992 \text{ sectors}$

$(\because \text{each cylinder contains } 16 \times 64 = 1024 \text{ sectors})$

$\therefore \text{Total} = 408 + 84992 = 85400 \text{ sectors}$

$\therefore$ The required cylinder number is $\langle 1284 \rangle$ which will contain the last sector of the file

45. Consider the following sequence of micro–operations

$\text{MBR} \leftarrow \text{PC}$

$\text{MAR} \leftarrow \text{X}$

$\text{PC} \leftarrow \text{Y}$

$\text{Memory} \leftarrow \text{MBR}$

Which one of the following is a possible operation performed by this sequence?

(A) Instruction fetch (B) Operand fetch

(C) Conditional branch (D) Initiation of interrupt service

Answer:-(D)

Exp:- PC content is stored in memory via MBR and PC gets new address from Y. It represents a function call (routine), which is matching with interrupt service initiation

46. The line graph L(G) of a simple graph G is defined as follows:

- There is exactly one vertex v(e) in L(G) for each edge e in G.
- For any two edges e and e' in G, L(G) has an edge between v(e) and v(e'), if and only if e and e' are incident with the same vertex in G.

Which of the following statements is/are **TRUE**?

(P) The line graph of a cycle is a cycle.

(Q) The line graph of a clique is a clique.

(R) The line graph of a planar graph is planar.

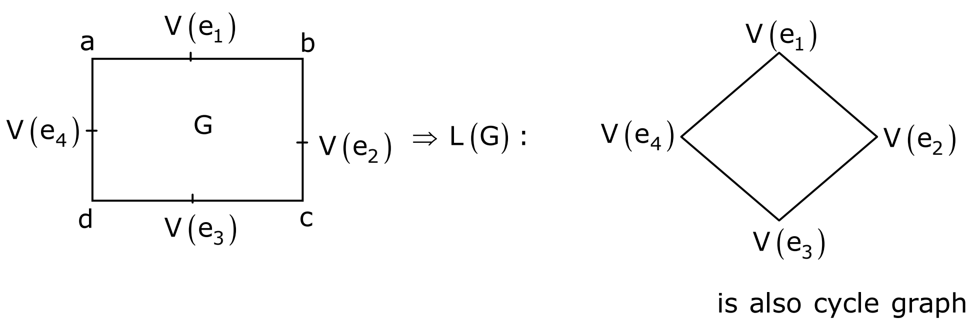
(S) The line graph of a tree is a tree.

(A) P only (B) P and R only

(C) R only (D) P, Q and S only

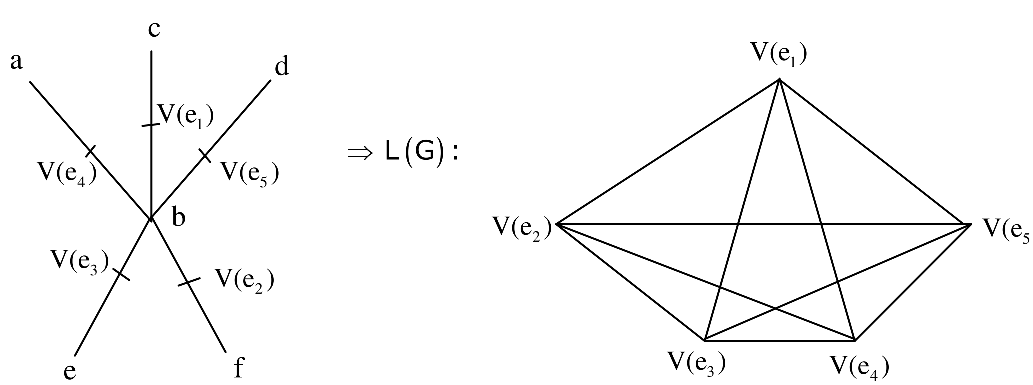
Answer: -(A)

Exp:- P) The line graph of a cycle is a cycle

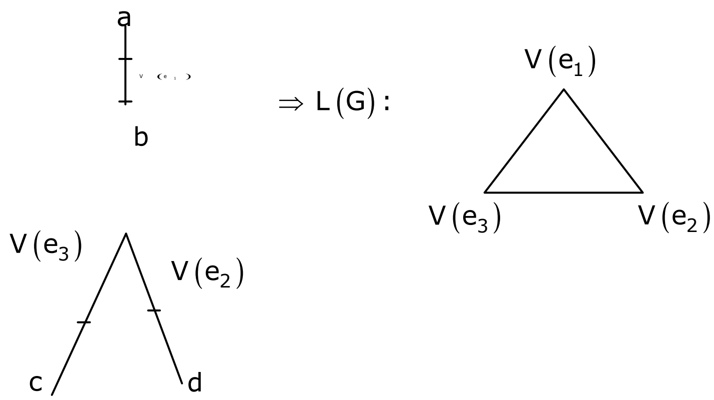


R) Line graph of planar graph need not be planar always. Consider the following example.

Consider the following planar graph (star graph)



S) Hence line graph of planar graph need not be planar(Here we got K_5 which is not planar).



The line graph of a tree need not be tree.

47. What is the logical translation of the following statement?

"None of my friends are perfect."

(A) $\exists x\left(F(x)\wedge\neg P(x)\right)$

(B) $\exists x\left(\neg F(x)\wedge P(x)\right)$

(C) $\exists x\left(\neg F(x)\wedge\neg P(x)\right)$

(D) $\neg\exists x\left(F(x)\wedge P(x)\right)$

Answer: -(D)

Exp:- "None of my friends are perfect"

$=\forall x\left(F(x)\rightarrow\neg P(x)\right)$

$=\forall x\left(\neg F(x)\vee\neg P(x)\right)$

$=\neg\exists x\left(F(x)\wedge P(x)\right)$

Common Data Questions: 48 & 49

The procedure given below is required to find and replace certain characters inside an input character string supplied in array A. The characters to be replaced are supplied in array oldc, while their respective replacement characters are supplied in array newc. Array A has a fixed length of five characters, while arrays oldc and newc contain three characters each. However, the procedure is flawed

```
void find_and_replace (char * A, char * oldc, char * newc) {
        for (int i = 0; i < 5; i++)
          for (int j = 0; j < 3; j++)
            if (A[i] == oldc[j])   A[i] = newc[j];
}
```

The procedure is tested with the following four test cases

(1) oldc = "abc", newc = "dab"

(2) oldc = "cde", newc = "bcd"

(3) oldc = "bca", newc = "cda"

(4) oldc = "abc", newc = "bac"

48. The tester now tests the program on all input strings of length five consisting of characters 'a', 'b', 'c', 'd' and 'e' with duplicates allowed. If the tester carries out this testing with the four test cases given above, how many test cases will be able to capture the flaw?

(A) Only one (B) Only two (C) Only three (D) All four

Answer:-(B)

Exp:- Flaw in this given procedure is that one character of Array 'A' can be replaced by more than one character of newc array, which should not be so.Test case (3) and (4) identifies this flaw as they are containing 'oldc' and 'newc' array characters arranged in specific manner. Following string can reflect flaw, if tested by test case (3).

initially i = j = 0

$$\underset{i=0}{A = "\underline{b}cda"} \qquad \underset{j=0}{\text{oldc} = "\underline{b}ca"} \qquad \underset{j=0}{\text{newc} = "\underline{c}da"}$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

b = b so replaced by c

Next i = 0 & j = 1

$$\underset{i=0}{A = "\underline{c}cda"} \qquad \underset{j=1}{\text{oldc} = "b\underline{c}a"} \qquad \underset{j=1}{\text{newc} = "c\underline{d}a"}$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

c = c so replaced by d

Likewise single character 'b' in A is replaced by 'c' and then by 'd'.
Same way test case (4) can also catch the flaw

49. If array A is made to hold the string "abcde", which of the above four test cases will be successful in exposing the flaw in this procedure?

(A) None (B) 2 only (C) 3 and 4 only (D) 4 only

Answer:-(C)

Exp:- Now for string "abcde" in array A, both test case (3) and (4) will be successful in finding the flaw, as explained in above question.

Common Data Questions: 50 & 51

The following code segment is executed on a processor which allows only register operands in its instructions. Each instruction can have almost two source operands and one destination operand. Assume that all variables are dead after this code segment

```
c = a + b;
d = c * a;
e = c + a;
x = c * c;
if (x > a) {
   y = a * a;
}
else {
   d = d * d;
   e = e * e;
}
```

50. Suppose the instruction set architecture of the processor has only two registers. The only allowed compiler optimization is code motion, which moves statements from one place to another while preserving correctness. What is the minimum number of spills to memory in the compiled code?

(A) 0 (B) 1 (C) 2 (D) 3

Answer:- (B)

Exp:- After applying the code motion optimization the statement d=c*a; and e=c+a; can be moved down to else block as d and e are not used anywhere before that and also value of a and c is not changing.

$c = a + b;$	$R_2 \leftarrow R_1 + R_2$
$x = c * c;$	$R_2 \leftarrow R_2 * R_2$ $[\text{spill}_c]$ 1 memory spill to store the value of c in memory
$\text{if } (x > a)$	CMP R_2 R_1
$\{\ y = a * a;\ \}$	$R_2 \leftarrow R_1 * R_1$

else{
 	
$d = c * a;$
$d = d * d;$
 	
$e = c + a;$
$e = e * e;$
}
	$R_2 \leftarrow [\text{spill}_c]$
	$R_2 \leftarrow R_2 * R_1$
	$R_2 \leftarrow R_2 * R_2$
	$R_2 \leftarrow [\text{spill}_c]$
	$R_2 \leftarrow R_2 + R_1$
	$R_2 \leftarrow R_2 * R_2$

In the above code total number of spills to memory is 1

51. What is the minimum number of registers needed in the instruction set architecture of the processor to compile this code segment without any spill to memory? Do not apply any optimization other than optimizing register allocation

(A) 3 (B) 4 (C) 5 (D) 6

Answer:- (B)

Exp:-

$c = a + b;$	$R_2 \leftarrow R_1 + R_2$
$d = c * a;$	$R_3 \leftarrow R_2 * R_1$
$e = c + a;$	$R_4 \leftarrow R_2 + R_1$
$x = c * c;$	$R_2 \leftarrow R_2 * R_2$

if $(x > a)$	CMP R_2 R_1
$\{ \ y = a * a; \ \}$	$R_1 \leftarrow R_1 * R_1$

else {
$d = d * d;$
$e = e * e;$
}
	$R_3 \leftarrow R_3 * R_3$
	$R_4 \leftarrow R_4 * R_4$

In the above code minimum number of registers needed are = 4

Linked Answer Questions: Q.52 to Q.55 Carry Two Marks Each

Statement for Linked Answer Questions: 52 & 53

Relation R has eight attributes ABCDEFGH. Fields of R contain only atomic values. $F = \{CH \to G, A \to BC, B \to CFH, E \to A, F \to EG\}$ is a set of functional dependencies (FDs) so that F^+ is exactly the set of FDs that hold for R

52. How many candidate keys does the relation R have?

(A) 3 (B) 4 (C) 5 (D) 6

Answer:-(B)

Exp:- Candidate keys are AD, BD, ED and FD

53. The relation R is

(A) in INF, but not in 2NF (B) in 2NF, but not in 3NF

(C) in 3NF, but not in BCNF (D) in BCNF

Answer:-(A)

Exp:- $A \to BC, B \to CFH$ and $F \to EG$ are partial dependencies. Hence it is in 1NF but not in 2NF

Statement for Linked Answer Questions: 54 & 55

A computer uses 46–bit virtual address, 32–bit physical address, and a three–level paged page table organization. The page table base register stores the base address of the first–level table (T_1) ,which occupies exactly one page. Each entry of T_1 stores the base address of a page of the second–level table (T_2). Each entry of T_2 stores the base address of a page of the third–level table (T_3) Each entry of T_3 stores a page table entry (PTE). The PTE is 32 bits in size. The processor used in the computer has a 1 MB 16 way set associative virtually indexed physically tagged cache. The cache block size is 64 bytes.

54. What is the size of a page in KB in this computer?

(A) 2 (B) 4 (C) 8 (D) 16

Answer:-(C)

Exp:- Let the page size be 2^X Bytes.

Then, the page offset = X bits

46-x	x

Now, we are using 3-level paging. First level page table is contained in one page. Each page table entry is 32-bit.

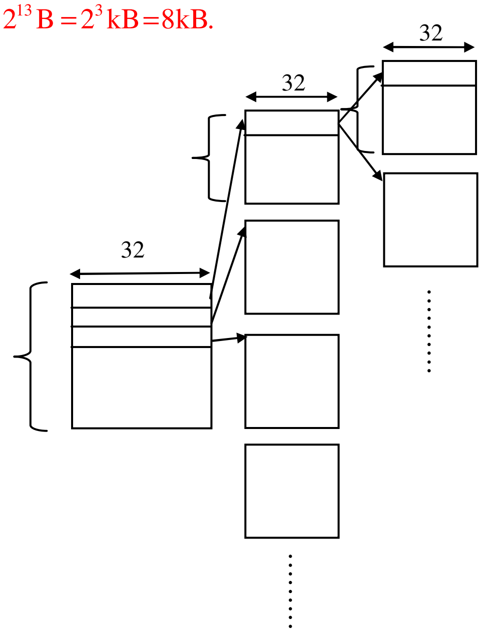
The size of T_3 is = $\frac{2^{46} * 2^2}{2^x} = 2^{46+2-x}$ [$\because$ PTE=32 bit = 4B = 2^2B]

The size of T_2 is = $\frac{2^{46+2-x} * 2^2}{2^x} = 2^{46+4-2x}$

The size of T_1 is = $\frac{2^{46+4-2x} * 2^2}{2^x} = 2^{46+6-3x} = 2^X$ [$\because T_1$ occupies exactly one page]

$\therefore\ 46+6-3x = x \ \Rightarrow x = 13$

So, page size = $2^{13}B = 2^3 kB = 8kB$.



55. What is the minimum number of page colours needed to guarantee that no two synonyms map to different sets in the processor cache of this computer?

(A) 2 (B) 4 (C) 8 (D) 16

Answer:- (C)

Exp:- As the page size is 2^{13} Bytes and page coloring is asked so we divide cache size by page size and group 16 pages in one set.

Number of pages in cache=1MB/8KB=128 pages

Number of set in cache=128/16=8 sets

Take any page of LAS, it will be mapped with cache on any one of these 8 sets (set association mapping).For any two synonym to map with same set they should be colored with same color of that respective set. So minimum we need 8 colors for this mapping.

Q. No. 56 – 60 Carry One Mark Each

56. Complete the sentence:

Universalism is to particularism as diffuseness is to ________

(A) specificity (B) neutrality (C) generality (D) adaptation

Answer:-(A)

Exp:- The relation is that of antonyms

57. Were you a bird, you ___________ in the sky.

(A) would fly (B) shall fly

(C) should fly (D) shall have flown

Answer:-(A)

58. Which one of the following options is the closest in meaning to the word given below?

Nadir

(A) Highest (B) Lowest (C) Medium (D) Integration

Answer:-(B)

Exp:- Nadir in the lowest point on a curve

59. Choose the grammatically INCORRECT sentence:

(A) He is of Asian origin (B) They belonged to Africa

(C) She is an European (D) They migrated from India to Australia

Answer:-(C)

60. What will be the maximum sum of 44, 42, 40, ... ?

(A) 502 (B) 504 (C) 506 (D) 500

Answer:-(C)

Exp:- The maximum sum is the sum of 44, 42,- - - -2.

The sum of 'n' terms of an AP

$$=\frac{n}{2}\left[2a+(n-1)d\right]$$

In this case, n = 22, a = 2 and d = 2

$$\therefore \text{Sum} = 11[4+21\times 2] = 11\times 46 = 506$$

Q. No. 61 – 65 Carry Two Marks Each

61. Out of all the 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7?

(A) 13/90 (B) 12/90 (C) 78/90 (D) 77/90

Answer:- (D)

Exp:- The number of 2 digit multiples of 7 = 13

$\therefore$ Probability of choosing a number

Not divisible by $7 = \frac{90-13}{90} = \frac{77}{90}$

62. A tourist covers half of his journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. The average of the tourist in km/h during his entire journey is

(A) 36 (B) 30 (C) 24 (D) 18

Answer:- (C)

Exp:- Let the total distance covered be 'D'

Now, average speed = $\frac{D}{\text{Total time taken}}$

$$= \frac{D}{\left(\frac{\frac{D}{2}}{60} + \frac{\frac{D}{4}}{30} + \frac{\frac{D}{4}}{10}\right)} = \frac{1}{\frac{1}{120} + \frac{1}{120} + \frac{1}{40}} = \frac{120}{5} = 24\,\text{km/hr}$$

63. Find the sum of the expression

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

(A) 7 (B) 8 (C) 9 (D) 10

Answer:- (B)

Exp:- The expression can be written as

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

$$= \frac{\sqrt{2}-\sqrt{1}}{\left(\sqrt{2}\right)^2-\left(\sqrt{1}\right)^2} + \frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} + \frac{\sqrt{4}-\sqrt{3}}{\left(\sqrt{4}\right)^2-\left(\sqrt{3}\right)^2} + \ldots\ldots + \frac{\sqrt{81}-\sqrt{80}}{\left(\sqrt{81}\right)^2-\left(\sqrt{80}\right)^2}$$

$$= \sqrt{81} - \sqrt{1} = 8$$

64. The current erection cost of a structure is Rs. 13,200. If the labour wages per day increase by 1/5 of the current wages and the working hours decrease by 1/24 of the current period, then the new cost of erection in Rs. is

(A) 16,500 (B) 15,180 (C) 11,000 (D) 10,120

Answer:- (B)

Exp:- Let 'W' be the labour wages, and 'T' be the working hours.

Now, total cost is a function of $W \times T$

Increase in wages = 20%

$\therefore$ Revised wages = 1.2 W

$$\text{Decrease in labour time} = \left(\frac{100}{24}\right)\%$$

$$\therefore \text{ Revised time} = \left(1-\frac{1}{24}\right)T = \frac{23}{24}T$$

$$\therefore \text{ Revised Total cost} = 1.2\times\frac{23}{24}WT = 1.15WT$$
$$= 1.15\times 13200 = 15180$$

65. After several defeats in wars, Robert Bruce went in exile and wanted to commit suicide. Just before committing suicide, he came across a spider attempting tirelessly to have its net. Time and again, the spider failed but that did not deter it to refrain from making attempts. Such attempts by the spider made Bruce curious. Thus, Bruce started observing the near-impossible goal of the spider to have the net. Ultimately, the spider succeeded in having its net despite several failures. Such act of the spider encouraged Bruce not to commit suicide. And then, Bruce went back again and won many a battle, and the rest is history.

Which one of the following assertions is best supported by the above information?

(A) Failure is the pillar of success (B) Honesty is the best policy

(C) Life begins and ends with adventures (D) No adversity justifies giving up hope

Answer:- (D)

Q. No. 1 – 25 Carry One Mark Each

1. Which of the following problems are decidable?
   1) Does a given program ever produce an output?
   2) If L is context-free language, then, is $\overline{L}$ also context-free?
   3) If L is regular language, then, is $\overline{L}$ also regular?
   4) If L is recursive language, then, is $\overline{L}$ also recursive?

   (A) 1,2,3,4 (B) 1,2 (C) 2,3,4 (D) 3,4

Answer:- (D)

Exp:- CFL's are not closed under complementation. Regular and recursive languages are closed under complementation.

2. Given the language L-{ab, aa, baa}, which of the following strings are in L*?
   1) abaabaaabaa
   2) aaaabaaaa
   3) baaaaabaaaab
   4) baaaaabaa

   (A) 1,2 and 3 (B) 2,3 and 4 (C) 1,2 and 4 (D) 1,3 and 4

Answer:-(C)

Exp:- $L = \{ab,\ aa,\ baa\}$

Let S1 = ab , S2 = aa and S3 =baa

abaabaaabaa can be written as S1S2S3S1S2

aaaabaaaa can be written as S1S1S3S1

baaaaabaa can be written as S3S2S1S2

3. In the IPv4 addressing format, the number of networks allowed under Class C addresses is

   (A) 2^{14} (B) 2^{7} (C) 2^{21} (B) 2^{24}

Answer:-(C)

Exp:- For class C address, size of network field is 24 bits. But first 3 bits are fixed as 110; hence total number of networks possible is 2^{21}

4. Which of the following transport layer protocols is used to support electronic mail?

   (A) SMTP (B) IP (C) TCP (D) UDP

Answer:-(C)

Exp:- E-mail uses SMTP, application layer protocol which intern uses TCP transport layer protocol.

5. Consider a random variable X that takes values + 1 and -1 with probability 0.5 each. The values of the cumulative distribution function F(x) at x = -1 and +1 are

   (A) 0 and 0.5 (B) 0 and 1

(C) 0.5 and 1 (D) 0.25 and 0.75

Answer:-(C)

Exp:- The cumulative distribution function

$$F(x)=P(X\le x)$$

$$F(-1)=P(X\le -1)=P(X=-1)=0.5$$

$$F(+1)=P(X\le +1)=P(X=-1)+P(X=+1)=0.5+0.5=1$$

6. Register renaming is done is pipelined processors

(A) as an alternative to register allocation at compile time

(B) for efficient access to function parameters and local variables

(C) to handle certain kinds of hazards

(D) as part of address translation

Answer:-(C)

Exp:- Register renaming is done to eliminate WAR/WAW hazards.

7. The amount of ROM needed to implement a 4 bit multiplier is

(A) 64 bits (B) 128 bits (C) 1 Kbits (D) 2 Kbits

Answer:-(D)

Exp:- For a 4 bit multiplier there are $2^4 \times 2^4 = 2^8 = 256$ combinations.

Output will contain 8 bits.

So the amount of ROM needed is $2^8 \times 8$ bits $= 2$ Kbits.

8. Let W(n) and A(n) denote respectively, the worst case and average case running time of an algorithm executed on an input of size n. Which of the following is **ALWAYS TRUE**?

(A) $A(n)=\Omega(W(n))$ (B) $A(n)=\Theta(W(n))$

(C) $A(n)=O(W(n))$ (D) $A(n)=o(W(n))$

Answer:-(C)

Exp:- The average case time can be lesser than or even equal to the worst case. So A(n) would be upper bounded by W(n) and it will not be strict upper bound as it can even be same (e.g. Bubble Sort and merge sort).

$$\therefore A(n)=O(W(n))$$

9. Let G be a simple undirected planar graph on 10 vertices with 15edges. If G is a connected graph, then the number of **bounded** faces in any embedding of G on the plane is equal to

(A) 3 (B) 4 (C) 5 (D) 6

Answer:-(D)

Exp:- We have the relation V-E+F=2, by this we will get the total number of faces,

F = 7. Out of 7 faces one is an unbounded face, so total 6 bounded faces.

10. The recurrence relation capturing the optimal execution time of the *Towers of Hanoi* problem with n discs is

(A) $T(n) = 2T(n-2) + 2$ (B) $T(n) = 2T(n-1) + n$

(C) $T(n) = 2T(n/2) + 1$ (D) $T(n) = 2T(n-1) + 1$

Answer:-(D)

Exp:- Let the three pegs be A,B and C, the goal is to move n pegs from A to C using peg B

The following sequence of steps are executed recursively

1.move $n-1$ discs from A to B. This leaves disc n alone on peg A --- **T(n-1)**

2.move disc n from A to C---------**1**

3.move $n-1$ discs from B to C so they sit on disc n----- **T(n-1)**

So, T(n) = 2T(n-1) +1

11. Which of the following statements are **TRUE** about an SQL query?

P : An SQL query can contain a HAVING clause even if it does not have a GROUP BY clause

Q : An SQL query can contain a HAVING clause only if it has GROUP BY clause

R : All attributes used in the GROUP BY clause must appear in the SELECT clause

S : Not all attributes used in the GROUP BY clause need to appear in the SELECT clause

(A) P and R (B) P and S (C) Q and R (D) Q and S

Answer:-(B)

Exp:- If we use a HAVING clause without a GROUP BY clause, the HAVING condition applies to all rows that satisfy the search condition. In other words, all rows that satisfy the search condition make up a single group. So, option P is true and Q is false.

S is also true as an example consider the following table and query.

Id	Name
1	Ramesh
2	Ramesh
3	Rajesh
4	Suresh

Select count (*)
From student
Group by Name

Output will be

Count (*)
2
1

1

12. Given the basic ER and relational models, which of the following is **INCORRECT**?

(A) An attribute of an entity can have more than one value

(B) An attribute of an entity can be composite

(C) In a row of a relational table, an attribute can have more than one value

(D) In a row of a relational table, an attribute can have exactly one value or a NULL value

Answer:-(C)

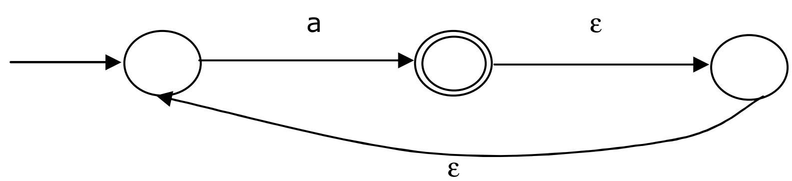
Exp:- The term 'entity' belongs to ER model and the term 'relational table' belongs to relational model.

Options A and B both are true since ER model supports both multivalued and composite attributes.

As multivalued attributes are not allowed in relational databases, in a row of a relational (table), an attribute cannot have more than one value.

13. What is the complement of the language accepted by the NFA show below?

Assume $\Sigma = \{a\}$ and ε is the empty string.



(A) $\varnothing$ (B) $\{\varepsilon\}$ (C) a^* (D) $\{a, \varepsilon\}$

Answer:- (B)

Exp:- Language accepted by NFA is a^+, so complement of this language is $\{\epsilon\}$

14. What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational"

(A) $\exists x\ (\text{real}(x) \vee \text{rational}(x))$ (B) $\forall x\ (\text{real}(x) \rightarrow \text{rational}(x))$

(C) $\exists x\ (\text{real}(x) \wedge \text{rational}(x))$ (D) $\exists x\ (\text{rational}(x) \rightarrow \text{real}(x))$

Answer:- (C)

Exp:- Option A: There exists x which is either real or rational and can be both.

Option B: All real numbers are rational

Option C: There exists a real number which is rational.

Option D: There exists some number which is not rational or which is real.

15. Let A be the 2 x 2 matrix with elements $a_{11} = a_{12} = a_{21} = +1$ and $a_{22} = -1$. Then the eigen values of the matrix A^{19} are

(A) 1024 and -1024 (B) $1024\sqrt{2}$ and $-1024\sqrt{2}$

(C) $4\sqrt{2}$ and $-4\sqrt{2}$ (D) $512\sqrt{2}$ and $-512\sqrt{2}$

Answer:-(D)

Exp:- Characteristic equation of A is $|A-\lambda I|=0$ where λ is the eigen value

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 2 = 0 \Rightarrow \lambda^2 = \pm\sqrt{2}$$

Every matrix satisfies its characteristic equation

Therefore $A^2 - 2I = 0 \Rightarrow A^2 = 2I$

$$A^{19} = A^{18} \times A = \left(A^2\right)^9 \times A = (2I)^9 \times A = 512 \times A$$

Hence eigen values of A^{19} are $\pm 512\sqrt{2}$

16. The protocol data unit (PDU) for the application layer in the Internet stack is

(A) Segment (B) Datagram (C) Message (D) Frame

Answer:-(C)

Exp:- The PDU for Datalink layer, Network layer , Transport layer and Application layer are frame, datagram, segment and message respectively.

17. Consider the function f(x) = sin(x) in the interval $x \in [\pi/4, 7\pi/4]$. The number and location (s) of the local minima of this function are

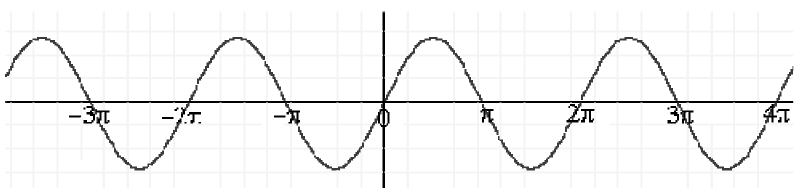
(A) One, at $\pi/2$ (B) One, at $3\pi/2$

(C) Two, at $\pi/2$ and $3\pi/2$ (D) Two, at $\pi/4$ and $3\pi/2$

Answer:-(B)

Exp:- Sin x has a maximum value of 1 at $\frac{\pi}{2}$, and a minimum value of –1 at $\frac{3\pi}{2}$ and at all angles conterminal with them.

The graph of $f(x) = \sin x$ is



$\therefore$ In the interval $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$, it has one local minimum at $x = \frac{3\pi}{2}$

18. A process executes the code

fork ();
fork ();
fork ();

The total number of **child** processes created is

(A) 3 (B) 4 (C) 7 (D) 8

Answer:- (C)

Exp:- If fork is called n times, there will be total 2^n running processes including the parent process. So, there will be 2^n-1 child processes.

19. The decimal value 0.5 in IEEE single precision floating point representation has

(A) fraction bits of 000…000 and exponent value of 0

(B) fraction bits of 000…000 and exponent value of -1

(C) fraction bits of 100…000 and exponent value of 0

(D) no exact representation

Answer:-(B)

Exp:- $(0.5)_{10} = (1.0)_2 \times 2^{-1}$

So, exponent = -1 and fraction is 000 - - - 000

20. The truth table

X	Y	f(X,Y)
0	0	0
0	1	0
1	0	1
1	1	1

represents the Boolean function

(A) X (B) X + Y (C) $X \oplus Y$ (D) Y

Answer:- (A)

Exp:- $XY' + XY = X(Y' + Y) = X$

21. The worst case running time to search for an element in a balanced binary search tree with $n2^n$ elements is

(A) $\Theta(n \log n)$ (B) $\Theta(n2^n)$ (C) $\Theta(n)$ (D) $\Theta(\log n)$

Answer:-(C)

Exp:- The worst case search time in a balanced BST on 'x' nodes is logx. So, if x = $n2^n$, then $\log(n2^n) = \log n + \log(2^n) = \log n + n = \theta(n)$

22. Assuming $P \neq NP$, which of the following is **TRUE**?

(A) NP-complete = NP (B) NP-complete $\cap P = \varnothing$

(C) NP-hard = NP (D) P = NP-complete

Answer:-(B)

Exp:- If P!=NP, then it implies that no NP-Complete problem can be solved in polynomialtime which implies that the set P and the set NPC are disjoint.

23. What will be the output of the following C program segment?

```
Char inChar = 'A' ;
switch (inChar ) {
case 'A' : printf ("Choice A\ n");
case 'B' :
case 'C' : print f("Choice B");
case 'D' :
case 'E' :
default  : printf ("No Choice") ; }
```

(A) No choice

(B) Choice A

(C) Choice A
Choice B No choice

(D) Program gives no output as it is erroneous

Answer:-(C)

Exp:- Since there is no 'break' statement , the program executes all the subsequent case statements after printing "choice A"

24. Which of the following is **TRUE?**

(A) Every relation is 3NF is also in BCNF

(B) A relation R is in 3NF if every non-prime attribute of R is fully functionally dependent on every key of R

(C) Every relation in BCNF is also in 3NF

(D) No relation can be in both BCNF and 3NF

Answer:-(C)

Exp:- Option A is false since BCNF is stricter than 3NF (it needs LHS of all FDs should be candidate key for 3NF condition)

Option B is false since the definition given here is of 2NF

Option C is true, since for a relation to be in BCNF it needs to be in 3NF, every relation in BCNF satisfies all the properties of 3NF.

Option D is false, since if a relation is in BCNF it will always be in 3NF.

25. Consider the following logical inferences.

I_1 : If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.

I_2 : If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.

Which of the following is **TRUE**?

(A) Both I_1 and I_2 are correct inferences

(B) I_1 is correct but I_2 is not a correct inference

(C) I_1 is not correct but I_2 is a correct inference

(D) Both I_1 and I_2 are not correct inferences

Answer:- (B)

Exp:-

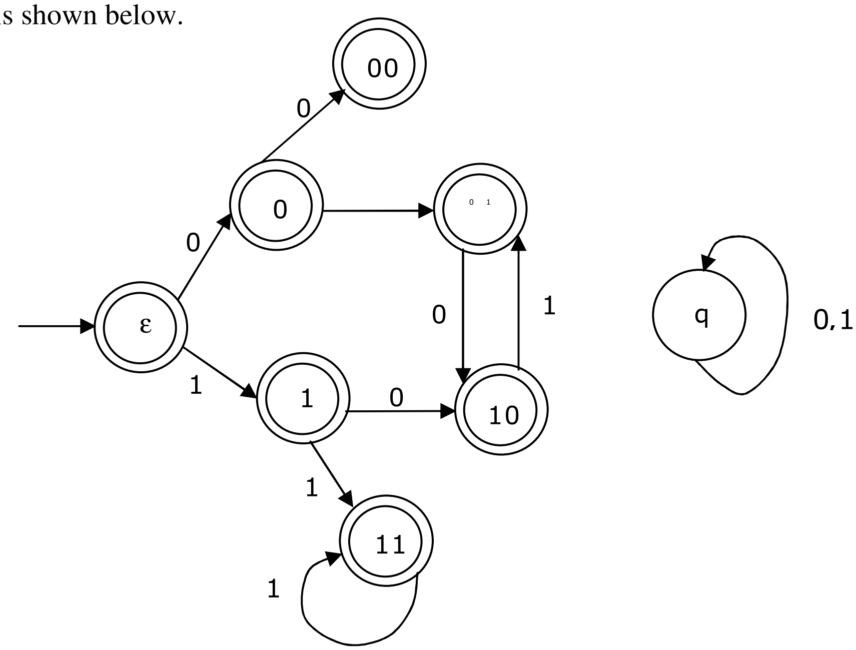
$$I_1: \quad R \rightarrow \sim C \approx \sim R \vee \sim C$$
$$\begin{array}{c} C \\ \hline \sim R \\ \hline \end{array} \quad (\text{there was no rain})$$

$$I_2: \quad R \rightarrow \sim C \approx \sim R \vee \sim C$$
$$\begin{array}{cc} \sim R & \sim R \\ \hline & \sim R \vee C \\ \hline \end{array}$$

$(I_1 \text{ is correct and } I_2 \text{ is not correct inference})$

Q. No. 26 – 51 Carry Two Marks Each

26. Consider the set of strings on {0,1} in which, *every substring of 3 symbols* has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially completed DFA that accepts this language is shown below.



The missing arcs in the DFA are

(A)

	00	**01**	**10**	**11**	**q**
00	1	0			
01				1	
10	0				
11			0		

(B)

	00	**01**	**10**	**11**	**q**
00		0			1
01		1			
10				0	
11		0			

(C)

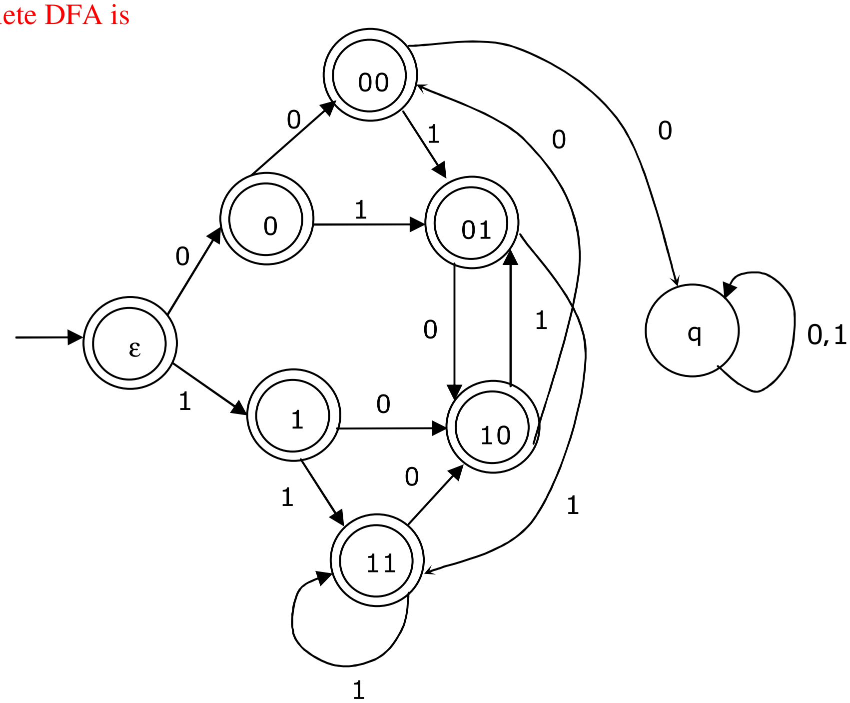
	00	**01**	**10**	**11**	**q**
00		1			0
01		1			
10			0		
11		0			

(D)

	00	**01**	**10**	**11**	**q**
00		1			0
01				1	
10	0				
11			0		

Answer:-(D)

Exp:- The complete DFA is



27. The height of a tree is defined as the number of edges on the longest path in the tree. The function shown in the pseudocode below is invoked as height (root) to compute the height of a binary tree rooted at the tree pointer root.

int height (treeptr n)

{ if (n== NULL) return -1;

if $\left(n \rightarrow \text{left} == \text{NULL}\right)$

```
        if (n → right == NULL) return 0;
        else return [B1];                                   // Box 1
    else {h1 = height (n → left);
        if (n → right == NULL) return (1 + h1);
        else {h2 = height (n → right);
                    return [B2] ;                           // Box 2
                    }
    }
}
```

The appropriate expressions for the two boxes B1 and B2 are

(A) $B1: (1 + \text{height}(n \to \text{right}))$

$B2: (1 + \max(h1, h2))$

(B) $B1: (\text{height}(n \to \text{right}))$

$B2: (1 + \max(h1, h2))$

(C) $B1: \text{height}(n \to \text{right})$

$B2: \max(h1, h2)$

(D) $B1: (1 + \text{height}(n \to \text{right}))$

$B2: \max(h1, h2)$

Answer:-(A)

Exp:- int height (treeptr n)

```
{       if (n = = nu11) return – 1;
                        → /*   If there is no node, return -1     */

        if (n → left == NULL) → /* If there is no left child for node 'n' */

                if (n → right == NULL) return O;
                        → /*If no left child & no right child for 'n', return */
                else return (1+height (n → right) );
                        → /*   If no left child, but there is a right child, then compute height
                        for right sub tree. Therefore total height is 1+ height (n → right) */

        else { → / *   If there exist left child node for node 'n' */

                h1 = height (n → left);
                        → / * First Find the height of left sub tree for node 'n' */

                If (n → right == NULL) return (1 + h1);
```

$\rightarrow$ /* If there exist left child and no right child and no right child for a node 'n', then total height
= height from (n to $n \rightarrow$ left) + left sub tree height
$=1 + \text{height}\ (n \rightarrow \text{left}) = 1 + h1$ */

else {h_2 = height $(n \rightarrow \text{right})$;

$\rightarrow$/* If there exist right child also, then compute height of right sub tree for a node 'n' */

return $(1 + \max\ (h_1, h_2))$;

$\rightarrow$ /* Total height for node 'n'=
1 + Max (Left Subtree height, Right sub tree height)
= 1 + Max (h_1, h_2) */

}

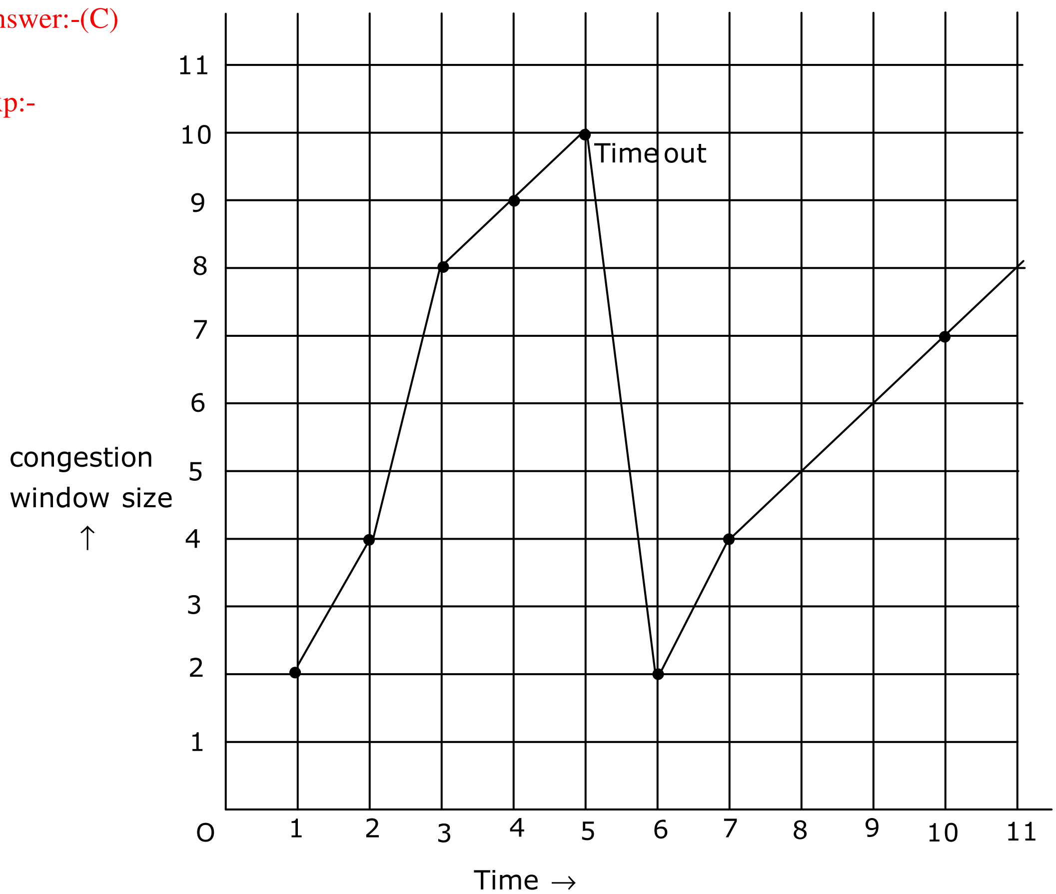
}

28. Consider an instance of TCP's Additive Increase Multiplicative decrease (AIMD) algorithm where the window size at the start of the slow start phase is 2 MSS and the threshold at the start of the first transmission is 8 MSS. Assume that a timeout occurs during the fifth transmission. Find the congestion window size at the end of the tenth transmission.

(A) 8MSS (B) 14MSS (C) 7MSS (D) 12MSS

Answer:-(C)

Exp:-



Given, initial threshold = 8

Time = 1, during 1st transmission,Congestion window size = 2 (slow start phase)

Time = 2, congestion window size = 4 (double the no. of acknowledgments)

Time = 3, congestion window size = 8 (Threshold meet)

Time = 4, congestion window size = 9, after threshold (increase by one Additive increase)

Time = 5, transmits 10 MSS, but time out occurs congestion window size = 10

Hence threshold = (Congestion window size)/2 = $^{10}\!/_{2} = 5$

Time = 6, transmits 2

Time = 7, transmits 4

Time = 8, transmits 5 (threshold is 5)

Time = 9, transmits 6, after threshold (increase by one Additive increase)

Time = 10, transmits 7

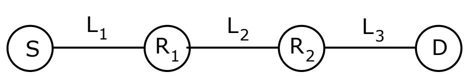
$\therefore$ During 10^{th} transmission, it transmits 7 segments hence at the end of the tenth transmission the size of congestion window is 7 MSS.

29. Consider a source computer (S) transmitting a file of size 10^6 bits to a destination computer (D) over a network of two routers (R_1 and R_2) and three links (L_1, L_2, and L_3). L_1 connects S to R_1; L_2 connects R_1 to R_2; and L_3 connects R_2 to D. Let each link be of length 100km. Assume signals travel over each line at a speed of 10^8 meters per second. Assume that the link bandwidth on each link is 1Mbps. Let the file be broken down into 1000 packets each of size 1000 bits. Find the total sum of transmission and propagation delays in transmitting the file from S to D?

(A) 1005ms (B) 1010ms (C) 3000ms (D) 3003ms

Answer:- (A)

Exp:-



Transmission delay for 1 packet from each of S, R_1 and R_2 will take 1ms

Propagation delay on each link L_1, L_2 and L_3 for one packet is 1ms

Therefore the sum of transmission delay and propagation delay on each link for one packet is 2ms.

The first packet reaches the destination at 6^{th}ms

The second packet reaches the destination at 7^{th}ms

So inductively we can say that 1000^{th} packet reaches the destination at 1005^{th} ms

30. Suppose R1 $(\underline{A}, B)$ and R_2 $(\underline{C}, D)$ are two relation schemas. Let r_1 and r_2 be the corresponding relation instances. B is a foreign key that refers to C in R_2. If data in r_1 and r_2 satisfy referential integrity constrains, which of the following is **ALWAYS TRUE**?

(A) $\Pi_B(r_1) - \Pi_C(r_2) = \varnothing$ (B) $\Pi_C(r_2) - \Pi_B(r_1) = \varnothing$

(C) $\Pi_B(r_1) = \Pi_C(r_2)$ (D) $\Pi_B(r_1) - \Pi_C(r_2) \neq \varnothing$

Answer:-(A)

Exp:- Since B is a foreign key referring C,values under B will be subset of values under C ($\Pi_B(r_1) \subseteq \Pi_C(r_2) \Rightarrow \Pi_B(r_1) - \Pi_C(r_2) = \varnothing$)

31. Consider the virtual page reference string

1,2,3,2,4,1,3,2,4,1

on a demand paged virtual memory system running on a computer system that has main memory size of 3 page frames which are initially empty. Let LRU, FIFO and OPTIMAL denote the number of page faults under the corresponding page replacement policy. Then

(A) OPTIMAL < LRU < FIFO (B) OPTIMAL < FIFO < LRU

(C) OPTIMAL = LRU (D) OPTIMAL = FIFO

Answer:- (B)

Exp:- FIFO

```
1  1  1  4  4  4
   2  2  2  1  1        → (6) faults
      3  3  3  2
```

Optimal

```
1  1  1  1  1
   2  2  4  4           → (5) faults
      3  3  2
```

LRU

```
1  1  1  4  4  4  2  2  2
   2  2  2  2  3  3  3  1      → (9) faults
      3  3  1  1  1  4  4
```

Optimal < FIFO < LRU

32. A file system with 300 GByte disk uses a file descriptor with 8 direct block addresses, 1 indirect block address and 1 doubly indirect block address. The size of each disk block is 128 Bytes and the size of each disk block address is 8 Bytes. The maximum possible file size in this file system is

(A) 3 KBytes (B) 35 KBytes

(C) 280 KBytes (D) dependent on the size of the disk

Answer:-(B)

Exp:- Each block size = 128 Bytes

Disk block address = 8 Bytes

$\therefore$ Each disk can contain = $\frac{128}{8} = 16$ addresses

Size due to 8 direct block addresses: 8 x 128

Size due to 1 indirect block address: 16 x 128

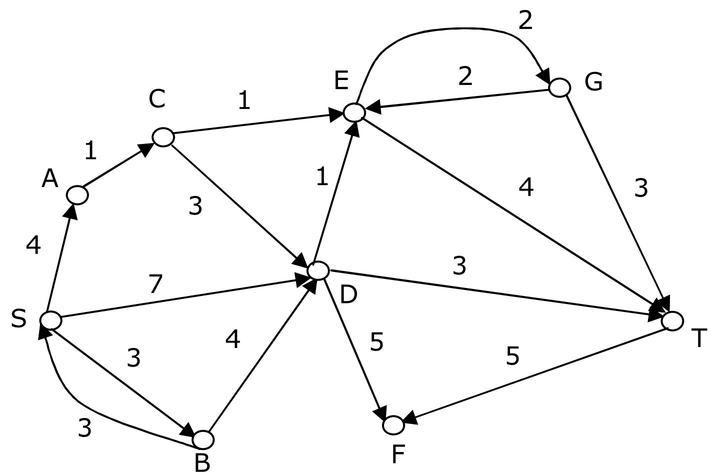
Size due to 1 doubly indirect block address: 16 x 16 x 128

Size due to 1 doubly indirect block address: 16 x 16 x 128

So, maximum possible file size:

$= 8\times128 + 16\times128 + 16\times16\times128 = 1024 + 2048 + 32768 = 35840 \text{ Bytes} = 35\,\text{KBytes}$

33. Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex *v* is updated only when a strictly shorter path to *v* is discovered.



(A) SDT (B) SBDT (C) SACDT (D) SACET

Answer:- (D)

Exp:- Let d[v] represent the shortest path distance computed from 'S'

Initially d[S]=0, d[A] = ∞, $d[B] = \infty.----, d[T] = \infty$

And let P[v] represent the predecessor of v in the shortest path from 'S' to 'v' and let P[v]=-1 denote that currently predecessor of 'v' has not been computed

$\rightarrow$ Let Q be the set of vertices for which shortest path distance has not been computed

$\rightarrow$ Let W be the set of vertices for which shortest path distance has not been computed

$\rightarrow$ So initially, $Q = \{S, A, B, C, D, E, F, G, T\}, W = \phi$

We will use the following procedure

Repeat until Q is empty

{

1 u = choose a vertex from Q with minimum d[u] value

2. $Q = Q - u$

3. update all the adjacent vertices of u

4. W = W U{u}

}

d[S] = 0, d[A] = ∞, d[B] = ∞ ,………, d[T] = ∞

Iteration 1

Step 1 : $u = S$

Step 2 : $Q = \{A, B, C, D, E, F, G, T\}$

Step 3: final values after adjustment

$$d[S] = 0, d[A] = 4, d[B] = 3, d[C] = \infty, d[D] = 7, d[E] = \infty - —, d[T] = \infty$$

$$P[A] = S, P[B] = S, P[C] = -1, P[D] = S, P[E] = -1 - —, P[T] = -1$$

Step 4 : $W = \{S\}$

Iteration 2:

Step 1: $u = B$

Step 2: $Q = \{A, C, D, E, F, G, T\}$

step 3: final values after adjustment

$$d[S] = 0, d[A] = 4, d[B] = 3, d[C] = \infty, d[D] = 7, d[E] = \infty - —, d[T] = \infty$$

$$P[A] = S, P[B] = S, P[C] = -1, P[D] = S, P[E] = -1 - —, P[T] = -1$$

Step 4: $W = \{S, B\}$

Iteration 3:

Step 1: $u = A$

Step 2: $Q = \{C, D, E, F, G, T\}$

step 3: final values after adjustment

$$d[S] = 0, d[A] = 4, d[B] = 3, d[C] = 5, d[D] = 7, d[E] = \infty - —, d[T] = \infty$$

$$P[A] = S, P[B] = S, P[C] = A, P[D] = S, P[E] = -1 - —, P[T] = -1$$

Step 4: $W = \{S, B, A\}$

Iteration 4:

Step 1: $u = C$

Step 2: $Q = \{D, E, F, G, T\}$

step 3: final values after adjustment

$$d[S] = 0, d[A] = 4, d[B] = 3, d[C] = 5, d[D] = 7, d[E] = 6, - - -, d[T] = \infty$$

$$P[A] = S, P[B] = S, P[C] = A, P[D] = S, P[E] = C, - - -, P[T] = -1$$

Step 4: $W = \{S, B, A, C\}$

Iteration 5:

$\text{Step 1}: u = E$

$\text{Step 2}: Q = \{D, F, G, T\}$

$\text{step 3}: \text{final values after adjustment}$

$$d[S]=0, d[A]=4, d[B]=3, d[C]=5, d[D]=7, d[E]=6, d[F]=\infty, d[G]=8, d[T]=10$$

$$P[A]=S, P[B]=S, P[C]=A, P[D]=S, P[E]=C, P[F]=-1, P[G]=E, P[T]=E$$

$\text{Step 4}: W = \{S, B, A, C, E\}$

After iteration 5, we can observe that P[T]=E , P[E]=C , P[C]=A , P[A]=S,
So the shortest path from S to T is SACET

34. A list of n strings, each of length n, is sorted into lexicographic order using the merge-sort algorithm. The worst case running time of this computation is

(A) $O(n \log n)$ (B) $O(n^2 \log n)$

(C) $O(n^2 + \log n)$ (D) $O(n^2)$

Answer:-(B)

Exp:- The height of the recursion tree using merge sort is logn and n^2 comparisons are done at each level, where at most n pairs of strings are compared at each level and n comparisons are required to compare any two strings, So the worst case running time is $O(n^2 \log n)$

35. Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to

(A) 15 (B) 30 (C) 90 (D) 360

Answer:- No option matching (marks to all)

Exp:- 4 vertices from 6 vertices can be chosen in in ${}^6c_4$. Number of cycles of length 4 that can be formed from those selected vertices is (4-1)!/2 (left or right/ up or down does not matter), so total number of 4 length cycles are $({}^6c_4.3!)/2 = 45$.

36. How many onto (or surjective) functions are there from an n-element $(n \geq 2)$ set to a 2-element set?

(A) 2^n (B) 2^n-1 (C) 2^n - 2 (D) $2(2^n$-2)

Answer:- (C)

Exp:- Total number of functions is 2^n, out of which there will be exactly two functions where all elements map to exactly one element, so total number of onto functions is 2^n-2

37. Consider the program given below, in a block-structured pseudo-language with lexical scoping and nesting of procedures permitted.

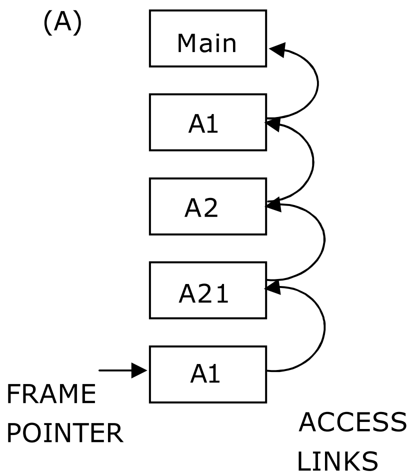
```
Program main;
  Var . . .
  Procedure A1;
    Var ….
```

```
    Call A2;
  End A1
  Procedure A2;
    Var . . .
    Procedure A21;
        Var . . .
       Call A1;
   End A21
   Call A21;
End A2
  Call A1;
End main.
```

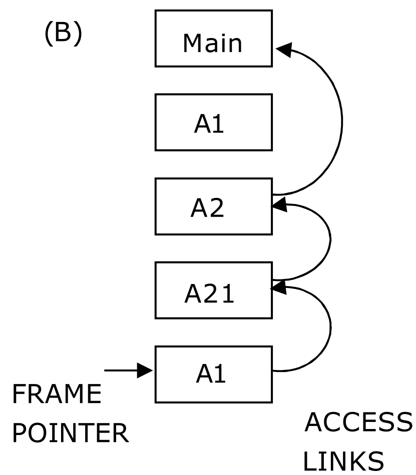
Consider the calling chain: Main $\rightarrow$ A1 $\rightarrow$ A2 $\rightarrow$ A21 $\rightarrow$ A1

The correct set of activation records along with their access links is given by

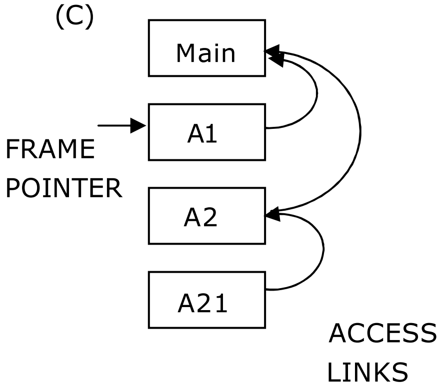
(A)



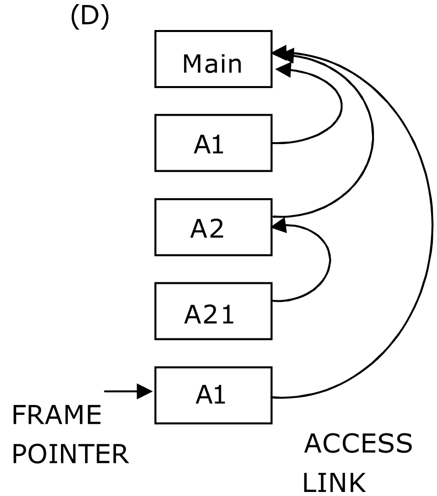
(B)



(C)



(D)



Answer:-(D)

Exp:- Access link is defined as link to activation record of closest lexically enclosing block in program text, so the closest enclosing blocks respectively for A1 ,A2 and A21 are main , main and A2

38. Suppose a circular queue of capacity (n – 1) elements is implemented with an array of *n* elements. Assume that the insertion and deletion operations are carried out using REAR and FRONT as array index variables, respectively. Initially, REAR = FRONT = 0. The conditions to detect queue full and queue empty are

(A) *full:* (REAR+1) mod n==FRONT
empty: REAR ==FRONT

(B) *full:*(REAR+1)mod n==FRONT
empty: (FRONT+1)mod n==REAR

(C) *full:* REAR==FRONT
empty: (REAR+1) mod n ==FRONT

(D) *full:*(FRONT+1)mod n==REAR
empty: REAR ==FRONT

Answer:- (A)

Exp:- The ***counter example*** for the condition *full* : REAR = FRONT is

Initially when the Queue is empty REAR=FRONT=0 by which the above *full* condition is satisfied which is false

The ***counter example*** for the condition *full* : (FRONT+1)mod n =REAR is

Initially when the Queue is empty REAR=FRONT=0 and let n=3, so after inserting one element REAR=1 and FRONT=0, at this point the condition *full* above is satisfied, but still there is place for one more element in Queue, so this condition is also false

The ***counter example*** for the condition *empty* : (REAR+1)mod n = FRONT is

Initially when the Queue is empty REAR=FRONT=0 and let n=2, so after inserting one element REAR=1 and FRONT=0, at this point the condition *empty* above is satisfied, but the queue of capacity n-1 is full here

The ***counter example*** for the condition *empty* : (FRONT+1)mod n =REAR is

Initially when the Queue is empty REAR=FRONT=0 and let n=2, so after inserting one element REAR=1 and FRONT=0, at this point the condition *empty* above is satisfied, but the queue of capacity n-1 is full here

39. An Internet Service Provider (ISP) has the following chunk of CIDR-based IP addresses available with it: 245.248.128.0/20. The ISP wants to give half of this chunk of addresses to Organization A, and a quarter to Organization B, while retaining the remaining with itself. Which of the following is a valid allocation of address to A and B?

(A) 245.248.136.0/21 and 245.248.128.0/22

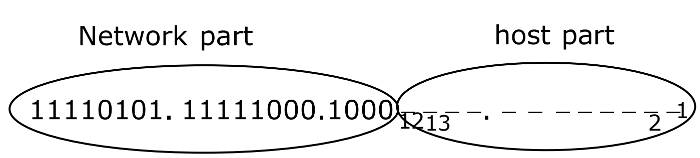
(B) 245.248.128.0/21 and 245.248.128.0/22

(C) 245.248.132.0/22 and 245.248.132.0/21

(D) 245.248.136.0/24 and 245.248.132.0/21

Answer:- (A)

Exp:-



Since half of 4096 host addresses must be given to organization A, we can set 12^{th} bit to 1 and include that bit into network part of organization A, so the valid allocation of addresses to A is 245.248.136.0/21

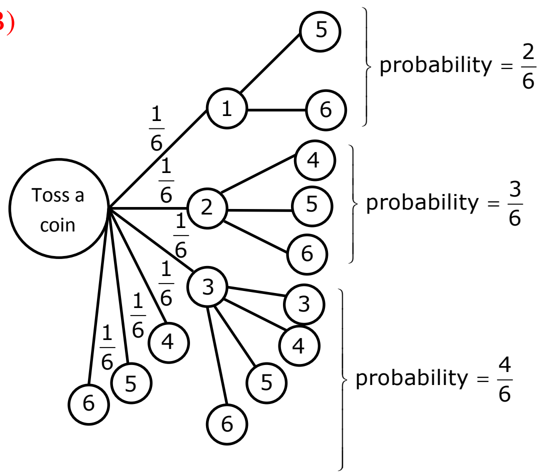
Now for organization B, 12^{th} bit is set to '0' but since we need only half of 2048 addresses, 13^{th} bit can be set to '0' and include that bit into network part of organization B so the valid allocation of addresses to B is 245.248.128.0/22

40. Suppose a fair six-sided die is rolled once. If the value on the die is 1, 2, or 3, the die is rolled a second time. What is the probability that the sum total of values that turn up is at least 6?

(A) 10/21 (B) 5/12 (C) 2/3 (D) 1/6

Answer:- (B)

Exp:-



$$\therefore \text{Required probability} = \frac{1}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{3}{6}+\frac{1}{6}\times\frac{4}{6}+\frac{1}{6}=\frac{15}{36}=\frac{5}{12}$$

41. Fetch_And_Add (X, i) is an atomic Read-Modify-Write instruction that reads the value of memory location X, increments it by the value i, and returns the old value of X. It is used in the pseudocode shown below to implement a busy-wait lock. L is an unsigned integer shared variable initialized to 0. The value of 0 corresponds to lock being available, while any non-zero value corresponds to the lock being not available.

```
AcquireLock(L){
    While (Fetch_And_Add(L,1))
                L = 1;
}
Release Lock(L){
    L = 0;
}
```

This implementation

(A) fails as L can overflow

(B) fails as L can take on a non-zero value when the lock is actually available

(C) works correctly but may starve some processes

(D) works correctly without starvation

Answer:- (B)

Exp:- 1. Acquire lock (L) {

2. While (Fetch_And_Add(L, 1))

3. L = 1.

}

4. Release Lock (L) {

5. L = 0;

6. }

Let P and Q be two concurrent processes in the system currently executing as follows

P executes 1,2,3 then Q executes 1 and 2 then P executes 4,5,6 then L=0 now Q executes 3 by which L will be set to 1 and thereafter no process can set

L to zero, by which all the processes could starve.

42. Consider the 3 process, P1, P2 and P3 shown in the table.

Process	Arrival time	Time units Required
P1	0	5
P2	1	7
P3	3	4

The completion order of the 3 processes under the policies FCFS and RR2 (round robin scheduling with CPU quantum of 2 time units) are

(A) **FCFS:** P1, P2, P3 **RR2:** P1, P2, P3

(B) **FCFS:** P1, P3, P2 **RR2:** P1, P3, P2

(C) **FCFS:** P1, P2, P3 **RR2:** P1, P3, P2

(D) **FCFS:** P1, P3, P2 **RR2:** P1, P2, P3

Answer:-(C)

Exp:- For FCFS Execution order will be order of Arrival time so it is P1,P2,P3

Next For RR with time quantum=2,the arrangement of Ready Queue will be as follows:

RQ: P1,P2,P1,P3,P2,P1,P3,P2

This RQ itself shows the order of execution on CPU(Using Gantt Chart) and here it gives the completion order as P1,P3,P2 in Round Robin algorithm.

43. What is the minimal form of the Karnaugh map shown below? Assume that X denotes a don't care term.

cd \ ab	00	01	11	10
00	1	X	X	1
01	X			1
11				
10	1			X

(A) $\bar{b}\bar{d}$
(B) $\bar{b}\bar{d}+\bar{b}\bar{c}$
(C) $\bar{b}\bar{d}+a\bar{b}\bar{c}d$
(D) $\bar{b}\bar{d}+\bar{b}\bar{c}+\bar{c}\bar{d}$

Answer:- (B)

Exp:-

cd \ ab	00	01	11	10
00	1	X		1
01	X			1
11				
10	1			X

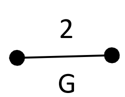
$d'b'+c'b'$

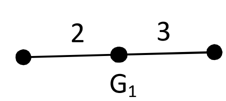
44. Let G be a weighted graph with edge weights greater than one and G' be the graph constructed by squaring the weights of edges in G. Let T and T' be the minimum spanning trees of G and G' respectively, with total weights t and t'. Which of the following statements is **TRUE**?

(A) T' = T with total weight $t' = t^2$
(B) T' = T with total weight $t' < t^2$
(C) T' $\neq$ T but total weight $t' = t^2$
(D) None of these

Answer:-(D)

Exp:-





Graph G is counter example for options (B) and (C) and Graph G_1 is counter example for option (A)

45. The bisection method is applied to compute a zero of the function $f(x) = x^4 - x^3 - x^2 - 4$ in the interval [1,9]. The method converges to a solution after ______ iterations.

(A) 1 (B) 3 (C) 5 (D) 7

Answer:- (B)

Exp:-

$$f(x) = x^4 - x^3 - x^2 - 4$$

$$f(1) < 0 \text{ and } f(9) > 0 \therefore x_0 = \frac{1+9}{2} = 5$$

$$f(5) > 0 \quad \therefore \text{ root lies in } [1,5]$$

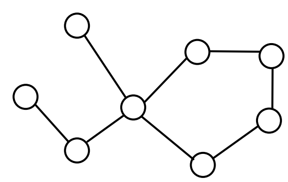
$$x_1 = \frac{1+5}{2} = 3$$

$$f(3) > 0 \therefore \text{ root lies in } [1,3]$$

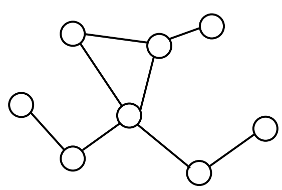
$$x_2 = \frac{1+3}{2} = 2$$

$$f(2) = 0 \therefore \text{ root is } 2$$

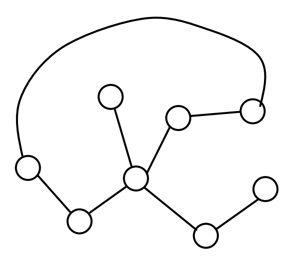
46. Which of the following graph is isomorphic to



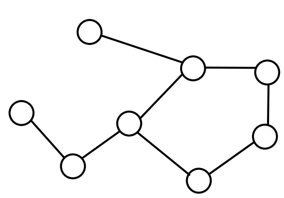
(A)



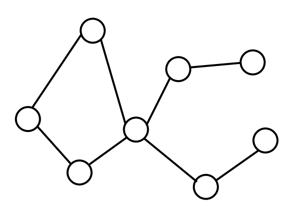
(B)



(C)



(D)



Answer:-(B)

Exp:- The graph in option (A) has a 3 length cycle whereas the original graph does not have a 3 length cycle

The graph in option (C) has vertex with degree 3 whereas the original graph does not have a vertex with degree 3

The graph in option (D) has a 4 length cycle whereas the original graph does not have a 4 length cycle

47. Consider the following transactions with data items P and Q initialized to zero:

T_1 : read (P) ;
read (Q) ;
if P = 0 then Q : = Q + 1 ;
write (Q).

T_2 : read (Q) ;
read (P)
if Q = 0 then P : = P + 1 ;
write (P).

Any non-serial interleaving of T1 and T2 for concurrent execution leads to

(A) a serializable schedule

(B) a schedule that is not conflict serializable

(C) a conflict serializable schedule

(D) a schedule for which precedence graph cannot be drawn

Answer:-(B)

Exp:- Let S be a non-serial schedule, without loss of generality assume that T1 has started earlier than T2. The first instruction of T1 is read(P) and the last instruction of T2 is write(P), so the precedence graph for S has an edge from T1 to T2, now since S is a non-serial schedule the first instruction of T2(read(Q)) should be executed before last instruction of T1(write(Q)) and since read and write are conflicting operations, the precedence graph for S also contains an edge from T2 to T1, So we will have a cycle in the precedence graph which implies that any non serial schedule with T1 as the earliest transaction will not be conflict serializable.

In a similar way we can show that if T2 is the earliest transaction then also the schedule is not conflict serializable.

Common Data Questions: 48 & 49

Consider the following relations A, B and C:

A

Id	**Name**	**Age**
12	Arun	60
15	Shreya	24
99	Rohit	11

B

Id	**Name**	**Age**
15	Shreya	24
25	Hari	40

98	Rohit	20
99	Rohit	11

C

Id	Phone	Area
10	220	02
99	2100	01

48. How many tuples does the result of the following SQL query contain?

SELECT A.Id
FROM A
WHERE A.Age > **ALL**(**SELECT** B.Age
FROM B
WHERE B.Name = 'Arun')

(A) 4 (B) 3 (C) 0 (D) 1

Answer:-(B)

Exp:- As the result of subquery is an empty table, '>ALL' comparison is true . Therefore, all the three row id's of A will be selected from table A.

49. How many tuples does the result of the following relational algebra expression contain? Assume that the schema of $A \cup B$ is the same as that of A.

$$(A \cup B) \bowtie_{A.Id>40 \vee C.Id<15} C$$

(A) 7 (B) 4 (C) 5 (D) 9

Answer:-(A)

Exp:- The final table is

AUB . Id	Name	Age	C.Id	Phone	Area
12	Arun	60	10	2200	02
15	Shreya	24	10	2200	02
25	Hari	40	10	2200	02
98	Rohit	20	10	2200	02
98	Rohit	20	99	2100	01
99	Rohit	11	10	2200	02
99	Rohit	11	99	2100	01

Common Data Questions: 50 & 51

Consider the following C code segment:

```
int a,   b,  c = 0;
void prtFun(void);
```

```
main( )
{       static int  a = 1;              /* Line 1 */
         prtFun( );
        a + = 1;
         prtFun( )
        printf("\n %d %d ", a, b);
}

void prtFun(void)
{       static int a=2;                 /* Line 2 */
        int b=1;
        a+=++b;
        printf("\n %d %d ", a, b);
}
```

50. What output will be generated by the given code segment if:
Line 1 is replaced by **auto int a = 1**;
Line 2 is replaced by **register int a = 2**;

(A)	(B)	(C)	(D)
3 1	4 2	4 2	4 2
4 1	6 1	6 2	4 2
4 2	6 1	2 0	2 0

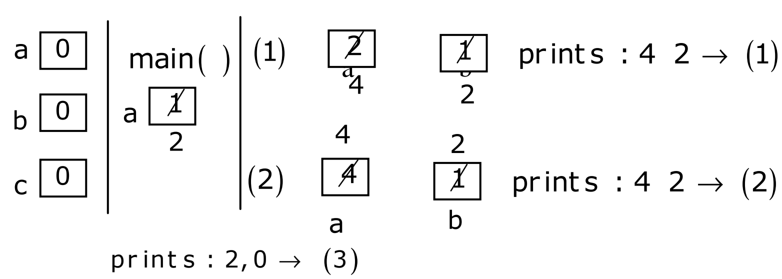
Answer:-(D)

Exp:- **Static local variables:** Scope is limited to function/block but life time is entire program.

Automatic local variables:

Storage allocated on function entry and automatically deleted or freed when the function is exited.

Register variables: Same as automatic variables except that the register variables will not have addresses Hence may not take the address of a register variable.

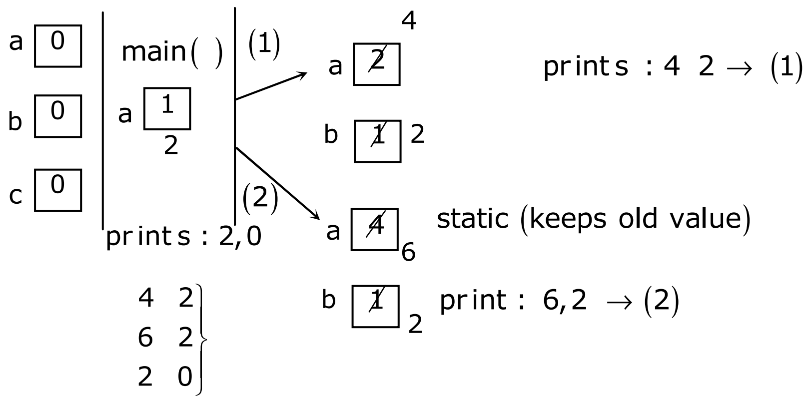


51. What output will be generated by the given code segment?

(A)	(B)	(C)	(D)
3 1	4 2	4 2	3 1
4 1	6 1	6 2	5 2
4 2	6 1	2 0	5 2

Answer:- (C)

Exp:-



Linked Answer Questions: Q.52 to Q.55 Carry Two Marks Each

Statement for Linked Answer Questions: 52 & 53

A computer has a 256 KByte, 4-way set associative, write back data cache with block size of 32 Bytes. The processor sends 32 bit addresses to the cache controller. Each cache tag directory entry contains, in addition to address tag, 2 valid bits, 1 modified bit and 1 replacement bit.

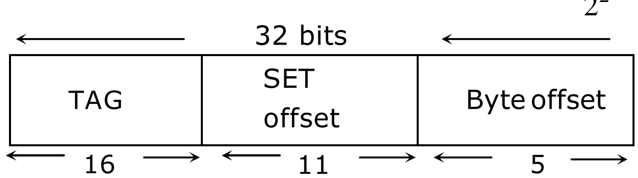
52. The number of bits in the tag field of an address is

(A) 11 (B) 14 (C) 16 (D) 27

Answer:- (C)

Exp:- Number of blocks $= \dfrac{256\,\text{KB}}{32\,\text{Bytes}} = \dfrac{2^{18}}{2^5} = 2^{13}$ blocks

As it is 4–way set associative, number of sets $= \dfrac{2^{13}}{2^2} = 2^{11}$



53. The size of the cache tag directory is

(A) 160 Kbits (B) 136 Kbits (C) 40 Kbits (D) 32 Kbits

Answer:-(A)

Exp:- TAG controller maintains 16 + 4 = 20 bits for every block

Hence, size of cache tag directory $= 20 \times 2^{13}$ bits $= 160$ K bits

Statement for Linked Answer Questions: 54 & 55

For the grammar below, a partial LL(1) parsing table is also presented along with the grammar. Entries that need to be filled are indicated as **E1, E2, and E3**. ε is the empty string, \$ indicates end of input, and | separates alternate right hand sides of productions.

$S \to a\,A\,b\,B \mid b\,A\,a\,B \mid \varepsilon$

$A \to S$

$B \to S$

	a	b	\$
S	**E1**	**E2**	$S \to \varepsilon$
A	$A \to S$	$A \to S$	error
B	$B \to S$	$B \to S$	**E3**

54. The First and Follow sets for the non-terminals A and B are

(A) FIRST(A) = {a, b, ε } = FIRST (B)
FOLLOW(A) = {a, b}
FOLLOW(B) = {a, b, \$}

(B) FIRST(A) = {a, b, \$}
FIRST(B) = {a, b, ε }
FOLLOW(A) = {a, b}
FOLLOW(B) ={\$}

(C) FIRST(A) = {a, b, ε } = FIRST(B)
FIRST(A) = {a, b}
FOLLOW(B) = $\varnothing$

(D) FIRST(A) = {a, b,} = FIRST(B)
FIRST(A) = {a, b}
FOLLOW(B) ={a, b}

Answer:- (A)

Exp:- $\text{First}(A) = \text{First}(S) = \text{First}(aAbB) \cup \text{First}(bAaB) \cup \text{First}(\in)$

$= \{a\} \cup \{b\} \cup \{\in\} = \{\in, a, b\}$

$\text{First}(B) = \text{First}(S) = \{\in, a, b\}$

$\text{Follow}(A) = \text{First}(bB) \cup \text{First}(aB) = \{a, b\}$

$\text{Follow}(B) = \text{Follow}(S) = \{\$\} \cup \text{Follow}(A) = \{\$, a, b\}$

55. The appropriate entries for E1, E2, and E3 are

(A) E1: $S \rightarrow aAbB,\ A \rightarrow S$
E2: $S \rightarrow bAaB,\ B \rightarrow S$
E3: $B \rightarrow S$

(B) E1: $S \rightarrow aAbB,\ S \rightarrow \varepsilon$
E2: $S \rightarrow bAaB,\ S \rightarrow \varepsilon$
E3: $S \rightarrow \varepsilon$

(C) E1: $S \rightarrow aAbB,\ S \rightarrow \varepsilon$
E2: $S \rightarrow bAaB,\ S \rightarrow \varepsilon$
E3: $B \rightarrow S$

(D) E1: $A \rightarrow S,\ S \rightarrow \varepsilon$
E2: $B \rightarrow S,\ S \rightarrow \varepsilon$
E3: $B \rightarrow S$

Answer:- (C)

Exp:-

$\text{First}(S) = \{\in, a, b\},\ \text{Follow}(S) = \{\$, a, b\}$

	a	b	\$
S	$S \rightarrow aAbB$ $S \rightarrow \in$	$S \rightarrow bAaB$ $S \rightarrow \in$	$S \rightarrow \in$

$B \rightarrow S$ to be placed in LL(1) Parsing table as follows:-

$\text{First}(S) = \{\in, a, b\},\ \text{Follow}(B) = \{\$, a, b\}$

	a	b	\$
B	$B \rightarrow S$	$B \rightarrow S$	$B \rightarrow S$

Q. No. 56 – 60 Carry One Mark Each

56. The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of Rs.50 per unit. The number of units to be produced by the firm such that the profit is maximized is

(A) 5 (B) 10 (C) 15 (D) 25

Answer:- (A)

Exp:-

$P = 50q - 5q^2$

$\frac{dp}{dq} = 50 - 10q;\ \frac{d^2p}{dq^2} < 0$

$\therefore$ p is maximum at $50 - 10q = 0$ or, $q = 5$

Else check with options

57. Choose the most appropriate alternative from the options given below to complete the following sentence:

Suresh's dog is the one ________ was hurt in the stampede.

(A) that (B) which (C) who (D) whom

Answer:- (A)

58. Choose the grammatically **INCORRECT** sentence:

(A) They gave us the money back less the service charges of Three Hundred rupees.

(B) This country's expenditure is not less than that of Bangladesh.

(C) The committee initially asked for a funding of Fifty Lakh rupees, but later settled for a lesser sum.

(D) This country's expenditure on educational reforms is very less

Answer:-(D)

59. Which one of the following options is the closest in meaning to the word given below?

Mitigate

(A) Diminish (B) Divulge (C) Dedicate (D) Denote

Answer:- (A)

60. Choose the most appropriate alternative from the options given below to complete the following sentence:

Despite several __________ the mission succeeded in its attempt to resolve the conflict.

(A) attempts (B) setbacks (C) meetings (D) delegations

Answer:- (B)

Q. No. 61 – 65 Carry Two Marks Each

61. Wanted Temporary, Part-time persons for the post of Field Interviewer to conduct personal interviews to collect and collate economic data. Requirements: High School-pass, must be available for Day, Evening and Saturday work. Transportation paid, expenses reimbursed.

Which one of the following is the best inference from the above advertisement?

(A) Gender-discriminatory (B) Xenophobic

(C) Not designed to make the post attractive (D) Not gender-discriminatory

Answer:- (C)

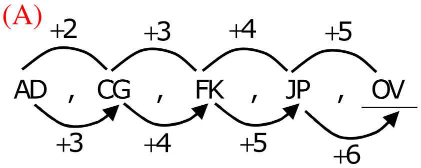
Exp:- Gender is not mentioned in the advertisement and (B) clearly eliminated

62. Given the sequence of terms, AD CG FK JP, the next term is

(A) OV (B) OW (C) PV (D) PW

Answer:- (A)

Exp:-



63. Which of the following assertions are CORRECT?

P: Adding 7 to each entry in a list adds 7 to the mean of the list

Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list

R: Doubling each entry in a list doubles the mean of the list

S: Doubling each entry in a list leaves the standard deviation of the list unchanged

(A) P, Q (B) Q, R (C) P, R (D) R, S

Answer:-(C)

Exp:- P and R always hold true

Else consider a sample set {1, 2, 3, 4} and check accordingly

64. An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.

The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is

(A) 0.288 (B) 0.334 (C) 0.667 (D) 0.720

Answer:-(B)

Exp:-

	x	y
Supply	60%	40%
Reliable	96%	72%
Overall	0.576	0.288

$$\therefore P(x) = \frac{0.288}{0.576 + 0.288} = 0.334$$

65. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

(A) 8 meters (B) 10 meters (C) 12 meters (D) 14 meters

Answer:-(B)

Exp:-

$$y = 2x - 0.1x^2$$

$$\frac{dy}{dx} = 2 - 0.2x$$

$$\frac{d^2y}{dx^2} < 0 \therefore y \text{ maximises at } 2 - 0.2x = 0$$

$$\Rightarrow x = 10$$

$$\therefore y = 20 - 10 = 10\text{m}$$

Q. No. 1 – 25 Carry One Mark Each

1. The simplified SOP (Sum of Product) form of the Boolean expression $\left(P+\overline{Q}+\overline{R}\right).\left(P+\overline{Q}+R\right).\left(P+Q+\overline{R}\right)$ is

(A) $\left(\overline{P}Q+\overline{R}\right)$ (B) $\left(P+\overline{Q}\,\overline{R}\right)$ (C) $\left(\overline{P}Q+R\right)$ (D) $\left(PQ+R\right)$

Answer: - (B)

Exp: -

P \ QR	00	01	11	10
0		1	1	1
1				

$$f=\left(P+\overline{R}\right)\left(P+\overline{Q}\right)$$

$$=P+\overline{Q}\,\overline{R}$$

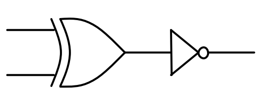
Alternate method

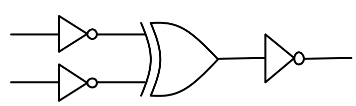
$$\left(P+\overline{Q}+\overline{R}\right).\left(P+\overline{Q}+R\right).\left(P+Q+\overline{R}\right)=\overline{\overline{\left(P+\overline{Q}+\overline{R}\right).\left(P+\overline{Q}+R\right).\left(P+Q+\overline{R}\right)}}$$

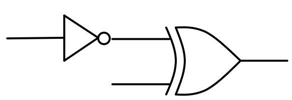
$$=\overline{\overline{P}QR+\overline{P}Q\overline{R}+\overline{P}\,\overline{Q}R}=\overline{\overline{P}Q\left(R+\overline{R}\right)+\overline{P}\,\overline{Q}R}=\overline{\overline{P}Q+\overline{P}\,\overline{Q}R}=\overline{\overline{P}\left(Q+\overline{Q}R\right)}$$

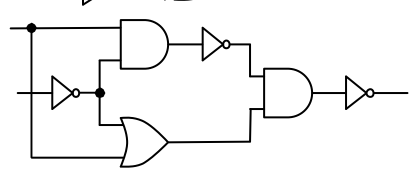
$$=\overline{\overline{P}\left(Q+R\right)}=P+\overline{Q}\;\overline{R}$$

2. Which one of the following circuits is NOT equivalent to a 2-input XNOR (exclusive NOR) gate?

(A) 

(B) 

(C) 

(D) 

Answer: - (D)

Exp: - All options except option 'D' gives EX-NOR gates

3. The minimum number of D flip-flops needed to design a mod-258 counter is

(A) 9 (B) 8 (C) 512 (D) 258

Answer: - (A)

Exp: - $2^n \geq 258 \Rightarrow n=9$

4. A thread is usually defined as a 'light weight process' because an operating system (OS) maintains smaller data structures for a thread than for a process. In relation to this, which of the followings is TRUE?

(A) On per-thread basis, the OS maintains only CPU register state

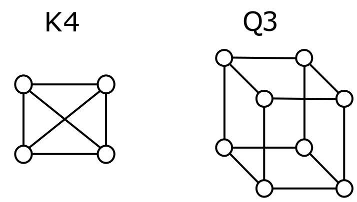
(B) The OS does not maintain a separate stack for each thread

(C) On per-thread basis, the OS does not maintain virtual memory state

(D) On per thread basis, the OS maintains only scheduling and accounting information

Answer: - (A)

5. K4 and Q3 are graphs with the following structures



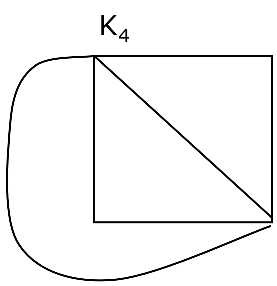
Which one of the following statements is TRUE in relation to these graphs?

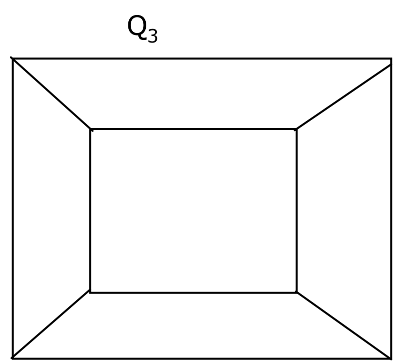
(A) K4 is planar while Q3 is not (B) Both K4 and Q3 are planar

(C) Q3 is planar while K4 is not (D) Neither K4 not Q3 is planar

Answer: - (B)

Exp: -





$\therefore$ Both K_4 and Q_3 are planar

6. If the difference between the expectation of the square of random variable $\left(E\left[X^2\right]\right)$ and the square of the expectation of the random variable $\left(E\left[X^2\right]\right)$ is denoted by R then

(A) $R = 0$ (B) $R<0$ (C) $R \geq 0$ (D) $R>0$

Answer: - (C)

7. The lexical analysis for a modern computer language such as Java needs the power of which one of the following machine models in a necessary and sufficient sense?

(A) Finite state automata

(B) Deterministic pushdown automata

(C) Non-Deterministic pushdown automata

(D) Turing machine

Answer: - (A)

Exp: - Lexical Analysis is implemented by finite automata

8. Let the page fault service time be 10ms in a computer with average memory access time being 20ns. If one page fault is generated for every 10^6 memory accesses, what is the effective access time for the memory?

(A) 21ns (B) 30ns (C) 23ns (D) 35ns

Answer: - (B)

Exp: - P = page fault rate

EA = p × page fault service time

$$+(1-p)\times \text{Memory access time}$$

$$=\frac{1}{10^6}\times 10\times 10^6+\left(1-\frac{1}{10^6}\right)\times 20 \cong 29.9\,\text{ns}$$

9. Consider a hypothetical processor with an instruction of type LW R1, 20(R2), which during execution reads a 32-bit word from memory and stores it in a 32-bit register R1. The effective address of the memory location is obtained by the addition of constant 20 and the contents of register R2. Which of the following best reflects the addressing mode implemented by this instruction for the operand in memory?

(A) Immediate Addressing (B) Register Addressing

(C) Register Indirect Scaled Addressing (D) Base Indexed Addressing

Answer: - (D)

Exp: - Here 20 will act as base and content of R_2 will be index

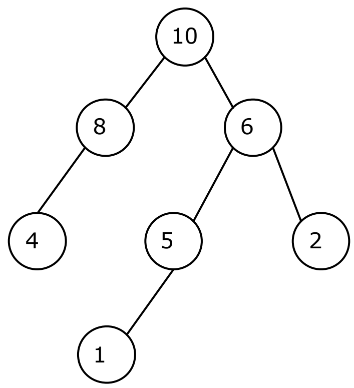
10. What does the following fragment of C-program print?

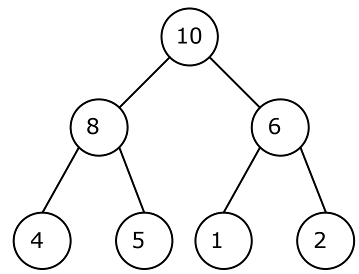
```
char c[ ] = "GATE2011";
char *p =c;
printf("%s", p+p[3]−p[1]);
```

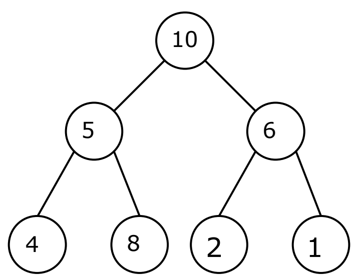
(A) GATE2011 (B) E2011 (C) 2011 (D) 011

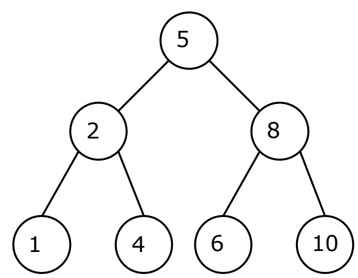
Answer: - (C)

11. A max-heap is a heap where the value of each parent is greater than or equal to the value of its children. Which of the following is a max-heap?

(A)


(B)


(C)


(D)


Answer: - (B)

Exp: - Heap is a complete binary tree

12. An algorithm to find the length of the longest monotonically increasing sequence of numbers in an array $A[0 : n-1]$ is given below.

Let L_i denote the length of the longest monotonically increasing sequence starting at index i in the array

Initialize $L_{n-1} = 1$

For all i such that $0 \le i \le n-2$

$$L_i = \begin{cases} 1 + L_{i+1} & \text{if } A[i] < A[i+1] \\ 1 & \text{Otherwise} \end{cases}$$

Finally the length of the longest monotonically increasing sequence is $\text{Max}(L_0, L_1, \ldots, L_{n-1})$. Which of the following statements is TRUE?

(A) The algorithm uses dynamic programming paradigm

(B) The algorithm has a linear complexity and uses branch and bound paradigm

(C) The algorithm has a non-linear polynomial complexity and uses branch and bound paradigm

(D) The algorithm uses divide and conquer paradigm.

Answer: - (A)

13. Let P be a regular language and Q be a context free language such that $Q \subseteq P$. (For example, let P be the language represented by the regular expression p*q* and Q be $\{p^n q^n \mid n \in N\}$). Then which of the following is ALWAYS regular?

(A) $P \cap Q$ (B) $P - Q$ (C) $\Sigma^* - P$ (D) $\Sigma^* - Q$

Answer: - (C)

Exp: - $\Sigma^* - P$ is the complement of P so it is always regular,

since regular languages are closed under complementation

14. In a compiler, keywords of a language are recognized during
    (A) parsing of the program (B) the code generation
    (C) the lexical analysis of the program (D) dataflow analysis

Answer: - (C)

Exp: - Any identifier is also a token so it is recognized in lexical Analysis

15. A layer-4 firewall (a device that can look at all protocol headers up to the transport layer) CANNOT
    (A) block entire HTTP traffic during 9:00PM and 5:00AM
    (B) block all ICMP traffic
    (C) stop incoming traffic from a specific IP address but allow outgoing traffic to the same IP address
    (D) block TCP traffic from a specific user on a multi-user system during 9:00PM and 5:00AM

Answer: - (A)

Exp: - Since it is a layer 4 firewall it cannot block application layer protocol like HTTP.

16. If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads?
    (A) 1/3 (B) 1/4 (C) 1/2 (D) 2/3

Answer: - (A)

Exp: - Sample space = $\{HH, HT, TH\}$

Required probability $= \frac{1}{3}$

17. Consider different activities related to email.
    m1: Send an email from a mail client to a mail server
    m2: Download an email from mailbox server to a mail client
    m3: Checking email in a web browser
    Which is the application level protocol used in each activity?
    (A) m1:HTTP m2:SMTP m3:POP (B) m1:SMTP m2:FTP m3:HTTP
    (C) m1: SMTP m2: POP m3: HTTP (D) m1: POP m2: SMTP m3:IMAP

Answer: - (C)

Exp: - Sending an email will be done through user agent and message transfer agent by SMTP, downloading an email from mail box is done through POP, checking email in a web browser is done through HTTP

18. A company needs to develop a strategy for software product development for which it has a choice of two programming languages L1 and L2. The number of lines of code (LOC) developed using L2 is estimated to be twice the LOC developed with L1. the product will have to be maintained for five years. Various parameters for the company are given in the table below.

Parameter	Language L1	Language L2
Man years needed for development	LOC / 10000	LOC / 10000
Development Cost per year	Rs. 10,00,000	Rs. 7,50,000
Maintenance time	5 years	5 years
Cost of maintenance per year	Rs. 1,00,000	Rs. 50,000

Total cost of the project includes cost of development and maintenance. What is the LOC for L1 for which the cost of the project using L1 is equal to the cost of the project using L2?

(A) 4000 (B) 5000 (C) 4333 (D) 4667

Answer: - (B)

Exp: - LOC $L_1 = x$

$L_2 = 2x$

Total cost of project

$$\frac{x}{10000}\times 1000000 + 5\times 100000 = \frac{2x}{10000}\times 750000 + 50000\times 5$$

$$100x + 500000 = 150x + 250000$$

$$\Rightarrow 50x = 500000 - 250000$$

$$\therefore \quad x = \frac{250000}{50} \Rightarrow x = 5000$$

19. Let the time taken to switch between user and kernel modes of execution be t_1 while the time taken to switch between two processes be t_2. Which of the following is TRUE?

(A) $t_1 > t_2$

(B) $t_1 = t_2$

(C) $t_1 < t_2$

(D) Nothing can be said about the relation between t_1 and t_2

Answer: - (C)

Exp: - Process switching also involves mode changing.

20. A company needs to develop digital signal processing software for one of its newest inventions. The software is expected to have 40000 lines of code. The company needs to determine the effort in person-months needed to develop this software using the basic COCOMO model. The multiplicative factor for this model is given as 2.8 for the software development on embedded systems, while the exponentiation factor is given as 1.20. What is the estimated effort in person-months?

(A) 234.25 (B) 932.50 (C) 287.80 (D) 122.40

Answer: - (A)

Exp: - Effort person per month

$= \alpha.(\text{kDSI})^{B}$

$\text{KDSI} = \text{Kilo LOC}$

$= 2.8 \times (40)^{1.20}$

$= 2.8 \times 83.6511$

$= 234.22$ person per month

21. Which of the following pairs have DIFFERENT expressive power?
    (A) Deterministic finite automata (DFA) and Non-deterministic finite automata (NFA)
    (B) Deterministic push down automata (DPDA) and Non-deterministic push down automata (NPDA)
    (C) Deterministic single-tape Turing machine and Non-deterministic single tape Turing machine
    (D) Single-tape Turing machine and multi-tape Turing machine

Answer: - (B)

Exp: - NPDA is more powerful than DPDA.

Hence answer is (B)

22. HTML (Hyper Text Markup Language) has language elements which permit certain actions other than describing the structure of the web document. Which one of the following actions is NOT supported by pure HTML (without any server or client side scripting) pages?
    (A) Embed web objects from different sites into the same page
    (B) Refresh the page automatically after a specified interval
    (C) Automatically redirect to another page upon download
    (D) Display the client time as part of the page

Answer: - (D)

23. Which of the following is NOT desired in a good Software Requirement Specifications (SRS) document?
    (A) Functional Requirements
    (B) Non Functional Requirements
    (C) Goals of Implementation
    (D) Algorithms for Software Implementation

Answer: - (D)

24. A computer handles several interrupt sources of which the following are relevant for this question.

    Interrupt from CPU temperature sensor

    Interrupt from Mouse

Interrupt from Keyboard

Interrupt from Hard Disk

(A) Interrupt from Hard Disk (B) Interrupt from Mouse

(C) Interrupt from Keyboard (D) Interrupt from CPU temp sensor

Answer: - (D)

25. Consider a relational table with a single record for each registered student with the following attributes.

1. Registration_Number: Unique registration number for each registered student
2. UID: Unique Identity number, unique at the national level for each citizen
3. BankAccount_Number: Unique account number at the bank. A student can have multiple accounts or joint accounts. This attributes stores the primary account number
4. Name: Name of the Student
5. Hostel_Room: Room number of the hostel

Which of the following options is INCORRECT?

(A) BankAccount_Number is a candidate key

(B) Registration_Number can be a primary key

(C) UID is a candidate key if all students are from the same country

(D) If S is a superkey such that $S \cap UID$ is NULL then $S \cup UID$ is also a superkey

Answer: - (A)

Exp: - In case two students hold joint account then BankAccount_Num will not uniquely determine other attributes.

Q. No. 26 – 51 Carry Two Marks Each

26. Which of the given options provides the increasing order of asymptotic complexityoffunctionsf_1,f_2,$f_3$andf_4?

$f_1(n) = 2^n;\ f_2(n) = n^{3/2};\ f_3(n) = n\log_2 n;\ f_4(n) = n^{\log_2 n}$

(A) f_3, f_2, f_4,f_1 (B) f_3, f_2, f_1,f_4 (C) f_2, f_3, f_1,f_4 (D) f_2, f_3, f_4,f_1

Answer: - (A)

Let $n = 1024$

$$f_1(n) = 2^{1024}$$

$$f_2(n) = 2^{15}$$

$$f_3(n) = 10 \times 2^{10}$$

$$f_4(n) = 1024^{10} = 2^{100}$$

$\therefore f_3, f_2, f_4, f_1$ is the rquired increasing order

27. Four matrices M_1, M_2, M_3 and M_4 are dimensions $p \times q$, $q \times r$, $r \times s$ and $s \times t$ respectively can be multiplied in several ways with different number of total scalar multiplications. For example When multiplied as $((M_1 \times M_2) \times (M_3 \times M_4))$ the total number of scalar multiplications is pqr+rst+prt. When multiplied as $(((M_1 \times M_2) \times M_3) \times M_4)$, the total number of scalar multiplications is pqr+prs+pst.

If p=10, q=100, r=20, s=5 and t=80, then the minimum number of scalar multiplications needed is

(A) 248000 (B) 44000 (C) 19000 (D) 25000

Answer: - (C)

Exp: - Multiply as $(M_1 \times (M_2 \times M_3)) \times M_4$

The total number of scalar multiplication is

$= qrs + pqs + pst$

$= 10000 + 5000 + 4000 = 19000$

28. Consider a relational table r with sufficient number of records, having attributes A_1, A_2,..., A_n and let $1 \le p \le n$. Two queries Q1 and Q2 are given below.

Q1 : $\pi_{A1...A_n}\left(\sigma_{A_p=c}(r)\right)$ where c is a const

Q2: $\pi_{A1...A_n}\left(\sigma_{c_1 \le A_p \le c_2}(r)\right)$ where c_1 and c_2 are constants

The database can be configured to do ordered indexing on A_p or hashing on A_p. Which of the following statements is TRUE?

(A) Ordered indexing will always outperform hashing for both queries

(B) Hashing will always outperform ordered indexing for both queries

(C) Hashing will outperform ordered indexing on Q1, but not on Q2

(D) Hashing will outperform ordered indexing on Q2, but not on Q1.

Answer: - (C)

29. Consider the matrix as given below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$$

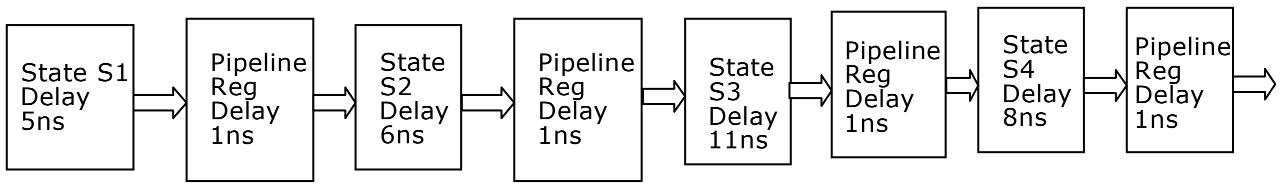
Which one of the following provides the CORRECT values of eigenvalues of the matrix?

(A) 1,4,3 (B) 3,7,3 (C) 7,3,2 (D) 1,2,3

Answer: - (A)

Exp: - Given matrix is upper triangular matrix and its diagonal elements are its eigen values = 1, 4, 3

30. Consider an instruction pipeline with four stages (S1, S2, S3 and S4) each with combinational circuit only. The pipeline registers are required between each stage and at the end of the last stage. Delays for the stages and for the pipeline registers are as given in the figure.



What is the approximate speed up of the pipeline in steady state under ideal conditions when compared to the corresponding non-pipeline implementation?

(A) 4.0 (B) 2.5 (C) 1.1 (D) 3.0

Answer: - (B)

Exp: - $\frac{(5+6+11+8)}{(11+1)} = \frac{30}{12} = 2.5$

31. Definition of a language L with alphabet $\{a\}$ is given as following

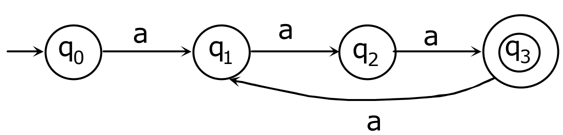
$L = \{a^{nk} \mid k > 0, \text{ and } n \text{ is a positive integer constant}\}$

What is the minimum number of states needed in a DFA to recognize L?

(A) k+1 (B) n+1 (C) 2^{n+1} (D) 2^{k+1}

Answer: - (B)

Exp: - Let n = 3 and k=1



$(n+1)$ states

32. An 8KB direct mapped write-back cache is organized as multiple blocks, each of size 32-bytes. The processor generates 32-bit addresses. The cache controller maintains the tag information for each cache block comprising of the following.

1 Valid bit

1 Modified bit

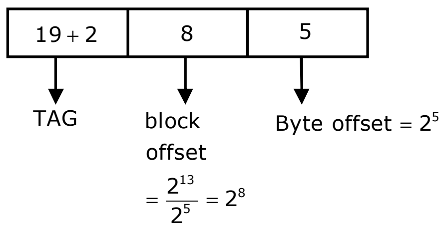
As many bits as the minimum needed to identify the memory block mapped in the cache.

What is the total size of memory needed at the cache controller to store meta-data (tags) for the cache?

(A) 4864 bits (B) 6144bits (C) 6656bits (D) 5376bits

Answer: - (D)

Exp: -



Required answer $= 256 \times (19+2) = 5376$ bits

33. An application loads 100 libraries at startup. Loading each library requires exactly one disk access. The seek time of the disk to a random location is given as 10ms. Rotational speed of disk is 6000rpm. If all 100 libraries are loaded from random locations on the disk, how long does it take to load all libraries? (The time to transfer data from the disk block once the head has been positioned at the start of the block may be neglected)

(A) 0.50s (B) 1.50s (C) 1.25s (D) 1.00s

Answer: - (B)

Exp: - 6000 rotations ________ 60 sec

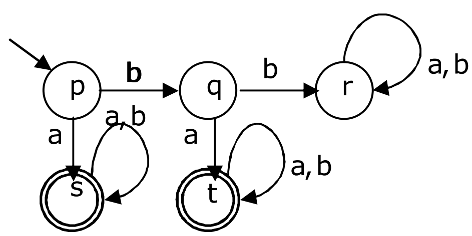
1 rotation ________ 10 ms

$\therefore$ Rotational latency = 5ms

Time for one disk access = 15 ms

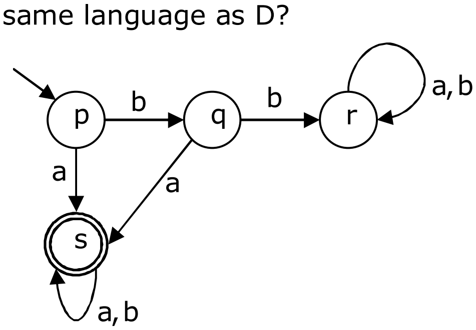
Time to load all libraries $= 15 \times 100 = 1500\,\text{ms} = 1.5\,\text{sec}$

34. A deterministic finite automation $(\text{DFA})\,D$ with alphabet $\Sigma = \{a, b\}$ is given below

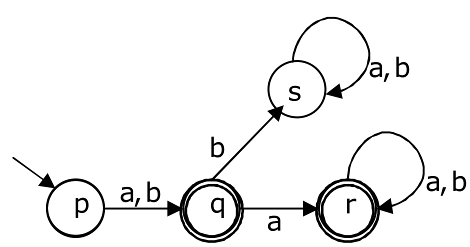


Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?

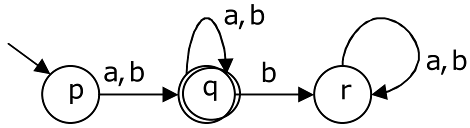
(A)



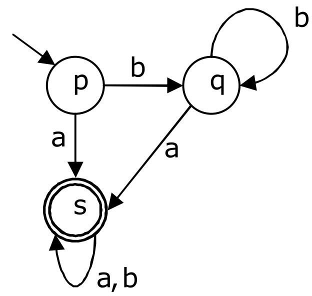
(B)



(C)



(D)



Answer: - (A)

Exp: - Options B and C will accept the string b

Option – D will accept the string "bba"

Both are invalid strings.

So the minimized DFA is option A

35. The following is comment written for a C function

/* This function computes the roots of a quadratic equation
a.x^2+b.x+c=0. The function stores two real roots
in *root1 and *root2 and returns the status of validity of
roots. It handles four different kinds of cases.
(i) When coefficient a is zero irrespective of discriminant
(ii) When discriminant is positive
(iii) When discrimanant is zero
(iv) When discrimanant is negative
Only in cases (ii) and (iii), the stored roots are valid.
Otherwise 0 is stored in the roots. the function returns 0 when
the roots are valid and -1 otherwise.
The functin also ensures root1>=root2.
int get_QuadRoots(float a, float b, float c, float *root1, float *root2);
*/

A software test engineer is assigned the job of doing black box testing. He comes up with the following test cases, many of which are redundant.

Test Case	Input set			Expected Output set		
	a	b	C	Root1	Root2	Return Value
T1	0.0	0.0	7.0	0.0	0.0	-1
T2	0.0	1.0	3.0	0.0	0.0	-1
T3	1.0	2.0	1.0	-1.0	-1.0	0
T4	4.0	-12.0	9.0	1.5	1.5	0
T5	1.0	-2.0	-3.0	3.0	-1.0	0
T6	1.0	1.0	4.0	0.0	0.0	-1

Which one of the following options provide the set of non-redundant tests using equivalence class partitioning approach from input perspective for black box testing?

(A) T1,T2,T3,T6 (B) T1,T3,T4,T5 (C) T2,T4,T5,T6 (D) T2,T3,T4,T5

Answer: - (C)

Exp: - T_1 and T_2 checking same condition a = 0 hence, any one of T_1 and T_2 is redundant.

T_3, T_4: in both case discriminant $(D)=b^2-4ac=0$. Hence any one of it is redundant.

T_5 : D>0

T_6 : D<0

36. Database table by name Loan_Records is given below.

Borrower	**Bank_Manager**	**Loan_ Amount**
Ramesh	Sunderajan	10000.00
Suresh	Ramgopal	5000.00
Mahesh	Sunderajan	7000.00

What is the output of the following SQL query?

```
SELECT count(*)
FROM(
(SELECT Borrower. Bank_Manager FROM Loan_Records) AS S
NATURAL JOIN
(SELECT Bank_Manager, Loan_Amount FROM Loan_Records) AS T
);
```

(A) 3 (B) 9 (C) 5 (D) 6

Answer: - (C)

Exp: -

S

Borrower	Bank_Manager
Ramesh	Sunderajan
Suresh	Ramgqpal
Mahesh	Sunderjan

T

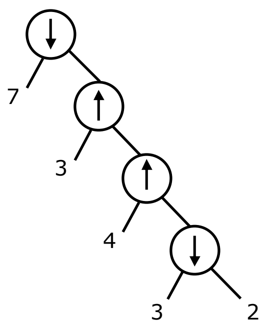
Bank_Manager	Loan_Amount
Sunderajan	10000.00
Ramgopal	5000.00
Sunderjan	7000.00

After executing the given query, the output would be

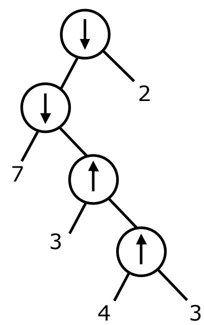
Borrower	**Bank_Manager**	**Load_Amount**
Ramesh	Sunderajan	10000.00
Ramesh	Sunderajan	7000.00
Suresh	Ramgopal	5000.00
Mahesh	Sunderajan	10000.00
Mahesh	Sunderajan	7000.00

37. Consider two binary operators '$\uparrow$' and '$\downarrow$' with the precedence of operator $\downarrow$ being lower than that of the operator $\uparrow$. Operator $\uparrow$ is right associative while operator $\downarrow$, is left associative. Which one of the following represents the parse tree for expression $(7 \downarrow 3 \uparrow 4 \uparrow 3 \downarrow 2)$?

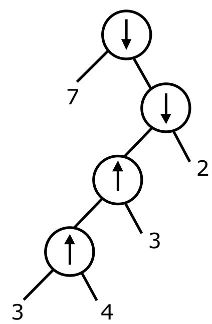
(A)



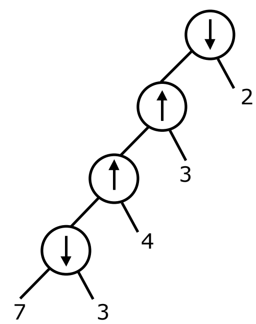
(B)



(C)



(D)



Answer: - (B)

Exp: - $7 \downarrow 3 \uparrow 4 \uparrow 3 \downarrow 2$

$\Rightarrow 7 \downarrow 3 \uparrow (4 \uparrow 3) \downarrow 2$ as $\uparrow$ is right associative

$\Rightarrow 7 \downarrow (3 \uparrow (4 \uparrow 3)) \downarrow 2$

$\Rightarrow (7 \downarrow (3 \uparrow (4 \uparrow 3))) \downarrow 2$ as $\downarrow$ is left associative

38. Consider the languages L1, L2 and L3 as given below

$L1 = \{0^p 1^q \mid p, q \in N\}$

$L2 = \{0^p 1^q \mid p, q \in N \text{ and } p = q\}$ and

$L3 = \{0^p 1^q 0^r \mid p, q, r \in N \text{ and } p = q = r\}$

Which of the following statements is NOT TRUE?

(A) Push Down Automata (PDA) can be used to recognize L1 and L2

(B) L1 is a regular language

(C) All the three languages are context free

(D) Turing machines can be used to recognize all the languages

Answer: - (C)

Exp: - L1: regular language

L2: context free language

L3: context sensitive language

39. On a non-pipelined sequential processor, a program segment, which is a part of the interrupt service routine, is given to transfer 500 bytes from an I/O device to memory.

Initialize the address register

Initialize the count to 500

LOOP: Load a byte from device

Store in memory at address given by address register

Increment the address register

Decrement the count

If count != 0 go to LOOP

Assume that each statement in this program is equivalent to a machine instruction which takes one clock cycle to execute if it is a non-load/store instruction. The load-store instructions take two clock cycles to execute.

The designer of the system also has an alternate approach of using the DMA controller to implement the same transfer. The DMA controller requires 20 clock cycles for initialization and other overheads. Each DMA transfer cycle takes two clock cycles to transfer one byte of data from the device to the memory.

What is the approximate speedup when the DMA controller based design is used in place of the interrupt driven program based input-output?

(A) 3.4 (B) 4.4 (C) 5.1 (D) 6.7

Answer: - (A)

Exp: - No. of clock cycles required by using load-store approach = 2 + 500 × 7 = 3502 and that of by using DMA = 20 + 500 × 2 = 1020

Required speed up $= \frac{3502}{1020} = 3.4$

40. We are given a set of n distinct elements and an unlabeled binary tree with n nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree?

(A) 0 (B) 1 (C) n! (D) $\frac{1}{n+1} . {}^{2n}C_n$

Answer: - (D)

41. Which one of the following options is CORRECT given three positive integers x, y and z, and a predicate

$P(x) = \neg(x = 1) \wedge \forall y \left(\exists z (x = y * z) \Rightarrow (y = x) \vee (y = 1)\right)$

(A) P(x) being true means that x is a prime number

(B) P(x) being true means that x is a number other than 1

(C) P(x) is always true irrespective of the value of x

(D) P(x) being true means that x has exactly two factors other than 1 and x

Answer: - (A)

42. Given $i = \sqrt{-1}$, what will be the evaluation of the definite integral $\int_0^{\pi/2} \frac{\cos x + i \sin x}{\cos x - i \sin x} dx$?

(A) 0 (B) 2 (C) $-i$ (D) i

Answer: - (D)

Exp: - $\int_0^{\pi/2} \frac{e^{ix}}{e^{-ix}} dx = \int_0^{\pi/2} e^{2ix} dx$

$$= \left(\frac{e^{2ix}}{2i}\right)_0^{\pi/2} = \frac{1}{2i}\left[e^{i\pi} - 1\right] = \frac{1}{2i}\left[\cos\pi + i\sin\pi - 1\right] = \frac{1}{2i}\left[-1 + 0 - 1\right] = \frac{-2}{2i} = \frac{-1}{i} \times \frac{i}{i} = \frac{-i}{-1} = i$$

43. Consider a database table T containing two columns X and Y each of type integer. After the creation of the table, one record (X= 1, Y=l) is inserted in the table.

Let MX and MY denote the respective maximum values of X and Y among all records in the table at any point in time. Using MX and MY, new records are inserted in the table 128 times with X and Y values being MX+1, 2*MY+1 respectively. It may be noted that each time after the insertion, values of MX and MY change.

What will be the output of the following SQL query after the steps mentioned above are carried out?

SELECT Y FROM T WHERE X=7;

(A) 127 (B) 255 (C) 129 (D) 257

Answer: - (A)

Exp: -

X	Y
1	1
2	3
3	7
4	15
5	31
6	63
7	127

44. Consider a finite sequence of random values $X = [x_1, x_2, ... x_n]$. Let μ_x be the mean and σ_x be the standard deviation of X .Let another finite sequence Y of equal length be derived from this as $y_i = a * x_i + b$, where a and b are positive constants. Let μ_y be the mean and σ_y be the standard deviation of this sequence. Which one of the following statements is INCORRECT?

(A) Index position of mode of X in X is the same as the index position of mode of Y in Y.

(B) Index position of median of X in X is the same as the index position of median of Y in Y.

(C) $\mu_y = a\mu_x + b$

(D) $\sigma_y = a\sigma_x + b$

Answer: - (D)

45. A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card?

(A) 1/5 (B) 4/25 (C) 1/4 (D) 2/5

Answer: - (A)

Exp: - (2,1), (3,2), (4,3), (5,4)

Required probability $= \frac{4}{5 \times 4} = \frac{4}{20} = \frac{1}{5}$

46. Consider the following table of arrival time and burst time for three processes P0, P1 and P2.

Process	**Arrival time**	**Burst Time**
P0	0 ms	9 ms
P1	1 ms	4ms
P2	2 ms	9ms

The pre-emptive shortest job first scheduling algorithm is used. Scheduling is carried out only at arrival or completion of processes. What is the average waiting time for the three processes?

(A) 5.0 ms (B) 4.33 ms (C) 6.33 ms (D) 7.33 ms

Answer: - (A)

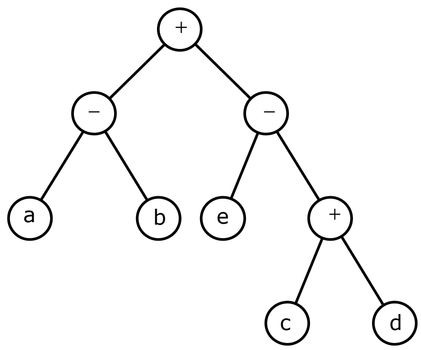
Exp: -

P_0	P_1	P_0	P_2

0 1 5 13 22

Average waiting time $= \dfrac{4+11}{3} = 5$ ms

47. Consider evaluating the following expression tree on a machine with load-store architecture in which memory can be accessed only through load and store instructions. The variables a, b, c, d and e are initially stored in memory. The binary operators used in this expression tree can be evaluated by the machine only when the operands are in registers. The instructions produce result only in a register. If no intermediate results can be stored in memory, what is the minimum number of registers needed to evaluate this expression?



(A) 2 (B) 9 (C) 5 (D) 3

Answer: - (D)

Exp: - Load R_1, a ; $R_1 \leftarrow M[a]$

Load R_2, b ; $R_2 \leftarrow M[b]$

Sub R_1, R_2 ; $R_1 \leftarrow R_1 - R_2$

Load R_2, c ; $R_2 \leftarrow M[c]$

Load R_3, d ; $R_3 \leftarrow M[d]$

Add R_2, R_3 ; $R_2 \leftarrow R_2 + R_3$

Load R_3, e ; $R_3 \leftarrow M[e]$

Sub R_3, R_2 : $R_3 \leftarrow R_3 - R_2$

Add R_1, R_3 ; $R_1 \leftarrow R_1 + R_3$

Total 3 Registers are required minimum

Common Data Questions: 48 & 49

Consider the following recursive C function that takes two arguments

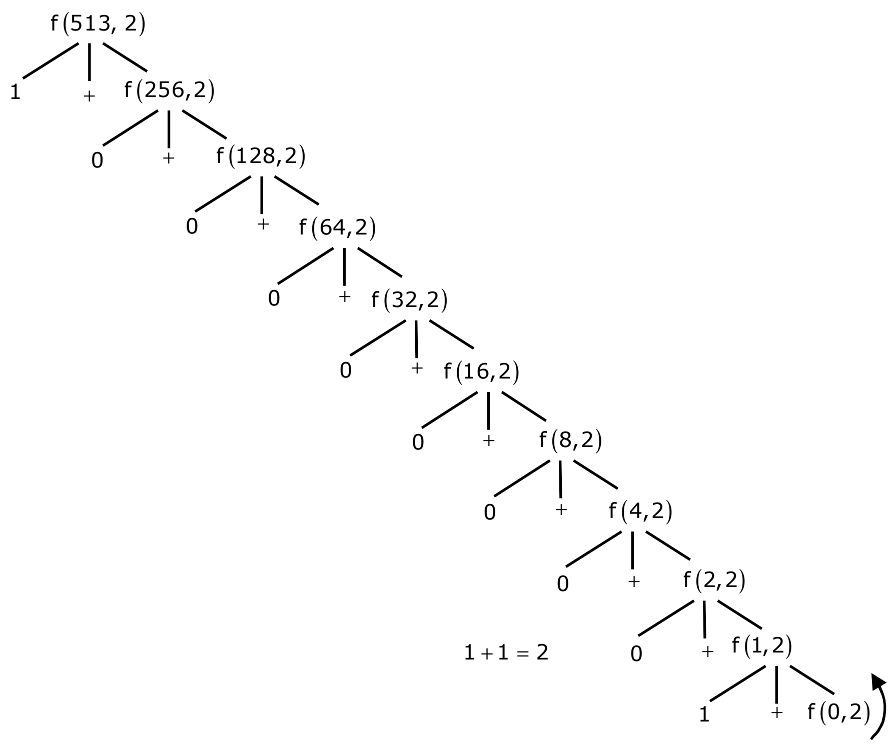
```
unsigned int foo(unsigned int n, unsigned int r) {
    if (n > 0) return (n%r) + foo (n/r, r));
    else return 0;
}
```

48. What is the return value of the function foo when it is called as foo (513, 2)?

(A) 9 (B) 8 (C) 5 (D) 2

Answer: - (D)

Exp: -

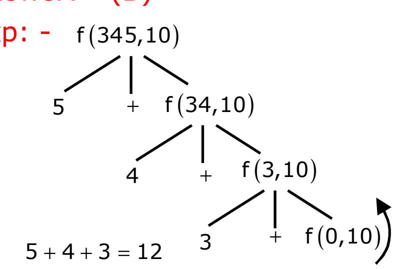


49 What is the return value of the function foo when it is called as foo (345, 10) ?

(A) 345 (B) 12 (C) 5 (D) 3

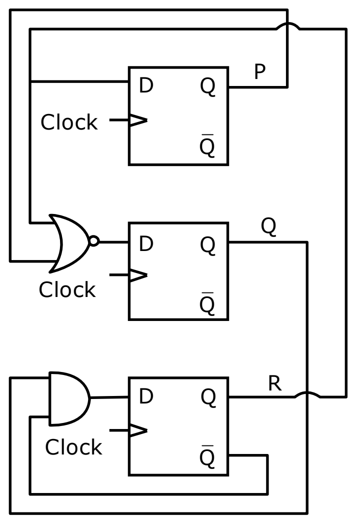
Answer: - (B)

Exp: -



Common Data Questions: 50 & 51

Consider the following circuit involving three D-type flip-flops used in a certain type of counter configuration.



50. If all the flip-flops were reset to 0 at power on, what is the total number of distinct outputs (states) represented by PQR generated by the counter?

(A) 3 (B) 4 (C) 5 (D) 6

Answer: - (B)

Exp: -

CLOCK	Inputs			Outputs		
	$D_1 = R$	$D_2 = \overline{(P+R)}$	$D_3 = Q\,\bar{R}$	P	Q	R
1	0	1	0	0	1	0
2	0	1	1	0	1	1
3	1	0	0	1	0	0
4	0	0	0	0	0	0

So Total number of distinct outputs is 4

51. If at some instance prior to the occurrence of the clock edge, P. Q and R have a value 0, 1 and 0 respectively, what shall be the value of PQR after the clock edge?

(A) 000 (B) 001 (C) 010 (D) 011

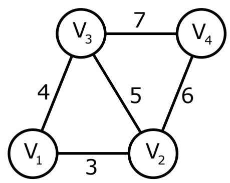
Answer: - (D)

Exp: -From the Table Shown in the explanation of question 50, if first state is 010 next State is 011

Linked Answer Questions: Q.52 to Q.55 Carry Two Marks Each

Statement for Linked Answer Questions: 52 & 53

An undirected graph G(V,E) contains $n\ (n > 2)$ nodes named $v_1, v_2, \dots. v_n$. Two nodes v_i, v_j are connected if and only if $0 < |i - j| \le 2$. Each edge (v_i, v_j) is assigned a weight i + j. A sample graph with n = 4 is shown below



52. What will be the cost of the minimum spanning tree (MST) of such a graph with n nodes?

(A) $\frac{1}{12}(11n^2 - 5n)$ (B) $n^2 - n + 1$

(C) $6n - 11$ (D) $2n + 1$

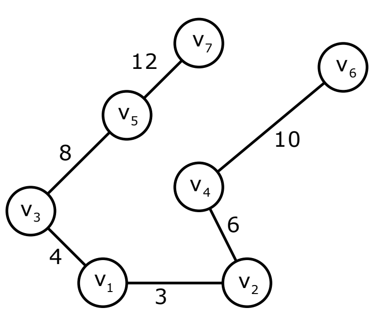
Answer: - (B)

53. The length of the path from v_5 to v_6 in the MST of previous question with n = 10 is

(A) 11 (B) 25 (C) 31 (D) 41

Answer: - (C)

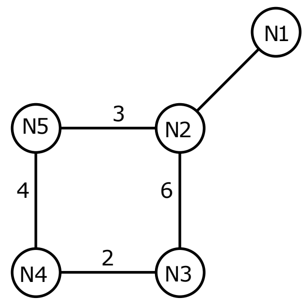
Exp: -



$12 + 8 + 4 + 3 + 6 + 10 = 31$

Statement for Linked Answer Questions: 54 & 55

Consider a network with five nodes, N1 to N5, as shown below



The net work uses a Distance Vector Routing protocol. Once the routes have stabilized, the distance vectors at different nodes are as following

N1 : $(0,1,7,8,4)$

N2 : $(1,0,6,7,3)$

N3 : $(7,6,0,2,6)$

N4 : $(8,7,2,0,4)$

N5 : $(4,3,6,4,0)$

Each distance vector is the distance of the best known path at that instance to nodes, N1 to N5, where the distance to itself is 0. Also, all links are symmetric and the cost is identical in both directions. In each round, all nodes exchange their distance vectors with their respective neighbors. Then all nodes update their distance vectors. In between two rounds, any change in cost of a link will cause the two incident nodes to change only that entry in their distance vectors

54. The cost of link N2-N3 reduces to 2 in (both directions). After the next round of updates, what will be the new distance vector at node, N3?

(A) (3. 2, 0, 2, 5) (B) (3, 2, 0, 2, 6)

(C) (7, 2, 0, 2, 5) (D) (7, 2, 0, 2, 6)

Answer: - (A)

Exp: -

$$\begin{array}{c|c|l} & N_3 & \\ \hline N_1 & 3 & \rightarrow 1+2 \\ \hline N_2 & 2 & \\ \hline N_3 & 0 & \\ \hline N_4 & 2 & \\ \hline N_5 & 5 & \rightarrow 2+3 \\ \hline \end{array}$$

55. After the update in the previous question, the link N1-N2 goes down. N2 will reflect this change immediately in its distance vector as cost, ∞. After the NEXT ROUND of update, what will be the cost to N1 in the distance vector of N3?

(A) 3 (B) 9 (C) 10 (D) ∞

Answer: - (C)

Exp: - N_3 has neighbors N_2 and N_4

N_2 has made entry ∞

N_4 has the distance of 8 to N_1

N_3 has the distance of 2 to N_4

So 2 + 8 = 10

Q. No. 56 – 60 Carry One Mark Each

56. If Log (P) = (1/2)Log (Q) = (1/3) Log (R), then which of the following options is TRUE?

(A) $P^2 = Q^3R^2$ (B) $Q^2 = PR$ (C) $Q^2 = R^3P$ (D) $R = P^2Q^2$

Answer: - (B)

Exp:- $\log P = \frac{1}{2}\log Q = \frac{1}{3}\log(R) = k$

$\therefore P = b^k, Q = b^{2k}, R = b^{3k}$

Now, $Q^2 = b^{4k} = b^{3k}\, b^k = PR$

57. Choose the most appropriate word(s) from the options given below to complete the following sentence.

I contemplated_________Singapore for my vacation but decided against it.

(A) To visit (B) having to visit (C) visiting (D)for a visit

Answer: - (C)

Exp: - Contemplate is a transitive verb and hence is followed by a gerund Hence the correct usage of contemplate is verb+ ing form.

58. Choose the most appropriate word from the options given below to complete the following sentence.

If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or______________.

(A) Hyperbolic (B) Restrained

(C) Argumentative (D) Indifferent

Answer: - (B)

Exp: - The tone of the sentence clearly indicates a word that is similar to understated is needed for the blank. Alternatively, the word should be antonym of strong (fail to make strong impression). Therefore, the best choice is restrained which means controlled/reserved/timid.

59. Choose the word from the options given below that is most nearly opposite in meaning to the given word: **Amalgamate**

(A) Merge (B) Split (C) Collect (D) Separate

Answer: - (B)

Exp: - Amalgamate means combine or unite to form one organization or structure. So the best option here is split. Separate on the other hand, although a close synonym, it is too general to be the best antonym in the given question while Merge is the synonym; Collect is not related.

60. Which of the following options is the closest in the meaning to the word below:

Inexplicable

(A) Incomprehensible (B) Indelible

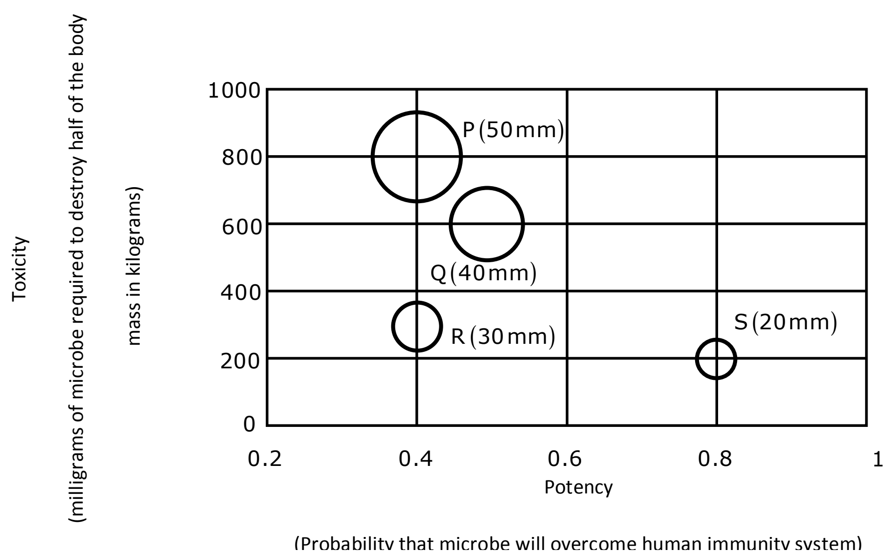
(C) Inextricable (D) Infallible

Answer: - (A)

Exp: - Inexplicable means not explicable; that cannot be explained, understood, or accounted for. So the best synonym here is incomprehensible.

Q. No. 61 – 65 Carry Two Marks Each

61. P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt?

(A) P (B) Q (C) R (D) S

Answer: - (D)

Exp: - By observation of the table, we can say S

	P	Q	R	S
Requirement	800	600	300	200
Potency	0.4	0.5	0.4	0.8

62. The variable cost (V) of manufacturing a product varies according to the equation V= 4q, where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation F = 100/q. How many units should be produced to minimize the total cost (V+F)?

(A) 5 (B) 4 (C) 7 (D) 6

Answer: (A)

Exp: - Checking with all options in formula: (4q+100/q) i.e. (V+F). Option A gives the minimum cost.

63. A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks required so that there will be no pending order at the end of the 5th day?

(A) 4 (B) 5 (C) 6 (D) 7

Answer: - (C)

Exp: - Let each truck carry 100 units.

2800 = 4n + e   n = normal

3000 = 10n + e   e = excess/pending

$$\therefore n = \frac{100}{3}, e = \frac{8000}{3}$$

$$5\text{days} \Rightarrow 500x = \frac{5.100}{3} + \frac{8000}{3}$$

$$\Rightarrow 500x = \frac{8500}{3} 17 \Rightarrow x > 5$$

Minimum possible = 6

64. A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much spirit is now left in the container?

(A) 7.58 litres (B) 7.84 litres (C) 7 litres (D) 7.29 litres

Answer: - (D)

Exp:- $10\left(\frac{10-1}{10}\right)^{3} = 10\left(\frac{9}{10}\right)^{3} = \frac{729}{1000}$

$\therefore \frac{729}{1000} \times 1 = 7.29$ litres

65. **Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.**

Based on the above passage which topic would not be included in a unit on bereavement?

(A) how to write a letter of condolence

(B) what emotional stages are passed through in the healing process

(C) what the leading causes of death are

(D) how to give support to a grieving friend

Answer: - (C)

Exp: - The given passage clearly deals with how to deal with bereavement and grief and so after the tragedy occurs and not about precautions. Therefore, irrespective of the causes of death, a school student rarely gets into details of causes—which is beyond the scope of the context. Rest all are important in dealing with grief.

Q. No. 1 – 25 Carry One Mark Each

1. Let G=(V, E) be a graph. Define $\xi(G)=\sum_d i_d \times d$, where i_d is the number of vertices of degree d in G. If S and T are two different trees with $\xi(S)=\xi(T)$, then

(A) $|S| = 2|T|$ (B) $|S| = |T| - 1$ (C) $|S| = |T|$ (D) $|S| = |T| + 1$

2. Newton-Raphson method is used to compute a root of the equation $x^2 - 13 = 0$ with 3.5 as the initial value. The approximation after one iteration is

(A) 3.575 (B) 3.676 (C) 3.667 (D) 3.607

3. What is the possible number of reflexive relations on a set of 5 elements?

(A) 2^{10} (B) 2^{15} (C) 2^{20} (D) 2^{25}

4. Consider the set S = {1, ω, ω^2}, where ω and ω^2 are cube roots of unity. If * denotes the multiplication operation, the structure (S, *) forms

(A) A group (B) A ring
(C) An integral domain (D) A field

5. What is the value of $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$?

(A) 0 (B) e^{-2} (C) $e^{-1/2}$ (D) 1

6. The minterm expansion of $f(P, Q, R) = PQ + Q\overline{R} + P\overline{R}$ is

(A) $m_2 + m_4 + m_6 + m_7$ (B) $m_0 + m_1 + m_3 + m_5$
(C) $m_0 + m_1 + m_6 + m_7$ (D) $m_2 + m_3 + m_4 + m_5$

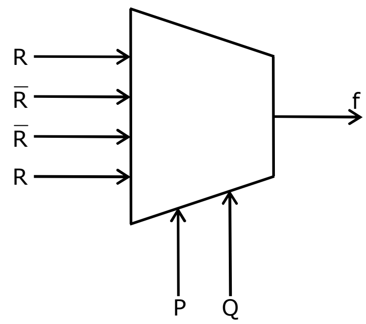
7. A main memory unit with a capacity of 4 megabytes is built using 1M×1-bit DRAM chips. Each DRAM chip has 1K rows of cells with 1K cells in each row. The time taken for a single refresh operation is 100 nanoseconds. The time required to perform one refresh operation on all the cells in the memory unit is

(A) 100 nanoseconds (B) $100*2^{10}$ nanoseconds
(C) $100*2^{20}$ nanoseconds (D) $3200*2^{20}$ nanoseconds

8. P is a 16-bit signed integer. The 2's complement representation of P is $(F87B)_{16}$. The 2's complement representation of 8*P is

(A) $(C3D8)_{16}$ (B) $(187B)_{16}$ (C) $(F878)_{16}$ (D) $(987B)_{16}$

9. The Boolean expression for the output f of the multiplexer shown below is



(A) $\overline{P \oplus Q \oplus R}$

(B) $P \oplus Q \oplus R$

(C) $P + Q + R$

(D) $\overline{P + Q + R}$

10. In a binary tree with n nodes, every node has an odd number of descendants. Every node is considered to be its own descendant. What is the number of nodes in the tree that have exactly one child?

(A) 0 (B) 1 (C) $(n-1)/2$ (D) n-1

11. What does the following program print?

```
#include < stdio.h >
void f (int * p, int * g){
        p = q;
        * p = 2;
}
int i = 0, j = 1;
int main ( ){
        f(&i, & j);
        printf ("%d %d \ n",  i, j);
        return 0;
}
```

(A) 2 2 (B) 2 1 (C) 0 1 (D) 0 2

12. Two alternative packages A and B are available for processing a database having 10^k records. Package A requires $0.0001n^2$ time units and package B requires $10n\log_{10}n$ time units to process n records. What is the smallest value of k for which package B will be preferred over A?

(A) 12 (B) 10 (C) 6 (D) 5

13. Which data structure in a compiler is used for managing information about variables and their attributes?

(A) Abstract syntax tree (B) Symbol table

(C) Semantic stack (D) Parse table

14. Which languages necessarily need heap allocation in the runtime environment?

(A) Those that support recursion (B) Those that use dynamic scoping

(C) Those that allow dynamic data structures (D) Those that use global variables

15. One of the header fields in an IP datagram is the Time to Live (TTL) field. Which of the following statements best explains the need for this field?

(A) It can be used to prioritize packets

(B) It can be used to reduce delays

(C) It can be used to optimize throughput

(D) It can be used to prevent packet looping

16. Which one of the following is not a client server application?

(A) Internet chat (B) Web browsing (C) E-mail (D) Ping

17. Let L1 be a recursive language. Let L2 and L3 be languages that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?

(A) L2 – L1 is recursively enumerable

(B) L1 – L3 is recursively enumerable

(C) $L2 \cap L1$ is recursively enumerable

(D) $L2 \cup L1$ is recursively enumerable

18. Consider a B^+-tree in which the maximum number of keys in a node is 5. What is the minimum number of keys in any non-root node?

(A) 1 (B) 2 (C) 3 (D) 4

19. A relational schema for a train reservation database is given below

Passenger (pid, pname, age)

Reservation (pid, cass, tid)

Table : Passenger

pid	'pname	Age
0	'Sachin'	65
1	'Rahul'	66
2	'Sourav'	67
3	'Anil'	69

Table : Reservation

pid	class	tid
0	'AC'	8200
1	'AC'	8201
2	'SC'	8201
5	'AC'	8203
1	'SC'	8204
3	'AC'	8202

What pids are returned by the following SQL query for the above instance of the tables?

```
SELECT   pid
FROM     Reservation
WHERE    class = 'AC'  AND
         EXISTS  (SELECT  *
                  FROM Passenger
                  WHERE  age > 65  AND
                  Passenger.pid = Reservation.pid)
```

(A) 1, 0 (B) 1, 2 (C) 1, 3 (D) 1, 5

20. Which of the following concurrency control protocols ensure both conflict serializability and freedom from deadlock?

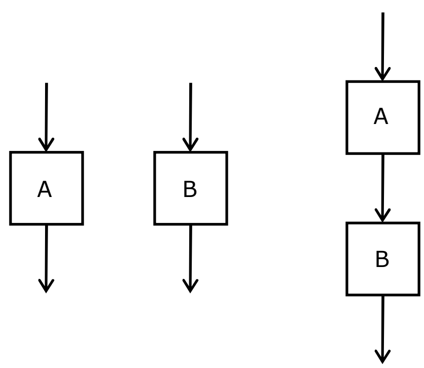
I. 2-phase locking

II. Time-stamp ordering

(A) I only (B) II only

(C) Both I and II (D) Neither I nor II

21. The cyclomatic complexity of each of the modules A and B shown below is 10. What is the cyclomatic complexity of the sequential integration shown on the right hand side?



(A) 19 (B) 21 (C) 20 (D) 10

22. What is the appropriate pairing of items in the two columns listing various activities encountered in a software life cycle?

P. Requirements Capture	1. Module Development and Integration
Q. Design	2. Domain Analysis
R. Implementation	3. Structural and Behavioral Modeling
S. Maintenance	4. Performance Tuning

(A) P-3, Q-2,R-4,S-1 (B) P-2, Q-3,R-1,S-4

(C) P-3, Q-2,R-1,S-4 (D) P-2, Q-3,R-4,S-1

23. Consider the methods used by processes P1 and P2 for accessing their critical sections whenever needed, as given below. The initial values of shared boolean variables S1 and S2 are randomly assigned.

Method used by PI	**Method used by P2**
while (S 1 = = S 2) ; Critica1 Section S1 = S2;	while (S 1 != S 2) ; Critica1 Section S2 = not (S1);

Which one of the following statements describes the properties achieved?

(A) Mutual exclusion but not progress

(B) Progress but not mutual exclusion

(C) Neither mutual exclusion nor progress

(D) Both mutual exclusion and progress

24. A system uses FIFO policy for page replacement. It has 4 page frames with no pages loaded to begin with. The system first accesses 100 distinct pages in some order and then accesses the same 100 pages but now in the reverse order. How many page faults will occur?

(A) 196 (B) 192 (C) 197 (D) 195

25. Which of the following statements are true?

I. Shortest remaining time first scheduling may cause starvation

II. Preemptive scheduling may cause starvation

III. Round robin is better than FCFS in terms of response time

(A) I only (B) I and III only (C) II and III only (D) I, II and III

Q. No. 26 – 55 Carry Two Marks Each

26. Consider a company that assembles computers. The probability of a faulty assembly of any computer is *p.* The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of *q.* What is the probability of a computer being declared faulty?

(A) $pq+(1-p)(1-q)$ (B) $(1-q)p$ (C) $(1-p)q$ (D) pq

27. What is the probability that divisor of 10^{99} is a multiple of 10^{96}?

(A) 1/625 (B) 4/625 (C) 12/625 (D) 16/625

28. The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences can not be the degree sequence of any graph?

I. 7, 6, 5, 4, 4, 3, 2, 1 II. 6, 6, 6, 6, 3, 3, 2, 2

III. 7, 6, 6, 4, 4, 3, 2, 2 IV. 8, 7, 7, 6, 4, 2, 1, 1

(A) I and II (B) III and IV (C) IV only (D) II and IV

29. Consider the following matrix

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

If the eigenvalues of A are 4 and 8, then

(A) $x = 4, y = 10$ (B) $x = 5, y = 8$ (C) $x = -3, y = 9$ (D) $x = -4, y = 10$

30. Suppose the predicate F(x, y, t) is used to represent the statement that person x can fool person y at time t. which one of the statements below expresses best the meaning of the formula $\forall x \exists y \exists t (\neg F(x, y, t))$?

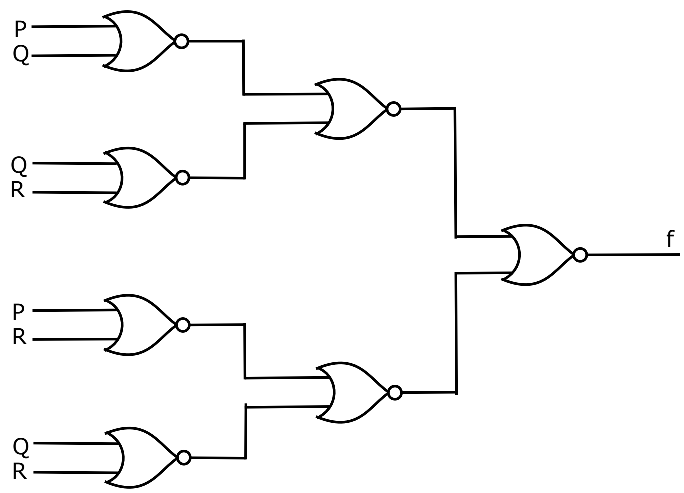
(A) Everyone can fool some person at some time

(B) No one can fool everyone all the time

(C) Everyone cannot fool some person all the time

(D) No one can fool some person at some time

31. What is the Boolean expression for the output f of the combinational logic circuit of NOR gates given below?



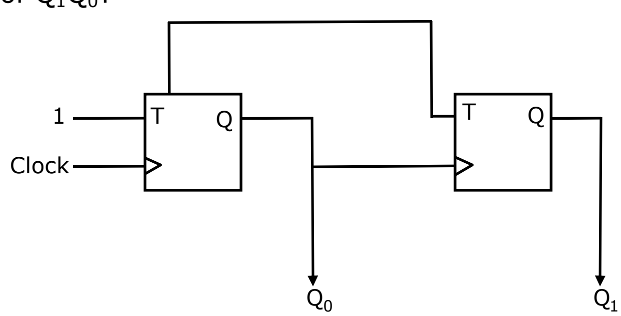
(A) $\overline{Q+R}$

(B) $\overline{P+Q}$

(C) $\overline{P+R}$

(D) $\overline{P+Q+R}$

32. In the sequential circuit shown below, if the initial value of the output Q_1Q_0 is 00, what are the next four values of Q_1Q_0?



(A) 11,10,01,00

(B) 10,11,01,00

(C) 10,00,01,11

(D) 11,10,00,01

33. A 5-stage pipelined processor has Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Write Operand (WO) stages. The IF, ID, OF and WO stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD and SUB instructions, 3 clock cycles for MUL instruction, and 6 clock cycles for DIV instruction respectively. Operand forwarding is used in the pipeline. What is the number of clock cycles needed to execute the following sequence of instructions?

Instruction	Meaning of instruction
$I_0 : \text{MUL } R_2, R_0, R_1$	$R_2 \leftarrow R_0 * R_1$
$I_1 : \text{DIV } R_5, R_3, R_4$	$R_5 \leftarrow R_3 / R_4$
$I_2 : \text{ADD } R_2, R_5, R_2$	$R_2 \leftarrow R_5 + R_2$
$I_3 : \text{SUB } R_5, R_2, R_6$	$R_5 \leftarrow R_2 - R_6$

(A) 13 (B) 15 (C) 17 (D) 19

34. The weight of a sequence a_0, $a_1, \ldots, a_{n-1}$ of real numbers is defined as $a_0 + a_1/2 + \ldots + a_{n-1}/2^{n-1}$. A subsequence of a sequence is obtained by deleting some elements from the sequence, keeping the order of the remaining elements the same. Let X denote the maximum possible weight of a subsequence of $a_0, a_1, \ldots, a_{n-1}$. Then X is equal to

(A) $\max(Y, a_0 + Y)$ (B) $\max(Y, a_0 + Y/2)$ (C) $\max(Y, a_0 + 2Y)$ (D) $a_0 + Y/2$

35. What is the value printed by the following C program?

```
#include <stdio.h>
int f(int * a, int n)
{
    if (n <= 0)return 0;
    else if(*a % 2 = = 0) return * a + f(a + 1, n - 1);
    else return * a - f(a + 1, n - 1);
}
int main ( )
{
    int a[ ] = {12, 7, 13, 4, 11, 6};
    printf ("%d",  f(a, 6));
    return 0;
}
```

(A) -9 (B) 5 (C) 15 (D) 19

36. The following C function takes a simply-linked list as input argument. It modifies the list by moving the last element to the front of the list and returns the modified list. Some part of the code is left blank.

```
typedef struct node {
          int value;
          struct node *next;
}     Node;
Node *move_to_front(Node *head) {
        Node *p, *q;
        if ((head = = NULL:  || (head->next = = NULL)) return head;
        q = NULL; p = head;
        while (p-> next !=NULL)  {
                q=P;
                p=p->next;
}
________________________________
return head;
}
```

Choose the correct alternative to replace the blank line.

(A) q = NULL; p->next = head; head = p;

(B) q->next = NULL; head = p; p->next = head;

(C) head = p; p->next = q; q->next = NULL;

(D) q->next = NULL; p->next = head; head = p;

37. The program below uses six temporary variables a, b, c, d, e, f.

$a = 1$

$b = 10$

$c = 20$

$d = a + b$

$e = c + d$

$f = c + e$

$b = c + e$

$e = b + f$

$d = 5 + e$

return $d + f$

Assuming that all operations take their operands from registers, what is the minimum number of registers needed to execute this program without spilling?

(A) 2 (B) 3 (C) 4 (D) 6

38. The grammar $S \rightarrow aSa|bS|c$ is

(A) LL(1) but not LR(1)

(B) LR(1) but not LR(1)

(C) Both LL(1) and LR(1)

(D) Neither LL(1) nor LR(1)

39. Let $L = \{w \in (0+1)^* \mid w \text{ has even number of 1s}\}$, i.e. L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L?

(A) $(0^*10^*1)^*$

(B) $0^*(10^*10^*)^*$

(C) $0^*(10^*1^*)^*0^*$

(D) $0^*1(10^*1)^*10^*$

40. Consider the languages $L1 = \{0^i1^j \mid i \neq j\}$. $L2 = \{0^i1^j \mid i = j\}$. $L3 = \{0^i1^j \mid i = 2j+1\}$. $L4 = \{0^i1^j \mid i \neq 2j\}$. Which one of the following statements is true?

(A) Only L2 is context free

(B) Only L2 and L3 are context free

(C) Only L1 and L2 are context free

(D) All are context free

41. Let w be any string of length n in {0, 1}*. Let L be the set of all substrings of w. What is the minimum number of states in a non-deterministic finite automaton that accepts L?

(A) n-1

(B) n

(C) n+1

(D) 2^{n-1}

42. Consider the following schedule for transactions T1, T2 and T3:

<u>T1</u>	<u>T2</u>	<u>T3</u>
Read(X)		
	Read(Y)	
		Read(Y)
	Write(Y)	
Write(X)		
		Write(X)
	Read(X)	
	Write(X)	

Which one of the schedules below is the correct serialization of the above?

(A) $T1 \rightarrow T3 \rightarrow T2$

(B) $T2 \rightarrow T1 \rightarrow T3$

(C) $T2 \rightarrow T3 \rightarrow T1$

(D) $T3 \rightarrow T1 \rightarrow T2$

43. The following functional dependencies hold for relations R(A, B, C) and S(B, D, E)

$B \rightarrow A$,
$A \rightarrow C$

The relation R contains 200tuples and the relation S contains 100tuples. What is the maximum number of tuples possible in the natural join $R \bowtie S$?

(A) 100

(B) 200

(C) 300

(D) 2000

44. The following program is to be tested for statement coverage:

```
begin
    if (a = = b)  {S1;  exit;}
    else if (c = = d)   {S2;}
         else {S3; exit;}
    S4;
end
```

The test cases T1, T2, T3 and T4 given below are expressed in terms of the properties satisfied by the values of variables a, b, c and d. The exact values are not given.

T1 : a, b, c and d are all equal

T2 : a, b, c and d are all distinct

T3 : a=b and c !=d

T4 : a !=b and c=d

Which of the test suites given below ensures coverage of statements S1, S2, S3 and S4?

(A) T1, T2, T3 (B) T2, T4 (C) T3, T4 (D) T1, T2, T4

45. The following program consists of 3 concurrent processes and 3 binary semaphores. The semaphores are initialized as S0=1, S1=0, S2=0.

Process P0	**Process P1**	**Process P2**

while (true) {
wait (S0);
print '0'
release (S1);
release (S2);
}
	wait (S1);
	Release (S0);
		wait (S2);
		release (S0);

How many times will process P0 print '0'?

(A) At least twice (B) Exactly twice (C) Exactly thrice (D) Exactly once

46. A system has n resources $R_0,\ldots,R_{n-1}$, and k processes $P_{0,\ldots}..P_{k-1}$. The implementation of the resource request logic of each process P_i. is as follows:

```
if (i% 2==0) {
   if (i<n) request R_i;
   if (i+2<n)request R_{i+2};
}
else {
   if (i<n) request R_{n-i};
   if (i+2<n)request R_{n-i-2};
}
```

In which one of the following situations is a deadlock possible?

(A) $n = 40, k = 26$ (B) $n = 21, k = 12$ (C) $n = 20, k = 10$ (D) $n = 41, k = 19$

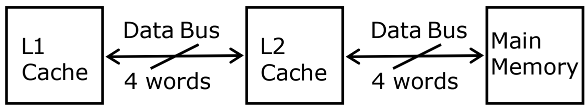
47. Suppose computers A and B have IP addresses 10.105.1.113 and 10.105.1.91 respectively and they both use the same net mask N. Which of the values of N given below should not be used if A and B should belong to the same network?

(A) 255.255.255.0 (B) 255.255.255.128

(C) 255.255.255.192 (D) 255.255.255.224

Common Data Questions: 48 & 49

A computer system has an L1 cache, an L2 cache, and a main memory unit connected as shown below. The block size in L1 cache is 4 words. The block size in L2 cache is 16 words. The memory access times are 2 nanoseconds. 20 nanoseconds and 200 nanoseconds for L1 cache, L2 cache and main memory unit respectively.



48. When there is a miss in L1 cache and a hit in L2 cache, a block is transferred from L2 cache to L1 cache. What is the time taken for this transfer?

(A) 2 nanoseconds (B) 20 nanoseconds

(C) 22 nanoseconds (D) 88 nanoseconds

49. When there is a miss in both L1 cache and L2 cache, first a block is transferred from main memory to L2 cache, and then a block is transferred from L2 cache to L1 cache. What is the total time taken for these transfers?

(A) 222 nanoseconds (B) 888 nanoseconds

(C) 902 nanoseconds (D) 968 nanoseconds

Common Data Questions: 50 & 51

Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the weight of the edge {i, j}.

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

50. What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?

(A) 7 (B) 8 (C) 9 (D) 10

51. What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges?

(A) 7 (B) 8 (C) 9 (D) 10

Linked Answer Questions: Q.52 to Q.55 Carry Two Marks Each

Statement for Linked Answer Questions: 52 & 53

A hash table of length 10 uses open addressing with hash function h(k)=k mod 10, and linear probing. After inserting 6 values into an empty hash table, the table is as shown below

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

52. Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

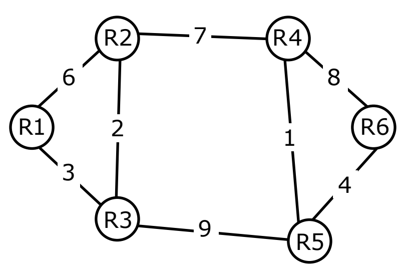
(A) 46, 42, 34, 52, 23, 33
(B) 34, 42, 23, 52, 33, 46
(C) 46, 34, 42, 23, 52, 33
(D) 42, 46, 33, 23, 34, 52

53. How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

(A) 10 (B) 20 (C) 30 (D) 40

Statement for Linked Answer Questions: 54 & 55

Consider a network with 6 routers R1 to R6 connected with links having weights as shown in the following diagram



54. All the routers use the distance vector based routing algorithm to update their routing tables. Each router starts with its routing table initialized to contain an entry for each neighbour with the weight of the respective connecting link. After all the routing tables stabilize, how many links in the network will never be used for carrying any data?

(A) 4 (B) 3 (C) 2 (D) 1

55. Suppose the weights of all unused links in the previous question are changed to 2 and the distance vector algorithm is used again until all routing tables stabilize. How many links will now remain unused?

(A) 0 (B) 1 (C) 2 (D) 3

Q. No. 56 – 60 Carry One Mark Each

56. Choose the most appropriate word from the options given below to the complete the following sentence:

His rather casual remarks on politics ___________ his lack of seriousness about the subject.

(A) masked (B) belied (C) betrayed (D) suppressed

57. Which of the following options is closest in meaning to the word Circuitous.

(A) cyclic (B) indirect (C) confusing (D) crooked

58. Choose the most appropriate word from the options given below to complete the following sentence:

If we manage to ____________ our natural resources, we would leave a better planet for our children.

(A) uphold (B) restrain (C) cherish (D) conserve

59. 25 persons are in a room. 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is:

(A) 2 (B) 17 (C) 13 (D) 3

60. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

Unemployed: Worker

(A) fallow: land (B) unaware: sleeper (C) wit: jester (D) renovated:house

Q. No. 61 – 65 Carry Two Marks Each

61. If 137+276=435 how much is 731+672?

(A) 534 (B) 1403 (C) 1623 (D) 1513

62. Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e. brothers and sisters). All were born on 1^{st} january. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:

i. Hari's age + Gita's age > Irfan's age + Saira's age

ii. The age difference between Gita and Saira is 1 year. However Gita is not the oldest and Saira is not the youngest.

iii. There are no twins.

In what order were they born (oldest first)?

(A) HSIG (B) SGHI (C) IGSH (D) IHSG

64. 5 skilled workers can build a wall in 20days: 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?

(A) 20 (B) 18 (C) 16 (D) 15

63. Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.

Which of the following statements best sums up the meaning of the above passage:

(A) Modern warfare has resulted in civil strife.

(B) Chemical agents are useful in modern warfare.

(C) Use of chemical agents in warfare would be undesirable

(D) People in military establishments like to use chemical agents in war.

65. Given digits 2,2,3,3,4,4,4,4 how many distinct 4 digit numbers greater than 3000 can be formed?

(A) 50 (B) 51 (C) 52 (D) 54

Q. No. 1 – 20 Carry One Mark Each

1. Which one of the following in NOT necessarily a property of a Group?
   (A) Commutativity (B) Associativity
   (C) Existence of inverse for every element (D) Existence of identity

2. What is the chromatic number of an n-vertex simple connected graph which does not contain any odd length cycle? Assume $n \geq 2$.
   (A) 2 (B) 3 (C) n-1 (D) n

3. Which one of the following is TRUE for any simple connected undirected graph with more than 2 vertices?
   (A) No two vertices have the same degree.
   (B) At least two vertices have the same degree.
   (C) At least three vertices have the same degree.
   (D) All vertices have the same degree.

4. Consider the binary relation R = {(x,y), (x,z), (z,x), (z,y)} on the set {x,y,z}. Which one of the following is TRUE?
   (A) R is symmetric but NOT antisymmetric
   (B) R is NOT symmetric but antisymmetric
   (C) R is both symmetric and antisymmetric
   (D) R is neither symmetric nor antisymmetric

5. $(1217)_8$ is equivalent to
   (A) $(1217)_{16}$ (B) $(028F)_{16}$ (C) $(2297)_{10}$ (D) $(0B17)_{16}$

6. What is the minimum number of gates required to implement the Boolean function (AB+C) if we have to use only 2-input NOR gates?
   (A) 2 (B) 3 (C) 4 (D) 5

7. How many 32K x 1 RAM chips are needed to provide a memory capacity of 256K-bytes?
   (A) 8 (B) 32 (C) 64 (D) 128

8. A CPU generally handles an interrupt by executing an interrupt service routine
   (A) As soon as an interrupt is raised
   (B) By checking the interrupt register at the end of fetch cycle.
   (C) By checking the interrupt register after finishing the execution of the current instruction.
   (D) By checking the interrupt register at fixed time intervals.

9. In which one of the following page replacement policies, Belady's anomaly may occur?

(A) FIFO (B) Optimal (C) LRU (D) MRU

10. The essential content(s) in each entry of a page table is / are

(A) Virtual page number

(B) Page frame number

(C) Both virtual page number and page frame number

(D) Access right information

11. What is the number of swaps required to sort n elements using selection sort, in the worst case?

(A) $\theta(n)$ (B) $\theta(n \log n)$ (C) $\theta(n^2)$ (D) $\theta(n^2 \log n)$

12. $S \rightarrow aSa \mid bSb \mid a \mid b$;The language generated by the above grammar over the alphabet {a,b} is the set of

(A) All palindromes.

(B) All odd length palindromes.

(C) Strings that begin and end with the same symbol

(D) All even length palindromes.

13. Which of the following statement(s) is / are correct regarding Bellman-Ford shortest path algorithm?

P. Always finds a negative weighted cycle, if one exists.

Q. Finds whether any negative weighted cycle is reachable from the source.

(A) P only (B) Q only

(C) both P and Q (D) Neither P nor Q

14. Let π_A be a problem that belongs to the class NP. Then which one of the following is TRUE?

(A) There is no polynomial time algorithm for π_A .

(B) If π_A can be solved deterministically in polynomial time, then P = NP.

(C) If π_A is NP-hard, then it is NP-complete.

(D) π_A may be undecidable.

15. Which one of the following languages over the alphabet {0,1} is described by the regular expression: (0+1)*0(0+1)*0(0+1)*?

(A) The set of all strings containing the substring 00.

(B) The set of all strings containing at most two 0's.

(C) The set of all strings containing at least two 0's.

(D) The set of all strings that begin and end with either 0 or 1.

16. Which one of the following is FALSE?

(A) There is unique minimal DFA for every regular language

(B) Every NFA can be converted to an equivalent PDA.

(C) Complement of every context-free language is recursive.

(D) Every nondeterministic PDA can be converted to an equivalent deterministic PDA.

17. Match all items in Group 1 with correct options from those given in Group 2.

Group 1		Group 2	
P.	Regular expression	1.	Syntax analysis
Q.	Pushdown automata	2.	Code generation
R.	Dataflow analysis	3.	Lexical analysis
S.	Register allocation	4.	Code optimization

(A) P-4. Q-1, R-2, S-3 (B) P-3, Q-1, R-4, S-2

(C) P-3, Q-4, R-1, S-2 (D) P-2, Q-1, R-4, S-3

18. Consider the program below:

```
# include < stdio.h >
int fun(int n, int * f_p) {
        int t, f;
        if (n <= 1) {
                * f_p = 1;
                return 1;
        }
        t = fun (n- 1, f_p);
        f = t+ * f_p;
        * f_p = t;
        return f;
}
int main() {
        int x = 15;
        printf ("% d\ n", fun(5, & x));
        return 0;
}
```

The value printed is

(A) 6 (B) 8 (C) 14 (D) 15

19. The coupling between different modules of a software is categorized as follows:

I. Content coupling
II. Common coupling
III. Control coupling
IV Stamp coupling
V. Data coupling

Coupling between modules can be ranked in the order of strongest (least desirable) to weakest (most desirable) as follows:

(A) I-II-III-IV-V (B) V-IV-III-II-I (C) I-III-V -II-IV (D) IV-II-V -III-I

20. Consider the HTML table definition given below:

```
< table border=1>
        <tr> <td rowspan=2> ab </td>
            <td colspan=2> cd </td>
        </tr>
        <tr> <td> ef </td>
            <td rowspan=2> gh </td>
        </tr>
        <tr> <td colspan=2> ik </td>
        </tr>
</table>
```

The number of rows in each column and the number of columns in each row are:

(A) $\langle 2,2,3 \rangle$ and $\langle 2,3,2 \rangle$
(B) $\langle 2,2,3 \rangle$ and $\langle 2,2,3 \rangle$
(C) $\langle 2,3,2 \rangle$ and $\langle 2,3,2 \rangle$
(D) $\langle 2,3,2 \rangle$ and $\langle 2,2,3 \rangle$

Q. No. 21 – 56 Carry Two Marks Each

21. An unbalanced dice (with 6 faces, numbered from 1 to 6) is thrown. The probability that the face value is odd is 90% of the probability that the face value is even. The probability of getting any even numbered face is the same.

If the probability that the face is even given that it is greater than 3 is 0.75, which one of the following options is closest to the probability that the face value exceeds 3?

(A) 0.453
(B) 0.468
(C) 0.485
(D) 0.492

22. For the composition table of a cyclic group shown below

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Which one of the following choices is correct?

(A) a, b are generators
(B) b, c are generators
(C) c, d are generators
(D) d, a are generators

23. Which one of the following is the most appropriate logical formula to represent the statement? “Gold and silver ornaments are precious”.

The following notations are used:

G(x): x is a gold ornament

S(x): x is a silver ornament

P(x): x is precious

(A) $\forall x\left(P(x) \rightarrow \left(G(x) \wedge S(x)\right)\right)$
(B) $\forall x\left(\left(G(x) \wedge S(x)\right) \rightarrow P(x)\right)$
(C) $\exists x\left(\left(G(x) \wedge S(x)\right) \rightarrow P(x)\right)$
(D) $\forall x\left(\left(G(x) \vee S(x)\right) \rightarrow P(x)\right)$

24. The binary operation $\square$ is defined as follows

P	Q	P$\square$Q
T	**T**	**T**
T	**F**	**T**
F	**T**	**F**
F	**F**	**T**

Which one of the following is equivalent to $P \vee Q$?

(A) $\neg Q \square \neg P$ (B) $P \square \neg Q$ (C) $\neg P \square Q$ (D) $\neg P \square \neg Q$

25. $\int_0^{\pi/4} (1 - \tan x)/(1 + \tan x)\, dx$ evaluates to

(A) 0 (B) 1 (C) ln 2 (D) $\frac{1}{2}$ln 2

26. Consider the following well-formed formulae:

I. $\neg \forall x (P(x))$ II. $\neg \exists x (P(x))$ III. $\neg \exists x (\neg P(x))$ IV. $\neg \exists x (\neg P(x))$

Which of the above are equivalent?

(A) I and III (B) I and IV (C) II and III (D) II and IV

27. Given the following state table of an FSM with two states A and B, one input and one output:

Present State A	Present State B	Input	Next State A	Next State B	Output
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	0	1	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	0	0	1

If the initial state is A = 0, B=0, what is the minimum length of an input string which will take the machine to the state A=0, B=1 with Output=1?

(A) 3 (B) 4 (C) 5 (D) 6

28. Consider a 4 stage pipeline processor. The number of cycles needed by the four instructions I1, I2, I3, I4 in stages S1, S2, S3, S4 is shown below:

	S1	S2	S3	S4
I1	2	1	1	1
I2	1	3	2	2
I3	2	1	1	3
I4	1	2	2	2

What is the number of cycles needed to execute the following loop?

For (i=1 to 2) {I1; I2; I3; I4;}

(A) 16 (B) 23 (C) 28 (D) 30

29. Consider a 4-way set associative cache (initially empty) with total 16 cache blocks. The main memory consists of 256 blocks and the request for memory blocks is in the following order:

0, 255, 1, 4, 3, 8, 133, 159, 216, 129, 63, 8, 48, 32, 73, 92, 155.

Which one of the following memory block will NOT be in cache if LRU replacement policy is used?

(A) 3 (B) 8 (C) 129 (D) 216

30. Consider a system with 4 types of resources R1 (3 units), R2 (2 units), R3 (3 units), R4 (2 units). A non-preemptive resource allocation policy is used. At any given instance, a request is not entertained if it cannot be completely satisfied. Three processes P1, P2, P3 request the sources as follows if executed independently.

Process P1:	Process P2:	Process P3:
t=0: requests 2 units of R2	t=0: requests 2 units of R3	t=0: requests 1 unit of R4
t=1: requests 1 unit of R3	t=2: requests 1 unit of R4	t=2: requests 2 units of R1
t=3: requests 2 units of R1	t=4: requests 1 unit of R1	t=5: releases 2 units of R1
t=5: releases 1 unit of R2 and 1 unit of R1.	t=6: releases 1 unit of R3	t=7: requests 1 unit of R2
t=7: releases 1 unit of R3	t=8: Finishes	t=8: requests 1 unit of R3
t=8: requests 2 units of R4		t=9: Finishes
t=10: Finishes		

Which one of the following statements is TRUE if all three processes run concurrently starting at time t=0?

(A) All processes will finish without any deadlock

(B) Only P1 and P2 will be in deadlock.

(C) Only P1 and P3 will be in a deadlock.

(D) All three processes will be in deadlock.

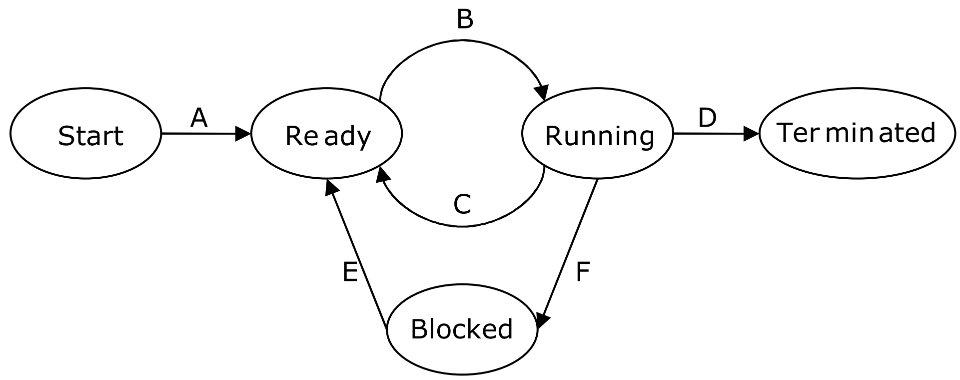
31. Consider a disk system with 100 cylinders. The requests to access the cylinders occur in following sequence:

4, 34, 10, 7, 19, 73, 2, 15, 6, 20

Assuming that the head is currently at cylinder 50, what is the time taken to satisfy all requests if it takes 1ms to move from one cylinder to adjacent one and shortest seek time first policy is used?

(A) 95ms (B) 119ms (C) 233ms (D) 276ms

32. In the following process state transition diagram for a uniprocessor system, assume that there are always some processes in the ready state:



Now consider the following statements:

I. If a process makes a transition D, it would result in another process making transition A immediately.

II. A process P_2 in blocked state can make transition E while another process P_1 is in running state.

III. The OS uses preemptive scheduling.

IV. The OS uses non-preemptive scheduling.

Which of the above statements are TRUE?

(A) I and II (B) I and III (C) II and III (D) II and IV

33. The enter_CS() and leave_CS() functions to implement critical section of a process are realized using test-and-set instruction as follows:

```
void enter_CS(X)
{
        while(test-and-set(X));
}
void leave_CS(X)
{
        X=0;
}
```

In the above solution, X is a memory location associated with the CS and is initialized to 0. Now consider the following statements:

I. The above solution to CS problem is deadlock-free

II. The solution is starvation free.

III. The processes enter CS in FIFO order.

IV More than one process can enter CS at the same time.

Which of the above statements is TRUE?

(A) I only (B) I and II (C) II and III (D) IV only

34. A multilevel page table is preferred in comparison to a single level page table for translating virtual address to physical address because

(A) It reduces the memory access time to read or write a memory location.

(B) It helps to reduce the size of page table needed to implement the virtual address space of a process.

(C) It is required by the translation lookaside buffer.

(D) It helps to reduce the number of page faults in page replacement algorithms.

35. The running time of an algorithm is represented by the following recurrence relation:

$$T(n) = \begin{cases} n & n \leq 3 \\ T\left(\frac{n}{3}\right) + cn & \text{otherwise} \end{cases}$$

Which one of the following represents the time complexity of the algorithm?

(A) $\theta(n)$ (B) $\theta(n \log n)$ (C) $\theta(n^2)$ (D) $\theta(n^2 \log n)$

36. The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function h(k) = k mod 10 and linear probing. What is the resultant hash table?

(A)

0	
1	
2	2
3	23
4	
5	15
6	
7	
8	18
9	

(B)

0	
1	
2	12
3	13
4	
5	5
6	
7	
8	18
9	

(C)

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

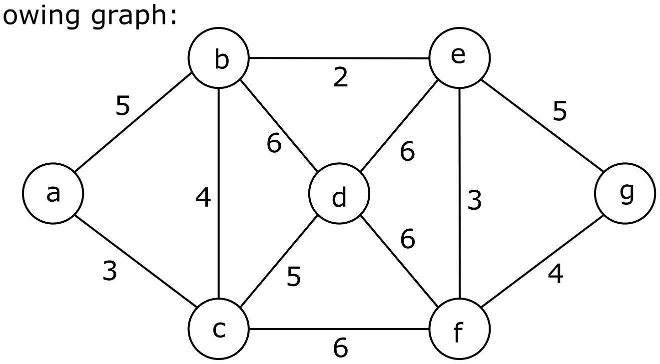
(D)

0	
1	
2	12,2
3	13,3,23
4	
5	5,15
6	
7	
8	18
9	

37. What is the maximum height of any AVL-tree with 7 nodes? Assume that the height of a tree with a single node is 0.

(A) 2 (B) 3 (C) 4 (D) 5

38. Consider the following graph:



Which one of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's algorithm?

(A) (b,e) (e,f) (a,c) (b,c) (f,g) (c,d)   (B) (b,e) (e,f) (a,c) (f,g) (b,c) (c,d)

(C) (b,e) (a,c) (e,f) (b,c) (f,g) (c,d)   (D) (b,e) (e,f) (b,c) (a,c) (f,g) (c,d)

39. In quick sort, for sorting n elements, the $(n/4)^{th}$ smallest element is selected as pivot using an O(n) time algorithm. What is the worst case time complexity of the quick sort?

(A) $\theta(n)$ (B) $\theta(n \log n)$ (C) $\theta(n^2)$ (D) $\theta(n^2 \log n)$

40. Let $L = L_1 \cap L_2$, where L_1 and L_2 are languages as defined below:

$L_1 = \{a^m\ b^m\ c\ a^n\ b^m \mid m, n \geq 0\}$

$L_2 = \{a^i b^j c^k \mid i, j, k \geq 0\}$

Then L is

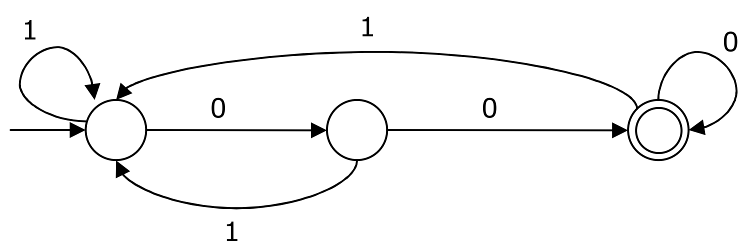
(A) Not recursive

(B) Regular

(C) Context free but not regular

(D) Recursively enumerable but not context free.

41.



The above DFA accepts the set of all strings over {0,1} that

(A) begin either with 0 or 1   (B) end with 0

(C) end with 00   (D) contain the substring 00.

42. Which of the following statements are TRUE?

I There exist parsing algorithms for some programming languages whose complexities are less than $\theta(n^3)$.

II A programming language which allows recursion can be implemented with static storage allocation.

III No L-attributed definition can be evaluated in the framework of bottom-up parsing.

IV Code improving transformations can be performed at both source language and intermediate code level.

(A) I and II   (B) I and IV   (C) III and IV   (D) I, III and IV

43. Consider two transactions T_1 and T_2, and four schedules S_1, S_2, S_3, S_4 of T_1 and T_2 as given below:

$T_1 : R_1[x] W_1[x] W_1[y]$
$T_2 : R_2[x] R_2[y] W_2[y]$
$S_1 : R_1[x] R_2[x] R_2[y] W_1[x] W_1[y] W_2[y]$
$S_2 : R_1[x] R_2[x] R_2[y] W_1[x] W_2[y] W_1[y]$
$S_3 : R_1[x] W_1[x] R_2[x] W_1[y] R_2[y] W_2[y]$
$S_4 : R_2[x] R_2[y] R_1[x] W_1[x] W_1[y] W_2[y]$

Which of the above schedules are conflict-serializable?

(A) S_1 and S_2 (B) S_2 and S_3 (C) S_3 only (D) S_4 only

44. The following key values are inserted into a B+ - tree in which order of the internal nodes is 3, and that of the leaf nodes is 2, in the sequence given below. The order of internal nodes is the maximum number of tree pointers in each node, and the order of leaf nodes is the maximum number of data items that can be stored in it. The B+ - tree is initially empty.

10, 3, 6, 8, 4, 2, 1

The maximum number of times leaf nodes would get split up as a result of these insertions is

(A) 2 (B) 3 (C) 4 (D) 5

45. Let R and S be relational schemes such that R={a,b,c} and S={c}. Now consider the following queries on the database:

I. $\pi_{R-S}(r) - \pi_{R-S}\left(\pi_{R-S}(r) \times s - \pi_{R-S,S}(r)\right)$

II. $\left\{t \mid t \in \pi_{R-S}(r) \wedge \forall u \in s\left(\exists v \in r\left(u = v[s] \wedge t = v[R-S]\right)\right)\right\}$

III. $\left\{t \mid t \in \pi_{R-S}(r) \wedge \forall v \in r\left(\exists u \in s\left(u = v[s] \wedge t = v[R-S]\right)\right)\right\}$

IV Select R.a, R.b
From R,S
Where R.c=S.c

Which of the above queries are equivalent?

(A) I and II (B) I and III (C) II and IV (D) III and IV

46. In the RSA public key cryptosystem, the private and public keys are (e,n) and (d,n) respectively, where n=p*q and p and q are large primes. Besides, n is public and p and q are private. Let M be an integer such that 0<M<n and $\phi(n) = (p-1)(q-1)$. Now consider the following equations.

I $M' = M^e \bmod n$
$M = (M')^d \bmod n$

II. $ed \equiv 1 \bmod n$

III. $ed \equiv 1 \bmod \phi(n)$

IV. $M' = M^e \bmod \phi(n)$
$M = (M')^d \bmod \phi(n)$

Which of the above equations correctly represent RSA cryptosystem?

(A) I and II (B) I and III (C) II and IV (D) III and IV

47. While opening a TCP connection, the initial sequence number is to be derived using a time-of-day (ToD) clock that keeps running even when the host is down. The low order 32 bits of the counter of the ToD clock is to be used for the initial sequence numbers. The clock counter increments once per millisecond. The maximum packet lifetime is given to be 64s.

Which one of the choices given below is closest to the minimum permissible rate at which sequence numbers used for packets of a connection can increase?

(A) 0.015/s (B) 0.064/s (C) 0.135/s (D) 0.327/s

48. Let G(x) be the generator polynomial used for CRC checking. What is the condition that should be satisfied by G(x) to detect odd number of bits in error?

(A) G(x) contains more than two terms

(B) G(x) does not divide $1+x^k$, for any k not exceeding the frame length

(C) 1+x is a factor of G(x)

(D) G(x) has an odd number of terms.

49. Which of the following statements are TRUE?

I The context diagram should depict the system as a single bubble.

II External entities should be identified clearly at all levels of DFDs.

III Control information should not be represented in a DFD.

IV A data store can be connected either to another data store or to an external entity.

(A) II and III (B) II and III (C) I and III (D) I, II and III

50. Consider the following statements about the cyclomatic complexity of the control flow graph of a program module. Which of these are TRUE?

I. The cyclomatic complexity of a module is equal to the maximum number of linearly independent circuits in the graph.

II. The cyclomatic complexity of a module is the number of decisions in the module plus one, where a decision is effectively any conditional statement in the module.

III. The cyclomatic complexity can also be used as a number of linearly independent paths that should be tested during path coverage testing.

(A) I and II (B) II and III (C) I and III (D) I, II and III

Common Data Questions: 51 & 52

A hard disk has 63 sectors per track, 10 platters each with 2 recording surfaces and 1000 cylinders. The address of a sector is given as a triple $\langle c,h,s \rangle$, where *c* is the cylinder number, *h* is the surface number and *s* is the sector number. Thus, the 0^{th} sector is addressed as $\langle 0,0,0 \rangle$, the 1^{st} sector as $\langle 0,0,1 \rangle$, and so on

51. The address <400, 16, 29> corre4sponds tp sector number:

(A) 505035 (B) 505036 (C) 505037 (D) 505038

52. The address of the 1039^{th} sector is

(A) $\langle 0,15,31\rangle$ (B) $\langle 0,16,30\rangle$ (C) $\langle 0,16,31\rangle$ (D) $\langle 0,17,31\rangle$

Common Data Questions: 53 & 54

A sub-sequence of a given sequence is just the given sequence with some elements (possibly none or all) left out. We are given two sequences X[m] and Y[n] of lengths m and n, respectively, with indexes of X and Y starting from 0.

53. We wish to find the length of the longest common sub-sequence (LCS) of X[m] and Y[n] as l(m,n), where an incomplete recursive definition for the function l(i,j) to compute the length of the LCS of X[m] and Y[n] is given below:

$l(i, j) = 0$, if either i=0 or j=0
= expr1, if i,j>0 and $X[i\text{-}1] = Y[j-1]$
= expr2, if i,j>0 and $X[i\text{-}1] = Y[j-1]$

Which one of the following options is correct?

(A) $\text{expr1} \equiv l(i-1, j)+1$
(B) $\text{expr1} \equiv l(i, j-1)$
(C) $\text{expr2} \equiv \max(l(i-1, j),\ l(i,j\text{-}1))$
(D) $\text{expr2} \equiv \max(l(i-1, j-1),\ l(i,j))$

54. The values of l(i,j) could be obtained by dynamic programming based on the correct recursive definition of l(i,j) of the form given above, using an array L[M,N], where M = m+1 and N=n+1, such that L[i,j] = l(i,j).

Which one of the following statements would be TRUE regarding the dynamic programming solution for the recursive definition of l(i,j)?

(A) All elements L should be initialized to 0 for the values of l(i,j) to be properly computed.
(B) The values of l(i,j) may be computed in a row major order or column major order of L(M,N).
(C) The values of l(i,j) cannot be computed in either row major order or column major order of L(M,N).
(D) L[p,q] needs to be computed before L[r,s] if either p<r or q<s.

Common Data Questions: 55 & 56

Consider the following relational schema:

Suppliers(sid:integer, sname:string, city:string, street:string)
Parts(pid:integer, pname:string, color:string)
Catalog(sid:integer, pid:integer, cost:real)

55. Consider the following relational query on the above database:

```
SELECT      S.sname
FROM        Suppliers S
WHERE S.sid NOT IN (SELECT C.sid
                     FROM Catalog C
                    WHERE C.pid NOT (SELECT  P.pid
                                     FROM    Parts P
                                     WHERE P.color<> 'blue'))
```

Assume that relations corresponding to the above schema are not empty. Which one of the following is the correct interpretation of the above query?

(A) Find the names of all suppliers who have supplied a non-blue part.

(B) Find the names of all suppliers who have not supplied a non-blue part.

(C) Find the names of all suppliers who have supplied only blue parts.

(D) Find the names of all suppliers who have not supplied only blue parts.

56. Assume that, in the suppliers relation above, each supplier and each street within a city has a unique name, and (sname, city) forms a candidate key. No other functional dependencies are implied other than those implied by primary and candidate keys. Which one of the following is TRUE about the above schema?

(A) The schema is in BCNF

(B) The schema is in 3NF but not in BCNF

(C) The schema is in 2NF but not in 3NF

(D) The schema is not in 2NF

Linked Answer Questions: Q.57 to Q.60 Carry Two Marks Each

Statement for Linked Answer Questions: 57 & 58

Frames of 1000 bits are sent over a 10^6 bps duplex link between two hosts. The propagation time is 25ms. Frames are to be transmitted into this link to maximally pack them in transit (within the link).

57. What is the minimum number of bits (l) that will be required to represent the sequence numbers distinctly? Assume that no time gap needs to be given between transmission of two frames.

(A) l=2 (B) l=3 (C) l=4 (D) l=5

58. Suppose that the sliding window protocol is used with the sender window size of 2^l, where l is the number of bits identified in the earlier part and acknowledgements are always piggy backed. After sending 2^l frames, what is the minimum time the sender will have to wait before starting transmission of the next frame? (Identify the closest choice ignoring the frame processing time.)

(A) 16ms (B) 18ms (C) 20ms (D) 22ms

Statement for Linked Answer Questions: 59 & 60

Consider a binary max-heap implemented using an array.

59. Which one of the following array represents a binary max-heap?

(A) $\{25,12,16,13,10,8,14\}$ (B) $\{25,14,13,16,10,8,12\}$

(C) $\{25,14,16,13,10,8,12\}$ (D) $\{25,14,12,13,10,8,16\}$

60. What is the content of the array after two delete operations on the correct answer to the previous question?

(A) $\{14,13,12,10,8\}$ (B) $\{14,12,13,8,10\}$

(C) $\{14,13,8,12,10\}$ (D) $\{14,13,12,8,10\}$

Q.1 – Q.20 Carry One Mark Each

1. $\lim_{x \to \infty} \frac{x - \sin x}{x + \cos x}$ equals

(A) 1 (B) -1 (C) ∞ (D) $-\infty$

2. If P, Q, R are subsets of the universal set U, then $(P \cap Q \cap R) \cup (P^c \cap Q \cap R) \cup Q^c \cup R^c$ is

(A) $Q^c \cup R^c$ (B) $P \cup Q^c \cup R^c$ (C) $P^c \cup Q^c \cup R^c$ (D) U

3. The following system of equations

$$x_1 + x_2 + 2x_3 = 1$$
$$x_1 + 2x_2 + 3x_3 = 2$$
$$x_1 + 4x_2 + \alpha x_3 = 4$$

has a unique solution. The only possible value(s) for α is/are

(A) 0 (B) either 0 or 1
(C) one of 0, 1 or -1 (D) any real number

4. In the IEEE floating point representation the hexadecimal value 0x00000000 corresponds to

(A) The normalized value 2^{-127} (B) The normalized value 2^{-126}
(C) The normalized value +0 (D) The special value +0

5. In the Karnaugh map shown below, X denotes a don't care term. What is the minimal form of the function represented by the Karnaugh map?

cd \ ab	00	01	11	10
00	1	1		1
01	X			
11	X			
10	1	1		X

(A) $\bar{b}.\bar{d} + \bar{a}.\bar{d}$ (B) $\bar{a}.\bar{b} + \bar{b}.\bar{d} + \bar{a}.b.\bar{d}$ (C) $\bar{b}.\bar{d} + \bar{a}.b.\bar{d}$ (D) $\bar{a}.\bar{b} + \bar{b}.\bar{d} + \bar{a}.\bar{d}$

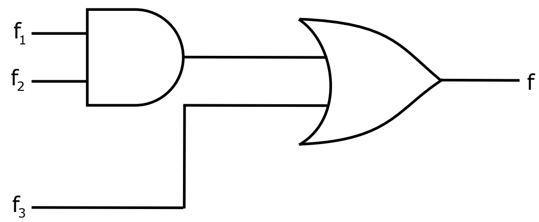
6. Let r denote number system radix. The only value(s) of r that satisfy the equation $\sqrt{121_r} = 11_r$ is / are

(A) decimal 10 (B) decimal 11
(C) decimal 10 and 11 (D) any value >2

7. The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity

(A) $\Theta(n)$ (B) $\Theta(m)$ (C) $\Theta(m+n)$ (D) $\Theta(mn)$

8. Given f_1, f_3 and f in canonical sum of products form (in decimal) for the circuit



$f_1 = \Sigma m(4,5,6,7,8)$
$f_3 = \Sigma m(1,6,15)$
$f = \Sigma m(1,6,8,15)$
then f_2 is

(A) $\Sigma m(4,6)$ (B) $\Sigma m(4,8)$ (C) $\Sigma m(6,8)$ (D) $\Sigma m(4,6,8)$

9. Which of the following is true for the language $\{a^p \mid p \text{ is a prime}\}$?

(A) It is not accepted by a Turing Machine
(B) It is regular but not context-free
(C) It is context-free but not regular
(D) It is neither regular nor context-free, but accepted by a Turing machine

10. Which of the following are decidable?

I. Whether the intersection of two regular languages is infinite
II. Whether a given context-free language is regular
III. Whether two push-down automata accept the same language
IV. Whether a given grammar is context-free

(A) I and II (B) I and IV (C) II and III (D)II and IV

11. Which of the following describes a handle (as applicable to LR-parsing) appropriately?

(A) It is the position in a sentential form where the next shift or reduce operation will occur
(B) It is non-terminal whose production will be used for reduction in the next step
(C) It is a production that may be used for reduction in a future step along with a position in the sentential form where the next shift or reduce operation will occur
(D) It is the production p that will be used for reduction in the next step along with a position in the sentential form where the right hand side of the production may be found

12. Some code optimizations are carried out on the intermediate code because
(A) They enhance the portability of the compiler to other target processors
(B) Program analysis is more accurate on intermediate code than on machine code
(C) The information from dataflow analysis cannot otherwise be used for optimization
(D) The information from the front end cannot otherwise be used for optimization

13. If L and $\bar{L}$ are recursively enumerable then L is
(A) regular (B) context-free
(C) context-sensitive (D) recursive

14. What is the maximum size of data that the application layer can pass on to the TCP layer below?
(A) Any size (B) 2^{16} bytes-size of TCP header
(C) 2^{16} bytes (D) 1500 bytes

15. Which of the following tuple relational calculus expression(s) is/are equivalent to $\forall t \in r\left(P(t)\right)$?

I. $\neg\exists t \in r\left(P(t)\right)$

II. $\exists t \notin r\left(P(t)\right)$

III. $\neg\exists t \in r\left(\neg P(t)\right)$

IV. $\exists t \notin r\left(\neg P(t)\right)$

(A) I only (B) II only
(C) III only (D) III and IV only

16. A clustering index is defined on the fields which are of type
(A) non-key and ordering (B) non-key and non-ordering
(C) key and ordering (D) key and non-ordering

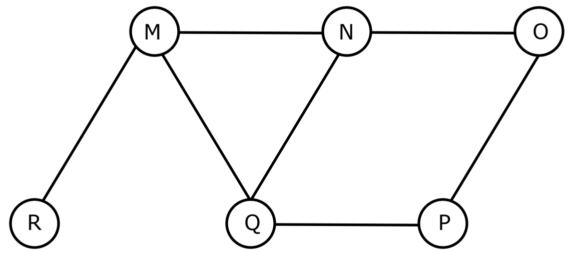
17. Which of the following system calls results in the sending of SYN packets?
(A) socket (B) bind (C) listen (D) connect

18. Which combination of the integer variables x, y and z makes the variable a get the value 4 in the following expression?

$a = (x > y)?((x > z)?x : z) : ((y > z)?y : z)$

(A) $x = 3, y = 4, z = 2$ (B) $x = 6, y = 5, z = 3$
(C) $x = 6, y = 3, z = 5$ (D) $x = 5, y = 4, z = 5$

19. The Breadth First Search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is



(A) MNOPQR (B) NQMPOR (C) QMNPRO (D) QMNPOR

20. The data blocks of a very large file in the Unix file system are allocated using
(A) contiguous allocation
(B) linked allocation
(C) indexed allocation
(D) an extension of indexed allocation

Q.21 – Q.75 Carry Two Marks Each

21. The minimum number of equal length subintervals needed to approximate $\int_1^2 xe^x dx$ to an accuracy of at least $\frac{1}{3} \times 10^{-6}$ using the trapezoidal rule is
(A) 1000e (B) 1000 (C) 100e (D) 100

22. The Newton-Raphson iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n}\right)$ can be used to compute the
(A) square of R (B) reciprocal of R
(C) square root of R (D) logarithm of R

23. Which of the following statements is true for every planar graph on n vertices?
(A) The graph is connected
(B) The graph is Eulerian
(C) The graph has a vertex-cover of size at most 3n/4
(D) The graph has an independent set of size at least n/3

24. Let $P = \sum_{\substack{1 \le i \le 2k \\ i \text{ odd}}} i$ and $Q = \sum_{\substack{1 \le i \le 2k \\ i \text{ even}}} i$, where k is a positive integer. Then
(A) $P = Q - K$ (B) $P = Q + K$ (C) $P = Q$ (D) $P = Q + 2K$

25. A point on a curve is said to be an extremum if it is a local minimum or a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is

(A) 0 (B) 1 (C) 2 (D) 3

26. If P, Q, R are Boolean variables, then

$(P + \overline{Q})(P.\overline{Q} + P.R)(\overline{P}.\overline{R} + \overline{Q})$

Simplifies to

(A) $P.\overline{Q}$ (B) $P.\overline{R}$ (C) $P.\overline{Q} + R$ (D) $P.\overline{R} + Q$

27. Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability that the studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability that the studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probability that she studies computer science on Wednesday?

(A) 0.24 (B) 0.36 (C) 0.4 (D) 0.6

28. How many of the following matrices have an eigenvalue 1?

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \text{and} \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$

(A) one (B) two (C) three (D) four

29. Let X be a random variable following normal distribution with mean +1 and variance 4. Let Y be another normal variable with mean -1 and variance unknown. If $P(X \le -1) = P(Y \ge 2)$, the standard deviation of Y is

(A) 3 (B) 2 (C) $\sqrt{2}$ (D) 1

30. Let fsa and pda be two predicates such that fsa(x) means x is a finite state automaton, and pda(y) means that y is a pushdown automaton. Let equivalent be another predicate such that equivalent (a, b) means a and b are equivalent. Which of the following first order logic statements represents the following:

Each finite state automaton has an equivalent pushdown automaton

(A) $(\forall x \text{ fsa}(x)) \Rightarrow (\exists y \text{ pda}(y) \wedge \text{equivalent}(x, y))$

(B) $\sim \forall y (\exists x \text{ fsa}(x) \Rightarrow \text{pda}(y) \wedge \text{equivalent}(x, y))$

(C) $\forall x \exists y (\text{fsa}(x) \wedge \text{pda}(y) \wedge \text{equivalent}(x, y))$

(D) $\forall x \exists y (\text{fsa}(y) \wedge \text{pda}(x) \wedge \text{equivalent}(x, y))$

31. P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim (\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(A) Only I and II
(B) Only I, II and III
(C) Only I, II and IV
(D) All of I, II III and IV

32. For a magnetic disk with concentric circular tracks, the seek latency is not linearly proportional to the seek distance due to

(A) non-uniform distribution of requests
(B) arm starting and stopping inertia
(C) higher capacity of tracks on the periphery of the platter
(D) use of unfair arm scheduling policies

33. Which of the following is/are true of the auto-increment addressing mode?

I. It is useful in creating self-relocating code

II. If it is included in an Instruction Set Architecture, then an additional ALU is required for effective address calculation

III. The amount of increment depends on the size of the data item accessed

(A) I only
(B) II only
(C) III only
(D) II and III only

34. Which of the following must be true for the RFE (Return From Exception) instruction on a general purpose processor?

I. It must be a trap instruction

II. It must be a privileged instruction

III. An exception cannot be allowed to occur during execution of an RFE instruction

(A) I only
(B) II only
(C) I and II only
(D) I, II and III only

35. For inclusion to hold between two cache levels L1 and L2 in a multi-level cache hierarchy, which of the following are necessary?

I. L1 must be a write-through cache

II. L2 must be a write-through cache

III. The associativity of L2 must be greater than that of L1

IV. The L2 cache must be at least as large as the L1 cache

(A) IV only
(B) I and IV only
(C) I, II and IV only
(D) I, II, III and IV

36. Which of the following are NOT true in a pipelined processor?

I. Bypassing can handle all RAW hazards

II. Register renaming can eliminate all register carried WAR hazards

III. Control hazard penalties can be eliminated by dynamic branch prediction

(A) I and II only (B) I and III only (C) II and III only (D) I, II and III

37. The use of multiple register windows with overlap causes a reduction in the number of memory accesses for

I. Function locals and parameters

II. Register saves and restores

III. Instruction fetches

(A) I only (B) II only (C) III only (D) I, II and III

38. In an instruction execution pipeline, the earliest that the data TLB (Translation Lookaside Buffer) can be accessed is

(A) Before effective address calculation has started

(B) During effective address calculation

(C) After effective address calculation has completed

(D) After data cache lookup has completed

39. Consider the following functions:

$f(n) = 2^n$

$g(n) = n!$

$h(n) = n^{\log n}$

Which of the following statements about the asymptotic behaviour of f(n), g(n), and h(n) is true?

(A) $f(n) = O(g(n));\ g(n) = O(h(n))$ (B) $f(n) = \Omega(g(n));\ g(n) = O(h(n))$

(C) $g(n) = O(f(n));\ h(n) = O(f(n))$ (D) $h(n) = O(f(n));\ g(n) = \Omega(f(n))$

40. The minimum number of comparisons required to determine if an integer appears more than n/2 times in a sorted array of n integers is

(A) $\Theta(n)$ (B) $\Theta(\log n)$ (C) $\Theta(\log^* n)$ (D) $\Theta(1)$

41. A B-tree of order 4 is built from scratch by 10 successive insertions. What is the maximum number of node splitting operations that may take place?

(A) 3 (B) 4 (C) 5 (D) 6

42. G is a graph on n vertices and 2n-2 edges. The edges of G can be partitioned into two edge-disjoint spanning trees. Which of the following is NOT true for G?

(A) For every subset of k vertices, the induced subgraph has at most 2k-2 edges

(B) The minimum cut in G has at least two edges

(C) There are two edge-disjoint paths between every pair of vertices

(D) There are two vertex-disjoint paths between every pair of vertices

43. Consider the Quicksort algorithm. Suppose there is a procedure for finding a pivot element which splits the list into two sub-lists each of which contains at least one-fifth of the elements. Let T(n) be the number of comparisons required to sort n elements. Then

(A) $T(n) \le 2T(n/5) + n$

(B) $T(n) \le T(n/5) + T(4n/5) + n$

(C) $T(n) \le 2T(4n/5) + n$

(D) $T(n) \le 2T(n/2) + n$

44. The subset-sum problem is defined as follows: Given a set S of n positive integers and a positive integer W, determine whether there is a subset of S Whose elements sum to W.

An algorithm Q solves this problem in O(nW) time. Which of the following statements is false?

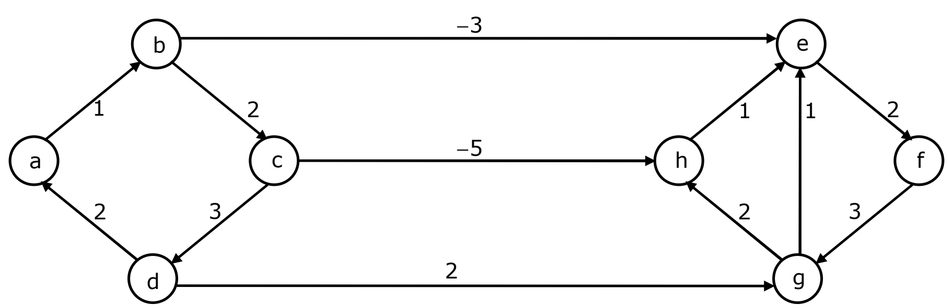
(A) Q solves the subset-sum problem in polynomial time when the input is encoded in unary

(B) Q solves the subset-sum problem in polynomial time when the input is encoded in binary

(C) The subset sum problem belongs to the class NP

(D) The subset sum problem is NP-hard

45.



Dijkstra's single source shortest path algorithm when run from vertex *a* in the above graph, computes the correct shortest path distance to

(A) only vertex a

(B) only vertices a, e, f, g, h

(C) only vertices a, b, c, d

(D) all the vertices

46. You are given the postorder traversal, P, of a binary search tree on the n elements 1, 2,….,n. You have to determine the unique binary search tree that has P as its postorder traversal. What is the time complexity of the most efficient algorithm for doing this?

(A) $\Theta(\log n)$

(B) $\Theta(n)$

(C) $\Theta(n \log n)$

(D) None of the above, as the tree cannot be uniquely determined

47. We have a binary heap on n elements and wish to insert n more elements (not necessarily one after another) into this heap. The total time required for this is

(A) $\Theta(\log n)$ (B) $\Theta(n)$ (C) $\Theta(n \log n)$ (D) $\Theta(n^2)$

48. Which of the following statements is false?

(A) Every NFA can be converted to an equivalent DFA

(B) Every non-deterministic Turing machine can be converted to an equivalent deterministic Turing machine

(C) Every regular language is also a context-free language

(D) Every subset of a recursively enumerable set is recursive

49. Given below are two finite state automata ($\rightarrow$ indicates the start state and F indicates a final state)

Y:

	a	b
$\rightarrow 1$	1	2
2(F)	2	1

Z:

	a	b
$\rightarrow 1$	2	2
2(F)	1	1

Which of the following represents the product automaton $Z \times Y$?

(A)

	a	b
$\rightarrow P$	S	R
Q	R	S
R(F)	Q	P
S	Q	P

(B)

	a	b
$\rightarrow P$	S	Q
Q	R	S
R(F)	Q	P
S	P	Q

(C)

	a	b
$\rightarrow P$	Q	S
Q	R	S
R(F)	Q	P
S	Q	P

(D)

	a	b
$\rightarrow P$	S	Q
Q	S	R
R(F)	Q	P
S	Q	P

50. Which of the following statements are true?

I. Every left-recursive grammar can be converted to a right-recursive grammar and vice-versa

II. All ε-productions can be removed from any context-free grammar by suitable transformations

III. The language generated by a context-free grammar all of whose productions are of the form $X \rightarrow w$ or $X \rightarrow wY$ (where, w is a string of terminals and Y is a non-terminal), is always regular

IV. The derivation trees of strings generated by a context-free grammar in Chomsky Normal Form are always binary trees

(A) I, II, III and IV (B) II, III and IV only

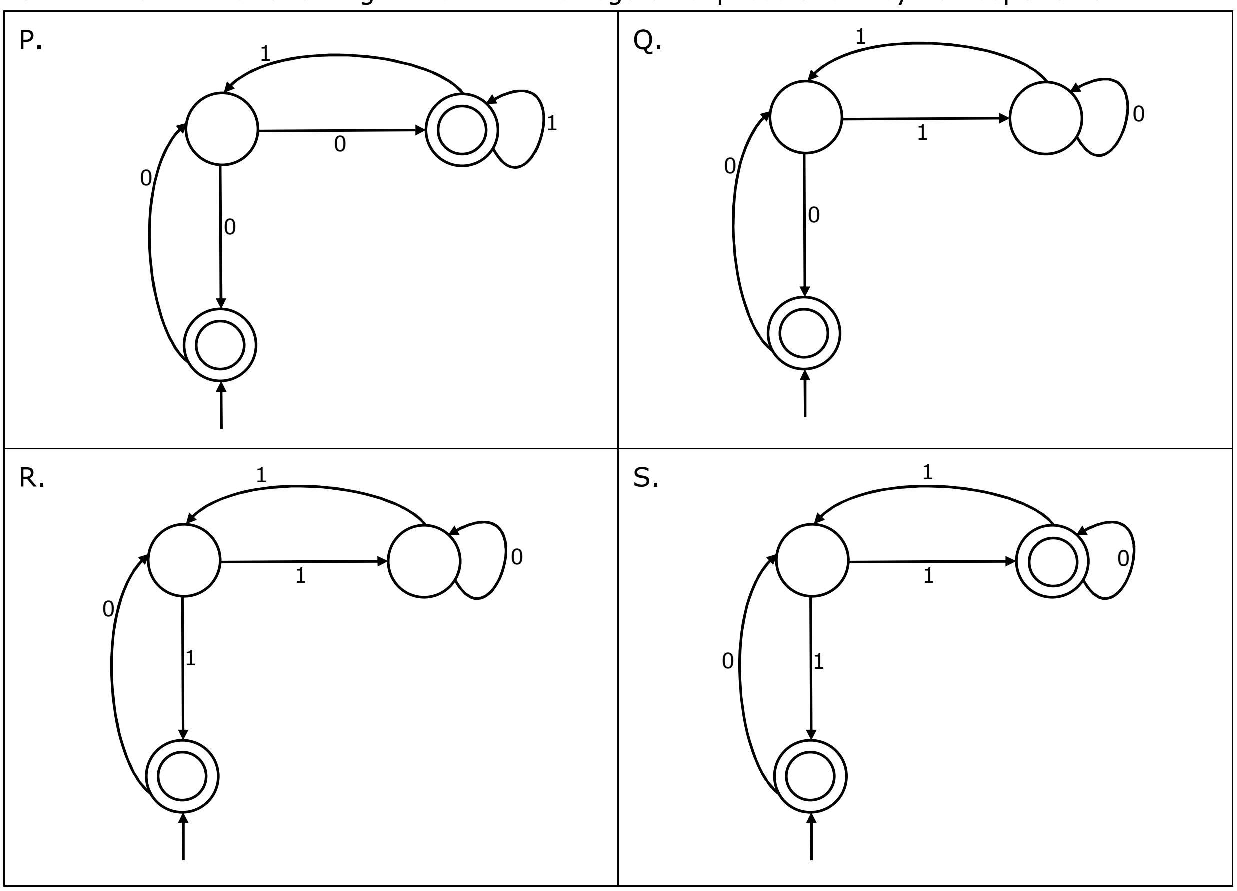
(C) I, III and IV only (D) I, II and IV only

51. Match the following:

E.	Checking that identifiers are declared before their use	P.	$L = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$
F.	Number of formal parameters in the declaration of a function agrees with the number of actual parameters in use of that function	Q.	$X \rightarrow XbX \mid XcX \mid dXf \mid g$
G.	Arithmetic expressions with matched pairs of parentheses	R.	$L = \{wcw \mid w \in (a \mid b)^*\}$
H.	Palindromes	S.	$X \rightarrow bXb \mid cXc \mid \varepsilon$

(A) E – P, F – R, G – Q, H – S
(B) E – R, F – P, G – S, H – Q
(C) E – R, F – P, G – Q, H – S
(D) E – P, F – R, G – S, H – Q

52. Match the following NFAs with the regular expressions they correspond to



1. $\varepsilon + 0(01^*1 + 00)^*01^*$
2. $\varepsilon + 0(10^*1 + 00)^*0$
3. $\varepsilon + 0(10^*1 + 10)^*1$
4. $\varepsilon + 0(10^*1 + 10)^*10^*$

(A) P – 2, Q – 1, R – 3, S – 4 (B) P – 1, Q – 3, R – 2, S – 4

(C) P – 1, Q – 2, R – 3, S – 4 (D) P – 3, Q – 2, R – 1, S – 4

53. Which of the following are regular sets?

I. $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$

II. $\{a^n b^m \mid n = 2m\}$

III. $\{a^n b^m \mid n \neq m\}$

IV. $\{xcy \mid x, y, \in \{a, b\}^*\}$

(A) I and IV only (B) I and III only (C) I only (D) IV only

54. Which of the following are true?

I. A programming language which does not permit global variables of any kind and has no nesting of procedures/functions, but permits recursion can be implemented with static storage allocation

II. Multi-level access link (or display) arrangement is needed to arrange activation records only if the programming language being implemented has nesting of procedures/functions

III. Recursion in programming languages cannot be implemented with dynamic storage allocation

IV. Nesting procedures/functions and recursion require a dynamic heap allocation scheme and cannot be implemented with a stack-based allocation scheme for activation records

V. Programming languages which permit a function to return a function as its result cannot be implemented with a stack-based storage allocation scheme for activation records

(A) II and V only (B) I, III and IV only

(C) I, II and V only (D) II, III and V only

55. An LALR(1) parser for a grammar G can have shift-reduce (S-R) conflicts if and only if

(A) The SLR(1) parser for G has S-R conflicts

(B) The LR(1) parser for G has S-R conflicts

(C) The LR(0) parser for G has S-R conflicts

(D) The LALR(1) parser for G has reduce-reduce conflicts

56. In the slow start phase of the TCP congestion control algorithm, the size of the congestion window

(A) does not increase (B) increases linearly

(C) increases quadratically (D) increases exponentially

57. If a class B network on the Internet has a subnet mask of 255.255.248.0, what is the maximum number of hosts per subnet?

(A) 1022 (B) 1023 (C) 2046 (D) 2047

58. A computer on a 10Mbps network is regulated by a token bucket. The token bucket is filled at a rate of 2Mbps. It is initially filled to capacity with 16Megabits. What is the maximum duration for which the computer can transmit at the full 10Mbps?

(A) 1.6 seconds (B) 2 seconds (C) 5 seconds (D) 8 seconds

59. A client process P needs to make a TCP connection to a server process S. Consider the following situation: the server process S executes a socket (), a bind () and a listen () system call in that order, following which it is preempted. Subsequently, the client process P executes a socket () system call followed by connect () system call to connect to the server process S. The server process has not executed any accept () system call. Which one of the following events could take place?

(A) connect () system call returns successfully

(B) connect () system call blocks

(C) connect () system call returns an error

(D) connect () system call results in a core dump

60. What is printed by the following C program?

```
int f(int x, int *py, int **ppz)
{
    int y, z;
    **ppz += 1; z = *ppz;
    *py += 2; y = *py;
    x += 3;
    return x + y + z;
}
```

```
void main( )
{
    int c, *b, **a;
    c = 4; b = &c; a = &b;
    printf("%d", f(c,b,a));
}
```

(A) 18 (B) 19 (C) 21 (D) 22

61. Choose the correct option to fill ? 1 and ? 2 so that the program below prints an input string in reverse order. Assume that the input string is terminated by a newline character.

```
void recerse(void){
    int c;
    if(?1)reverse( );
    ?2
}
main( ){
    printf("Enter Text"); printf("\n");
    reverse( );printf("\n");
}
```

(A) ?1 is (getchar( ) != '\n')

?2 is getchar(c);

(B) ?1 is (c = getchar( )) != '\n')

?2 is getchar(c);

(C) ?1 is (c != '\n')

?2 is putchar(c);

(D) ?1 is ((c = getchar( )) != '\n')

?2 is putchar(c);

62. The following C function takes a single-linked list of integers as a parameter and rearranges the elements of the list. The function is called with the list containing the integers 1,2,3,4,5,6,7 in the given order. What will be the contents of the list after the function completes execution?

```
struct node {
    int value;
    struct node *next;
};
Void rearrange(struct node *list){
    struct node *p, *q;
    int temp;
    if(!list || !list->next)return;
    p = list; q = list->next;
    while(q){
        temp = p->value;p->value = q->value;
        q->value = temp;p = q->next;
        q = p?p->next : 0;
    }
}
```

(A) 1,2,3,4,5,6,7 (B) 2,1,4,3,6,5,7 (C) 1,3,2,5,4,7,6 (D) 2,3,4,5,6,7,1

63. The P and V operations on counting semaphores, where s is a counting semaphore, are defined as follows:

```
P(s) : s = s – 1;
       ifs < 0 then wait;
V(s) : s = s + 1;
       ifs <= 0 then wakeup a process waiting on s;
```

Assume that P_b and V_b the wait and signal operations on binary semaphores are provided. Two binary semaphores x_b and y_b are used to implement the semaphore operations P(s) and V(s) as follows:

$P(s):\ \ P_b(x_b);$
$\quad s = s - 1;$
$\quad \text{if} \ \ (s < 0) \ \ \{$
$\qquad V_b(x_b);$
$\qquad P_b(y_b);$
$\quad \}$
$\quad \text{else} \ \ V_b(x_b);$

$V(s):\ \ P_b(x_b);$
$\quad s = s + 1;$
$\quad \text{if} \ \ (s <= 0) \ \ V_b(y_b);$
$\qquad V_b(x_b);$

The initial values of x_b and y_b are respectively

(A) 0 and 0 (B) 0 and 1 (C) 1 and 0 (D) 1 and 1

64. Which of the following statements about synchronous and asynchronous I/O is NOT true?
(A) An ISR is invoked on completion of I/O in synchronous I/O but not in asynchronous I/O
(B) In both synchronous and asynchronous I/O, an ISR (Interrupt Service Routine) is invoked after completion of the I/O
(C) A process making a synchronous I/O call waits until I/O is complete, but a process making an asynchronous I/O call does not wait for completion of the I/O
(D) In the case of synchronous I/O, the process waiting for the completion of I/O is woken up by the ISR that is invoked after the completion of I/O

65. Which of the following is NOT true of deadlock prevention and deadlock avoidance schemes?
(A) In deadlock prevention, the request for resources is always granted if the resulting state is safe
(B) In deadlock avoidance, the request for resources is always granted if the result state is safe
(C) Deadlock avoidance is less restrictive than deadlock prevention
(D) Deadlock avoidance requires knowledge of resource requirements a priori

66. A process executes the following code

$\text{for}(i = 0;\ \ i < n;\ \ i++)\,\text{for}(\ \ );$

The total number of child processes created is

(A) n (B) $2^n - 1$ (C) 2^n (D) $2^{n+1} - 1$

67. A processor uses 36 bit physical addresses and 32 bit virtual addresses, with a page frame size of 4 Kbytes. Each page table entry is of size 4 bytes. A three level page table is used for virtual to physical address translation, where the virtual address is used as follows

- Bits 30-31 are used to index into the first level page table
- Bits 21-29 are used to index into the second level page table
- Bits 12-20 are used to index into the third level page table, and
- Bits 0-11 are used as offset within the page

The number of bits required for addressing the next level page table (or page frame) in the page table entry of the first, second and third level page tables are respectively

(A) 20, 20 and 20 (B) 24, 24 and 24 (C) 24, 24 and 20 (D) 25, 25 and 24

68. Let R and S be two relations with the following schema

$R(\underline{P}, \underline{Q}, R1, R2, R3)$

$S(\underline{P}, \underline{Q}, S1, S2)$

Where {P, Q} is the key for both schemas. Which of the following queries are equivalent?

I. $\Pi_P(R \bowtie S)$

II. $\Pi_P(R) \bowtie \Pi_P(S)$

III. $\Pi_P(\Pi_{P,Q}(R) \cap \Pi_{P,Q}(S))$

IV. $\Pi_P(\Pi_{P,Q}(R) - (\Pi_{P,Q}(R) - \Pi_{P,Q}(S)))$

(A) Only I and II (B) Only I and III

(C) Only I, II and III (D) Only I, III and IV

69. Consider the following relational schemes for a library database:

Book (Title, Author, Catalog_no, Publisher, Year, Price)

Collection (Title, Author, Catalog_no)

with in the following functional dependencies:

I. Title Author $\rightarrow$ Catalog_no

II. Catalog_no $\rightarrow$ Title Author Publisher Year

III. Publisher Title Year $\rightarrow$ Price

Assume {Author, Title} is the key for both schemes. Which of the following statements is true?

(A) Both Book and Collection are in BCNF

(B) Both Book and Collection are in 3NF only

(C) Book is in 2NF and Collection is in 3NF

(D) Both Book and Collection are in 2NF only

70. Consider a file of 16384 records. Each record is 32 bytes long and its key field is of size 6 bytes. The file is ordered on a non-key field, and the file organization is unspanned. The file is stored in a file system with block size 1024 bytes, and the size of a block pointer is 10 bytes. If the secondary index is built on the key field of the file, and a multi-level index scheme is used to store the secondary index, the number of first-level and second-level blocks in the multi-level index are respectively

(A) 8 and 0 (B) 128 and 6 (C) 256 and 4 (D) 512 and 5

Common Data for Questions: 71 72 and 73

Consider a machine with a 2-way set associative data cache of size 64Kbytes and block size 16bytes. The cache is managed using 32 bit virtual addresses and the page size is 4Kbyts. A program to be run on this machine begins as follows:

```
double ARR[1024] [1024];
int i, j ;
  /* Initialize array ARR to 0.0 */
   for(i = 0;i < 1024; i++)
     for(j = 0; j < 1024; j++)
       ARR[i] [j] = 0.0;
```

The size of double is 8Bytes. Array ARR is located in memory starting at the beginning of virtual page 0xFF000 and stored in row major order. The cache is initially empty and no pre-fetching is done. The only data memory references made by the program are those to array ARR

71. The total size of the tags in the cache directory is

(A) 32Kbits (B) 34Kbits (C) 64Kbits (D) 68Kbits

72. Which of the following array elements has the same cache index as ARR [0] [0]?

(A) ARR [0] [4] (B) ARR [4] [0] (C) ARR [0] [5] (D) ARR [5] [0]

73. The cache hit ratio for this initialization loop is

(A) 0% (B) 25% (C) 50% (D) 75%

Common Data for Questions: 74 and 75

Consider the following C functions:

```
int  f1  ( int n )
{
   if (n == 0| |n == 1)
      return n;
   else
      return(2 * f1(n – 1) + 3 * f1(n – 2));
}
int  f2  ( int n )
{
  int  i;
  int  X[N],  Y[N],  Z[N];

  X[0] = Y[0] = Z[0] = 0;
  X[1] = 1;  Y[1] = 2;  Z[1] = 3;
  for(i = 2;  i <= n;  i ++) {
     X[i] = Y[i – 1] + Z[i – 2];
     Y[i] = 2 * X[i];
     Z[i] = 3 * X[i];
   }
   return  X[n];
}
```

74. The running time of f1 (n) and f2 (n) are

(A) $\Theta(n)$ and $\Theta(n)$

(B) $\Theta(2^n)$ and $\Theta(n)$

(C) $\Theta(n)$ and $\Theta(2^n)$

(D) $\Theta(2^n)$ and $\Theta(2^n)$

75. F1 (8) and f2 (8) return the values

(A) 1661 and 1640

(B) 59 and 59

(C) 1640 and 1640

(D) 1640 and 1661

Linked Answer Questions: Q.76 to 85 Carry Two Marks Each

Statement for Linked Answer Questions: 76 & 77

Delayed branching can help in the handling of control hazards

76. For all delayed conditional branch instructions, irrespective of whether the condition evaluates to true or false

(A) The instruction following the conditional branch instruction in memory is executed

(B) The first instruction in the fall through path is executed

(C) The first instruction in the taken path is executed

(D) The branch takes longer to execute than any other instruction

77. The following code is to run on a pipelined processor with one branch delay slot:

I1 : ADD $R2 \leftarrow R7 + R8$

I2 : SUB $R4 \leftarrow R5 - R6$

I3 : ADD $R1 \leftarrow R2 + R3$

I4 : STORE Memory $[R4] \leftarrow R1$

BRANCH to Label if $R1 == 0$

Which of the instructions I1, I2, I3 or I4 can legitimately occupy the delay slot without any other program modification?

(A) I1 (B) I2 (C) I3 (D) I4

Statement for Linked Answer Questions: 78 & 79

Let x_n denote the number of binary strings of length n that contain no consecutive 0s.

78. Which of the following recurrences does x_n satisfy?

(A) $x_n = 2x_{n-1}$ (B) $x_n = x_{\lfloor n/2 \rfloor} + 1$ (C) $x_n = x_{\lfloor n/2 \rfloor} + n$ (D) $x_n = x_{n-1} + x_{n-2}$

79. The value of x_5 is

(A) 5 (B) 7 (C) 8 (D) 16

Statement for Linked Answer Questions: 80 & 81

The subset-sum problem is defined as follows. Given a set of n positive integers, $S = \{a_1, a_2, a_3, \ldots, a_n\}$, and positive integer W, is there a subset of S whose elements sum to W? A dynamic program for solving this problem uses a 2-dimensional Boolean array, X, with n rows and W+1 columns.

$X[i, j], 1 \le i \le n, 0 \le j \le W,$ is TRUE if and only if there is a subset of $\{a_1, a_2, ..., a_i\}$ whose elements sum to j.

80. Which of the following is valid for $2 \le i \le n$ and $a_i \le j \le W$?

(A) $X[i, j] = X[i-1, j] \vee X[i, j-a_i]$ (B) $X[i, j] = X[i-1, j] \vee X[i-1, j-a_i]$

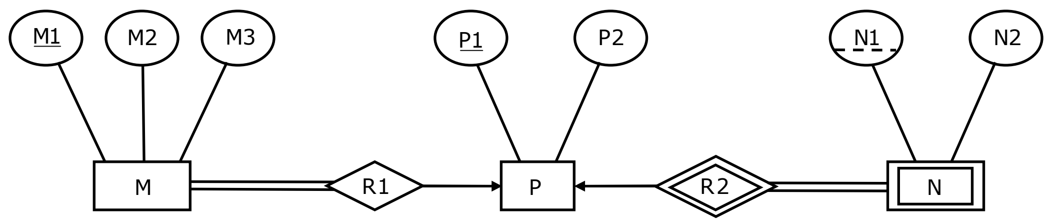
(C) $X[i, j] = X[i-1, j] \wedge X[i, j-a_i]$ (D) $X[i, j] = X[i-1, j] \wedge X[i-1, j-a_i]$

81. Which entry of the array X, if TRUE, implies that there is a subset whose elements sum to W?

(A) $X[1, W]$ (B) $X[n, 0]$ (C) $X[n, W]$ (D) $X[n-1, n]$

Statement for Linked Answer Questions: 82 & 83

Consider the following ER diagram



82. The minimum number of tables needed to represent M, N, P, R1, R2 is

(A) 2 (B) 3 (C) 4 (D) 5

83. Which of the following is a correct attribute set for one of the tables for the correct answer to the above question?

(A) $\{M1, M2, M3, P1\}$ (B) $\{M1, P1, N1, N2\}$ (C) $\{M1, P1, N1\}$ (D) $\{M1, P1\}$

Statement for Linked Answer Questions: 84 & 85

Consider the following C program that attempts to locate an element x in an array Y[ ] using binary search. The program is erroneous.

```
f(int Y[10], int x) {
   int u, j, k;
   i = 0; j = 9;
   do {
         k = (i + j) / 2;
         if (Y[k] < x) i = k; else j = k;
      } while ((Y[k] != x) && (i < j));
   if (Y[k] == x) printf("x is in the array");
   else printf("x is not in the array");
}
```

84. On which of the following contents of Y and x does the program fail?

(A) Y is $[1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10]$ and $x < 10$

(B) Y is $[1\ 3\ 5\ 7\ 9\ \ 11\ \ 13\ \ 15\ \ 17\ 19]$ and $x < 1$

(C) Y is $[2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2]$ and $x > 2$

(D) Y is $[2\ 4\ 6\ 8\ 10\ 12\ 14\ 16\ 18\ 20]$ and $2 < x < 20$ and x is even

85. The correction needed in the program to make it work properly is

(A) Change line 6 to: $\text{if}\,(Y[k] < x)\ \ i = k + 1;\ \ \text{else}\ j = k - 1;$

(B) Change line 6 to: $\text{if}\,(Y[k] < x)\ \ i = k - 1;\ \ \text{else}\ j = k + 1;$

(C) Change line 6 to: $\text{if}\,(Y[k] <= x)\ \ i = k;\ \ \text{else}\ j = k;$

(D) Change line 7 to: $\}\ \text{while}\ ((Y[k] == x)\ \&\&\ (i < j));$

Q.1 – Q.20 Carry One Mark Each

1. Consider the following two statements about the function $f(x) = |x|$:

   P. $f(x)$ is continuous for all real values of x

   Q. $f(x)$ is differentiable for all real values of x

   Which of the following is TRUE?

   (A) P is true and Q is false. (B) P is false and Q is true.

   (C) Both P and Q are true. (D) Both P and Q are false.

2. Let S be a set of n elements. The number of ordered pairs in the largest and the smallest equivalence relations on S are:

   (A) n and n (B) n^2 and n (C) n^2 and 0 (D) n and 1

3. What is the maximum number of different Boolean functions involving n Boolean variables?

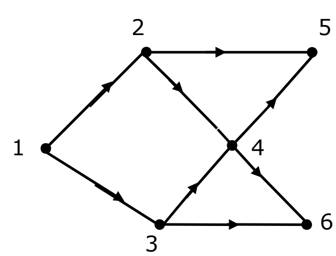
   (A) n^2 (B) 2^n (C) 2^{2^n} (D) 2^{n^2}

4. Let G be the non-planar graph with the minimum possible number of edges. Then G has

   (A) 9 edges and 5 vertices (B) 9 edges and 6 vertices

   (C) 10 edges and 5 vertices (D) 10 edges and 6 vertices

5. Consider the DAG with $V = \{1, 2, 3, 4, 5, 6\}$, shown below.



   Which of the following is NOT a topological ordering?

   (A) 1 2 3 4 5 6 (B) 1 3 2 4 5 6 (C) 1 3 2 4 6 5 (D) 3 2 4 1 6 5

6. Which of the following problems is undecidable?

   (A) Membership problem for CFGs. (B) Ambiguity problem for CFGs.

   (C) Finiteness problem for FSAs. (D) Equivalence problem for FSAs.

7. Which of the following is TRUE?

(A) Every subset of a regular set is regular.

(B) Every finite subset of a non-regular set is regular.

(C) The union of two non-regular sets is not regular.

(D) Infinite union of finite sets is regular.

8. How many 3-to-8 line decoders with an enable input are needed to construct a 6-to-64 line decoder without using any other logic gates?

(A) 7 (B) 8 (C) 9 (D) 10

9. Consider the following Boolean function of four variables:

$$f(w,x,y,z) = \sum(1,3,4,6,9,11,12,14)$$

The function is:

(A) independent of one variables. (B) independent of two variables.

(C) independent of three variables. (D) dependent on all the variables.

10. Consider a 4-way set associative cache consisting of 128 lines with a line size of 64 words. The CPU generates a 20-bit address of a word in main memory. The number of bits in the TAG, LINE and WORD fields are respectively:

(A) 9, 6, 5 (B) 7, 7, 6 (C) 7, 5, 8 (D) 9, 5, 6

11. Consider a disk pack with 16 surfaces, 128 tracks per surface and 256 sectors per track. 512 bytes of data are stored in a bit serial manner in a sector. The capacity of the disk pack and the number of bits required to specify a particular sector in the disk are respectively:

(A) 256 Mbyte, 19 bits (B) 256 Mbyte, 28 bits

(C) 512 Mbyte, 20 bits (D) 64 Gbyte, 28 bits

12. The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes in a binary tree of height h is:

(A) $2^h - 1$ (B) $2^{h-1} - 1$ (C) $2^{h+1} - 1$ (D) 2^{h+1}

13. The maximum number of binary trees that can be formed with three unlabeled nodes is:

(A) 1 (B) 5 (C) 4 (D) 3

14. Which of the following sorting algorithms has the lowest worst-case complexity?

(A) Merge sort (B) Bubble sort (C) Quick sort (D) Selection sort

15. Consider the following segment of C-code:

```
int j, n;
   j = 1;
   while (j <=n)
       j = j*2;
```

The number of comparisons made in the execution of the loop for any n > 0 is:

(A) $\lceil \log_2 n \rceil + 1$ (B) n (C) $\lceil \log_2 n \rceil$ (D) $\lfloor \log_2 n \rfloor + 1$

16. Group 1 contains some CPU scheduling algorithms and Group 2 contains some applications. Match entries in Group 1 to entries in Group 2.

Group I	**Group II**
(P) Gang Scheduling	(1) Guaranteed Scheduling
(Q) Rate Monotonic Scheduling	(2) Real-time Scheduling
(R) Fair Share Scheduling	(3) Thread Scheduling

(A) P - 3 Q - 2 R - 1 (B) P - 1 Q - 2 R - 3
(C) P - 2 Q - 3 R - 1 (D) P - 1 Q - 3 R - 2

17. Consider the following statements about user level threads and kernel level threads. Which one of the following statements is FALSE?

(A) Context switch time is longer for kernel level threads than for user level threads.
(B) User level threads do not need any hardware support.
(C) Related kernel level threads can be scheduled on different processors in a multi-processor system.
(D) Blocking one kernel level thread blocks all related threads.

18. Which one of the following is a top-down parser?

(A) Recursive descent parser. (B) Operator precedence parser.
(C) An LR(k) parser. (D) An LALR(k) parser.

19. In Ethernet when Manchester encoding is used, the bit rate is:

(A) Half the baud rate. (B) Twice the baud rate.
(C) Same as the baud rate. (D) None of the above

20. Which one of the following uses UDP as the transport protocol?

(A) HTTP (B) Telnet (C) DNS (D) SMTP

Q.21 – Q.75 Carry Two Marks Each

21. How many different non-isomorphic Abelian groups of order 4 are there?

(A) 2 (B) 3 (C) 4 (D) 5

22. Let *Graph(x)* be a predicate which denotes that *x* is a graph. Let *Connected(x)* be a predicate which denotes that *x* is connected. Which of the following first order logic sentences DOES NOT represent the statement: *"Not every graph is connected"?*

(A) $\neg\forall x\left(Graph(x)\Rightarrow Connected(x)\right)$ (B) $\exists x\left(Graph(x)\wedge\neg Connected(x)\right)$

(C) $\neg\forall x\left(\neg Graph(x)\vee Connected(x)\right)$ (D) $\forall x\left(Graph(x)\Rightarrow\neg Connected(x)\right)$

23. Which of the following graphs has an Eulerian circuit?

(A) Any *k*-regular graph where *k* is an even number.

(B) A complete graph on 90 vertices.

(C) The complement of a cycle on 25 vertices.

(D) None of the above

24. Suppose we uniformly and randomly select a permutation from the 20! Permutations of 1, 2,3 ,…..,20. What is the probability that 2 appears at an earlier position than any other even number in the selected permutation?

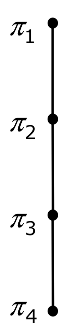
(A) $\frac{1}{2}$ (B) $\frac{1}{10}$ (C) $\frac{9!}{20!}$ (D) None of these

25. Let A be a 4 $\times$ 4 matrix with eigenvalues -5, -2, 1, 4. Which of the following is an eigenvalue of $\begin{bmatrix} A & I \\ I & A \end{bmatrix}$, where I is the 4 $\times$ 4 identity matrix?

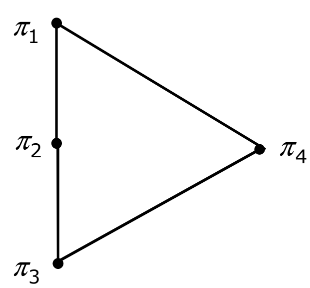
(A) -5 (B) -7 (C) 2 (D) 1

26. Consider the set $S=\{a,b,c,d\}$. Consider the following 4 partitions π_1,π_2,π_3,π_4 on $S:\pi_1=\{\overline{abcd}\},\pi_2=\{\overline{ab},\overline{cd}\},\pi_3=\{\overline{abc},\overline{d}\},\pi_4=\{\overline{a},\overline{b},\overline{c},\overline{d}\}$. Let p be the partial order on the set of partitions $S'=\{\pi_1,\pi_2,\pi_3,\pi_4\}$ defined as follows: π_i p π_j if and only if π_i refines π_j. The poset diagram for (S',p) is:

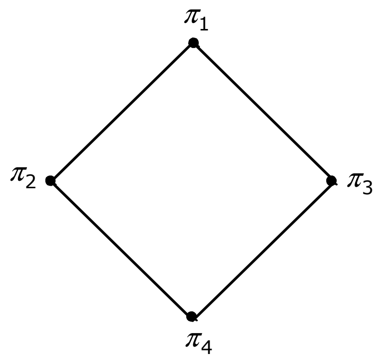
(A)



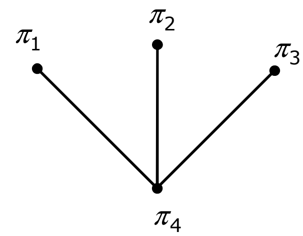
(B)



(C)



(D)



27. Consider the set of (column) vectors defined by $X = \left\{x \in R^3 \middle| x_1 + x_2 + x_3 = 0, \text{ where } x^T = [x_1, x_2, x_3]^T\right\}$. Which of the following is TRUE?

(A) $\left\{[1,-1,0]^T, [1,0,-1]^T\right\}$ is a basis for the subspace X.

(B) $\left\{[1,-1,0]^T, [1,0,-1]^T\right\}$ is a linearly independent set, but it does not span X and therefore is not a basis of X.

(C) X is not a subspace of R^3

(D) None of the above

28. Consider the series $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}, x_0 = 0.5$ obtained from the Newton-Raphson method. The series converges to

(A) 1.5 (B) $\sqrt{2}$ (C) 1.6 (D) 1.4

29. A minimum state deterministic finite automaton accepting the language $L = \left\{w \middle| w \in \{0,1\}^*, \text{ number of 0s and 1s in } w \text{ are divisible by 3 and 5, respectively}\right\}$ has

(A) 15 states (B) 11 states (C) 10 states (D) 9 states

30. The language $L = \left\{0^i 2 1^i \middle| i \geq 0\right\}$ over the alphabet $\{0,1,2\}$ is:

(A) not recursive

(B) is recursive and is a deterministic CFL.

(C) is a regular language.

(D) is not a deterministic CFL but a CFL.

31. Which of the following languages is regular?

(A) $\left\{ww^R \middle| w \in \{0,1\}^+\right\}$ (B) $\left\{ww^R x \middle| x, w \in \{0,1\}^+\right\}$

(C) $\left\{wxw^R \middle| x, w \in \{0,1\}^+\right\}$ (D) $\left\{xww^R \middle| x, w \in \{0,1\}^+\right\}$

32. Let $f(w,x,y,z) = \sum(0,4,5,7,8,9,13,15)$. Which of the following expressions are NOT equivalent to f ?

(P) $x'y'z' + w'xy' + wy'z + xz$

(Q) $w'y'z' + wx'y' + xz$

(R) $w'y'z' + wx'y' + xyz + xy'z$

(S) $x'y'z' + wx'y' + w'y$

(A) P only (B) Q and S (C) R and S (D) S only

33. Define the connective * for the Boolean variables X and Y as: X * Y = XY + $X'Y'$. Let $Z = X * Y$. Consider the following expressions P, Q and R.

$\pi P : X = Y * Z$ $Q : Y = X * Z$ $R : X * Y * Z = 1$

Which of the following is TRUE?

(A) Only P and Q are valid. (B) Only Q and R are valid.

(C) Only P and R are valid. (D) All P, Q, R are valid.

34. Suppose only one multiplexer and one inverter are allowed to be used to implement any Boolean function of n variables. What is the minimum size of the multiplexer needed?

(A) 2^n line to 1 line (B) 2^{n+1} line to 1 line

(C) 2^{n-1} line to 1 line (D) 2^{n-2} line to 1 line

35. In a look-ahead carry generator, the carry generate function G_i and the carry propagate function P_i for inputs A_i and B_i are given by:

$P_i = A_i \oplus B_i$ and $G_i = A_i B_i$

The expressions for the sum bit S_i and the carry bit C_{i+1} of the look-ahead carry adder are given by:

$S_i = P_i \oplus C_i$ and $C_{i+1} = G_i + P_i C_i$, where C_0 is the input carry.

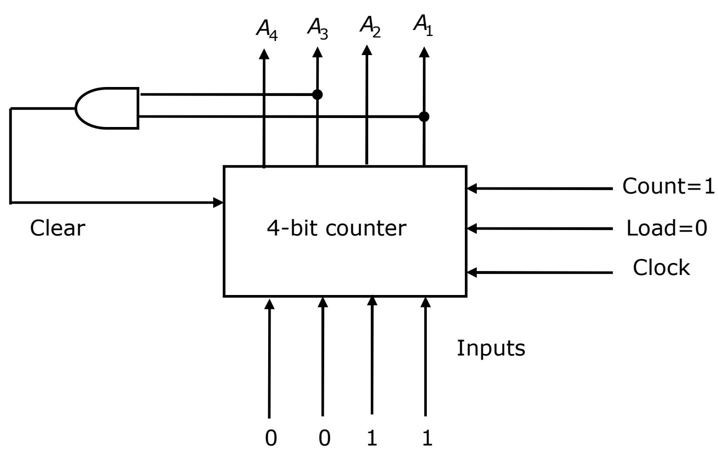
Consider a two-level logic implementation of the look-ahead carry generator. Assume that all P_i and G_i are available for the carry generator circuit and that the AND and OR gates can have any number of inputs. The number of AND gates and OR gates needed to implement the look-ahead carry generator for a 4-bit adder with S_3, S_2, S_1, S_0 and C_4 as its outputs are respectively:

(A) 6, 3 (B) 10, 4 (C) 6, 4 (D) 10, 5

36. The control signal functions of a 4-bit binary counter are given below (where X is "don't care"):

Clear	Clock	Load	Count	Function
1	X	X	X	Clear to 0
0	X	0	0	No change
0	↑	1	X	Load input
0	↑	0	1	Count next

The counter is connected as follows:



Assume that the counter and gate delays are negligible. If the counter starts at 0, then it cycles through the following sequence:

(A) 0, 3, 4
(B) 0, 3, 4, 5
(C) 0, 1, 2, 3, 4
(D) 0, 1, 2, 3, 4, 5

37. Consider a pipelined processor with the following four stages:

IF: Instruction Fetch

ID: Instruction Decode and Operand Fetch

EX: Execute

WB: Write Back

The IF, ID and WB stages take one clock cycle each to complete the operation. The number of clock cycles for the EX stage depends on the instruction. The ADD and SUB instructions need 1 clock cycle and the MUL instruction needs 3 clock cycles in the EX stage. Operand forwarding is used in the pipelined processor. What is the number of clock cycles taken to complete the following sequence of instructions?

ADD	R2, R1, R0	R2 ← R1 + R0
MUL	R4, R3, R2	R4 ← R3 * R2
SUB	R6, R5, R4	R6 ← R5 – R4

(A) 7 (B) 8 (C) 10 (D) 14

38. The following postfix expression with single digit operands is evaluated using a stack:

8 2 3 ^ / 2 3 * + 5 1 * -

Note that ^ is the exponentiation operator. The top two elements of the stack after the first * is evaluated are:

(A) 6, 1 (B) 5, 7 (C) 3, 2 (D) 1, 5

39. The inorder and preorder traversal of a binary tree are

`d b e a f c g` and `a b d e c f g`, respectively

The postorder traversal of the binary tree is:

(A) d e b f g c a (B) e d b g f c a (C) e d b f g c a (D) d e f g b c a

40. Consider a hash table of size seven, with starting index zero, and a hash function $(3x + 4) \bmod 7$. Assuming the hash table is initially empty, which of the following is the contents of the table when the sequence 1, 3, 8, 10 is inserted into the table using closed hashing? Note that – denotes an empty location in the table.

(A) 8, –, –, –, –, –, 10
(B) 1, 8, 10, –, –, –, 3
(C) 1, –, –, –, –, –, 3
(D) 1, 10, 8, –, –, –, 3

41. In an unweighted, undirected connected graph, the shortest path from a node S to every other node is computed most efficiently, in terms of *time complexity,* by

(A) Dijkstra's algorithm starting from S.
(B) Warshall's algorithm
(C) Performing a DFS starting from S.
(D) Performing a BFS starting from S.

42. Consider the following C function:

```
int f(int n)
{static int r = 0;
 if (n <= 0) return 1;
  if (n > 3)
     {r = n;
      return f(n-2)+2;
     }
  return f(n-1)+r;
}
```

What is the value of $f(5)$?

(A) 5 (B) 7 (C) 9 (D) 18

43. A complete n-ary tree is a tree in which each node has n children or no children. Let I be the number of internal nodes and L be the number of leaves in a complete n-ary tree. If $L = 41$, and $I = 10$, what is the value of n?

(A) 3 (B) 4 (C) 5 (D) 6

44. In the following C function, let $n \geq m$.

```
int gcd(n,m)
 {
    if (n%m ==0) return m;
       n = n%m;
    return gcd(m,n);
 }
```

How many recursive calls are made by this function?

(A) $\Theta(\log_2 n)$ (B) $\Omega(n)$ (C) $\Theta(\log_2 \log_2 n)$ (D) $\Theta(\sqrt{n})$

45. What is the time *complexity* of the following recursive function:

```
int DoSomething (int n) {
    if (n <= 2)
        return 1;
    else
        return (DoSomething (floor(sqrt(n))) + n);
}
```

(A) $\Theta(n^2)$ (B) $\Theta(n \log_2 n)$ (C) $\Theta(\log_2 n)$ (D) $\Theta(\log_2 \log_2 n)$

46. Consider the following C program segment where CellNode represents a node in a binary tree:

```
struct CellNode  {
   struct CellNOde *leftChild;
     int element;
   struct CellNode *rightChild;
};
int GetValue (struct CellNode *ptr)  {
  int value = 0;
  if (ptr != NULL)                {
     if ((ptr->leftChild == NULL) &&
           (ptr->rightChild == NULL))
        value = 1;
else
        value = value + GetValue(ptr->leftChild)
```

```
                                  + GetValue(ptr->rightChild);
    }
return(value);
}
```

The value returned by GetValue when a pointer to the root of a binary tree is passed as its argument is:

(A) the number of nodes in the tree

(B) the number of internal nodes in the tree

(C) the number of leaf nodes in the tree

(D) the height of the tree

47. Consider the process of inserting an element into a *Max Heap*, where the *Max Heap* is represented by an *array*. Suppose we perform a binary search on the path from the new leaf to the root to find the position for the newly inserted element, the number of *comparisons* performed is:

(A) $\Theta(\log_2 n)$ (B) $\Theta(\log_2 \log_2 n)$ (C) $\Theta(n)$ (D) $\Theta(n \log_2 n)$

48. Which of the following is TRUE about formulae in Conjunctive Normal Form?

(A) For any formula, there is a truth assignment for which at least half the clauses evaluate to true.

(B) For any formula, there is a truth assignment for which all the clauses evaluate to true.

(C) There is a formula such that for each truth assignment, at most one-fourth of the clauses evaluate to true.

(D) None of the above.

49. Let w be the minimum weight among all edge weights in an undirected connected graph. Let e be a specific edge of weight w. Which of the following is FALSE?

(A) There is a minimum spanning tree containing e.

(B) If e is not in a minimum spanning tree T, then in the cycle formed by adding e to T, all edges have the same weight.

(C) Every minimum spanning tree has an edge of weight w.

(D) e is present in every minimum spanning tree.

50. An array of n numbers is given, where n is an even number. The maximum as well as the minimum of these n numbers needs to be determined. Which of the following is TRUE about the number of comparisons needed?

(A) At least $2n - c$ comparisons, for some constant c, are needed.

(B) At most $1.5n - 2$ comparisons are needed.

(C) At least $n \log_2 n$ comparisons are needed.

(D) None of the above.

51. Consider the following C code segment:

```
int IsPrime(n)
      {
        int i,n;
        for(i=2;i<=sqrt(n);i++)
          if(n%i == 0)
            {printf("Not Prime\n"); return 0;}
        return 1;
       }
```

Let $T(n)$ denote the number of times the *for* loop is executed by the program on input *n.* Which of the following is TRUE?

(A) $T(n) = O(\sqrt{n})$ and $T(n) = \Omega(\sqrt{n})$
(B) $T(n) = O(\sqrt{n})$ and $T(n) = \Omega(1)$
(C) $T(n) = O(n)$ and $T(n) = \Omega(\sqrt{n})$
(D) None of the above

52. Consider the grammar with non-terminals $N = \{S, C, S_1\}$, terminals $T = \{a, b, i, t, e\}$, with S as the start symbol, and the following set of rules:

$$S \to iCtSS_1 \mid a$$
$$S_1 \to eS \mid \varepsilon$$
$$C \to b$$

The grammar is NOT LL(1) because:

(A) it is left recursive
(B) it is right recursive
(C) it is ambiguous
(D) it is not context-free.

53. Consider the following two statements:

P: Every regular grammar is LL(1)

Q: Every regular set has a LR(1) grammar

Which of the following is TRUE?

(A) Both P and Q are true
(B) P is true and Q is false
(C) P is false and Q is true
(D) Both P and Q are false

54. In a simplified computer the instructions are:

OP R_j, R_i - Performs R_j OP R_i and stores the result in register R_i.

OP m, R_i - Performs *val* OP R_i and stores the result in R_i. *val* denotes the content of memory location *m.*

MOV m, R_i - Moves the content of memory location *m* to register R_i.

MOV R_i, m - Moves the content of register R_i to memory location *m.*

The computer has only to registers, and OP is either ADD or SUB. Consider the following basic block:

$t_1 = a + b$
$t_2 = c + d$
$t_3 = e - t_2$
$t_4 = t_1 - t_3$

Assume that all operands are initially in memory. The final value of the computation should be in memory. What is the minimum number of MOV instructions in the code generated for this basic block?

(A) 2 (B) 3 (C) 5 (D) 6

55. An operating system uses Shortest Remaining Time first (SRT) process scheduling algorithm. Consider the arrival times and execution times for the following processes:

Process	Execution time	Arrival time
P1	20	0
P2	25	15
P3	10	30
P4	15	45

What is the total waiting time for process P2?

(A) 5 (B) 15 (C) 40 (D) 55

56. A virtual memory system uses First In First Out (FIFO) page replacement policy and allocates a fixed number of frames to a process. Consider the following statements:

P: Increasing the number of page frames allocated to a process sometimes increases the page fault rate.

Q: Some programs do not exhibit locality of reference.

Which one of the following is TRUE?

(A) Both P and Q are true, and Q is the reason for P

(B) Both P and Q are true, but Q is not the reason for P.

(C) P is false, but Q is true

(D) Both P and Q are false.

57. A single processor system has three resource types X, Y and Z, which are shared by three processes. There are 5 units of each resource type. Consider the following scenario, where the column ***alloc*** denotes the number of units of each resource type allocated to each process, and the column ***request*** denotes the number of units of each resource type requested by a process in order to complete execution. Which of these processes will finish **LAST**?

	alloc			request		
	X	Y	Z	X	Y	Z
P0	1	2	1	1	0	3
P1	2	0	1	0	1	2
P2	2	2	1	1	2	0

(A) P0 (B) P1 (C) P2

(D) None of the above, since the system is in a deadlock.

58. Two processes, P1 and P2, need to access a critical section of code. Consider the following synchronization construct used by the processes:

```
/*      P1     */
while (true)   {
     wants1 = true;
     while (wants2==true);
     /* Critical
        Section   */
     wants1=false;
}
/*  Remainder section  */
```

```
/*      P2     */
while (true)   {
     wants2 = true;
     while (wants1==true);
     /* Critical
        Section   */
     Wants2=false;
}
/*  Remainder section  */
```

Here, `wants1` and `wants2` are shared variables, which are initialized to `false`.

Which one of the following statements is **TRUE** about the above construct?

(A) It does not ensure mutual exclusion.

(B) It does not ensure bounded waiting.

(C) It requires that processes enter the critical section in strict alternation.

(D) It does not prevent deadlocks, but ensures mutual exclusion.

59. Information about a collection of students is given by the relation **studinfo**(<u>studId,</u> name, sex). The relation **enroll**(<u>studId, courseId</u>) gives which student has enrolled for (or taken) what course(s). Assume that every course is taken by at least one male and at least one female student. What does the following relational algebra expression represent?

$$\Pi_{\text{courseId}}\left(\left(\Pi_{\text{studId}}\left(\sigma_{sex=\text{"female"}}\left(\text{studInfo}\right)\right)\times\Pi_{\text{courseId}}\left(\text{enroll}\right)\right)-\text{enroll}\right)$$

(A) Courses in which all the female students are enrolled.

(B) Courses in which a proper subset of female students are enrolled.

(C) Courses in which only male students are enrolled.

(D) None of the above

60. Consider the relation **employee**(name, sex, supervisorName) with *name* as the key. *supervisorName* gives the name of the supervisor of the employee under consideration. What does the following Tuple Relational Calculus query produce?

$$\{\text{e.name} \mid \text{employee}(\text{e}) \wedge$$

$$(\forall x)\left[\neg\text{employee}(\text{x}) \vee x.\text{supervisorName} \neq \text{e.name} \;\vee\; \text{x.sex} = \text{"male"}\right]\}$$

(A) Names of employees with a male supervisor.

(B) Names of employees with no immediate male subordinates.

(C) Names of employees with no immediate female subordinates.

(D) Names of employees with a female supervisor.

61. Consider the table **employee**(empId, name, department, salary) and the two queries Q_1, Q_2 below. Assuming that department 5 has more than one employee, and we want to find the employees who get higher salary than anyone in the department 5, which one of the statements is **TRUE** for any arbitrary employee table?

Q_1 : Select e.empId

From employee e

Where not exists

(Select * From employee s where s.department = "5" and s.salary >=e.salary)

Q_2 : Select e.empId

From employee e

Where e.salary > Any

(Select distinct salary From employee s Where s.department = "5")

(A) Q_1 is the correct query

(B) Q_2 is the correct query

(C) Both Q_1 and Q_2 produce the same answer.

(D) Neither Q_1 nor Q_2 is the correct query

62. Which one of the following statements if **FALSE?**

(A) Any relation with two attributes is in BCNF

(B) A relation in which every key has only one attribute is in 2NF

(C) A prime attribute can be transitively dependent on a key in a 3 NF relation.

(D) A prime attribute can be transitively dependent on a key in a BCNF relation.

63. The order of a leaf node in a B^+-tree is the maximum number of (value, data record pointer) pairs it can hold. Given that the block size is 1K bytes, data record pointer is 7 bytes long, the value field is 9 bytes long and a block pointer is 6 bytes long, what is the order of the leaf node?

(A) 63 (B) 64 (C) 67 (D) 68

64. Consider the following schedules involving two transactions. Which one of the following statements is **TRUE**?

$S_1 : r_1(X); r_1(Y); r_2(X); r_2(Y); w_2(Y); w_1(X)$

$S_2 : r_1(X); r_2(X); r_2(Y); w_2(Y); r_1(Y); w_1(X)$

(A) Both S_1 and S_2 are conflict serializable.

(B) S_1 is conflict serializable and S_2 is not conflict serializable.

(C) S_1 is not conflict serializable and S_2 is conflict serializable.

(D) Both S_1 and S_2 are not conflict serializable.

65. There are n stations in a slotted LAN. Each station attempts to transmit with a probability p in each time slot. What is the probability that **ONLY** one station transmits in a given time slot?

(A) $np(1-p)^{n-1}$ (B) $(1-p)^{n-1}$ (C) $p(1-p)^{n-1}$ (D) $1-(1-p)^{n-1}$

66. In a token ring network the transmission speed is 10^7 bps and the propagation speed is 200 metres/μs. The 1-bit delay in this network is equivalent to:

(A) 500 metres of cable. (B) 200 metres of cable.

(C) 20 metres of cable. (D) 50 metres of cable.

67. The address of a class B host is to be split into subnets with a 6-bit subnet number. What is the maximum number of subnets and the maximum number of hosts in each subnet?

(A) 62 subnets and 262142 hosts. (B) 64 subnets and 262142 hosts.

(C) 62 subnets and 1022 hosts. (D) 64 subnets and 1024 hosts.

68. The message 11001001 is to be transmitted using the CRC polynomial x^3+1 to protect it from errors. The message that should be transmitted is:

(A) 11001001000 (B) 11001001011

(C) 11001010 (D) 110010010011

69. The distance between two stations M and N is L kilometers. All frames are K bits long. The propagation delay per kilometer is t seconds. Let R bits/second be the channel capacity. Assuming that processing delay is negligible, the *minimum* number of bits for the sequence number field in a frame for maximum utilization, when the *sliding window protocol* is used, is:

(A) $\left\lceil \log_2 \frac{2LtR+2K}{K} \right\rceil$ (B) $\left\lceil \log_2 \frac{2LtR}{K} \right\rceil$

(C) $\left\lceil \log_2 \frac{2LtR+K}{K} \right\rceil$ (D) $\left\lceil \log_2 \frac{2LtR+K}{2K} \right\rceil$

70. Match the following:

(P) SMTP	(1) Application layer
(Q) BGP	(2) Transport layer
(R) TCP	(3) Data link layer
(S) PPP	(4) Network layer
	(5) Physical layer

(A) P - 2 Q - 1 R - 3 S - 5
(B) P - 1 Q - 4 R - 2 S - 3
(C) P - 1 Q - 4 R - 2 S - 5
(D) P - 2 Q - 4 R - 1 S - 3

Common Data Questions

Common Data for Questions 71, 72, 73:
Consider the following program segment. Here R1, R2 and R3 are the general purpose registers.

	Instruction	Operation	Instruction size (no.of words)
	MOV R1, (3000)	R1 ← m[3000]	2
LOOP:	MOV R2, (R3)	R2 ← M[R3]	1
	ADD R2, R1	R2 ← R1 + R2	1
	MOV (R3), R2	M[R3] ← R2	1
	INC R3	R3 ← R3 + 1	1
	DEC R1	R1 ← R1 − 1	1
	BNZ LOOP	Branch on not zero	2
	HALT	Stop	1

Assume that the content of memory location 3000 is 10 and the content of the register R3 is 2000. The content of each of the memory locations from 2000 to 2010 is 100. The program is loaded from the memory location 1000. All the numbers are in decimal.

71. Assume that the memory is word addressable. The number of memory references for accessing the data in executing the program completely is:

(A) 10 (B) 11 (C) 20 (D) 21

72. Assume that the memory is word addressable. After the execution of this program, the content of memory location 2010 is:

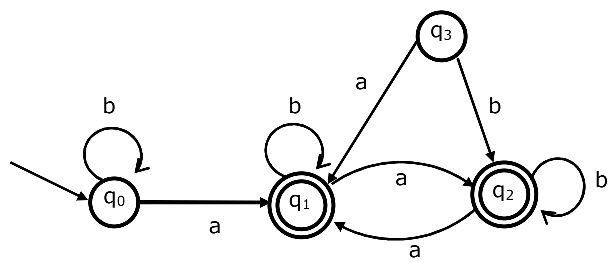
(A) 100 (B) 101 (C) 102 (D) 110

73. Assume that the memory is byte addressable and the word size is 32 bits. If an interrupt occurs during the execution of the instruction "INC R3", what return address will be pushed on to the stack?

(A) 1005 (B) 1020 (C) 1024 (D) 1040

Common Data for Questions 74, 75:

Consider the following Finite State Automaton:



74. The language accepted by this automaton is given by the regular expression

(A) $b^*ab^*ab^*ab^*$ (B) $(a+b)^*$ (C) $b^*a(a+b)^*$ (D) $b^*ab^*ab^*$

75. The minimum state automaton equivalent to the above FSA has the following number of states

(A) 1 (B) 2 (C) 3 (D) 4

Linked Answer Questions: Q.76 to Q.85 Carry Two Marks Each

Statement for Linked Answer Questions 76 & 77:

Suppose the letters *a, b, c, d, e, f* have probabilities $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},\frac{1}{32},\frac{1}{32}$, respectively.

76. Which of the following is the Huffman code for the letter *a, b, c, d, e, f?*

(A) 0, 10, 110, 1110, 11110, 11111

(B) 11, 10, 011, 010, 001, 000

(C) 11, 10, 01, 001, 0001, 0000

(D) 110, 100, 010, 000, 001, 111

77. What is the average length of the correct answer to Q.76?

(A) 3 (B) 2.1875 (C) 2.25 (D) 1.9375

Statement for Linked Answer Questions 78 & 79:

Consider the CFG with $\{S,A,B\}$ as the non-terminal alphabet, $\{a,b\}$ as the terminal alphabet, S as the start symbol and the following set of production rules:

$$\begin{array}{ll} S \to aB & S \to bA \\ B \to b & A \to a \\ B \to bS & A \to aS \\ B \to aBB & S \to bAA \end{array}$$

78. Which of the following strings is generated by the grammar?
    (A) aaaabb (B) aabbbb (C) aabbab (D) abbbba

79. For the correct answer strings to Q.78, how many derivation trees are there?
    (A) 1 (B) 2 (C) 3 (D) 4

Statement for Linked Answer Questions 80 & 81:

Consider a machine with a byte addressable main memory of 2^{16} bytes. Assume that a direct mapped data cache consisting of 32 lines of 64 bytes each is used in the system. A 50×50 two-dimensional array of bytes is stored in the main memory starting from memory location 1100H. Assume that the data cache is initially empty. The complete array is accessed twice. Assume that the contents of the data cache do not change in between the two accesses.

80. How many data cache misses will occur in total?
    (A) 48 (B) 50 (C) 56 (D) 59

81. Which of the following lines of the data cache will be replaced by new blocks in accessing the array for the second time?
    (A) line 4 to line 11 (B) line 4 to line 12
    (C) line 0 to line 7 (D) line 0 to line 8

Statement for Linked Answer Questions 82 & 83:

A process has been allocated 3 page frames. Assume that none of the pages of the process are available in the memory initially. The process makes the following sequence of page references (reference string): **1, 2, 1, 3, 7, 4, 5, 6, 3, 1**

82. If optimal page replacement policy is used, how many page faults occur for the above reference string?
    (A) 7 (B) 8 (C) 9 (D) 10

83. Least Recently Used (LRU) page replacement policy is a practical approximation to optimal page replacement. For the above reference string, how many more page faults occur with LRU than with the optimal page replacement policy?
    (A) 0 (B) 1 (C) 2 (D) 3

Statement for Linked Answer Questions 84 & 85:

Suppose that a robot is placed on the Cartesian plane. At each step it is allowed to move either one unit up or one unit right, i.e., if it is at (i, j) then it can move to either $(i+1, j)$ or $(i, j+1)$.

84. How many distinct paths are there for the robot to reach the point (10,10) starting from the initial position (0,0)?

(A) $\binom{20}{10}$ (B) 2^{20}

(C) 2^{10} (D) None of the above

85. Suppose that the robot is not allowed to traverse the line segment from (4,4) to (5,4). With this constraint, how many distinct paths are there for the robot to reach (10,10) starting from (0,0)?

(A) 2^{9} (B) 2^{19}

(C) $\binom{8}{4} \times \binom{11}{5}$ (D) $\binom{20}{10} - \binom{8}{4} \times \binom{11}{5}$